Conservation of Energy

What You Know

- You understand the concept of energy.
- You know how to define a system so you can consider energy flows into and out of the system.
- You've seen how work involves a transfer of mechanical energy.
- You recognize kinetic energy and understand how the work–kinetic energy theorem relates the net work done on a system to the change in its kinetic energy.

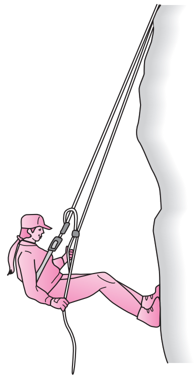
What You're Learning

- You'll see how forces come in two types: *conservative forces* and *nonconservative forces*.
- You'll learn that energy transferred as a result of work done against conservative forces ends up stored as *potential energy*.
- You'll learn expressions for potential energy associated with gravitational and elastic forces.
- You'll learn to treat the sum of kinetic energy and potential energy as a system's *total mechanical energy*.
- You'll see that mechanical energy is conserved in the absence of nonconservative forces.
- You'll learn to use the conservation-of-mechanical-energy principle to solve problems that would otherwise be difficult because they involve varying acceleration.
- You'll see how to evaluate situations where nonconservative forces result in a loss of mechanical energy.
- You'll see how *potential energy curves* describe a wide variety of systems, including molecules.

How You'll Use It

- In Chapter 8, you'll use the calculus expression for potential energy to explore the potential energy associated with the gravitational force over distances significant in space flight and astronomy.
- You'll also see how conservation of energy leads you to understand the physics of simple orbits and how energy conservation leads to the concept of *escape speed*.
- In Chapter 9, you'll apply conservation of mechanical energy to so-called *elastic collisions*, which conserve mechanical energy.
- Energy will continue to play an important role as you explore Newtonian physics further in Parts 1 and 2.
- In Part 3, you'll see how to extend the conservation-of-energy principle to account for heat as well as mechanical work.
- In Part 4, you'll see that electric and magnetic fields are repositories of potential energy.

How many different energy conversions take place as the Yellowstone River plunges over Yellowstone Falls?



(a)

(b)

FIGURE 7.1 Both the rock climber and the mover do work, but only the climber can recover that work as kinetic energy.

The rock climber of Fig. 7.1*a* does work as she ascends the vertical cliff. So does the mover of Fig. 7.1*b*, as he pushes a heavy chest across the floor. But there's a difference. If the rock climber lets go, down she goes, gaining kinetic energy as she falls. If the mover lets go of the chest, though, he and the chest stay right where they are.

This contrast highlights a distinction between two types of forces, called *conservative* and *nonconservative*. That distinction will help us develop one of the most important principles in physics: **conservation of energy**. The introduction to Chapter 6 briefly mentioned three forms of energy: kinetic energy, potential energy, and internal energy—although there we worked quantitatively only with kinetic energy. Here we'll develop the concept of potential energy and show how it's associated with conservative forces. Nonconservative forces, in contrast, are associated with irreversible transformations of mechanical energy into internal energy. We'll take a brief look at such transformations here and formulate a broad statement of energy conservation. In Chapters 16–19 we'll elaborate on internal energy and see how it's related to temperature, and we'll expand our statement of energy conservation to include not only work but also heat as means of energy transfer.

7.1 Conservative and Nonconservative Forces

Both the climber and the mover in Fig. 7.1 are doing work against forces—gravity for the climber and friction for the mover. The difference is this: If the climber lets go, the gravitational force "gives back" the energy she supplied by doing work, which then manifests itself as the kinetic energy of her fall. But the frictional force doesn't "give back" the energy supplied by the mover, in the sense that this energy can't be recovered as kinetic energy.

A **conservative force** is a force like gravity or a spring that "gives back" energy that was transferred by doing work. A more precise description of what it means for a force to be conservative follows from considering the work involved as an object moves over a closed path—one that ends where it started. Suppose our rock climber ascends a cliff of height h and then descends to her starting point. As she climbs, the gravitational force is directed opposite to her motion, so gravity does negative work $-mgh$ (recall Fig. 6.4). When she descends, the gravitational force is in the same direction as her motion, so the gravitational work is $+mgh$. The total work that gravity does on the climber as she traverses the closed path up and down the cliff is therefore zero.

Now consider the mover in Fig. 7.1*b*. Suppose he pushes the chest across a room, discovers it's the wrong room, and pushes it back to the door. Like the climber, the mover and chest describe a closed path. But the frictional force always acts to oppose the motion of the chest. The mover needs to apply a force to oppose friction—that is, in the same direction as the chest's motion—so he ends up doing positive work as he crosses the room in both directions. Therefore, the total work he does is positive even when he moves the chest over a closed path. That's the nature of the frictional force, and, in contrast to the conservative gravitational force the climber had to deal with, this makes the frictional force **nonconservative**.

You'll notice that we didn't talk here about the work done by friction but rather the work done by the mover in opposing friction. That's because frictional work is a rather subtle concept, which we'll touch on later in the chapter. Nevertheless, our two examples make the distinction between conservative and nonconservative forces quite clear: Only for *conservative* forces is the work done in moving over a closed path equal to zero. That property provides a precise mathematical definition of a conservative force:

> When the total work done by a force $\vec{F}$ acting as an object moves over any closed path is zero, the force is conservative. Mathematically,
>
> $$\oint \vec{F} \cdot d\vec{r} = 0 \qquad \text{(conservative force)} \tag{7.1}$$

The expression given in Equation 7.1 comes from the most general formula for work, Equation 6.11: $W = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r}$. The circle on the integral sign indicates that the integral is to be taken over a *closed* path.

Equation 7.1 suggests a related property of conservative forces. Suppose a conservative force acts on an object in the region shown in Fig. 7.2. Move the object along the straight path from point A to point B, and designate the work done by the conservative force as W_{AB}. Since the work done over any closed path is zero, the work W_{BA} done in moving back from B to A must be $-W_{AB}$, whether we return along the straight path, the curved path, or any other path. So, going from A to B involves work W_{AB}, regardless of the path taken. In other words:

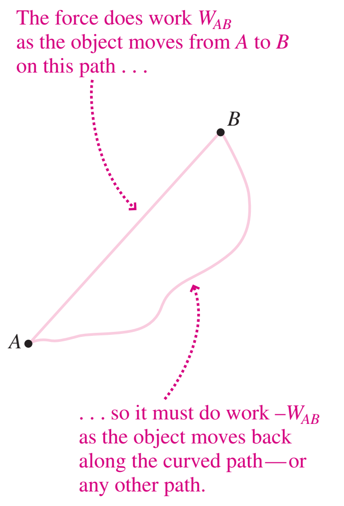


FIGURE 7.2 The work done by a conservative force is independent of path.

The work done by a conservative force in moving between two points is independent of the path taken; mathematically, $\int_A^B \vec{F} \cdot d\vec{r}$ depends only on the endpoints A and B, not on the path between them.

Important examples of conservative forces include gravity and the static electric force. The force of an ideal spring—fundamentally an electric force—is also conservative. Nonconservative forces include friction, drag forces, and the electric force in the presence of time-varying magnetic effects, which we'll encounter in Chapter 27.

GOT IT? 7.1 Suppose it takes the same amount of work to push a trunk straight across a rough floor as it does to lift a weight the same distance straight upward. If both trunk and weight are moved instead on identically shaped curved paths between the same two points as before, is the work (a) still the same for both, (b) greater for the weight, or (c) greater for the trunk?

7.2 Potential Energy

The climber in Fig. 7.1*a* did work ascending the cliff, and the energy transferred as she did that work was somehow stored, in that she could get it back in the form of kinetic energy. She's acutely aware of that stored energy, since it gives her the potential for a dangerous fall. *Potential* is an appropriate word here: The stored energy is **potential energy**, in the sense that it has the potential to be converted into kinetic energy.

We'll give potential energy the symbol U, and we begin by defining *changes* in potential energy. Specifically:

The change ΔU_{AB} in potential energy associated with a conservative force is the negative of the work done by that force as it acts over any path from point A to point B:

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r} \qquad \text{(potential energy)} \qquad (7.2)$$

Here $\int_A^B \vec{F} \cdot d\vec{r}$ is the work done by the force $\vec{F}$, as defined in Equation 6.11. But why the minus sign? Because, if a conservative force does *negative* work (as does gravity on a weight being lifted), then energy is stored and ΔU must be *positive*. Another way to think about this is to consider the work *you* would have to do in order to just counter a conservative force like gravity. If $\vec{F}$ is the conservative force (e.g., gravity, pointing down), then you'd have to apply a force $-\vec{F}$ (e.g., upward), and the work you do would be $\int_A^B (-\vec{F}) \cdot d\vec{r}$ or $-\int_A^B \vec{F} \cdot d\vec{r}$, which is the right-hand side of Equation 7.2. Your work represents a transfer of energy, which here ends up stored as potential energy. So another way of interpreting Equation 7.2 is to say that the change in potential energy is equal to the work an external agent would have to do in just countering a conservative force.

Changes in potential energy are all that ever matter physically; the actual value of potential energy is meaningless. Often, though, it's convenient to establish a reference point at which the potential energy is defined to be zero. When we say "the potential energy U," we really mean the potential-energy difference ΔU between that reference point and whatever other point we're considering. Our rock climber, for example, might find it convenient to take the

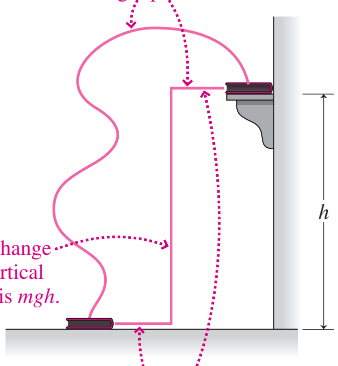


FIGURE 7.3 A good choice of path makes it easier to calculate the potential-energy change.

zero of potential energy at the base of the cliff. But the choice is purely for convenience; only potential-energy *differences* really matter. We'll often drop the subscript AB and write simply ΔU for a potential-energy difference. Keeping the subscript is important, though, when we need to be clear about whether we're going from A to B or from B to A.

Equation 7.2 is a completely general definition of potential energy, applicable in all circumstances. Often, though, we can consider a path where force and displacement are parallel (or antiparallel). Then Equation 7.2 simplifies to

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx \qquad (7.2a)$$

where x_1 and x_2 are the starting and ending points on the x-axis, taken to coincide with the path. When the force is constant, this equation simplifies further to

$$\Delta U = -F(x_2 - x_1) \qquad (7.2b)$$

✓TIP Understand Your Equations

Equation 7.2b provides a very simple expression for potential-energy changes, but it applies *only* when the force is constant. Equation 7.2b is a special case of Equation 7.2a that follows because a constant force can be taken outside the integral.

