FREEMIND

Date ______

Page ______

UNIT 1 :

CHAPTER 1.1

→ Mathematical logic : Introduction, Statement, Notations And Connectivities

→ Basic logical Operation

→ Truth Table

→ Contradictions

→ Algebra of prepositions

→ Logical Implications

→ Normal Form

→ Functionality Complete set of Connections

* DISCRETE MATHEMATICS :

→ It Is A collection of mathematical topics that Examine And Use finite And countably Infinite mathematical Objects.

→ It Is Important In computer Science because It Includes computing machines where Information Is stored And manipulated In discrete fashion.

•) APPLICATIONS :

→ Logic In programming And Software development.

→ Computer graphics Use Linear Algebra which Is part of discrete mathematics.

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Eg = Number of Students In class = 72
→ Hight of Students = Continuous Values
Eg = 5.56 ft, 7.82 Ft.
→ Today Is Friday = T / F, 0 / 1 } discrete values

* PROPOSITIONS =

→ A proposition Is the statement which can either be true Or false but not both.
→ Notations = Any Alphabetic form
Eg = p, q, r, s, - - - -.

* PROPOSITIONAL LOGIC =

→ Propositional logic Is the area of Logics which deals with propositions.
(Propositions are the statements which can either be true Or False but not both.)
→ Eg = "Elephants Are bigger than mouse."

Is It A statement ? Y
Is Is True Or False ? Y
Is It Proposition ? Y

→ Rules = (i) It should be a statement
(ii) It should be true Or false not both.

FREEMIND

Date

Page

* NEGATION =

→ Negation Is Opposite to proposition.

Eg = p : Today Is Friday

¬p : Today Is ~~Not~~ not Friday.

Or

It Is ~~No~~ not the state that today is Friday.

¬(¬p) → Today Is Friday.

☞ Note = It Is not necessary that we need to convert positive statement to negative statement by the help of Negation.

→ It Is Also possible that we can convert negative statement with positive statement by the help Of negation.

* TRUTH TABLE =

→ It Is the method of relationships between the propositions.

Eg =

p	¬p
T	F
F	T

Q. "520 < 111", Is this a proposition?

→ Is this A statement? Y

Is It True Or False? F

Is this A proposition? Y

FREEMIND

Date

Page

Q " y < 5 ", Is this a proposition ?
→ Is this statement ? Y
Is It true Or false ? NULL
Is this a proposition ? N

→ Here, y's value Is not defined hence, we can not say It Is true Or False.
Therefore NULL.
Thus, not a ~~posito~~ proposition because It Is having two rules
(i) It should be statement.
(ii) It should be true Or false but not both.

* WELL - FORMED FORMULAES =

→ Any Expression that Obeys syntactical rules of propositional logic, It Is termed As well-formed formulaes.

RULES =

(i) → It should be a capital letter with no blank space.

Eg = $(P \rightarrow Q)$

(ii) → Brackets should be present.

Eg = $\underbrace{A \land B, \ A \lor B, \ A \rightarrow B, \ A \longleftrightarrow B}_{\text{Are Also well formed formulaes.}}$

☞ **Note** = **Predicate** = It cannot be true or false until unless the value is not assigned to the variable.

Eg = $\boxed{P(x) : x > 8}$

Until unless we are not assigning the values to the variable x, hence we cannot predict the value to be true or false.

PROPOSITIONAL FUNCTION =

Q. $P(x) : x > 3$, then what is truth value of $P(4)$

⇒ Since, $\boxed{P(x) : x > 3}$ is a propositional function

$\boxed{P(4) : 4 > 3}$ True

Hence, the truth value of propositional function $P(x) : x > 3$ at $P(4)$ is true.

☞ **Note** = Proposition can be true or false but predicate cannot be true or false unless and until values is assigned to it.

* QUANTIFIERS =

→ A quantifier is an operator which is used to create proposition from propositional function.

→ There Are two types Of Quantifiers

ⓐ UNIVERSAL QUANTIFIER:-

→ P (x) Is true for all values In the Universe of discourse.

$\boxed{P(x)\ \forall x\ P(x)}$

→ $\boxed{\text{Notation} - \forall}$

ⓑ EXISENTIAL QUANTIFIER :-

→ There Exist A value of x In the Universe Of discourse such that P(x) Is true.

$\boxed{P(x)\ \exists\ x P(x)}$

→ $\boxed{\text{Notation} = \exists}$

Q. What Is the truth value Of $\forall x\, P(x)$, where P (x) Is the statement "$x^2 < 10$" And Universe Of exceeding 4?

→ Given,

∵ $\boxed{\text{Propositional function} = P(x) : x^2 < 10}$

P(0) : 0 < 10 ⟶ True

P(1) : 1 < 10 ⟶ True

P(2) : 4 < 10 ⟶ True

P(3) : 9 < 10 ⟶ True

P(4) : 16 < 10 ⟶ False

→ Hence, the truth value becomes false because At x = 4, the propositional value becomes false.

