Test of Significance.

~~μ = 20 persons~~
$\mu = 20$ minutes $\longrightarrow$

Identify the parameter of interest (mean / variance / S.D.)

① Set a Hypothesis (Claim)

1) Null Hypothesis H_0: Change has no effect
$\mu = 20$ minutes after change

2) Alternative Hypothesis H_1: Change has +ve effect
$\mu > 20$ minutes after the change

② Threshold

Known as Significance level $\alpha : 0.05$ (Greek letter α)

Take a sample visiting, calculate mean & S.D. for sample.

If we assume that null hypothesis is true, what is the probability of getting a sample with the statistics we get?

If that probability is lower than α, we reject the null hypothesis & we say we have evidence for the alternative.

③ Take a sample $n = 100$, Measure sample mean, sample S.D.
$\bar{X} = 25$ minutes.

④ Calculate p-value (probability value)
p-value $= P(\text{sample mean } \bar{X} \geq 25 \mid H_0 \text{ is true})$

⑤ p-value $< \alpha$, then Reject H_0
p-value $\geq \alpha$, then do not Reject H_0.

③ Compute a test statistic which is a function of the sample set of observations.

✓ Derive the distribution of the test statistic under the null hypothesis assumption.

✓ No hypothesis is ~~correct~~ perfect. There are inherent errors since it is based on observations which are random.

✓ The Performance of a hypothesis test depends on
(i) Extent of variability in data.
(2) No. of observations (Sample Size)
(3) Test statistic (function of observations)
(4) Test criterion (threshold)

E15915
Mar @ 2024
306

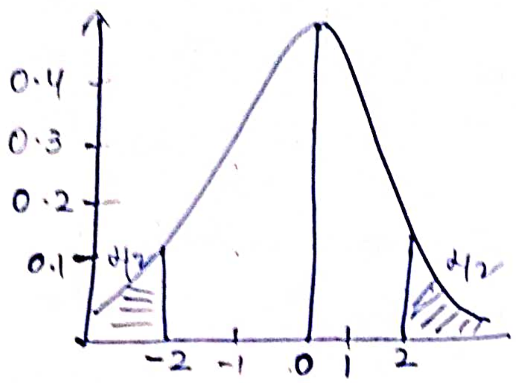
...sided & One-sided tests

...o sided test

$H_0 : \mu = 0$

$H_1 : \mu \neq 0$

- Test statistic standard normal RV z



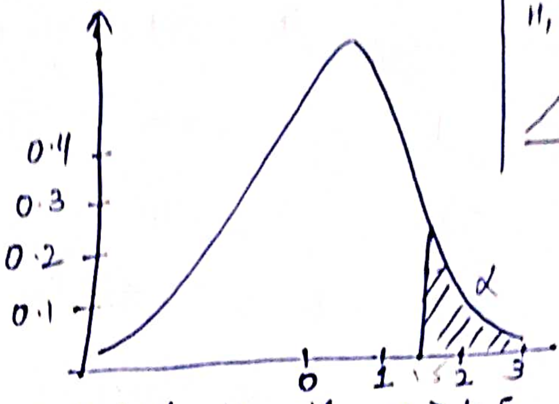
- Reject H_0 if $z < -2$ or $z > 2$

- One sided test $\bar{x}$

$H_0 : \mu = 0$

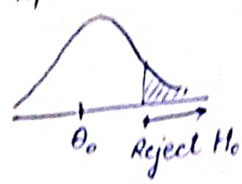
$H_1 : \mu > 0$ / $\mu < 0$

- Test statistic standard normal RV z



$H_0 : \theta = \theta_0$

$H_1 : \theta > \theta_0$



- Reject H_0 if $z > 1.5$

Two Types of errors (Type I & Type II)

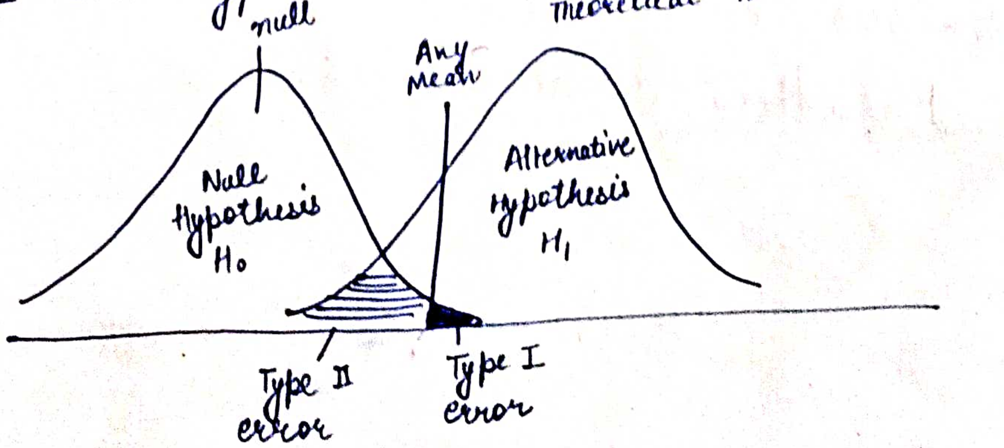
Decision → Truth ↓	H_0 is not rejected	H_0 is Rejected
H_0 is true	Correct decision $Pr = 1 - \alpha$	Type I error $Pr = \alpha$
H_1 is true	Type II error $Pr = \beta$	Correct Decision $Pr = 1 - \beta$

- Pr : Probability

Typically the Type I error probability α (also called as level of significance of the test) is controlled by choosing the criterion from the distribution of the test statistic under the null hypothesis.

Errors in Hypothesis testing

- Type I and Type II error probabilities.



[St]atistical Test Power $= 1 -$ Type II error probability.
[Tr]ade off - If we decrease Type I error probability, then Type II error probability will increase.

