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UNIT 2

CHAPTER 2.1

SET =

⇒ Set Is collection of Unordered distinct Objects

Eg - $\boxed{A = \{a, e, i, o, u\}}$ $a \in A$

$\underbrace{\qquad\qquad}_{\text{Lowercase Elements}}$

Eg = Set of Odd positive Integer less than 10 can be denoted by

$\boxed{O = \{1, 3, 5, 7, 9\}}$

TYPES OF REPRESENTATION OF SET =

(i) Tabular Form

⇒ $A = \{a, b, c, d\}$

(ii) Roaster / Builder Form

⇒ $\{x \mid x \in N, x \text{ Is multiple of } 3\}$

Standard Notations

ϕ = Null Set

N = Natural Numbers

Z = Integers

└── $\{ ---- -1, 0, 1, ---- \}$

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TYPES OF SETS =

(i) **Finite Sets** = Countable Values
Eg = {a, e, i, o, u}

(ii) **Infinite Sets** = Uncountable Values
Eg = { x | x ∈ N where x Is multiple of 2 }

(iii) **Equality of Sets** = Equal number of values In both sets
Eg = A = B , $\boxed{x \in A, \ x \in B}$

→ If set consists of specific ~~&~~ numbers of different Elements then It Is termed As finite set.
→ If set consists of Infinite number of Elements Or counting different Elements does not come to An End , then It Is termed As Infinite set.
→ Two sets Are Equal when both ~~of them~~ ~~they~~ have same Elements And x ∈ A and x ∈ B & termed As Equality of sets.

SUBSET :

→ If All the Elements of set A Are present In set B then It Is called subset Or we can say set A Is contained Inside set B.

Eg = $A = \{x, y\}$
$B = \{x, y, z\}$
As A Is contained In B $\therefore A \subseteq B$

* Subset Is denoted by "$\subseteq$".

ALL SUBSET OF A SET =

- It consists Of All possible sets Including It's Element And Null set (ϕ).

Q Find All subsets of set $A = \{1, 2, 3, 4\}$

⇒ Subsets

$\{\ \}$
$\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$
$\{1, 2\}$
$\{1, 2, 3\}$ $\{2, 3, 4\}$, $\{1, 3, 4\}$ $\{1, 2, 4\}$
$\{1, 2, 3, 4\}$
~~{1, 2}~~ $\{1, 3\}$ $\{1, 4\}$
$\{2, 3\}$, $\{2, 4\}$
$\{3, 4\}$

Proper Subset =

→ If set B contains Atleast One Element that Is not present In set A.

Eg - $A = \{12, 24\}$ $B = \{12, 24, 36\}$
$\therefore A / q.,$ $\boxed{A \subset B}$

→ Proper Subset Is denoted by "$\subset$".

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IMPROPER SUBSET =

→ The set which contains all Elements of set Original set Is termed As Improper subset.

Eg = $A = \{1, 2, 4\}$ $B = \{1, 2, 4\}$

then, $A \subseteq B$

Note = If a set has n Elements then number of subset of given set Is "2^n" And number of proper subset of given subset Is "$2^n - 1$".

Q Write the following sets In builder form.

(a) $A = \{2, 4, 6, 8, 10, 12, 14\}$

⇒ $A = \{x \mid x \in$ Multiple of 2 where x Is less than 15$\}$

(b) $k = \{3, 6, 9, 12, 15, 18\}$

⇒ $k = \{x \mid x \in$ Multiple of 3 where x Is less than 19.$\}$

(c) L = { Punjab, Haryana, UP, Bihar }

⇒ L = $\{x \mid x \in$ States of India$\}$

Q Write the following In tabular form.

(a) $A = \{x : x^2 = 9\}$

⇒ $A = \{-3, 3\}$

(b) $B = \{x : x$ Is the multiple of 3 And $0 < x < 20\}$

$\Rightarrow x = \{3, 6, 9, 12, 15, 18\}$

(c) $C = \{x : x$ Is positive Even Integer$\}$

$\Rightarrow x = \{2, 4, 6, 8, 10, ----\}$

(d) $D = \{x : x$ Is multiple of $5\}$

$\Rightarrow x = \{5, 10, 15, 20, ----\}$

Q If $A = \{x : 3x = 6\}$, does $A = 2$?

$\boxed{A = \{2\} \neq A = 2}$

Q Determine power set $P(A)$ of set $A = \{1, 2, 3\}$.

$\Rightarrow P(A) = 2^3 = 8$

$\Rightarrow P(A) = \{\ \}$
$\{1\}\ \{2\}\ \{3\}$
$\{1, 2\}\ \{1, 3\}$
$\{2, 3\}$
$\{1, 2, 3\}$

CARTESSIAN PRODUCT OF TWO SETS :-

→ The cartessian product of two set P And Q. In that Order Is the set of All Ordered pair whose first member belongs to set P And second member belongs to set Q And Is denoted by $P \times Q$ i.e.;

$$\boxed{P \times Q = \{(x, y),\ x \in P \text{ and } y \in Q\}}$$

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Q. Let $P = \{a, b, c\}$ And $Q = \{k, l, m, n\}$.
Determine cartessian product of P and Q.

⇒ $P \times Q = \{(a, k)\ (a, l)\ (a, m)\ (a, n)$
$(b, k)\ (b, l)\ (b, m)\ (b, n)$
$(c, k)\ (c, l)\ (c, m)\ (c, n)\}$

Q. If $A = \{1, 2, 4, 5\}$
$B = \{a, b, c\}$
$C = \{a, 5\}$

Compute $A \cup C$, $(A \cup C) \times B$

⇒ $A \cup C = \{1, 2, 4, 5, a\}$

⇒ $(A \cup C) \times B = \{(1, a)\ (1, b)\ (1, c)$
$(2, a)\ (2, b)\ (2, c)$
$(4, a)\ (4, b)\ (4, c)$
$(5, a)\ (5, b)\ (5, c)$
$(a, a)\ (a, b)\ (a, c)\}$

Q. (a) If $A = \{+, -\}$ And $B = \{00, 01, 10, 11\}$
find $A \times B$

⇒ $A \times B = \{(+, 00), (+, 01), (+, 10), (+, 11),$
$(-, 00), (-, 01), (-, 10), (-, 11)\}$

(b) Determine power set of $A = \{a, b, c, d\}$

⇒ $2^n = 2^4 = 16$

$\{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\},$
$\{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\},$
$\{a, c, d\}, \{a, b, d\}\}$

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(c) "Show that cartissian product of $B \times A$ Is not Equal to cartissian product of $A \times B$ where $A = \{1, 2\}$ And $B = \{a, b, c\}$.

$\Rightarrow$ $A = \{1, 2\}$

$B = \{a, b, c\}$

$\therefore$ A/q,

$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\} \quad \text{— (i)}$$

$$B \times A = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\} \quad \text{— (ii)}$$

From Equation (i) And (ii)

$$\boxed{A \times B \neq B \times A}$$

(d) What Is the cartissian product $A \times B \times C$ when $A = \{0, 1\}$ $B = \{1, 2\}$ $C = \{0, 1, 2\}$?

$\Rightarrow$ $A \times B = \{(0,1), (0,2), (1,1), (1,2)\}$

$$A \times B \times C = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$$

#. CARDINALITY OF A SET =

→ The total number of Unique Elements In the set Is termed As cardianallity of the set.

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Eg = Q A = {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}

Hence,

Cardianallity $n(A) = 7$

Q Let A be the set of All non-negative Even Integers i.e.,

$A = \{0, 2, 4, 6, 8, 10, ---\}$

As A Is An Infinite set, hence the cardianallity of the set Is Infinite.

The cardianallity of the Empty set Is "ϕ" Is 0.

i.e., $\boxed{n(\phi) = 0}$

Q. Find the cardinal number of the following set

{x : x Is A letter In the word "BASEBALL"}

$\Rightarrow$ $\boxed{n(x) = 5}$

OPERATIONS ON SET =

(i) UNION = Union Is the set that contain those Elements that Are Either "A" Or "B" Or In both.

Eg = $A = \{a, b, c, d\}$ $B = \{a, b, c, e, F\}$

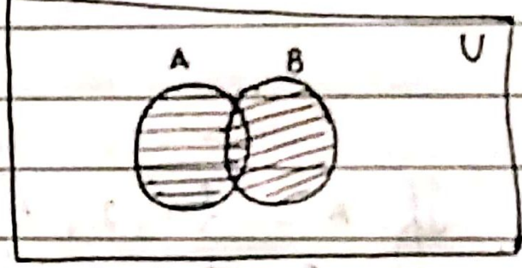
$\boxed{A \cup B = \{a, b, c, d, e, F\}}$

Q. Show that $A \cup B = A$, If $A = \{a, e, i, o, u\}$ And $B = \{a, i, u\}$

$\boxed{A \cup B = \{a, e, i, o, u\}}$ — (i)

∴ From Equation (i)

$\boxed{A \cup B = A}$

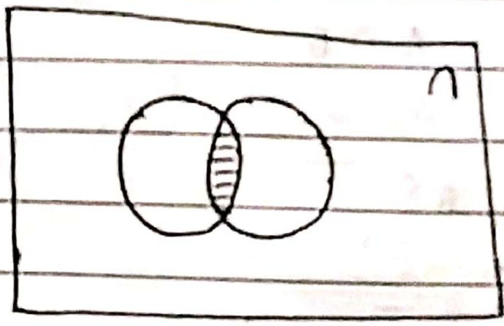


(ii) INTERSECTION =

→ It Is the set containing those Elements which are In both i.e., A And B.

Eg = $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 6, 7\}$

$\boxed{A \cap B = \{1, 3\}}$



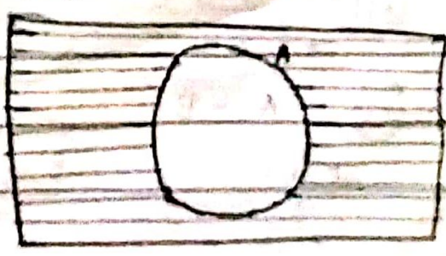
(iii) COMPLIMENT =

→ The compliment of a set Is Equal to U − A.

Eg = $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3, 4\}$

∴ $(U - A) = \{1, 5, 6\}$

$\Rightarrow \boxed{\bar{A} = \{1, 5, 6\}}$

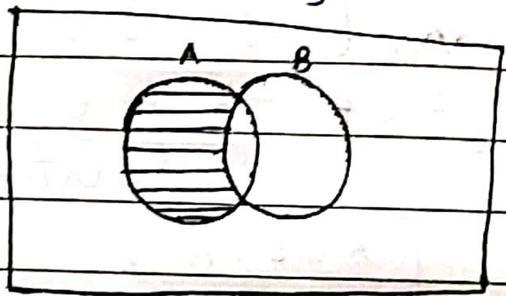


(iv) DIFFERENCE (A – B)

→ It Is the set containing those Elements which Are In A but not In B.

