




# Effect of a porous medium on flow and mixed convection heat transfer of nanofluids with variable properties in a trapezoidal enclosure

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## Abstract

In the present study, the flow field and heat transfer of a water–copper nanofluid with variable properties in a trapezoidal enclosure saturated with porous media are studied. The governing equations are solved by finite volume method and the SIMPLER algorithm. The nanofluid flow is assumed to be laminar, steady and incompressible. Simulations are performed for sidewall (trapezoid legs) angles of 30°, 45° and 60° with respect to horizontal axis, Reynolds numbers from 10 to 1000, Darcy numbers of  $10^{-2}$ ,  $10^{-3}$ ,  $10^{-4}$  and volume fractions of 0 to 0.04 of nanoparticles. Numerical results show that the average Nusselt number increases with increasing volume fraction of nanoparticles for all studied Darcy numbers. The convection and motion of the nanofluid decrease by reducing the Darcy number which leads to a reduction in the velocity and local Nusselt number. The average Nusselt number increases by increasing the Darcy number for all aspect ratios. Also, the average Nusselt number increases with increasing Reynolds number for all Darcy numbers, aspect ratios and volume fractions of nanoparticles.

**Keywords** Porous medium · Nanofluid · Trapezoid enclosure · Numerical solution · Mixed convection · Darcy number

## Introduction

The fluid flow and heat transfer in an enclosure filled with porous media have widely attracted interest of many researchers. Insulation materials, geophysical applications, heating and cooling in buildings, underground heat pump systems and solar energy are examples of fluid flows in porous mediums [1–4]. On the one hand, it has always been desirable to have access to smaller dimensions and higher returns for better heat transfer in industrial equipment such as electronic components and heat exchangers. On the other hand, it has always been desirable to have access to smaller dimensions and higher efficiency for higher heat transfer in industrial equipments such as electronic components and heat exchangers. In recent years, the study of material properties in a nanoscale has been more considered due to that the nanoparticles exhibit different behaviors from the macroscale. Dispersion of nanoparticles into common fluids such as water, oil and ethylene glycol may change the properties of colloid and improve heat transfer [5–21].

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Mixed convection has many applications in nuclear reactors, industrial lubricants, heat exchangers, metal melting industries, etc., due to the combined effects of natural and forced convections. In particular, the cooling of elliptical heat resources is considered in the electronic industry and solar collectors [22].

Mansour et al. [23] studied mixed convection of a nanofluid in a cavity with a moving cold upper wall, cold sidewalls and a bottom wall in which constant heat flux was applied. According to their results, the fluid flow decreases by increasing the volume fraction of nanoparticles, but the average Nusselt increases.

Ghasemi and Aminossadati [24] studied the mixed convection of aluminum oxide–water in a triangular enclosure with horizontal insulated wall, vertical cold and moving wall in the up or down directions and hot inclined wall. According to their results, heat transfer increases by increasing the volume fraction of nanoparticles and moving the vertical wall in both directions for the whole studied range of the Richardson number.

Sheikhzadeh et al. [25] investigated the heat transfer and fluid flow of aluminum oxide–water nanofluid in a square box with moving upper wall, insulated horizontal walls and vertical walls with hot and cold temperatures on the right and left sides, respectively, numerically using finite volume method. Based on their results, by considering the variable properties of the thermal conductivity and fluid viscosity coefficients, the average Nusselt number changes compared to that for constant properties. In addition, they reported that the difference is larger for low Richardson numbers (0.01 and 0.1) than high Richardson numbers (10 and 100).

Pishkar and Ghasemi [26] studied mixed convection heat transfer and fluid flow in a horizontal channel with a blade for a water–copper nanofluid numerically. They found that the heat transfer increases by increasing the volume fraction and Reynolds number. This increase is higher at higher Reynolds numbers for a constant volume fraction.

Chamkha and Abu-Nada [27] investigated mixed convection heat transfer and fluid flow in a square enclosure with lateral insulated walls, hot upper and cold bottom walls for two cases. In the first case, only the upper horizontal wall was movable, and in the second case, the upper and lower horizontal walls moved in the opposite direction. They revealed that the average Nusselt number increases by increasing the volume fraction and decreasing the Richardson number.

Abbasian et al. [28] investigated mixed convection heat transfer and fluid flow of the water–copper nanofluid in a square enclosure in which the horizontal walls were

insulated and the lateral walls had sinusoidal temperature boundary condition. Based on their results, the average Nusselt number increases with the increase in phase difference and the Richardson number for a constant volume fraction.

Oztop et al. [29] studied natural convection heat transfer and fluid flow in an opened cavity filled with porous media numerically. They demonstrated that the Nusselt number increases by increasing the Darcy number for all Rayleigh numbers.

Bourantas et al. [30] studied natural convection and heat transfer of a nanofluid with constant properties in square enclosures in porous media numerically. The bottom wall was under constant heat flux partially, and the other walls were cold. Based on their results, the average Nusselt number increases with increasing Rayleigh number for all Darcy numbers at a constant volume fraction.

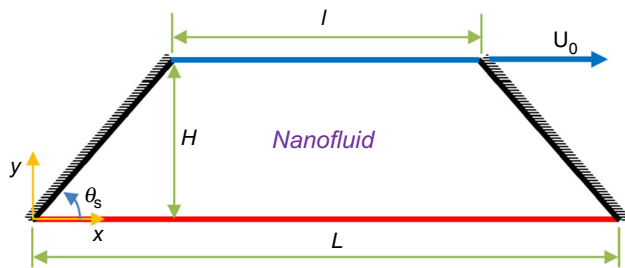
Hajipour and Dehkordi [31] investigated mixed convection heat transfer of aluminum oxide–water in a channel filled with porous material experimentally and numerically. Their results differed from the previous studies.

Nielda and Kuznetsov [32] investigated the effect of Brownian motion and thermophoresis on forced convection of a nanofluid in a channel filled with porous media. Their results showed that Brownian motion and thermophoresis have opposite effects on the average Nusselt number.

In the present study, fluid flow and mixed convection heat transfer of a nanofluid with variable properties in a trapezoidal cavity filled with porous media are investigated numerically. The sidewalls of the enclosure are insulated, and the bottom and top walls are hot and cold, respectively. To analyze the mixed convection, a computer program written in FORTRAN, based on the finite volume method and SIMPLER algorithm, is used. This method has been used in the previous works [33–41] and was confirmed by the researchers. The Brinkman–Forchheimer model is used to model the porous media. The studies are conducted for various volume fractions of nanoparticles, Richardson and Darcy numbers and different aspect ratios.

