Homework PAE 3

For this assignment, we are required to calculate the link budget of our satellite by using as a reference the attached OA paper.

The exact variables that we need to compute are:

- The received power;
- SNR (Signal/Noise Ratio);
- The range of elevation angles where reception will be feasible.

The formulas that we will need:

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi R)^2}$$

$$P(dBm) = 10\log\frac{P(mW)}{1mW}$$

$$P_n = k\, T_p\, B_n$$

Formulas for received power, noise power and P(mW) to P(dBm).

$$\mathrm{SNR} = \frac{P_{\text{exp.signal}}}{P_{\text{noise}}}$$

Signal to Noise Ratio (for dB, subtract the value for the powers in dBm).

$$FSPL = 20\log_{10}(d) + 20\log_{10}(f) + 20\log_{10}\left(\frac{4\pi}{c}\right) - G_t - G_r$$

Free-Space Path Losses formula.

Now, we will take a look at the know values and what we need to calculate.

Universal constants:

$$k_B = 1.380649\text{x}10^{-23} J.K^{-1}$$

$$c = 3 \times 10^8 m/s$$

Antenna parameters:

Gain of the satellite's antenna: 0 dB.
Gain of the NanoSat Lab ground station antenna: 14.14 dB

The satellite's antenna is a monopole (MON), and its gain is considered to be 0 dB.

The NanoSat Lab's GS antenna is a four-patch antenna, thus having a directivity and so its gain is calculated at around 14.14 dB.

Signal parameters:

Since we are using LoRa transmission, the transmitted power will be close to its maximum capacity, **which is 22 dBm**.

The frequency that we will be transmitting at **is 868 MHz**, a commonly used frequency in Europe for research/educational satellites.

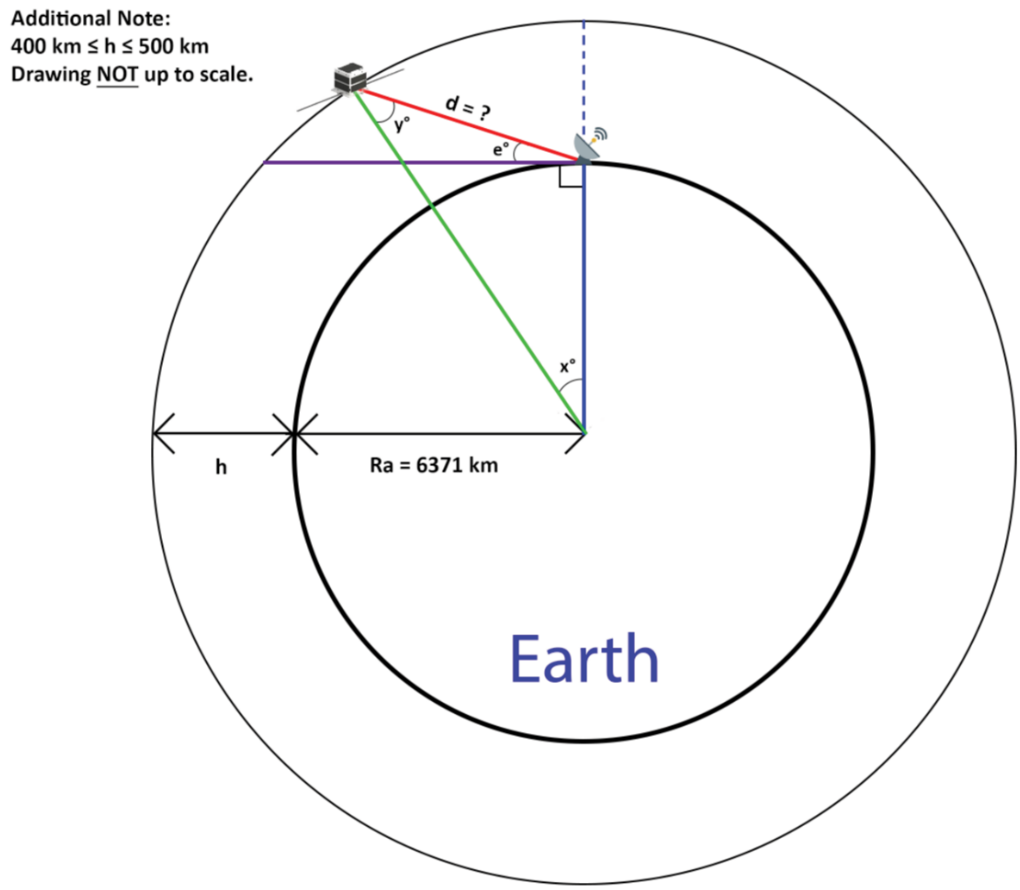
Considering that the LoRa modulation can correct for the Doppler shift, and the maximum that it can correct is directly proportional with the BW (bandwidth), we will need to choose a BW big enough in order to correct the maximum Doppler shift that we can experience. However, since the noise power is also proportional to the BW, we will need to also pay attention to choose the smallest favorable frequency, so that we minimize the noise.

