

Fundamentals of Robotic Kinematics and Power Systems

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Matrix Multiplication and Linear Equations

Learning Objectives

- Represent linear systems using matrices.
- Apply matrix inversion to solve equations.
- Connect matrix methods with robotic transformations.

1.1 Introduction to Matrix Representation

Matrix algebra forms the mathematical foundation of robotic motion, transformations, and control. All forward and inverse kinematic equations rely on matrix multiplication.

1.2 Linear System Representation

A linear system of equations can be elegantly represented using matrix notation:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases} \Rightarrow AX = B$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}.$$

If A is invertible (square and non-singular), then the solution is given by:

$$X = A^{-1}B$$

Key Concept

In robotics, matrix A often represents transformation matrices or Jacobian matrices that relate joint velocities to end-effector velocities. Matrix inversion is directly related to solving for joint angles given desired end-effector positions in inverse kinematics problems.

1.3 Application in Robotic Transformations

The power of matrix representation becomes evident in homogeneous transformation matrices used in robotics:

$$T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

where R is the rotation matrix and p is the position vector.

Kinematics and Inverse Kinematics

Learning Objectives

- Derive forward and inverse kinematic equations for planar manipulators.
- Solve matrix-form inverse kinematics.
- Interpret multiple physical solutions and their implications.

2.1 Introduction to Robotic Manipulators

A **manipulator** is the mechanical arm or linkage system designed to interact with the environment by moving, positioning, and orienting objects. It is the part of the robot that performs the actual 'work', such as picking up a component, welding, and assembling the parts. It is analogous to human arm Its key components are:

- **Base:** Structural support and mounting point.
- **Links:** Rigid members connecting joints.
- **Joints:** Provide relative motion between links, either rotational (revolute) or prismatic.
- **Actuators:** Motors that drive joint motion.
- **End-Effector:** The tool or gripper at the arm's end performing the desired task.

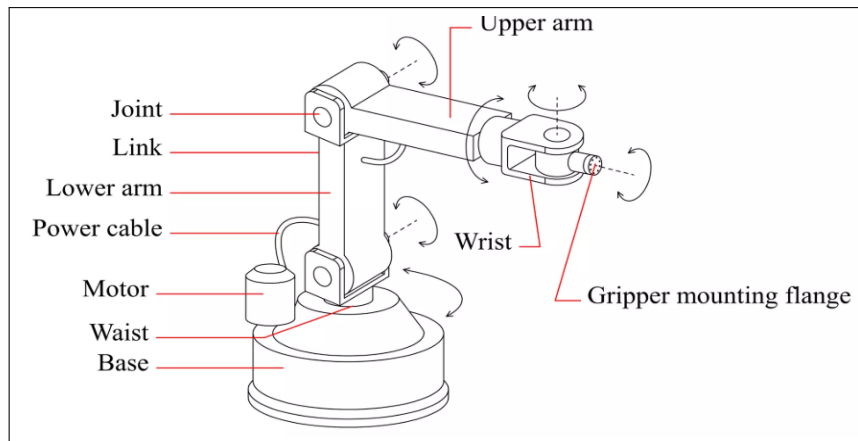


Figure 2.1: A Visual Representation of a Robotic rover showing Base, Links, Joints, Actuators, End Effectors and other components.

2.1.1 Workspace and Joint Space

The **workspace** (also called reachable space or working envelope) of a manipulator is the total volume or region in space that the robot's end-effector can reach. It defines the area or volume where the manipulator can actually operate.

The **joint space** of a robot manipulator is the mathematical space formed by all possible values of its joint variables.