Gravitational Potential Energy

We're frequently moving things up and down, causing changes in potential energy. Figure 7.3 shows two possible paths for a book that's lifted from the floor to a shelf of height h. Since the gravitational force is conservative, we can use either path to calculate the potential-energy change. It's easiest to use the path consisting of straight segments. No work or potential-energy change is associated with the horizontal motion, since the gravitational force is perpendicular to the motion. For the vertical lift, the force of gravity is constant and Equation 7.2b gives immediately $\Delta U = mgh$, where the minus sign in Equation 7.2b cancels with the minus sign associated with the *downward* direction of gravity. This result is quite general: When a mass m undergoes a vertical displacement Δy near Earth's surface, gravitational potential energy changes by

$$\Delta U = mg\,\Delta y \qquad \text{(gravitational potential energy)} \qquad (7.3)$$

The quantity Δy can be positive or negative, depending on whether the object moves up or down; correspondingly, the potential energy can either increase or decrease. We emphasize that Equation 7.3 applies *near Earth's surface*—that is, for distances small compared with Earth's radius. That assumption allows us to treat the gravitational force as constant over the path. We'll explore the more general case in Chapter 8.

We've found the *change* in potential energy associated with raising the book, but what about the potential energy itself? That depends on where we define the zero of potential energy. If we choose $U = 0$ at the floor, then $U = mgh$ on the shelf. But we could just as well take $U = 0$ at the shelf; then potential energy when the book is on the floor would be $-mgh$. Negative potential energies arise frequently, and that's OK because only *differences* in potential energy really matter. Figure 7.4 shows a plot of potential energy versus height with $U = 0$ taken at the floor. The *linear* increase in potential energy with height reflects the *constant* gravitational force.

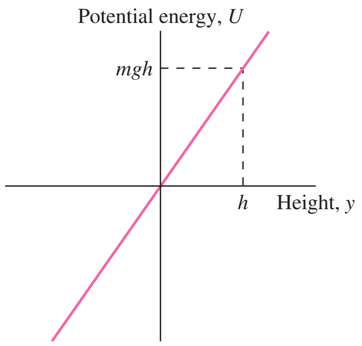


FIGURE 7.4 Gravitational force is constant, so potential energy increases linearly with height.

EXAMPLE 7.1 Gravitational Potential Energy: Riding the Elevator

A 55-kg engineer leaves her office on the 33rd floor of a skyscraper and takes an elevator up to the 59th floor. Later she descends to street level. If the engineer chooses the zero of potential energy at her office and if the distance from one floor to the next is 3.5 m, what's the potential energy when the engineer is (a) in her office, (b) on the 59th floor, and (c) at street level?

INTERPRET This is a problem about gravitational potential energy relative to a specified point of zero energy—namely, the engineer's office.

DEVELOP Equation 7.3, $\Delta U = mg\,\Delta y$, gives the change in gravitational energy associated with a change Δy in vertical position. We're given positions in floors, not meters, so we need to convert using the given factor 3.5 m per floor.

EVALUATE (a) When the engineer is in her office, the potential energy is zero, since she defined it that way. (b) The 59th floor is $59 - 33 = 26$ floors higher, so the potential energy when she's there is

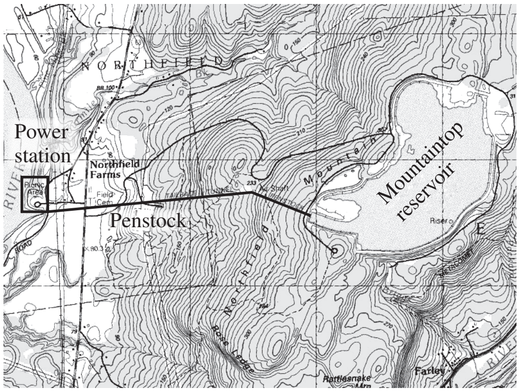
$$U_{59} = mg\,\Delta y = (55\text{ kg})(9.8\text{ m/s}^2)(26\text{ floors})(3.5\text{ m/floor}) = 49\text{ kJ}$$

Here we can write U rather than ΔU because we're calculating the potential-energy *change* from the place where $U = 0$. (c) The street level is 32 floors *below* the engineer's office, so

$$U_{\text{street}} = mg\,\Delta y = (55\text{ kg})(9.8\text{ m/s}^2)(-32\text{ floors})(3.5\text{ m/floor}) = -60\text{ kJ}$$

ASSESS Makes sense: When the engineer goes *up*, the potential energy relative to her office is positive; when she goes *down*, it's negative. And the distance down is a bit farther, so the magnitude of the change is greater going down. ■

APPLICATION **Pumped Storage**



Electricity is a wonderfully versatile form of energy, but it's not easy to store. Large electric power plants are most efficient when operated continuously, yet the demand for power fluctuates. Renewable energy sources like wind and solar vary, not necessarily with demand. Energy storage can help in both cases. Today, the only practical way to store large amounts of excess electrical energy is to convert it to gravitational potential energy. In so-called pumped-storage facilities, surplus electric power pumps water from a lower reservoir to a higher one, thereby increasing gravitational potential energy. When power demand is high, water runs back down, turning the pump motors into generators that produce electricity. The map here shows the Northfield Mountain Pumped Storage Project in Massachusetts, including the mountaintop reservoir, the location of the power station 214 m below on the Deerfield River, and the *penstock*, the pipe that conveys water in both directions between the power station and the reservoir. You can explore this facility quantitatively in Problem 29.

Elastic Potential Energy

When you stretch or compress a spring or other elastic object, you do work against the spring force, and that work ends up stored as **elastic potential energy**. For an ideal spring, the force is $F = -kx$, where x is the distance the spring is stretched from equilibrium, and the minus sign shows that the force opposes the stretching or compression. Since the force varies with position, we use Equation 7.2a to evaluate the potential energy:

$$\Delta U = -\int_{x_1}^{x_2} F(x)\,dx = -\int_{x_1}^{x_2} (-kx)\,dx = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$

where x_1 and x_2 are the initial and final values of the stretch. If we take $U = 0$ when $x = 0$ (that is, when the spring is neither stretched nor compressed) then we can use this result to write the potential energy at an arbitrary stretch (or compression) x as

$$U = \tfrac{1}{2}kx^2 \qquad \text{(elastic potential energy)} \qquad (7.4)$$

Comparison with Equation 6.10, $W = \frac{1}{2}kx^2$, shows that this is equal to the work done in stretching the spring. Thus the energy transferred by doing work gets stored as potential energy. Figure 7.5 shows potential energy as a function of the stretch or compression of a spring. The *parabolic* shape of the potential-energy curve reflects the *linear* change of the spring force with stretch or compression.

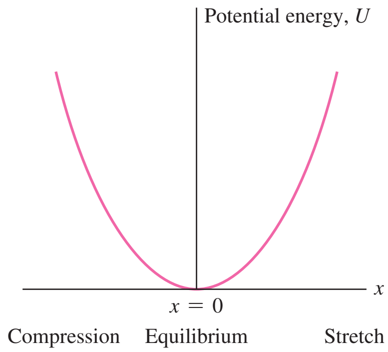


FIGURE 7.5 The potential-energy curve for a spring is a parabola.

EXAMPLE 7.2 Energy Storage: Springs versus Gasoline

A car's suspension consists of springs with an overall effective spring constant of 120 kN/m. How much would you have to compress the springs to store the same amount of energy as in 1 gram of gasoline?

INTERPRET This problem is about the energy stored in a spring, as compared with the chemical energy of gasoline.

DEVELOP Equation 7.4, $U = \frac{1}{2}kx^2$, gives a spring's stored energy when it's been compressed a distance x. Here we want that energy to equal the energy in 1 gram of gasoline. We can get that value from the "Energy Content of Fuels" table in Appendix C, which lists 44 MJ/kg for gasoline.

EVALUATE At 44 MJ/kg, the energy in 1 g of gasoline is 44 kJ. Setting this equal to the spring energy $\frac{1}{2}kx^2$ and solving for x, we get

$$x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{(2)(44\text{ kJ})}{120\text{ kN/m}}} = 86\text{ cm}$$

ASSESS This answer is absurd. A car's springs couldn't compress anywhere near that far before the underside of the car hit the ground. And 1 g isn't much gasoline. This example shows that springs, though useful energy-storage devices, can't possibly compete with chemical fuels. ■

EXAMPLE 7.3 Elastic Potential Energy: A Climbing Rope

Ropes used in rock climbing are "springy" so that they cushion a fall. A particular rope exerts a force $F = -kx + bx^2$, where $k = 223$ N/m, $b = 4.10$ N/m^2, and x is the stretch. Find the potential energy stored in this rope when it's been stretched 2.62 m, taking $U = 0$ at $x = 0$.

INTERPRET Like Example 7.2, this one is about elastic potential energy. But this one isn't so easy because the rope isn't a simple $F = -kx$ spring for which we already have a potential-energy formula.

DEVELOP Because the rope force varies with stretch, we'll have to integrate. Since force and displacement are in the same direction, we can use Equation 7.2a, $\Delta U = -\int_{x_1}^{x_2} F(x)\,dx$. But that's not so much a formula as a strategy for deriving one.

EVALUATE Applying Equation 7.2 to this particular rope, we have

$$\begin{aligned} U &= -\int_{x_1}^{x_2} F(x)\,dx = -\int_0^x (-kx + bx^2)\,dx = \tfrac{1}{2}kx^2 - \tfrac{1}{3}bx^3\Big|_0^x \\ &= \tfrac{1}{2}kx^2 - \tfrac{1}{3}bx^3 \\ &= (\tfrac{1}{2})(223\text{ N/m})(2.62\text{ m})^2 - (\tfrac{1}{3})(4.1\text{ N/m}^2)(2.62\text{ m})^3 \\ &= 741\text{ J} \end{aligned}$$

ASSESS This result is about 3% less than the potential energy $U = \frac{1}{2}kx^2$ of an ideal spring with the same spring constant. This shows the effect of the extra term $+bx^2$, whose positive sign reduces the restoring force and thus the work needed to stretch the spring. ■

GOT IT? 7.2 Gravitational force actually decreases with height, but that decrease is negligible near Earth's surface. To account for the decrease, would the exact value for the potential-energy change associated with a height change h be (a) greater than, (b) less than, or (c) equal to mgh, where g is the gravitational acceleration at Earth's surface?

Where's the Stored Energy and What's the System?

In discussing the climber of Fig. 7.1*a*, the book of Fig. 7.3, and the engineer of Example 7.1, we were careful not to use phrases like "the climber's potential energy," "the potential energy of the book," or "the engineer's potential energy." After all, the climber herself hasn't changed in going from the bottom to the top of the cliff; nor is the book any different after you've returned it to the shelf. So it doesn't make a lot of sense to say that potential energy is somehow a property of these objects. Indeed, the idea of potential energy requires that two (or more) objects interact via a force. In the examples of the climber, the book, and the engineer, that force is gravity—and the pairs of interacting objects are, correspondingly, the climber and Earth, the book and Earth, and the engineer and Earth. So to characterize potential energy, we need in each case to consider a system consisting of at least two objects. In each example the *configuration* of that system changes, because the relative positions of the objects making up the system are altered. In each case, one member of the system—climber, book, or engineer—has moved relative to Earth. So potential energy is energy associated with the *configuration of a system*. It really makes no sense to talk about the potential energy of a single, structureless object. That's in contrast with kinetic energy, which is associated with the motion of a system that might be as simple as a single object.