The typical values for the BW are: 125 kHz, 250 kHz and 500 kHz. By inputting in the formula for the maximum Doppler shift a velocity of the satellite of 7.8 km/s, we get that for f0 = 868 MHz the maximum doppler shift is: Δf = 22.56 kHz. Taking into account the fact that for a BW of 125 kHz can compensate up to ±31.25 kHz of frequency shift, this will be our value for the BW. So, **B_n = 125 kHz**.

In order to calculate the noise temperature, we will reference the paper directly:

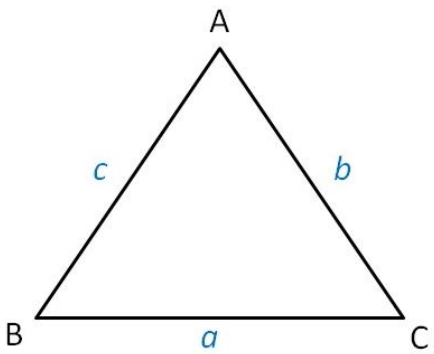
- **for the uplink, a temperature of 290K** is considered the worst case;
- for the downlink, the temperature can be calculated as the sum of the sky and the ground temperature; for the ground, 2320K would be a median estimation, and for the sky that value would be 20K, so, in total, **for the downlink, we will consider a noise temperature of 2340K**.

That only leaves us to compute the distance between the GS (ground station) and the satellite. Obviously, this distance is not constant and is strictly dependent on the elevation of the satellite. We will use the following drawing as a geometric model:



What we need to find out is d =? (the red line).

We will primarily use the sines law in the colored triangle, law shown below:



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The triangle has:

- One angle of $90^\circ + e^\circ$, where e° is the elevation in degrees;
- One angle of x°;
- One angle of y°;
- The blue side which is = R_a = 6371 km;
- The green side which is = Ra + h, Ra = 6371 km and 400 km ≤ h ≤ 500 km;
- The red side which is what we need to compute.

We have:

$$\frac{h + Ra}{\sin(90^\circ + e^\circ)} = \frac{Ra}{\sin(x^\circ)}$$

=>

$$x^\circ = \sin^{-1}\left(\frac{Ra}{(h + Ra)\sin(90^\circ + e^\circ)}\right)$$

So:

