

HOMEWORK PAE 1: Orbits

Study case 1: 3CAT-2 Satellite

1. From analyzing its orbital parameters, we can deduce that, since the Perigee and Apogee are around 360-380 km, the satellite is orbiting in a LEO orbit.
2. The eccentricity of the orbit is 0.0009841.
3. The exact values listed for the perigee and apogee of the orbit are 366 km and 380 km respectively.
Note: Observing its movement, I saw that it reached an altitude of 405 km. How is that possible given the value for the apogee?
4. The orbital period of the satellite is 1h 32m 00s (92.0 min).

5. **(Read no. 6 for correction)**

In order to calculate the separation between consecutive ground tracks, we need to find out what type of orbit the satellite is following according to its relative position to the relevant celestial bodies (Earth, Sun).

Considering the fact that the inclination is 97.390° (close to 98°), the orbital period is 92 mins (between 90 and 100 minutes) and the altitude is around 400 km, we can deduce that the satellite is following a SSO (Sun-Synchronous Orbit).

From here, it all comes down to a simple calculation:

Using the given formula:

$$\Delta\Omega_2 = -2\pi \frac{T}{T_e} [\text{rad/orbit}]$$

we can find out the rotation during orbit by substituting the variables as such:

$T = 92$ mins (the orbital period)

$T_e = 1440$ mins (the orbital period of Earth around the Sun)

So, we have:

$$\Delta\Omega_2 = -2\pi * 92/1440 = 0.401425 \text{ rad/orbit} = 22.99995^\circ/\text{orbit} \approx 23^\circ/\text{orbit}$$

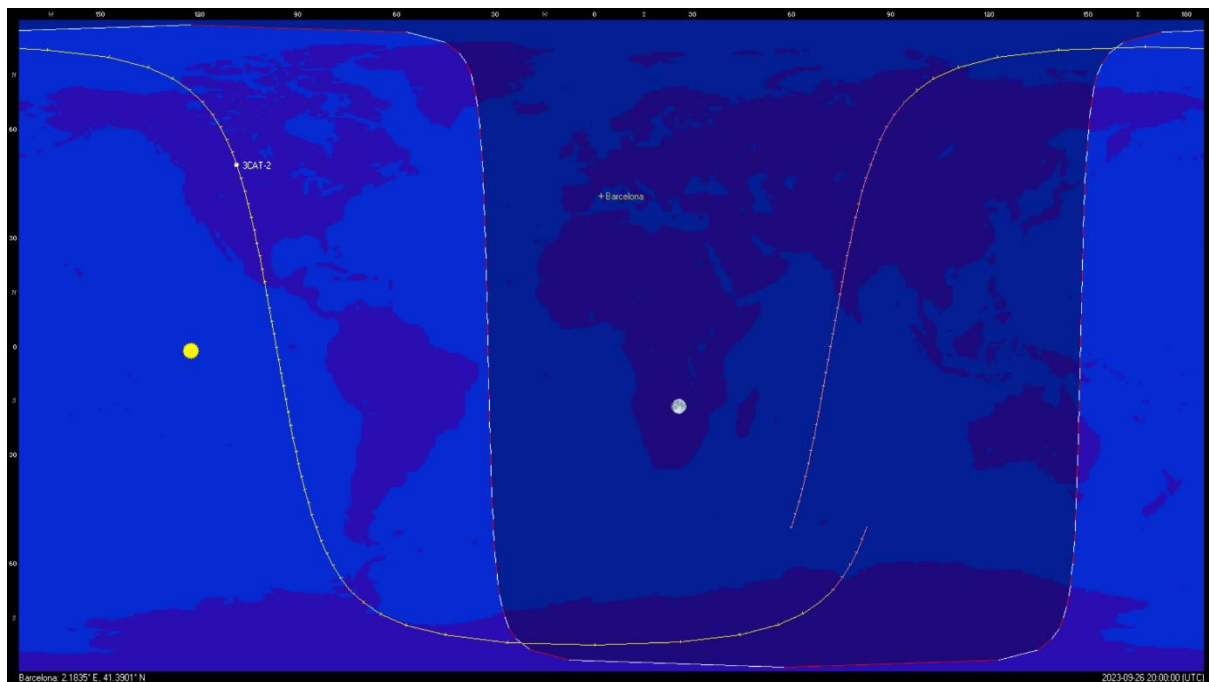
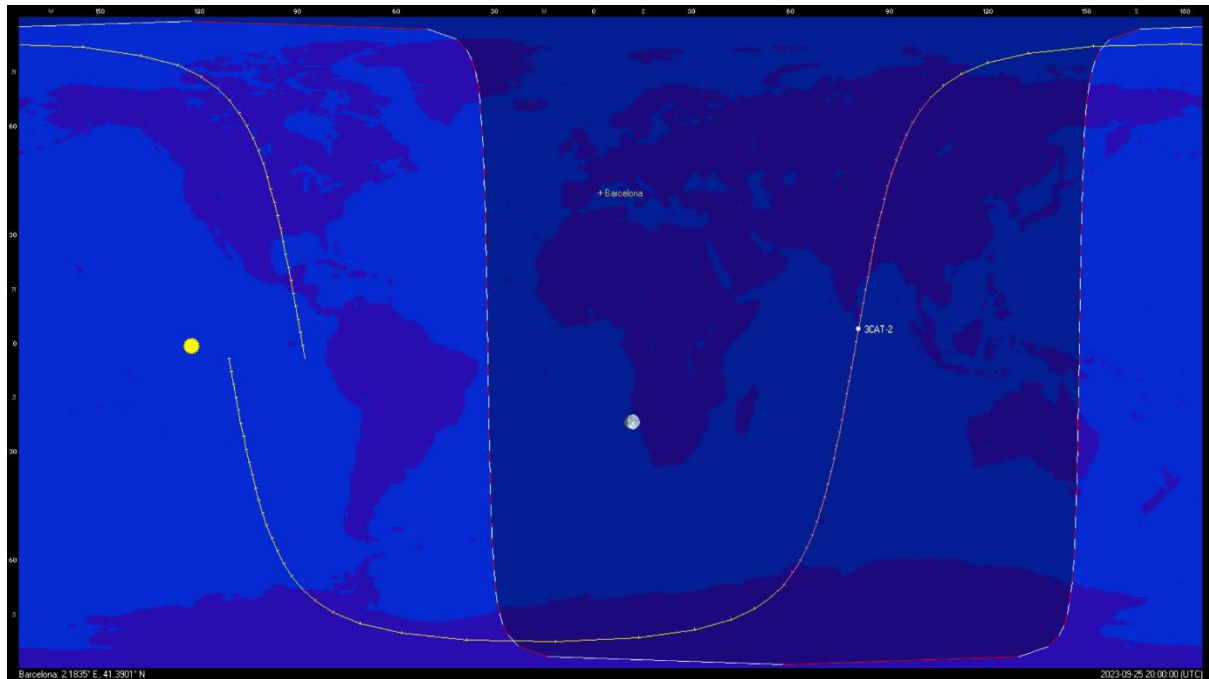
If we want to convert that to km to find out the separation between ground tracks, we can use:

