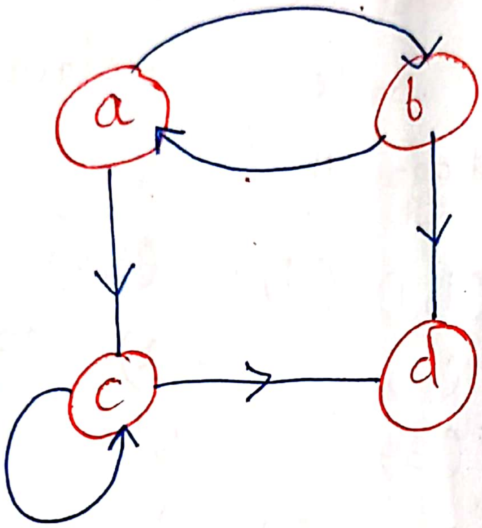
Q-2. Consider the digraph as shown below. Write relation as set of ordered pairs and check for equivalence or partial ordering. (3)



Sol.

→ from the diagram, R = { (a,b) (b,a) (a,c) (c,c) (c,d) (b,d) }

Reflexive :→ $(a,a) \notin R$

∴ R is not reflexive.

Symmetric :→ $(a,c) \in R$ but $(c,a) \notin R$

∴ R is not symmetric

Antisymmetric :→ $(a,b) \in R$ $(b,a) \in R$ but $a \neq b$ always.

∴ R is not ~~bra~~ antisymmetric

Transitive :→ $(a,c) \in R$ $(c,d) \in R$ but $(a,d) \notin R$

∴ R is not transitive.

Hence R is neither an equivalence relation nor a partial ordering relation.

Q— Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and R be a relation on A defined by "$(x,y) \in R$ if y is divisible by x". Is R equivalence relation? Is R partial order relation? Draw its diagraph?

Ans:-> Here $R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)\ (1,7)\ (1,8)\ (2,2)\ (2,4)\ (2,6)\ (2,8)\ (3,3)\ (3,6)\ (4,4)\ (4,8)\ (5,5)\ (6,6)\ (7,7)\ (8,8)\}$

Reflexive :-> $(a,a) \in R$ it is reflexive.

Antisymmetric :-> R is antisymmetric

Transitive :-> R is transitive

Hence R is partial order relation.

But R is not symmetric $(1,2) \in R$ but $(2,1) \notin R$.

$\therefore$ R is not an equivalence relation.