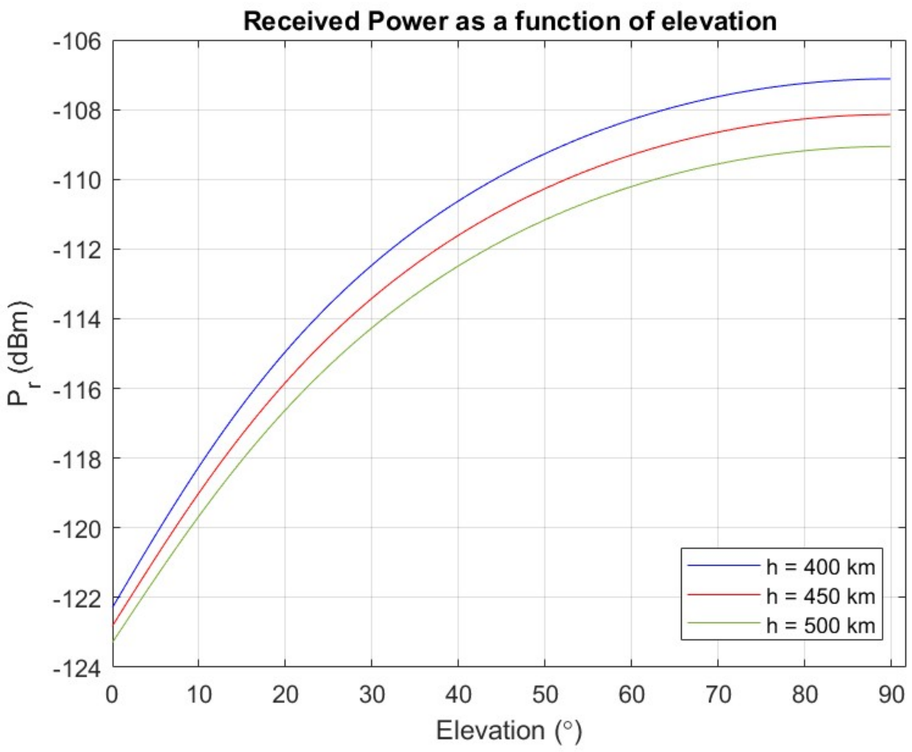
$$y^\circ = 180^\circ - 90^\circ - e^\circ - x^\circ = 90^\circ - e^\circ - x^\circ$$

<u>And, finally:</u>

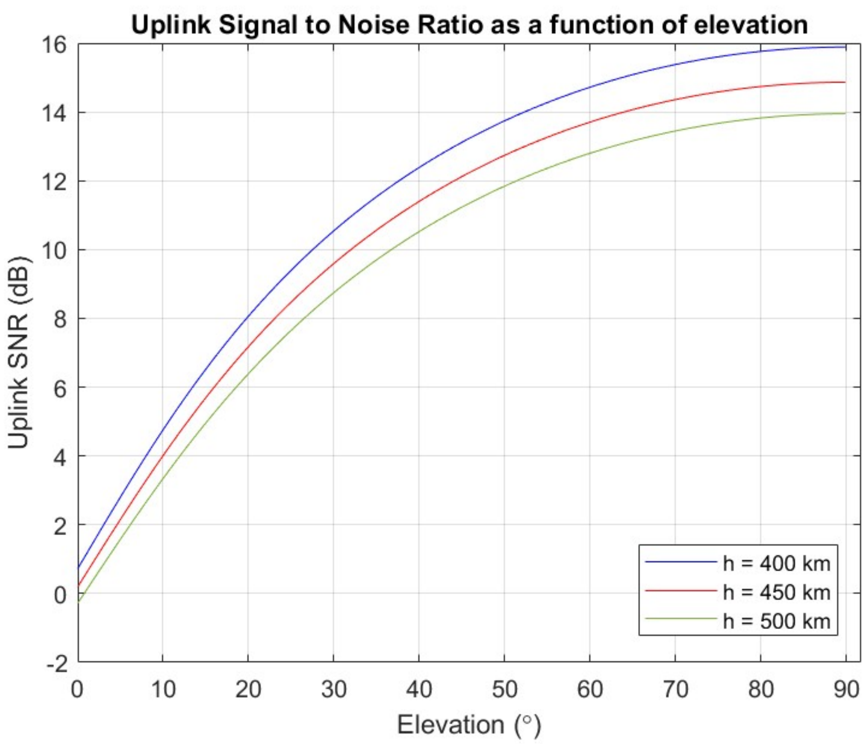
$$d = (h + Ra) \times \frac{\sin(90^\circ - e^\circ - x^\circ)}{\sin(90^\circ + e^\circ)}$$

