Scanner

Selected Fun Problems of the ACM Programming Contest SS25

04./05.06.2025

David Knöpp

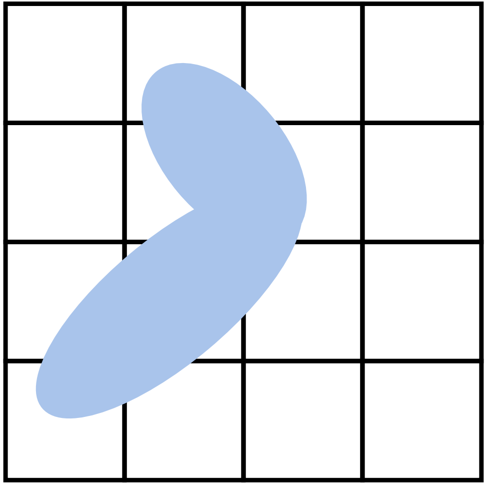
About questions

- Please ask questions if you don’t understand an important aspect
- Otherwise, please save your questions for after the presentation

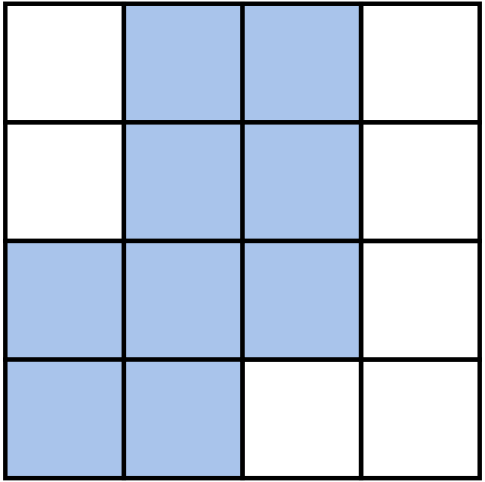
Contents

1 Problem Definition

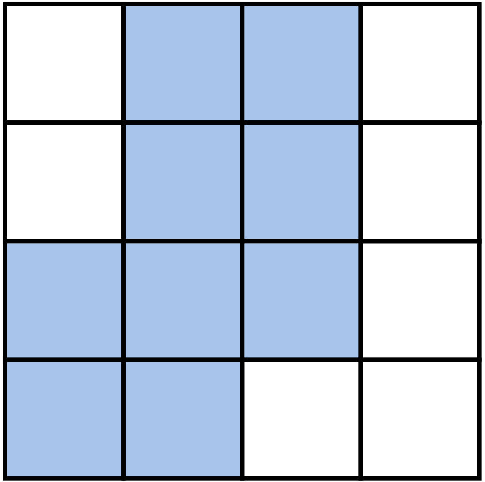
Result Matrix



Result Matrix



Result Matrix

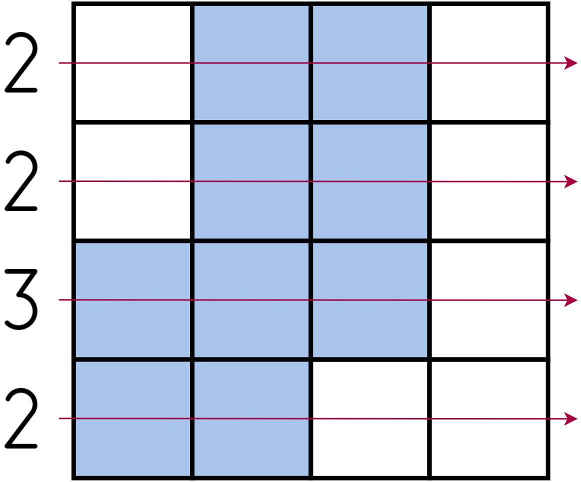


```
. # # .
. # # .
# # # .
# # . .
```

This is our actual output.

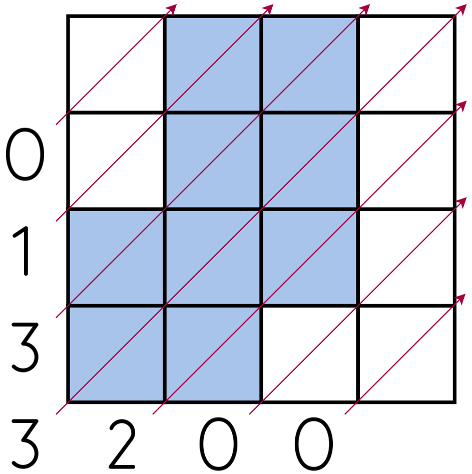
But what is our input?

