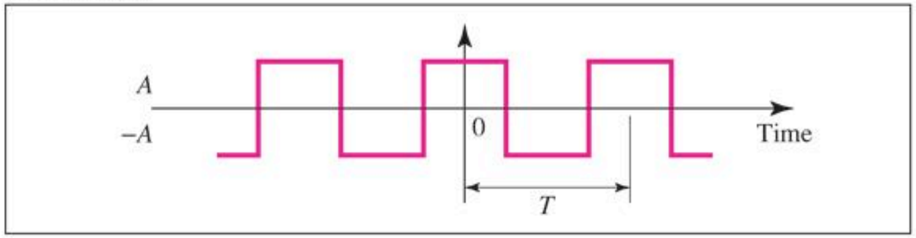
Signature

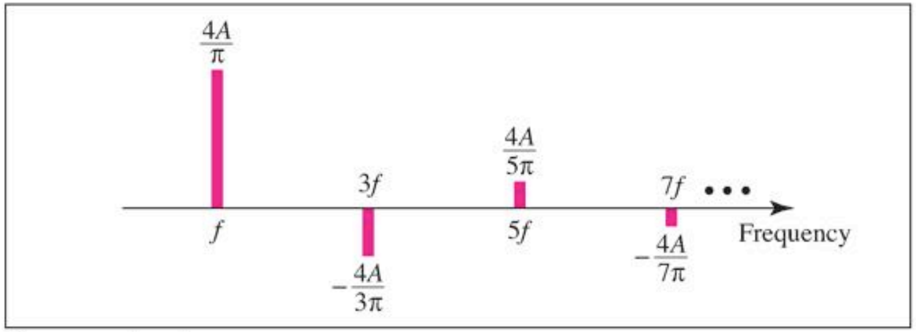
Examples of Signals and the Fourier Series Representation

Time domain



$A_0 = 0 \qquad A_n = \begin{cases} \dfrac{4A}{n\pi} & \text{for } n = 1, 5, 9, \ldots \\ -\dfrac{4A}{n\pi} & \text{for } n = 3, 7, 11, \ldots \end{cases} \qquad B_n = 0$

$$s(t) = \frac{4A}{\pi}\cos(2\pi ft) - \frac{4A}{3\pi}\cos(2\pi 3ft) + \frac{4A}{5\pi}\cos(2\pi 5ft) - \frac{4A}{7\pi}\cos(2\pi 7ft) + \cdots$$