## Governing equations and boundary conditions

The schematic of the problem is shown in Fig. 1. The bottom wall of the enclosure is at the temperature  $T_h$ , the upper one is at the temperature  $T_c$ , and the sidewalls are insulated. The enclosure is filled with porous material. The nanofluid flow is assumed to be laminar, steady and incompressible. Simulations are performed for sidewall (trapezoid legs) angles of  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  with respect to



**Fig. 1** Geometry of a two-dimensional trapezoidal cavity with insulated side walls, top moving wall and hot bottom wall filled with porous media

**Table 1** Thermophysical properties of base fluid (at 300 K) and nanoparticles [42]

Property	Water (the base fluid)	Copper
$c_p$ (J/kg K)	4179	385
$\rho$ (kg/m <sup>3</sup> )	997.1	8933
$k$ (W/m K)	0.613	401
$\beta$ (K <sup>-1</sup> )	$21 \times 10^{-5}$	$1.67 \times 10^{-5}$
$\mu$ (W/m K)	0.001003	–

horizontal axis, Reynolds numbers from 10 to 1000, Darcy numbers of  $10^{-2}$ ,  $10^{-3}$ ,  $10^{-4}$  and volume fractions of 0–0.04.

Thermophysical properties of water as base fluid and copper nanoparticles are presented in Table 1.

The governing equations, including continuity, momentum and energy equations for Newtonian fluid and two-dimensional, laminar and steady flow, are defined as follows. The Brinkman–Forchheimer model is used to model the porous media.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{1}{\rho_{nf}} \left[ \frac{\partial}{\partial x} \left( \mu_{nf} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{nf} \frac{\partial u}{\partial y} \right) \right] - \frac{v_{nf}}{\varepsilon} u \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{1}{\rho_{nf}} \left[ \frac{\partial}{\partial x} \left( \mu_{nf} \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_{nf} \frac{\partial v}{\partial y} \right) \right] + \frac{(\rho\beta)_{nf}}{\rho_{nf}} g(T - T_c) - \frac{v_{nf}}{\varepsilon} v \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = -\frac{1}{(\rho c_p)_{nf}} \left[ \frac{\partial}{\partial x} \left( k_{nf} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_{nf} \frac{\partial T}{\partial y} \right) \right] \quad (4)$$

Since the problem is solved as a dimensionless manner, it is first necessary to non-dimensionalize the governing equations and boundary conditions. The quantities used for non-dimensionalizing are introduced in Eq. (5):

$$X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad V = \frac{v}{U_0}, \quad U = \frac{u}{U_0}, \quad \theta = \frac{T - T_c}{T_h - T_c} \quad (5)$$

$$P = \frac{p}{\rho_f U_0^2},$$

$$\text{Gr} = \frac{g\beta_f H^3 (T_h - T_c)}{v_f^2}, \quad \text{Re} = \frac{U_0 H}{v_f}, \quad \text{Ri} = \frac{\text{Gr}}{\text{Re}^2} = \frac{10^4}{\text{Re}^2}, \quad \text{Pr} = \frac{v_f}{\alpha_f}, \quad \text{Da} = \frac{\varepsilon}{H^2}$$

where  $\varepsilon$  represents the porosity of the medium and  $\text{Da}$  is dimensionless Darcy number. Reduction in the Darcy number indicates that the porosity of the media decreases. Using the dimensionless quantities, the equations for mass conservation, momentum and energy conservation are obtained:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\rho_{nf} v_f \text{Re}} \left[ \frac{\partial}{\partial X} \left( \mu_{nf} \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu_{nf} \frac{\partial U}{\partial Y} \right) \right] - \frac{v_{nf}}{v_f} \frac{U}{\text{Da} H^2 \text{Re}_f} \quad (7)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\rho_{nf} v_f \text{Re}} \left[ \frac{\partial}{\partial X} \left( \mu_{nf} \frac{\partial V}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu_{nf} \frac{\partial V}{\partial Y} \right) \right] - \frac{v_{nf}}{v_f} \frac{V}{\text{Da} H^2 \text{Re}_f} + \frac{(\rho\beta)_{nf}}{\rho_{nf} \beta_f} \text{Ri} \theta \quad (8)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\text{Re} \text{Pr} \alpha_f (\rho c_p)_{nf}} \left[ \frac{\partial}{\partial X} \left( k_{nf} \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left( k_{nf} \frac{\partial \theta}{\partial Y} \right) \right] \quad (9)$$

According to the geometry of the problem, the boundary conditions are:

$$q'' = -k_{\text{nf}} \frac{T_h - T_c}{H} \frac{\partial \theta}{\partial n} \bigg|_{\text{wall}} \quad (20)$$

On the bottom wall

$$U = V = 0, \theta = 1$$

On the top wall

$$U = 1, V = 0, \theta = 0 \quad (10)$$

On the side walls ( $n$  represents the normal direction)  $U = V = 0, \frac{\partial \theta}{\partial n} = 0$

The nanofluid properties, including the density, thermal capacity, volumetric expansion coefficient, diffusion coefficient, the viscosity [43] and thermal conductivity coefficient [44], are obtained from relations (11)–(16), respectively.

$$\rho_{\text{nf}} = (1 - \phi)\rho_f + \phi\rho_s \quad (11)$$

$$(\rho c_p)_{\text{nf}} = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_s \quad (12)$$

$$(\rho\beta)_{\text{nf}} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \quad (13)$$

$$\alpha_{\text{nf}} = \frac{k_{\text{nf}}}{(\rho c_p)_{\text{nf}}} \quad (14)$$

$$\mu_{\text{nf}} = \mu_f(1 - \phi)^{-2.5} \quad (15)$$

$$\frac{k_{\text{eff}}}{k_f} = 1 + \frac{k_p A_p}{k_f A_f} + Ck_p Pe \frac{A_p}{k_f A_f} \quad (16)$$

The effect of Brownian motion is appeared by considering the effect of temperature on thermal conductivity coefficient. The amount of thermal conductivity of the nanofluid by considering this effect is more than that when the effect of this motion is ignored. In (16),  $C = 36,000$ ,  $\frac{A_p}{A_f} = \frac{d_f \phi}{d_p(1 - \phi)}$ ,  $Pe = \frac{U_p d_p}{\alpha_f}$  [44]. The Brownian motion velocity ( $U_p$ ) is obtained from Eq. (17):

$$U_p = \frac{2k_B T}{\pi \mu_f d_p^2} \quad (17)$$

Here,  $d_f$  and  $d_p$  are the diameters of water molecules and copper particles, respectively, which are  $2 \times 10^{-10}$  and  $100 \times 10^{-9}$ . It should be noted that this particular model is used for spherical nanoparticles. The volume fraction of nanoparticles should be between 1 and 8%, and the base fluid can be water and ethylene glycol.

