Hamasaiid model derivations

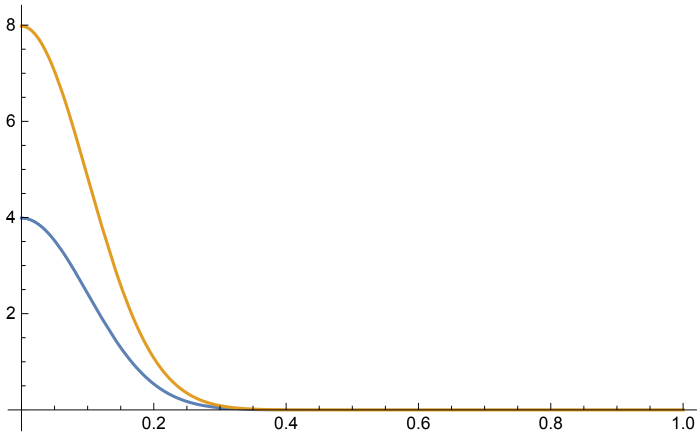
Surface profile asperity peak height Distributions, Real(blue), Orange(Hamasaiid)

```
Quit[]
```

In[∘]:=

$$\text{Show}\left[\text{Plot}\left[\left\{\frac{e^{-\frac{(y)^2}{2(\sigma)^2}}}{\sqrt{2\pi}\,\sigma} /. \sigma\to 0.1,\ 2\,\frac{e^{-\frac{(y)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma} /. \sigma\to 0.1\right\}, \{y, 0, 1\}, \text{PlotRange}\to \text{All}\right]\right]$$

Out[∘]=



Real distribution

In[∘]:= `Quit[]`

Height distribution $\phi_0(y)$

In[∘]:=

$$\phi_0 = \frac{e^{-\frac{(y)^2}{2(\sigma_0)^2}}}{\sqrt{2\pi}\,\sigma_0}$$

Out[∘]=

$$\frac{e^{-\frac{y^2}{2\sigma_0^2}}}{\sqrt{2\pi}\,\sigma_0}$$

Arithmetic average of absolute height deviation, R_0

In[∘]:=
```
R0 = Simplify[Integrate[ϕ0 * Abs[y], {y, -Infinity, Infinity}], {σ0 > 0, σ0 ∈ Reals}]
```

Out[∘]= $\sqrt{\frac{2}{\pi}}\ \sigma_0$

Mean μ_0

In[∘]:=
```
μ0 = 0
```

Out[∘]= 0

Standard deviation σ_0

In[∘]:=
```
σ0 =
  Simplify[Sqrt[Integrate[ϕ0 * (y - μ0) ^2, {y, -Infinity, Infinity}]], {σ0 > 0, σ0 ∈ Reals}]
```

Out[∘]= σ_0

Surface area of bases of valleys/peaks + gaps, A_0, given by number of valleys/peaks, N_0, height distribution ϕ_0, and valley/peak slope m_0

In[∘]:=
```
A0 = Simplify[
   ϵ * N0 * 2 * (Integrate[ϕ0 * Pi * (y / m0) ^2, {y, 0, Infinity}]), {σ0 > 0, σ0 ∈ Reals}]
```

Out[∘]= $\frac{\pi \in N_0\, \sigma_0^2}{m_0^2}$

Density of peaks/valleys over surface area, n_0

In[∘]:=
```
n0 = N0 / A0
```

Out[∘]= $\frac{m_0^2}{\pi \in \sigma_0^2}$

Average diameter of circular base, B_0

In[ ]:= `B0 = 2 / m0 * Simplify[Integrate[ϕ0 * Abs[y], {y, -Infinity, Infinity}], {σ0 > 0, σ0 ∈ Reals}]`

Out[ ]= $$\frac{2\sqrt{\frac{2}{\pi}}\,\sigma_0}{m_0}$$

Slope, m_0, given by mean peak spacing $L_0 = B_0$

In[ ]:= $$\text{Solve}\left[L_0 == \frac{2\sqrt{\frac{2}{\pi}}\,\sigma_0}{m_0},\ m_0\right]$$

Out[ ]= $$\left\{\left\{m_0 \to \frac{2\sqrt{\frac{2}{\pi}}\,\sigma_0}{L_0}\right\}\right\}$$

Hamasaiid distribution

In[ ]:= `Quit[]`

Height distribution $\phi_1(y)$

In[ ]:= $$\phi_1 = 2\,\frac{e^{-\frac{(y)^2}{2\,(\sigma_0)^2}}}{\sqrt{2\,\pi}\,\sigma_0}$$

Out[ ]= $$\frac{e^{-\frac{y^2}{2\,\sigma_0^2}}\sqrt{\frac{2}{\pi}}}{\sigma_0}$$

Arithmetic average of absolute height deviation, R_1

In[ ]:= `R1 = Simplify[Integrate[ϕ1 * y, {y, 0, Infinity}], {σ0 > 0, σ0 ∈ Reals}]`