Frequency domain

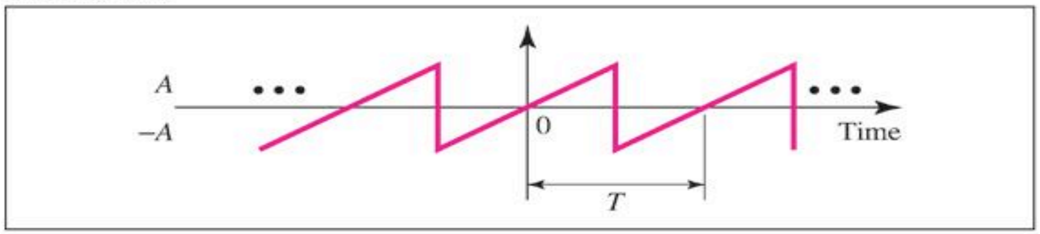
Home work

Homework

Signature

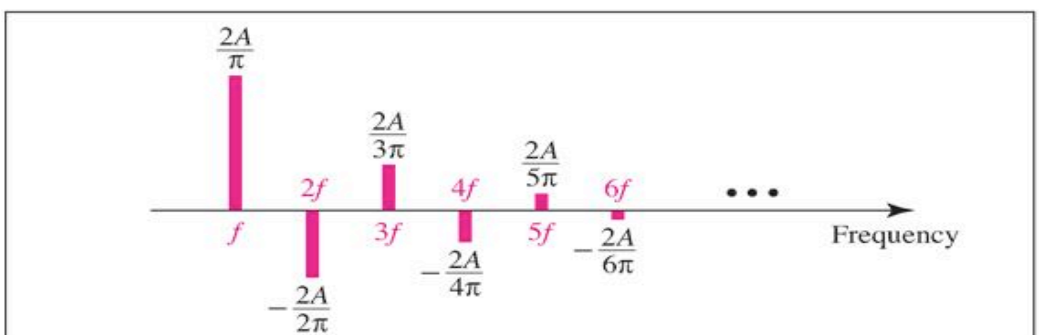
Sawtooth Signal

Time domain



$A_0 = 0 \qquad A_n = 0 \qquad B_n = \begin{cases} \dfrac{2A}{n\pi} & \text{for } n \text{ odd} \\ -\dfrac{2A}{n\pi} & \text{for } n \text{ even} \end{cases}$

$$s(t) = \frac{2A}{\pi}\sin(2\pi ft) - \frac{2A}{2\pi}\sin(2\pi 2ft) + \frac{2A}{3\pi}\sin(2\pi 3ft) - \frac{2A}{4\pi}\sin(2\pi 4ft) + \cdots$$



Frequency domain

Fourier Transform Pair (คู่ผลการแปลงฟูเรียร์)

Time domain → frequency domain

Fourier Transform: แปลงไป → Time domain

frequency domain

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Analysis

Inverse Fourier Transform: แปลงกลับ → frequency domain

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

Synthesis

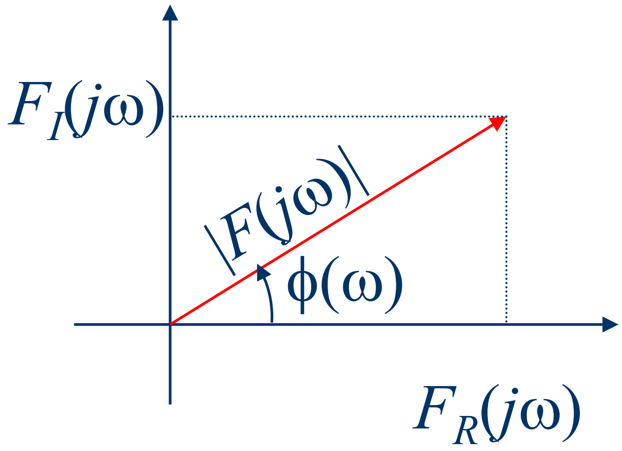
Continuous Spectra

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(j\omega) = F_R(j\omega) + jF_I(j\omega)$$

$$= \underbrace{|F(j\omega)|}_{\text{Magnitude}} e^{\overbrace{j\phi(\omega)}^{}}$$

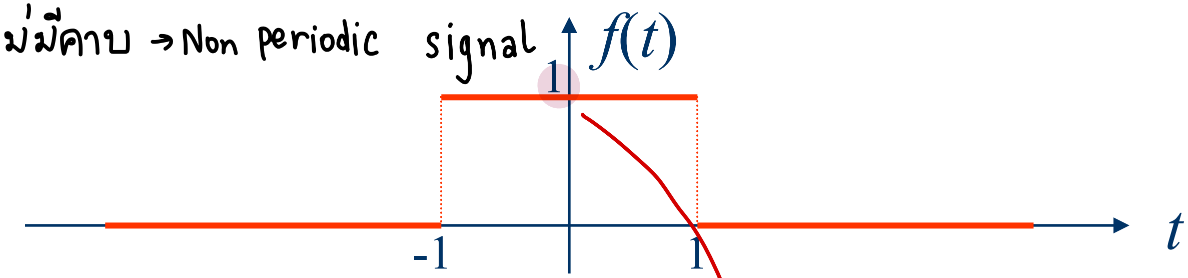
Phase



Example

Time domain → ใช้ frequency
periodic → ใช้ Series
non-periodic → ใช้ Transform

ไม่มีคาบ → Non periodic signal



$$F(j\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-1}^{1} 1\, e^{-j\omega t}dt = \frac{1}{-j\omega}e^{-j\omega t}\Big|_{-1}^{1}$$

$$=\frac{j}{\omega}\left(e^{-j\omega}-e^{j\omega}\right) = \frac{2\sin\omega}{\omega} = 2\,\mathrm{sinc}(\omega)$$

