

```

Print[
  "#####";
Print["##### Problem
  1 #####"];
a = -1.0; b = 1.;
 $\alpha = 10^{28}$ ;  $\gamma = 10^{28}$ ;
ua = ub = 0.;
 $\mu[x_] := 1$ ;  $\beta[x_] := 0.5 * 300 x^8$ ;  $\sigma[x_] := 2 x^2$ ;
f[x_] := 10 Exp[x^14];
  [funkcja eksponencjalna
(* 1. Mathematica Solution of the Problem *)
u = NDSolveValue[{- $\mu[x]$  y'[x] -  $\mu'[x]$  y[x] +  $\beta[x]$  y'[x] +  $\sigma[x]$  y[x] == f[x],
  [rozwiązanie numeryczne równania różniczkowego
    y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
  [wykres [wypełnienie [oś
    PlotRange -> Full, Frame -> True, GridLines -> Automatic,
    [zakres wykresu [compl... [ramka [prawda [linie siatki [automatyczny
    PlotLabel -> Style[Framed["u=u(x)"]], Background -> LightYellow]];
    [etykieta grafiki [styl [z ramką [tło [jasność
Print["||u||0 = ",  $\sqrt{NIntegrate[u[x]^2, \{x, a, b\}]}$ , " ||u||1 = ",
   $\sqrt{NIntegrate[u[x]^2 + u'[x]^2, \{x, a, b\}]}$ ];
Print["#####
  #####"];
Print["#####
  #####"];
Print["##### Problem 2
  #####"];
a = -1.0; b = 1.;
ua = ub = 0.;
 $\mu[x_] := 1$ ;  $\beta[x_] := 70 x^2$ ;  $\sigma[x_] := 0 * 40 x^2$ ;
f[x_] := 40 x^1;
(* 2. Mathematica Solution of the Problem *)
u = NDSolveValue[{- $\mu[x]$  y'[x] -  $\mu'[x]$  y[x] +  $\beta[x]$  y'[x] +  $\sigma[x]$  y[x] == f[x],
  [rozwiązanie numeryczne równania różniczkowego
    y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
  [wykres [wypełnienie [oś
    PlotRange -> Full, Frame -> True, GridLines -> Automatic,
    [zakres wykresu [compl... [ramka [prawda [linie siatki [automatyczny
    PlotLabel -> Style[Framed["u=u(x)"]], Background -> LightYellow]];
    [styl [z ramką [tło [jasność
Print["||u||0 = ",  $\sqrt{NIntegrate[u[x]^2, \{x, a, b\}]}$ , " ||u||1 = ",