Getting the input



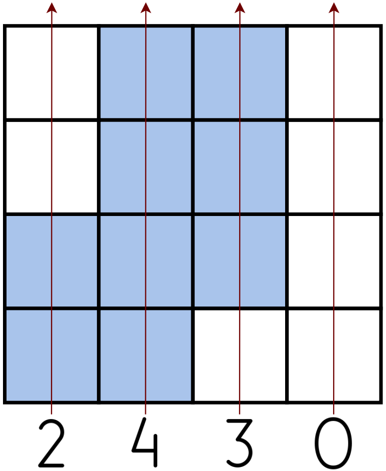
2 2 3 2

Getting the input



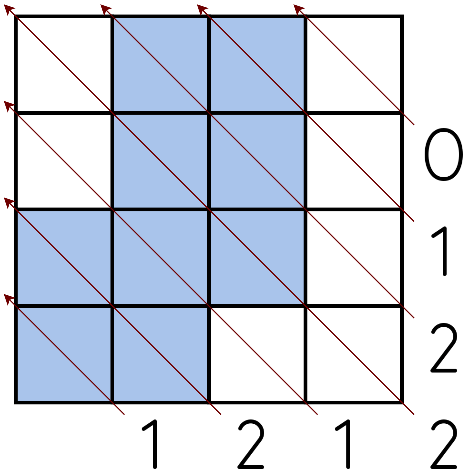

```
2 2 3 2
0 1 3 3 2 0 0
```

Getting the input




```
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
```

Getting the input




```
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0
```

Getting the input

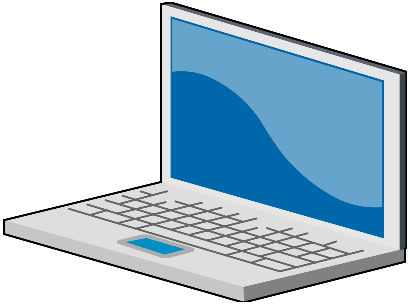
The number at the top represents the number of matrices that will follow.

In our case, it's just one.

```
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0
```

And that's our input!

Live Demo

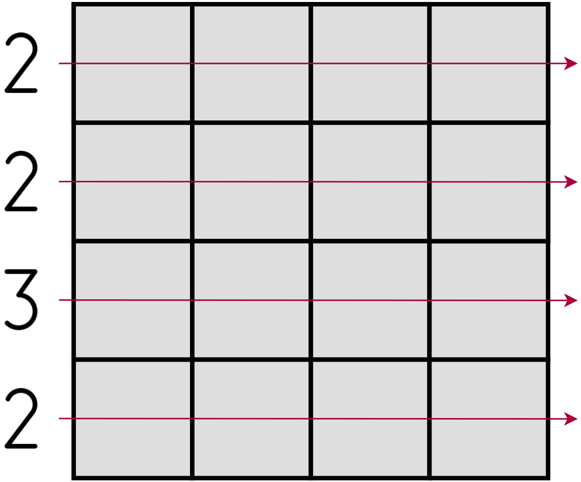


2 Solution

Tools

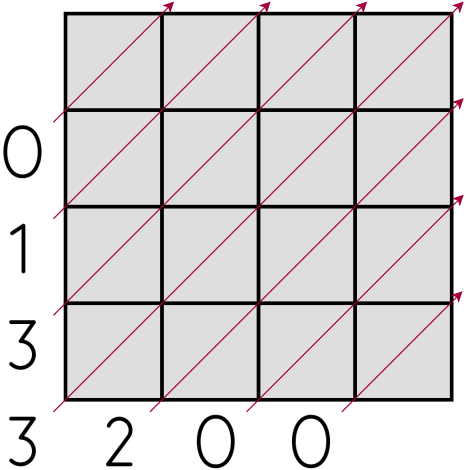
- Python
  - Personal experience
- Numpy
  - Convenient and efficient matrix operations
  - Personal experience

