# Homework 1

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## 1.1

$$\mu = 2 \cdot 10^{-2} \text{ Nsm}^{-2}$$
$$V = 61.0 \text{ cms}^{-1} = 0.61 \text{ ms}^{-1}$$
$$Y = 2 \text{ mm} = 2 \cdot 10^{-3} \text{ m}$$

For flow between two plates, one moving and one stationary:

$$\tau_{xy} = -\mu \frac{\partial v_x}{\partial y} \tag{1}$$

where $\tau$ is the momentum flux in the positive y-direction. For steady state:

$$\begin{aligned} -\mu \frac{\partial v_x}{\partial y} &= -\mu \frac{V}{Y} \\ &= -2 \cdot 10^{-2} \text{ Nsm}^{-2} \cdot \frac{0.61 \text{ ms}^{-1}}{2 \cdot 10^{-3} \text{ m}} \\ &= -6.1 \text{ Nm}^{-2} \end{aligned} \tag{2}$$

The direction of momentum transfer is from the top plate to the bottom plate (-y direction)

## 1.2

$$v_x = 3y - y^3$$
$$\rho = 10^3 \text{ kgm}^{-3}$$
$$v = 7 \cdot 10^{-7} \text{ m}^2\text{s}^{-1}$$
$$\mu = v\rho = 7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1}$$

**a)**

$$\frac{\partial v_x}{\partial y} = 3 \ - \ 3y^2 \tag{3}$$

$$\left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0} = 3 \text{ cms}^{-1}$$

$$= 3 \cdot 10^{-2} \text{ ms}^{-1}$$

$$\tau_{xy}\Big|_{x=x_1,y=0} = -\mu \left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0}$$

$$= -7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1} \ \cdot \ 3 \cdot 10^{-2} \text{ ms}^{-1}$$

$$= -2.1 \cdot 10^{-5} \text{ kgs}^{-2}$$

The shear stress at $x = x_1, y = 0$ is $-2.1 \cdot 10^{-5}$ kgs$^{-2}$

**b)**

$$\frac{\partial v_x}{\partial y} = 3 \ - \ 3y^2 \text{ cms}^{-1}$$

$$\left.\frac{\partial v_x}{\partial y}\right|_{y=0.8mm} = 1.08 \text{ cms}^{-1}$$

$$= 1.08 \cdot 10^{-2} \text{ ms}^{-1}$$

$$\tau_{xy}\Big|_{y=0.8mm} = -\mu \left.\frac{\partial v_x}{\partial y}\right|_{x=x_1,y=0.8mm}$$

$$= -7 \cdot 10^{-4} \text{ kgm}^{-1}\text{s}^{-1} \ \cdot \ 1.08 \cdot 10^{-2} \text{ ms}^{-1}$$

$$= -7.56 \cdot 10^{-6} \text{ kgs}^{-2}$$

The shear stress at $x = x_1, y = 0.8$ is $-7.56 \cdot 10^{-6}$ kgs$^{-2}$

**c)**

Momentum flux in the x-direction:

$$\tau_{yx} = -\mu \frac{\partial v_y}{\partial x} \tag{4}$$

Since the velocity profile has no component in y-direction, momentum flux in x-direction is 0

