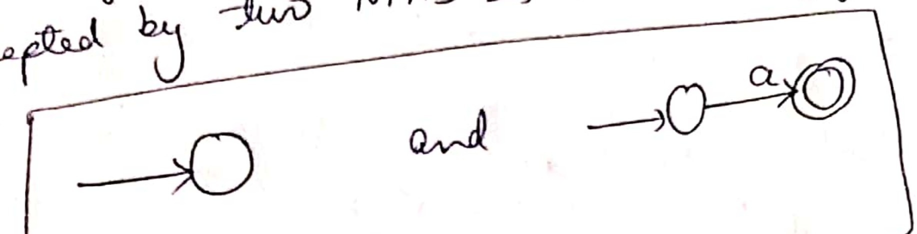
Kleen's Theorem - Part 1 :-

For any alphabet Σ, every regular language over Σ can be accepted by a finite automata.

Proof:- We know that every regular language over Σ can be accepted by an NFA.

We will prove the theorem by structural Induction.

→ The language ϕ and $\{a\}$ (where $a \in \Sigma$) can be accepted by two NFA's as shown in figure (i)



(i)

→ The Induction hypothesis is that L_1 & L_2 are both Regular languages over Σ.

→ For both $i = 1$ & $i = 2$, L_i can be accepted by an NFA

$$M_i = (Q_i, \Sigma, q_i, A_i, \delta_i)$$

→ We can assume, by renaming states if necessary, that Q_1 & Q_2 are disjoint.

⇒ In Induction step we must show that there are NFA's accepting the three languages:

(i) $L(M_1) \cup L(M_2)$

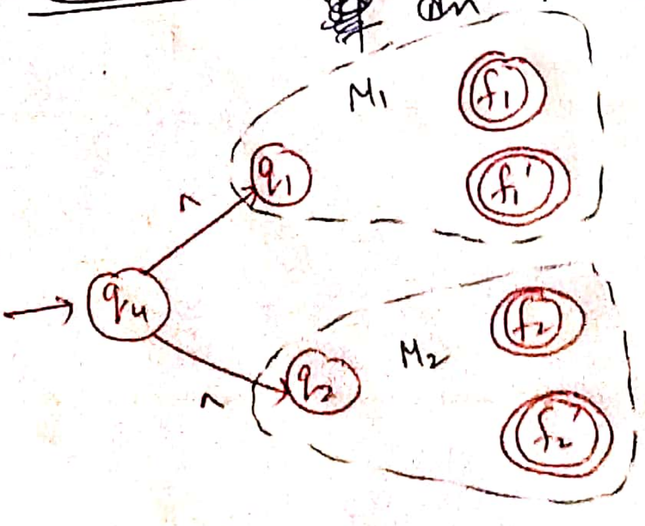
(ii) $L(M_1)\, L(M_2)$

(iii) $L(M_1)^*$

→ In each case, we will give an informal definition & a diagram showing the idea of the construction.

For simplicity, each diagram shows two NFAs M_1 & M_2 as having two accepting states, both distinct from the initial state.

⇒ **UNION** :- An NFA M_u accepting $L(M_1) \cup L(M_2)$



$M_u = (Q_u, \Sigma, q_u, A_u, \delta_u)$

q_u → Initial states. (take one additional state)

δ_u → includes all the ones in M_1 & M_2 as well as Λ-transitions from q_u to q_1 & q_2, the initial state of M_1 & M_2