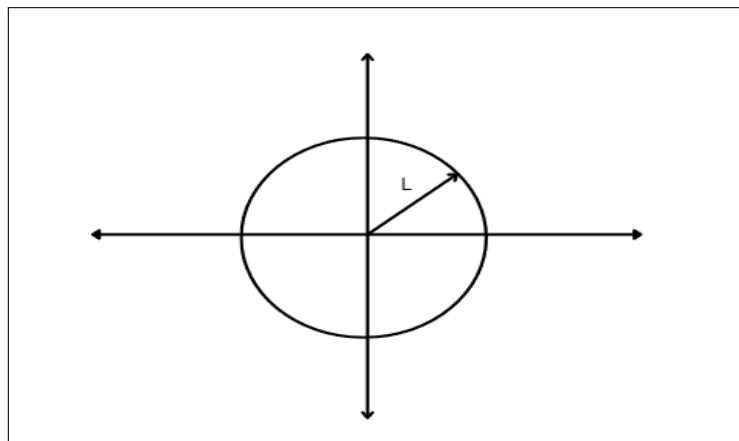


Figure 2.2: Workspace of a single arm where the perimeter of the circle shows its workspace

2.1.2 Degrees of Freedom (DOF)

In robotics, the **degrees of freedom** refer to the number of independent movements or parameters that define the configuration or position of the manipulator in space.

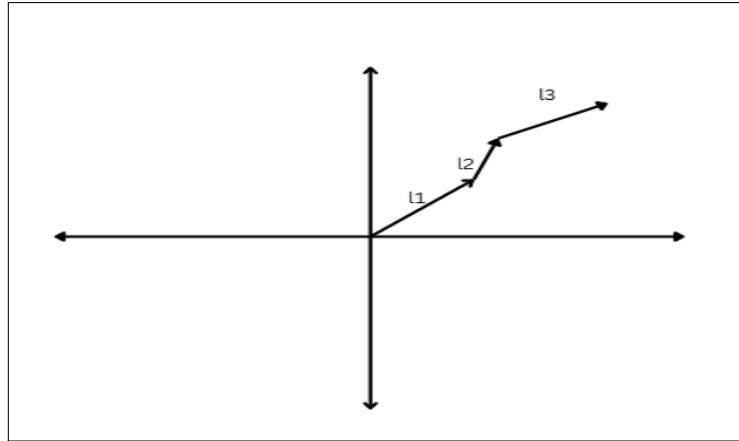


Figure 2.3: Image Showing the robotic arm with Degree of Freedom 3

2.1.3 Safe Limit

The **safe limit** defines the maximum permissible range of motion, speed, force, or torque ensuring the manipulator operates within mechanical and control constraints, keeping humans and equipment safe.

2.1.4 Major Divisions of Robotic Components

Voltaic logic is used across the following core components:

- **Sensors:** Detect physical quantities (e.g., position, force, temperature) and convert them to electrical signals for feedback control.
- **Processors:** Execute control algorithms, interpret sensor data, and coordinate actuator behavior.
- **Actuators:** Convert electrical signals into mechanical motion, enabling joint or end-effector movement.

2.2 Actuator Technologies in Robotics

2.2.1 Planetary Gear Motors

Planetary gear motors provide compact, high-torque output through their unique gear arrangement. They offer excellent efficiency and are common in robotics applications requiring high torque-to-size ratio.

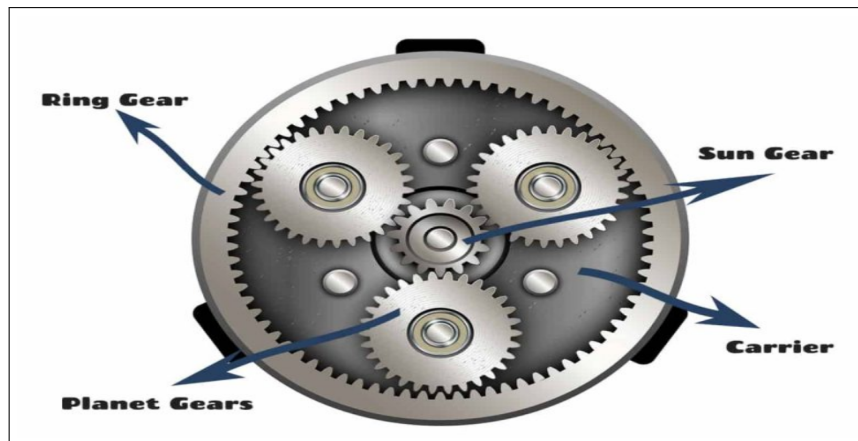


Figure 2.4: Placeholder – Diagram of a Planetary Gear Motor showing sun gear, planet gears, and ring gear arrangement.

