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Homework 2

github repository link - https://github.com/Volodimirich/DSP/tree/master/HomeWork2

Task №1

Design a low pass FIR filter with parameters: passband Fpass=5 MHz; stopband Fstop=6MHz; attenuation at least 60dB in the stopband (out-of-band attenuation). Let the sampling frequency be Fs=50MHz. Determine the design with the lowest computational complexity.

Provide code.

Solution:

Nyquist frequency is equal to $F_N = 50\ MHz / 2 = 25 MHz$.

Then normalized passband and stop frequency are equal:

$F_{pass} = \frac{5 * 10^6}{25 * 10^6} = 0.2,\ F_{stop} = \frac{6 * 10^6}{25 * 10^6} = 0.24$

The function that checks that the filter conditions are fulfilled is as follows:

```
# This function check filter correctness
def filter_analyse(b, graph = False):
      result = True
      w, h = signal.freqz(b)
      w_n = w/max(w)
      h_n = 20 * np.log10 (abs(h))

      if graph:
          plot_graph(w_n, h_n)

      max_passband = h_n[0]
      min_passband = h_n[0]

      for x, y in zip(w_n, h_n):
         #If x < Fpass then amplitude must be greater than -3dB
         if x < 0.2:
              if y < -3:
                  return False
              min_passband = min_passband if y < min_passband else y
              max_passband = max_passband if y > max_passband else y
              #Ripple not greater than 1db
              if (max_passband - min_passband) > 1:
                  return False
```

```
        #If x > Fstop then amplitude must be less than attenuation
        if x > 0.24 and y > -60:
            return False

    return True
```

Then the function performing the brute-force has the following form:

```
def filter_brute_force():
      res_fin = 1000
      data = (0, 0)
      windows = ['boxcar', 'triang', 'blackman', 'hamming', 'hann',
                 'bartlett', 'flattop', 'parzen', 'bohman',
                 'blackmanharris', 'nuttall', 'barthann', 'cosine',
                 'exponential', 'tukey', 'taylor']
      for shift in range(100):
          for window_name in windows:
              res = 100
              while True:
                  b = signal.firwin(res, 0.2 + 0.0001 * shift ,
                      pass_zero = 'lowpass', window = window_name)
                  if filter_analyse(b):
                      if res < res_fin:
                          data = (res, window_name, 0.2+ 0.0001 * shift)
                          res_fin = res
                       break
                  else:
                       res += 1
                  if res > 500:
                       break

      return data
```

In this function we went through the possible windows from the existing ones, and the shift where -6dB is located. By going through the possible values and selecting the best parameter by the order parameter we get a filter with the following parameters:

Order - 149, Window type - blackman, Shift - 0.2063

In this case our plot has the next form:

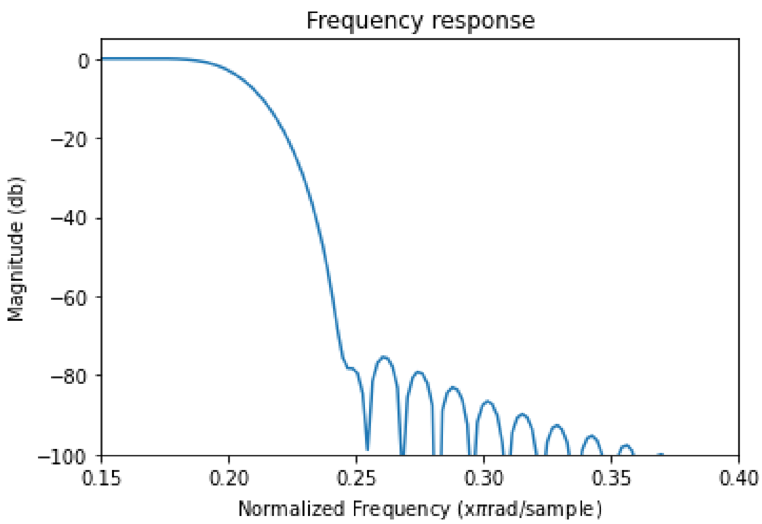

Figure 1: Fir1 filter with best parameters

Also let’s analyse firls filter:

```
order_min = 1000
for i in range(1, 200):
      for order in range(101, 201, 2):
      res = filter_analyse(signal.firls(order,[0, 0.001 * i, 0.24, 1],
[1, 1, 0, 0]))
      if res:
            if order_min > order:
                  data = (order, 0.001 * i)
                  order_min = order
            break
```