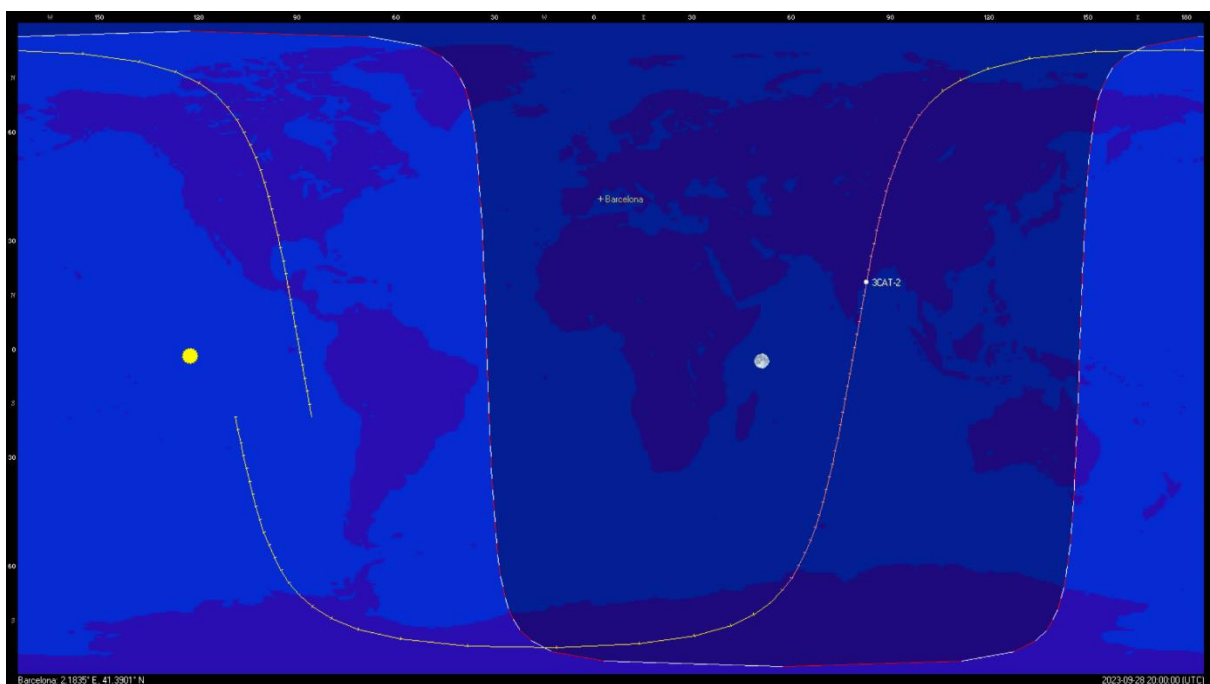
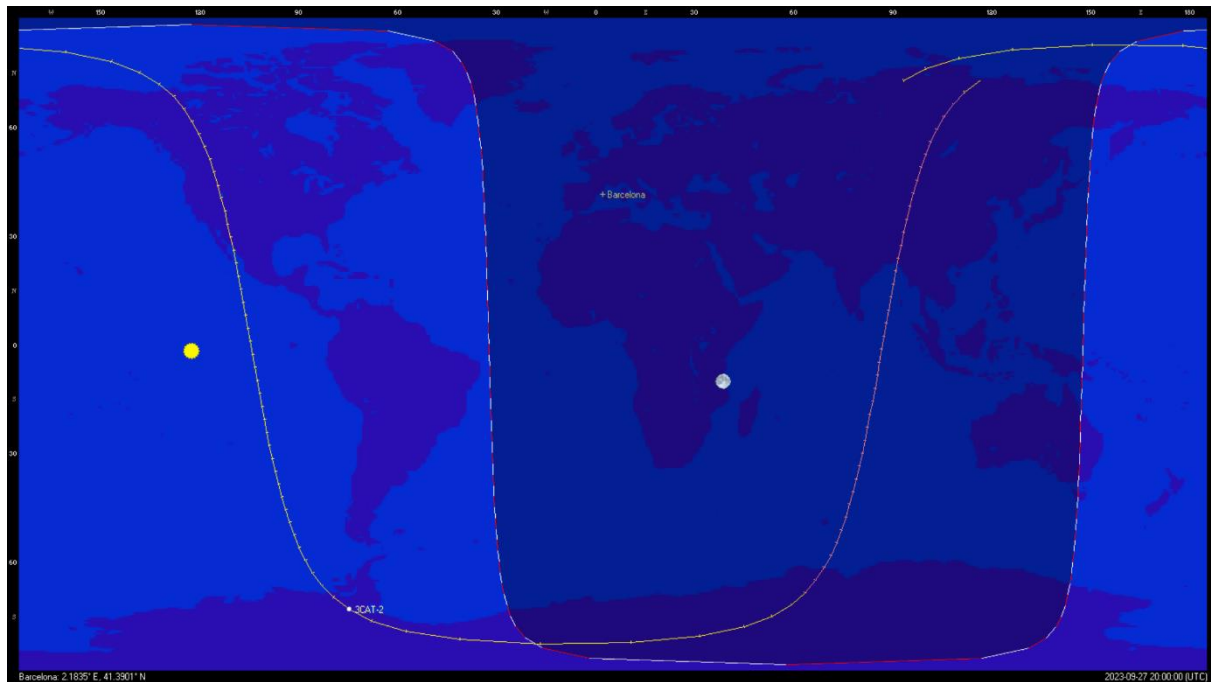
$$D \text{ (km)} = \Delta\Omega_2 (^\circ) / 360^\circ * 40,750 \text{ km.}$$