Note = We need to calculate all the propositional values in the given range. If the single value becomes false, then the whole propositional function is stated to be false.

* LOGICAL OPERATORS (CONNECTIVITIES)

(i) Conjunction (AND)
(ii) Disjunction (OR)
(iii) EX - OR (XOR)
(iv) If - then ($A \rightarrow B$)
(v) Biconditional Statement ($A \longleftrightarrow B$)

→ Connectivities are used to form one compound new proposition by connecting two or more existing proposition.

→ Connectivities are also termed as logical operators.

Q. P : Today is Friday.
Q : It is Raining.

→ $P \wedge Q$) — Conjunction

Today is Friday ~~AND~~ and It is raining.

•) CONJUNCTION =

→ Let P and Q are in conjunction then it is denoted as =

$\boxed{P \wedge Q}$

where,
Both the propositions needs to be true for conjunction value to be true.
All the other conditions are false conjunctional value.

⇒

P	Q	P ∧ Q
T	T	T
T	F	F
F	T	F
F	F	F

•) DISJUNCTION =

→ Disjunction is also called OR connectivity.

→ The two propositions P and Q are said to be in disjunction when $\boxed{P \vee Q}$

where atleast for disjunctional value to be true one ~~pro~~ statement should be true

→ All other conditions will give false value.

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

Q. Find the conjunction of the propositions P And Q where

P : Ribecas's PC has more than 16GB free hard-disk space.

Q : The processor In Ribecas's PC runs faster than 1GHz.

→ To find $\boxed{P \wedge Q}$

$P \wedge Q$ = Ribecas's PC has more than 16 GB free hard-disk space And the processor runs faster than 1GHz.

Q. Translate the statement "Students who have taken calculas Or Introductory Computer science can take this class" In A statement In propositional logic using the propositions.

→ P : Students who have taken calculas can ~~attend~~ take this class.

Q : Students who have taken Introductory computer science can take this class.

The given statement Is In disjunction which has been translated Into P and Q.

•) <u>NEGATION</u> =

→ To Interchange (Oppose) the propositional function Is termed As negation.

→ The symbols of notation = ¬ , ~ .

Eg = P : Harshida plays football.
¬P = Harshida does not plays football.

•) IMPLICATION =

→ If the given propositional function is $\boxed{P \rightarrow Q}$, then, for the value of P being true and Q being false will only give false value.
All other conditions will have true value.

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	T	T

P	Q	Q → P
T	T	T
T	F	T
F	T	F
F	F	T

In this table if Q's & value is true and P's value is false then only the resultant is false because here, Q → P.

Eg = (a) If I Am Elated then I will lower the taxes.
(b) If you get 100% In finals then you will get grade A.

Q. Let p be the statement
p : Maria learns discrete Mathematics And q be the statement ~~m~~
q : Maria will find A good job.
Express $p \to q$ As a statement In English.
→ $p \to q$: If Maria learns discrete Mathematics then ~~m~~ Maria will find A good job.

Q. What Is the value of variable x after the statement
If $2+2=4$ then $x := x+1$
If $x=0$ before this statement Is Executed.
→ $2+2=4$, Is having true value, then
$x = 0+1$
⇒ $\boxed{x = 1}$

* BICONDITIONAL STATEMENTS =
→ ~~If~~ It gives the value If And Only If.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F

Q. Let the statements be

p : You can take the flight.

q : You can buy the ticket.

→ $p \leftrightarrow q$: If you can buy the ticket then you ~~If you~~ can take the flight.

☞ Note = When p And q have same truth value then the Output comes true, then It Is termed As biconditional statement.

*. CONVERSE, CONTRAPOSITIVE, INVERSE =

(i) CONVERSE =

→ The proposition $q \to p$ Is the converse of $p \to q$

(ii) CONTRAPOSITIVE =

→ The contrapositive of $p \to q$ Is $\neg q \to \neg p$.

(iii) INVERSE =

→ The Inverse of $p \to q$ Is $\neg p \to \neg q$ And the vice versa Is Also true.

Q. Find the contrapositive converse And Inverse of the conditional statement.

"The home team wins whenever It Is raining."

→ Converse = $\boxed{p \to q = q \to p}$

If It Is raining then the home team wins.

Date
Page

Inverse = $\boxed{p \to q = \neg p \to \neg q}$

→ If the home team does not win then It Is not raining.

*. TRUTH TABLES OF COMPOUND PROPOSITIONS

Q. Construct the truth table of following compound propositions.

$(p \vee \neg q) \to (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \to (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

*. PERCEDENCE OF LOGICAL OPERATORS =

→ Percedence Of Operator means which Operator comes first And which comes next.