Example




```
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0
```

Example



1

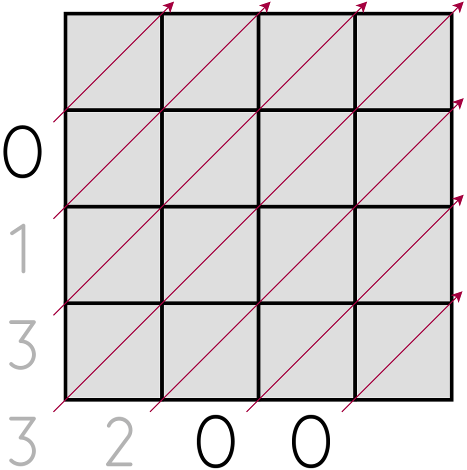
2 2 3 2

0 1 3 3 2 0 0

2 4 3 0

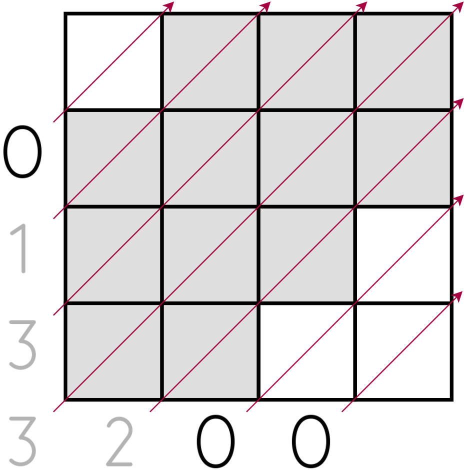
1 2 1 2 2 1 0

Example



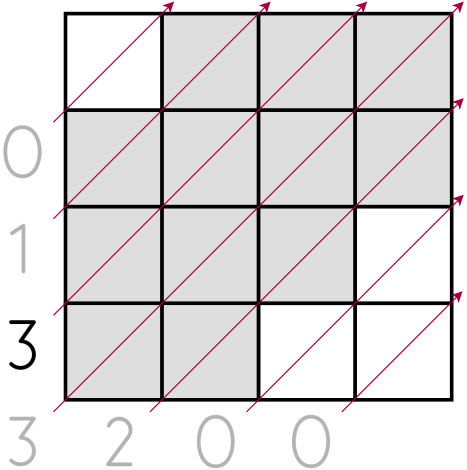

```
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0
```

Example



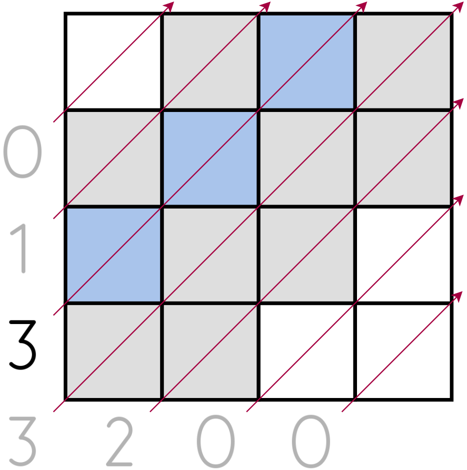

```
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0
```

Example



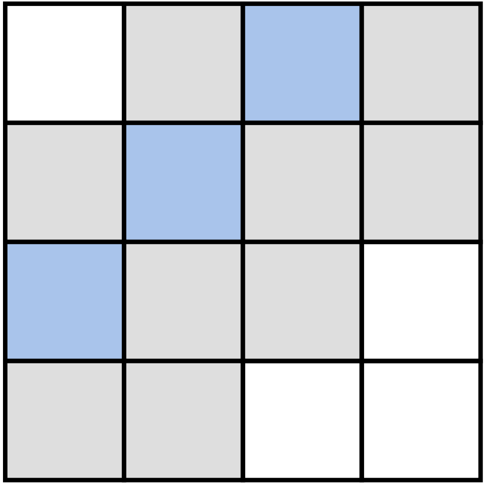
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0

Example



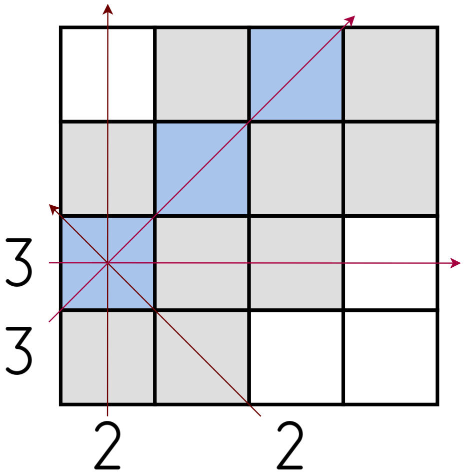

```
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0
```

