(1)

Proposition: Either true or false

(1) law of excluded Middle

(2) law of Contradiction

Atomic proposition:- Any Proposition which can not be divided further is said to be atomic. usually atomic proposition are donated by p, q, r, s, t eg:- 2+2 = 5

Compound Proposition:- One or More atomic proposition Combined to form a Compound proposition using Connectives / Operators (~, ∧, ∨, →, ↔)

Operators in Proposition:- (1) Negation (~/¬):- Atomic Operator.

p	~p
F	T
T	F

(2) Disjunction (∨) (or) → Binary Operator

p	q	p ∨ q
F	F	F
F	T	T
T	F	T
T	T	T

if p is true then whether p∨q will be true.

(Premises, Conclusion) (Valid)

if (p∨q) is true then whether p is true or not (Not Valid)

③ $\neg(p \vee q) \rightarrow p$ (true (invalid) (because p is false here,
or not)

$(p \vee q) \rightarrow \neg p$ (false) (q true)   this is valid ④

$(p \vee q) \rightarrow \neg q$ (false) (p true)   (this is valid) ⑤

$$\frac{\begin{array}{c} (p \vee q) \\ \neg p \end{array}}{q}$$

what is disjunction syllogism → if disjunction of two values are true and one of them is false then second has to be true.

Mathematical logic

Statement & Notation:- It is a declarative sentence & Assertive sentences that is either true or false, but not both.

eg:- Sun rises from east (true)

② Einstein is the scientist (true) Means statement

③ lamborgini is a best car (this is not statement) declarative sentence means not

Types of sentences	Statement	Truth value
① Assertive / declarative (Shahrukh Khan is an actor)	✓	T
② Exclamatory Wow! Nice car	X	X
③ Imperative (order, request, command) (Help me in work)	X	X

(4) Interrogative Sentence
Do you like Coffee? × ×

(5) Open Sentence (its a opinion)
Shahrukh is Best actor. × ×
Red is a Beautiful Color

State which of following Sentence are statement. Justify Also write truth values.

(1) Sun is a Star → } Declarative, Statement, Truth value is true
(2) Please, help me } Not, No, No

Truth value :- Truth value of a Statement is denoted by "T" if statement is true.
→ Truth value of a statement is denoted by F if statement is false.
→ Usually Statements are denoted by letter P, Q, R or P_1, P_2

eg:- The Sun is a star → true T.V → T
$2+2 = 5$ False T.V → F

→ Composite or Compound Statement :- A statement involving more than one statement is called Composite or Compound statement.

Simple Statement → A statement involving only one statement.

eg:- The simple statement which form a Compound

(4)

Elementary statement are called component of statements.

(5) Logical Connectives :- 2 or more simple statements can be joined by any of five terms.

And, or, if then, if & only if

eg:- (1) India is a country → Truth value T

(2) Mumbai is capital of India → ,, F

(3) Their statement is false → cannot define

(4) Close the door → ,, ,,

(5) 2N students are clever → Truth value depend on content.

⇒ Connectives → Logical Connectives

Any word or expression used to connect two or more statements is called connectives (5 types)

eg:- Today is hot & it is Rainy

(1) Negation $(\neg, \sim)$:- If P is proposition then ~ is called Negation P and is denoted by $\neg P$.

eg:- P: Rose is Red

$\neg P$: Rose is not Red

P	$\neg P$
T	F
F	T

(2) Conjunction :- (AND) or Join

Let P and q be the two propositions P & q becomes true when P is true and q is true.

OR

The Compound Statement formed by Connecting the two statements using the Connective "and" is Called Conjunction of 2 Statements.

OR

Conjunction of 2 statement P and Q is denoted by $(P \wedge Q)$

Truth table $\wedge = (AND)$

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \wedge Q$
0	0	0
0	1	0
1	0	0
1	1	1

Delhi is India and 2+2=4 → T

Delhi is in India and 2+2=5=F

Deli is in

(3) Disjunction :- (OR) (V)

let p and q be the two propositions, P or q becomes false when p is false and q is false, otherwis it is true.

OR

The Compound statement formed by Connecting the two statements using the Connective "OR" is Called Disjunction of 2 statements.

OR

Disjunction of two statements P and Q is denoted by $P \vee Q$.

Truth table $\vee = OR$

(6)

V = OR

Delhi is in india or 2+2 = 4 T

" " 2+2=5 T

" " 2+2=4 T

" " 2+2=1 T

P	Q	P V Q
T	T	T
T	F	T
F	T	T
F	F	F

P	Q
0	0
0	1
1	0
1	1

(ii) Conditional :- (if - then) or ($\Rightarrow$) ($\rightarrow$)

also Called Implication

let p and q be the 2 proposition $p \rightarrow q$ is false when p is true & q is false.