Any Mean written on the graph is the threshold value.

$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$n = k - 1$

For example,
For a given application the burning rate of a solid propellant should be 50 cm/s.
✓ 25 samples of solid propellant are taken and their burning rate noted. The average burning rate is computed to be 51.3 cm/s. The S.D. in the burning rate is known to be 2 cm/s.

0.05 >
0.0012

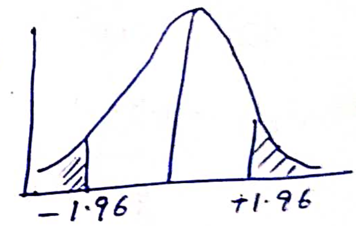
✓ Null hypothesis : $\mu = 50$ cm/s.
✓ Alternate hypothesis : $\mu \neq 50$ cm/s (Two-sided test)

0.99940

✓ Test statistic $z = \frac{\bar{x} - 50}{2/\sqrt{25}} \sim N(0,1)$; $z = 3.25$

$$z = \frac{\bar{x} - 50}{\sigma/\sqrt{N}}.$$

$3.25 > 1.96$



✓ Critical value for $\alpha = 0.05$ is ± 1.96
✓ Decision : Reject Null hypothesis ($\because 3.25 > 1.96$)

Test for differences in Means : (2-sample t-test)
✓ Two groups of teachers of similar capabilities are trained by two methods A & B. Is method B more effective than Method A?
✓ Group 1 : $\bar{x}_1 = 70$, $s_1 = 3.3665$, ✓ Group 2 : $\bar{x}_1 = 74$, $s_1 = 5.3955$
✓ Null hypothesis : $\mu_1 - \mu_2 = 0$
✓ Alternative hypothesis : $\mu_1 - \mu_2 < 0$ - one sided test
✓ Test statistic (assuming unknown but equal variances for two groups)

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_p^2}{N_1} + \frac{S_p^2}{N_2}}} \sim t_{N_1+N_2-2} \; ; \quad S_p = \frac{(N_1-1)s_1^2 + (N_2-1)s_2^2}{N_1 + N_2 - 2}, \quad t = -1.989$$

✓ Critical value for $\alpha = 0.05$ is -1.73, t
✓ Decision : Reject null hypothesis (Method B is better)

for Differences in Variances:

✓ variability in yields from two different processes are to be compared to decide whether they are identical or not.

✓ 50 samples for each process taken. Yield variances are found to be $s_1^2 = 2.05$, $s_2^2 = 7.64$

✓ Null hypothesis : $\frac{\sigma_1^2}{\sigma_2^2} = 1$ (Population variances are equal)

✓ Alternate hypothesis : $\frac{\sigma_1^2}{\sigma_2^2} \neq 1 \rightarrow$ two sided test.

✓ Test statistic (assuming unknown but equal variances for 2 groups)

$$f = \frac{s_1^2}{s_2^2} \sim F(N_1 - 1, N_2 - 1) \; ; \; f = 0.27$$

✓ Critical value for $\alpha = 0.025$ is 0.567 & $\alpha = 0.975$ is 1.762

✓ Decision : Reject null hypothesis (Process 2 has higher variability)

There is 2.5% probability that f distribution < 0.567

Type of test	Characteristic	Example
z-test $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$	Sum of independent normal variables	Test for a mean or comparison b/w two gps. means (variance known)
t-test $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$	Ratio of a standard normal variable & χ^2 variables with p DF.	Test for a mean or comparison b/w gp. means (variance unknown)
χ^2-test		Test for variance
F-test		Test for comparing variances.

Test concerning sample mean

- $H_0 : \mu = \mu_0$
- $H_1 : \mu \neq \mu_0$.

$$P\left(-Z_{\alpha/2} < \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} < Z_{\alpha/2}\right) = 1 - \alpha$$

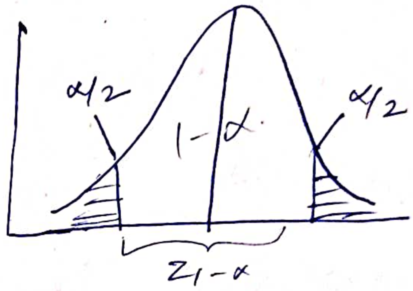
one sided test

$$P\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} < Z_\alpha\right) = 1 - \alpha.$$

searchers are interested in the mean age of a certain population. A Random sample of 10 individuals drawn from the population of interest has a mean of 27. Assuming that population is approximately normally distributed with variance 20, can we conclude that mean is different from 30 years ($\alpha = 0.05$). If the p-value is 0.0340.

Sol. $n = 10$

$\bar{x} = 27$, $\sigma^2 = 20$, $\alpha = 0.05$



$H_0 : \mu = 30$

$H_1 : \mu \neq 30$

Test statistic $Z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

$$Z = \frac{\bar{x} - 30}{\sqrt{20}/\sqrt{10}}$$

$$Z = \frac{\bar{x} - 30}{\sqrt{2}} = \frac{27 - 30}{\sqrt{2}} = \frac{-3}{\sqrt{2}} = \frac{-3}{1.414}$$

$$Z = -2.121$$

we Reject H_0 if $Z > Z_{1-0.025} = Z_{0.975}$

or $Z < -Z_{1-0.025} = -Z_{0.975}$

$1 - 0.034$

0.966

$Z_{0.975} = 1.96$ (from table)

$P(-2.121 < Z < 2.121) = 0.966$

$\cancel{-1.96 > -2.121}$

$\underbrace{-2.121}_{Z} < \underbrace{-1.96}_{Z_{1-\alpha}}$

If $H_0 : \mu = 30$

$H_a : \mu < 30$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{27 - 30}{\sqrt{20}/\sqrt{10}} = \frac{-3}{\sqrt{2}} = -2.121$$

$P(-2.121 < Z_\alpha) = P(-2.121 <$

Reject H_0 if $Z < -Z_{1-\alpha}$

$Z_{1-\alpha} = 1.645$

$-2.121 < -1.645$

Reject H_0.

tailed test

The population of all verbal GRE scores are known to have a S.D. of 8.5. The UW Psychology department hopes to receive applicants with a verbal GRE scores over 210. This year, the mean verbal GRE scores for the 42 applicants was 212.79. Using a value of $\alpha = 0.05$ is this new mean significantly greater than the desired mean of 210?