↓

sinc function $= \frac{\sin x}{x}$

Signature

$$= \frac{1}{-j\omega} e^{-j\omega t}\Big|_{-1}^{1}$$

$$= \left(\frac{1}{-j\omega} e^{-j\omega(1)}\right) - \left(\frac{1}{-j\omega} e^{-j\omega(-1)}\right)$$

$$= \frac{e^{-j\omega}}{-j\omega} - \frac{e^{j\omega}}{-j\omega}$$

$j = i = \sqrt{-1}$

$$= \frac{j}{j} \cdot \frac{1}{-j\omega} (e^{-j\omega} - e^{j\omega})$$

$$\frac{j}{-j^2\omega} \rightarrow -(-1)^2\omega \qquad \frac{j}{\omega}$$

$$= \frac{j}{\omega} (e^{-j\omega} - e^{j\omega})$$

$$= \frac{2j}{2j} \cdot \frac{j}{\omega} (e^{-j\omega} - e^{j\omega})$$

$$= \frac{2j^2}{2\omega j} (e^{-j\omega} - e^{j\omega})$$

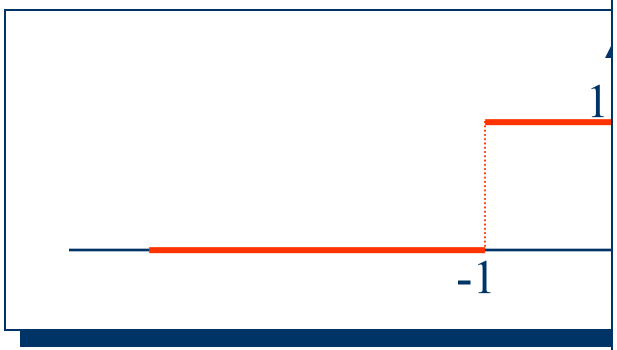
$$= \frac{-2}{2\omega j} (e^{-j\omega} - e^{j\omega})$$