   $\sqrt{NIntegrate[u[x]^2 + u'[x]^2, \{x, a, b\}]}$ ];
Print["#####
  #####"];

```



```

Print["#####          #####          #####
#####          #####"];
Print["#####          #####          #####
#####          #####"];
Print["#####          #####          Problem 5
#####          #####"];
a = -1.0;
b = 1.;
ua = ub = 0.;
 $\mu[x_] := 1;$   $\beta[x_] := 100 (x - 0.25)^3;$   $\sigma[x_] := 40 x^2;$ 
 $f[x_] := 40 (x + 0.3)^5;$ 

u = NDSolveValue[{- $\mu[x] y'[x] - \mu'[x] y[x] + \beta[x] y'[x] + \sigma[x] y[x] == f[x],$ 
  [rozwiązanie numeryczne równania różniczkowego]
  y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
  [wykres] [wypełnienie] [oś]
  PlotRange -> Full, Frame -> True, GridLines -> Automatic,
  [zakres wykresu] [compl...] [ramka] [prawda] [linię siatki] [automatyczny]
  PlotLabel -> Style[Framed["u=u(x)"], Background -> LightYellow]];
  [etykieta grafiki] [styl] [z ramką] [tło] [jasność]
Print["||u||0 = ",  $\sqrt{\text{NIntegrate}[u[x]^2, \{x, a, b\}]}$ , " ||u||1 = ",
   $\sqrt{\text{NIntegrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$ ];
Print["#####          #####          #####
#####          #####"];
Print["#####          #####          #####
#####          #####"];
Print["#####          #####          Problem 6
#####          #####"];
a = -1.0;
b = 1.;
ua = ub = 0.;

 $\mu[x_] := 1.;$   $\beta[x_] := 1500. x^8;$   $\sigma[x_] := (10. + 2. x^2);$ 
 $f[x_] := 1. * 10. * \text{Exp}[(x + 0.15)^{14}];$ 
  [funkcja eksponencjalna]

u = NDSolveValue[{- $\mu[x] y'[x] - \mu'[x] y[x] + \beta[x] y'[x] + \sigma[x] y[x] == f[x],$ 
  [rozwiązanie numeryczne równania różniczkowego]
  y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