Operator	Percedence
$\neg$	1
$\wedge$	2
$\vee$	3
$\to$	4
$\leftrightarrow$	5

Q. $p \vee \neg q \wedge r \to s \leftrightarrow t$

$(p \vee_{3} ((\neg q) \wedge r) \xrightarrow[4]{} s) \xleftrightarrow[5]{} t)$

Q. Construct the truth table Using precedence.

$p \to (\sim q \wedge \sim p) \longleftrightarrow \sim q$

p	q	$\sim p$	$\sim q$	$(\sim q \wedge \sim p)$	$p \to (\sim q \wedge \sim p)$	r
T	T	F	F	F	F	T
T	F	F	T	F	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

* TRUTH TABLE OR, AND, XOR =

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

x	y	$x \vee y$	$x \wedge y$	$x \oplus y$
0	1	1	0	1
1	1	1	1	0
1	0	1	0	0
0	0	0	0	1

Q. Find the bitwise $\wedge$, bitwise $\vee$ And bitwise $\oplus$ Of the following strings =

01101 10110 , 11000 11101

x	y	$x \wedge y$	$x \vee y$	$x \oplus y$
0	1	0	1	1
1	1	1	1	0
1	0	0	1	1
0	0	0	0	0
1	0	0	1	1
1	1	1	1	0
0	1	0	1	1
1	1	1	1	0
1	0	0	1	1
0	1	0	1	1

* TAUTOLOGY :

→ Tautology states that the values In the final table Are always true.

T	T	T
T	F	T
F	T	T
F	F	T

→ A propositional function which Is true In Every possible case Is called tautology.

* CONTRADICTION :

→ The propositional function which Is false In every possible case, Is termed As Contradiction.

T	T	F
T	F	F
F	T	F
F	F	F

* CONTIGENCY =

→ The propositional values which has sometimes value true And sometimes false.

T	T	T
T	F	F
F	T	T
F	F	F

* LOGICAL EQUIVALENCE =

→ Two statements (~~Eg~~ propositions) Are always ~~true~~ logically Equivalent If their Equivalence Is always true.

Or

Equivalence Ensures that the truth value Is always Identical for both tables.

→ Symbol = $\equiv$

Table T1 $\equiv$ Table T2

	T

	F

Date

Page

Q. $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$\because$ $\boxed{LHS = RHS}$

$\therefore$ $\boxed{\neg(p \vee q) \equiv \neg p \wedge \neg q}$

Q $(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$

p	q	$(p \rightarrow q)$	$\neg p$	$\neg q$	$(\neg q \rightarrow \neg p)$
T	T	T	F	F	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

$\because$ $\boxed{LHS = RHS}$

$\boxed{(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)}$

Q. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$(q \wedge r)$	$p \vee (q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

$\therefore$ $\boxed{LHS = RHS}$

$$\boxed{p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)}$$

H.W. Which of the following is not equivalent to $p \leftrightarrow q$

Q.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(a) $(\neg p \vee q) \wedge (p \vee \neg q)$

p	q	$\neg p$	$(\neg p \vee q)$	$\neg q$	$(p \vee \neg q)$	$(\neg p \vee q) \wedge (p \vee \neg q)$
T	T	F	T	F	T	T
T	F	F	F	T	T	F
F	T	T	T	F	F	F
F	F	T	T	T	T	T

∵ $\boxed{LHS = RHS}$

$$\boxed{p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)}$$

(b) $(\neg p \vee q) \wedge (q \rightarrow p)$

p	q	$\neg p$	$\neg p \vee q$	$q \rightarrow p$	$(\neg p \vee q) \wedge (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

∵ $\boxed{LHS = RHS}$

$$\boxed{p \leftrightarrow q \equiv (\neg p \vee q) \wedge (q \rightarrow p)}$$

(c) $(\neg p \wedge q) \vee (p \wedge \neg q)$

p	q	$\neg p$	$(\neg p \wedge q)$	$\neg q$	$p \wedge \neg q$	$(\neg p \wedge q) \vee (p \wedge \neg q)$
T	T	F	F	F	F	F
T	F	F	F	T	T	T
F	T	T	T	F	F	T
F	F	T	F	T	F	F

∵ $\boxed{LHS \neq RHS}$

$$\boxed{p \leftrightarrow q \not\equiv (\neg p \wedge q) \vee (p \wedge \neg q)}$$

(d) $(\neg p \wedge \neg q) \vee (p \wedge q)$

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \wedge q$	$(\neg p \wedge \neg q) \vee (p \wedge q)$
T	T	F	F	F	T	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	T	F	T

∵ $\boxed{LHS = RHS}$

$$\boxed{p \leftrightarrow q \equiv (\neg p \wedge \neg q) \vee (p \wedge q)}$$

FREEMIND

Date

Page

Conclusion =

(i) Propositions = Should be
(a) Statement
(b) Either true or false but not both.
(or 0 & 1)

(ii) Propositional logic = These represents logics between propositions.

(iii) Quantifiers = These are used to convert propositions from propositional function (Operators).

Universal Quantifier

$\boxed{P(x) \ \forall x \ P(x)}$

For All values In Universe of discourse.

Existential Quantifier

$\boxed{P(x) \ \exists x \ P(x)}$

For A value In Universe of discourse.