So, the separation between consecutive ground tracks is:

$$D \text{ (km)} = 23^\circ/360^\circ * 40,750 \text{ km} = 2,603.472 \text{ km.}$$

6. Using the simulation tool, we can observe that the relative position of the Sun and the satellite is constant, once every 3 days, at the same local time. These screenshots see the relative position of the Sun and the satellite at 20:00:00 UTC for 4 consecutive days (date and time shown bottom right):





As we can clearly see, when the sun is in the same position (done by fixating the local time to 20:00:00n UTC), the satellite is not in the same position every day (which would be a SSO), but once every 3 days. This is called a repetitive orbit with a 3-day cycle and it's great when it comes to the power and thermal design implications due to its periodic nature.

So, the earlier assumption was wrong and the calculations need fixing (the values might not be far off, since the orbit is still periodic, but the method is not correct).

7. I simulated multiple days of passes, and here are my observations:

- The satellite appears to pass the Barcelona station only in these intervals:

12 AM – 12:30 AM;

1:30 AM – 2:30 AM;

2:30 AM – 3:30 AM;

11:30 AM – 12:30 PM;

1:30 PM – 2:30 PM;

2:30 PM – 3:30 PM;

- The maximum elevation appears to be around 78.9°;
- **For a satellite elevation >0°:**
 - there are approx. 4.72 passes/day (236 passes/50 days);
 - there is a 50/50 split of ascending and descending passes (usually 2 ascending, then 2 descending, and so on, as expected);
 - the minimum range appears to be around 375 km, very close to the value of the perigee;
 - considering the range, elevation and illumination, it appears that around half of all the passes are useful, having an elevation of at least 8° (ideally it would be 10°), a range of less than 1500 km and a positive Sun Elevation;
- **For a satellite elevation >5°:**
 - there are approx. 3.48 passes/day (174 passes/50 days);
 - there is a 50/50 split of ascending and descending passes (as expected);
 - the minimum range appears to again be around 375 km;
 - considering the range and illumination, it appears that still around half of all the passes are useful;
- **For a satellite elevation >10°:**
 - there are approx. 2.46 passes/day (132 passes/50 days);
 - there is a 50/50 split of ascending and descending passes (as expected);
 - the minimum range appears to again be around 375 km;
 - considering the range, elevation and illumination, it appears that still around half of all the passes are useful;
- **For a satellite elevation >20°:**
 - there are approx. 1.68 passes/day (84 passes/50 days);
 - there is a 50/50 split of ascending and descending passes (as expected);
 - the minimum range appears to again be around 375 km;
 - considering the range and illumination, it appears that still around half of all the passes are useful, only this time is only due to the lighting conditions because the range is almost always <1000 km.

Study case 2: The HISPASAT 1C, 1D and 1E Satellites

1. From analyzing its classic “8” shape, as well as its elevation, we can safely say that the HISPASAT 1D follows a GEO orbit (Geosynchronous Equatorial Orbit).
2. The eccentricity of the orbit is 0.0002523.
3. The perigee is situated at 35 776 km, while the apogee is situated at 35 797 km.
4. The orbital period is 23h 56m 05s (1436.8 min), exactly the same as the orbital period of the Earth around the Sun (as expected).
5. For a GEO orbit, we can’t talk about “consecutive ground tracks”, but for a Geosynchronous orbit with an inclination different than 0 like this one we can talk about the width of its orbit (of the “figure-eight” pattern).

Using the formula:

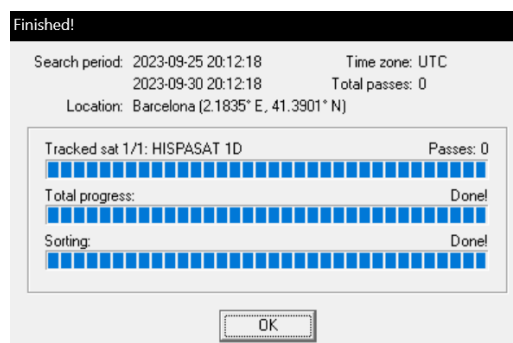
$$d = R * \sin(\theta),$$

where d is the width, θ is the inclination and R is Earth’s Radius, we find:

$$d = 6371 \text{ km} * \sin(0.046 \text{ rad}) = 297.561 \text{ km}.$$

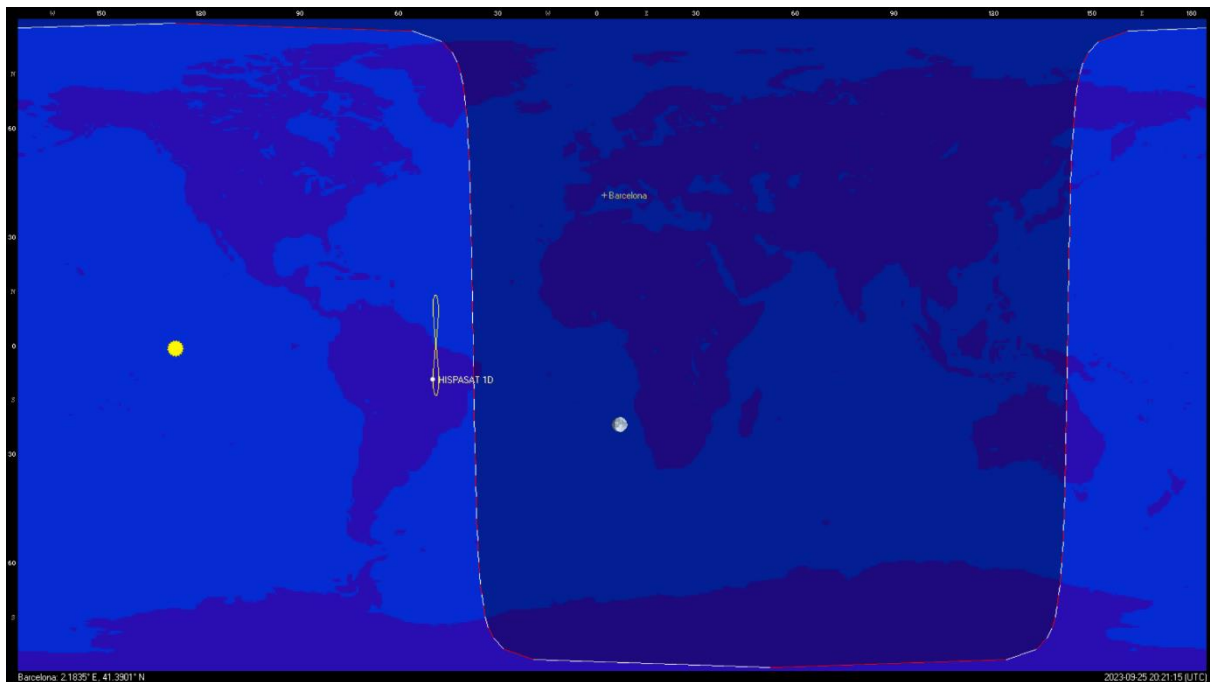
6. Just by observing its orbit, we can see that the satellite never comes in contact with the Barcelona station, the orbit being situated over Brazil and the Atlantic Ocean, close to the Equator.

But I did run the simulation for 5 days, just to prove it:



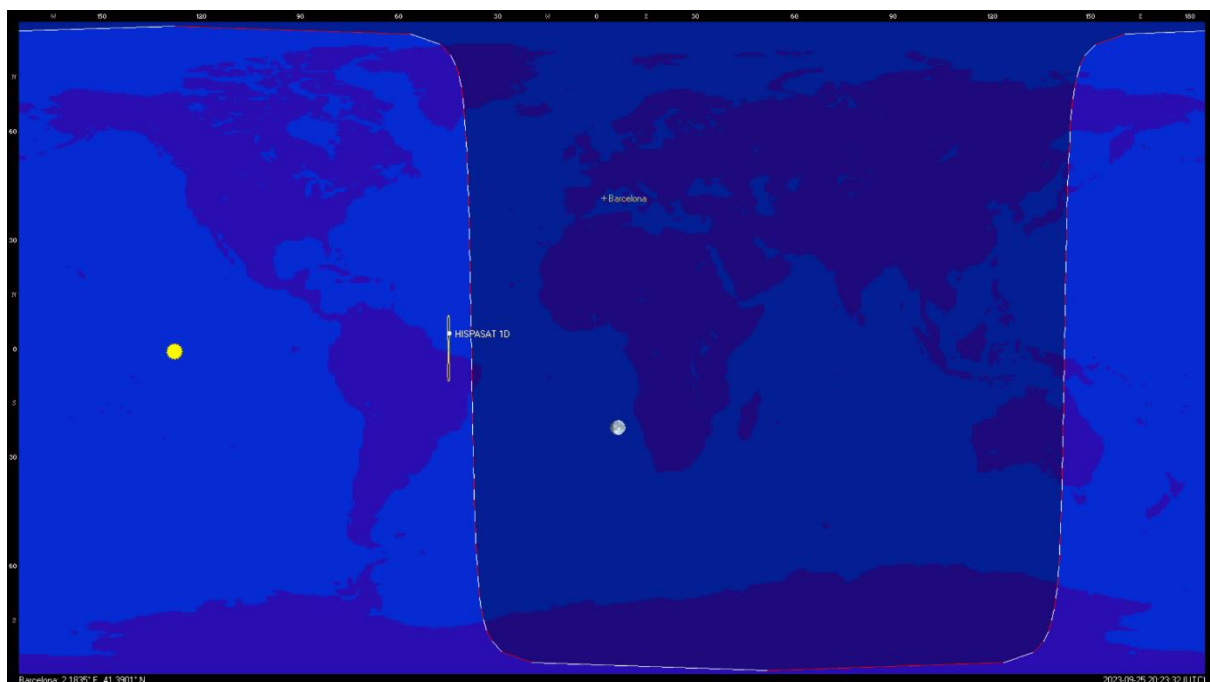
As you can see, 0 passes over Barcelona.

Also, to answer the question, the satellite doesn’t appear fully stationary as seen from Earth because it is very difficult for a GEO orbit to be “perfect” due to multiple factors, and so there is such a thing called “inclination” which determines the satellite’s “8-shaped” orbit.