In this case we are trying to find a minimum order value with a changing shift from 0 were magnitude is still 1.
In this case we have:

Order - 123, Shift - 0.173

In this case our plot has the next form:

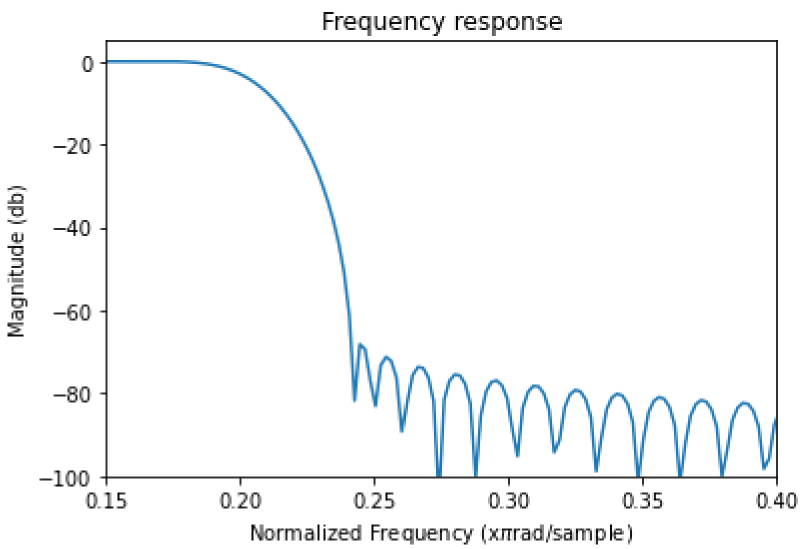

Figure 2: Firls filter with the best parameters

Fir2 gives us the same result as the firls. Then the best order, which we were able to achieve - 123.

Task №2

Using the impulse invariance method for analog to digital filter conversion, calculate the Chebyshev lowpass digital filter with parameters: passband 20MHz; passband ripple 0.2dB; stopband (out-of-band) attenuation 60dB; sampling frequency Fs = 100MHz.

a) Plot the impulse response for both analog and digital systems.
b) Plot the magnitude response for analog and digital systems in the frequency domain.

Provide code.

Solution:

In this task we don't have stopband frequency. Let's set this value as stop_freq = 42MHz. Just now we need filter order:

```
sample_freq = 100 * 10^6;
nyquist_freq = sample_freq / 2;
ripple = 0.2;
stopband = 60;
passband = 20 * 10^6;
passband_normalized = passband / nyquist_freq;
stop_freq = 42 * 10^6;
stop_normalized = stop_freq / nyquist_freq;
order = cheb1ord(passband_normalized, stop_normalized, ripple,
stopband);
```

As a result of this code we can understand that filter order = 4. After a little lab code modernization we can plot impulse responses for analog and digital systems.

```
[bz,az] = impinvar(b, a, sample_freq);
[r,p] = residue(b,a);
t = linspace(0, 40/sample_freq, 1000);
h = real(r.'*exp(p.*t)/sample_freq);
#' useless comment, little fix because of ' in operation
figure(1);
plot(t, h);
hold;
impz(bz, az, [], sample_freq);
legend('Analog', 'Digital');
hold off;
```

The plot is as follows:

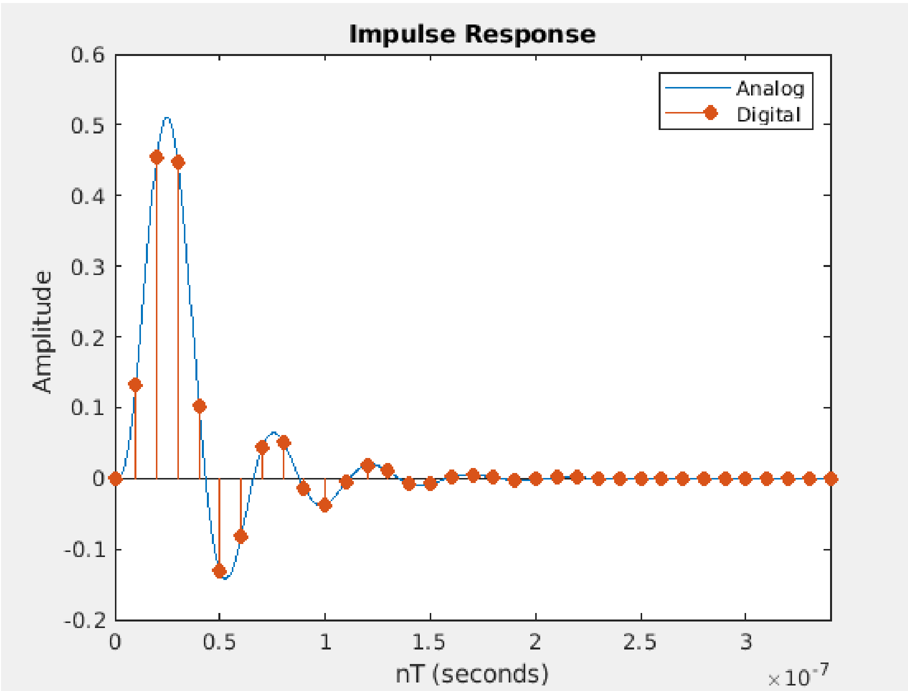

Figure 1: Impulse response

The code for point b was also obtained by slightly modifying the lab work code.

```
figure(2);
[H,W] = freqz(bz, az);
F = linspace(0,sample_freq/2, length(W));
semilogy(F, abs(H))
hold on;
[H] = freqs(b,a,W*sample_freq);
semilogy(F, abs(H));
legend("Digital", "Analog");
```

```
hold off;
```

The plot is as follows:

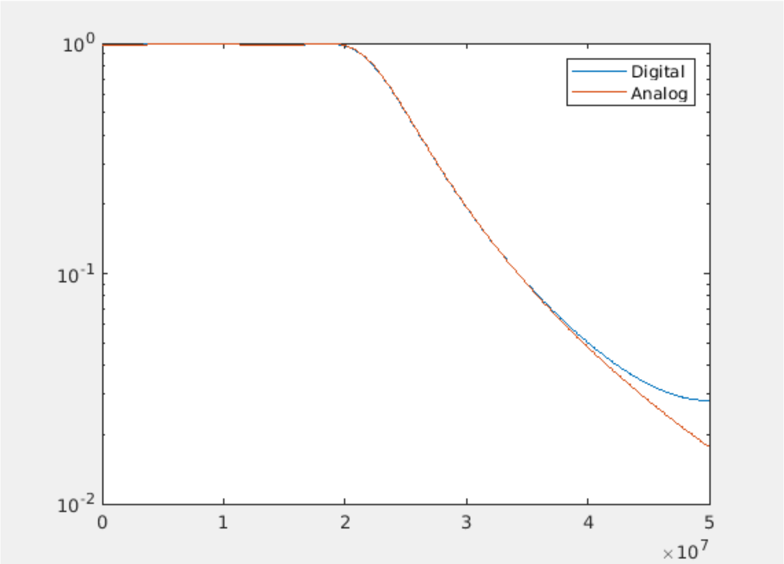

Figure 2: Frequency response

Task №3

Implement a digital prototype of the analog filter with the transfer function

$$H(s) = \frac{s+2.5}{s^2+2.5s+4}$$

using the Bilinear Transformation. The sample clock frequency is Fs =20Hz.

a) Determine the Linear Difference Equation of the digital filter.

b) Plot impulse and frequency responses for digital and analog filters.

Provide code.

Solution:

$$\frac{s+2.5}{s^2+2.5s+4} = \frac{2s+5}{2s^2+5s+8} = \frac{A}{B} \qquad \text{Bilinear transformation: } s = \frac{2(z-1)}{T(z+1)}$$

On the first step, let's calculate B part:

$$2s^2+5s+8 = 2\frac{4(z-1)^2}{T^2(z+1)^2}+5\frac{2(z-1)}{T(z+1)}+8 = \frac{8(z-1)^2+10(z-1)T(z+1)+8T^2(z+1)^2}{T^2(z+1)^2} =$$

$$\frac{8z^2-16z+8+10z^2T-10T+8T^2z^2+16T^2z+8T^2}{T^2(z+1)^2} = \frac{(8+10T+8T^2)z^2+(16T^2-16)z+8-10T+8T^2}{T^2(z+1)^2}$$