(iv) **Well-Formed Formulaes** =
→ The syntatical rules obeys propositional logic representation In propositions.

Rules =
(a) Should be capital letter, no blank space.
(ii) Brackets should be there.

(v) **Predicates** = Unless Until values are n[ot] Assigned to the variables to assign true or false.

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Date
Page

(vi) Connectivities = These Are Also called logical Operators.

These Are connectivity between two Or more propositions to form compound proposition.

- Conjunction: $\wedge$
- Disjunction: $\vee$
- Negation: $\neg$, $\sim$
- Inverse: $\cancel{p \to q} = p \to q$, $\cancel{\ } = q \to p$
- Bi-Conditional: $p \leftrightarrow q$

(vii) Tautology = All the values In the final table Are true.

(viii) Contrapositive = $p \to q = \neg q \to \neg p$

~~vii~~ (ix) Converse = $p \to q = q \to p$

(x) ~~Implication~~ = Inverse = $p \to q = \neg p \to \neg q$

(xi) Contradiction = All Values In final table Is false.

(xii) Contingency = Some values Are true some Are false.

(xiii) Logical Equivalence = The propositional final truth values of All the tables Are logically Equal.

ALGEBRA OF PROPOSITIONS =

(i) IDEMPOTENT LAW =

(a) $p \vee p \equiv p$

(b) $p \wedge p \equiv p$

(ii) ASSOCIATIVE LAW =

(a) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

(b) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(iii) COMMUTATIVE LAW =

(a) $p \vee q \equiv q \vee p$

(b) $p \wedge q \equiv q \wedge p$

(iv) DISTRIBUTIVE LAW =

(a) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(b) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(v) IDENTITY LAW =

(a) $P \vee F \equiv P$

(b) $P \wedge F \equiv F$

(c) $P \vee T \equiv T$

(d) $P \wedge T \equiv P$

(vi) COMPLEMENT LAW =

(a) $p \vee \neg p \equiv T$

(b) $p \wedge \neg p \equiv F$

$\boxed{\neg T \equiv F}$

$\boxed{\neg F \equiv T}$

Scanned with CamScanner

FREEMIND

Date

Page

(vii) IVOLUTION LAW :

$\neg \neg p \equiv p$

(viii) DE-MORGAN'S LAW :

(a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

(b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

(ix) ABSORPTION LAW :

(a) $p \vee (p \wedge q) \equiv p$

(b) $p \wedge (p \vee q) \equiv p$

mapped

Q! $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

LHS =

$\neg((p \vee \neg p) \wedge (p \vee q))$ ⟶ By distributive law

$\neg(p \wedge q)$

$\boxed{\neg p \vee \neg q}$ ⟶ $\boxed{\text{RHS}}$

mapped

Q. $p \to (q \to r) \equiv (p \wedge q) \to r$

LHS =

$\boxed{p \to q = \neg p \vee q}$

$p \to (q \to r)$

$\neg p \vee (q \to r)$

$\neg p \vee (\neg q \vee r)$

$\cancel{(\neg p \vee \neg q) \vee (\neg p \vee r)}$

$(\neg p \vee \neg q) \vee r$ ⟶ By Associative law.

$\neg(p \wedge q) \vee r$ ⟶ By de-morgans law

$\boxed{(p \wedge q) \to r}$ ⟶ RHS

Q. $p \wedge r (\Leftrightarrow s) \vee (s \Leftrightarrow p)$

Since, $\boxed{p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$\Rightarrow \boxed{p \wedge ((r \to s) \wedge (s \to r)) \vee ((s \to p) \wedge (p \to s))}$

Q. $R \vee (S \longleftrightarrow T)$

$\Rightarrow$ ~~R v~~ Since, $\boxed{p \longleftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$\boxed{R \vee ((S \to T) \vee (T \to S))}$

$\because \boxed{p \to q \equiv \neg p \vee q}$

$\boxed{R \vee ((\neg S \vee T) \vee (\neg T \vee S))}$

$p \leftrightarrow q$

$(p \to q) \wedge (q \to \ldots$

$q \to p$

$(q \to p) \wedge (p \to q$

Q. $p \longleftrightarrow q$

$\Rightarrow$ Since, $\boxed{p \longleftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$\Rightarrow (p \to q) \wedge (q \to p)$

