

习题1

(1) 由闭式解的定义并带入相关数据得 $w^* = \frac{1+1}{1 \times 1} = \frac{1}{2}$ $b^* = \frac{1}{3}$

(2)

$$(w_E, b_E) = \operatorname{argmin} \sum_{i=1}^m \frac{|y_i - wx_i - b|^2}{(1 + w^2)}$$

$$\frac{\partial E(w_E, b_E)}{\partial w} = \sum_{i=1}^m \frac{-2x_i(y_i - wx_i - b)(1 + w^2) - 2w(y_i - wx_i - b)^2}{(1 + w^2)^2}$$

$$\frac{\partial E(w_E, b_E)}{\partial b} = \frac{2mb - 2 \sum_{i=1}^m (y_i - wx_i)}{1 + w^2}$$

令上面两式等于0,并带入相关数据得

$$b_E = \frac{1}{3}$$

$$w_E = \sqrt{\frac{13}{9}} - \frac{2}{3} = 0.53518375$$

(3)

欧氏距离:

$$(w_E, b_E) = \operatorname{argmin} \sum_{i=1}^m \frac{|y_i - wx_i - b|}{\sqrt{1 + w^2}}$$

当 $w^* = \frac{1+1}{1 \times 1} = \frac{1}{2}$ $b^* = \frac{1}{3}$ 时

$$\sum_{i=1}^m \frac{|y_i - wx_i - b|}{\sqrt{1 + w^2}} = 0.596284794$$

存在 $w = 0.5, b = 0.5$ 使得

$$\sum_{i=1}^m \frac{|y_i - wx_i - b|}{\sqrt{1 + w^2}} = 0.4472135955 < 0.596284794$$

所以 $w^* = \frac{1+1}{1 \times 1} = \frac{1}{2}$ $b^* = \frac{1}{3}$ 不是该问题的解

习题2

采用softmax函数, 令

$$p(y = i|x) = \frac{e^{z_i}}{\sum_{m=1}^K e^{z_m}}$$

习题3

```
1 import pandas as pd
2 import numpy as np
3 import math
4
5
6 # 构造对数几率函数
7 def sigmoid(z):
8     return 1 / (1 + math.exp(-z))
9
10
11 # 读取文件
12 train_feature = pd.read_csv("train_feature.csv", delimiter=",")
13 train_target = pd.read_csv("train_target.csv", delimiter=",")
14 val_feature = pd.read_csv("val_feature.csv", delimiter=",")
15 val_target = pd.read_csv("val_target.csv", delimiter=",")
16
17 # 构造训练集相关数组
18 x_train = np.array(train_feature.loc[:, :])
19 y_train = np.array(train_target.loc[:, :])
20 X_hat = np.append(x_train, np.ones([600, 1]), axis=1)
21 # 构造测试集相关数组
22 x_val = np.array(val_feature.loc[:, :])
23 y_val = np.array(val_target.loc[:, :])
24 X_val = np.append(x_val, np.ones([200, 1]), axis=1)
25
26 # 正例数量
27 T_num = np.sum(y_val == 1)
28 # 反例数量
29 F_num = np.sum(y_val == 0)
30
31 # 闭式解获得beta
32 Beta = np.dot(np.linalg.inv(np.dot(X_hat.T, X_hat)), np.dot(X_hat.T,
33 y_train))
34
35 TP = 0
36 FP = 0
37 TN = 0
38 # 计算过程
39 for i in range(200):
40     z = sigmoid(np.dot(Beta.T, X_val[i]))
41     if z >= 0.5 and y_val[i] == 1:
42         TP += 1
43     if z >= 0.5 and y_val[i] == 0:
44         FP += 1
45     if z < 0.5 and y_val[i] == 0:
46         TN += 1
47
48 P = TP / (TP + FP)
49 R = TP / T_num
50 Accuracy = (TP + TN) / (T_num + F_num)
51 print("闭式解")
52 print("Accuracy:", Accuracy)
53 print("Precision:", P)
54 print("Recall:", R)
```

```

55 # 数值方法解得beta
56 Beta_N = np.ones([11,1])
57
58
59 grad_1 = np.zeros([1, 11])
60 grad_2 = np.zeros([11, 11])
61
62
63 for k in range(20):
64     for i in range(600):
65         temp11 = np.dot(X_hat, Beta_N)
66         p1 = math.exp(temp11[i][0]) / (1 + math.exp(temp11[i][0]))
67         grad_1 += X_hat[i:i+1, 0:12] * (float(y_train[i]) - p1)
68         grad_2 += np.dot(X_hat[i:i+1, 0:12].T, X_hat[i:i+1, 0:12]) * p1 * (1
- p1)
69     temp22 = np.dot(np.linalg.inv(grad_2), -grad_1.T)
70     Beta_N = Beta_N - temp22
71
72 TP = 0
73 FP = 0
74 TN = 0
75 for i in range(200):
76     z = sigmoid(np.dot(Beta_N.T, X_val[i]))
77     if z >= 0.5 and y_val[i] == 1:
78         TP += 1
79     if z >= 0.5 and y_val[i] == 0:
80         FP += 1
81     if z < 0.5 and y_val[i] == 0:
82         TN += 1
83
84 P = TP / (TP + FP)
85 R = TP / T_num
86 Accuracy = (TP + TN) / (T_num + F_num)
87 print("牛顿法: ")
88 print("Accuracy:", Accuracy)
89 print("Precision:", P)
90 print("Recall:", R)

```

结果如下:

```

闭式解
Accuracy: 0.74
Precision: 0.6666666666666666
Recall: 1.0
牛顿法:
Accuracy: 1.0
Precision: 1.0
Recall: 1.0

```

