Homework 1

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1.1

$$\mu = 2 \cdot 10^{-2} \text{ Nsm}^{-2}$$
$$V = 61.0 \text{ cms}^{-1} = 0.61 \text{ ms}^{-1}$$
$$Y = 2 \text{ mm} = 2 \cdot 10^{-3} \text{ m}$$

For flow between two plates, one moving and one stationary:

$$\tau_{xy} = -\mu \frac{\partial v_x}{\partial y} \tag{1}$$

where τ is the momentum flux in the positive y-direction. For steady state:

$$\begin{aligned} -\mu \frac{\partial v_x}{\partial y} &= -\mu \frac{V}{Y} \\ &= -2 \cdot 10^{-2} \text{ Nsm}^{-2} \cdot \frac{0.61 \text{ ms}^{-1}}{2 \cdot 10^{-3} \text{ m}} \\ &= -6.1 \text{ Nm}^{-2} \end{aligned} \tag{2}$$

The direction of momentum transfer is from the top plate to the bottom plate (-y direction)

1.2

$$v_x = 3y - y^3$$
$$\rho = 10^3 \text{ kgm}^{-3}$$
$$v = 7 \cdot 10^{-7} \text{ m}^2\text{s}^{-1}$$
$$\mu = v\rho = 7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1}$$

a)

$$\begin{aligned}
\frac{\partial v_x}{\partial y} &= 3 \ - \ 3y^2 && (3) \\
\left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0} &= 3 \text{ cms}^{-1} \\
&= 3 \cdot 10^{-2} \text{ ms}^{-1} \\
\tau_{xy}\Big|_{x=x_1,y=0} &= -\mu \left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0} \\
&= -7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1} \ \cdot \ 3 \cdot 10^{-2} \text{ ms}^{-1} \\
&= -2.1 \cdot 10^{-5} \text{ kgs}^{-2}
\end{aligned}$$

The shear stress at $x = x_1, y = 0$ is $-2.1 \cdot 10^{-5}$ kgs^{-2}

b)

$$\begin{aligned}
\frac{\partial v_x}{\partial y} &= 3 \ - \ 3y^2 \text{ cms}^{-1} \\
\left.\frac{\partial v_x}{\partial y}\right|_{y=0.8mm} &= 1.08 \text{ cms}^{-1} \\
&= 1.08 \cdot 10^{-2} \text{ ms}^{-1} \\
\tau_{xy}\Big|_{y=0.8mm} &= -\mu \left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0.8mm} \\
&= -7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1} \ \cdot \ 1.08 \cdot 10^{-2} \text{ ms}^{-1} \\
&= -7.56 \cdot 10^{-6} \text{ kgs}^{-2}
\end{aligned}$$

The shear stress at $x = x_1, y = 0.8$ is $-7.56 \cdot 10^{-6}$ kgs^{-2}

c)

Momentum flux in the x-direction:

$$\tau_{yx} = -\mu \frac{\partial v_y}{\partial x} \tag{4}$$

Since the velocity profile has no component in y-direction, momentum flux in x-direction is 0

1.4

References