



UNIVERSITAT_{DE}
BARCELONA

OPTIMIZATION

MASTER IN FUNDAMENTAL PRINCIPLES OF DATA SCIENCE

OPTIMIZATION PROBLEM 6

CONJUGATE GRADIENT METHOD PROBLEM II



DATA SCIENCE @ UNIVERSITAT DE BARCELONA

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1 Problem description

Consider the conjugate gradient method applied to the minimization of

$$f(\mathbf{x}) = \frac{1}{2} \mathbf{x}^T A \mathbf{x} - \mathbf{b}^T \mathbf{x}$$

where A is a positive definite and symmetric matrix.

Show that the iterate \mathbf{x}_k minimizes f over

$$\mathbf{x}_0 + \langle \mathbf{v}_0, A\mathbf{v}_0, \dots, A_{k-1}\mathbf{v}_0 \rangle$$

where $\mathbf{v}_0 = \nabla f(\mathbf{x}_0)$, and $\langle \mathbf{v}_0, A\mathbf{v}_0, \dots, A_{k-1}\mathbf{v}_0 \rangle$ is the subspace generated by $\mathbf{v}_0, A\mathbf{v}_0, \dots, A_{k-1}\mathbf{v}_0$.

2 Solution