  [wykres] [wypełnienie] [oś]
  PlotRange -> Full, Frame -> True, GridLines -> Automatic,
  [zakres wykresu] [compl...] [ramka] [prawda] [linię siatki] [automatyczny]
  PlotLabel -> Style[Framed["u=u(x)"], Background -> LightYellow]];
  [etykieta grafiki] [styl] [z ramką] [tło] [jasność]

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Print["||u||0 = ",  $\sqrt{\text{NIntegrate}[u[x]^2, \{x, a, b\}]}$  , " ||u||1 = ",
 $\sqrt{\text{NIntegrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$  ];
Print["#####
#####"];
Print["#####
#####"];
Print["##### Problem 7
#####"];
a = -1.0;
b = 1.;
ua = ub = 0.;
 $\mu[x_] := 1.; \beta[x_] := -1500. x^8; \sigma[x_] := (10. + 2. x^2);$ 
 $f[x_] := 1. * 10. * \text{Exp}[(x + 0.15)^{14}];$ 
      [funkcja eksponencjalna]
u = NDSolveValue[{- $\mu[x] y'[x] - \mu'[x] y[x] + \beta[x] y'[x] + \sigma[x] y[x] = f[x]$ ,
      [rozwiązanie numeryczne równania różniczkowego]
      y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
      [wykres] [wypełnienie] [oś]
      PlotRange -> Full, Frame -> True, GridLines -> Automatic,
      [zakres wykresu] [compl...ramka] [prawda] [linie siatki] [automatyczny]
      PlotLabel -> Style[Framed["u=u(x)"], Background -> LightYellow]];
      [etykieta grafiki] [styl] [z ramką] [tło] [jasność]
Print["||u||0 = ",  $\sqrt{\text{NIntegrate}[u[x]^2, \{x, a, b\}]}$  , " ||u||1 = ",
 $\sqrt{\text{NIntegrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$  ];
Print["#####
#####"];
Print["#####
#####"];
Print["##### Problem 8
#####"];
a = -1.0;
b = 1.;
ua = ub = 0.;
 $\mu[x_] := 1.; \beta[x_] := -1500. x^8; \sigma[x_] := (80. + 2. x^2);$ 
 $f[x_] := 100 * \text{Exp}[(x + 0.15)^{14}];$ 
