(1)

$$f(x) = \frac{1}{2} x^T M x + q^T x + c$$

M invertible

Newton

$$x_{k+1} = x_k - \nabla^2 f(x_k)^{-1} \nabla f(x_k)$$

$$x_{k+1} = x_k - H_k^{-1} \nabla f(x_k) \qquad \nabla f(x_k) - \nabla f(x_{k-1}) = H_k (x_k - x_{k-1}) \quad \text{: Q-N method}$$

a) Newton $\Rightarrow$ Q-N

show that : $\nabla f(x_k) - \nabla f(x_{k-1}) = \nabla^2 f(x_k) (x_k - x_{k-1})$

$$\begin{aligned} \nabla f(x) &= Mx - q \\ \nabla^2 f(x) &= M \end{aligned} \Rightarrow \quad Mx_k - q - Mx_{k-1} + q = M (x_k - x_{k-1})$$

$$M(x_k - x_{k-1}) = M(x_k - x_{k-1})$$

TRUE

b) Q-N $\Rightarrow$ Newton

show that: $\cancel{\dfrac{\nabla f(x_k) - \nabla f(x_{k-1})}{x_k - x_{k-1}}}\ \nabla f(x_k)$

If f is not nondegenerative $\Rightarrow \neg$(Newton $\Rightarrow$ Q-N)

If f is not nondegenerative $\Rightarrow$ Newton method doesn't work since

$\nabla^2 f(x)^{-1} = M^{-1}$, but Q-N still can work :

There exists H_k such that $\nabla f(x_k) - \nabla f(x_{k-1}) = H_k (x_k - x_{k-1})$

$$M(x_k - x_{k-1}) = H_k (x_k - x_{k-1})$$

e.g. $\quad M = H_k$

Therefore : if f is not nondegenerative $\Rightarrow \neg$(Newton $\Rightarrow$ Q-N),

so Q-N $\Rightarrow$ Newton if f is nondegenerative

② Proove that there exists $c \in \mathbb{R}$ such that:

$$(x_k, f(x_k)) + c \cdot \big((x_{k-1}, f(x_{k-1})) - (x_k, f(x_k))\big) = (x_{k+1}, 0)$$

$$x_k + c \cdot (x_{k-1} - x_k) = x_{k+1}$$

$$f(x_k) + c \cdot (f(x_{k-1}) - f(x_k)) = 0$$

$$x_k + c \cdot (x_{k-1} - x_k) = x_k - f(x_k) \cdot \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$$

$$c = \frac{f(x_k)}{f(x_k) - f(x_{k-1})}$$

$$f(x_k) + \frac{f(x_k)}{f(x_k) - f(x_{k-1})} \cdot \big(f(x_{k-1}) - f(x_k)\big) = 0$$

$$f(x_k) - f(x_k) = 0$$

$$0 = 0 \quad \square$$