Out[ ]= $$\sqrt{\frac{2}{\pi}}\,\sigma_0$$

Mean, μ_1

In[ ]:=

```
μ1 = R1
```

Out[ ]= $\sqrt{\frac{2}{\pi}}\ \sigma_0$

Standard deviation from mean μ_1, σ_1

In[ ]:=

```
σ1 = Simplify[Sqrt[Integrate[ϕ1 * (y - μ1) ^2, {y, 0, Infinity}]], {σ0 > 0, σ0 ∈ Reals}]
```

Out[ ]= $\sqrt{\frac{-2+\pi}{\pi}}\ \sigma_0$

Surface area of bases of peaks + gaps, A_1, given by number of valleys and peaks, N_1, height distribution ϕ_1, and peak slope m_1

In[ ]:=

```
A1 =
  Simplify[ϵ * N1 * (Integrate[ϕ1 * Pi * (y / m1) ^2, {y, 0, Infinity}]), {σ0 > 0, σ0 ∈ Reals}]
```

Out[ ]= $\frac{\pi \in N_1\ \sigma_0^2}{m_1^2}$

Density of peaks over surface area, n_1

In[ ]:=

```
n1 = N1 / A1
```

Out[ ]= $\frac{m_1^2}{\pi \in \sigma_0^2}$

Average diameter of circular base, B_1

In[ ]:=

```
B1 = 2 / m1 * Simplify[Integrate[ϕ1 * y, {y, 0, Infinity}], {σ0 > 0, σ0 ∈ Reals}]
```

Out[ ]= $\frac{2\sqrt{\frac{2}{\pi}}\ \sigma_0}{m_1}$

Slope, m_1, given by mean peak spacing, L_0

In[∘]:= $\text{Solve}\left[L_0 == \frac{2\sqrt{\frac{2}{\pi}}\,\sigma_0}{m_1}, \{m_1\}\right]$

Out[∘]= $\left\{\left\{m_1 \to \frac{2\sqrt{\frac{2}{\pi}}\,\sigma_0}{L_0}\right\}\right\}$

In[∘]:= $m_1 = \frac{2\sqrt{\frac{2}{\pi}}\,\sigma_0}{L_0}$

Out[∘]= $\frac{2\sqrt{\frac{2}{\pi}}\,\sigma_0}{L_0}$

Updated n_1

In[∘]:= n_1

Out[∘]= $\frac{8}{\pi^2 \in L_0^2}$

Density of microcontact spots, $n_{s,1}$

In[∘]:= `ns,1 = n1 * Simplify[Integrate[ϕ1, {y, Y, Infinity}], {σ0 > 0, σ0 ∈ Reals}]`

Out[∘]= $\frac{8\,\text{Erfc}\left[\frac{Y}{\sqrt{2}\,\sigma_0}\right]}{\pi^2 \in L_0^2}$

Average projected radius of contact area, $a_{s,1}$, which exceeds entrapped air thickness, Y

In[∘]:= `as,1 = Simplify[Integrate[(y - Y) / m1 * ϕ1, {y, Y, Infinity}], {σ0 > 0, σ0 ∈ Reals}]`

Out[∘]= $\frac{1}{4} L_0 \left(2\,e^{-\frac{Y^2}{2\sigma_0^2}} + \frac{\sqrt{2\pi}\,Y\left(-1 + \text{Erf}\left[\frac{Y}{\sqrt{2}\,\sigma_0}\right]\right)}{\sigma_0}\right)$

Average radius of circular base, b_s

```
In[]:= bs = L0 / 2
Out[]= L0
       2
```

Plot Abbott-Firestone curve, $L_0 = 128\,e-6,\ \sigma_0 = 0.58\,e-6,\ \epsilon = 1.5$

```
In[]:= Quit[]
```

Measured Abott-firestone by Hamasaiid et al.

```
In[]:=
 data = {{0, 1.8463428571428571`*^-6}, {0.01109350237717907`, 1.6251428571428572`*^-6},
    {0.026941362916006323`, 1.471657142857143`*^-6},
    {0.053882725832012646`, 1.309142857142857`*^-6},
    {0.08716323296354991`, 1.182742857142857`*^-6}, {0.12678288431061804`,
     1.0608571428571428`*^-6}, {0.16640253565768623`, 9.660571428571426`*^-7},
    {0.20919175911251978`, 8.848`*^-7}, {0.2488114104595879`, 8.215999999999999`*^-7},
    {0.3090332805071315`, 7.403428571428573`*^-7},
    {0.38193343898573695`, 6.500571428571428`*^-7},
    {0.4548335974643424`, 5.507428571428572`*^-7},
    {0.5309033280507132`, 4.6045714285714254`*^-7},
    {0.6307448494453249`, 3.2954285714285667`*^-7},
    {0.7036450079239303`, 2.3925714285714286`*^-7},
    {0.7876386687797148`, 1.3994285714285725`*^-7},
    {0.8478605388272582`, 6.320000000000009`*^-8},
    {0.9033280507131537`, 1.8057142857142762`*^-8}, {0.96513470681458`, 0}};
```

