MASCARET

Validation Manual

Version v8p5
December 1, 2023

Contents

1. Steady_State

1.1 Purpose

This test shows the capability of MASCARET kernels to well compute a steady state in the very simple case of a rectangular channel.
A comparison of the result of the steady kernel and the converged results of the transcritical kernel, both implicit and explicit form is also done with the analytical solution.

1.2 Description

1.2.1 Geometry

The domain is a rectangular channel of 10000 m long and 100 m width with a slope of 0.05%.
The geometry is described by 2 cross section located at $X = 0\ m$ and $X = 10000\ m$.

1.2.2 Initial condition

No initial condition for the steady kernel (not requested).
An initial constant water level $Z = 8\ m$ with a constant discharge of $Q = 1000\ m^3.s^{-1}$ for the other simulation.

1.2.3 Boundary condition

A constant discharge upstream : $Q = 1000\ m^3.s^{-1}$
A constant level downstream : $H = 3\ m$

1.2.4 Physical parameters

The roughness coefficient is chosen so that the normal height is $h_n = 5\ m$ hence a value $K = 30.6\ m^{1/3}.s^{-1}$ (application of the Strickler formula).

1.2.5 Numerical parameters

The mesh is define with a space step of 100 m.
The vertical discretization of the Cross Sections is homogeneous and equal to 1 m.
No time step parameters for the steady kernel.
For the transcritical kernel
The simulation duration is set to 2000 time step.
The time step is variable with a wishes Courant number of 0.8 for the explicit form and a wishes Courant number of 2 for the implicit form.

The initial time step is chosen to 1 s.

1.3 Results

1.3.1 Analytic solution

In steady state, the Saint-Venant equation system is reduced to the dynamic equation, which is itself simplified. In addition, in a uniform channel, of rectangular geometry, the expression of the friction term is further simplified when this friction is assumed to be zero on the banks: the hydraulic radius is then equal to the water height h, that is ie the variable to be determined. The solution is then written:

$$\frac{\partial h}{\partial x} = \frac{I - \dfrac{q^2}{K^2 h^{10/3}}}{1 - \dfrac{q^2}{g h^3}} \tag{1.1}$$

with

I: Bottom slope

K: Strickler friction coefficient

q: Discharge per width unit

This differential equation on h is solved using a Runge-Kutta method (10 m step, 4^{th} order)

Abscissae	Analytic Solution	Steady Kernel	Transcritical Kernel Explicit	Transcritical Kernel Implicit

Table 1.1: Numerical comparison of analytic and computed water height

Figures 1.1 and 1.2 show the comparison of water height with the different kernel. We could also see in figure 1.1 that the water height upstream tends to approach asymptotically the normal height h_n which is theoretically exact

