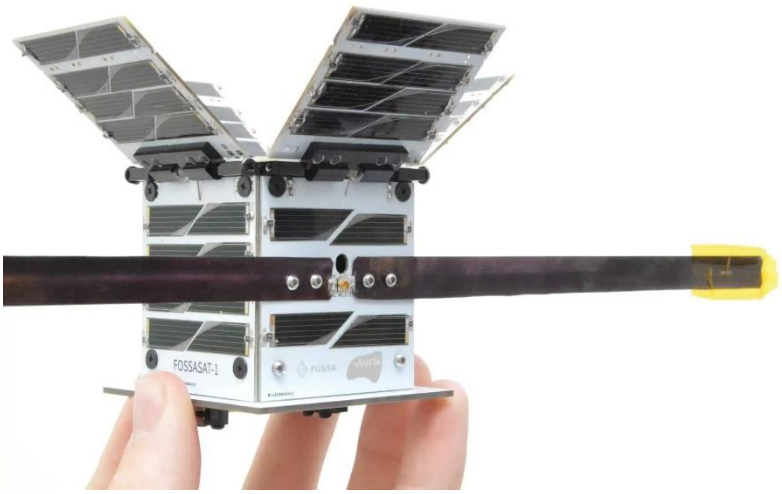
Homework PAE 2

Exercise 2: Considering the following reference picture:



We can see that the antenna is about 1/3 of the total height of one face of the cube and covers the entire width of the cube (I'm exaggerating the proportions a bit just to make sure the panels fit).

We will consider the 2 cases:

- a "normal" face – without the antenna (fig. 1)
- the face with the antenna (fig. 2)