Helper function to transpose x and y axi

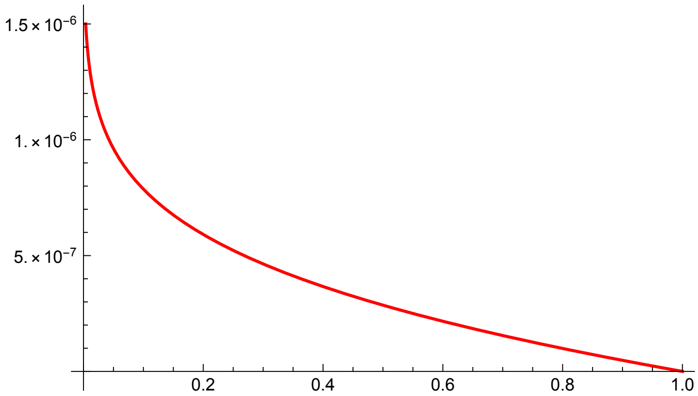
```
In[]:= axisFlip = # /. {x_Line | x_GraphicsComplex :> MapAt[#~Reverse~2 &, x, 1],
        x : (PlotRange → _) :> x~Reverse~2} &;
```

Modeled a_s / b_s

In[ ]:=

$$\text{abfs} = \text{Plot}\left[\frac{1}{2}\left(2\,e^{-\frac{Y^2}{2\,\sigma_0^2}} + \frac{\sqrt{2\,\pi}\,Y\left(-1+\text{Erf}\left[\frac{Y}{\sqrt{2}\,\sigma_0}\right]\right)}{\sigma_0}\right) /.\ \{\sigma_0 \to 0.578*\text{^}-6,\ L_0 \to 128.5*\text{^}-6\},\right.$$
$$\left.\{Y, 0, 1.5*\text{^}-6\},\ \text{PlotStyle} \to \text{Red}\right]\ //\ \text{axisFlip}$$

Out[ ]=

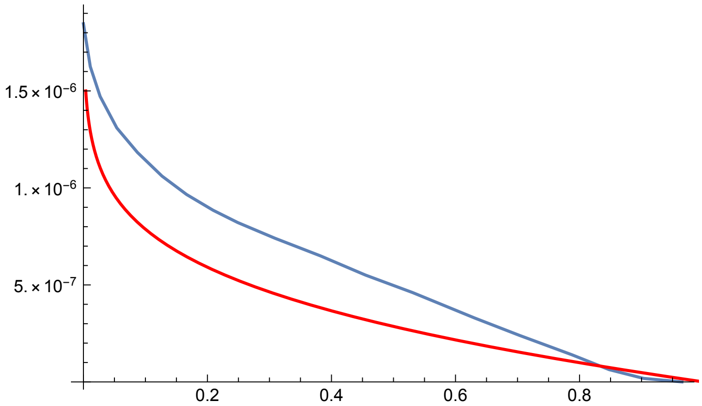


Measured Abbott firestone(Red) vs model (Blue)

In[ ]:=

```
Show[ListLinePlot[data], abfs]
```

Out[ ]=



Plot HTC(y)

In[ ]:=

```
Quit[]
```

Initialize values for $a_{s,2}$, $n_{s,2}$, $a_{s,1}$, $n_{s,1}$

In[∘]:=
$$a_{s,1} = \frac{1}{4} L_0 \left(2\, e^{-\frac{Y^2}{2\sigma_0^2}} + \frac{\sqrt{2\pi}\, Y \left(-1 + \mathrm{Erf}\left[\frac{Y}{\sqrt{2}\,\sigma_0}\right]\right)}{\sigma_0} \right);$$

$$n_{s,1} = \frac{8\, \mathrm{Erfc}\left[\frac{Y}{\sqrt{2}\,\sigma_0}\right]}{\pi^2\, \epsilon\, L_0^2};$$

In[∘]:= $b_s = L_0 / 2;$

Effective heat transfer coefficient λ_s

In[∘]:=
$\lambda_s = 2 * \lambda_1 * \lambda_2 / (\lambda_1 + \lambda_2)$

Out[∘]=
$$\frac{2\, \lambda_1\, \lambda_2}{\lambda_1 + \lambda_2}$$

Peak heat transfer coefficient, h

In[∘]:=
$$\mathrm{TraditionalForm}\left[h_1 = \frac{2\, \lambda_s\, n_{s,1}\, a_{s,1}}{\left(1 - \frac{a_{s,1}}{b_s}\right)^{1.5`}}\right];$$

