SMUR – Unit I – Chap 1 – Intro to Stats 09/02/24

Mean ⇒ average of the given observations.

Mean for grouped = $\frac{\Sigma f_i x_i}{\Sigma f_i}$

Mean for ungrouped = $\frac{\text{Sum of observation}}{\text{Total no. of observation.}}$

(Ques.)

Marks Obtained (x_i)	No. of students (f_i)	$f_i x_i$
10	1	10
20	1	20
36	3	108
40	4	160
50	3	150
56	2	112
60	4	240
70	4	280
72	1	72
80	1	80
88	2	176
92	3	276
95	1	95
	$\Sigma f_i = 30.$	$\Sigma f_i x_i = 1779$

Mean = $\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1779}{30} = 59.3.$

279
270
9

Three methods to find mean –

(1) Direct method.

class interval	f_i	x_i	$f_i x_i$
10 – 25	2	17.5	35
25 – 40	3	32.5	97.5
40 – 55	7	47.5	332.5
55 – 70	6	52.5	375
70 – 85	6	77.5	465
85 – 100	6	92.5	555
	$\Sigma f_i = 30$		Σ 1960.

$x_i = \frac{25+10}{2}$

$\frac{35}{2}$

$= 17.5$

$$\text{mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{186\cancel{0}}{3\cancel{0}} = 62.$$

(ii) Assumed mean method :-

(a) choose the assumed mean 'a' in the (x_i) which lies in the centre.

(b) find difference b/w x_i & a & name it as d_i.

(c) calculate $f_i d_i$

$$\bar{d} = \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$\bar{x} = a + \bar{d}$$

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

Ques.)

CI	f_i	x_i	d_i (62.5 − 17.5)	$f_i d_i$
10 - 25	2	17.5	−45	−90
25 - 40	3	32.5	−30	−90
40 - 55	7	47.5	−15	−105
55 - 70	6	$\boxed{62.5}$	0	0
70 - 85	6	77.5	15	90
85 - 100	6	92.5	30.	180.
	$\Sigma f_i = 30$			$\Sigma f_i d_i = -15$

Here a = "62.5"

Now $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

$$= 62.5 + \left(\frac{-15}{30}\right)$$

$$= 62.5 - 0.5$$

$$\bar{x} = 62.$$

(III) Step deviation method.

One more column is added to the table.

$$u_i = \left(\frac{x_i - a}{h}\right)$$

$a \rightarrow$ assumed mean

$h \rightarrow$ height of class or class size

Ques.)

C.I.	f_i	x_i	d_i $(62.5 - x_i)$	$u_i = \frac{d_i}{h}$ $\frac{d_i}{15}$	$f_i u_i$
10 – 25	2	17.5	−45	−3	−6
25 – 40	3	32.5	−30	−2	−6
40 – 55	7	47.5	−15	−1	−7
55 – 70	6	$\boxed{62.5}$	0	0	0
70 – 85	6	77.5	15	1	6
85 – 100	6	92.5	30	2	12
	$\Sigma = 30$				$\Sigma f_i u_i = -1$

$$\bar{u} = \frac{\Sigma f_i u_i}{\Sigma f_i} \qquad \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\bar{x} = 62.5 + \cancel{15}^{1}\left(\frac{-1}{\cancel{30}_{2}}\right)$$

$$= 62.5 - \frac{1}{2}$$

$$\bar{x} = 62$$

2250
450
2700
225
−175
50

Practice Problems

Ques.)

Daily expenditure	f_i	x_i	$f_i x_i$	d_i $(x_i - 225)$	$f_i d_i$	u_i $\frac{d_i}{50}$	$f_i u_i$
100 – 150	4	125	500	−100	−400	−2	−8
150 – 200	5	175	875	−50	−250	−1	−5
200 – 250	12	$\boxed{225}$	2700	0	0	0	0
250 – 300	2	275	550	50	100	1	2
300 – 350	2	325	650	100	200	2	4
	$\Sigma f_i = 25$		$\Sigma f_i x_i = 5275$		$\Sigma f_i d_i$ (−350)		6 −8 −5 = 6 − 13 = −7

31
2700
875
500
650
550
5275

(I) Direct.