      [funkcja eksponencjalna]
u = NDSolveValue[{- $\mu[x] y'[x] - \mu'[x] y[x] + \beta[x] y'[x] + \sigma[x] y[x] = f[x]$ ,
      [rozwiązanie numeryczne równania różniczkowego]
      y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
      [wykres] [wypełnienie] [oś]
      PlotRange -> Full, Frame -> True, GridLines -> Automatic,
      [zakres wykresu] [compl...ramka] [prawda] [linie siatki] [automatyczny]

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styl z ramką tło jasnożółty

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PlotRange → Full , Frame → True, GridLines → Automatic,
  [wynies] [wyplnienie] [os]
  [compl...] [ramka] [prawda] [linie siatki] [automatyczny]
PlotLabel → Style[Framed["u=u(x)"] , Background → LightYellow]] ;
  [styl] [z ramką] [tło] [jasność]
Print["||u||0 = ",  $\sqrt{\text{NIntegrate}[u[x]^2, \{x, a, b\}]}$  , " ||u||1 = ",
 $\sqrt{\text{NIntegrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$  ];
Print["#####"];
Print["#####"];
Print["##### Problem 11"];
Print["#####"];
a = -1.0;
b = 1.;
ua = ub = 0.;
μ[x_] := 1.; β[x_] := 1500. x^8; σ[x_] := (80. + 2. x^2);
f[x_] := 100 * Exp[(x - 0.15)^7] * x;
  [funkcja eksponencjalna]
u = NDSolveValue[{-μ[x] y'[x] - μ'[x] y[x] + β[x] y'[x] + σ[x] y[x] == f[x] ,
  [rozwiązanie numeryczne równania różniczkowego]
  y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}, Filling → Axis,
  [wykres] [wyplnienie] [os]
  PlotRange → Full , Frame → True, GridLines → Automatic,
  [compl...] [ramka] [prawda] [linie siatki] [automatyczny]
  PlotLabel → Style[Framed["u=u(x)"] , Background → LightYellow]] ;
  [styl] [z ramką] [tło] [jasność]
Print["||u||0 = ",  $\sqrt{\text{NIntegrate}[u[x]^2, \{x, a, b\}]}$  , " ||u||1 = ",
