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CORRECTION TO "PUISEUX'S THEOREM REVISITED"

P.M. COHN
Department of Mathematics, University College London, Gower Street, London WC1E 6BT, United Kingdom

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I am indebted to A. Benhissi for drawing my attention to a gap in the proof of [2, Lemma 2]. This proof is incomplete and should be amended as follows:

The first sentence in line 2 of the proof of [2, Lemma 2] should read:

We shall use induction on n.

Replace the last 6 lines of the proof by:

this only leaves the case $C = N + Ax$, where N is nilpotent. Let the minimal equation of C be

$$C^r + a_1 C^{r-1} + \cdots + a_r I = 0 .$$

Since C is nilpotent mod x, each a_i is divisible by x; let k_i be the exponent of the precise power of x dividing a_i and write $x = y^h$, where h is the least rational number such that $hk_i \geq i$, thus $h = \max\{i/k_i \mid i = 1, 2, \ldots, r\}$. If we write $a_i = y^i b_i$, then $b_i \in k[\![y]\!]$ and for some i, $b_i \not\equiv 0 \pmod{y}$.

We now transform C as follows. Let u_1 be any vector in n-space over $k((y))$ such that $u_1, Cu_1, \ldots, C^{r-1}u_1$ are linearly independent and define $u_2, \ldots, u_r$ by

$$Cu_i = yu_{i+1} \quad (i = 1, 2, \ldots, r-1) . \tag{1}$$

It follows that $Cu_r = y^{-1}C^2 u_{r-1} = \cdots = y^{1-r}C^r u_1 = -y^{1-r}(a_1 y^{r-1} u_r + a_2 y^{r-2} u_{r-1} + \cdots a_r u_1)$, hence

$$Cu_r = -y(b_1 u_r + b_2 u_{r-1} + \cdots + b_r u_1) . \tag{2}$$

If $r < n$, we can choose a vector v such that $u_1, \ldots, u_r, v, Cv, \ldots, C^{r-1}v$ are linearly independent, and writing $u_{r+1} = v$, we can repeat the above process. After $[n/r]$ steps we have a basis of n-space and relative to this basis all the entries of C are multiples of y, by (1), (2). Thus $C = yD$ and the minimal equation for D is

$$D^r + b_1 D^{r-1} + \cdots + b_r I = 0 .$$

Here not all the coefficients are divisible by y, so D is not nilpotent and now an induction on n completes the proof.

I am also grateful to M.Ojanguren for pointing out that Puiseux's theorem fails to extend to finite characteristic. He observes that in characteristic p the equation $t^p + tx + x = 0$ in t over the rational function field $k(x)$ has no solution that is expressible as a fractional power series in x. See also [2, Chapître V, Corps commutatifs, §4, Exercice 2, p.AV. 143f].

References

[1] N. Bourbaki, Algèbre (Hermann, Paris, 1959).
[2] P.M. Cohn, Puiseux's theorem revisited, J. Pure Appl. Algebra 31 (1984) 1–4.