Proposition or Sentence
An expression consisting of some symbols, letters or words is called a proposition (or sentence) if it is true or false.

For example:-

1. Jaipur is capital of Rajasthan.
2. $2+3=5$
3. $9<6$
4. Mumbai is in America.
5. "Wish you a happy life" is not a proposition because True or False is not ~~the~~ ~~co~~ certain.

Truth Value:
If any proposition is True then its truth value is denoted by T.
If any proposition is False then its truth value is denoted by F.

Example:-

1 is less than 3 : T
14 is odd number : F

Types of Proposition.
1. Simple Proposition

The proposition having one subject and one predicate is called a simple proposition.

Example:-

1. This flower is pink.
2. Every even number is divisible by 2.

2. Compound Proposition

Two or more simple proposition when combined by various connectivities into a single composite sentence is called compound proposition.

Example:

1. The earth is round and revolves around the sun.
2. A triangle is equilateral iff three sides are equal.

Logical Connectives

The particular words and symbol used to join two or more proposition into a single composite form or compound proposition are called logical connectives.

Logical connectives words	Symbol	Use
→ And/conjunction/join	$\wedge$	$p \wedge q$
→ or/disjunction/meet	$\vee$	$p \vee q$
→ Negation	$\sim$	$\sim p$
→ Equivalent	$\longleftrightarrow$	$p \longleftrightarrow q$
→ Conditional 'if -- then --'	$\Rightarrow$	$p \Rightarrow q$
→ Biconditional 'if and only if' (iff)	$\Leftrightarrow$	$p \Leftrightarrow q$
→ NAND (NOT + AND)	$\uparrow$	$p \uparrow q$
→ NOR (NOT + OR)	$\downarrow$	$p \downarrow q$
→ XOR	$\oplus$	$p \oplus q$

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Basic Logical Operation

1. Conjunction → Any two proposition can be combined by the word "And" to form a compound proposition said to be the conjunction.

Truth Table :-

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Example:-

1. Delhi is in India and $2+2=4$ T
2. Delhi is in India and $2+2=5$ F
3. Delhi is in Russia and $2+2=4$ F

2. Disjunction → Any two proposition can be combined by the word "or" to form a compound composition is said to be disjunction.

Truth Table :-

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Examples :-

1. Delhi is in India or 2+2=4 T
2. Delhi is in India or 2+2=5 T
3. Delhi is in Russia or 2+2=5 F

3. Negation → The negation proposition of any given proposition p is the proposition whose truth value is opposite to p.

Truth Table:-

p	$\sim p$
T	F
F	T

⇒ Tautologies → A proposition is said to be a tautology if it contains only T in last column of truth table.

⇒ Contradiction → A proposition is said to be a contradiction if it contains only F in last column of truth table.

Ques Show that the following proposition is tautology.
$\{(p \vee \sim q) \wedge (\sim p \vee \sim q)\} \vee q$

p	q	$\sim p$	$\sim q$	$\overset{A}{p \vee \sim q}$	$\overset{B}{\sim p \vee \sim q}$	$\overset{C}{A \wedge B}$	$C \vee q$
T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	T
F	F	T	T	T	T	T	T

Last column contains all T values, therefore, it is a tautology.

Ques Prove that the proposition $p \vee \sim(q \wedge r)$ and $(p \vee \sim q) \vee \sim r$ are equivalent.

p	q	r	$(q \wedge r)$	$\sim(q \wedge r)$	$p \vee \sim(q \wedge r)$	$p \vee \sim q$	$(p \vee \sim q) \vee \sim r$
T	T	T	T	F	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	F	F	F	F
F	T	F	F	T	T	F	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

As both columns are ~~so~~ identical,
$\therefore$ they are equivalent

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Date: / / 2022

Page No.

Ques Determine whether following proposition is contradiction or tautology.

$(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

p	q	~p	~q	A p v q	B p v ~q	C ~p v q	D ~p v ~q	A∧B
T	T	F	F	T	T	T	F	F
T	F	F	T	T	T	F	T	F
F	T	T	F	T	F	T	T	F
F	F	T	T	F	T	T	T	F

It is a contradiction

Ques Determine whether it is tautology or contradiction.

