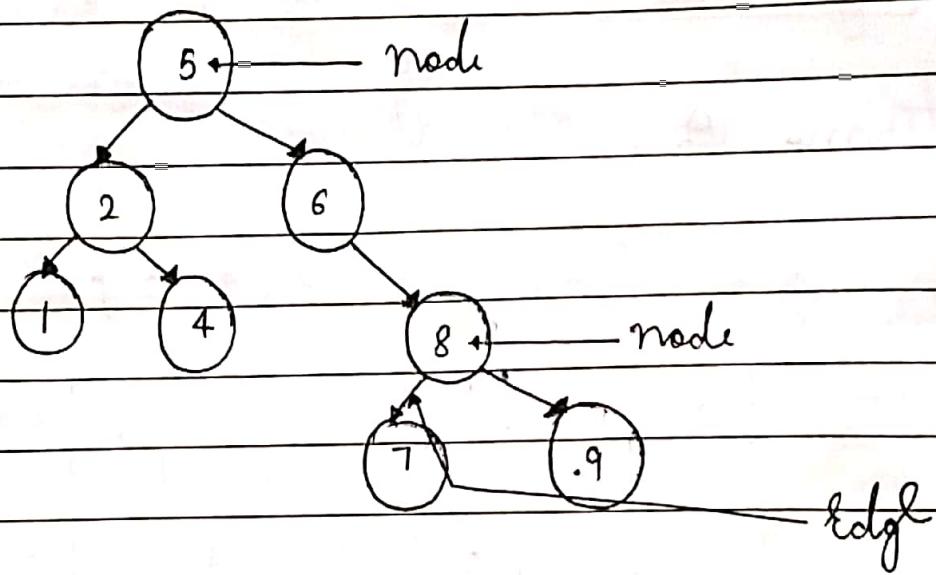


UNIT - 3

CHAPTER 3.1

TREE =

→ Tree Is A discrete structure that represents hierarchical relationships between Individual Elements or nodes.



SIBLING =

→ A node with same parent.

ANCESTOR =

→ Parent node of a child node Is termed its ancestor.

DESCENDENT =

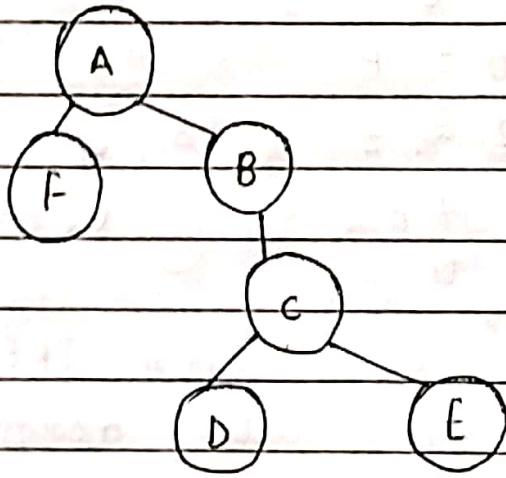
→ All the hierarchical children of parent node are called as the descendants of the parent node.

LEAF =

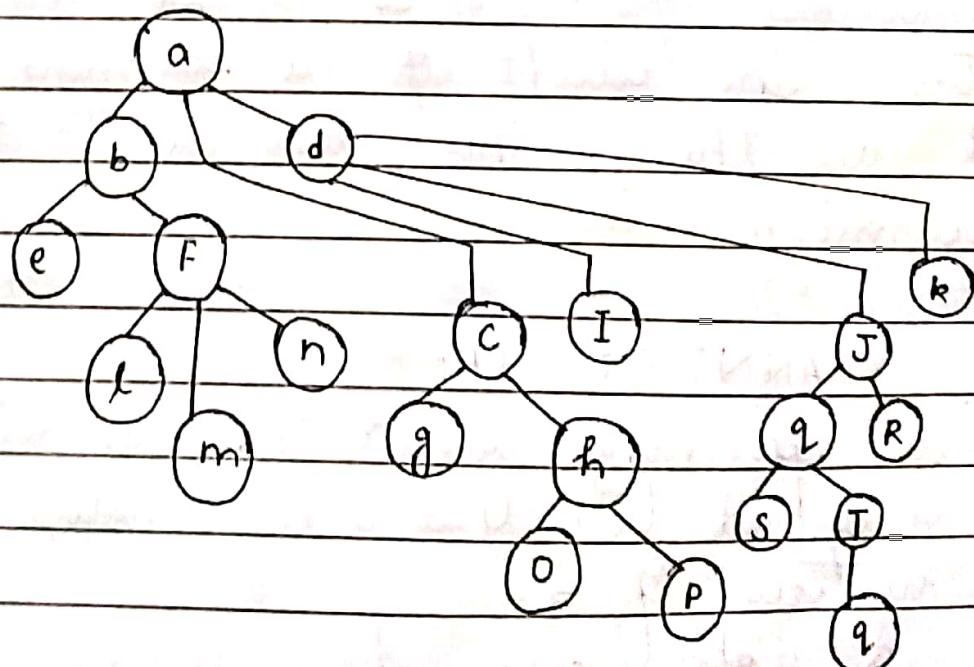
→ A node with no children is termed as leaf node.

INTERNAL NODES =

→ Any node which is not a leaf that is termed as Internal nodes.



Here, A, F, B, C are the Internal nodes.



i) Which vertex is root?

→ a vertex is the root.

ii) Which vertices / vertex are Internal nodes?

→ b, c, d, F, g, h, J, q, S, t are Internal nodes.

iii) Leaf Nodes = e, l, m, n, j, o, p, s, q, r

iv) Children of J = q, r

v) Parent of H = c

vi) Siblings of o = p

vii) Ancestors of m = F, b, a

viii) Descends of B = e, F, l, m, n

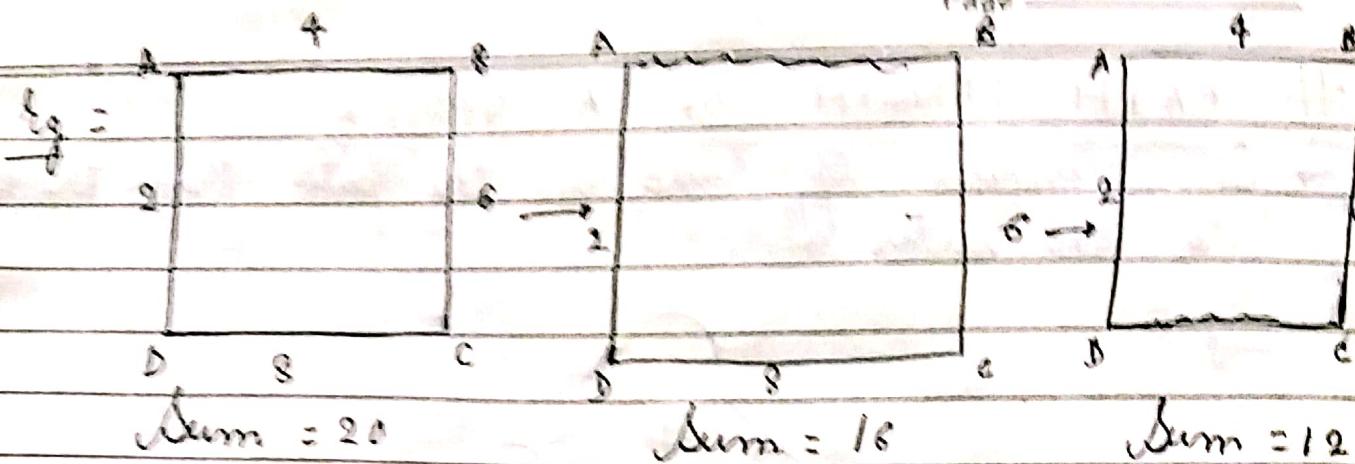
MINIMUM SPANNING TREE =

→ A spanning tree with assigned weight less than or equal to the weight of every possible spanning tree of a weighted, connected and undirected graph G is called minimum spanning tree. The weight of a spanning tree is the sum of all the weights assigned to each edge of spanning tree.

SPANNING TREE =

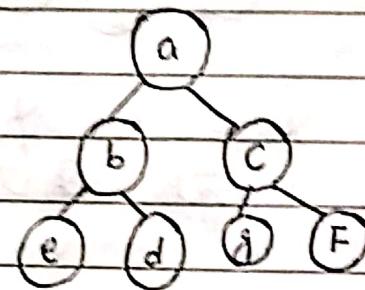
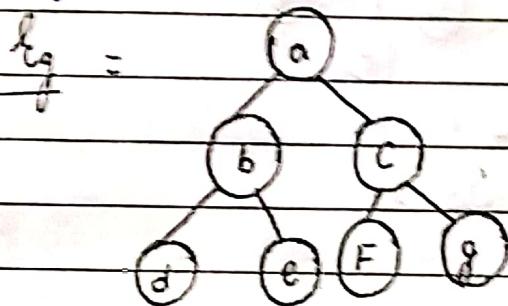
→ A spanning tree of the connected undirected graph G is a tree that minimally includes all of the vertices of G.

→ A graph may have many spanning tree.



ORDERED TREES =

→ If In a tree at each level an ordering is defined then such a tree is called an ordered tree.



ROOTED TREES =

→ If a directed tree has exactly one node or vertex called root whose incoming degree is zero and all other vertices having incoming degree one then the tree is called rooted tree.

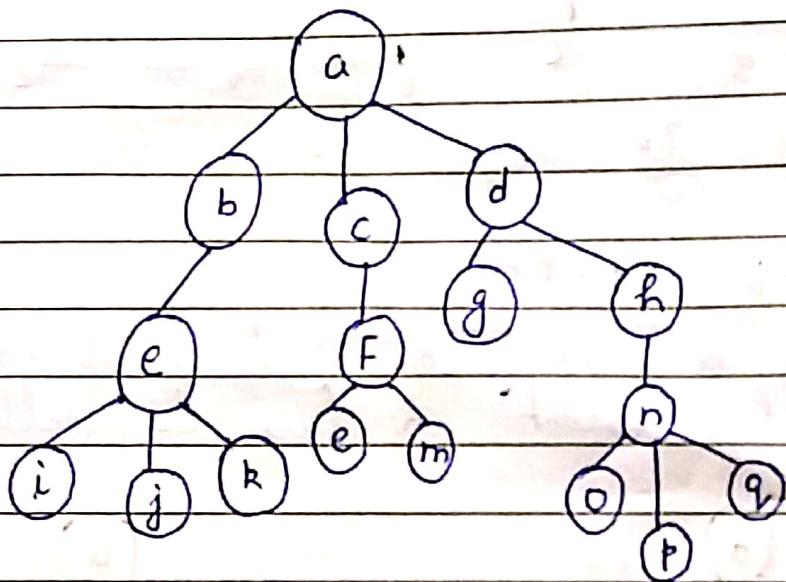
→ A tree with no node is also a rooted tree (empty tree).

→ A single node with no children is called rooted tree.

PATH LENGTH OF A VERTEX =

→ The number of edges in the path from the root to the vertex.

$$\text{Eq} =$$



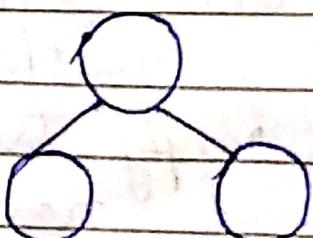
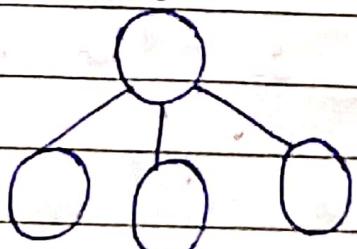
i) Path length of node I is 1.

ii) Path length of node F is 2.

FOREST =

→ Let A be the root And the corresponding edges connecting the node are deleted from a tree we obtain a set of disjoint trees. This set of disjoint trees is called a forest.

