SET THEORY

⇒ Set → A collection of well distinct objects
Eg $A = \{c, f, l, p\}$

Objects of set are elements of set
Eg 1. Set of even number
2. Set of prime number

Notation : Set are denoted by capital letters.

⇒ Roster Form
– Tabulation form
– Elements of set are listed within { } and are seperated by comma.

1. N denote set of first three natural no.
$N = \{1, 2, 3\}$

2. V denote set of all vowels
$V = \{a, e, i, o, u\}$

3. A be set of letters in the word 'BOOK'
$A = \{b, o, \cancel{o}, k\}$

Note: Order of elements doesnot matter but elements should not be repeated.

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⇒ Set builder form
A rule or statement is written within a pair of brackets so that the set is well defined.
All the elements of set must process a single property to become member of that set.

$P = \{1, 2, 3, 4, 5\}$
$P = \{x \mid x \in N, 1 \leq x \leq 5\}$

Ques Represent the following sets in tabular form:-
(i) $A = \{x \mid x = 3p$ where $1 \leq p \leq 5$ and p is a natural number$\}$
$A = \{3, 6, 9, 12, 15\}$

(ii) $B = \{x \mid x^2 - 3x + 2 = 0\}$
$B = \{2, 1\}$

$P = 2$
$S = -3$
$x^2 - 2x - x + 2 = 0$
$x(x-2) - 1(x-2) = 0$
$(x-2)(x-1) = 0$
$x = 2, 1$

⇒ Cardinality
- Cardinal Number
- No. of distinct elements of set
- denoted by $n(A)$

$A = \{a, d, g, l, p\}$ ⇒ $n(A) = 5$

$B = \{a, a, d, b, d\}$ ⇒ $n(B) = 3$

$C = \{a, \{c, d\}, k\}$ ⇒ $n(C) = 3$

=) Types of Set

1. Null Set
- Empty Set
- No elements
- Denoted by ϕ or $\{ \}$
- Cardinal Number $= 0$

$\phi \neq \{\phi\}$

Null set — set having null value

2. Singleton Set
- Only one element is present in set
$A = \{a\}$ $B = \{5\}$
- Cardinal No. $= 1$

3. Finite Set
$n(A) = n$

4. Infinite Set
$n(A) = \infty$

5. Equal Sets
- $n(A) = n(B)$
- Elements are also same

$A = \{a, b, c\}$
$B = \{c, b, a\}$

6. Equivalent Set
- $n(A) = n(B)$

$A = \{a, b, c\}$
$B = \{1, 2, 3\}$

⇒ Subset

- If all the elements of A is contained in B then

$A \subset B$ → superset
↓
Subset

$\subseteq$
↓
sign is used when cardinal no. of both sets are same

Eg $A = \{1, 5, 6\}$
$B = \{1, 2, 3, 4, 5, 6\}$

Note: ϕ is a subset of every set
⇒ $\phi \subset A$, $\phi \subset B$

- $A \subseteq A$
- $A \subset B \Rightarrow x \in A$ and $x \in B$
- $A \not\subset B \Rightarrow x \in A$ but $x \notin B$

⇒ Power Set

- The family consisting of all the subsets of a set is called power set of that set.
- denoted by P(A)

$A = \{1, 2, 3\}$
$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

$P(A) = \{x \mid x \subseteq A\}$

$n(A) = 3$
$n[P(A)] = 2^3 = 8$

Note: No. of proper subset $= 2^n - 1$
n here is cardinal no. of set.

Ques Determine the power sets of the following:-

(i) $X = \{a\}$

$P(X) = \{\phi, \{a\}\}$

(ii) $A = \{\phi, \{\phi\}\}$

$P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

(iii) (a, b)

Let $A = \{a, b\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

=> Universal Set

All objects under discussion

Eg $U = \{x \mid x \text{ is an integer}, x \leq 8\}$

$A = \{1, 2, 4\}$

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

=> Disjoint Sets

Two sets A and B are said to be disjoint if

$A \cap B = \phi$

Eg $A = \{1, 2, 3\}$

$B = \{a, b, c\}$

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⇒ Ordered Pairs

An element of the form (a, b) is called an ordered pair.