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{\cancel{5275}^{211}}{\cancel{25}} = 211$$

(II) Assumed. mean.

$$\bar{x} = \bar{a} + \frac{\Sigma f_i d_i}{\Sigma f_i} \Rightarrow 225 + \left(\frac{\cancel{-350}^{14}}{\cancel{25}}\right)$$

$$= 225 - 14 = 211$$

(III) Step Deviation

$$\bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) = 225 + \cancel{50}^{2} \times \frac{-7}{\cancel{25}} = 225 - 14 = 211$$

Median ⇒ middlemost value in the data.

for odd no. of obs.

$$\text{median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

for even no. of obs.

$$\text{median} = \frac{\left(\frac{n}{2}\right)^{th}\text{term} + \left(\frac{n}{2}+1\right)^{th}}{2}.$$

How to calculate - arrange in ascending order.

Q) Median (Grouped data)

- median class वो होता which divides data into 2 halves.
- then find cummulative frequency.
- Now, locate nearest freq. to $n/2$ and corresponding class is median class.

$$\boxed{M = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h}$$

l → lower limit of med. class

Ques)

Monthly consumption	No. of Consumers.	c.f.
65 - 85	4	4
85 - 105	5	9
105 - 125	13	22
l = 125 - 145	20	42
145 - 165	14	56
165 - 185	8	64
185 - 205	4	68.
	$\Sigma f_i = 68.$	

cf, 24., → median class. 20(h)

Now, $n/2 = 68/2 = 34$ is nearer to 42.

$$\text{Med.} = l + \frac{\left(\frac{n}{2} - cf\right) \times h}{f} \Rightarrow 125 + \frac{(34-22) \times 20}{20}$$

$$= 125 + 12$$

Median = 137.

• **Mode** ⇒ The mode is the value ~~that~~ is repeatedly occurring in the dataset.

For ungrouped data

↳ max. number of occurring integer in data set.

For grouped data

↳ calculate maxi. frequency. (modal class).

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h.$$

Ques.)

Marks Obtained	No. of students
10 – 20	5 f_0
$\boxed{20 - 30}$	$\boxed{12}$ f_1
30 – 40	8 f_2
40 - 50	5

highest freq = 12 f_1

Modal class = 20–30.
l = 20

Now,

$$\text{mode} = 20 + \left(\frac{12-5}{2(12)-5-8}\right) \times 10$$

$$= 20 + \left(\frac{7}{24-13}\right) \times 10 = 20 + \frac{7}{11} \times 10$$

$$= 20 + \frac{70}{11} \Rightarrow 20 + 6.37$$

Mode = 26.37

Ques.)

Marks	No. of students
0–10	2
10–20	3
20–30	5
30–40	4 f_0
l $\boxed{40-50}$	$\boxed{19}$ f_1
50–60	17 f_2
60–70	18
70–80	12
80–90	15
90–100	5

$h = 10$.

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \frac{(19-4)}{(2(19)-4-17)} \times 10$$

$$= 40 + \frac{15 \times 10}{38-21}$$

$$= 40 + \frac{150}{17}$$

$$= 40 + 8.82$$

Mode = 48.82

Ques.)

Size of family	no. of family	
1-3	7	f_0
3-5	8	f_1 — modal class.
5-7	2	f_2
7-9	2	
9-11	1	

$h = 2$, l

$$\text{mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h = 3 + \left(\frac{8-7}{16-7-2}\right) \times 2$$

$$= 3 + \frac{2}{7} = 3 + \frac{2}{7} = 3 + 0.285$$

$$\text{mode} = 3.285$$

Relationship b/w mode, median & mean.

$$\boxed{\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}}$$

Dispersion ⇒ Statistical dispersion means the ~~output~~ extent to which the numerical value is likely to vary about an average.

Types ⇒ ① Absolute ② Relative.

① Absolute : expresses the variation in terms of the average of deviations of observation like standard deviation. It includes :- Range, Variance, SD, Quartile Deviation

② Relative : used to compare the distribution of two or more data sets. It includes :- coefficient of range, coeffi of variance. coefficient of SD, coff of Q.D.

Coefficient of Dispersion

Range. $CD = \frac{x_{max} - x_{min}}{x_{max} + x_{min}}$

SD $CD = SD/mean$

~~Quartile~~ $CD = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Mean Deviation. $CD = \frac{\text{Mean Deviation}}{\text{Average}}$

① Range ⇒ Difference between highest & lowest value.

Range = highest − lowest.

for. eg.

5 students in class.

heights = 150, 160, 175, 190, 200

Range = 200 − 150

$\boxed{\text{Range} = 50}$.

