ARDEN'S THEOREM (FA → RE)

If P and Q are two Regular expressions over Σ, and if P does not contain ε, then the following equation in R given by $R = Q + RP$ has a unique solution $R = QP^*$. 104.

Proof:

$$R = Q + RP \quad \text{—— (i)}$$

Replace R with QP^*

$$R = Q + QP^*P \quad \text{—— (ii)}$$

$$R = Q(\varepsilon + P^*P) \quad \text{—— } (\because \varepsilon + R^*R = R^*)$$

$$\Rightarrow QP^*$$

Proof: this is the unique solution.

$$R = Q + RP \quad \text{—— (i)}$$

Replace R with $Q + RP$

$$R = Q + (Q + RP)P$$

$$\Rightarrow Q + QP + RP^2 \Rightarrow Q + QP + Q + [Q + RP]P^2$$

$$\Rightarrow Q + QP + QP^2 + RP^3$$

$$\vdots$$

$$= Q + QP + QP^2 + \dots QP^n + RP^{n+1}$$

Replace with $[R = QP^*]$

$$= Q + QP + QP^2 + \dots QP^n + QP^* P^{n+1}$$

$$= Q\,[\underbrace{\varepsilon + P + P^2 + \dots P^n + P^* P^{n+1}}]$$

$$= Q \quad P^*$$

Example Proof using Identities

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Ques Prove that $(1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1)$ is equal to $0^*1(0+10^*1)^*$.

Ans

$$LHS = (1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1)$$

$$\Rightarrow (1+00^*1)\left[\varepsilon+(0+10^*1)^*(0+10^*1)\right]$$

$$\boxed{(\because \varepsilon + R^*R = R^*)}$$

$$\Rightarrow (1+00^*1)(0+10^*1)^*$$

$$\Rightarrow (\varepsilon.1+00^*1)(0+10^*1)^*$$

$$\boxed{(\because \varepsilon.R = R)}$$

$$\Rightarrow \underbrace{(\varepsilon+00^*)}_{\boxed{\varepsilon+RR^* = R^*}}1(0+10^*1)^*$$

$$\Rightarrow 0^*1(0+10^*1)^*$$