$\Rightarrow$ Since, $\boxed{p \to q \equiv \neg p \vee q}$

$\Rightarrow (\neg p \vee q) \wedge (\neg q \vee p)$

$\because \boxed{\neg (p \vee q) \equiv \neg p \wedge \neg q}$

$\because \neg p \vee q \equiv \neg (p \wedge \neg q)$

$\boxed{\neg (p \wedge \neg q) \wedge (\neg (q \wedge \neg p))}$

Q. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

LHS =

$\Rightarrow \neg(p \lor (\neg p \land q))$

By de-Morgan's law

$\boxed{\neg(p \lor q) \equiv \neg p \land \neg q}$

$\Rightarrow \neg p \land \neg(\neg p \land q)$

$\Rightarrow \neg p \land (\neg\neg p \lor \neg q)$

By Involution law

$\boxed{\neg\neg p \equiv p}$

$\Rightarrow \neg p \land (p \lor \neg q)$

By the help of distributive law

$\boxed{p \land (q \lor r) \equiv (p \land q) \lor (p \land r)}$

$\Rightarrow (\neg p \land p) \lor (\neg p \land \neg q)$

By the help of Complement law

$\boxed{\neg p \land p \equiv F}$

$\Rightarrow F \lor (\neg p \land \neg q)$

By using Identity law

$\boxed{p \lor F \equiv p}$

And By the help of Commutative law

$\boxed{p \lor q \equiv q \lor p}$

$\boxed{p \lor F \equiv F \lor p} \equiv$

$\boxed{F \lor (\neg p \land \neg q) \equiv \neg p \land \neg q}$

FREEMIND

Date ______

Page ______

* Conclusion =

(i) Idempotent law = (a) $p \vee p \equiv p$, (b) $p \wedge p \equiv p$

(ii) Associative law = (a) $(p \vee q) \vee r = \cancel{(p \vee r) \vee}$ $p \vee (q \vee r)$

(b) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

(iii) Commutative law =

(a) $p \vee q = q \vee p$, (b) $p \wedge q = q \wedge p$

(iv) Distributive law =

(a) $p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$

(b) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

(v) De-Morgan's law

(a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

(b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

* PRINCIPLES OF DUALITY =

→ Two formulas A1 and A2 are said to be dual of each other if either one can be obtained from the other by replacing AND ($\wedge$) with OR ($\vee$) and vice versa.
Also if the formulas contains true or false then we will replace true by false and vice versa to obtain the dual.

Note : (i) The two connectives AND($\wedge$) and OR($\vee$) are called dual of each other.
(ii) Like AND($\wedge$) and OR($\vee$), NAND($\uparrow$

FREEMIND

Date

Page

" And NOR($\downarrow$) Are Also called dual of each other.

Eg : (i) $p \wedge (q \wedge r)$

$\boxed{p \vee (q \vee r)}$

(ii) $\boxed{\neg p \vee \neg q \equiv \neg p \wedge \neg q}$

(iii) $(p \wedge \neg q) \vee (\neg p \wedge q)$

$\boxed{(p \vee \neg q) \wedge (\neg p \vee q)}$

(iv) $(p \uparrow q) \uparrow (p \uparrow q)$

$\boxed{(p \downarrow q) \downarrow (p \downarrow q)}$

Q. $((\neg p \vee q) \wedge (q \wedge \neg S)) \vee (p \vee F)$

$\boxed{((\neg p \wedge q) \vee (q \vee \neg S)) \wedge (p \wedge T)}$

* FUNCTIONALLY COMPLETE SET OF CONNECTIVES =

→ A set of connectives Is called functionally complete If every formula can be Expressed In terms of Equivalent formulae containing the connectives from this set.

Q. Write Equivalent formulae for

$p \wedge r\ (\Leftrightarrow s)\ \vee (s \Leftrightarrow p)$

$\because \boxed{(p \Leftrightarrow q) \equiv (p \to q) \wedge (q \to p)}$

Hy, Similarly,

$p \wedge r\ (\Leftrightarrow s)\ \vee\ ((s \to p) \wedge (p \to s))$

$\boxed{p\ ((r \to s) \wedge (s \to r))\ \vee ((S \to p) \wedge (p \to s))}$

Q. Write an equivalent formulae $R \vee (S \Leftrightarrow T)$ which does not involve Biconditional and Implication.

$\Rightarrow\ \because \boxed{p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$R \vee ((S \to T) \wedge (T \to S))$

$\because \boxed{p \to q \equiv (\neg p \vee q)}$

$\boxed{R \vee ((\neg S \vee T) \wedge (\neg T \vee S))}$

Q. Show that $(\neg p, \wedge)$ is functionally complete

$(p \Leftrightarrow q)$

$\because \boxed{p \Leftrightarrow q \equiv (p \to q) \wedge (q \to p)}$

$\Rightarrow (p \to q) \wedge (q \to p)$

$\because \boxed{p \to q \equiv \neg p \vee q}$

FREEMIND

Date

Page

$\Rightarrow (\neg p \vee q) \wedge (\neg q \vee p)$

$\boxed{p \vee q \equiv \neg(\neg p \wedge \neg q)}$

$\neg p \vee q \equiv \neg(\neg\neg p \wedge \neg q)$

$\boxed{\neg p \vee q \equiv \neg(p \wedge \neg q)}$

Similarly,

$(\neg q \vee p) \equiv \neg(\neg\neg q \wedge \neg p)$

$\boxed{(\neg q \vee p) \equiv \neg(q \wedge \neg p)}$

$\therefore$ A/q,

$\boxed{\neg(p \wedge \neg q) \wedge (\neg(q \wedge \neg p))}$

Q Express $p \leftrightarrow Q$ In terms of $\{\neg, \wedge\}$.