After that calculate A part:

$$2s+5 = 2\frac{2(z-1)}{T(z+1)}+5 = \frac{4(z-1)+5T(z+1)}{T(z+1)} = \frac{4(z-1)T(z+1)+5T^2(z+1)^2}{T^2(z+1)^2} =$$
$$\frac{4Tz^2-4T+5T^2z^2+10T^2z+5T^2}{T^2(z+1)^2} = \frac{(4T+5T^2)z^2+10T^2z+5T^2-4T}{T^2(z+1)^2})$$

Then we have:

$$\frac{A}{B} = \frac{(4T+5T^2)z^2+10T^2z+5T^2-4T}{(8+10T+8T^2)z^2+(16T^2-16)z+8-10T+8T^2}$$

According to $T = \frac{1}{F} = \frac{1}{20}$ we have:

$$\frac{A}{B} = \frac{0.2125z^2+0.025z-0.1875}{8.52z^2-15,96z+7.52} = \frac{0.0249+0.00293z^{-1}-0.022z^{-2}}{1-1.8732z^{-1}+0.8827z^{-2}} = \frac{Y(z)}{X(z)}$$

$$0.0249\,X(z) + 0.00293\,X(z)\,z^{-1} - 0.022\,X(z)\,z^{-2} = Y(z) - 1.8732\,Y(z)\,z^{-1} + 0.8827\,Y(z)\,z^{-2}$$

Then linear difference equation has the following form

$$y[n] = -1.8732\,y[n-1] + 0.8827\,y[n-2] - 0.0249\,x[n] + 0.00293\,x[n-1] - 0.022\,x[n-2]$$

```
clock_freq = 20;
nyquist_freq = clock_freq / 2;
b = [0, 1, 2.5];
a = [1, 2.5, 4];
bz = [0.0249, 0.0029, -0.022];
az = [1, -1.8732, 0.8826];

[freq_resp, ang_resp] = freqz(bz,az);
x_vals = linspace(0, nyquist_freq, length(freq_resp));

figure(1);
semilogy(x_vals,abs(freq_resp))
hold on;
[H] = freqs(b,a, ang_resp*clock_freq);
semilogy(x_vals, abs(H))

legend("Digital", "Analog")
hold off;
fvtool(bz, az)
```

