Optimization

master in fundamental principles of data science

Optimization Problem 10

Simplex problem

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1 Problem description

Prove that the number of faces of dimension p of a n-dimensional simplex is equal to

$$\binom{n+1}{p+1} = \frac{(n+1)!}{(p+1)!(n-p)!}$$

2 Solution

By definition, a n-dimensional simplex S is a convex polyhedron with $n+1$ vertices. Moreover, we have that the faces of a simplex are a lower dimensional simplex and that $0 \leq p \leq n$. Therefore, a face of dimension p of a n-dimensional is a simplex made up by $p+1$ vertices of the $n+1$ vertices from the original simplex.

The number of faces of dimension p is given by the amount of **combinations** of $p+1$ vertices that we can make by selecting them among the $n+1$ available vertices. We can express the previous statement using combinatorics, which results in the following expression:

$$C_{p+1}^{n+1} = \binom{n+1}{p+1} = \frac{(n+1)!}{(p+1)!\big((n+1)-(p+1)\big)!} = \frac{(n+1)!}{(p+1)!(n-p)!}$$