A_u → $(A_1 \cup A_2)$

$Q_u = Q_1 \cup Q_2 \cup q_u$

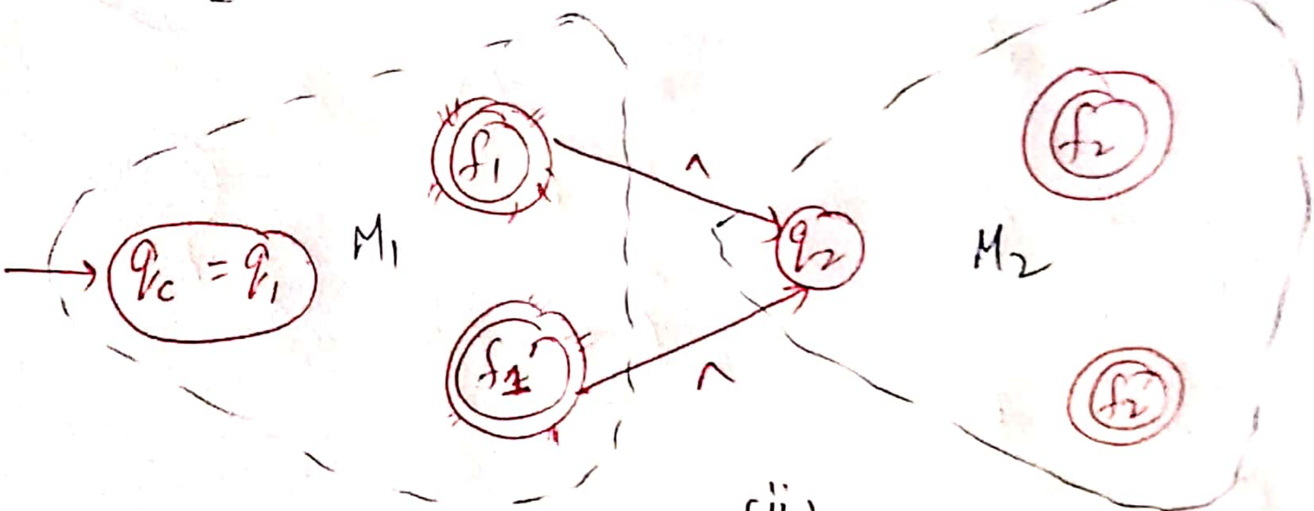
$\Sigma = \Sigma$

q_u = Initial State

$A_u = A_1 \cup A_2$

$\delta_u \Rightarrow$ $(M_1$ & $M_2)$ and include Λ- transition from q_u.

→ <u>Concatination:-</u>



(ii)

M_c accepting $L(M_1)L(M_2)$ as shown in fig(ii).

No New states need to be added to those M_1 & M_2.

$M_c = (Q_c, \Sigma, q_c, A_c, \delta_c)$

→ $q_c = q_1$ (Initial state)

→ δ_c = M_1 & M_2 and a new Λ-transition from every element of <u>A_1 to q_2</u>.

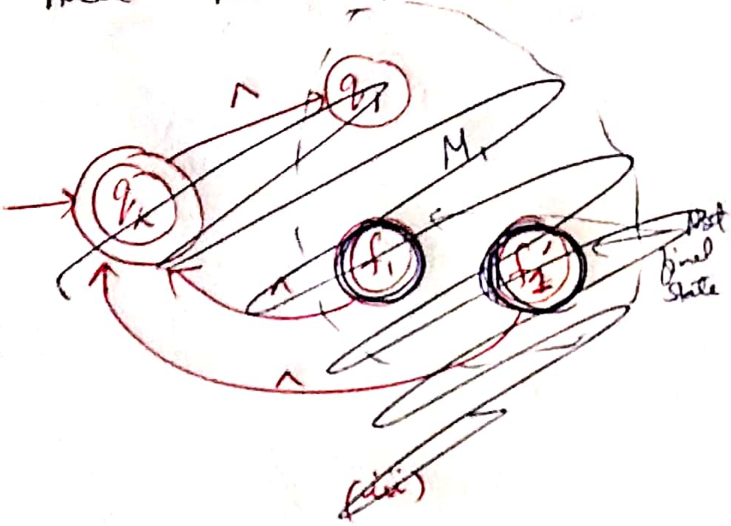
→ $A_c = A_2$.

→ $\Sigma = \Sigma$

→ $Q_c = Q_1 \cup Q_2$

→ Kleen's Closure (*) :→

final NFA, M_K accepting $L(M_1)^*$



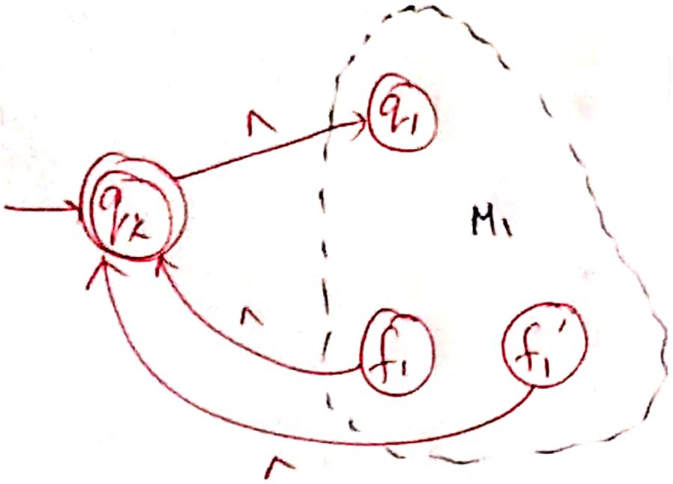
$M_K = (Q_K, \Sigma, q_K, A_K, \delta_K)$

$Q_K = Q_1 \cup q_K$ (New State)

$q_K \Rightarrow$ Initial State

$A_K \Rightarrow q_K$

$\delta_K \to$ Transition are those M_1 & Λ transition from q_K to q_1



$\Sigma \Rightarrow \Sigma$

By Induction hypothesis, if follows. $\boxed{x_1 \in L(M_1)^*}$. ~~Therefore~~

Hence Proved.