2.2.2 Stepper Motors

Stepper motor are electromechanical devices that convert electrical pulses into discrete mechanical steps, enabling precise angular control without requiring feedback sensors for many applications.

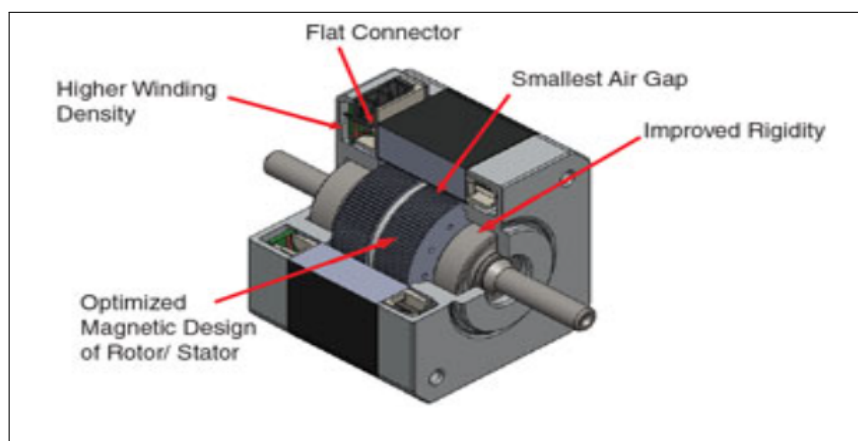


Figure 2.5: Placeholder – Diagram of a Stepper Motor showing stator coils and rotor teeth arrangement.

2.3 Forward Kinematics

Forward kinematics is the process of calculating the position and orientation of the robot's end-effector (tool tip) when the joint angles are known.

Input: Joint variables

For a revolute joint \rightarrow angle (θ)

Output:

End-effector position (x, y, z)

2.3.1 Forward Kinematics of a Two-Link Planar Arm

The forward kinematics problem determines the end-effector position given the joint angles:

$$x = l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2), \quad y = l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

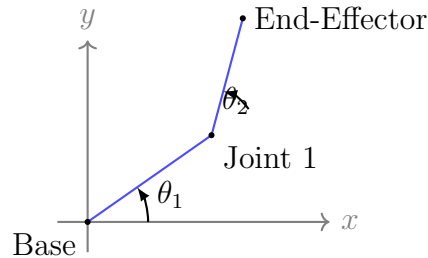


Figure 2.6: Two-link planar manipulator with joint angles θ_1 and θ_2 , link lengths l_1 and l_2 .

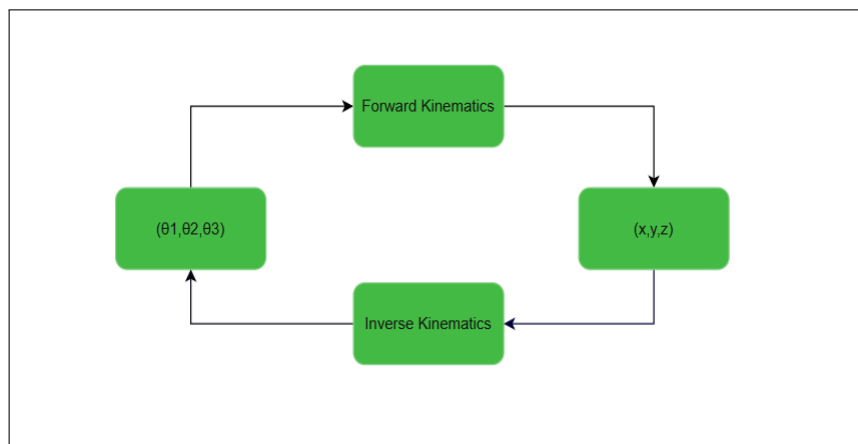


Figure 2.7: A Flow chart representing how Forward and Inverse Kinematics

2.4 Inverse Kinematics

Inverse kinematics is the process of calculating the required joint angles when the end-effector position and orientation are known.

