Nash theorem, Nash equilibria

Vojtěch Šára, Marek Douša

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1 Basics; notation, definitions

Normal form game of $n \in \mathbb{N}$ players is $G = (P, A, w)$, where P is a finite set n players.

$$A = A_1 \times \cdots \times A_n$$

Is a set of action profiles. Action profile is $(a_1, a_2, ...a_n)$ where $a_i \in A_i$ =set of actions of player i.

$u = (u_1, ..., u_n)$ where $u_i : A \to \mathbb{R}$ is the utility function of i - how good this action is for player i.

Every player i selects an action $a_i \in A_i$ - all players select at once. Every player wants to maximize u_i

EXAMPLE:
Rock paper scissors:

$$P = \{1, 2\}, A_1 = \{ROCK, PAPER, SCISSORS\} = A_2, A = A_1 \times A_2$$

$$M = (M_a)_a \in A, M_a = (u_1(a), u_2(a)) : \begin{pmatrix} 1/2 & R & P & S \\ R & 0,0 & -1,1 & 1,-1 \\ P & 1,-1 & 0,0 & 1,-1 \\ S & -1,1 & 1,-1 & 0,0 \end{pmatrix}$$

A **strategy** is a prescription how to select an action. There are two types:

1. Pure strategy - Always select the same action
2.