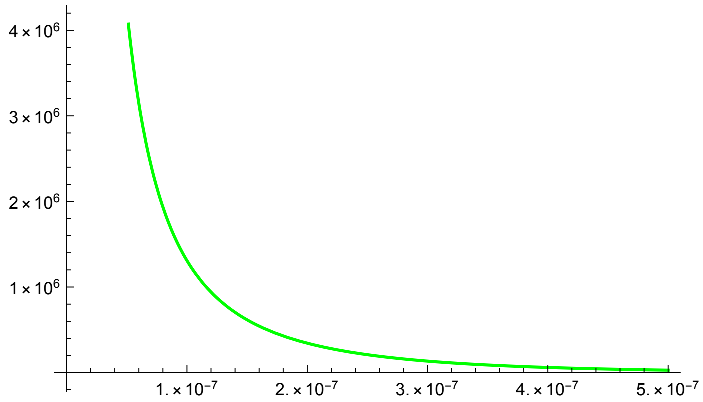
$$h(Y) = \left(8\, \lambda_1\, \lambda_2 \left(\frac{\sqrt{2\pi}\; Y \left(\mathrm{erf}\left(\frac{Y}{\sqrt{2}\;\sigma_0}\right) - 1\right)}{\sigma_0} + 2\, e^{-\frac{Y^2}{2\sigma_0^2}}\right) \mathrm{erfc}\left(\frac{Y}{\sqrt{2}\;\sigma_0}\right)\right) \Big/ \left(\pi^2 (\lambda_1 + \lambda_2)\, L_0\, \epsilon \left(\frac{1}{2}\left(-\frac{\sqrt{2\pi}\; Y \left(\mathrm{erf}\left(\frac{Y}{\sqrt{2}\;\sigma_0}\right) - 1\right)}{\sigma_0} - 2\, e^{-\frac{Y^2}{2\sigma_0^2}}\right) + 1\right)^{1.5`}\right);$$

Plot peak heat transfer coefficient, h as a function of air gap width Y

In[ ]:=
$$\text{hplot1} = \text{Plot}\left[\left(8\,\lambda_1\,\lambda_2\left(\frac{\sqrt{2\pi}\,Y\left(\text{erf}\left(\frac{Y}{\sqrt{2}\,\sigma_0}\right)-1\right)}{\sigma_0}+2\,e^{-\frac{Y^2}{2\sigma_0^2}}\right)\text{erfc}\left(\frac{Y}{\sqrt{2}\,\sigma_0}\right)\right)\Big/\left(\pi^2\,(\lambda_1+\lambda_2)\,L_0\,\epsilon\left(\frac{1}{2}\left(-\frac{\sqrt{2\pi}\,Y\left(\text{erf}\left(\frac{Y}{\sqrt{2}\,\sigma_0}\right)-1\right)}{\sigma_0}-2\,e^{-\frac{Y^2}{2\sigma_0^2}}\right)+1\right)^{1.5`}\right)\,/.$$

```
{λ1 → 29, λ2 → 70, σ0 → 0.578*^-6, ϵ → 1.5, L0 → 128.5*^-6},
{Y, 0, 0.5*^-6}, PlotStyle → Green];
```

```
Show[hplot1 , PlotRange → Automatic]
```

Out[ ]=


Peak heat transfer coefficient h using values by Hamasaiid (without capillary pressure)

In[ ]:=
$$h_1 = \frac{2\,\lambda_s\,n_{s,1}\,a_{s,1}}{\left(1-\frac{a_{s,1}}{b_s}\right)^{1.5`}}\;/.$$

```
{λ1 → 29, λ2 → 70, σ0 → 0.578*^-6, ϵ → 1.5, L0 → 128.5*^-6, Y → 0.164*^-6}
```

Out[ ]= 516 922.

Calculate initial air gap thickness Y_0

In[ ]:=
```
Quit[]
```

Ideal gas law for air thickness

In[ ]:= $(P_1 - P_\gamma)\ V_1 / T_1 == P_0\ V_0 / T_0$

Out[ ]= $$\frac{(P_1 - P_\gamma)\ V_1}{T_1} == \frac{P_0\ V_0}{T_0}$$

Re-initialize slopes

$$m_1 = \frac{2\sqrt{\frac{2}{\pi}}\ \sigma_0}{L_0};$$

Re-initialize distribution functions

$$\phi_1 = 2\frac{e^{-\frac{(y)^2}{2\,(\sigma_0)^2}}}{\sqrt{2\,\pi}\ \sigma_0};$$

Initial volume for cone-shaped air pocket, V_0

In[ ]:= $V_{0,1}$ =

```
Simplify[Integrate[{1/3*Pi/m1^2 *φ1*y^3}, {y, 0, Infinity}], {σ0 > 0, σ0 ∈ Reals}]
```

