CURIOSA

219. Rotations. The circles in Fig. 1 represent eight discs of equal size, all but A remaining fixed in position. A rotates clockwise without slipping about the other discs until it comes back to its initial position, keeping contact with at least one of the other discs throughout. How many revolutions has it made about its own axis?

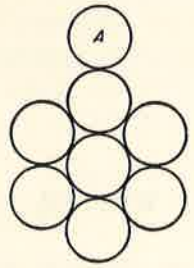

Fig. 1

Explanation: Suppose the disc A to rotate about B, a disc of the same size. Consider the point P. Clearly, that point will reach the position Q, with the circle A in the dotted position below. By then, the halfway point A has made one complete revolution about its own axis. The number of revolutions for a complete circuit is then 2.

When, in the first figure, A first touches the first circle, it has rolled about an arc of 60 degrees, one-sixth of the way around a circle, hence one-third of a turn about its own axis. A traverses 12 such arc segments. Hence the total number of turns about its own axis is 4.

A more effective way of regarding this type of rotation is the following: In Fig. 2 suppose A rolls along the straight line to the dotted position on the right, where the distance RR' is equal to the circumference of B. Then suppose that the line RR' with the circle A rigidly attached and tangent to it at R' be turned about the circle B until A gets back to its initial position. This adds one more revolution.

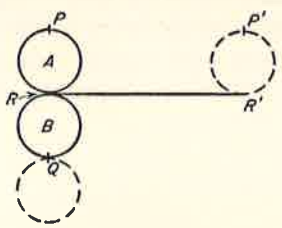

Fig. 2

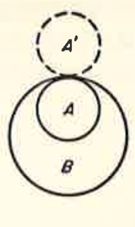

Fig. 3

From this point of view it should be a simple matter to answer the following questions:

1. In Fig. 3, A, with the radius a, rolls clockwise within the rigid circle B, with radius b, until A returns to its initial position. By then, how many revolutions has A made about its own axis?
2. Suppose A is fixed and B rotates. How many revolutions does B make by the time it returns to its initial position?
3. Suppose A is external to B, in the position A', and one rotates about the other. How many revolutions about its own axis does that make?

HARRY LANGMAN

PRINCE RUPERT'S PROBLEM AND ITS EXTENSION BY PIETER NIEUWLAND

BY D. J. E. SCHREK

IN SEVERAL well-known German works[1] a note is to be found on the following problem:

"To perforate a cube in such a way that a second cube of the same size may pass through the hole." All writers agree in that this problem was first proposed by Prince Rupert, Count Palatine of the Rhine and Duke of Bavaria, so that it is proper to name the problem after him.

Prince Rupert, Ruprecht, or Robert (1619–1682), is well known in the history of the world; his life is described in many standard biographical works[2] and numerous monographs on his life and letters have been published.[3]

He was the son of Frederick V, Elector Palatine and surnamed the Winter King, because of his tragical ephemeral reign as king of Bohemia. His mother was Elizabeth, daughter of James I of England, so that he was a nephew of Charles I. He was educated in the Netherlands and from his will we learn that he possessed a house near Rhenen in that country. In the Civil War he was one of Charles's most loyal supporters and one of the bravest. Until after the battle of Naseby (1645), he was commander of the Royalist forces and, by his boldness and recklessness, the dread of the parliamentary troops. Nor did he give up the struggle after Charles's defeat and execution. For several years as a naval commander he combated Cromwell's fleet. After the Restoration (1660) Rupert came back to England. By this time he had become quite an Englishman in life and thought and served his country again in the Anglo-Dutch wars. Prince Rupert took a great interest in scientific research. In after years he labored at his own forge and in his own laboratory at Windsor Castle, of which he was appointed governor.

[1] *Encyklopädie der Mathematischen Wissenschaften*, v. III, 1, 2, p. 1133–1134. M. Cantor, *Geschichte der Mathematik*, v. 3, p. 25. Max Simon, *Über die Entwicklung der Elementargeometrie im XIX. Jahrhundert*, Leipzig, 1906, p. 213.

[2] *Dictionary of National Biography*, v. 49, p. 405–417.

[3] Among them Eliot Warburton, *Memoirs of Prince Rupert and the Cavaliers*, Paris, 1849, may be recommended.

Some of Rupert's inventions and discoveries were improvements of war material, as is expressed by the writer of this funeral ode:

"Thou prideless thunderer, that stooped so low,
To forge the very bolts thy arm should throw,
Whilst the same eyes great Rupert did admire
Shining in fields and sooty at the fire:
At once the Mars and Vulcan of the war."

On the other hand, he applied himself to purely scientific research. He invented an alloy, still called Prince's metal, and studied the curious properties of quickly cooled drops of glass ("glass tears," Rupert drops).