$a \rightarrow$ 1st element

$b \rightarrow$ 2nd element

Equality of ordered pair

Let (a, b) and (c, d) be any two ordered pairs

$(a, b) = (c, d)$

when $a = c$

and $b = d$

⇒ Product of two sets

Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$

$A \times B = \{(1, a), (2, a), (3, a), (1, b), (2, b), (3, b)\}$

$B \times B = \{a, b\} \times \{a, b\}$
$= \{(a, a), (a, b), (b, a), (b, b)\}$

Ques Find the values of $A = \{1\}, B = \{a, b\}, C = \{2, 3\}$

(i) $A \times B \times C$

$A \times B = \{1\} \times \{a, b\} = \{(1, a), (1, b)\}$

$A \times B \times C = \{(1, a), (1, b)\} \times \{2, 3\}$
$= \{(1, a, 2), (1, b, 2), (1, a, 3), (1, b, 2)\}$

(ii) $B^2 = \{a, b\} \times \{a, b\} = \{(a, a), (a, b), (b, a), (b, b)\}$

(iii) $B^2 \times 4 = \{(a, a, 4), (a, b, 4), (b, a, 4), (b, b, 4)\}$

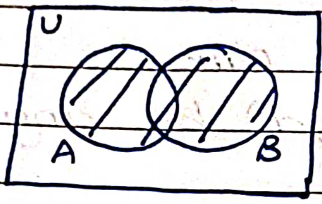
⇒ Operations on Sets

Union

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$A = \{1, 2, 3, 4\}$ $B = \{2, 4, 5, 6\}$

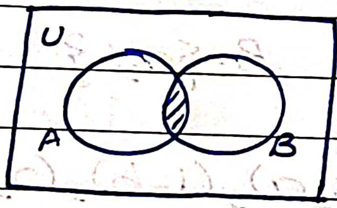
$A \cup B = \{1, 2, 3, 4, 5, 6\}$



Intersection

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$A \cap B = \{2, 4\}$



Difference of two sets

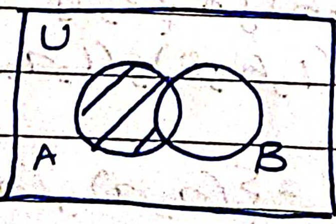
$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

→ Difference of B from A

$A - B = \{1, 3\}$

$A - B = A - (A \cap B)$

$A - B \neq B - A$



A − B

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Complement of set

- Denoted by A^c or $\bar{A}$ or A'

$A' = \{x \mid x \in U \text{ and } x \notin A\}$

$A' = U - A$

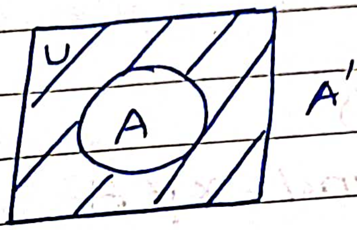
$(A')' = A$

$U' = \phi$

$A' \cup A = U$

$\phi' = U$

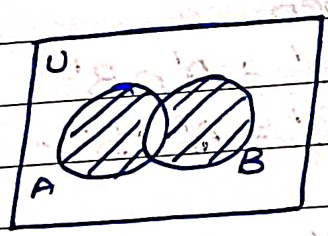
$A' \cap A = \phi$



⇒ **Symmetric Difference**

- denoted by $A \oplus B$ or $A \Delta B$

$A \oplus B = (A-B) \cup (B-A)$



$A = \{1, 2, 3, 4\}$

$B = \{2, 3, 5, 6\}$

$A \oplus B = \{1, 4, 5, 6\}$

$A \oplus B = (A \cup B) - (A \cap B)$

⇒ **Key points**

1. $x \in (A \cup B)$

   $\Rightarrow x \in A$ or $x \in B$

2. $x \in (A \cap B)$

   $\Rightarrow x \in A$ and $x \in B$

3. $x \notin (A \cup B)$

   $\Rightarrow x \notin A$ and $x \notin B$

4. $x \notin (A \cap B)$
$\Rightarrow x \notin A$ or $x \notin B$

5. $x \in A$ OR $x \in A'$
$\Rightarrow x \notin A'$ $\Rightarrow x \notin A$

6. $x \in (A - B)$
$\Rightarrow x \in A$ and $x \notin B$.

