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Skolkovo Institute of Science and Technology

MASTER'S THESIS

## Computational modeling of magnetic materials

Master's Educational Program: Materials Science

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Student \_\_\_\_\_

Dmitry Volkov  
June 18, 2021

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Research Advisor: \_\_\_\_\_

Artem R. Oganov  
Full Professor  
Habilitation, PhD

Moscow 2021

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Skolkovo Institute of Science and Technology

МАГИСТЕРСКАЯ ДИССЕРТАЦИЯ

## Компьютерное моделирование магнитных материалов

Магистерская образовательная программа: Материаловедение

Студент

Дмитрий Волков

Июнь 18, 2021

Научный руководитель:

Артём Ромаевич Оганов

Профессор РАН

д-р физ.-мат. наук

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# **Computational modeling of magnetic materials**

Dmitry Volkov

Submitted to the Skolkovo Institute of Science and Technology  
on June 18, 2021

## **Abstract**

Modern technology would be impossible without hard magnetic materials which play an important role in the advancement of industrial and scientific growth. They are invariably used in power generation and transmission, analogue and digital data storage, medical appliances like magnetic resonance imaging (MRI), magnetic therapy and drug delivery, sensors and actuators, scientific instruments, etc.

Despite this in the past 25 years, there has been no significant achievement in the discovery of new hard magnets and all the existing ones contain rare-earth elements (REEs) in their composition. Also interesting fact that due to the World Trade Organisation data, more than 90% of world rare-earth-elements mining situated in China. This situation makes rare-earth elements critical materials, and also, it's mean that all magnets production in the world strongly dependent on China suppliers. This situation is not satisfying for many countries which economy are strongly relay on these materials and also lead to a challenging task for specialists in material science to find a possible composition of strong hard magnets that will not contain REEs.

In the last few years, there have been a few attempts at systematically predicting the existence of new magnets applying various computational techniques. Unfortunately, all existing modern methods can exact only some elementary properties (stability, magnetic moment per cell, density of states, magnetic-crystalline anisotropy, etc.), staying absolutely blind to the critical temperature. As a consequence, most of the discovery computational efforts are usually spent on the materials with unsatisfying  $T_C$  and therefore with low technological potential. Hence Curie temperature is one of the most important optimization parameters in a computational search for novel magnetic materials.

To sum up in this work will be performed a computational search for the novel composition of ferromagnetic materials not containing REEs and estimation of their critical temperature by the newly developed method in order to find the most promising materials from the technological viewpoint.

Research Advisor:

Name: Artem R. Oganov

Degree: Habilitation, PhD

Title: Full Professor

# **Компьютерное моделирование магнитных материалов**

**Дмитрий Волков**

Представлено в Сколковский институт науки и технологий

Июнь 18, 2021

## **Реферат**

В наши дни сложно переоценить роль магнитных материалов в индустриальном и научном развитии. Повсеместное использование ферромагнетиков в тяжелой промышленности, цифровых приборах, накопителях (жесткие диски) и медицинском оборудовании (МРТ) делают этот класс материалов критически важным.

Несмотря на это, в последние 25 лет не было сделано никаких значительных открытий в области поиска новых ферромагнетиков. В то же время, наиболее широко используемые на сегодняшний день составы магнитных материалов неизменно включают в себя редкоземельные элементы (РЗЭ). Принимая во внимание, чрезвычайно неравномерное распределение добычи РЗЭ, на более чем 90% сконцентрированной в Китае, сложившаяся ситуация является не удовлетворительной для многих стран, экономика которых напрямую зависит от импорта этих материалов. Совокупность вышеперечисленных фактов ясно обозначает задачу поиска эффективной альтернативы уже существующим магнитам, состав которых не включал бы в себя критические или дорогие элементы.

Безусловно, в последние несколько лет было предпринято несколько попыток систематического поиска новых магнитов с использованием современных вычислительных алгоритмов, но, к сожалению, все разработанные на данный момент методы могут определять только некоторые элементарные свойства материала такие как: стабильность, магнитный момент на ячейку и т. д., при этом не давая данных о критической температуре. Как следствие, большая часть вычислительных ресурсов тратится не рационально: на материалы с неудовлетворительно низкой критической температурой и как следствие малым или отсутствующим технологическим потенциалом. Основываясь на вышесказанном, можно заключить, что на сегодняшний день температура Кюри является чрезвычайно важным параметром при оптимизации в вычислительном поиске новых магнитных материалов и объясняет актуальность разработки эффективного метода ее определения.

В представленной работе будет проведен вычислительный поиск новых структур ферромагнитных материалов, отвечающих обозначенным физическим и экономическим критериям с использованием эволюционного алгоритма. Для обнаруженных структур будет проведен расчет их критической температуры и магнитной анизотропии с целью их ранжирования и определения наиболее перспективных с технологической точки зрения.

Научный руководитель:

Имя: Артём Ромаевич Оганов

Ученое звание, степень: д-р физ.-мат. наук

Должность: Профессор РАН

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# Chapter 1

## Introduction

### 1.1 Background

Ferromagnetic materials (FM) known to humanity for more than 2000 years. It's been a long time since the first samples of “adamants” also known as “lodestones” amazed our ancestors by their unique properties. Since then, this class of materials was thoroughly analyzed and described by many generations of brightest scientists. But yet even in the 21st century, we still have many questions to answer to understand this mysterious phenomenon fully. This work will be attempted to disclose one of the properties characteristic of magnetic structures. Namely, magnetic temperature depending ordering, also known as critical temperature or Curie temperature. With the main goal of applying developed methodology for the search of new promising structures.

Uniqueness of FM materials comes from the ability to retain their magnetism even after an applied external magnetic field is disappearing. Such properties’ explanation lies in their high magnetocrystalline anisotropy providing stability of their magnetization direction concerning their crystal axes. Thus magnetic anisotropy is basically the source of coercivity and hysteresis, which distinguishes hard (permanent) magnets from soft magnetic materials. On the contrary to electromagnets, FM materials can create and support a magnetic field  $H$  in free space without the continuous expenditure of electric or other forms of energy, thus playing an essential role in the modern world.

FM materials not only becoming more common by replacing electromagnets for many purposes, including small motors, engines, and generators, they are also increasingly becoming irreplaceable components of many mass-market consumer goods like electrical devices and data storage, medical devices, industrial products, and are also part of countless new applications. As a result, demands for cheap and strong magnets have increased nearly a hundred-fold in the past few decades most people would be hard-pressed to identify where these magnets are found. Possible applications can be grouped into three main categories, namely uniform, nonuniform, and steady as it is shown in Table 1.1.

Table 1.1: Summary of the ferromagnetic materials applications.

Field	Magnetic effect	Type	Examples
Uniform	Zeeman splitting	Static	Magnetic resonance imaging (MRI)
	Torque		Alignment of magnetic powder
	Hall effect, magnetoresistance		Sensors, read-heads
	Force on conductor Induced emf	Dynamic	Motors, actuators, loudspeakers Generators, microphones
Nonuniform	Force on charged particles	Static	Beam control, radiation sources (microwave, uv; X-ray)
	Force on magnet	Dynamic	Bearings, couplings, Maglev
	Force on paramagnet		Mineral separation
Time varying	Varying field Force on iron Eddy currents	Dynamic	Magnetometers Switchable clamps, holding m Metal separation, brakes

Since the discovery of lodestone ( $\text{Fe}_3\text{O}_4$ ) ferromagnetic materials passed a great path. Carbon steels used till the late 19th century were replaced with Alnico alloys which are still used in the industry. The revolutionary introduction of rare-earth magnets in the second part of 20th century began with the discovery of their large magnetocrystalline anisotropy. First produced samples of  $\text{Sm}_5\text{Gd}$  reported in 1960 lead to an industrial breakthrough and further development in this field till the discovery of the most usable nowadays Nb-Fe-B compositions [1]. A summary of different ferromagnetic materials with the corresponding energy product can be found in Figure 1.1. The remarkable increase in the energy product in the magnetic materials was accompanied with the tremendous decrease in the volume of the magnets providing the same amount of magnetic energy. The current commercially available hard magnets are mainly ferrites, Alnico,  $\text{SmCo}_5$  and  $\text{Sm}_2\text{Tm}_{17}$  ( $\text{Tm}=\text{Co, Fe, Zr, and Cu}$ ),  $\text{SmFeN}$  and  $\text{NdFeB}$ .

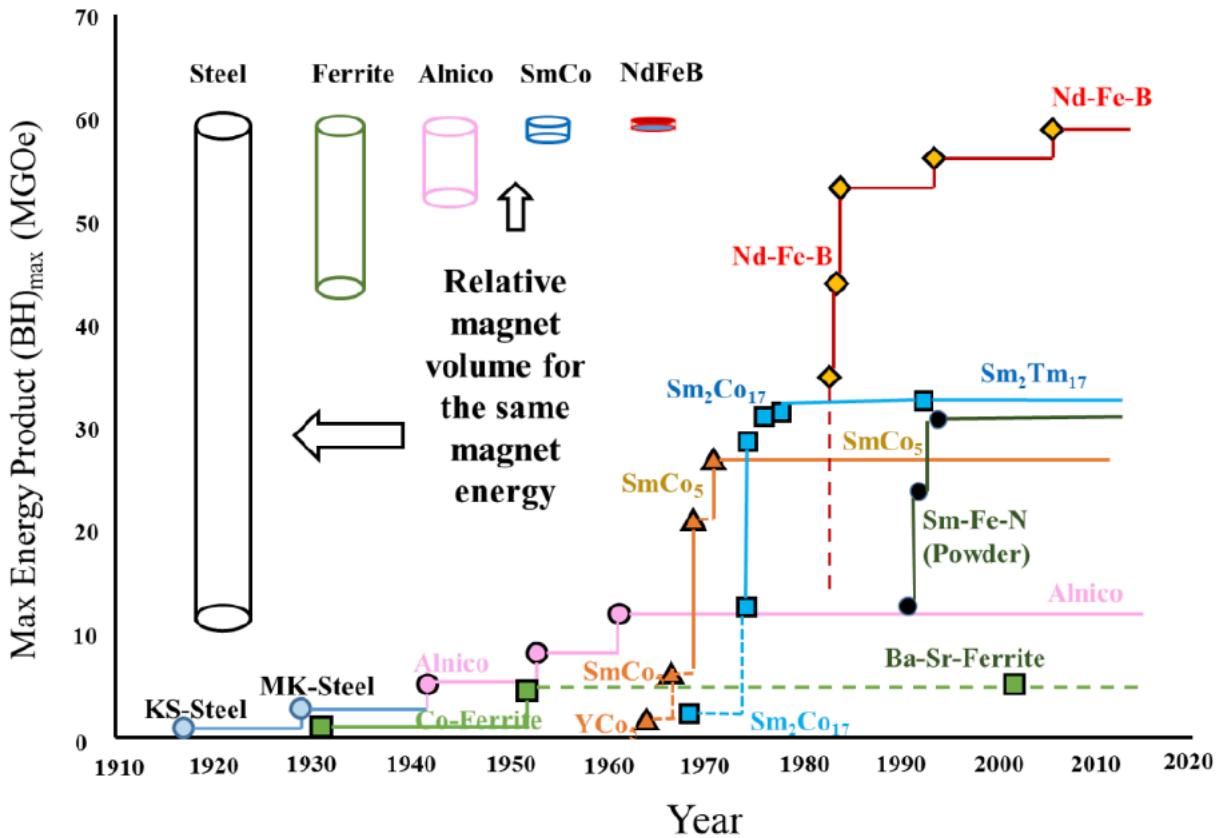


Figure 1.1: Development in energy product of permanent magnetic materials.

Nowadays, the rare-earth (REEs) FM became the most critical magnets in the market. Their dominating role might be explained due to their excellent magnetic properties at room temperature and their relatively low cost per energy-density unit. However, supply chain issues in recent years led to both prices and availability unpredictable issues.

The demand for hard magnetic materials becomes even bigger taking into account their numerous commercial applications in data storage, engines, actuators, generators, wind turbines, and hybrid cars. Not only increased demand but limited export from China has led to increased prices for these materials in the last years. These problems are forcing both industry and science to pay attention to the opportunities of developing new types of FM materials.

## 1.2 Aim and Goals

The ultimate goal of this study is to search for new technologically promising magnetic compositions. Considering the capabilities of modern evolutionary algorithms that allow identifying stable structures with a high magnetic moment, the problem of automated determination of the critical magnetic temperature (Curie point) remains unsolved. To solve the indicated problem and em-

power the end-user with a straightforward methodology for its calculation, the following research tasks are indicated:

1. Development and implementation of the critical magnetic temperature calculation methodology based on DFT approaches followed by Monte Carlo simulations.
2. Training of several classical ML models to solve a regression problem of critical temperature estimation based on the available experimental data.
3. Testing of the developed methods on compositions with known experimental and theoretical values of Curie temperature.
4. Computational search for new compositions with high magnetic properties using evolutionary algorithm USPEX.
5. Benchmarking of the newly find structures concerning their critical temperature to determine the most promising from the technological viewpoint.

### 1.3 Outline

Chapter 1 contains the current section with a problem background and formulation of the project objectives. Chapter 2 contains a general overview of the ferromagnetic materials properties and theoretical models used for this study. Chapter 3 describes the methodology used in this work, namely the construction of several machine learning (ML) models over available experimental data and calculations based on density function theory (DFT) followed by Monte-Carlo (MC) simulations. Chapter 4 is dedicated to the discussion of the results obtained by both methods. Finally, chapter 5 summarizes the research outcome and outlines possible future studies.

# Chapter 2

## Literature review

### 2.1 General Introduction to Ferromagnetics

#### 2.1.1 Origin of Ferromagnetism

In all materials, the atomic masses are from the atomic nucleus, while the electrical, chemical and magnetic properties are determined by the electronic structure. Electrons are distributed in specific shells at definite distances from the nucleus, which is due to the different energy level in each shell. In the FM elements, including  $\alpha$ -Fe, Co, Ni and Gd, there are three important features:

1. There must be an unfilled inner electron shell within the atom.
2. There must be uncompensated electronic spins in the unfilled inner shell.
3. The atoms must form a crystal lattice having a lattice constant at least 3 times the radius of the unfilled electron shell.

Electrons have an intrinsic property called spin that contributes to their magnetic moment. The classical model of electron spin is the electron spinning on its axis, i.e. either spin-up or spin-down. Macroscopic magnetic properties of materials are a consequence of magnetic moments associated with individual electrons, which is the sum of spin moment Figure 2.1a and orbital moment Figure 2.1b.

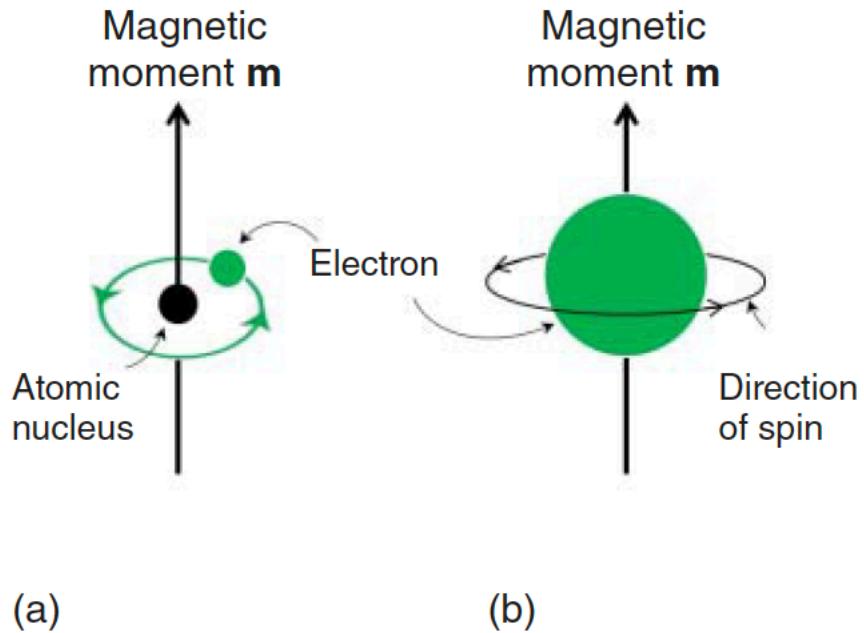


Figure 2.1: Diagram showing the magnetic moment associated with (a) orbital motion and (b) spin motion of an electron.

The magnetic susceptibility  $\chi$  is a dimensionless proportionality constant that indicates the degree of magnetization of a material in response to an applied magnetic field, and is given by:

$$\chi = \frac{M}{H} \quad (2.1)$$

where  $\chi$  is the magnetic susceptibility,  $M$  is the magnetization of the material, and  $H$  is the applied field.

Depending on the magnetic ordering and the sign, magnitude and temperature dependence of the magnetic susceptibility, the magnetic materials are classified into diamagnetic, paramagnetic, ferromagnetic (FM), antiferromagnetic (AFM) and ferrimagnetic (FiM) materials.

### 2.1.2 T-symmetry

Generally T-symmetry or Time Reversal symmetry is the symmetry of physical laws under a time reversal transformation ( $\tau$ ).

$$\tau : t \rightarrow -t \quad (2.2)$$

The origin of ferromagnetism (FM) is time reversal symmetry breaking due to the existence of unpaired spin of electrons and the associated current [2]. This statement can be understood by considering that if time is reversed, the spin or current flow direction will thus be reversed and this results in magnetization direction reversion, i.e. time reversal symmetry breaking.

$$\tau : \mu = -\mu \quad (2.3)$$

In other words as it is illustrated on Figure 2.2 the local magnetic moment  $\mathbf{m}$  may be represented classically by a charge that dynamically traces an orbit, as indicated by the arrowheads. A spatial inversion produces no change, but time reversal switches the orbit and thus  $\mathbf{m}$ .

