(1) Imp Prove De-Morgan's Law (1)

(a) $(A \cup B)^c = A^c \cap B^c$

Let

$x \in (A \cup B)^c$

$= x \notin A \cup B$

$\Rightarrow x \notin A$ and $x \notin B$

$= x \in A^c$ and $x \in B^c$

$= x \in (A^c \cap B^c)$.

$\therefore (A \cup B)^c \subset A^c \cap B^c$ —— (1)

Again let $x \in A^c \cap B^c$

$\Rightarrow x \in A^c$ and $x \in B^c$

$\Rightarrow x \notin A$ and $x \notin B$

$= x \notin A \cup B$

$\Rightarrow x \in (A \cup B)^c$.

$\therefore A^c \cap B^c \subset (A \cup B)^c$ —— (2)

Combining (1) and (2), we get

$$\boxed{A^c \cap B^c = (A \cup B)^c}$$

(2) To prove $(A \cap B)^c = A^c \cup B^c$

$x \in (A \cap B)^c = x \notin A \cap B$

$= x \notin A$ or $x \notin B$

$= x \in A^c$ or $x \in B^c$

$= x \in (A^c \cup B^c$

$\therefore (A \cap B)^c \subset A^c \cup B^c$ —— (1)

Again Let $x \in A^c \cup B^c$

$\Rightarrow x \notin A$ or $x \notin B$

$= x \notin A \cap B$

$= x \in (A \cap B)^c$

$\therefore A^c \cup B^c \subset (A \cap B)^c$ —— (2)

Combining (1) and (2)

$(A \cap B)^c = A^c \cup B^c$

Combination :→

→ A combination is a selection of some or all, objects from a set of given objects, where order of objects does not matter.

→ It is represented by $^nC_r = \frac{n!}{r!(n-r)!}$

(i) $^{10}C_6 = \frac{10!}{6! \times (10-6)!}$

$= \frac{10 \times 9 \times 8 \times 7 \times 6!}{6! \times 4 \times 3 \times 2 \times 1}$

$= 10 \times 3 \times 7 = 210$

(ii) $^{52}C_4 = \frac{52!}{4! \times (52-4)!}$

$= \frac{52 \times 51 \times 50 \times 49 \times 48!}{4 \times 3 \times 2 \times 1 \times 48!}$

$= 270725$

Q → How many 16 - bit strings are th[ere] containing exactly 5 0's?

Ans :→ A 16 - bit string having exactly five 0's is determined if we tell which bits are 0's. This can be done in $^{16}C_5$ ways.

∴ the total no. of 16 bit strings is

$$= {}^{16}C_5 = \frac{16!}{5! \times (16-5)!}$$

$$= \frac{16 \times 15 \times 14 \times 13 \times 12 \times 11!}{5 \times 4 \times 3 \times 2 \times 1 \times 11!}$$

$$= \underline{4368}$$

Imp

Q → from 10 programmers in how many ways can 5 be selected when

a) A particular programmer is included every time.

b) A particular programmer is not included at all.

Ans → We have to select 5 programmers from the 10 programmers. So, the no. of ways to select them is $^{10}C_5$

$$= \frac{10!}{5! \times (10-5)!} = 252$$

(a) When a particular programmer is included every time then the remaining = $5-1 = 4$ programmers can be selected from the remaining $= 10-1 = 9$ programmers. This can be done in $^{9}C_4$ ways. $= \frac{9!}{4!\,(9-4)!}$

$= 126$

(b) When a particular programmer is not included at all, then the five programmers can be selected from the remaining $= 10-1 = 9$ programmers

This can be done as:

$$^{9}C_5 = \frac{9!}{5!\,(9-5)!} = 126$$