Sol. $n = 42$, $\bar{x} = 212.79$, $\mu = 210$
$\sigma = 8.5$

To find the probability of finding a mean above 212.79 we find out z-score

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{212.79 - 210}{8.5/\sqrt{42}} = 2.13$$

This will be a one-tailed test because we're only rejecting H_0 if our observed mean ~~μ~~ > 210.

$H_0: \mu = 210$

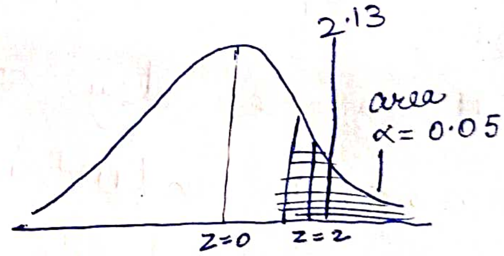
$H_1: \mu > 210$

For $\alpha = 0.05$, $z = 1.64$

The critical value of z is 1.64.

Our observed value of z is $2.13 > 1.64$

We therefore reject H_0.



We can also calculate the p-value for our observed mean & compare it to alpha.

For this one-tailed test, the p-value is the area under the normal distribution above our observed value of z.

z	Area b/w mean & z	Area beyond z
2.13	0.4834	0.0166

p-value $p = 0.0166$, $\alpha = 0.05$

$0.0166 < 0.05$

H_0 rejected.

9382
8174
2546
7097
2032
7012
7232
7529

Two tailed z - test

Suppose you start up a company that has developed a drug that is supposed to increase IQ. You know that the S.D. of IQ in the general population is 15. You test your drug on 36 patients and obtain a mean IQ of 97.65. Using an α of 0.05 is this IQ significantly different than the population mean of 100?

Sol. $\sigma = 15$, $n = 36$, $\bar{x} = 97.65$, $\mu = 100$

$H_0: \mu = 100$

$H_1: \mu \neq 100$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{97.65 - 100}{15/\sqrt{36}} = -0.94$$

✓ We reject the null hypothesis if our observed mean is either significantly larger or smaller than 100.

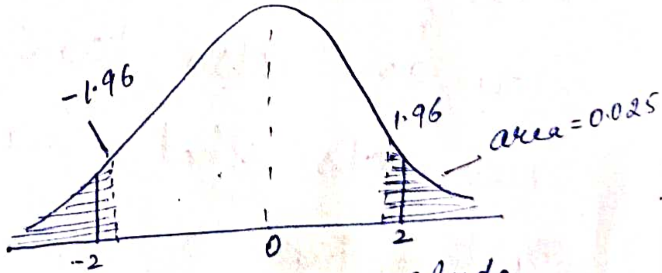
✓ Our critical values of z are therefore the two values that span the middle 95% of the area under the standard normal distribution. This means that the areas in each of the two tails is $\frac{0.05}{2} = 0.025$.

9788000232

Z	Area b/w mean & z	Area beyond z
1.96	0.4750	0.025

$\Rightarrow$ Critical value of $z = 1.96$

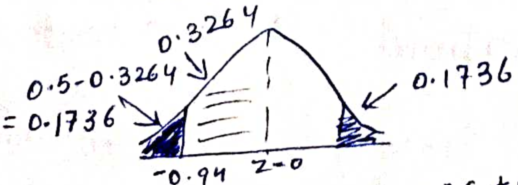
✓ The Rejection region contains values of z less than -1.96 & greater than 1.96. Our observed value of z falls outside the rejection region, so we fail to reject H_0, and conclude that our drug did not have a significant effect on IQ.



From table

Z	Area b/w mean & z	Area beyond z
-0.94	-0.3264	~~0.8264~~ 0.17361

value -0.94
0.17361



1 - 0.8264 = 0.1736

Total area = 0.1736 + 0.1736

P-value = 0.3472 > 0.05

we fail to reject H_0.

Suppose the arousal of hot cats has a population that is normally distributed with a S.D. of 6. Tomorrow we sample 49 hot cats from this population and obtain a mean arousal of 46.44 & a S.D. of 5.6968.
Using an $\alpha = 0.01$, is this observed mean significantly less than an expected arousal of 47?

Q2 Suppose the jewelry of exams has a population that is normally distributed with a S.D. of 5. We are walking down the street & sample 9 exams from this population & obtain a mean jewelry of 28.95 & a S.D. of 6.3802.
Using an $\alpha = 0.01$, is this observed mean significantly different than an expected jewelry of 27?

304
404
405

22

$\sigma = 6$, $n = 49$, $\bar{x} = 46.44$, $s = 5.6968$

$\alpha = 0.01$, $\mu = 47$

$H_0 : \mu = 47$

$H_1 : \mu < 47$ (One tailed test)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{46.44 - 47}{6/\sqrt{49}} = \frac{-0.56}{6/7} = -0.6533$$

$p = 0.2578 > 0.01$

we fail to reject H_0.