Out[ ]= $$\left\{\frac{\pi^{3/2}\ L_0^2\ \sigma_0}{6\sqrt{2}}\right\}$$

Final volume for cone – shaped air pocket, V_1

In[ ]:= $V_{1,1} = 1/3 * \mathrm{Pi} * Y_{0,1}\verb|^|3/m_1\verb|^|2$

Out[ ]= $$\frac{\pi^2\ L_0^2\ Y_{0,1}^3}{24\ \sigma_0^2}$$

Solve initial air layer thickness, Y_0

In[ ]:= $$\mathrm{Simplify}\left[\mathrm{Solve}\left[(P_1 - P_\gamma)\ / T_1 * \frac{\pi^2\ L_0^2\ Y_{0,1}^3}{24\ \sigma_0^2} == \frac{P_0\ V_{0,1}}{T_0},\ \{Y_{0,1}\}\right]\right]$$

Out[ ]= $$\left\{\left\{Y_{0,1} \to \frac{\sqrt{2}\ P_0^{1/3}\ \sigma_0^{1/3}}{\pi^{1/6}\left(\frac{(P_1-P_\gamma)\ T_0}{T_1\ \sigma_0^2}\right)^{1/3}}\right\},\ \left\{Y_{0,1} \to -\frac{(-1)^{1/3}\sqrt{2}\ P_0^{1/3}\ \sigma_0^{1/3}}{\pi^{1/6}\left(\frac{(P_1-P_\gamma)\ T_0}{T_1\ \sigma_0^2}\right)^{1/3}}\right\},\ \left\{Y_{0,1} \to \frac{(-1)^{2/3}\sqrt{2}\ P_0^{1/3}\ \sigma_0^{1/3}}{\pi^{1/6}\left(\frac{(P_1-P_\gamma)\ T_0}{T_1\ \sigma_0^2}\right)^{1/3}}\right\}\right\}$$

In[ ]:= $Y_{0,1} = \frac{\sqrt{2}\, P_0^{1/3}\, \sigma_0^{1/3}}{\pi^{1/6} \left(\frac{(P_1 - P_\gamma)\, T_0}{T_1\, \sigma_0^2}\right)^{1/3}}$

Out[ ]= $$\frac{\sqrt{2}\, P_0^{1/3}\, \sigma_0^{1/3}}{\pi^{1/6} \left(\frac{(P_1 - P_\gamma)\, T_0}{T_1\, \sigma_0^2}\right)^{1/3}}$$

Determine Y_t, Y as a function of time

In[ ]:= Quit[]

Fitting coefficient, C_0

In[ ]:= $C_{0,1} = V_{al,1} * n_{s,1} * 1/2$

Out[ ]= $$\frac{1}{3\, \pi^{5/6}\, \in} \sqrt{2}\, \mathrm{Erfc}\left[\frac{P_0^{1/3}}{\pi^{1/6} \left(\frac{(P_1 - P_\gamma)\, T_0}{T_1\, \sigma_0^2}\right)^{1/3} \sigma_0^{2/3}}\right] \left(\pi^{1/3} + \frac{\left(-1 + e^{-\frac{P_0^{2/3}}{\pi^{1/3} \left(\frac{(P_1 - P_\gamma)\, T_0}{T_1}\right)^{2/3}}}\right) P_0^{2/3}}{\left(\frac{(P_1 - P_\gamma)\, T_0}{T_1}\right)^{2/3}}\right) \sigma_0$$

Volume of aluminum inside asperities, V_{al}

In[ ]:= V

```
Val,1 = Simplify[1/3 * Pi * 1/(m1)^2 * Integrate[ϕ1 * y^3, {y, 0, Infinity}] -
    1/3 * Pi * (Y0,1 / m1)^2 * Integrate[ϕ1 * y, {y, 0, Y0,1}], {σ0 > 0, σ0 ∈ Reals}]
```

Out[ ]= $$\frac{\pi^{7/6}\, L_0^2 \left(\pi^{1/3} + \frac{\left(-1 + e^{-\frac{P_0^{2/3}}{\pi^{1/3} \left(\frac{(P_1 - P_\gamma)\, T_0}{T_1}\right)^{2/3}}}\right) P_0^{2/3}}{\left(\frac{(P_1 - P_\gamma)\, T_0}{T_1}\right)^{2/3}}\right) \sigma_0}{6\sqrt{2}}$$

Time dependent air gap thickness, Y_t

In[∘]:= $Y_{t,1} = Y_{0,1} + 2 * C_{0,1} * h_1 * (T_2 - T_3) \big/ (\rho * L * S) * t$