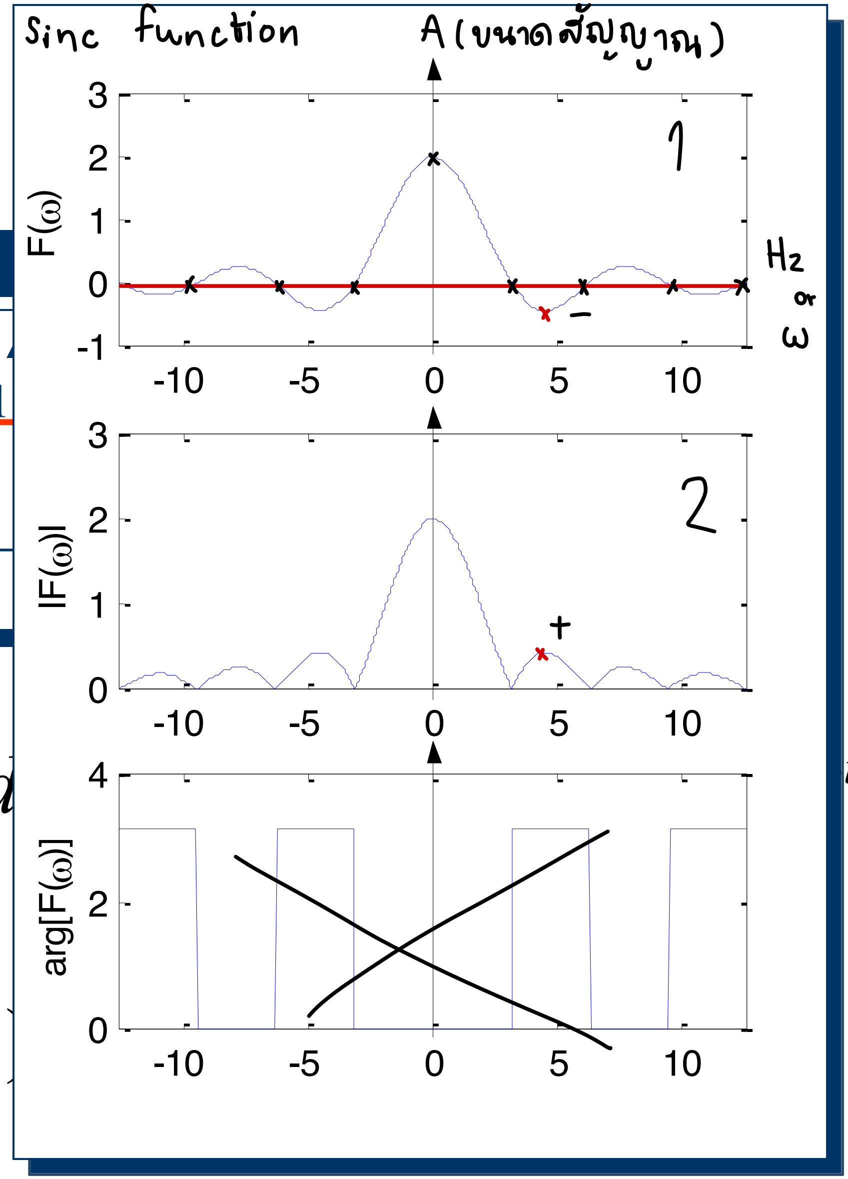
$$= \frac{2}{\omega} \frac{(e^{j\omega} - e^{-j\omega})}{2j} \leftarrow \sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$= \frac{2 \sin \omega}{\omega}$$

$\frac{2\sin\omega}{\omega} \xrightarrow{\text{diff}} 2\cos\omega$

Example



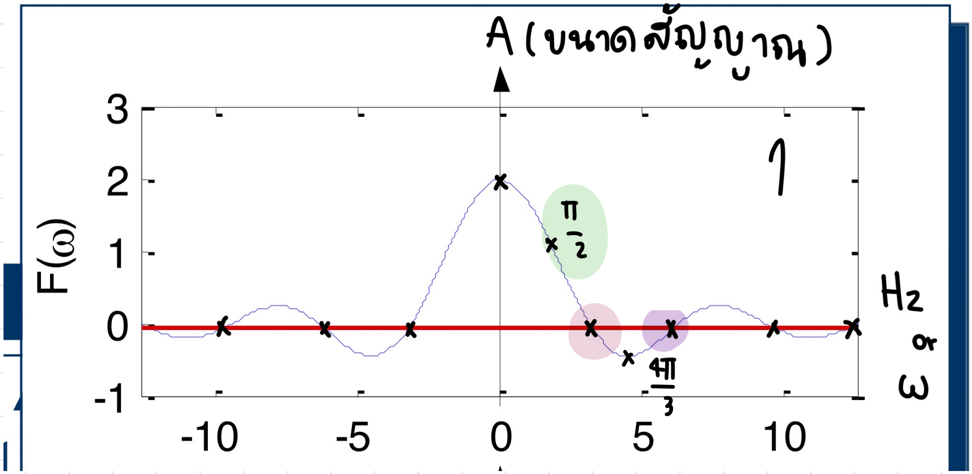


$$F(j\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}d$$

$$t \Big|_{-1}^{1}$$

$$= \frac{j}{\omega}(e^{-j\omega} - e^{j\omega}$$

Signature



$$\frac{2\sin\omega}{\omega} \rightarrow 2\cos\omega$$

$$\omega = 0 \rightarrow 2\cos(0) \rightarrow 2$$

$\sin 0 \rightarrow \omega = \pi$	$\sin 0 \rightarrow \omega = 2\pi$	$\frac{\pi}{2}$
$\omega = 2\pi f$	$f = 1$	$F(\omega) = \dfrac{2\sin(\frac{\pi}{2})}{\frac{\pi}{2}}$
$\pi = 2\pi f$	↓	
$f = \frac{1}{2}$	แกน x	$\frac{4}{\pi}$

