Graphs

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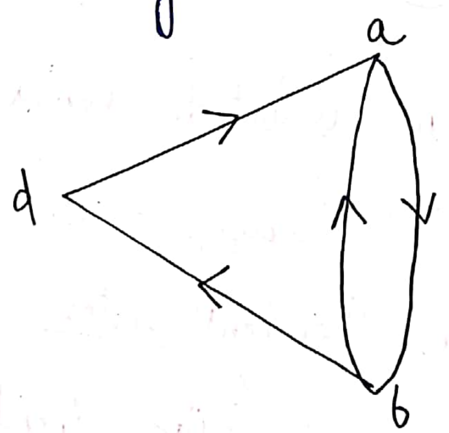
→ **Graph** :→ The graph consist of points or nodes called vertices which are connected to each other by way of lines called edges.

→ These lines may be directed or undirected.

• **Directed Graph** :→

A directed graph is defined as an ordered pair (V, E) where V is a set and E is a binary relation on V.

The elements in V are called vertices.
The ordered pairs in E are called edges.
The following graph is a directed graph.

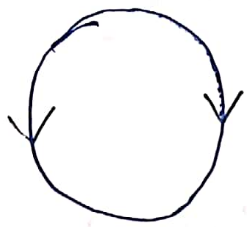


directed graph.

Here the vertices are a, b, d and the edges are (a, b) (b, a) (b, d) (d, a)

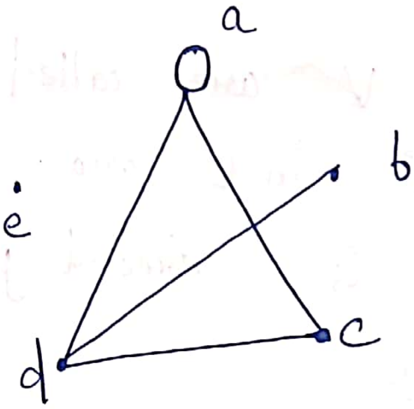
→ The vertex a is called the initial vertex and the vertex b is called the terminal vertex of the edge (a, b).

• An edge that is incident from and into the same vertex is called a loop or self-loop.



loop

• A vertex is said to be an isolated vertex if there is no edge incident with it.



→ Here, vertex 'e' is an isolated vertex.

→ The vertex 'a' has self-loop.

→ The vertex 'b' is a Pendent vertex

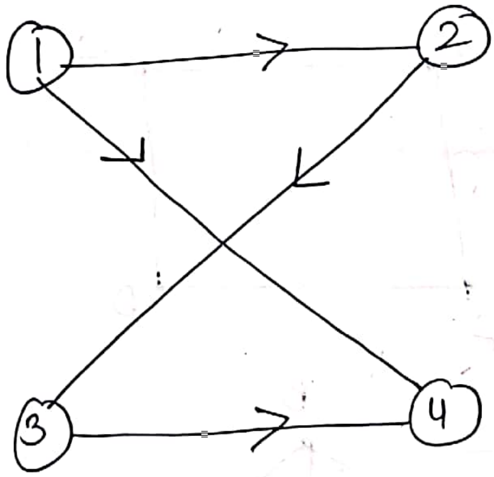
that has only one edge incident on it.

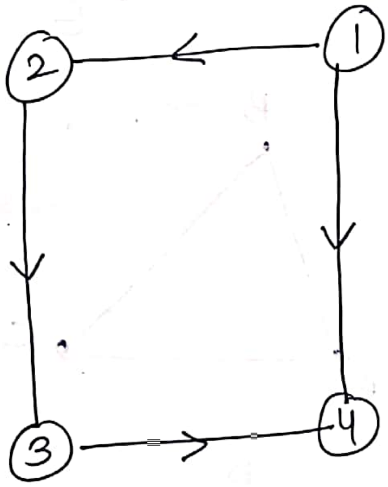
Undirected Graphs :→ It consists of a set of vertices V and a set of edges E. The edge set contains the unordered pairs of vertices.

eg. Let $V = \{1, 2, 3, 4\}$

and $E = \{(1,2)\ (1,4)\ (3,4)\ (2,3)\}$

→ The graph can be drawn in various ways:→ 2 of which are as follows:

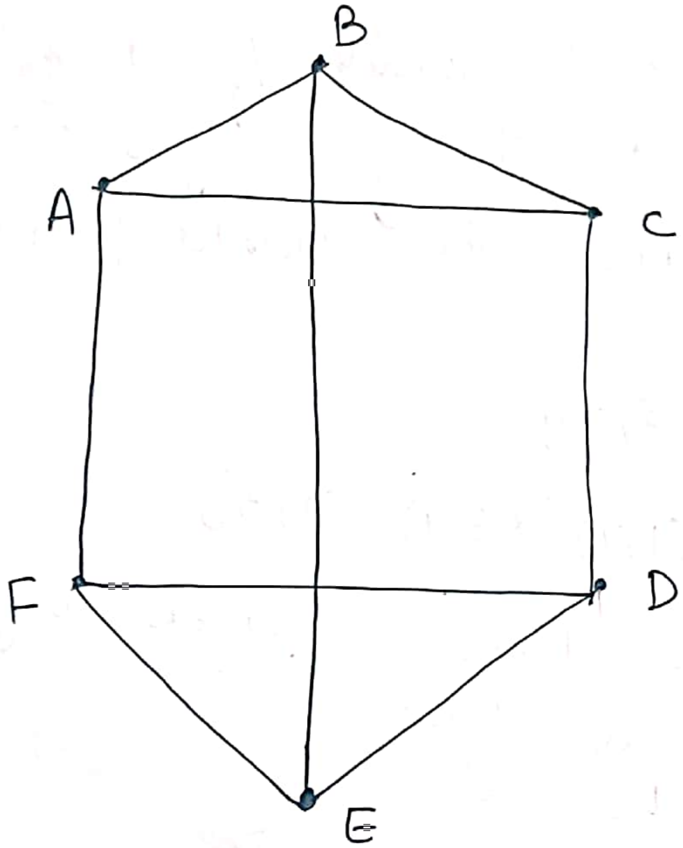




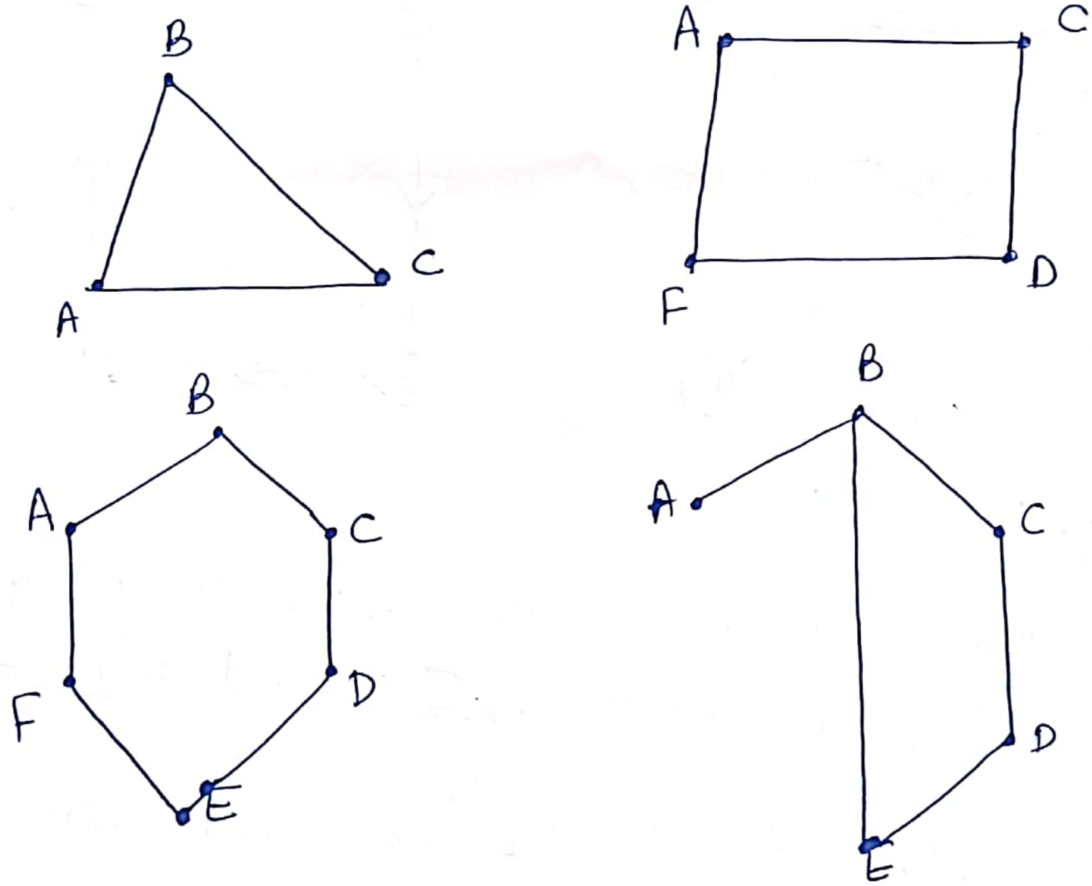
- **Subgraph** :→

A subgraph of a graph $G = (V, E)$ is a graph $G' = (V', E')$ in which $V' \subseteq V$ and $E' \subseteq E$ (Subset) and each edge of G' has same end vertices in G' as in graph G.

Ex:→ Consider the graph G as shown below. Show the different subgraphs of this graph.



Sol:→

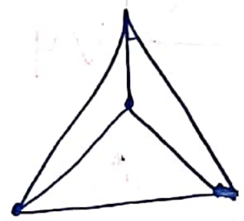


• Complement of a Subgraph

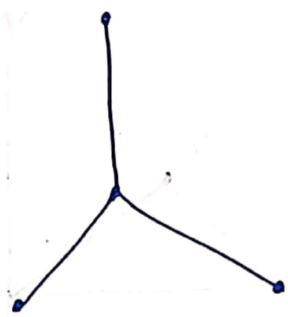
→ Let $G = (V, E)$ be a graph and S be a subgraph of G. If edges of S be deleted from the graph G, the graph so obtained is complement of subgraph S. It is denoted by $\bar{S}$.

$\therefore$ $\bar{S} = G - S$

Consider the graph

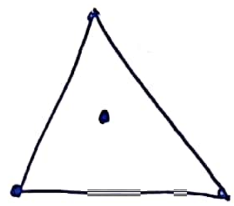


and its subgraph



then the complement of subgraph S is

$\bar{S} =$



<u>Note</u> :→ The no. of vertices do not change in the complement of a subgraph.

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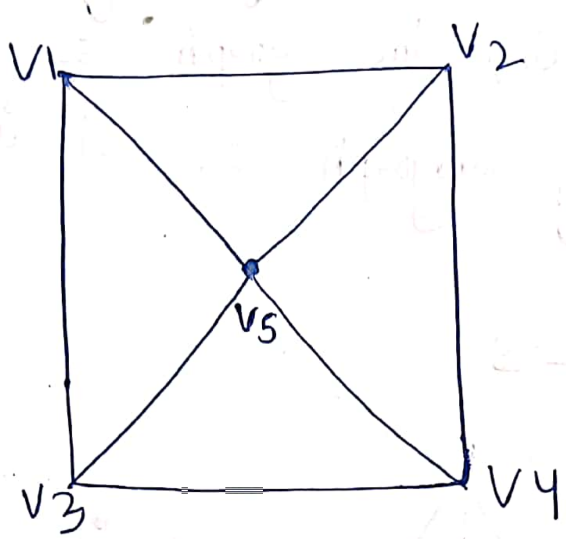
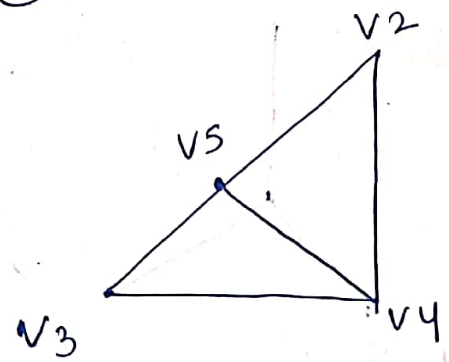
Q :→ Consider the graphs as shown below. Determine the subgraphs.

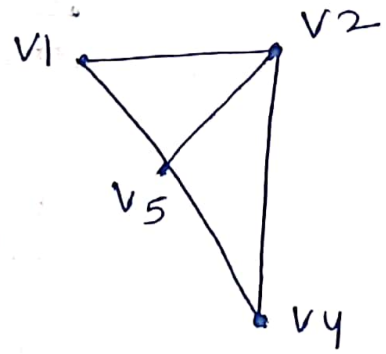
(1) $G - v_1$ (2) $G - v_3$ 3) $G - v_5$.