Q2 $\sigma = 5$, $n = 9$, $\bar{x} = 28.95$, $s = 6.3802$, $\alpha = 0.01$

$H_0 : \mu = 27$

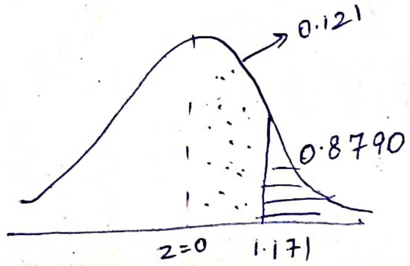
$H_1 : \mu \neq 27$ (Two - tailed test)

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{28.95 - 27}{5/\sqrt{9}} = \frac{1.95 \times 3}{5} = 1.17$$

z	~~Area b/w mean & z~~	Area beyond z
1.17		0.87900

$$\begin{array}{r} 1.000 \\ 0.879 \\ \hline 0.121 \end{array}$$

p-value $= 0.242001$



For -ve z

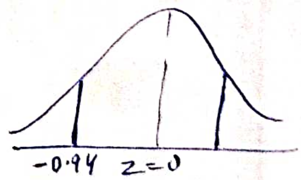
For +ve z

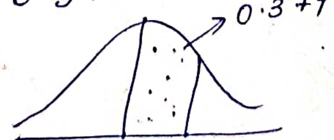
$z = 1.17$

value $= 0.87900$.

Area $z = 0$ to $1.17 = 0.879 - 0.5$

$= 0.379$





Area ~~b~~ $z > 1.17 = 0.5 - 0.379$

$= 0.121$

Total area $= 0.121 + 0.121 = 0.242$

p value $= 0.242 > 0.01$

we fail to reject H_0.

Degrees of freedom : The Maximum number of logically independent values, which may vary in a data sample.

Df = sample size − No. of parameters we need to calculate during an analysis.

Types of t-tests

A t-test is a hypothesis test of the mean of one or two normally distributed populations.

	Purpose
1-sample t-test	Tests whether the mean of a single population is equal to a target value.
2-sample t-test	Tests whether the difference b/w means of two independent populations is equal to a target value.
3 paired t-test	Test whether the mean of the differences between dependent or paired observations is equal to a target value.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$, degrees of freedom $= n-1$

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}}$$

$t_{tab} < |t_{cal}|$
H_0 rejected.

Q1 The following data represents haemoglobin values in gm/dl for 10 patients 10.5, 9, 6.5, 8, 11, 7, 7.5, 8.5, 9.5, 12. Is the mean value for patients significantly differ from the mean value of general population (12 gm/dl). Evaluate the role of chance ($\alpha = 0.05$)

Sol.
$$\bar{x} = \frac{10.5 + 9 + 6.5 + 8 + 11 + 7 + 7.5 + 8.5 + 9.5 + 12}{10}$$

n-1 degrees of freedom.

$H_0 : \mu = 12$
$H_1 : \mu \neq 12$

$$\bar{x} = 8.95$$

$$s = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n-1}} = 1.802005$$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{8.95 - 12}{1.802002/\sqrt{10}} = -5.35234$$

$t_{9,\,0.05} = 2.262 < |-5.35234|$ $t_{tab} < t_{cal}$

two tailed test H_0 rejected.

Hypothesis t-tests on the difference b/w two population means $(\mu_1 - \mu_2)$.

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad , \quad S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}}$$

$$df = n_1 + n_2 - 2.$$

Q1 The following data represents weight in kg for 10 males and 12 females.

Male : 80, 75, 95, 55, 60, 70, 75, 72, 80, 65

Female : 60, 70, 50, 85, 45, 60, 80, 65, 70, 62, 77, 82

Two independent samples. Is there a statistically significant difference b/w the mean weight of males & females, where $\alpha = 0.01$.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$n_1 + n_2 - 2 = 10 + 12 - 2 = 20.$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$ (two tail)

$t = 1.074$

$t_{20, 0.01} = 2.845 > 1.074$

$t_{tabulated} > t_{calculated}$

H_0 ~~does~~ not rejected.

We fail to reject H_0

Based on field experiments, a new variety green gram is expected to given an yield of 12.0 ~~to~~ quintals per hectare. The variety was tested on 10 randomly selected farmers fields. The yields (quintals / hectare) were recorded as 14.3, 12.6, 13.7, 10.9, 13.7, 10.9, 13.7, 12.0, 11.4, 12.0, 12.6, 13.1. Do the results conform the expectation?

Sol. Null hypothesis $H_0 : \mu = 12$

Alternative Hypothesis $H_1 : \mu \neq 12$

Level of significance : 5%.

Test statistic, $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$

$$\bar{x} = \frac{\sum x}{n} = \frac{126.3}{10} = 12.63$$

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = 1.0853$$

$$t = \frac{12.63 - 12}{1.0853/\sqrt{12}} = \frac{+0.63 \times 2\sqrt{3}}{1.0853} = 1.836$$

$n-1 = 9$ degrees of freedom.

$t_{(0.05, 9)} = 2.262$

$t_{cal} \nless t_{tab}$

We fail to reject null hypothesis H_0.

A Group of 5 patients treated with medicine A is of weight 42, 39, 38, 60 and 41 kgs. Second group of 7 patients from the same hospital treated with medicine B is of weight 38, 42, 56, 64, 68, 69 & 62 kgs. Find whether there is any difference between medicines?

Sol. $H_0 : \mu_1 = \mu_2$ (there is no significant difference between the medicines A and B as regards on increase in weight).

$H_1 : \mu_1 \neq \mu_2$

Level of significance = 5%.

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad , \quad S_p = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

$$t = \frac{|44 - 57|}{\sqrt{125.6\left(\frac{1}{7} + \frac{1}{75}\right)}} = 1.98$$

$$t_{tab\,(5+7-2)} = t_{tab\,(10)} = 2.228$$

$2.228 > 1.98$

$t_{tab} > t_{cal}$.

We fail to reject H_0.

Q3 The summary of the results of an yield trial on onion with two methods differ with regards to onion yield. The onion yield is given in kg/plot.