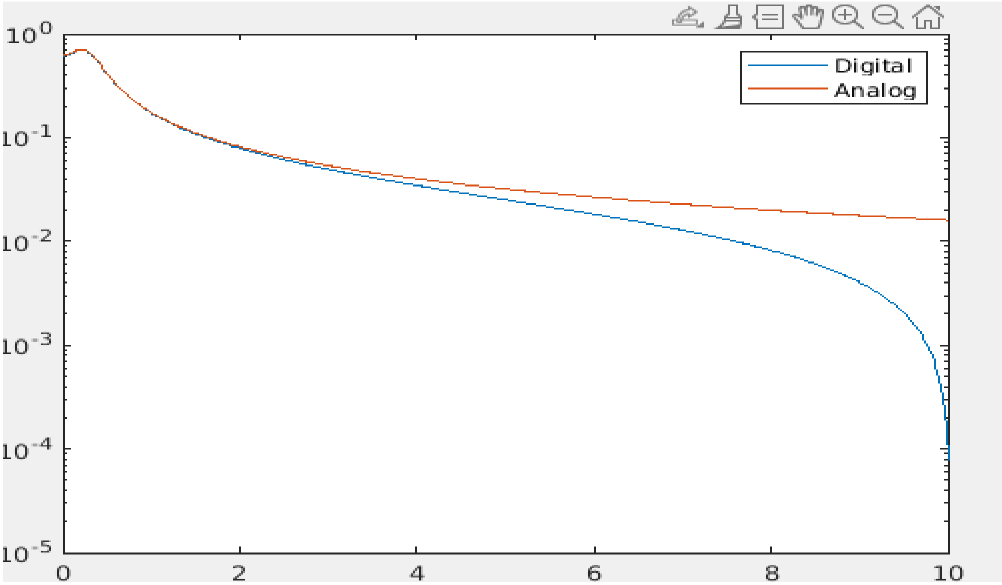

Figure 1: Frequency response

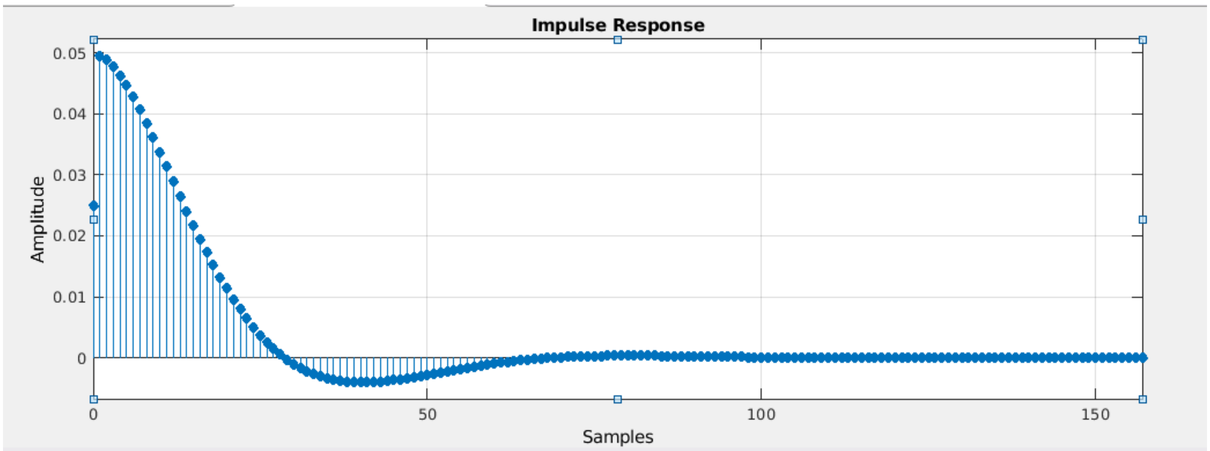

Figure2: Impulse response

Task №4

A filter has the transfer function

$$H(z) = 3 + 4z^{-1} + 6z^{-2} + 8z^{-3}$$

Determine the impulse response of the filter with the modified frequency response

$$F(w) = H(w - \frac{\pi}{4})$$

Solution:

Let's use definition $z = re^{iw}$

$$H(w-\frac{\pi}{4}) = 3 + 4re^{-(w-\frac{\pi}{4})i} + 6re^{-2(w-\frac{\pi}{4})i} + 8re^{-3(w-\frac{\pi}{4})i} = 3 + 4re^{\frac{\pi i}{4}}e^{-wi} + 6re^{\frac{\pi i}{2}}e^{-2wi} + 8re^{\frac{3\pi i}{4}}e^{-3wi}$$

After that we need to calculate IDTFT:

$$x[w] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X[w]e^{wni}dw$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}(3+4re^{\frac{\pi i}{4}}e^{-wi}+6re^{\frac{\pi i}{2}}e^{-2wi}+8re^{\frac{3\pi i}{4}}e^{-3wi})e^{wni}dw = \frac{3}{2\pi}\int_{-\pi}^{\pi} e^{nwi}dw+ \frac{2r}{\pi}\int_{-\pi}^{\pi} e^{(n-1)wi}dw + \frac{3r}{\pi}\int_{-\pi}^{\pi} e^{(n-2)wi}dw + \frac{4r}{\pi}\int_{-\pi}^{\pi} e^{(n-3)wi}dw$$

Let’s calculate part of our equation:

$$\int_{-\pi}^{\pi} e^{nwi}dw$$

$$\int_{-\pi}^{\pi} e^{wki}dw = \left.\frac{e^{wki}}{ki}\right|_{-\pi}^{\pi} = \left.\frac{\cos wk + i\sin wk}{ki}\right|_{-\pi}^{\pi} = \frac{\cos \pi k + i\sin \pi k}{ki} - \frac{\cos(-\pi)k + i\sin(-\pi)k}{ki} = \frac{\cos \pi k + i\sin \pi k}{ki} - \frac{\cos \pi k - i\sin \pi k}{ki} = \frac{2i\sin \pi k}{ki} = \frac{2sin\pi k}{k}$$