② Standard Deviation ⇒ It is the measure shows how much variation from mean, exist in data

for ungrouped data

SD ⇒ Population.

$$\boxed{\sigma = \sqrt{\frac{1}{N} \times \Sigma(x_i - \mu)^2}}$$

μ → mean

N → No. of observations

x_i → ith observation of population.

SD ⇒ Sample.

$$\boxed{\sigma = \sqrt{\frac{1}{N-1} \Sigma(x_i - \mu)^2}}$$

for grouped data

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \mu)^2}{N}} \quad \text{(Population)}$$

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \mu)^2}{N-1}} \quad \text{(Sample)}.$$

• Group freq. distribution in standard Deviation

$$\sigma = \sqrt{\frac{\Sigma f_i (d_i)^2}{n} - \left(\frac{\Sigma f_i d_i}{n}\right)^2}$$

$d_i = \frac{x_i - A}{c}$ — x_i: mid pt, A: assumed mean, c: class width

Ques.)

CI	f_i	x_i	$f_i x_i$	$x_i - \mu$	$(x_i-\mu)^2$	$f_i (x_i-\mu)^2$
0-10	27	5	135	-11.82	139.71	3772.17
10-20	10	15	150	-1.82	3.31	33.1
20-30	7	25	175	8.18	66.912	468.38
30-40	5	35	175	18.18	330.31	1652.55
40-50	4	45	180	28.18	794.11	3176.44
50-60	2	55	110	38.18	1457.71	2915.42
	$\Sigma f_i = 55$		$\Sigma f_i x_i = 925$		$\Sigma = 2792.26$	Σ 12018.06

$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{925}{55}$

$\bar{x} = 16.8181$

$= 16.82$

Now,

$\Sigma f_i (x_i - \mu)^2 = 12018.06$

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \mu)^2}{\Sigma f_i - 1}}$$

$$= \sqrt{\frac{12018.06}{54}} = \sqrt{222.556}$$

$\sigma = 14.91$

(8) variance ⇒ It can be obtained by sum of square of distance of each term in the distribution from the mean & dividing the total no. of terms in distribution.

ungrouped

Population. $\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N}$

Sample $\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N-1}$

grouped.

$\sigma^2 = \frac{\Sigma f_i (x_i - \mu)^2}{N}$

$\sigma^2 = \frac{\Sigma f_i (x_i - \mu)^2}{N-1}$

Practice Problem on S.D & V.

Ques.

interval	modified	f_i	x_i	$f_i x_i$
5001-10000	5000.5-10000.5	12	7500.5	90006
10001-15000	1000.5-15000.5	17	12500.5	212508.5
15001-20000	15000.5-20000.5	13	17500.5	227506.5
20001-25000	20000.5-25000.5	12	22500.5	270006
250001-30000	25000.5-30000.5	26	27500.5	715013
		Σ 80		Σ1515040

$$\sigma = \sqrt{\frac{\Sigma f_i (x_i - \mu)^2}{N-1}}$$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{\cancel{1515040}}{\cancel{80}} \quad 18938$$

$\bar{x} = 18938.$

$$\sigma = \sqrt{\frac{12(18938-7500.5)^2 + 17(12500.5-18938)^2 + 13(17500.5-18938)^2 + \ldots}{80-1.}}$$

$$= \sqrt{\frac{1987084.6 + \ldots}{79}} \Rightarrow 7255.03.$$

$\sigma = 7255.03$

Ques.

x_i	f_i	x_if_i	(x_i-u)	$(x_i-u)^2$	$\Sigma f_i(x_i-u)^2$
60	2	120	-4	16	32
61	1	61	-3	9	9
62	12	744	-2	4	48
63	29	1827	-1	1	29
64	25	1600	0	0	0
65	12	780	1	1	12
66	10	660	2	4	40
67	4	268	3	9	~~27~~ 36
68	5	340	4	16	~~64~~ 80.
	$\Sigma 100$	$\Sigma 6400.$			$\Sigma = 286$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{64\cancel{00}}{1\cancel{00}}$$

$\bar{x} = 64.$

$$\sigma = \sqrt{\frac{f_1(x_1-u)^2}{N}} = \sqrt{\frac{286}{100}} = \sqrt{2.86}$$

$\sigma = 1.69$.

Ques.