In 1660 "The Royal Society of London for the Promotion of Natural Knowledge," with the proud device "Nullius in Verba," By No One's Words (we swear), had been founded by Charles II. Prince Rupert was among the first elected Fellows; the Charter-book bears his signature on the first page, together with that of his royal cousin,[4] and we are told that Prince Rupert took much interest in some branches of science and in the work of the Society.[5]

Rupert's biographies tell us that he promulgated his discoveries and inventions in the *Philosophical Transactions of the Royal Society*. It is not sure whether he did so with the present problem. I have not been able to find it in the *Transactions* of the years 1665 up to 1682, the year of Rupert's death. Having applied to the Hon. Secretary of the Royal Society I learned that in spite of much trouble, his staff had not been able to trace any information about this question.

The problem was taken up by no less an authority than John Wallis (1616–1703). In his "De Algebra Tractatus"[6] he tells us under the heading "Perforatio Cubi, alterum ipsi aequalem recipiens":

"Rupertus, Princeps Palatinus, dum in Aula Regis Angliae Caroli II versabatur, Vir magno ingenio & sagacitate, affirmavit aliquando, omnino fieri posse (et posito pignore se facturum suscepit) ut, aequalium cuborum, per foramen in eorum altero factum, transeat alter. Quod & ipsum, audio, praestitisse. Id quomodo fiat, jam sumus

[4] *The signatures in the first Journal-book and Charter-book of the Royal Society, being a facsimile of the signatures of the founders, patrons, and Fellows of the Society from the year 1660 down to the present time*, London, 1912.

[5] *The Record of the Royal Society of London*, 4th ed., London, 1940, p. 21.

[6] *Opera Mathematica*, v. II, Oxoniae 1693, Caput CIX, p. 470–471.

ostensuri"[7] and he enunciates the problem in the following words: "Problema. Duorum cuborum inter se aequalium alterum sic Excavare, ut per eum transeat reliquus Integer."[8] Some time afterwards the same question was studied in another part of the British Isles. Gibson in his historical work on Cork[9] quotes an older similar work[10] in which the author mentions one Philip Ronayne, who lived near Cork in the beginning of the 18th century. The Ronaynes were an old Cork family with a very long history, which has been treated by William John Knight LL.D., in a series of articles.[11] We learn that Philip Ronayne has distinguished himself by several essays in the most sublime parts of mathematics. He wrote a *Treatise of Algebra* in two books, the second edition of which was printed in London in 1726. A copy of that edition is in the British Museum; it is very rare now. Smith says that it passed through several editions and that it was "much read and esteemed by all the philomaths of the present time."

Now Smith affirms emphatically that Philip Ronayne solved the problem of the interpenetrating cubes and that he demonstrated its possibility both geometrically and algebraically. Moreover, Smith declares that he saw a model of the cubes, made by a Daniel Voster, a Dutchman, who kept a school in Cork. This scanty information is all we know about it.[12]

Half a century later we find the problem again in a work by the well-known Dutch mathematician Jan Henri van Swinden (1746–1823).[13]

Van Swinden refers to both Rupert and Ronayne. In modern times the problem reappears from time to time in mathematical journals. Apart from Hennessy's already mentioned article, we find it

[7] "Rupert, Prince Palatine, staying in the palace of Charles II, King of England, a man of great wit and boldness, affirmed one day that it was quite possible (and having laid a wager undertook making) that, a hole being made in one of two equal cubes, the other might pass through; which I learn he has achieved. In which way this is done we shall show."

[8] "Problem. To perforate one of two equal cubes in such a way that the intact cube may pass through."

[9] Gibson, *History of the County and City of Cork*, v. II, p. 408.

[10] Smith, *History of Cork*.

[11] *Journal of the Cork Historical and Archaeological Society*, v. 22 and 23, particularly v. 23, p. 99.

[12] See H. Hennessy "Ronayne's Cubes," *Phil. Mag.*, Series 5, v. 39, January–June 1895 p. 183–187.

[13] J. H. van Swinden, *Grondbeginsels der Meetkunde*, 2nd ed., Amsterdam, 1816, p. 512–513. This work was highly valued in Germany, where it was re-edited, translated, and enlarged by C. F. A. Jacobi. This edition ordinarily quoted as *Van Swinden-Jacobi, Elemente der Geometrie*, is the best known abroad. In it the problem is on p. 394.

it an interesting note by two young German mathematicians.[14] Recently U. Graf inserted a fine picture of a model in a work on mathematical education that he wrote in collaboration with W. Lietzmann,[15] another in the *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht*[16] and treated the problem in a little book on recreational mathematics, *Kabarett der Mathematik,* which I have not been able to consult.

So far the original problem. But there is another side to the question. We shall see that not only a cube of *equal* size, but a somewhat *larger* one may pass through a hole in the other. This being the case, the question arises: which is the maximum size of the first cube? It was another Dutch mathematician, Pieter Nieuwland (1764–1794), who studied this problem, as we shall see in the last part of this paper.

Passing to the problem itself we shall find it useful to distinguish three different phases and to treat each of them separately:

1. The proof that a solution is possible *at all,* by considering a special solution of the case of equal cubes.
2. A further discussion of the general problem of the equal cubes.
3. The problem of a *larger* cube passing through the other, finding the edge of the largest possible cube and other computations.