The natural heat transfer coefficient is:

$$h_{\text{nf}} = \frac{q''}{T_h - T_c} \quad (18)$$

The Nusselt number, whose length scale is measured based on the size of the enclosure edge, is defined as:

$$Nu = \frac{h_{\text{nf}} H}{k_f} \quad (19)$$

The heat flux of walls per unit area is:

By placing the relations (18) and (20) into (19), the Nusselt number is obtained using relation (21):

$$Nu = - \left( \frac{k_{\text{nf}}}{k_f} \right) \frac{\partial \theta}{\partial n} \bigg|_{\text{wall}} \quad (21)$$

The average Nusselt number on the hot wall is:

$$Nu_{\text{Avg}} = \frac{1}{L} \int_0^1 Nu_x dX \quad (22)$$

## Validation of the numerical scheme

In order to validate the results of the computer program, numerical simulations are performed and the results are compared with the numerical results. To verify the present results, the geometry presented by Chamkha and Abu-Nada [27] is considered for the simulations. The results are compared in Table 2. As can be seen, the relative differences between the values of the Nusselt number are negligible and, therefore, the accuracy of the present simulations is assured.

**Table 2** Comparison of the average Nusselt number in mixed convection [27]

Ri	$\phi$	Chamkha and Abu-Nada [27]	Present work	Difference percentage
0.01	0.02	32.80	33.14	1.04
	0.1	36.90	36.40	1.36
1	0.02	4.92	4.76	3.25
	0.1	4.95	4.84	2.22
10	0.02	1.72	1.68	2.32
	0.1	2.01	1.93	3.98

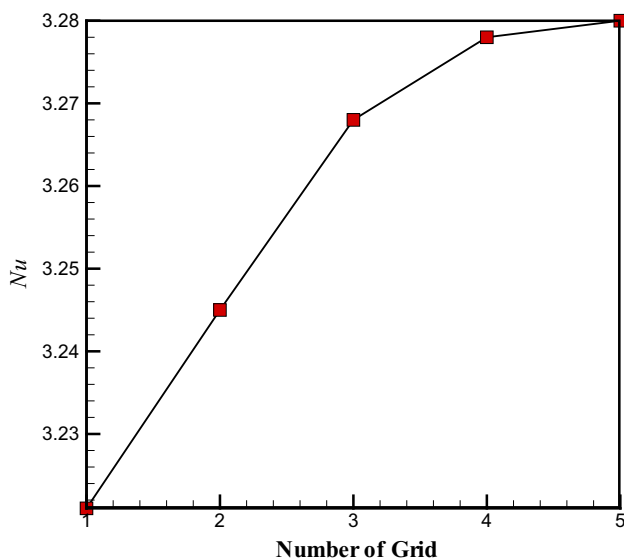
## Grid study

In order to find an appropriate grid resolution, the average Nusselt number of water–copper nanofluid is calculated for the grid resolutions of  $221 \times 111$ ,  $241 \times 121$ ,  $261 \times 131$ ,  $281 \times 141$  and  $301 \times 151$  for  $\theta_s = 45^\circ$ , the Reynolds number of 100, the Darcy number  $10^{-2}$  and volume fraction of 0.02. The results are compared in Table 3. It is found that the grid resolution of  $281 \times 141$  is appropriate. In Fig. 2, the average Nusselt number versus of number of grid for  $\theta_s = 45^\circ$ ,  $Re = 100$ ,  $Da = 10^{-2}$  and  $\varphi = 0.02$  is presented.

The convergence criterion for pressure, velocity and temperature is obtained from Eq. (23) in which  $M$  and  $N$  are the number of points in the direction of  $x$  and  $y$ , and  $\zeta$  represents the variable that is solved.  $k$  is the number of iterations, and the maximum error rate is  $10^{-6}$ .

**Table 3** The average Nusselt number on the hot wall for water–copper nanofluid for  $\theta_s = 45^\circ$ ,  $Re = 100$ ,  $Da = 10^{-2}$  and  $\varphi = 0.02$

Number of grid	Grid resolution	$Nu_{Avg}$
1	$221 \times 111$	3.221
2	$241 \times 121$	3.245
3	$261 \times 131$	3.268
4	$281 \times 141$	3.278
5	$301 \times 151$	3.280



**Fig. 2** The average Nusselt number versus of number of grid for  $\theta_s = 45^\circ$ ,  $Re = 100$ ,  $Da = 10^{-2}$  and  $\varphi = 0.02$

$$\text{Error} = \frac{\sum_{i=1}^M \sum_{j=1}^N \left| \zeta_{i,j}^{k+1} - \zeta_{i,j}^k \right|}{\sum_{i=1}^M \sum_{j=1}^N \left| \zeta_{i,j}^{k+1} \right|} \leq 10^{-6} \quad (23)$$

## Results

### Variation in vertical velocity component

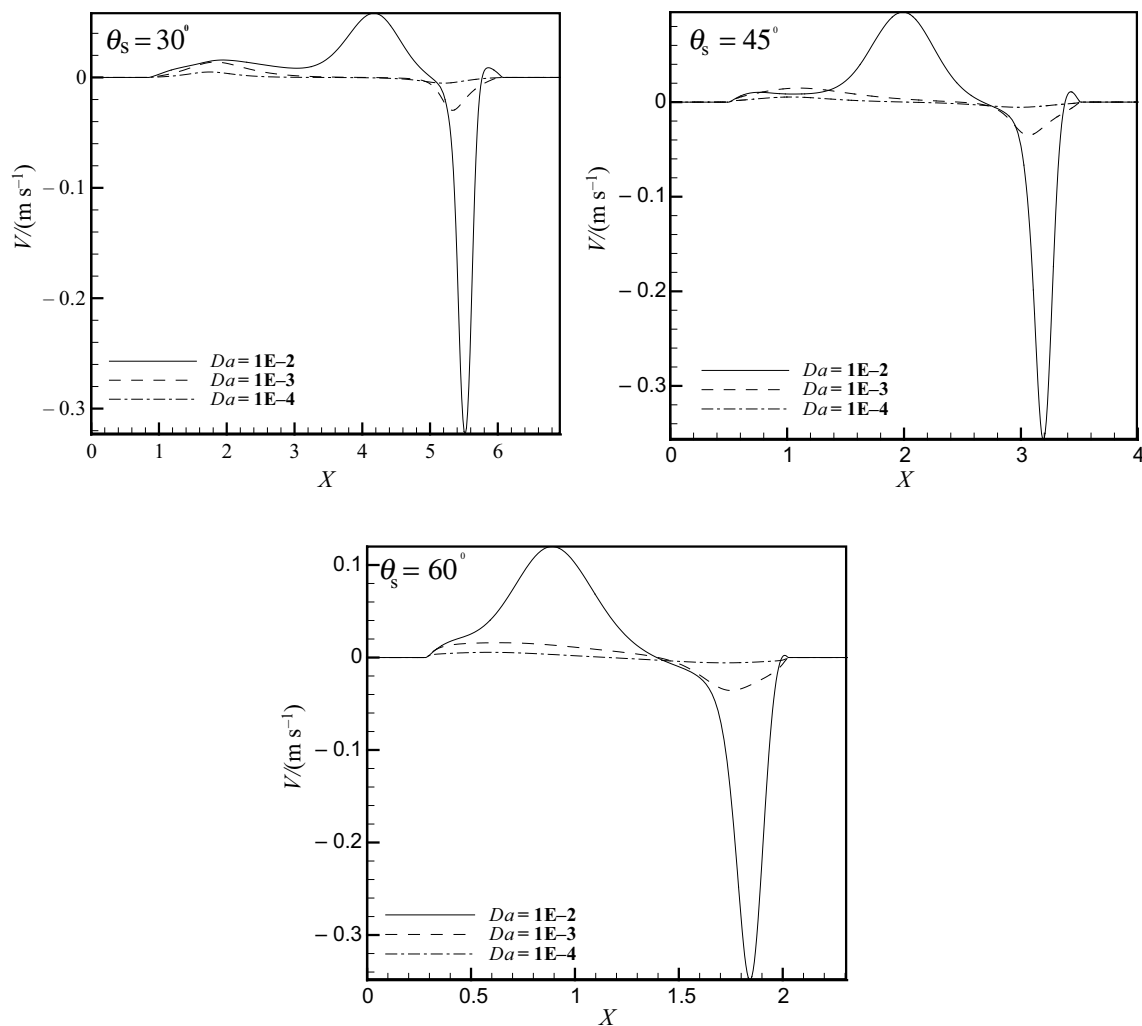
In Fig. 3, the variations in vertical velocity component for the central height versus  $x$  are shown for various Darcy numbers at  $Re = 2000$  and  $\varphi = 0.02$ . This quantity can be a measure of the convection and movement of the nanofluid. The variation in vertical velocity component is higher at  $Da = 10^{-2}$  for all leg angles, and the largest amount is related to the right region of the cavity.

