GD: $x_{k+1} = x_k - \alpha \nabla f(x_k)$

assume: $f$ is L-Lipschitz $\quad$ PL: $\frac{1}{2} \|\nabla f(x)\|^2 \geq \mu (f(x) - f^*)$ $\quad \mu > 0$

$$f(x_{k+1}) \leq f(x_k) + \langle \nabla f(x_k), x_{k+1} - x_k \rangle + \frac{L}{2} \|x_{k+1} - x_k\|^2$$

$$\alpha = \frac{1}{L}: \quad f(x_{k+1}) - f(x_k) \leq \langle \nabla f(x_k), -\frac{1}{L} \nabla f(x_k) \rangle + \frac{L}{2} \cdot \frac{1}{L^2} \|\nabla f(x_k)\|^2$$

$$f(x_{k+1}) - f(x_k) \leq -\frac{1}{2L} \|\nabla f(x_k)\|^2 \overset{PL}{\leq} -\frac{\mu}{L} (f(x) - f^*)$$

$$f(x_{k+1}) - f^* \leq (1 - \frac{\mu}{L})(f(x_k) - f^*) \leq (1 - \frac{\mu}{L})^{k+1} \cdot \underset{f(x_0) - f^*}{\underset{\shortparallel}{C}}$$