So where is potential energy stored? In the system of interacting objects. Potential energy is inherently a property of a system and can't be assigned to individual objects.

In the case of gravity, we can go further and say that the energy is stored in the *gravitational field*—a concept that we'll introduce in the next chapter. It's the gravitational field that changes, not the individual objects, when we change the configuration of a system whose components interact via gravity.

What about a spring? We *can* talk about "the potential energy of a spring" because any flexible object, including a spring, necessarily comprises a system of interacting parts. In the case of a spring, the individual molecules in the spring ultimately interact via electric forces, and the associated *electric field* is what changes as the spring stretches or compresses. And, as we'll see quantitatively in Chapter 23, it's in the electric field that the potential energy resides. When we talk about "elastic potential energy" we're really describing potential energy stored in molecular electric fields.

7.3 Conservation of Mechanical Energy

The work–kinetic energy theorem, developed in Section 6.3, shows that the change ΔK in an object's kinetic energy is equal to the net work done on it:

$$\Delta K = W_{\text{net}}$$

Here we'll consider the case where the only forces acting are conservative; then, as our interpretation of Equation 7.2 shows, the work done is the negative of the potential-energy change: $W_{\text{net}} = -\Delta U$. As a result, we have $\Delta K = -\Delta U$, or

$$\Delta K + \Delta U = 0$$

What does this equation tell us? It says that any change ΔK in kinetic energy K must be compensated by an opposite change ΔU in potential energy U in order that the two changes sum to zero. If kinetic energy goes up, then potential energy goes down by the same amount, and vice versa. In other words, the total **mechanical energy**, defined as the sum of kinetic and potential energy, does not change.

Remember that at this point we're considering the case where only conservative forces act. For that case, we've just shown that mechanical energy is conserved. This principle, called **conservation of mechanical energy**, is expressed mathematically in the two equivalent ways we've just discussed:

$$\Delta K + \Delta U = 0 \tag{7.5}$$

and, equivalently, (conservation of mechanical energy)

$$K + U = \text{constant} = K_0 + U_0 \tag{7.6}$$

Here K_0 and U_0 are the kinetic and potential energy when an object is at some point, and K and U are their values when it's at any other point. Equations 7.5 and 7.6 both say the same thing: In the absence of nonconservative forces, the total mechanical energy $K + U$ doesn't change. Individually, K and U can change, as energy is transformed from kinetic to potential and vice versa—but when only conservative forces are acting, then the total mechanical energy remains unchanged.

The work–kinetic energy theorem—which itself follows from Newton's second law—is what lies behind the principle of mechanical energy conservation. Although we derived the work–kinetic energy theorem by considering a single object, the principle of mechanical energy conservation holds for any isolated system of macroscopic objects, no matter how complex, as long as its constituents interact only via conservative forces. Individual constituents of a complex system may exchange kinetic energy as, for example, they undergo collisions. Furthermore, the system's potential energy may change as the configuration of the system changes—but add all the constituents' kinetic energies and the potential energy contained in the entire system, and you'll find that the sum remains unchanged.

Keep in mind that we're considering here only isolated systems. If energy is transferred to the system from outside, by external forces doing work, then the system's mechanical energy increases. And if the system does work on its environment, then its mechanical energy decreases. Ultimately, however, energy is always conserved, and if you make the system large enough to encompass all interacting objects, and if those objects interact only via conservative forces, then the system's mechanical energy will be strictly conserved.

Conservation of mechanical energy is a powerful principle. Throughout physics, from the subatomic realm through practical problems in engineering and on to astrophysics, the principle of energy conservation is widely used in solving problems that would be intractable without it. Here we consider its use in macroscopic systems subject only to conservative forces; later we'll expand the principle to more general cases.

PROBLEM-SOLVING STRATEGY 7.1 Conservation of Mechanical Energy

When you're using energy conservation to solve problems, Equation 7.6 basically tells it all. Our IDEA problem-solving strategy adapts well to such problems.

INTERPRET First, interpret the problem to be sure that conservation of mechanical energy applies. Are all the forces conservative? If so, mechanical energy is conserved. Next, identify a point at which you know both the kinetic and the potential energy; then you know the total mechanical energy, which is what's conserved. If the problem doesn't do so and it's not implicit in the equations you use, you may need to identify the zero of potential energy—although that's your own arbitrary choice. You also need to identify the quantity the problem is asking for, and the situation in which it has the value you're after. The quantity may be the energy itself or a related quantity like height, speed, or spring compression. In some situations, you may have to deal with several types of potential energy—such as gravitational and elastic potential energy—appearing in the same problem.

DEVELOP Draw your object first in the situation where you know the energies and then in the situation that contains the unknown. It's helpful to draw simple bar charts suggesting the relative sizes of the potential- and kinetic-energy terms; we'll show you how in several examples. Then you're ready to set up the quantitative statement of mechanical energy conservation, Equation 7.6: $K + U = K_0 + U_0$. Consider which of the four terms you know or can calculate from the given information. You'll probably need secondary equations like the expressions for kinetic energy and for various forms of potential energy. Consider how the quantity you're trying to find is related to an energy.

EVALUATE Write Equation 7.6 for your specific problem, including expressions for kinetic or potential energy that contain the quantity you're after. Solving is then a matter of algebra.

ASSESS As usual, ask whether your answer makes physical sense. Does it have the right units? Are the numbers reasonable? Do the signs make sense? Is your answer consistent with the bar charts in your drawing?

EXAMPLE 7.4 Energy Conservation: Tranquilizing an Elephant

A biologist uses a spring-loaded gun to shoot tranquilizer darts into an elephant. The gun's spring has $k = 940$ N/m and is compressed a distance $x_0 = 25$ cm before firing a 38-g dart. Assuming the gun is pointed horizontally, at what speed does the dart leave the gun?

INTERPRET We're dealing with a spring, assumed ideal, so conservation of mechanical energy applies. We identify the initial state—dart at rest, spring fully compressed—as the point where we know both kinetic and potential energy. The state we're then interested in is when the dart just leaves the gun, when potential energy has been converted to kinetic energy and before gravity has changed its vertical position.

DEVELOP In Fig. 7.6 we've sketched the two states, giving the potential and kinetic energy for each. We've also sketched bar graphs showing the relative sizes of the energies. To use the statement of energy conservation, Equation 7.6, we also need expressions for the kinetic energy ($\frac{1}{2}mv^2$) and the spring potential energy ($\frac{1}{2}kx^2$; Equation 7.4). Incidentally, using Equation 7.4 implicitly sets the zero of elastic potential energy when the spring

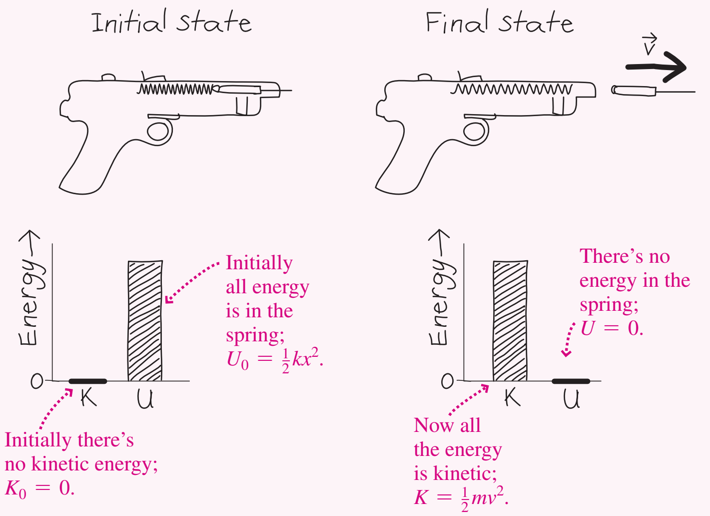


FIGURE 7.6 Our sketches for Example 7.4, showing bar charts for the initial and final states.

is in its equilibrium position. We might as well set the zero of gravitational energy at the height of the gun, since there's no change in the dart's vertical position between our initial and final states.

EVALUATE We're now ready to write Equation 7.6, $K + U = K_0 + U_0$. We know three of the terms in this equation: The initial kinetic energy K_0 is 0, since the dart is initially at rest. The initial potential energy is that of the compressed spring, $U_0 = \frac{1}{2}kx_0^2$. The final potential energy is $U = 0$ because the spring is now in its equilibrium position and we've taken the gravitational potential energy to be zero. What we don't know is the final kinetic energy, but we do know that it's given by $K = \frac{1}{2}mv^2$. So Equation 7.6 becomes $\frac{1}{2}mv^2 + 0 = 0 + \frac{1}{2}kx^2$, which solves to give

$$v = \sqrt{\frac{k}{m}}x_0 = \left(\sqrt{\frac{940\ \text{N/m}}{0.038\ \text{kg}}}\right)(0.25\ \text{m}) = 39\ \text{m/s}$$

ASSESS Take a look at the answer in algebraic form; it says that a stiffer spring or a greater compression will give a higher dart speed. Increasing the dart mass, on the other hand, will decrease the speed. All this makes good physical sense. And the outcome shows quantitatively what our bar charts suggest—that the dart's energy starts out all potential and ends up all kinetic. ■

Example 7.4 shows the power of the conservation-of-energy principle. If you had tried to find the answer using Newton's law, you would have been stymied by the fact that the spring force and thus the acceleration of the dart vary continuously. But you don't need to worry about those details; all you want is the final speed, and energy conservation gets you there, shortcutting the detailed application of $\vec{F} = m\vec{a}$.

EXAMPLE 7.5 Conservation of Energy: A Spring and Gravity

The spring in Fig. 7.7 has $k = 140$ N/m. A 50-g block is placed against the spring, which is compressed 11 cm. When the block is released, how high up the slope does it rise? Neglect friction.