By replacing k with the corresponding expression with n we obtain:

$$h[n] = \frac{3sin\pi n}{\pi n}+\frac{4rsin\pi(n-1)}{\pi(n-1)}e^{\frac{\pi i}{4}}+\frac{6rsin\pi(n-2)}{\pi(n-2)}e^{\frac{\pi i}{2}}+\frac{8rsin\pi(n-3)}{\pi(n-3)}e^{\frac{3\pi i}{4}} = 3\cdot sinc(n) + 4r\cdot sinc(n-1)e^{\frac{\pi i}{4}} + 6r\cdot sinc(n-2)e^{\frac{\pi i}{2}} + 8r\cdot sinc(n-3)e^{\frac{3\pi i}{4}}$$

Then:

$$h[n] = 3\delta[n] + 4r\delta[n-1]e^{\frac{\pi i}{4}} + 6r\delta[n-2]e^{\frac{\pi i}{2}} + 8r\delta[n-3]e^{\frac{3\pi i}{4}}$$

Task №5

For a linear system with the transfer function

$$H(z) = \frac{1z+2}{3z^3+4z^2+5z+6}$$

a) Calculate the difference equation relating the input x[n] to the output y[n]
b) Design block diagram realizations (Direct-Form 1 and Direct-Form 2)
c) Plot impulse and frequency responses

Provide code.

Solution:

$$H(z) = \frac{1z+2}{3z^3+4z^2+5z+6} = \frac{1z^{-2}+2z^{-3}}{3+4z^{-1}+5z^{-2}+6z^{-3}} = \frac{\frac{1}{3}z^{-2}+\frac{2}{3}z^{-3}}{1+\frac{4}{3}z^{-1}+\frac{5}{3}z^{-2}+2z^{-3}} = \frac{Y(z)}{X(z)}$$

$$Y(z) + \frac{4}{3}Y(z)z^{-1} + \frac{5}{3}Y(z)z^{-2} + 2Y(z)z^{-3} = \frac{1}{3}X(z)z^{-2} + \frac{2}{3}X(z)z^{-3}$$

$$Y(z) = -\frac{4}{3}Y(z)z^{-1} - \frac{5}{3}Y(z)z^{-2} - 2Y(z)z^{-3} + \frac{1}{3}X(z)z^{-2} + \frac{2}{3}X(z)z^{-3}$$

$$y[n] = -\frac{4}{3}y[n-1] - \frac{5}{3}y[n-2] - 2y[n-3] + \frac{1}{3}x[n-2] + \frac{2}{3}x[n-3]$$