 $\sqrt{\text{NIntegrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$  ];
Print["#####"];
Print["#####"];
Print["##### Problem 12"];
Print["#####"];
a = -1.0;
b = 1.;
ua = ub = 0.;
μ[x_] := -1.; β[x_] := 1500. x^8; σ[x_] := (80. + 2. x^2);
f[x_] := 100 * Exp[(x - 0.15)^7] * x;
  [funkcja eksponencjalna]
u = NDSolveValue[{-μ[x] y'[x] - μ'[x] y[x] + β[x] y'[x] + σ[x] y[x] == f[x] ,
  [rozwiązanie numeryczne równania różniczkowego]
  y[a] == ua, y[b] == ub}, y, {x, a, b}];

```

```
Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
    wykres                               wypełnienie   Łoś
PlotRange -> Full , Frame -> True, GridLines -> Automatic,
    compl... ramka      prawda linie siatki     automatyczny
PlotLabel -> Style[Framed["u=u(x)"]], Background -> LightYellow]];
    styl      z ramką          tło              jasnożółty

Print["||u||_0 = ", Sqrt[NIntegrate[u[x]^2, {x, a, b}]] , " ||u||_1 = ",
    Sqrt[NIntegrate[u[x]^2 + u'[x]^2, {x, a, b}]]];

Print["#####"];
Print["#####"];
Print["#####"];
Print["#####"];
Print["##### Problem 13"];
Print["#####"];

a = -1.0;
b = 1.;
ua = ub = 0.;
μ[x_] := -1.; β[x_] := 1500. x^8; σ[x_] := (80. x^2);
f[x_] := 100 * Exp[(x - 0.15)^7] * x;
    funkcja eksponencjalna
u = NDSolveValue[{-μ[x] y'[x] - μ'[x] y[x] + β[x] y'[x] + σ[x] y[x] == f[x],
    rozwiązanie numeryczne równania różniczkowego
y[a] == ua, y[b] == ub}, y, {x, a, b}];

Print[wsymm = Plot[u[x], {x, a, b}], Filling -> Axis,
    wykres                               wypełnienie   Łoś
PlotRange -> Full , Frame -> True, GridLines -> Automatic,
    compl... ramka      prawda linie siatki     automatyczny
PlotLabel -> Style[Framed["u=u(x)"]], Background -> LightYellow]];