Input: Position (x, y, z) Output: Joint angles $(\theta_1, \theta_2, \theta_3)$

2.4.1 Inverse Kinematics: Problem Formulation

The inverse kinematics problem is more challenging: given desired end-effector position (x, y) , find the joint angles (θ_1, θ_2) that achieve this position.

Example 2.1 – Comprehensive Inverse Kinematics of a Two-Link Planar Manipulator

Problem Statement

Given: A two-link planar manipulator with equal link lengths $l_1 = l_2 = 1$ and desired end-effector position:

$$(x, y) = \left(\frac{2 - \sqrt{2}}{2\sqrt{2}}, \frac{2 - \sqrt{6}}{2\sqrt{2}} \right) \Rightarrow (x, y) \approx (0.2071, -0.1589)$$

Goal: Determine all possible joint angle configurations (θ_1, θ_2) that achieve this position.

Step 1: Establish Kinematic Equations

From the forward kinematics equations:

$$x = l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) = \cos \theta_1 + \cos(\theta_1 + \theta_2) \quad (2.1)$$

$$y = l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) = \sin \theta_1 + \sin(\theta_1 + \theta_2) \quad (2.2)$$

Step 2: Distance Constraint and Elbow Angle

Squaring and adding equations (1) and (2):

$$\begin{aligned} x^2 + y^2 &= [\cos \theta_1 + \cos(\theta_1 + \theta_2)]^2 + [\sin \theta_1 + \sin(\theta_1 + \theta_2)]^2 \\ &= \cos^2 \theta_1 + 2 \cos \theta_1 \cos(\theta_1 + \theta_2) + \cos^2(\theta_1 + \theta_2) \\ &\quad + \sin^2 \theta_1 + 2 \sin \theta_1 \sin(\theta_1 + \theta_2) + \sin^2(\theta_1 + \theta_2) \\ &= 2 + 2[\cos \theta_1 \cos(\theta_1 + \theta_2) + \sin \theta_1 \sin(\theta_1 + \theta_2)] \end{aligned}$$

Using trigonometric identity:

$$\cos A \cos B + \sin A \sin B = \cos(A - B) = \cos(-\theta_2) = \cos \theta_2$$

Therefore:

$$x^2 + y^2 = 2 + 2 \cos \theta_2$$

Solving for $\cos \theta_2$:

$$\cos \theta_2 = \frac{x^2 + y^2 - 2}{2}$$

Substituting numerical values:

$$x^2 + y^2 \approx (0.2071)^2 + (-0.1589)^2 \approx 0.0429 + 0.0253 = 0.0682$$

$$\cos \theta_2 = \frac{0.0682 - 2}{2} = \frac{-1.9318}{2} = -0.9659$$

Example 2.1 – Continued (Page 2 of 3)**Step 3: Solve for θ_2**

From $\cos \theta_2 = -0.9659$, we find:

$$\theta_2 = \cos^{-1}(-0.9659) = \pm 165^\circ \quad (\text{or} \quad \pm 2.8798 \text{ radians})$$

This gives two possible elbow configurations:

- $\theta_2 = +165^\circ$: **Elbow-down configuration**
- $\theta_2 = -165^\circ$: **Elbow-up configuration**

Step 4: Solve for θ_1 using Matrix Method

We can rewrite the original equations in matrix form. Let:

$$\alpha = \theta_1$$

$$\beta = \theta_1 + \theta_2$$

Then equations become:

$$\cos \alpha + \cos \beta = x$$

$$\sin \alpha + \sin \beta = y$$

Using sum-to-product identities:

$$2 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) = x$$

$$2 \sin \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\alpha - \beta}{2} \right) = y$$