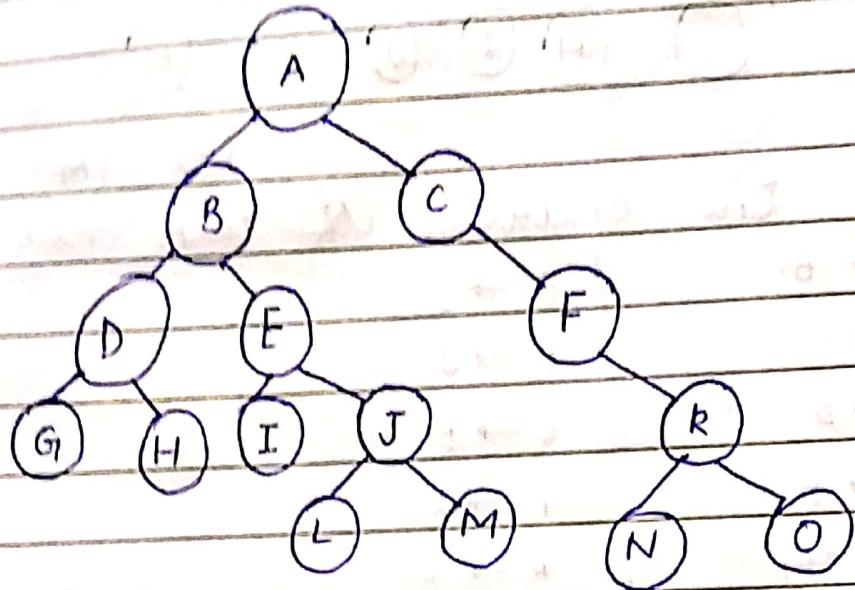
$$\text{Eq} =$$



FOREST

BINARY TREE

→ If the Outdegree = degree of binary node Is less than Or equal to two then the tree Is called binary tree.



i) Which Is the root = A

ii) Which node Are leaf = G, H, I, L, M, N, O

iii) Name parent node of each node =

B → A

I → F

C → A

J → F

D → B

L → I

F → B

M → J

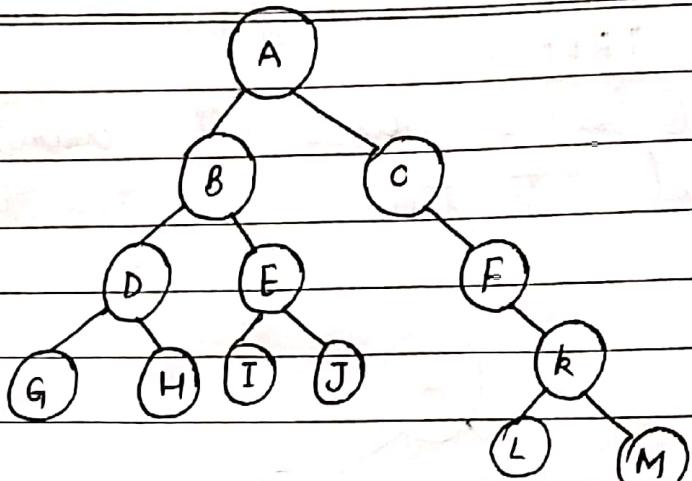
G → D

K → F

H → D

N → K

O → K



i) List the children of each node

$A \rightarrow B$	$E \rightarrow I$
$A \rightarrow C$	$E \rightarrow J$
$B \rightarrow D$	$C \rightarrow F$
$B \rightarrow E$	$F \rightarrow K$
$D \rightarrow G$	$K \rightarrow L$
$D \rightarrow H$	$K \rightarrow M$

ii) List the sibling

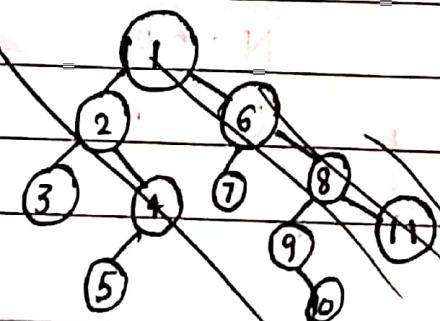
$\rightarrow (B, C), (D, E), (G, H), (I, J), (L, M)$

iii) Find depth of each node

$A \rightarrow 0$	$D \rightarrow 2$	$G \rightarrow 3$	$J \rightarrow 3$	$M \rightarrow 4$
$B \rightarrow 1$	$E \rightarrow 2$	$H \rightarrow 3$	$K \rightarrow 3$	
$C \rightarrow 1$	$F \rightarrow 2$	$I \rightarrow 3$	$L \rightarrow 4$	

Max. depth = 4

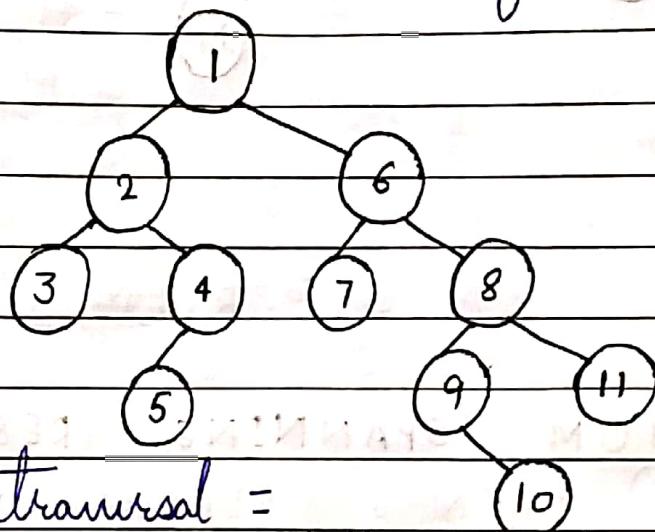
Max level = 4



TRAVERSING IN TREE =

→ Traversing means to visit all the nodes of the tree.

- i) Pre-Order traversal = Root → Left → Right
- ii) Post-Order traversal = Left → Right → Root
- iii) In-Order traversal = Left → Root → Right

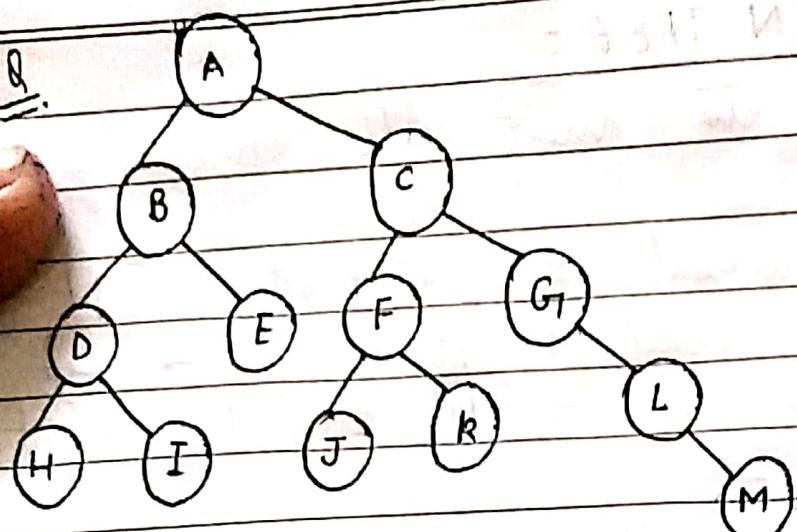


Pre-Order traversal =
Root → Left → Right

⇒ 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Post-Order traversal = Left → Right → Root
⇒ 3, 5, 4, 2, 7, 10, 9, 11, 8, 6, 1,

In-Order traversal = Left → Root → Right
⇒ 3, 2, 5, 4, 1, 7, 6, 9, 10, 8, 11



Pre-Order =

R - L - R

A, B, D, H, I, E, C, F, J, K,
G, L, M

Post Order =

L - R - R

H, I, D, E, B, J, K, F,
M, L, G, C, A

In-Order =

L - R - R

H, D, I, B, E, A, J, F, K, ~~M~~ C, G, L, M.

MINIMUM SPANNING TREES ALGORITHM

i) PRIM'S ALGORITHM

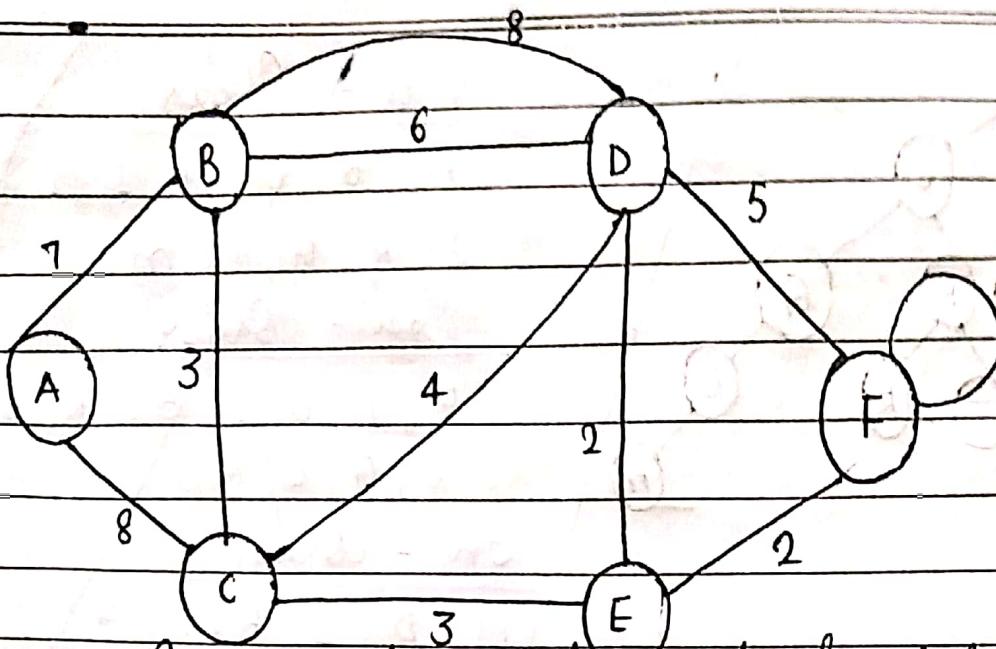
ii) KRUSKAL'S ALGORITHM

- Note :-
-) In MST $v = v'$ and $E' \subset E$
 -) Normal $G(v, E)$, MST $G'(v', E')$
 -) $|E'| = |V| - 1$

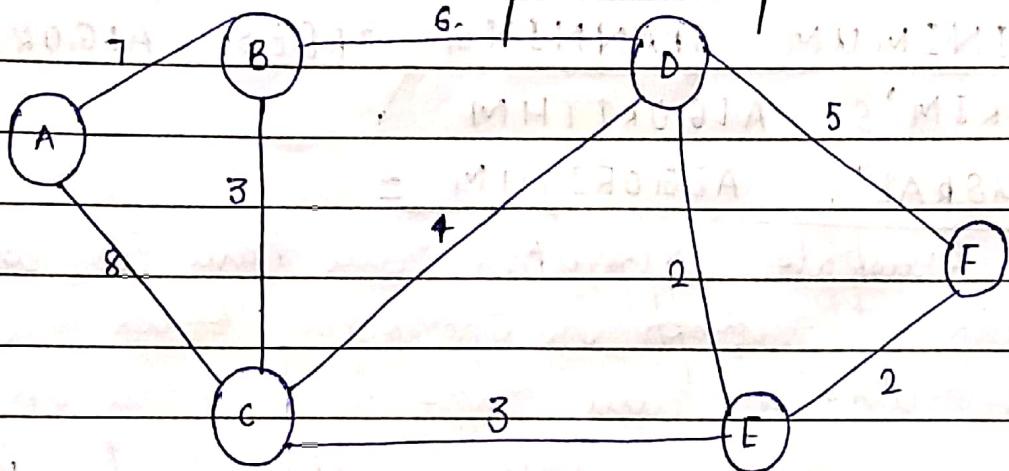
* PRIM'S ALGORITHM =

→ Prim's Algorithm is a greedy algorithm that is used to find the MST from a graph.

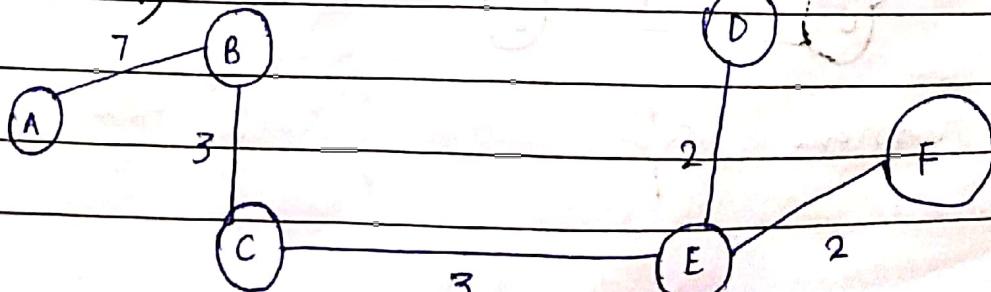
→ It starts with a single node and explores all the adjacent nodes with all the connected edges at every step.



Step 1 = Remove the loop and parallel edges.