$\Rightarrow p \leftrightarrow Q \equiv (p \rightarrow Q) \wedge (Q \rightarrow p)$

$\because$ $\boxed{p \rightarrow Q \equiv \neg p \vee Q}$

$\therefore (\neg p \vee Q) \wedge (\neg Q \vee p)$

$\because$ $\boxed{(\neg p \vee Q) \equiv \neg(p \wedge \neg Q)}$

(By de-Morgan's Law)

$\boxed{\neg(p \wedge \neg Q) \wedge (\neg(Q \wedge \neg P))}$

* NORMAL FORMS =

→ Let A (P1, P2, P3, ----, Pn) be a statement formulae then the construction of truth table may not be practical always so we consider alternate procedure known as reduction to normal form.

(i) DISJUNCTION NORMAL FORM =

→ A statement form which consists of disjunction between conjunction is called DNF.

Eg = (a) $(p \wedge q) \vee r$

(b) $(p \wedge \neg q) \vee (\neg p \vee r) \vee (r \wedge \neg q)$

(ii) CONJUNCTION NORMAL FORM =

→ A statement which consists of conjunction between disjunction is called CNF.

Q. Reduce to DNF

$(p \rightarrow q) \wedge (\neg p \wedge q)$

$\Rightarrow \boxed{(p \rightarrow q) = \neg p \vee q}$

$\Rightarrow (\neg p \vee q) \wedge (\neg p \wedge q)$

$\Rightarrow (\neg p \wedge (\neg p \wedge q)) \vee (q \wedge (\neg p \wedge q))$

$\Rightarrow (\neg p \wedge \neg p \wedge q) \vee (q \wedge \neg p \wedge q)$

$\Rightarrow \boxed{(\neg p \wedge q) \vee (\neg p \wedge q)}$ a.

Q. Reduce to CNF.

$(p \wedge q) \vee (\neg p \wedge q \wedge r)$

$\Rightarrow (p \vee (\neg p \wedge q \wedge r)) \wedge (q \vee (\neg p \wedge q \wedge r))$

$\Rightarrow ((p \vee \neg p) \wedge (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p) \wedge (q \wedge q) \wedge (q \vee r))$

$\Rightarrow \boxed{((p) \wedge (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p) \wedge (q) \wedge (q \vee r))}$

Q. Obtain the CNF of the form and obtain the tautology as well.

$((p \to q) \wedge (q \vee (p \wedge r))$

$\Rightarrow \boxed{p \to q \equiv (\neg p \vee q)}$

$\Rightarrow ((\neg p \vee q) \wedge (q \vee (p \wedge r)))$

$\Rightarrow \boxed{((\neg p \vee q) \wedge ((q \vee p) \wedge (q \vee r))}$

~~¬p A~~

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	r	$p \to q$	$p \wedge r$	$q \vee (p \wedge r)$	$(p \to q) \wedge (q \vee (p \wedge r))$
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	F
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	F	F
F	F	F	T	F	F	F

Q. Obtain the DNF of

$$p \vee (\neg p \to q \vee (q \to \neg r))$$

$\Rightarrow \because$ $\boxed{p \to q = \neg p \vee q}$

$$p \vee (\neg p \to q \vee (\neg q \vee \neg r))$$

$$\Rightarrow p \vee (\neg p \to ((q \vee \neg q) \vee (q \vee \neg r))$$

$$\Rightarrow p \vee (\neg p \to (q \vee (q \vee \neg r))$$

$\because$ $\boxed{p \to q \equiv \neg p \vee q}$

$$\Rightarrow p \vee (p \to \vee (q \vee (q \vee \neg r))$$

$$\Rightarrow p \vee (p \vee q) \vee (p \vee (q \vee \neg r))$$

$$\Rightarrow (p \vee p \vee q) \vee (p \vee p \vee (q \vee \neg r))$$

$$\Rightarrow (p \vee q) \vee (p \vee (q \vee \neg r))$$

$$\boxed{(\neg p \wedge \neg q) \vee (\neg p \wedge \neg(\neg q \wedge r))}$$

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FREEMIND

Date

Page

Q Find CNF of $p \wedge (p \rightarrow q)$

$\Rightarrow$ $\because$ $\boxed{p \rightarrow q \equiv \neg p \vee q}$

$\Rightarrow p \wedge (\neg p \vee q)$

$(p \wedge \neg p) \vee (p \wedge q)$

$\Rightarrow p \vee (p \wedge q)$

$(p \vee p) \wedge (p \vee q)$

$p \wedge (p \vee q)$

$\because$ $\boxed{p \equiv p \vee F}$ (By ~~Com~~ Identity algebra of proposition)

$\therefore$ $\boxed{(p \vee F) \wedge (p \vee q)}$

FREEMIND

Date

Page

* ARGUMENT =

→ Argument means collection of statements.