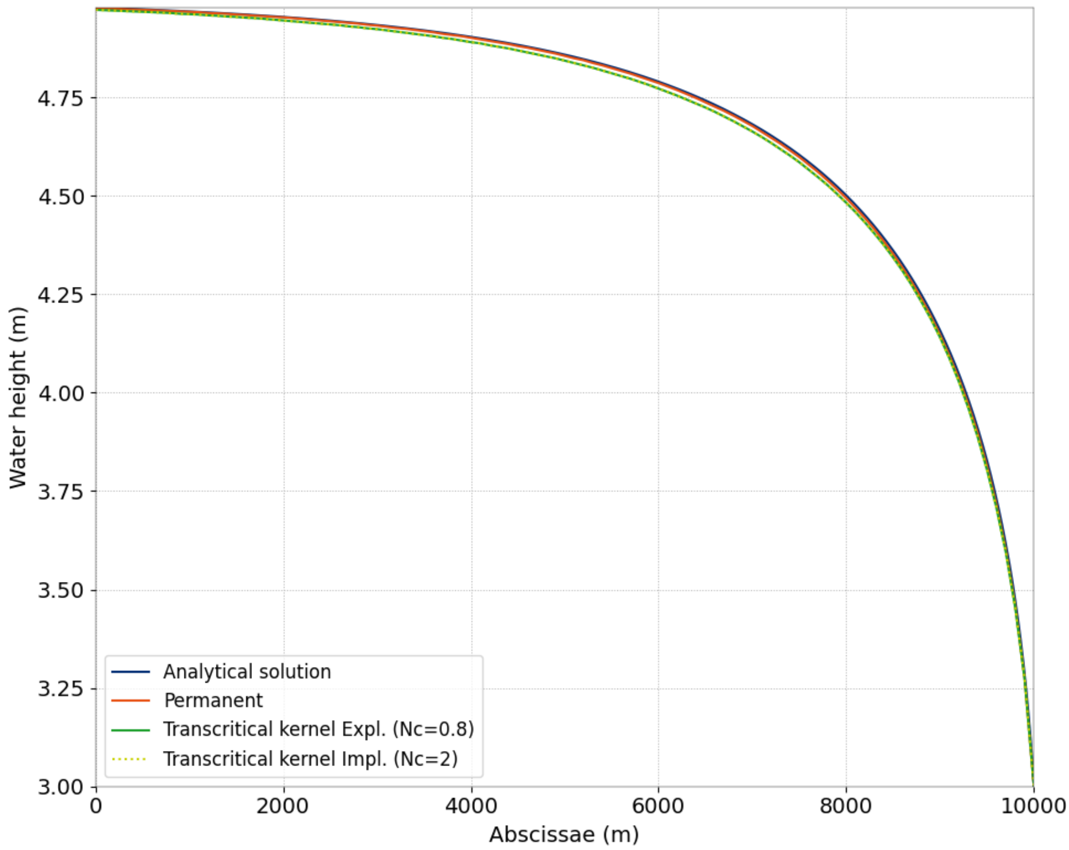

Figure 1.1: Comparison of analytic and computed water height

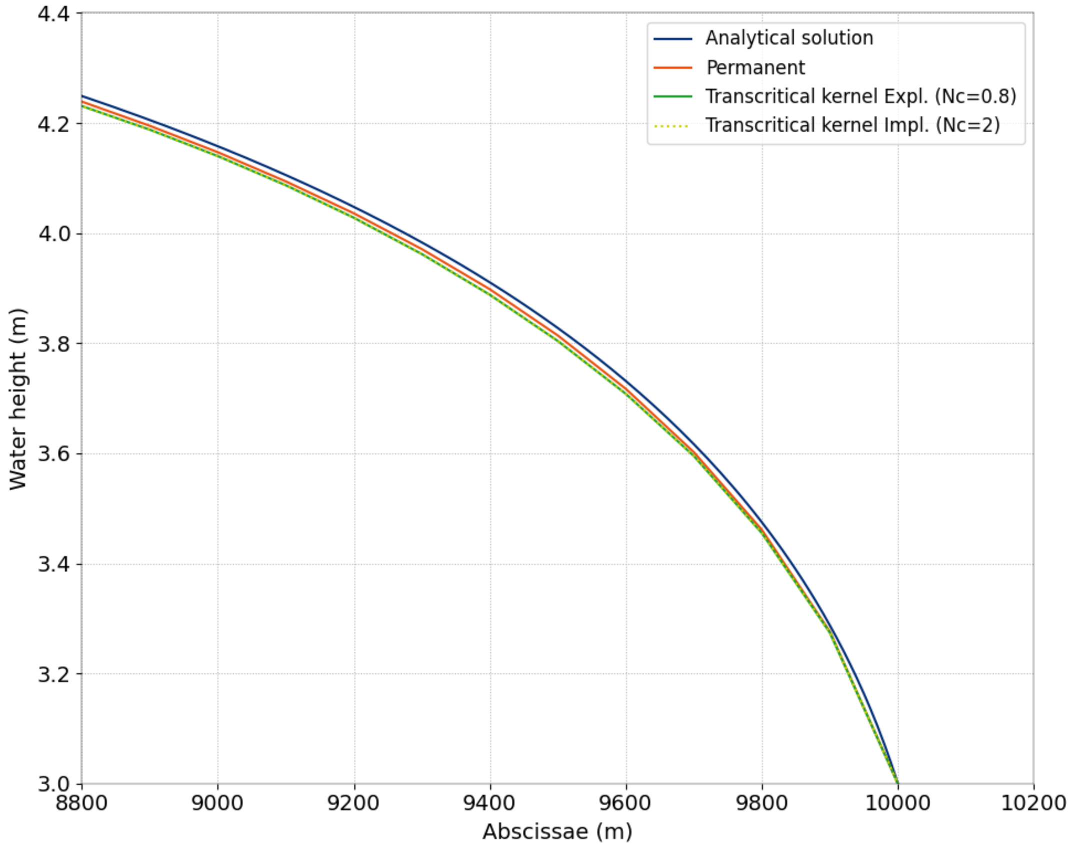

Figure 1.2: Zoom of figure 1.1 between 9000 *m* and 10000 *m*

1.4 Conclusion

The treatment of gravity-friction terms in the dynamic equation is good.
The comparison of all solutions to the analytical one show the good agreement of all kernels.

2. Non_Hydrostatic

2.1 Purpose

This test shows the capability of the transcritical kernel of MASCARET to deal with non-hydrostatic terms to model the Favre's waves.
These secondary waves appear during a sudden variation of flow in a channel and are in the form of undulations superimposed on the body of the Saint-Venant wave.

2.2 Description

2.2.1 Geometry

The domain is a rectangular channel of 30 m long and 40 cm width with a slope of 4% .

2.2.2 Initial condition

The steady state corresponding to the normal height $h_n = 0.2m$

2.2.3 Boundary condition

A constant discharge upstream : $Q = 0.035\ m^3.s^{-1}$
A hydrograph to simulate a flow trigger downstream on a very short time, from $Q = 0.035\ m^3.s^{-1}$ to $Q = 0\ m^3.s^{-1}$ in 0.07 s

2.2.4 Physical parameters

The roughness coefficient is chosen so that the normal height is $h_n = 0.2\ m$ hence a value $K = 102\ m^{1/3}.s^{-1}$ (application of the Strickler formula).

2.2.5 Numerical parameters

The mesh is define with a space step of 5 cm.
The vertical discretization of the Cross Sections is homogeneous and equal to 1 cm.
The simulation time is set to 6 s.
The time step is variable with a wishes Courant number of 0.8.
The initial time step is chosen to 0.01 s.

2.3 Results

The evolution in time of the water level with and without take in account the non-hydrostatic terms and the comparison to experimental measurements at 2 m of the downstream limit of the domain is presented on figure 2.1.