Eg = $A = \{1, 2, 3, 4, 5, 6\}$, $B = \{3, 4, 5, 6, 7, 8\}$

⇒ $\boxed{A - B = \{1, 2\}}$



A – B Is shaded

Q. If $A = \{1, 2, 5, 6\}$

$B = \{2, 5, 7\}$

$C = \{1, 3, 5, 7, 9\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Find

(i) $A \cap B$

⇒ $\boxed{A \cap B = \{2, 5\}}$

(ii) A^c

⇒ $A^c = \bar{A}$

$\boxed{\bar{A} = U - A}$

⇒ $\bar{A} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 5, 6\}$

$\boxed{\bar{A} = \{3, 4, 7, 8, 9\}}$

(iii) $B \cup C$

⇒ $\boxed{B \cup C = \{1, 2, 3, 5, 7, 9\}}$

(iv) $A - B$

⇒ $A - B = \{1, 2, 5, 6\} - \{2, 5, 7\}$

⇒ $\boxed{A - B = \{1, 6\}}$

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(v) $(A \cup C) - B$

$\Rightarrow \boxed{(A \cup C) = \{1, 2, 3, 5, 6, 7, 9\}}$

$\Rightarrow (A \cup C) - B = \{1, 2, 3, 5, 6, 7, 9\} - \{2, 5, 7\}$

$\Rightarrow \boxed{(A \cup C) - B = \{1, 3, 6, 9\}}$

(vi) $(A \cup B)^c$

$\Rightarrow \boxed{A \cup B \ \{1, 2, 5, 6, 7\}}$

$U - (A \cup B)^c$

$\Rightarrow (A \cup B)^c = (\overline{A \cup B})$

$\boxed{\overline{A \cup B} = \{3, 4, 8, 9\}}$

ALGEBRA OF SETS

(i) IDEMPOTENT LAW :

$A \cup A \equiv A$

$A \cap A \equiv A$

(ii) ASSOCIATIVE LAW :

$(A \cup B) \cup C \equiv A \cup (B \cup C)$

$(A \cap B) \cap C = A \cap (B \cap C)$

(iii) COMMUTATIVE LAW =

$A \cup B = B \cup A$

$A \cap B = B \cap A$

(iv) DISTRIBUTIVE LAW :

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(v) DE-MORGAN'S LAW :

$(A \cup B)' = A' \cap B'$

$(A \cap B)' \equiv A' \cup B'$

(vi) IDENTITY LAW =

(a) $A \cup \phi \equiv A$

(b) $A \cap U \equiv A$

(c) $A \cup U \equiv U$

(d) $A \cap \phi \equiv \phi$

(vii) INVOLUTION LAW =

(a) $(A')' \equiv A$

(viii) COMPLEMENT LAW =

(a) $A \cup A' \equiv U$

(b) $A \cap A' \equiv \phi$

(c) $U' \equiv \phi$

(d) $\phi' \equiv U$

PRINCIPLE OF INCLUSION =

•) Theorem - 1 =

→ Let p and q be any two non-disjoint set, then

$$\boxed{|P \cup Q| = |P| + |Q| - |P \cap Q|}$$

•) Theorem - 2 =

→ Let p, q and r are three finite sets then

$$\boxed{|P \cup Q \cup R| = |P| + |Q| + |R| - |P \cap Q| - |P \cap R| - |Q \cap R| + |P \cap Q \cap R|}$$

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Statements =

→ Let A and B be any finite sets then

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

→ To find the number of elements in $A \cup B$ we add $n(A)$ and $n(B)$ and then we subtract $n(A \cap B)$ that is we include $n(A)$ and $n(B)$ and we exclude $n(A \cap B)$, this principle is called Inclusion - Exclusion principle.

Q. Out of 1200 students at a college 582 took Economics, 627 took English, 543 took Maths, 217 took both Economics and English, 307 took both Economics and Maths, 250 took both Maths and English, 222 took all three courses. How many took none of the three.

⇒ $|A| = 582$, $|B| = 627$, $|C| = 543$,

$|A \cap B| = 217$, $|A \cap C| = 307$, $|B \cap C| = 250$,

$|A \cap B \cap C| = 222$

∵ A/q,

$$\boxed{|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|}$$

⇒ $|A \cup B \cup C| = 582 + 627 + 543 - 217 - 307 - 250 + 222$

$\boxed{|A \cup B \cup C| = 1200}$ (Taken Course)

∴ A/q, Not taken any of the three course =

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Q Among the first 1000 positive Integers determine the Integers which they are not divisible by 5 nor by 7 nor by 9 ?

$\Rightarrow |A| = \frac{1000}{5} = 200$

$\Rightarrow |B| = \frac{1000}{7} = 142$

$\Rightarrow |C| = \frac{1000}{9} = 111$

$\Rightarrow |A \cap B| = \frac{1000}{5 \times 7} = 28$

$\Rightarrow |A \cap C| = \frac{1000}{5 \times 9} = 22$

$\Rightarrow |B \cap C| = \frac{1000}{7 \times 9} = 15$

$\Rightarrow |A \cap B \cap C| = \frac{1000}{5 \times 7 \times 9} = 3$

$\therefore$ A/q,

$$\boxed{|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|}$$

$\Rightarrow |A \cup B \cup C| = 200 + 142 + 111 - 28 - 25 - 15 + 3$

$\boxed{|A \cup B \cup C| = 391}$

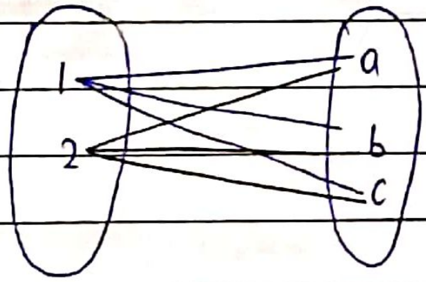
$\therefore \quad 1000 - 391$

$\Rightarrow \boxed{609}$

#. RELATION AND FUNCTION :

•) **Relation** = Let A and B be two known Empty set then a relation from A to B is a subset of A × B.

Eg = (1, 2) (a, b, c)



(1, a), (1, b), (1, c),
(2, a), (2, b), (2, c)

$\boxed{R \subseteq A \times B}$

#. Point : (i) $R_{max} = A \times B$

(ii) $R_{min} = \phi$

(iii) Total Number of Relations = $2^{m \cdot n}$

where,

m = Number of Elements In set A
n = Number of Elements In set B

Q. If m = 2, n = 3, find total number of relations.

⇒ ∴ A/q,

$\boxed{\text{Total number of relations} = 2^{m \cdot n}}$

∴ A/q,

$\boxed{\text{Total Number of relations} = 2^{6}}$

BINARY RELATIONS

→ It Is A set Of Ordered pair where first Element Is from set A And second Element Is from set B

Eg = $A = \{a, b\}$

$B = \{1, 2, 3\}$

$$\boxed{A \times B = \{(a,1)(b,2)(a,3)\}}$$

Types Of Relation

(i) Reflexive Relation =

→ Let R be relation In set A then R Is called reflexive relation If;

$$\boxed{(a, a) \in R \ \forall a \in A}$$

Eg = Let $A = \{1, 2, 3, 4\}$

$$R1 = \underbrace{[(1,1), (2,4)\ (3,3)\ (4,1)\ (4,4)]}_{\text{Non - Reflexive}}$$

$$R2 = \underbrace{[(1,1)\ (2,2)\ (2,3)\ (3,3)\ (4,4)]}_{\text{Reflexive}}$$

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(ii) Irreflexive Relation =

→ Relation R on set A is said to be Irreflexive if

$$\boxed{(a, a) \notin R}$$

Eg Let $A = \{1, 2, 3, 4\}$

$R_1 = \underbrace{[(1, 2)\ (2, 1)\ (3, 3)\ (4, 4)]}_{\text{Not Irreflexive}}$

$R_2 = \underbrace{[(1, 2)\ (1, 3),\ (2, 4)]}_{\text{Irreflexive}}$

(iii) Symmetric Relation =

→ Let R be a relation in set A then R is said to be symmetric relation if;

$$\boxed{(a, b) \in R \Rightarrow (b, a) \in R}$$

Eg = Let $A = \{1, 2, 3, 4\}$

$R_1 = \underbrace{\{(1, 2)\ (2, 1)\ (3, 4),\ (4, 3)\}}_{\text{Symmetric}}$

$R_2 = \underbrace{\{(1, 3)\ (3, 1)\ (3, 4)\ (4, 4)\}}_{\text{Not Symmetric}}$

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$R3 = \{ \underbrace{(1,1)\ (2,2)\ (3,3)\ (4,4)}_{\text{Symmetric relation}} \}$ —

Note : The diagonal elements are also having symmetric relation.

(iv) Anti - Symmetric Relation :

→ A relation R on a set A is said to be Anti - Symmetric if

$$\boxed{(a,b) \in R \text{ and } (b,a) \in R}$$

$$\Rightarrow \boxed{a = b}$$

Eg : Let $A = \{1, 2, 3, 4\}$

$$\boxed{R = \{(1,2)\ (3,3)\ (4,3)\}}$$

Q. Can a relation be symmetric and anti-symmetric?

→ Yes, in case of diagonal.

Eg : $\{(1,1)\ (2,2)\ (3,3)\}$

(v) Asymmetric Relation :

→ A relation R on set A is said to be Asymmetric relation if

$$\boxed{(a,a) \in R\ ,\ (b,a) \notin R}$$

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Eg = Let $A = \{1, 2, 3, 4\}$

$$R1 = \underbrace{\{(1,2)\ (2,3),\ (3,1)\ (4,3)\}}_{\text{Asymmetric}}$$

$$R2 = \underbrace{\{(1,2)\ (2,1)\ (2,3)(4,3)\}}_{\text{Not Asymmetric}}$$

TRANSITIVE RELATION =

→ A relation R on set A Is said to be transitive If,

$$\boxed{(a,b) \in R\ ,\ (b,c) \in R}$$
$$\Rightarrow \boxed{(a,c) \in R}$$

Eg : $\{(1,2)\ (2,1)\ (1,1),\ (2,2)\}$

INVERSE RELATION =

→ Let R be a relation from set A to set B then the Inverse relation R^{-1} from set B to set A Is defined by

$$\boxed{\{(b,a) : (a,b) \in R\}}$$

Eg = Let $A = \{1, 2, 3\}$ $B = \{a, b\}$

$R = \{(1,a)\ (1,b)\ (3,a),\ (2,b)\}$

$$\boxed{R^{-1} = \{(a,1)\ (b,1)\ (a,3)\ (b,2)\}}$$

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EQUIVALENCE RELATION =

→ Equivalence relation must satisfy three conditions.

(i) Reflexive Relation

$$\boxed{(a,a) \in R \ \forall \ a \in A}$$

(ii) Symmetric Relation

$$\boxed{(a,b) \in R \Rightarrow (b,a) \in R.}$$

(iii) Transitive Relation

$$\boxed{(a,b) \in R, \ (b,c) \in R, \ (a,c) \in R}$$

PARTIAL ORDER RELATION =

→ Must follow: (i) Reflexive Relation

$$\boxed{(a,a) \in R \ \forall \ a \in A}$$

(ii) ~~Asymm~~ Anti-Symmetric Relation

$$\boxed{\begin{array}{c}(a,b) \in R \text{ And } (b,a) \in R \\ \Rightarrow a = b\end{array}}$$

(iii) Transitive Relation

$$\boxed{(a,b) \in R, \ (b,c) \in R, \ (a,c) \in R}$$

If $A = \{0, 1, 2, 3\}$,

$R_2 = \{(0,0), (0,2), (2,0), (2,2), (2,3), (3,2), (3,3)\}$

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$\because$ (i) Reflexive

$R = \{(0,0), (1,1), (2,2), (3,3), \ldots\}$

$\because$ $R \neq R_2$

$\therefore$ Not Reflexive.

