# Homework PAE 6

**Exercise 1**.

- To determine the weight (or mass) of the satellite, we simply place it on a kitchen scale;

- Ensure the scale is calibrated and can measure with sufficient precision;

- We remember this value as m.

**Exercise 2**.

Density $\rho$ is given by:

$\rho = m/V$

Where:

- m is the mass of the satellite (from step 1).

- V is the volume of the satellite. For a cube with a side length s = 5 cm, V = s^3 = 125 cm^3.

**Exercise 3**.

The moment of inertia I for a cube rotating about an axis that goes through the center of the cube and is parallel to one of its sides is:

I = (1/6) * m * s^2

Where:

- m is the mass of the cube. (step 1)

- s is the side length of the cube. (5 cm)

Effects of Changes in Density and Dimensions:

- If the density is doubled while keeping the dimensions the same, the mass will double. This will result in the moment of inertia doubling as well.

- If the dimensions are doubled while keeping the density the same, the volume (and thus the mass) will increase by a factor of 8, and the moment of inertia will increase by a factor of 32.

Implications for an ADCS (Attitude Determination and Control System):

- A change in moment of inertia will affect the satellite's rotational dynamics. For a given torque, a larger moment of inertia will result in a slower angular acceleration (based on $\tau = I \alpha$, where $\tau$ is torque and $\alpha$ is angular acceleration).

- The ADCS will need to account for these changes in dynamics when performing maneuvers. For example, with a larger moment of inertia, more torque might be required to achieve the same change in angular velocity in a given amount of time.

**Exercise 4**.

Taking into consideration that the satellite does not have a uniformly distributed mass, determining the center of mass analytically requires:

- Break Down the Satellite into Simple Shapes: Decompose the satellite into smaller parts that have uniform density or known mass.

- Determine Mass and Center of Mass of Each Part: For each part, determine its mass mi and its center of mass ri (given as a position vector).

- Compute the Weighted Average of the Position Vectors: The center of mass R of the entire satellite is:

$R = \Sigma(mi * ri) / \Sigma mi$

Here, the summations run over all the parts of the satellite.

**Exercise 5**.

One common method is the pendulum method:

- Suspend the satellite from a point and let it oscillate like a pendulum. The line of the suspension point to the center of oscillation is a line that goes through the center of mass.

- Repeat this by suspending the satellite from a different point. Where the lines from the two suspension points intersect is the center of mass.

For the moment of inertia:

- Using the same pendulum setup, you can determine the moment of inertia by measuring the period of oscillation and knowing the mass and the distance from the suspension point to the center of mass.

**Exercise 6**.

Vibration modes in a cube can be thought of as the number of half-wavelengths (antinodes) fitting along the dimension of the cube. For a cube of side length 'L', the basic modes in each direction can be:

- Fundamental (1 half-wavelength along one side)
- 2nd Harmonic (2 half-wavelengths)
- 3rd Harmonic (3 half-wavelengths)

... and so on.

For each mode, the wavelength `λ` in terms of cube side length 'L' can be determined. For example:

- Fundamental: $\lambda = 2L$ (because only half the wavelength fits along 'L')
- 2nd Harmonic: $\lambda = L$
- 3rd Harmonic: $\lambda = 2L/3$

... and so on.

Using the speed of sound `v` in the material, the frequency `f` for each mode can be computed as:

$f = v / \lambda$

For each material (pine tree wood, aluminum, steel), you need to know:

- Young's modulus `E` (elasticity of the material)
- Density `d` (mass per unit volume)

Using the given formula:

$v = \mathrm{sqrt}(E/d)$

Compute the speed of sound `v` in each material.

Using the previously determined `v` for each material and `λ` for each mode, compute the resonance frequency `f` using:

$f = v / \lambda$

We would draw the cube and represent the antinodes (points of maximum vibration) and nodes (points of no vibration) for each mode. The fundamental mode will have an antinode in the middle of the cube's side, the 2nd harmonic will have nodes at 1/3 and 2/3 along the side, and so on.

Using the values of `E` and `d` for pine tree wood, aluminum, and steel, we would repeat the above steps to determine the resonance frequencies for each material.