I. The Possibility of Finding a Solution. The special case alluded to above is that of a movement of the solid cube along the diagonal PQ of the perforated, four of its edges being parallel to that diagonal. Then two faces of the solid cube are perpendicular to PQ, and if the perforated cube is projected on a plane perpendicular to P, the projections of those two faces are coinciding squares; on the other hand, this square is also the projection of the solid cube itself. The projection of the perforated cube on that same plane (isometric projection) is a regular hexagon, as is well known from elementary geometry and so the question is reduced to a problem of plane geometry: that of a square lying within a regular hexagon.

This is the way in which John Wallis[17] proceeds: he shows that the square can be easily constructed within the hexagon without any contact with the sides. There is, however, a second indirect method,

[14] F. Koch and J. Reisacher, "Die Aufgabe, einen Würfel durch einen andern durchzuschieben," *Arch. d. Math. u. Physik,* Series 3, v. 10, 1906, p. 335–336.

[15] W. Lietzmann and U. Graf, *Mathematik in Erziehung und Unterricht,* v. 2, p. 168.

[16] U. Graf, "Die Durchbohrung eines Würfels mit einem Würfel," *Zeitschr. f. math. u. naturw. Unterr.,* v. 72, 1941.

[17] John Wallis, *Opera Mathematica, loc. cit.,* v. II.

consisting in inscribing a square in the hexagon and showing that the side of that square is longer than the edge of the original cube. This way[18] is preferable in my view.

This plane geometry problem is an easy one indeed. Let the side of the hexagon be y, that of the square g, the edge of the cube a. Then (Fig. 1), DE being parallel to BC, we have:

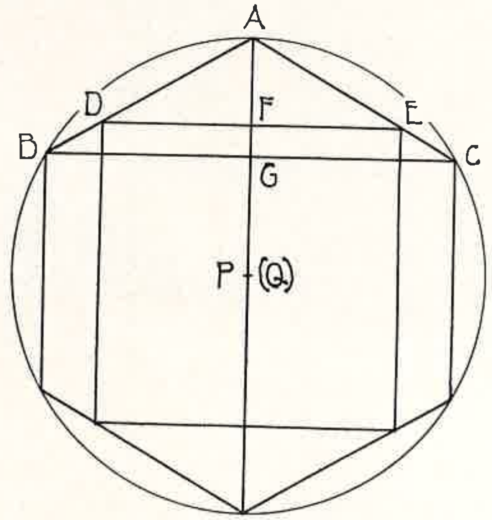

Fig. 1

$$DE:BC = AF:AG$$

$$g:y\sqrt{3} = (y - {}^1/_2 g):{}^1/_2 y,$$

whence

$$g = y(3 - \sqrt{3}) = 1.2679492y. \qquad (1)$$

Now BC, which is the diagonal of a base is also equal to $a\sqrt{2}$, hence

$$y\sqrt{3} = a\sqrt{2}. \qquad (2)$$

From (1) and (2) we deduce:

$$g = a(\sqrt{6} - \sqrt{2}) = 1{,}0352761a.$$

Consequently $g > a$. In other words: a square that *has* the side a may be freely constructed within the hexagon. Q.E.D.

[18] Koch and Reisacher, *loc. cit.*

II. A Further Inquiry about the Equal Cubes. Having proved the possibility of the perforation, we may now consider the problem in a more general manner and, literally and metaphorically, from another point of view. Let a be the edge of the square serving as the base of the perforated cube. Then draw two lines like AB, parallel to the same diagonal and at a distance of $^1/_2a$ from it (Fig. 2). This figure may be conceived as the horizontal projection of the perforated cube; it is clear, then, that ABC is the base of a right triangular prism and

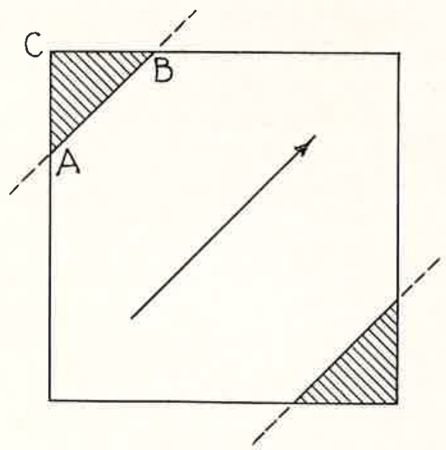

Fig. 2
Horizontal projection

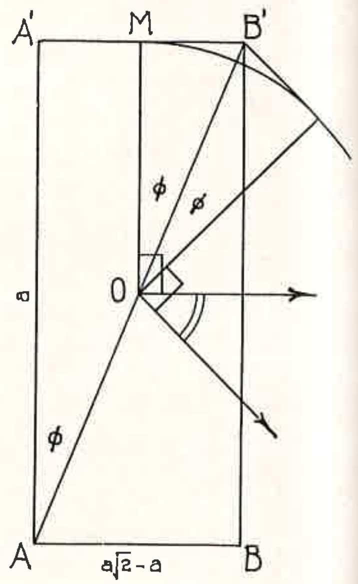

Fig. 3
Vertical projection

that $AB = a(\sqrt{2} - 1)$. If now the solid cube moves horizontally while the bases of both cubes remain in the same plane in the direction of the arrow, the two prisms will evidently be all that remains of the cube and it is easy to see that they do not cohere. We shall now prove that they do cohere if the solid cube passes in a *slanting* direction.