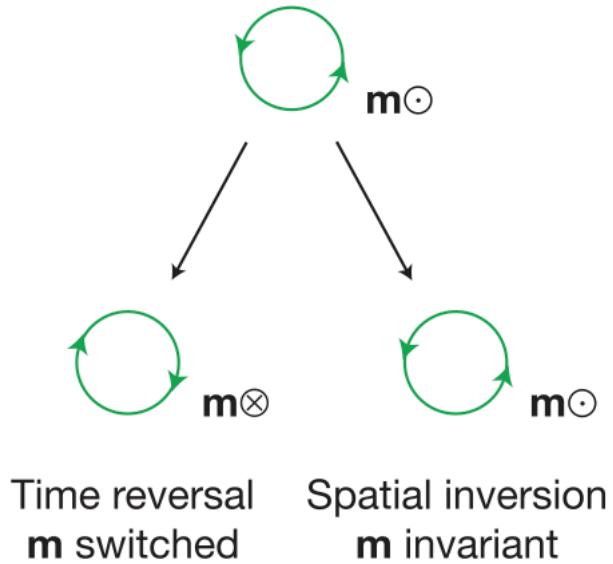


Figure 2.2: Illustration of time reversion symmetry breaking [2]

Thus, magnetic ordering always breaks at least one symmetry of the crystal, the invariance under time inversion. Invariance under rotations around axes not coinciding with the ordered moments also disappears. This symmetry braking is not related to asymmetries in the spin Hamiltonian. Spin Hamiltonians are constructed to describe the behaviour of the degrees of freedom of atoms or ions occupying lattice sites. Therefore, they have the full symmetry of the lattice. For instance, in a cubic lattice the bilinear term describing the interaction between magnetic moments can be written in the Heisenberg form:

$$H = -\frac{J_{ex}}{2} \sum_{i,k} (S_x^{(i)} S_x^{(j)} + S_y^{(i)} S_y^{(j)} + S_z^{(i)} S_z^{(j)}) \quad (2.4)$$

Here only nearest-neighbour interaction is included. Nearest neighbours are equivalent in a cubic lattice, hence the unique exchange parameter  $J_{ex}$ . In a tetragonal lattice,  $z$  is not equivalent with  $x$  and  $y$ , the interaction Hamiltonian will include two exchange parameters and have the form:

$$H = -\frac{J_{ex}}{2} \sum_{i,k} [J_{ex} (S_x^{(i)} S_x^{(j)} + S_y^{(i)} S_y^{(j)}) + J' S_z^{(i)} S_z^{(j)}] \quad (2.5)$$

Evidently, anisotropic exchange is allowed. Disregarding single-ion magneto crystalline anisotropy, if  $J'/J > 1$  the latter Hamiltonian describes a magnet with easy-axis anisotropy, if  $J'/J < 1$  with easy-plane anisotropy. In the case of extreme anisotropy we find the Hamiltonians which define two important models:

$$H_{Ising} = -\frac{J_{ex}}{2} \sum_{i,k} S_z^{(i)} S_z^{(j)} \quad (2.6)$$

$$H_{XY} = -\frac{J_{ex}}{2} \sum_{i,k} (S_x^{(i)} S_x^{(j)} + S_y^{(i)} S_y^{(j)}) \quad (2.7)$$

As the Ising and XY models exclude some components of the spin vectors, the entities they describe can be looked upon as one- and two-dimensional moments, respectively. However, this does not automatically make them low-dimensional models. The dimensionality is determined by the meaning of the indices  $i$  and  $j$ . Those indicate sites in a lattice of well defined dimension. Thus one can study three-dimensional Ising models or one-dimensional XY models; there is no contradiction in terms here.

### 2.1.3 Types of magnetism

In terms of magnetization, materials can be classified into paramagnetic, diamagnetic, ferrimagnetic (FiM), ferromagnetic (FM) and antiferromagnetic (AFM) as it is shown in Figure 2.3.

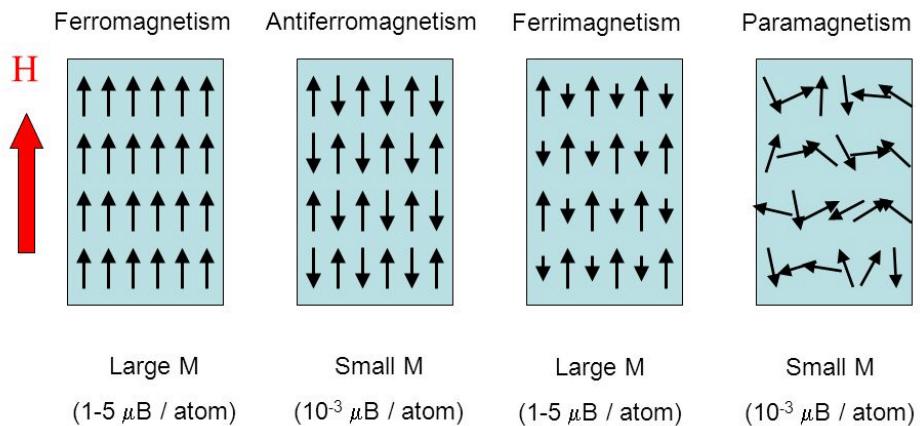


Figure 2.3: Schematic illustration of various types of magnetism.

A FM material usually forms permanent magnet or is attracted to magnets, and it undergoes phase change to paramagnetic above Curie temperature ( $T_C$ ) through a second-order phase transition. FM (including FiM) is the strongest type; it is the only type that creates forces strong enough to be felt and is responsible for the common phenomena of magnetism encountered in everyday life. An AFM has magnetization microscopically but has no macroscopic magnetization due to the cancelation of antiparallel spins of alternative layers. The temperature from an AFM to

paramagnetic phase transition is called Néel temperature. FiM is a relatively weak FM, such as  $\text{CoFe}_2\text{O}_4$  spinel. A diamagnetic generates weak but negative magnetization when a magnetic field is applied.

Figure 2.4 houses characteristic types of magnetization of pure elements in a solid-state and low temperature.

The periodic table displays the magnetic properties of elements. Elements are categorized into three main groups: paramagnetic (para), diamagnetic (dia), and ferromagnetic (Ferro). Specific sub-categories include Antiferromagnetic (AF) and Ferrimagnetic (Ferr). The table includes the following groups:

- Para (para):** Li, Be, Na, Mg, K, Ca, Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zr, Nb, Mo, Tc, Re, Rh, Pd, Ag, Hf, Tu, W, Re, Os, Ir, Pt, Au.
- Diamagnetic (dia):** He, B, C, N, O, Al, Si, P, S, Cl, Ar, Ga, Ge, As, Se, Br, Kr, Cd, In, Sn, Sb, Te, I, Xe, Hg, Tl, Pb, Bi, Po, At, Rn.
- Ferromagnetic (Ferro):** Fe, Ni, Co.
- Antiferromagnetic (AF):** Cr, Mn, Fe, Co, Ni, Cu, Zr, Nb, Mo, Tc, Re, Rh, Pd, Ag, Hf, Tu, W, Re, Os, Ir, Pt, Au.
- Ferrimagnetic (Ferr):** Fe, Ni, Co.
- Others:** Ce, Pr, Nd, Sm, Eu, Gd, Tb, Dy, Ho, Er, Tm, Yb, Lu, Tb, Pa, U, Np, Pu, Am, Cm, Bk, Cf, Es, Fm, Md, No, Lw.

Figure 2.4: The magnetic properties of pure elements in a solid state at low temperature.

A magnetic solid, made up of atoms with magnetic moment, has quantum exchange interactions that tend to align the magnetic moments at low temperature. When  $T < T_C$ , the macroscopic magnetization arises and retains even in the absence of a magnetic field. The magnetic moments tend to align in the same direction without the aid of an external magnetic field. This is known as the FM phase. In the FM category, materials are divided into strong and weak ferromagnets. By introducing the total particle number  $N = N \uparrow + N \downarrow$  and the spin polarization  $s = N \uparrow - N \downarrow$ , a strong ferromagnet has almost  $s = 100\%$  at the Fermi energy as shown in Figure 2.5a, while a weak ferromagnet gets a smaller spin polarization and paramagnet has no net spin polarization Figure 2.5 (b, c).

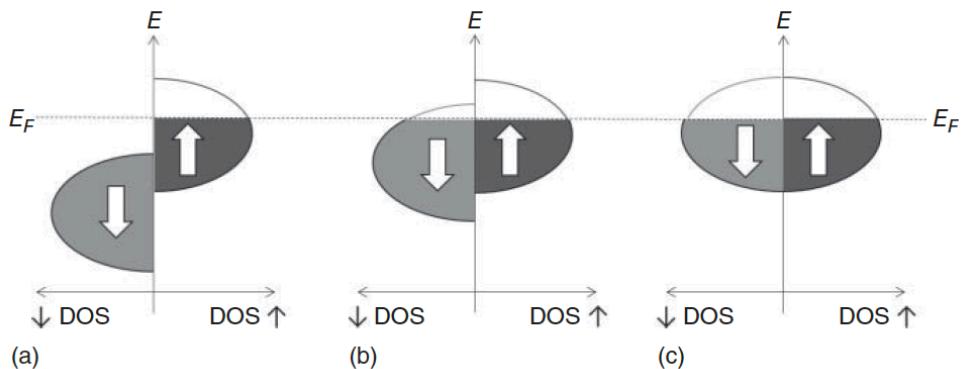


Figure 2.5: Schematic densities of states (DOSS) for ferromagnetism: (a) strong ferromagnet, (b) weak ferromagnet, and (c) paramagnet.

Meanwhile, there is another type of magnets that possess magnetization microscopically,

but have no macroscopic magnetization due to the cancelation of antiparallel spins of neighboring pairs – this is known as AFM phase. A magnet that exhibits no macroscopic magnetization at high temperature (when  $H = 0$ ) is known as the paramagnetic phase, in which the magnetic moment induced by the applied H-field is rather weak. Figure 2.6 shows the relationship between magnetization and temperature of a ferromagnet, where one can see that the magnetization starts decreasing close to  $T_C$ , and when  $T > T_C$ , magnetic moments align randomly resulting in zero macroscopic magnetization.

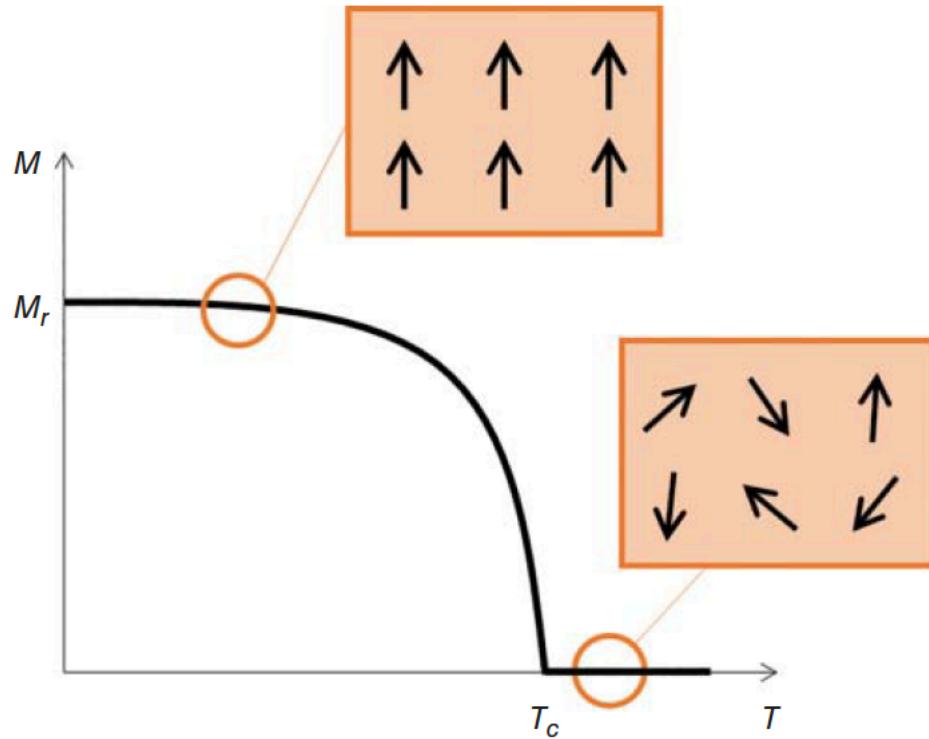


Figure 2.6: Illustration of magnetization versus temperature.

Any atoms or ions with existing unpaired electrons exhibit ferromagnetism such as Fe, Co, and Ni. Magnetization can exist, even very weak, in non-FM materials such as graphene as long as their defects or edge structures induce dangling bonds with unpaired electrons [3, 4]. However, those FM materials with practical application potentials are usually compounds containing Fe, Co, or Ni.

#### 2.1.4 Ferromagnetic Phase Transition

General description of FM to paramagnetic phase transitions might be given with free energy approach (Landau Free-Energy Theory) by writing out a series expansion for the free energy close to Curie temperature ( $T_C$ ). The magnetic free energy is defined as a power series in the order of magnetization  $M$ . By only considering the symmetries, the series must only contain terms that respect the symmetry of the order parameter:

$$F(M) = F_0 + \alpha M^2 + \beta M^4 + \dots \quad (2.8)$$

$F$  is minimized with respect to  $M$ :

$$\frac{\partial f}{\partial M} = 2\alpha M + 4\beta M^3 = 0 \quad (2.9)$$

$$\begin{cases} M = 0, \\ M^2 = -\frac{\alpha}{2\beta}. \end{cases} \quad (2.10)$$

In Figure 2.7a, the symmetry changes precisely at  $T = T_C$  and the transition occurs when  $\alpha$  changes its sign. When  $T < T_C$ , there are two energy minima  $M = \pm M_s$  that indicate  $\alpha < 0$  and  $\beta > 0$ . When  $T > T_C$ , a single energy minimum  $M = 0$  indicates  $\alpha > 0$  and  $\beta > 0$ . The temperature dependence of  $\alpha$  can be identified as:

$$\alpha(T) = \alpha_0 \left( \frac{T - T_C}{T_C} \right) \quad (2.11)$$

With the presence of an external magnetic field  $H$ , a linear term is added as:

$$F(M) = \alpha M^2 + \beta M^4 + \dots - \mu H M \quad (2.12)$$

As shown in Figure 2.7b, when an external magnetic field breaks the symmetry, the global minimum of  $F(M)$  changes from two minima to a single one.

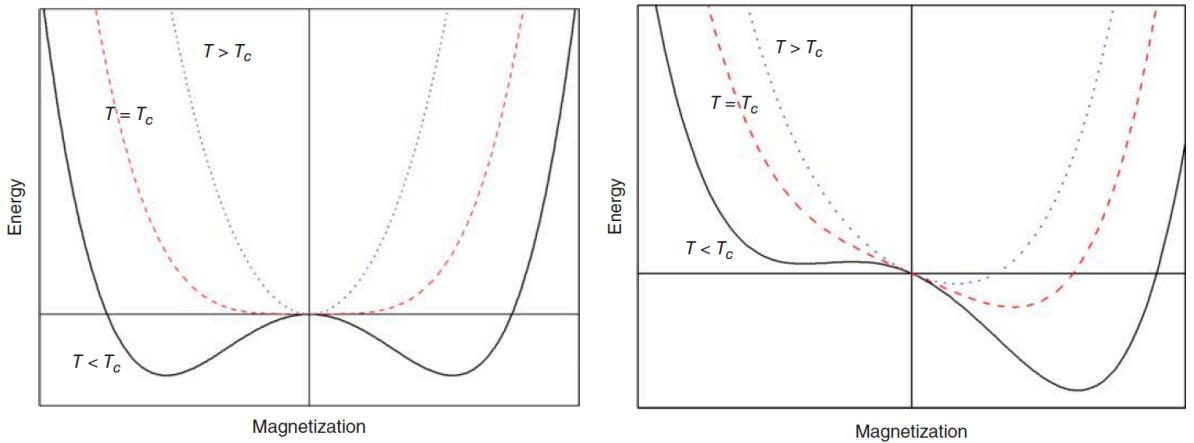


Figure 2.7: Sketch of Landau free energy

Therefore, we can see that even when  $T > T_C$ , a paramagnetic phase of material, which has no macroscopic magnetization, can also be attracted with the presence of an externally applied magnetic field.

## 2.2 Domain and Domain Walls

In FM, materials, exchange, and dipolar couplings are competing. The equality between both interactions leads to a state in which the local macroscopic magnetization is extremely small. Still, an almost parallel alignment of the moments is preserved at short distances. In the absence of magnetocrystalline anisotropy, a gradual rotation of the moments takes place. Overall, the material splits into a number of isolated zones, called magnetic domains. Within each domain, the moments are parallel as required by coupling, the magnetization is directed along a precise so-called easy direction, as required by anisotropy. From one zone to another, the direction of magnetization changes so that the total magnetization disappears. Between two neighboring domains, a gradual rotation of the moments takes place which defines a Bloch wall. A Bloch wall has a special parameter characteristic thickness,  $\delta$ , for which its value of energy per unit area,  $\gamma$ , is minimum.  $\delta$  and  $\gamma$  are given by :

$$\delta = \pi \sqrt{A/K} \quad (2.13)$$

$$\gamma = 4\sqrt{AK} \quad (2.14)$$

here  $A$ , the exchange constant expressed in  $J/m$ , characterises the strength of the exchange coupling and  $K$  is the anisotropy constant expressed in  $J/m^3$ . Most typically  $\delta$  varies from 50 nm in the 3d metals to 3 nm in high-anisotropy compounds ( $\text{SmCo}_5$ ,  $\text{Nd}_2\text{Fe}_{14}\text{B}$ ).

