

Asymptotic Notation

It represent space & time Complexity.

1) Big-Oh (O) notation :- (Upper Bound)

$$f(n) = O(g(n))$$

iff $f(n) \leq c * g(n) \quad \forall n \geq n_0$
where, c & $n_0 \rightarrow$ Constant.

a) $f(n) = 3n + 2$

$$f(n) \leq c * g(n) \quad \forall n \geq n_0$$

$$3n + 2 \leq 4 * n \quad \forall n \geq 2$$

$$c = 4, n_0 = 2, g(n) = n$$

$$f(n) = O(n)$$

b) $f(n) = 2000$

$$f(n) \leq c * g(n) \quad \forall n \geq n_0$$

$$2000 * n^0 \leq 2001 * n^0 \quad \forall n \geq 1$$

$c = 2001, n_0 = 1$

$$g(n) = n^0$$

$$f(n) = O(n^0)$$

$$f(n) = O(1)$$

We know $n^0 = 1$
that

$$O(1) < O(n) < O(n^2)$$

2) $\Omega(n)$:- (Lower Bound)

$$f(n) = \Omega(g(n))$$

$$\text{iff } f(n) \geq c * g(n) \quad \forall n \geq n_0$$

where $c, n_0 \rightarrow \text{Constant}$

a) $f(n) = 3n + 2$

$$f(n) \geq c * g(n) \quad \forall n \geq n_0$$

$$3n + 2 \geq 3 * n \quad \forall n \geq 1$$

$$\boxed{c = 3, g(n) = n, n_0 = 1}$$

$$f(n) = \Omega(n)$$

(b) $f(n) = 2n - 3$

$$f(n) \geq c * g(n) \quad \forall n \geq n_0$$

$$2n - 3 \geq 1 * n \quad \forall n \geq 3$$

$$\boxed{c = 1, n_0 = 3, g(n) = n}$$

$$f(n) = \Omega(n)$$

3) Theta (Θ) Notation :- (Average Case)

$$f(n) = \Theta(g(n))$$

$$c_1 * g(n) \leq f(n) \leq c_2 * g(n) \quad \forall n \geq n_0$$

where $c_1, c_2, n_0 \rightarrow \text{constant}$

a) $f(n) = 3n + 2$

$$3 * n \leq 3n + 2 \leq 4 * n \quad \forall n \geq 2$$

$$c_1 = 3, c_2 = 4, g(n) = n$$

$$f(n) = \Theta(n)$$

4) Little Oh (o) :-

$$f(n) = o(g(n))$$

$$\text{iff } f(n) = O(g(n)) \text{ \& } f(n) \neq \Omega(g(n))$$

* Rate of growth \rightarrow

$\log n, n, n \log n, n^2, n^3, 2^n$
 \longrightarrow Time order of complexity