AI1103 - Assignment 1

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Download all python codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_4/simulation_4.py

and latex-tikz codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_4/latex_4.tex

1 GATE EC, Q.23

Two independent random variables X and Y are uniformly distributed in the interval [-1,1].The probability that max $\{X,Y\}$ is less than $\frac{1}{2}$ is

A) 3/4
B) 9/16
C) 1/4
D) 2/3

2 Solution

Lemma 2.1. *CDF of the random variable X is :*

$$F_X(x) = \begin{cases} 0 & x \leq -1 \\ \frac{1}{2}(x+1) & -1 < x < 1 \\ 1 & x \geq 1 \end{cases} \tag{2.0.1}$$

Proof. Given X is uniformly distributed in [-1,1] i.e. $X \sim$ U(-1,1)
PDF of X :

$$f_X(x) = \begin{cases} 0 & x \leq -1 \\ \frac{1}{2} & -1 \leq x \leq 1 \\ 0 & x \geq 1 \end{cases} \tag{2.0.2}$$

For $-1 \leq x \leq 1$

$$F_X(x) = \Pr(X \leq x) \tag{2.0.3}$$
$$= \int_{-1}^{x} \frac{1}{2} dx \tag{2.0.4}$$
$$= \frac{1}{2}(x+1) \tag{2.0.5}$$

Hence (2.0.1) is proved ◻

Lemma 2.2. *CDF of the random variable Y is :*

$$F_X(x) = \begin{cases} 0 & x \leq -1 \\ \frac{1}{2}(x+1) & -1 < x < 1 \\ 1 & x \geq 1 \end{cases} \tag{2.0.6}$$

Proof. Given Y is uniformly distributed in [-1,1] i.e. $Y\sim$ U(-1,1)
PDF of Y :

$$f_Y(y) = \begin{cases} 0 & y \leq -1 \\ \frac{1}{2} & -1 \leq y \leq 1 \\ 0 & y \geq 1 \end{cases} \tag{2.0.7}$$

For $-1 \leq y \leq 1$

$$F_Y(y) = P(Y \leq y) \tag{2.0.8}$$
$$= \int_{-1}^{y} \frac{1}{2} dy \tag{2.0.9}$$
$$= \frac{1}{2}(y+1) \tag{2.0.10}$$

Hence (2.0.6) is proved ◻

Lemma 2.3.

$$\Pr\left(max\{X,Y\} < \frac{1}{2}\right) = \frac{9}{16} \tag{2.0.11}$$

Proof. $max\{X,Y\} < \frac{1}{2} \implies X < \frac{1}{2}\ ,\ Y < \frac{1}{2}$
Given X and Y are independent,

$$Pr(X < \frac{1}{2}, Y < \frac{1}{2}) \tag{2.0.12}$$
$$= \Pr\left(X < \frac{1}{2}\right) \times \Pr\left(Y < \frac{1}{2}\right) \tag{2.0.13}$$
$$= F_X\left(\frac{1}{2}\right) \times F_Y\left(\frac{1}{2}\right) \tag{2.0.14}$$
$$= \frac{3}{2} \times \frac{1}{2} \times \frac{3}{2} \times \frac{1}{2} \tag{2.0.15}$$
$$= \frac{9}{16} \tag{2.0.16}$$

Hence (2.0.11) is proved ◻

Option B is correct

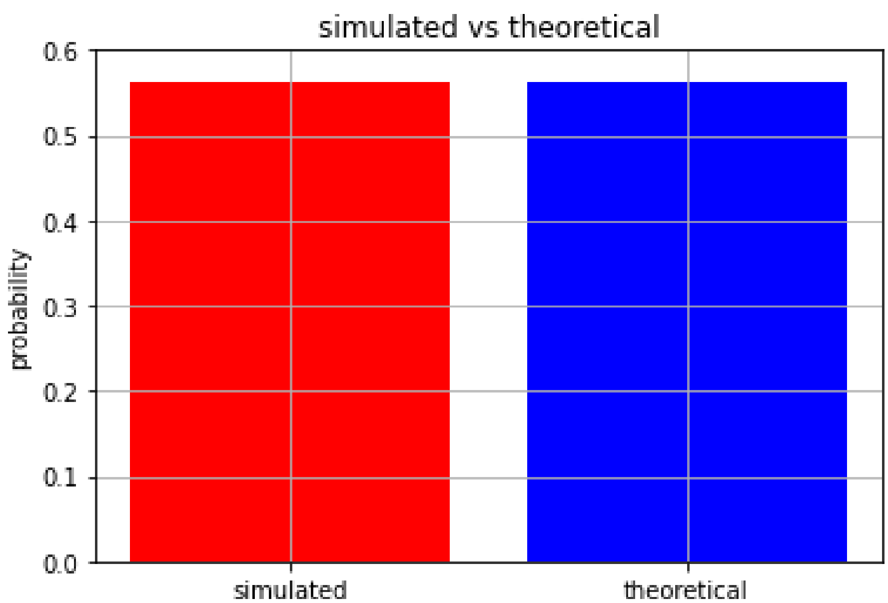

Fig. 4: simulated vs theoretical