$q \vee (p \wedge \sim q) \vee (\sim p \vee \sim q)$

p	q	~p	~q	p ∧ ~q	~p v ~q	q v (p∧~q) v (~pv~q)
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

It is a tautology

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Conditional Statement

Many statements are of the form "if p then q" such statement are said to be conditional statement and denoted by $p \Rightarrow q$ or $p \rightarrow q$

CAD Truth Table:-

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional Statement

p if and only if q $p \Leftrightarrow q$ or $p \leftrightarrow q$

Truth Table:-

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Converse of Conditional Statement

$p \rightarrow q$ is conditional statement

$\therefore q \rightarrow p$ is converse.

Example: If X is a labourer then he is poor.

Converse: If X is poor then he is a labourer.

Inverse of Conditional Statement

$p \rightarrow q$ is conditional statement.

$\therefore \sim p \rightarrow \sim q$ is its inverse

Example: If Harsh read book then he will get knowledge.

Inverse: If Harsh not read book then he will not get knowledge.

Contrapositive of Conditional Statement.

$p \rightarrow q$ is conditional statement.

$\sim q \rightarrow \sim p$ is contrapositive.

Example: If $f(x)$ is differentiable then it is continou.

Contrapositive: If $f(x)$ is not continou then it is not differentiable.

Ques Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

It is a tautology

Ques: Is this tautology. $p \rightarrow (p \wedge (q \rightarrow p))$

p	q	$q \rightarrow p$	$p \wedge (q \rightarrow p)$	$p \rightarrow (p \wedge (q \rightarrow p))$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

It is a tautology

Ques Prove that following is tautology or not.
$(p \vee q \vee r) \leftrightarrow [(((p \rightarrow q) \rightarrow q) \rightarrow r) \rightarrow r]$.

p	q	r	$p \vee q \vee r$ (D)	$p \rightarrow q$ (A)	$A \rightarrow q$ (B)	$B \rightarrow r$ (C)	$C \rightarrow r$ (E)	$D \leftrightarrow E$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	T	F	T	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T	T
F	F	T	T	T	F	T	T	T
F	F	F	F	T	F	T	F	T

It is a tautology

Ques Show that $p \Rightarrow (q \Rightarrow r) = (p \wedge q) \Rightarrow r$

p	q	r	$q \to r$ (A)	$p \to A$ (B)	$p \wedge q$ (C)	$C \to r$ (D)
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	T	F	T
F	F	F	T	T	F	T

$B = D$

$\Rightarrow$ They are equal

Proved

Ques Show they are equivalent.

$p \vee (p \wedge q) \Leftrightarrow p$ and p

p	q	$p \wedge q$	$p \vee (p \wedge q)$ (A)	$A \Leftrightarrow p$	$p \wedge p$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	F	T	F
F	F	F	F	T	F

Important Laws

1. Idempotent Law

$p \wedge p \Leftrightarrow p$ and $p \vee p \Leftrightarrow p$

2. Commutative law

$p \wedge q \Leftrightarrow q \wedge p$ and $p \vee q \Leftrightarrow q \vee p$

3. Associative law

$(p \vee q) \vee r = p \vee (q \vee r)$
$(p \wedge q) \wedge r = p \wedge (q \wedge r)$

4. De-Morgan law

$\sim(p \wedge q) = \sim p \vee \sim q$
$\sim(p \vee q) = \sim p \wedge \sim q$

5. $p \rightarrow q = \sim p \vee q$

Normal Form

Let $A(p_1, p_2, \ldots p_n)$ be a statement formula then the construction of truth table may not be practical always. So, we consider alternate procedure known as reduction to normal form.

