UNIT – (1)

Statement & notations. (PROPOSITIONS)

→ proposition or statement is a declarative sentance that is either T or F
" London is in Denmark " (false), "2<4" (True)

Predicate

→ A predicate is an expression of one or more variables defined on specific domain. A predicate with variable can be made a proposition by either assigning a value to the variable or by quantifying the variable

Example – let E(x,y) denote "x=y"
let A(a,b,c) denote " a+b+c=0".

Well formed formula

Is a predicate holding any of the following –

1. All propositional constand and variable ar WFF.
2. if X is a variable and Y is a WFF, ∀x Y and ∃x Y are also WFF
3. True and false values are WFF.
4. Each atomic formula is a WFF.
5. All Connectives Connecting WFF are WFF.

QUANTIFIERS

The variable of predicates is quantified by Quantifiers. Are of 2 types → Universal and Existential Quantifiers.

(I) UNIVERSAL

Universal quantifiers states that the statement within its scope are true for every value of the specific variable.

it is denoted by the symbol $\forall$.

$\forall x P(x)$ is read as for every value of x, $P(x)$ is true

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A: (II) Existential

Existential quatifier states that the statement within its scope are true for some values of the specific variable. It is denoted as symbol $\exists$

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A: $\exists x P(x)$ is read as for some values of x, $P(x)$ is true

CONNECTIVES

OR ($\vee$), AND ($\wedge$), Negation ($\sim$), Implication / if-then ($\longrightarrow$), if and only if ($\Longleftrightarrow$)

($P \rightarrow Q$) → conditional statement

Forms of CS:
① Converse $Q \rightarrow P$
② Inverse $\sim P \rightarrow \sim Q$
③ Contrapositive $\sim Q \rightarrow \sim P$

XOR operation – ($\veebar$)

A	B	$A \veebar B$
True	True	False
True	False	True ~~false~~
False	True	True
False	False	False

input values should be exactly True or false.

NAND operation - ($\uparrow$)

A	B	$A \uparrow B$
T	T	F
T	F	T
F	T	T
F	F	T

NOR operation ($\downarrow$)

A	B	$A \downarrow B$
T	T	F
T	F	F
F	T	F
F	F	T

logical Equivalence → Two propositions A & B are said to be logically equivalent if truth values of A are equal ~~and~~ to value of B for all possible input.

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Tautologies → formula which is always true for every value

Prove $[(A \to B) \land A] \to B$

A	B	$A \to B$	$(A \to B) \land A$	$\to B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Contradictions → always false for every value.

Prove $(A \lor B) \land [(\sim A) \land (\sim B)]$

A	B	$A \lor B$	$\sim A$	$\sim B$	$(\sim A \land \sim B)$	$(A \lor B) \land (\sim A \land \sim B)$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	T	F	F	F
F	F	F	T	T	T	F

Contingency → formula which have both true and false value

Prove $(A \lor B) \land (\sim A)$

A	B	$A \lor B$	$\sim A$	$(A \lor B) \land (\sim A)$
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

LOGICAL IMPLICATION →

Relationship b/w 2 statements or sentences. The relation translates verbally into "logically implies" or "if / then" $(\Rightarrow)$

logical implication is relationship btw different propositions where the second is a logical consequence of the first.

PROPOSITIONAL EQUIVALENCES –

Two statements X and Y are logically equivalent if any of following 2 conditions hold –

The truth tables of each statement have the same truth values.

LAWS OF LOGICAL Equivalence –

(I) IDEMPOTENT LAW →

we only use it for single statement. If we combine 2 same statement with symbol $\wedge$ (and) and $\vee$ (OR), then resultant statement is statement itself.

$$P \vee P \equiv P \quad , \quad P \wedge P \equiv P$$

(II) COMMUTATIVE LAW →

if we combine 2 statement with symbol $\wedge$ or $\vee$ then resultant will be same even if we change the position

$$P \vee Q \equiv Q \vee P$$

$$P \wedge Q \equiv Q \wedge P$$

(III) ASSOCIATIVE LAW →

$$P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$$

$$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$$

(VII) Absorption law

$P \vee (P \wedge Q) \equiv P$

$P \wedge (P \vee Q) \equiv P$

(VIII) Negation law

$P \vee \sim P \equiv T$

$P \wedge \sim P \equiv F$

{ $\vee$ - or, $\wedge$ - and }

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(IV) Distributive law

$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

(V) Complement law / Negation law

$P \vee \sim P \equiv T$ and $P \wedge \sim P \equiv F$

$\sim T \equiv F$ and $\sim F \equiv T$.