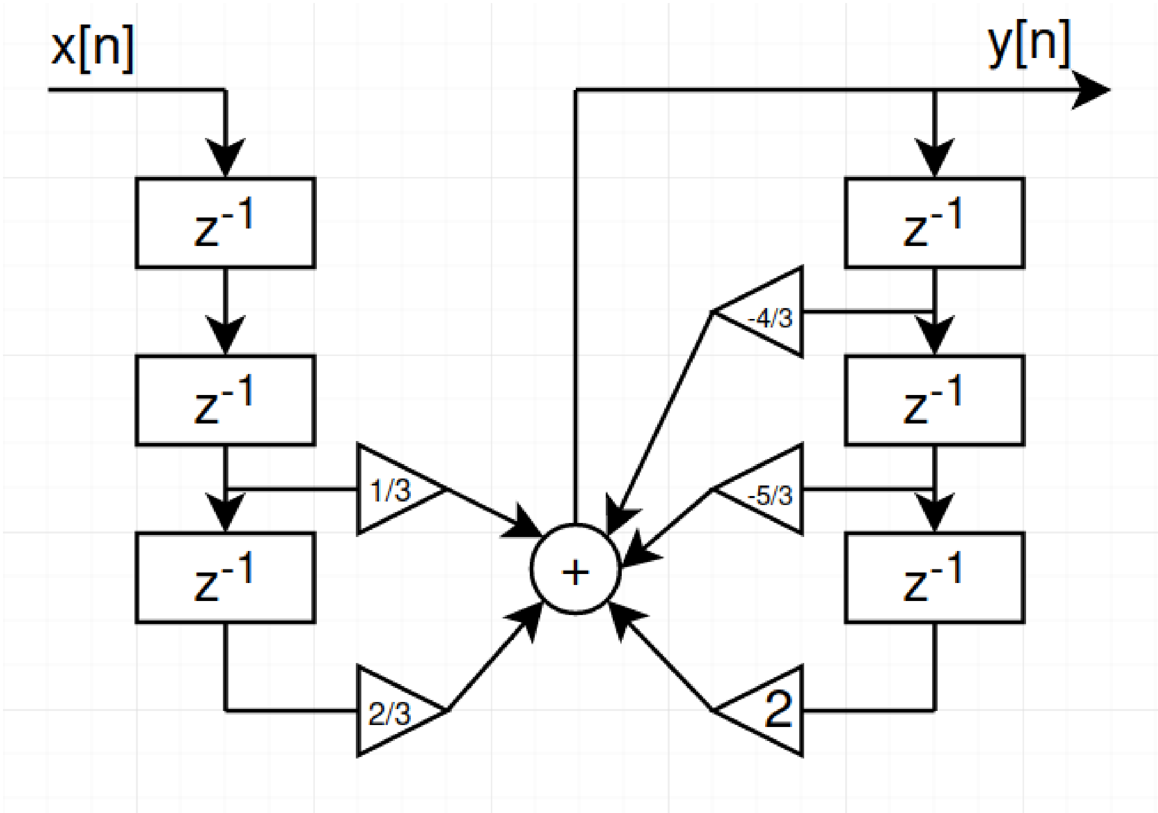

Figure 1: Direct-Form 1

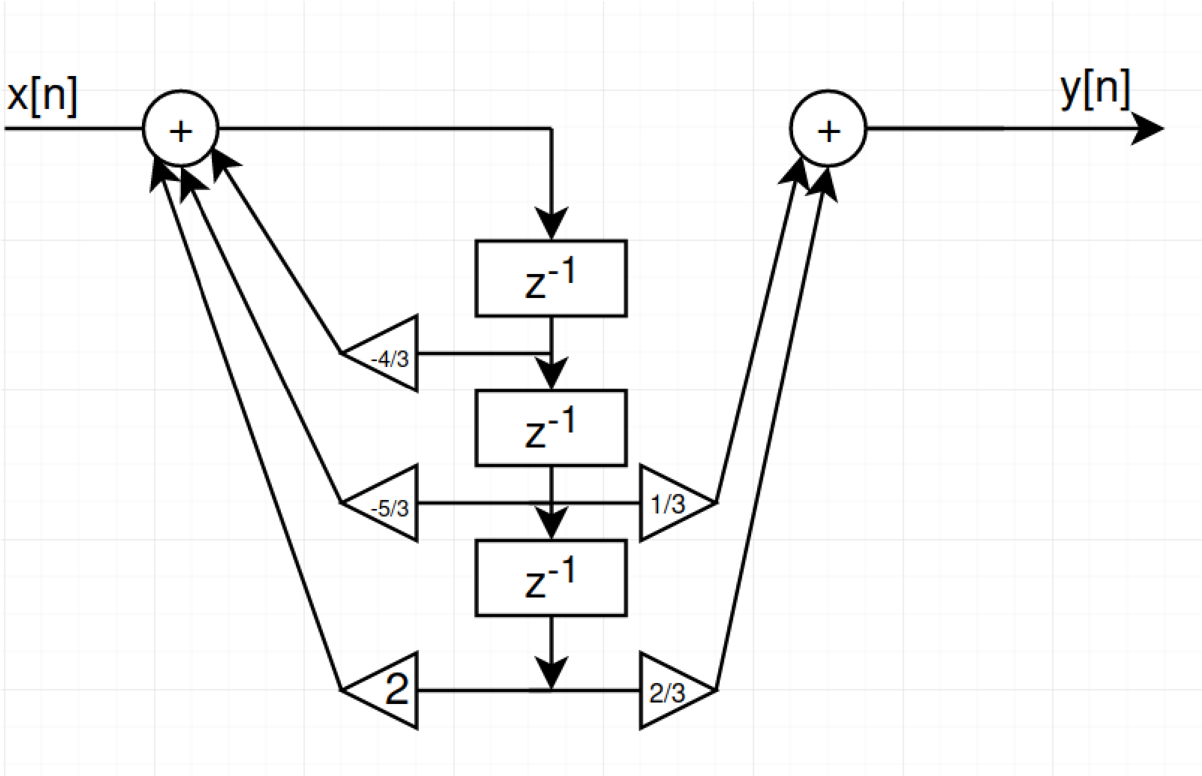

Figure 2: Direct-Form 2

According to $H(z) = \frac{\frac{1}{3}z^{-2} + \frac{2}{3}z^{-3}}{1 + \frac{4}{3}z^{-1} + \frac{5}{3}z^{-2} + 2z^{-3}}$ we can calculate impulse and frequency response with using next code:

```
fvtool([0 0 1/3 2/3],[1 4/3 5/3 2])
```