Table 2.1: Domain wall thickness

Magnetic material	Domain wall width (nm)
Co	14
$\text{Nd}_2\text{Fe}_{14}\text{B}$	3.9
$\text{Sm}_2\text{Fe}_{17}\text{N}_3$	3.6
$\text{Sm}_2\text{Co}_{17}$	8.6
FePd	11.4

In the general case, splitting of a FM material into domains separated by walls occurs spontaneously. In a homogeneous material, the wall energy does not depend on the wall position. Under an applied external field  $H$ , the initial magnetization variation is to minimize the Zeeman energy and the dipolar energy. In a system of cubic symmetry, the susceptibility is equal to the inverse of the demagnetizing field slope; it is defined by the sample shape and not connected to the intrinsic material properties. During this process, the magnetization variation occurs by growing the domains aligned by the applied field at the expense of others. The material is in the single domain state at a larger field, and magnetization variation occurs by moment rotation.

FM materials also exhibit domain structure where each domain has its own magnetization direction that can be switched by external magnetic field. There are easy axes in FM materials that are the directions magnetic moments should follow. For example, for a cubic structured FM material such as Fe, the  $\langle 100 \rangle$  directions are usually the easy axes, while for a hexagonal-structured FM material such as Co, the  $\langle 0001 \rangle$  are the easy axes. Due to this difference, in a cubic structure, both  $90^\circ$  and  $180^\circ$  magnetic domains can be formed (Figure 2.8a), while in a hexagonal structured FM material, the domains are usually aligned with an angle of  $180^\circ$  (Figure 2.8b).

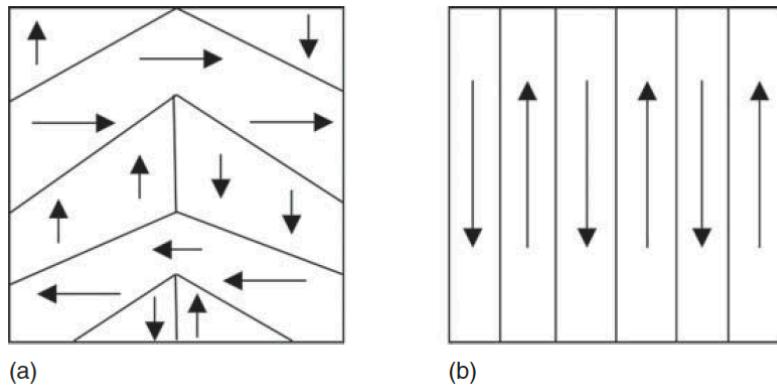


Figure 2.8: Domain structures in cubic (a) and hexagonal (b) structured FM materials.

The easy axes of cubic-structured Ni are  $\langle 111 \rangle$ , so it can form  $180^\circ$ ,  $71^\circ$ , and  $109^\circ$  magnetic domains; these are possible angles between all  $\langle 111 \rangle$  easy axes. The magnetization switching can be realized by not only external magnetic field but also mechanical stress.

FM domain wall is only one to a few atomic layers. Figure 2.9a illustrates the  $180^\circ$  domain wall structure in FM materials, where the more common one is the Bloch wall, but in thinner films a Néel wall is often favored Figure 2.9b. By contrast, Figure 2.9c demonstrates a narrow domain wall (Ising type) structure usually as the case of a ferroelectric domain structure.

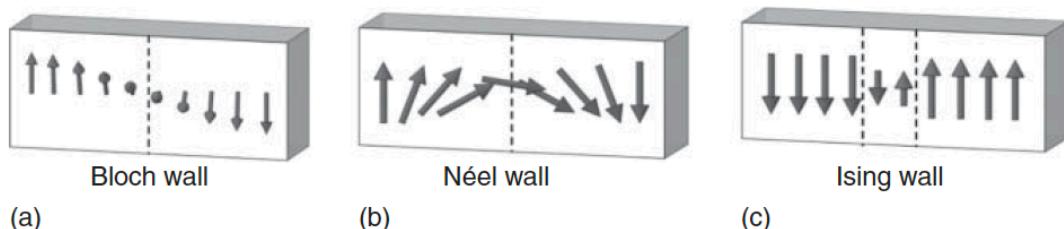


Figure 2.9: Domain wall structures of (a) Bloch-type FM domain walls, (b) Néel-type FM domain walls, and (c) Ising-type ferroelectric domain walls.

### 2.2.1 Magnetic anisotropy

Magnetic anisotropy is defined as the directional dependence of the magnetic properties for materials. Specifically, the preferential direction for its magnetic moment in the absence of an applied magnetic field. Strong easy-axis anisotropy is a prerequisite for hard magnetism, while near-zero anisotropy is desirable for soft magnets. Generally, the tendency for magnetization to lie along an easy axis is represented by the energy density term:

$$E_a = K_1 \sin^2 \theta \quad (2.15)$$

where  $\theta$  is the angle between the magnetic field and the anisotropy axis, and  $K_1$  is the anisotropy constant, which ranges from  $1k\text{J}/\text{m}^3$  to more than  $20M\text{J}/\text{m}^3$ . There are several sources of magnetic anisotropy:

- Magnetocrystalline anisotropy: intrinsic property due mainly to spin-orbit coupling;
- Shape anisotropy: induced by the nonspherical shape of the grains;
- Stress anisotropy: created by applied mechanical stress due to the existence of magnetostriction, which could alter the domain structure;
- Exchange anisotropy: occurs when the interaction between antiferromagnet and a ferromagnet occurs at their interface;
- Anisotropy induced by grain alignment and stress through magnetic annealing, irradiation, and plastic deformation:
  - Magnetic annealing: thermomagnetic treatment (heat treatment in a magnetic field) could introduce anisotropy in certain alloys;
  - Irradiation: when the materials are irradiated by neutron at high temperature in a magnetic field, the direction irradiated will become an easy axis;
  - Plastic deformation: plastic tension/compression would cause the specimen volume in tension/compression parallel to the deformation axis, which is the preferred axis to magnetize.

## 2.3 Magnetocrystalline anisotropy

The magnetocrystalline anisotropy primarily arises from spin-orbit coupling. When an external field tries to reorient the spin of an electron, the orbit of that electron also tends to be reoriented.

But the orbit is strongly coupled to the lattice and therefore resists the attempt to rotate the spin axis. The energy required to rotate the spin system of a domain away from the easy direction, anisotropy energy, is the energy required to overcome the spin-orbit coupling. The strength of the magnetocrystalline anisotropy in any particular crystal is measured by the magnitude of the anisotropy constant  $K_1$ ,  $K_2$ , etc.  $L1_0$  structure in this research is in tetragonal symmetry, conventional expression for the anisotropy energy in tetragonal symmetry is:

$$E_a = K_1 \sin^2 \theta + K_2 \sin^4 \theta + K'_2 \sin^4 \theta \cos 4\varphi + K_3 \sin^6 \theta + K'_3 \sin^6 \theta \sin 6\varphi \quad (2.16)$$

where  $K_i$  are the anisotropy constants,  $\theta$  is the angle between the magnetic field and the anisotropy axis,  $\varphi$  is the angle between the magnetization and the field. The magnitude of the magnetocrystalline anisotropy generally decreases with temperature more rapidly than the magnetization vanishes at the Curie temperature. Since the anisotropy contributes strongly to the coercive field, it has a great influence on industrial uses of FM materials. Materials with high magnetic anisotropy usually have high coercivity; that is they are hard to demagnetize. The high anisotropy of rare earth metals is mainly responsible for the strength of rare earth magnets, which are widely used in permanent magnets.

## 2.4 Heisenberg spin model

Magnetic systems are fundamentally quantum mechanical by nature therefore the exchange interaction - one of the most important properties of such systems also is a purely quantum mechanical effect. In addition to the properties at the electronic level, the properties of magnetic materials are heavily influenced by thermal effects which are typically difficult to incorporate into standard density functional theory (DFT) approaches. Therefore, models of magnetic materials should combine quantum mechanical properties with accurate thermodynamic theories. The simplest model of magnetism using this approach is the Ising model, which allows the atomic moments one of two allowed states along a fixed axis. The Ising model is quite useful as a descriptive system, but due to a large amount of approximation, the applicability of this model in relation to real materials is limited. The more accurate theory is the Heisenberg spin model which describes the essential physics of magnetic material at the atomic level, where the energetics of a system of interacting atomic moments is given by a spin Hamiltonian. The spin Hamiltonian typically has the form:

$$\mathcal{H} = \mathcal{H}_{exc} + \mathcal{H}_{ani} + \mathcal{H}_{app} \quad (2.17)$$

denoting terms for the exchange interaction ( $\mathcal{H}_{exc}$ ), magnetic anisotropy ( $\mathcal{H}_{exc}$ ), and externally applied magnetic fields ( $\mathcal{H}_{app}$ ) respectively. The dominant term in the spin Hamiltonian is

the Heisenberg exchange energy, which arises due to the symmetry of the electron wavefunction and the Pauli exclusion principle which governs the orientation of electronic spins in overlapping electron orbitals. Due to its electrostatic origin, the associated energies of the exchange interaction are typically up to 1000 times larger than the next largest contribution and gives rise to magnetic ordering temperatures in the range 300–1300 K [5]. The exchange energy for a system of interacting atomic moments is given by the expression:

$$\mathcal{H}_{exc} = - \sum_{i \neq j} J_{ij} S_i \cdot S_j \quad (2.18)$$

Here  $J_{ij}$  is the exchange interaction between atomic sites  $i$  and  $j$ ,  $S_i$  is a unit vector denoting the local spin moment direction,  $S_j$  is the spin moment direction of neighboring atoms. The unit vectors are taken from the actual atomic moment  $\mu_s$  and given by  $S_i = \mu_s / |\mu_s|$ . It is important to note here the significance of the sign of  $J_{ij}$ . For ferromagnetic materials where neighboring spins align in parallel,  $J_{ij} > 0$ , and for antiferromagnetic materials where the spins prefer to align antiparallel  $J_{ij} < 0$ . Due to the strong distance dependence of the exchange interaction, for quite accurate calculation of the given equation, it is often enough to include only nearest neighbors because the impact of others is neglectable small. In some cases, it allows to significantly reduces required computational resources still giving good results for many materials of interest, especially for simple ferromagnetic systems. In reality, the exchange interaction can extend to several atomic spacings, representing hundreds of pairwise interactions. According to the more fundamental approach in complex systems exchange interaction forms a tensor with components:

$$J_{ij}^M = \begin{vmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{vmatrix} \quad (2.19)$$

which is capable of describing anisotropic exchange interactions. In this case exchange energy:

$$\mathcal{H}_{exc} = - \sum_{i \neq j} [S_x^i S_y^j S_z^j] \begin{vmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{vmatrix} \begin{bmatrix} S_x^j \\ S_y^j \\ S_z^j \end{bmatrix} \quad (2.20)$$

Described model will be used in this work for the Curie temperature calculation using DFT followed by Monte Carlo simulation.

## 2.5 Monte Carlo simulations

Metropolis Monte-Carlo

## Chapter 3

# Materials and Methods

### 3.1 Data driven estimation of critical temperature

This work was mainly inspired by the publications related to the ML-driven estimation of the critical temperature for the superconducting [6] and ferromagnetic materials [7]. In the first work [6] were described several ML models for the estimation of the superconducting critical temperature over the 18000 samples dataset. The ML pipeline presented in this work consists of two models working sequentially. The first model classifies superconductors into three groups (Fe-based, cuprates, and low  $T_C$  superconductors). While the second one solves the regression problem for the particular class, in other words predicting the  $T_C$ . As it was reported classification models that worked solely on chemical composition information showed strong predictive power with an accuracy score of 92%. Regression Random Forest model worked already with compositional and structural descriptors. It showed  $R^2$  score of 0.85, 0.80 and 0.74 for these classes respectively

The second work [7] reported the construction of several ML models in a similar way to predict the critical temperature of FM with descriptors based only on chemical composition. The construction of proposed models did not involve any electronic structure calculations and has been entirely trained over available 2500 records of experimental data. The Random Forest algorithm gave the best-reported result with a mean absolute error (MAE) of 57 K and  $R^2$  value of 0.81 over the test set containing 767 compounds. Also, the authors report that they made several attempts to include structural information into the descriptors. Still, such an approach did not outperform the model's predictive power containing data obtained from chemical composition only.

#### 3.1.1 Data and predictors

The success of any ML method ultimately depends on access to reliable and plentiful data. Curie temperature data used in this work was extracted from the database provided by the authors of the described previously publication [7]. It houses 5092 experimental values of the critical temperature for ferromagnetic materials from various sources [8, 9].

### 3.1.2 Features generation

The problem which we are working on is an estimation of Curie temperature ( $T_C$ ) from the sample chemical composition. It might be classified as supervised ML for the regression. The goal of supervised ML is to find a function  $f(X)$  that maps the feature vector  $X$ , to the target variable  $y(T_C)$  and allows to interpolate a set of available data to the target property in the most accurate way. In our case feature vector ( $X$ ) contains only the information which might be extracted from the chemical composition of a given sample. Typically, ML employs an intermediate step between compiling raw data (chemical composition) and applying a machine learning algorithm. This step converts data from a raw format into a numerical representation which is more convenient for ML algorithms and visualization. There are no rigorous rules on how it should be done, but the performance of any ML models ultimately depends on the ability to choose the most relevant descriptors. In this work features were constructed according to the following criteria:

- Generation of features should not affect the size of the dataset (one should avoid the cases when descriptor could not be constructed for all the samples in dataset and missing values appears)
- They should be easy to compute, ideally some already known properties of compounds
- Features that allow distinguishing different materials with different chemical composition.

To accomplish this criterion were tested several existing algorithms from references [10–13], and finally made a decision to use ones described in ref. [14, 15] employing the Materials Agnostic Platform for Informatics and Exploration (Magpie). All produced descriptors might be divided into three groups:

1. Attributes which might be determined from the periodic law: Number ( $N$ ), Period ( $P$ ), Group ( $G$ ), Atomic weight ( $AW$ ) and at some sense Mendeleev Number ( $MN$ ).
2. Properties of a particular elements in a composition: Magnetic moment ( $\mu$ ), Bandgap ( $KS$ ), Melting Temperature ( $T_M$ ), Electronegativity ( $\varepsilon$ ), Covalent radius ( $R$ ), lattice volume ( $V$ )
3. Attributes of electronic structure, namely the fraction of electrons from the  $s$ ,  $p$ ,  $d$ , and  $f$  shells

For all categories of descriptors listed above was applied statistical functions defined below:

Table 3.1: Property statistics functions

N	Function	Symbol
1	Minimum	$x_{min}$
2	Maximum	$x_{max}$
3	Range	$\Delta x$
4	Mode	$\tilde{x}$
5	Mean	$\bar{x}$
6	Mean absolute deviation	$\langle x \rangle$

For instance, property Atomic Weight ( $AW$ ) has features: Minimum Atomic Weight ( $AW_{min}$ ), maximum Atomic Weight ( $AW_{max}$ ), range Atomic Weight ( $\Delta AW$ ), mode Atomic Weight ( $\widetilde{AW}$ ), mean Atomic Weight ( $\overline{AW}$ ), mean absolute deviation Atomic Weight  $\langle AW \rangle$ .

High dimensional descriptor  $v_{chem} = \{x_H, x_{He}, x_{Li}, x_{Be}\dots\}$  describes the atomic fraction of every element in a particular sample. For example,  $Mn_2O_3$  might be represented by the vector  $\{\dots, 0, 2/5, 0, \dots, 0, 3/5, 0, \dots\}$  with 3/5 assigned to the element in 8th position (Oxygen) and 2/5 to 25th (Manganese) other positions are equal to zero. The final dimensionality of generated descriptors is 215. All the constructed descriptors presented in Table 2.