Method I	Method II
$n_1 = 12$	$n_2 = 12$
$\bar{x}_1 = 25.25$	$\bar{x}_2 = 28.83$
$SS_1 = 186.25$	$SS_2 = 737.6667$

$H_0 : \mu_1 = \mu_2$ (the two propagation method do not differ with regard to onion yield)

$H_1 : \mu_1 \neq \mu_2$ (two tail test)

Level of Significance = 5%

$$t = \frac{|\bar{x}_1 - \bar{x}_2|}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

$$S^2 = \frac{\left[\sum x_1^2 - \frac{(\sum x_1)^2}{n_1}\right] + \left[\sum x_2^2 - \frac{(\sum x_2)^2}{n_2}\right]}{n_1 + n_2 - 2}$$

$$S^2 = \frac{SS_1 + SS_2}{n_1 + n_2 - 2} = \frac{186.25 + 737.6667}{12 + 12 - 2} = 41.9962$$

$$t = \frac{|25.25 - 28.83|}{\sqrt{41.9962\left(\frac{1}{12} + \frac{1}{12}\right)}} = \frac{3.58}{\sqrt{6.9994}} = 1.353$$

$t_{cal} = 1.353$

$t_{tab} = t_{0.05, 22} = 2.064$

$t_{tab} > t_{cal}$

H_0 not rejected.

now, we have studied two ~~spe~~ sample hypothesis tests that are used for analyzing the data of two groups.

Hypothesis testing can be done for multiple groups - that is more than two groups of data.

The following test that we can use when we have multiple groups:

1) χ^2 - test for categorical variables that determine whether there is a difference in the population proportions between two or more groups.

2) The one way analysis of variance (ANOVA) for numerical variables that determine whether there is a difference in the means among more than two groups.

χ^2 - test is based on a comparison of the actual count (or frequency) in each cell, the intersection of a row and column, with the frequency that would be expected to occur if the null-hypothesis were true. The expected frequency for each cell is obtained by multiplying the row total of that cell by the column total of the cell and dividing by the total sample size.

$$\text{Expected frequency} = \frac{(\text{Row total})(\text{Column total})}{\text{Sample size.}}$$

Chi-square Test (χ^2 – test)

It is an important ~~tool~~ test of hypothesis which fall in non-parametric test. This test was first introduced by Karl Pearson in the year 1900.

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where O refers to the observed frequencies and E refers to the expected frequencies.

Uses of Chi-square test

A) A test of independence

This test is helpful in detecting the association between two or more attributes. Suppose we have N observations classified according to two attributes.

If $\chi^2_{cal} < \chi^2_{tab}$.

we fail to reject H_0, and it is conculded that the two attributes have no association that means they are independent & vice-versa.

Q1 From the data given below, find out whether there is any relationship between gender and the preference of colour

Colour	Male	Female	Total
Red	25	45	70
Blue	45	25	70
Green	50	10	60
Total	120	80	200

For $\nu = 2$, $\chi^2_{0.05} = 5.991$

H_0 : There is no relationship b/w gender and preference of colour.

H_1 : There is no relationship b/w gender and preference of colour.

(Male) $E = \dfrac{70 \times 120}{200}$

$= 42$

(Female) $E = \dfrac{70 \times 80}{200}$

$= 28$

Colour	Gender	O	E	O − E	$(O-E)^2$	$(O-E)^2/E$
Red	M	25	42	−17	289	6.88
	F	45	28	17	289	10.32
Blue	M	45	42	3	9	0.21
	F	25	28	−3	9	0.32
Green	M	50	~~42~~ 36	14	196	5.44
	F	10	~~28~~ 24	−14	196	8.16

$\chi^2 = 31.33$

Degrees of freedom = (No. of rows − 1) (No. of columns − 1)
$= (3-1)(2-1) = 2$

$\chi^2_{2,\,0.05} = 5.99$

$\chi^2_{tab} > \chi^2_{cal}$

H_0 is rejected.

Q3 Two hundred bolts were selected at random from the output of each of the five machines. The number of defective bolts found were 5, 9, 13, 7 & 6. Is there a significant difference among the machines? Use 5% level of significance. (Given for $\nu = 4$, $\chi^2_{0.05} = 9.488$)

As there are 5 machines, the total no. of defective bolts should be equally distributed among these machines.

So Expected no. of defective bolts for each machine (E) $= \dfrac{\text{Sum of defective bolts}}{\text{No. of machines producing these defective bolts}}$

In an anti malaria campaign in a certain area, quinine was administered to 812 persons out of a total population of 3248. The number of fever cases is shown below:

Treatment	Fever (A)	No fever (a)	Total
Quinine (B)	140 (AB)	30 (aB)	170 (B)
No Quinine (b)	60 (Ab)	20 (ab)	80 (b)
Total	200 (A)	50 (a)	250 (N)

Discuss the usefullness of quinine in checking malaria.

For $\nu = 1$, $\chi^2_{0.05} = 3.84$

H_0 : Quinine is not effective in checking malaria.

H_1 : Quinine is effective in checking malaria.

Applying χ^2 test

$E_{AB} = \dfrac{200 \times 170}{250} = 136.$ $\qquad E_{ab} = \dfrac{80 \times 50}{250} = 64$

O	E	(O − E)	$(O-E)^2$	$(O-E)^2/E$
140	136	4	16	0.118
60	64	−4	16	0.250
30	34 $\frac{170 \times 50}{250}$	−4	16	0.471
20	16	4	16	1.000
				1.839

$\chi^2_{cal} = 1.839$ $\qquad$ Degrees of freedom = (# Rows − 1)(# Columns − 1) = (2−1)(2−1)

$\chi^2_{cal} < \chi^2_{tab}$

We fail to reject H_0.

ANOVA (Analysis of Variance)

It is an analytic tool used in statistics that splits the aggregate variability found inside a data set into two parts

(1) Systemic factors

↓

Have a statistical influence on the given data set

(2) Random factors

↓

Do not have

I have an option to compare the means pairwise in t-test but it will increase Type I error unnecessarily in making decision. But ANOVA helps me to compare means simultaneously ~~with~~ ~~to~~ draw inferece conclusion.

