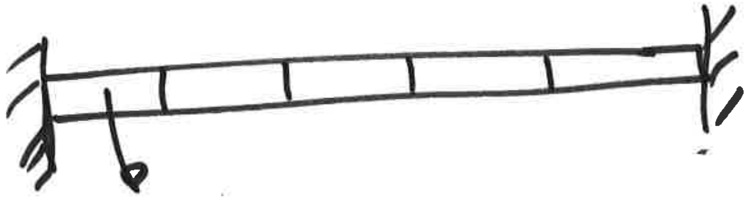
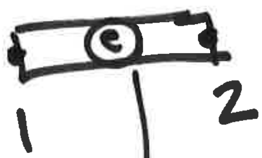




1. $[K^{e}]\{d^{e}\} = \{f^{e}\}$

2. Assembly of all elements

singular ← $[K^{G}]\{d^{G}\} = \{F^{G}\}$

3. Boundary Conditions

non-singular

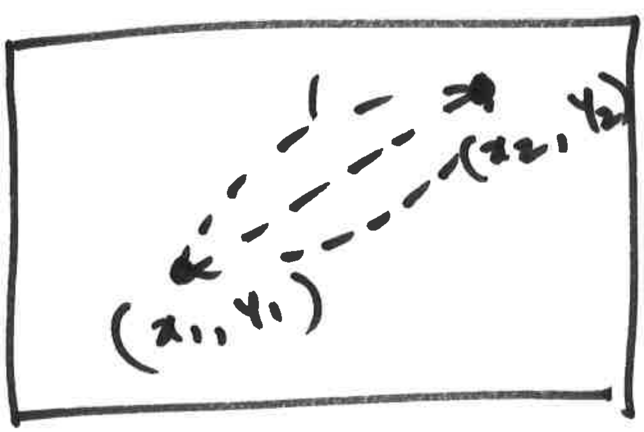
4. Displacements ⟶ stresses

$$I = \int_a^b F(x, y, y')\, dx \rightarrow \text{only one value}$$

$I[y(x)] \rightarrow$ only one value

We need to find $y(x)$ such way that either $I[y(x)]$ is maximum or minimum

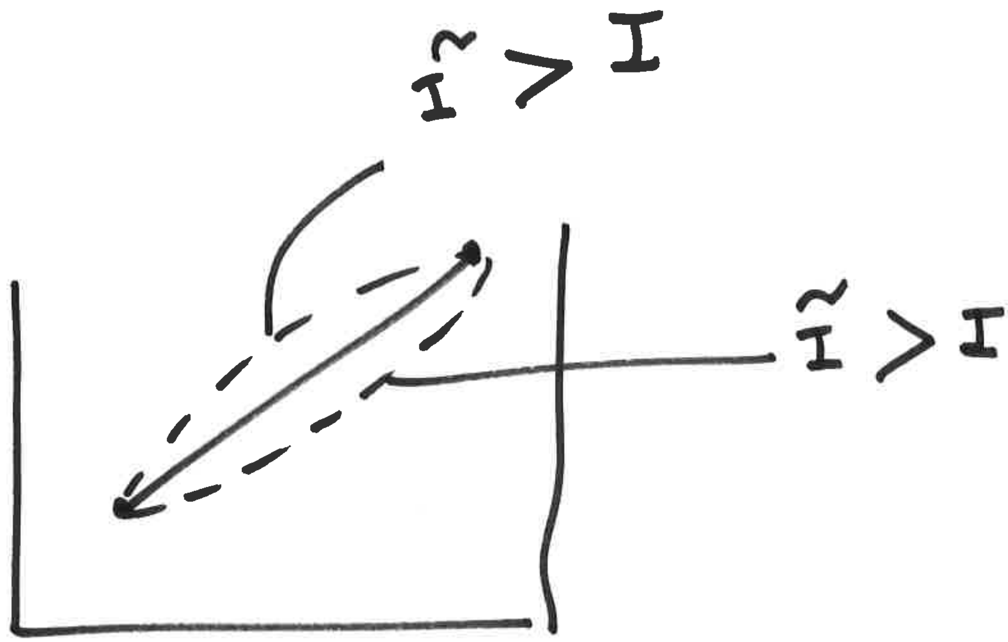
$\boxed{\text{Functional}}$

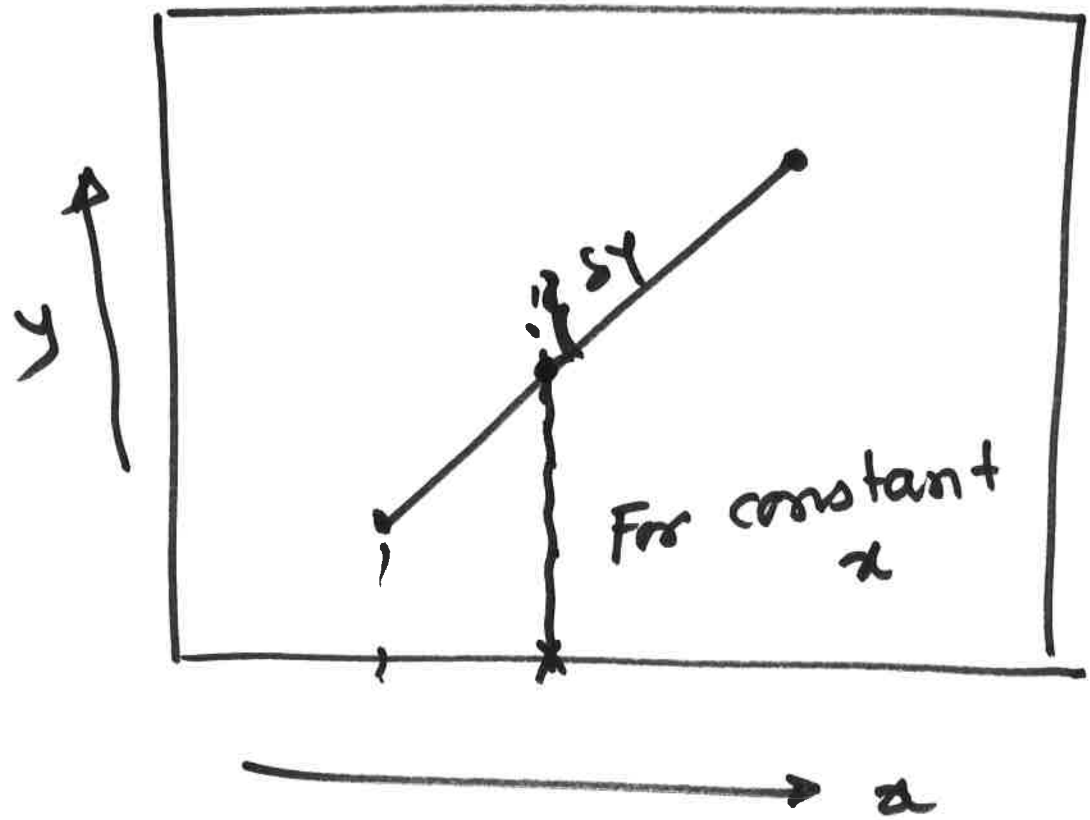


$$\delta I = 0$$

$$\delta^2 I > 0 \qquad \text{minimum}$$

$$\delta^2 I < 0 \qquad \text{maximum}$$





$$Y + \Delta Y = f(x + \Delta x)$$

due to change in Δx

$$\delta\left(\frac{dy}{dx}\right) \equiv \delta y'$$

$$\Rightarrow \left[\frac{d(\delta y)}{dx}\right]$$

$$\delta \int_a^b y \, dx = \int_a^b \delta y \, dx$$

4

$$\int_a^b u v' \, dx = -\int_a^b u' v \, dx + [u v]_a^b$$

$$\frac{d}{dx}(uv) = uv' + vu'$$

$$\int \frac{d}{dx}(uv) = \int uv' + \int vu'$$

$$uv = \int uv' \, dx + \int vu' \, dx$$

$$\int_a^b w'''' \, v \, dx = \int v'' w'' \, dx - [v' w''']_a^b + [v w''']_a^b$$

