ARMA, ARIMA & SARIMA Modeling

In this notebook we do the following: review AR and MA concepts, build ARMA models for stationary data, extend to ARIMA for non-stationary series via differencing, introduce SARIMA to handle seasonal effects, and walk through model identification, estimation, validation, and forecasting.

In [10]:

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import seasonal_decompose, STL
from statsmodels.tsa.stattools import adfuller
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.arima.model import ARIMA
import seaborn as sns
import statsmodels.api as sm
```

In [2]:

```python
# load monthly temperature data
monthly = pd.read_csv(
    'https://zenodo.org/records/10951538/files/arima_temp.csv?download=1',
    skipfooter=2, header=0, index_col=0, names=['month','temp'], engine='pyt
)
monthly.index = pd.to_datetime(monthly.index)
```

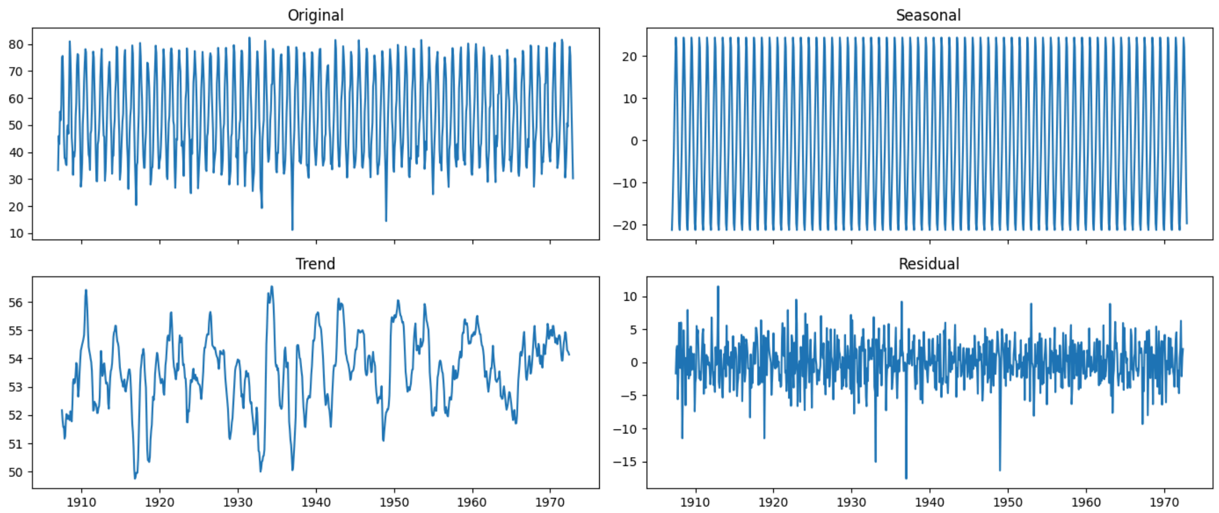
Discussion

We load 1907–1972 monthly avg. temperatures. The index is a DatetimeIndex; this will let us resample and plot seasonality. Initial .head() and .describe() confirm 792 observations and no glaring missing values.

Decomposition

In [3]:

