i) $\|\lambda x+(1-\lambda)y\| \overset{3.}{\leq} \|\lambda x\|+\|(1-\lambda)y\| \overset{2.}{=} |\lambda|\|x\|+|(1-\lambda)|\|y\| \overset{\lambda\in[0,1]}{=} \lambda\|x\|+(1-\lambda)\|y\|$

ii) ~~$f(x)=y$~~

$$f(\lambda x+(1-\lambda)y) \overset{f(x)=g(Ax)}{=} g(A(\lambda x+(1-\lambda)y)) \overset{A\text{-lin. op.}}{=} g(\lambda Ax+(1-\lambda)Ay) \overset{g\text{-convex}}{\leq}$$

$$\leq \lambda g(Ax)+(1-\lambda)g(Ay) = \lambda f(x)+(1-\lambda)\cancel{g}f(y)$$

iii) x^* - local ~~minimum~~ minimizer $\Rightarrow \exists \varepsilon: \forall y: \|x^*-y\| \leq \varepsilon \Rightarrow f(x^*) \leq f(y)$

Assume $\exists z \in D(f): f(z) < f(x^*)$, then $\forall \lambda \in (0,1]: \lambda f(z)+(1-\lambda)f(x^*) < f(x^*)$

$$f(x^*) > \lambda f(z)+(1-\lambda)f(x^*) \overset{f\text{-convex}}{\geq} f(\lambda z+(1-\lambda)x^*)\text{, but for } \lambda = \frac{\varepsilon}{\|z-x^*\|^2}$$

$$f(\lambda z+(1-\lambda)x^*) = f\left(\frac{\varepsilon z}{\|z-x^*\|}+x^*-\frac{\varepsilon x^*}{\|z-x^*\|}\right) = f(\underbrace{x^*+\varepsilon\frac{z-x^*}{\|z-x^*\|}}_{v})$$

so $f(x^*) > f(v)$. But $\|x^*-v\| \leq \varepsilon \Rightarrow f(x^*) \leq f(v)$

Contradiction $\Rightarrow$ ~~If~~ $\neg\exists z \in D(f): f(z) < f(x^*) \Rightarrow$

$\Rightarrow \forall z \in D(f): f(z) \geq f(x^*) \Rightarrow x^*$ - global ~~minimum~~ minimizer

$x^* = \bar{x}$

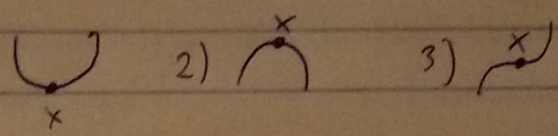
iv) $\nabla f(x^*) = 0$

f-convex $\Leftrightarrow \forall_y : f(y) \geq f(x^*) + \nabla f(x^*)^T (y-x) = f(x^*) \Rightarrow x^*$ - global minimizer

Stationary point → gradient = 0

Gradient - best linear approximation, if $\nabla f(x) = 0 \Rightarrow$ function is „close" to constant function in x's ~~nei~~ neighbourhood

intuitively 3 cases: 1) 2) 3)



only „1)" may be part of convex function. It can ~~bee~~ be seen that

$$x \text{ is local minimizer} \xRightarrow[f\text{-convex}]{} x \text{ is global minimizer}$$

v) DONE :)