Regular Expressions

- Regular Expressions are used for representing certain sets of strings in an algebric fashion.

Roles

$\boxed{a, b, c \ldots \wedge, \phi}$

1. Any terminal symbol i.e symbols $\varepsilon, \Sigma,$ including $\wedge$ and ϕ are regular expressions.
   $\{\varepsilon, \Sigma, \wedge, \phi\}$

2. The union of two regular expressions is also a regular expression R_1, R_2 : $R_1 \cup R_2 \Rightarrow (R_1 + R_2)$

3. Concatation is also RE. : $R_1, R_2 \Rightarrow (R_1 \cdot R_2)$

4. Closure of RE is RE : $R \rightarrow R^*$

$a^* = \{\varepsilon / \wedge, a, aa, aaa \ldots\}$

$a^+ = \{a, aa, aaa \ldots\}$

$a^+ \cdot a = a^*$

ARDEN'S THEOREM (FA → RE)

(rev E.)

Examples:

describe the following sets as Regular Expressions:

1) $\{0, 1, 2\}$: 0 or 1 or 2

$R = 0 + 1 + 2$

2) $\{\Lambda, ab\}$

$R = \underbrace{\Lambda}\ ab$

this is present.

3) $\{abb, a, b, bba\}$: abb or a or b or bba

$R = abb + a + b + bba$.

4) $\{\Lambda, 0, 00, 000\}$.. : $R = 0^*$ all strings that can be formed with 0 including empty symbol.

5) $\{1, 11, 111, 1111, \ldots\}$: $R = 1^+$

THEOREM (FA → RE)

3

Identities of Regular Expression

1) $\phi + R = R$

ϕ = empty set $\therefore$ $\phi \cup R = R$

2) $\phi.R + R\phi = \phi$

3) $\epsilon R = R\epsilon = R$

$\{(\wedge = \epsilon)\}$

4) $\epsilon^* = \epsilon$ and $\phi^* = \epsilon$

5) $R + R = R$

6) $R^* . R^* = R^*$

7) $RR^* = R^* R$

8) $(R^*)^* = R^*$

9) $\epsilon + RR^* = \epsilon + R^* R = R^*$

10) $(PQ^*)P = P(QP)^*$

11) $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

12) $R(P+Q) = RP + RQ$ and $(P+Q)R = PR + QR$.

THEOREM (FA

rev Σ,

Eg Design RE for following language over

1) Language accepting string of length exactly 2.

$L_1 = \{aa, ab, ba, bb\}$

$R_1 = aa + ab + ba + bb$

$a(a+b) + b(a+b) \Rightarrow \boxed{(a+b)(a+b)}$

2) Language accepting of length atleast 2.

$L_2 = \{aa, ab, ba, bb, aaa, bbb, abab \ldots\}$

abba

$R_2 = \underbrace{(a+b)(a+b)}_{\text{atleast } 2}(a+b)^*$

anything more than length 2. e^*)

3) Language accepting string of length atmost 2.

$L_3 = \{\varepsilon, a, b, aa, bb, ab, ba\}$

$R_3 = \varepsilon + a + b + aa + ab + bb + ba.$

$= (\varepsilon + a + b)(\varepsilon + a + b)$

)+RP]

①

$\frac{1}{2}$ $\frac{1}{\uparrow}$ $\frac{1}{\uparrow}$ ab ba

a, b, c → 0, 1, 2

If

$(a+b+c)^* a (a+b+c)^* b (a+b+c)^* + (a+b+c)^* b\ c$

$(0+1)^* 1\ 0|1\ 0|1$ _ _ _ _ _ _ _

$(0+1)^* 1 (0+1)(0+1)^9$ _ _ _

$(0+1)^9$

101

Σ^1

Σ^2

Σ^3

$(0^* 1^* 00)^* 0^* 1^*$

1 0 1

$\varepsilon, 0, 000$ $\varepsilon, 0$.

1 1 $\boxed{1\ 0\ 1}$ 0100

$0^* = \{$

$\{a^n b^{\boxed{2m+1}} : n \geq 0 \ \& \ m \geq 0.\}$

P.

$(0^* 1 00)^*$

$a^0 b^{2\times 0+1}$

$a^0 b^1$

$a^1 b^3$

$a^2 b^5$

$a^3 b^7$

a^4

ε, 0100, 01000100, 0.

$(aa)^*$ $(bb)^* b$.

0, aa, aaaa bb b

bbbb b.

bbbbbb b.

ARDENS

...ressions

ab ba

$(a+b)^*$

ε, a, b, aa, bbb, ab

$\varepsilon^* R$

$P +$