Regular Expression

- **Priority of operators of RE :**

( ), a^*, a^+, $\cdot$, $+$.

- **Definition :**

1.) RE is said to be valid iff it can be derived from RE by a finite no. of application of the rule r^*, r^+, $r_1 . r_2$, $r_1 + r_2$.

2.) if Σ is a given alphabet then, ϕ, ε/λ, $a \in \Sigma$ are REs.

1. $a + \phi = a$
2. $a + \varepsilon = \{a, \varepsilon\}$

ϕ = null
ε = string of length 0

3. $r = a+b$, $L(r) = \{a, b\}$
4. $r = a \cdot b$, $L(r) = \{a \cdot b\}$
5. $r = a+b+c$, $L(r) = \{a, b, c\}$
6. $r = (ab+a) \cdot b$, $L(r) = \{abb, ab\}$
7. $r = a^+$, $L(r) = \{a, aa, aaa, aaaa \dots\}$
8. $r = a^*$, $L(r) = \{\varepsilon, a, aa, aaa, \dots\}$
9. $r = (a+ba)(b+a)$, $L(r) = \{ab, aa, bab, baa\}$
10. $r = (a+\varepsilon)(\underbrace{b+\phi}_{b})$, $L(r) = \{ab, b\}$
11. $r = (a+b)(a+b)$, $L(r) = \{aa, ab, ba, bb\}$
12. $r = (a+b)^*$ $L(r)$

3

$L(r) \Rightarrow (a+b)^0 = \varepsilon$

$(a+b)^1 = \{a, b\}$

$(a+b)^2 = \{aa, ab, ba, bb\}$

$(a+b)^3 = \{\cancel{aaa, b} \ldots\}$

$\therefore \{\varepsilon, ab, aabb, \ldots\}$

13) $r = (a+b)^* \cdot (a+b)$

$\{\varepsilon, \ldots\} \cdot (a+b) = (a+b)^+$

14) $r = a^* \cdot a^*$

$\{\varepsilon, a, aa \ldots\}$ $\varepsilon(a, aa, aaa)$

$= a^*$

15) $r = (ab)^* = \{\varepsilon, ab, abab, ababab \ldots\}$

$(ab)^0$

$(ab)^1$

$(ab)^2 = (ab)(ab)$

$(ab)^3 = (ababab)$

eg 1 : $r_1 = a^*$

$r_2 = a^* + (aa)^*$

(a) $L(r_1) \subseteq L(r_2)$

(b) $L(r_1) \supseteq L(r_2)$

✓ (c) $L(r_1) = L(r_2)$

(d) $L(r_1) \neq L(r_2)$

Ans: Same language can be generated by one or more Regular Expressions.

Ques 2: ϕ = empty set / null set . = 0

ε = empty string.

(a) $r = \varepsilon^*$, $L(r) = \{\varepsilon^0, \varepsilon^1, \varepsilon^2 \ldots\}$

$= \{\varepsilon, \varepsilon, \varepsilon \ldots\}$

to remove duplicacy in set $= \{\varepsilon\}$

(b) $r = \varepsilon^+$, $L(r) = \{\varepsilon^1, \varepsilon^2 \ldots\} = \{\varepsilon\}$

(c) $r = \phi^*$, $L(r) = \{\phi^0, \phi^1, \phi^2 \dots\} = \{\varepsilon\}$

$\{\varepsilon, x, x \dots\}$

$A = \{a, b\}$

eg :- $A^0 = \{\varepsilon\}$ → string of length 0

$A^1 = \{a, b\}$

$A^2 = \{aa, ab, ba, bb\}$

(d) $r = \phi^+$, $L(r) = \{\phi^1, \phi^2 \dots\} = \cancel{\{\phi\}}\ \phi$

Questions :

1. $r^+ \cup r^* = r^*$
2. $r^+ \cap r^* = r^+$
3. $r^* . r^+ = r^+$
4. $(r^*)^* = \{a^0, a^1, a^2 \dots\}^* = \{\varepsilon, a, aa \dots\}^*$

   $\Rightarrow \{\varepsilon^*, a^*, aa^* \dots\} = \{\varepsilon, a, aa \dots\}$

   $\Rightarrow r^*$
5. $(r^*)^+ = r^*$.