Out[∘]=

$$\frac{\sqrt{2}\, P_0^{1/3}\, \sigma_0^{1/3}}{\pi^{1/6} \left(\frac{(P_1 - P_\gamma)\, T_0}{T_1\, \sigma_0^2}\right)^{1/3}} + \left(16\sqrt{2}\, t\, \mathrm{Erfc}\left[\frac{P_0^{1/3}}{\pi^{1/6}\left(\frac{(P_1-P_\gamma)\,T_0}{T_1\,\sigma_0^2}\right)^{1/3} \sigma_0^{2/3}}\right]^2 \left(\pi^{1/3} + \frac{\left(-1 + e^{-\frac{P_0^{2/3}}{\pi^{1/3}\left(\frac{(P_1-P_\gamma)\,T_0}{T_1}\right)^{2/3}}}\right) P_0^{2/3}}{\left(\frac{(P_1-P_\gamma)\,T_0}{T_1}\right)^{2/3}}\right) (T_2 - T_3)\, \lambda_1\, \lambda_2 \left(2\, e^{-\frac{P_0^{2/3}}{\pi^{1/3}\left(\frac{(P_1-P_\gamma)\,T_0}{T_1\,\sigma_0^2}\right)^{2/3} \sigma_0^{4/3}}} + \frac{2\,\pi^{1/3}\left(-1 + \mathrm{Erf}\left[\frac{P_0^{1/3}}{\pi^{1/6}\left(\frac{(P_1-P_\gamma)\,T_0}{T_1\,\sigma_0^2}\right)^{1/3}\sigma_0^{2/3}}\right]\right) P_0^{1/3}}{\left(\frac{(P_1-P_\gamma)\,T_0}{T_1\,\sigma_0^2}\right)^{1/3}\sigma_0^{2/3}}\right) \sigma_0\right) \Bigg/$$

$$\left(3\, L\, \pi^{17/6}\, S\, \epsilon^2\, \rho\, L_0\, (\lambda_1 + \lambda_2) \left(1 + \frac{1}{2}\left(-2\, e^{-\frac{P_0^{2/3}}{\pi^{1/3}\left(\frac{(P_1-P_\gamma)\,T_0}{T_1\,\sigma_0^2}\right)^{2/3}\sigma_0^{4/3}}} - \left(2\,\pi^{1/3}\left(-1 + \mathrm{Erf}\left[\frac{P_0^{1/3}}{\pi^{1/6}\left(\frac{(P_1-P_\gamma)\,T_0}{T_1\,\sigma_0^2}\right)^{1/3}\sigma_0^{2/3}}\right]\right) P_0^{1/3}\right) \Bigg/ \left(\left(\frac{(P_1 - P_\gamma)\,T_0}{T_1\,\sigma_0^2}\right)^{1/3} \sigma_0^{2/3}\right)\right)\right)^{1.5}\right)$$

Determine *h* as a function of time

In[∘]:= `Quit[]`

Calculate relevant quantities

In[ ]:=

$$\phi_1 = 2\,\frac{e^{-\frac{(y)^2}{2\,(\sigma_0)^2}}}{\sqrt{2\,\pi}\;\sigma_0};$$

$$Y_{0,1} = \frac{\sqrt{2}\;P_0^{1/3}\,\sigma_0^{1/3}}{\pi^{1/6}\left(\frac{(P_1-P_y)\,T_0}{T_1\,\sigma_0^2}\right)^{1/3}};$$

```
C0,1 = Val,1 * ns,1 * 1 / 2;
Val,1 = Simplify[1 / 3 * Pi * 1 / (m1) ^2 * Integrate[ϕ1 * y^3, {y, 0, Infinity}] -
    1 / 3 * Pi * (Y0,1 / m1) ^2 * Integrate[ϕ1 * y, {y, 0, Y0,1}], {σ0 > 0, σ0 ∈ Reals}];
Yt,1 = Y0,1 + 2 * C0,1 * h1 * (T2 - T3) / (ρ * L * S) * t;
```

$$a_{t,1} = \frac{1}{4}\,L_0\left(2\,e^{-\frac{Y_{t,1}{}^2}{2\,\sigma_0^2}} + \frac{\sqrt{2\,\pi}\;Y_{t,1}\left(-1+\mathrm{Erf}\left[\frac{Y_{t,1}}{\sqrt{2}\;\sigma_0}\right]\right)}{\sigma_0}\right);$$

$$n_{t,1} = \frac{8\,\mathrm{Erfc}\left[\frac{Y_{t,1}}{\sqrt{2}\;\sigma_0}\right]}{\pi^2\,\epsilon\,L_0^2};$$

$$h_{t,1} = \frac{2\,\lambda_s\,n_{t,1}\,a_{t,1}}{\left(1-\frac{a_{t,1}}{b_s}\right)^{1.5`}};$$

$$a_{s,1} = \frac{1}{4}\,L_0\left(2\,e^{-\frac{Y_{0,1}{}^2}{2\,\sigma_0^2}} + \frac{\sqrt{2\,\pi}\;Y_{0,1}\left(-1+\mathrm{Erf}\left[\frac{Y_{0,1}}{\sqrt{2}\;\sigma_0}\right]\right)}{\sigma_0}\right);$$

$$n_{s,1} = \frac{8\,\mathrm{Erfc}\left[\frac{Y_{0,1}}{\sqrt{2}\;\sigma_0}\right]}{\pi^2\,\epsilon\,L_0^2};$$

$$h_1 = \frac{2\,\lambda_s\,n_{s,1}\,a_{s,1}}{\left(1-\frac{a_{s,1}}{b_s}\right)^{1.5`}};$$

