Optimization

MASTER IN FUNDAMENTAL PRINCIPLES OF DATA SCIENCE

# Optimization Problem 4
## Quadratic method minimization problem

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# 1   Problem description

Let $f$ be a real function on $\mathbb{R}^n$. Also let $x_0 \in \mathbb{R}^n$, $z \in \mathbb{R}^n$, and $\theta \in \mathbb{R}$. Define

$$F(\theta) = f(x_0 + \theta z)$$

and suppose that we are looking for the minimum of $F$ (that is, for the minimum of $f$ in the direction $z$ through the point $x_0$). Let $x_0 + \theta_1 z$, $x_0 + \theta_2 z$ and $x_0 + \theta_3 z$ be three points where $f$ is evaluated. Show that the minimum predicted by applying the quadratic approximation method is $x_0 + \theta^* z$, where

$$\theta^* = \frac{[\theta_2^2 - \theta_3^2]F(\theta_1) + [\theta_3^2 - \theta_1^2]F(\theta_2) + [\theta_1^2 - \theta_2^2]F(\theta_3)}{2[(\theta_2 - \theta_3)F(\theta_1) + (\theta_3 - \theta_1)F(\theta_2) + (\theta_1 - \theta_2)F(\theta_3)]}$$

and it is indeed the minimum of the parabola passing through the above three points if

$$\frac{(\theta_2 - \theta_3)F(\theta_1) + (\theta_3 - \theta_1)F(\theta_2) + (\theta_1 - \theta_2)F(\theta_3)}{(\theta_2 - \theta_3)(\theta_3 - \theta_1)(\theta_1 - \theta_2)} < 0$$

# 2   Solution