

AI1103 - Assignment 2

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Download all python codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_2/simulation_2.py

and latex-tikz codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_2/Latex_2.tex

1 PROBLEM 5.28

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x , has the following form, where k is some unknown constant.

$$\Pr(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5 - x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases} \quad (1.0.1)$$

- Find the value of k .
- What is the probability that you study at least two hours ? Exactly two hours? At-most two hours?

2 SOLUTION

Expanding the given form , we get :

Probability of studying x number of hours is as follows when x varies from 0 to 4, and is 0 for all other values of x .

x	0	1	2	3	4
$\Pr(X = x)$	0.1	$1k$	$2k$	$2k$	k

TABLE 2: Probabilities in terms of k

We know by definition,

$$\sum_{x=0}^4 \Pr(X = x) = 1 \quad (2.0.1)$$

By substituting the probabilities in (2.0.1) we get,

$$\Rightarrow 0.1 + k + 2k + 2k + k = 1 \quad (2.0.2)$$

$$\Rightarrow 6k = 0.9 \quad (2.0.3)$$

Therefore, from (2.0.3)

$$k = 0.15 \quad (2.0.4)$$

Therefore the probability for x hours of study where $0 \leq x \leq 4$, is as follows :

x	0	1	2	3	4
$\Pr(X = x)$	0.1	0.15	0.3	0.3	0.15

TABLE 2: Probabilities after finding k

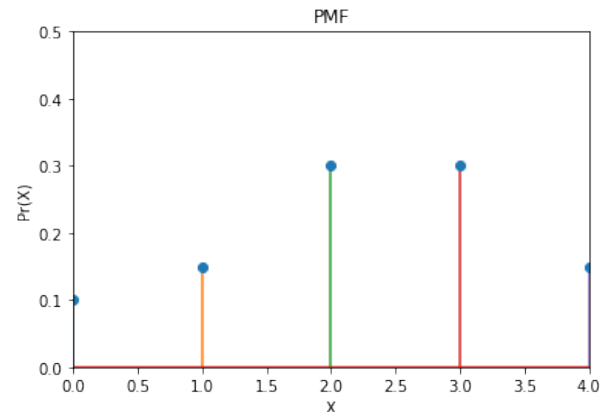


Fig. 2: Probability Mass Function (PMF)

We know that, Cumulative Distributive Function (CDF)

$$F(x) = \Pr(X \leq x) \quad (2.0.5)$$

x	0	1	2	3	4
$F(X)$	0.1	0.25	0.55	0.85	1

TABLE 2: CDF

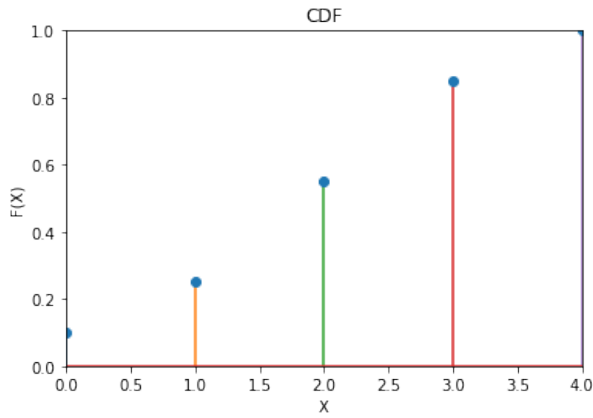


Fig. 2: Cumulative Distributive Function (CDF)

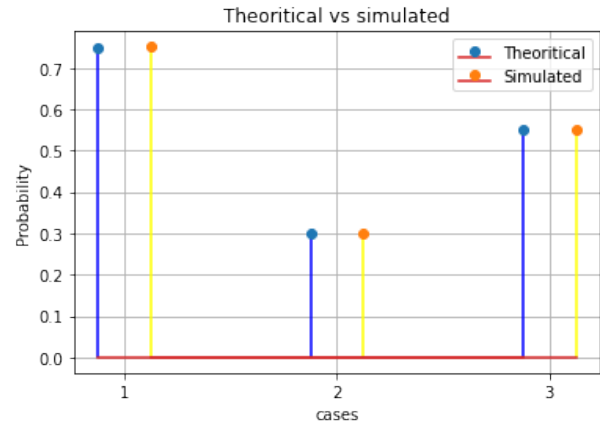


Fig. 3: Comparison of theoretical and simulation values

1) Probability of studying at least two hours

$$= \sum_{x=2}^4 \Pr(X = x) \quad (2.0.6)$$

$$= \Pr(X = 2) + \Pr(X = 3) + \Pr(X = 4) \quad (2.0.7)$$

$$= \Pr(1 < X \leq 4) \quad (2.0.8)$$

$$= F(4) - F(1) \quad (2.0.9)$$

Substituting CDF values of (2) in (2.0.9)

$$= 1 - 0.25 \quad (2.0.10)$$

$$= 0.75 \quad (2.0.11)$$

2) Probability of studying exact two hours

$$= \Pr(X = 2) \quad (2.0.12)$$

$$= 0.3 \quad (2.0.13)$$

3) Probability of studying at most two hours

$$= \sum_{x=0}^2 \Pr(X = x) \quad (2.0.14)$$

$$= \Pr(X = 0) + \Pr(X = 1) + \Pr(X = 2) \quad (2.0.15)$$

$$= F(2) \quad (2.0.16)$$

By substituting probabilities in (2.0.16)

$$= 0.55 \quad (2.0.17)$$

Final solution :

$\Pr(X \geq 2)$	$\Pr(X = 2)$	$\Pr(X \leq 2)$
0.75	0.3	0.55
Case1	Case2	Case3
at-least 2 hrs	Exactly 2 hrs	at-most 2 hrs