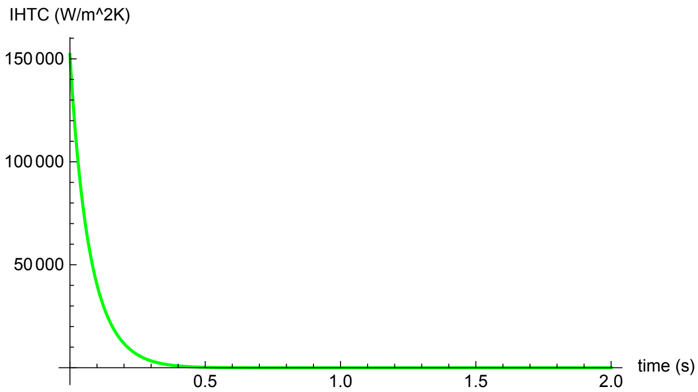
```
λs = 2 * λ1 * λ2 / (λ1 + λ2);
bs = L0 / 2;
```

$$m_1 = \frac{2\sqrt{\frac{2}{\pi}}\;\sigma_0}{L_0};$$

Hamasaiid model for time-evolution of Al-alloy IHTC

In[∘]:= Plot$\left[\dfrac{2\,\lambda_s\,n_{t,1}\,a_{t,1}}{\left(1-\frac{a_{t,1}}{b_s}\right)^{1.5`}}\right.$ /. { σ_0 → 0.578*^-6, L_0 → 128.7*^-6, λ_1 → 29,
λ_2 → 109, ϵ → 1.5, P_γ → 0.87 * 26*^6, P_1 → 26*^6, P_0 → 1.013*^5, T_0 → 300,
T_1 → 860, T_2 → 853, T_3 → 453, ρ → 2810, S → 4*^-3, L → 389*^3}, {t, 0, 2},
PlotStyle → Green, PlotRange → Full, AxesLabel → {"time (s)", "IHTC (W/m^2K)"}]

Out[∘]=

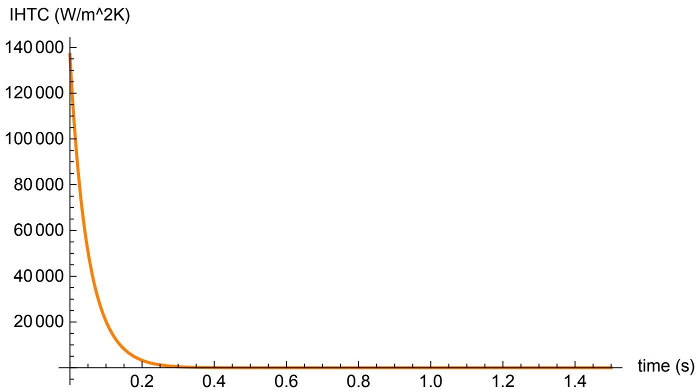


Hamasaiid model for time evolution of Mg-alloy IHTC

In[∘]:=

Plot$\left[\right.$Evaluate$\left[\dfrac{2\,\lambda_s\,n_{t,1}\,a_{t,1}}{\left(1-\frac{a_{t,1}}{b_s}\right)^{1.5`}}\right.$ /. { σ_0 → 0.578*^-6, L_0 → 128*^-6,
λ_1 → 29, λ_2 → 70, ϵ → 1.5, P_γ → 0.87 * 26*^6, P_1 → 26*^6, P_0 → 1.013*^5,
T_0 → 300, T_1 → 860, T_2 → 853, T_3 → 453, ρ → 1810, S → 4*^-3, L → 370*^3}],
{t, 0, 1.5}, PlotStyle → Orange, PlotRange → Full,
AxesLabel → {"time (s)", "IHTC (W/m^2K)"}]

Out[∘]=



Estimate peak IHTC for Al

In[∘]:= $h_{t,1}$ /. { σ_0 → 0.578*^-6, L_0 → 128.7*^-6, λ_1 → 29, λ_2 → 109,
ϵ → 1.5, P_γ → 0.87 * 26*^6, P_1 → 26*^6, P_0 → 1.013*^5, T_0 → 300,
T_1 → 860, T_2 → 853, T_3 → 453, ρ → 2810, S → 4*^-3, L → 389*^3, t → 0}

Out[∘]= 152 095.

