

## Problem set 2 - Asset Pricing A.Y. 2018–19 - LUISS

The problem set is a group work. Problem set must be **typed** and submitted using the **e-learning platform** by the due date below. Submit also any script, or excel file, you have used (if the number of files is large, please zip the files in a single compressed file).

**Available online: Deadline: October 16 - 2PM**

### Problem 1

Consider the definitions of yield to maturity, yield curve, forward rate and holding-period return that we have examined in class. In this problem you are asked to study the so called expectation theory of the term structure of interest rates, and its implications in terms of forward rates and holding period returns. In the data file `bondprice.dat.txt` you will find the time-series for prices of constant maturity US government zero-coupon bonds with maturities 1, 2, 3, 4, and 5 years. For example, the series `price1` corresponds to the price of a bond with maturity that is constant over time and equal to 1 year. Use this data to:

- compute monthly yields to maturity at 1, 2, 3, 4 and 5 years horizon. Plot the yield curve at 3 different dates of your choice. Explain briefly the classic interpretation of the curve, and comment on its slope.
- compute monthly time series of forward rates from year 1 to 2, from 2 to 3, from 3 to 4 and from 4 to 5. Explain what is the interpretation of a forward rate, and how you have computed them.
- compute monthly holding period returns from investing in the five different constant maturity zero-coupon bonds, and monthly holding period excess returns using the yield of the 1 year zero as risk-free rate.
- Prepare a table that reports, for maturity equal to 1, 2, 3, 4 and 5 the average holding period return, and the corresponding standard error and standard deviation. What do you conclude from this table? Is the expectation theory of the term structure of interest rates supported by the data?
- Run the following OLS regression

$$y_{t+N}^{(1)} - y_t^{(1)} = a + b(f_t^{(N+1)} - y_t^{(1)}) + \epsilon_{t+N},$$

where  $N = 1, 2, 3, 4$ ,  $y_t^{(N)}$  is the  $N$ -year bond yield at date  $t$  and  $f_t^{(N)}$  denotes the  $N$ -period ahead forward rate. Prepare a table that reports

- estimate of the intercept  $a$  and standard error,
- estimate of slope coefficient  $b$  and standard error,
- adjusted  $R^2$ .

Comment your results and explain if these results support the expectation theory of the term structure of interest rates.

- Run the following OLS regression

$$hpr_{t+1}^{(N)} - y_t^{(1)} = a + b(f_t^{(N+1)} - y_t^{(1)}) + \epsilon_{t+1},$$

where  $N = 1, 2, 3, 4$ , and  $hpr_{t+1}^{(N)}$  denotes the one-year holding period return at date  $t + 1$  on a  $N$ -year bond. Prepare a table that reports

- estimate of the intercept  $a$  and standard error,
- estimate of slope coefficient  $b$  and standard error,
- adjusted  $R^2$ .

Comment your results and explain if these results support the expectation theory of the term structure of interest rates.

## Problem 2

Consider a quadratic specification for the utility function:

$$u(c) = -\frac{1}{2}(c^* - c)^2,$$

where  $c^*$  is a positive constant level of consumption. Plot this utility function and its marginal utility. What is the range of consumption levels that makes economic sense? Compute the coefficient of relative risk aversion (i.e., use the standard formula  $RRA = -\frac{C u_{cc}}{u_c}$  to compute the relative risk aversion coefficient). Then evaluate the risk aversion at  $c = 0$ ,  $c = c^*/2$ ,  $c = c^*$ , and  $c = 2c^*$ .

### Problem 3

Consider the standard present value formula when the periodic interest rate is constant and equal to  $r$ :

$$P_t = \sum_{k=1}^{\infty} \frac{E_t D_{t+k}}{(1+r)^k}.$$

Use this formula to discuss the effects of various kind of news about future dividends on prices and returns:

1. An increase in  $D_{t+1}$  by \$1 announced at time  $t$ .
2. An increase in  $D_{t+k}$  by \$1 announced at time  $t+2$ .
3. An increase in  $D_{t+1}, D_{t+2}, \dots$  by \$1, at all future dates, announced at time  $t$ .
4. An increase in  $D_{t+k}$  by \$1 combined with a decrease of  $D_{t+k+1}$  by  $\$1 + r$ , both announced at time  $t+2$ .

Plot the time path of prices, dividends and returns in each case.