Basic Counting Principles

(1)

There are mainly 2 counting principles namely (i) Sum Rule (2) Product Rule

These 2 principles form the basis of permutations and combinations and hence Known as basic Counting Principles.

- **Sum Rule** :→ If there are 2 jobs such that they can be performed independently in m and n ways. Then no. of ways in which either of the 2 jobs can be performed is m+n.

- **Product Rule** :→ If there are 2 jobs such that one of them can be done in m ways and when it has been done, second job can be done in n ways, then the 2 jobs can be done in m×n ways.

Q:→ In a class there are 10 boys and 8 girls. The teacher wants to select either a boy or girl to represent the class in a function. In how many ways the teacher can make this selection?

Ans:> The teacher can select a boy in 10 ways and a girl in 8 ways.

∴ By sum rule, the no. of ways of selecting either a boy or a girl $= 10 + 8 = 18$

Q:> In a class there are 10 boys and 8 girls. The teacher wants to select a boy and a girl to represent the class in a function. In how many ways can the teacher make this selection?

→ ~~The teacher~~ Can make this selection by

① selecting a boy among 10 boys

② " " girl among 8 boys

The first job can be done in 10 ways and the second job can be done in 8 ways. ∴ by product rule, the total no. of ways of selecting a boy and a girl $= 10 \times 8 = 80$

Permutations and Combinations

① **Factorial n:>**

The product of first n natural number is called factorial n. It is denoted by $n!$ or $\underline{n|}$.

② It can be written as: $n! = n(n-1)! = n(n-1)(n-2)! \ldots$

$= n(n-1)(n-2)(n-3)\cdots 1$

We have $1! = 1$ and $0! = 1$

Q- find the value of $\frac{10!}{8!}$

$= \frac{10 \times 9 \times \cancel{8!}}{\cancel{8!}} = 90$

Q:→ Determine the value of $\frac{n!}{(n-1)!}$

Here $\frac{n!}{(n-1)!} = \frac{n(n-1)!}{(n-1)!} = n$

- **Permutation** :→

A permutation is an arrangement of a no. of objects in some definite order taken some or all at a time. The total no. of permutations of n distinct objects taken r at a time is denoted by ^nP_r or $P(n, r)$ where $1 \le r \le n$.

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

This is the permutation of n different objects taken r at a time given by as mentioned above.

Q:→ Determine the value of following:

(1) $^4P_2 = \frac{4!}{(4-2)!} = \frac{4 \times 3 \times \cancel{2!}}{\cancel{2!}} = 12$

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(2) $^9P_3 = \frac{9!}{(9-3)!} = \frac{9 \times 8 \times 7 \times \cancel{6!}}{\cancel{6!}} = 504$

Q→2. Determine the value of n if

(1) $4 \times {}^nP_3 = {}^{n+1}P_3 = \frac{4 \times n!}{(n-3)!}$

$= \frac{(n+1)!}{(n+1-3)!}$

$= \frac{4 \times n!}{(n-3)!} = \frac{(n+1) \times n!}{(n-2)(n-3)!}$

$= 4(n-2) = (n+1)$

$\Rightarrow 4n - 8 = (n+1)$

$= 3n = 9$

$\Rightarrow \boxed{n = 3}$

(2) $6 \times {}^nP_3 = 3 \times {}^{n+1}P_3$

$= \frac{6 \times n!}{(n-3)!} = \frac{3 \times (n+1)!}{(n+1-3)!}$

$$= \frac{6 \times n!}{(n-3)!} = \frac{3(n+1)(n!)}{(n-2)(n-3)!}$$

$$= 6(n-2) = 3\times(n+1) \Rightarrow 6n - 12 = 3n+3$$

$$= 6n - 3n = 12 + 3 \Rightarrow 3n = 15$$

$$= n = 5$$

Q:→ How many variable names of 8 letters can be formed from the letters a, b, c, d, e, f, g, h if no letter is repeated?

Ans:→ There are 9 letters and 8 are to be selected.

∴ Total no. of variable names of 8 letters is $^9P_8 = \frac{9!}{(9-8)!} = \frac{9!}{1!} = 9!$

* Imp. Q:→ There are 10 persons called on an interview. Each one is capable to be selected for the job. How many permutations are there to be select 4 from the 10?

Ans:→ There are 10 persons and 4 are to be selected.