    styl      z ramką          tło              jasnożółty

Print["||u||_0 = ", Sqrt[NIntegrate[u[x]^2, {x, a, b}]] , " ||u||_1 = ",
    Sqrt[NIntegrate[u[x]^2 + u'[x]^2, {x, a, b}]]];

Print["#####"];
Print["#####"];
Print["#####"];
Print["#####"];
Print["##### Problem 14"];
Print["#####"];

a = -1.0;
b = 1.;
ua = ub = 0.;
μ[x_] := -1.; β[x_] := 1500. x^8; σ[x_] := -(480. x^2);
f[x_] := 200 * Exp[(x - 0.15)^7] * x;
    funkcja eksponencjalna
u = NDSolveValue[{-μ[x] y'[x] - μ'[x] y[x] + β[x] y'[x] + σ[x] y[x] == f[x],
    rozwiązanie numeryczne równania różniczkowego
y[a] == ua, y[b] == ub}, y, {x, a, b}];
```

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Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
    wykres      wypełnienie  Łoś
    PlotRange -> Full, Frame -> True, GridLines -> Automatic,
    compl...  ramka  prawda  linie siatki  automatyczny
    PlotLabel -> Style[Framed["u=u(x)"], Background -> LightYellow]];
    styl  z ramką  tło  jasność
Print["||u||_0 = ",  $\sqrt{\text{NIntegrate}[u[x]^2, \{x, a, b\}]}$ , " ||u||_1 = ",
 $\sqrt{\text{NIntegrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$ ];
Print["#####  #####  #####
#####
#####"];
Print["#####  #####  #####
#####
#####"];
Print["#####  #####  Problem 15
#####
#####"];
a = -1.0;
b = 1.;
ua = ub = 0.;
 $\mu[x_] := -1.; \beta[x_] := 100. x^7; \sigma[x_] := (480. x^2);$ 
f[x_] := 20 * Exp[(x - 0.15)^7] * x;
    funkcja eksponencjalna
u = NDSolveValue[{- $\mu[x] y'[x] - \mu'[x] y[x] + \beta[x] y'[x] + \sigma[x] y[x] == f[x]$ ,
    rozwiązanie numeryczne równania różniczkowego
    y[a] == ua, y[b] == ub}, y, {x, a, b}];

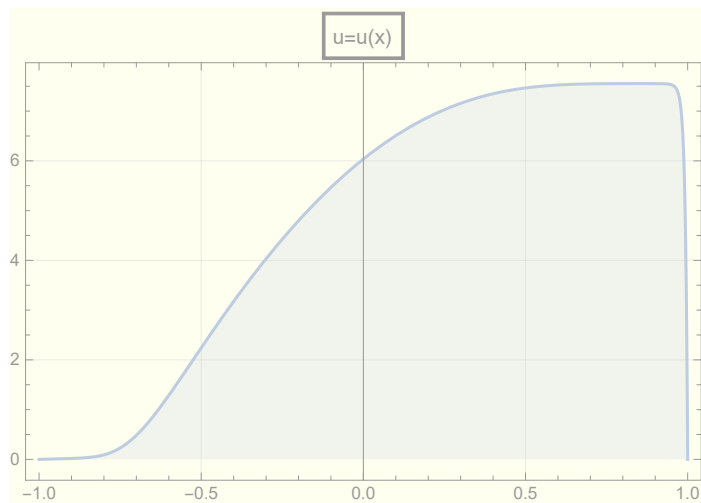
Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
    wykres      wypełnienie  Łoś
    PlotRange -> Full, Frame -> True, GridLines -> Automatic,
    compl...  ramka  prawda  linie siatki  automatyczny
    PlotLabel -> Style[Framed["u=u(x)"], Background -> LightYellow]];
    styl  z ramką  tło  jasność
Print["||u||_0 = ",  $\sqrt{\text{NIntegrate}[u[x]^2, \{x, a, b\}]}$ , " ||u||_1 = ",
 $\sqrt{\text{NIntegrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$ ];
Print["#####  #####  #####
#####
#####"];
Print["#####  #####  #####
#####
#####"];
Print["#####  #####  Problem 16
#####
#####"];
a = -1.0;
b = 1.;
ua = ub = 0.;
 $\mu[x_] := -1.; \beta[x_] := 20. \text{Sin}[3 \text{Pi} * x / 2]; \sigma[x_] := 40. x^4;$ 
f[x_] := 7;
u = NDSolveValue[{- $\mu[x] y'[x] - \mu'[x] y[x] + \beta[x] y'[x] + \sigma[x] y[x] == f[x]$ ,
    rozwiązanie numeryczne równania różniczkowego
    y[a] == ua, y[b] == ub}, y, {x, a, b}];

```



```
Print[wsymm = Plot[u[x], {x, a, b}, Filling -> Axis,
      wykres      wypełnienie  oś
      PlotRange -> Full, Frame -> True, GridLines -> Automatic,
      compl... ramka  prawda  linie siatki  automatyczny
      PlotLabel -> Style[Framed["u=u(x)"], Background -> LightYellow]];
      styl  z ramką  tło  jasność
Print["||u||0 = ",  $\sqrt{N \text{Integrate}[u[x]^2, \{x, a, b\}]}$ , " ||u||1 = ",
       $\sqrt{N \text{Integrate}[u[x]^2 + u'[x]^2, \{x, a, b\}]}$ ];
```

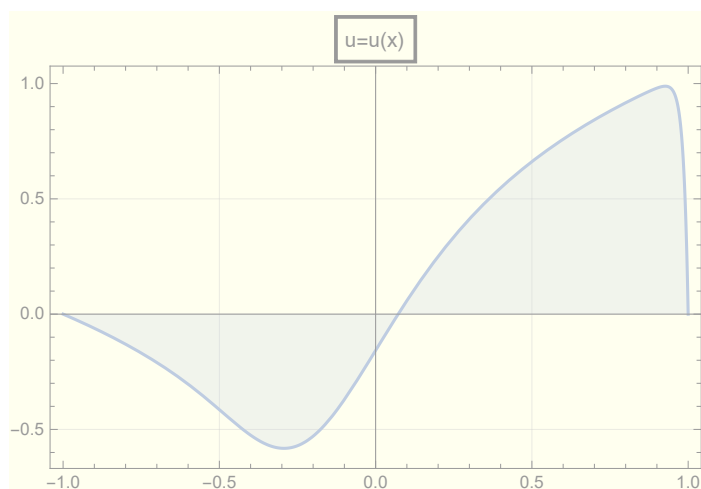
```
=====
Problem 1 =====
```



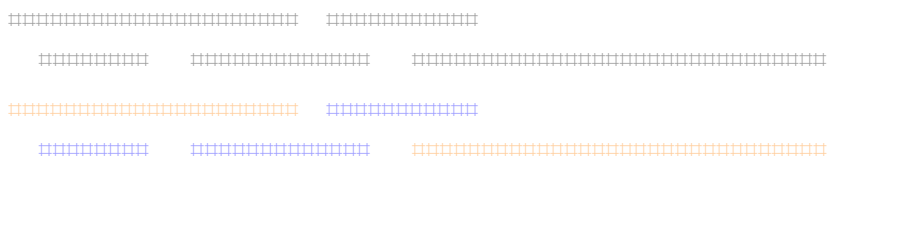
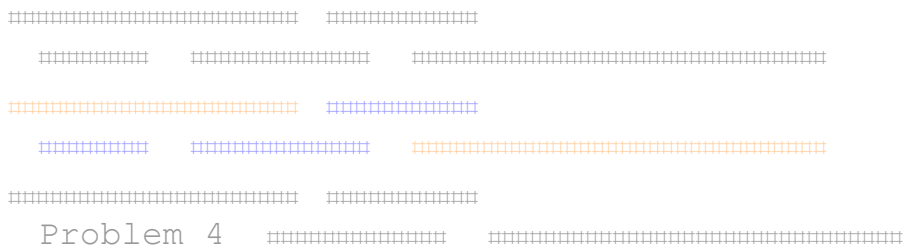
```
||u||0 = 7.89224 ||u||1 = 63.4669
```

```
=====
=====
=====
=====
=====
=====
```

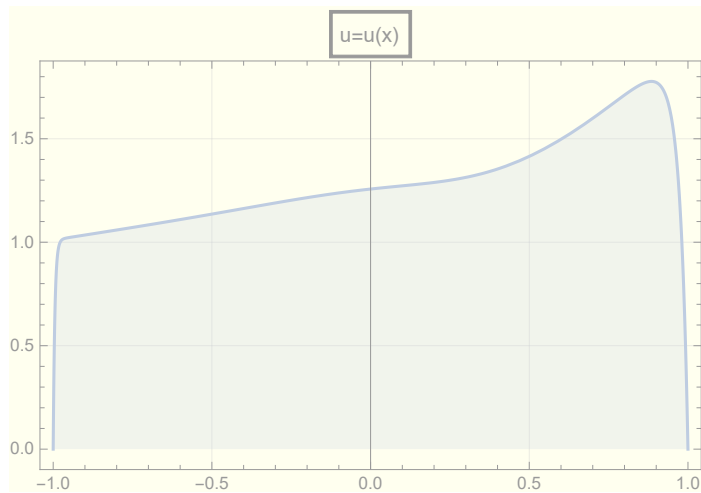
```
Problem 2 =====