$\therefore$ Neither Equivalence relation nor partial Order relation

DOMAIN, CO-DOMAIN, RANGE :

→ Domain = First Element of the set

→ Co-domain = Second set

→ Range = Second Element of the set

Q Let $A = \{1, 2, 3, 5\}$

$B = \{4, 6, 9\}$

Find domain, co-domain and range where

$R = \{(x, y)$ difference between x and y is odd $x \in A$ and $y \in B\}$

$\Rightarrow R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

(i) Domain = $\{1, 2, 3, 5\}$

(ii) Co-domain = B

(iii) Range = $\{4, 6, 9\}$

Q. Consider the digraph as shown below: Write relation as set of ordered pairs and check for equivalence or partial ordering.

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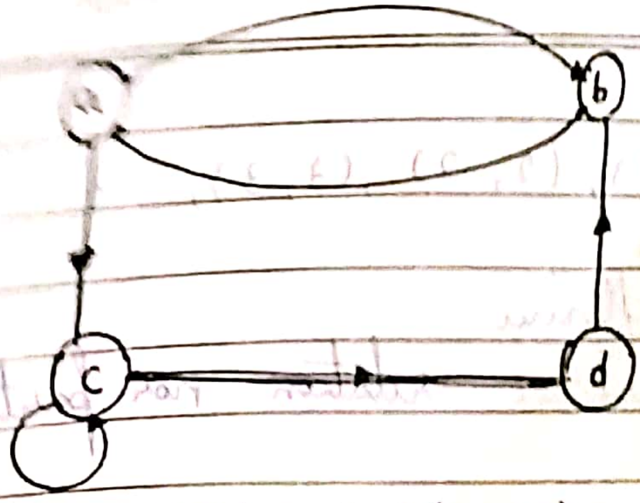


R = { (a, b), (b, a), (a, c), (c, c), (c, d), (d, b) }

∴ It Is not reflexive

Hence, neither Equivalence relation Or partial Order relation.

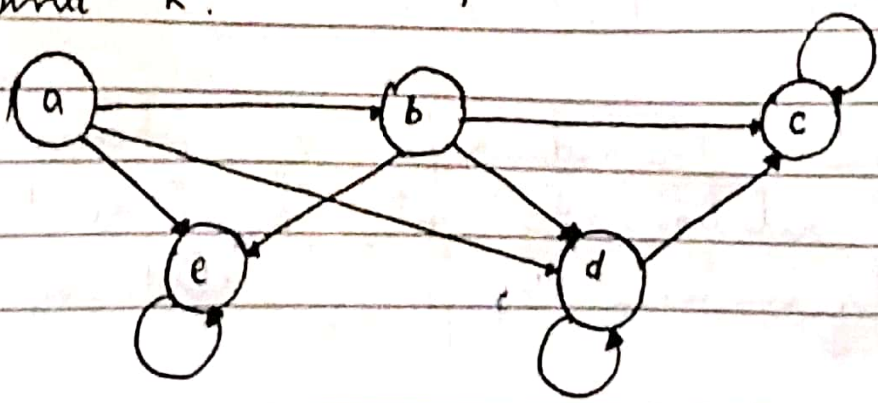
COMPLEMENT OF A RELATION :

$$\boxed{R^c = A \times A - R}$$

where, $A \times A$ = Universal relation

R = Given ~~part~~ Ordered relation.

Q. Let A = { a, b, c, d, e } And R be A relation on A, whose corresponding digraph Is given below. Find $\overline{R}$.



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$\because$ A/q,

$\Rightarrow R = \{(a,b), (a,d), (a,e), (b,c), (b,d), (c,c), (d,c), (d,d), (e,e)\}$

$\Rightarrow A \times A = \{a,b,c,d,e\} \times \{a,b,c,d,e\}$
$\{(a,a), (a,b), (a,c), (a,d), (a,e), (b,a), (b,b), (b,c), (b,d), (b,e), (c,a), (c,b), (c,c), (c,d), (c,e), (d,a), (d,b), (d,c), (d,d), (d,e), (e,a), (e,b), (e,c), (e,d), (e,e)\}$

$\therefore$ A/q,

$$\boxed{R^c = A \times A - R}$$

$$\boxed{\begin{aligned}\Rightarrow R^c = \{&(a,a), (a,c), (b,a), (b,b), (c,a), (c,b), (c,d), (c,e), (d,a), (d,b), (d,e),\\ &(e,a), (e,b), (e,c), (e,d)\}\end{aligned}}$$

Q. Let $A = \{1,2,3,4,5,6,7,8\}$ and R be a relation on A defined by "$(x,y) \in R$ if y is divisible by x". Is R equivalence relation? Is R partial order relation? Draw its diagraph.

$\Rightarrow R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (1,7), (1,8), (2,2), (2,4), (2,6), (2,8), (3,3), (3,6), (4,4), (4,8), (5,5), (6,6), (7,7), (8,8)\}$

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(i) Reflexive

∵ It follows

$$\boxed{(a, a) \in R \quad \forall\, a \in A}$$

(ii) ~~Asymmetric~~ Anti - Symmetric

∵

$$\boxed{\begin{gathered}(a, b) \in R \quad \text{and} \quad (b, a) \in R \\ \Rightarrow \; a = b\end{gathered}}$$

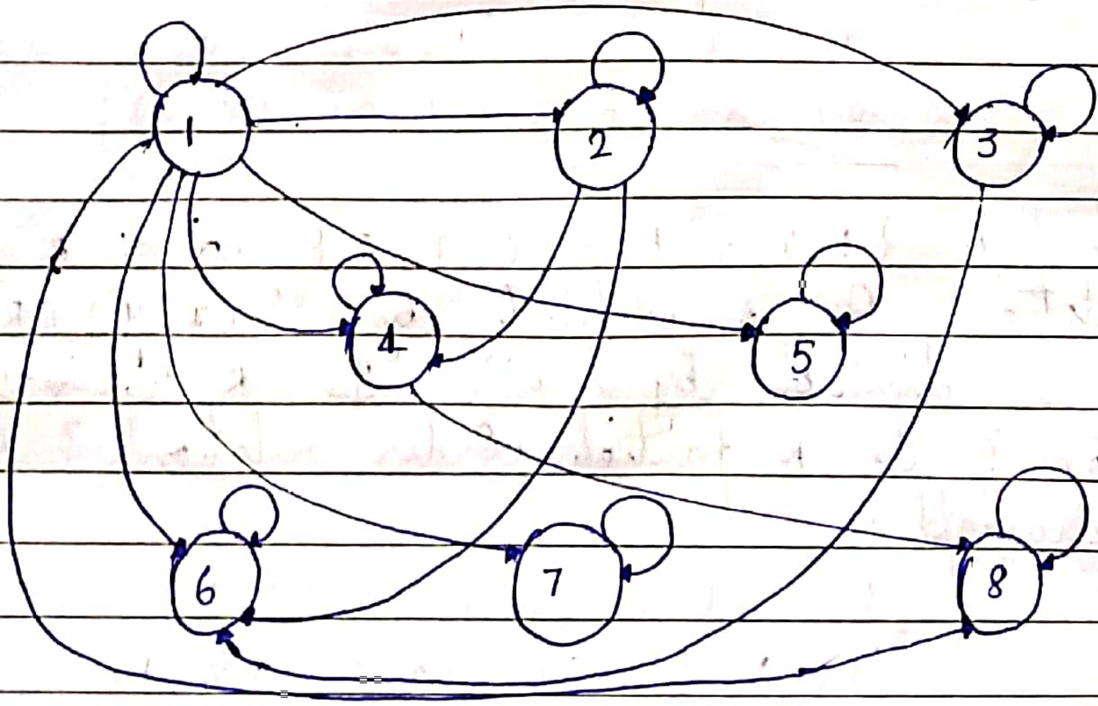
(iii) Transitive

∵

$$\boxed{(a, b) \in R, \; (b, c) \in R, \Rightarrow (a, c) \in R}$$

∴ It is partial Order relation.

→



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Q. Let R be the relation on the +ve Integers N defined by Equation $x + 3y = 12$ i.e;

$R = \{(x, y) : x + 3y = 12\}$

(i) $R = \{(3, 3), (6, 2), (9, 1)\}$

(ii) Domain = First Element

$\{3, 6, 9\}$

Co-domain = Second Set

$\{3, 2, 1\}$

Range = Second Element

$\{3, 2, 1\}$

(iii) $\boxed{R^{-1} = \{(3, 3), (2, 6), (1, 9)\}}$

PARTITION OF SETS =

→ $A = \{1, 2, 3, 4\}$

$a_1 = \{1, 2\}$, $a_2 = \{3, 4\}$

$\therefore A/q,$

Condition =

→ $\boxed{a_1 \cup a_2 = A}$

→ $\boxed{a_1 \cap a_2 = \phi.}$

Eg = $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$a_1 = \{1, 2, 3\}$ $a_2 = \{4, 5, 6, 7, 8\}$

$\therefore A/q,$

$\boxed{a_1 \cup a_2 = \{1, 2, 3, 4, 5, 6, 7, 8\} = A}$

$\boxed{\therefore a_1 \cap a_2 = \phi}$

Hence, partition of sets Is possible.

Q. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$a_1 = \{1, 2, 3\}$, $a_2 = \{6, 7, 8\}$

$\therefore A/a,$

$a_1 \cup a_2 = \{1, 2, 3, 6, 7, 8\} \neq A$ because 4, 5 Is not present.

Hence, partition of set Is not possible.

Q. HS Note = We need to check condition 1 And condition 2 for partitioning of set to be possible.

COMPATIBLE RELATION =

→ Compa Condition 1 = Reflexive = $(a, a) \in R \ \forall\ a \in A$.
Condition 2 = Symmetric = $(a, b) \in R \Rightarrow (b, a) \in R$.

HS Note = Compatible relation cannot be Equivalence relation.

Q. $A = \{x : x$ Is An English Alphabet$\}$

a = book , b = kite , c = then

Show that It Is compatible Or not.

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⇒ Reflexive = $(a, a) \in R \ \forall \ a \in A$

∴ Here,

$(a, a) \in R$ ∵ R Is An English Alphabet

$\boxed{(book, book) \in R \ \forall \ book \in R}$

∴ Reflexive

Symmetric = $\boxed{(a, b) \in R \Rightarrow (b, a) \in R}$

∴ $\boxed{(book, kite) \in R \Rightarrow (kite, book) \in R}$

∴ Symmetric

Hence, It Is compatible relation.

COMPOSITION OF RELATIONS =

→ R O R , S O R

Here, R And S Are relations.

Q. $R_1 = \{(1, 2), (3, 4), (5, 6), (7, 8)\}$

$R_2 = \{(2, 1), (6, 7), (9, 10), (8, 11)\}$

⇒ $\boxed{R_1 \circ R_2 = \{(1, 1), (5, 7), (7, 11)\}}$

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Q. $R = \{(2,2), (3,4)\}$ $\quad S = \{(1,2), (2,1)\}$

(i) $R \circ S = \{(2,1)\}$

(ii) $S \circ R = \{(1,2)\}$

CLOSURE OF RELATION =

→ Condition 1 = Reflexive = denoted by Δ.