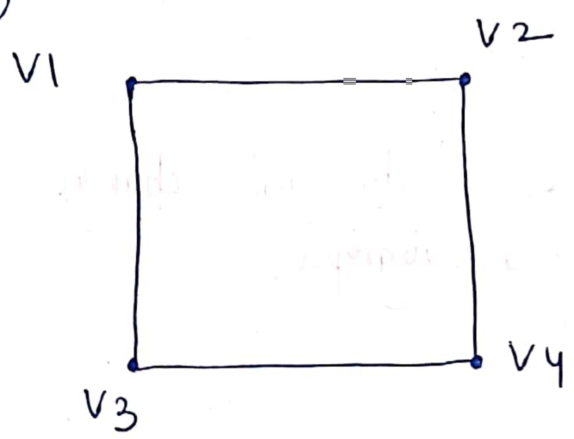
Ans :→ (1) $G - v_1$



(2) $G - v_3$

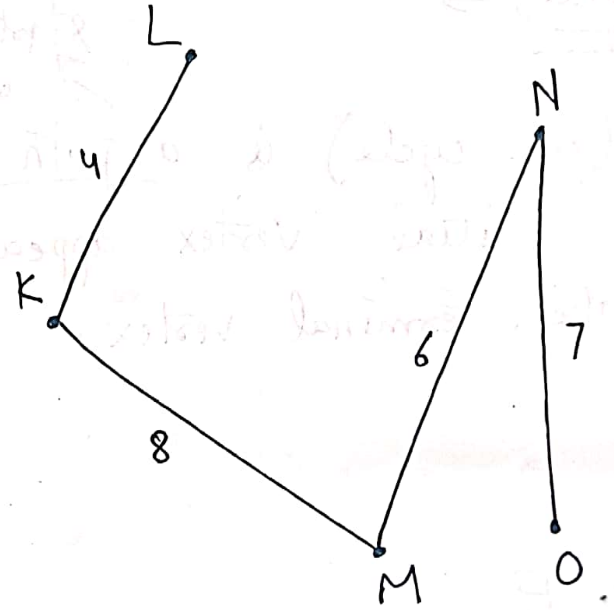


(3) $G - v_5$



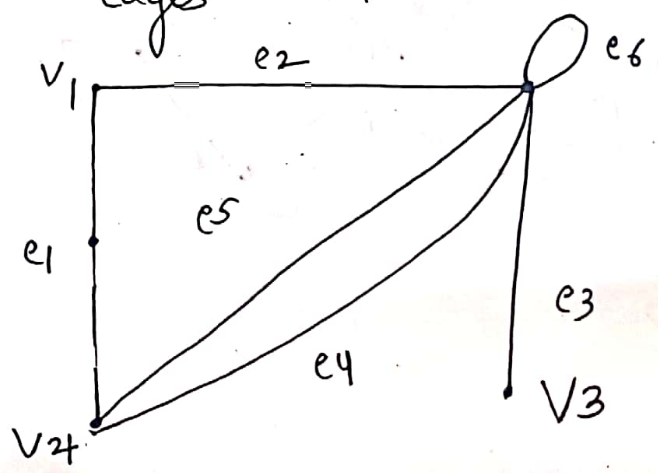
1. Weighted Graphs :→

A graph $G = (V, E)$ is called a weighted graph if each edge of graph G is assigned a positive number w called the weight of the edge. ex :→



• Multigraph :→

A multigraph $G = (V, E)$ consist of a set of vertices V and a set of edges E such that edge set E may contain multiple edges and self loops. eg :→



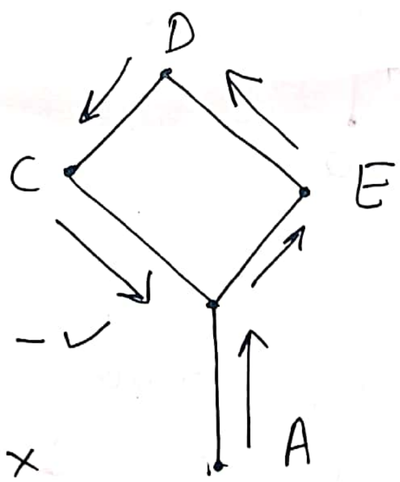
- **Euler path or chain :→**

An Euler path (or chain) through a graph is a path whose edge list contains each edge of the graph exactly once.

- **Euler circuit (or cycle) :→**

→ An Euler circuit (or cycle) is a path through a graph in which initial vertex appears second time as the terminal vertex.

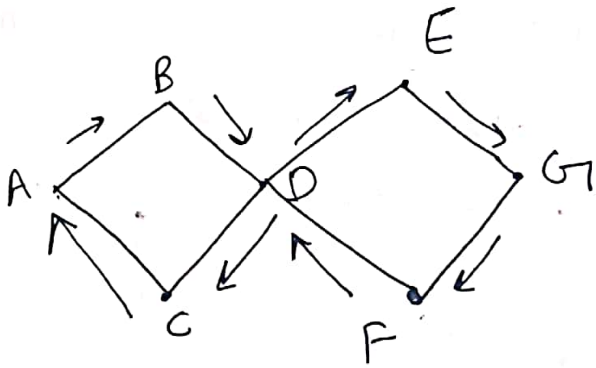
start / end same



→ Euler path – ✓

Euler ckt – ✗

Eulerian graph – ✗



Euler path – ✓

Euler circuit – ✓

Eulerian graph – ✓

Euler graph :→ is a graph that posseses an Euler circuit. An Euler circuit uses every edge exactly once but vertices may be repeated.

- Hamiltonian path (or chain) :→

A hamiltonian path (or chain) through a graph is a path whose vertex list contain each vertex of the graph exactly once, except if path is a circuit.

- Hamoltonian circuit (or cycle) :→

→ A hamoltonian circuit (or cycle) is a path in which the initial vertex appears a second time as the terminal vertex.

- Hamiltonian Graph :→ is a graph that posseses a Hamiltonian path. A Hamiltonian path uses each vertex exactly once but edges may not be included.

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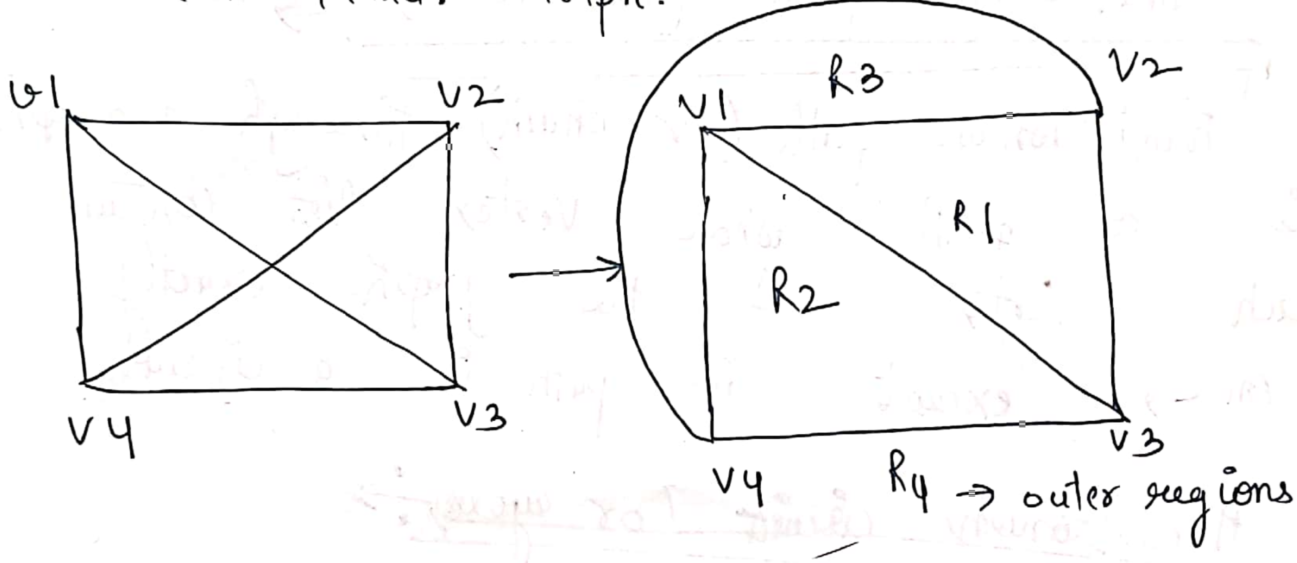
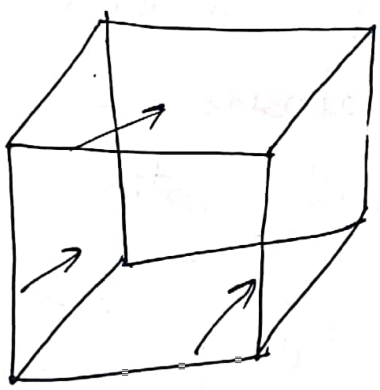
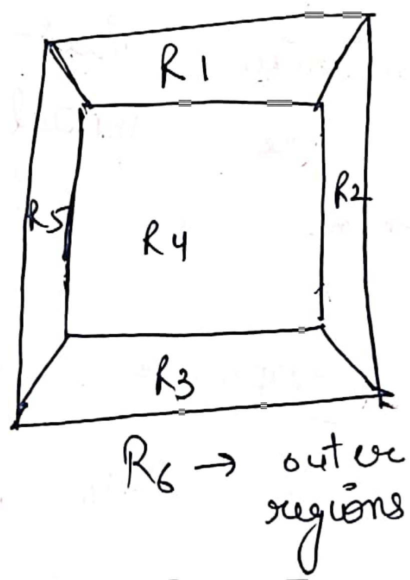
Planar Graph :->

A Graph is called Planar if it can be drawn in the plane without any edges crossing. And a graph that cannot be drawn on a plane without a crossover b/w its edges is called Non- Planar Graph.







