Homework 1

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August 26, 2017

1.1

$$\begin{aligned}
\mu &= 2 \cdot 10^{-2}\ \mathrm{Nsm^{-2}} \\
V &= 61.0\ \mathrm{cms^{-1}} = 0.61\ \mathrm{ms^{-1}} \\
Y &= 2\ \mathrm{mm} = 2 \cdot 10^{-3}\ \mathrm{m}
\end{aligned}$$

For flow between two plates, one moving and one stationary:

$$\tau_{xy} = -\mu \frac{\partial v_x}{\partial y} \tag{1}$$

where τ is the momentum flux in the positive y-direction. For steady state:

$$\begin{aligned}
-\mu \frac{\partial v_x}{\partial y} &= -\mu \frac{V}{Y} \\
&= -2 \cdot 10^{-2}\ \mathrm{Nsm^{-2}} \ \cdot\ \frac{0.61\ \mathrm{ms^{-1}}}{2 \cdot 10^{-3}\ \mathrm{m}} \\
&= -6.1\ \mathrm{Nm^{-2}}
\end{aligned} \tag{2}$$

The direction of momentum transfer is from the top plate to the bottom plate (-y direction)

1.2

$$\begin{aligned}
v_x &= 3y\ -\ y^3 \\
\rho &= 10^3\ \mathrm{kgm^{-3}} \\
v &= 7 \cdot 10^{-7}\ \mathrm{m^2s^{-1}} \\
\mu &= v\rho = 7 \cdot 10^{-4}\ \mathrm{kgm^{-1}s^{-1}}
\end{aligned}$$

a)

$$\frac{\partial v_x}{\partial y} = 3 \; - \; 3y^2 \tag{3}$$

$$\left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0} = 3 \text{ cms}^{-1}$$

$$= 3 \cdot 10^{-2} \text{ ms}^{-1}$$

$$\tau_{xy}\Big|_{x=x_1,y=0} = -\mu \left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0}$$

$$= -7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1} \;\cdot\; 3 \cdot 10^{-2} \text{ ms}^{-1}$$

$$= -2.1 \cdot 10^{-5} \text{ kgs}^{-2}$$

The shear stress at $x = x_1, y = 0$ is $-2.1 \cdot 10^{-5}$ kgs^{-2}

b)

$$\frac{\partial v_x}{\partial y} = 3 \; - \; 3y^2 \text{ cms}^{-1}$$

$$\left.\frac{\partial v_x}{\partial y}\right|_{y=0.8mm} = 1.08 \text{ cms}^{-1}$$

$$= 1.08 \cdot 10^{-2} \text{ ms}^{-1}$$

$$\tau_{xy}\Big|_{y=0.8mm} = -\mu \left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0.8mm}$$

$$= -7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1} \;\cdot\; 1.08 \cdot 10^{-2} \text{ ms}^{-1}$$

$$= -7.56 \cdot 10^{-6} \text{ kgs}^{-2}$$

The shear stress at $x = x_1, y = 0.8$ is $-7.56 \cdot 10^{-6}$ kgs^{-2}

c)

Momentum flux in the x-direction:

$$\tau_{yx} = -\mu \frac{\partial v_y}{\partial x} \tag{4}$$

Since the velocity profile has no component in y-direction, momentum flux in x-direction is 0