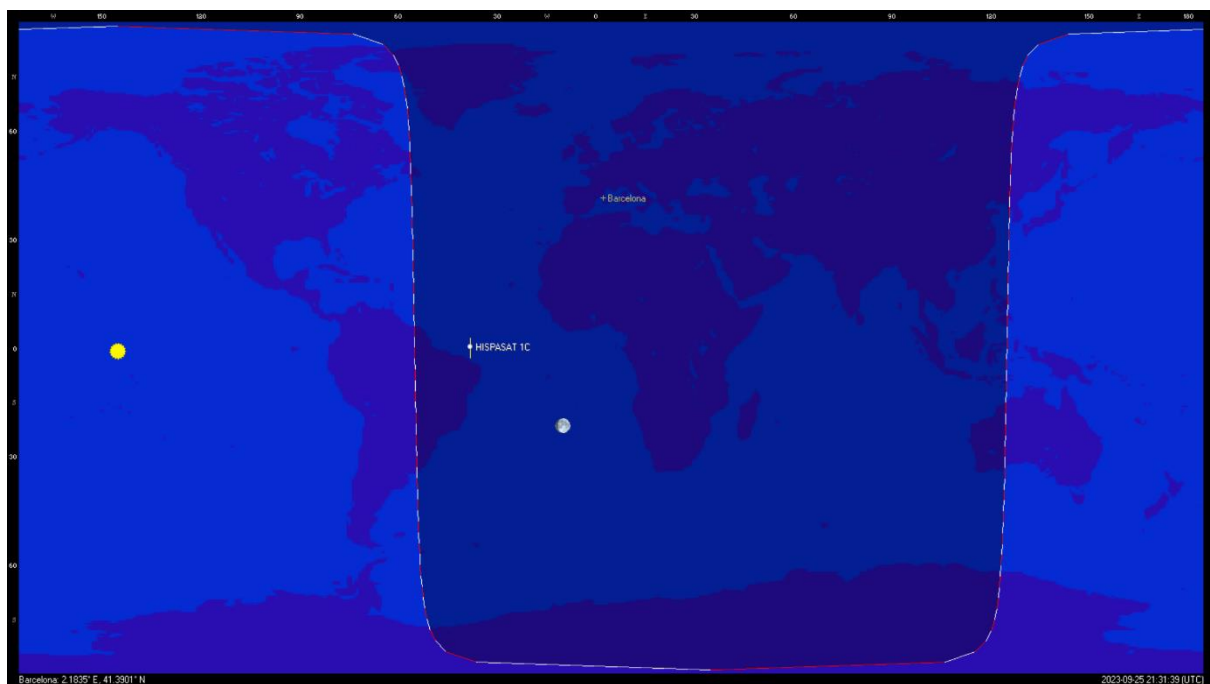
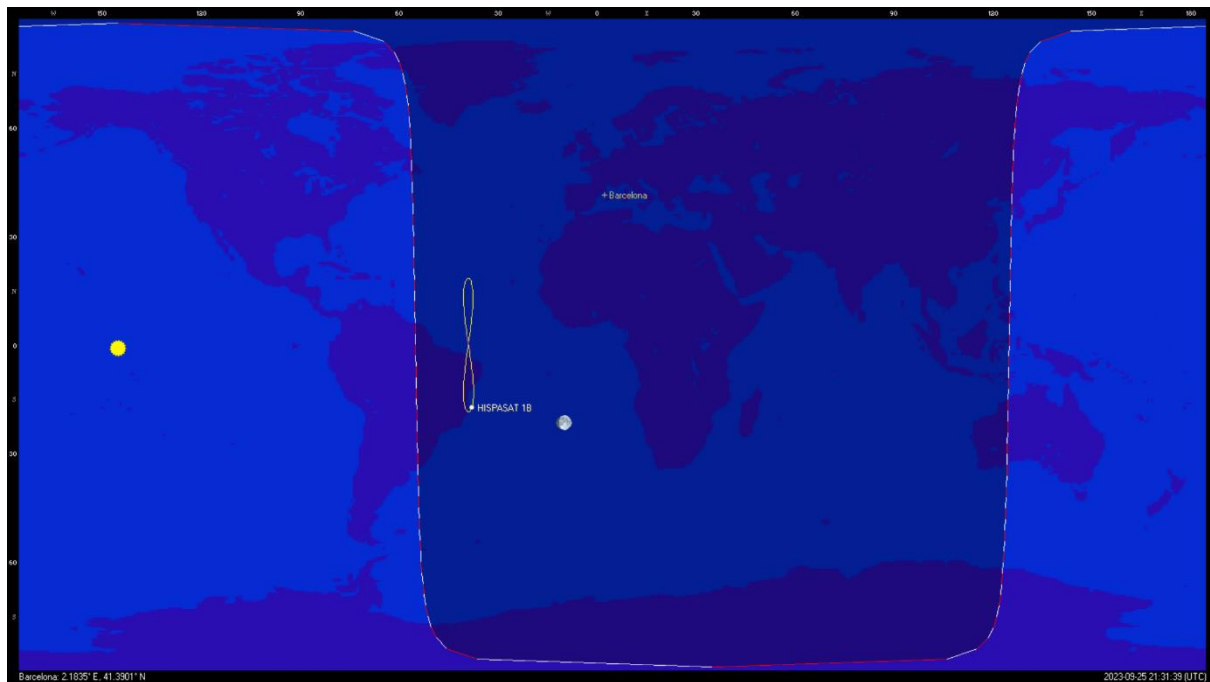
In the picture what I've said can be clearly seen.