1. Disjunction Normal Form → Disjunction b/w the conjunctions.

Example: $(p \wedge q) \vee r$

$(p \wedge \sim q) \vee (\sim p \wedge \sim r) \vee (r \wedge \sim q)$

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Ques Obtain DNF of $(p \rightarrow q) \wedge (\sim p \wedge q)$

$(\sim p \vee q) \wedge (\sim p \wedge q)$

Apply distributive law,

$(\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

$(\sim p \wedge q) \vee (q \wedge \sim p)$

2. Conjunction Normal Form

Conjunction b/w disjunction

Ex $p \wedge q$ $\quad (\sim p \vee q) \wedge (\sim p \vee r)$

Obtain CNF of $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

$[p \vee (\sim p \wedge q \wedge r)] \wedge [q \vee (\sim p \wedge q \wedge r)]$

$[(p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q \wedge q) \wedge (q \vee r)]$

$[(p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge q \wedge (q \vee r)]$

Ans

Data : / / 2022
Page No.

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Ques Obtain DNF of $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r))$

$\Rightarrow p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r))$

$\Rightarrow p \vee (\sim p \rightarrow (q \vee (\sim q \vee \sim r))$

$\Rightarrow p \vee (\sim p \rightarrow (q \vee \sim q) \vee (q \vee \sim r))$

$\Rightarrow p \vee (p \vee (q \vee \sim r))$

$\Rightarrow p \vee (p \vee q) \vee (p \vee \sim r)$

$\Rightarrow p \vee p \vee q \vee p \vee \sim r$

$\Rightarrow p \vee q \vee \sim r$ Ans.

Ques Obtain CNF of $(p \rightarrow q) \wedge (q \vee (p \wedge r))$

$\Rightarrow (\sim p \vee q) \wedge (q \vee p) \wedge (q \vee r)$

p	q	r	$\sim p$	A: $\sim p \vee q$	B: $q \vee p$	C: $q \vee r$	$A \wedge B \wedge C$
T	T	T	F	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	F	T	T	F
T	F	F	F	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	F
F	F	F	T	T	F	F	F

Not a tautology.

Ques Find CNF of $p \wedge (p \rightarrow q)$

$\Rightarrow p \wedge (p \rightarrow q)$

$\Rightarrow p \wedge (\sim p \vee q)$

$\Rightarrow \underbrace{(p \wedge \sim p)}_{\downarrow \; 1} \vee (p \wedge q) \Rightarrow p \wedge q$

Ques Obtain CNF of $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

$\Rightarrow (p \vee r) \wedge (q \rightarrow p) \wedge (p \rightarrow q)$

$\Rightarrow (p \vee r) \wedge (\sim q \vee p) \wedge (\sim p \vee q)$

Ques Obtain DNF of $\sim(p \vee q) \rightarrow (p \wedge q)$

$\Rightarrow (p \vee q) \vee (p \wedge q)$

$\Rightarrow \cancel{(p \vee q} [(p \vee p) \wedge (p \vee q)] \vee [(q \vee p) \wedge (q \vee q)]$

$\Rightarrow [p \wedge (p \vee q)] \vee [(q \vee p) \wedge q]$

$\Rightarrow [(p \wedge p) \vee (p \wedge q)] \vee [(q \wedge q) \vee (p \wedge q)]$

$\Rightarrow p \vee p$

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Ques Obtain DNF of $\sim(p \vee q) \rightarrow (p \wedge q)$

$\Rightarrow (p \vee q) \vee (p \wedge q)$

$\Rightarrow p \vee q \vee p \wedge q$

$\Rightarrow p \vee q$ Ans

Argument → A process by which a conclusion is obtained from given set of premises.

(i) Premises → Given group of proposition.

(ii) Conclusion → Proposition getting by given premises.

Example:

"I will become famous or I will be writer"

I will not be a writer

I will become famous

Valid Argument → Let $p_1, p_2 -- p_n$ are premises and Q is conclusion then the argument is valid iff $A(p_1, p_2, ... p_n) \rightarrow Q$ is a tautology.

Falacy Argument → An argument is called falacy if it is not valid.

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Ques Test the validity of argument.
If it rain, Ram will be sick
It didnot rain.
∴ Ram was not sick

p = If it rain
q = Ram will be sick
~p = It didnot rain
~q = Ram was not sick

$[(p \rightarrow q) \wedge \sim p] \rightarrow \sim q$

	p	q	$\overset{A}{p \rightarrow q}$	$A \overset{B}{\wedge} \sim p$	$B \rightarrow \sim q$
F	T	T	T	F	T
T	T	F	F	F	T
F	F	T	T	T	F
T	F	F	T	T	T

Not a valid argument

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Date : / / 2022

Page No.