Diameters	No. of circles	x_i	x_if_i	(x_i-u)	$(x_i-u)^2$
32.5 – 36.5	15	34.5	517.5	-9	81
36.5 – 40.5	17	38.5	654.5	-5	25
40.5 – 44.5	21	42.5	892.5	-1	1
44.5 – 48.5	22	46.5	1023	3	9
48.5 – 52.5	25	50.5	1262.5	7	49
	$\Sigma = 100$		$\Sigma 4350$		

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{4350}{100} = 43.5 \qquad \bar{x} = 43.5$$

$$\sigma = \sqrt{\frac{\Sigma f_i(x_i-u)^2}{N}} = \sqrt{\frac{15(81)+17(25)+21(1)+22(9)+25(49)}{100}}$$

$$= \sqrt{\frac{1215+425+21+198+1225}{100}} = \sqrt{\frac{3084}{100}} = \sqrt{30.84}$$

$\sigma = 5.55 \qquad \sigma^2 = 30.84.$

- **Quartile** ⇒ When we divide data into 4 parts then we get 3 quatiles. i.e. 25%, 50%, 75%.

for ungrouped data

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ term}$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{th} \text{ term}$$

for grouped data.

$$Q_1 = \left(\frac{n}{4}\right)^{th} \text{ term}$$

$$Q_3 = 3\left(\frac{n}{4}\right)^{th} \text{ term}$$

$$\boxed{\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}}$$

> first arrange into ascending order

> then calculate Q_1 & Q_3

> now calculate median for Q_1 & Q_3.

> Calculate QD.

$$Q_r = l_1 + \left(\frac{r\left(\frac{n}{4}\right) - c.f.}{f}\right) \times (l_2 - l_1)$$

Ques. Quatile Deviation.

Days	No. of vehicles	increasing
1	20	5
2	15	2 10] Q_1
3	18	3 15
4	5	17
5	10	18
6	17	19
7	21	20
8	19	8 21] Q_3
9	25	9 25
10	28	28

$n = 10$ terms.

$$Q_1 = \frac{11}{4} = \underline{\underline{2.75}}$$

$$Q_3 = \frac{33}{4} = \underline{\underline{8.25}}$$

$$Q_1 = \frac{2^{nd} + 3^{rd}}{2} = \frac{10+15}{2} = \frac{25}{2}.$$

$$\boxed{Q_1 = 12.5}.$$

$$Q_3 = \frac{8^{th} + 9^{th}}{2} = \frac{21+25}{2} = \frac{46}{2} = 23 \quad \boxed{Q_3 = 23}$$

$$Q.D. = \frac{Q_3 - Q_1}{2} = \frac{23 - 12.5}{2} = \frac{10.5}{2} = 5.25$$

$$\boxed{Q.D = 5.25}$$

Ques.

Marks	Frequency	C.f.
0-10	10	10
10-20	20	30
20-30	30	60
30-40	50	110
40-50	40	150
50-60	30	180
	Σ: 180	

$n = 180$.

$$Q_1 = \left(\frac{180}{4}\right) = 45$$

$$Q_3 = 3\times\left(\frac{180}{4}\right) = 135.$$

Now,

$$Q_1 = l_1 + \frac{\left(\frac{n}{4} - c.f.\right)}{f} \times (l_2 - l_1)$$

$$= 20 + \frac{\left(\left(\frac{180}{4}\right) - 30\right)}{30} \times 10$$

$$= 20 + \frac{(45-30)}{30} \times 10$$

$$= 20 + \frac{15}{3} \; 5$$

$$\boxed{Q_1 = 25}$$

$$Q_3 = l_1 + \frac{\left(3\left(\frac{n}{4}\right) - c.f.\right)}{f} \times l_2 - l_1$$

$$= 40 + \frac{\left(3\left(\frac{180}{4}\right) - 110\right)}{40} \times 10$$

$$= 40 + \frac{135-110}{4}$$

$$= 40 + \frac{25}{4}$$

$$\boxed{Q_3 = 46.25}.$$

$$Q.D = \frac{Q_3 - Q_1}{2} = \frac{46.25 - 25}{2}$$

$$= \frac{21.25}{2}$$

$$\boxed{QD = 10.625}$$

Ques.