→ Condition 2 : Symmetric = $R \cup R^{-1}$

→ Condition 3 = Transitive - R^* denoted by

$$R^* = \bigcup_{n=1}^{\infty} R \rightarrow R^1 \cup \underbrace{R^2}_{R \circ R} \cup \underbrace{R^3}_{R \circ (R \circ R)}$$

Q. $A = \{1, 2, 3\}$

$R = \{(2,1), (3,1), (2,2), (3,2)\}$

(i) $\Delta = \{(1,1), (2,2), (3,3)\}$

$$\Rightarrow \boxed{R \cup \Delta = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}}$$

(ii) $R^{-1} = \{(1,2), (1,3), (2,2), (2,3)\}$

$\therefore R \cup R^{-1} = \{(1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2)\}$

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(iii) Transitive =

$R = \{(2,1), (3,1), (2,2), (3,2)\}$

$\Rightarrow R \circ R = \{(2,1), (3,1), (2,2), (3,2)\}$
$\{(2,1), (3,1), (2,2), (3,2)\}$

$\Rightarrow \boxed{R \circ R = \{(2,1), (2,2), (3,1), (3,2)\} = R^2}$

$R \circ (R \circ R) = \{(2,1), (3,1), (2,2), (3,2)\}$
$\{(2,1), (2,2), (3,1), (3,2)\}$

$\Rightarrow \{(2,1), (2,2), (3,1), (3,2)\} = R^3$

$R \circ (R \circ R)$

Q $R = \{(1,2), (2,3), (5,6), (3,9)\}$ u.
Find $R \circ R$, $R \circ (R \circ R)$

$\Rightarrow R \circ R = \{(1,2), (2,3), (5,6), (3,9)\}$
$\{(1,2), (2,3), (5,6), (3,9)\}$

$\boxed{\{(1,3), (2,9)\}}$

$\Rightarrow R \circ (R \circ R) = \{(1,2), (2,3), (5,6), (3,9)\}$
$\{(1,3), (2,9)\}$
$= \boxed{\{(1,9)\}}$

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CONGRUENCE OF RELATIONS =

→ "Congruence" means similarity Or Equal In Every Aspect.

Congruence of two triangle Is written As $\Delta_1 \cong \Delta_2$.

Eg = A•————•B $\boxed{AB \cong CD}$ (Congruent line segment)

C•————•D

***

THEOREM =

The congruence relation Is An Equivalence

→ $x \bmod y = \frac{x}{y} = R$

Eg = $\boxed{15 \bmod 10 = 5}$

Here, $x = 15$, $y = 10$ And $R = 5$

$$\boxed{a \equiv b \pmod m}$$

$a = 15$, $b = 10$, $m = 5$

$$\boxed{15 \equiv 10 \pmod 5}$$

Condition 1 = $a \bmod m = b \bmod m$

Condition 2 = $a - b$ Is divisible by m.

•) Reflexive : $a \equiv b \pmod{m}$

$a \equiv a \pmod{m}$

$a - a = 0$ Is divisible by m.

∴ This Is A reflexive relation.

•) Symmetric : $a \equiv b \pmod{m}$

$b \equiv a \pmod{m}$

$\boxed{a - b = k \cdot m}$

$\boxed{-(b - a) = k \cdot m}$

$b - a = (-k)\, m$

$\boxed{b - a = k1 \cdot m}$

$\boxed{b \equiv a \pmod{m}}$

Transitivity : $a \equiv b \pmod{m}$

$b \equiv c \pmod{m}$

$\Downarrow$

$\boxed{a \equiv c \pmod{m}}$

• $\boxed{a - b = mk1}$ —— i

$\boxed{b - c = mk2}$ —— ii

∴ A/q, i + ii

$a - \cancel{b} + \cancel{b} - c = m(k1 + k2)$

$a - c = m(k3)$

$\therefore \boxed{a \equiv c \pmod m}$

Q Let R be a relation on set $\{1, 2, 3, 4\}$ with $R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$. Find the reflexive, symmetric and transitive closure of R.

$\Rightarrow$ $R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$\Delta = \{(1,1), (2,2), (3,3), (4,4)\}$

$$\boxed{R \cup \Delta = \{(1,1), (1,4), (2,2), (2,3), (3,1), (3,3), (3,4), (4,4)\}}$$

Symmetric $= R \cup R^{-1}$

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$R^{-1} = \{(1,1), (4,1), (3,2), (1,3), (4,3)\}$

$\therefore$ A/q,

$\Rightarrow$ $$\boxed{R \cup R^{-1} = \{(1,1), (1,3), (1,4), (2,3), (3,1), (3,2), (3,4), (4,1), (4,3)\}}$$

Transitive =

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$R \circ R = R^2 = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}^2$

$R^2 = \{(1,1), (1,4), (2,1), (2,4), (3,1), (3,4)\}$

$R \circ R \circ R = R \circ (R \circ R)$

$\{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$\{(1,1), (1,4), (2,1), (2,4), (3,1), (3,4)\}$

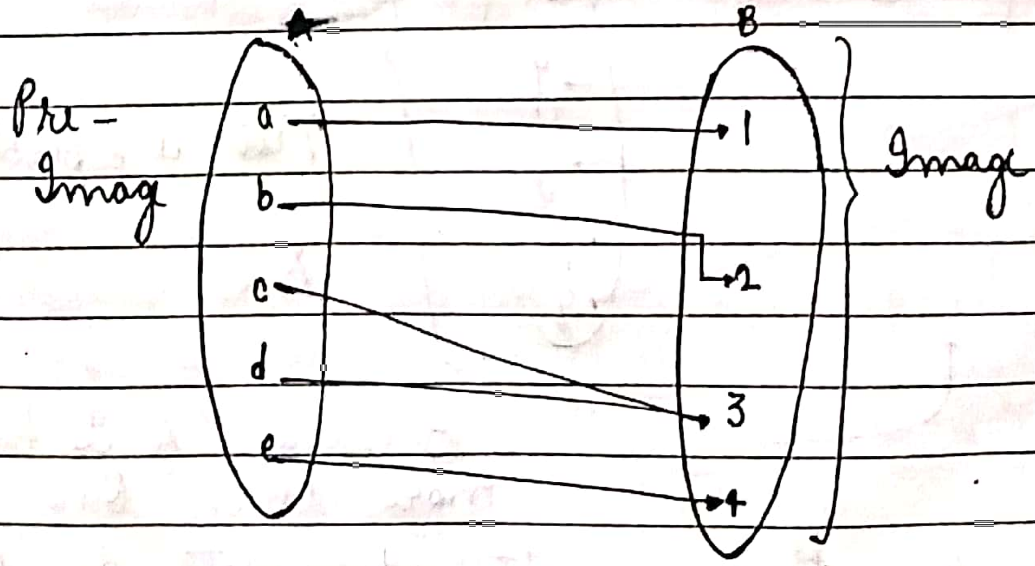
$$\boxed{R^3 = \{(1,1), (1,4), (2,1), (2,4), (3,1), (3,4)\}}$$

$$\boxed{R^* = R1 \cup R2}$$

$\therefore R^* = \{(1,1), (1,4), (2,1), (2,4), (3,1), (3,4)\} \cup \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$

$$\Rightarrow \boxed{R^* = \{(1,1), (1,4), (2,1), (2,3), (2,4), (3,1), (3,4)\}}$$

FUNCTIONS :



→ If $A \to B$ is A function from a to b then,

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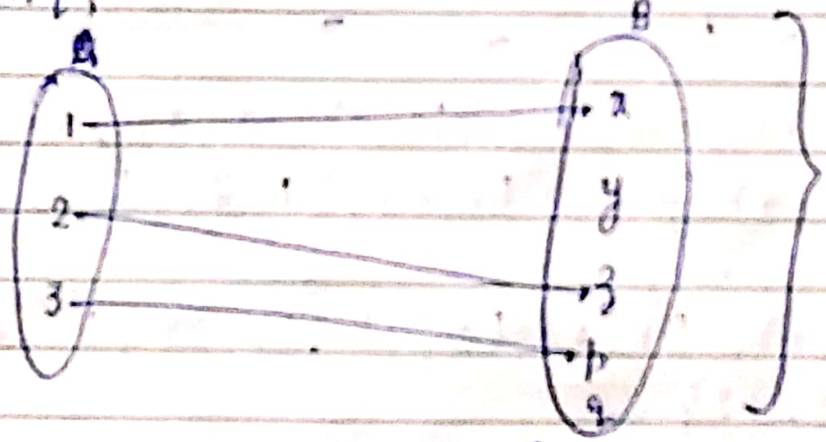
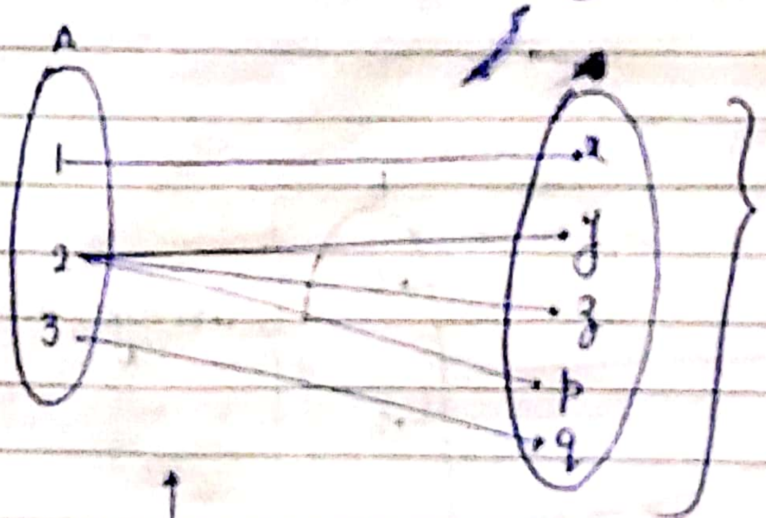
(i) Each Element of 'a' has One And Only One Image In b.

(ii) Different Elements of 'a' can have same Image In 'b'.

Q Let $A = \{1, 2, 3\}$, $B = \{x, y, z, p, q\}$
$\therefore A/q$.



Acceptable ✓



Not acceptable ×

Because A Is having more than One Image In B. Hence, It Is not Accepted According to properties

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DOMAIN AND RANGE OF A FUNCTION :-

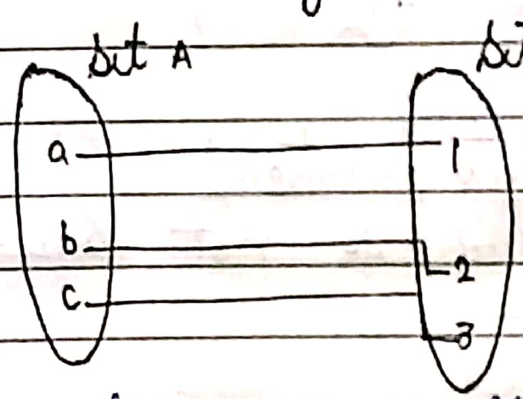
→ Let $A = \{1, 2, 3, 4\}$, $B = \{x, y, z, p, q\}$

A/q,

$\boxed{\text{domain} = \{1, 2, 3, 4\}}$

TYPES OF FUNCTIONS :-

(i) One - One (Injective Function) :-



$F(x1) = F(x2)$

$\boxed{x1 = x2}$

→ The function F Is called One - One Or Injective If different Element In x Is having different Integer In y.