Dividing the second by the first:

$$\tan \left(\frac{\alpha + \beta}{2} \right) = \frac{y}{x}$$

$$\frac{\alpha + \beta}{2} = \tan^{-1} \left(\frac{y}{x} \right) = \tan^{-1} \left(\frac{-0.1589}{0.2071} \right) \approx -37.5^\circ$$

Therefore:

$$\alpha + \beta = 2\theta_1 + \theta_2 = -75^\circ$$

Example 2.1 – Continued (Page 3 of 3)**Step 5: Find Complete Solutions**

We now solve the system:

$$\begin{aligned} 2\theta_1 + \theta_2 &= -75^\circ \\ \theta_2 &= \pm 165^\circ \end{aligned}$$

Case 1: Elbow-down configuration ($\theta_2 = +165^\circ$)

$$\begin{aligned} 2\theta_1 + 165^\circ &= -75^\circ \Rightarrow 2\theta_1 = -240^\circ \Rightarrow \theta_1 = -120^\circ \\ \Rightarrow (\theta_1, \theta_2) &= (-120^\circ, +165^\circ) \end{aligned}$$

Case 2: Elbow-up configuration ($\theta_2 = -165^\circ$)

$$\begin{aligned} 2\theta_1 - 165^\circ &= -75^\circ \Rightarrow 2\theta_1 = 90^\circ \Rightarrow \theta_1 = 45^\circ \\ \Rightarrow (\theta_1, \theta_2) &= (45^\circ, -165^\circ) \end{aligned}$$

Step 6: Verification and Physical Interpretation

Let's verify both solutions using forward kinematics:

Solution 1: $(\theta_1, \theta_2) = (-120^\circ, +165^\circ)$

$$\begin{aligned} x &= \cos(-120^\circ) + \cos(-120^\circ + 165^\circ) = \cos(-120^\circ) + \cos(45^\circ) \\ &= -0.5 + 0.7071 = 0.2071 \quad \checkmark \\ y &= \sin(-120^\circ) + \sin(45^\circ) = -0.8660 + 0.7071 = -0.1589 \quad \checkmark \end{aligned}$$

Solution 2: $(\theta_1, \theta_2) = (45^\circ, -165^\circ)$

$$\begin{aligned} x &= \cos(45^\circ) + \cos(45^\circ - 165^\circ) = 0.7071 + \cos(-120^\circ) \\ &= 0.7071 - 0.5 = 0.2071 \quad \checkmark \\ y &= \sin(45^\circ) + \sin(-120^\circ) = 0.7071 - 0.8660 = -0.1589 \quad \checkmark \end{aligned}$$

Conclusion

The inverse kinematics problem for this two-link manipulator has **two distinct solutions**, representing different arm configurations that reach the same end-effector position. This multiplicity is characteristic of redundant manipulators and has important implications for path planning and obstacle avoidance in practical robotics applications.

Chapter 2 Summary

- **Forward kinematics** computes end-effector position (x, y) from given joint angles (θ_1, θ_2) .
- **Inverse kinematics** determines joint angles (θ_1, θ_2) from desired end-effector position (x, y) .
- Multiple configurations can reach the same position, leading to **multiple solutions**.
- The choice between solutions depends on practical constraints like joint limits, obstacle avoidance, and energy efficiency.

Battery and Power Systems in Robotics

Learning Objectives

- Calculate power requirements for robotic systems.
- Select appropriate battery capacity based on operational needs.
- Understand C-rating and its implications for battery performance.
- Apply safety margins in power system design.

3.1 Power Fundamentals

The electrical power in a system is given by:

$$P = VI$$

where P is power in watts (W), V is voltage in volts (V), and I is current in amperes (A).

For mechanical systems, power relates force and velocity:

$$P = Fv$$

where F is force in newtons (N) and v is velocity in meters per second (m/s).

3.2 Battery Capacity Calculation

Example 3.1 – Mobile Robot Power Requirements

Scenario: A 15 kg robot moving at 1 m/s with friction coefficient $\mu = 1$, powered by a 24 V battery system.