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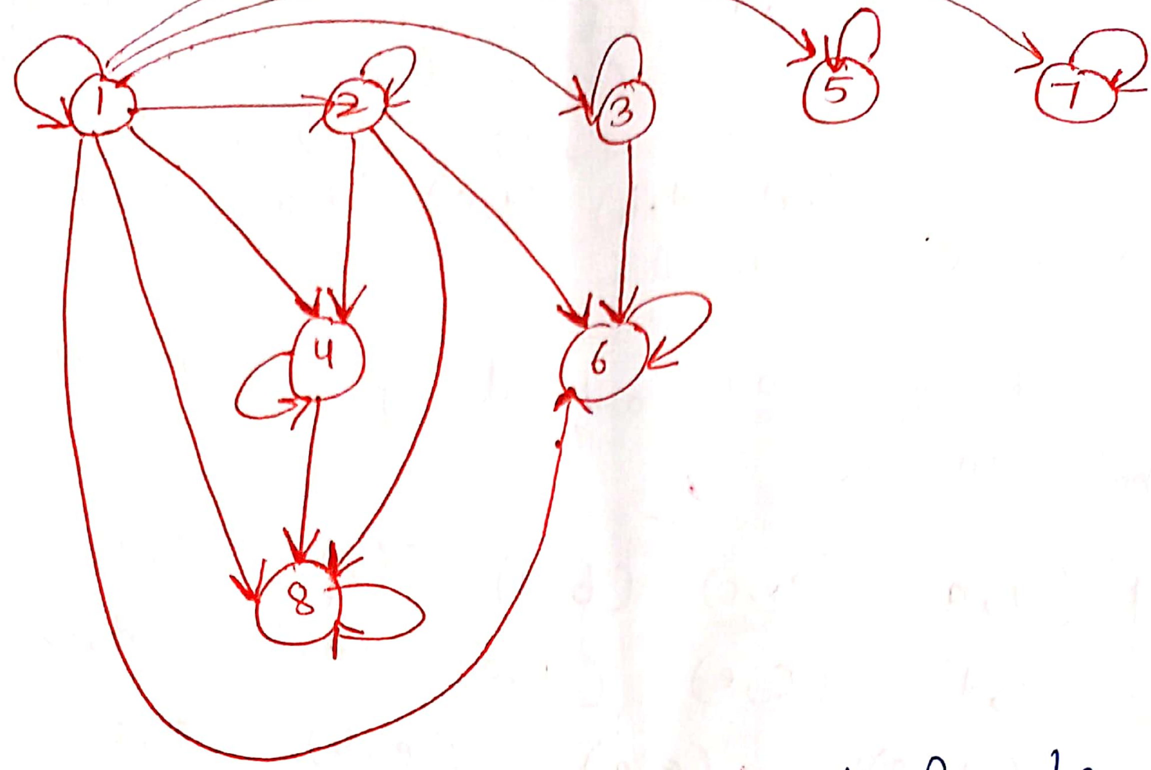
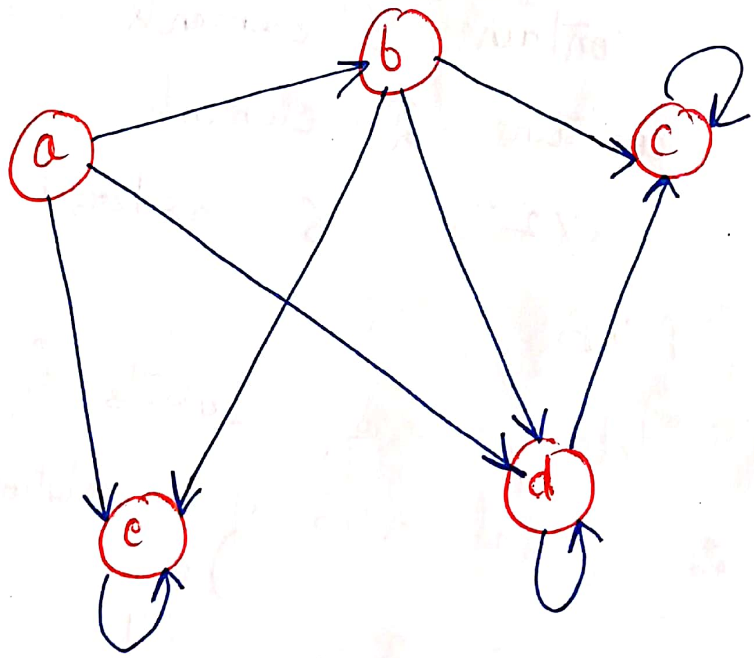


Q:→ Let $A = \{a, b, c, d, e\}$ and R be a relation on A whose corresponding diagraph is given below. find R.

Ans:→



from the diagraph,

$R = \{(a,b)\ (a,e)\ (a,d)\ (b,c)\ (b,d)\ (b,e)\ (c,c)\ (d,d)\ (e,e)\}$

Also $A \times A = \{ (a,a)\ (a,b)\ (a,c)\ (a,d)\ (a,e)$
$(b,a)\ (b,b)\ (b,c)\ (b,d)\ (b,e)$
$(c,a)\ (c,b)\ (c,c)\ (c,d)\ (c,e)$
$(e,a)\ (e,b)\ (e,c)\ (e,d)\ (e,e)$
$(d,a)\ (d,b)\ (d,c)\ (d,d)\ (d,e) \}$ set reflexive

$\therefore \quad \bar{R} = A \times A - R$

The set of all ordered pairs in $A \times A$ but not in R

$= \{ (a,a)\ (a,c)\ (b,a)\ (b,b)\ (c,a)\ (c,b)$
$(c,d)\ (c,e)\ (d,a)\ (d,b)\ (d,c)$
$(d,e)\ (e,a)\ (e,b)\ (e,c)$
$(e,d) \}$

Q:> find the no of relations from $A = \{a, b, c\}$ to $B = \{(1,2)\}$

Ans:> A contains 3 elements, B contains 2 elements

So $3 \times 2 = 6$ ordered pairs in $A \times B$

Hence, total no. of subsets of $A \times B$ is $2^6 = 64$.