Example



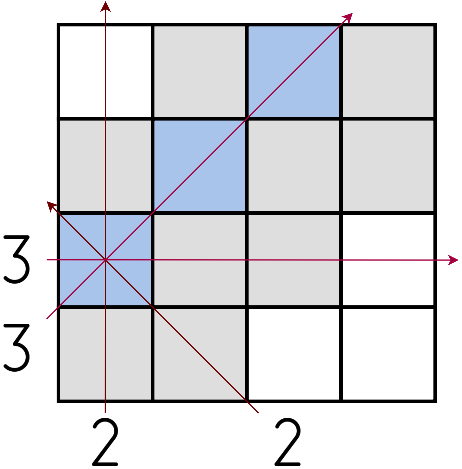
```
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0
```

Example



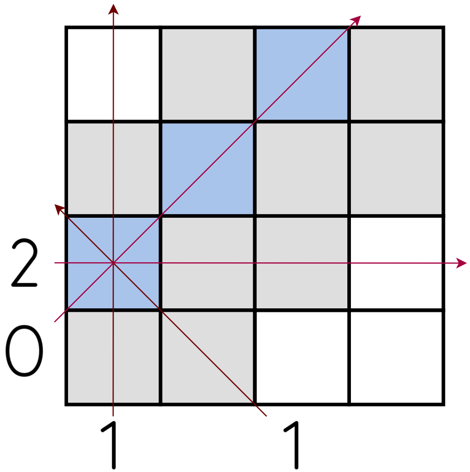
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0

Example



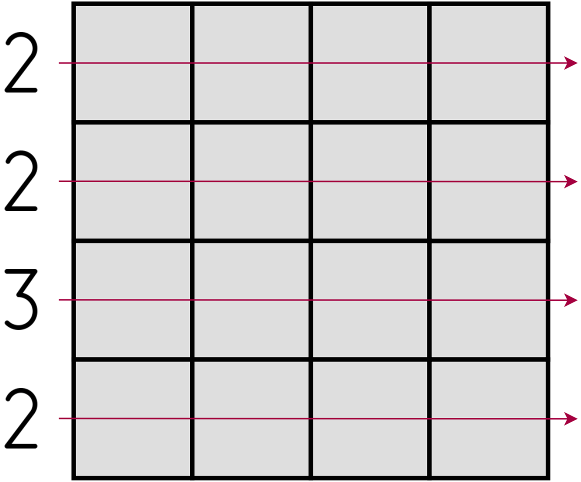
1
2 2 3 2
0 1 3 3 2 0 0
2 4 3 0
1 2 1 2 2 1 0