∴ Total no. of permutations to select 4 persons $= {}^{10}P_4$.

$$= \frac{10!}{(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!} = 5040.$$

- Permutation with Restrictions

- The no. of permutations of n different objects taken r at a time in which p particular objects do not occur is ${}^{n-p}P_r$.

- The no. of permutations of n different objects taken r at a time in which p particular objects are present is ${}^{n-p}P_{r-p} \times {}^{r}P_p$.

Q:→ How many 6-digit numbers can be formed by using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 if every number is to start with '30' with no digit repeated.

Ans:→ All the numbers begin with 30. So, we have to choose 4-digits from the remaining 7-digits.

∴ Total no. of numbers that begins with '30' is

$${}^{7}P_4 = \frac{7!}{(7-4)!} = \frac{7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

$$= 840$$

Permutations when all of the objects are not distinct.

→ The number of permutations of n objects, of which n_1 objects are of one kind and n_2 objects of another kind, when all are ~~at~~ taken at a time is $\dfrac{n!}{n_1!\, n_2!}$

Q→ Determine the no. of permutations that can be made out of the letters of the word 'PROGRAMMING'.

Sol→ There are 11 letters in the word 'PROGRAMMING' out of which G's, M's and R's are 2 each.

∴ The total no. of permutations is

$$\frac{11!}{2! \times 2! \times 2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2!}}{2 \times 1 \times 2 \times 1 \times \cancel{2!}}$$

$$= 4989600$$

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1. Boolean Algebra is a algebra of logic. It deals with variables that have 2 discrete values 0 (false) and 1 (True).

(1)

Properties of Boolean Algebra

1. Commutative

(1) $a+b = b+a$

(2) $a * b = b * a$

2. Distributive Properties

(1) $a + (b * c) = (a+b) * (a+c)$

(2) $a * (b+c) = (a * b) + (a * c)$

3. Identity Properties :→

(1) $a + 0 = a$

(2) $a * 1 = a$

4. Complemented Laws :→

(1) $a + a' = 1$

(2) $a * a' = 0$

- Boolean Algebra was invented by George Boole and it is applicable in building basic electronic circuits and design of digital computers.

- Boolean Expression:→

→ A boolean expression always produces a Boolean value.

→ It is composed of a combination of Boolean constants (T/F), boolean variables and logical connectives.

Eg. — $AB'C$ is a boolean expression.

- Boolean Identities

- Double Complement Law

$$\neg(\neg A) = A$$

- Complement Law

$A + \neg A = I$ (OR Form)

$A . \neg A = 0$ (AND form)

- Idempotent Law

$A + A = A$ (OR form)

$A . A = A$ (AND form)

- Identity Law

$A + 0 = A$ (OR form)

$A . I = A$ (AND form)

Dominance law

$A + I = I$ (OR form)

$A \cdot 0 = 0$ (AND form)

- ## Commutative law

$A + B = B + A$ (OR form)

$A \cdot B = B \cdot A$ (AND form)

- ## Associative law

$A + (B + C) = (A + B) + C$ (OR form)

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$ (AND form)

- ## Absorption law

$A \cdot (A + B) = A$

$A + (A \cdot B) = A$

- ## Simplification law

$A \cdot (\neg A + B) = A \cdot B$

$A + (\neg A \cdot B) = A + B$

- ## Distributive law

$A + (B \cdot C) = (A + B) \cdot (A + C)$

$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

- ## De-Morgan's law

$\neg (A \cdot B) = \neg A + \neg B$

$\neg (A + B) = \neg A \cdot \neg B$

(2)

Representation :→

- Canonical forms

→ There are 2 kinds of Canonical forms :→

- Sum of minterms (SOM) form)
  or Sum of Products (SOP)
- The product of maxterms (POM) form

SoP

(1) A sum of product form expression contain's product terms (AND terms) which are sum (OR together) that's why called as SOP.

(2) A product term which contains all the literals (variables) either in complemented or uncomplemented form is called minterms.

	Sequence	Minterm	
000	0	$a'b'c'$	— m_0
001	1	$a'b'c$	— m_1
010	2	$a'bc'$	— m_2
011	3	$a'bc$	— m_3
100	4	$ab'c'$	— m_4
101	5	$ab'c$	— m_5
110	6	abc'	— m_6
111	7	abc	— m_7

eg.