Planar graph

Theorem :-> A complete Graph of 5 vertices is Non- Planar.

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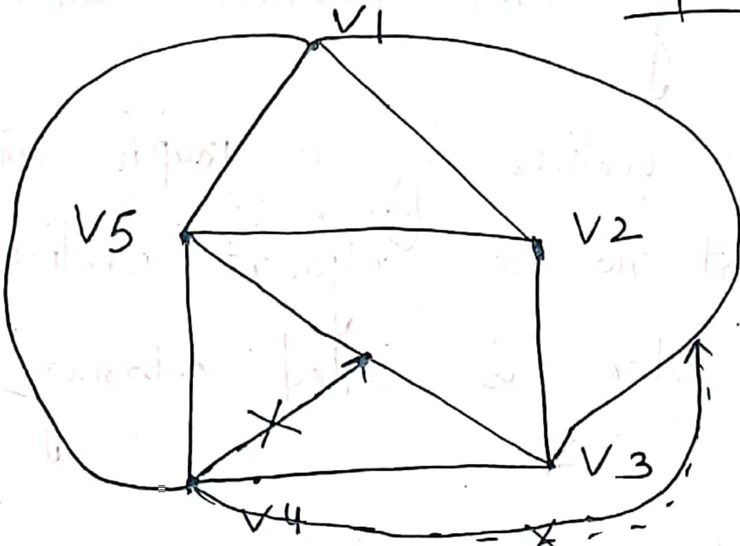
Complete Graph :→ edge b/w every vertex

→ all edges should be connected.

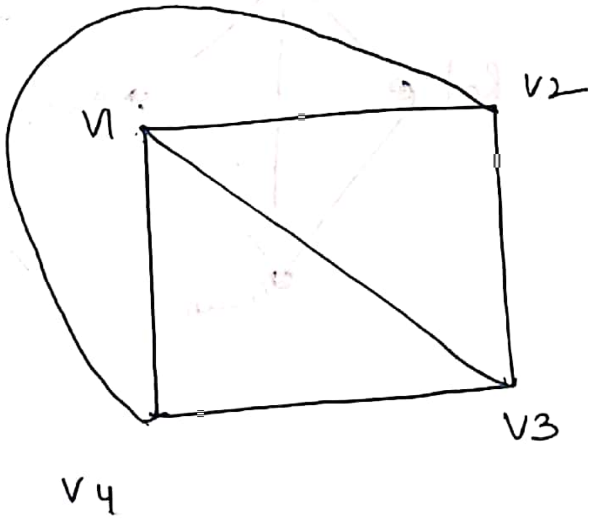


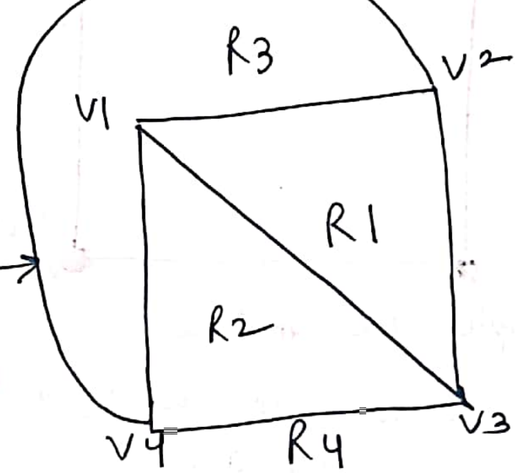
We will connect V1 to V4 and V1 to V3 so that intersection shouldn't be there

This is a non-planar graph.
We cannot draw a planar graph with 5 vertices.

Euler's formula :→ Let G be a connected Planar Graph with 'e' edges and vertices 'v'. Let 'r' be the no. of regions in Planar representation of G. Then,

$$\boxed{r = e - v + 2}$$





→ ① first draw the regions for this graph

② $e = 6, \quad v = 4$

$r = 6 - 4 + 2$

$\boxed{r = 4}$

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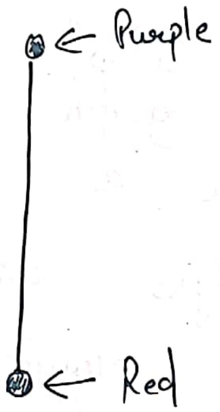
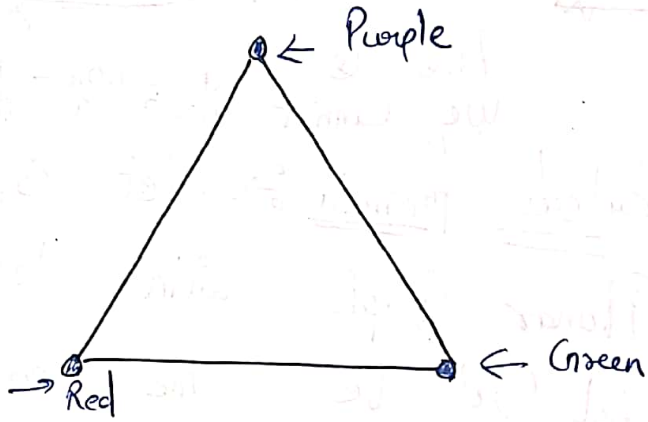
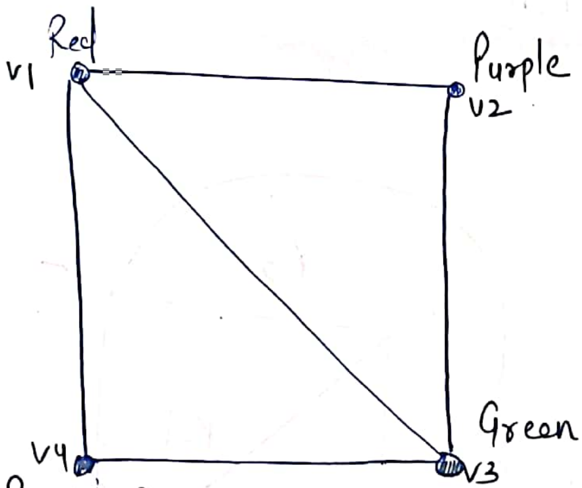
Graph coloring and chromatic number

→ Painting all the vertices of a graph with colors such that no 2 adjacent vertices have the same color is called coloring of graph.