* CONCLUSIONS =

→ Conclusion Is the proposition that Is Asserted On the basis of other propositions of Argument.

* PREMISES =

→ Propositions which Are Assumed for Accepting the conclusion of Premises.

* VALID ARGUMENT =

→ An Argument Is valid If conclusion Is true whenever All premises Are true.

* FALACY ARGUMENT =

→ An Argument Is called falacy Argument If it Is not A valid Argument.

Q. Show that the following rule Is valid Or not

$$p \therefore p \vee q$$

⇒

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

→ Valid

* TYPES OF INFERENCE RULES = (ELEMENTARY VALID ARGUMENT FORMS)

(i) **Modus Ponens Rules** =

$$\frac{\begin{array}{c} p \to q \\ p \end{array}}{q}$$

(ii) **Modus Tollens Rules** =

$$\frac{\begin{array}{c} p \to q \\ \neg p \end{array}}{\neg q}$$

(iii) **Hypothetical Syllogism** =

$$\frac{\begin{array}{c} p \to q \\ q \to r \end{array}}{p \to r}$$

(iv) **Disjunction Syllogism** =

$$\frac{\begin{array}{c} p \vee q \\ \neg p \end{array}}{q}$$

(v) **Addition** -

$$\frac{p}{p \vee q}$$

(vi) **Simplification** =

$$\frac{p \wedge q}{p} \quad \text{Or} \quad \frac{p \wedge q}{q}$$

(vii) Conjunction =

$$\begin{array}{l} p \\ q \\ \hline p \wedge q \end{array}$$

(viii) Resolution =

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline q \vee r \end{array}$$

Q. Show that the rule Modus Ponens Is Valid Or Not

$$\begin{array}{l} p \to q \\ p \\ \hline q \end{array}$$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

→ Valid

Q. Show that hypothetical syllogism Is valid

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

Date ______

Page ______

p	q	r	$p \to q$	$q \to r$	$p \to r$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

Valid

Q Show that Modus Tollens Rule Is Valid Or Not

$$\begin{array}{l} p \to q \\ \neg p \\ \hline \neg q \end{array}$$

p	q	$\neg p$	$p \to q$	$\neg q$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

← Valid

Q Show that disjunction syllogism Is valid Or not.

$$\begin{array}{l} p \vee q \\ \neg p \\ \hline q \end{array}$$

Date
Page

p	q	$p \vee q$	$\neg p$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	T

← Valid

Q. Simplification

$$\frac{p \wedge q}{p} \quad \text{Or} \quad \frac{p \wedge q}{q}$$

(i) (ii)

For (i) and (ii) = (i)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

← Valid

Q. Conjunction =

$$\frac{\begin{array}{c} p \\ q \end{array}}{p \wedge q}$$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

← Valid

Date ______

Page ______

Q "Consider the following Argument And deter-mine wether It Is valid Or not

S_1 = ~~If~~ If I will get selected In IS Examination then I will not be able to go to London

S_2 = Since I Am going to London, I will not select In IS Examination.

⇒ p = I will get selected In IS Examination

q = I Am going to London

$$\begin{array}{l} p \rightarrow \neg q \\ q \\ \hline \neg p \end{array}$$

p	q	$\neg p$	$p \rightarrow q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Valid

Q. Consider the following Argument And determine wether It Is valid Or not.

S_1 = Either I will get good marks Or I will not graduate.

S_2 = If I did not graduate I will go to Canada.

S_3 = I get good marks

S_4 = Then I would not go to Canada.

→ p = I will get good marks
q = I will graduate
r = I will go to Canada

Q

$p \vee \neg q$

$\neg q \to r$

p

$\neg r$

p	q	r	$\neg q$	$p \vee \neg q$	$\neg q \to r$	$\neg r$
T	T	T	F	T	T	F
T	T	F	F	T	T	T
T	F	T	T	T	T	F
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	T	F	F	F	T	T
F	F	T	T	T	T	F
F	F	F	T	T	F	T

Q State which rule of Inference Is Used

Q1: It Is below freezing now therefore It Is below freezing or raining now

→ P: It Is below freezing now
Q: It Is raining now.

p

$p \vee q$ By Addition rule

Date
Page

Q. "Show that premises is valid
It is not sunny this Afternoon And It is colder than Yesterday. We will go swimming Only If It is sunny.
If we do not go swimming then we will take A trip And If we take A trip then we will be home by sunset.
Lt to the conclusion we will be home by sunset.