6. $(r^+)^* = r^*$.

7. $((r^*)^+)^* \cdot r^+ = r^*$

8. $(a+b)^* = (a^*+b)^* = (a+b^*)^*$

9. $(a+b)^* = (a^*+b^*)^*$

10. $\underset{\{a\}\checkmark}{(a+b)^*} \neq \underset{\{a\}\to\times}{(a.b)^*}$

11. $\underset{\{a\}\checkmark \; \{b\}\checkmark}{(a+b)^*} \neq \underset{\{a\}\times \; \{b\}\checkmark}{(a^* . b)^*} \neq \underset{\{b\}\times}{(a . b^*)^*} = (a^* b^*)^*$.

* <u>if you want ba from $(a^* b^*)^*$</u>

$$(a^* b^*)^2 \Rightarrow (\underset{\varepsilon}{a^*}\ \underset{b}{b^*})(\underset{a}{a^*}\ \underset{\varepsilon}{b^*})$$

How to design RE from Language (7)

(1) Start with ab:

{ab, aba, abb, abba . . . . }

$\Rightarrow ab(a+b)^*$

(2) Start with bba:

$\Rightarrow bba(a+b)^*$

(3) ends with abb:

$\Rightarrow (a+b)^* abb$

(4) contains substring abb.

$\Rightarrow (a+b)^* \; aab \; (a+b)^*$.

(5) starts and ends with a

- if a is starting and ending ~~with~~
  a
- if in starting & ending
  a (a+b)* a

$\Rightarrow a + a(a+b)^*a$

(6) start and ends with same symbol

$\Rightarrow a + a(a+b)^* a + b + b(a+b)^* b.$

(7) starts and end with different symbol.

$\Rightarrow a(a+b)^* b + b(a+b)^* a.$

(8) $|W| = 3$

$(a+b)(a+b)(a+b)$

(9) $|w| \geq 3$

$(a+b)^3 (a+b)^*$

(10) $|W| \leq 3$

$\Rightarrow \{\underbrace{\varepsilon}_{0}, \underbrace{a, b}_{1}, aa, bb, ba, ab, \ldots\}$

$\Rightarrow \underbrace{\varepsilon}_{0} + \underbrace{(a+b)}_{1} + \underbrace{(a+b)^2}_{2} + \underbrace{(a+b)^3}_{3}$

$\Rightarrow (a+b+\varepsilon)$

(1) $|W|_a = 2$

$b^* \; a \; b^* \; a \; b^*$

(2) $|W|_a \geq 2$

$(a+b)^* \; a \; (a+b)^* \; a \; (a+b)^*$

(3) $|W|_a \leq 2$

$\varepsilon + b^* a b^* + b^* a \; b^* a \; b^*$

(4) Symbol from left end is 1

$(a+b)^2 \; b \, (a+b)^*$

(5) $|W| = 0 \pmod 3$ — 0, 3, 6, 9, ...

$[\underbrace{(a+b)}_{\text{in combination of length 3}}{}^3]^*$ → for 0

(6) $|W| = 2 \pmod 3$

$(a+b)^2 \; [(a+b)^3]^*$

7) $|w|_b = 0 \pmod 2$ $\qquad b = 0, 2, 4, 6, 8 \ldots$

$a^* + (a^* b a^* b a^*)^*$

8) $|w|_a = 1 \pmod 3$

✓ $b^* a b^* \cdot (b^* a b^* a b^* a b^*)^*$

9) $|w|_b = 2 \pmod 3$

bb

7) $a^* (bb)^* . a^*$ $\qquad$ bb bb bb

$a^* \downarrow \epsilon$, $bb \downarrow 1$

~~consecutive~~

8) $(\cancel{b^* a . (aaa)^* b^*})^*$

1
4
7
10
13 $a^* bb a^* \cdot (a^* b a^* b a^* b a^*)^*$