The maximum and minimum velocity changes increase by increasing the leg angle of the trapezoidal enclosure. For example, maximum velocity is 0.31 and 0.325 for the leg angles of  $30^\circ$  and  $60^\circ$ , respectively, representing an increase of 5%. By reducing the Darcy number or reducing the porosity of the medium, the changes of the velocity become much more smooth compared to  $Da = 10^{-2}$ , as the values of the vertical component of the velocity approach zero. These variations in velocity indicate a change in the behavior of the nanofluid from mixed convection to natural one and conduction by decreasing the Darcy number.

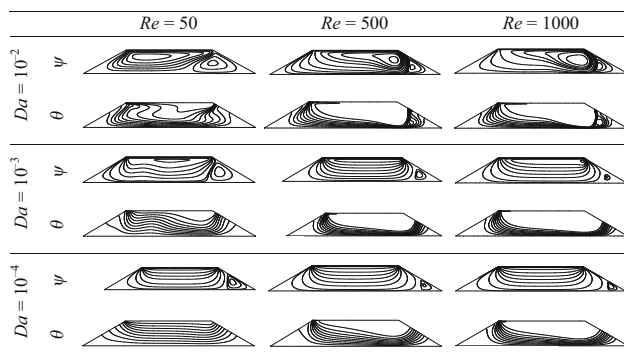
### Evaluation of streamlines and isotherms

In Fig. 4, streamlines and isothermal lines are shown for copper–water nanofluid for different Darcy and Reynolds numbers at  $\varphi = 0.02$ . Considering this figure, the streamlines appear as two vortices for all Reynolds numbers. The large initial vortex is affected by the lid-driven. This initial vortex has an enough velocity to move the fluid in the right-hand corner of the cavity. Hence, the secondary vortex is generated due to the effect of this velocity. As the Reynolds number increases, which actually indicates that the lid-driven is higher, the center of the first vortex moves to the right-hand side of the enclosure and causes smaller and stronger secondary vortex to form in the corner of the cavity.

As mentioned above, this behavior is observed at all Reynolds numbers. The curvature of the streamlines in the initial vortex decreases, and the secondary vortex becomes smaller by decreasing the Darcy number. The reason is a reduction in the convection and motion of nanofluid by decreasing the Darcy number. At the Darcy number of  $10^{-4}$ , streamlines become parallel to each other, indicating a change in the behavior of the nanofluid from mixed



**Fig. 3** The variation in vertical velocity component at mid-height versus  $x$  for different Darcy numbers at  $Re = 2000$  and  $\varphi = 0.02$

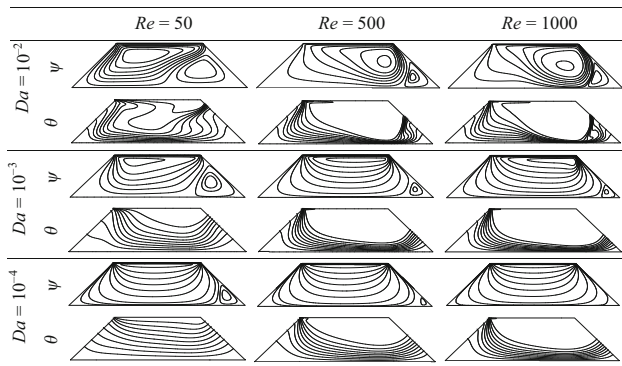


**Fig. 4** Streamlines and isothermal lines for copper-water nanofluid for different Darcy and Reynolds numbers at  $\varphi = 0.02$

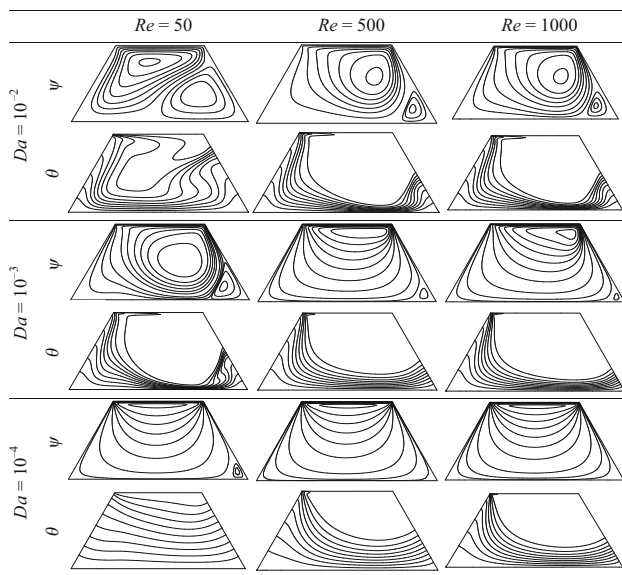
convection to natural convection and conduction. At the Darcy number of  $10^{-2}$ , isothermal lines have more curvature than other Darcy numbers. The curvature indicates that the porosity of the medium is such that the nanofluid can maintain its movement and convection. At this Darcy

number, the density of the isothermal lines in the lower part of the trapezoid enclosure increases, especially in the right corner, by increasing the Reynolds number. The density of isothermal lines is greater in the vicinity of large and small vortices. Higher density of isothermal lines represents a higher temperature gradient and hence the higher the heat transfer in these regions. Since this behavior is proportional to the increase of the Reynolds number and hence the increase in the nanofluid convection, this behavior is reasonable. The curvature of isothermal lines decreases by decreasing the Darcy number (decreasing the porosity of the media) for constant Reynolds numbers. This behavior is due to the dominance of conduction. However, the density of isothermal lines increases with increasing Reynolds number in the lower part of the enclosure for small Darcy numbers. In Figs. 5 and 6, the streamlines and isothermal lines for the copper-water nanofluid for different Darcy and Reynolds numbers at  $\varphi = 0.02$  and leg angles of  $45^\circ$  and  $60^\circ$  are presented.