```python
# additive decomposition
decomp = seasonal_decompose(monthly.temp, model='additive', period=12)
fig, axes = plt.subplots(2,2, figsize=(14,6), sharex=True)
axes[0,0].plot(monthly.temp); axes[0,0].set_title('Original')
axes[0,1].plot(decomp.seasonal); axes[0,1].set_title('Seasonal')
axes[1,0].plot(decomp.trend); axes[1,0].set_title('Trend')
axes[1,1].plot(decomp.resid); axes[1,1].set_title('Residual')
plt.tight_layout();
```



Discussion • The annual cycle is very regular, peaking mid-year. • Trend component is relatively flat but shows slight upward drift in mid-century. • Residuals look roughly zero-mean but may still contain autocorrelation

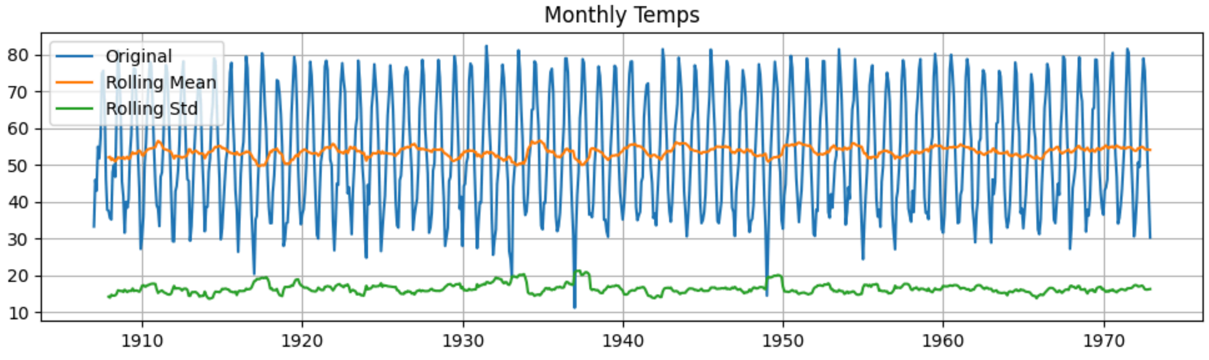
Stationarity Check

In [4]:

```python
def adf_test(series, title=''):
    result = adfuller(series.dropna(), autolag='AIC')
    print(f"{title} ADF p-value = {result[1]:.3f}")
    rolmean = series.rolling(12).mean()
    rolstd  = series.rolling(12).std()
    plt.figure(figsize=(12,3))
    plt.plot(series, label='Original')
    plt.plot(rolmean, label='Rolling Mean')
    plt.plot(rolstd,  label='Rolling Std')
    plt.legend(); plt.title(title); plt.grid(); plt.show()

adf_test(monthly.temp, 'Monthly Temps')
```

```
Monthly Temps ADF p-value = 0.000
```



Discussion

• ADF strongly rejects non-stationarity (p≈0), yet clear seasonality remains. Rolling mean/std are flat on annual scale but oscillate within each year—so periodicity doesn’t imply a unit root.

Remove Seasonality

In [5]:

```
monthly['temp_d12'] = monthly.temp.diff(12)
data = monthly.temp_d12.dropna()
```

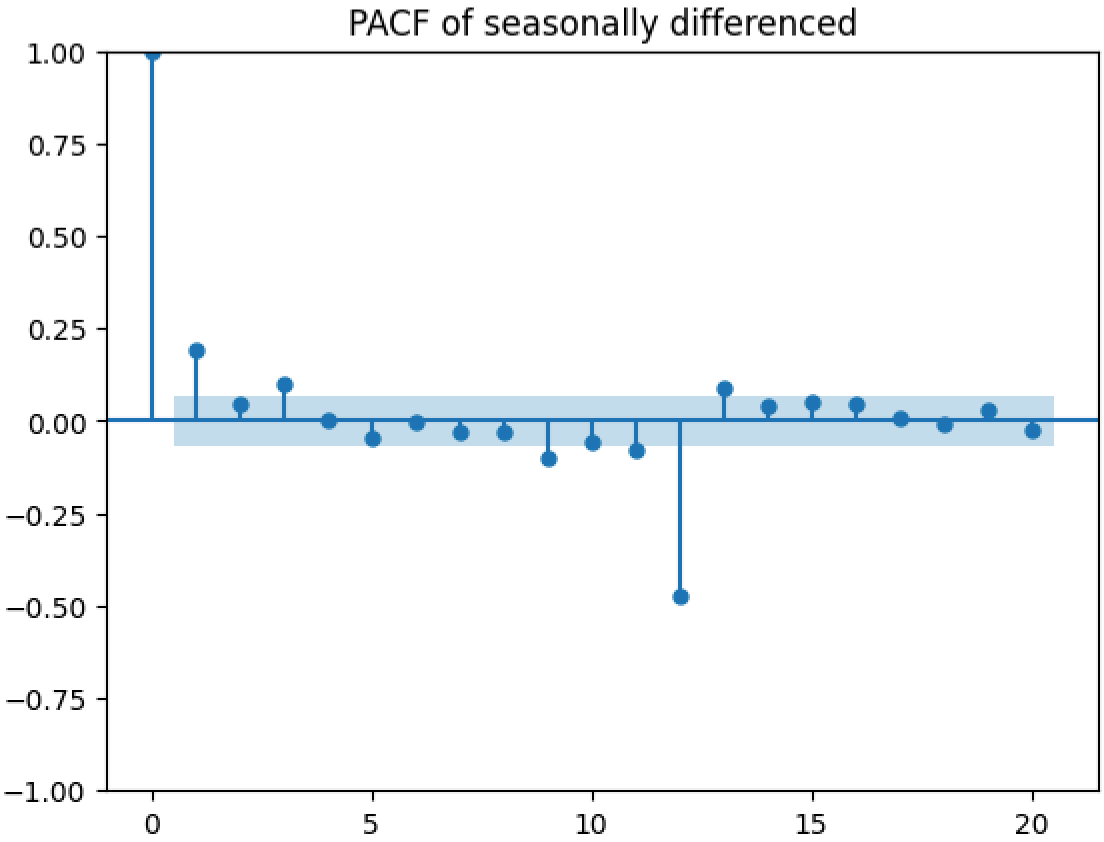
Discussion

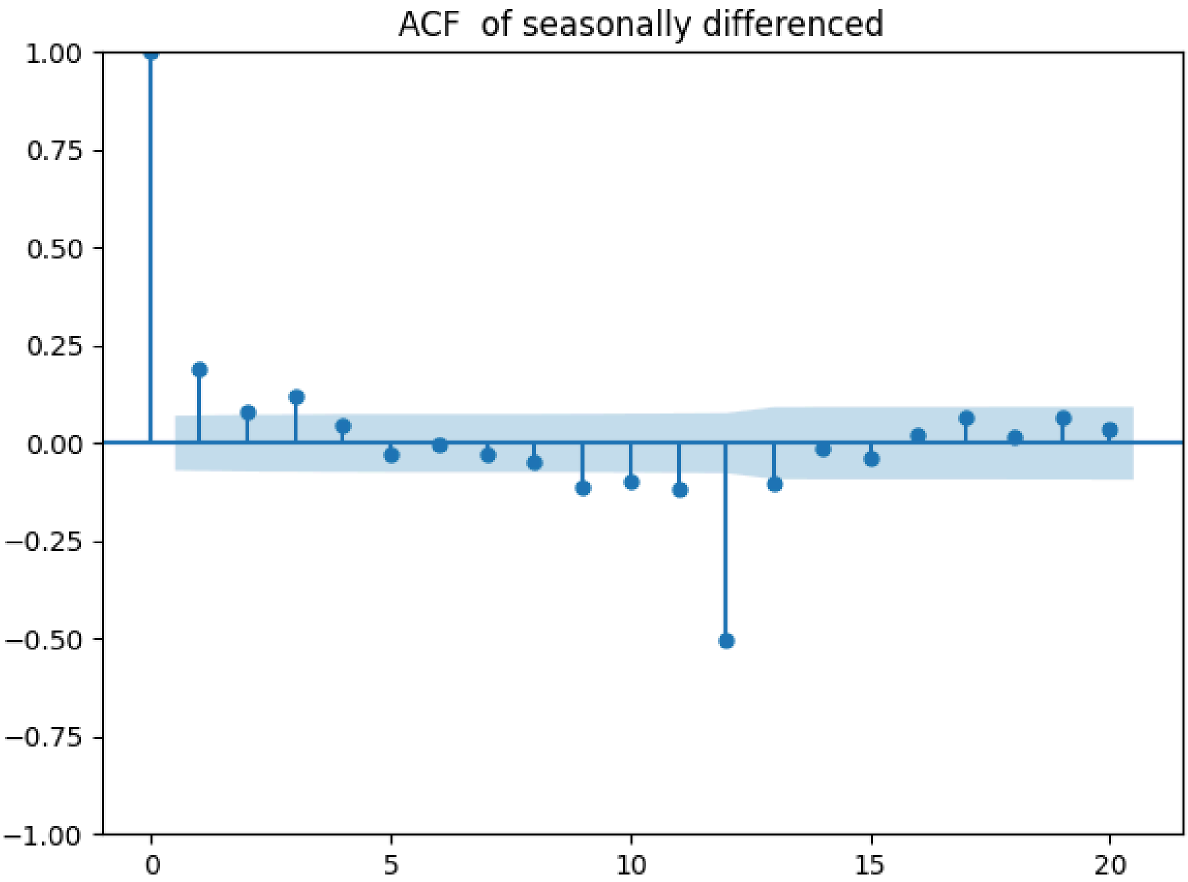
• We perform a 12-month seasonal differencing. • Now our series should have no persistent periodic swing but still may need one more difference for trend.

**Identify p & q for ARMA

In [6]:

```
plot_pacf(data, lags=20, title='PACF of seasonally differenced')
plot_acf (data, lags=20, title='ACF  of seasonally differenced')
plt.tight_layout();
```





Discussion

• PACF cuts off after lag 2 (with a smaller spike at 3), so candidate p∈{1,2,3}. • ACF shows significant spikes at lags 1 and 3, so q∈{1,3}. • Seasonal lags (12,24) are still present—ignore them when choosing p, q

**Fit ARMA (as ARIMA with d=0))

In [7]:

```python
train = data[:-36]
test  = data[-36:]
model_arma = ARIMA(train, order=(2,0,3)).fit()
print(model_arma.summary())
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
statsmodels/tsa/base/tsa_model.py:473: ValueWarning: No frequency informatio
n was provided, so inferred frequency MS will be used.
  self._init_dates(dates, freq)
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
statsmodels/tsa/base/tsa_model.py:473: ValueWarning: No frequency informatio
n was provided, so inferred frequency MS will be used.
  self._init_dates(dates, freq)
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
statsmodels/tsa/base/tsa_model.py:473: ValueWarning: No frequency informatio
n was provided, so inferred frequency MS will be used.
  self._init_dates(dates, freq)
```

```
                               SARIMAX Results                                
============================================================================
==
Dep. Variable:               temp_d12   No. Observations:                  7
44
Model:                 ARIMA(2, 0, 3)   Log Likelihood               -2244.4
94
Date:                Mon, 21 Apr 2025   AIC                             4502.9
87
Time:                        18:33:24   BIC                             4535.2
72
Sample:                    01-01-1908   HQIC                            4515.4
32
                         - 12-01-1969                                         
Covariance Type:                  opg                                         
============================================================================
==
                 coef    std err          z      P>|z|      [0.025      0.97
5]
----------------------------------------------------------------------------
--
const          0.0295      0.258      0.114      0.909      -0.476       0.5
35
ar.L1          0.8911      0.027     32.609      0.000       0.838       0.9
45
ar.L2         -0.7790      0.025    -30.871      0.000      -0.828      -0.7
30
ma.L1         -0.7610      0.072    -10.545      0.000      -0.902      -0.6
20
ma.L2          0.7600      0.109      7.000      0.000       0.547       0.9
73
ma.L3          0.2393      0.048      4.969      0.000       0.145       0.3
34
sigma2        24.1867      2.784      8.686      0.000      18.729      29.6
44
============================================================================
=======
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):                
36.49
Prob(Q):                              0.92   Prob(JB):                        
0.00
Heteroskedasticity (H):               0.74   Skew:                            
0.04
Prob(H) (two-sided):                  0.02   Kurtosis:                        
4.08
============================================================================
=======

