

CHANGE MAKING PROBLEM :-

COIN CHANGE PROBLEM I-

- (i) find out the ways you can make the change of the given amount using the given coins.
- (ii) find minimum no. of coins to get the given value.

Condition:- Infinite supply of coins.

Ex:- Coins $\Rightarrow \{1, 2, 3\}$ $w = 5$

(1,1,1,1) (1,1,1,2) (1,2,2) (1,1,3) (2,3)

Sol.ⁿ (i) \Rightarrow No. of ways $\Rightarrow 5$

(ii) \Rightarrow Minimum no. of coin to get ~~max~~ given value = 2
(2,3)

- Rule
- ① Exclude the coin (previous value)
 - ② Include " " $(6-3)=3 \rightarrow$ go to 3rd column within same row & take that value
 - ③ Add ① & ②

Example:- Coins $\Rightarrow \{2, 3, 5, 10\}$
 $w = 15$

(i) \Rightarrow No. of ways \rightarrow

(i) \ (j) w	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Coins																
2	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0
3	1	0	1	0+1=1	1+0=1	0+1=1	1+1=2	0+1	2	2	2	2	3	2	3	3
5	1	0	1	1	1	2	2	2	3	3	4	4	5	5	6	7
10	1	0	0	1	1	2	2	2	3	3	5	4	6	6	7	7+2=9

1 way to make sum zero & i.e if we don't consider respective coin

- ① Exclude the coin \Rightarrow pick above value. i.e 1
 - ② Include " " $\Rightarrow [i,j] = [3,6] \Rightarrow 6-3 = 3 \rightarrow$ go to 3rd column within same row & pick that value = 1
 - ③ Add ① & ② $\Rightarrow 1+1 = 2$
- // by all

(i) No. of ways = 9 Ans.

Algorithm is (finding No. of ways)
// if coin value $> w$, then just copy the above value.

1) Initialize value = 1 for 0th column

2) for ($i=0$; $i \leq \text{coin.length}$; $i++$)

{
 for ($j=1$; $j \leq \text{amount}$; $j++$)

{
 if ($i==0$) // Row = 0

$a[0][j] = a[i][j - \text{coin}[i]]$

 else if ($\text{coin}[i] > j$)

$a[i][j] = a[i-1][j]$;

 else

$a[i][j] = a[i-1][j] + a[i][j - \text{coin}[i]]$

 }

}

}

(ii)

Example:- Coins $\rightarrow \{2, 3, 5\}$

$$W=8$$

(Find Minimum No. of Coins? to make $W=8$)

When sum is not there \rightarrow Consider ∞

		W \rightarrow								
Coin	(i)	(w)								
		0	1	2	3	4	5	6	7	8
2	i=0	0	∞	1	∞	2	∞	3	∞	4
3	i=1	0	∞	1	1	2	2	2	3	3
5	i=2	0	∞	1	1	2	1	2	2	2

Min. no. of Coins are Zero
To make sum Zero
i.e. (zero)

Ans. 0

Algorithm:- (Minimum No. of Coins)

// if $\text{coin}[i] > w$ just copy above value.

1) Initialize 0th column = 0 (i.e. min. no. of zero coin required to make sum = 0)

2) // Initialize 1st row [2]

for (int i = 0 ; i ≤ coin.length ; i++)

{ for (j = 1 ; j ≤ sum ; j++)

{ if (i == 0)

{ if (j % coin[0] == 0)

$a[i][j] = j / \text{coin}[0]$

else

$a[i][j] = \infty$

}

else if (coin[i] > j)

{

$a[i][j] = a[i-1][j]$

}

else

{

$a[i][j] = \min\{a[i-1][j], 1 + a[i][j - \text{coin}[i]]\}$

}

}

$O(nw)$

Ⓟ