Firm Optimization: Monopoly

The objective is to model the channel through which the riskiness of land, as an input, is transmitted into the portfolios of households. Beginning with the uncertain input, land, $D_t = (1-\phi_t)D_{t-1} - \theta t + \sigma\epsilon_t$. The modelling restrictions are as follows: $\bullet D_0 >> 0 \bullet \epsilon_t$ is IID white noise

The monopoly firm has the output function of the type $h_t = min\{g(k_t, l_t), \phi_t D_t\}$, where $g(k_t, l_t)$ is the general production function of the intermediate input, ϕ_t is the rate of utilisation of land input by the firm at time t, and h_t is the output of house. Additionally, $g(k_t, l_t) < f(k_t, l_t)$.

Then the monopoly firm will maximise the present value of the lifetime expected profits. Imposing an ad-hoc inverse demand function that is highly inelastic, that is the usual case for monopoly firm, so that, $P_t = P(h_t) = a - b \cdot h_t$. This is the demand for the houses from the households. Let the concave cost function for the firm in the long run be simply $C(h(t)) = \alpha h_t^2$.

The tractable property of the Leontief production function that models the perfect complementary nature of land with the intermediate building inputs implies that $h_t = \phi_t D_t \quad \forall t$. Then the expected profit of the firm at time t, based on the information set till time $t-1$ is,

$\mathbb{E}[\pi_t|\Omega_{t-1}] = \mathbb{E}\left[(a - b\phi_t D_t) \cdot \phi_t D_t - \alpha\phi_t^2 D_t^2\right]$

$\therefore \mathbb{E}[\pi_t|\Omega_{t-1}] = \mathbb{E}\left[a\phi_t D_t - \phi_t^2 D_t^2(b+\alpha)\right]$

Thus, the optimization of the monopoly firm is to maximise the lifetime expected profits,

$$\max_{\{\phi_t\}_{t=0}^{\infty}} \Sigma_{t=0}^{\infty} \beta^t \mathbb{E}\left[a\phi_t D_t - \phi_t^2 D_t^2(b+\alpha)\right]$$

subject to the risky input $D_t = (1-\phi_t)D_{t-1} - \theta t + \sigma\epsilon_t$

Firm Optimization: Perfect Competition

There is a perfect competition sector that produces the output commodity. The perfectly competitive firms have the production function, $f(k_t, l_t) = k_t^{\alpha} l_t^{1-\alpha}$ and the amount of this output that is used in the production of houses is given by $g(k_t, l_t) < f(k_t, l_t)$. This output is used for the household consumption c_t, for capital accumulation and for $g(k_t, l_t)$. Then, the capital flow in the economy is given as follows, $k_t = f(k_{t-1}, l_{t-1}) - \phi_{t-1}D_{t-1} - c_{t-1} + (1-\delta)k_{t-1}$, since it is known from the Leontief Production function of the monopoly builder that $g(k_t, l_t) = \phi_t D_t$ and in the continuum of households, $\Sigma_i c_{it} = c_t \quad \forall t$. Then the perfectly competitive firms minimise their costs, $w_t l_t + r_t k_t$ subject to the output constraint, $\phi_t D_t < f(k_t, l_t)$, there must be a slackness condition since the firms must produce at least enough for the housing firm.

Household Optimization

There is a continuum of identical households (as of version 0.1) that own houses for their personal use and also derive a wealth accumulation from the ownership. The inelastic inverse market demand function for the stock of houses is given by $P(h_t) = a - b \cdot h_t \quad \forall t$, then the demand for housing stock by the ith household is $P_i(h_t) = w_{it}P(h_t) \implies \Sigma_i w_{it} = 1 \quad \forall t$.

The demand for housing stock by the ith household at time t is given by, $P_i(h_t) = w_{it} \cdot (a - bh_t)$ and the ith household has stock of housing equal to $h_{it} = w_{it} \cdot h_t$. Then the market demand at time t is simply $P(h_t) = \Sigma_i P_i(h_t) = 1 \cdot (a - bh_t)$, and this is used by the monopoly firm, where, $\Sigma_i w_{it} = 1 \quad \forall t$. The ith household derives utility from the consumption of the general good, c_{it} and from the consumption of housing, $c_{it}^h \leq w_{it}h_t \quad \forall t$.

The evolution of wealth [1] of the ith household is given as follows,

$W_{it} = W_{it-1} \cdot \Big(w_{it-1}h_{t-1}\frac{P_t}{P_{t-1}} + (1 - w_{it-1})(1+r)B_{t-1}\Big) - c_{it-1} - c_{it-1}^h + \omega_{t-1}$

The wealth equation states that the actual consumption of housing units does not indicate that the stock of housing depreciates for the household, but rather that all maintenance work done on the housing stock is paid for from the wealth accumulation.

The household lifetime utility optimization is then to $\max_{\{c_t, w_t, c_t^h, l_t\}} \Sigma_{t=0}^{\infty} \frac{min\{c_t^q(1-l_t)^{(1-q)}, c_t^h\}^{1-\sigma} - 1}{1-\sigma}$, subject to the wealth evolution.