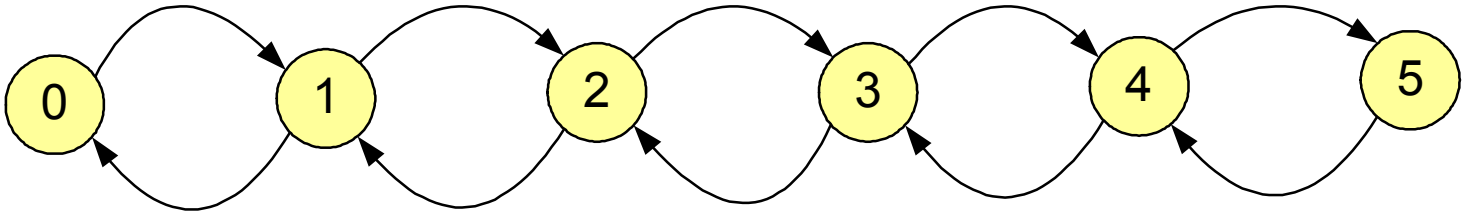
A gasoline station has **three self-service pumps**.

- The time required for a customer to pump gas has an exponential distribution with a mean of **3 minutes**.
- In addition to the space at the pumps, there is room for two or more cars to wait.
- Cars arrive at random according to a Poisson process at an average rate of **60/hour** when there is room at the pumps.
- When all pumps are in use, the arrival rate drops to **40/hour**.
- When one customer is waiting, the arrival rate drops to **20/hour**.
- When both waiting spaces are full, no arrivals occur.

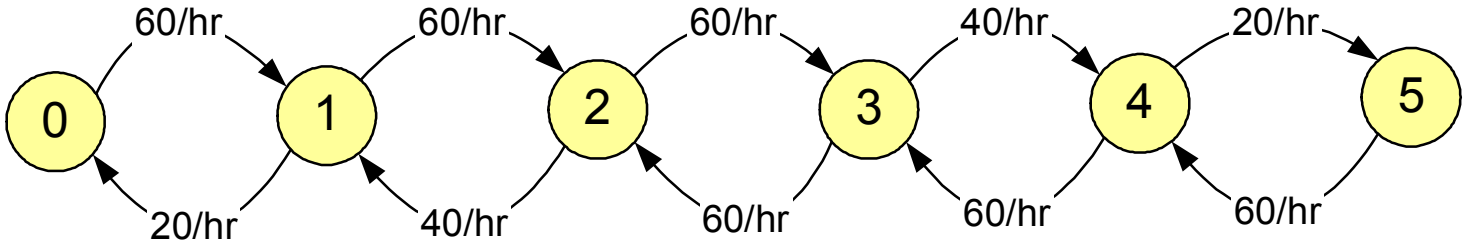
Calculate

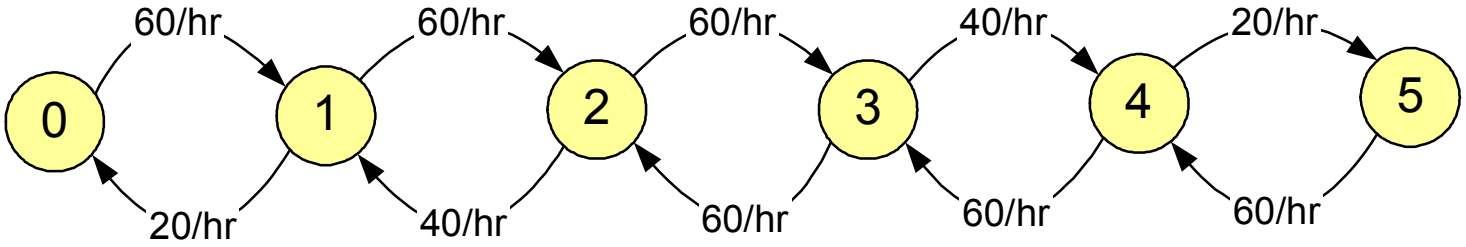
- steady-state probabilities
- average number of customers either waiting or in service
- proportion of customers who are lost because of balking.

Birth-Death Model:



What are the "birth" and "death" rates?


0
1
2
3
4
5
60/hr
60/hr
60/hr
40/hr
20/hr
20/hr
40/hr
60/hr
60/hr
60/hr



$$\frac{1}{\pi_0} = 1 + \left(\frac{60}{20}\right) + \left(\frac{60}{20}\times\frac{60}{40}\right) + \left(\frac{60}{20}\times\frac{60}{40}\times\frac{60}{60}\right) + \left(\frac{60}{20}\times\frac{60}{40}\times\frac{60}{60}\times\frac{40}{60}\right)$$
$$+ \left(\frac{60}{20}\times\frac{60}{40}\times\frac{60}{60}\times\frac{40}{60}\times\frac{20}{60}\right)$$
$$= 1 + 3 + \left(3\times\frac{3}{2}\right) + \left(3\times\frac{3}{2}\times 1\right) + \left(3\times\frac{3}{2}\times 1\times\frac{2}{3}\right) + \left(3\times\frac{3}{2}\times 1\times\frac{2}{3}\times\frac{1}{3}\right)$$
$$= 1 + 3 + 4.5 + 4.5 + 3 + 1$$
$$= 17$$

$$\textit{so that } \pi_0 = \frac{1}{17}, \pi_1 = \pi_4 = 3\pi_0, \pi_2 = \pi_3 = 4.5\pi_0, \pi_5 = \pi_0$$

Steady-state distribution

i	π_i
0	**0.0588235**
1	**0.176471**
2	**0.264706**
3	**0.264706**
4	**0.176471**
5	**0.0588235**

Average arrival rate:

$\pi_0 \times 60/hr =$ 3.52941

$\pi_1 \times 60/hr =$ 10.5882

$\pi_2 \times 60/hr =$ 15.8824

$\pi_3 \times 40/hr =$ 10.5882

$\pi_4 \times 20/hr =$ 3.52941

$\pi_5 \times 0/hr =$ 0.00000

Total: **44.1176/hr**