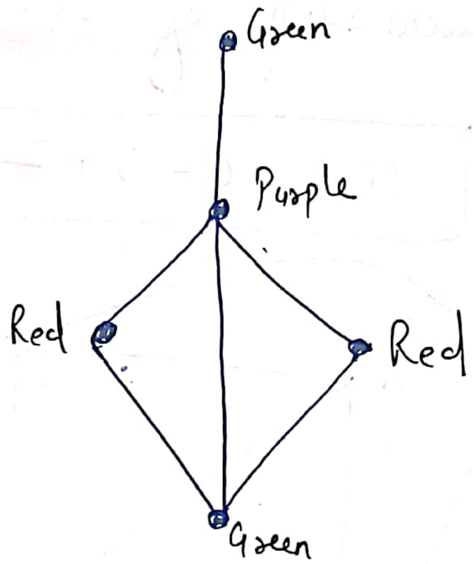
Example :-



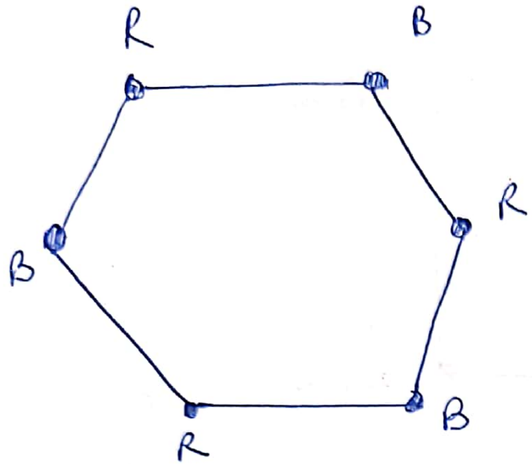


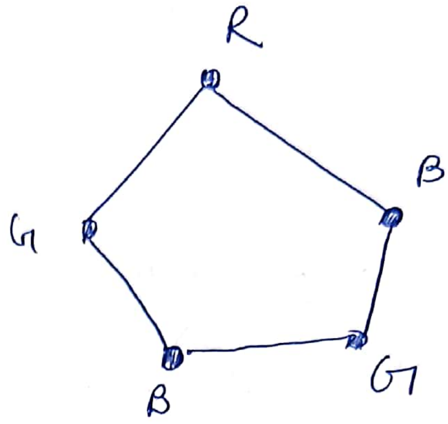


Purple (we can take same color as v_2 and v_4 are not connected)

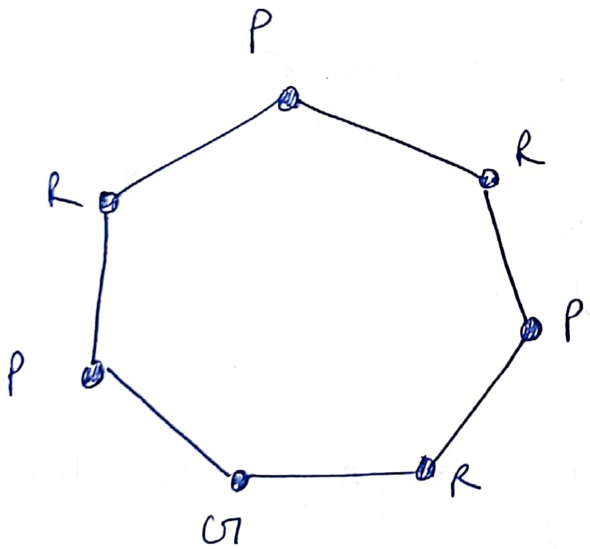


If a graph is circuit (closed) with n vertices then
(i) it is 2-chromatic if n is even.
(ii) it is 3-chromatic if n is odd.



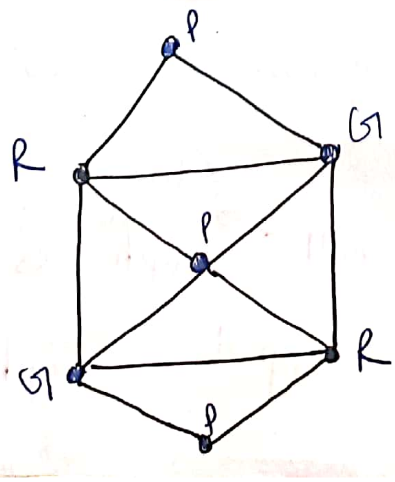


Q:→ Determine the chromatic no. of graph.



Vertices are odd, so it will be 3-chromatic.

Q:→ Find the chromatic no. of a graph.



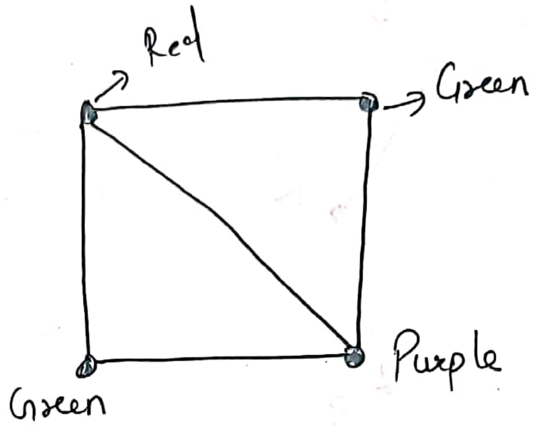
3-Chromatic graph

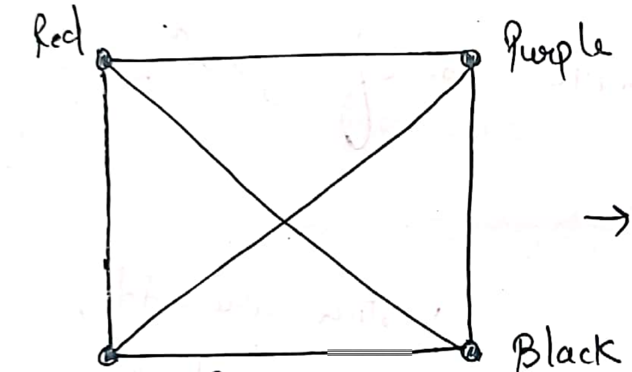
Chromatic number :→

The least no. of colors required for coloring of a graph G is called its chromatic no.

eg :→

(2 - Chromatic Graph)





→ 4- chromatic Graph.

(Complete graph)

Note :→

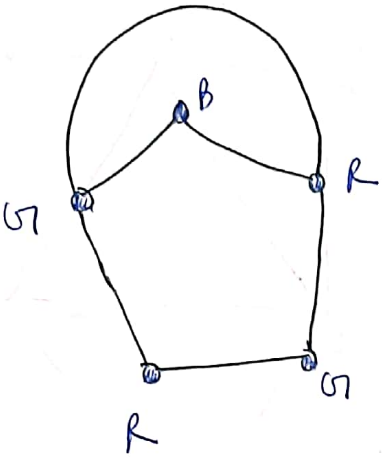
1. The chromatic no. of graph G is denoted by $\chi(G)$
   ↓
   (Kai)

2. if $\chi(G) = k$, the graph is called as K – chromatic

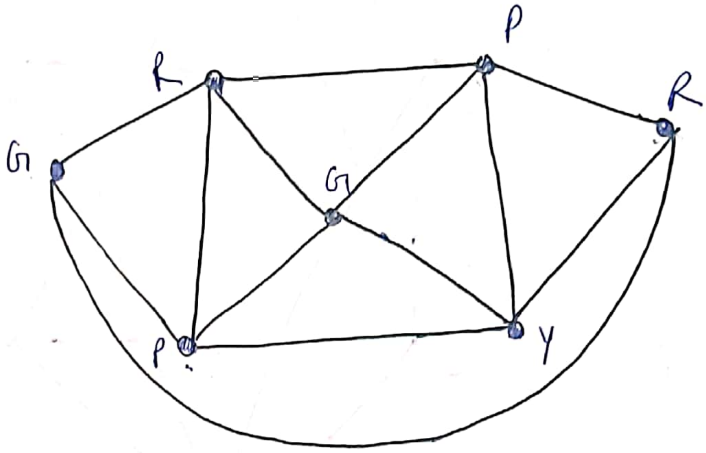
3. Chromatic no. of null graph (having only 1 vertex) is 1.

