i) $f_1, f_2, f_3 \dots$ smooth with parameters $L_1, L_2, L_3 \dots$

$$f = \sum_{i=1}^{m} f_i$$

$$\|\nabla f(x) - \nabla f(y)\| = \left\|\sum_{i=1}^{m} \nabla f_i(x) - \sum_{i=1}^{m} \nabla f_i(y)\right\| = \left\|\sum_{i=1}^{m} \nabla f_i(x) - \nabla f_i(y)\right\| \leq$$

$$\leq \sum_{i=1}^{m} \|\nabla f_i(x) - \nabla f_i(y) \leq \sum_{i=1}^{m} L_i \|x-y\|$$

triangular inequality

$\Rightarrow$ f smooth with param. $\sum_{i=1}^{m} L_i$

f_i smooth with param. L_1

ii) $g = f \circ A$ $\qquad \nabla g(x) = \nabla f(Ax) \cdot A$

$$\|\nabla g(x) - \nabla g(y)\| = \|\nabla f(Ax) \cdot A - \nabla f(Ay) \cdot A\| = \|(\nabla f(Ax) - \nabla f(Ay)) \cdot A\| \leq$$

$$\leq \|\nabla f(Ax) - \nabla f(Ay)\| \cdot \|A\|_{op} \leq (L \cdot \|Ax - Ay\|) \cdot \|A\| \leq (L \cdot \|A\|_{op} \cdot \|x-y\|) \cdot \|A\| =$$

$$= L \cdot \|A\|^2 \cdot \|x-y\|$$

iii) $\log(1+x)$ converges slower than $\log(2+x)$

iv) $$X_{k+1} = X_k - \alpha f'(X_k) = g(X_k)$$

$$f'(x) = \frac{x - g(x)}{\alpha} \qquad f(x) = \frac{1}{\alpha}\left(\frac{1}{2}x^2 - \int g(x)dx\right)$$

sufficient condition on g: $\forall k$: $|X_{k+1} - g(X_{k+1})| < |X_k - g(X_k)|$

f – convex and smooth

a) convex $\forall \beta \in \langle 0,1 \rangle$:

$$f(\beta x + (1-\beta)y) = \frac{1}{\alpha}\left(\frac{1}{2}(\beta x + (1-\beta)y)^2 - \int g(\beta x + (1-\beta)y)\right) =$$

$$= \frac{1}{\alpha}\left(\frac{1}{2}(\beta^2 x^2 + 2\beta(1-\beta)xy + (1-\beta)^2 y^2) - \int g(\beta x + (1-\beta)y)\right)$$

we need $$\leq \frac{1}{\alpha}\left(\frac{1}{2}(\beta^2 x^2)\right) - \int g(\beta x) + \frac{1}{2}(1-\beta)^2 y^2 - \int g((1-\beta)y)$$

$$\beta(1-\beta)xy - \int g(\beta x + (1-\beta)y) \leq -\int g(\beta x) - \int g((1-\beta)y)$$

$$\beta(1-\beta)xy \leq \int g(\beta x + (1-\beta)y) - \int g(\beta x) - \int g((1-\beta)y)$$

b) smooth

$$\forall x,y: \left|\frac{x - g(x)}{\alpha} - \frac{y - g(y)}{\alpha}\right| \leq L|x-y|$$

$$\frac{1}{\alpha}|x - g(x) - (y - g(y))| \leq L|x-y|$$

linear convergence?

$$f(y) \geq f(x) + f'(x)\cdot(y-x) + \frac{\mu}{2}(y-x)^2$$

$$\frac{1}{\alpha}\left(\frac{1}{2}y^2 - \int g(y)\right) \geq \frac{1}{\alpha}\left(\frac{1}{2}x^2 - \int g(x)\right) + \frac{x-g(x)}{\alpha}(y-x) + \frac{\mu}{2}(y-x)^2 \quad /\cdot 2\alpha$$

$$y^2 - 2\int g(y) \geq x^2 - 2\int g(x) + 2(y-x)(x-g(x)) + \alpha\mu(y-x)^2$$

$$2\left(\int g(x) - \int g(y) + g(x)(y-x)\right) \geq x^2 - y^2 + 2xy - 2x^2 + \alpha\mu(y-x)^2$$

$$2\int g(x) - g(y) \quad + 2g(x)(y-x) \geq -(x-y)^2 + \alpha\mu(y-x)^2$$

$$g(x)(y-x) + \int g(x) - g(y) \geq (x-y)^2 \cdot \frac{\alpha\mu - 1}{2}$$

Satisfied for (2)?

$g(x) = \log(1+x)$ $\quad \int g(x) = x\log(1+x) + \log(1+x) - x$

$\forall x,y$:

$$(y-x)\log(1+x) + x\log(1+x) + \log(1+x) - x - y\log(1+y) - \log(1+y) + y \geq (x-y)^2\frac{\alpha\mu-1}{2}$$

$$(y+1)\left(\log(1+x) - \log(1+y)\right) \geq (x-y)^2\frac{\alpha\mu-1}{2} + x - y$$

$$(y+1)\log\frac{1+x}{1+y} \geq (x-y)^2\underbrace{\frac{\alpha\mu-1}{2}}_{\leq 0} + x - y$$

Not satisfied $(x,y) = (-0{,}5;\ -0{,}70)$

for $g(x) = \log(2+x)$:

$$2(y+2)\log\frac{2+x}{2+y} \geq (x-y)^2\frac{\alpha\mu-1}{2} + x - y$$

~~[illegible]~~ Satisfied for all x,y near the optimum