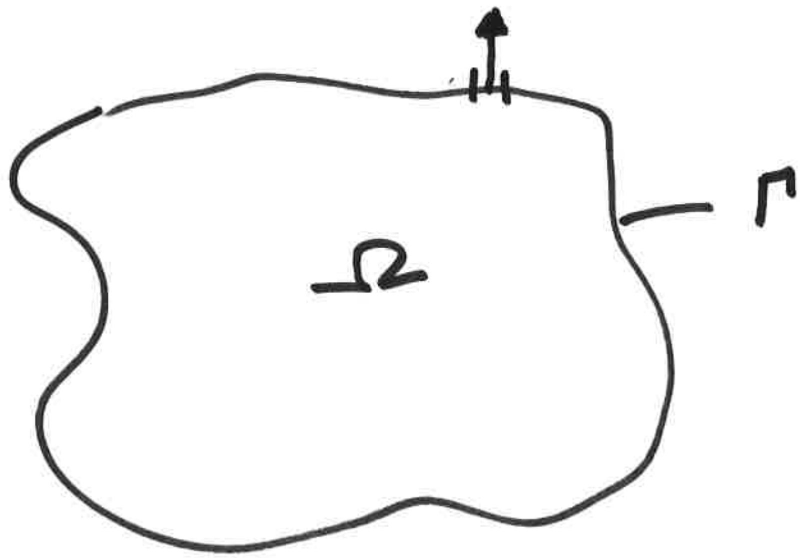
weighted

$$\begin{tikzcd}
v \arrow[dr, "+"] & w'''' \\
v' \arrow[dr, "-"] & w''' \\
v'' & w'' \arrow[l, bend left, "+"]
\end{tikzcd}$$

$$\begin{tikzcd}
u \arrow[dr, "+"] & v' \\
u' & v \arrow[l, bend left, "-"]
\end{tikzcd} \longrightarrow -\int u'v + [uv]_a^b$$

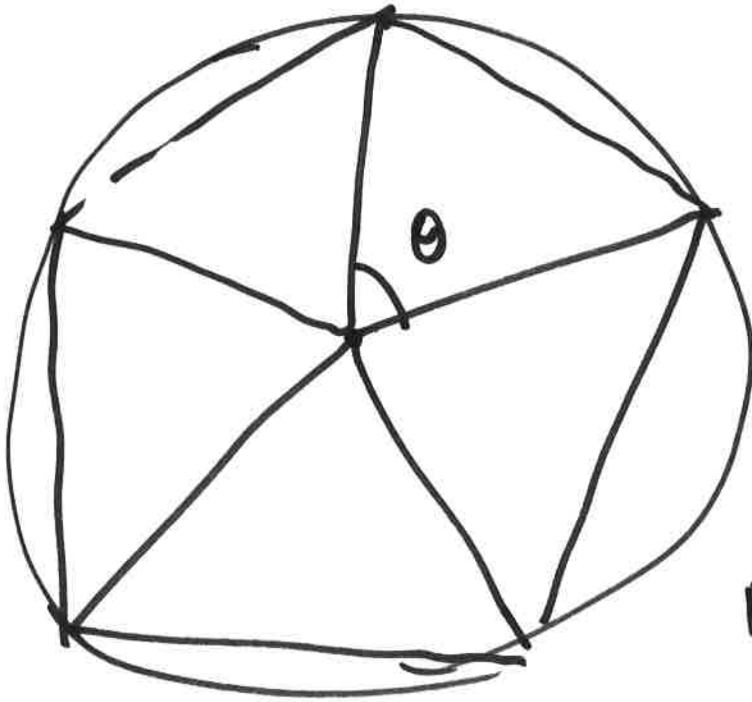
$$\int_{\Omega} \nabla F \, dx \, dy = \oint_{\Gamma} \hat{n} F \, ds$$