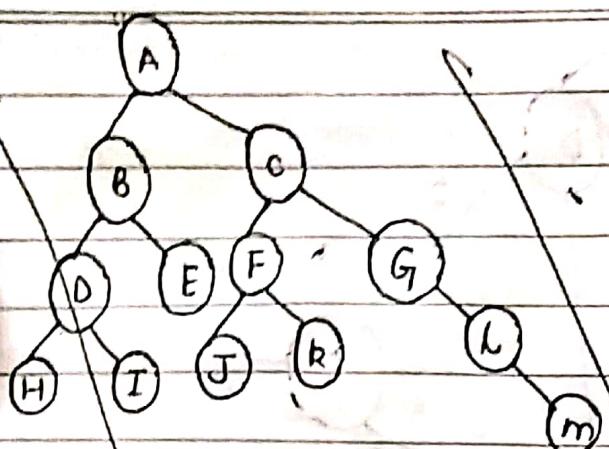
Step 2 = Choose any vertex as the root node and chuck all the outgoing edges from this node. (By comparing all the previous weighted edges, the edge which has minimum weight without making loop will be considered.)



$$G'(v') = 6$$

$$G'(E') = 5$$

$$\therefore G'(v') - G'(E') = G'(E')$$



Pre - Order

A, B, D, H, I, F, C, F,
J, K, G, L, m

Post - Order

H, I, D, E, B, J, K, F, m,
L, G, C, A

In - Order

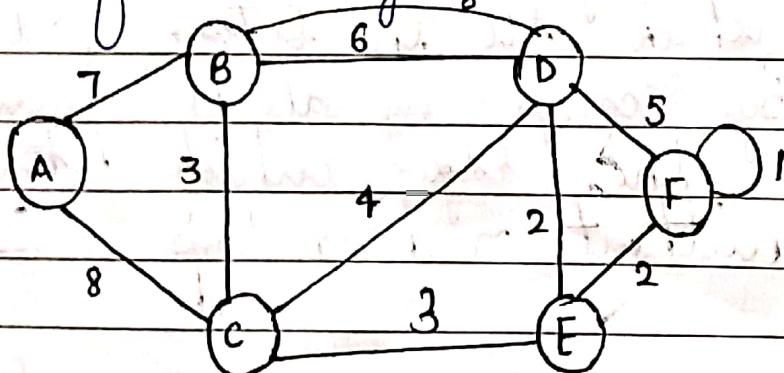
H, D, I, B, E, A, J, F, K, E,
C, G, L, m

* MINIMUM SPANNING TREES ALGORITHM =

(i) PRIM'S ALGORITHM

* KRUSKAL'S ALGORITHM =

→ In kruskals algorithm we have to sort all the edges of the graph in increasing order. Then it keeps on adding new edges and nodes in the minimum spanning tree if the newly added edge does not form a cycle.



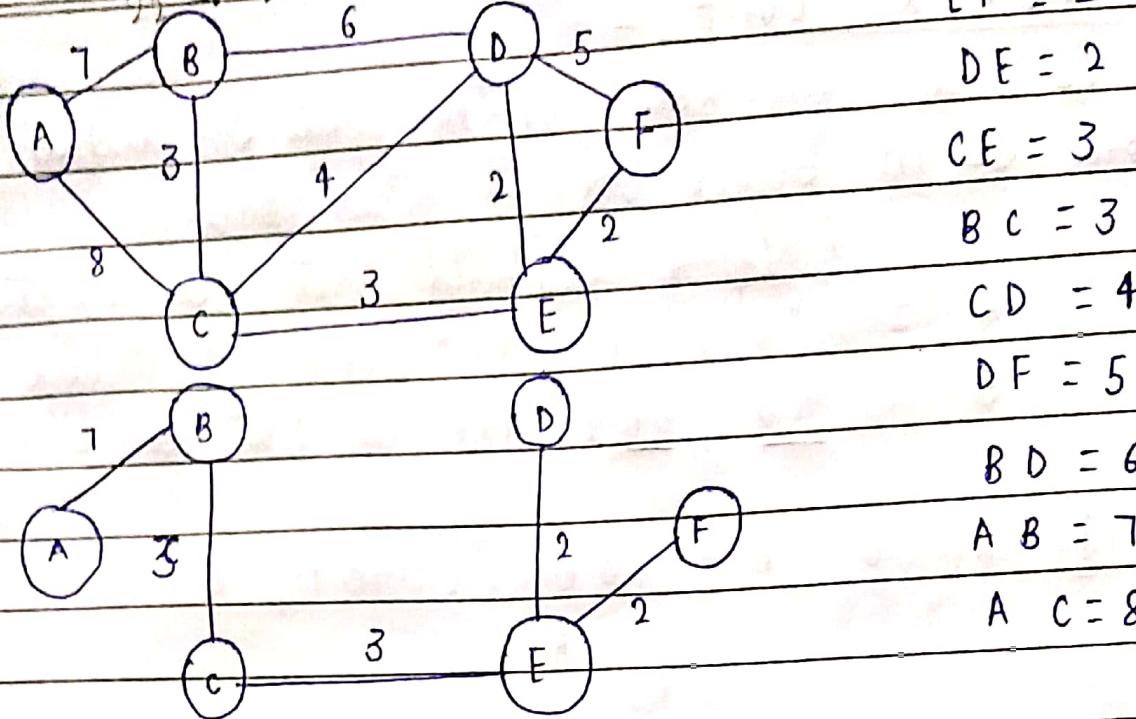
Tip' = Remove loops and parallel edges

18 → IOT → 806 → 7.00 → FREEMIKE

18 → Soft Skills → 110 → 3 Date 4/20

19 → CC → 303 → 11.15 → 12.45
20 → MAD → 309A → 3.00 → 4.00 Page 00

20 → DM → 303 → 4.20 → 11.00



$$E' = 5$$

$$V' = 6$$

$$\therefore G(E') = G(V') - 1$$

PROPERTIES OF MST (Minimum Spanning Tree)

- The disconnected graph does not have spanning tree.
- A complete undirected graph can have n^{n-2} number of spanning trees where n is the number of vertices.
- Adding one edge to spanning tree will create a loop.
- The spanning tree must not form a cycle i.e; no edge is traversed twice.
- If each edge has a distinct weight then there will be only a unique minimum spanning tree.

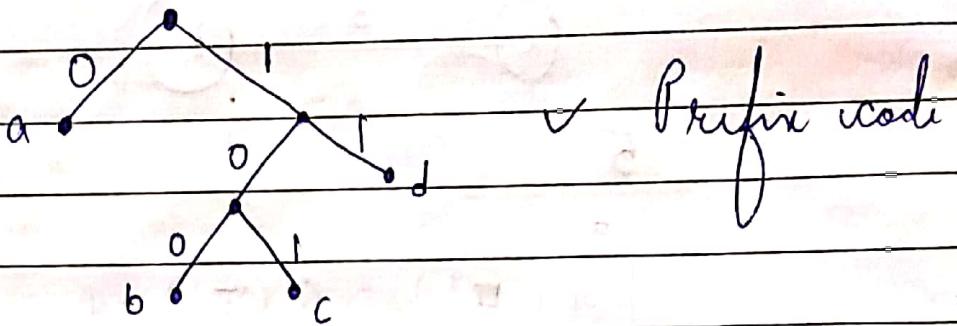
* PREFIX CODE =

→ It is a variable length code in which no code word is a prefix of another one.

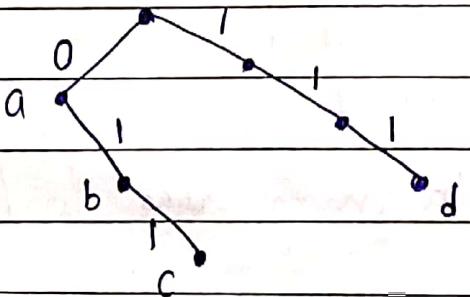
Should not be repeated

A = 00, 11, 110, 111 ✓ Prefix code
 B = 00, 001, 111 ✗ Not a Prefix code

Q. i) a = 0, b = 100, c = 101, d = 11



ii) a = 0, b = 01, c = 011, d = 1111, 11011



(because a, b and c lies on same trajectory.)

HUFFMANN CODING =

- It is a technique of compressing the data to reduce its size without losing any of the detail.
- It was first developed by David Huffman.
- It is useful to compress the data in which there

Are "frequently occurring characters"

Note: It is basically used to compress long bits
to short bits.

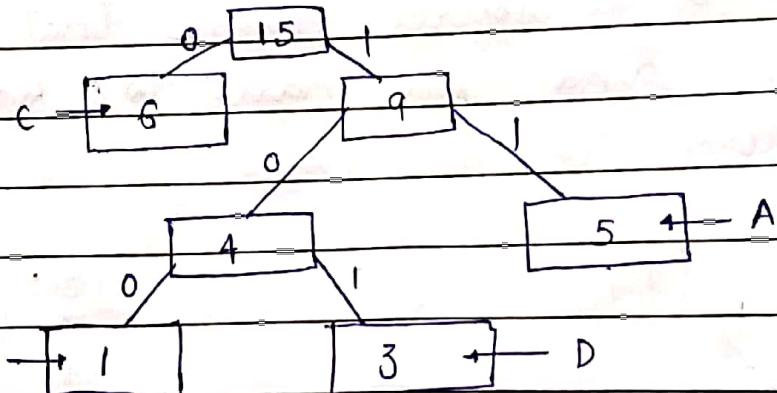
B	C	A	A	D	D	C	C	A	C	A	C
Char	freq	code									

size = frequency count *
code length

A	5	1111	$5 \times 2 = 10$
B	1	100	$1 \times 3 = 3$
C	6	0	$6 \times 1 = 6$
D	3	101	$3 \times 3 = 9$
			28 bits

$$4 \times 8 = 32 \text{ bits}$$

$$15 \text{ bits}$$



Q. How many bits are required for encoding the message 'mississippi'? Below is the frequency count.

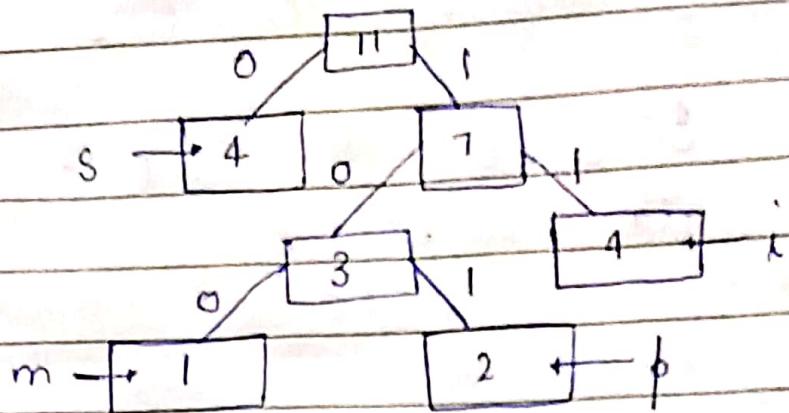
Char	freq	Code	Size
m	1	100	$1 \times 3 = 3$
i	4	11	$4 \times 2 = 8$
s	4	0	$4 \times 1 = 4$
p	2	101	$2 \times 3 = 6$
			32 bits

15 bits

8 bits

10 bits

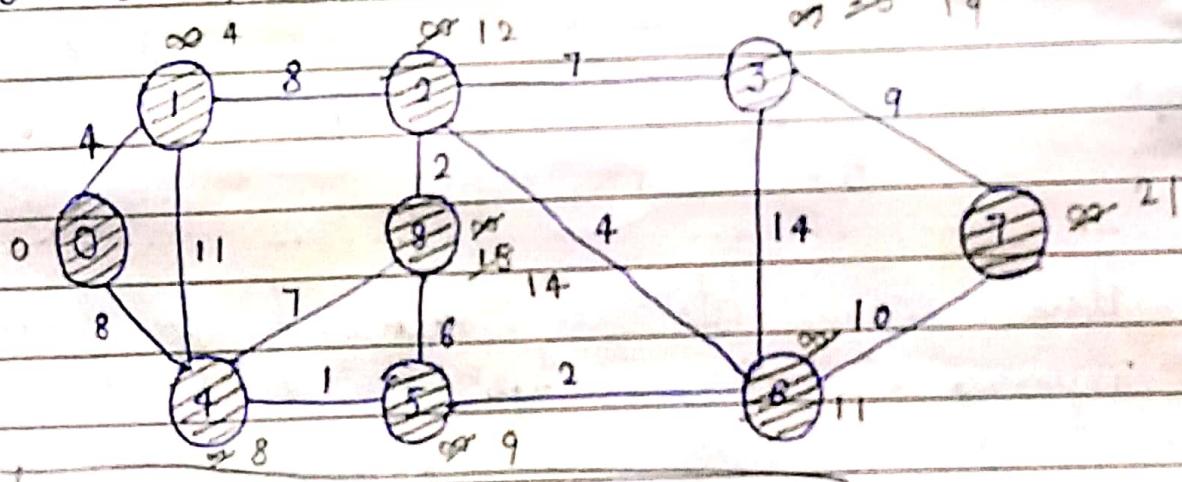
11 bits



#. SHORTEST DISTANCE IN WEIGHTED GRAPHS

i) DIJKSTRA's ALGORITHM (Single source shortest path)

→ It is a single source shortest path algorithm.
Here, single source means that Only One source is given And we have to find the shortest path from source to all nodes.