→ Premises =

p = It is sunny this Afternoon.
q = It is colder than Yesterday.
r = We will go swimming.
s = ~~It is Sunny~~ We will take a trip.
t = We will be home by sunset.

$$\neg p \wedge q$$
$$r \rightarrow p$$
$$\neg r \rightarrow s$$
$$s \rightarrow t$$

Conclusion = t

1. $\neg p \wedge q$ (By simplification Inference rule)
2. $\neg p$
3. $r \rightarrow p$
4. $\neg r$

$\left.\begin{matrix} \neg p \\ r \rightarrow p \\ \neg r \end{matrix}\right\}$ By Modus Tollens

5. $\neg r \rightarrow s$ (By Modus Ponens)
6. s
7. $s \rightarrow t$

} By Modus Ponens

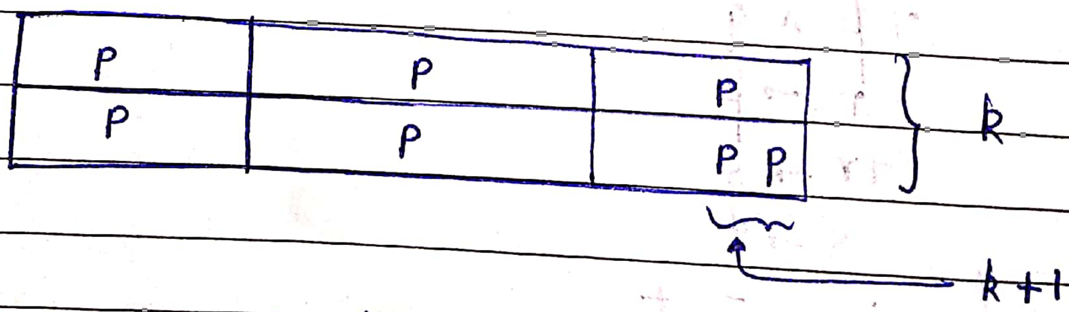
8. t

~~PEG~~ PIGEONHOLE PRINCIPLE =

→ Pigeonhole principle states that If their are more pigeons than pigeonhole then their must be Atleast One pigeonhole with Atleast two pigeons In It.

THEOREM =

→ If k Is A positive Integer And $k+1$ Or more Objects are placed Into k box then their Is One box containing two Or more Objects In It.



n = pigeons
m - pigeonholes
where, $\boxed{n < m}$

Date

Page

→ This principle Is Also termed As Dirichlet Drawer Principle.

(i) If A Is the Average number of pigeons per hole, where A Is not An Integer then Atleast One pigeonhole contains

ciel[A].

(Smallest Integer greater than Or Equal to A.)

$$\boxed{\text{ciel}[A] \geq A}$$

(ii) Remaining pigeonhole contains atmost Floor [A]

(Largest Integer less than Or Equal to A.)

$$\boxed{\text{Floor}[A] \leq A}$$

FLOOR AND CIEL FUNCTIONS =

(i) FLOOR FUNCTION =

→ The greatest Integer that Is less than Or Equal to x.

where x = Average number of pigeons per hole.

(ii) CEILEING FUNCTION =

FREEMIND

Date

Page

→ The ~~great~~ greatest Integer that Is greater than or Equal to x.

where, x Is the Average number of pigeons per hole.

Q. How many students must be In A class to guarantee that Atleast two students receive the same score On the final Exam, If the Exam Is graded On the scale of 1 to 100 points.

⇒ 0 ⟶ 100 = 101 total

∴ $\boxed{101 + 1 = 102}$

Q. Find the minimum number of students In class so that two students Are born In same month and same date.

⇒ Total number Of Months = 12

∴ A/q,

12 + 1

⇒ $\boxed{13}$

Total ~~Nur~~ number of days = 365

∴ A/q,

365 + 1

⇒ $\boxed{366}$

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GENERALISED PIGEONHOLE PRINCIPLE :

→ If n pigeonhole are Occupied by $k(n+1)$ or more pigeons then Atleast One pigeonhole Is Occupied by $k+1$ or more pigeons.

Eg = pigeonholes $= 3$

Pegions $= 7$

$\therefore n = 3$

$m = 7$

$\because$ A/q, $\boxed{kn + 1 = m}$

$\Rightarrow 3k + 1 = 7$

$\Rightarrow 3k = 6$

$\boxed{k = 2}$

$\because$ A/q, ~~$3 \times 2 + 1 =$~~

$\boxed{k + 1 = 2 + 1 = 3}$ Ans.

Q. Find the number of teachers on A college to be sure than 4 of them are born in same month.

→ Number of months $= 12$ ~~$= m$~~ n

$\therefore$ A/q,

~~$n = 4$~~ $k + 1 = 4 \Rightarrow \boxed{k = 3}$

$\boxed{kn + 1 = m}$ $= 3 \times 12 + 1$

~~$\Rightarrow 3n + 1 = 12$~~ $\boxed{37}$ Ans.

Date

Page

Q. A box contain 10 blue balls, 20 red balls, 8 green balls, 15 yellow balls, 25 white balls. How many balls must be choosen to ensure that we have 12 balls of same colour.

⇒ ∵ A/q,

$$k + 1 = 12$$

$$\Rightarrow \boxed{k = 11}$$

$n = 5$ (given)

∵ A/q,

$$kn + 1 = 11 \times 5 + 1$$

$$\Rightarrow 55 + 1$$

$$\Rightarrow \boxed{56}$$

a.