**Fig. 5** Streamlines and isotherms of water-copper nanofluid at  $\phi = 0.02$  for different Reynolds and Darcy numbers and  $\theta_s = 45^\circ$



**Fig. 6** Streamlines and isotherms of water-copper nanofluid at  $\phi = 0.02$  for different Reynolds and Darcy numbers and  $\theta_s = 60^\circ$

At leg angles of  $45^\circ$  and  $60^\circ$ , the behavior of streamlines and isotherms is similar to the angle of  $30^\circ$ . At constant Reynolds numbers, the curvature of the streamlines is reduced and they tend to be parallel to each other with decreasing the Darcy number, which is equivalent to decreasing of the porosity of the media. This behavior shows the decrease in nanofluid movement in the enclosure. Also, in these two angles, the center of the initial vortex is shifted to the right side of the enclosure and the second vortex becomes smaller by increasing the Reynolds number at a constant Darcy number. At  $45^\circ$  and  $60^\circ$ , the density of isothermal lines increases close to the bottom of the hot wall and on the right side of the enclosure for different Reynolds and Darcy numbers.

## Variation in the Nusselt number

In Figs. 7–9, the variations in the average Nusselt number as a function of the volume fraction of nanoparticles are shown at the leg angles of  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  for different Darcy and Reynolds numbers. The average Nusselt number increases by increasing the volume fraction of the nanoparticles for different leg angles and Darcy numbers. In fact, the thermal conductivity of the nanofluid and the heat transfer increase with increasing volume fraction of nanoparticles. At a constant leg angle, the average Nusselt number decreases with the reduction in the Darcy number (reduction in the convection and the motion of the nanofluid). The variations in the average Nusselt number are close to each other in terms of volume fraction at Reynolds numbers of 10, 50 and 100, but the variation in average Nusselt number increases by increasing the Reynolds number to 500 and 1000, which leads to the overcoming of forced convection on natural one.

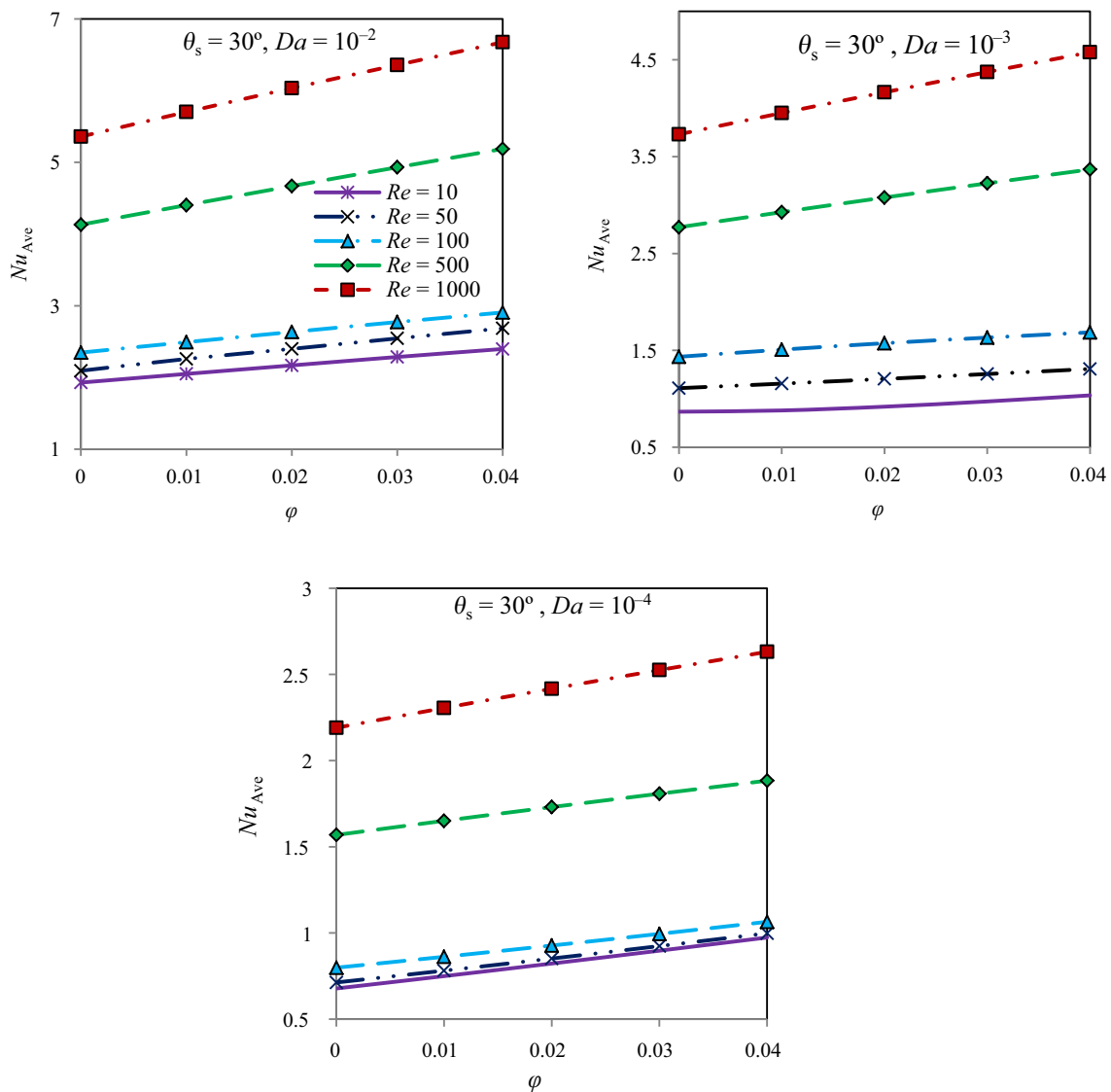
At  $\theta_s = 30^\circ$  and  $Da = 10^{-2}$ , the average Nusselt number increases by 1.78 times with the increase in Reynolds number from 10 to 1000. This increase is the same for different volume fractions. For  $Da = 10^{-3}$ , the average Nusselt number increases by 3.53 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.02. Also, for  $Da = 10^{-4}$ , the average Nusselt number increases by 2.23 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.0.

