AI1103 - Assignment 2

G Vojeswitha - AI20BTECH11024

Download all python codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_2/simulation_2.py

and latex-tikz codes from

https://github.com/Vojeswitha05/Probability_AI1103/blob/main/Assignment_2/Latex_2.tex

1 Problem 5.28

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x, has the following form, where k is some unknown constant.

$$\Pr(X=x)=\begin{cases} 0.1, & \text{if} x=0 \\ kx, & \text{if } x=1 or 2 \\ k(5-x), & \text{if } x=3 or 4 \\ 0, & \text{otherwise} \end{cases} \quad (1.0.1)$$

a) Find the value of k.
b) What is the probability that you study at least two hours ? Exactly two hours? At-most two hours?

2 Solution

Expanding the given form , we get :
Probability of studying x number of hours is as follows when x varies from 0 to 4, and is 0 for all other values of x.

x	0	1	2	3	4
$\Pr(X=x)$	0.1	1k	2k	2k	k

TABLE 2: Probabilities in terms of k

We know by definition,

$$\sum_{x=0}^{4}\Pr(X=x)=1 \quad (2.0.1)$$

By substituting the probabilities in (2.0.1) we get,

$$\implies 0.1+k+2k+2k+k=1 \quad (2.0.2)$$

$$\implies 6k=0.9 \quad (2.0.3)$$

Therefore, from (2.0.3)

$$k=0.15 \quad (2.0.4)$$

Therefore the probability for x hours of study where $0 \le x \le 4$, is as follows :

x	0	1	2	3	4
$\Pr(X=x)$	0.1	0.15	0.3	0.3	0.15

TABLE 2: Probabilities after finding k

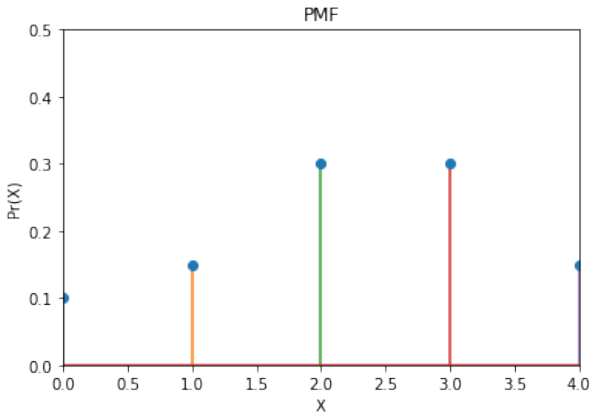

Fig. 2: Probability Mass Function (PMF)

We know that, Cumulative Distributive Function (CDF)

$$F(x)=\Pr(X\le x) \quad (2.0.5)$$

x	0	1	2	3	4
$F(X)$	0.1	0.25	0.55	0.85	1

TABLE 2: CDF

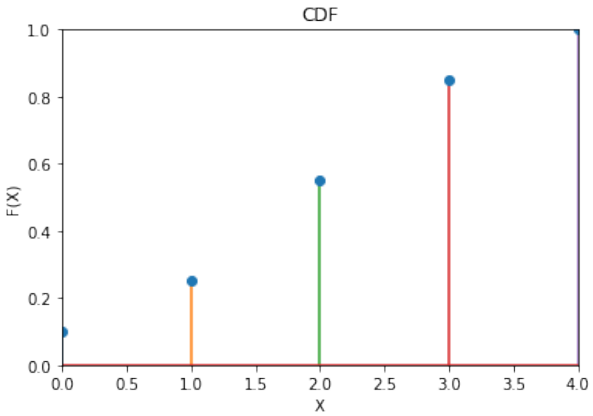

Fig. 2: Cumulative Distributive Function (CDF)

1) Probability of studying at least two hours

$$= \sum_{k=2}^{4} \Pr(X = k) \tag{2.0.6}$$

$$= \Pr(1 < X \leq 4) \tag{2.0.7}$$

$$= F(4) - F(1) \tag{2.0.8}$$

Substituting CDF values of (2) in (2.0.8)

$$= 1 - 0.25 \tag{2.0.9}$$

$$= 0.75 \tag{2.0.10}$$

2) Probability of studying exact two hours

$$= \Pr(X = 2) \tag{2.0.11}$$

$$= 0.3 \tag{2.0.12}$$

3) Probability of studying at most two hours

$$= \sum_{x=0}^{2} \Pr(X = x) = \Pr(X \leq 2) \tag{2.0.13}$$

$$= F(2) \tag{2.0.14}$$

By substituting probabilities in (2.0.14)

$$= 0.55 \tag{2.0.15}$$

Final solution :

$\Pr(X \geq 2)$	$\Pr(X = 2)$	$\Pr(X \leq 2)$
0.75	0.3	0.55
Case1	Case2	Case3

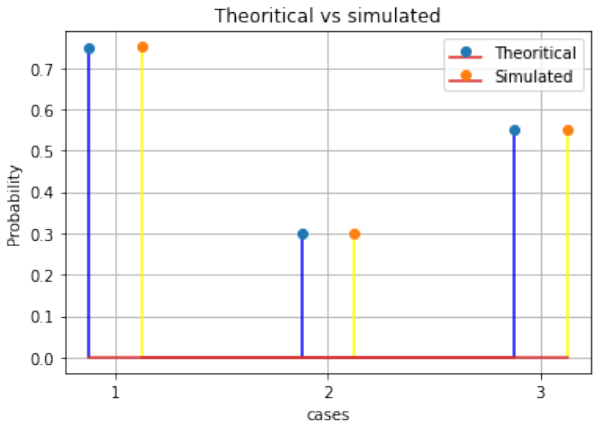

Fig. 3: Comparison of theoretical and simulation values