## Problem set 1 - Asset Pricing A.Y. 2018–19 - LUISS

The problem set is a group work. Problem set must be **typed** and submitted using the e-learning platform by the due date below. Submit also any script, or excel file, you have used (if the number of files is large, please zip the files in a single compressed file).

Available online: September 21 - 11AM - Deadline: September 28 - 11AM

## Problem 1

In this problem set, you are ask to study the relationship between oil prices and stock prices. Intuitively, if oil prices decline because of an increase in supply, the economy should benefit and stock prices should go up. On the contrary, if oil prices decline because demand shrinks, then this is a signal of weak aggregate demand and stock prices should go down. You can find data for this problem set on the e-learning page. Note that the data have been downloaded from Thomson Reuters Datastream, at daily frequency, for the period 12/31/2006 to 9/20/2017. Refer to Datastream for details regarding the data and to the Excel sheet readme for the name of the variables.

- 1. Plot the oil price and the stock market price index in two separate figures. Can you identify any clear pattern between the two time-series? Explain briefly.
- 2. Compute daily log changes in oil prices and in the stock market index (henceforth "returns") and report mean, standard deviation, skewness, kurtosis and first order auto-correlation. Provide a short comment regarding your results.
- 3. Compute the VaR and the ES with 1% and 5% confidence level for oil and stock market returns. Comment on the methodology you have used to compute the value-at-risk and the expected shortfall. What would the VaR have been if you had assumed normal returns with mean and standard deviations equal to the unconditional values over the full sample?
- 4. Compute the unconditional correlation between the two return series. How do you interpret the numbers you found?
- 5. Compute time-varying correlations between the two returns series using a rolling window of 20 days. Plot your results and comment your findings. What is the average of the time-varying correlations?

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6. Some economists have suggested that it is possible to estimate a proxy for changes in the aggregate demand of oil estimating the following equation:

$$\Delta p_{oil,t} = c + \Delta p_{copper,t} + \Delta p_{dollar,t} + \Delta r_{10,t} + \epsilon_t, \tag{1}$$

where  $\Delta$  denote the log daily change,  $p_{dollar}$  is the dollar/euro exchange rate and  $r_{10}$  is the yield on the 10-year US Treasury bond. The fitted values of the above regressions can be considered a proxy for changes in oil prices that do not depend on changes in its supply. Estimate the equation and report your results using the sample 6/30/2011 to 6/30/2014. How do you interpret these results? Comment on the sign and significance of the coefficients, as well as on the  $R^2$ . Do you agree that the fitted values from this regression might be interpreted as changes in aggregate demand. Comment briefly.

- 7. Use the coefficient estimates from (1) to forecast what the price of oil should have been if only aggregate demand changed in the period that goes from 6/30/2014 to the end of the available sample (*Hint*: keep the coefficient estimates from the sample 6/30/2011 to 6/30/2014 and apply them to the data for the sample 6/30/2014 to the more recent observation. This is a sort of out-of-sample forecast based on the model.). Plot your forecast values together with the actual values. How much of the decline in oil prices can be attributed to changes in aggregate demand?
- 8. Compute the time-varying correlations (using the same 20-day rolling window) between stock market returns and demand-related changes in oil prices (i.e., the fitted values from (1)) for the sample 6/30/2011 to latest available observation. Plot your results results and comment briefly.
- 9. Compute the time-varying correlations (using the same 20-day rolling window) between stock market returns and supply-related changes in oil prices (i.e., the residuals values from (1)) for the sample 6/30/2011 to the latest available observation. Plot your results results and comment briefly.
- 10. Consider now an augmented version of (1) that includes a proxy for volatility in the markets measured by the daily log changes in the VIX volatility index:

$$\Delta p_{oil,t} = c + \Delta p_{copper,t} + \Delta p_{dollar,t} + \Delta r_{10,t} + \Delta VIX_t + \epsilon_t. \tag{2}$$

Estimate this equation for the sample 6/30/2011 to latest available observation. Report and comments your results.

11. Plot the time-varying correlations (using the same 20-day rolling window) between stock market returns and supply- and demand-related changes in oil prices (i.e., fitted and residuals values from (2)). Are there any differences with respect to the results you have obtained using (1)? Explain briefly commenting which of the two specifications you prefer and why.

## Problem 2

Consider a consumer that wants to maximize her utility that depends on the consumption of two goods: apples X and peaches Y. The price of apples is  $P_X$  and the price of peaches is  $P_Y$ . The consumer has a disposable income equal to W and cannot save. Suppose the utility function is equal to  $U(X,Y)=X^{\alpha}Y^{1-\alpha}$ . Find the demand curves of X and Y. Compute the derivative of each demand curve with respect to  $P_X$ ,  $P_Y$ ,  $\alpha$  and W and comment your results.

## **Problem 3**

Assume that R is lognormally distributed, with mean  $\mu$  and variance  $\sigma^2$ . Therefore,  $r = \log R$  is normally distributed, with mean  $\mu$  and variance  $\sigma^2$ . Assume further that  $\mu = 0$ . Prove that:  $E(R) = e^{0.5\sigma^2}$  and that  $\sigma^2(R) = e^{\sigma^2(r)}[e^{\sigma^2(r)} - 1]$  (Hint: Use the density function of the normal distribution and the definition of expected value in continuos time.).