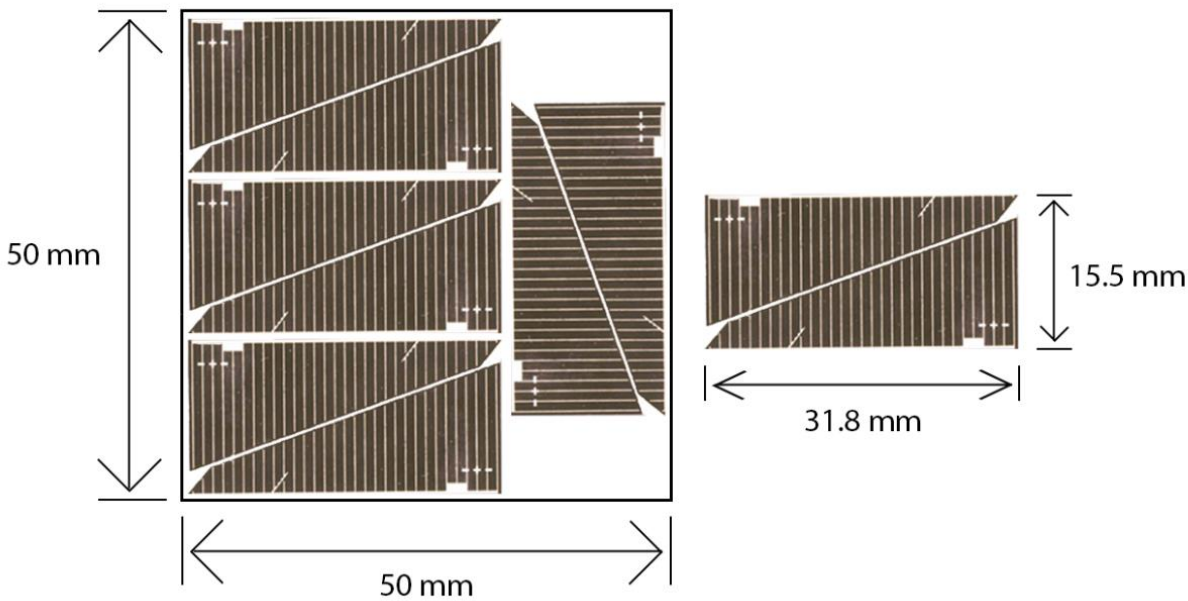


Fig. 1: A "normal" face – without the antenna

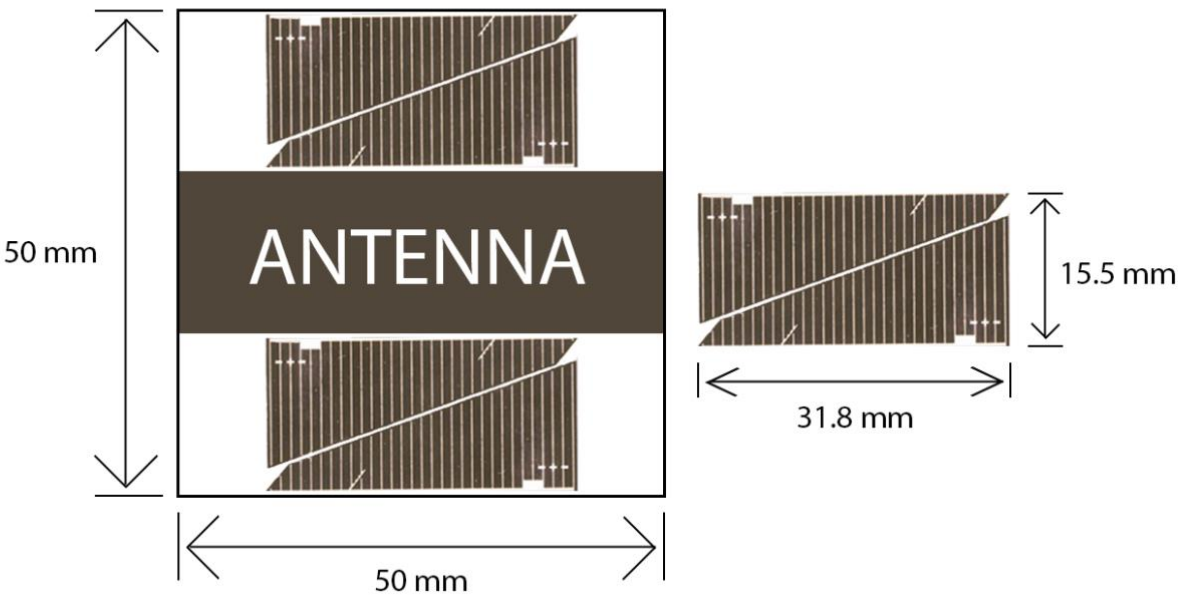


Fig. 2: The face with the antenna

It is obvious to see that, in both cases, the way the panels are arranged is the most optimal, so further calculations of area are redundant.

In case no. 1, **the "normal" face fits exactly 8 solar cells**, which will be our maximum number.

In case no. 2, **the face with the antenna fits exactly 4 solar cells**, which will be our minimum number.

P.S.: If the antenna would have been moved to the side in the second case, another rectangle of 2 solar cells could be put. But, since it doesn't seem like a good idea to have the antenna on the side due to multiple factors like weight distribution and because the reference picture has the antenna in the middle, we'll stick with this model (it's the "minimum" number, anyway, so it's fine).

Exercises 3 and 4:

For all the power calculation that we'll be doing in this paper we will consider that all the solar cells are connected in such a way so that the total generated power is the sum of the generated power of each of the cells. Also, because we are considering the End Of Life generated power, we will take the efficiency as 24% (not 27%).

We will use the sun constant S = 1400 W/m^2. Also, we will consider the conversion efficiency of the MPPT = 95%, the boost converter energy = 90% and other losses with a factor of 95%, giving us a total factor of losses F = 81.225%. The packaging efficiency will also be taken into account, being set at 85%. Considering the formula:

$$P_{EOL} = S \cdot \cos\varphi \cdot \eta_{SC} \cdot F \cdot P_F \cdot A_{SA}$$

for each solar array, we will have the P_{EOL} = 231.9786 * cos(ϕ) * A_{SA} , where ϕ is the incident angle of the sunbeams and A_{SA} is the area of the solar array.

<u>THE MINIMUM GENERATED POWER:</u>

The minimum generated power which, as stated in the guidelines for this homework, happens when the side with the antenna is facing the sun at a 90 degrees angle, is easily extractable from the data sheet of the solar cells.

P_{fm} = 231.9786 * cos(90°) * 4*2.277*10^(-4) W = 0.211286 W

In this case, the 4 adjacent faces with the illuminated one will produce 0 W each, because the sunrays are parallel to them, and because cos(90°) = 0 (the angle between the plane's normal and the sunrays), P = 0. Furthermore, the face opposite the illuminated one is completely in the shadow, so it also produces 0 W (self-explanatory).

So, the minimum generated power is P_{tm} = P_{fm} = 0.211286 W ≈ 0.211 W (for simplicity).

<u>THE "NORMAL" GENERATED POWER:</u>

The "normal" generated power which, as stated in the guidelines, happens when one of the other 5 faces is facing the sun at a 90 degrees angle, is calculated equivalently.

P_{fN} = 231.9786 * cos(90°) * 8*2.277*10^(-4) W = 0.422572 W.

Again, the other 5 faces generate no power for the same reason, so the total "normal" generated power is P_{tN} = P_{fN} = 0.422572 W ≈ 0.422 W (for simplicity).

THE MAXIMUM GENERATED POWER:

The way the guidelines describe it, the maximum generated power that needs to be calculated is when the cube's faces are parallel with the XY, XZ and YZ planes.

To my understanding, these planes contain the direction to the sun, the direction of motion and a 3rd direction perpendicular to the orbit. In that case, when the cube's faces are parallel with the XY, XZ and YZ planes for sure one of the faces is directly perpendicular to the sunrays.