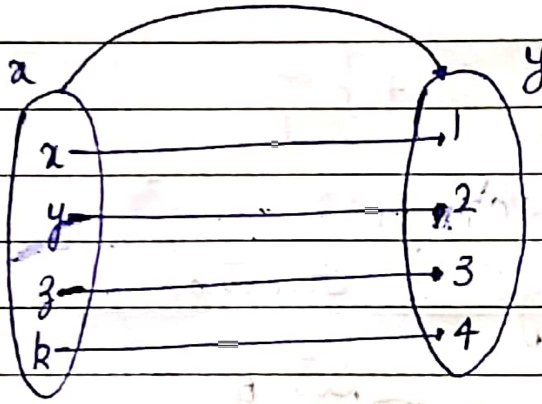
Or

We can say that Every Element of x has A Unique Image In the co-domain y And There Is no Element of y which Is Image of more than One Element of domain.

Q. Consider $x = \{x, y, z, k\}$
$y = \{1, 2, 3, 4\}$ And F Is function

(from x to y .

→ $F = \{(x,1), (y,2), (z,3), (k,4)\}$



One - One function

(ii) Surjective (Onto function)

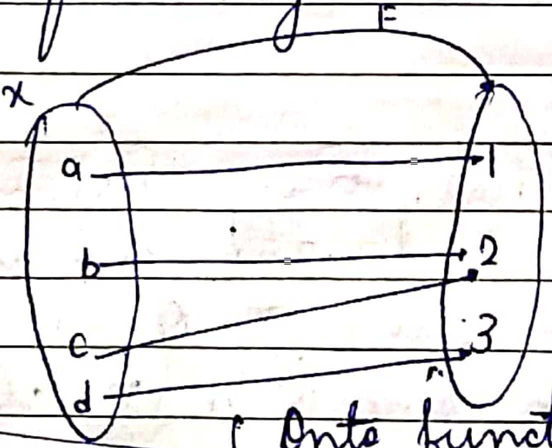
→ ~~The function which Is not One - One Is termed As Onto~~ .

Or

~~The Pre Image~~ Elements Of ~~y can have~~

Every Element Of x ~~can have~~ Is having non-Unique Image In the co-domain y.

Eg =



Each Element of B Is mapped with atleast one Element of A.

(Onto function)

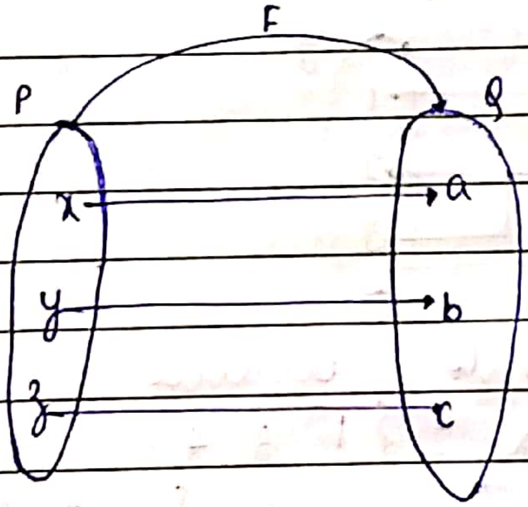
$$\boxed{\begin{array}{ccc} \text{Range} & = & \text{Co-domain} \\ (1, 2, 3) & & (1, 2, 3) \end{array}}$$

(iii) Bijective (One - One And Onto function) -

→ A function which Is both One - One (Injective) And Onto (Surjective) Is termed As Bijective.

Eg = $P = \{x, y, z\}$ $Q = \{a, b, c\}$

$F = \{(x, a), (y, b), (z, c)\}$



Bijective
→ Because One-One
→ Onto

(iv) Into function =

→ The function Is said to be Into If the range Is not Equal to co - domain of y.

$\boxed{\text{Range} \neq \text{Co-domain}}$

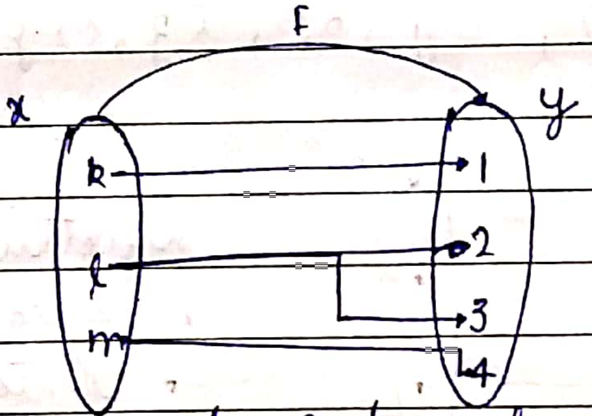
Note = If A function Is not Onto then It Is called Into function.

(v) One - One Into function =

→ The function F Is called One-One Into function If It Is One-One but not Onto.

Q. $x = \{k, l, m\}$, $Y = \{1, 2, 3, 4\}$ And $F : x \rightarrow Y$ such that

$F = \{(k, 1), (l, 3), (m, 4)\}$



∴ It Is not Onto because

Range = $\{1, 3, 4\}$

Co-domain = $\{1, 2, 3, 4\}$

∴ $\boxed{\text{Range} \neq \text{Co-domain}}$

But ~~Into~~ One-One because Each Element Of ~~co~~ x Is mapped with Unique Element In co-domain y.

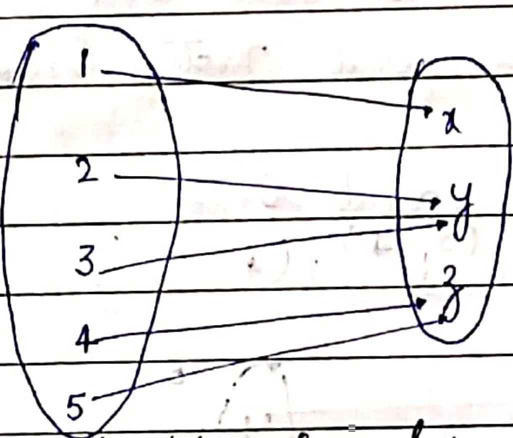
∴ One-One but not Onto

Hence, One-One Into function.

(vi) Many - One function =

→ The function F Is said to be many to One If there Exist two Or more than two different Element In x having same Image In Y.

Eg :-

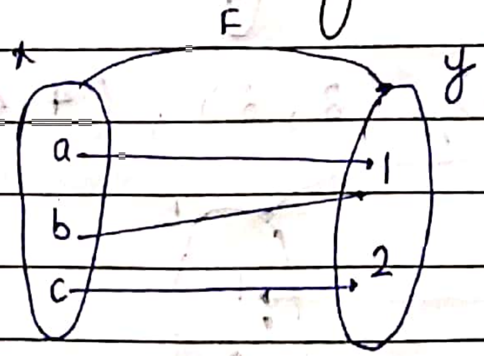


(Many One function)

Note :- If the function Is not One - One that means It Is called many - One.

(vii) Many - One Into function =

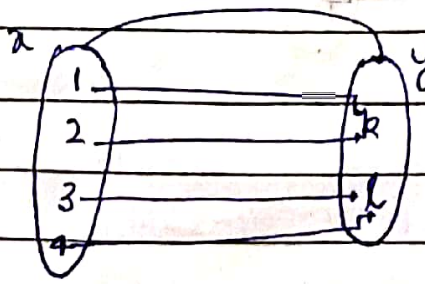
Eg :-



(Many - One Into function)

(viii) Many - One - Onto function =

Eg :-



(Many - One Onto function)

Q. Let $A = B = \{1, 2, 3, 4\}$. Define functions $F : A \to B$ (If possible) such that

(i) F Is One-One And Onto

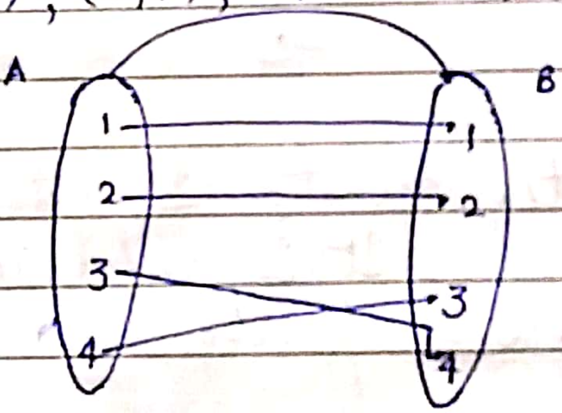
(ii) F Is neither One-One nor Onto.

(iii) F Is Onto but not One One.

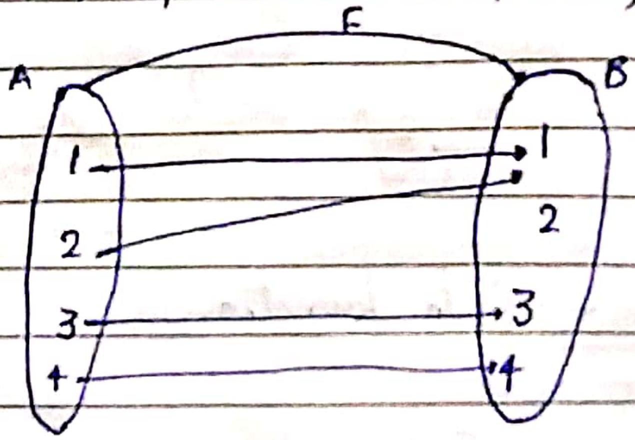
(iv) F Is One-One And not Onto

(i) F Is One-One And Onto

$F = \{(1,1), (2,2), (3,4), (4,3)\}$



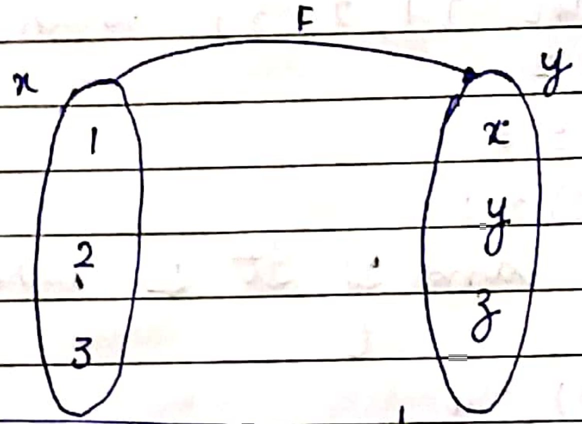
(ii) $F = \{(1,1), (2,1), (3,3), (4,4)\}$



(iii) Not possible

(iv) Not possible

INVERSE OF A FUNCTION =



$\boxed{x = F^{-1}(y)}$ $\boxed{F(x) = y}$

Q Let $F : R \rightarrow R$, $F(x) = 2x - 3$

→ Let, $F(x) = y$

$y = 2x - 3$

$x = \frac{y+3}{2}$

$F^{-1}(y) = x$

$\Rightarrow \boxed{F^{-1}(y) = \frac{y+3}{2}}$

→ A One-to-One correspondance Is called Invertible because we can define An Inverse Of the function.