4. Chromatic no. of complete graph K of n vertices is n.

What are chromatic no's of the graph G and H as shown in figure.

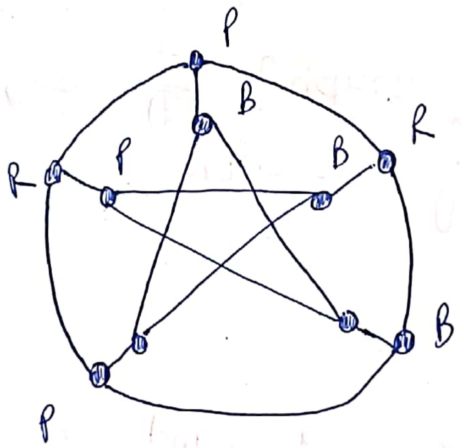


3- chromatic.



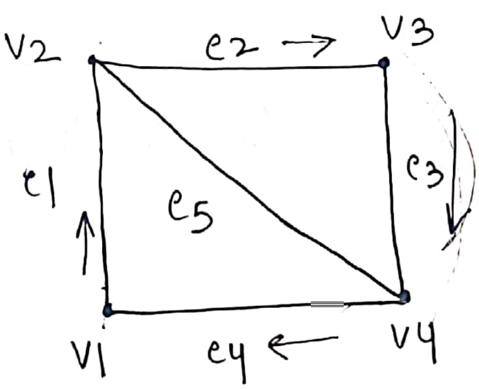
4- chromatic

Q:→ find the chromatic no of graph.



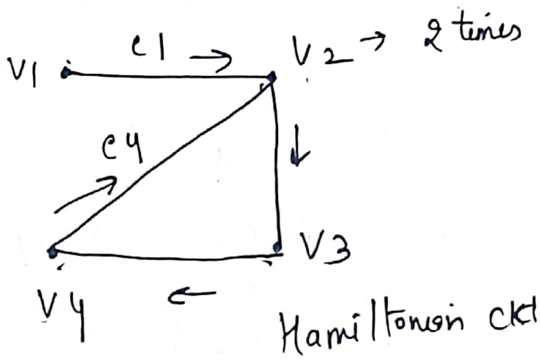
→ 3 chromatic.

Example of Hamiltonian Graph:→



→ Hamiltonian ckt - ✓

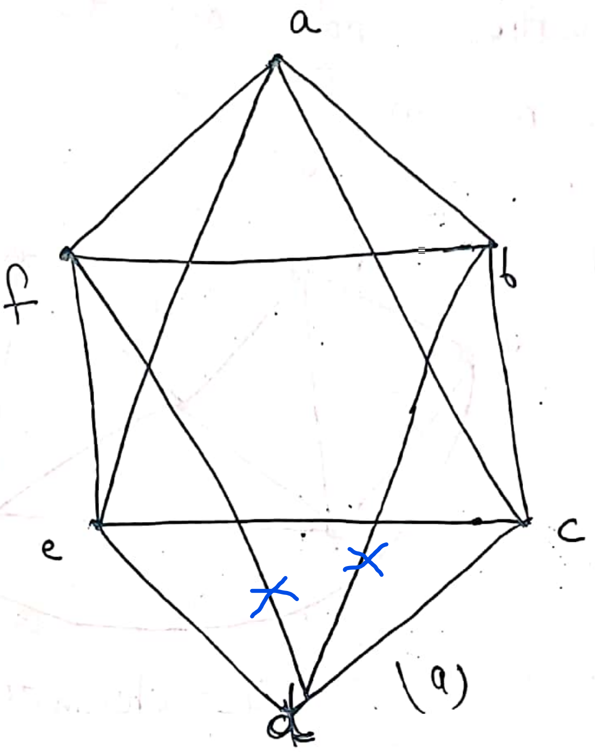
" Graph - ✓



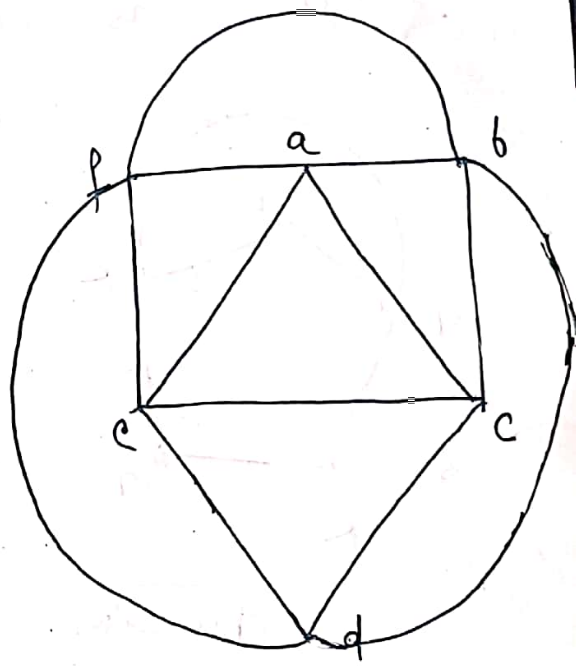
Hamiltonian ckt - X

" Graph - X

Q.-



(a)



Consider the graph as shown above:

(i) Is it a complete graph?

b) Is it planar graph? If so, find the no. of regions.

(i) Since the edge b/w a and d is not present in the given graph. It cannot be a complete graph.

(b) It can be re-drawn in which no 2 edges cross

$V=6 \qquad E=12$

$V-E+R=2$

$6-12+R=2 \qquad \boxed{R=8}$

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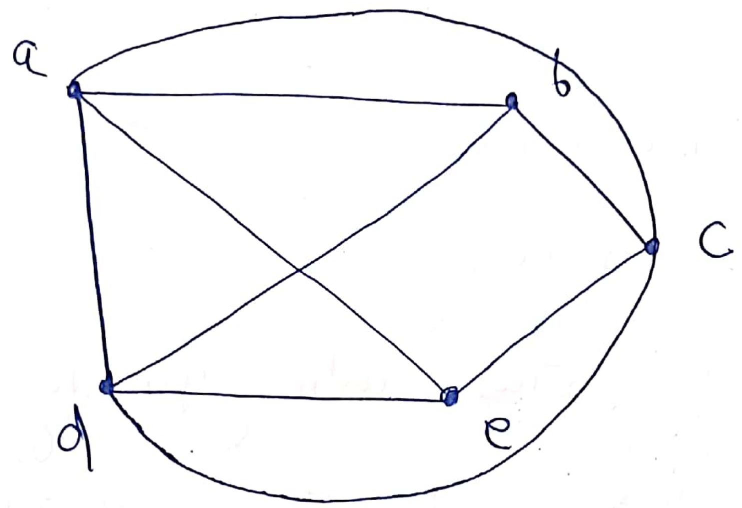
Non-planar graph

Example:-



→ **Walk and Paths in a Graph:→**

→ **Walk:→** is a finite alternating sequence $v_1 e_1 v_2 e_2 v_3 e_3 \ldots e_n v_n$ of vertices and edges, beginning and ending with same or different vertices.

→ **Length of a walk:→** The no. of edges is called length of walk.

→ **Closed and open walk:→** A walk is said to be closed if its origin and terminuous vertex $(v_0 = v_n)$ is equal, otherwise it is called as open walk.