Example

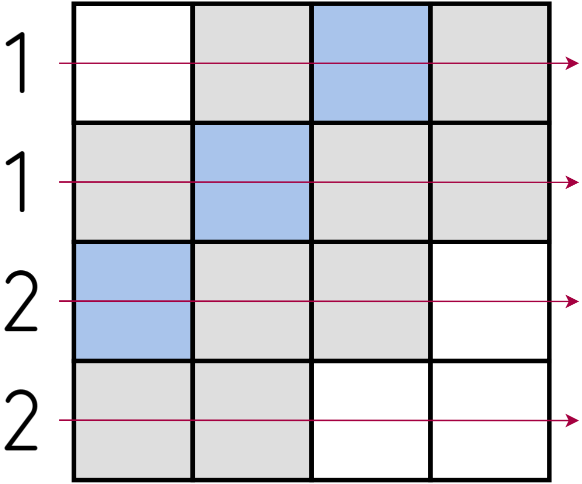


1
1 1 2 2
0 1 0 3 2 0 0
1 3 2 0
1 1 1 1 2 0 0

Example

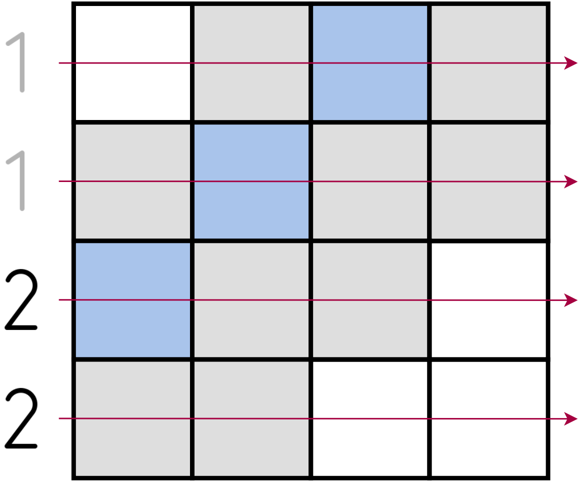


Example



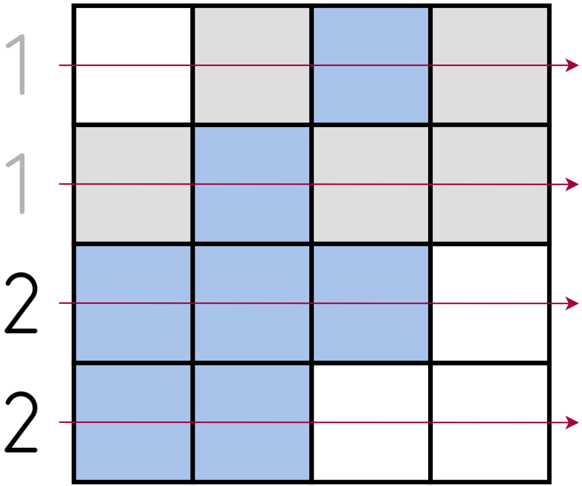
1
1 1 2 2
0 1 0 3 2 0 0
1 3 2 0
1 1 1 1 2 0 0

Example



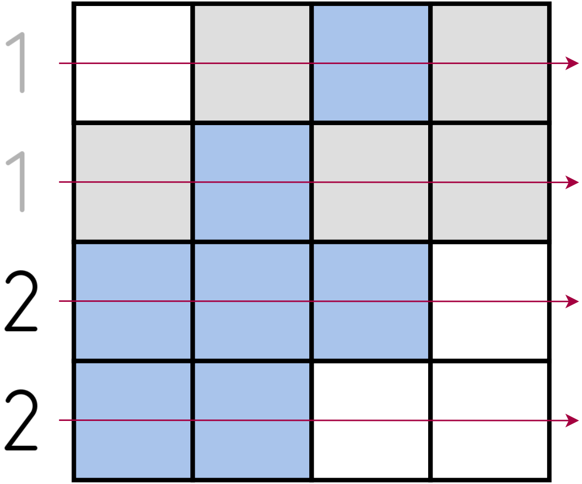

```
1
1 1 2 2
0 1 0 3 2 0 0
1 3 2 0
1 1 1 1 2 0 0
```

Example




```
1
1 1 2 2
0 1 0 3 2 0 0
1 3 2 0
1 1 1 1 2 0 0
```

Example



1

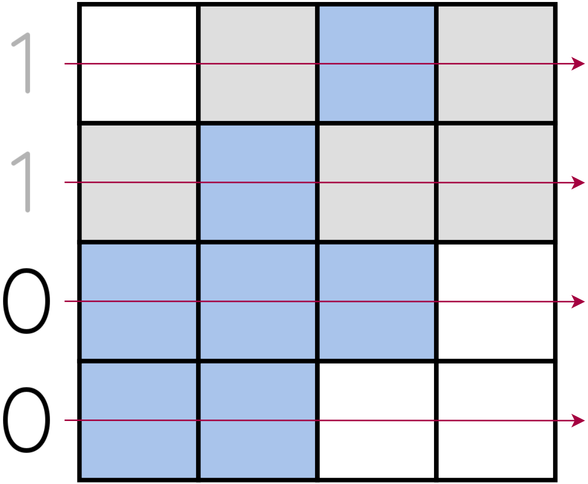
1 1 2 2

0 1 0 3 2 0 0

1 3 2 0

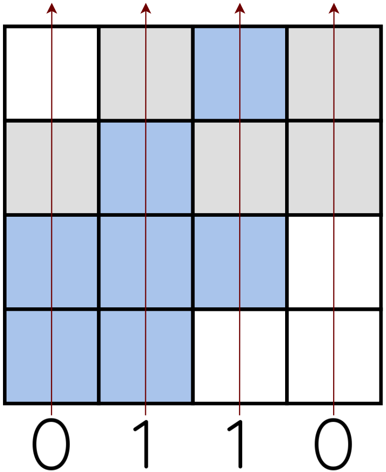
1 1 1 1 2 0 0

Example



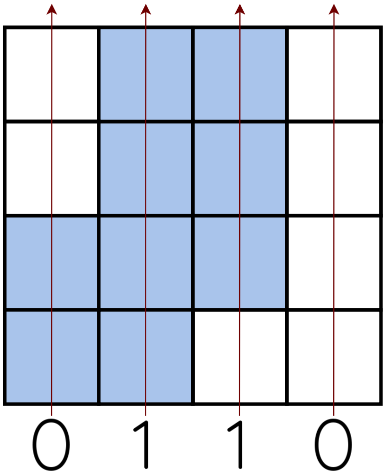

```
1
1 1 0 0
0 1 0 1 0 0 0
0 1 1 0
0 0 0 0 2 0 0
```