Let $F(a,b,c) = a'b'c' + ab'c + abc' + abc$

or $F(a,b,c) = m_0 + m_5 + m_6 + m_7$

Hence

$F(a,b,c) = \Sigma(0,5,6,7) = \Sigma(0,5,6,7)$

Now we will find the complement of $F(a,b,c)$

$F'(a,b,c) = a'bc + a'b'c + a'bc' + ab'c'$

or $F(a,b,c) = m_3 + m_1 + m_2 + m_4$

Hence

$F'(a,b,c) = \Sigma(3,1,2,4)$

$= \Sigma(1,2,3,4)$

- **POS form** or POM (Product of Maxterms)

- A maxterm is addition of all variables taken either in their direct or complemented form.

A	B	C		Maxterm
0	0	0	$x+y+z$	M_0
0	0	1	$x+y+z'$	M_1
0	1	0	$x+y'+z$	M_2
0	1	1	$x+y'+z'$	M_3
1	0	0	$x'+y+z$	M_4
1	0	1	$x'+y+z'$	M_5
1	1	0	$x'+y'+z$	M_6
1	1	1	$x'+y'+z'$	M_7

Eg.

$$F(x,y,z) = (x+y+z).(x+y+z').(x+y'+z).(x'+y+z)$$

$$F(x,y,z) = M_0 . M_1 . M_2 . M_4$$

Hence

$$F(x,y,z) = \Pi(0,1,2,4)$$

$$F''(x,y,z) = (x+y'+z').(x'+y+z').(x'+y'+z).(x'+y'+z')$$

$$F(x,y,z) = M_3 . M_5 . M_6 . M_7$$

Hence $F'(x,y,z) = \Pi(3,5,6,7)$

Consider the Boolean expression

$f_1(a,b,c) = ((ab)'c)'((a'+c)(b'+c'))'$ into SOP form

Ans: $f_1(a,b,c) = ((ab')c)'((a'+c)(b'+c'))'$

$(a.b)' \rightarrow a'+b'$

$= ((ab)''+c')((a'+c)' + (b'+c')')$ DeMorgan's Law

$= (ab+c')((a'+c)' + (b'+c')')$ Involution

$= (ab+c')(a''c' + b''c'')$ - DeMorgan's

$= (ab+c')(ac'+bc)$

$= abac' + abbc + c'ac' + c'bc$

$= aabc' + abbc + ac'c' + bc'c$

Commutative

$= abc' + abc + ac' + 0$ (Idempotent $aa=a$)

$= abc' + abc + ac'$ (complement $c'c=0$)

$= abc' + ac' + abc$

$= ac' + abc$ (Absorption)

Q:→ Consider the Boolean expression $f(x,y,z) = x(y'z)'$. Reduce it to SoP.

Ans:→

$$f(x,y,z) = x(y'z)'$$

$$= x(y''+z') \quad \text{De Morgan's}$$

$$= x(y+z') \quad \text{Involution } \boxed{(a')' = a}$$

$$= xy + xz'$$

$$= xy(z+z') + x(y+y')z'$$

$$\left(\begin{array}{c} a+a' = 1 \\ \text{Complement law} \end{array} \right)$$

$$= xyz + \underbrace{xyz' + xyz'}_{} + xy'z'$$

$$= xyz + xyz' + xy'z'$$

$$a + a = a$$

Idempotent law

Imp.

→ Prove the law of Boolean algebra for the following statements

(i) The zero element is unique

(2) The unit element is unique.

Proof :→

① if 0_1 and 0_2 are 2 zero elements of B, we have to show $0_1 = 0_2$.

Now 0_1 is zero element of B and $0_2 \in B$, then

$$0_2 + 0_1 = 0_2 = 0_1 + 0_2 \quad \text{—— ①}$$

Similarly, 0_2 is the zero element of B and $0_1 \in B$, then

$$0_1 + 0_2 = 0_1 = 0_2 + 0_1 \quad \text{—— ②}$$

from ① and ②, $\boxed{0_1 = 0_2}$

Hence, zero element is unique.

(2) Let I_1 and I_2 be 2 unit elements of B, we have to show that $I_1 = I_2$

Now I_1 is unit element of B and $I_2 \in B$

then $I_2 + I_1 = I_2 = I_1 + I_2$ — (1)

Similarly, I_2 is unit element of B and $I_1 + B$, then

$I_1 + I_2 = I_1 = I_2 + I_1$ — (2)

from (1) and (2)

$$\boxed{I_1 = I_2}$$