We want to display analysis results using both logarithmic and linear scale (and maybe some other scales). This document discusses the implementations of `LinearScale` and `LogarithmicScale`.

So far, zoom is implemented in the way that it changes the size of container (`JComponent`) containing graph (of either verification result or time course). The scale should adjust acordingly. Furthermore, we know dimension of given model value (including time).

Therefore, scale is a transformation between intervals $[b_m, t_m]$ (domain of model value) and $[b_v = 0, t_v]$ (size of contaier), i.e. a function f and its inverse, such that $f(b_m) = 0$ and $f(t_m) = t_v$.

1 Linear Scale

In the case of linear scale f is linear. As the inverse is linear too, it is easier to consider $f^{-1}(x) = ax+b$. Hence $b_m = f^{-1}(0) = b$, and $t_m = f^{-1}(t_v) = at_v+b_m$, implying $a = \frac{t_m - v_m}{t_v}$.

2 Logarithmic Scale

In logarithmic scale f is an logarithm of a certain base. However, we want $f(b_m) = 0$. If $f = \log_a$, this condition holds only if $b_m = 1$. Therefore, we have to identify b_m with one. This is done by putting $f(x) = \log_a(x - b_m + 1)$ (division is not possible since it would not work for negative b_m).

$$\log_a(t_m - b_m + 1) = t_v \quad \Longrightarrow \quad \frac{\ln(t_m - b_m + 1)}{\ln a} = t_v \quad \Longrightarrow \quad \ln a = \frac{\ln(t_m - b_m + 1)}{t_v}$$

$$f : x \to \frac{\ln(x - b_m + 1)}{\ln a}$$
$$f^{-1} : x \to e^{x \ln a} - 1 + b_m$$

Since java does not implement general logarithm, `LogarithmicScale` stores the value of $\ln a$. Furthermore, for additional precision, functions $\mathrm{log1p}(x) = \ln(x+1)$ and $\mathrm{expm1}(x) = e^x - 1$ are used when possible.