Example



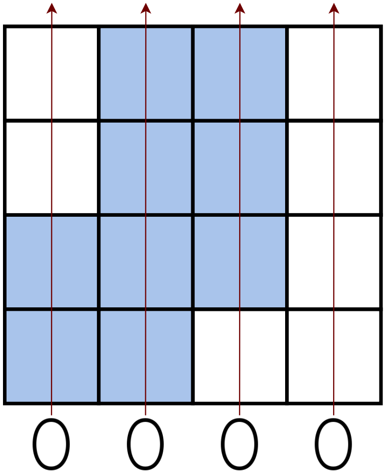

```
1
1 1 0 0
0 1 0 1 0 0 0
0 1 1 0
0 0 0 0 2 0 0
```

Example



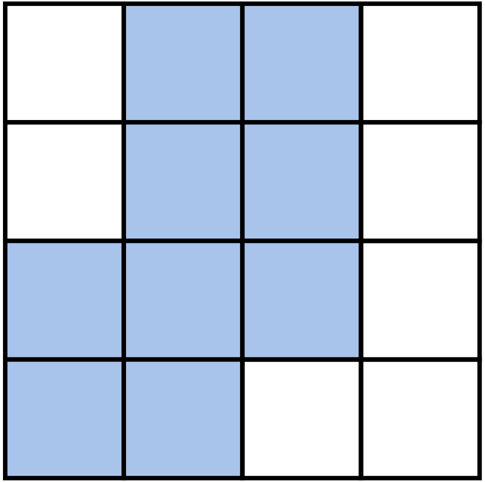

```
1
1 1 0 0
0 1 0 1 0 0 0
0 1 1 0
0 0 0 0 2 0 0
```

Example



1
0 0 0 0
0 0 0 0 0 0 0
0 0 0 0
0 0 0 0 0 0 0

Example



```
1
0 0 0 0
0 0 0 0 0 0 0
0 0 0 0
0 0 0 0 0 0 0
```

Code Snippets

- Snippets of the actual Python-program
- Highly simplified - treat it like pseudo-code

Fill known cells


```
def compare_and_fill(sensor_data_point, arr):
        # number of unassigned elements
        n_of_unassigned = n_of_unassigned(arr)

        # Declare all unassigned as empty
        if sensor_data_point == 0:
            for cell in arr:
                if cell == UNASSIGNED:
                    cell = EMPTY

        # Declare all unassigned as full
        elif sensor_data_point == n_of_unassigned:
            for cell in arr:
                if cell == UNASSIGNED:
                    cell = FULL
                    update_sensor_data(cell.x, cell.y)
```

Main Loop


```
while(not is_done()):
            diag_lr = get_diagonal_lr(matrix)
            diag_rl = get_diagonal_rl(matrix)

            # Horizontals
            for i in range(height):
                compare_and_fill(sensor_data_horizontal[i], matrix[i,:])
            # LR-Diags
            for i in range(height + width - 1):
                compare_and_fill(sensor_data_diagonal_lr[i], diag_lr[i])
            # Verticals
            for i in range(width):
                compare_and_fill(sensor_data_vertical[i], matrix[:,i])
            # RL-Diags
            for i in range(height + width - 1):
                compare_and_fill(sensor_data_diagonal_rl[i], diag_rl[i])
```

Are we done?

- Does the algorithm always find a solution?
- What if there is no solution?
- What if there are multiple solutions?

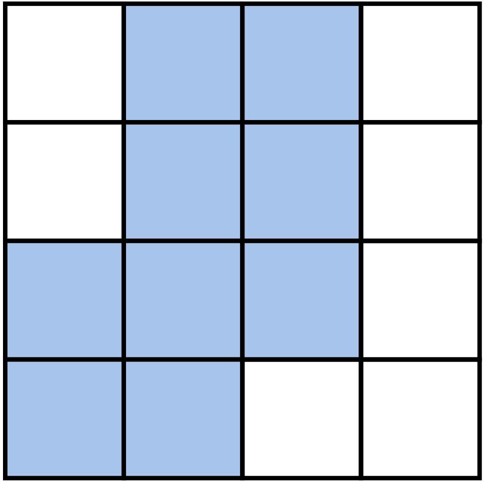
Are we done?

- Does the algorithm always find a solution?
- What if there is no solution?
- What if there are multiple solutions?

Let’s first answer a different question:

- How do we know whether we found a solution?