Table 3.2: List of descriptors used in this work

N	Features	Symbol	Dim.
1	Atomic fraction vector	$v_{chem}$	90
2	Number	$AW_{min}, AW_{max}, \Delta AW, \widetilde{AW}, \overline{AW}, \langle AW \rangle$	6
3	Period	$P_{min}, P_{max}, \Delta P, \widetilde{P}, \overline{P}, \langle P \rangle$	6
4	Group	$G_{min}, G_{max}, \Delta G, \widetilde{G}, \overline{G}, \langle G \rangle$	6
5	Mendeleev Number	$MN_{min}, MN_{max}, \Delta MN, \widetilde{MN}, \overline{MN}, \langle MN \rangle$	6
6	Atomic weight	$AW_{min}, AW_{max}, \Delta AW, \widetilde{AW}, \overline{AW}, \langle AW \rangle$	6
7	Lattice Volume	$V_{min}, V_{max}, \Delta V, \widetilde{V}, \overline{V}, \langle V \rangle$	6
8	Magnetic moment	$\mu_{min}, \mu_{max}, \Delta \mu, \widetilde{\mu}, \overline{\mu}, \langle \mu \rangle$	6
9	Bandgap	$KS_{min}, KS_{max}, \Delta KS, \widetilde{KS}, \overline{KS}, \langle KS \rangle$	6
10	Melting Temperature	$T_{min}^M, T_{max}^M, \Delta T^M, \widetilde{T^M}, \overline{T^M}, \langle T^M \rangle$	6
11	Electronegativity	$\varepsilon_{min}, \varepsilon_{max}, \Delta \varepsilon, \widetilde{\varepsilon}, \overline{\varepsilon}, \langle \varepsilon \rangle$	6
12	Covalent radius	$R_{min}, R_{max}, \Delta R, \widetilde{R}, \overline{R}, \langle R \rangle$	6
13	s-shell electrons (filled/unfilled)	$N_{min}^s, N_{max}^s, \Delta N^s, \widetilde{N^s}, \overline{N^s}, \langle N^s \rangle$	12
14	p-shell electrons (filled/unfilled)	$N_{min}^p, N_{max}^p, \Delta N^p, \widetilde{N^p}, \overline{N^p}, \langle N^p \rangle$	12
15	d-shell electrons (filled/unfilled)	$N_{min}^d, N_{max}^d, \Delta N^d, \widetilde{N^d}, \overline{N^d}, \langle N^d \rangle$	12
16	f-shell electrons (filled/unfilled)	$N_{min}^f, N_{max}^f, \Delta N^f, \widetilde{N^f}, \overline{N^f}, \langle N^f \rangle$	12

### 3.1.3 Data pre-processing

Initially, the dataset contained 5092 samples. However, this data needed pre-processing since it included many compositions for which in literature was reported different values of  $T_C$ . There are several reasons for these occurrences. In some compounds magnetism is related to the quality of the sample, so the experiments performed by different groups and over different periods of time may report sufficiently different values of  $T_C$ . A second reason related to the polymorphism, namely to the existence of compounds with the same stoichiometry but different crystal structure and hence different values of  $T_C$  (e.g Fe-bcc and Fe-fcc). Since at this stage the aim of this project was to construct the ML model based only on the chemical composition, structural information or details of the experiment were not included in the descriptors. In other words, feature vectors ( $X$ ) of two samples with the same stoichiometry are indistinguishable even thou the target property ( $y$ ) differs and it is not possible to extract any information about polymorphism or sample quality. As such it was made a decision to establish criteria to exact a single value of  $T_C$  for the samples with the same stoichiometry. In this work was used the median of the distribution instead of mean or maximum value since it is less dependent on the possible outliers. For example, if for the same stoichiometry was reported 3 different values of  $T_C$  305, 330, and 600K they were replaced with a single median value of  $T_C = 330K$ . Of course, such an approach has drawbacks. In cases when we have an even number of samples with the same stoichiometries (e.g. 4 or 6)  $T_C$  also depends on a half sum of the distribution central values. But after all, it allows us to operate with more reliable values since most of them are associated with real experimental measurements but not with a mean value of several experiments. After this operation, the size of the dataset decreased almost by half and became 2557 samples, which might be considered as relatively small from the point of view of ML algorithms. Distribution of  $T_C$  temperatures in the pre-processed data shown on Figure 3.1.

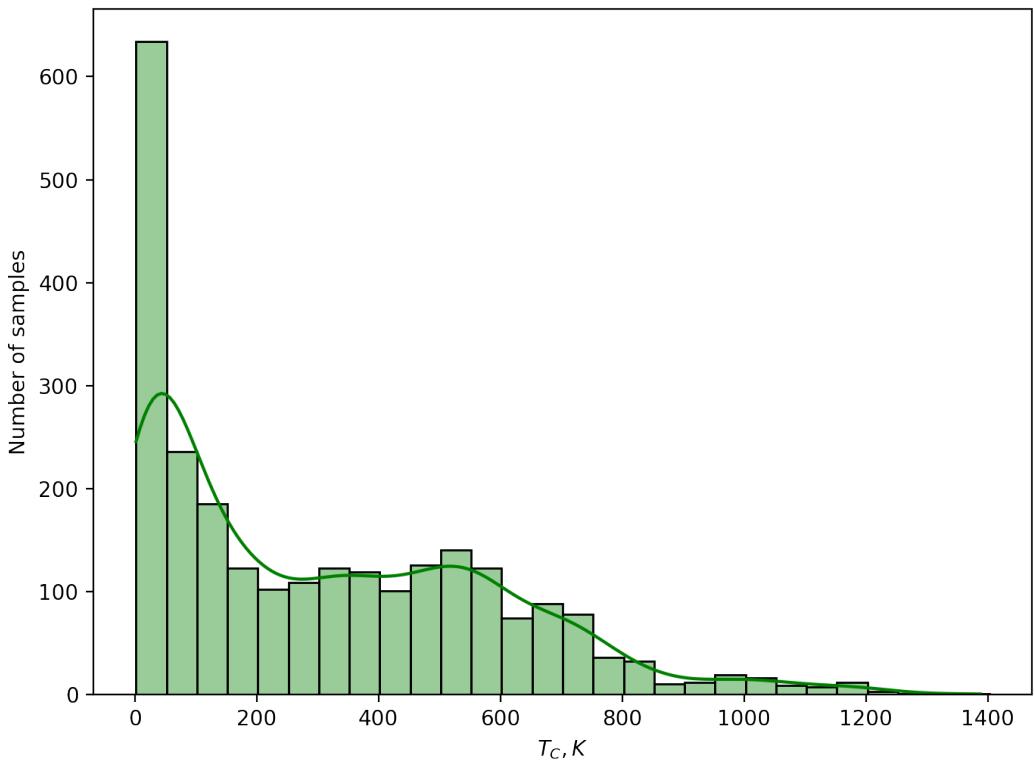


Figure 3.1: Distribution of critical temperature over 2557 samples.

### 3.1.4 Machine-learning models

#### Models evaluation

As it was already mentioned the size of our data is relatively small. This fact means that we were not able to follow the standard procedure of the model training and evaluation which requires splitting of the data into three equal and mutually exclusive partitions namely training, testing, and validation sets. In case when the dataset is not so big such split makes each of the subsets too small decreasing the overall ability to learn and thus accuracy. That's why the entire dataset was divided only into 2 not equal parts: 1st is a training set which included 2/3 of the data (1714 samples) and 2nd is testing set consisting of the remaining 834 samples. Schematically this process shown in Figure 3.2.

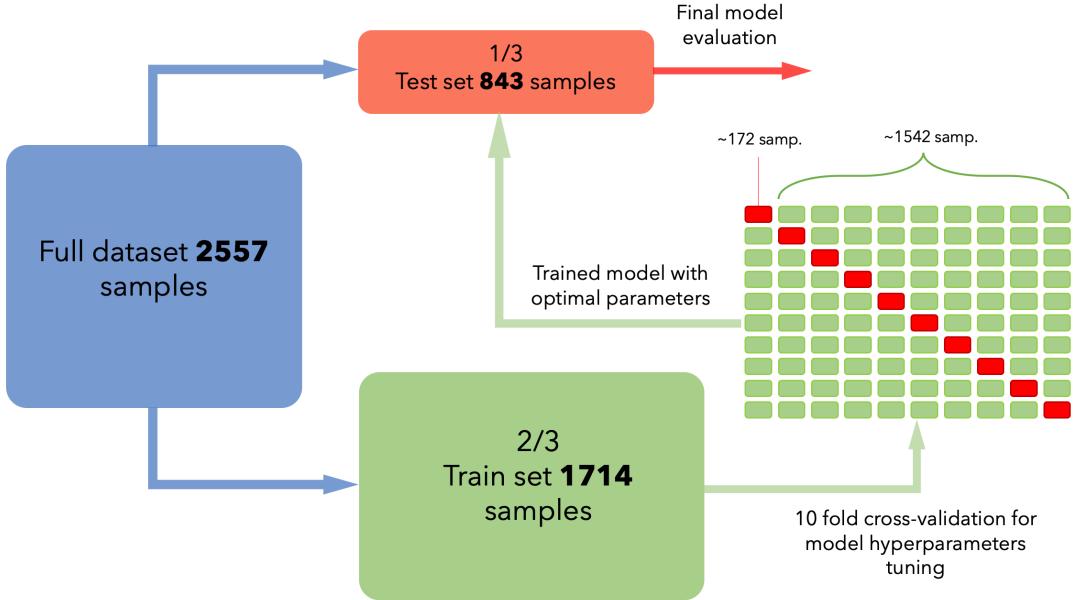


Figure 3.2: Schematic process of cross-validation process and final model evaluation on the test subset.

Hyperparameters optimization for all models was performed through the standard K-fold cross-validation procedure. The training set is split into K subsets (in this work K=10) then the model is trained over the K-1 sets and evaluated on the remaining one, thus the cross-validation error is an average error of the K evaluation rounds. The best model (model with optimum hyperparameters) is a model with the lowest cross-validation error. Finally, the test set was used for estimation the generalization error of the model, namely for assessing how well the given model performs on never-seen-before data.

Quantitatively model's performance were measured by the coefficient of determination  $R^2$ :

$$R^2 = 1 - \frac{\sum_i (y_i - f(x_i))^2}{\sum_i (y_i - \mu)^2} \quad (3.1)$$

and root mean squared error (RMSE):

$$RMSE = \sqrt{\frac{\sum_i (y_i - f(x_i))^2}{n}} \quad (3.2)$$

Here:  $y_i$  - target property;  $f(x_i)$  - predicted value;  $\mu$  - mean

For the perfect predictor, namely  $y_i = f(x_i)$ , these values should be equal to 1 and 0 respectively.  $RMSE$  and  $R^2$  were specified as the main optimization parameters for all trained models. Also, for the all models were estimated and reported maximum error but this parameter was not chosen as an optimization one. In the construction of ML models were compared four

different algorithms:

1. Kernel Ridge Regression (KRR)
2. Random Forest (RF)
3. Extreme Gradient Boosting (XGBoost)
4. Sure Independence Screening plus Sparsifying Operator (SISSO)

The first three models come from Python distributions Scikit-Learn [16] and XGBoost [17] respectively. SISSO is a separate algorithm on Fortran 90 [18, 19]. KRR, RF, and XGBoost were trained and evaluated following the 10-fold cross-validation and testing procedure described previously. SISSO was trained on entire dataset without accurate validation, due to the lack of computational resources. Hyperparameters tuning for every model will be described explicitly.

### **Kernel ridge regression.**

The first model which was chosen as a baseline for particular work is a Kernel Ridge Regression (KRR). Simple Ridge Regression was not considered since the sufficiently large dimensionality of the descriptors might significantly affect the performance of this model. Kernel ridge regression combines ridge regression (linear least squares with  $l_2$ -norm regularization) with the kernel trick. Which in compare to Ridge Regression might be written as follows: Ridge regression:

$$y = w^T \phi(x) \quad (3.3)$$

The idea is to minimize loss function (estimate the best weights):

$$J(w) = \sum_{i=1}^n (y_i - w^T x_i)^2 \quad (3.4)$$

Not kernilized solution of this equation for the nonlinear regression might be written as follows:

$$w = (\phi^T \phi + \lambda I)^{-1} \phi^T y \quad (3.5)$$

Kernel Trick allows as to sufficiently decrease the complexity of this solution, since the dot product  $\phi \phi^T$  can be found just as  $K(x_m, x_n)$  function:

$$w = (\phi^T (\phi \phi^T + \lambda I))^{-1} y \quad (3.6)$$

$$K(x_m, x_n) = \phi \phi^T \quad (3.7)$$

It thus learns a linear function in the space induced by the respective kernel and the data. For non-linear kernels, this corresponds to a non-linear function in the original space. In this work KRR algorithm was tuned with respect to the regularization parameter  $\lambda$  and different Kernels, namely:

$$K(x_m, x_n) = x_m^T x_n \quad (3.8)$$

$$K(x_m, x_n) = (x_m^T x_n + r)^d \quad (3.9)$$

$$K(x_m, x_n) = \exp\left(-\frac{\|x_m - x_n\|^2}{2\sigma}\right) \quad (3.10)$$

$$K(x_m, x_n) = \exp(-\alpha \|x_m - x_n\|^2) \quad (3.11)$$

It turns out that for all non-linear Kernels optimal value of optimization parameter  $\lambda$  is equal to 0.10 (tested in range 0.05 - 1). Best  $R^2$  and  $RMSE$  scores were achieved by the Laplacian Kernel which is also the most computationally demanding one (required almost 5 times more computational resources than others). The lowest value of maximum error was given by the simplest linear Kernel. Performance of different KRR models shown on scatter plots in Figure 3.3, and also in table 3.3.

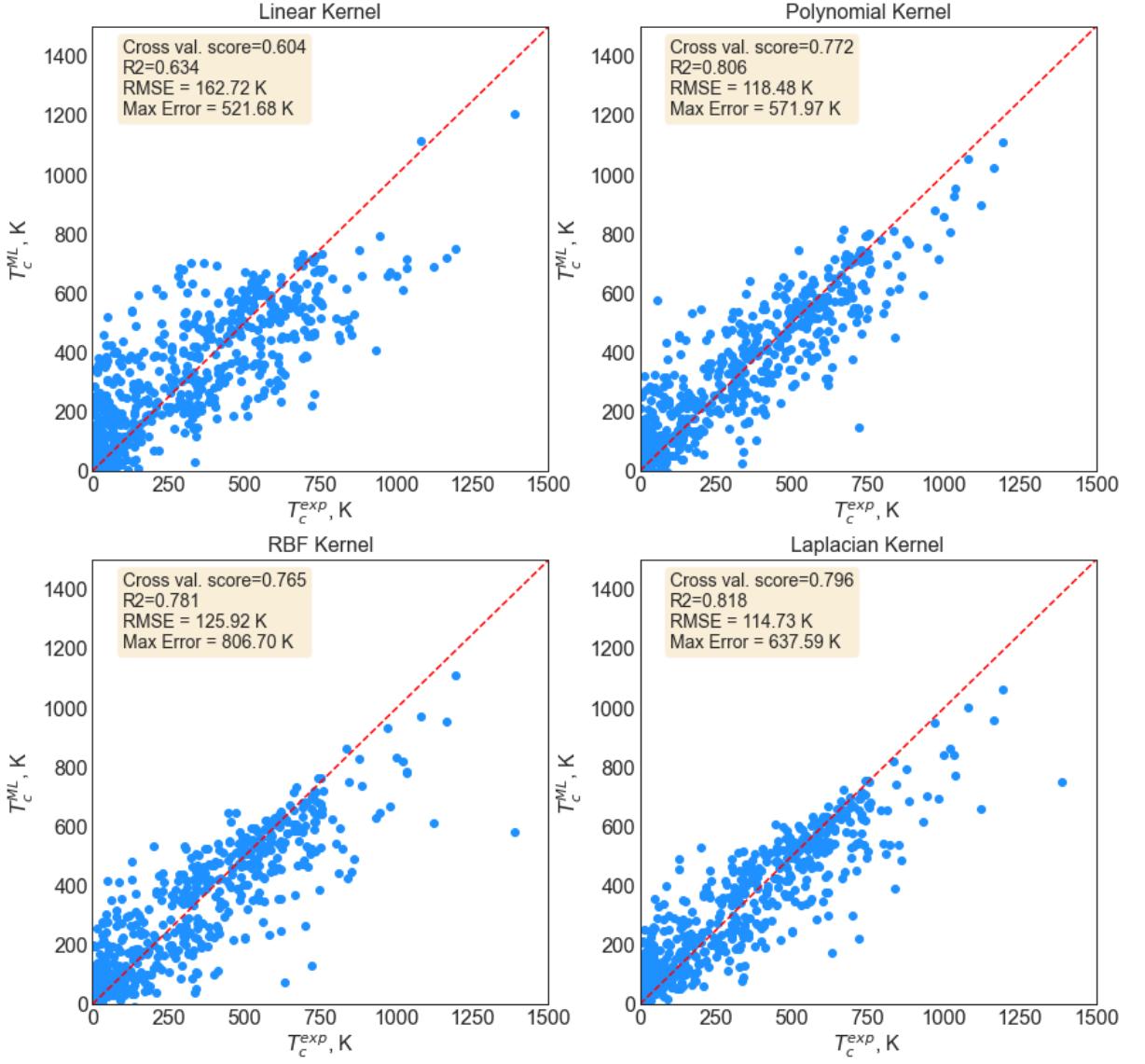


Figure 3.3: Comparison of experimental ( $T_c^{exp}$ ) and predicted values ( $T_c^{ML}$ ) for different Kernels.

Table 3.3: KRR models performance with respect to different Kernels

Metrics/Kernel	Linear	Polynomial	RBF	Laplacian
R2 (test)	0.616	0.759	0.756	0.793
R2 (cross.val)	0.601	0.768	0.754	0.787
RMSE (K)	161.9	131.4	132.3	121.7
Max Error (K)	598.1	783.4	795.6	642.05

### Random forest algorithm

Random forest (RF) is one of the most powerful, versatile, and widely used ML methods. RF, like its name implies, consists of a large number of individual decision trees (see Figure 3.4) that operate as an ensemble. Each individual tree in the RF trains only on a random part of the training

set samples and also only on the randomly selected subspace of descriptors (i.e. every tree in RF learns on different samples and features). This approach allows to build a large number of relatively uncorrelated models (trees) which operating as a committee will outperform any of the individual constituent models.