Independent Variable

Treatment variable

Dependent Variable (Response variable)

Factor

Mean for particular factor (One) for more than two groups.

Assumptions.

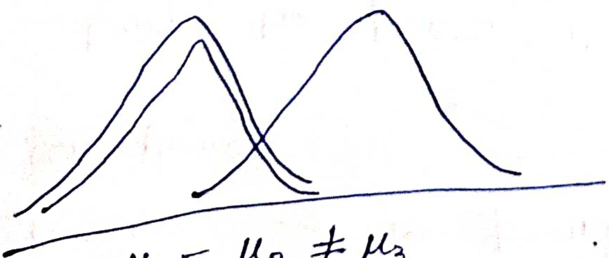
✓ Populations are normally distributed.

✓ Populations have equal variance.

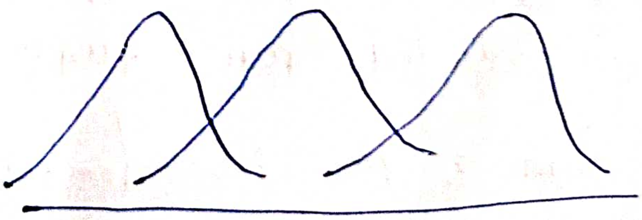
✓ Samples are randomly & independently drawn.

$H_0 : \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_c$

H_1 : Not all μ_j are the same.



$\mu_1 = \mu_2 \neq \mu_3$



$\mu_1 \neq \mu_2 \neq \mu_3$

Total variation, $SST = SSA + SSW$.

SST – Total sum of squares.

SSA = Sum of squares Among Groups

SSW = Sum of squares within Groups.

$$SST = \sum_{j=1}^{c} \sum_{i=1}^{n_j} (X_{ij} - \bar{\bar{X}})^2$$

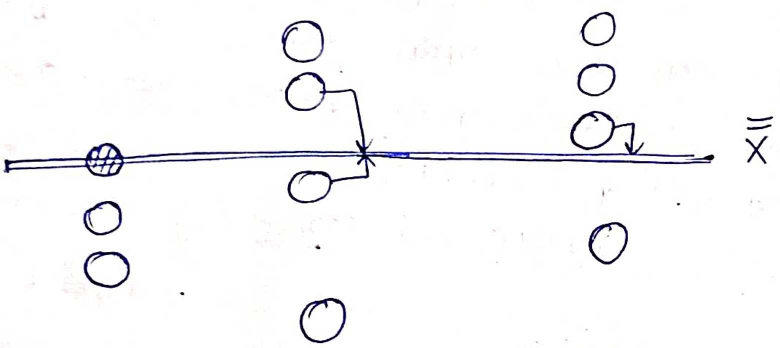
SST = Total sum of squares

c = no. of groups

n_j = no. of observations in group j.

X_{ij} = ith observation from group j.

$\bar{\bar{X}}$ = Grand mean



$$SST = (X_{11} - \bar{\bar{X}})^2 + (X_{12} - \bar{\bar{X}})^2 + (X_{13} - \bar{\bar{X}})^2 + \cdots + (X_{21} - \bar{\bar{X}})^2 + (X_{22} - \bar{\bar{X}})^2 + \ldots$$

SSA = Sum of squares among groups

c = no. of groups

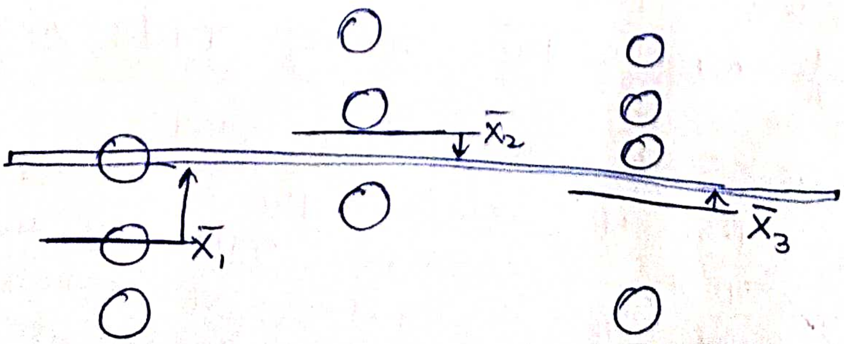
n_j = Sample size from group j

X_j = Sample mean from group j.

$\bar{\bar{X}}$ = Grand mean

$$SSA = \sum_{j=1}^{c} n_j (\bar{X}_j - \bar{\bar{X}})^2$$

$$SSA = n(\bar{X}_1 - \bar{\bar{X}})^2 + n_2(\bar{X}_2 - \bar{\bar{X}})^2 + \ldots$$



Among $= \frac{SSA}{c-1} = \frac{SSA}{\text{degrees of freedom}}$

SSW = Sum of squares within groups

c = no. of groups

n_j = Sample size from group j.

$\bar{X}_j$ = Sample mean from group j.