(VI) De Morgan's law →

$\sim (P \wedge Q) \equiv \sim P \vee \sim Q$

$\sim (P \vee Q) \equiv \sim P \wedge \sim Q$

Normal Forms →

Problem of finding whether given statement is tautology or constradiction is called the Decision problem. For Dicision problem, construction of truth table may not be practical always. We consider an alternate procedure known as the reduction to normal forms -

1. Disjunctive Normal form (DNF) → $(p \vee q)$
2. Conjuctive Normal form → $(p \wedge q)$

Inverse → $p \to q$ is $\sim p \to \sim q$

converse → $p \to q$ is $q \to p$

contra-positive → $p \to q$ is $\sim q \to \sim p$

Duality Principal →

for any true statement, the dual statement obtained by interchaging unions into intersections and universal set into Null set is also true.

Example → $(A \cap B) \cup C$ is $(A \cup B) \cap C$

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Structure of Arguments —

An argument can be defined as a sequence of statements, collection of permises and a conclusion.

Premises $p_1, p_2, p_3 ---- p_n$

conclusion q

if $(p_1 \wedge p_2 \wedge p_3 ---- \wedge p_n) \rightarrow q$ indicates a topology

(i) Valid Argument —

when if all their premises are true, then their conclusions will also be true.

Example - 1. " If tomorrow is holiday, I will go to mall."
2. " if tomorrow is holiday"
3. ∴ " I will go to mall".

Rules for Inference —

we can construct more complicated valid argument with the help of using simple arguments, which work as the building blocks. there are some simple arguments that have been established as valid and very important. These types of argument are known as the rules of inference.

① Modus Ponens

(i) $P \rightarrow Q$

(ii) P

(iii) ----------

(iv) $\therefore Q$

② Modus Tollens

(i) $P \rightarrow Q$

(ii) $\sim Q$

(iii) ----------

(iv) $\therefore \sim P$

③ Hypothetical syllogism

(i) $P \rightarrow Q$

(ii) $Q \rightarrow R$

(iii) ------

(iv) $\therefore P \rightarrow R$

④ Disjunction syllogism

(i) $\sim P$

(ii) $P \vee Q$

(iii) ------

(iv) $\therefore Q$

⑤ Additional

(i) P

(ii) ----

(iii) $\therefore P \vee Q$

⑥ Simplification

(i) $P \wedge Q$

(ii) -----

(iii) $\therefore P$

⑦ conjution

(i) P

(ii) Q

(iii) ------

(iv) $\therefore P \wedge Q$

⑧ Resolution

(i) $P \vee Q$

(ii) $\sim P \vee R$

(iii) -------

(iv) $\therefore Q \vee R$

⑨ Constructive Dilemma

$(P \rightarrow Q) \wedge (R \rightarrow S)$ and $P \vee R$

1. $(P \rightarrow Q) \wedge (R \rightarrow S)$
2. $P \vee R$
3. --------
4. $\therefore Q \vee S$

⑩ Destructive Dilemma

1. $(P \rightarrow Q) \wedge (R \rightarrow S)$
2. $\sim Q \vee \sim S$
3. --------
4. $\therefore \sim P \vee \sim R$

Some other rules are $\longrightarrow$

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① Universal Instantiation

1. $\forall x\, P(x)$
2. — — — — —
3. $\therefore P(c)$, for any c

② Universal Generalization

1. $P(c)$ for any arbitrary c
2. — — — — — —
3. $\therefore \forall x\, P(x)$

③ Existential Instantiation

1. $\exists x\, P(x)$
2. — — — —
3. $\therefore P(c)$, for some element c

④ Existential Generalization

1. $P(c)$ for some element c
2. — — — — — —
3. $\therefore \exists x\, P(x)$

Example → ① P = My fiance come to meet me

Q = I will be happy

R = I will go to office

S = I will complete my work

1. Premises : $P \to Q$, $\sim P \to R$, $R \to S$
2. Conclusion : $\sim Q \to S$

Step	Reason
$P \to Q$	Premise
$\sim Q \to \sim P$	Contrapositive of (1)
$\sim P \to R$	Premise
$\sim Q \to R$	Hypothetical syllogism by (2) and (3)
$R \to S$	Premise
$\sim Q \to S$	Hypothetical syllogism

② P = An employee in my office

$P \to Q$
$\sim P \to R$
$R \to S$

$\{P \to Q\}$
$\{\sim P \to S\}$

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Q : completed his daily work
R : completed his monthly files

2. Premises $\exists x \left(P(x) \wedge \sim Q(x) \right)$
$\forall x \; (P(x) \to R(x))$

Q. conclusion $\exists x (R(x) \wedge \sim Q(x))$

Steps	Reason
$\exists x (P(x) \wedge \sim Q(x))$	Premises
$P(a) \wedge \sim Q(a)$	Existential Instalation
$P(a)$	simplified by (2)
$\forall x \; (P(x) \to R(x))$	Premises
$P(a) \to R(a)$	Universal Instalation
$R(a)$	Modus Ponens by (3) and (5)
$\sim Q(a)$	Simplified by (2)
$R(a) \wedge \sim Q(a)$	Conjution by 6 and 7
$\exists (R(a) \wedge \sim Q(a))$	Existential Generalization

The Pigeonhole Principle and Application

Theorem –

1) If "A" is the average number of pigeons per hole, where A is not an integer then.