$\Rightarrow$ De Morgan's Law
$(A \cup B)' = A' \cap B'$
$(A \cap B)' = A' \cup B'$

Ques. It states that if A and B are any two sets then
(i) $(A \cup B)' = A' \cap B'$
(ii) $(A \cap B)' = A' \cup B'$

Sol. (i) $(A \cup B)' = A' \cap B'$

$x \in (A \cup B)'$
$\Rightarrow x \notin (A \cup B)$
$\Rightarrow x \notin A$ and $x \notin B$
$\Rightarrow x \in A'$ and $x \in B'$
$\Rightarrow x \in (A' \cap B')$
$(A \cup B)' \subset A' \cap B'$ — ①

$y \in A' \cap B'$
$\Rightarrow y \in A'$ and $y \in B'$
$\Rightarrow y \notin A$ and $y \notin B$
$\Rightarrow y \notin (A \cup B)$
$\Rightarrow y \in (A \cup B)'$
$(A' \cap B') \subset (A \cup B)'$ — ②

From ① + ② $\Rightarrow (A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$
$\Rightarrow x \in (A \cap B)'$
$\Rightarrow x \notin (A \cap B)$
$\Rightarrow x \notin A$ or $x \notin B$
$\Rightarrow x \in A'$ or $x \in B'$ $\Rightarrow x \in (A' \cup B')$
~~AAR~~ ~~$(A \cap B)' = (A' \cup B')$~~

Ques. To prove $A = B$

Let $x \in A \xrightarrow{\cancel{\text{and}}} x \in B$

$A \subset B$ — (1)

Let $y \in B \xrightarrow{\cancel{\text{and}}} y \in A$

$B \subset A$ — (2)

From (1) and (2)

$A = B$

$(A \cap B)' \subset (A' \cup B')$ — (1)

$y \in A' \cup B'$

$y \in A'$ or $y \in B'$

$y \notin A$ or $y \notin B$

$y \notin (A \cap B)$

$y \in (A \cap B)'$

$\Rightarrow (A' \cup B') \subset (A \cap B)'$ — (2)

From (1) and (2)

$(A \cap B)' = A' \cup B'$

Ques. If A, B and C are any three non-empty sets, prove that

$$(A-B)\times C = (A\times C)-(B\times C)$$

Sol. $(x,y) \in (A-B)\times C$

$\Rightarrow x \in A$ and $x \notin B$ and $y \in C$

$\Rightarrow (x \in A$ and $y \in C)$ and $(x \notin B$ and $y \in C)$

$\Rightarrow (x,y) \in (A\times C)$ and $(x,y) \notin (B\times C)$

$\Rightarrow (x,y) \in (A\times C)-(B\times C)$

$(A-B)\times C \subset (A\times C)-(B\times C)$ — (1)

Let $(x,y) \in (A\times C)-(B\times C)$

~~$\Rightarrow x \in (A\times C)$ and $x\, y \notin$~~

$\Rightarrow (x,y) \in (A\times C)$ and $(x,y) \notin (B\times C)$

$\Rightarrow (x \in A$ and $y \in C)$ and $(x \notin B$ and $y \in C)$

$\Rightarrow (x \in A$ and $x \notin B)$ and $y \in C$

$\Rightarrow x \in (A-B)$ and $y \in C$

$\Rightarrow (x,y) \in (A-B)\times C$

~~$\Rightarrow (A-B)\times C \in (A\times$~~

$\Rightarrow (A\times C)-(B\times C) \subset (A-B)\times C$ — (2)

From (1) and (2)

$$(A-B)\times C = (A\times C)-(B\times C)$$

Proved

Ques If $A = \{1,4\}$, $B = \{4,5\}$, $C = \{5,7\}$ Then determine $(A \times B) \cap (A \times C)$

Sol $(A \times B) = \{(1,4), (1,5), (4,4), (4,5)\}$
$(A \times C) = \{(1,5), (1,7), (4,5), (4,7)\}$
$(A \times B) \cap (A \times C) = \{(1,5), (4,5)\}$

