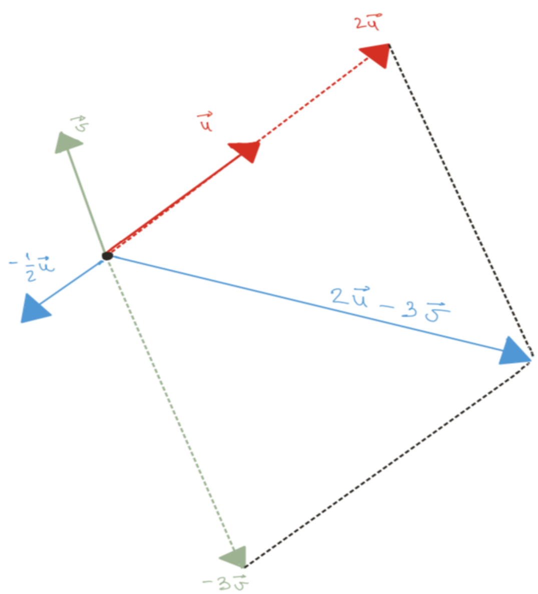
Basis Vectors

I want you to imagine the origin and 2 random vectors **u** and **v** starting at the origin (it is important that **u** and **v** don't lie on the same line). As already discussed in the section about vectors, the *two operations that all vectors necessarily have are*:

1) *Multiplication by a constant, aka scaling.*
2) *Addition.*

The result of the above operations is always a vector.

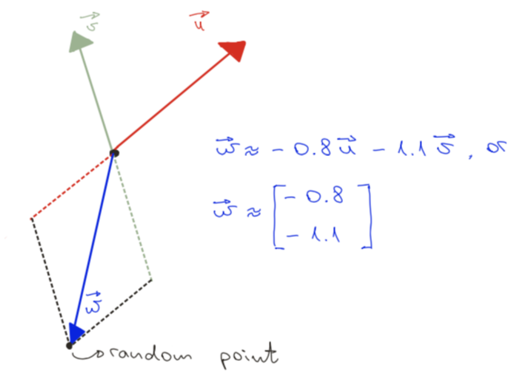
In your mind I want you to perform those two operations on **u** and **v** in many possible crazy ways. For example, 2**u** - 3**v** or -0.5**u** or -5**u** + 4**v**, etc. I have tried first 2 and here is what I get:



Definition: An expression constructed from a set of vectors by using the 2 operations above is called a linear combination.

For example, -5**u** + 4**v** is a linear combination of **u** and **v**.

Do you see that every vector in the plane where **u** and **v** lie can be obtained by applying those two operations to **u** and **v**? If you are not 100% convinced, take a look at the following picture where I managed to approximately represent a random vector **w** on the plane as a linear combination of **u** and **v**.



What if **u** and **v** lie on the same line? In this case it is impossible to obtain a vector that is not on the same line using the operations above.

Before taking a look at the definition of basis vectors take a look at the following examples.

Example:

Imagine a 2 dimensional plane passing through the origin. As we just saw, every vector in this plane can be represented as a linear combination of 2 vectors that don't lie on the same line.

Example:

Imagine a line passing through the origin and a random vector **w** on a line. I think that it is not difficult to see that if you scale v appropriately, you can get any vector on that line.

Note:

In math literature 2 dimensional plane can be referred to as 2 dimensional vector space and a line can be referred to as 1 dimensional vector space.

Definition:

A set of vectors in a vector space is called a basis if every other vector in that vector space can be uniquely represented as a linear combination of that set of vectors.

In the examples above, **u** and **v** (if they don't lie on the same line) are the basis vectors of 2 dimensional vector space (plane) and **w** is a basis vector of 1 dimensional vector space (line).