(OR)

The Compound statement formed by Connecting the two statements using the Connective "if then" is called Conditional statement)

OR

Conditional of two statement p and q is denoted by $p \rightarrow q$

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

Trick $p \rightarrow q$
Check where False in p So Output will always be true. But if true in q then False otherwise True

$p \rightarrow$ antecedent or hypothesis

$q \rightarrow$ Consequent or Conclusion

$p \rightarrow q$ reads as p implies q; or p only if q eg:- I am hungry q = I will eat p→q ⇒ if I am hungry then I will eat.

Bi- Condition :- If p and q are given statement then the statements iff ⟺ " if and only if "q" written as $p \longleftrightarrow q$ called

Bi- Conditional

p	q	$p \longleftrightarrow q$
T	T	
T	F	
F	T	
F	F	T

True

only true if both false or both true.

⇒ Tautologies & Contradiction

$$((p \overset{OR}{\vee} \neg q) \wedge (\neg p \overset{OR}{\vee} \neg q)\} \vee q$$

A proposition is said to be a tautologies if contains only T (true) in last column of truth table & " " only f (false in last column of truth table ie Contradiction.

① eg:- $Q \vee (P \wedge Q) \vee (\neg p \vee \neg Q)$ answer

Solution

P	Q	¬P	¬Q	P ∨ ¬Q	¬P ∨ ¬Q	(P ∨ ¬Q) ∧ (¬P ∨ ¬Q)	
T	T	F	F	T	F	F	T
T	F	F	T	T	T	F	T
F	T	T	F	F	T	T	T
F	F	T	T	T	T	T	T

tautology

④

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(Q) Prove the proposition $p \vee \neg(q \wedge r)$ and $(p \vee \neg q) \vee \neg r$ are equivalent. $2^3 = 8$

p	q	r	$q \wedge r$	$\neg(q \wedge r)$	$p \vee \neg(q \wedge r)$	$(p \vee \neg q)$	$\neg q$	$\neg r$	
T	T	T	T	F	T	T	F	F	T
T	T	F	F	T	T	T	F	T	T
T	F	T	T	F	T	T	T	F	T
F	T	T	F	T	F	F	F	F	F
T	F	F	F	T	T	T	T	T	T
F	T	F	F	T	T	F	F	T	T
F	F	T	F	T	T	T	T	F	T
F	F	F	F	T	T	T	T	T	T

Equal

example → $(p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$

Bi conditional Statement :- A statement "p if and only if q" such statement are said to be bi-conditional statement and denoted by $p \Leftrightarrow q$ or $p \leftrightarrow q$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

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Converse, inverse and Contrapositive of Conditional Statement.

(1) Converse of Conditional Statement :- Let $p \to q$ be a Conditional Statement then $q \to p$ is called its Converse.

eg:- If x is labourer then he is poor.

<u>Converse</u>

if x is poor then he is labourer

(2) Inverse ------ :- Let $p \to q$ be a Conditional Statement then $\sim p \to \sim q$ is called its inverse.

eg:- if Harsa Read book then he will get knowledge:-

<u>Inverse:-</u>

if Harsa not Read book then he will not get knowledge.

(3) Contrapositive of Conditional Statement :- let $p \to q$ be a Conditional Statement then $\sim q \to \sim p$ is called Contrapositive.

eg:- If f(x) is differentiable then it is Continuous.

Contrapositive:-

if f(x) is not Continuous then it is not differentiable.

example

(1) $(p \wedge q) \rightarrow (p \vee q)$ is a tautology. $\overset{x}{}$

p	q	$p\wedge q$	$p \vee q$	x
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

(2) ~~$p \rightarrow (p \wedge q ($~~

$p \rightarrow (p \wedge (q \rightarrow p))$

p	q	$q \rightarrow p$	$p \wedge (q \rightarrow p)$	$p \rightarrow (p \wedge (q \rightarrow p))$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

b

Q for Students :-

$(p \vee q \vee r) \Longleftrightarrow [(((p \overset{a}{\rightarrow} q) \rightarrow q) \rightarrow r) \rightarrow r]$

p	q	r	$(p \vee q \vee r)$	$p \rightarrow q$	a
T	T	T	T	T	T
T	F	T	T	F	T
T	T	F	T	T	
F	T	F	T	T	
F	T	T	T	T	
T	F	F	T	F	T
F	F	F	F	T	
F	T	T	T	T	

Normal form (1)

Some Important law

(1) Idempotent law

$$p \wedge p \Leftrightarrow p \ \& \ p \vee p \Leftrightarrow p$$

(2) Commutative law

$$p \wedge q \Leftrightarrow q \wedge p \text{ and } p \vee q \Leftrightarrow q \vee p$$