In fact, let $ABB'A'$ (Fig. 3) be the rectangle of which AB (Fig. 2) is the base and O the center of that rectangle.

Imagine a square (side $= a$) with the same center O and turning around O. The middle M of the upper side will then describe an arc

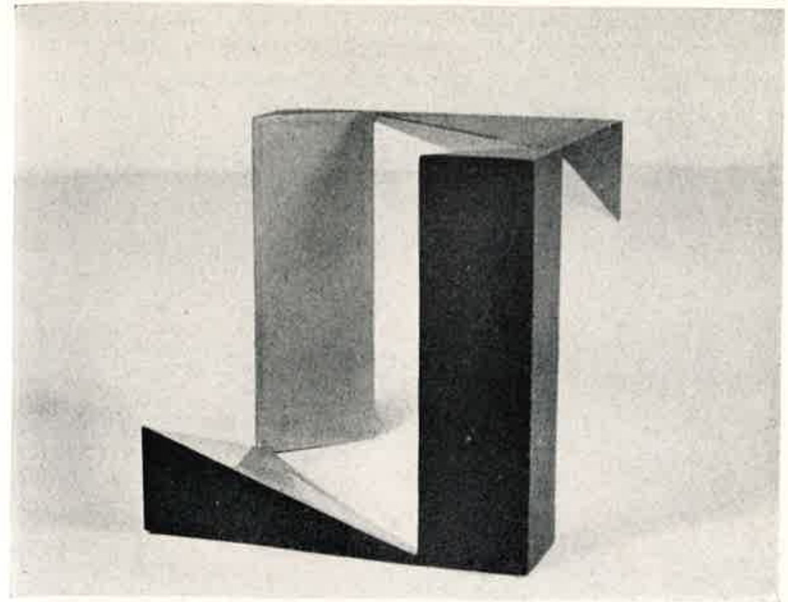

Fig. 4a

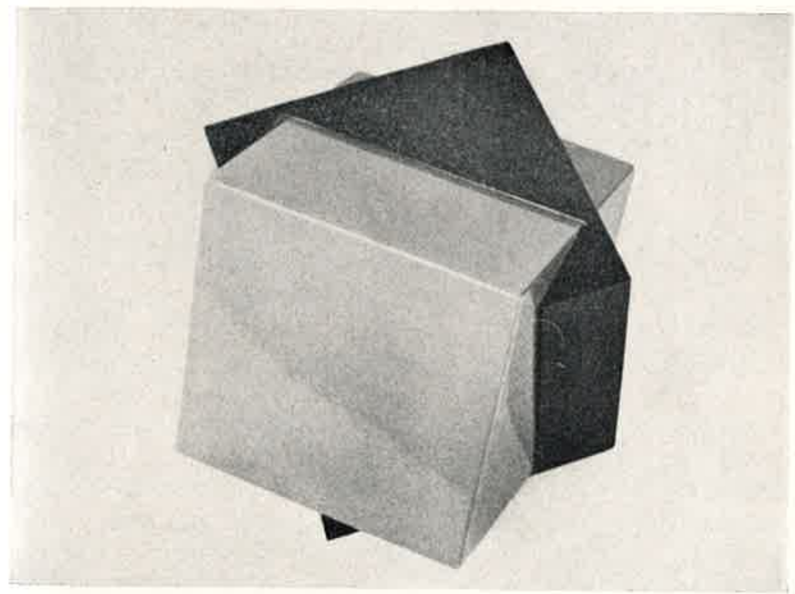

Fig. 4b

of a circle with center O and radius $OM = {}^1/_2 a$, and the upper side (that is, the edge of the solid cube) will remain tangent to that circle. At last that tangent will pass through B' a second time. In the meantime the two prisms remain connected and the remnant of the perforated cube constitutes a single solid (Figs. 4a and 4b). The angle φ in Fig. 3 may be computed without difficulty. For evidently tan $\varphi = MB'/MO = \sqrt{2} - 1 = 0{,}4142136$, so that $\varphi = 22° 30'$, a result immediately to be found without a table by means of a regular octagon.

Utrecht, Holland

(*To be concluded*)

BOOK REVIEWS

Eidted by Carl B. Boyer

Brooklyn College

THE MATHEMATICAL BASIS OF THE ARTS. By Joseph Schillinger New York, Philosophical Library, 1948. x + 696 p. Price, $12.00.

This is a companion work to the late Joseph Schillinger's two-volume *Schillinger System of Musical Composition*, Carl Fischer, New York, 1946. The relationship between the *System* and the *Basis* (for convenience the two works will be designated in this way) is rather curious. Of the 24 chapters of the *Basis*, 5 have been taken, with minor omissions, from the *System*. But 4 of these chapters deal with ideas and procedures that are not peculiarly applicable to music. They are "Nature of Esthetic Symbols," "Quadrant Rotation," "Coordinate Expansion," and "Composition of Density."