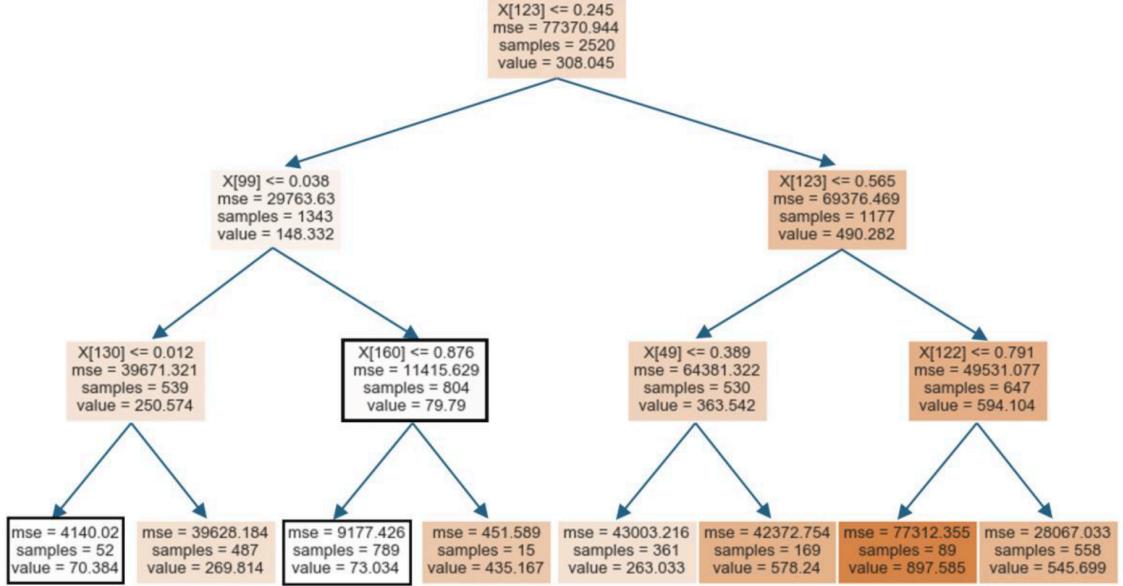


Figure 3.4: Single Tree model with depth 4 for the regression problem under consideration.

There are several important advantages that make it especially suitable for the problem under consideration. First, it can find complicated non-linear dependencies from the data. Compare to the number of other methods (e.g. linear regression), it does not make any assumptions or fitting about the functional form of the relationship between the predictors and the target variable. The second important advantage of this method is that, by combining information from individual independent predictors (trees), it can determine the importance of each particular feature, thus making the whole model more interpretable, and allows us to extract useful insights from the trained model. For example, predictor importance was employed for the decreasing of the feature space dimensionality. Predictors were sorted according to their importance and firstly were removed 25% of less important features (Q75) and then another 25% (Q50). New models were trained and evaluated on the reduced features spaces (Q50, Q75) as it is shown in Table 3.4. Hyperparameters that were optimized are maximum tree depth as shown in Figure 3.5 (a) and the number of trees Figure 3.5(b). Convergence was achieved after 100 trees in an ensemble with a depth equal to 10.

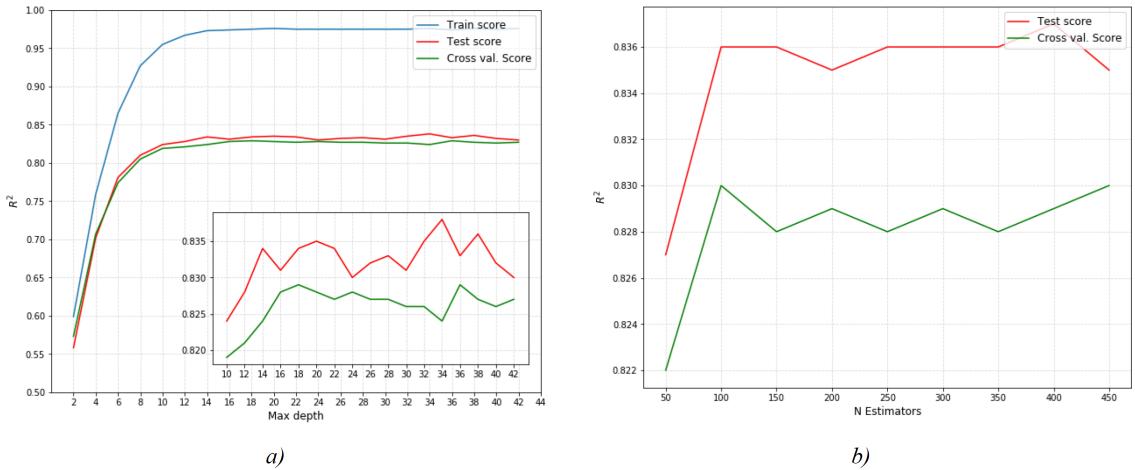


Figure 3.5: RF model hyper parameters tuning a) Convergence with respect to the tree depth b) Convergence with respect to the number of trees.

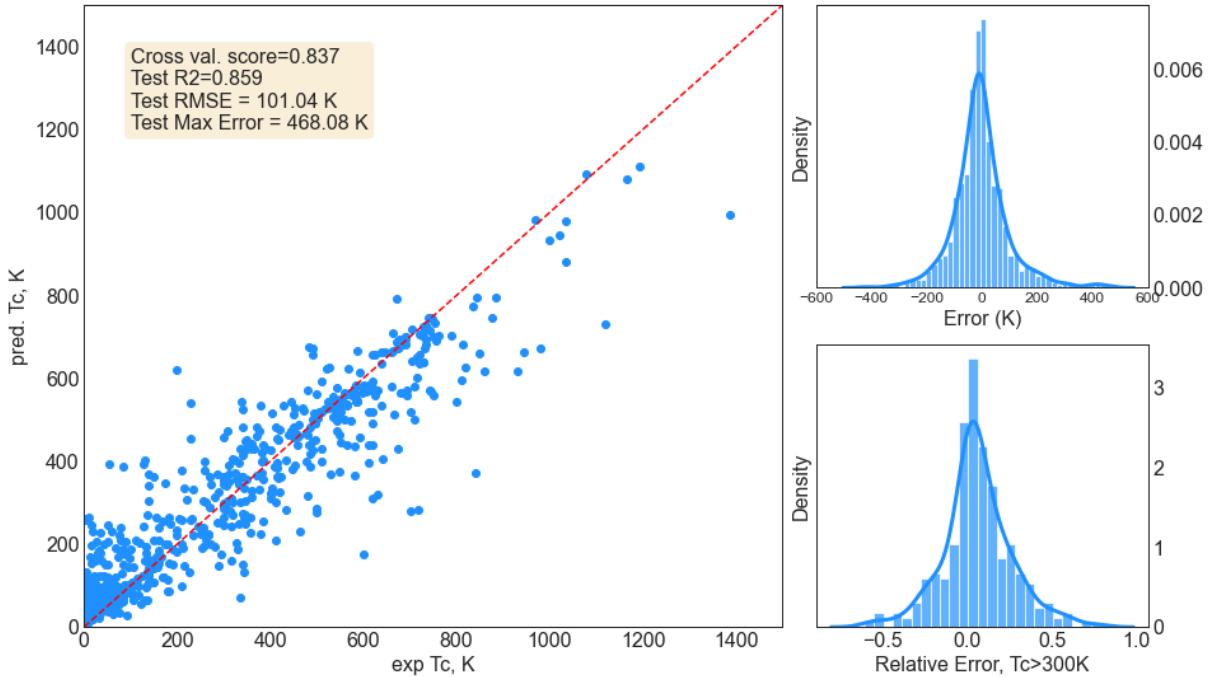


Figure 3.6: Optimized RF model performance, left comparison of predicted and experimental data, right errors and relative errors for the compounds with  $T_c > 300K$ .

Table 3.4: RF models performance with respect to the features space

Metrics	All	Q75	Q50
$R^2$ (test)	0.833	0.819	0.810
$R^2$ (cross.val)	0.828	0.816	0.807
RMSE (K)	109.4	113.6	131.3
Max Error (K)	516.3	540.5	601.3

As it might be seen from the table 3.4 in general RF outperform the best KRR model and shows good predictive power. Error distribution (Figure 3.6 right top) shows the relation between experimental and predicted values of critical temperature ( $T_C^{exp} - T_C^{pred}$ ). In our case It is highly symmetrical around zero and might be considered as Gaussian type. From this fact we can conclude that RF model has no systematic bias towards overestimation or underestimation. The distribution of relative errors  $(T_C^{exp} - T_C^{pred})/T_C^{exp}$  showed only for the compounds with  $T_C > 300K$  since for the compounds with low TC relatively small absolute error may lead to a huge relative one (i.e. for  $T_C^{exp} = 1K ; T_C^{pred} = 15K$ , 1400% error). Relative error distribution already not so symmetrical but it might be explained due to the low number (136) of samples in it.

## XGBoost algoritm

XGBoost is another ensemble algorithm based on decision trees, which utilizes a gradient boosting (GB) framework. Initially, it was designed for small-to-medium structured or tabular datasets, which is exactly the case in this work. Gradient boosting (GB) is an ML technique very similar to RF. It produces the prediction model as an ensemble of weak prediction models. The algorithm starts with a single leaf which represents an initial guess for the target variable ( $y$ ) for the samples, if it is not specified explicitly initial guess is always the average value of ( $y$ ). The general difference from the simple RF algorithm that GB builds a new tree, based on the errors made by the previous regressor and combines it with the results of the original leaf to make a new prediction. Trees depth in this algorithm controlled by the number of leaves (for regression problems and relatively small datasets  $<5000$  the empirical value of leaves between 8 and 20). To scale the contribution of each predictor GB use a special parameter called learning rate which is the value between 0 and 1. This parameter is a coefficient that shows the contribution (in RF we couldn't affect it) of each individual tree to the overall result and also controls the total amount of estimators. Empirical evidence shows that taking lots of small steps in the right direction (use small learning rate) results in a better prediction with testing dataset i.e. lower the variance but also require additional resources. In this work learning rate was set to 0.1 to keep the balance between accuracy and a reasonable amount of computational time. XGBoost parameter tuning was done according to the general recommended approach:

1. Firstly, at fixed learning rate was estimated the optimal number of trees as it is shown in Figure 3.7 (a).
2. Secondly was tuned tree-specific parameters in our case only trees depth see Figure 3.7 (b)
3. Were determined optimal regularization parameters ( $\gamma, \alpha$ ). These values were set to 1 and 0.5 respectively.

#### 4. Optimization of the learning rate

Therefore, final hyperparameters were as follows: Max depth=8, Number of trees=250, learning rate=0.1,  $\gamma = 1$ ,  $\alpha = 0.5$ .

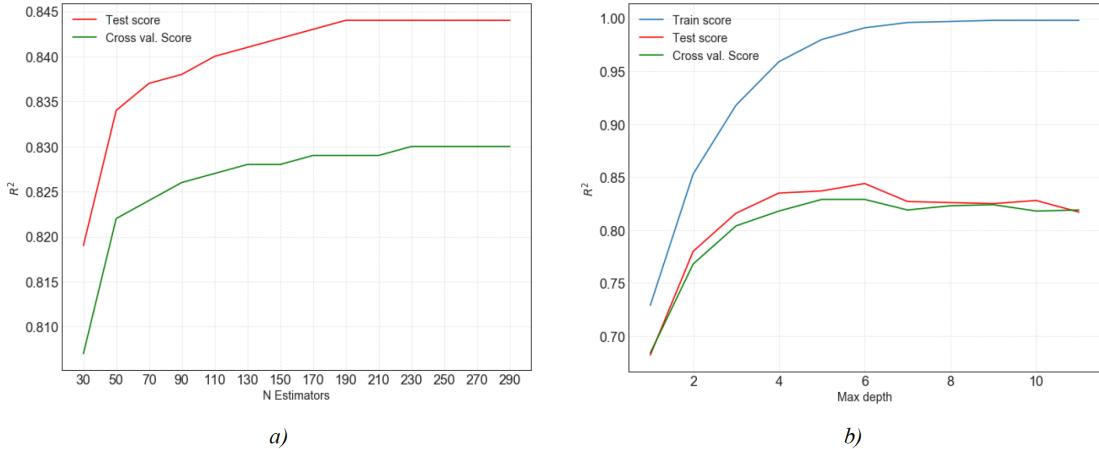


Figure 3.7: XGBoost model hyperparameters tuning: a) Convergence with respect to the number of trees b) Convergence with respect to the tree depth.

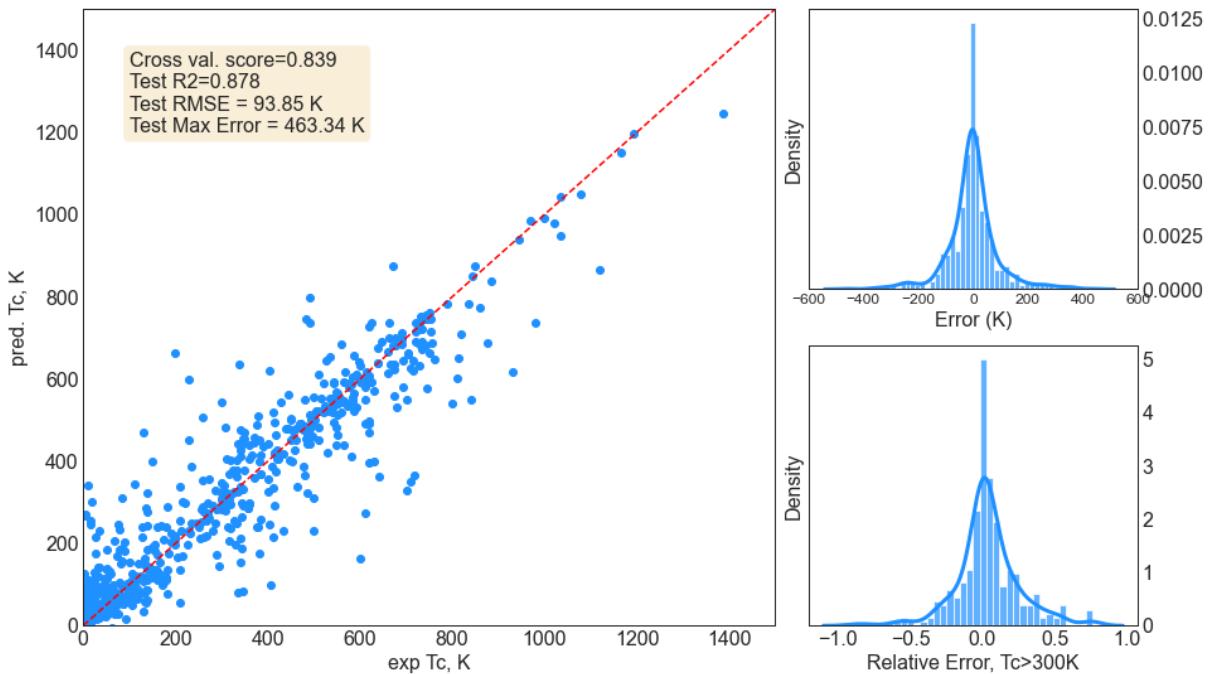


Figure 3.8: Optimized XGBoost performance, left comparison of predicted and experimental data, right errors and relative errors for the compounds with  $T_C > 300\text{K}$ .

Table 3.5: XGBoost performance with respect to the features space

Metrics	All	Q75	Q50
$R^2$ (test)	0.845	0.833	0.823
$R^2$ (cross.val)	0.837	0.825	0.817
RMSE (K)	105.6	111.8	123.6
Max Error (K)	595.6	623.5	677.3

From the Table 3.5 we can conclude that XGBoost in general shows higher accuracy than the RF and KRR. Errors distribution (Figure 3.8 top right) is a Gaussian type and highly symmetrical around zero, so this model also has no biases towards the overestimation or underestimation of the  $T_C$ .

Surprisingly maximum error of this model is sufficiently larger than that one estimated from RF. Finally, we can conclude that XGBoost is the most accurate model out of all considered in this work with respect to the  $R^2$  and  $RMSE$  optimization parameters. The mean absolute error ( $MAE$ ) for this model is 69K.

### Sure independence screening and sparsifying operator (SISSO)

The sure independence screening and sparsifying operator (SISSO) is a systematic approach for discovering descriptors for materials' properties, within the framework of compressed-sensing-based dimensionality reduction. It gives an ability to handle huge and correlated features spaces and converges to the optimal solution building a combination of features relevant to the materials' target property. There are several works [18–20] which report the ability of this algorithm to work efficiently both with regression and classification problems and also to achieve a stable results with relatively small training sets. The output of SISSO is a mathematical model, in the form of explicit, analytic functions of the input physical quantities. This aspect makes this model highly interpretable and gives an opportunity to inspect the equations and obtain some useful physical insights about the problem under consideration. An input 20-dimensional features space for SISSO were build-out of the 60 most important descriptors which were determined from RF. Were tested different combinations of these features in order to minimize the overall correlation between them. Finally, was constructed a 20-dimensional feature space as it is shown in the correlation matrix (Figure 3.9).

