1. Porous medium heated from below

Regard a system of Darcy's equation for flow in a porous medium saturated with water and a convective heat equation. In addition to the simple formulations in the lecture, we include two additional effects:

- Heat expansion leading to a decrease of the water density $\rho(T)$ and therefore a tendency for less dense ("lighter") water to move up in the force field of gravity \vec{g}
- Convective heat transport: heat not only moves due to heat conduction, but also along with the moving water.

Here we provide the so-called Boussinesq approximation which keeps the temperature dependency of the density only in the gravity term:

$$\begin{aligned} \vec{q} &= -\rho_{ref} k (\nabla p - \rho(T) \vec{g}) \\ \partial_t \rho_{ref} &+ \nabla \cdot \vec{q} = 0 \\ \partial_t (cT) &- \nabla \cdot (\lambda \nabla T - c \vec{q}) = 0 \end{aligned}$$

The unknowns and parameters are as follows:

p		pressure
T		temperature
$ec{q}$		water mass flux
$\rho(T)$	$ \rho_{ref} - \alpha (T - T_{ref}) $	density
k	100	permeability
$ec{g}$	(0,-1)	gravity vector
c	0.001	heat capacity
α	0.01	heat expansion coefficient
λ	0.01	heat conductivity
ρ_{ref}	1	reference density
T_{ref}	0	reference temperature
P_{ref}	0	reference pressure

The values of the constants have been choosen as to ensure that the tasks posed below will work.

Regard the domain $\Omega = (0,300) \times (0,150)$ Let $\Gamma_{top} = (0,300) \times 150$, $\Gamma_{bot} = (0,300) \times 0$, We set Dirichlet boundary conditions

$$P = 0$$
 on Γ_{top}
 $T = 0$ on Γ_{top}
 $T = 0$ on Γ_{hot}

and homogeneous Neumann boundary conditions for all other boundaries.

Investigate two things:

- Stationary distribution under the above boundary conditions
- What happens if we set $T = T_{heat} > 0$ on Γ_{bot} ?

1.1. Implementation:

- Implement the finite volume discretization of the problem. Feel free to use the VoronoiFVM code. You can consult the (unfortunately unpublished) preprint https://dx.doi.org/10.20347/WIAS.PREPRINT.741. The discrete problem can be described using specific implementations for storage, reaction and flux, in VoronoiFVM.jl, and from the examples provided in the last lectures it should be possible to derive the implementation for the backward Euler time stepping scheme¹
- Calculate the steady state for $T_{heat} = 0$

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¹I verified that this works.

• Calculate the transient solutions for $T_{heat} = 0.5, T_{heat} = 1$, $T_{heat} = 2, T_{heat} = 5, T_{heat} = 10$ In order to watch the evolution, consider to use 'scalarplot' from 'GridVisualize.jl' with the 'PlutoVista.jl' backend as used in the lecture notebooks.

1.2. Optionally:

• Use alternative timestepping methods from DifferentialEquations.jl (Please come back to me if you want to try this out – in fact the more sophisticated schemes allow for faster solution.)

1.3. Report:

- Introduce the problem and some information on the physical background
- Discuss the finite volume space discretization approach
- Discuss possibilities for the time discretization. Are there any obstacles for implementing the explicit Euler method?
- Discuss possible solution methods for the discretized problem
- Present simulation results. Do you see qualitative differences in the transient behavior for growing T_{heat} ?
- Discuss ways to improve performance
- Discuss any of the optional topics