It is true that most of the *System* is extremely technical. But the single chapter, "Production of Music," reprinted in the *Basis*, by no means exhausts the musical material that might be understandable and interesting to the general reader. Parts of the several chapters on melody and musical scales in the *System* might well have been included also, and the three chapters on "Semantic (Connotative) Composition" in Book XI are really refreshing.

On the other hand, all of the mathematical techniques fundamental to the *System* are explained in far greater detail in the *Basis*, so that the musician who had wrestled with the *System* might find a measure of elucidation in the new book. The *Basis* also contains a few new techniques, and an extensive application of all the mathematics to visual art to straight lines and circular arcs in particular.

Without question, Schillinger's greatest asset was his ability to conceive of the combining of many smaller units into gigantic wholes. (He had even planned to write 20 books, "scientific, technical, and popular," upon his theories, but lived only long enough to complete two of them.) Not only has he listed many existing or potential art forms based upon sight and sound and even upon smell, taste, and touch, but he showed also correlations between the elements proper to valid in the sense indicated above. This is readily done by means of simple counter examples.

(13). (*a*) Take $f = t$, $g = t^2$, and let R be the interval from 0 to 1 on the t-axis. Then

$$\int_R f^2 g^2\, dR = 1/7 \quad\text{and}\quad \int_R f^2\, dR \cdot \int_R g^2\, dR = 1/15.$$

(*b*) Take $f = \sin t$, $g = \cos t$, and let R be the interval from 0 to π. Then

$$\int_R f^2 g^2\, dR = \pi/8 \quad\text{and}\quad \int_R f^2\, dR \cdot \int_R g^2\, dR = \pi^2/4.$$

Neither $i\ m\ s \leq m\ i\ s$ nor $i\ m\ s \geq m\ i\ s$ is valid.

(14). (*a*) Take f and g as in (*a*) above and let R again be the interval (01). Then

$$\int_R f^2 g^2\, dR = 1/7 \quad\text{and}\quad [\int_R f\, dR]^2 \cdot [\int_R g\, dR]^2 = 1/36.$$

(*b*) Take f and g as in (*b*) above and let R be the interval from 0 to $\pi/2$. Then

$$\int_R f^2 g^2\, dR = \pi/16 \text{ and } [\int_R f\, dR]^2 \cdot [\int_R g\, dR]^2 = 1.$$

It follows that neither $i\ m\ s \leq m\ s\ i$ nor $i\ m\ s \geq m\ s\ i$ is valid.

(15). The examples used in (14), (*a*) and (*b*), show that neither $i\ m\ s \leq s\ i\ m$ nor $i\ m\ s \geq s\ i\ m$ is valid.

(34). The same examples lead to the conclusion that neither $m\ i\ s \leq m\ s\ i$ nor $m\ i\ s \geq m\ s\ i$ is valid.

(45). (*a*) Take $f = \sin t$, $g = \cos t$, and R the interval from 0 to $\pi/2$. Then

$$[\int_R f\, dR]^2 \cdot [\int_R g\, dR]^2 = 1 \text{ and } [\int_R fg\, dR]^2 = 1/4;$$

(*b*) Take $f = t$, $g = t^2$ and let R be the interval (01). Then

$$[\int_R f\, dR]^2 \cdot [\int_R g\, dR]^2 = 1/36 \text{ and } [\int_R fg\, dR]^2 = 1/16.$$

It follows that neither $m\ s\ i \leq s\ i\ m$ nor $m\ s\ i \geq s\ i\ m$ is valid.

It is characteristic of a negative result such as has here been established, that it raises a more fundamental question than it has answered. Observing the fact that for the case (45), the examples used for (*a*) and (*b*) are those used for (*b*) and (*a*), respectively, in some of the other cases, we are led to ask for conditions on the functions f and g as well as on the domain R, sufficient, or necessary, or necessary and sufficient for the validity of the inequalities (13), (14), (15), (34), and (45).

SWARTHMORE COLLEGE

PRINCE RUPERT'S PROBLEM AND ITS EXTENSION BY PIETER NIEUWLAND

BY D. J. E. SCHREK

(*Concluded from Page 80, March–June, 1950*).

Summary. There is an infinitude of directions in which the solid cube may be passed through the perforated one. Using the term inclination for the angle between this direction and the horizontal plane, we may say that its limits are 0° and 45° (these values not included). In the special Case I the inclination is 35° 15′ 52″, being the angle of a diagonal of a cube with a face, and in fact 0 < 35° 15′ 52″ < 45°. Once more we see that Wallis's solution refers to a single case out of an infinitude of cases.