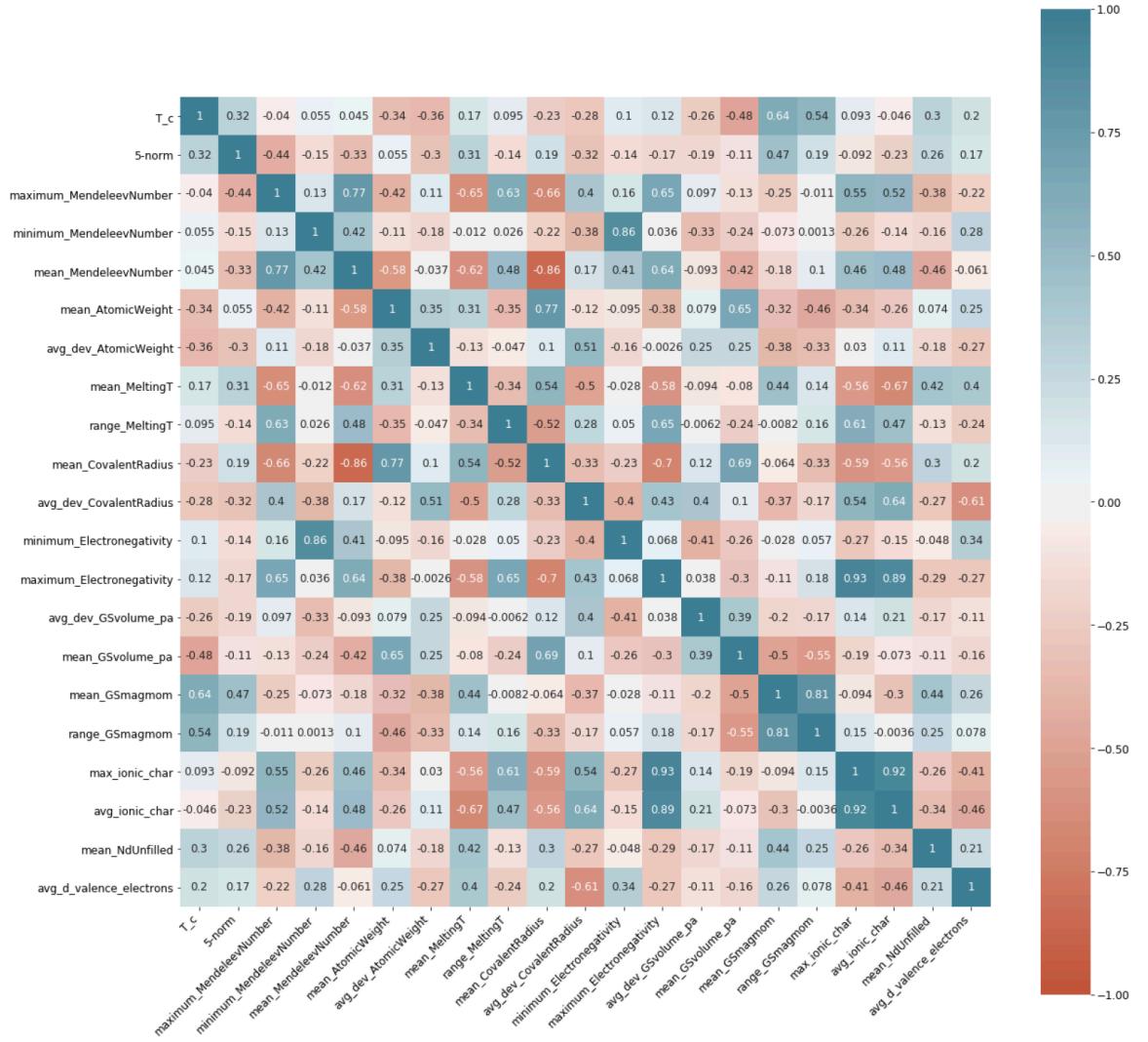


Figure 3.9: Correlation matrix for the most valuable descriptors in RF.  
These descriptors were used as input for the SISSO algorithm.

After that SISSO was trained on the entire dataset. Initially, for a baseline model, feature space rung parameter were set to 2, and calculations were performed up to the 7-dimensional descriptor (see Figure 3.10a). As it was expected RMSE and maximum error has a tendency to decrease with the increased size of produced descriptors (see Figure 3.10c). Then for the second rung on feature space were increased by one and calculations were also started up to the 7-dimensional descriptor. Unfortunately, only a 3-dimensional (see Figure 3.10b) descriptor was calculated.

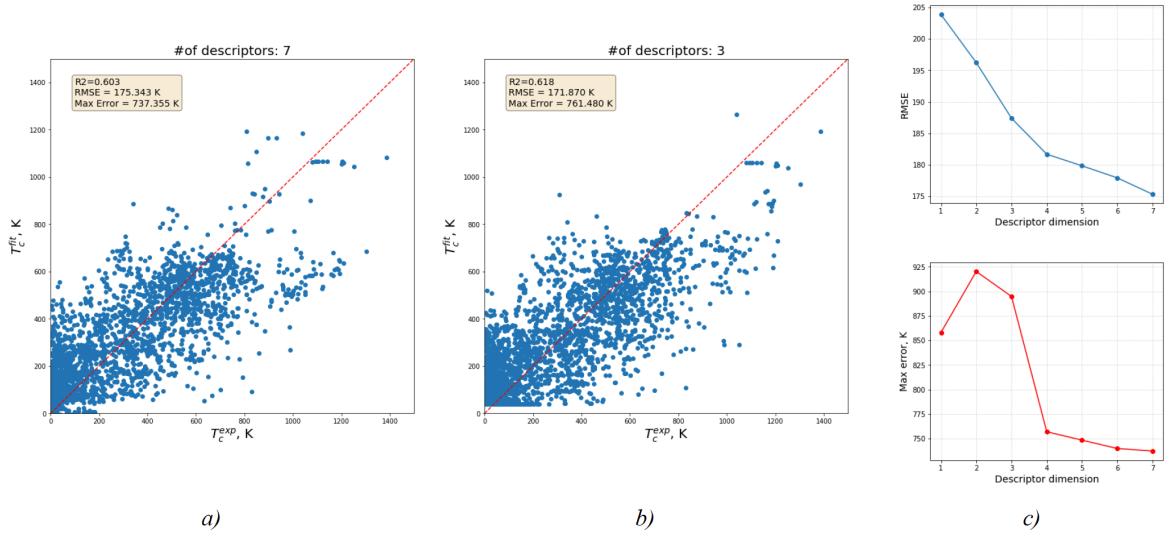


Figure 3.10: SISSO model performance: a) Rung of feature space=3, subspace size=100, b) Rung of feature space=2, subspace size=10, c) RMSE and Max error metrics with respect to the descriptors dimensionality for the feature space with rung 2.

As it is shown  $R^2$  value and  $RMSE$  metrics for SISSO worse than for the other algorithms. But probably the potential of SISSO is still not fully disclosed and it is possible to achieve a way better result. The only problem is that this algorithm is rather computationally demanding and requires a lot more resources than its available now for the author of this work. Also, it is interesting to check which features were produced. Here is shown 3 features which were generated during the second run (rung of features space equal to 3):

$$X_1 = \frac{\bar{\mu} \cdot \varepsilon_{max} \cdot MN_{max}}{\bar{V}^2 \cdot \bar{T}_M \cdot \Delta T_M} \quad (3.12)$$

$$X_2 = \frac{\bar{N}^d \cdot MN_{min}}{\exp(\Delta\mu)} - \frac{\bar{N}^d \cdot \bar{MN}}{\exp(\varepsilon_{min})} \quad (3.13)$$

$$X_3 = \frac{\left( N^V - \bar{N}^d \right)^2 \bar{\mu}^2}{\varepsilon_{min}^3} \quad (3.14)$$

As it might be seen the most frequently used primary features is a magnetic moment ( $\bar{\mu}, \Delta\mu$ ) and electronegativity ( $\varepsilon_{min}, \varepsilon_{max}$ ), these features were included in all tree generated descriptors. Features related to the electronic structure namely number of valence electrons  $N^V$  and number of electrons on d-shell ( $\bar{N}^d$ ) as well as Mendeleev number ( $MN_{min}, MN_{max}, \bar{MN}$ ), were included in 2 out of three descriptors. Properties like melting temperature ( $\bar{T}_M, \Delta T_M$ ) and lattice volume ( $\bar{V}$ ) were used only for one descriptor.

### 3.1.5 Physical understanding of the most important descriptors

Building an efficient model that could make an accurate prediction is an important part of the ML pipeline. But what is even more important is to interpret your model properly and find what kind of physical insights and understanding of a particular problem might be extracted from it. That's why in some cases simpler models like decision trees or Random Forest are more preferable due to their high interpretability and transparency. Figure 3.11 shows the histogram of the top 15 most valuable features which were extracted from our trained RF model. Actually, this feature is not strongly different between 3 of our models (KRR, RF and XGBoost), that's why it's enough to consider only one such histogram.

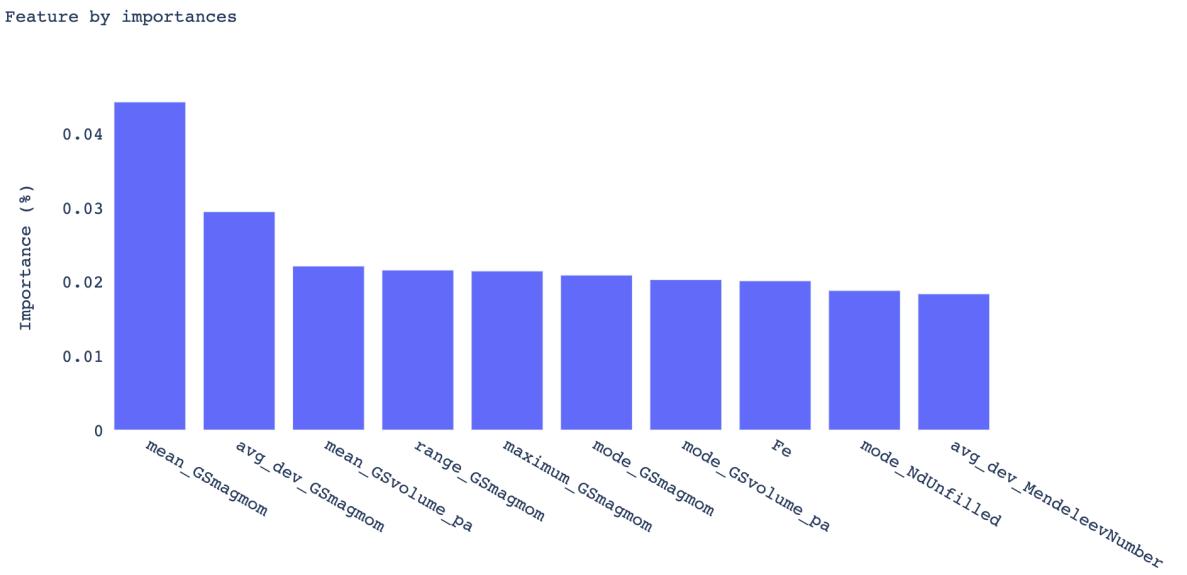


Figure 3.11: RF feature importance histogram.

Five descriptors out of fifteen related to the magnetic moment, namely:  $\mu_{max}$ ,  $\Delta\mu$ ,  $\bar{\mu}$ ,  $\tilde{\mu}$ ,  $\langle \mu \rangle$ . This fact is quite understandable since by definition target property ( $T_C$ ) is a point of sharp changes in the magnetic properties. There are 2 descriptors corresponding to the atomic fraction of particular elements, namely Iron (Fe) and Cobalt (Co) it is also might be explained due to their strong ferromagnetic nature, other important elements like Nickel (Ni) and Manganese (Mn) are 25th and 40th respectively. There 2 features related to the number of electrons on d-shell  $\overline{N^d}$ ,  $\widetilde{N^d}$  and again it's a well-known fact that partially filled d-shell usually correlated with nonzero total spin and thus magnetism. Also, there 2 descriptors related to lattice volume  $\overline{V}$ ,  $\widetilde{V}$ , this already not so trivial, but probably related to the phenomena of the exchange coupling which is strongly dependent on the number of nearest neighbours in the crystal structure. Comparing the most valuable features extracted from KRR, RF, and XGBoost with the features computed from SISSO we may say that they have a lot in common despite the fact that all these algorithms use different mathematical

approaches and approximations.

## 3.2 DFT + MC calculations of critical temperature

One of the foundation stones of this particular work is developing a stable and efficient method for estimating ferromagnetics critical temperature (Curie temperature). In a framework of described in section 2.4 Heisenberg spin model, it is straightforward that the main parameter which should be found to conduct this calculation is exchanged interaction  $J_{ij}$ . Finding this parameter allows to carry out Monte Carlo simulation for the precise description of the system's thermodynamic properties or apply less computational demanding but yet quite accurate Curie-Weiss approximation. Steps required for these purposes are schematically illustrated in figure 3.17 and described in more detail in upcoming sections.

### 3.2.1 Obtain initial geometry

On a stage of development and validation of algorithm, this work used FM structures with known magnetic properties, and critical temperature preferably obtained both from experimental and theoretical studies. In particular was used structural information provided by the Materials Project and NOVOMAG databases, which have already proven as a reliable tool for computational materials exploring.

### 3.2.2 Generation of spin configurations

From obtained structural information of a parent FM cell, one may generate various derivative structures with specified AFM properties (i.e., ratio of spin-up and spin-down atoms is 1:1) utilizing existing algorithms for derivative structure generation. For these purposes, currently used open-source python library, *pymatgen* [21] with integrated derivative structure generator *enumlib* [22–25]. Based on symmetry analysis its generates different AFM spin configurations from the parental FM. One of the big advantages of the current derivative structure enumerator is the possibility of eliminating the heuristics-based approach. On the contrary of producing a huge number of randomly oriented spin configurations with the hope of getting the one that is close enough to FM ground state, it makes a rigorous and thus reliable search for all possible spin states. The number of obtained spin configurations we may consider almost exhaustive, which is also proven in a series of studies by comparing with experimental data.

### 3.2.3 Energy calculations

Initial FM cell, as well as various generated AFM structures, are then relaxed using collinear density functional theory, and their energies are accurately calculated.

Spin-polarized DFT calculations are carried out using generalized gradient approximation (GGA) as implemented in the Vienna Ab initio Simulation Package (VASP) [26] with projector augmented-wave (PAW) [27] method applying the Perdew–Burke–Ernzenhof (PBE) [28] exchange-correlation functional. For all calculations was used sufficiently large value of cut-off energy up to 1.5 of the maximum provided in pseudopotential file.

Because the energy differences between various close to ground state spin configurations are usually on several milli-electron volts (meV), it's essential to conduct computations with sufficiently high accuracy but also keep computational demanding on a reasonable limit. To keep the balance between efficient usage of computational resources and accuracy requirements, stages of relaxation and static energy calculations use a step-by-step approach (with increasing accuracy). This way allows to quickly determine unsuitable for the future calculations structures and exclude them saving resources.

For the relaxation stage was employed gamma-centered grid with the default specified in VASP reciprocal density of  $64 \text{ \AA}^{-3}$  while the static calculations utilize significantly higher reciprocal density of  $300 \text{ \AA}^{-3}$ . This type of mesh is known to be a pretty flexible (and probably the most commonly used) flavor of the automatic grid generation mode and is shown to achieve faster energy convergence compared to standard Monkhorst-Pack grids.

The parameter of electronic convergence (energy difference between successive electronic steps) is varying from  $10^{-4} \text{ eV}$  for the initial relaxation stages up to  $10^{-6} \text{ eV}$  for the finalizing static runs.

Worth mentioning that at this step, it is essential to ensure that among all produced spin configurations, the ground state is found to be FM. Otherwise (AFM structure is ground state), it is needed to treat this material differently, and the workflow cannot be continued. Another important step is to check if all the structures kept their local magnetization unchanged (or at least these changes are not significant). Quite often, especially structures with low initial magnetic moments exhibit total disappearance of magnetization in AFM state. Such behavior makes them absolutely nonmagnetic and thus not representative for further Hamiltonian fitting.

For the case of structures with strongly correlated d-shell electrons, it was also tested Hubbard correction approach (GGA + U) using the Dudarev [29] formulation which is proven to give accurate results for such systems.

The effective U values for transition metals calibrated by performing a fitting to experimental binary formation enthalpies have been taken from the Materials Project website [30]. This is a common practice and has already shown reasonable results in a high-throughput screening study performed in ref. [31]. Of course, usage of Huburd correction values without proper calibration, for instance, by linear response method [32], may lead to unsatisfying results, especially for structures containing elements with relatively low magnetization. Unfortunately, the computational demand

of such calibration is sufficiently large and not applicable in high-throughput calculations.

### 3.2.4 Hamiltonian fitting

#### Building the system

The energies calculated at the previous stage for the relaxed parental FM and derivative AFM configurations make it possible to determine the exchange coupling parameters ( $J_{ij}$ ). For these purposes initially carried out mapping of magnetic neighbors in every generated spin configuration utilizing *pymatgen* [21] and *siman* [33] libraries . This stage performed not only mapping of magnetic nearest (mNN) and next-nearest magnetic neighbors (mNNN), located in the first and second coordination spheres respectively, the search is extended to the higher-order coordination spheres up to the number of generated spin configurations. Hence for any FM system with a number of generated AFM spin configurations equal to  $N$ , mapping was done up to the  $N - th$  coordination sphere in every structure (e.g., for 6 generated AFM structures, mapping of magnetic neighbors performed up to the 6-th coordination spheres).