→ Atleast one pigeon hole contains ceil [A] (smallest integer greater than or equal to A) pigeons

→ Remaining pigeon holes contains at most floor [A] pigeon

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Defn — if k is a +ve integer and k+1 or more objects are placed in k boxes, then there is atleast one box containing two or more of the Objects.

Generalized pigeonhole Priciple — if 'n' object are placed into k (n > k) boxes, then there is atleast one box containing atleast [n/k] objects or $\left\lceil \frac{n-1}{k} \right\rceil + 1$.

Example — Min no of student in class so that 4 of them are born in same month

$k+1 = 4$, $n = 12$
$(k = 3)$

$kn + 1 = 12 \times 3 + 1 = \boxed{37}$

All Questions and Remaining Concepts ⟶

(Q1) Prove that $P \leftrightarrow Q$ is logical equivalent to $(P \to Q) \land (Q \to P)$

Ans) To prove —

$$P \leftrightarrow Q \equiv (P \to Q) \land (Q \to P)$$

(and)

P	Q	$P \leftrightarrow Q$	$P \to Q$	$Q \to P$	$(P \to Q) \land (Q \to P)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Hence proved that they are logically equivalent

∨ – OR
∧ – and

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(Q2) Prove that this expression is a tautology or not.

(i) $P \rightarrow (P \wedge (Q \rightarrow P))$

(ii) $P \vee Q \vee R \longleftrightarrow ((((P \rightarrow Q) \rightarrow Q) \rightarrow R) \rightarrow R)$

(Ans) (i) $P \rightarrow (P \wedge (Q \rightarrow P))$

P	Q	$Q \rightarrow P$	$P \wedge (Q \rightarrow P)$	$P \rightarrow P \wedge (Q \rightarrow P)$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

Yes, it is a tautology

(2)

P	Q	R	(A) $P \vee Q$	(B) $A \vee R$	(C) $P \rightarrow Q$	(D) $C \rightarrow Q$	(E) $D \rightarrow R$	F $(E \rightarrow R)$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	F	T
T	F	T	T	T	F	T	T	T
T	F	F	T	T	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	F	T
F	F	T	F	T	T	F	T	T
F	F	F	F	F	T	F	T	F

$B \leftrightarrow F$
T
T
T
T
T
T
T
T
T

yes it is a tautology.

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$\vee \to$ OR
$\wedge$ — and

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(Que) Determine WFF or not ?

1. $((\sim P \wedge Q))$ No — after ( ) this will become WFF
2. $((P \to Q) \to (\wedge Q))$ No — ( ) " "
3. $(P \to (Q \to R))$ Yes
4. $(P \to (P \vee Q))$ Yes
5. $(P \to Q)$ Yes
6. $(((P \wedge Q) \to Q))$ NO
7. $(((P \to Q) \wedge (Q \to R)) \leftrightarrow (P \to R))$ NO.

(Ques) ① $((P \to Q) \wedge (Q \to R)) \to (P \to R)$
② $(P \to (Q \vee R)) \to ((P \wedge Q) \to R)$

(i) satisfiable (ii) valid (iii) unsatisfiable but not valid

P	Q	R	A: $P \to Q$	B: $Q \to R$	C: $A \wedge B$	D: $P \to R$	$C \to D$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

This is only valid.

For next we have

$\wedge$ – and
$\vee$ – or

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P	Q	R	A: $Q \vee R$	B: $P \to A$	C: $P \wedge Q$	D: $C \to R$	$B \to D$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	F
T	F	T	T	T	F	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	F	T	T
F	T	F	T	T	F	T	T
F	F	T	T	T	F	T	T
F	F	F	F	T	F	T	T

It is satisfiable but not valid, It is contigency.

(Que) $\sim(\sim P \wedge Q) \wedge (P \vee Q) \equiv P$

LHS $\sim(\sim P \wedge Q) \wedge (P \vee Q)$

$(P \vee \sim Q) \wedge (P \vee Q)$ → Distributive

$P \vee (\sim Q \wedge Q)$

$P \vee F \equiv P$ → RHS

(Que) $P \to (q \to R) \equiv \sim(P \wedge Q) \vee R \equiv (P \wedge Q) \to R$

LHS - $P \to (Q \to R)$

$P \to (\sim Q \vee R)$ → Implification law

$\sim P \vee (\sim Q \vee R)$

$(\sim P \vee \sim Q) \vee R$ → Associative law.