Let, $u = 0, v = 1$

$\therefore d[0] = \infty, c[0, 1] = 4, d[1] = \infty$

11/11/21

0 1 1 < 01

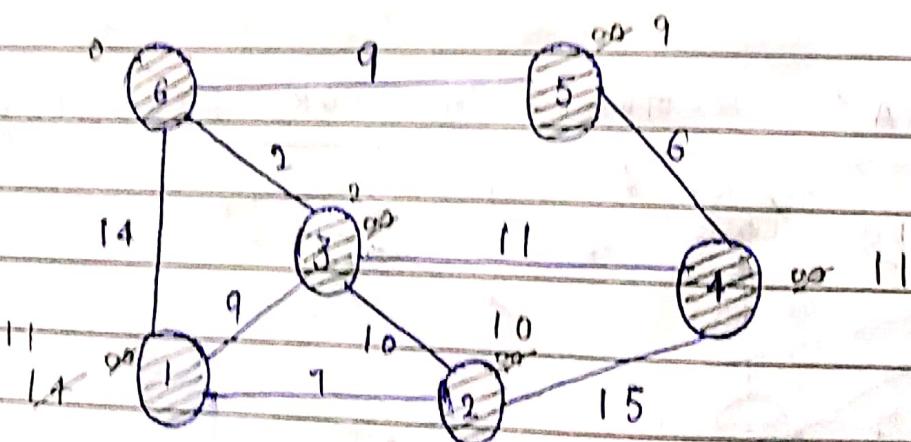
{ 1 (01) }

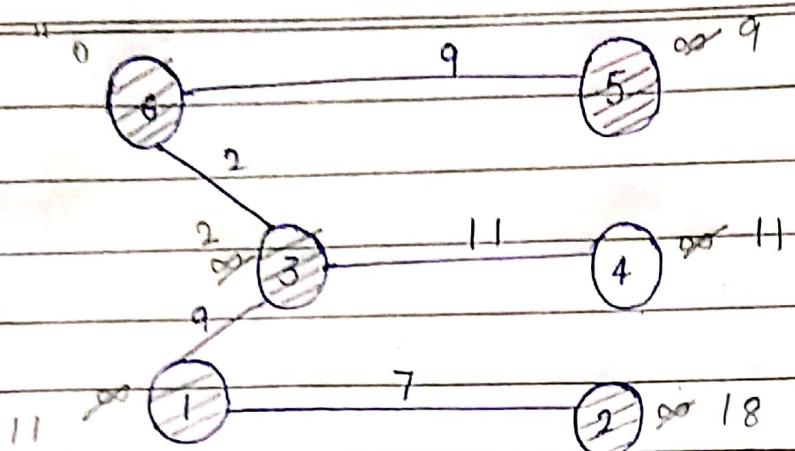
{ d [V] = 4 }

{ 111 = 4 }

11/11 for today. Visit and make them visit.

	0	1	2	3	4	5	6	7 8
0	0	4	19	19	8	9	11	21 14
1	00	0	8	15	00	00	00	00 00
2	00	00	0	7	00	00	00	00 2
3	00	00	00	0	00	00	00	00 00
4	00	00	00	00	0	01	3	13 11
5	00	00	00	00	00	0	2	12 6
6	00	00	00	00	00	00	00	10 00
7	00	00	00	00	00	00	00	0 00
8	00	00	00	00	00	00	00	00 0

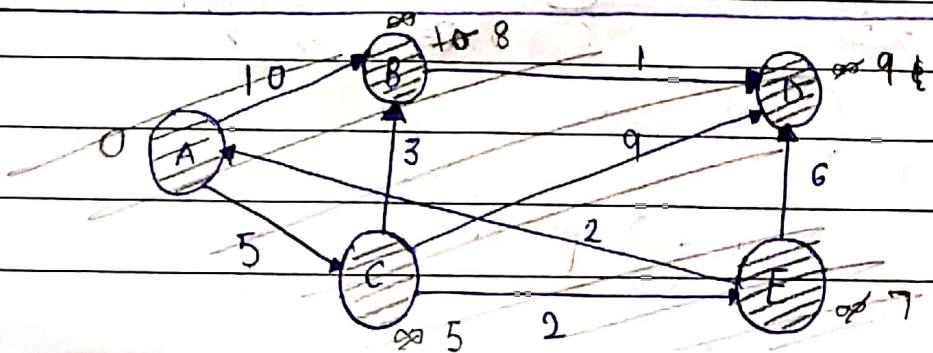


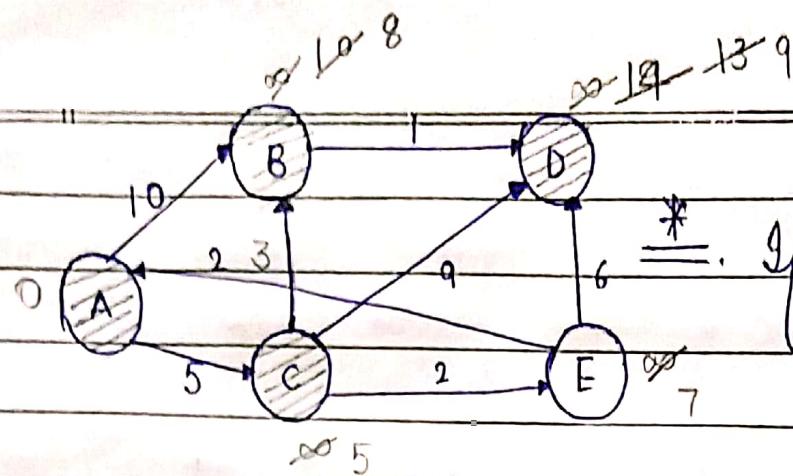


#	Source	1	2	3	4	5
0	6	∞	∞	∞	∞	∞
1	0	∞	∞	2	∞	9
2	0	11	∞	2	11	9
3	0	11	∞	2	11	9
4	0	11	18	2	11	9
5	0	11	18	2	11	9

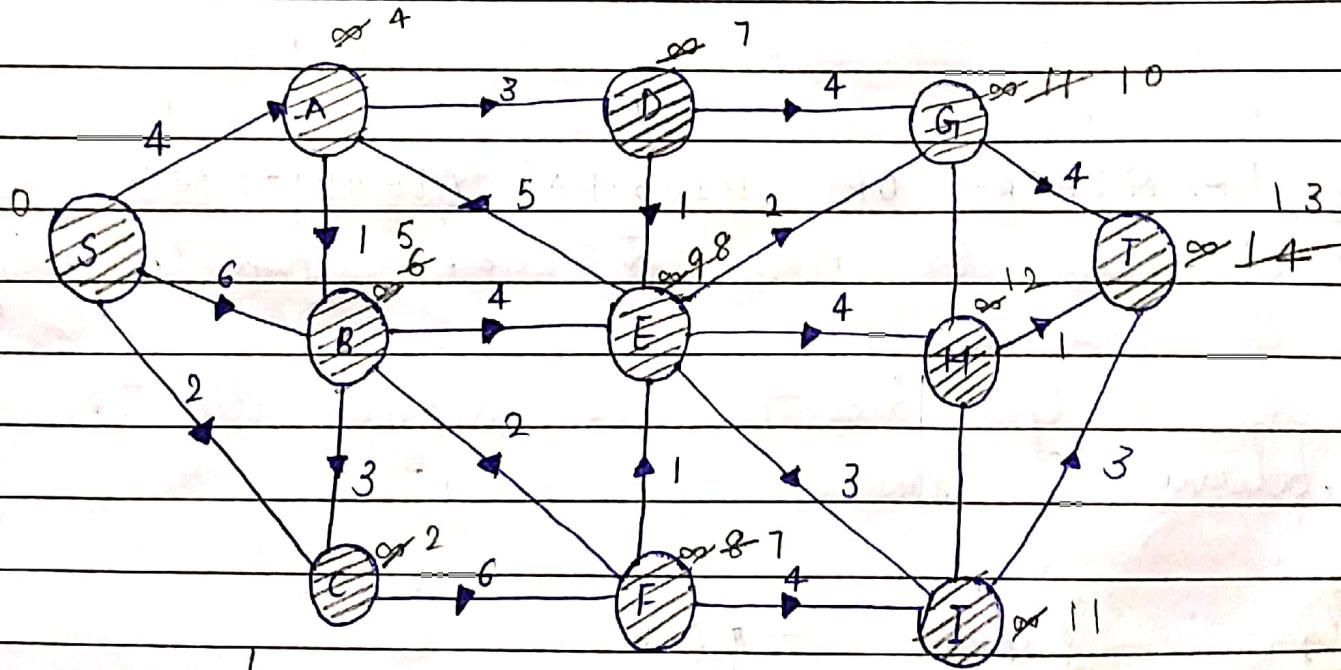
Note = a node which is already used as source for some other node cannot be updated.

DIJKSTRA ALGORITHM FOR DIRECTED GRAPH





	A	B	C	D	E
A	0	∞	∞	∞	∞
C		10	5	∞	∞
E		8		14	7
B		8			13
D				9	



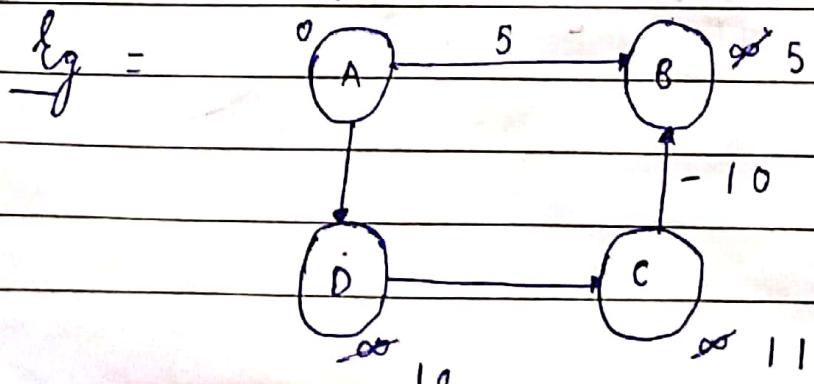
	S	A	B	C	D	E	F	G	H	I	T
S	0	∞									
C		4	6	2	∞						
A		4	6	1	∞	8	∞	∞	∞	∞	∞

	S	A	B	C	D	E	F	G	H	I	T
D					7	9	7	∞	∞	∞	∞
F					8	7	11	∞	∞	∞	
E					8		11	∞	11	∞	
G							10	12	11	∞	
I								12	11	14	
H								12			14
T											13

$S \rightarrow C \rightarrow A \rightarrow B \rightarrow D \rightarrow F \rightarrow E \rightarrow G \rightarrow I \rightarrow H \rightarrow T$

* DRAWBACK OF DIJSTRA'S ALGORITHM:

- It may or may not work when the weight of edges are negative.
- As, this algorithm is designed on the basis of positive (+ve) weight.



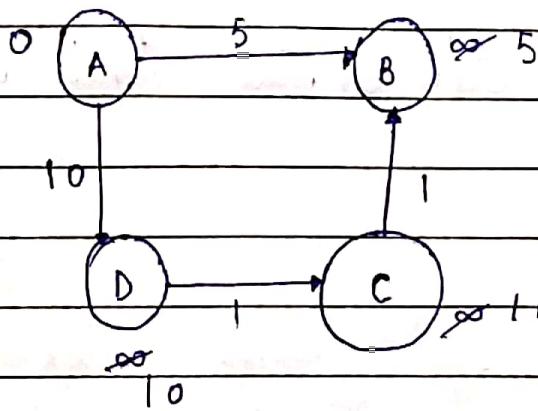
→ Suppose we want to update the distance from vertex $c(u)$ to vertex $B(v) = 11 - 10 = 1$

→ But we cannot update it because this node is already visited.

So, In this case Dijkstra algorithm has given wrong distance.

This is not always true.

→ Suppose If we change the value as -1 from vertex $c(u)$ to vertex $B(v) = 11 - 1 = 10$



GRAPH

→ The graph consists of points or nodes called vertices which are connected to each other by way of lines called edges.

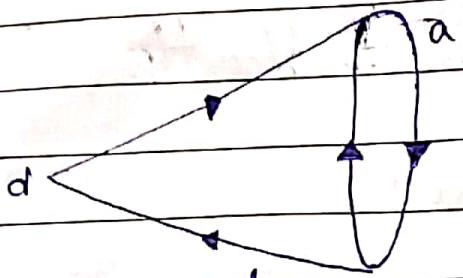
→ These lines may be directed or undirected.

i) DIRECTED GRAPH =

→ A directed graph is defined as an ordered pair (V, E) where V is a set and E is a binary

relation on V :-
The elements in V are called vertices.

The ordered pair in E are called edges.



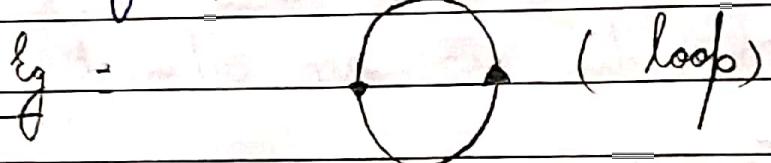
(directed graph)

→ here the vertices b are $(a, b), (b, a), (b, d), (d, a)$

→ The vertex a is called the Initial and vertex d is called the Terminal

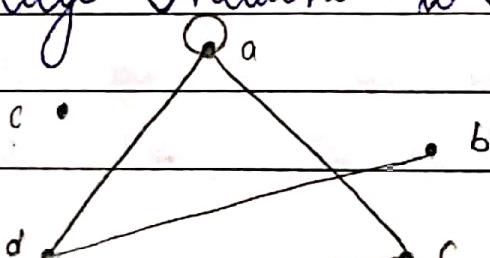
② LOOP IN GRAPH :-

→ An edge that is incident from same vertex and has reflection on same vertex is called loop or self-loop.



③ ISOLATED VERTEX :-

→ A vertex is said to be isolated vertex if there is no edge incident to it.



→ blue, vertex 'e' Is An Isolated vertex.

→ blue, a Is having self loop.

→ blue, b Is a Pendant vertex.