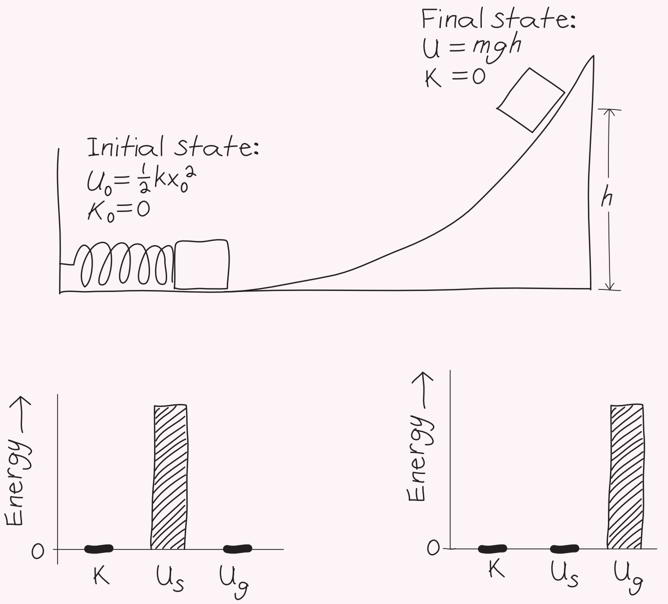


FIGURE 7.7 Our sketches for Example 7.5.

INTERPRET This example is similar to Example 7.4, but now we have changes in both elastic and gravitational potential energy. Since friction is negligible, we can consider that only conservative forces act, in which case we can apply conservation of mechanical energy. We identify the initial state as the block at rest against the compressed spring; the final state is the block momentarily at rest at its topmost point on the slope. We'll take the zero of gravitational potential energy at the bottom.

DEVELOP Figure 7.7 shows the initial and final states, along with bar charts for each. We've drawn separate bars for the spring and gravitational potential energies, U_s and U_g. Now apply Equation 7.6, $K + U = K_0 + U_0$.

EVALUATE In both states the block is at rest, so kinetic energy is zero. In the initial state we know the potential energy U_0: It's the spring energy $\frac{1}{2}kx^2$. We don't know the final-state potential energy, but we do know that it's gravitational energy—and with the zero of potential energy at the bottom, it's $U = mgh$. With $K = K_0 = 0$, $U_0 = \frac{1}{2}kx^2$, and $U = mgh$, Equation 7.6 reads $0 + mgh = 0 + \frac{1}{2}kx^2$. We then solve for the unknown h to get

$$h = \frac{kx^2}{2mg} = \frac{(140\ \text{N/m})(0.11\ \text{m})^2}{(2)(0.050\ \text{kg})(9.8\ \text{m/s}^2)} = 1.7\ \text{m}$$

ASSESS Again, the answer in algebraic form makes sense; the stiffer the spring or the more it's compressed, the higher the block will go. But if the block is more massive or gravity is stronger, then the block won't get as far.

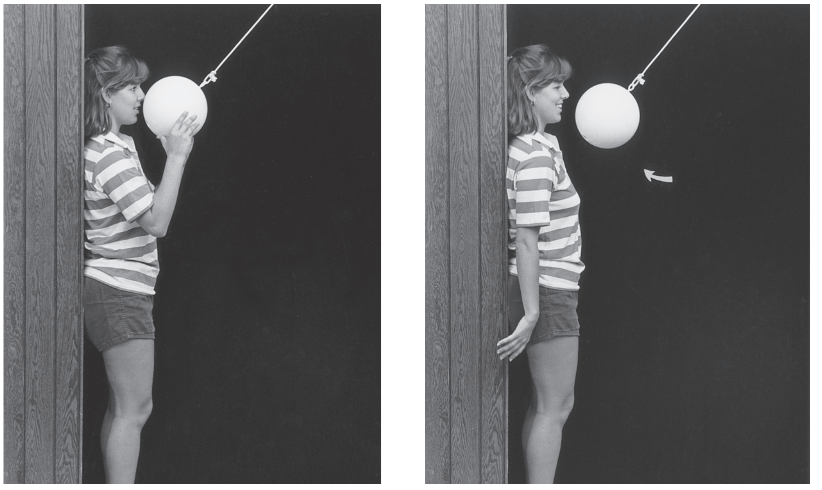
✓TIP Save Steps

You might be tempted to solve first for the block's speed when it leaves the spring and then equate $\frac{1}{2}mv^2$ to mgh to find the height. You could—but conservation of mechanical energy shortcuts all the details, getting you right from the initial to the final state. As long as energy is conserved, you don't need to worry about what happens in between.

■

Video Tutor Demo | **Chin Basher?**

GOT IT? 7.3 A bowling ball is tied to the end of a long rope and suspended from the ceiling. A student stands at one side of the room and holds the ball to her nose, then releases it from rest. Should she duck as it swings back? Explain.



7.4 Nonconservative Forces

In the examples in Section 7.3, we assumed that mechanical energy was strictly conserved. In the everyday world of friction and other nonconservative forces, however, conservation of mechanical energy is sometimes a reasonable approximation and sometimes not. When it's not, we have to consider energy transformations associated with nonconservative forces.

Friction is a nonconservative force. Recall from Chapter 5 that friction is actually a complex phenomenon, involving the making and breaking of microscopic bonds between two surfaces in contact (review Fig. 5.18). Associated with these bonds are myriad force application points, and different points may undergo different displacements depending on the strengths of the temporary bonds. For these reasons it's difficult to calculate, or even to define unambiguously, the work done by friction.

What friction and other nonconservative forces do, however, is unambiguous: They convert the kinetic energy of macroscopic objects into kinetic energy associated with the random motions of individual molecules. Although we're still talking about kinetic energy, there's a huge difference between the kinetic energy of a macroscopic object like a moving car, with all its parts participating in a common motion, versus the random motions of molecules going helter-skelter in every direction with a range of speeds. We'll explore that difference in Chapter 19, where we'll find that, among other profound implications, it places serious constraints on our ability to extract energy from fuels.

You'll also see, in Chapter 18, that molecular energy may include potential energy associated with stretching of spring-like molecular bonds. The combination of molecular kinetic and potential energy is called **internal energy** or **thermal energy**, and we give it the symbol E_{int}. Here "internal" implies that this energy is contained within an object and that it isn't as obvious as the kinetic energy associated with overall motion of the entire object. The alternative term "thermal" hints that internal energy is associated with temperature, heat, and related phenomena. We'll see in Chapters 16–19 that temperature is a measure of the internal energy per molecule, and that what you probably think of as "heat" is actually internal energy. In physics, "heat" has a very specific meaning: It designates another way of transferring energy to a system, in addition to the mechanical work we've considered in Chapters 6 and 7.

So friction and other nonconservative forces convert mechanical energy into internal energy. How much internal energy? Both theory and experiment give a simple answer: The amount of mechanical energy converted to internal energy is given by the product of the nonconservative force with the distance over which it acts. With friction, that means $\Delta E_{\text{int}} = f_k d$, where d is the distance over which the frictional force acts. (Here we write *kinetic* friction f_k explicitly because *static* friction f_s does not convert mechanical energy to internal energy because there's no relative motion involved.) Since the increase in internal energy comes at the expense of mechanical energy $K + U$, we can write

$$\Delta K + \Delta U = -\Delta E_{\text{int}} = -f_k d \tag{7.7}$$

Example 7.6 describes a system in which friction converts mechanical energy to internal energy.

EXAMPLE 7.6 Nonconservative Forces: A Sliding Block

A block of mass m is launched from a spring of constant k that's initially compressed a distance x_0. After leaving the spring, the block slides on a horizontal surface with frictional coefficient μ. Find an expression for the distance the block slides before coming to rest.

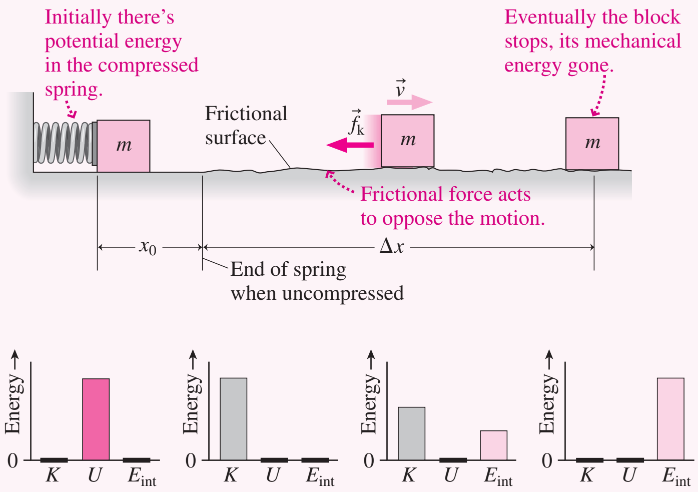


FIGURE 7.8 Intermediate bar charts show gradual conversion of mechanical energy into internal energy.

INTERPRET The presence of friction means that mechanical energy isn't conserved. But we can still identify the kinetic and potential energy in the initial state: The kinetic energy is zero and the potential energy is that of the spring. In the final state, there's no mechanical energy at all. The nonconservative frictional force converts the block's mechanical energy into internal energy of the block and the surface it's sliding on. The block comes to rest when all its mechanical energy has been converted.

DEVELOP Figure 7.8 shows the situation. With $K_0 = 0$, we determine the total initial energy from Equation 7.4, $U_0 = \frac{1}{2}kx_0^2$. As the block slides a distance d, Equation 7.7 shows that the frictional force converts mechanical energy equal to $f_k d$ into internal energy. All the mechanical energy will be gone, therefore, when $f_k d = \frac{1}{2}kx_0^2$. Here the frictional force has magnitude $f_k = \mu n = \mu mg$, where in this case of a horizontal surface the normal force n has the same magnitude as the weight mg. So our statement that all the mechanical energy gets converted to internal energy becomes $\frac{1}{2}kx_0^2 = \mu mgd$.

EVALUATE We solve this equation for the unknown distance d to get $d = kx_0^2/2\mu mg$. Since we weren't given numbers, there's nothing further to evaluate.

ASSESS Make sense? The stiffer the spring or the more it's compressed, the farther the block goes. The greater the friction or the normal force mg, the sooner the block stops. If $\mu = 0$, mechanical energy is once again conserved; then our result shows that the block would slide forever. ■

GOT IT? 7.4 For which of the following systems is (1) mechanical energy conserved and (2) total energy conserved? (a) the system is isolated, and all forces among its constituents are conservative; (b) the system is not isolated, and work is done on it by external forces; (c) the system is isolated, and some forces among its constituents are not conservative

7.5 Conservation of Energy

We often speak of energy being "lost" due to friction, or to air resistance, or to electrical resistance in power transmission. But that energy isn't really lost; instead, as we've just seen for friction, it's converted to internal energy. Physically, the internal energy manifests itself by warming the system. So the energy really is still there; it's just that we can't get it back as the kinetic energy of macroscopic objects.

Accounting for internal energy leads to a broader statement of energy conservation. Rearranging the first equality of Equation 7.7 lets us write

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

This equation shows that the sum of the kinetic, potential, and internal energy of an isolated system doesn't change even though energy may be converted among these three different forms. You can see this conservation of energy graphically in Fig. 7.8, which plots all three forms of energy for the situation of Example 7.6.