Estimate peak IHTC for Mg

In[∘]:= $h_{t,1}$ /. { $\sigma_0 \to 0.578*^{\wedge}-6$, $L_0 \to 128.5*^{\wedge}-6$, $\lambda_1 \to 29$, $\lambda_2 \to 70$, $\epsilon \to 1.5$, $P_\gamma \to 0.87 * 26*^{\wedge}6$, $P_1 \to 26*^{\wedge}6$, $P_0 \to 1.013*^{\wedge}5$, $T_0 \to 300$, $T_1 \to 860$, $T_2 \to 853$, $T_3 \to 453$, $\rho \to 1810$, $S \to 4*^{\wedge}-3$, $L \to 370*^{\wedge}3$, $t \to 0$}

Out[∘]= 136 366.

Surface energy

In[∘]:= Quit[]

Young's Equation

$\gamma_1 * \mathrm{Cos}[\theta] = \gamma_1 - \gamma_{12}$

Capillary Force in notch (good wetting)

In[∘]:= $F_c = 2 * \gamma_1 * \mathrm{Sin}[\theta + \phi] / (r_0 - x * \mathrm{Cot}[\phi])$

Out[∘]= $$\frac{2\,\mathrm{Sin}[\theta+\phi]\,\gamma_1}{-x\,\mathrm{Cot}[\phi] + r_0}$$

Absolute values of capillary pressure model by Chao Yuan et al.

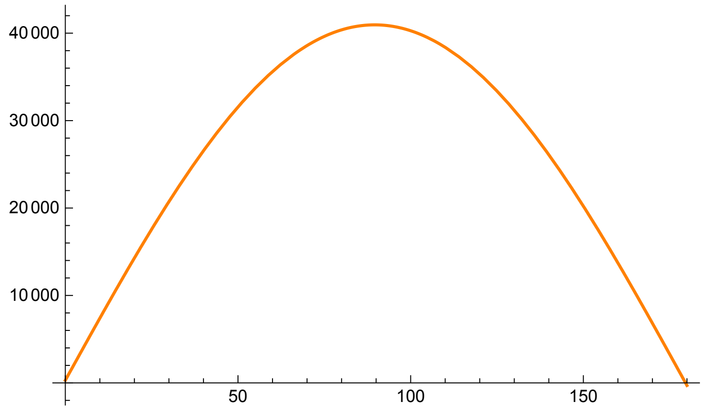
In[∘]:=

$$P_\gamma = 2 * \gamma * \mathrm{Sin}[\theta + \phi] / (Y * \mathrm{Cot}[\phi]) \;/.\; \left\{\gamma \to 0.9,\ \phi \to \mathrm{ArcTan}\left[\frac{2\sqrt{\frac{2}{\pi}}\ 0.578*^{\wedge}-6}{128.7*^{\wedge}-6}\right],\right.$$
$$\left. Y \to 3.5*^{\wedge}-7,\ \sigma_0 \to 0.578*^{\wedge}-6,\ L_0 \to 128.7*^{\wedge}-6,\ \theta \to \mathrm{Pi}/6\right\}$$

Out[∘]= 18 656.9

In[∘]:=
```
Plot[2 * γ * Sin[θ Degree + ϕ] / (Y * Cot[ϕ]) /.
  {γ → 1, ϕ → ArcTan[(2 Sqrt[2/π] 0.578*^-6)/(128.7*^-6)], Y → 3.5*^-7, σ0 → 0.578*^-6, L0 → 128.7*^-6},
 {θ, 0, 180}, PlotStyle → Orange, PlotRange → Automatic]
```

Out[∘]=



Water hammer pressure

Aluminum gate

In[∘]:=
```
P = ρ * c * V * Sin[x] /. {ρ → 2810, c → 3840, V → 1.931, x → 3.5 °};
N[P]
```

Out[∘]= 1.27202×10^6

Mercury gate

In[∘]:=
```
P = ρ * c * V * Sin[x] /. {ρ → 1810, c → 3357.19, V → 2.867, x → 3.5 °}
```

Out[∘]= 1.06355×10^6

Wave velocity aluminum

In[∘]:=
```
c = Evaluate[(E / ρ) ^ (1 / 2) /. {ρ → 2800, E → 41.3*^9}] // N
```

Out[∘]= 3840.57

Wave velocity magnesium

In[∘]:=
```
c = Evaluate[(E / ρ) ^ (1 / 2) /. {ρ → 1810, E → 20.4*^9}] // N
```

Out[∘]= 3357.19

Stagnation pressure

In[ ]:=
```
P = 1/2 * ρ * V^2 /. {ρ → 2570, V → 4}
```

Out[ ]=
```
20560
```