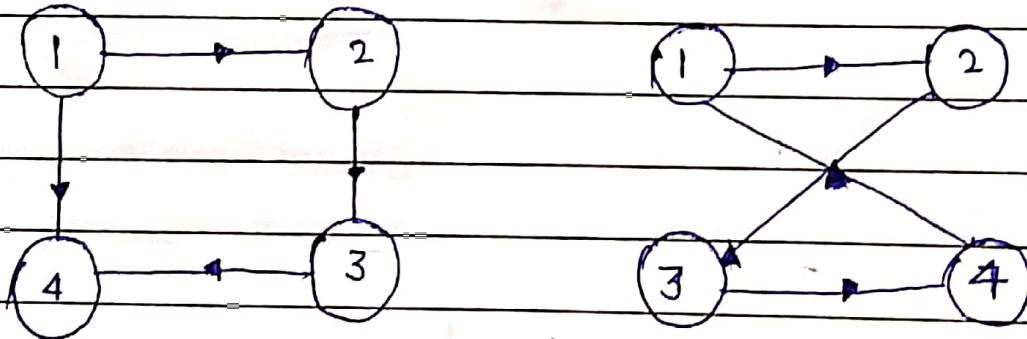
that has Only One Edge Incident to It.

④ UNDIRECTED GRAPH =

→ It consists of A set of vertices V and A set of edges E . The edge set contains the Unordered pairs of vertices.

$Ex = Let V = \{1, 2, 3, 4\}$ and $E = \{(1, 2), (1, 4), (3, 4), (2, 3)\}$

The graph can be drawn In various ways one of which Is



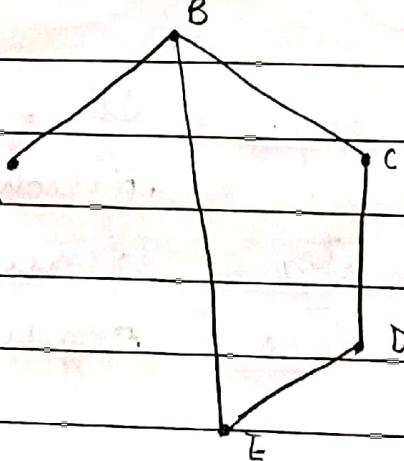
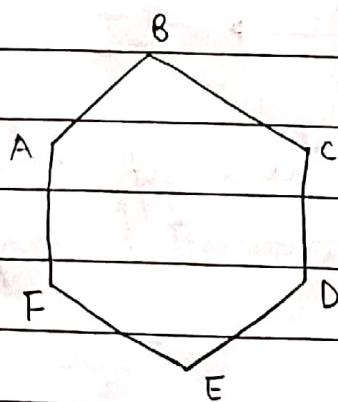
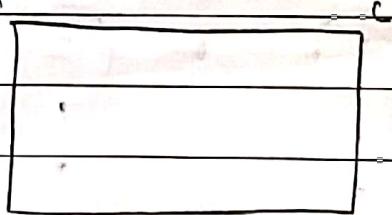
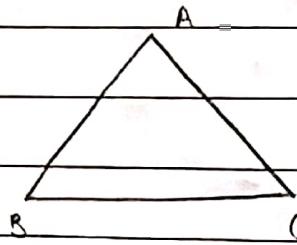
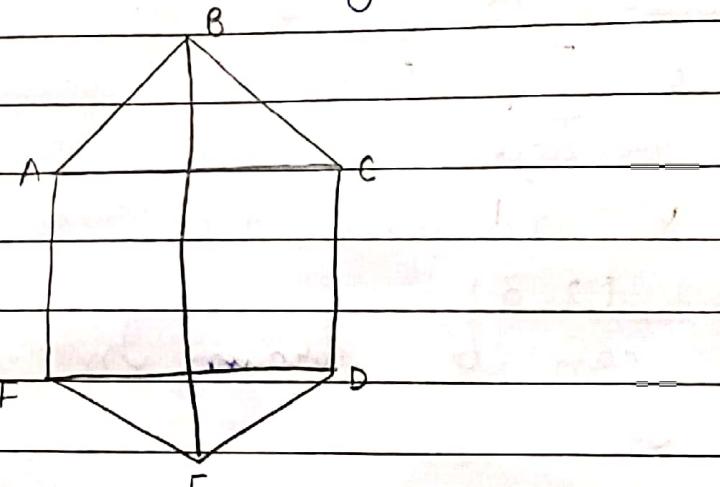
-) Mixed Graph = It Is the mixture of both directed and undirected graph.
-) Odd vertex = degree Odd
-) Even vertex = degree Even

SUBGRAPH =

→ A subgraph of a graph $G = (V, E)$ Is a graph

$G' = (V', E')$ In which $V' \subseteq V$ and $E' \subseteq E$ And each edge of G' and G has same end vertices. In G' as in graph G .

Q. Consider the graph G as shown below. Show the subgraphs in the given graph.

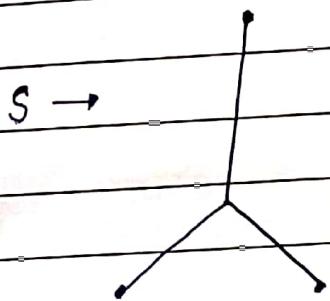
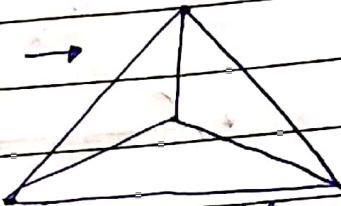


COMPLEMENT OF A SUBGRAPH :-

→ Let $G_1 = (V, E)$ be a graph and S be a subgraph of edges of S be deleted from graph G_1 , the graph so obtained is the complement of subgraph S . It is denoted by \bar{S} .

$$\therefore \bar{S} = G_1 - S$$

Q. Consider the graph G → and its subgraph

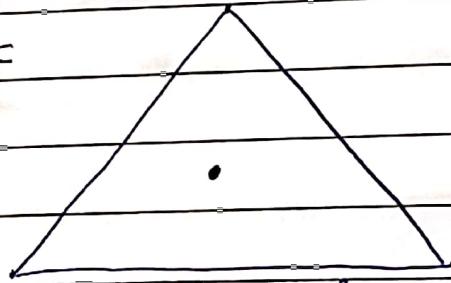


then create the complement of

subgraph S .

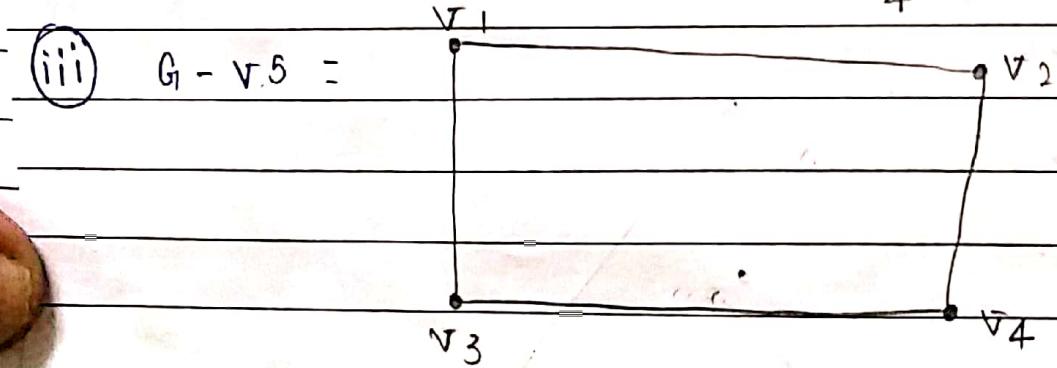
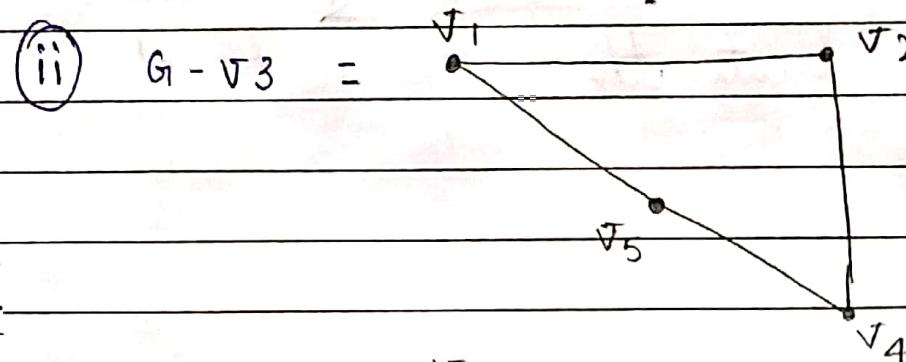
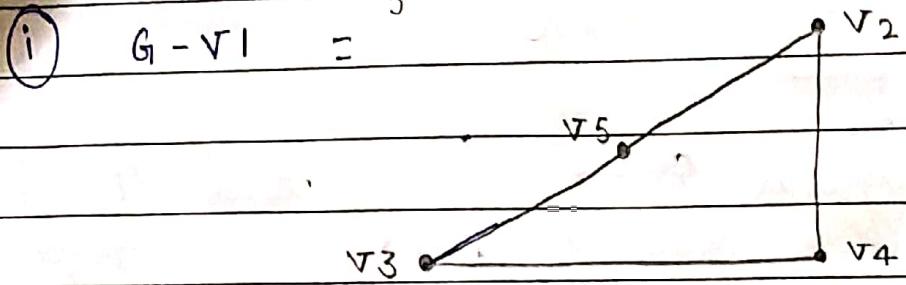
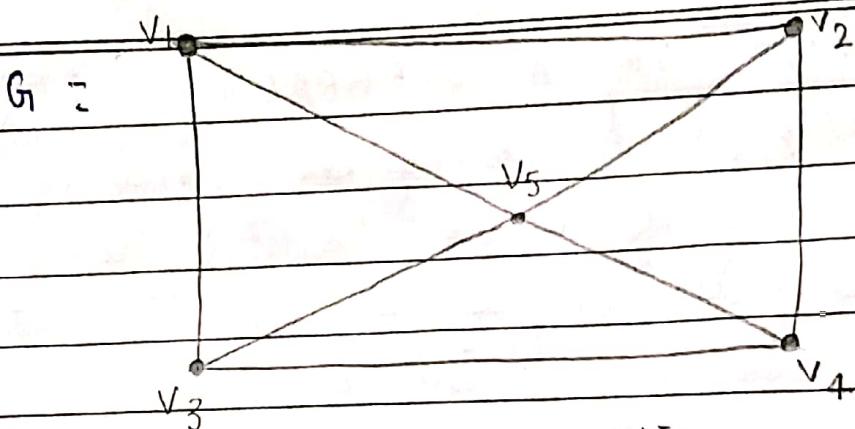
$$\bar{S} = G - S$$

$$\text{So, } \bar{S} =$$



Note : The number of vertices do not change in the complement of a subgraph.

Consider the graphs shown below. Determine the complement of a subgraph.



ORDER OF A GRAPH =

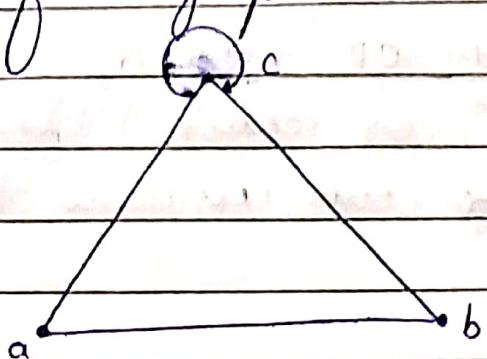
→ Let G be a graph then the number of the vertices in the graph G is called Order of a graph.

SIZE OF A GRAPH =

→ Let G be a graph then the number of edges in a graph is called the size of a graph.

Q. Consider the graph as shown below. Determine the =

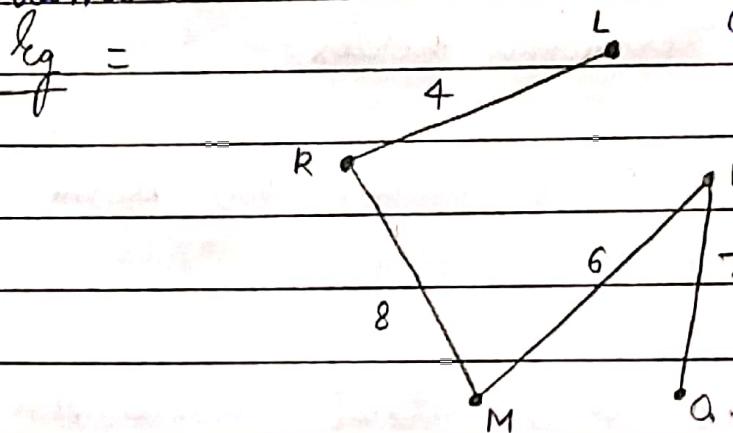
i) Order of a graph = 3



ii) Size of a graph = 4

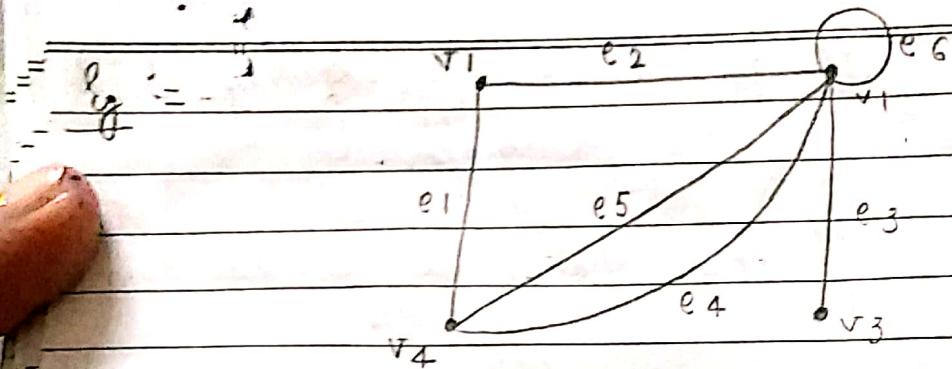
WEIGHTED GRAPH

→ A graph $G = (V, E)$ is called a weighted graph if each edge of graph G is assigned a positive number w called the weight of the edge.