$\therefore$ total no of relations from A to B = 64.

Q:→ (a) if A is a set containing n elements find the no of relations from A to A.

(b) if A = (1,2) find all possible relations from A to A.

Ans:→ We know that no. of relations from A to B is 2^{mn}.

∴ total no. of relations from A to A is $2^{n \times n} = 2^{n^2}$.

b) A contains 2 elements ∴ total no of relations $= 2^{2^2} = 4^2 = 16$

The various relations are

ϕ, {(1,2) (2,1) (1,1) (2,2)}

{(1,2) (2,1)}

{(1,2) (1,1)}

{(1,2) (2,2)}

{(2,1) (1,1)}

{(2,1) (2,2)}

{(1,1) (2,2)}

{(1,2) (2,1) (1,1)}

{(1,2) (1,1) (2,2)}

{(2,1) (2,2) (1,1)}

{(1,2) (2,1) (2,2)}

{(1,2) (2,1) (1,1) (2,2)}

{(1, 1) (2, 2) (1, 2)} {(2, 1) (1, 1) (1, 2)}

Q-24. Let R be the relation on the positive integers N defined by the equation $x+3y=12$; that is

$$R = \{(x, y) : x+3y = 12\}$$

(a) Write R as a set of ordered pairs.

(b) find (i) domain of R (ii) range of R (iii) R^{-1}.

Ans:→ (a) (9, 1) (6, 2) (3, 3) = R

(b) (i) {9, 6, 3}

(ii) (1, 2, 3)

(iii) {(1, 9) (2, 6) (3, 3)} = R^{-1}

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Equivalence Relation :→ A relation R on set A is said to be equivalence if it is reflexive, symmetric and transitive. (1)

Ex :→ $A = \{0, 1, 2, 3\}$

$R_1 = \{(0,0)\ (1,1)\ (2,2)\ (3,3)\}$

→ Is R_1 an equivalence relation?

→ Yes

→ Reflexive ✓

Symmetric ✓

Transitive ✓

$R_2 = \{(0,0)\ (0,2)\ (2,0)\ (2,2)\ (2,3)\ (3,2)\ (3,3)\}$

Is R_2 an equivalence relation?

Is R_2 reflexive?

→ No, (1,1) is not present.

So, this relation is not an equivalence relation.

Compatible relation :→

A binary relation on a set which is reflexive and symmetric is called a compatible relation.

→ Every equivalence relation is a compatible relation But every compatible relation may not be an equivalence relation.

b- Let $A = \{ x : x$ is a English word$\}$ compatible relation. Show R is a

) Reflexive :→ Every English word has letters same as itself. ∴ R is reflexive

) Symmetric :→ if a has one or more letters same as that of b, then b also has one or more letters same as a.

Hence R is symmetric.

∴ R is a compatible relation.

But R is not transitive

Take $a =$ book, $b =$ Kite, $c =$ ten (any Eng word)

Here $(a, b) =$ (book, Kite) $\in R$ (as book and Kite has 'K' common)

$(b, c) =$ (Kite, ten) $\in R$. But $(a, c) =$ (book, ten) $\notin R$

∴ R is not an equivalence relation.

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→ Find the Domain & Range of the relation.

$R = \{(x, y) : y = x + \frac{6}{x}$ where $x, y \in N$ & $x < 6\}$

$R : N \longrightarrow N$

Convert set builder to roaster form.

$R = \{(1, 7)\ (2, 5)\ (3, 5)\}$.

$x = 1$

$y = 1 + \frac{6}{1}$

$= 7$

Domain $= \{1, 2, 3\}$

Range $= \{7, 5\}$

$x = 2$

$y = 2 + \frac{\cancel{6}\,3}{\cancel{2}}$

$= 5$

Codomain $= \underline{N}$

3:→ $R = \{(x, y) : x, y \in W,$
$x^2 + y^2 = 25\}$

$x = 3$

$y = \cancel{3} + \frac{\cancel{6}\,2}{\cancel{3}}$

$= 5$

find the domain, Range, Codomain

$R = \{(0, 5)\ (5, 0)\ (3, 4)\ (4, 3)\}$

$(2)^2 + y^2 = 25$

$0 + y^2 = 25$

$\boxed{y = 5}$

$1 + y^2 = 25$

$y^2 = 24$ X

$y^2 = 21$ X

$3^2 + y^2 = 25$

$y^2 = 25 - 9$

$= 16$

$y = 4$

domain $= \{0, 3, 4, 5\}$

Range $= \{0, 3, 4, 5\}$

$\boxed{\text{Codomain} = W}$

Partial Order Relation (2)