Ques Investigate the validity of all following argument.

$p \rightarrow r$

$\sim p \rightarrow q$

$q \rightarrow s$

$\therefore \sim r \rightarrow s$

$$[(p \rightarrow r) \wedge (\sim p \rightarrow q) \wedge (q \rightarrow s)] \rightarrow [\sim r \rightarrow s]$$

p	q	r	s	~p	~r	A p → r	B ~p → q	C q → s	D ~r → s	A∧B∧C	E
T	T	T	T	F	F	T	T	T	T	T	T
T	T	T	F	F	F	T	T	F	T	F	T
T	T	F	T	F	T	F	T	T	T	F	T
T	T	F	F	F	T	F	T	F	F	F	T
T	F	T	T	F	F	T	T	T	T	T	T
T	F	T	F	F	F	T	T	T	T	T	T
T	F	F	T	F	T	F	T	T	T	F	T
T	F	F	F	F	T	F	T	T	F	F	T
F	T	T	T	T	F	T	T	T	T	T	T
F	T	T	F	T	F	T	T	F	T	F	T
F	T	F	T	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F	F	T
F	F	T	T	T	F	T	F	T	T	F	T
F	F	T	F	T	F	T	F	T	T	F	T
F	F	F	T	T	T	T	F	T	T	F	T
F	F	F	F	T	T	T	F	T	F	F	T

Valid

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Ques Examine the validity of argument.
"If prices are high then wages are high.
Prices are high or there are price controls.
If there are price controls then there is not an inflation, there is an inflation therefore ~~pa~~ wages are high.

p = Prices are high
q = Wages are high
r = There are price controls
s = There is inflation
~p = Prices are not high
~q = Wages are not high
~r = There are no price control
~s = There is no inflation.

$$[(p \rightarrow q) \wedge (p \vee r) \wedge (r \rightarrow \sim s) \wedge s] \rightarrow q$$

p	q	r	s	~s	p→q	p∨r	r→~s	A∧B∧C∧s	D→q
T	T	T	T	F	T	T	F	F	T
T	T	T	F	T	T	T	T	F	T
T	T	F	T	F	T	T	T	T	T
T	T	F	F	T	T	T	T	F	T
T	F	T	T	F	F	T	F	F	T
T	F	T	F	T	F	T	T	F	T
T	F	F	T	F	F	T	T	F	T
T	F	F	F	T	F	T	T	F	T
F	T	T	T	F	T	T	F	F	T
F	T	T	F	T	T	T	T	F	T
F	T	F	T	F	T	F	T	F	T
F	T	F	F	T	T	F	T	F	T
F	F	T	T	F	T	T	F	F	T
F	F	T	F	T	T	T	T	F	T
F	F	F	T	F	T	F	T	F	T
F	F	F	F	T	T	F	T	F	T

Valid

Ques. Test the validity of the following argument:
"If my brother stands first in class. I will give him a watch. Either he stands first or I was out of station. I did not give my brother a watch this time. Therefore I was out of station.

Sol. p = My brother stands first in class.
q = I will give him a watch
r = I was out of slation
$\sim q$ = I did not give my brother a watch.

$$[(p \rightarrow q) \wedge (p \vee r) \wedge \sim q] \rightarrow r$$

p	q	r	$\sim q$	$p \rightarrow q$	$p \vee r$	$A \wedge B \wedge \sim q$	$C \rightarrow r$
T	T	T	F	T	T	F	T
T	T	F	F	T	T	F	T
T	F	T	T	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	F	T	T	F	T
F	T	F	F	T	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	F	F	T

Valid Argument

Ques. Test the validity of the following argument:
"If it is good pen, then it is Parker pen.
It is a parker pen therefore it is a good pen"

Sol. p = It is good pen
q = It is parker pen.

$[(p \rightarrow q) \wedge q] \rightarrow p$

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

Not Valid