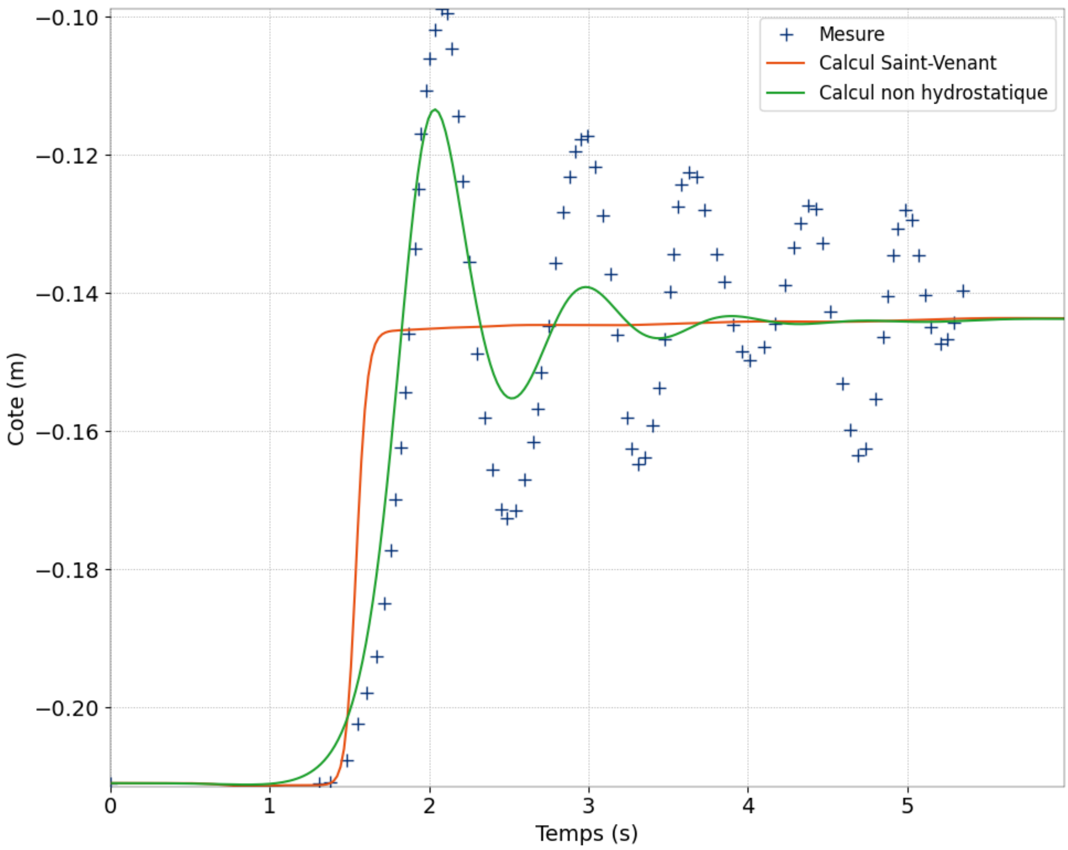

Figure 2.1: Water level evolution at $x = 28m$

2.4 Conclusion

The transcritical kernel of MASCARET is able to model the non hydrostatic term of the Favre's waves which superimpose to the main Saint-Venant wave.
Calculated amplitude is lower and damping is faster compared to experimental data.
A high order scheme tends to solve this problem [1].

3. Singular_Head_Loss

3.1 Purpose

The purpose of this test case is to validate the steady computation kernel and the transient kernel in transcritical mode, in the case of a rectangular channel with a singular head loss.

3.2 Description

3.2.1 Geometry

The domain is a rectangular channel of 4990 m length and 1 m width with nul slope. The geometry is described by 2 cross section located at $X = 0$ m and $X = 4990$ m . The local Head Loss is located a the abscissa $X = 2500$ m.

3.2.2 Initial condition

No initial condition for the steady kernel (not requested).
An initial linear varying water level (between $Z = 1.5$ m upstream and $Z = 1$ m downstream) with a constant discharge of $Q = 1$ $m^3.s^{-1}$ for the transcritical kernel.

3.2.3 Boundary condition

A constant discharge upstream : $Q = 1$ $m^3.s^{-1}$
A constant level downstream : $Z = 1$ m

3.2.4 Physical parameters

The roughness is taken in account with a Strickler coefficient value $K = 90$ $m^{1/3}.s^{-1}$
The local Head Loss coefficient is : 0.5

3.2.5 Numerical parameters

The mesh is define with a space step of 10 m. The domain is thus devided in 499 elements
The vertical discretization of the Cross Sections is homogeneous and equal to 2 m.

3.3 Results

3.3.1 Analytic solution

With a singular Head Loss, the momentum equation is written

$$\frac{\partial}{\partial x}\left(\frac{Q^2}{Lh} + \frac{1}{2}gLh^2\right) = -\frac{\partial}{\partial x}(\alpha V_{amont}^2) \times gLh \tag{3.1}$$

with

Q : total discharge ($m^3.s^{-1}$)

L : width of the domain (m)

h : water height (m)

$V_{upstream}$: velocity at the upstream cross section of the head loss ($m.s^{-1}$)

α: head loss coefficient

By discretizing this equation with finite difference method, we obtain ($L = 1\ m$) :

$$\left[\frac{2}{h_1 + h_2} \times \frac{Q_1^2 - Q_2^2}{\Delta x}\right] + \left[\frac{(Q_1 + Q_2)^2}{(h_1 + h_2)^2} + g \times \frac{h_1 + h_2}{2}\right]\frac{\Delta h}{\Delta x} = -\frac{\alpha}{2\Delta x}\left(\frac{Q_1}{h_1}\right)^2 \times \frac{h_1 + h_2}{2} \tag{3.2}$$