Mapping procedure returns a series of coefficients ( $n_{ij}$ ) which, with the addition of calculated energies ( $E_{FM/AFM}$ ), allows building a system of equations. Produced system includes  $N + 1$  equations (1 FM +  $N$  AFM configurations) and  $N + 1$  variables ( $E_g, J_1, \dots, J_n$ ).

$$\begin{cases} E_g + n_{11}J_1 + \dots + n_{1N}J_n = E_{FM} \\ E_g + n_{21}J_1 + \dots + n_{2N}J_n = E_{AFM1} \\ \vdots \\ E_g + n_{N1}J_1 + \dots + n_{NN}J_n = E_{AFM_N} \end{cases} \quad (3.15)$$

Its might be illustrated on the toy example from figure 3.12. Suppose we have a 2D structure with a spin-up red atoms and spin-down blue atoms. As a 1st nearest neighbors of our spin-up central atom, we have 4 atoms with an antiparallel spin which means that sign of their interaction will be negative. Building the second coordination sphere we can see that all 2nd neighbors have the same spin with the central one, therefore, the sign of their interaction will be positive.

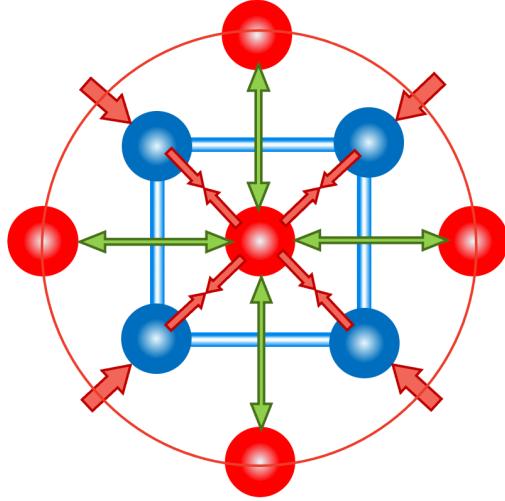


Figure 3.12: Schematic determination of  $J_1$  and  $J_2$  coefficients in 2D case.

Hence the total energy of the given ferromagnetic system can be written as:

$$E_{AFM} = E_g - 4J_1 + 4J_2 \quad (3.16)$$

Where  $E_g$  – corresponds for the part of energy given by the geometry itself.

### Solving the system

Extracting values of exchange couplings from the produced systems were done in two different methodologies.

In the case of a nonzero determinant, it is possible to obtain an exact solution directly solving the system.

$$\det \begin{pmatrix} n_{11} & \dots & n_{1N} \\ \dots & & \dots \\ n_{21} & \dots & n_{NN} \end{pmatrix} \neq 0 \quad (3.17)$$

Otherwise, the less stable (with the highest energy) AFM configuration is excluded from the calculations until the equations system becomes solvable or runs out of the structures. This method has obvious disadvantages. Firstly quite frequently from the mapped coefficients, it's impossible to build any solvable system, and the calculations fail. Secondly, excluding less favorable spin configuration from the solution, one by one, frequently leads to a minimal possible system  $2 \times 2$ , which returns only the  $J_1$  parameter and thus makes the results poorly representative for the further  $T_C$  estimation.

The second methodology utilizes a common strategy applicable for overdetermined systems, namely the least-squares method. In this study, fitting was done starting from the  $N \times N$

system, continually decreasing the number of taken into account variables up to the  $N \times 2$  ( $E_g, J_1$ ). Such an approach allows to better distinguish strong and weak exchange correlations in the materials showing how every group of magnetic atoms located on a particular coordination sphere contribute to the final exchange values. Worth mentioning that contrary to the direct system solving this approach is extremely sensitive to the quality of all the produced AFM structures. In other words, any poorly relaxed or disordered physically meaningless structure that might appear in the system won't be excluded at any stage and strongly affect all the produced results. For clarity let's consider a couple of examples, namely FM Mn and Eu oxides.

### Example 1

For the case of EuO were generated 3 unique AFM cells (see figure 3.13). Based on the energies of given structures obtained from the spin-polarized DFT calculations, one may build a system of 3 equations that has 3 variables ( $E_g, J_1, J_2$ ). Since the coefficients obtained for a given system form a nonzero determinant, this system can be solved exactly.

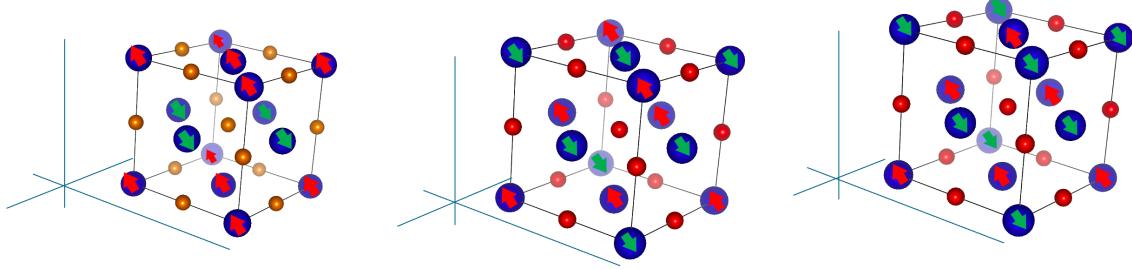


Figure 3.13: Three different spin configurations produced for EuO that allows to calculate the  $J_1$  and  $J_2$  exchange couplings.

$$\begin{cases} E_g + 4J_1 - 6J_2 = E_{AFM1} \\ E_g + 0J_1 + 6J_2 = E_{AFM2} \\ E_g - 4J_1 - 2J_2 = E_{AFM3} \end{cases} \quad (3.18)$$

$$\begin{aligned} E_g &= -41.63324 \text{ eV} \\ J_1 &= 1.18 \text{ meV} \\ J_2 &= 0.42 \text{ meV} \\ T_C^{GGA} &= 69 \text{ K} \\ T_C^{\text{exp.}} &= 67 \text{ K} \end{aligned} \quad (3.19)$$

Solution of the system allows calculating exchange coupling parameters for the first and second coordination spheres ( $J_1, J_2$ ). As it might be seen from the calculated values, in this case, exchange couplings have the order of meV. This fact once again points out the critical importance of extremely accurate relaxation and energy estimation steps. The calculated value of critical temperature 67K is in perfect agreement with the experimental value of 69K.

## Example 2

For the case of MnO were considered 3 spin configurations (parental FM and two generated AFM). In the same manner, for the given structures were performed accurate relaxation followed by single-point energy calculations.

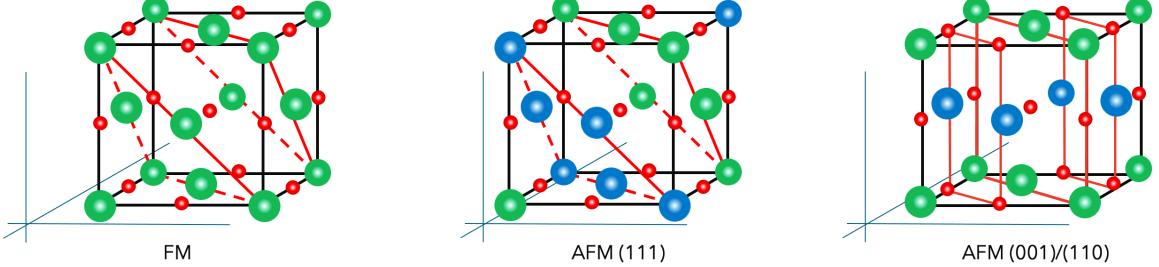


Figure 3.14: Three different spin configurations produced for MnO that allows to calculate the  $J_1$  and  $J_2$  exchange couplings.

$$\begin{cases} E_g + 6J_1 + 3J_2 = E_{FM} \\ E_g - 2J_1 + 3J_2 = E_{AFM1} \\ E_g + 0J_1 - 3J_2 = E_{AFM2} \end{cases} \quad (3.20)$$

$$\begin{aligned} J_1 &= 9.50 \text{ meV} \\ J_2 &= 14.91 \text{ meV} \\ T_C^{GGA+U} &= 249 \text{ K} \\ T_C^{exp.} &= 118 \text{ K} \end{aligned} \quad (3.21)$$

Obtained values of exchange coupling for the first ( $J_1$ ) and second coordination spheres ( $J_2$ ) on the order of several meV. In this case, the estimated value of critical temperature  $T_C^{GGA+U} = 249K$  overestimates the experimental one  $T_C^{exp.} = 118K$  more than two times. Nevertheless, results still might be considered valuable since they allow to distinguish structures with doubtful technological potential.

One more important point here is the quite a noticeable difference between the experimental values obtained for the system under consideration by different groups. For instance  $90K$  in ref. [34], while ref. [35] reported  $118K$  and the most recent results of  $85K$  in ref. [36]. This variability also raises how it is correct to compare theoretically estimated values with experimentally obtained. Since samples of the same material might have a different quality and even small defects might strongly affect the magnetic properties. While varying experimental techniques and differ in equipment might lead to even bigger divergence. This fact was also briefly discussed during the description of the data pre-processing in section 3.1.3

### 3.2.5 Monte Carlo simulation

As a final step of the workflow values of exchange coupling parameters ( $J_{ij}$ ) used to study the material transition from ferromagnetic (FM) to paramagnetic (PM) state. It is done by the source of

constrained Monte Carlo (MC) metropolis algorithm as implemented in Vampire software which allows to carry out calculations of the equilibrium temperature-dependent magnetic properties. Vampire contains a predefined functionality to estimate critical temperature by performing a temperature sweep and calculating the average magnetization, returning the classic M-T curve.

For all the systems, exchange energy was considered isotropic (equal in all directions). To neglect finite-size and surface effects in a bulk material was chosen sufficiently large dimensionality of  $10 \times 10 \times 10 \text{ nm}^3$  and defined periodic boundary conditions in all three spatial directions. Temperature range of modeling was specified from 0 to  $1500 \text{ K}$  with an increment of  $25 \text{ K}$  for each temperature done  $10^5$  equilibration time steps which is needed to reach thermal equilibrium in the system followed by the  $10^5$  loop-time steps, which is already counted to the average of magnetization as it is illustrated in figure 3.15. Quite a large number of equilibration steps explained due to known equilibration-temperature dependence which tends to drastically slow down close to the Curie point. Specified numbers chosen heuristically but yet yields reasonable results for the most of systems. The bigger number of averaging steps, in general, lead to more smooth data but obviously requests more computational resources.

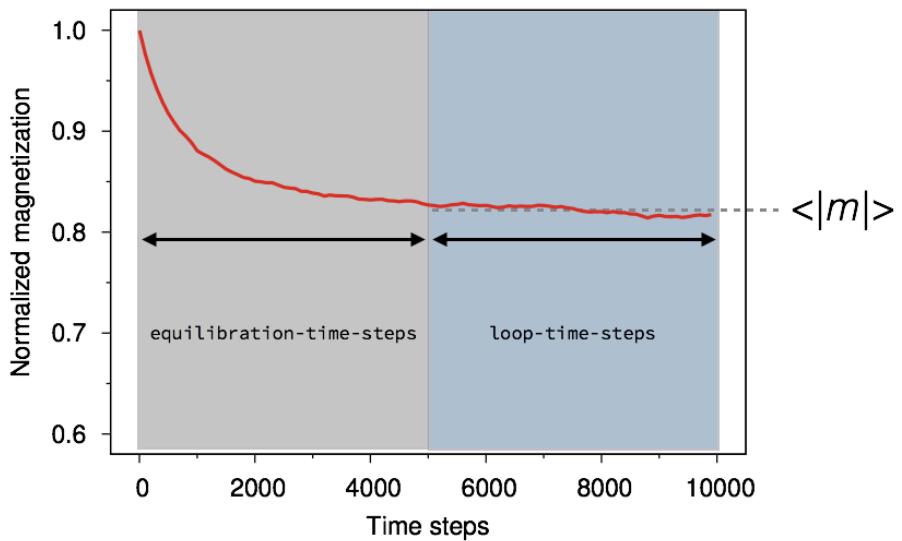


Figure 3.15: Time-dependent magnetisation caused by a stepwise increase in temperature.

As a result of running MC simulation returns data for the magnetization-temperature curve as it is shown in figure 3.16, from which one may determine the critical temperature as point with a maximum gradient.

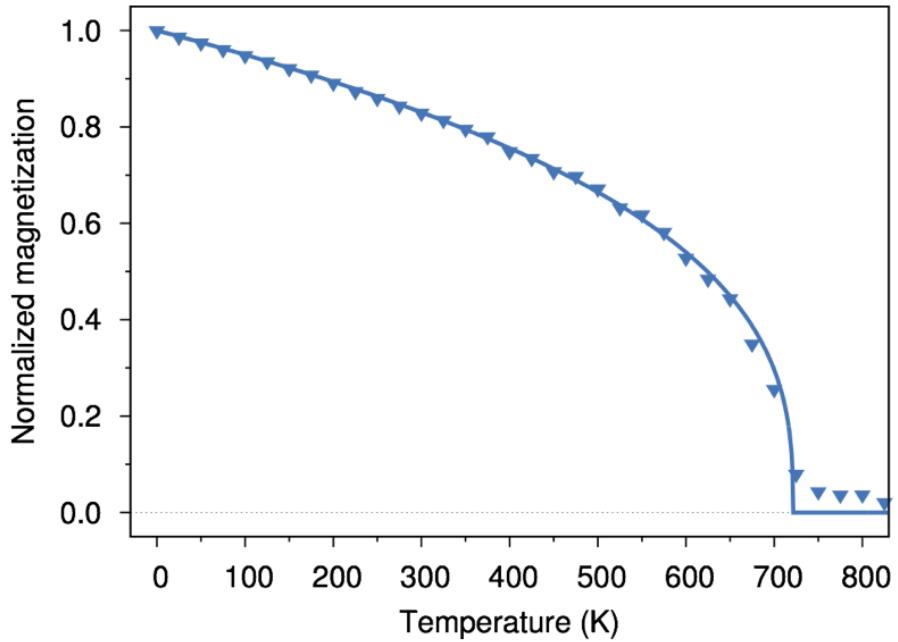


Figure 3.16: MC magnetization temperature curve.

### 3.2.6 End-to-end pipeline details

All pieces of the code for integration with side packages and data pre/post-processing produced during the work were formed into a complete end-to-end Python-based package. The main goal of the prepared package was to automate all the stages of the critical temperature calculations, eliminating the need for user control and making it usable for any FM material. Schematically algorithm shown in figure 3.17.

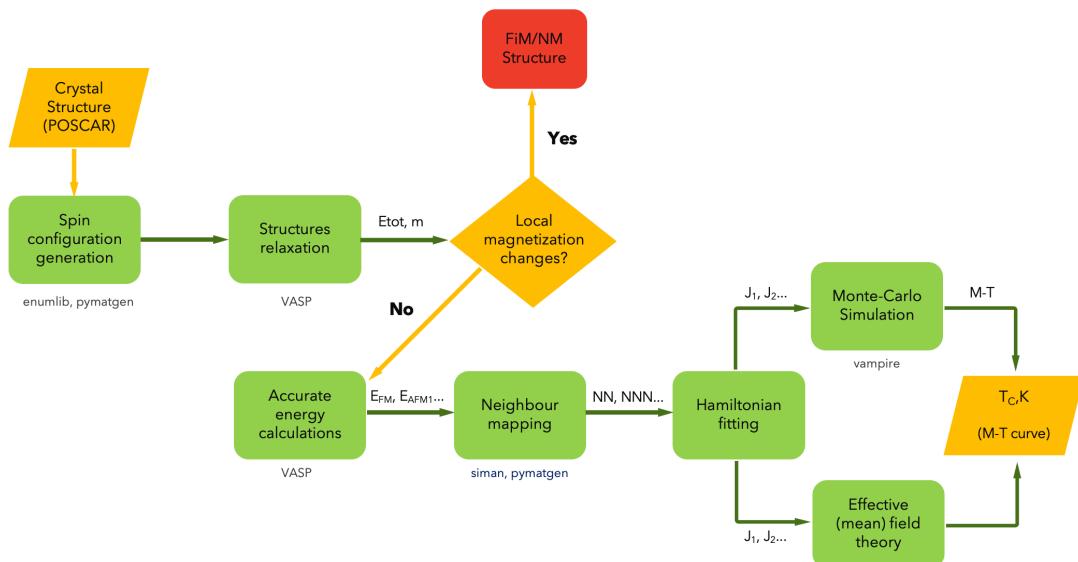


Figure 3.17: End-to-end pipeline scheme.

The package was built with the philosophy to be suitable even for people without special knowledge in electronic structure calculations keeping the entry barrier as low as possible. In the architecture of the code, we tried to save a balance between being simple for the novice yet maintaining flexibility for advanced users who know exactly what it does. As minimum input information its only needs to provide crystal structure data in a form of a commonly used POSCAR type file. All the required parameters for calculations by the DFT method are set by default. If necessary, the user can always change them by editing a JSON-type file using the keywords specified by the VASP manual (for the INCAR and KPOINTS files). This adjustment is more than welcomed if the user has any specific knowledge about the particular system, which may speed up the convergence or, in general, lead to more accurate results, but not mandatory.

At the moment, for the code to work correctly, user need installed *python 3.6* or later with a series of easily accessible through the Python Package Index (PyPI) repository libraries (*pymatgen*, *ase*, *siman*, *numpy*, *scipy*). Structure enumeration process required additional compilation of *enumlib* and its submodule *symlib* using any available fortran compiler (i.e., gfortran). Also, it is necessary to have installed program for *ab-initio* calculation *vasp-5.4.4* with a possibility to run in several threads.

The built package currently did not include the stage of MC simulation since they become excessively slow in a python implementation due to its interpretable nature. Therefore at the final stage, the critical temperature is estimated by the Curie–Weiss law, and for more accurate simulations, it's still necessary to run *Vampire* software explicitly. In the future iteration of the project, this disadvantage expected to be fixed.