Step 1: Calculate Required Force

$$F = \mu N = \mu mg = 1 \times 15 \times 9.8 = 147 \text{ N}$$

Step 2: Calculate Mechanical Power

$$P_{\text{mech}} = Fv = 147 \times 1 = 147 \text{ W}$$

Step 3: Calculate Electrical Current Assuming 100% efficiency for simplicity:

$$I = \frac{P}{V} = \frac{147}{24} \approx 6.13 \text{ A}$$

Step 4: Calculate Battery Capacity For 40 minutes of operation:

$$t = 40 \text{ minutes} = \frac{40}{60} \approx 0.667 \text{ hours}$$

$$Q = I \times t = 6.13 \times 0.667 \approx 4.09 \text{ Ah} = 4090 \text{ mAh}$$

Step 5: Apply Safety Margin With 1.5× safety margin:

$$Q_{\text{actual}} = 1.5 \times 4090 \approx 6135 \text{ mAh}$$

Recommendation: Choose a 24 V, 6200 mAh battery.

3.3 C-Rating and Discharge Characteristics

C rating is a measure of how quickly a battery can safely discharge relative to its capacity. It indicates the discharge or charge current as a multiple of the battery's rated capacity.

$$I_{\text{max}} = C_r \times Q_{\text{Ah}}$$

where C_r is the C-rating and Q_{Ah} is the capacity in ampere-hours.

The discharge time can be estimated as:

$$t_{\text{discharge}} = \frac{1}{C_r} \text{ hours} \quad \text{or} \quad t_{\text{discharge}} = \frac{60}{C_r} \text{ minutes}$$

C-Rating Interpretation

- $C_r = 1$: Battery can be discharged over 1 hour
- $C_r = 10$: Battery can be discharged over 6 minutes
- $C_r = 0.5$: Battery can be discharged over 2 hours

Higher C-ratings indicate batteries capable of delivering higher peak currents.

Example 3.2 – Calculation of C-rating of the battery

Scenario: A drone operates at 12V for 10 minutes and its standard operational current is 30A. Find its C-rating for the standard operational current and if for first 30 seconds, the current required is 75A and for remaining time current required is 30A, then find C-rating for it.

Step 1: Calculate the C-rating of the battery at standard Current

$$C_r = \frac{I}{t_d} = \frac{60}{10} = 6$$

Step 2: Calculate the battery capacity for the second case

$$\text{Capacity} = I_1 \times t_1 + I_2 \times t_2 = 75A \times 0.5\text{min} + 30A \times 9.5\text{min} = 5375 \text{ mAh}$$

Step 3: Calculate the discharge time

$$t = \frac{\text{Capacity}}{I} = \frac{5375}{75000} \approx 0.0717 \text{ h}$$

Step 4: Calculate C-rating of the battery

$$C_r = \frac{1}{t_d} = \frac{1}{0.0717} = 14$$

3.4 Motor Performance Characteristics

3.4.1 Stall Torque vs. Operating Torque

- **Stall Torque:** : It is the maximum torque a motor can produce when its output shaft is not rotating. It occurs at zero speed with maximum current flow. Beyond this point, the motor cannot generate additional torque to start motion.(motor stalled)
- **Operating Torque:** It is the torque at which a motor runs efficiently under normal

working conditions. It is lower than the stall torque and corresponds to rated speed and power. Operating torque ensures stable performance without overheating or overload. (typically 60–80% of stall torque)

3.4.2 Operating Voltage

The **operating voltage** is the range of electrical voltage required for a device to function correctly. It ensures proper performance without damage or malfunction. Values below or above this range can cause instability or failure.

3.4.3 Power Rating

Power rating is the maximum amount of power a device can handle or deliver safely during operation. It defines the upper performance limit without causing damage. Exceeding the power rating can lead to overheating or failure.

3.4.4 Drawn Current

Drawn current is the amount of electrical current consumed by a device from its power source during operation. It depends on the load and operating conditions. Higher load or torque generally increases the drawn current.