$$\omega = \frac{3\pi}{2} \rightarrow F(\omega) = \frac{4\pi}{3} \rightarrow -0.42$$

$$F(\omega) = 1.27$$

Example: Multiplication by a Sinusoid

Time domain → $x(t) = p_\tau(t)\cos(\omega_0 t)$ sinusoidal burst

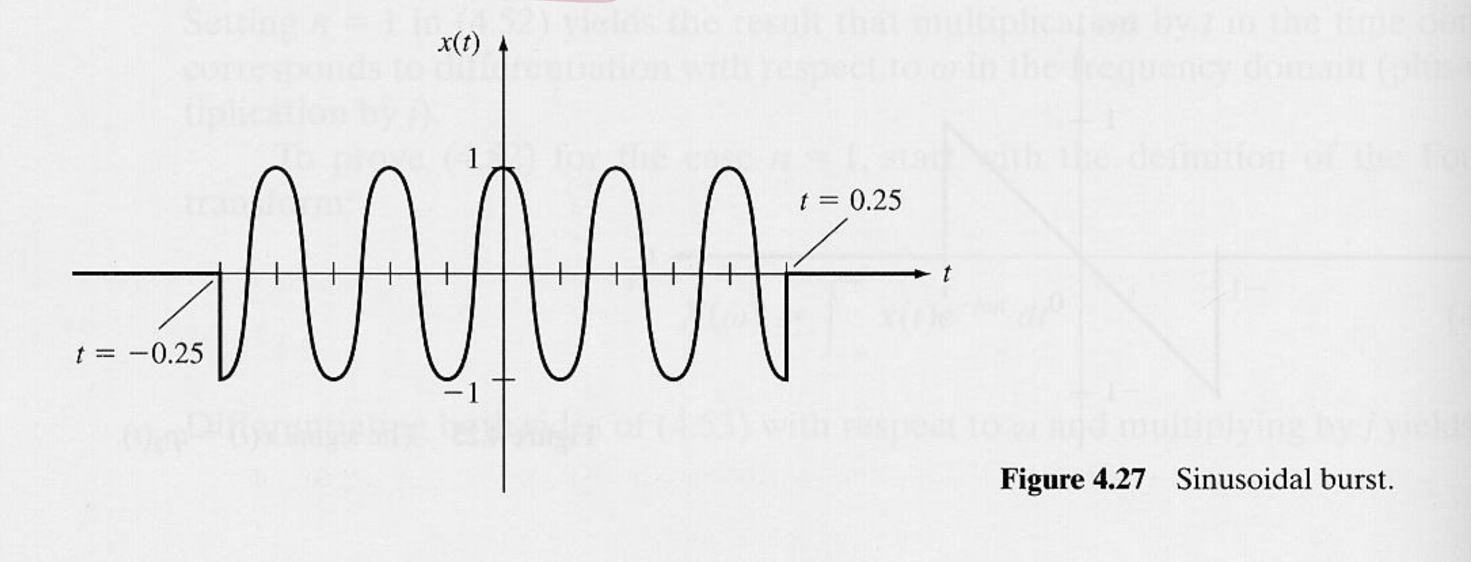


Figure 4.27 Sinusoidal burst.