```



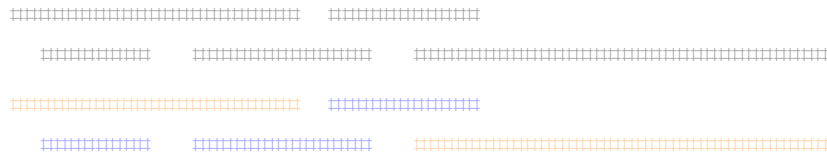
```
||u||0 = 0.757434 ||u||1 = 6.21677
```



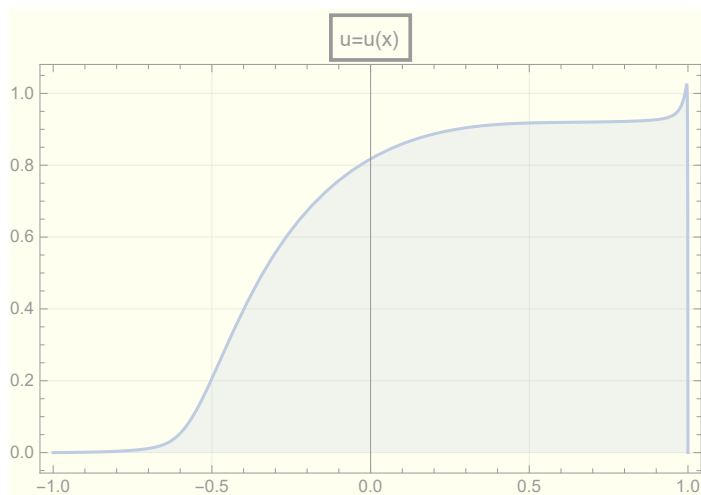
Problem 5



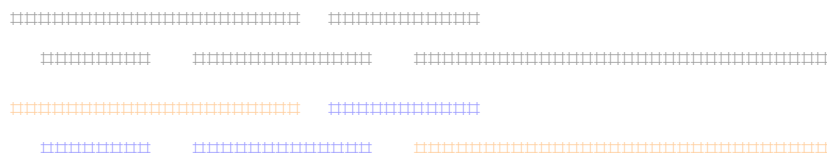
$$\|u\|_0 = 1.84237 \quad \|u\|_1 = 13.065$$



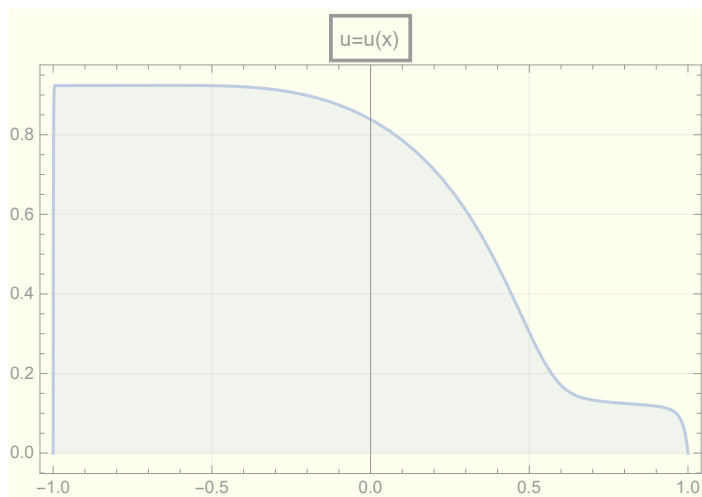
Problem 6



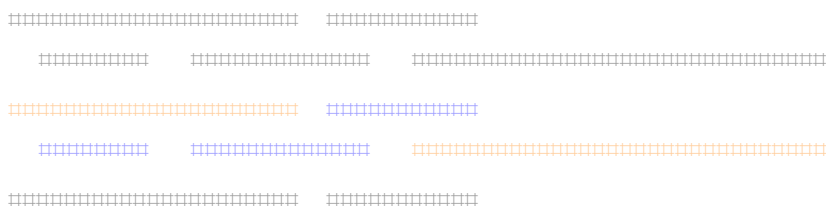
$$\|u\|_0 = 1.00459 \quad \|u\|_1 = 28.5453$$



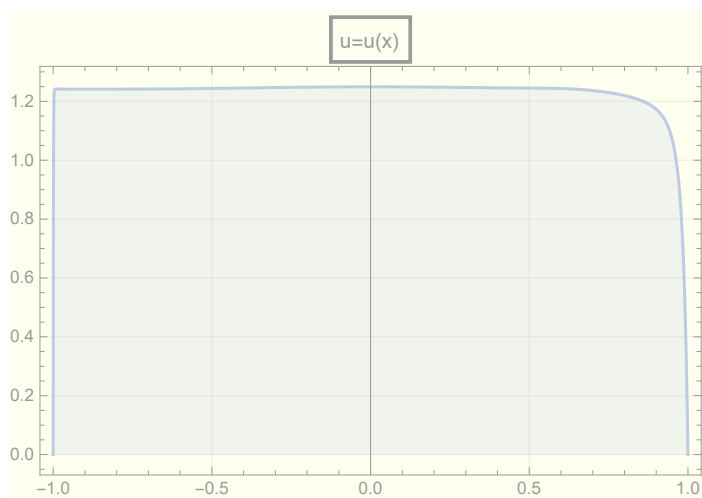
Problem 7



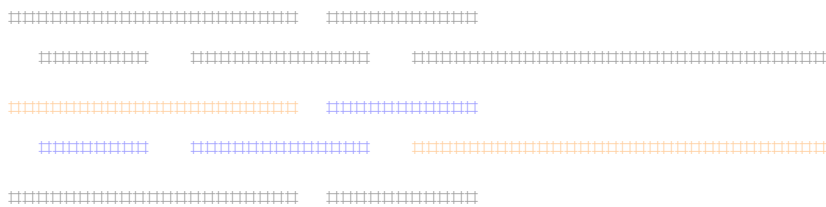
$$\|u\|_0 = 1.02651 \quad \|u\|_1 = 25.2419$$



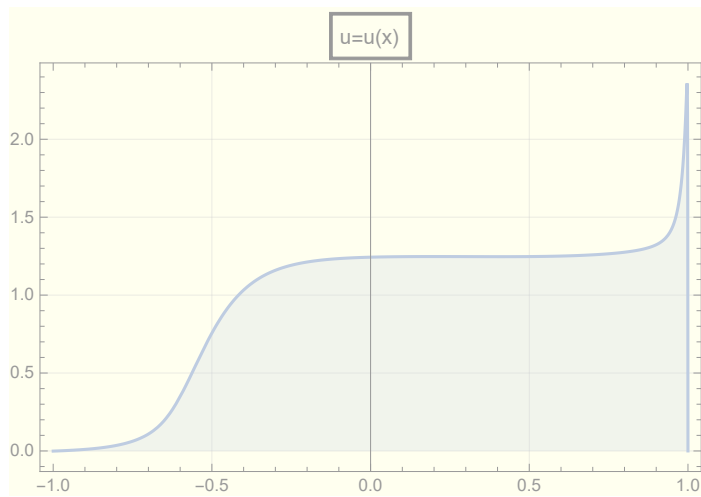
Problem 8



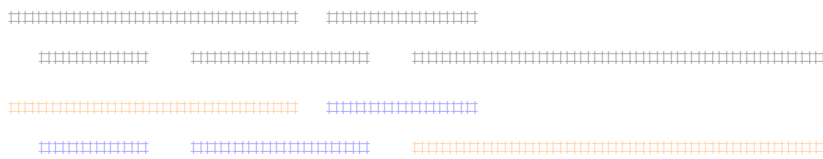
$$\|u\|_0 = 1.74008 \quad \|u\|_1 = 34.4889$$



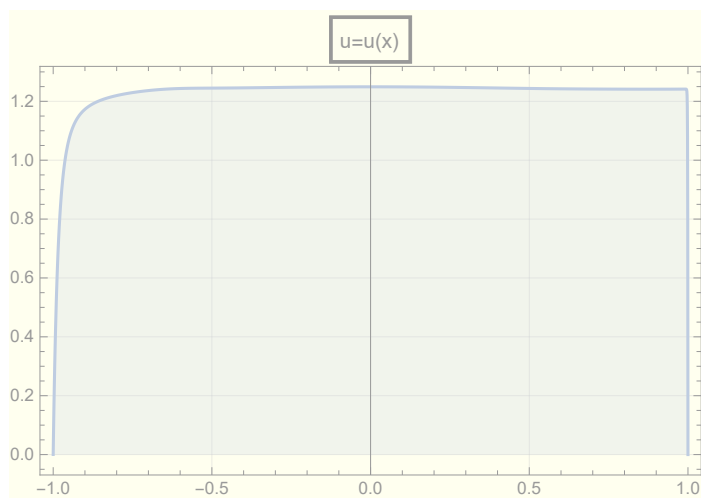
Problem 9



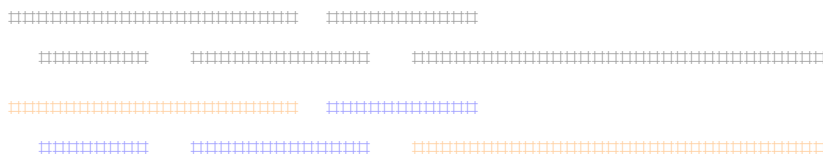
$$\|u\|_0 = 1.53347 \quad \|u\|_1 = 68.6188$$



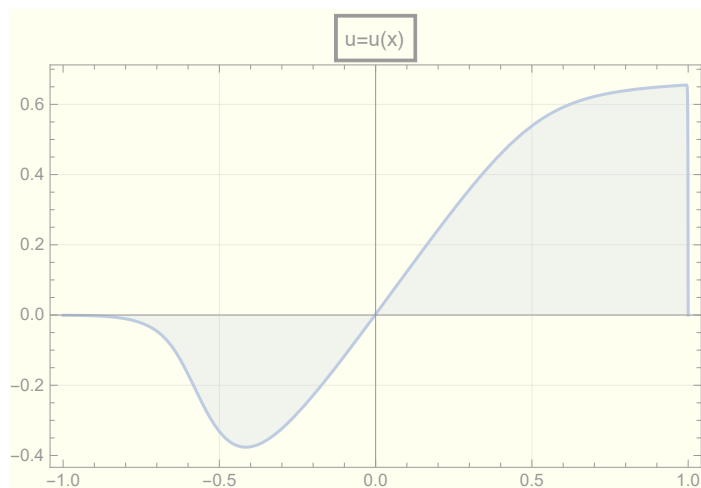
Problem 10



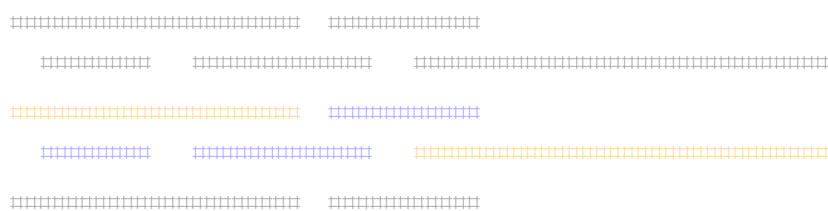
$$\|u\|_0 = 1.74008 \quad \|u\|_1 = 34.4889$$



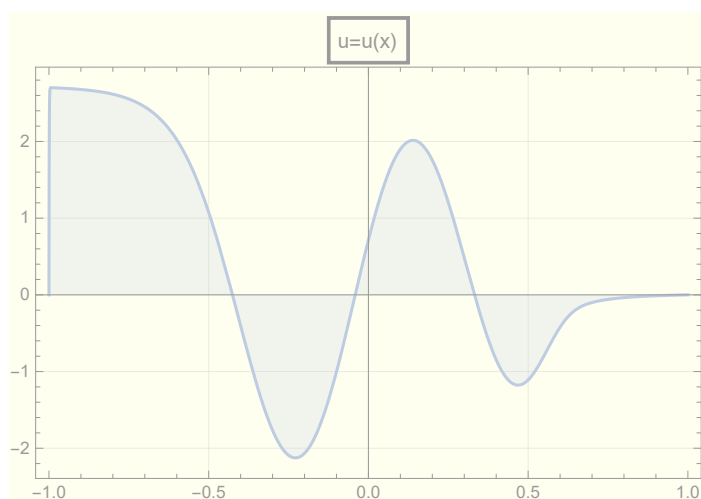
Problem 11



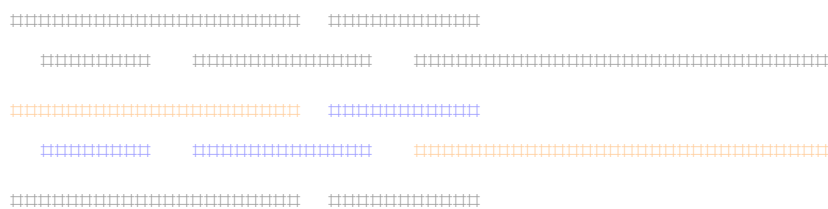
$$\|u\|_0 = 0.541533 \quad \|u\|_1 = 17.9267$$



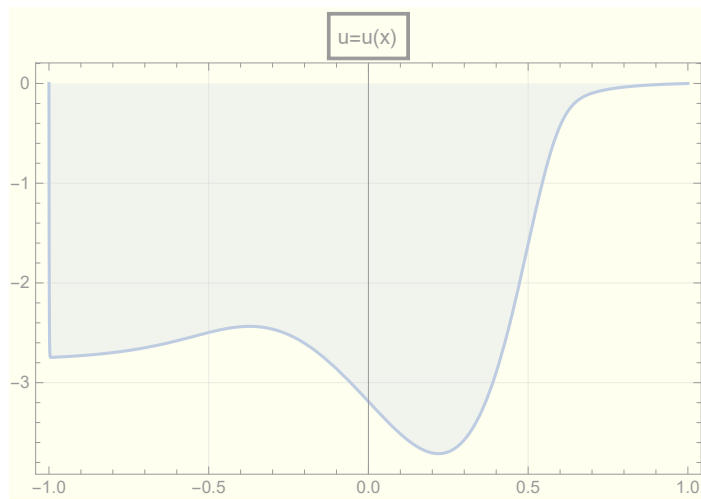
Problem 12



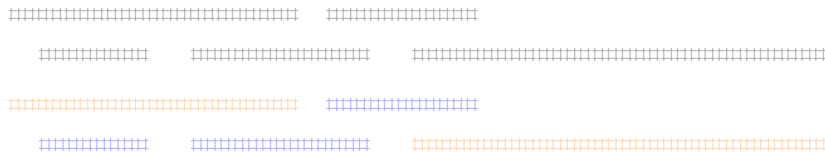
$$\|u\|_0 = 2.16755 \quad \|u\|_1 = 74.7838$$



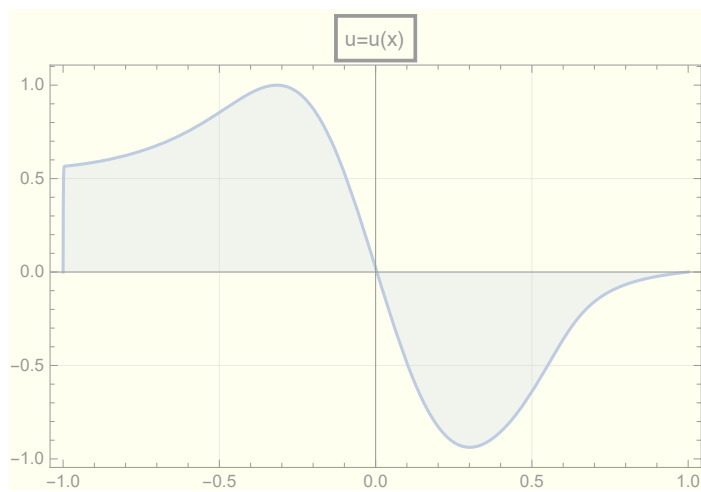
Problem 13



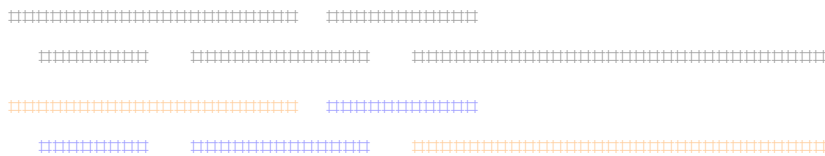
$$\|u\|_0 = 3.54264 \quad \|u\|_1 = 75.2955$$



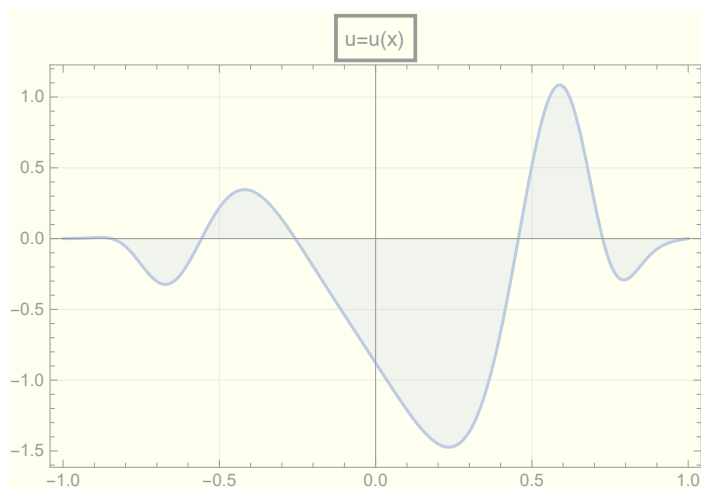
Problem 14



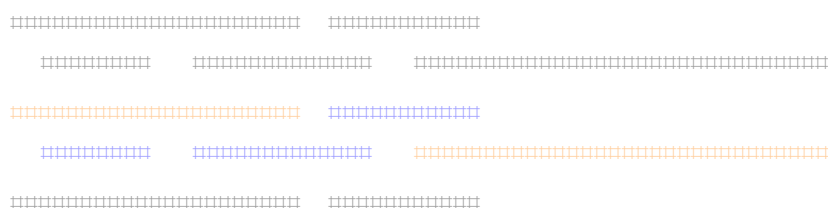
$$\|u\|_0 = 0.933146 \quad \|u\|_1 = 15.7777$$



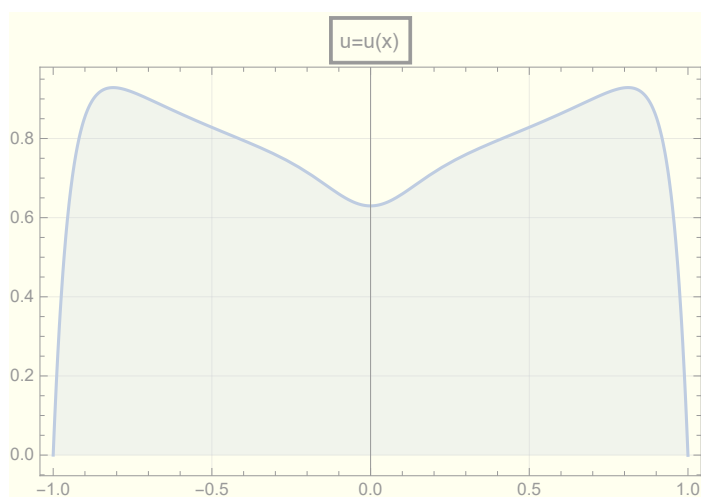
Problem 15



$$\|u\|_0 = 0.94049 \quad \|u\|_1 = 6.76564$$



Problem 16



$$\|u\|_0 = 1.11649 \quad \|u\|_1 = 4.61997$$