→ A relation R on a set A is called a partial relation if it is
- reflexive
- anti symmetric
- transitive

$A = \{1, 2, 3\}$

$R_1 = \{ \}$ ✗

$R_2 = \{(1,1)\ (2,2)\ (3,3)\}$ ✓

$R_3 = \{(1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)\}$ ✗

$R_4 = \{(1,1)\ (2,2)\ (3,3)\ (1,3)\ (2,3)\}$ ✓

$R_5 = \{(1,1)\ (1,2)\ (2,3)\ (1,3)\}$ ✗

$R_6 = A \times A$

$\{(1,1)\ \underline{(1,2)}\ (1,3)$
$\underline{(2,1)}\ (2,2)\ (2,3)\ \ldots\}$

→ Not antisymmetric
→ Not Partial order.

Domain Range and Codomain

Let $A = \{1, 2, 3, 5\}$

$B = \{4, 6, 9\}$

R is a relation from A to B

Set Builder

$R = \{(x, y) :$ difference b/w x and y is odd, $x \in A$ and $y \in B\}$

Roster

$R = \{(1,4)\ (1,6)\ (2,9)\ (3,4)\ (3,6)\ (5,4)\ (5,6)\}$

Domain $\Rightarrow \{1, 2, 3, 5\}$

↓

1st elements in the ordered pair of R.

Range $\Rightarrow \{4, 6, 9\}$

→ 2nd elements in the ordered pairs of R.

Codomain (is always the second set)

a. B.

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Domain and Range of a relation

Domain is the set of all first coordinates of the ordered pairs belonging to R.

Similarly the range of R is set of all second co-ordinates of ordered pairs belonging to R eg.

→ Consider $A = \{1, 3, 5, 7\}$ $B = \{2, 4, 6, 8, 10\}$ and $R = \{(1, 8) (3, 6) (5, 2) (1, 4)$ be a relation from A to B.

Then

Domain of $R = \{1, 3, 5\}$

Range of $R = \{2, 4, 6, 8\}$

- Equivalence Relation :→

→ A relation R on A is said to be an equivalence relation if it is

- (1) it is reflexive
- (2) it is symmetric
- (3) it is transitive

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Composition of Relations

Let $R = \{(1,2)\ (3,4)\ (2,2)\}$

$S = \{(4,2)\ (2,5)\ (3,1)\ (1,3)\}$

find RoS, SoR, $Ro(SoR)$, RoR, SoS.

Ans:→ Given $R = \{(1,2)\ (3,4)\ (2,2)\}$

$S = \{(4,2)\ (2,5)\ (3,1)\ (1,3)\}$

(i) $RoS = \{(1,5), (3,2), (2,5)\}$

2) $SoR = \{(4,2)\ (3,2)\ (1,4)\}$

3) $R = \{(1,2)\ (3,4)\ (2,2)\}$

$SoR = \{(4,2)\ (3,2)\ (1,4)\}$

$Ro(SoR) = \{(3,2)\}$

4) $RoR = R^2$

$= \{(1,2)\ (2,2)\}$

5) $SoS = S^2$

$S^2 = \{(4,5)\ (3,3)\ (1,1)\}$

Composition of Relations :→

Let A, B and C be the sets.

Let R is the relation from A to B

i.e. $R \subseteq A \times B$

S is the relation from B to C i.e.

$S \subseteq B \times C$

The composition of R and S denoted by RoS where

$$RoS = \{(a,c) \in A \times C : \text{for some } b \in B, (a,b) \in R \text{ and } (b,c) \in S\}$$

Eg :→ $A = \{1, 2, 3\}$, $B = \{p, q, r\}$

$C = \{x, y, z\}$

$R = \{(1,p) \; (1,r) \; (2,p) \; (2,q)\}$

$S = \{(p,y) \; (q,x) \; (q,y) \; (r,z)\}$

Compute RoS.

$$\boxed{RoS = \{(1,y) \; (1,z) \; (2,y) \; (2,x)\}}$$