AI1103 - Assignment 1

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Download all python codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_4/simulation_4.py

and latex-tikz codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_4/latex_4.tex

1 GATE EC, Q.23

Two independent random variables X and Y are uniformly distributed in the interval [-1,1].The probability that max $[X,Y]$ is less than $\frac{1}{2}$ is

A) 3/4
B) 9/16
C) 1/4
D) 2/3

2 Solution

X ~ U(-1,1)

$$F_X(x) = P(X < x) \tag{2.0.1}$$
$$= \int_{-1}^{x} \frac{1}{2} dx \tag{2.0.2}$$
$$= \frac{1}{2}(x+1) \tag{2.0.3}$$

$$F_X(x) = \begin{cases} 0 & x \leq -1 \\ \frac{1}{2}(x+1) & -1 < x < 1 \\ 1 & x \geq 1 \end{cases} \tag{2.0.4}$$

Y ~ U(-1,1)

$$F_Y(y) = P(Y < y) \tag{2.0.5}$$
$$= \int_{-1}^{x} \frac{1}{2} dy \tag{2.0.6}$$
$$= \frac{1}{2}(y+1) \tag{2.0.7}$$

$$F_Y(y) = \begin{cases} 0 & y \leq -1 \\ \frac{1}{2}(y+1) & -1 < y < 1 \\ 1 & y \geq 1 \end{cases} \tag{2.0.8}$$

$Pr\Big(\text{max(X,Y)}\Big) < \frac{1}{2}$ implies that $X < \frac{1}{2}$ & $Y < \frac{1}{2}$

Given X and Y are independent, so

$$Pr(X < \frac{1}{2}, Y < \frac{1}{2}) \tag{2.0.9}$$
$$= Pr(X < \frac{1}{2}) \times Pr(Y < \frac{1}{2}) \tag{2.0.10}$$
$$= F_X(\frac{1}{2}) \times F_Y(\frac{1}{2}) \tag{2.0.11}$$
$$= \frac{3}{2} \times \frac{1}{2} \times \frac{3}{2} \times \frac{1}{2} \tag{2.0.12}$$
$$= \frac{9}{16} \tag{2.0.13}$$

Option B is correct

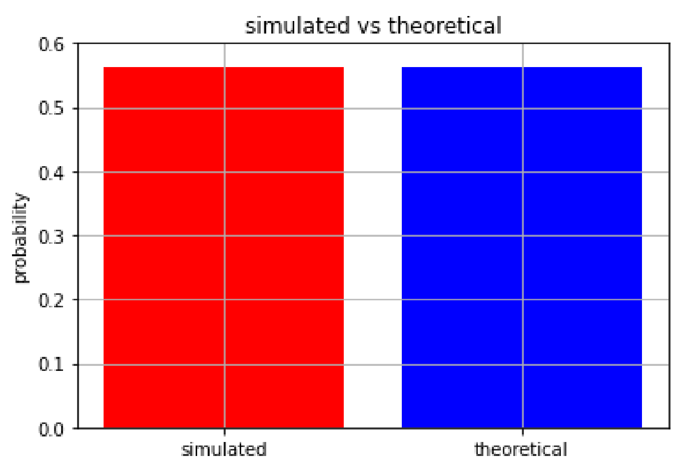

Fig. 4: simulated vs theoretical