Index 1 correspond to upstream section, index 2 correspond to downstream section of the head loss.
Numerical application:

$Q_1 = 1\ m^3.s^{-1}$

$Q_2 = 1\ m^3.s^{-1}$

$h_1 = 1.2453\ m$

$h_2 = 1.2272\ m$

$\Delta x = 10\ m$

$\alpha = 0.5$

Those values gives $\Delta h_{analytique} = 0.0172\ m$
Figure 3.1 show the comparison of water height with the different kernel. Both kernel gives similar results. The computed head loss are:

$\Delta h_{calculee}$ avec le noyau permanent 7.0 : 0.018 m

$\Delta h_{calculee}$ avec le noyau transitoire transcritique : 0.018 m

The relative error is : 5%.

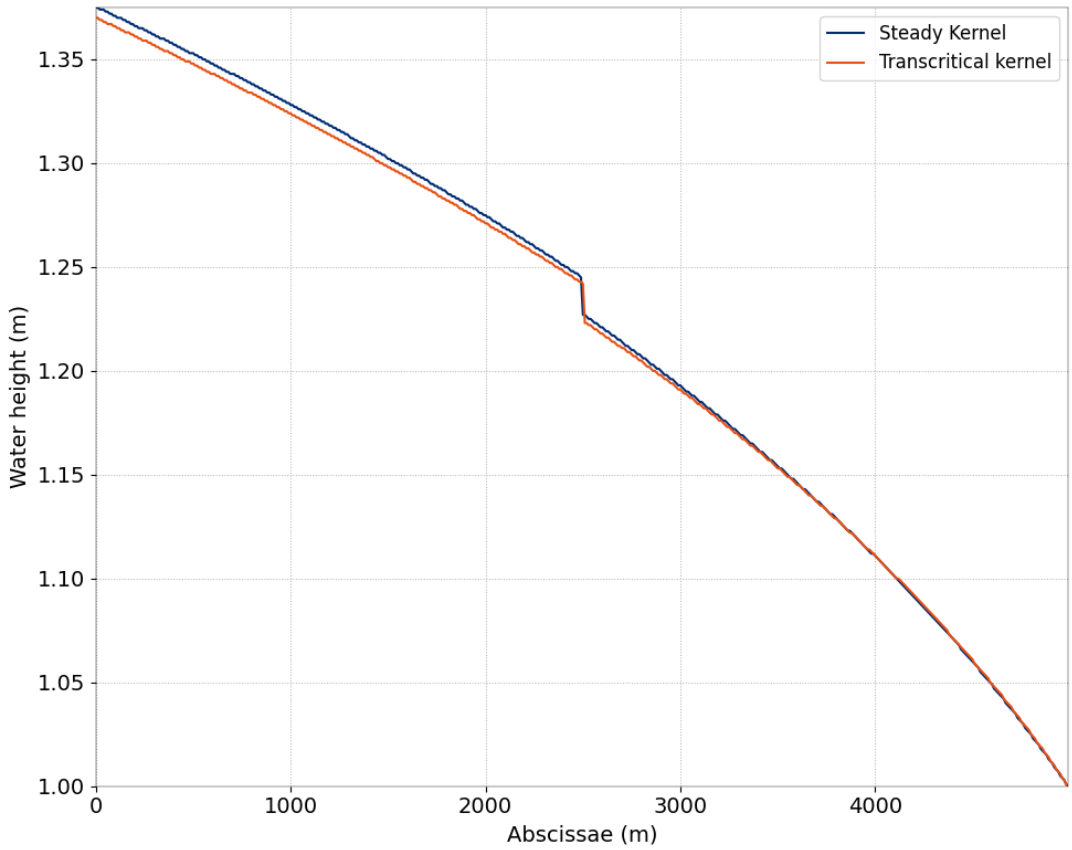

Figure 3.1: Comparison of computed water height

3.4 Conclusion

The treatment of local head loss is good.
The differences between the two kernels are nearly 0.

[1] Goutal N. Bristeau M.-O. and Sainte-Marie J. Numerical simulations of a non-hydrostatic shallow water model.