Frequency domain $X(\omega) = \frac{1}{2}\left[\tau \operatorname{sinc}\left(\frac{\tau(\omega+\omega_0)}{2}\right) + \tau \operatorname{sinc}\left(\frac{\tau(\omega-\omega_0)}{2}\right)\right]$

Sine wave → กราฟแท่ง

Square wave → sinc function

Signature

Example: Multiplication by a Sinusoid – Cont’d

$$X(\omega) = \frac{1}{2}\left[\tau \operatorname{sinc}\left(\frac{\tau(\omega+\omega_0)}{2}\right) + \tau \operatorname{sinc}\left(\frac{\tau(\omega-\omega_0)}{2}\right)\right]$$

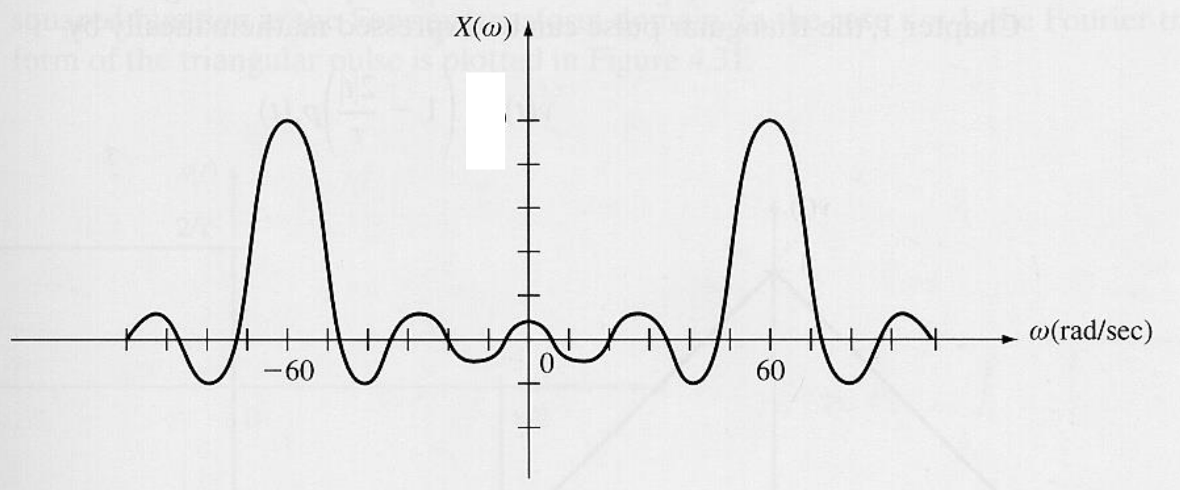


Figure 4.28 Fourier transform of the sinusoidal burst $x(t) = p_{0.5}(t)\cos 60t$.

$$\begin{cases} \omega_0 = 60\ rad/\sec \\ \tau = 0.5 \end{cases}$$

Example: Fourier Transform of the Rectangular Pulse

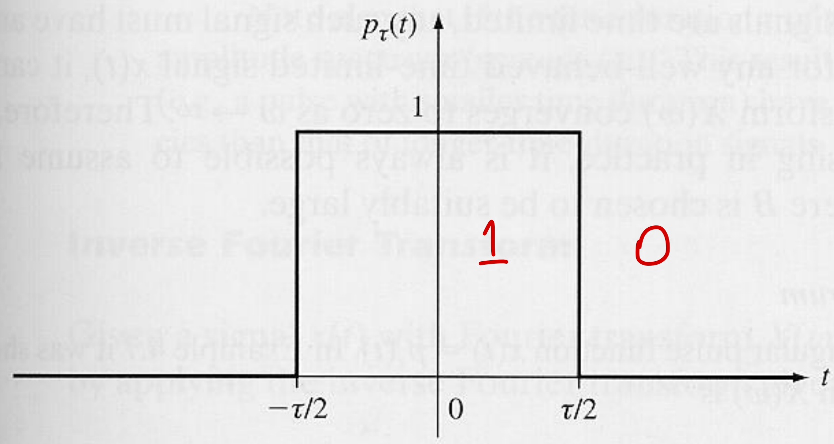


Figure 4.18 Rectangular pulse of duration τ seconds.