(3) Associative law

$$(p \vee q) \vee r = p \vee (q \vee r)$$

$$\text{and } (p \wedge q) \wedge r = p \wedge (q \wedge r)$$

(4) De-Morgan law

$$\neg(p \overset{\to \text{and}}{\wedge} q) \Leftrightarrow \neg p \overset{\to \text{or}}{\vee} \neg q$$

$$\text{and } \neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$$

(5) $p \to q \Leftrightarrow \neg p \vee q$

(1) Disjunction NF :→ Consist of disjunction b/w Conjunction:-

(1) $(p \overset{\to \text{and}}{\wedge} q) \overset{\to \text{or disjunction}}{\vee} r$ (conjunction)

(2) $(p \wedge \neg q) \vee (\neg p \wedge r) \overset{\to \text{or}}{\vee} (r \wedge \neg q)$

Q:- Obtain DNF $(p \to q) \wedge (\neg p \wedge q)$

5th law

$$(\neg p \vee q) \wedge (\neg p \wedge q)$$

$$\cancel{(\neg p \wedge \neg p) \vee}$$

$$(\neg p \wedge (\neg p \wedge q)) \vee (q \wedge \neg p \wedge q)$$

$$\boxed{\text{1st law} \to (\neg p \wedge q) \vee (q \wedge \neg p)}$$

Normal forms

Obtain CNF $(p \wedge q) \vee (\neg p \wedge q \wedge r)$

~~(p∧q~~ ~~p∧~~

$(p \vee (\neg p \wedge q \wedge r)) \vee (q \vee (\neg p \wedge q \wedge r))$

~~p∧~~ $(p \vee \neg p) \wedge (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p$

$(p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p$

$p \vee (\neg p \rightarrow (q \vee (q \rightarrow \neg r)))$

$p \vee (\neg p \rightarrow (q \vee (\neg q \vee \neg r)))$

$p \vee (\neg p \rightarrow ((q \vee \neg q) \vee (q \vee \neg r)))$

~~p∨¬p∨r~~ $p \vee (\neg p \rightarrow (1 \vee (q \vee \neg r)))$

$p \vee (\underline{\neg p \rightarrow (q \vee \neg r)}) \rightarrow = p$

$p \vee ~~\neg~~ (p \vee (q \vee \neg r))$

~~p∨ (p∨q ∨ p)~~

$p \vee (p \vee (q \vee \neg r)$

$p \vee (p \vee q \vee p \vee \neg r)$

$p \vee ($

Normal forms

(ii) Conjunction Normal form

$A \vee \neg A$

A

(i) $p \wedge q$

(2) $(\neg p \vee q) \wedge (\neg p \vee r)$ (1) $p \vee p$ $p \to p = p$

eg:- $(p \wedge q) \vee (\neg p \wedge q \wedge r)$ $p \wedge p$ $p \wedge p = p$

$p \vee p = p$

$p \vee (\neg p \wedge q \wedge r) \wedge q \vee (\neg p \wedge q \wedge r)$

$(p \vee \neg p) \wedge (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p \wedge (q \vee q) \wedge (q \vee r)$

$(q \vee \neg p) \wedge$ ~~...~~ $\wedge (q \vee r)$

$\therefore (p \vee q) \wedge (p \vee r) \wedge (q \vee \neg p)$

Normal form 4
Obtain CNF of $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$

$p \vee (\sim p \rightarrow (q \vee (\sim q \vee \sim r)))$

$p \vee (\sim p \rightarrow (q \vee \sim q \vee q \vee \sim r))$

$p \vee (\sim p \rightarrow (q \vee \sim r))$

$p \vee (p \vee (q \vee \sim r))$

$p \vee (p \vee q \vee p \vee \sim r)$

$p \vee (p \vee q \vee \sim r)$

$(p \vee p) \vee (p \vee q) \vee (p \vee \sim r)$

$p \vee (p \vee q \vee \sim r)$

Argument :- It is a process by which a conclusion is obtained from given set of premises.

(i) Premises :- Given group of preposition is called premises.

(ii) Conclusion :- The preposition getting by given premises is called conclusion.

eg :- " I will become famous or I will be a writer".
I will not be a writer.
I will become famous.

Valid Argument :- let $P_1, P_2, P_3 ---, P_n$ are premises and Q is conclusion then the argument is valid iff $A(P_1, P_2, -- P_n) \rightarrow Q$ is tautology.

Falacy Argument :- An argument is called falacy if it is not valid.

aural form 5 :- if it rain, Ram will be sick.

Conclusion It did not rain

→ ∴ Ram was not sick.

if means →

let p = if it rain

q = Ram will be sick

$\sim p$ = it did not rain.

$\sim q$ = Ram was not sick.

$((p \to q) \wedge \sim p) \to \sim q$

p	q	$p \to q$	$\sim p$	$(p \to q) \wedge \sim p$	$\sim q$	
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Not valid Argument

(2) Investigate the validity of argument

$p \to s$

$\sim p \to q$

$q \to s$

$\therefore \sim s \to s$
