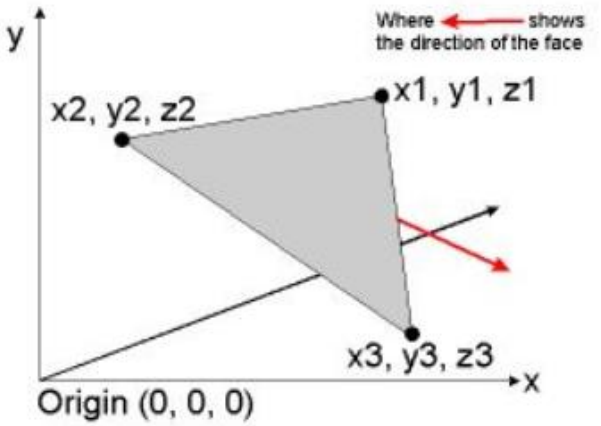
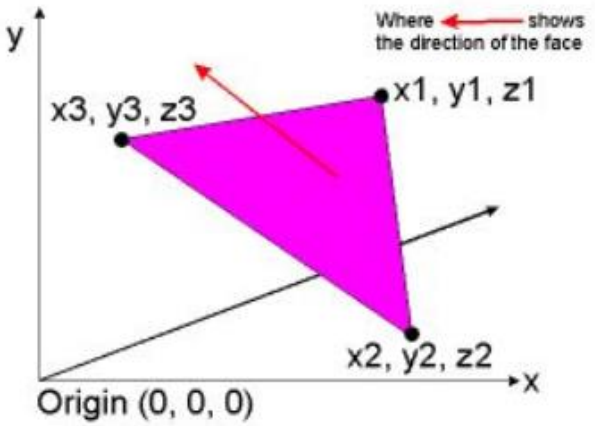
How to get the facing direction of a triangle given the positions of its vertices?

To answer this question we need to find the normal vector for this triangle. A surface normal for a triangle can be calculated by taking the vector cross product of two edges of that triangle





Example:

a = (3, -3, 1) b = (4, 9, 2)

$$\text{a X b} = \begin{vmatrix} i & j & k \\ a1 & a2 & a3 \\ b1 & b2 & b3 \end{vmatrix}$$

$$\begin{aligned} \mathbf{a} \times \mathbf{b} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 1 \\ 4 & 9 & 2 \end{vmatrix} \\ &= \mathbf{i}(-3 \cdot 2 - 1 \cdot 9) - \mathbf{j}(3 \cdot 2 - 1 \cdot 4) + \mathbf{k}(3 \cdot 9 + 3 \cdot 4) \\ &= -15\mathbf{i} - 2\mathbf{j} + 39\mathbf{k} \end{aligned}$$

Facing direction vector -> -15i -2j +39k

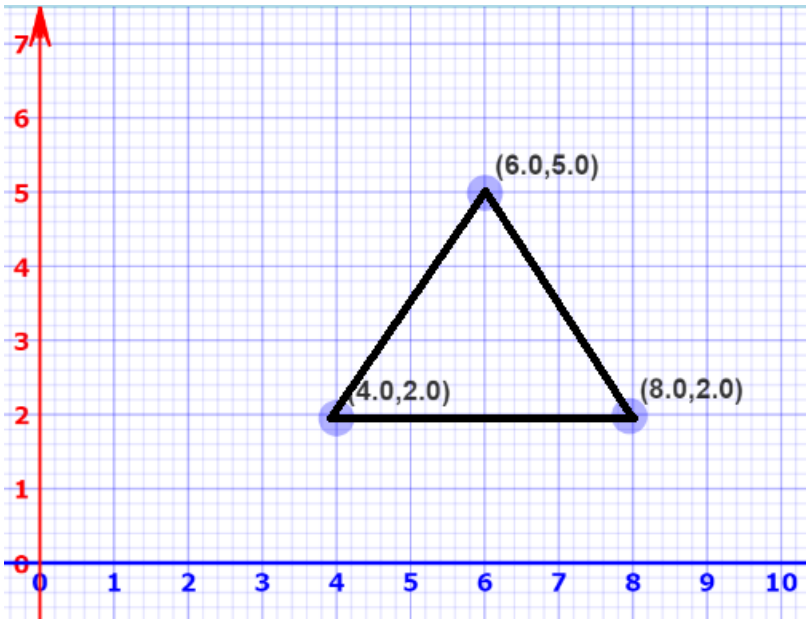
How to calculate the area of a triangle given the positions of its vertices?

By applying the math formula below we can find the area of a triangle by its vertices

$$\frac{Ax(By - Cy) + Bx(Cy - Ay) + Cx(Ay - By)}{2}$$

Example:

A(4,2) B(8,2) C(6,5)



$$A = \frac{b * h}{2}$$

$$A = \frac{4 * 3}{2} = 6$$

$$\frac{(\ 4(2 - 5) + 8(5 - 2) + 6(2 - 2)\ )}{2} = \frac{4(-3) + 8(3) + 6(0)}{2}$$

$$\frac{-12 + 24}{-2} = 6$$