Where: $x^\circ = \sin^{-1}\left(\frac{Ra}{(h + Ra)\sin(90^\circ + e^\circ)}\right)$

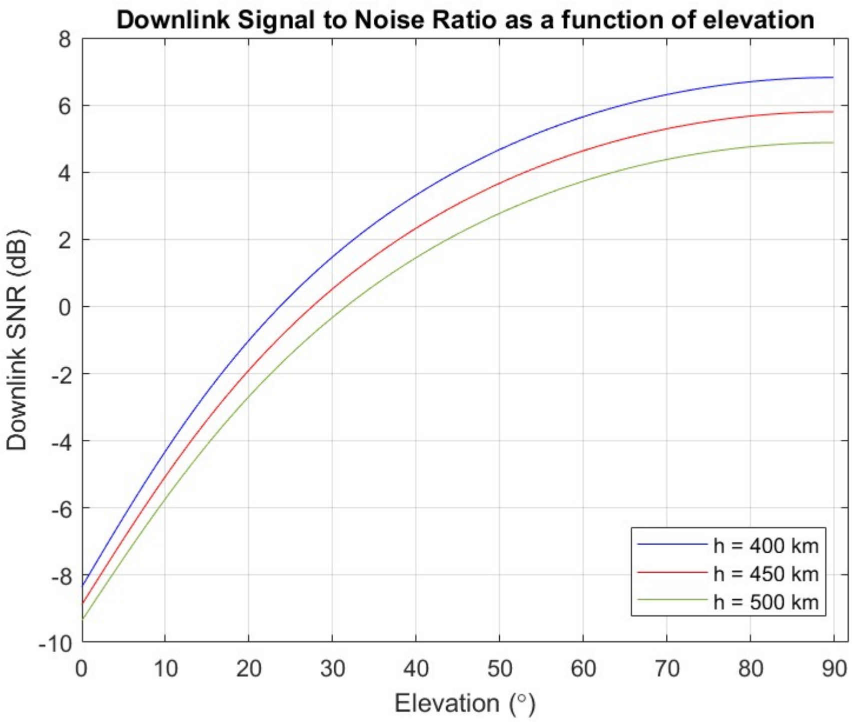
As I have previously stated, 400 km ≤ h ≤ 500 km. We will now look and analyse 3 graphs made in MATLAB. The purpose of the graphs is stated in their title. Each of the graphs contains 3 functions, as I have made h be equal to 400, 450 and 500 km respectively.



As we can see, the received power for all 3 heights is in approx. in the (-122, -108) dBm interval. This is similar to what the reference paper shows, even being an improvement.



The Uplink SNR is for basically every elevation, which is a really good result.



The Downlink SNR is lower (since the downlink noise temperature is considerably higher), but still positive for most of the interval.

If we consider the sensitivity values from the reference paper which are:

- approx. -123 dBm for SF7;
- approx. -126 dBm for SF8;
- approx. -129 dBm for SF9;
- approx. -132 dBm for SF10;
- approx. -134.5 dBm for SF11;
- approx. -137 dBm for SF12,

we will find that, if we want as low of a Spreading Factor as possible (specifically SF7), to be absolutely certain that the sensitivity is low enough, we will have to consider elevations higher than 5° to be accommodate for all heights. Since the value is so low, we could even consider elevations higher than 10° to ensure that, in all cases, the antenna can feasibly catch the signal. For any other SF higher than SF7, we can consider any elevation. If we also want to have a positive SNR for both uplink and downlink, we can consider elevations higher than 35-40°.

Note: The sensitivity values might need recalculating for our specific case.