scalar function

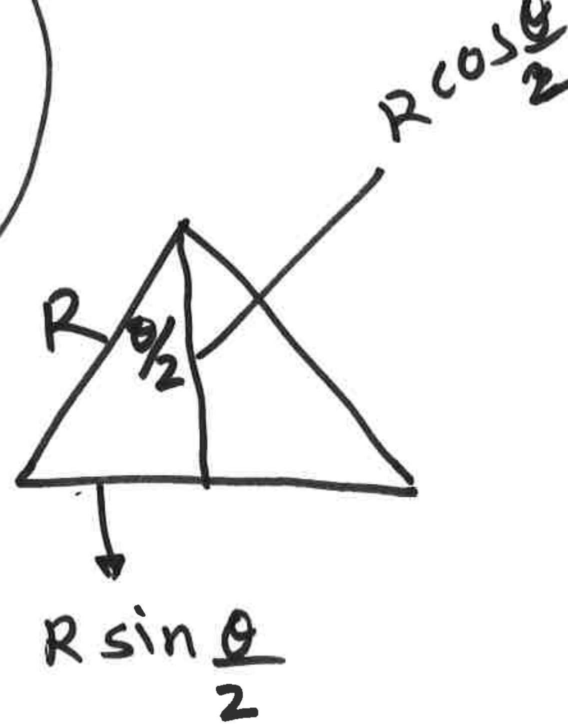


$$\int_{\Omega} \operatorname{div}(G) \, dx \, dy = \oint_{\Gamma} \hat{n} \cdot G \, ds$$

②



$$\theta = \frac{2\pi}{n}$$



$A_{\triangle le} =$

$$\frac{1}{\cancel{2}} \cancel{2} R^2 \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{\theta}{2}\right)$$

$$A_{\triangle le} = \cancel{\frac{1}{2}} R^2 \sin\left(\frac{\theta}{2}\right) \cos(\theta/2)$$

$$A_{\triangle le} = R^2 \sin\left(\frac{\pi}{n}\right) \cos\left(\frac{\pi}{n}\right)$$

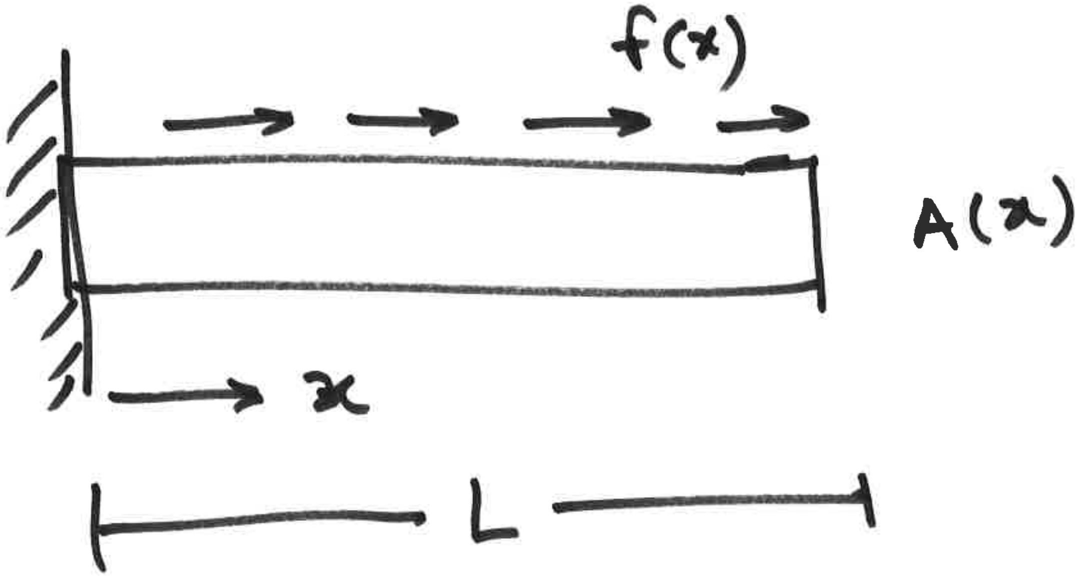
$$A_{app} = \sum_{i=1}^{n} R^2 \sin\left(\frac{\pi}{n}\right) \cos\left(\frac{\pi}{n}\right)$$