So far we've considered only isolated systems, in which all forces are internal to the system. For Example 7.6 to be about an isolated system, for instance, that system had to include the spring, the block, and the surface on which the block slides. What if a system isn't isolated? Then external forces may do work on it, increasing its energy. Or the system may do work on its environment, decreasing its energy. In that case we can generalize Equation 7.7 to read

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W_{\text{ext}} \tag{7.8}$$

where W_{ext} is the work done on the system by forces acting from outside. If W_{ext} is positive, then this external work adds energy to the system; if it's negative, then the system does work on its surroundings, and its total energy decreases. Recall that doing work is the *mechanical* means of transferring energy; in Chapters 16–18 we'll introduce *heat* as a nonmechanical energy-transfer mechanism, and we'll develop a statement like Equation 7.8 that includes energy transfers by both work and heat.

Energy Conservation: The Big Picture

So far we've considered kinetic energy, potential energy, and internal energy, and we've explored energy transfer by mechanical work and by dissipative forces like friction. We've also hinted at energy transfer by heat, to be defined in Chapter 16. But there are other forms of energy, and other energy-transfer mechanisms. In Part 3, you'll explore electromagnetism, and you'll see how energy can be stored in both electric and magnetic fields; their combination into electromagnetic waves results in energy transfer by *electromagnetic radiation*—the process that delivers life-sustaining energy from Sun to Earth and that also carries your cell phone conversations and data. Electromagnetic fields interact with matter, so energy transfers among electromagnetic, mechanical, and internal energy are important processes in the everyday physics of both natural and technological systems. But again, for any isolated system, such transfers only interchange *types* of energy and don't change the total *amount* of energy. Energy, it seems, is strictly conserved.

In Newtonian physics, conservation of energy stands alongside the equally fundamental principle of conservation of mass (the statement that the total mass of an isolated system can't change). A closer look, however, shows that neither principle stands by itself. If you measure precisely enough the mass of a system before it emits energy, and again afterward, you'll find that the mass has decreased. Einstein's equation $E = mc^2$ describes this effect, which ultimately shows that mass and energy are interchangeable. So Einstein replaces the separate conservation laws for mass and energy with a single statement: **conservation of mass–energy**. You'll see how mass–energy interchangeability arises when we study relativity in Chapter 33. Until then, we'll be dealing in the realm of Newtonian physics, where it's an excellent approximation to assume that energy and mass are separately conserved.

GOT IT? 7.5 Consider Earth and its atmosphere as a system. Which of the following processes conserves the total energy of this system? (a) a volcano erupts, spewing hot gases and particulate matter high into the atmosphere; (b) a small asteroid plunges into Earth's atmosphere, heating and exploding high over the planet; (c) over geologic time, two continents collide, and the one that is subducted under the other heats up and undergoes melting; (d) a solar flare delivers high-energy particles to Earth's upper atmosphere, lighting the atmosphere with colorful auroras; (e) a hurricane revs up its winds, extracting energy from water vapor evaporated from warm tropical seas; (f) coal burns in numerous power plants, and uranium fissions in nuclear reactors, with both processes sending electrical energy into the world's power grids and dumping warmed water into the environment

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7.6 Potential-Energy Curves

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PhET: Energy Skate Park

Figure 7.9 shows a frictionless roller-coaster track. How fast must a car be coasting at point A if it's to reach point D? Conservation of mechanical energy provides the answer. To get to D, the car must clear peak C. Clearing C requires that the total energy exceed the potential energy at C; that is, $\frac{1}{2}mv_A^2 + mgh_A > mgh_C$, where we've taken the zero of potential energy with the car at the bottom of the track. Solving for v_A gives $v_A > \sqrt{2g(h_C - h_A)}$. If v_A satisfies this inequality, the car will reach C with some kinetic energy remaining and will coast over the peak.

Figure 7.9 is a drawing of the actual roller-coaster track. But because gravitational potential energy is directly proportional to height, it's also a plot of potential energy versus position: a **potential-energy curve**. Conceptual Example 7.1 shows how we can study the car's motion by plotting total energy on the same graph as the potential-energy curve.

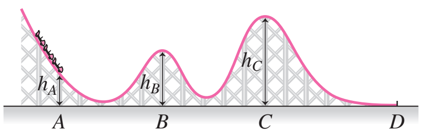


FIGURE 7.9 A roller-coaster track.

CONCEPTUAL EXAMPLE 7.1 Potential-Energy Curves

Figure 7.10 plots potential energy for our roller-coaster system, along with three possible values for the total mechanical energy. Since mechanical energy is conserved in the absence of nonconservative forces, the total-energy curve is a horizontal line. Use these graphs to describe the motion of a roller-coaster car, initially at point A and moving to the right.

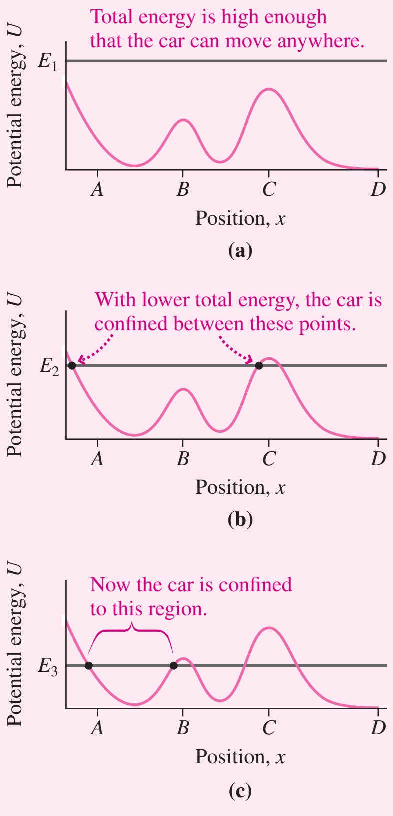


FIGURE 7.10 Potential and total energy for a roller coaster.

EVALUATE We're assuming there are no nonconservative forces (an approximation for a real roller coaster), so mechanical energy is conserved. In each figure, the sum of kinetic and potential energy therefore remains equal to the value set by the line indicating the total energy. When the roller-coaster car rises, potential energy increases and kinetic energy consequently decreases. But as long as potential energy remains below the total energy, the car still has kinetic energy and is still moving. Anywhere potential energy equals the total energy, the car has no kinetic energy and is momentarily at rest.

In Fig. 7.10*a* the car's total energy exceeds the maximum potential energy. Therefore, it can move anywhere from its initial position at A. Since it's initially moving to the right, it will clear peaks B and C and will end up at D still moving to the right—and, since D is lower than A, it will be moving faster than it was at A.

In Fig. 7.10*b* the highest peak in the potential-energy curve exceeds the total energy; so does the very leftmost portion of the curve. Therefore, the car will move rightward from A, clearing peak B, but will come to a stop just before peak C, a so-called **turning point** where potential energy equals the total energy. Then it will roll back down to the left, again clearing peak B and climbing to another turning point where the potential-energy curve and total-energy line again intersect. Absent friction, it will run back and forth between the two turning points.

In Fig. 7.10*c* the total energy is lower, and the car can't clear peak B. So now it will run back and forth between the two turning points we've marked.

ASSESS Make sense? Yes: The higher the total energy, the larger the extent of the car's allowed motion. That's because, for a given potential energy, the car it has more energy available in the form of kinetic energy.

MAKING THE CONNECTION Find a condition on the speed at A that will allow the car to move beyond peak B.

EVALUATE With total energy equal to U_B, the car could just barely clear peak B. The initial energy is $\frac{1}{2}mv_A^2 + mgh_A$, where v_A and h_A are the car's speed and height at A, and where we've taken the zero of potential energy at the bottom of the curve. Requiring that this quantity exceed $U_B = mgh_B$ then gives $v_A > \sqrt{2g(h_B - h_A)}$.

Even though the car in Figs. 7.10*b* and *c* can't get to D, the total energy still exceeds the potential energy at D. But the car is blocked from reaching D by the **potential barrier** of peak C. We say that it's **trapped** in a **potential well** between its turning points.

Potential-energy curves are useful even with nongravitational forces where there's no direct correspondence with hills and valleys. The terminology used here—potential barriers, wells, and trapping—remains appropriate in such cases and indeed is widely used throughout physics.

Figure 7.11 shows the potential energy of a system comprising a pair of hydrogen atoms, as a function of their separation. This energy is associated with attractive and repulsive electrical forces involving the electrons and the nuclei of the two atoms. The potential-energy curve exhibits a potential well, showing that the atoms can form a **bound system** in which they're unable to separate fully. That bound system is a hydrogen molecule (H_2). The minimum energy, -7.6×10^{-19} J, corresponds to the molecule's equilibrium separation of 0.074 nm. It's convenient to define the zero of potential energy when the atoms are infinitely far apart; Fig. 7.11 then shows that any total energy less than zero results in a bound system. But if the total energy is greater than zero, the atoms are free to move arbitrarily far apart, so they don't form a molecule.

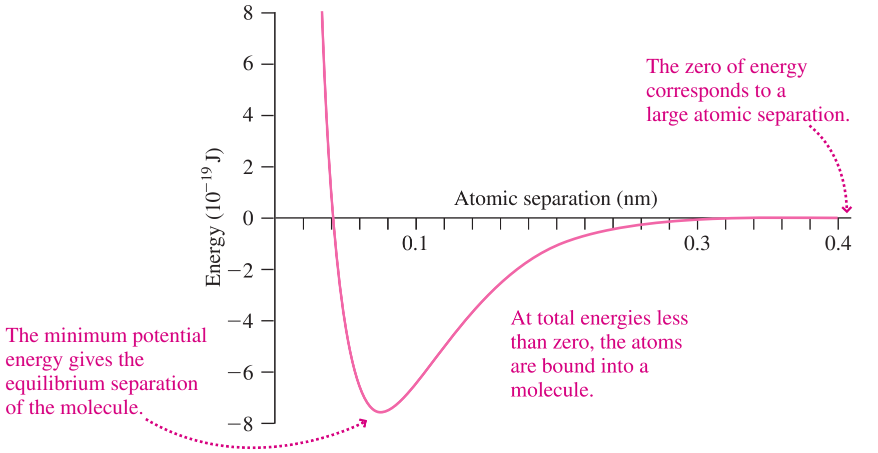


FIGURE 7.11 Potential-energy curve for two hydrogen atoms.

EXAMPLE 7.7 Molecular Energy: Finding Atomic Separation

Very near the bottom of the potential well in Fig. 7.11, the potential energy of the two-atom system given approximately by $U = U_0 + a(x - x_0)^2$, where $U_0 = -0.760$ aJ, $a = 286$ aJ/nm^2, and $x_0 = 0.0741$ nm is the equilibrium separation. What range of atomic separations is allowed if the total energy is -0.717 aJ?