→ A function Is not Invertible If It Is not A One-to-One correspondance because the Inverse Of such A function does not Exists.

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Q. Let F be a function
$a: \{a, b, c\}$ to $\{1, 2, 3\}$ such that
$F(a) = 2$
$F(b) = 3$
$F(c) = 1$
If F Invertible And If It Is what Is It's Inverse?

→ $F^{-1}(2) = a$
$F^{-1}(3) = b$
$F^{-1}(1) = c$

Q. Let $F: 3 \rightarrow 2$ $F(x) = x+1$
Is F Invertible And If It Is what Is Its Inverse?

→ $F(x) = x+1$
Let, $F(x) = y$
$y = x+1$
$\Rightarrow x = y-1$
$\boxed{F^{-1}(y) = x}$
$\therefore \boxed{F^{-1}(y) = y-1}$

Q. Let F be the function from R to R with $F(x) = x^2$ is F Invertible?

→ Because $F(-2) = F(2) = 4$, F is not One-to-

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Ans. If an inverse function were defined, it would have to assign 2 elements to 4.
$\therefore$ Hence F is not invertible.

COMPOSITION OF FUNCTION =

$\rightarrow$ Let $F: A \rightarrow B$ and $g: B \rightarrow C$ be 2 functions.

$g \circ F = A \rightarrow C$ defined by $(g \circ F)(x) = g(F(x))$ the composition of F and g.

Q. Let $F = \{1, 2, 3\} \rightarrow \{a, b\}$ is a function defined by $F(1) = a$, $F(2) = a$, $F(3) = b$ and $g = \{a, b\} \rightarrow \{5, 6, 4\}$ is defined by $g(a) = 5$, $g(b) = 7$. Find $g \circ F$ and $F \circ g$.

$\rightarrow$ $R_F = \{a, b\}$

$D_g = \{a, b\}$

$\therefore$ $R_F \subseteq D_g$ $\therefore$ $g \circ F$ is defined

And

$g \circ F = \{1, 2, 3\} \rightarrow \{5, 6, 7\}$

$(g \circ F)(1) = gF(1) = g(a) = 5$

$(g \circ F)(2) = gF(2) = g(a) = 5$

$(g \circ F)(3) = gF(3) = g(b) = 7$

To find $F \circ g$, $R_g = \{5, 6, 7\}$, $D_F = \{1, 2, 3\}$

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$\because R_g \not\subset D_F$

$\therefore F \circ g$ Is not defined.

Q. Let $F = \{1, 2, 3\} \to \{a, b\}$ be a function defined by $F(1) = a$, $F(2) = b$, $F(3) = c$. Let $g\{a, b\} \to \{5, 6, 7\}$ be A function defined by $g(a) = 5$, $g(b) = 7$. Find $g \circ F$ And $F \circ g$.

→ $F = \{1, 2, 3\} \to \{a, b\}$ And

$g = \{a, b\} \to \{5, 6, 7\}$

$$\boxed{R_F = \{a, b\} = D_g}$$

$\therefore g \circ F$ Is defined

$$\boxed{g \circ F = \{1, 2, 3\} \to \{5, 6, 7\}}$$

But $\boxed{R_g \neq D_F}$

$\therefore F \circ g$ Is not defined

$g \circ F(1) = g(F(1)) = g(a) = 5$

$g \circ F(2) = g(F(2)) = g(b) = 7$

$g \circ F(3) = g(F(3)) = g(b) = 7$

$$\therefore \boxed{g \circ F = \{5, 7\}}$$

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Q. Let $F = R \to R$ be defined by $F(x) = x^3$ and $g = R \to R$ be defined by $g(x) = 3x+1$. Find $(g o F)(x)$, $F o g(x)$. Is $g o F = F o g$?

$\to$ $\because$ $D_F = R_g$

$\therefore$ $g o F = R \to R$ (defined)

$g o F(x) = g(F(x)) = g(x^3) = 3x^3 + 1$ — (i)

IIly,

$\boxed{D_g = R_F}$

$\therefore$ $F o g = R \to R$ (defined)

$F(g(x)) = F(3x+1) = (3x+1)^3$ — (ii)

From equation (i) and (ii)

$\boxed{F o g \neq g o F}$

Q. If, $F(x) = x^2$, $g(x) = x^3$. Find $F o (g o (F o g))$.

$\to$ $F o (g o (F o (x^3)$

$F o (g o (x^6)$

$\Rightarrow$ $F o (x^{18})$

$\boxed{x^{36}}$

Q. If $F(x) = 4x - 1$, $g(x) = x^2 + 2$ then find $(F o (F o g))(1)$.

$\Rightarrow$ $F o (F o (x^2+2)(1)$

$\Rightarrow$ $F o (4(x^2+2)-1)(1)$

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$\Rightarrow F\circ(4x^2+8-1)(1)$

$\Rightarrow F\circ(4x^2+7)(1)$

$(4(4x^2+7)-1)\ F(1)$ ← Value of x to be substituted finally..

$(16x^2+28-1)$

$\Rightarrow 16\times1+27$

$\Rightarrow (16+27)$

$\Rightarrow \boxed{43}$

Q. If $F(x)=\dfrac{x}{x+1}$, $g(x)=\dfrac{1}{(x-1)}$ find $F\circ g(x)$.

$\Rightarrow F\circ\left(\dfrac{1}{(x-1)}\right)$

$\Rightarrow F\circ g(x)=\dfrac{\dfrac{1}{(x-1)}}{\dfrac{1}{(x-1)}+1}$

$\Rightarrow F\circ g(x)=\left(\dfrac{\dfrac{1}{\cancel{(x-1)}}}{\dfrac{1+(x-1)}{\cancel{(x-1)}}}\right)$

$\Rightarrow F\circ g(x)=\dfrac{1}{1+(x-1)}$

$\Rightarrow F\circ g(x)=\dfrac{1}{\cancel{1}+x-\cancel{1}}$

$\Rightarrow \boxed{F\circ g(x)=\dfrac{1}{x}}$

Q. If $F(x) = x+5$, $g(x) = x^2$, find $(g \circ F)(x)$, where $F = R \to R$, $g = R \to R$ Is given

→ ∵ $\cancel{D_F = R_g}$

$$\boxed{R_F = D_g}$$

∴ $g \circ F$ Is possible.

∴ A/q,

$g(x+5) = g(x+5)^2$

$\Rightarrow (g \circ F) = x^2 + 25 + 10x$

$\Rightarrow \boxed{(g \circ F)(x) = x^2 + 25 + 10}$

very very Important

ALGEBRIC STRUCTURES

→ If there Exists A system such that It consists of the non-Empty set And One Or more Operations, On that set then that system Is called An Algebric Expression structure.

→ It Is generally denoted by $(A, op_1, op_2, op_3, \ldots, op_n)$

where, A = Non-Empty set And

$op_1, op_2, op_3, \ldots, op_n$ = Operations On A.

→ An Algebric system Is Also called An

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algebraic structure, because the operations on set define the structure on the elements of A.

* BINARY OPERATIONS =

→ Consider a non-empty set A and a function such that

$F : A \longrightarrow A$, then F is called binary operation on A whose domain is the set of ordered pair of elements of A.

→ If * is a binary operation on A then it can be written as $a * b$.

→ It can be denoted by any of the symbols like $+, -, *, \oplus, \Delta, \square, \vee, \wedge$ etc.

TABLES OF OPERATIONS =

→ Consider a non-empty finite set

$A = \{a_1, a_2, a_3, ----, a_n\}$

→ A binary operation * on A can be defined by means of table as shown below -

*	a_1	a_2	a_3	----	a_n
a_1	$a_1 * a_1$	$a_1 * a_2$	$a_1 * a_3$	----	$a_1 * a_n$
a_2	$a_2 * a_1$	$a_2 * a_2$			
a_3	⋮				
⋮	⋮				
a_n	$a_n * a_1$	- -	- -		$a_n * a_n$

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Q. Consider the set $a = \{1, 2, 3\}$ and a binary operation $*$ on set a defined by

$$a * b = 2a + 2b.$$

Represent the operation $*$ as a table on a.

$a \rightarrow$, $b \downarrow$

*	1	2	3
1	4	6	8
2	6	8	10
3	8	10	12

For $a = 1$, $b = 1$
$2 \times 1 + 2 \times 1 = 4$

For $a = 2$, $b = 1$
$2 \times 2 + 2 \times 1 = 6$

For $a = 3$, $b = 1$
$2 \times 3 + 2 \times 1 = 8$

For $a = 1$, $b = 2$
$2 \times 1 + 2 \times 2 = 6$

For $a = 2$, $b = 2$
$2 \times 2 + 2 \times 2 = 8$

For $a = 3$, $b = 2$
$2 \times 3 + 2 \times 2 = 10$

For $a = 1$, $b = 3$
$2 \times 1 + 2 \times 3 = 8$

For $a = 2$, $b = 3$
$2 \times 2 + 2 \times 3 = 10$

For $a = 3$, $b = 3$
$2 \times 3 + 2 \times 3 = 12$

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PROPERTIES OF BINARY OPERATION 2

(i) CLOSURE PROPERTY =

→ Consider a non-Empty set A And a binary operation * on A then A Is closed Under operation * If,

$$\boxed{a * b \in A}$$

where, a And b are Elements of A.

Q. Consider the set $A = \{-1, 0, 1\}$. Determine wether A Is closed Under

(i) Addition

(ii) Multiplication

(i) Given set $A = \{-1, 0, 1\}$

binary operation = +

A/q,

Closure property = $\boxed{a + b \in A}$

Let, $a = -1$, $b = 0$

$\therefore a + b = -1 + 0 = -1 \in A$

Let, $a = 1$, $b = 0$

$\therefore a + b = 1 + 0 = 1 \in A$

Let, $a = 1$, $b = 1$

$\therefore a + b = 1 + 1 = 2 \notin A$

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$\therefore$ ~~It Is~~ The set $A = \{-1, 0, 1\}$ Is not closed Under binary Operation Addition.

(ii) Given set $A = \{-1, 0, 1\}$
binary Operation $= *$

$\therefore$ A/q,
Closure Property $= \boxed{a * b \in A}$

Let, $a = -1$, $b = -1$
$\therefore a * b = (-1) * (-1) = 1 \in A$.

Let, $a = 0$, $b = -1$
$\therefore a * b = (0) * (-1) = 0 \in A$

Let, $a = 0$, $b = 1$
$\therefore a * b = (0) * (1) = 0 \in A$

$\because$ For All cases of $a, b \in A$, $a * b \in A$
$\therefore$ The set $A = \{-1, 0, 1\}$ Is closed Under binary Operation $*$.

Q. Consider the set $A = \{1, 3, 5, 7, 9, ---\}$. The set of Odd positive Integers. Determine wether A Is closed Under
(i) Addition
(ii) Multiplication

(i) Given set $= A = \{1, 3, 5, 7, 9, ---\}$
Binary Operation $= +$

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∴ A/q,

Closure Property = $\boxed{a + b \in A}$

Let $a = 1$, $b = 1$

∴ $a + b = 1 + 1 = 2 \notin A$

∴ Binary operation + Is not possessed Under closure property.