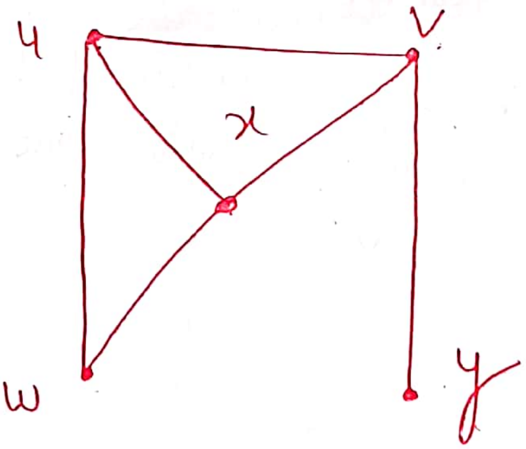
→ **Path:→** A walk is called Path if all vertices are not repeated.

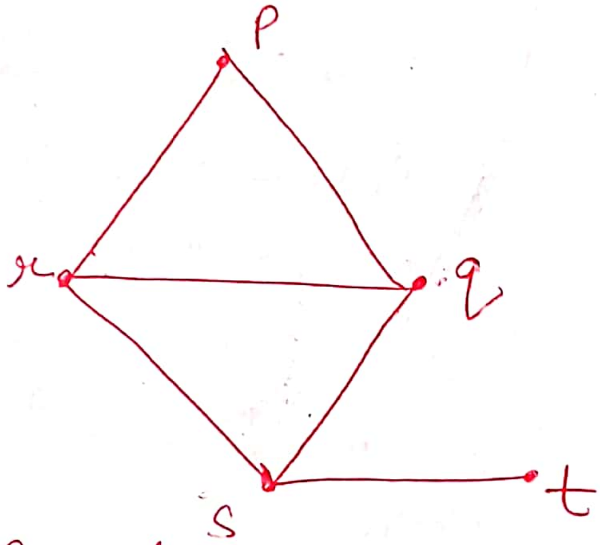
Isomorphism of Graph :-

Two Graphs G and G' are isomorphic if -

1. no. of vertices are same
2. no. of edges are same.
3. An equal no. of vertices with given degree.
4. Vertex correspondance and edge correspondance valid.

Example's





1. No. of vertices are same (5 vertices)
2. Edges are also same (6 edges)
3. An equal no. of vertices with given degree

u = 3	p = 2
v = 3	q = 3
w = 2	r = 3
x = 3	s = 3
y = 1	K = 1

(4) y — t

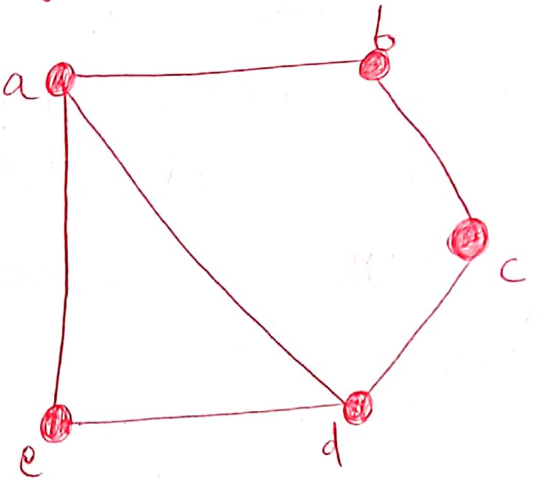
w — p

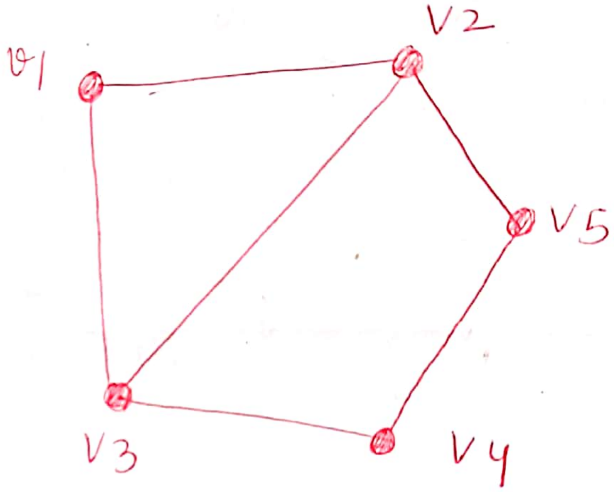
v — s ⇒ Both are isomorphic

u — r

x — q

Q:→ Show that the graphs shown in the diagram below are isomorphic.





Ans:→ (1) no of vertices = 5 (both cases)

(2) no. of edges = 6 edges

(3) degree →

a = 3 (no. of edges)

b = 2

c = 2

d = 3

e = 3

$v_1 = 2$

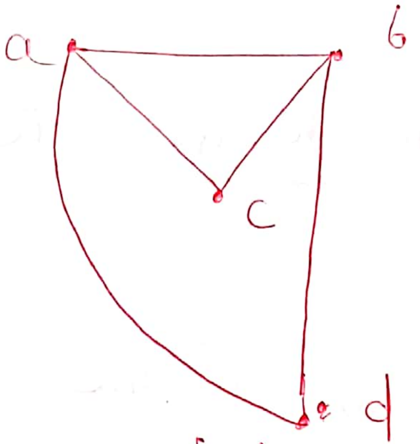
$v_2 = 3$

$v_5 = 2$ ⇒ Both are isomorphic

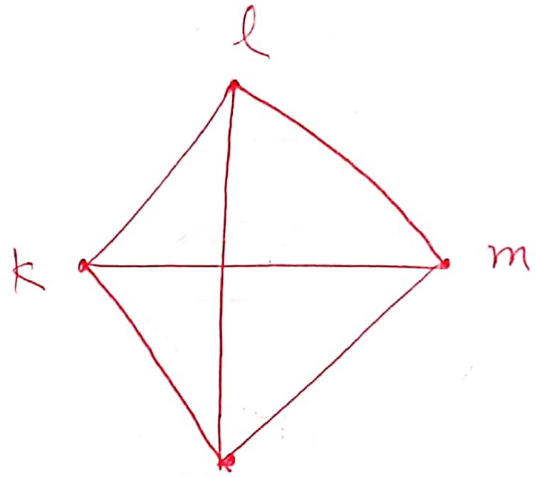
$v_4 = 2$

$v_3 = 3$

Q:→ Show that the graphs are not isomorphic



(a)



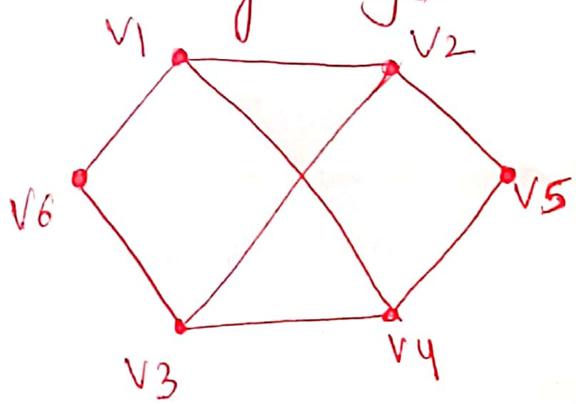
(b)

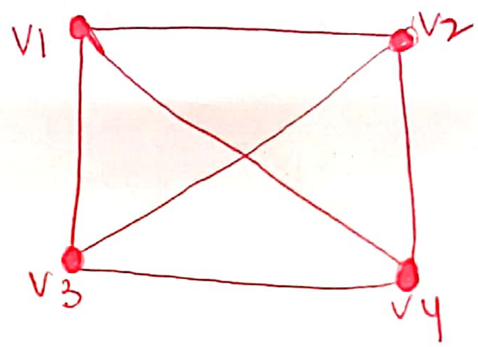
Ans:→ The graphs are not isomorphic because the vertices of the graph is having degree 3 but the graph (a) contains 2 vertices having degree less than three.