MULTIGRAPH

→ A multigraph $G = (V, E)$ consists of a set of vertices V and a set of edges E such that edge set E may contain multiple edges and



EULER'S PATH OR CHAIN

→ An Euler path (or chain) through a graph is a path whose edge list contains each edge of the graph exactly once.

Note = No vertex is visited twice.

EULER'S CIRCUIT OR CYCLE

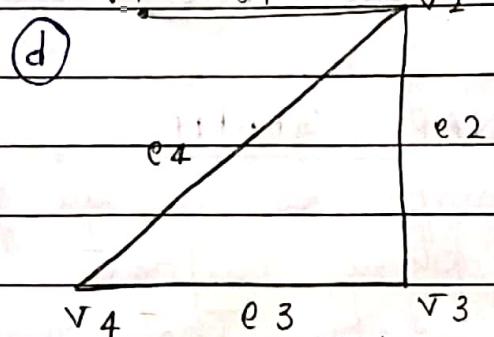
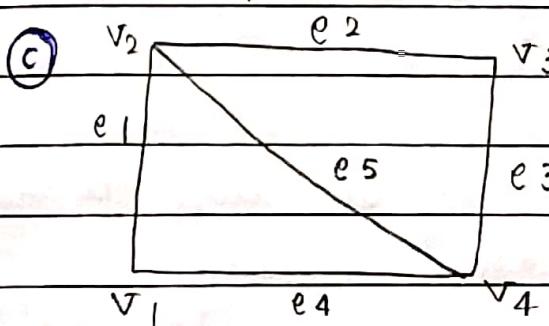
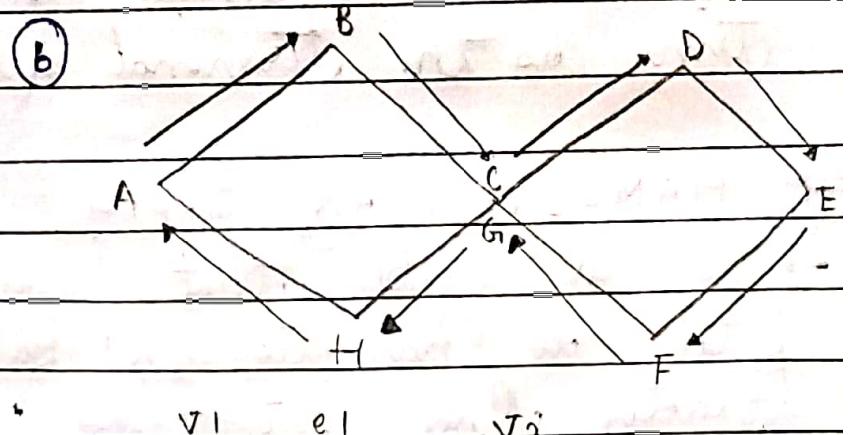
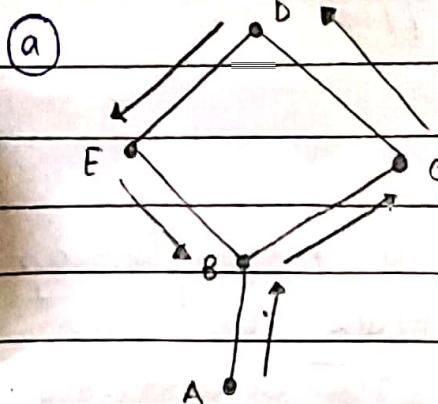
→ An Euler circuit (or cycle) is a path through a graph in which initial vertex appears second time as the terminal vertex.

Note = Start and end vertex are same.

EULER GRAPH =

→ Euler's graph is a graph that passes an Euler's circuit. Euler's circuit uses every edge exactly once but vertex may be repeated.

Q. Consider the graph(s) shown below. Determine Euler's path, Euler's circuit and Euler's graph of them.



	(a)	(b)	(c)	(d)
Euler's path	Yes	Yes	Yes	No
Euler's circuit	No	Yes	Yes	No
Euler's graph	No	Yes	Yes	No

HAMILTONIAN PATH (OR CHAIN) =

→ A hamiltonian path (or chain) through a graph is a path vertex list containing each vertex of the graph exactly once, except path is a circuit.

HAMILTONIAN CIRCUIT (OR CYCLE) =

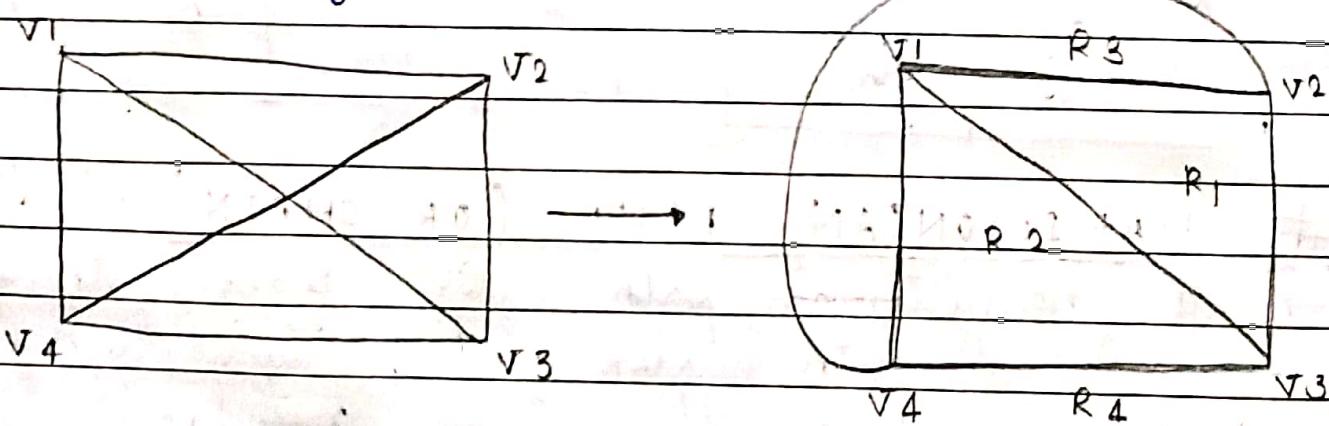
→ A hamiltonian circuit (or cycle) is a path in which the initial vertex appears at second time as the terminal vertex.

HAMILTONIAN GRAPH =

→ Is a graph that possesses a hamiltonian path. A hamiltonian path uses each vertex exactly once but edges may not be included.

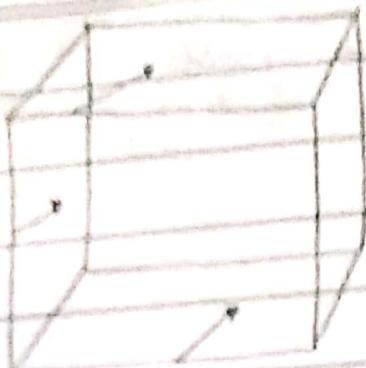
* PLANAR GRAPH =

→ A graph is called planar if it can be drawn in the plane without any edges crossing. And, a graph that cannot be drawn on a plane without a crossover between its edges is called non-planar graph.

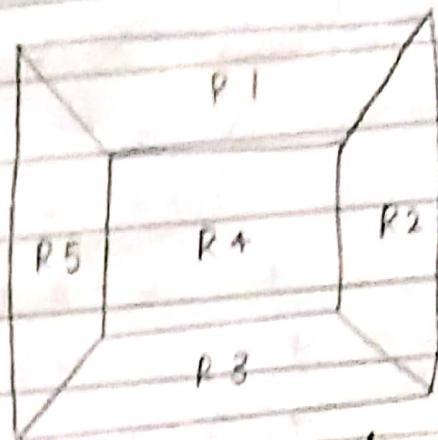


(Non-planar graph)

R₄ → Outer region
(Planar Graph)



(Non-planar graph)



(Planar graph)

R6 → Outer regions

Theorem = A complete graph of 5 vertices Is non-planar.

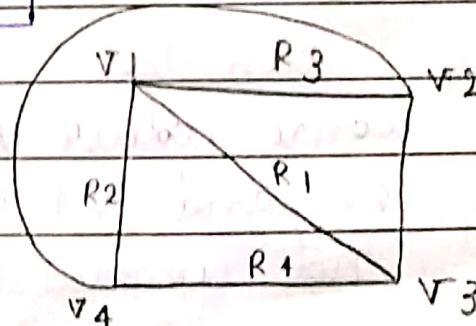
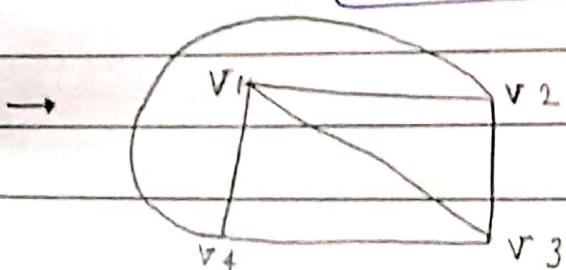
COMPLETE GRAPH =

- Edge between every vertex
- All edges should be connected.
- like will connect v_1 to v_4 And v_1 to v_3
So that intersection should not be there.

* FULER'S FORMULA =

- Let G be a connected planar graph with ' e ' edges and vertices ' v '. Let ' r ' be the number of regions in planar representation of G . Then,

$$r = e - v + 2$$



i) First draw regions for this graph
 ii) $e = 6, v = 4$

$$\therefore r = e - v + 2$$

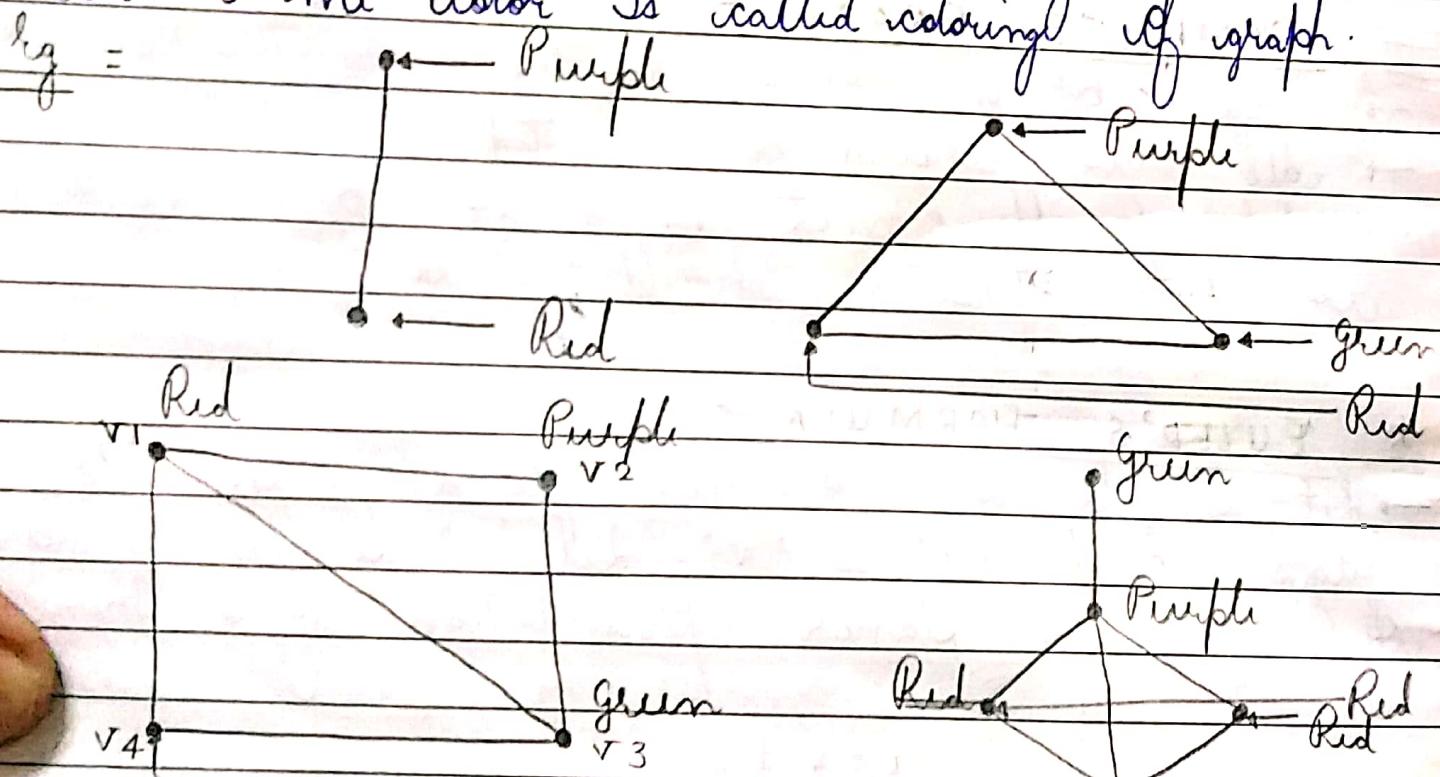
$$r = 6 - 4 + 2$$

$$\Rightarrow r = 2 + 2$$

$$\Rightarrow r = 4$$

GRAPH COLORING AND CHROMATIC NUMBER

→ Painting all the vertices of a graph with colors such that no two adjacent vertices have the same color is called coloring of graph.