Class	fi	c.f.
0-10	5	5
10-20	3	8
20-30	4	12
30-40	3	15
40-50	3	18
50-60	4	22
60-70	7	27
70-80	9	38
80-90	7	45
90-100	8	53
	Σ=53	

$n = 53$.

$$Q_1 = \frac{53}{4} = (13.25)^{th} \text{ term}$$

$$Q_3 = 3\times 13.25 = (39.75)^{th} \text{ term}.$$

$$Q.D = \frac{Q_3 - Q_1}{2} = \frac{82.5 - 34.167}{2}$$

$$\boxed{QD = 24.1665}.$$

$$Q_1 = l_1 + \frac{\left(\frac{n}{4} - C.F.\right)}{f} \times (l_2 - l_1)$$

$$= 30 + \frac{(13.25 - 12)}{3} \times 10$$

$$= 30 + \cancel{1.25}\ \frac{12.5}{3}$$

$$Q_1 = 30 + 4.167$$

$$\boxed{Q_1 = 34.167}$$

$$Q_3 = l_1 + \frac{\left(3\left(\frac{n}{4}\right) - c.f.\right)}{f} \times (l_2 - l_1)$$

$$= 80 + \frac{(39.75 - 38)}{7} \times 10$$

$$= 80 + \frac{17.5}{7}$$

$$= 80 + 2.5$$

$$\boxed{Q_3 = 82.5}.$$

Prob ⇒ It denotes the ~~probo~~ possibility of the outcomes of any random event. for eg. when we flip a coin in air, what is the possibility of getting a head? → Head & tail are the possible outcomes. thus. 1/2. will be answer.

It is measure of the likelihood of an event to happen. It measure the certainty of event.

P(E) = No. of favourable outcomes/No. of total outcomes.

$$\boxed{P(E) = n(E)/n(S)}$$

Ques. A die is thrown once. What is prob. of getting 3?

Possible outcomes = $\underbrace{1, 2, 3, 4, 5, 6}$

No. of Possible outcomes = 6 [n(S)]

favourable outcome = 3

No. of f.o. = 1 [n(E)]

Sol. $P(E) = \frac{n(E)}{n(S)} = \frac{1}{6}$..

Various Terms ⇒

① Random Experiment – An exp whose result can't be predicted until it is noticed is called random exp.

② Sample Space – set of all possible result of a random exp.

③ Random Variable – variable which denotes the possible outcomes of random experiment.

(i) Discrete	(ii) continuous
It takes only those distinct values which are countable.	It could take an infinite number of possible values.

(4) Independent event - When prob of occurrence of one event has no impact on the prob of another event, then both termed as Independent of each other.

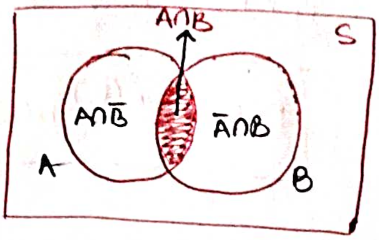
(5) Expected value - mean of the random variable.

$1+2+3+4+5+6$ 15 21

$\frac{21}{6} = 3.5$.

Law of Addition of Prob / Addition Rule of Prob

It states that prob. of 2 events is the sum of prob. of two events that will happen, minus the prob. of both event that will happen.



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Ques. An urn contain 10 black & 10 white. Find the prob of drawing two balls of the same color.

Prob of drawing 2 black balls $= \frac{^{10}C_2}{^{20}C_2}$

" 2 red balls $= \frac{^{10}C_2}{^{20}C_2}$

Prob. of drawing two balls of same color $= \frac{^{10}C_2}{^{20}C_2} + \frac{^{10}C_2}{^{20}C_2}$

$= 2\left(\frac{^{10}C_2}{^{20}C_2}\right) \Rightarrow 2\left(\frac{\frac{10\times 9}{2\times 1}}{\frac{20\times 19}{2\times 1}}\right)$

$= \cancel{2}\,\frac{\cancel{10}\times 9}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{20}\times 19}$

$\boxed{P(E) = \frac{9}{19}}$

Ques. A → 4 White, 2 Black

B → 3 white, 3 Black

Find prob of drawing a white ball?

Two mutually exclusive cases.

First bag & second bag. → $\frac{1}{2}$.

(A) Drawing ball from A bag.

$\left(\frac{1}{2} \times \frac{4}{6}\right)$

(B) Drawing ball from B bag.

$\left(\frac{1}{2} \times \frac{3}{6}\right)$

∴ events are mutually excl. the reqd. prob.

$= \left(\frac{1}{2}\times\frac{4}{6}\right) + \left(\frac{1}{2}\times\frac{3}{6}\right) = \frac{1}{3} + \frac{1}{4}$

$\boxed{P(E) = \frac{7}{12}}$.