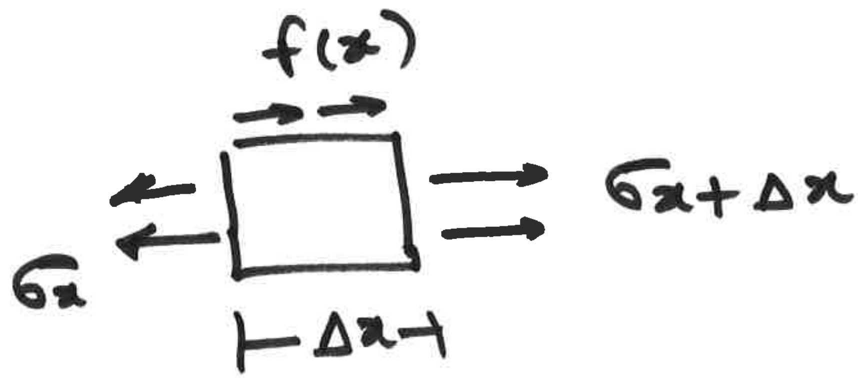
$$A_{app} = n R^2 \sin\left(\frac{\pi}{n}\right) \cos\left(\pi/n\right)$$

$$\lim_{n \to \infty} A_{app} = ?$$

$$\lim_{1/n \to 0} \pi R^2 \frac{\sin\left(\pi/n\right)}{\left(\frac{\pi}{n}\right)} \overset{1}{\cos\left(\frac{\pi}{n}\right)}$$

$$\boxed{A_{app} = \pi R^2}$$





$$\sigma_{x+\Delta x} A - \sigma_x A + f(x)\,\Delta x = 0$$

$$\frac{\left(\sigma_{x+\Delta x} A - \sigma_x A\right)}{\Delta x} + f(x) = 0$$

$$\lim_{\Delta x \to 0} \qquad \frac{\partial(\sigma A)}{\partial x} + f(x) = 0$$

$$\sigma = E\varepsilon$$

$$\sigma = E\frac{du}{dx}$$

$$\boxed{\frac{d}{dx}\left(AE\frac{du}{dx}\right) + f(x) = 0}$$

$\boxed{u(0) = 0}$ → essential boundary

$$\sigma\Big|_{x=L} = 0$$

$\boxed{E\frac{du}{dx}\Big|_{x=L} = 0}$ → natur

$$u_N = \sum c_i\phi_i + \phi_0$$

ϕ_0 as well as ϕ_i are known functions of x

$$\frac{d}{dx}\left(AE\,\frac{d(C_i\phi_i+\phi_0)}{dx}\right)+f(x)$$

$$= R(x, C_i) \neq 0$$

strong $\text{sol}^n \longrightarrow$ at every x

$$R(x, C_i) = 0$$

$$\int_0^L W\,R(x, C_i) = 0$$

$\searrow$ Weighted Residual method

$$\int_0^L \left[\frac{d}{dx}\left(AE\,\frac{du_N}{dx}\right)+f(x)\right] w\,dx = 0$$

$\cancel{w}\ d\big(\ \big)\ \cancel{w}$

$w + d'$

$w' \quad d$

$$-\int_0^L w' \left(AE \frac{du_N}{dx} \right) dx + \left[w \left(AE \frac{du_N}{dx} \right) \right]_0^L + \int_0^L w\, f(x)\, dx = 0$$

weak form