All the described code available at the GitHub repository ([https://github.com/Volkov-da/curie\\_calculator](https://github.com/Volkov-da/curie_calculator)), which is planned to be maintained and updated.

## Chapter 4

# Results

### 4.1 ML models results

In section 3.1 was described the construction of several ML models aimed at predicting ferromagnetic materials critical temperature based exclusively on their chemical composition. The final summary of the best models' performance in comparison with reference work is presented in table 4.1.

Table 4.1: Summary of the best ML models performance

Metrics	SISSO	KRR	RF	XGB	RF (ref.)	KRR (ref.)
$R^2(test)$	0.618	0.793	0.833	0.845	0.87	0.72
$R^2(cross.val)$	-	0.787	0.828	0.837	0.81	-
MAE (K)	138	84	71	69	57	-
RMSE (K)	171	121	109	105	-	-
Max Error (K)	761	642	516	595	-	-

The highest accuracy demonstrated by the XGBoost model with  $R^2$  and mean absolute error of 0.845 and 69 K respectively on the test set. But the lowest maximum error is shown by the RF. It is worth to mention that descriptors used in this work are different from the one used in the reference. A serious drawback of considered features is the absence of any structural information which makes it impossible to distinguish between two compositions with similar stoichiometry. But after all this work might be considered as a second small step (after ref. [7] toward the ML-driven computational search for novel magnetic materials.

All the generic python code produced in this part of the work stored at GitHub repository ([https://github.com/Volkov-da/curie\\_calculator/tree/master/ML\\_model](https://github.com/Volkov-da/curie_calculator/tree/master/ML_model)). The whole process of data cleaning, preprocessing as well as model training and validation stored in a form of briefly commented jupyter-notebook (file *ML\_training.ipynb*).

The pipeline showed the best performance (MinMaxScaler + XGBoost) was additionally re-trained over the entire dataset and stored in easy reproducible JSON format.

## 4.2 DFT calculations results

Using the framework described in section 3.2 were calculated values of critical temperature for 20 binary structures containing pure metals, oxides, borides, etc., and 2 gadolinium containing ternary structures. Obtained values were compared with experimental ones as well as with the results from the theoretical calculations of other groups. Relative error was calculated as follows:

$$\Delta = \frac{|T_{exp} - T_{DFT}|}{T_{exp}} \quad (4.1)$$

As it might be seen from the table 4.2 in general GGA + U approach outperforms GGA. Mean relative error for the unary structures in the case of GGA is 39.5% for GGA + U is 30.9%, for oxides 40.4% and 33.5%, for other binary structures (borides, etc.) it is 42.4% and 25.1% respectively.

From the plot 4.1 we may conclude that in general GGA tends to underestimate critical temperature with the exclusions of 4 structures, namely: *MnO*, *NiO*, *MnAs*, *EuS*.

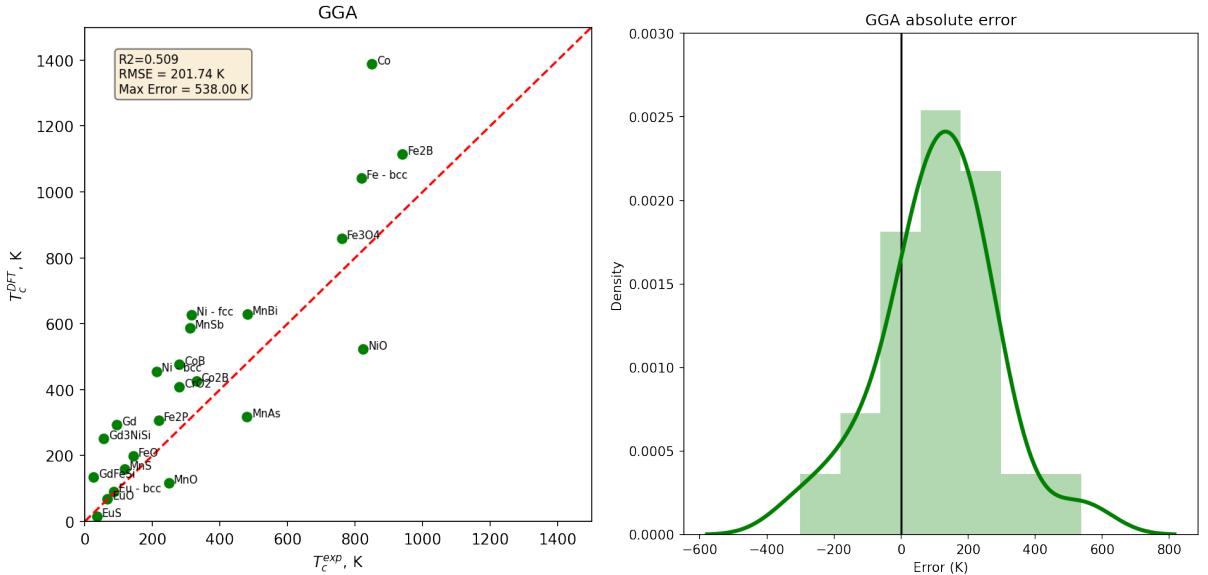


Figure 4.1: Left: Comparison of calculated by GGA results with experimental. Right: Absolute errors distribution.

Maximum relative errors of 125 % are shown in the case of *EuS*, but anyway, the resulting value we may consider pretty reasonable due to the small absolute difference (16 K and 36 K respectively). A maximum absolute error of 538 K appears in the case of cobalt, the FM material with the highest known critical temperature. The lowest absolute difference in calculated and experimental values was shown by three Eu containing structures its 5 K for *Eu-bcc*, 20 K for *EuS* and 2 K for *EuO* which is so far also the material with the lowest relative error of only 5.5%.

Results obtained during the GGA+U calculations presented in figure 4.2. The distribution of

errors is closer to zero compared to GGA results, and there is no clear tendency towards over/under-estimation of Curie point. Maximum relative error of 53.4% in this case shown by  $MnO$ , while a bit bigger maximum absolute error of 608 K once again by cobalt. Lowest absolute difference of 15 K and 23 K as well as the lowest relative error of 9.4% and 11.6% were shown by  $MnS$  and  $FeO$  respectively. Eu-containing structures were not considered in GGA+U calculations due to the absence of a predefined U value. Once again worth mentioning that due to the automated way of calculating, none of Hubbard correction values was calibrated explicitly. Hence we may expect even better results in the case of proper calibration with a linear response method for each particular structure.

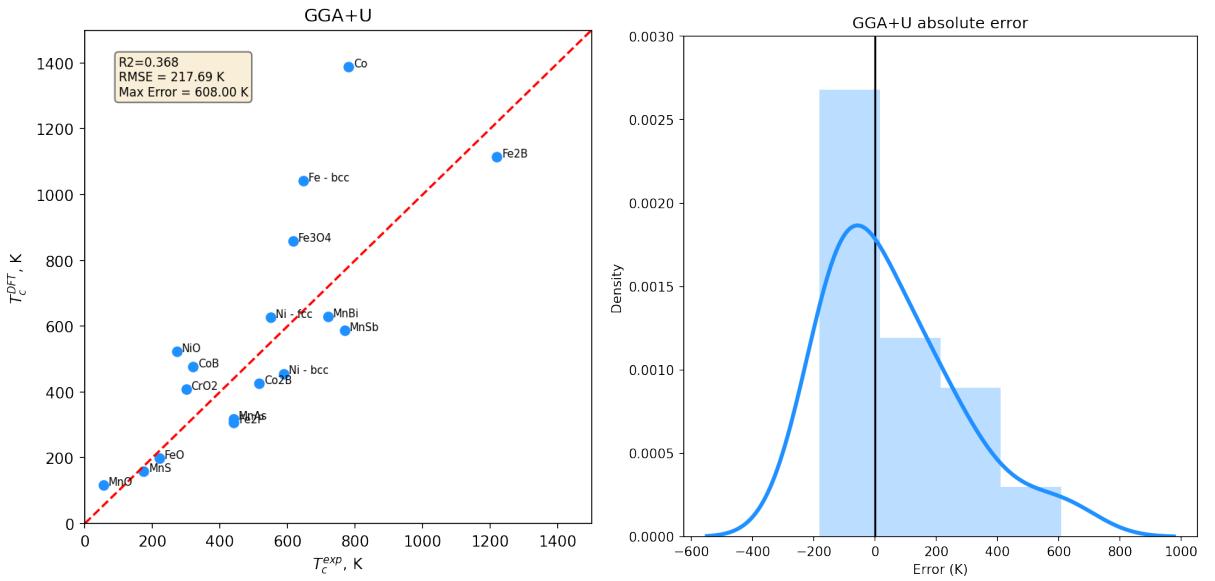


Figure 4.2: Left: Comparison of calculated by GGA+U results with experimental. Right: Absolute errors distribution.

Results obtained during GGA and GGA + U calculations with relative errors compared to the experimental data presented in the table 4.2.

Table 4.2: Comparison of calculated by GGA and GGA+U results with experimental.

Material	ref.	exp.	GGA	GGA+U	$\Delta^{GGA}$ , %	$\Delta^{GGA+U}$ , %
Fe-bcc	950	1043	820	648	21.4	37.9
Co	1311	1388	850	780	38.8	43.8
Ni-fcc	300	627	315	550	49.8	12.3
Ni-bcc	250	456	212	590	53.5	29.4
Gd	293	294	94	-	68.0	-
Eu-bcc	111	91	86	-	5.5	-
EuO	35	69	67	-	2.9	-
Fe <sub>3</sub> O <sub>4</sub>	-	858	760	616	11.4	28.2
CrO <sub>2</sub>	305	408	280	300	31.4	26.5
MnO	240 [39]	118 [35]	249	55	111.0	53.4
NiO	393 [39]	523 [39]	824	272	57.6	48.0
Fe <sub>2</sub> B	1000	1115	940	1220	15.7	9.4
CoB	411	477	280	320	41.3	32.9
CoB <sub>2</sub>	450	426	330	516	22.5	21.1
Fe <sub>2</sub> P	-	306	219	441	28.4	44.1
MnBi	-	630	481	720	23.7	14.3
MnSb	-	587	311	769	47.0	31
MnAs	-	318	480	440	50.9	38.4
EuS	-	16	36	-	125.0	-
Gd <sub>2</sub> NiSi	215	251	56	-	77.7	-
GdFeSi	145	135	25	-	81.5	-

### 4.3 Novel structures study

At a final stage of all the performed work develop methods for critical temperature calculations we applied in a direct exploration of newly predicted magnetic structures. As mentioned before, nowadays, it's already possible to perform a computational search of such structures but optimizing them only concerning stability and magnetic moment. So as a final step of this work, a developed in section 3.2 algorithm based on DFT + MC calculations and the best-trained ML model from section 3.1.4 was applied to benchmark new structures with respect to their Curie temperature. Namely were tested 55 promising magnetic compositions obtained during coevolutionary (Mendelevian) search.

Distribution of estimated critical temperatures presented in the figure 4.3. As it is shown,

the correlation between the results produced by the two described methodology is quite weak. Generally, it might be explained due to the nature of the descriptors used in ML model training. Since prepared features were exclusively based on chemical information without considering structural data, the whole predictive model tends to predict high values of critical temperature for the compositions with the close to known in model stoichiometry. Thus even for previously unknown Fe and Co-containing structures ML model always expect values of critical temperature higher than 400 K in the same fashion as for the known from training data compositions containing these elements.

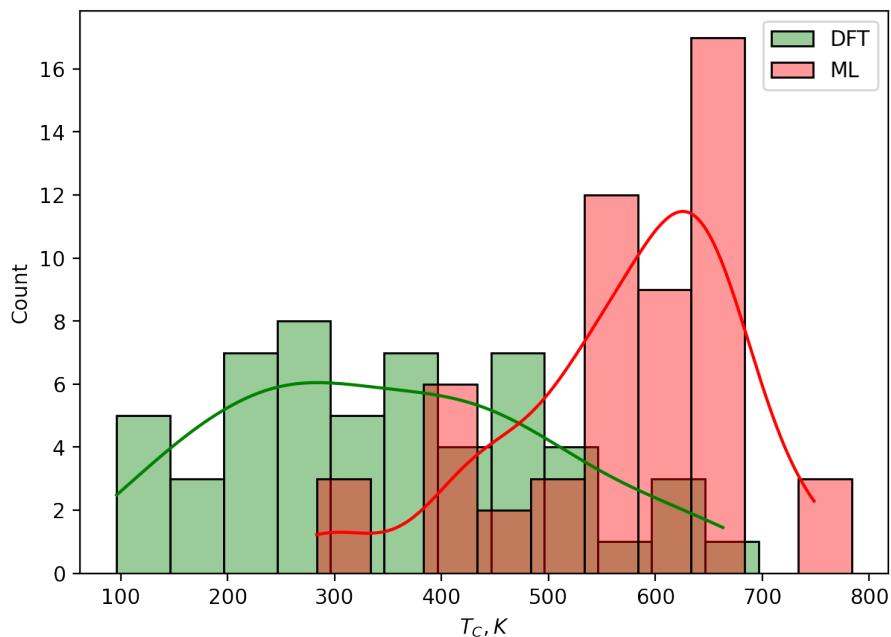


Figure 4.3: Histogram of estimated critical temperature using ML and DFT.

Estimation given for the new structures by DFT + MC approach (green histogram) has a more uniform distribution and closer in shape to the experimental critical temperature distribution obtained for known FM materials from figure 3.1. Of course, such a comparison might seem incorrect but intuitively its looks like the more new structures will be tested the more these distributions should have in common.

According to the produced results tree newly predicted structures were classified as a materials with high critical temperature namely  $C2/m - Fe_6Mo$  with  $T_C^{DFT} = 630K$  and  $T_C^{ML} = 479K$ , stable  $Cmm\bar{m} - HfCo_5$  with  $T_C^{DFT} = 633K$  and  $T_C^{ML} = 425K$  and metastable  $F\bar{4}3m - HfCo_5$  with  $T_C^{DFT} = 663K$  and  $T_C^{ML} = 425K$ .

## Chapter 5

# Discussion

### 5.1 Results reasoning

Results obtained in this work, in general, were quite expected. Estimated values of critical temperature obtained by GGA and GGA + U on average differed from the experimental on 40% and 30%, respectively. The reasoning for such deviation comes from the following sources of errors. First of all, despite the fact that DFT theoretically is exact in practice accuracy of calculations is strongly dependent on the choice of exchange-correlation functional. General gradient approximation (GGA) used in this work tends to show good accuracy for the majority of materials properties still has known shortages in magnetic calculations. The estimation of magnetic exchange coupling was discussed in several theoretical studies. For instance, in ref. [29] was reported statistical error of 40% for the number of transitional metals complexes containing Fe, Co, Mn. At the same time, other work reported an even bigger divergence of almost 11 times compared to experimental values [30]. Secondly used in this work Heisenberg spin model still leave out a lot of complexity of real material due to a number of simplifications and approximations. Thirdly inaccuracies might appear on a stage of Monte Carlo simulations. All the materials were treated as purely isotropic, which might lead to a certain level of errors. Also, we cannot fully deny the possible contribution of size effects since the finite size of modeled cube. Finally, it is worth admitting that taking into account all possible factors by theoretical methods is beyond our control. Magnetism is an extremely complex phenomenon in which properties should be modeled on all size scales starting from electronic and atomistic, ending up on a macroscale level. Even a small amount of unaccounted impurities or defects can cause a significant scatter in the experimental results, but at the same time not be reflected in the existing theoretical models.

### 5.2 Future Work

The project presented in this master's thesis is planned to be continued. Part of the work related to DFT + MC is planned to be additionally tested concerning exchange-correlation energy functionals. The nearest future planned to run tests using Local-density approximations (LDA) and LDA + U functionals. Additionally, it is planned to develop an end-to-end methodology for magnetic

anisotropy energy calculations which further might be considered for more accurate Monte Carlo simulations. The prepared python package is also planned to be maintained and improved with respect to parallelization and memory usage.

Part of the work related to the data-driven estimation of critical temperature also seems to be extremely promising. Despite the absence of structural information in descriptors, it already shows good predictive power. Inclusion of such information into the descriptors is already well-established practice by the source of various fingerprints. At the same time, the usage of modern architectures of Neural Networks (i.e., Graph Convolutional NN) may lead to stunning results frequently outperforming classical electronic structures calculations and obviously using much fewer computational resources. The only stopping factor here is the absence of any reliable and sufficiently big magnetic structures database at the moment. Fortunately, this only the matter of time, and sooner or later author of this work or one of many groups in the world will prepare it.

## 5.3 Conclusion

1. In this work, a methodology for calculating the critical magnetic temperature using DFT and subsequent Monte Carlo simulations based on the Ising spin model was developed and tested. Results obtained during these calculations showed an average divergence of 30% with experimental values but yet in the majority of cases allows to distinguish materials with high critical temperature.
2. As a competitive approach, several regression machine learning models have been trained on the descriptors solely based on the chemical composition of the material. Nevertheless, the best-trained model showed high predictive power with the  $R^2$  value of 0.87 and MAE of 57 K, making this approach comparable in accuracy to demanding electronic structure calculations.
3. Both approaches have been applied to determine the critical temperature of 55 newly predicted structures with promising magnetic properties. For three of the studied structures, a high value of the critical temperature was estimated, which allows us to assume their possible technological potential.
4. Two declared methodologies are implemented as a python library. A simple interface and setting most of the required numeric parameters by default minimizes the need for end-user management and, in most cases, fully automates the calculations.

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