At leg angles of  $45^\circ$  and  $60^\circ$ , the behavior of the average Nusselt number is similar to that for the leg angle of  $30^\circ$  by increasing the volume fraction of the nanoparticles and different Darcy and Reynolds numbers.

At  $\theta_s = 45^\circ$  and  $Da = 10^{-2}$ , the average Nusselt number increases by 2.23 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.01. At  $Da = 10^{-3}$ , the average Nusselt number increases by 4.08 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.01. Also, for  $Da = 10^{-4}$ , the average Nusselt number increases by 2.66 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.0.

At  $\theta_s = 60^\circ$  and  $Da = 10^{-2}$ , the average Nusselt number increases by 2.28 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.04. For  $Da = 10^{-3}$ , the average Nusselt number increases by 3.95 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.01. Also, for  $Da = 10^{-4}$  the average Nusselt number increases by 2.34 times with the increase in Reynolds number from 10 to 1000 for the volume fraction of 0.0.

The average Nusselt number increases by increasing the leg angle of the trapezoid for all Reynolds numbers.

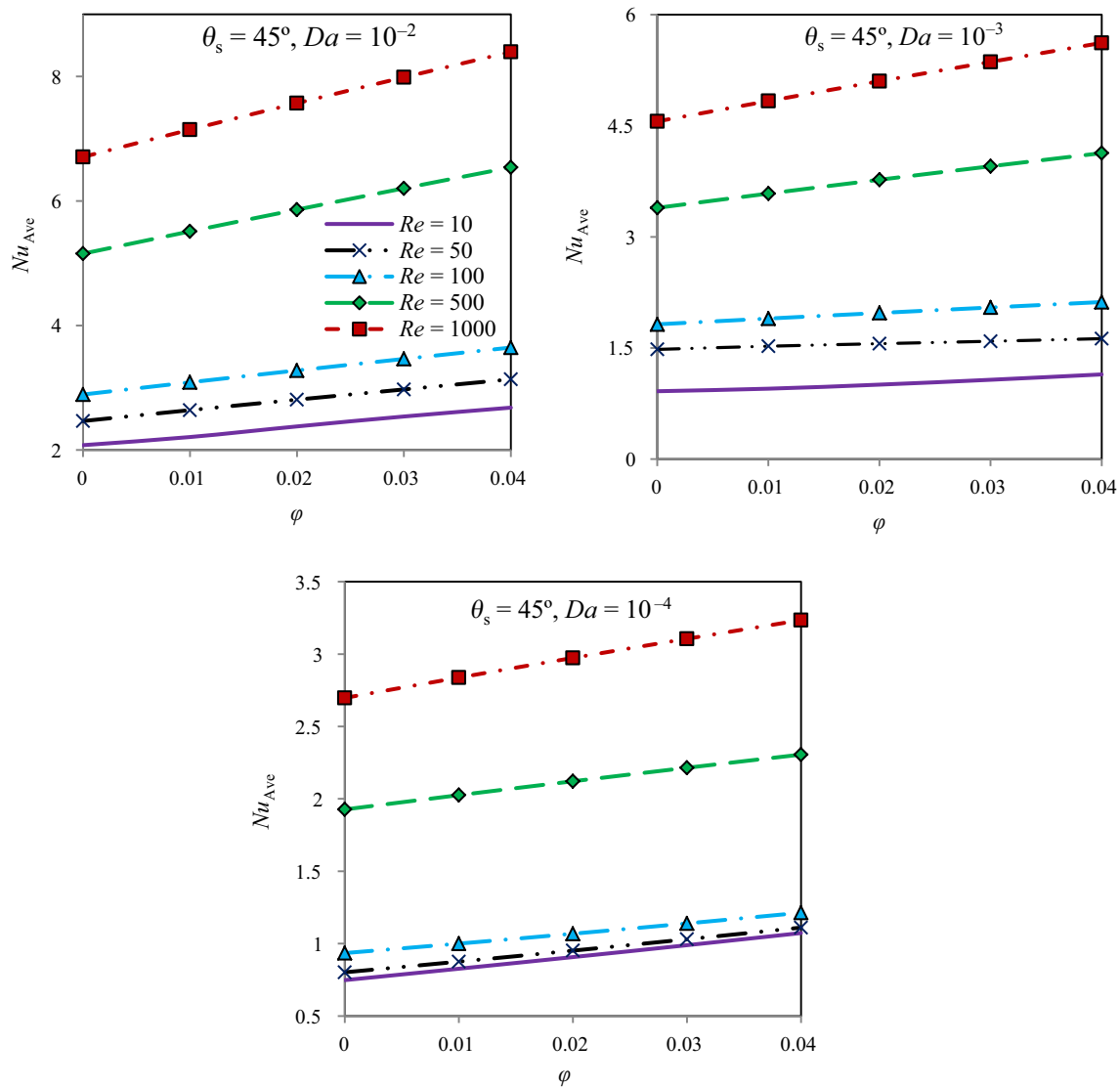


**Fig. 7** The average Nusselt number versus the volume fraction of nanoparticles at  $\theta_s = 30^\circ$  for various Darcy numbers

However, the enclosure becomes smaller and the motion and convection of the nanofluid are further limited with the increase in the leg angle of the cavity. In fact, with an increase in the leg angle, though the nanofluid has less space to move, the temperature gradient increases in the enclosure. In other words, the local Nusselt number increases and therefore the average Nusselt number

increases. For example, for  $Da = 10^{-2}$  at leg angles of  $30^\circ$ ,  $45^\circ$  and  $60^\circ$  at Reynolds number of 1000 and zero volume fraction, the average Nusselt number is 5.36, 6.71 and 8.50 and at the volume fraction of 0.04% is equal to 6.86, 8.40 and 10.90, respectively. These results show the increase in the Nusselt number and its variation with the volume fraction by increasing the leg angle.





**Fig. 8** The average Nusselt number versus the volume fraction of nanoparticles at  $\theta_s = 45^\circ$  for various Darcy numbers