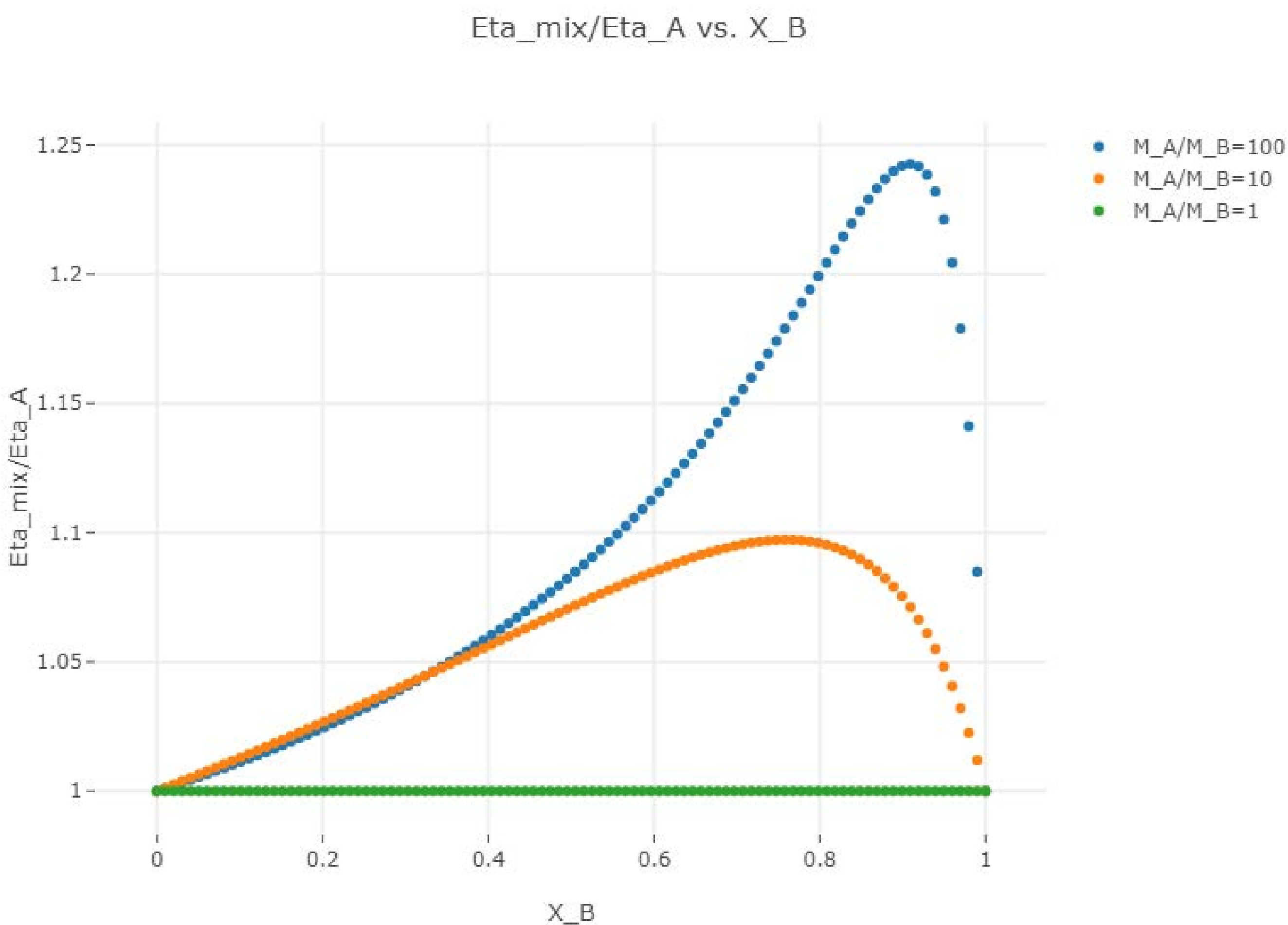
## 1.4

$$\eta_{mix} = \sum_{i=1}^{n} \frac{x_i \eta_i}{\sum_{j=1}^{n} \ x_j \phi_{ij}} \tag{5}$$

$$\phi_{ij} = \frac{1}{\sqrt{8}} \left[1 + \frac{M_i}{M_j}\right]^{-1/2} \left[1 \ + \ \left[\frac{\eta_i}{\eta_j}\right]^{1/2} \left[\frac{M_j}{M_i}\right]^{1/4}\right]^2 \tag{6}$$

For a binary gas with $\eta_A = \eta_B$:

$$\frac{\eta_{mix}}{\eta_A} = \frac{(1-x_B)}{1-x_B+x_B\left(\frac{1}{\sqrt{8}}\left[1+\frac{M_A}{M_B}\right]^{-1/2}\left[1\;+\;\left[\frac{M_B}{M_A}\right]^{1/4}\right]^2\right)} + \frac{x_B}{x_B+(1-x_B)\left(\frac{1}{\sqrt{8}}\left[1+\frac{M_B}{M_A}\right]^{-1/2}\left[1\;+\;\left[\frac{M_A}{M_B}\right]^{1/4}\right]^2\right)} \tag{7}$$



## 1.8

$$\begin{aligned}
T &= 2273 \text{ K} \\
T_M &= 1575 \text{ K} \\
M &= 52 \text{ gmol}^{-1} = 5.2 \cdot 10^{-2} \text{ kgmol}^{-1} \\
\rho &= 7100 \text{ kgm}^{-3} \\
\delta &= 0.272 \text{ nm}
\end{aligned}$$

$$\begin{aligned}
\frac{\epsilon}{K_B} &= 5.20 T_M \text{ K} \\
&= 5.20 \ \cdot \ 1575 \text{ K} \\
&= 8.19 \cdot 10^3 K
\end{aligned} \tag{8}$$

$$\begin{aligned}
T^* &= \frac{K_B T}{\epsilon} \\
&= \frac{2273 \text{ K}}{8.19 \cdot 10^3 \text{ K}} \\
&= 0.2775
\end{aligned} \tag{9}$$

From Figure 1.10 in the book:

$$\eta^*(V^*)^2 = 2.05$$

$$\begin{aligned} V^* &= \frac{1}{n\delta^3} \\ &= \frac{1}{\left[\frac{6.023\cdot10^{23}\text{ atoms}}{5.2\cdot10^{-2}\text{ kg}}\right]\left[\frac{7100\text{ kg}}{\text{m}^3}\right]\left(0.272\cdot10^{-9}\text{ m}\right)^3} \\ &= 0.604 \end{aligned} \tag{10}$$

$$\eta^* = \frac{2.05}{0.604^2} = 5.62$$

$$\begin{aligned} \eta &= \frac{\eta^*(MRT)^{1/2}}{\delta^2 N_A} \\ &= \frac{5.62\ \cdot\ \left(5.2\cdot10^{-2}\text{ kgmol}^{-1}\ \cdot\ 8.3144\text{ Jmol}^{-1}\text{K}^{-1}\ \cdot\ 2273\text{ K}\right)^{1/2}}{\left(0.272\cdot10^{-9}\text{ m}\right)^2\ \cdot\ 6.023\cdot10^{23}\text{ mol}^{-1}} \\ &= 3.954\cdot10^{-3}\text{ kgm}^{-1}\text{s}^{-1} \end{aligned} \tag{11}$$

The viscosity for Cr at $2273K$ is approximately $3.954\cdot10^{-3}\text{ kgm}^{-1}\text{s}^{-1}$