Volatility Protocol - Model-Free Methodology

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1 Overview

The model-free methodology estimates the expected 14-day volatility for any token for which liquid options markets exist (e.g. ETH and BTC). It uses real-time out-of-the-money weekly call and put options data. The index is a weighted average of a specific subset of the options expiring on the Friday before and the Friday after the present time plus 14 days.

For each volatility feed an inverse volatility feed is also created whose symbol is denoted by prepending an "i" to the symbol for the original index (e.g. volETH and ivolETH respectively). When volatility moves up the inverse feed moves down by an equivalent percentage, and vice versa. This document uses ETH as an example and shows how the volatility feed for ETH and its inverse are created. We calculate volETH using a similar methodology to that which underpins the VIX® Index, the premier volatility benchmark for the U.S. stock market.

(Note: This methodology works for any N-day advance expected volatility by selecting the closest option chains expiring before and after the target date (as discussed below in Section 2, Step 1) and by appropriate modification of the temporal weights in formula 3. For expository purposes this paper describes the 14-day advance volatility calculation. Any N-day advance indices published by Volatility Group will use the method described in this paper with appropriate modifications to the selection of the near-expiration options, the next-expiration options, and T_1, T_2, and T_{14} in formula 3. See Section 2 for details on these quantities.)

2 Method of Calculation

This method builds upon the conceptual work of [3] and modifies the algorithm created by [1] with a few significant changes. See [2] for the derivation of the valuation of a variance swap on which the formula defining the VIX® is based. The present, modified procedure is as follows:

1. Let t_{14} be fourteen days from the time of the index calculation. The options expiring at 8am UTC on the Friday preceding t_{14} are called the *near-expiration* options. The options expiring at 8am UTC on the first Friday after t_0 are called the *next-expiration* options.

2. Separately, for both the set of near-expiration options and next-expiration options:

   (a) Determine the *forward strike* K^*. This is defined as the strike for which the prices for the call and put options at that strike have the smallest absolute difference among all the pairs of options.

   (b) Determine the *forward level*:

   $$F = K^* + e^{RT}(C^* - P^*) \tag{1}$$

   where:

   - K^* is the forward strike as defined in the previous step.
   - C^* is the price for the call option with strike K^*.
   - P^* is the price for the put options with strike K^*.
   - R is the annual risk-free interest rate.
   - T is the time to expiration in years.

   (c) Determine the *at-the-money strike price* K_0. This is defined as the greatest strike price less than or equal to the forward level F.

   (d) Take the midpoint prices of all put options with strike less than K_0, the midpoint prices of all the call options with strike greater than K_0, and the average of the midpoint prices of the two options at the strike K_0. We call this subset of options the *admitted* options and the midpoints we designate as $Q(K)$ for each admitted option at strike K.

   (e) We now calculate σ^2 defined as:

   $$\sigma^2 = \frac{2e^{RT}}{T} \sum_i \frac{\Delta K_i}{K_i^2} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_0} - 1\right)^2 \tag{2}$$

   where:

   - The sum is over all admitted options.
   - $Q(K_i)$ is the midpoint price for option with strike K_i.
   - ΔK_i is half the difference between the strikes on either side of K_i unless K_i is the least or greatest admitted strike in which case ΔK_i is the difference between the adjacent strike.
   - As defined previously, T, R, F, K_0 are time to expiration, risk-free interest rate, the forward level, and the at-the-money strike respectively.

3. Take the weighted average of the values σ_1^2 and σ_2^2 obtained in Step 2 above for the near-expiration options and the next-expiration options:

$$volETH = 100 \times \sqrt{\left[T_1\sigma_1^2\left[\frac{T_2 - T_{14}}{T_2 - T_1}\right] + T_2\sigma_2^2\left[\frac{T_{14} - T_1}{T_2 - T_1}\right]\right] \times \frac{365}{14}} \quad (3)$$

where:

- T_1 and T_2 are times until near and next expiration in (fractional) years.
- $T_{14} = \frac{14}{365}$.

We now describe two details regarding the above definition.

1. All midpoint values are defined as being the mean of the least ask price and the greatest bid price. If a bid price is zero then remove this option from all further calculations. If there is a non-zero bid price and no ask price the mark price is used in place of the midpoint. If the midpoint is 1.5x greater than the mark price then the mark price is used in place of the midpoint.

2. In step 1c above, in the rare case there is only one valid option price with strike K_0, either call or put, use the price of the single available option in place of the average. In the even more rare case where neither the put option price nor the call option price with strike K_0 are valid then the index is undefined. By arbitrage, both of these situations are very short-lived.

3 Example

Here we present an example calculation to clarify the concepts in the previous section, following the numbering of that section.

1. Suppose the time of calculation is Monday, February 1, 2021 at 2pm UTC. The volatility index calculation estimates the volatility for February 15, 2021 at 2pm UTC. The near-expiration options are those expiring on February 12, 2021 at 8am UTC. The next-expiration options are those expiring on February 19, 2021 at 8am UTC.

2. We now illustrate the calculation for σ_1^2 using the near-expiration options. The process for the next-expiration options yielding σ_2^2 is identical.

   (a) For the near-expiration options we determine the forward strike $K^* = 1360$ because at that strike price the absolute difference between the call price and the put price, 30.175, is least among all available option pairs.

Options expiring 2/25/21 (near-expiration)			
Strike	Call midpoint	Put midpoint	Absolute Difference
1120	null	27.855	null
1200	187.365	49.73	137.635
1280	131.955	81.555	50.4
1360	95.485	125.55	30.175
1440	69.955	null	null
1520	52.05	null	null
1600	38.79	311.055	272.265

(b) Since $K^* = 1360$ we see that $C^* = 95.485$ and $P^* = 125.55$ allowing us to calculate the forward level F using Equation 1. For example if the risk-free rate $R = .0056$ and the time to expiration $T = .02898$ then $F = 1329.82$.

(c) Since $F = 1329.82$ we obtain $K_0 = 1280$ because this strike is the largest strike price less than or equal to F.

(d) The admitted put options are those with strike price less than K_0.

Admitted Put Options	
Strike	Put midpoint
1120	27.855
1200	49.73

The admitted call options are those with strike price greater than K_0.

Admitted Call Options	
Strike	Call midpoint
1360	95.485
1440	69.955
1520	52.05
1600	38.79

We average the call and put midpoints at K_0.

Admitted options at K_0			
Strike	Call midpoint	Put midpoint	Average midpoint
1280	131.955	81.555	106.755

(e) Substituting this data for all the admitted options into Equation 2 yields σ_1^2.

3. Given values calculated for σ_1^2 and σ_2^2 in the previous step, Equation 3 and the easily calculated time parameters now yield the index value.

References

[1] M. Brenner and D. Galai. “New financial instruments for hedge changes in volatility”. In: *Financial Analysts Journal* 45 (4 1989), pp. 61–65.

[2] John C. Hull. *Options, Futures, and Other Derivatives, Ninth Edition.* Pearson Press, 2015. ISBN: 978-0-13-345631-8.

[3] R. E. Whaley. “Derivatives on market volatility: Hedging tools long overdue”. In: *The journal of Derivatives* 1 (1 1993), pp. 71–84.