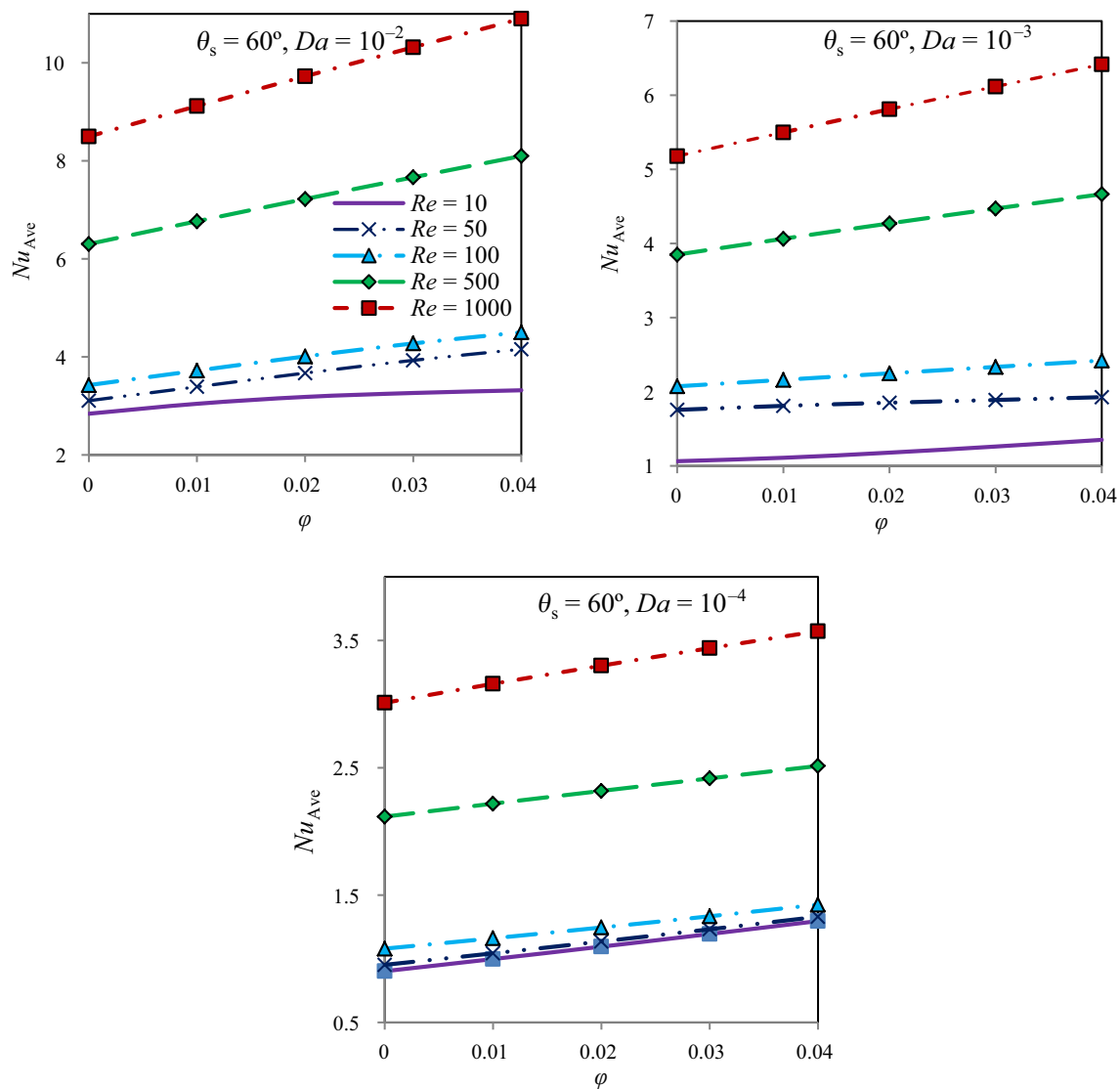
### Variation in the average Nusselt number with the Reynolds number

In order to evaluate the changes in the Nusselt number, the average Nusselt number is shown in Fig. 10 as a function of the Reynolds number for different Darcy numbers, volume fractions and leg angles.

For all cases examined, the average Nusselt number increases with the Reynolds number due to the increase in

nanofluid convection. At Reynolds number of 10 for Darcy numbers of  $10^{-3}$  and  $10^{-4}$ , the average Nusselt number is almost the same. This indicates that the porosity of the media has little effect at low Reynolds numbers unless the porosity increases.

The difference in the average Nusselt number increases with the Reynolds number for different volume fractions, leg angles and Darcy numbers. The highest difference is observed at the Reynolds number of 1000. Also, this



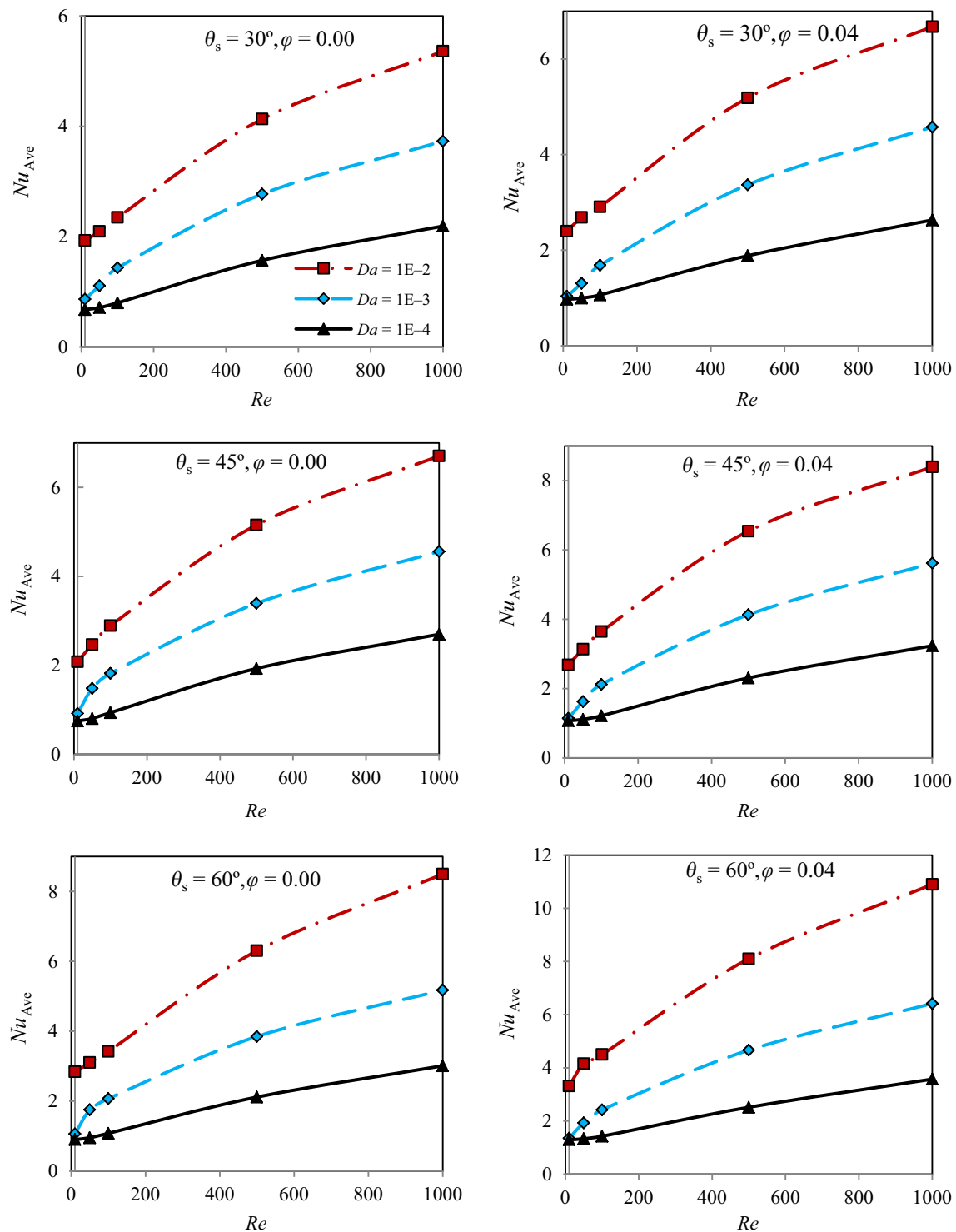
**Fig. 9** The average Nusselt number versus the volume fraction of nanoparticles at  $\theta_s = 60^\circ$  for various Darcy numbers

difference in the Nusselt number is more considerable at larger leg angles.