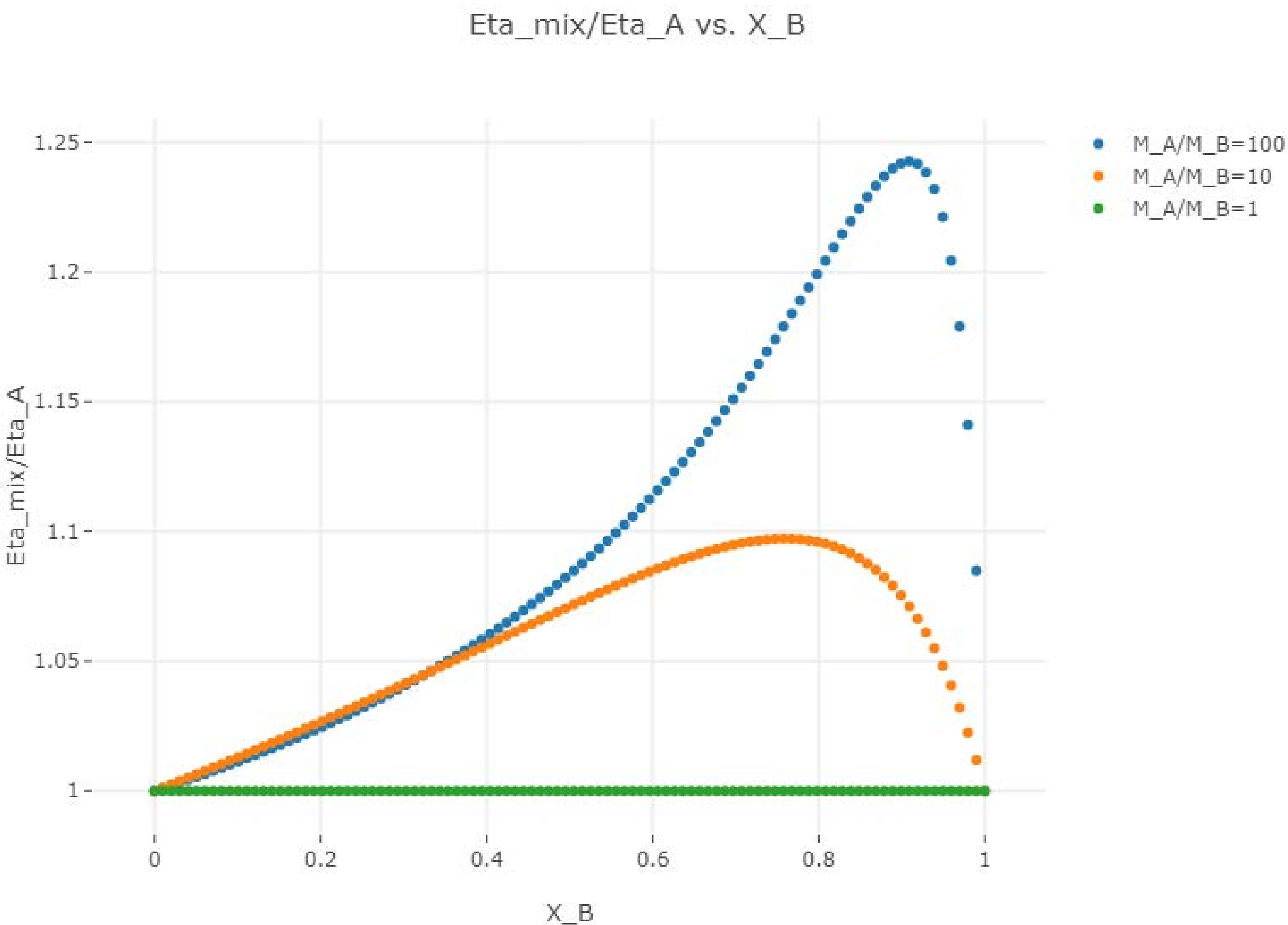
1.4

$$\eta_{mix} = \sum_{i=1}^{n} \frac{x_i \eta_i}{\sum_{j=1}^{n} x_j \phi_{ij}} \tag{5}$$

$$\phi_{ij} = \frac{1}{\sqrt{8}} \left[1 + \frac{M_i}{M_j}\right]^{-1/2} \left[1 \; + \; \left[\frac{\eta_i}{\eta_j}\right]^{1/2} \left[\frac{M_j}{M_i}\right]^{1/4}\right]^2 \tag{6}$$

For a binary gas with $\eta_A = \eta_B$:

$$\frac{\eta_{mix}}{\eta_A} = \frac{(1-x_B)}{1-x_B+x_B\left(\frac{1}{\sqrt{8}}\left[1+\frac{M_A}{M_B}\right]^{-1/2}\left[1\ +\ \left[\frac{M_B}{M_A}\right]^{1/4}\right]^2\right)} + \frac{x_B}{x_B+(1-x_B)\left(\frac{1}{\sqrt{8}}\left[1+\frac{M_B}{M_A}\right]^{-1/2}\left[1\ +\ \left[\frac{M_A}{M_B}\right]^{1/4}\right]^2\right)} \tag{7}$$



1.8

$$
\begin{aligned}
T &= 2273 \text{ K} \\
T_M &= 1575 \text{ K} \\
M &= 52 \text{ gmol}^{-1} = 5.2 \cdot 10^{-2} \text{ kgmol}^{-1} \\
\rho &= 7100 \text{ kgm}^{-3} \\
\delta &= 0.272 \text{ nm}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\epsilon}{K_B} &= 5.20 T_M \text{ K} \\
&= 5.20 \ \cdot \ 1575 \text{ K} \\
&= 8.19 \cdot 10^3 K
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
T^* &= \frac{K_B T}{\epsilon} \\
&= \frac{2273 \text{ K}}{8.19 \cdot 10^3 \text{ K}} \\
&= 0.2775
\end{aligned} \tag{9}
$$

From Figure 1.10 in the book:

$$\eta^*(V^*)^2 = 2.05$$

$$\begin{aligned} V^* &= \frac{1}{n\delta^3} \\ &= \frac{1}{\left[\frac{6.023\cdot 10^{23}\ \text{atoms}}{5.2\cdot 10^{-2}\ \text{kg}}\right]\left[\frac{7100\ \text{kg}}{\text{m}^3}\right]\left(0.272\cdot 10^{-9}\ \text{m}\right)^3} \\ &= 0.604 \end{aligned} \tag{10}$$

$$\eta^* = \frac{2.05}{0.604^2} = 5.62$$

$$\begin{aligned} \eta &= \frac{\eta^*(MTRT)^{1/2}}{\delta^2 N_A} \\ &= \frac{5.62\ \cdot\ \left(5.2\cdot 10^{-2}\ \text{kgmol}^{-1}\ \cdot\ 8.3144\ \text{Jmol}^{-1}\text{K}^{-1}\ \cdot\ 2273\ \text{K}\right)^{1/2}}{\left(0.272\cdot 10^{-9}\ \text{m}\right)^2\ \cdot\ 6.023\cdot 10^{23}\ \text{mol}^{-1}} \\ &= 3.954\cdot 10^{-3}\ \text{kgm}^{-1}\text{s}^{-1} \end{aligned} \tag{11}$$

References