(ii) ~~Mult~~ Given Set = {1, 3, 5, 7, 9, ----}
Binary Operation = *

∴ A/q,

Closure Property = $a * b \in A$

~~Let~~ ∵ Multiplication of two Odd numbers Are Always Odd

∴ $\forall\ a, b \in A$, $\boxed{a * b \in A}$

∴ Binary operation * Is closed for the given set {1, 3, 5, 7, 9, ----}

(ii) ASSOCIATIVE PROPERTY :-

→ Consider A non-Empty set And A binary Operation * On the given set then the Operation * Is said to be associative If

$\boxed{a * (b * c) = (a * b) * c}\ \forall\ a, b, c \in A$

Q "Consider the binary Operation $*$ On the set $\mathbb{Q}$, the set of Rational numbers defined by

$$a*b = a+b-ab \quad \forall \; a,b \in \mathbb{Q}$$

Determine wether $*$ Is Associative Or not.

$\rightarrow$ ∵ $\underbrace{\boxed{a*(b*c) = (a*b)*c}}_{\text{Associative property}} \quad \forall \; a,b,c \in A$

LHS =

$a*(b*c)$

∵ $a*b = a+b-ab$

∴ $\boxed{b*c = b+c-bc}$

$\Rightarrow a*(b+c-bc)$ $\Big\}$ ~~$a*b = a+b-ab$~~ ∴ $a*(b+c-bc) = a+(b+c-bc) - a*(b+c-bc)$

$a+(b+c-bc) - a(b+c-bc)$

$\boxed{a+b+c-bc-ab-ac+abc}$ —— (i)

RHS =

$(a*b)*c$

∵ $a*b = a+b-bc$

∴ $\boxed{(a*b)*c = (a*b)+c-(a*b)c}$

∴ $(a*b)+c-(a*b)*c$

∴ $\cancel{(a*b)}$ ∵ $\boxed{a*b = a+b-ab}$

$\Rightarrow (a+b-ab)+c-(a+b-ab)c$

$\Rightarrow \boxed{a+b-ab+c-ac-bc+abc}$ —— (ii)

From Equation (i) And (ii)

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$\boxed{LHS = RHS}$

∴ The binary operation for the given set holds Assoccativity.

Q. ~~For~~ Consider a set $A = \{-1, 0, 1\}$. Determine wether A Is closed Under
(i) Addition
(ii) Multiplication

(i) Under Addition Associativity Is As follows

$\boxed{a + (b+c) = (a+b) + c}$ $\forall\, a, b, c \in A$

$\because A = \{-1, 0, 1\} \longrightarrow$ Given

∴ A | a,

LHS =

$a + (b + c)$

Case (i) = Let, $a = -1$, $b = 1$, $c = 1$

$\Rightarrow -1 + (1 + 1)$

$\Rightarrow -1 + 2$

$1 \in A$

Case (ii) = Let, $a = 1$, $b = 1$, $c = 0$

$1 + (1 + 0) = 2 \notin A$

RHS

$(a + b) + c$

Case (i) = Let, $a = -1$, $b = 1$, $c = 1$

$(-1 + 1) + 1 = 1$

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Case (ii) = ~~a =~~ Let, $a = 1$, $b = 1$, $c = 0$

$(1+1)+0 = 2$

$\therefore$ $\boxed{LHS = RHS}$

$\therefore$ Addition As A binary Operation Is possible.

(ii) Multiplication =

$\boxed{a * (b * c) = (a * b) * c}$

LHS =

Case (i) = ~~a = -1, b = 1~~ Let, $a = -1$, $b = 1$, $c = 1$

$-1 * (1 * 1) = -1$

Case (ii) = Let, $a = 1$, $b = 1$, $c = 0$

$1 * (1 * 0) = 0$

RHS =

Case (i) = Let, $a = -1$, $b = 1$, $c = 1$

$(-1 * 1) * 1 = -1$

Case (ii) = Let, $a = 1$, $b = 1$, $c = 0$

$(1 * 1) * 0 = 0$

$\therefore$ $\boxed{LHS = RHS}$

$\therefore$ Multiplication As A binary Operation for the given set $A = \{-1, 0, 1\}$ Is possible.

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(iii) Commutative Law =

→ Consider a non-empty set A and a binary operation $*$ on A is commutative if for every

$(a, b) \in A$ we have,

$$\boxed{a * b = b * a}$$

Q. Consider the binary operation $*$ on Q, the set of rational numbers defined by

$$a * b = a^2 + b^2 \quad \forall (a, b) \in Q.$$

Determine whether $*$ is commutative or not.

⇒ ∵ A/q,

Commutativity = $\forall (a, b) \in A$

$$\boxed{a * b = b * a}$$

LHS =

$a * b = \boxed{a^2 + b^2}$ —— (i)

RHS =

$\boxed{b * a = b^2 + a^2}$ —— (ii)

∴ From equation (i) and (ii)

$$\boxed{LHS = RHS}$$

∴ $*$ is commutative for

$$\boxed{a * b = a^2 + b^2}$$

Q. Consider $S = \{a, b, c, d\}$ and $*$ be the binary operation on S defined as shown

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In the following table :

*	a	b	c	d
a	a	b	c	d
b	b	a	a	b
c	c	b	a	a
d	d	a	a	a

(i) Determine whether * Is Associative.
(ii) Determine whether * Is ~~A~~ commutative.

(i) For Associativity =

$\forall\ a, b, c \in A$

$\boxed{a * (b * c) = (a * b) * c}$

LHS =

$a * (b * c)$

$\because b * c = a$ (Given)

$a * a$

$\because a * a = a$ (Given)

$\therefore \boxed{a * (b * c) = a}$ (i)

RHS =

$(a * b) * c$

$\because a * b = b$ (Given)

$b * c$

$\because b * c = a$ (Given)

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$\therefore$ $\boxed{(a*b)*c = a}$ —— ii

$\therefore$ From Equation i And ii

$\boxed{LHS = RHS}$

$\therefore$ It does not holds ~~Assotia~~ Associativity for b, c ∈ A.

ii For Commutativity =

$\forall\ a, b \in A$

$\boxed{a*b = b*a}$

LHS =

$\boxed{a*b = b}$ —— (Given) —— i

RHS =

$\boxed{b*a = b}$ —— (Given) —— ii

From Equation i And ii

$\boxed{LHS = RHS}$

but,

$\forall\ b, c \in A$

$\boxed{b*c = c*b}$

LHS =

$\boxed{b*c = a}$ —— (Given) —— i

RHS =

$\boxed{c*b = b}$ —— (Given) —— ii

$\therefore$ i $\neq$ ii

$\therefore$ $\boxed{LHS \neq RHS}$

$\therefore$ It Is non- Commutative

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Q. Consider the binary operation $*$ and Q, set of rational numbers defined by

$$a * b = \frac{ab}{2} \quad \forall (a, b) \in Q$$

(i) Determine whether $*$ is Associative.
(ii) Determine whether $*$ is commutative.

(i) For Associativity

$$\boxed{a * (b * c) = (a * b) * c}$$

LHS =

$a * (b * c)$

$\because b * c = \frac{bc}{2}$ —— (given)

$\therefore \frac{a * (bc)}{2}$

$\because a * (bc) = \frac{abc}{2}$ —— (given)

$$\therefore \boxed{\frac{abc}{4}} \text{ —— (i)}$$

RHS =

$(a * b) * c$

$\because a * b = \frac{ab}{2}$ —— (given)

$\frac{(ab) * c}{2}$

$\because (ab) * c = \frac{abc}{2}$ —— (given)

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$$\boxed{\frac{abc}{4}} \quad \text{—— (ii)}$$

∴ A/q,
From Equation (i) And (ii)

$\boxed{LHS = RHS}$

∴ It holds Associativity.

(ii) For Commutativity =

$\forall\ a, b \in A$

$$\boxed{a * b = b * a}$$

LHS =

$$a * b = \frac{ab}{2} \quad \text{—— (i)}$$

RHS =

$$b * a = \frac{ba}{2} \quad \text{—— (ii)}$$

∴ A/q,
From Equation (i) And (ii)

$\boxed{LHS = RHS}$

∴ It holds commutativity

(iv) IDENTITY PROPERTY =

→ Consider A non-empty set A And A binary Operation * On A then the Operation * has An Identity property If there exists

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An Element e ~~and~~ In a such that

$$\boxed{\underbrace{a * e}_{} = \underbrace{e * a}_{}} \quad \forall \ a \in A$$

(Left Identity)

(Right Identity)

Q. Consider the binary Operation $*$ On I, the set of positive Integers defined by $a * b = \frac{ab}{2}$

Determine the Identity for binary Operation $*$ If Exists.

$\Rightarrow$ $\forall \ a \in A$

$$\boxed{a * e = e * a}$$

LHS

$\boxed{a * e = a}$ (We know)

$a * e = \frac{ae}{2}$ ——— Given $a * b = \frac{ab}{2}$

$\Rightarrow \cancel{a} = \frac{\cancel{a} e}{2}$

$\boxed{e = 2}$ ——— (i)

RHS =

$\boxed{e * a = \frac{ea}{2}}$ ——— Given $a * b = \frac{ab}{2}$

$\cancel{a} = \frac{e\cancel{a}}{2}$

$\Rightarrow \boxed{e = 2}$ ——— (ii)

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∴ From Equation (i) And Equation (ii)

$\boxed{LHS = RHS}$

∴ 2 Is Identity Element

(v) IDEMPOTENT PROPERTY =

→ Consider A non-empty set A . A binary Operation * On A then the Operation * has the Idempotent property

$$\boxed{a * a = a} \quad \forall \, a \in A$$

(vi) DISTRIBUTIVE LAW =

$$\boxed{a * (b + c) = (a * b) + (a * c)}$$

And

$$\boxed{(b + c) * a = (b * a) + (c * a)}$$

$\forall \, a, b, c \in A$

(vii) CANCELLATION PROPERTY =

$$\boxed{a * b = a * c \Rightarrow b = c}$$

And

$$\boxed{b * a = c * a \Rightarrow b = c}$$

$\forall \, a, b, c \in A$

very very Important

SEMI - GROUP =

→ Let Us consider An Algebric system $(a, *)$ where * Is A binary Operation On a,

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then the system (a, *) Is said to be a semi-group If It satisfies the following properties =

(i) The Operation * Is A closed Operation On set A.

(ii) The Operation * Is An Associative Operation.

Q Consider An Algebric system (A, *) where $A = \{1, 3, 5, 7, ----, \infty\}$ the set of All positive Odd Integers And * Is a binary Operation means multiplication.
Determine wether (a, *) Is A semi-Group.

→ For a set being a Semi-Group, It has to follow two properties

(i) Closure =

$\boxed{a * b \in A}$ $\forall\ a, b \in A$

Here, multiplication of Odd numbers Is Always Odd.
Hence, It Is closure.

(ii) Associative =

$\boxed{a*(b*c) = (a*b)*c}$ $\forall\ a, b, c \in A$

Case (i) $a = 1,\ b = 3,\ c = 5$

LHS =

$1 \times (3 \times 5) = 15$ —— (a)

RHS =

$(1 \times 3) \times 5 = 15$ —— (b)

∴ From Equation (a) And (b)

$\boxed{LHS = RHS}$

Hence, the given set $A = \{1, 3, 5, 7, \ldots \infty\}$ Is Associative As well.