The general case of the two equal cubes having been exposed, a special question remains, which Hennessy[19] has paid much attention to. The remains of the mutilated cube consist of the above-mentioned two prisms and two triangular parts, called by him the *flanges*. These flanges have sharp edges (*knife edges*, Hennessy calls them) at the inside and are connected with the prisms by four junctions. It is the thickness of these junctions that Hennessy is interested in. It is clear that in a model made of some material these junctions are the weakest parts and it is but natural to ask: *What* should the inclination be, that the thickness of the flange at the points of junction be as great as possible?

In Fig. 5 this maximum thickness at the point of junction is $B'C$. In fact, the segment cut off from BB' by the tangent to the arc of the circle will be the greatest possible if the point C of intersection of BB' and the arc is the point of contact itself. But then the solution is simple, for we have:

$$\sin\theta = \frac{OD}{DC} = \frac{{}^1/_2 a(\sqrt{2}-1)}{{}^1/_2 a} = \sqrt{2} - 1 = 0.4142136,$$

hence $$\theta = 24°\ 28'\ 12''.$$

Hennessy finds the same answer in quite another way—a rather cumbersome way—by means of the Calculus.

[19] Hennessy, *loc. cit.*, p. 186–187.

III. The Problem of the Larger Cube. Up to now the solid cube had the same size as the excavated one: It varied in *position*, but not in *size*. But now all that has been found suggests a new question: Can the solid cube be *larger* than the perforated and, if so, how much larger? It can indeed. If we enlarge the distance between the parallels in Fig. 2 and compare Figs. 2 and 3 with Fig. 6, the latter is altered in two respects. On the one hand AB is no longer $a\sqrt{2} - a$, but $a\sqrt{2} - x$, where x is the edge of the solid cube. On the other hand, $AB' = x$, for in order to get the largest cube the thickness of the junction must be zero. In Fig. 6 we have:

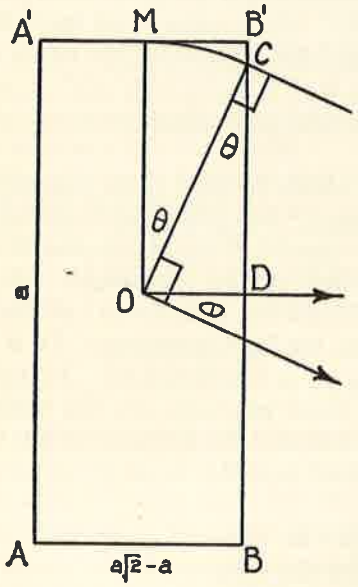

Fig. 5

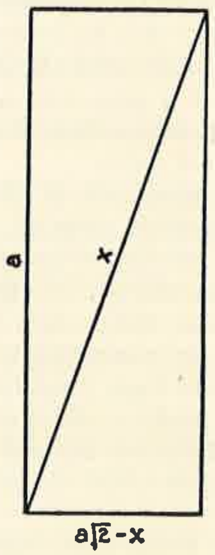

Fig. 6

$$x^2 = a^2 + (a\sqrt{2} - x)^2$$
$$0 = a^2 + 2a^2 - 2ax\sqrt{2},$$

hence $$x = {}^3/_4 a\sqrt{2} = 1{,}060660a.$$

Needless to say that this solution is only theoretical; realized in some material the junction must have some thickness.

All writers on the subject agree that this maximum value was first found by Pieter Nieuwland (1764–1794)[20]. Born near Amsterdam as the son of a carpenter, he was distinguished by his precocity. His father, though a humble artisan, had some knowledge of geometry and taught the boy the Elements of Euclid. An Amsterdam gentleman who, during his stay in the countryside, was struck by the talents of the child, enabled him to study arts as well as science; he was gifted for both and his knowledge of foreign languages, classical and modern, was amazing. Later on he applied himself especially to mathematics, physics, and astronomy and became the beloved pupil of J. H. van Swinden; both were active members of "Felix Meritis," the then prominent Amsterdam Society "for the promotion of Arts

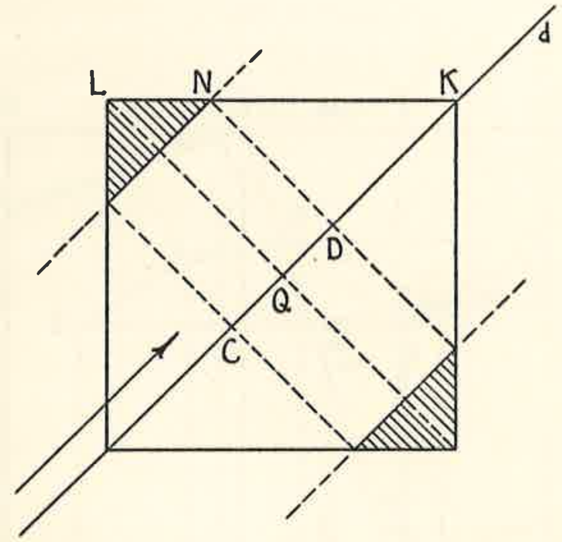

Fig. 7

and Science." After having been lecturer in nautical science in Amsterdam, he was appointed Professor in Leyden University in 1793, but died a year later.