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (compl
ex-step).
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
statsmodels/base/model.py:607: ConvergenceWarning: Maximum Likelihood optimi
zation failed to converge. Check mle_retvals
  warnings.warn("Maximum Likelihood optimization failed to "
```

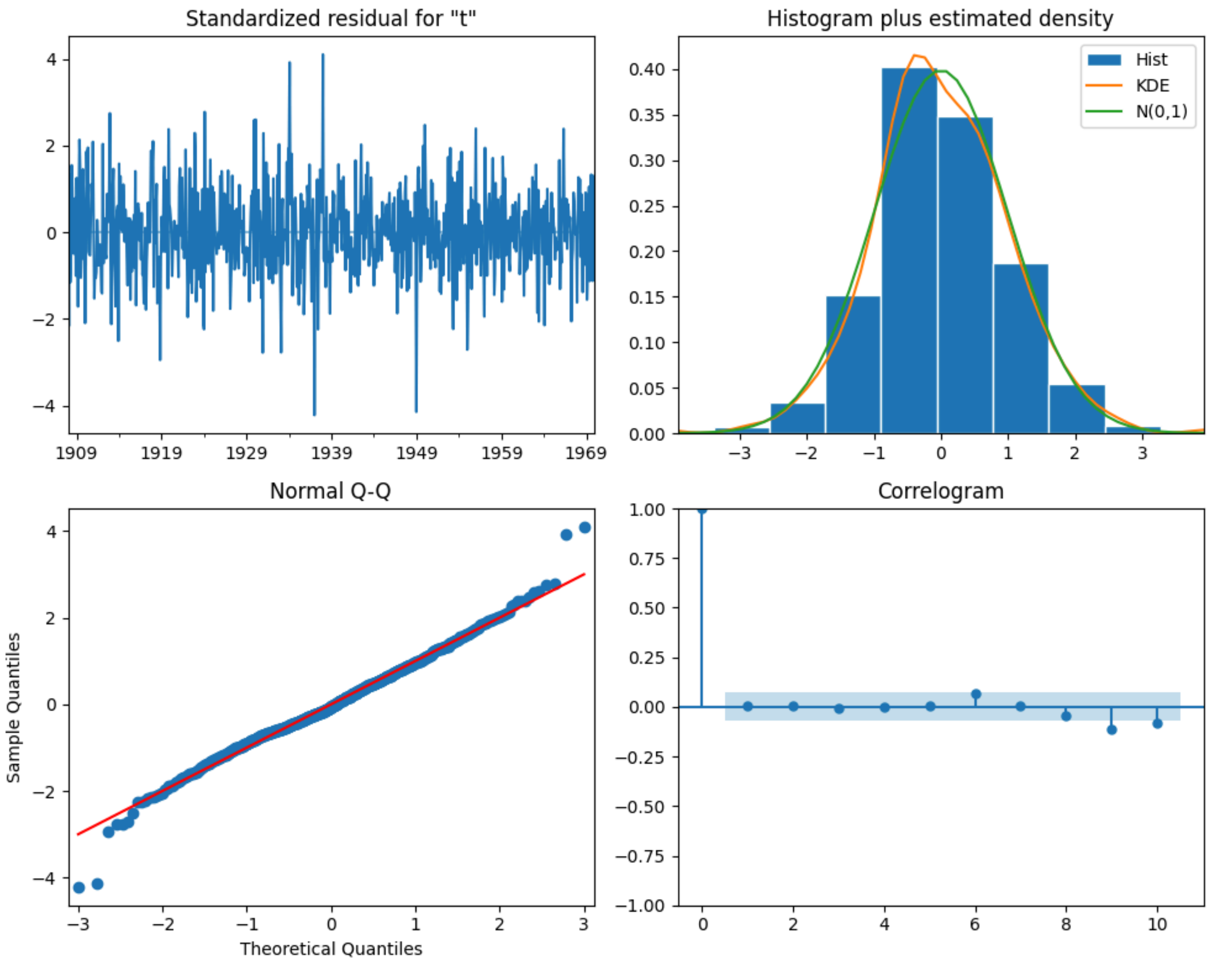
Discussion

• We've chosen ARMA(2,3) (i.e. ARIMA(2,0,3)) as our first candidate. • Coefficients with p≈0.8 for AR(2) and q≈… for MA indicate how past values and past shocks drive the process.

Residual Diagnostics

In [8]:

```
resid = model_arma.resid
fig = model_arma.plot_diagnostics(figsize=(10,8))
plt.tight_layout();
```



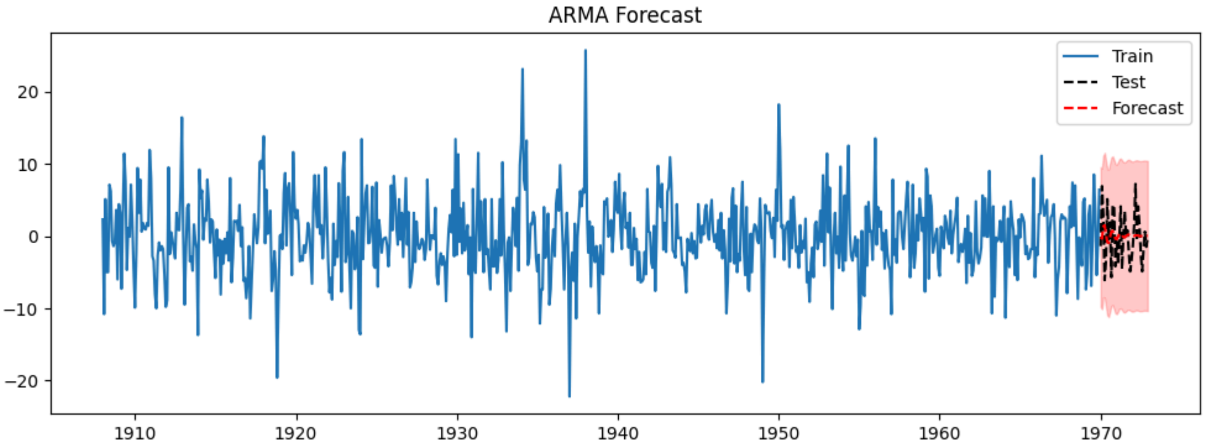
Discussion • Residuals should look like white noise: no remaining autocorrelation, roughly Gaussian. • A good model shows no structure in ACF of residuals, and QQ-plot near 45° line. • If diagnostics fail, we'd revisit p and q.

Forecast with ARMA

In [9]:

```
pred = model_arma.get_forecast(steps=36)
fc   = pred.predicted_mean
ci   = pred.conf_int()

plt.figure(figsize=(12,4))
plt.plot(train.index, train, label='Train')
plt.plot(test .index, test , 'k--', label='Test')
plt.plot(test .index, fc   , 'r--', label='Forecast')
plt.fill_between(test.index, ci.iloc[:,0], ci.iloc[:,1], color='r', alpha=0.
plt.legend(); plt.title('ARMA Forecast'); plt.show()
```



Discussion

- ARIMA(2,1,3)(0,1,1)[12] automatically handles both seasonality and trend.
- ‘d=1’ removes linear drift; seasonal_order takes care of annual cycle in one step.

**SARIMA & AutoARIMA

In [3]:
```
import numpy as np
import pmdarima as pm

print(np.__version__)
print(pm.__version__)

# should run without error now
auto = pm.auto_arima(
    monthly['temp'],
    seasonal=True, m=12,
    start_p=0, start_q=0, max_p=3, max_q=3,
    start_P=0, start_Q=0, max_P=2, max_Q=2,
    d=None, D=1, trace=True, stepwise=True
)
print(auto.summary())
```

```
1.26.4
2.0.4
Performing stepwise search to minimize aic
 ARIMA(0,0,0)(0,1,0)[12] intercept   : AIC=4796.839, Time=0.06 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```

```
 ARIMA(1,0,0)(1,1,0)[12] intercept   : AIC=4546.179, Time=0.35 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```

```
 ARIMA(0,0,1)(0,1,1)[12] intercept   : AIC=inf, Time=1.11 sec
 ARIMA(0,0,0)(0,1,0)[12]             : AIC=4794.867, Time=0.03 sec
 ARIMA(1,0,0)(0,1,0)[12] intercept   : AIC=4769.737, Time=0.09 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```

```
 ARIMA(1,0,0)(2,1,0)[12] intercept   : AIC=4451.142, Time=0.87 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```