INTERPRET This problem sounds complicated, with strange units and talk of molecular energies. But it's about just what's shown in Figs. 7.10 and 7.11. Specifically, we're given the total energy and asked to find the turning points—the points where the line representing total energy intersects the potential-energy curve. If the units look strange, remember the SI prefixes (there's a table inside the front cover), which we use to avoid writing large powers of 10. Here 1 aJ $= 10^{-18}$ J and 1 nm $= 10^{-9}$ m.

DEVELOP Figure 7.12 is a plot of the potential-energy curve from the function we've been given. The straight line represents the total energy E. The turning points are the values of atomic separation where the two curves intersect. We could read them off the graph, or we can solve algebraically by setting the total energy equal to the potential energy.

EVALUATE With the potential energy given by $U = U_0 + a(x - x_0)^2$ and the total energy E, the two turning points occur when $E = U_0 + a(x - x_0)^2$. We could solve directly for x, but then we'd have to use the quadratic formula. Solving for $x - x_0$ is easier:

$$x - x_0 = \pm\sqrt{\frac{E - U_0}{a}} = \pm\sqrt{\frac{-0.717 \text{ aJ} - (-0.760 \text{ aJ})}{286 \text{ aJ/nm}^2}}$$
$$= \pm 0.0123 \text{ nm}$$

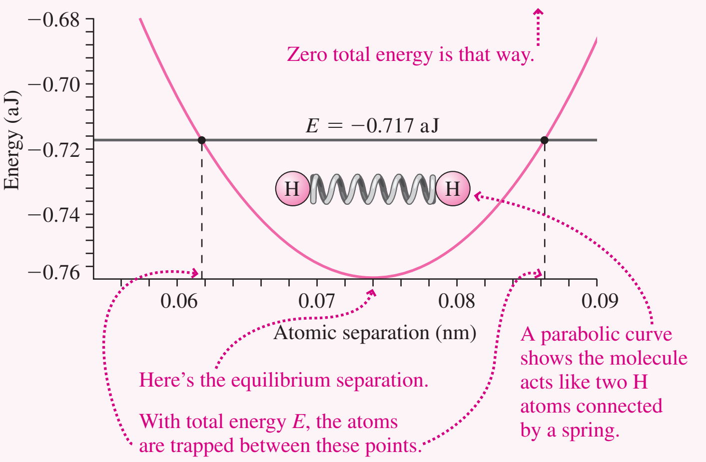


FIGURE 7.12 Analyzing the hydrogen molecule.

Then the turning points are at $x_0 \pm 0.0123$ nm—namely, 0.0864 nm and 0.0618 nm.

ASSESS Make sense? A look at Fig. 7.12 shows that we've correctly located the turning points. The fact that its potential-energy curve is parabolic (like a spring's $U = \frac{1}{2}kx^2$) shows that the molecule can be modeled approximately as two atoms joined by a spring. Chemists frequently use such models and even talk of the "spring constant" of the bond joining atoms into a molecule. ■

Force and Potential Energy

The roller-coaster track in Fig. 7.9 traces the potential-energy curve for a car on the track. But it also shows the force acting to accelerate the car: Where the graph is steep—that is, where the potential energy is changing rapidly—the force is greatest. At the peaks and valleys, the force is zero. So it's the *slope* of the potential-energy curve that tells us about the force (Fig. 7.13).

Just how strong is this force? Consider a small change Δx, so small that the force is essentially constant over this distance. Then we can use Equation 7.2b to write $\Delta U = -F_x\,\Delta x$, or $F_x = -\Delta U/\Delta x$. In the limit $\Delta x \rightarrow 0$, $\Delta U/\Delta x$ becomes the derivative, and we have

$$F_x = -\frac{dU}{dx} \qquad (7.9)$$

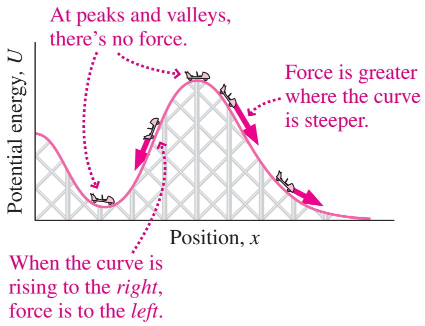
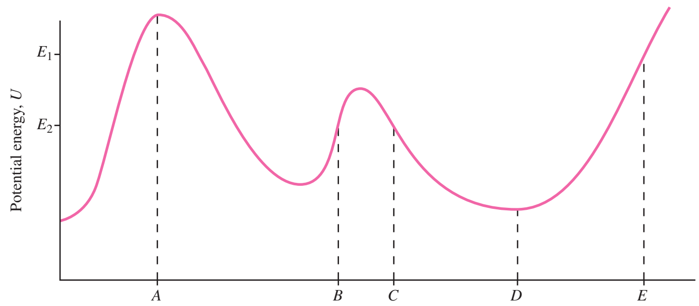


FIGURE 7.13 Force depends on the *slope* of the potential-energy curve.

This equation makes mathematical as well as physical sense. We've already written potential energy as the *integral* of force over distance, so it's no surprise that force is the *derivative* of potential energy. Equation 7.9 gives the force component in the x-direction only. In a three-dimensional situation, we'd have to take derivatives of potential energy with respect to y and z to find the full force vector.

Why the minus sign in Equation 7.9? You can see the answer in the molecular energy curve of Fig. 7.11, where pushing the atoms too close together—moving to the *left* of equilibrium—results in a repulsive force to the *right*, and pulling them apart—moving to the *right*—gives an attractive force to the *left*. You can see the same thing for the roller coaster in Fig. 7.13. In both cases the forces tend to drive the system back toward a minimum-energy state. We'll explore such minimum-energy equilibrium states further in Chapter 12.

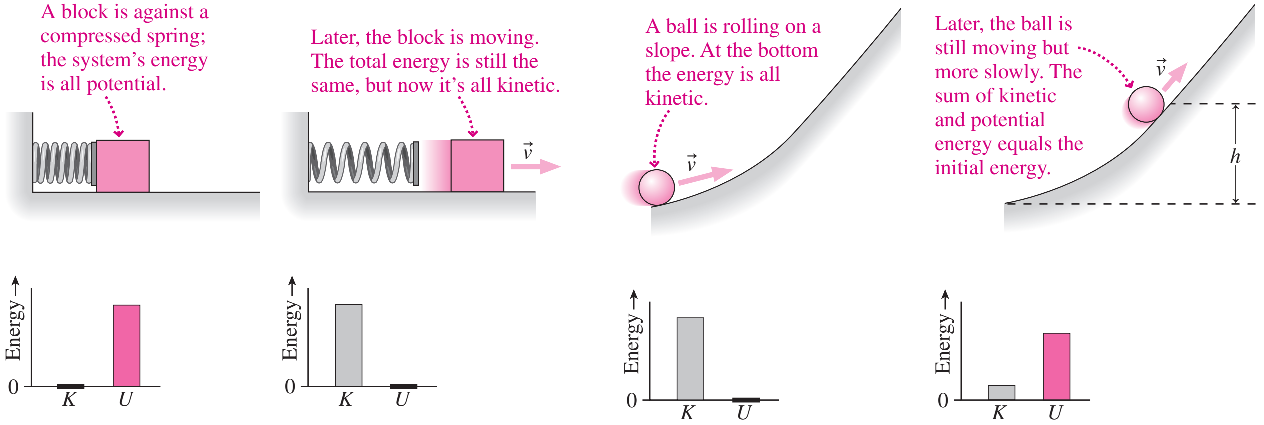
GOT IT? 7.6 The figure shows the potential energy associated with an electron in a microelectronic device. From among the labeled points, find (1) the point where the force on the electron is greatest, (2) the rightmost position possible if the electron has total energy E_1, (3) the leftmost position possible if the electron has total energy E_2 and starts out to the right of D, (4) a point where the force on the electron is zero, and (5) a point where the force on the electron points to the left. In some cases there may be more than one answer.



CHAPTER 7 SUMMARY

Big Ideas

The big idea here is conservation of energy. This chapter emphasizes the special case of systems subject only to conservative forces, in which case the total mechanical energy—the sum of kinetic and potential energy—cannot change. Energy may change from kinetic to potential, and vice versa, but the total remains constant. Applying conservation of mechanical energy requires the concept of potential energy—energy stored in a system as a result of work done against conservative forces.



If nonconservative forces act in a system, then mechanical energy isn't conserved; instead, mechanical energy gets converted to internal energy.

Key Concepts and Equations

The important new concept here is potential energy, defined as the negative of the work done by a conservative force. Only the change ΔU has physical significance. Expressions for potential energy include:

$$\Delta U_{AB} = -\int_A^B \vec{F} \cdot d\vec{r}$$

This one is the most general, but it's mathematically involved. The force can vary over an arbitrary path between points *A* and *B*.

$$\Delta U = -\int_{x_1}^{x_2} F(x)dx$$

This is a special case, when force and displacement are in the same direction and force may vary with position.

$$\Delta U = -F\,(x_2 - x_1)$$

This is the most specialized case, where the force is constant.

Given the concept of potential energy, the principle of conservation of mechanical energy follows from the work–kinetic energy theorem of Chapter 6. Here's the mathematical statement of mechanical energy conservation:

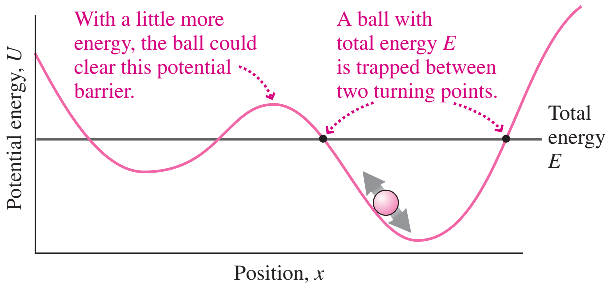
$$K + U = K_0 + U_0$$

K and U are the kinetic and potential energy at some point where we don't know one of these quantities.

The total mechanical energy is conserved, as indicated by the equal sign.

K_0 and U_0 are the kinetic and potential energy at some point where both are known. $K_0 + U_0$ is the *total mechanical energy*.

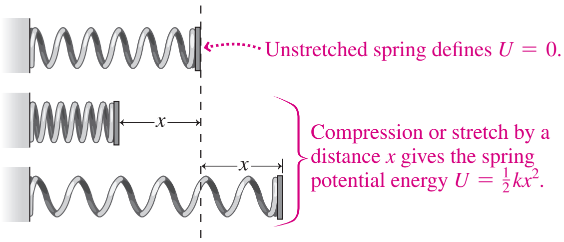
We can describe a wide range of systems—from molecules to roller coasters to planets—in terms of **potential-energy curves**. Knowing the total energy then lets us find **turning points** that determine the range of motion available to the system.