Ques Let A, B, C be any three sets prove that
$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol Let $(x, y) \in A \times (B \cap C)$
$\Rightarrow x \in A$ and $y \in (B \cap C)$
$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$
$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$
$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$
$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

$\Rightarrow A \times (B \cap C) \subset (A \times B) \cap (A \times C)$ — (1)

Let $(x, y) \in (A \times B) \cap (A \times C)$
$\Rightarrow (x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$
$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$
$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$
$\Rightarrow x \in A$ and $y \in (B \cap C)$
$\Rightarrow (x, y) \in (A \times (B \cap C))$

$\Rightarrow (A \times B) \cap (A \times C) \subset A \times (B \cap C)$ — (2)

From (1) and (2)

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Proved

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Ques If A, B, C, D are any four sets then prove that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$.

Sol. Let $(x, y) \in (A \cap B) \times (C \cap D)$

$\Rightarrow x \in A \cap B$ and $y \in C \cap D$

$\Rightarrow (x \in A$ and $x \in B)$ and $(y \in C$ and $y \in D)$

$\Rightarrow (x \in A$ and $y \in C)$ and $(x \in B$ and $y \in D)$

$\Rightarrow (x, y) \in (A \times C)$ and $(x, y) \in (B \times D)$

$\Rightarrow (x, y) \in (A \times C) \cap (B \times D)$

$\Rightarrow (A \cap B) \times (C \cap D) \subset (A \times C) \cap (B \times D)$ — (1)

Let $(x, y) \in (A \times C) \cap (B \times D)$

$\Rightarrow (x, y) \in A \times C$ and $(x, y) \in B \times D$

$\Rightarrow (x \in A$ and $y \in C)$ and $(x \in B$ and $y \in D)$

$\Rightarrow (x \in A$ and $x \in B)$ and $(y \in C$ and $y \in D)$

$\Rightarrow x \in (A \cap B)$ and $y \in (C \cap D)$

$\Rightarrow (x, y) \in (A \cap B) \times (C \cap D)$

$\Rightarrow (A \times C) \cap (B \times D) \subset (A \cap B) \times (C \cap D)$ — (2)

$\Rightarrow (A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$

Proved

Ques. If A & B are any two sets then prove that $A \cap (B-A) = \phi$

Sol. Let $x \in A \cap (B-A)$

$\Rightarrow x \in A$ and $x \in (B-A)$

$\Rightarrow x \in A$ and $(x \in B$ and $x \notin A)$

$\Rightarrow (x \in A$ and $x \notin A)$ and $x \in B$

$\Rightarrow (x \in \phi)$ and $x \in B$

$\Rightarrow x \in B \cap \phi$

$\Rightarrow x \in \phi$

Hence $A \cap (B-A) = \phi$

Remember:-

* ~~n(A∪~~ $n(A \cap B) = n(A) + n(B) - n(A \cup B)$
* $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
* $n(A \cup B) = n(A-B) + n(B-A) + n(A \cap B)$
* $n(A) = n(A-B) + n(A \cap B)$
* $n(B) = n(B-A) + n(A \cap B)$

For disjoint sets, $n(A \cup B) = n(A) + n(B)$

$n(A \cap B) = \phi$

$\Rightarrow$ Principle of Inclusion-Exclusion

An approach that derives the method for finding the number of elements in the union of two finite sets.

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Ques Among a group of students, 49 study Physics, 37 study English and 21 study Biology. If 9 of these students study ~~Maths~~ Physics and English, 5 study English and Biology, 4 study ~~English~~ Physics and Biology and 3 study Physics, English and Biology. Find number of students in the group.

Sol. $n(P) = 49$

$n(E) = 37$

$n(B) = 21$

$n(P \cap E) = 9$

$n(E \cap B) = 5$

$n(P \cap B) = 4$

$n(P \cap B \cap E) = 3$

$n(P \cup B \cup E) = n(P) + n(E) + n(B) - n(P \cap E) - n(E \cap B) - n(P \cap B) + n(P \cap B \cap E)$

$= 49 + 37 + 21 - 9 - 5 - 4 + 3$

$= 92$ Ans