7. In the picture below, the new satellite path can be seen:



As we can observe, the inclination has changed, making the path be less wide. From the point of view of the satellite operator, the fact that the orbit of the satellite has changed drastically is a point of concern, since most of the calculations and measurements are now obsolete. They either need to recalculate and recalibrate everything or activate the satellite's thrusters in order to attempt getting it on the original orbit.

8. The pictures below show HISPASAT 1B and 1C and their paths.



From what the mission states, the HISPASAT satellites were launched in order to “strengthen communication ties between the American continent (particularly South America) and the Iberian Peninsula for both governmental and private uses” (source: Wikipedia.com), fact that can be seen considering their orbits around the South American continent.

Study case 3: The GPS Satellites

1. From analyzing its orbital parameters, we can deduce that, since the Perigee and Apogee are around 19,900 – 20,400 km, the GPS BII-09 satellite is in the MEO orbit, which is expected for a GPS satellite.
2. The eccentricity of the orbit is 0.0090146.
3. The exact values for the perigee and apogee respectively are 19 941 and 20 420 km.
4. The orbital period is 11h 57m 54s (717.90 min), which is almost exactly half of a sidereal day (23 h 56m 4.091s).
5. It is known that the distance between 2 consecutive ground tracks of a GPS satellite is about 20,375 km ($R \cdot \pi$), the method of calculating being:

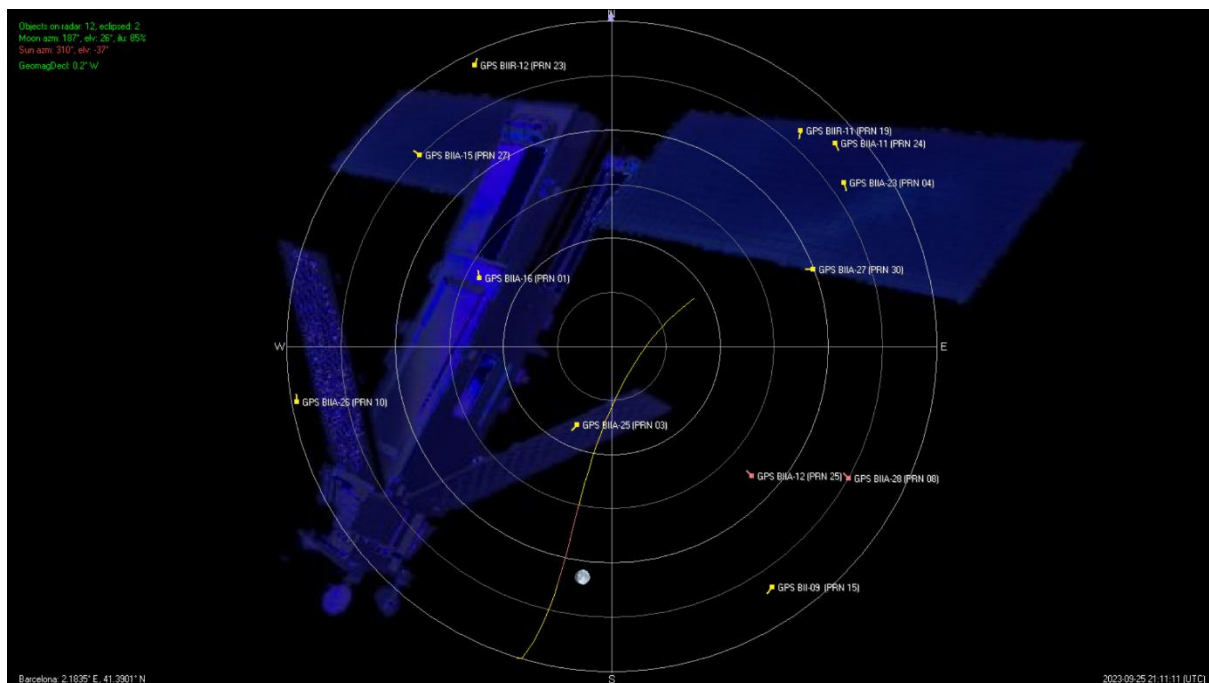
A GPS orbit is a 2:1 ratio Earth-Synchronous orbit, so using the given formula:

$$n |\Delta\Omega| = m \cdot 2\pi, \text{ where } n \text{ is the number of orbits}$$
$$m \text{ is the number of Earth revolutions (days)}$$

we find out that $|\Delta\Omega| = \pi$, so the distance between 2 consecutive ground tracks is

$$D \text{ (km)} = \pi/2\pi \cdot 40,750 \text{ km} = 1/2 \cdot 40,750 \text{ km} = 20.375 \text{ km}.$$

6. We have the Radar view as such:



We can see that, for elevation angles bigger than:

- 15°, there are 12 satellites in view;
- 30°, there are 11 satellites in view;
- 45°, there are 10 satellites in view;
- 60°, there are 8 satellites in view.

Obviously, the number drops as you search for higher elevations.

The system is made so that, at any time, you can see multiple satellites from anywhere. That is needed to have precise and fast calculations for all GPS devices on Earth.

Most satellites are seen at a higher angle elevation from Barcelona because of 2 main reasons:

1. The system is designed so that, at higher latitudes, a lot of satellites can still be seen, so satellites need to be at a high angle elevation;
2. At higher elevations, we are literally looking at a “wider” stretch of sky, due to how perception works.

Study case 4: Molniya Satellites

1. Due to its recognizable name, shape and especially inclination, we can safely say that the Molniya 3-50 satellite is following a HEO orbit (Highly Elliptical Orbit).
2. The value of the orbit's eccentricity is 0.7005231.
3. The values of the perigee and apogee are 1 576 km and 38 787 km respectively.
4. The orbital period is 11h 57m 56s (717.93 min), which is almost exactly half of a sidereal day (23 h 56m 4.091s).
5. To calculate the separation between consecutive ground tracks near the apogee region:

6.

$$D \text{ (km)} = R * (2\pi / |\Delta\Omega|),$$

where:

- R is the Earth's radius, roughly 6371 km.
- The satellite will see a shift in the ground track of approximately 90 degrees in longitude during the half period when it's near apogee.

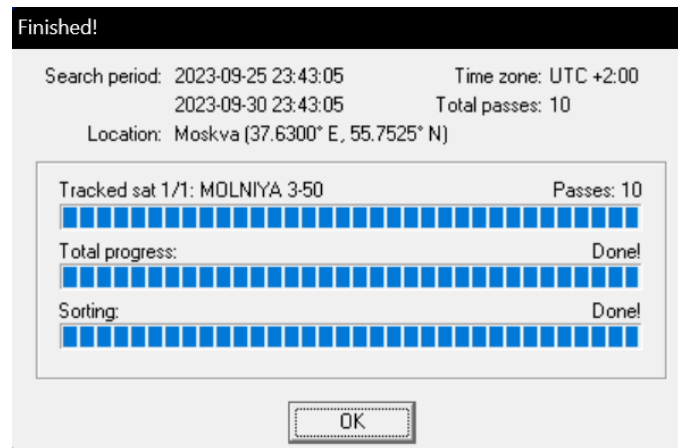
Plugging in the values:

$$D \approx 7,957.5 \text{ km}$$

P.S.: I did Google that.

7. A Molniya Orbit is not designed relative to the Sun. Its main purpose is providing coverage in the northernmost areas of the globe, since all geostationary satellites are around the equator and thus provide poor coverage to these areas.
So, designing Molniya satellite is more complex, considering the fact that the Sun illumination is not periodic, so more factors have to be taken into account.

8. As we can see, the Molniya 3-50 satellite passes Moscow 10 times in 5 days:



- That is an average of 2 passes/day for a satellite elevation $>5^\circ$;
- They tend to take place with around a 12h periodicity;
- The maximum elevation appears to be around 89.5° , which is really close to 90° ;
- The minimum range is about 5200 km in any point of the contact;
- The duration of contact is about 8 – 10 h.

All of this data proves that the Molniya satellites are made specifically for the northern regions of the globe (especially Russia, since all of them are launched by them), and the duration being so high is necessary since there need to be a lot of them visible at any point in time considering that Russia basically relies on them for communications.