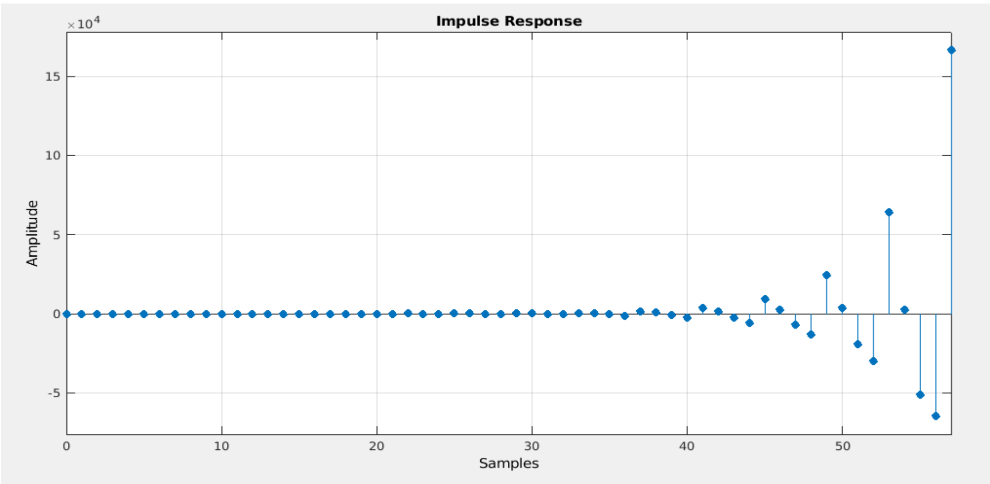

Figure 3: Impulse response

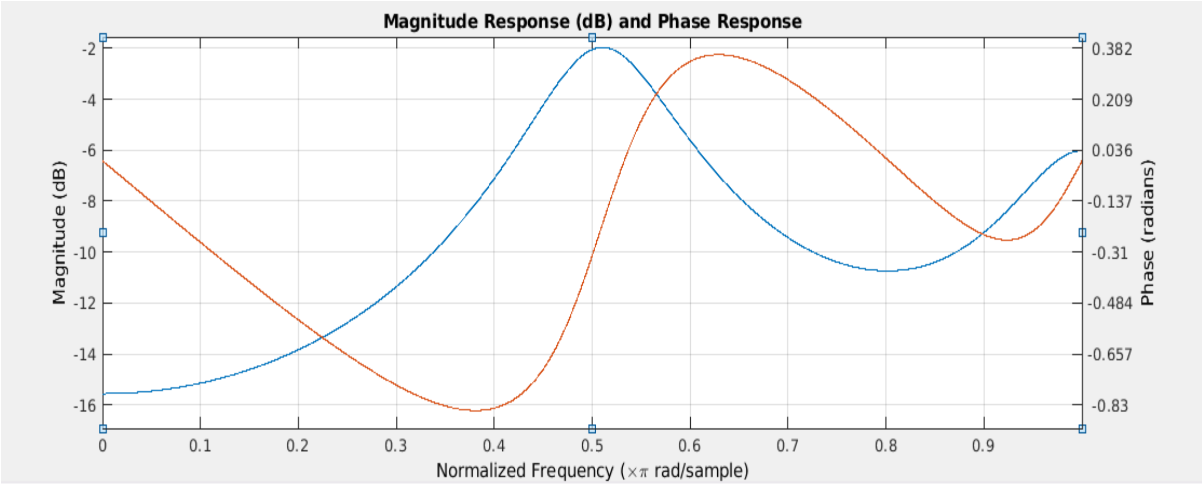

Figure 4: Frequency response