```
 ARIMA(1,0,0)(2,1,1)[12] intercept   : AIC=inf, Time=2.44 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,0)(1,1,1)[12] intercept   : AIC=inf, Time=1.11 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(0,0,0)(2,1,0)[12] intercept   : AIC=4477.926, Time=0.50 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(2,0,0)(2,1,0)[12] intercept   : AIC=4452.084, Time=0.97 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,1)(2,1,0)[12] intercept   : AIC=4450.271, Time=1.27 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,1)(1,1,0)[12] intercept   : AIC=4545.537, Time=0.44 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,1)(2,1,1)[12] intercept   : AIC=inf, Time=3.77 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,1)(1,1,1)[12] intercept   : AIC=inf, Time=1.74 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(0,0,1)(2,1,0)[12] intercept   : AIC=4453.370, Time=0.62 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(2,0,1)(2,1,0)[12] intercept   : AIC=4451.722, Time=2.20 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,2)(2,1,0)[12] intercept   : AIC=4451.531, Time=1.32 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(0,0,2)(2,1,0)[12] intercept   : AIC=4453.816, Time=0.76 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(2,0,2)(2,1,0)[12] intercept   : AIC=4452.909, Time=1.53 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,1)(2,1,0)[12]             : AIC=4448.358, Time=0.51 sec
 ARIMA(1,0,1)(1,1,0)[12]             : AIC=4543.587, Time=0.19 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,1)(2,1,1)[12]             : AIC=inf, Time=2.24 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,1)(1,1,1)[12]             : AIC=inf, Time=0.91 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(0,0,1)(2,1,0)[12]             : AIC=4451.504, Time=0.22 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,0)(2,1,0)[12]             : AIC=4449.262, Time=0.24 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(2,0,1)(2,1,0)[12]             : AIC=4449.809, Time=0.83 sec
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(1,0,2)(2,1,0)[12]             : AIC=4449.618, Time=0.56 sec
 ARIMA(0,0,0)(2,1,0)[12]             : AIC=4476.106, Time=0.15 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```

```
 ARIMA(0,0,2)(2,1,0)[12]             : AIC=4451.938, Time=0.25 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```

```
 ARIMA(2,0,0)(2,1,0)[12]             : AIC=4450.192, Time=0.27 sec
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
 ARIMA(2,0,2)(2,1,0)[12]             : AIC=4450.996, Time=0.69 sec

Best model:  ARIMA(1,0,1)(2,1,0)[12]
Total fit time: 28.244 seconds
                                      SARIMAX Results
============================================================================
===============
Dep. Variable:                                  y   No. Observations:
792
Model:             SARIMAX(1, 0, 1)x(2, 1, [], 12)   Log Likelihood
-2219.179
Date:                            Mon, 21 Apr 2025   AIC
4448.358
Time:                                    18:45:14   BIC
4471.655
Sample:                                01-01-1907   HQIC
4457.318
                                     - 12-01-1972
Covariance Type:                              opg
============================================================================
==
                 coef    std err          z      P>|z|      [0.025      0.97
5]
----------------------------------------------------------------------------
--
ar.L1          0.5606      0.147      3.821      0.000       0.273       0.8
48
ma.L1         -0.3914      0.164     -2.391      0.017      -0.712      -0.0
71
ar.S.L12      -0.6780      0.028    -24.014      0.000      -0.733      -0.6
23
ar.S.L24      -0.3467      0.029    -12.111      0.000      -0.403      -0.2
91
sigma2        17.1812      0.633     27.149      0.000      15.941      18.4
22
============================================================================
=======
Ljung-Box (L1) (Q):                  0.04   Jarque-Bera (JB):
153.83
Prob(Q):                             0.83   Prob(JB):
0.00
Heteroskedasticity (H):              0.68   Skew:
-0.37
Prob(H) (two-sided):                 0.00   Kurtosis:
5.05
============================================================================
=======