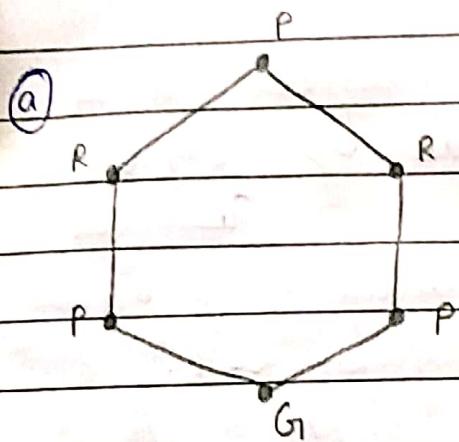


(We can take Purple
 - (but some colours like
 v2 and v4 are
 not connected))

Note = If a graph is circuit with n vertices then

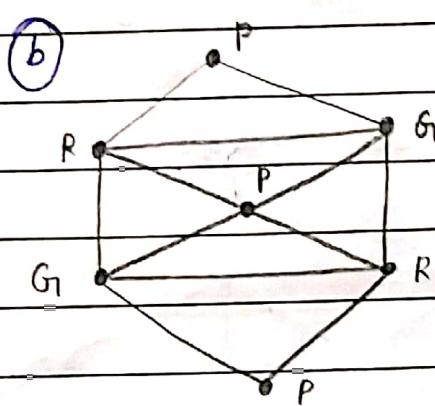
- If n is even.
- If n is odd.

Determine the chromatic number of graph =



These are even number of vertices.

Hence, chromatic number is 2.



The number of vertices is odd.

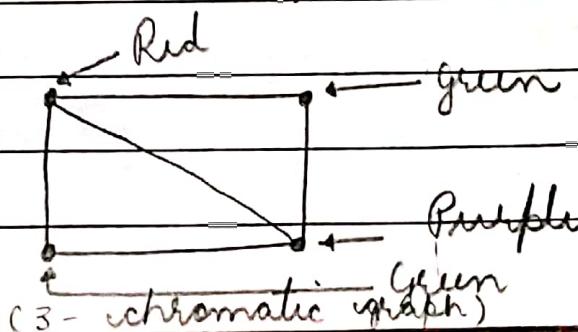
Hence, chromatic number is 3.

CHROMATIC NUMBER =

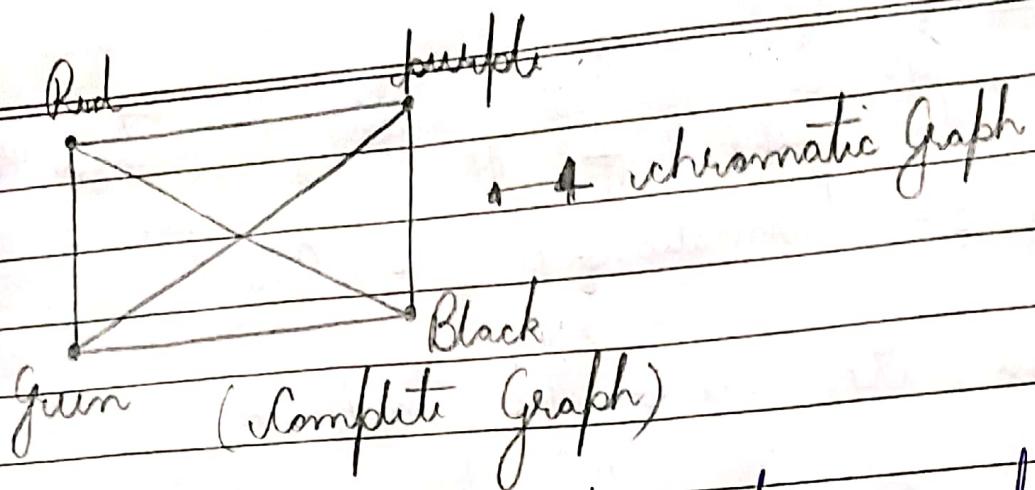
→ The least number of colors required for coloring of a graph G is called its chromatic number.

$\text{Ex} =$ Red

(2 - chromatic graph)



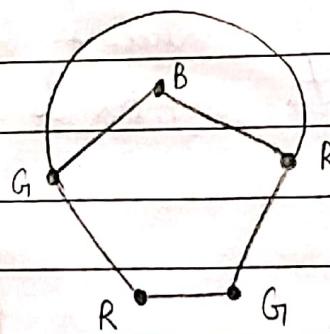
(3 - chromatic graph)



Note = •) The chromatic number of a graph G_1 is denoted by $\chi(G_1)$ (Zai).

-) If $\chi(G_1) = k$, the graph is called k -chromatic.
-) Chromatic number of null graph is 1. Only 1 vertex.
-) Chromatic number of complete graph K_n of n vertices is n .

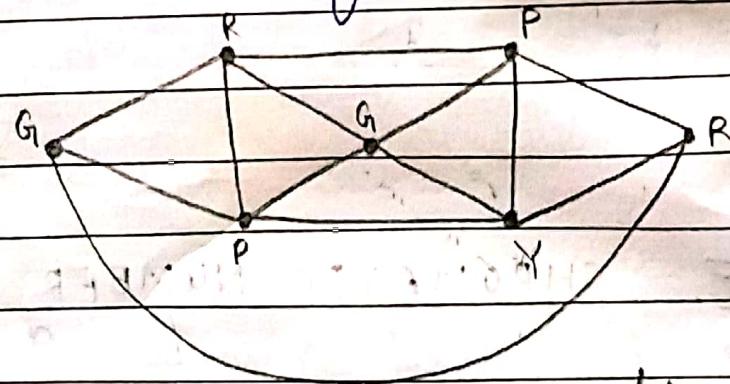
Q. Find the chromatic number of graph.



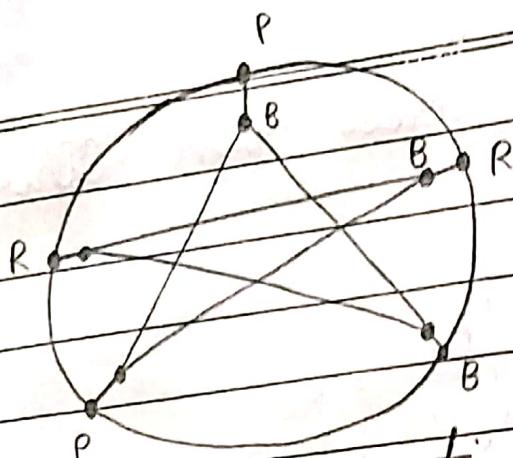
(3 - Chromatic)

→ Colour (G₁, B, R)

→ Odd number of vertices



4 - Chromatic



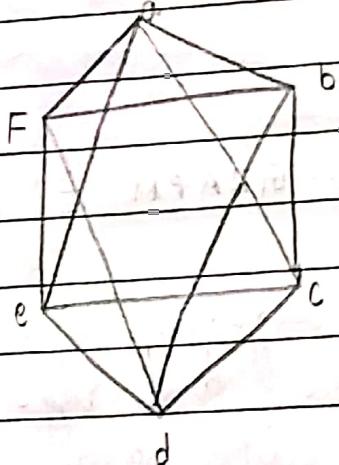
(3-chromatic)

Number of Unique
colours used
(R, B, P)

Q Consider the graph as shown in figure

i Is it a complete graph?

ii Is it a planar graph? If yes, find the number of regions.



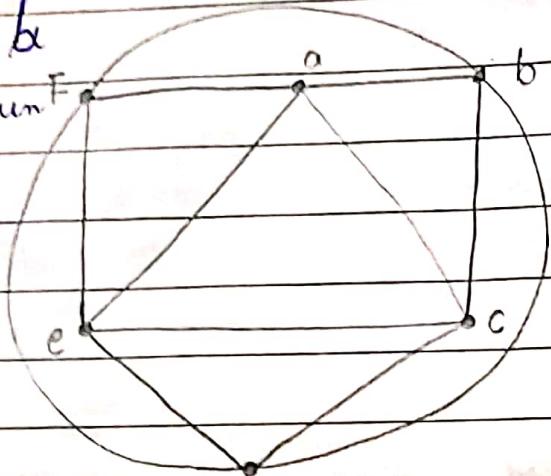
i No, It is not a complete graph as
(o, d), (f, c), (b, e) edges are not
present in the given graph.

ii Yes, It can be re-drawn in which no

no two edges will cross each other. So, It is a planar graph.

It can be

drawn



\therefore We have to find the region of the graph

$$\because r = e - v + 2$$

$$\therefore A/q, e = 12, v = 6$$

$$\therefore r = 12 - 6 + 2$$

$$r = 8$$

a

WALK AND PATH IN A GRAPH

(i) WALK =

\rightarrow It is a finite alternating sequence $v_1 e_1 v_2 e_2 v_3 e_3 \dots v_n$ of vertices and edges, beginning and ending with same or different vertices.

(ii)

LENGTH OF A WALK = The number of edges in a walk is called length of a walk.

iii) CLOSED AND OPEN WALK = A walk is said to be closed if its origin and terminus vertex ($v_0 = v_m$) is equal, otherwise it is called its open walk.

iv) PATH = A walk is said to be a path if all the vertices are not repeated.

ISOMORPHISM OF A GRAPH =

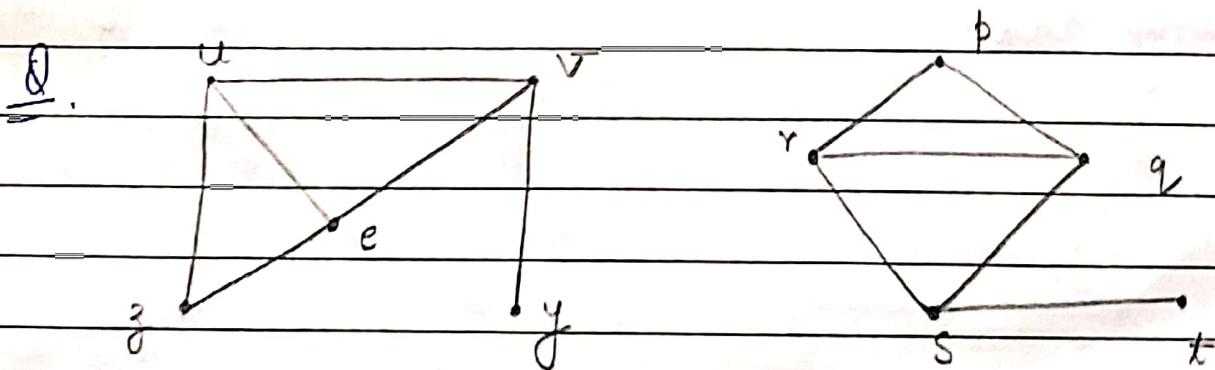
→ Two graphs G and G' are said to be isomorphic if it satisfies

i) Number of vertices are same.

ii) Number of edges are same.

iii) Degree of each vertex in both graphs would be same.

iv) Vertex correspondance and edge correspondance needs to be valid.