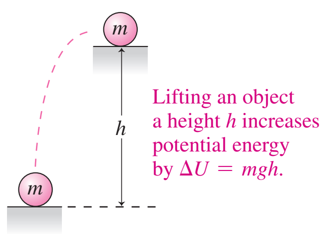


Applications

Two important cases of potential energy are the elastic potential energy of a spring, $U = \frac{1}{2}kx^2$, and the gravitational potential energy change, $\Delta U = mgh$, associated with lifting an object of mass m through a height h.

The former is limited to ideal springs for which $F = -kx$, the latter to the proximity of Earth's surface, where the variation of gravity with height is negligible.





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For Thought and Discussion

1. Figure 7.14 shows force vectors at different points in space for two forces. Which is conservative and which nonconservative? Explain.

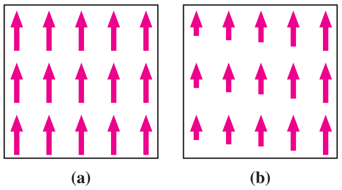

FIGURE 7.14 For Thought and Discussion 1; Problem 30

2. Is the conservation-of-mechanical-energy principle related to Newton's laws, or is it an entirely separate physical principle? Discuss.
3. Why can't we define a potential energy associated with friction?
4. Can potential energy be negative? Can kinetic energy? Can total mechanical energy? Explain.
5. If the potential energy is zero at a given point, must the force also be zero at that point? Give an example.
6. If the force is zero at a given point, must the potential energy also be zero at that point? Give an example.
7. If the difference in potential energy between two points is zero, does that necessarily mean that an object moving between those points experiences no force?
8. A tightrope walker follows an essentially horizontal rope between two mountain peaks of equal altitude. A climber descends from one peak and climbs the other. Compare the work done by the gravitational force on the tightrope walker and the climber.
9. If conservation of energy is a law of nature, why do we have programs—like mileage requirements for cars or insulation standards for buildings—designed to encourage energy conservation?

Exercises and Problems

Exercises

Section 7.1 Conservative and Nonconservative Forces

10. Determine the work you would have to do to move a block of mass m from point 1 to point 2 at constant speed over the two paths shown in Fig. 7.15. The coefficient of friction has the constant value μ over the surface. *Note:* The diagram lies in a horizontal plane.

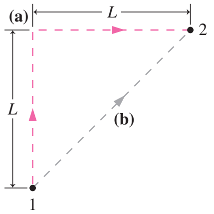

FIGURE 7.15 Exercises 10 and 11

11. Now take Fig. 7.15 to lie in a vertical plane, and find the work done by the gravitational force as an object moves from point 1 to point 2 over each of the paths shown.

Section 7.2 Potential Energy

12. Rework Example 7.1, now taking the zero of potential energy at street level.
13. Find the potential energy associated with a 70-kg hiker (a) atop New Hampshire's Mount Washington, 1900 m above sea level, and (b) in Death Valley, California, 86 m below sea level. Take the zero of potential energy at sea level.
14. You fly from Boston's Logan Airport, at sea level, to Denver, altitude 1.6 km. Taking your mass as 65 kg and the zero of potential energy at Boston, what's the gravitational potential energy when you're (a) at the plane's 11-km cruising altitude and (b) in Denver?
15. The potential energy associated with a 60-kg hiker ascending 1250-m-high Camel's Hump mountain in Vermont is -240 kJ; the zero of potential energy is taken at the mountaintop. What's her altitude?
16. How much energy can be stored in a spring with $k = 320$ N/m if the maximum allowed stretch is 18 cm?
17. How far would you have to stretch a spring with $k = 1.4$ kN/m for it to store 210 J of energy?
18. **BIO** A biophysicist grabs the ends of a DNA strand with optical tweezers and stretches it 26 μm. How much energy is stored in the stretched molecule if its spring constant is 0.046 pN/μm?

Section 7.3 Conservation of Mechanical Energy

19. A skier starts down a frictionless 32° slope. After a vertical drop of 25 m, the slope temporarily levels out and then slopes down at 20°, dropping an additional 38 m vertically before leveling out again. Find the skier's speed on the two level stretches.
20. A 10,000-kg Navy jet lands on an aircraft carrier and snags a cable to slow it down. The cable is attached to a spring with $k = 40$ kN/m. If the spring stretches 25 m to stop the plane, what was its landing speed?
21. A 120-g arrow is shot vertically from a bow whose effective spring constant is 430 N/m. If the bow is drawn 71 cm before shooting, to what height does the arrow rise?
22. In a railroad yard, a 35,000-kg boxcar moving at 7.5 m/s is stopped by a spring-loaded bumper mounted at the end of the level track. If $k = 2.8$ MN/m, how far does the spring compress in stopping the boxcar?
23. You work for a toy company, and you're designing a spring-launched model rocket. The launching apparatus has room for a spring that can be compressed 14 cm, and the rocket's mass is 65 g. If the rocket is to reach an altitude of 35 m, what should you specify for the spring constant?

Section 7.4 Nonconservative Forces

24. A 54-kg ice skater pushes off the wall of the rink, giving herself an initial speed of 3.2 m/s. She then coasts with no further effort. If the frictional coefficient between skates and ice is 0.023, how far does she go?
25. You push a 33-kg table across a 6.2-m-wide room. In the process, 1.5 kJ of mechanical energy gets converted to internal energy of the table/floor system. What's the coefficient of kinetic friction between table and floor?

Section 7.6 Potential-Energy Curves

26. A particle slides along the frictionless track shown in Fig. 7.16, starting at rest from point A. Find (a) its speed at B, (b) its speed at C, and (c) the approximate location of its right-hand turning point.

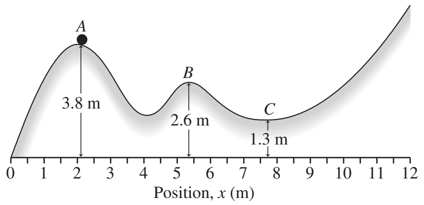

FIGURE 7.16 Exercise 26

27. A particle slides back and forth on a frictionless track whose height as a function of horizontal position x is $y = ax^2$, where $a = 0.92\ \text{m}^{-1}$. If the particle's maximum speed is 8.5 m/s, find its turning points.
28. A particle is trapped in a potential well described by $U(x) = 16x^2 - b$, with U in joules, x in meters, and $b = 4.0$ J. Find the force on the particle when it's at (a) $x = 2.1$ m, (b) $x = 0$, and (c) $x = -1.4$ m.

Problems

29. ENV The reservoir at Northfield Mountain Pumped Storage Project is 214 m above the pump/generators and holds 2.1×10^{10} kg of water (see Application on p. 113). The generators can produce electrical energy at the rate of 1.08 GW. Find (a) the gravitational potential energy stored, taking zero potential energy at the generators, and (b) the length of time the station can generate power before the reservoir is drained.
30. The force in Fig. 7.14*a* is given by $\vec{F}_a = F_0\,\hat{\jmath}$, where F_0 is a constant. The force in Fig. 7.14*b* is given by $\vec{F}_b = F_0(x/a)\,\hat{\jmath}$, where the origin is at the lower left corner of the box, a is the width of the square box, and x increases horizontally to the right. Determine the work you would have to do to move an object around the perimeter of each each box, going clockwise at constant speed, starting at the lower left corner.
31. A 1.50-kg brick measures 20.0 cm × 8.00 cm × 5.50 cm. Taking the zero of potential energy when the brick lies on its broadest face, what's the potential energy (a) when the brick is standing on end and (b) when it's balanced on its 8-cm edge? (*Note:* You can treat the brick as though all its mass is concentrated at its center.)
32. A carbon monoxide molecule can be modeled as a carbon atom and an oxygen atom connected by a spring. If a displacement of the carbon by 1.46 pm from its equilibrium position relative to the oxygen increases the molecule's potential energy by 0.0125 eV, what's the spring constant?
33. A more accurate expression for the force law of the rope in Example 7.3 is $F = -kx + bx^2 - cx^3$, where k and b have the values given in Example 7.3 and $c = 3.1\ \text{N/m}^3$. Find the energy stored in stretching the rope 2.62 m. By what percentage does your result differ from that of Example 7.3?
34. BIO For small stretches, the Achilles tendon can be modeled as an ideal spring. Experiments using a particular tendon showed that it stretched 2.66 mm when a 125-kg mass was hung from it. (a) Find the spring constant of this tendon. (b) How much would it have to stretch to store 50.0 J of energy?
35. The force exerted by an unusual spring when it's compressed a distance x from equilibrium is $F = -kx - cx^3$, where $k = 220$ N/m and $c = 3.1\ \text{N/m}^3$. Find the stored energy when it's been compressed 15 cm.
36. The force on a particle is given by $\vec{F} = A\,\hat{\imath}/x^2$, where A is a positive constant. (a) Find the potential-energy difference between two points x_1 and x_2, where $x_1 > x_2$. (b) Show that the potential-energy difference remains finite even when $x_1 \rightarrow \infty$.
37. A particle moves along the x-axis under the influence of a force $F = ax^2 + b$, where a and b are constants. Find the potential energy as a function of position, taking $U = 0$ at $x = 0$.
38. As a highway engineer, you're asked to design a runaway truck lane on a mountain road. The lane will head uphill at 30° and should be able to accommodate a 16,000-kg truck with failed brakes entering the lane at 110 km/h. How long should you make the lane? Neglect friction.
39. A spring of constant k, compressed a distance x, is used to launch a mass m up a frictionless slope at angle θ. Find an expression for the maximum distance along the slope that the mass moves after leaving the spring.
40. A child is on a swing whose 3.2-m-long chains make a maximum angle of 50° with the vertical. What's the child's maximum speed?
41. With $x - x_0 = h$ and $a = g$, Equation 2.11 gives the speed of an object thrown downward with initial speed v_0 after it's dropped a distance h: $v = \sqrt{v_0^2 + 2gh}$. Use conservation of mechanical energy to derive the same result.
42. BIO The *nuchal ligament* is a cord-like structure that runs along the back of the neck and supports much of the head's weight in animals like horses and cows. The ligament is extremely stiff for small stretches, but loosens as it stretches further, thus functioning as a biological shock absorber. Figure 7.17 shows the force–distance curve for a particular nuchal ligament; the curve can be modeled approximately by the expression $F(x) = 0.43x - 0.033x^2 + 0.00086x^3$, with F in kN and x in cm. Find the energy stored in the ligament when it's been stretched (a) 7.5 cm and (b) 15 cm.

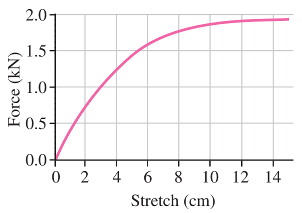

FIGURE 7.17 Problem 42

43. A 200-g block slides back and forth on a frictionless surface between two springs, as shown in Fig. 7.18. The left-hand spring has $k = 130$ N/m and its maximum compression is 16 cm. The right-hand spring has $k = 280$ N/m. Find (a) the maximum compression of the right-hand spring and (b) the speed of the block as it moves between the springs.