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (compl
ex-step).
```

Discussion

- auto_arima finds SARIMAX(1,0,1)x(2,1,0)[12] as best by AIC.

- This automates our manual grid search while still relying on our seasonal differencing choice.

Model Comparison & Grid Search

In [4]:

```
# small grid example
from itertools import product
orders = list(product([1,2],[0,1],[1,3]))
seasonals = [(0,1,1,12),(1,1,1,12)]
results = []
for o in orders:
    for s in seasonals:
        mod = ARIMA(monthly.temp, order=o, seasonal_order=s).fit()
        results.append((o,s,mod.aic))
pd.DataFrame(results, columns=['order','seasonal','AIC']).sort_values('AIC')
```

```
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```

Out[4]:

	order	seasonal	AIC
0	(1, 0, 1)	(0, 1, 1, 12)	4266.668838
10	(2, 0, 3)	(0, 1, 1, 12)	4267.441111
1	(1, 0, 1)	(1, 1, 1, 12)	4268.233301
8	(2, 0, 1)	(0, 1, 1, 12)	4268.303681
2	(1, 0, 3)	(0, 1, 1, 12)	4268.561543

*Discussion**

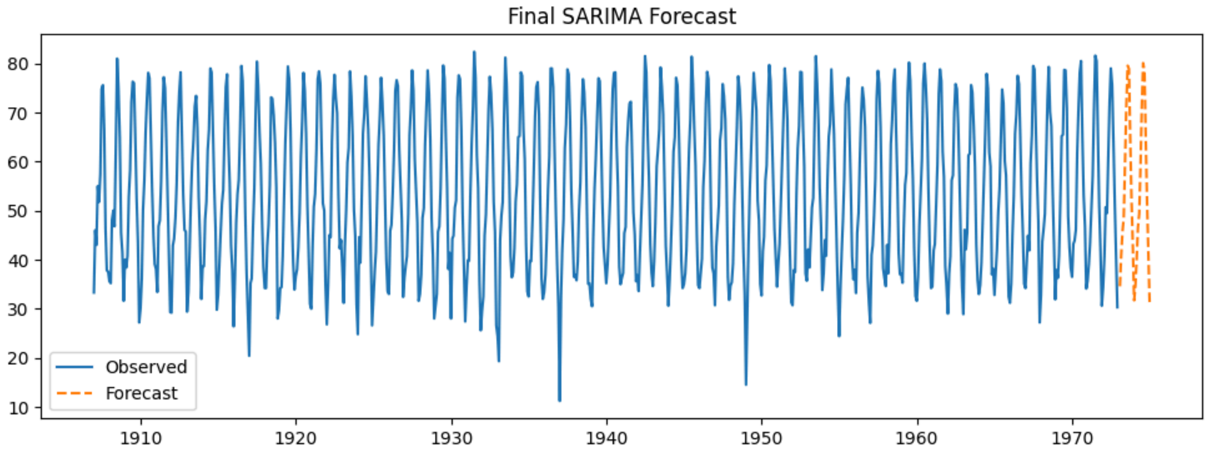
- We compare a handful of (p,d,q)×(P,D,Q,12) combinations by AIC.
- This sanity-check matches our earlier picks and gives us confidence in the final model.

Final Forecast & Evaluation

In [6]:

```python
# forecast next 24 months
fcast = auto.predict(n_periods=24)
future = pd.date_range(monthly.index[-1]+pd.offsets.MonthBegin(),
                       periods=24, freq='ME')
plt.figure(figsize=(12,4))
plt.plot(monthly.temp, label='Observed')
plt.plot(future, fcast,  label='Forecast', linestyle='--')
plt.legend(); plt.title('Final SARIMA Forecast');
```

```
/Users/mchildress/ts_basics/time_series_basics/lib/python3.10/site-packages/
sklearn/utils/deprecation.py:151: FutureWarning: 'force_all_finite' was rena
med to 'ensure_all_finite' in 1.6 and will be removed in 1.8.
  warnings.warn(
```



Discussion

- We project two years ahead with our SARIMA model.
- Confidence bands (not shown) will widen over forecast horizon.

- In practice we’d back-test on hold-out sets to ensure robust performance.