Van Swinden executed Nieuwland's will as far as science was concerned. Among his scientific papers is the correct solution of our problem, which Van Swinden inserted into his geometry[21].

[20] Most biographies of this remarkable poet and scientist are in Dutch. The reader may consult, however, *Nouvelle Biographie Generale*, v. 38. Paris, 1862, Col. 70–71. *Biographie Universelle*, v. 4, Paris, 1838, p. 386–387.

[21] Van Swinden, *loc. cit.*, p. 608–610. Van Swinden-Jacobi, *loc. cit.*, p. 542.

What is in this case the inclination of the maximum cube? In order to answer this and similar questions we use Figs. 7 and 8, Fig. 8 being a diagonal section of the perforated cube along d. Clearly ΔEGH is the section of a "flange" and E the middle of a "knife edge," as Hennessy calls them. If we substitute $^3/_4 a\sqrt{2}$ for x we find for the lengths of the lines the values indicated in Fig. 8.

For the inclination we have in this case:

$$\tan \psi = \frac{EF}{CF} = {}^1/_4\sqrt{2},$$

hence

$$\psi = 19° \, 28' \, 16''.$$

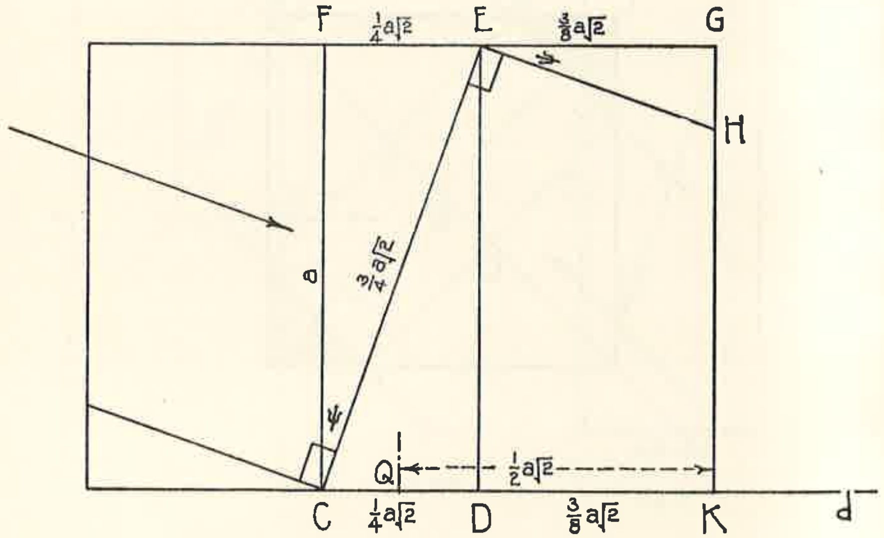

Fig. 8
(Vertical)

Koch and Reisacher conclude their note with the important remark (without proof), that the edges of the cube are divided in *rational proportions.*

Indeed they are, as may easily be proved.

(*a*) *Horizontal Edges.* It is clear (Figs. 7 and 8) that:

$$LN:NK = QD:DK = {}^1/_8 a\sqrt{2}:{}^3/_8 a\sqrt{2},$$

so that

$$LN:NK = 1:3.$$

(*b*) *Vertical Edges.* From the similar triangles CFE and EGH we deduce:

$$CF:FE = EG:GH,$$
$$a:{}^1/_4 a\sqrt{2} = {}^3/_8 a\sqrt{2}:GH,$$

hence

$$GH = {}^3/_{16}a,$$

so that

$$GH:HK = 3:13.$$

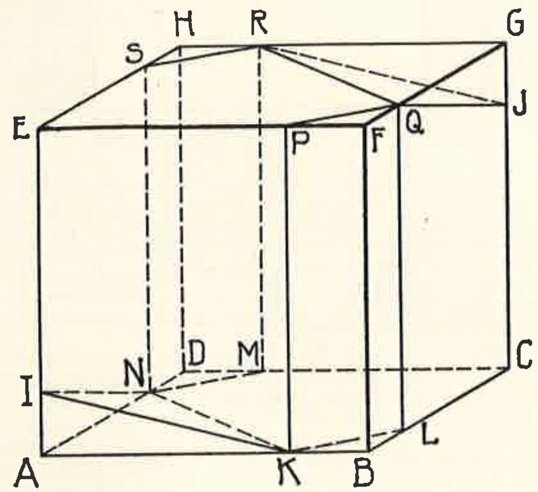

Fig. 9

These answers enable us to make an exact drawing of the perforated cube (Fig. 9). They also make possible drawing a layout. In Fig. 10 the remaining parts of the faces of the cube are shaded; the faces of the whole inside are left out.