X_{ij} = ith observation in group j

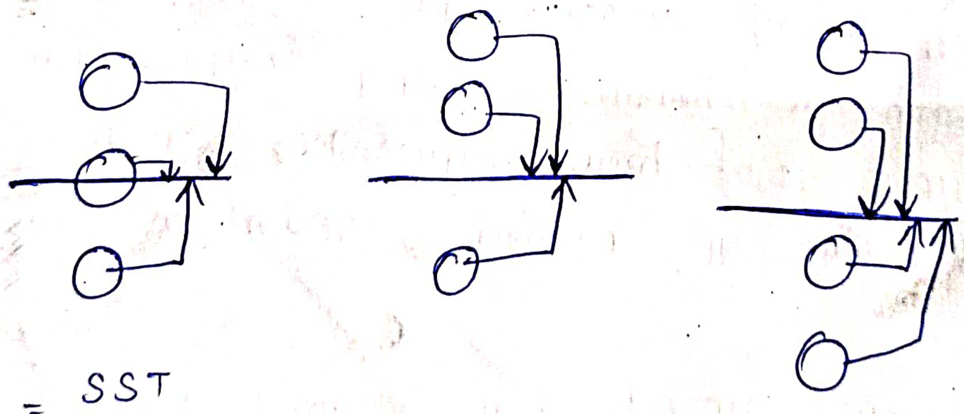
$$SSW = \sum_{j=1}^{c} \sum_{i=1}^{n_j} (X_{ij} - \bar{X}_j)^2$$

Mean square within $= \frac{SSW}{n-c} = \frac{SSW}{\text{degrees of freedom}}$

Total df $= df_A + df_W$

$n-1 = c-1 + df_W \Rightarrow df_W = n-c$

$$SSW = (X_{11} - \bar{X}_1)^2 + (X_{12} - \bar{X}_1)^2 + \cdots$$
$$(X_{21} - \bar{X}_2)^2 + (X_{22} - \bar{X}_2)^2 + \cdots$$



$$MST = \frac{SST}{n-1}$$

Source of Variation	df	Sum of Squares	Mean Square (Variance)	
Among Groups	$c-1$	SSA	$MSA = \frac{SSA}{c-1}$	$F_{stat} = \frac{MSA}{MSW}$
Within Groups	$n-c$	SSW	$MSW = \frac{SSW}{n-c}$	
Total	$n-1$	SST		

...... Manager wants to check the defects happening in three different shifts in the mobile assembly factory. He has analyzed the data of last five years and the average defects (including minor & major) in each year are reported in the table. At 0.05 significance level, is there a difference in mean distance?

SSA – SSA

Shift 1	Shift 2	Shift 3
254	234	200
263	218	222
241	235	197
237	227	206
251	216	204
1246	1130	1029
$\bar{x}_1 = 249.2$	$\bar{x}_2 = 226$	$\bar{x}_3 = 205.8$

Total sum = 3405

$$\bar{\bar{X}} = \frac{3405}{15} = 227$$

$n_1 = 5$, $n_2 = 5$, $n_3 = 5$, $c = 3$, $n = 15$

$$SSA = 5(249.2 - 227)^2 + 5(226 - 227)^2 + 5(205.8 - 227)^2$$

$$= 5 \times 492.84 + 5 \times 1 + 5 \times 449.44$$

$$= 2464.2 + 5 + 2247.2$$

$$= 4716.4$$

$$SSW = (254 - 249.2)^2 + (263 - 249.2)^2 + \dots + (251 - 249.2)^2$$
$$+ (234 - 226)^2 + (218 - 226)^2 + \dots + (216 - 227)^2 +$$
$$(200 - 205.8)^2 + (222 - 205.8)^2 + \dots + (204 - 205.8)^2$$

$$= 1119.6$$

$$MSA = \frac{SSA}{c-1} = \frac{4716.4}{2} = 2358.2, \quad MSW = \frac{SSW}{n-c} = \frac{1119.6}{12} = 93.3$$

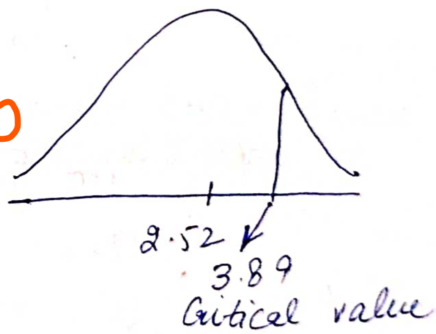
$$F_{stat} = \frac{MSA}{MSW} = \frac{2358.2}{93.3} = 25.275455$$

$$\cancel{F_{n-1,\,0.05} = F_{14,\,0.05} =}$$

$$c-1 = 2, \quad n-c = 12$$

$$F_{2,12} = 3.89$$

$$\alpha = 0.05$$

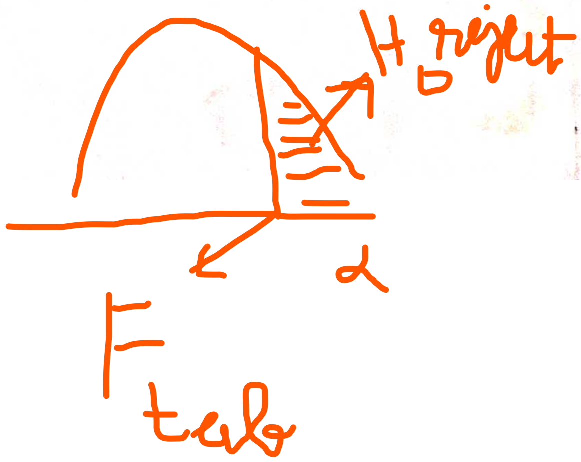


$$F_{tab} < F_{cal}$$

reject H_0.

If Ftab > Fcal
If Ftab > 25.275455

Ho reject not



We have studied variability in the data, specifically variability among the groups & variability within the groups.

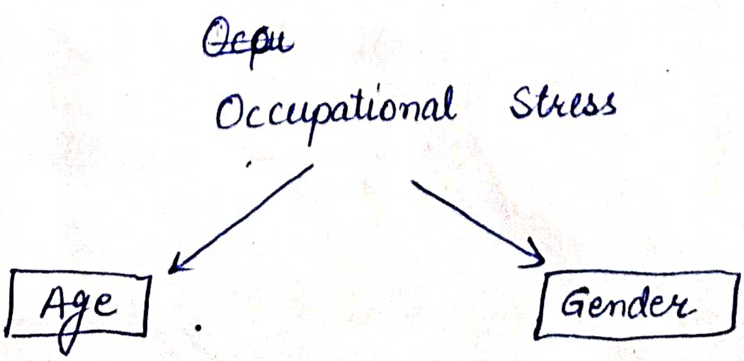
Analysis of Variance

- ANOVA is a technique of testing hypotheses about the significant difference in several population means.
- In ANOVA, the total variation in the sample data can be on account of two components, namely, variance b/w the sample and variance within the samples.
- Variance b/w the samples is attributed to the difference among the sample means. This variance is due to some assignable causes.
- Variance within the samples is the difference due to chance or experimental errors.