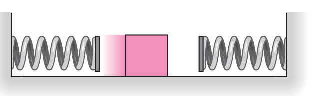

FIGURE 7.18 Problem 43

44. Automotive standards call for bumpers that sustain essentially no damage in a 4-km/h collision with a stationary object. As an automotive engineer, you'd like to improve on that. You've developed a spring-mounted bumper with effective spring

constant 1.3 MN/m. The springs can compress up to 5.0 cm before damage occurs. For a 1400-kg car, what do you claim as the maximum collision speed?

45. CH A block slides on the frictionless loop-the-loop track shown in Fig. 7.19. Find the minimum height h at which it can start from rest and still make it around the loop.

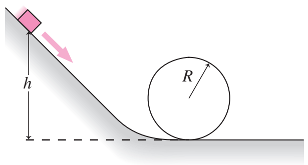

FIGURE 7.19 Problem 45

46. The maximum speed of the pendulum bob in a grandfather clock is 0.55 m/s. If the pendulum makes a maximum angle of 8.0° with the vertical, what's the pendulum's length?
47. A mass m is dropped from height h above the top of a spring of constant k mounted vertically on the floor. Show that the spring's maximum compression is given by $(mg/k)(1 + \sqrt{1 + 2kh/mg})$.
48. A particle with total energy 3.5 J is trapped in a potential well described by $U = 7.0 - 8.0x + 1.7x^2$, where U is in joules and x in meters. Find its turning points.
49. (a) Derive an expression for the potential energy of an object subject to a force $F_x = ax - bx^3$, where $a = 5$ N/m and $b = 2$ N/m^3, taking $U = 0$ at $x = 0$. (b) Graph the potential-energy curve for $x > 0$ and use it to find the turning points for an object whose total energy is -1 J.
50. CH In ionic solids such as NaCl (salt), the potential energy of a pair of ions takes the form $U = b/r^n - a/r$, where r is the separation of the ions. For NaCl, a and b have the SI values 4.04×10^{-28} and 5.52×10^{-98}, respectively, and $n = 8.22$. Find the equilibrium separation in NaCl.
51. Repeat Exercise 19 for the case when the coefficient of kinetic friction on both slopes is 0.11, while the level stretches remain frictionless.
52. ENV As an energy-efficiency consultant, you're asked to assess a pumped-storage facility. Its reservoir sits 140 m above its generating station and holds 8.5×10^9 kg of water. The power plant generates 330 MW of electric power while draining the reservoir over an 8.0-h period. Its efficiency is the percentage of the stored potential energy that gets converted to electricity. What efficiency do you report?
53. A spring of constant $k = 340$ N/m is used to launch a 1.5-kg block along a horizontal surface whose coefficient of sliding friction is 0.27. If the spring is compressed 18 cm, how far does the block slide?
54. A bug slides back and forth in a bowl 15 cm deep, starting from rest at the top, as shown in Fig. 7.20. The bowl is frictionless except for a 1.4-cm-wide sticky patch on its flat bottom, where the coefficient of friction is 0.89. How many times does the bug cross the sticky region?

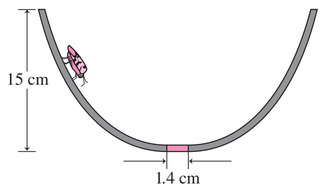

FIGURE 7.20 Problem 54

55. CH A 190-g block is launched by compressing a spring of constant $k = 200$ N/m by 15 cm. The spring is mounted horizontally, and the surface directly under it is frictionless. But beyond the equilibrium position of the spring end, the surface has frictional coefficient $\mu = 0.27$. This frictional surface extends 85 cm, followed by a frictionless curved rise, as shown in Fig. 7.21. After it's launched, where does the block finally come to rest? Measure from the left end of the frictional zone.

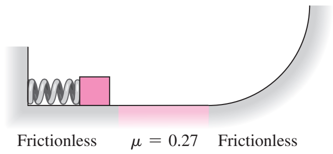

FIGURE 7.21 Problem 55

56. A block slides down a frictionless incline that terminates in a 45° ramp, as shown in Fig. 7.22. Find an expression for the horizontal range x shown in the figure as a function of the heights h_1 and h_2.

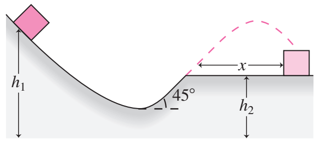

FIGURE 7.22 Problem 56

57. CH An 840-kg roller-coaster car is launched from a giant spring with $k = 31$ kN/m into a frictionless loop-the-loop track of radius 6.2 m, as shown in Fig. 7.23. What's the minimum spring compression that will ensure the car stays on the track?

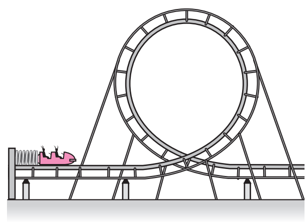

FIGURE 7.23 Problem 57

58. A particle slides back and forth in a frictionless bowl whose height is given by $h = 0.18x^2$, with x and h in meters. Find the x coordinates of its turning points if the particle's maximum speed is 47 cm/s.
59. A child sleds down a frictionless hill whose vertical drop is 7.2 m. At the bottom is a level but rough stretch where the coefficient of kinetic friction is 0.51. How far does she slide across the level stretch?
60. CH A bug lands on top of the frictionless, spherical head of a bald man. It begins to slide down his head (Fig. 7.24). Show that the bug leaves the head when it has dropped a vertical distance one-third of the head's radius.

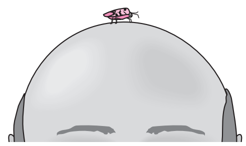

FIGURE 7.24 Problem 60

61. A particle of mass m is subject to a force $\vec{F} = (a\sqrt{x})\,\hat{\imath}$, where a is a constant. The particle is initially at rest at the origin and is given a slight nudge in the positive x-direction. Find an expression for its speed as a function of position x.
62. CH A block of weight 4.5 N is launched up a 30° inclined plane 2.0 m long by a spring with $k = 2.0$ kN/m and maximum compression 10 cm. The coefficient of kinetic friction is 0.50. Does the block reach the top of the incline? If so, how much kinetic energy does it have there? If not, how close to the top, along the incline, does it get?
63. Your engineering department is asked to evaluate the performance of a new 370-hp sports car. You know that 27% of the engine's power can be converted to kinetic energy of the 1200-kg car, and that the power delivered is independent of the car's velocity. What do you report for the time it will take to accelerate from rest to 60 mi/h on a level road?
64. Your roommate is writing a science fiction novel and asks your advice about a plot point. Her characters are mining ore on the Moon and launching it toward Earth. Bins with 1500 kg of ore will be launched by a large spring, to be compressed 17 m. It takes a speed of 2.4 km/s to escape the Moon's gravity. What do you tell her is an appropriate spring constant?
65. You have a summer job at your university's zoology department, where you'll be working with an animal behavior expert. She's assigned you to study videos of different animals leaping into the air. Your task is to compare their power outputs as they jump. You'll have the mass m of each animal from data collected in the field. From the videos, you'll be able to measure both the vertical distance d over which the animal accelerates when it pushes off the ground and the maximum height h it reaches. Your task is to find an algebraic expression for power in terms of these parameters.
66. CH Biomechanical engineers developing artificial limbs for prosthetic and robotic applications have developed a two-spring design for their replacement Achilles tendon. The first spring has constant k and the second ak, where $a > 1$. When the artificial tendon is stretched from $x = 0$ to $x = x_1$, only the first spring is engaged. For $x > x_1$, a mechanism engages the second spring, giving a configuration like that described in part (a) of Chapter 4's Problem 62. Find an expression for the energy stored in the artificial tendon when it's stretched a distance $2x_1$.
67. DATA Blocks with different masses are pushed against a spring one at a time, compressing it different amounts. Each is then launched onto an essentially frictionless horizontal surface that then curves upward, still frictionless (like Fig. 7.21 but without the frictional part). The table below shows the masses, spring compressions, and maximum vertical height each block achieves. Determine a quantity that, when you plot h against it, should yield a straight line. Plot the data, determine a best-fit line, and use its slope to determine the spring constant.

Mass m (g)	50.0	85.2	126	50.0	85.2
Compression x (cm)	2.40	3.17	5.40	4.29	1.83
Height h (cm)	10.3	11.2	19.8	35.2	3.81

Passage Problems

Nuclear fusion is the process that powers the Sun. Fusion occurs when two low-mass atomic nuclei fuse together to make a larger nucleus, in the process releasing substantial energy. This is hard to achieve because atomic nuclei carry positive electric charge, and their electrical repulsion makes it difficult to get them close enough for the short-range nuclear force to bind them into a single nucleus. Figure 7.25 shows the potential-energy curve for fusion of two deuterons (heavy hydrogen nuclei). The energy is measured in million electron volts (MeV), a unit commonly used in nuclear physics, and the separation is in femtometers ($1\text{ fm} = 10^{-15}\text{ m}$).

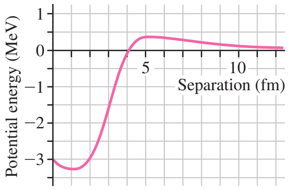

FIGURE 7.25 Potential energy for two deuterons (Passage Problems 68–71)

68. The force between the deuterons is zero at approximately
    a. 3 fm.
    b. 4 fm.
    c. 5 fm.
    d. the force is never zero.
69. In order for initially two widely separated deuterons to get close enough to fuse, their kinetic energy must be about
    a. 0.1 MeV.
    b. 3 MeV.
    c. −3 MeV.
    d. 0.3 MeV.
70. The energy available in fusion is the energy difference between that of widely separated deuterons and the bound deutrons after they've "fallen" into the deep potential well shown in the figure. That energy is about
    a. 0.3 MeV.
    b. 1 MeV.
    c. 3.3 MeV.
    d. 3.6 MeV.
71. When two deuterons are 4 fm apart, the force acting on them
    a. is repulsive.
    b. is attractive.
    c. is zero.
    d. can't be determined from the graph.

Answers to Chapter Questions

Answer to Chapter Opening Question

Potential energy turns into kinetic energy, sound, and internal energy.

Answers to GOT IT? Questions

7.1 (c) On the curved paths, the work is greater for the trunk. The gravitational force is conservative, so the work is independent of path. But the frictional force isn't conservative, and the longer path means more work needs to be done.
7.2 (b) The potential-energy change will be slightly less because at greater heights, the gravitational force is lower and so, therefore, is the work done in traversing a given distance.
7.3 No. Mechanical energy is conserved, so if the ball is released from rest, it cannot climb higher than its initial height.
7.4 (1) (a) only; (2) (a) and (c)
7.5 (a), (c), (e), (f)
7.6 (1) *B*; (2) *E*; (3) *C*; (4) *A* or *D*; (5) *B* or *E*