$\sim(P \wedge Q) \vee R$ → RHS → De morgan's law

$\equiv (P \wedge Q) \to R$ — implification law

(Que) $\sim(P \vee (\sim P \wedge Q)) \equiv \sim P \wedge \sim Q$

RHS ⇒ $\sim((P \vee \sim P) \wedge (P \vee Q))$ distributive

$\sim(T \wedge (P \vee Q))$

$\sim P \wedge \sim Q$ = RHS Hence proof

$\{\wedge - \text{and},\ \vee - \text{or}\}$

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(Que) $(P \to R) \vee (Q \to R) \equiv (P \wedge Q) \to R$

LHS $\quad (\sim P \vee R) \vee (\sim Q \vee R)$ Implification law

$(\sim P \vee \sim Q) \vee R$ distributive

$\sim (P \wedge Q) \vee R$

$(P \wedge Q) \to R \equiv RHS$

Hence proof

(Que) $(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q) \equiv P \vee Q$

$(P \wedge Q) \vee (\sim P \wedge Q) \vee (P \wedge \sim Q)$

$(P \vee \sim P) \wedge Q \quad \vee (P \wedge \sim Q)$ distributive

$T \wedge Q \quad \vee (P \wedge \sim Q)$

$Q \quad \vee (P \wedge \sim Q)$

$(Q \vee P) \wedge (Q \vee \sim Q)$

$(Q \vee P) \wedge T$

$(P \vee Q) \equiv RHS$

(Que) $P \wedge \sim (Q \wedge R)$

$P \wedge (\sim Q \vee \sim R)$ De-morgan's law

$(P \wedge \sim Q) \vee (P \wedge \sim R)$ [Associative law]

$\cancel{(\sim R \vee Q) \wedge (\sim}$

(Que) $(P \to Q) \wedge (Q \to R)$

$(\sim P \vee Q) \wedge (\sim Q \vee R)$

$((\sim P \vee Q) \wedge (\sim Q)) \vee ((\sim P \vee Q) \wedge R)$

$((\sim P \wedge \sim Q) \vee \underbrace{(Q \wedge \sim Q)}_{F}) \vee ((\sim P \wedge R) \vee (Q \wedge R))$

$(\sim P \wedge \sim Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$

(Que) obtain CNF of

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$\vee$ – OR
$\wedge$ – and

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$(P \to Q) \wedge (Q \vee (P \wedge R))$

$(\sim P \vee Q) \wedge ((Q \vee P) \wedge (Q \vee R))$

$(\sim P \vee Q) \wedge (Q \vee P) \wedge (Q \vee R)$

P	Q	R	$\sim P$	(A) $\sim P \vee Q$	(B) $(Q \vee P)$	(C) $(Q \vee R)$	(D) $(A \wedge B)$	$(D \wedge C)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	F	T	T	F	F
T	F	F	F	F	T	F	F	F
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	F	T	F	F
F	F	F	T	T	F	F	F	F

Not a tautology.

(Q) obtain CNF $\longrightarrow$

(i) $P \wedge (P \to Q)$

$P \wedge (\sim P \vee Q)$

(ii) $(\sim P \to R) \wedge (Q \leftrightarrow P)$

$(P \vee R) \wedge ((\sim Q \vee P) \wedge (\sim P \vee Q))$

$(P \vee R) \wedge (\sim Q \vee P) \wedge (\sim P \vee Q)$

(iii) $\sim [(P \vee Q) \leftrightarrow (P \wedge Q)]$

$\sim [\sim (P \vee Q) \vee (P \wedge Q) \wedge \sim (P \wedge Q) \vee (P \vee Q)]$

$(P \vee Q) \wedge \sim (P \wedge Q)$ $\qquad \sim A \leftrightarrow B$ $\qquad \{A = P \vee Q, B = P \wedge Q\}$

$(A \vee B) \wedge (\sim B \vee \sim A)$

$[(P \vee Q) \vee (P \wedge Q)] \wedge [\sim (P \wedge Q) \vee \sim (P \vee Q)]$

$(P \vee Q) \wedge [(\sim P \vee \sim Q) \wedge (\sim P \vee \sim Q)]$

$\equiv (P \vee Q) \wedge (\sim P \vee \sim Q)$

{ $\wedge$ - and }

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(xvi) (Q2) $Q \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$ is a tautology

~~$P \wedge \sim Q$~~

$(Q \vee P) \wedge (Q \vee \sim Q)$

$(Q \vee P) \vee (\sim P \wedge \sim Q)$

$(Q \vee P) \vee \sim (P \vee Q)$

$(P \vee Q)$ — A

$A \vee \sim (A)$

(T) is a tautology

★ Argument & Inference Theory :-

(Q) what is argument ?

A process by which a conclusion is obtain from the given set of premises.

Premises – Given group of propositions.

Conclusion –

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