$$\rightarrow n(v) = n(v')$$

$$\rightarrow n(E) = n(E')$$

→ An equal number of vertices with given

$$u = 3$$

$$v = 3$$

$$x = 3$$

$$y = 1$$

$$z = 2$$

$$p = 2$$

$$q = 3$$

$$r = 3$$

$$s = 3$$

$$t = 1$$

iv

$$y \rightarrow t$$

$$z \rightarrow p$$

$$v \rightarrow s$$

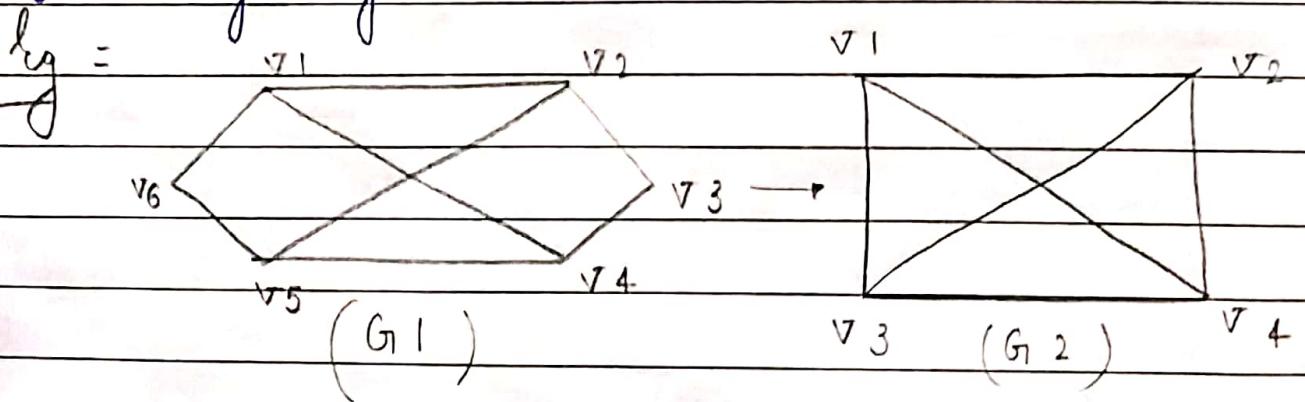
$$u \rightarrow r$$

$$x \rightarrow q$$

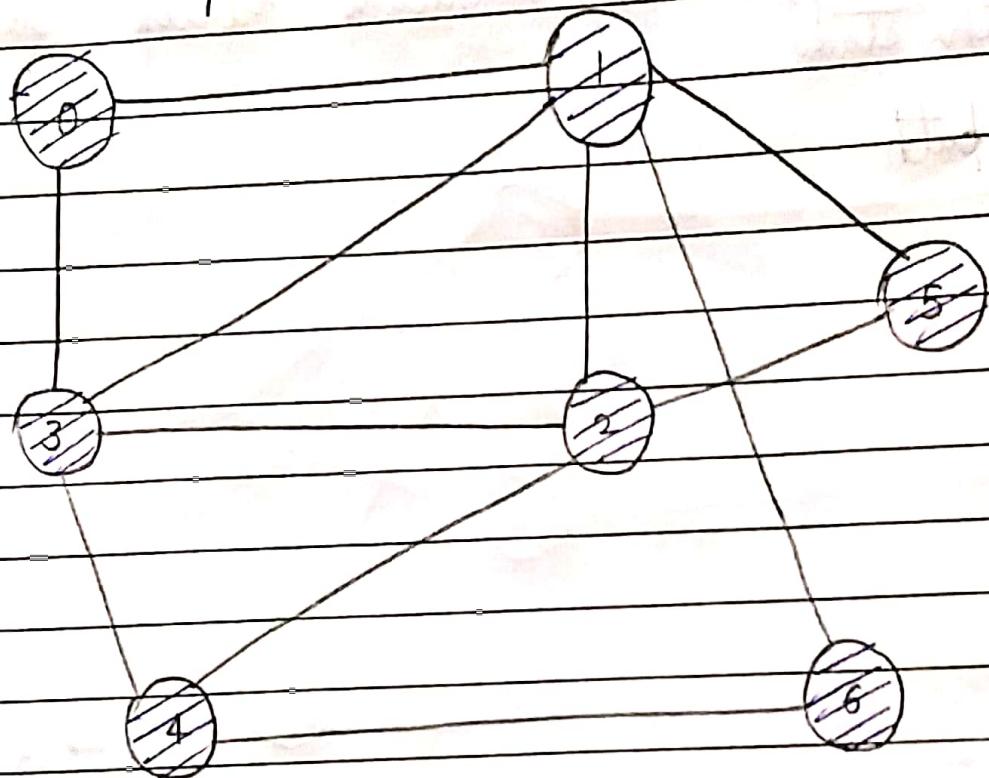
Both are Isomorphic.

HOMOMORPHISM OF A GRAPH

→ Two graph G_1 and G_2 are said to be homomorphic graph if G_2 can be obtained from G_1 by a sequence of subdivisions of edges G_1 .
 In other words, we can introduce vertices of degree 1 in any edge.



- GRAPH TRAVERSALS =
1. BFS (level order), Breadth - first search
 2. DFS depth first search
- BFS path



adjacent nodes =

0 → 1, 3

1 → 0, 3, 2, 5, 6

2 → 1, 5, 4, 3 → 1, 0, 2, 4

3 → 0, 2, 4, 5

4 → 3, 2

5 → 1, 2

6 → 1, 2, 3, 4

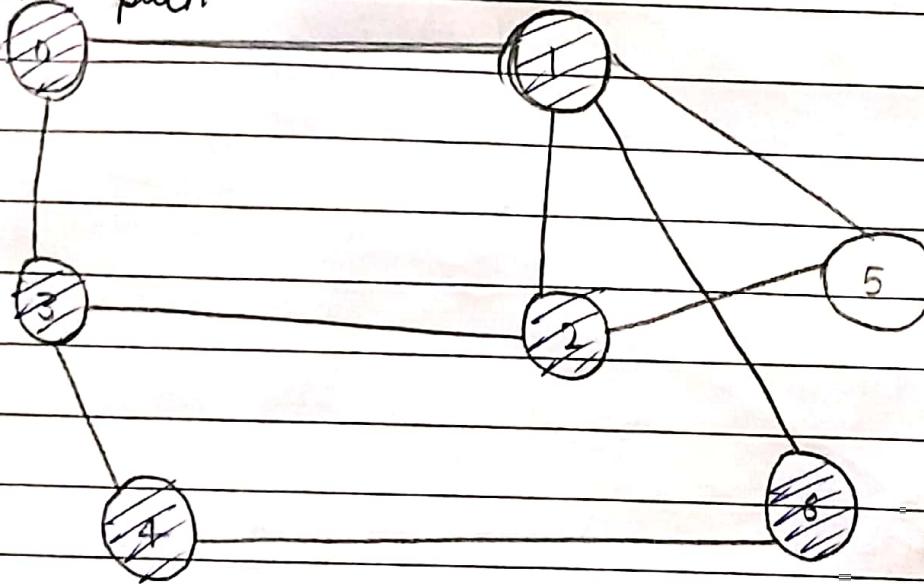
0	1	3	2	5	6	4
---	---	---	---	---	---	---

Result = 0 1 3 2 5 6 4

② D F S =

- Stack Is Used.
- It works In LIFO manner.
- We will go deeper till reach the dead end And we have to backtrack.
- Choose any One Adjacent vertex And push Into the stack.

Q. DFS path



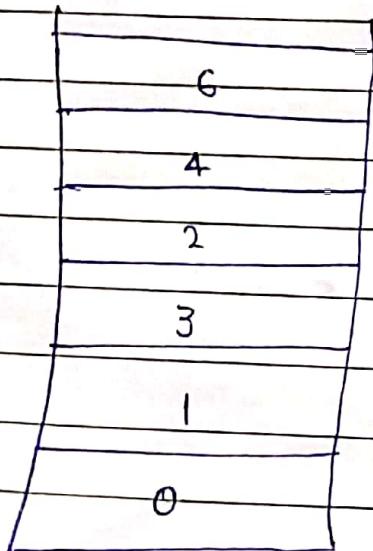
$1 \rightarrow 3, 2, 5, 6$

$3 \rightarrow 2, 4$

$2 \rightarrow 1, 3, 5, 4$

$4 \rightarrow 2, 6$

$6 \rightarrow 1, 4$



Result = 0, 1, 3, 2, 4, 6, 5

We cannot move further from 6

- Now we will start backtracking.
- 6 will be popped from the stack.
- Now go to 4 but 4 is not having any Unvisited vertex. It will be popped out.
- Go to 2, It has 5 vertex as Unvisited vertex. Visit 5.
- Backtrack from 5 to 2, then, 2 will be deleted from the stack.
- Pop Out 3, 1 and 0 afterwards.
- Now stack is empty and this is the indication that we have to stop.

PERMUTATIONS AND COMBINATIONS

i) SUM RULE = Independent
 $(m+n)$

ii) PRODUCT RULE = Sequences
 $(m \times n)$

Q. In a class there are 10 boys and 8 girls.
Teacher has to select either a boy or a girl.

→ ∵ Independent

∴ Number of subsequences = $(m+n)$
∴ A/q,

$$(10+8) = 18 \text{ students}$$

$$\frac{10!}{8!} = \frac{10 \times 9 \times 8!}{8!} = 90$$

Note = $nPr = \frac{n!}{(n-r)!}$

Q. What will be the value of =

$$i) \frac{4 \times n!}{(n-3)!} = \frac{(n+1)!}{(n-2)!}$$

$$\Rightarrow \frac{4 \times n!}{(n-3)!} = \frac{(n+1)(n!)^2}{(n+1-3)!}$$

$$\Rightarrow \frac{4}{(n-3)!} = \frac{(n+1)}{(n-2)(n-3)!}$$

$$\therefore 4(n-2) \therefore (n+1) - 7 = 3$$

$$\text{ii) } \frac{6 \times n!}{(n-3)!} = 3 \times \frac{(n+1)!}{(n-2)!}$$

$$\Rightarrow \frac{6 \times n!}{(n-3)!} = \frac{3 \times (n+1)(n!)!}{(n-2)(n-3)!}$$

$$\Rightarrow 6(n-2) = 3(n+1)$$

$$\Rightarrow 6n - 12 = 3n + 3$$

$$\Rightarrow 6n - 3n = 3 + 12$$

$$\Rightarrow 3n = 15$$

$$\boxed{n = 5}$$

Q. There are 10 persons called for an Interview. Each is capable for the job. How many permutations must be required to get selected?

$$\Rightarrow {}^{10}P_4$$

$$\therefore nPr = \frac{n!}{(n-r)!}$$

$\therefore A/q,$

$$\Rightarrow {}^{10}P_4 = \frac{10!}{(10-4)!} = \frac{10!}{6!}$$

$$\Rightarrow {}^{10}P_4 = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!} = 5040$$

BOOLEAN ALGEBRA =

→ Discovered by George Boole.

→ Introduced to design digital circuit.

#. SOP (Sum of Product) = 209

→ zero is a complement.

→ a product term which contains all the literals either in complemented or uncomplemented form is called minterm.

Minterm

$$0\ 0\ 0 \quad a' b' c' \longrightarrow m_0$$

$$0\ 0\ 1 \quad a' b' c \longrightarrow m_1$$

$$0\ 1\ 0 \quad a' b\ c' \longrightarrow m_2$$

$$0\ 1\ 1 \quad a' b\ c \longrightarrow m_3$$

$$1\ 0\ 0 \quad a\ b' c' \longrightarrow m_4$$

$$1\ 0\ 1 \quad a\ b' c \longrightarrow m_5$$

$$1\ 1\ 0 \quad a\ b\ c' \longrightarrow m_6$$

$$1\ 1\ 1 \quad a\ b\ c \longrightarrow m_7$$

$$\therefore F(a, b, c) = (a' b' c' + a b' c + a b c' + a b c)$$

$$F(a, b, c) = m_0 + m_5 + m_6 + m_7$$

(hypothetical scenario)

$$\text{Complement} = \sum (0, 1, 2, 3, 4, 5, 6, 7) - \sum (0, 5, 6, 7)$$

$$\text{Complement} = \sum (1, 2, 3, 4)$$

pos (Product of sum) =
 → In product of sum, one is complement

~~Maxterm~~

$$0\ 0\ 0 \quad a\ b\ c \longrightarrow m_0$$

$$0\ 0\ 1 \quad a\ b\ c' \longrightarrow m_1$$

$$0\ 1\ 0 \quad a\ b'\ c \longrightarrow m_2$$

$$0\ 1\ 1 \quad a\ b'\ c' \longrightarrow m_3$$

$$1\ 0\ 0 \quad a'\ b\ c \longrightarrow m_4$$

$$1\ 0\ 1 \quad a'\ b\ c' \longrightarrow m_5$$

$$1\ 1\ 0 \quad a'\ b'\ c \longrightarrow m_6$$

$$1\ 1\ 1 \quad a'\ b'\ c' \longrightarrow m_7$$

$$\therefore F(a, b, c) = (a+b+c) \cdot (a+b+c') \cdot (a+b'+c)$$

$$F(a, b, c) = m_0 \cdot m_1 \cdot m_2$$

A/q.

$$\text{Complement} = \sum (0, 1, 2, 3, 4, 5, 6, 7) -$$

$$\sum (0, 1, 2)$$

$$\Rightarrow \text{Complement} = \sum (3, 4, 5, 6, 7)$$

Consider the boolean expression

$$F(x, y, z) = x(y'z)' \cdot \text{Reduce it to}$$

$$F(x, y, z) = xyz'$$

Minterm

0 0 0

a' b' c' → m₀

0 0 1

a' b' c → m₁

0 1 0

a' b c' → m₂

0 1 1

a' b c → m₃

1 0 0

a b' c' → m₄

1 0 1

a b' c → m₅

1 1 0

a b c' → m₆

1 1 1

a b c → m₇

∴ A/q,

$$F(x, y, z) = xyz' \approx a b c' = \prod m_7$$

The