The question may be raised if special problems like this are worthy of study. No doubt some people will answer in the negative. Van Swinden[22] himself says about it: "The study of regular polyhedra may sometimes give rise to quaint problems," and even more destruc-

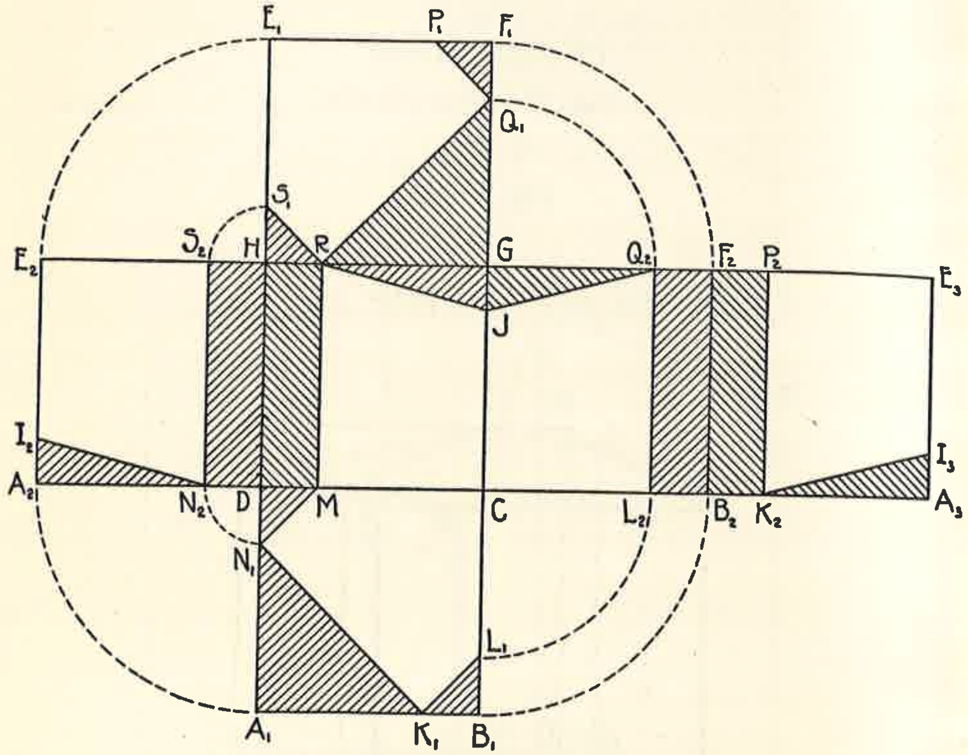

Fig. 10

tive is Montucla's criticism in his *History*[23]. He uses a metaphor, saying: "Cette théorie des corps réguliers pourroit être aujourd'hui comparée à ces anciennes mines, où non seulement on ne fouille plus, mais dont le produit a presque entièrement perdu sa valeur. Les

[22] Van Swinden, *loc. cit.*, p. 512.
[23] Montucla, *Histoire des Mathematiques*, v. 1, Paris, 1758, p. 220.

Géomètres la regardent tout au plus comme un objet d'amusement, ou capable de fournir quelque problème singulier," and in a footnote he adds: "Un problème de ce genre est celui de percer un cube, de manière qu'un autre cube égal puisse passer au travers; il a été proposé et résolu par le Prince Rupert, Frère [Cousin] de Charles II, Roi d'Angleterre, et l'on peut en voir la solution dans Wallis, Tome II." Nevertheless, the present writer believes that a survey of this "objet d'amusement" may be of some interest and has been encouraged by correspondents who spoke of it as an "intriguing question" and "a charming problem."

In conclusion, I wish to thank all those who by their help and valuable hints have assisted me. Th. A. Conroy, M.A., librarian of University College, Cork, took much trouble to provide me with information about Ronayne and the Hon. Secretary of the Royal Society and his staff did the same about Prince Rupert. I am also indebted to Dr. W. Lietzmann (Göttingen) and Dr. U. Graf (Wuppertal-Vohwinkel) for their permission to borrow the pictures of the interpenetrating cubes from their work *Mathematik in Erziehung und Unterricht* and to the staff of the Library of Utrecht University for their valuable assistance in every respect.

UTRECHT

CURIOSA

243. A Story for the Nursery. It seems that Noah appointed a monkey to be lookout from the crow's nest of the ark. From this point of vantage the monkey could see the pairs of animals, Mr. and Mrs. Elephant, Mr. and Mrs. Giraffe, and many other couples taking a stroll in the clear sunlight on the spacious promenade deck of the ark. Officer Monk noticed that, with one exception, the wedded pairs were accompanied by lively offsprings. Only Mr. and Mrs. Adder were not blessed with children. When Officer Monk asked Mr. Adder about this anomaly, on the side, the answer was "Since we are *adders* we cannot *multiply*."

Sometime later the ark floated through a lot of excellent driftwood and fine lumber. Quite a goodly quantity of this debris was hauled on board and from it many useful pieces of furniture were manufactured. In due time Officer Monk noticed with unrepressed astonishment that, on the occasion of the conventional stroll, Mr. and Mrs. Adder were accompanied by several baby adders. The Officer left his perch and hastened to make inquiry of Mr. Adder concerning this seeming miracle. "Oh," explained Mr. Adder, "you made us a log table and now we can multiply."

HORACE S. UHLER