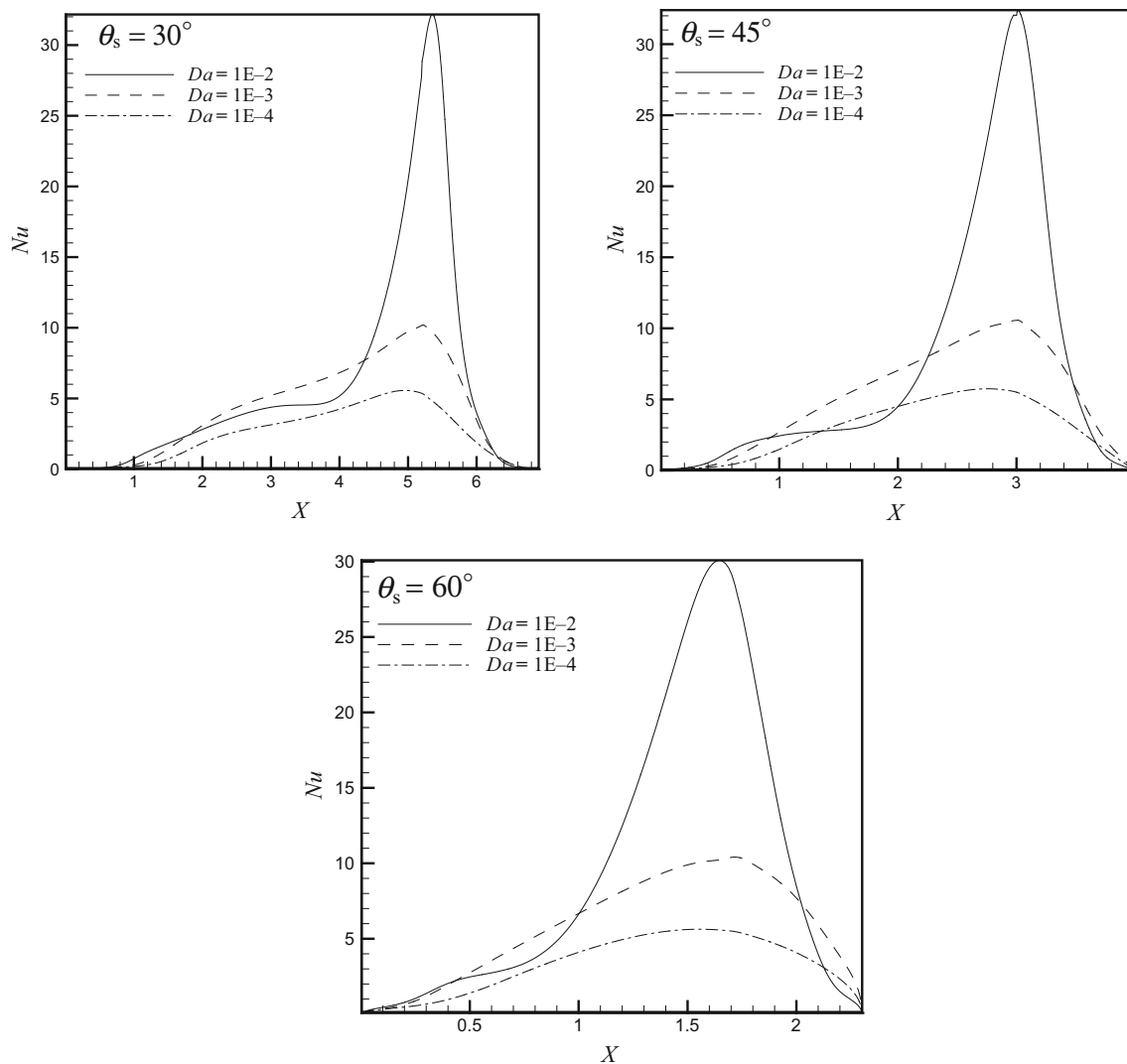
### Variation in local Nusselt number

In Fig. 11, the variation in the local Nusselt number on the lower hot wall is shown in terms of  $x$  for various Darcy numbers at  $Re = 1000$  and  $\phi = 0.02$ . The maximum amount of local Nusselt number is almost identical at

different leg angles. By increasing the leg angle, the maximum range of local Nusselt number increases. This increase in the maximum range of local Nusselt number by increasing the leg angle leads to an increase in the average Nusselt number. By decreasing the Darcy number, the variation curve of the local Nusselt number becomes smoother and its values drop sharply. As previously mentioned, the cause of this behavior is due to the limitation in



**Fig. 10** The average Nusselt number versus the Reynolds number for different volume fractions and leg angles



**Fig. 11** Local Nusselt number on a lower hot wall in terms of  $x$  for various Darcy numbers at  $Re = 1000$  and  $\phi = 0.02$

the convection and motion of the nanofluid with the reduction in the Darcy number.

## Conclusions

In the present study, the flow field and heat transfer of a water-copper nanofluid with variable properties in a trapezoidal enclosure saturated with porous media are studied. The study is conducted for aspect ratios of  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ , Reynolds numbers from 10 to 1000, Darcy numbers of  $10^{-2}$ ,  $10^{-3}$ ,  $10^{-4}$  and volume fractions of 0 to 0.04 of nanoparticles. The following results were obtained:

1. Numerical results show that the average Nusselt number increases with increasing volume fraction of nanoparticles for all studied Darcy numbers.
2. The convection and motion of the nanofluid decrease by reducing the Darcy number which leads to a reduction in the velocity and local Nusselt number.
3. The average Nusselt number increases by increasing the Darcy number for all aspect ratios.
4. The average Nusselt number increases with increasing Reynolds number for all Darcy numbers, aspect ratios and volume fractions of nanoparticles.
5. At leg angles of  $45^\circ$  and  $60^\circ$ , the behavior of streamlines and isotherms is similar to the angle of  $30^\circ$ .
6. The average Nusselt number increases by increasing the leg angle of the trapezoid for all Reynolds numbers.

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