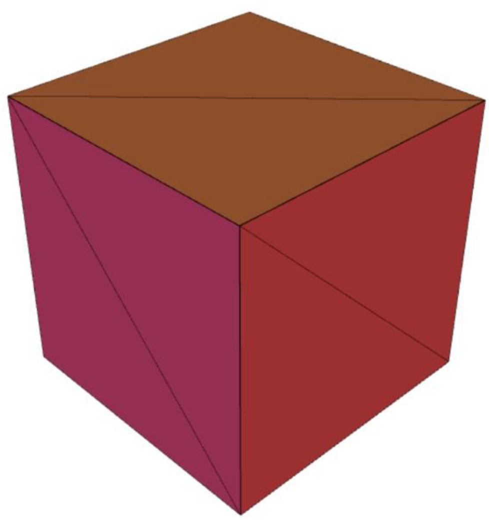
So, the maximum generated power is the "normal" one because, apart from the face with the antenna, the other 5 faces are identical and produce the same power.

But, since it's called the "Maximum Generated Power", I will assume any orientation of the satellite regarding the Sun.

Since, at most, 3 faces of the cube can get sunlight at once because of how spatial geometry works, we can consider a 3D model of a cube and draw some conclusion by pure observation.

Note: Obviously, to calculate the maximum generated power we will consider that none of the 3 faces that are being illuminated are the face with the antenna, which produces less power.

Using this: https://www.practicedrawingthis.com/3d/object.html?model=cube-001 free 3D model of a cube, my first assumption would be that the maximum generated power of the satellite would happen if the sunrays "enter" the cube through one of its corners, like so (imagine you are the Sun 😊):



In that case, it's actually pretty easy to calculate the total generated power of the cube.

Due to the sunrays "slicing" the cube by it's diagonal, it is pretty easy to visualize that the angle between the normal of all the 3 planes and the sunrays is 45°

So, in this case:

Each face produces $P_{fM} = P_{tN}*\cos(45°) \approx 0.298803$ W

(the maximum generated power of that face, the case when the rays are perpendicular, times the cos of the incident angle, per the formula from the course).

So, the total maximum generated power:

$P_{tM} = 3*P_{fM} = 0.89641W \approx 0.896$ W (for simplicity).

Note: Because of the fact that cos(x) is a concave function in the [0, π/2] interval and all the angles of the illuminated faces are dependent of each other with a (π/2 – x) type of dependency, the maximum happens for x = π/4 = 45°. The demonstration is quite a bit longer, so we'll just consider $P_{tM} = 0.896$ W.

Now, to find the average power generated by the solar cells, as instructed we will consider that all the 3 orientations have the same probability of happening (1/3 ≈ 33.33%), and we will plug them into this simple formula:

$P_{avg} = 1/3*P_{tm} + 1/3*P_{tN} + 1/3*P_{tM} = 0.5096$ W ≈ 0.51 W.

Note: This is only a simplified way of calculating, but because we are taking the absolute minimum, the absolute maximum and the "normal" into account (which happens the most often, since there are 5/6 panels that deliver the same power when facing the Sun), this way gives us a pretty good estimate of the real average generated power without any need for more complicated calculations.

Exercise 5:

Using the formula:

Orbital period

$$T = \frac{2\pi a^{\frac{3}{2}}}{k^{\frac{1}{2}}} = \left(\frac{a}{R}\right)^{\frac{3}{2}} T_E, \qquad a = R + \frac{h_{apog} + h_{perig}}{2}$$

$$T_E = 84{,}4\ min, R = 6366\ km\ (Earth's\ Radius),$$
$$\mathrm{k} = \mathrm{GM_T} = 3{,}986 \cdot 10^{14}\ \mathrm{m^3/s^2}$$

we can compute the orbital period for our satellite like so:

T = 2π*(6366000+500000)^(3/2)/(3.986*10^14)^(1/2) = 5,654.54 s = 94.2 mins.

In order to calculate the fraction of the orbit in eclipse, we will use the following algorithm:

1. **Eclipse Cone Half-Angle**:
   As before, when the Earth is between the satellite and the Sun, it casts a shadow. The half-angle α of this eclipse cone can be computed using:
   $\sin(\alpha) = \frac{R_e}{R_e+h}$
   Where R_e is the Earth's radius (roughly 6378 km) and h is the altitude of the satellite above the Earth.
2. **Angular Extent of the Eclipse**:
   Given the half-angle α of the eclipse cone, the angular extent of the eclipse (in radians) as seen from the center of the Earth is:
   $\theta = 2\alpha$
3. **Fraction of Orbit in Eclipse**:
   Since there are 2π radians in a full circle, the fraction f of the orbit the satellite spends in eclipse is:
   $f = \frac{\theta}{2\pi} = \frac{2\alpha}{2\pi} = \frac{\alpha}{\pi}$

So:

$\alpha = \arcsin(R_e / (R_e + h)) = 67.99901° \approx 70° \approx 1.22173$ rad

$f = 1.22173/\pi \approx 0.388$

So, the time spent in eclipse:

$T_{eclipse} = 94.2 * 0.388 \approx 36.54$ mins.

To calculate the maximum power consumed by the satellite, we will use this formula:

$P_{consumed} = E_{consumed} / t_{consumed} = C*V*DOD / t_{consumed} \approx 0.486$ W

So, if I want to make sure the satellite never loses the subsystems during an eclipse period or that its battery never goes under 10%, the subsystems need to be designed in such a way so that they don't consume more than 0.486 W during eclipse.