Homomorphism of Graphs

→ Two graphs G_1 and G_2 are called homomorphic graphs if G_2 can be obtained from G_1 by a sequence of subdivisions of edges of G_1. In other words, we can introduce vertices of degree 2 in any edge.

Q:→





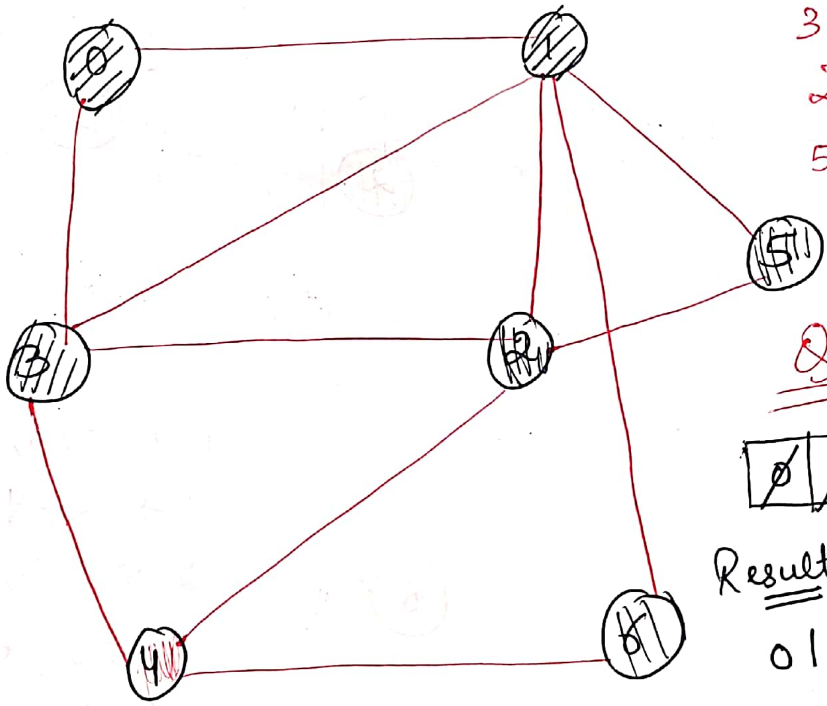
→ The 2 graphs are homomorphic because G_1 can be obtained from G_2 by introducing vertices of degree 2 on edges (V_1, V_3) and (V_2, V_4).

Graph Traversals

BFS (level order) Breadth-first Search

DFS – Depth first search

① BFS



Adjacent nodes

0 → 1, 3

1 → ~~0~~, 3, 2, ~~4~~ 5, 6

3 → 1, 0, 4, 2

2 → 1, 5, 4, 3

5 → 1, 2,

6 → 1, 4

Queue :→

~~0~~	~~1~~	~~3~~	~~2~~	~~5~~	~~6~~	~~4~~

Result :→

0 1 3 2 5 6 4

Take any node and start traversing.

→ For BFS, queue data structure is used.

→ it works in FIFO manner.

→ Take 0 as root node, insert 0 in queue, delete 0 and print 0

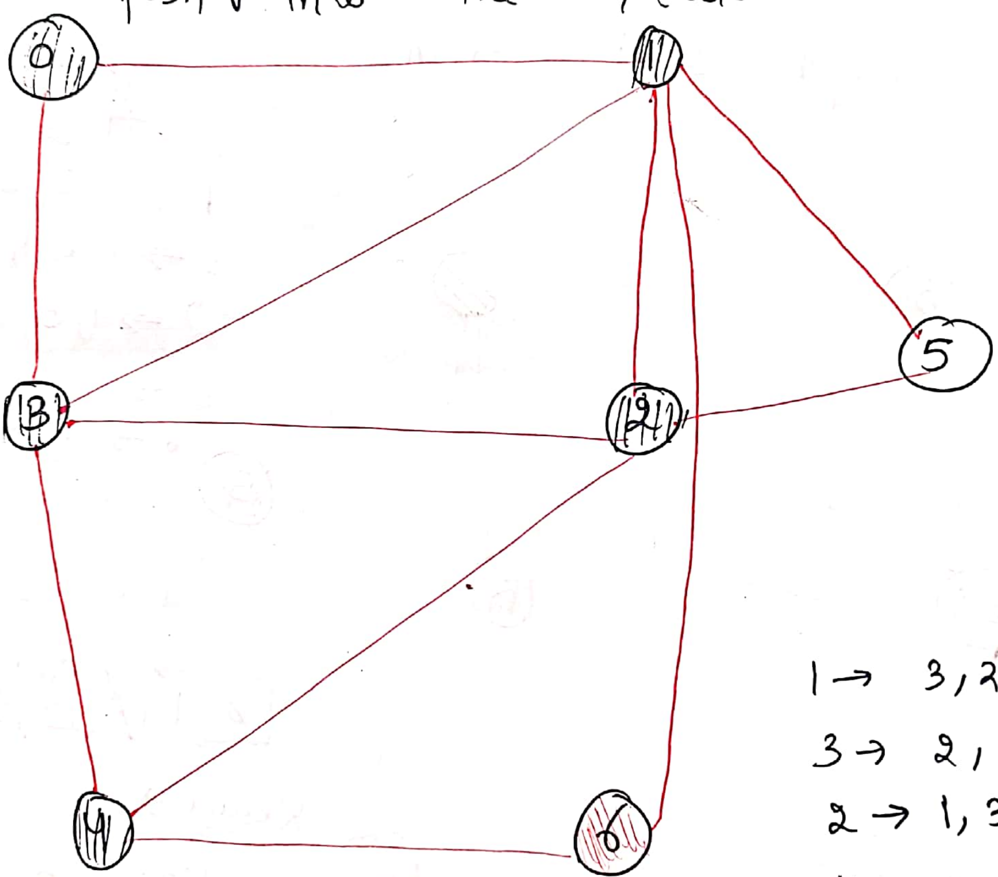
② DFS

→ Stack is used.

→ it works in LIFO manner.

→ we will go deeper till we reach the dead end. and we have to backtrack.

→ Choose any one adjacent vertice and push into the stack.



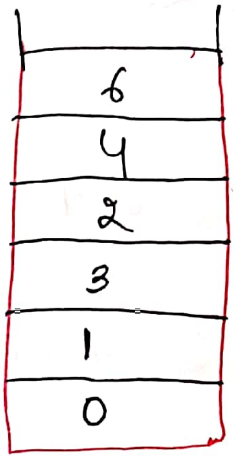
1 → 3, 2, 5, 6

3 → 2, 4

2 → 1, 3, 5, 4

4 → 2, 6

6 → 1, 4



Result :→

0, 1, 3, 2, 4, 6, 5

we cannot move further from 6.

→ Now we will start backtracking

→ 6 will be popped from the stack.

→ Now, go to 4 but 4 is not having any unvisited vertice. It will be popped out.

→ Go to 2, it has 5 vertex as unvisited.

→ 5 will be pushed into the stack.

→ 5 is also not having any unvisited vertex. Delete 5.

→ Backtrack from 5 to 2, then, 2 will be deleted from stack.

→ Pop out 3, 1 and 0 afterwards

→ Now stack is empty and this is the indication that we have to stop.