What is Two Way ANNOVA?

In some real-life situation, an analyst needs to explore two or more treatments simultaneously. This type of experimental design is referred to as factorial design. In a factorial design, two or more treatment variables are studied simultaneously.

Shubham 8392 @gmail.com



(2) paper 10%
(1) plague 10%
(3) print out
(4) Hard binding

Null and Alternative Hypotheses in Two-way ANNOVA

Row Effect

H_0 : All the row means are equal.

H_1 : All the row means are not equal.

Column Effect

H_0 : All the column means are equal.

H_1 : All the column means are not equal.

Interaction Effect

H_0 : Interaction effects are zero.

H_1 : Interaction effect is not zero.

We have to calculate

Sum of squares between columns : SSC

Sum of squares between rows : SSR

Sum of squares interaction : SSI

Sum of squares of errors : SSE

Total : SST

Sources	Degrees of freedom	Sum of Squares	Mean square	F
A	$a-1$	SSA	$SSR/R-1$	MSR/MSE
B	$b-1$	SSB	$SSC/C-1$	MSC/MSE
AB	$(a-1)(b-1)$	SSI	$SSI/(R-1)(C-1)$	MSI/MSE
Error	$N-ab$	SSE	$SSE/N-RC$	
Total	$N-1$	SST		

b : No. of variables along column

a : No. of variables along row.

A Quality Manager wants to determine whether the brand of laundary detergent used and the temperature of water affects the amount of dirt removed from the laundary.

Water Temperature

	10°C	25 °C	50 °C	Total
A	4	7	10	21
	5	9	12	26
	6	8	11	25
	5	12	9	26
Total	20 (5)	36 (9)	42 (10.5 ~ 10)	98 (8)
B	6	13	12	31
	6	15	13	34
	4	12	10	26
	4	12	13	29
Total	20 (5)	52 (13)	48 (12)	120 (10)
	40 (5)	88 (11)	90 (11)	$\frac{218}{24}$ ~ (9)

$$SSE = \cancel{\sum_i (x_i - \bar{x}_1)^2 + (x_i - \bar{x}} \sum_i (x_i - \bar{x}_i)^2$$

$$= (4-5)^2 + (5-5)^2 + (6-5)^2 + (5-5)^2 + (7-9)^2 + (9-9)^2 + (8-9)^2 + (12-9)^2 + (10-10)^2 + (12-10)^2 + (11-10)^2 + (9-10)^2 + (6-5)^2 + (6-5)^2 + (4-5)^2 + (4-5)^2 + (13-13)^2 + (15-13)^2 + (12-13)^2 + (12-13)^2 + (12-12)^2 + (13-12)^2 + (10-12)^2 + (13-12)^2$$

$$= 1+0+1+0+4+0+1+9+0+4+1+1+1+1+1+1+0+4+1+1+0+1+4+1$$

$$= 38$$

Degrees of freedom $= N - ab = 24 - 2 \times 3 = 18$

$$SSA = r \cdot b \sum (\bar{y}_i - \bar{\bar{y}})^2$$
$$= 4 \times 3 \left[(8-9)^2 + (10-9)^2 \right] = 4 \times 3 \times 2 = 24$$

Degrees of freedom $= a - 1 = 2 - 1 = 1$

$$SSB = r \cdot a \sum (\bar{y}_j - \bar{y})^2$$
$$= 4 \times 2 \left[(5-9)^2 + (11-9)^2 + (11-9)^2 \right]$$
$$= 8 \left[16 + 4 + 4 \right] = 8 \times 24 = 192$$

Degrees of freedom $= b - 1 = 3 - 1 = 2$

$$SSAB = r \times \sum_i \sum_j (\bar{y}_{ij} - \bar{y}_i - \bar{y}_j + \bar{\bar{y}})$$
$$= 4 \times (5-8-5+9)^2 + 4 \times (9-8-11+9)^2 + 4 \times (10-8-11+9)^2 + 4 \times (12-10-11+9)^2$$
$$+ 4 \times (5-10-5+9)^2 + 4 \times (13-10-11+9)^2 + 4 \times (1) + 4 \times (1) + 4 \times (0)$$
$$= 4 \times (1) + 4 \times (1) + 4 \times (0)$$
$$= 4 + 4 + 4 + 4 = 16$$

Degrees of freedom $= (a-1) \times (b-1) = (2-1) \times (3-1) = 2$

Sources	Degrees of freedom	Sum of Squares	Mean square	F
A	$a-1 = 2-1 = 1$	$SSA = 24$	$\frac{SSA}{a-1} = 24$	$MSA/MSE = 11.368$
B	$b-1 = 3-1 = 2$	$SSB = 192$	$\frac{SSB}{b-1} = 96$	$MSC/MSE = 45.476$
AB	$(a-1)(b-1) = 2$	$SSI = 16$	$\frac{SSI}{(a-1)(b-1)} = 8$	$MSI/MSE = 3.789$
Error	$N - ab = 24 - 6 = 18$	$SSE = 38$	$\frac{SSE}{N-ab} = 2.111$	
	$N-1 = 24-1 = 23$			

$F_{18, 1} = 3.01$ $\quad F_{tab} < F_{cal}$

$F_{18, 2} = 2.62$ $\quad H_0$ rejected.

1