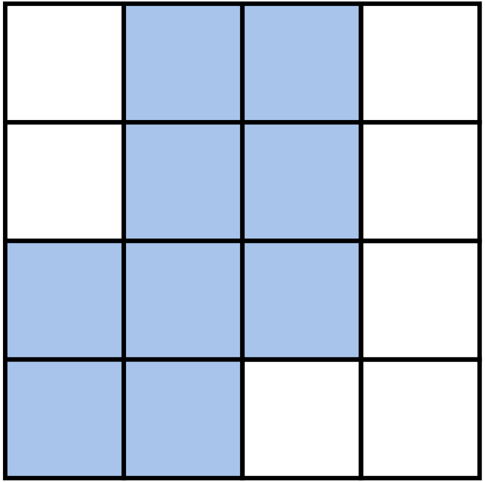
How do we know whether we found a solution?



```
1
0 0 0 0
0 0 0 0 0 0 0
0 0 0 0
0 0 0 0 0 0 0
```

All input_datas-entries are zero

Invalid state



```
1
0 0 0 0
0 0 0 0 0 0 0
0 1 0 0
0 0 0 0 0 0 0
```

Every cell is assigned, but there is one input_data-entry left!

⇒ contradiction

Termination Condition

1. Check whether all `input_data` is zero

```
def is_data_used(sensor_data):
        return not np.any(sensor_data != 0)
```

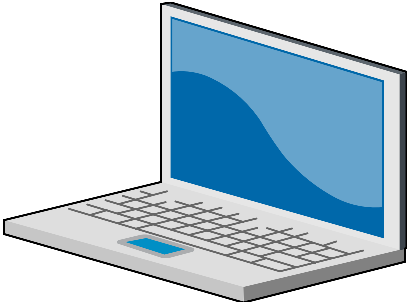
2. Check whether all cells are assigned

```
def is_all_assigned(matrix):
        return not np.any(matrix.cell == UNASSIGNED)
```

3. Check whether we're done

```
def is_done():
        return is_data_used(sensor_data) or is_all_assigned(matrix)
```

Demo: No Solution



Multiple solutions

- We always only assign `EMPTY` or `FULL` to a cell if we know it's state for certain
- Therefore, with our current method, we can only detect certain solutions
- If multiple solutions exist for a given input, we should get stuck

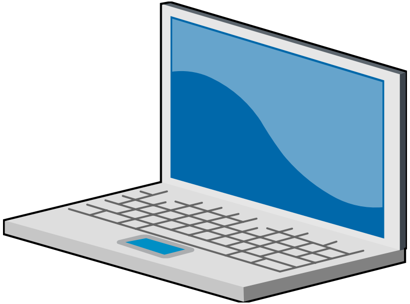
Multiple Solutions: Local Search


```python
# value is either FULL or EMPTY
def search_in_branch(idx, value, matrix, sensor_data):
        # save old data
        old_matrix = matrix.copy()
        old_sensor_data = sensor_data.copy()

        # assign variable
        matrix[idx] = value
        if value == FULL:
            update_sensor_data(idx[1], idx[0])
        fill_loop()

        # re-assign old data
        matrix = old_matrix
        sensor_data = old_sensor_data
```

Multiple Solutions: Local Search


```
# ... inside fill_loop()
if not has_change_occured:
            # indices of unassigned fields
            indices_of_unassigned = np.argwhere(matrix.cell == UNASSIGNED)

            for idx in indices_of_unassigned:
                # recursive calls
                for assignment in [EMPTY, FULL]:
                    search_in_branch(idx, value, matrix, sensor_data)
                    if solutions_found > 1:
                        # the solution is ambiguous -> leave loop
                        return
```

Demo: Local Search



Are we done?

Does the algorithm always find a solution?

Answer: No!

- A full assignment is valid only if all entries of `sensor_data` become zero
- We can get stuck ⇒ perform local search

What if there is no solution?

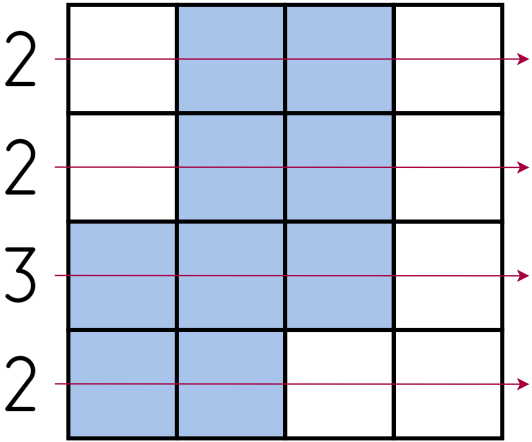
Answer: Data will be contradictory

What if there are multiple solutions?