$\therefore$ The given set holds the property
(i) Closure
(ii) Associative

$\therefore$ The given set $A = \{1, 3, 5, 7, \ldots, \infty\}$ Is A semi group.

Q. Let N be A set of positive Integers And let $*$ be the binary operation of LCM On N. Find semi-group :-

(a) $4 * 6$, $3 * 5$, $9 * 18$, $1 * 6$

$\therefore$ LCM of $4 * 6 = 12$
LCM of $3 * 5 = 15$
LCM of $9 * 18 = 18$
LCM of $1 * 6 = 6$

$\therefore$ $\boxed{R = \{12, 15, 18, 6\} \in N}$

(b) Is $(N, *)$ Is A semi-Group.

$N = \{1, 2, 3, 4, \ldots, \infty\}$ — Given

(i) Closure :-

$\boxed{a * b \in A \ \forall \ a, b \in A}$

positive. Product of two positive Integer Is Always

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Hence, It Is closure.

(ii) Associative =

$$\boxed{a * (b * c) = (a * b) * c} \quad \forall a, b, c \in A$$

∴ Product of two Or more positive Integers Always positive And It's LCM Is Also same. Hence, If It follows Associativity As well.

∴ The given set N Is semi-Group for binary Operation $*$.

(c) Is N ~~Co~~ commutative?

→ For commutativity

$$\boxed{a * b = b * a} \quad \forall a, b \in A$$

∴ $N = \{1, 2, 3, 4, 5, \ldots, \infty\}$

Product of All positive Integers Is always positive

Case (i) = Let $a = 1$, $b = 5$

LHS =

$a * b = \underbrace{1 \times 5}_{LCM} = 5$ —— (a)

RHS =

$b * a = \underbrace{5 \times 1}_{LCM} = 5$ —— (b)

From Equation (a) And (b)

LHS = RHS

Hence, the given set N Is commutative.

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(d) Find <u>Identity</u> Element of N.

→ ∵ $\boxed{a * e = e * a = a}$ $\forall\, a \in A$

~~For~~ Here,

~~fa~~ ~~To~~ for $a \in N$, consider $a * 1 = $ LCM of a And $1 = a$

And

$$a * 1 = a = 1 * a$$

∴ 1 Is the Identity Element of N.

Q. Consider the set Q of rational numbers And let $*$ be operation on Q defined by

$$a * b = a + b - ab$$

(i) Find $3*4$, $2*(-5)$, $7*\frac{1}{2}$

⇒ $3 * 4 = 3 + 4 - 3 * 4 = 7 - 12 = -5$

⇒ $2 * (-5) = 2 + (-5) - 2(-5) = -3 + 10 = 7$

⇒ $7 * \left(\frac{1}{2}\right) = 7 + \frac{1}{2} - \frac{7}{2} = \frac{15}{2} - \frac{7}{2} = \frac{8}{2} = 4$

(ii) Is $(Q, *)$ A Semi-Group?

(a) <u>Closure</u> = $\boxed{a * b \in Q}$ $\forall\, a, b \in Q$

∴ A/q, $\boxed{a * b = a + b - ab}$

where $a, b \in Q$

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(b) Associative =

$$\boxed{a * (b * c) = (a * b) * c}$$

LHS =

$\Rightarrow a * (b * c)$

$\Rightarrow a * (b + c - bc)$ —— $\because \boxed{a * b = a + b - ab}$

$\Rightarrow a + (b + c - bc) - a(b + c - bc)$

$\boxed{a + b + c - bc - ab - ac + abc}$ —— (i)

RHS =

$\Rightarrow (a * b) * c$

$\Rightarrow (a + b - ab) * c$ —— $\boxed{a * b = a + b - ab}$

$\Rightarrow (a + b - ab) + c - (a + b - ab)c$

$\Rightarrow a + b - ab + c - ac - bc + abc$

$\boxed{a + b + c - bc - ab - ac + abc}$ —— (ii)

From equation (i) and (ii)

$\boxed{\text{LHS} = \text{RHS}}$

Hence, Associative

Hence, $a * b = a + b - ab$ be Operation On S Is A semi-group.

(iii) Commutative =

$\boxed{a * b = b * a}$ $\forall\, a, b \in A$

LHS =

$a * b = \boxed{a + b - ab}$ —— (i)

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RHS =

$b*a = \boxed{b+a-ba}$ —— (iii)

$\boxed{a+b-ab}$ —— (ii)

From Equation (i) And (ii)

$\boxed{LHS = RHS}$

Hence, commutative

(iv) Identity =

$\boxed{a*e = e*a = a}$ $\forall\ a \in A$

LHS =

$a*e = a+e-ae$

$\Rightarrow a = \cancel{a}+e-ae$ $\qquad \because \boxed{a*e=a}$

$\Rightarrow \boxed{e(1-a) = 0}$

$\boxed{e = 0 \text{ for } a \neq 1}$ —— (a)

RHS =

$e*a = e+a-ea$

$\Rightarrow a = e+\cancel{a}-ea$ $\qquad \because \boxed{e*a=a}$

$\Rightarrow e-ea = 0$

$\Rightarrow e(1-a) = 0$

$\boxed{e = 0 \text{ for } a \neq 1}$ —— (b)

$\therefore$ From Equation (a) And (b)

$\boxed{LHS = RHS}$

Hence, Identity At $e = 0$ for $a \neq 1$

MONOIDS = (Semi-Group + Identity)

→ Let us consider an Algebric system (A, o) where o is the binary operation on A, then the system (A, o) is said to be a monoid if it satisfies following properties =

(i) The operation o is a closed operation on set A.

(ii) The operation o is an associative operation.

(iii) Their exists an identity element with respect to (w.r.t) operation o.

Eg = (N, x), (Z, +), (Q, +) etc.

Q. Consider an Algebric system (I, +) where the set I = {0, 1, 2, 3, 4} the set of natural numbers including 0 and + is the addition operation. Determine wether (I, +) is a monoid.

→ Closure =

$\boxed{a + b \in A}$ $\forall$ a, b $\in$ A

∵ Sum (Addition) of two positive integers is always a positive integer.

Hence, Closure.

Eg = Let, a = 0, b = 1

∴ $\boxed{a + b = 1 \in I}$ $\forall$ 0, 1 $\in$ I

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Association =

$\boxed{a+(b+c) = (a+b)+c}$ $\forall\ a, b, c \in A$

~~LHS~~ Case = Let $a = 0,\ b = 1,\ c = 2$

LHS

$a + (b+c)$

$\Rightarrow 0 + (1+2)$

$\Rightarrow 3$ —— (i)

RHS =

$(a+b)+c$

$\Rightarrow (0+1)+2$

$\Rightarrow 1+2$

$\Rightarrow 3$ —— (ii)

$\therefore$ From equation (i) And (ii)

$\boxed{LHS = RHS}$

Hence, ~~(a)+~~ Association for As $\boxed{a+(b+c) = (a+b)+c}$ $\forall\ a, b, c \in I$

~~Association~~

Identity

$\boxed{a * e = e * a}$ $\forall\ a \in A$

LHS =

$a + e = a$ $\quad\therefore \boxed{a + e = a}$

$\boxed{e = 0}$ —— (i)

RHS =

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$e + \cancel{a} = \cancel{a}$ ∴ $\boxed{e + a = a}$

$\Rightarrow \boxed{e = 0}$ —— (ii)

From Equation (i) And (ii)

$\boxed{LHS = RHS}$

Hence, It's Identity Element Is 0.

∴ The following set satisfies closure, Associativeness And Identity.

Hence, It the following set Is monoid.

GROUP = $\boxed{\text{Monoid} + a * a^{-1} = a^{-1} * a = e}$

→ (i) The Operation $*$ Is A closed Operation.
(ii) The Operation $*$ Is An Associative Operation
(iii) Their Exists An Identity Element w.r.t Operation o.
(iv) For Every $a \in G$, their Exists An Element $a^{-1} \in G$ such that

$$\boxed{a^{-1} * a = a * a^{-1} = e}$$

Q. Consider An Algebric system $(Q, *)$ where Q Is the set of rational numbers.
Let $*$ be the binary Operation defined by

$\boxed{a * b = a + b - ab}$ $\forall\ a, b \in Q$.

Determine wether $(Q, *)$ Is a Group.

→ Closure =

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$\therefore \boxed{a * b \in A} \ \forall\ a, b \in A$

$\therefore \boxed{a * b = a + b - ab} \ \forall\ a, b \in A$

Hence, closure.

Associative :

$\boxed{a * (b * c) = (a * b) * c} \ \forall\ a, b, c \in A$

LHS =

$a * (b * c)$

$\therefore \boxed{b * c = b + c - bc}$ $\quad \because a * b = a + b - ab$

$\Rightarrow a * (b + c - bc)$

$\Rightarrow a + (b + c - bc) - a(b + c - bc)$

$\Rightarrow \boxed{a + b + c - bc - ab - ac + abc}$ — (i)

RHS =

$(a * b) * c$

$\therefore \boxed{a * b = a + b - ab}$

$\Rightarrow (a + b - ab) * c$

$\Rightarrow (a + b - ab) + c - (a + b - ab)c$

$\Rightarrow a + b - ab + c - ac - bc + abc$

$\Rightarrow \boxed{a + b + c - bc - ab - ac + abc}$ — (ii)

$\therefore$ From equation (i) and (ii)

$\boxed{LHS = RHS}$

Hence, It holds Associativity

Identity

$\boxed{a * e = e * a} \ \forall\ a \in A$

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LHS =

$a * e = a + e - ae$ $\qquad \because \boxed{a * e = a}$

$\Rightarrow \cancel{a} = \cancel{a} + e - ae$

$\Rightarrow \cancel{e(1-} \; e - ae = 0$

$\Rightarrow \boxed{e(1-a) = 0}$

$\because \boxed{e = 0 \text{ for } a \neq 1}$ — (i)

RHS =

$e * a = e + a - ea$ $\qquad \because \boxed{e * a = a}$

$\Rightarrow a = e + a - ea$

$\Rightarrow e - ea = 0$

$\Rightarrow \boxed{e(1-a) = 0}$

$\boxed{e = 0 \text{ for } a \neq 1}$ — (ii)

$\therefore$ From equation (i) and (ii)

LHS = RHS

Hence, Identity at $e = 0$

Now, $\boxed{a * a^{-1} = a^{-1} * a}$

LHS =

$a * a^{-1} = a + a^{-1} - aa^{-1}$

$\because \boxed{a * a^{-1} = 1}$

$\Rightarrow 1 = a + a^{-1} - 1$

$\boxed{a + a^{-1} = 2}$ — (i)

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RHS =

$a^{-1} * a = a^{-1} + a - a^{-1} \cdot a$

$\because$ ~~a~~ $\boxed{a^{-1} \cdot a = 1}$

$\Rightarrow 1 = a^{-1} + a - 1$

$\Rightarrow 2 = a^{-1} + a$

$\boxed{a + a^{-1} = 2}$ — ii

From Equation i And ii

$\boxed{LHS = RHS}$

Hence, follows

~~$e\ a = a * a^{-1} = a^{-1} \cdot a$~~

$\boxed{a * a^{-1} = a^{-1} * a}$

$\therefore$ The given Equation of

$a * b = a + b - ab$ for the given binary operation $(Q, *)$ Is the group.