Conditional Probability ⇒ Possibility of an event or outcome happening, based on the existence of a previous event. Calculated by multiplying the probability of these preceding event by renewed prob. of succeeding or condn event.

The prob of occurence of any event A when another event B in relation to A has already occured is known as condn prob. $P(A/B)$.

$$P(A/B) = N(A\cap B)/N(B)$$

or

$$P(B/A) = N(A\cap B)/N(A)$$

$$= \frac{\left(\frac{N(A\cap B)}{N}\right)}{\left(\frac{N(A)}{N}\right)} \cdot \frac{P(A\cap B)}{P(A)}.$$

It can also be written as.

$$P(A\cap B) = P(B)\,P(A/B) \quad \text{if } P(B)\neq 0$$

$$P(A\cap B) = P(A)\,P(B/A)$$

Ques. 2 dies thrown simultaneously and sum obtained is 7. What is prob. that the number 3 has appeared at least once?

Sample space will be $6\times 6 = 36$. $\{(1,1)(1,2)(1,3)\ldots\ldots(6,6)\}$

Event A → combn in which 3 appeared.

Event B → combn of numbers (sum = 7).

$A = \{(1,3)(2,3)(3,3)(4,3)(5,3)(6,3)$
$(3,1)(3,2)(3,4)(3,5)(3,6)\}$ $\quad P(A) = 11/36$

$B = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$ $\quad P(B) = 6/36$.

$A\cap B = (3,4)(4,3)$ $\quad P(A\cap B) = 2/36$.

Applying condn prob,

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{2/36}{6/36} = \frac{1}{3}.$$

multiplication law of Prob. ⇒ If there are two independent events the respective probabilities of which are known, then the prob. that both will happen is product of prob. of their happening. $P(A \cap B) = P(B) P(A)$

If A & B are 2 independent events for a random experiment, then the prob. of simultaneous occurence of two independent events will be equal to the product of their prob.

$$P(A \cap B) = P(A) \cdot P(B).$$

from condⁿ prob

$$P(A \cap B) = P(A) \cdot P(B/A)$$

∵ A & B independent $P(B/A) = P(B)$

$$\therefore \quad P(A \cap B) = P(A) P(B)$$

Ques. An urn contain 20 red & 10 blue. 2 balls are drawn at random without replacement. what is prob. that both the balls are drawn red?

A & B denote the events that first & second ball drawn.

$P(A \cap B) = ?$

$$P(A) = P(\text{red at first}) = 20/30$$

$$P(A) = \frac{2}{3}.$$

Now, $20 - 1 = 19$ balls left in red balls & $30 - 1 = 29$ left.

$$P(B/A) = \frac{19}{29}$$

$$P(A \cap B) = P(A) \cdot P(B/A) = \frac{20}{30} \times \frac{19}{29}$$

$$= \frac{38}{87}.$$

Bayes' theorem ⇒ describes the prob. of occurence of an event related to any condⁿ. It is also considered for the case of conditional prob.

Let $E_1, E_2, E_3 \ldots E_n$ be a set of events associated with sample space S, where all the events have non-zero prob. of occurence & they form a partition of S. Let A be an event associated with S, then acc. to Bayes theorem

$$P(E_i/A) = \frac{P(E_i)\cdot P(A/E_i)}{\sum_{k=1}^{n} P(E_k)\cdot P(A/E_k)}$$

eg 1.) Bag I – 4W, 6B Bag II – 4W, 3B. One ball is drawn at random & it is black. Find prob. that it was from bag I.

Let E_1 be event of choosing Bag I
E_2 " Bag II

$$P(E_1) = P(E_2) = \frac{1}{2}$$

Also, $P(A/E_1) = 6/10$ $P(A/E_2) = 3/7$.

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\left(\frac{1}{2}\times\frac{6}{10}\right)}{\left(\frac{1}{2}\times\frac{6}{10}\right)+\left(\frac{1}{2}\times\frac{3}{7}\right)} \Rightarrow \frac{\frac{3}{10}}{\left(\frac{3}{10}\right)+\left(\frac{3}{20}\right)}$$

28
14
42

$$= \frac{(3/10)}{(3/10)+(3/14)} = \frac{\frac{3}{10}}{\frac{42+30}{140}} = \frac{3}{10}\times\frac{140}{72}$$

24

$$= \frac{14}{24} \quad \frac{7}{12}$$

$$\boxed{P(E_1/A) = \frac{7}{12}}$$ Ans.