3.4.5 Static and Dynamic Output

- **Static voltage:** Constant voltage level in a circuit without fluctuation.
- **Static output:** Steady-state output under constant input conditions.
- **Dynamic voltage:** Voltage varying with time due to load or operating changes.
- **Dynamic output:** Time-varying system response before reaching equilibrium.

3.4.6 Margin of Safe Operation

Safety Buffer between the components Maximum ratings and its actual operating condition, Ensuring reliability and durability.

$$\text{Margin} = \frac{\text{Maximum Rated Output}}{\text{Application Requirement}} \geq 1.5$$

A safety margin of 1.5–2.0 is recommended for reliable operation.

3.5 Mechanical Advantage in Robotic Systems

It is the ratio of the output force produced by a machine to the input force applied to it.

It shows how efficiently a machine helps lift or move a load using less effort.

For gear systems and lever mechanisms:

$$\text{Mechanical Advantage} = \frac{\text{Output Force}}{\text{Input Force}} = \frac{\text{Load}}{\text{Effort}}$$

For lever systems specifically:

$$M.A. = \frac{\text{Effort Arm Length}}{\text{Load Arm Length}}$$

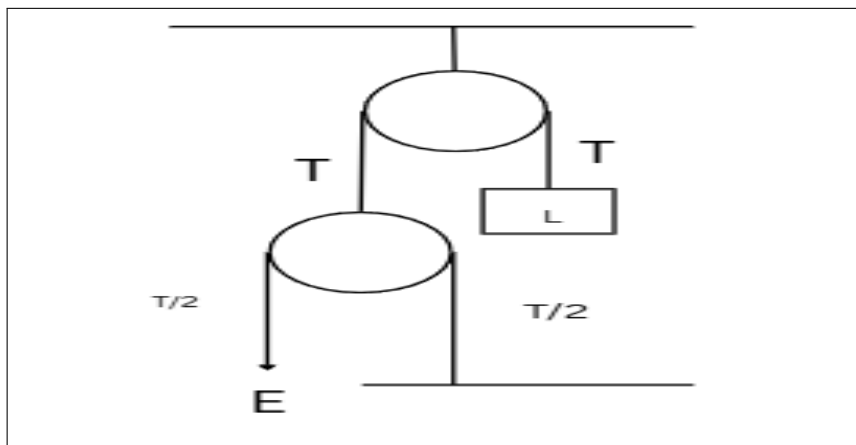


Figure 3.1: A Pulley System with mechanical Strength 2

Example 3.3- Calculation of Mechanical Strength of the pulley system described in Fig 3.1

Scenario: To Calculate the Mechanical Advantage of the Fig 3.1 Pulley System.

Step 1: Calculate the Mechanical Advantage

$$M.A = \frac{L}{E} = \frac{T}{T/2} = 2$$

Mechanical Advantage for Different Values

- $M.A > 1$: The machine provides a force advantage, meaning it amplifies the input force.
- $M.A < 1$: The machine provides a speed or distance advantage, meaning it increases the output speed or distance at the expense of force.
- $M.A = 1$: The machine offers no force gain, and its primary function is to change the direction of the force.

3.6 System Efficiency

Overall system efficiency accounts for losses in transmission and conversion:

$$\eta_{\text{total}} = \eta_{\text{motor}} \times \eta_{\text{gearbox}} \times \eta_{\text{transmission}} \times \dots$$

$$\eta = \frac{\text{Useful Output Power}}{\text{Input Power}} \times 100\%$$

Typical efficiencies:

- DC Motors: 70–85%
- Planetary Gearboxes: 85–95%
- Belt Drives: 90–98%
- Lead Screws: 20–80% (depends on pitch and lubrication)

Chapter 3 Summary

- Always calculate both mechanical and electrical power requirements
- Select batteries with adequate capacity AND appropriate C-rating
- Apply safety margins of 1.5–2.0 for reliable operation
- Consider system efficiency when sizing components
- Planetary gear systems provide excellent torque multiplication in compact packages
- Stepper motors offer precise positioning without feedback systems