Answer: We get stuck ⇒ perform local search

3 Discussion

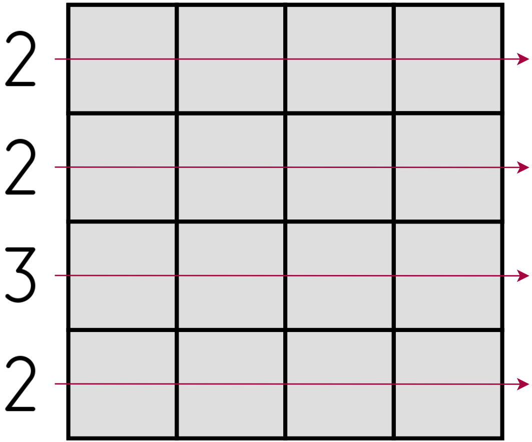
Interpretation as Linear System of Equations



$\rightarrow$

$$
\begin{aligned}
2 &= 0 + 1 + 1 + 0 \\
2 &= 0 + 1 + 1 + 0 \\
3 &= 1 + 1 + 1 + 0 \\
2 &= 1 + 1 + 0 + 0
\end{aligned}
$$

Interpretation as Linear System of Equations



$\rightarrow$

More general:

$$
\begin{aligned}
2 &= x_0 + x_1 + x_2 + x_3 \\
2 &= x_4 + x_5 + x_6 + x_7 \\
3 &= x_8 + x_9 + x_{10} + x_{11} \\
2 &= x_{12} + x_{13} + x_{14} + x_{15}
\end{aligned}
$$

Interpretation as Linear System of Equations

For a matrix of dimension $(n \times n)$, we get

- $\boldsymbol{2^n}$ variables $x_0, \ldots, x_{2^n - 1} \in \{0, 1\}$
- $n + n + (2n - 1) + (2n - 1) = \boldsymbol{6n - 2}$ linearly independent equations

Interpretation as Linear System of Equations

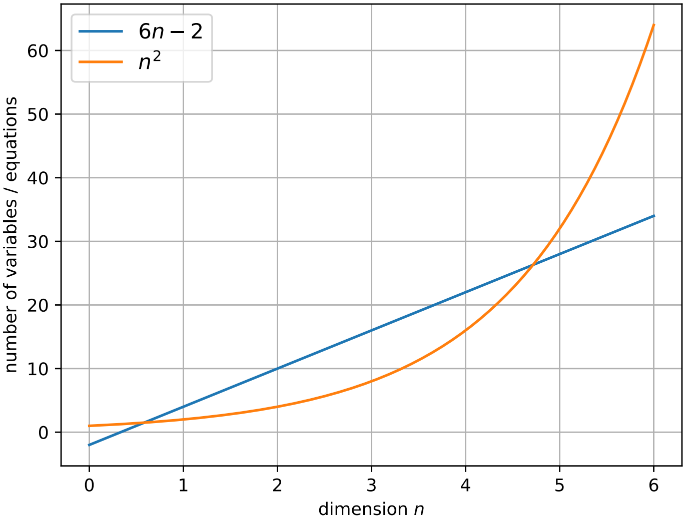
For a matrix of dimension $(n \times n)$, we get

- $\boldsymbol{2^n}$ variables $x_0, \ldots, x_{2^n-1} \in \{0, 1\}$
- $n + n + (2n - 1) + (2n - 1) = \boldsymbol{6n - 2}$ linearly independent equations

Question: For which $n \in \mathbb{N}$ is the linear system of equations underdetermined?

$\Rightarrow$ Solve $2^n > 6n - 2$

Interpretation as Linear System of Equations



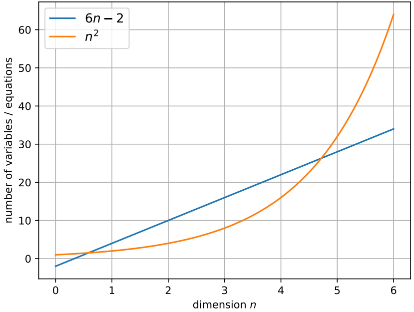
Interpretation as Linear System of Equations

Theory indicates:

- For all $n \geq 5$, the LSE is underdetermined $\Rightarrow$ Possibly no solution

Experiment confirms this!

- For all valid inputs of $n < 5$, a solution is found
- For some $n \geq 5$, the algorithm gets stuck (we have seen an example) $\Rightarrow$ perform local search



4 Summary

Summary

TODO