$$X(\omega) = 2\int_0^{\tau/2} (1)\cos(\omega t)\,dt = \frac{2}{\omega}\left[\sin(\omega t)\right]_{t=0}^{t=\tau/2} = \frac{2}{\omega}\sin\left(\frac{\omega\tau}{2}\right)$$

$$= \tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$$

Example: Fourier Transform of the Rectangular Pulse – Cont'd

$$X(\omega) = \tau \operatorname{sinc}\left(\frac{\omega\tau}{2}\right)$$

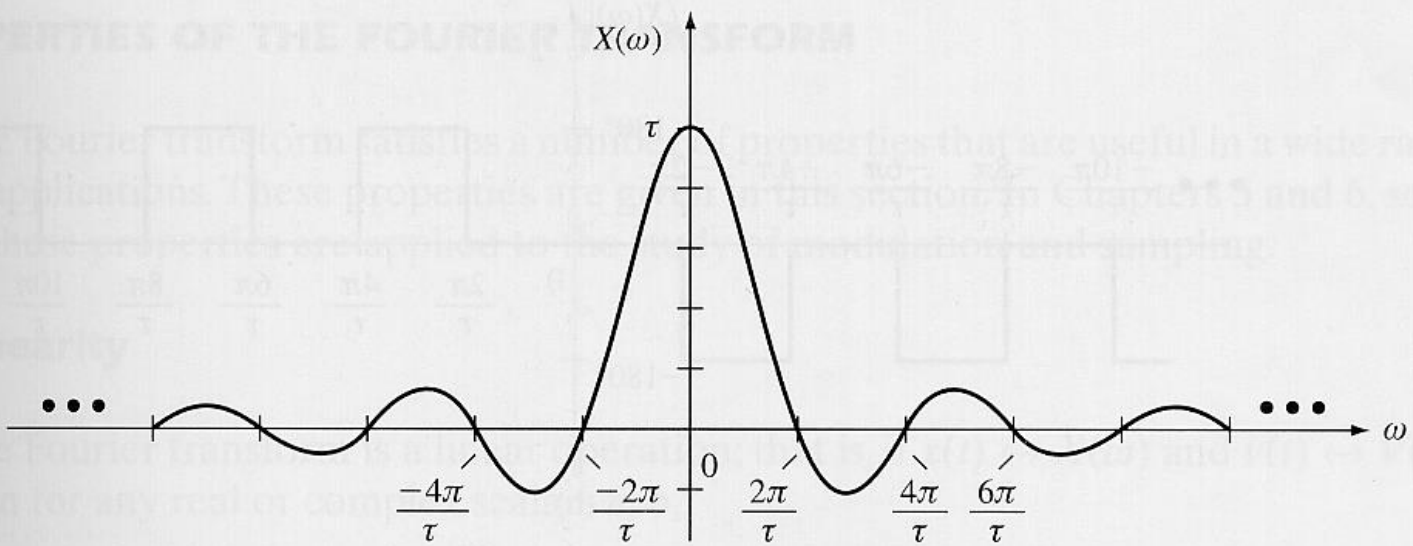


Figure 4.19 Fourier transform of the τ-second rectangular pulse.

$$\sin \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \qquad \cos \omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2}$$

$$V(\omega) = \int_{-\infty}^{\infty} v(t)\underline{\cos(\omega_c t)\, e^{-j\omega t} dt}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} v(t)(e^{j\omega_c t} + e^{-j\omega_c t})e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} v(t)e^{-j(\omega+\omega_c)t} dt + \frac{1}{2}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} v(t)e^{-j(\omega-\omega_c)t} dt$$

$$= \frac{V_o}{2}\left[\frac{e^{-j(\omega+\omega_c)t}}{-j(\omega+\omega_c)}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}} + \frac{V_o}{2}\left[\frac{e^{-j(\omega-\omega_c)t}}{-j(\omega-\omega_c)}\right]_{-\frac{\tau}{2}}^{\frac{\tau}{2}}$$

$$= \frac{V_o}{2}\left[\frac{e^{-j\frac{\tau}{2}(\omega+\omega_c)} - e^{j\frac{\tau}{2}(\omega+\omega_c)}}{-j(\omega+\omega_c)}\right] + \frac{V_o}{2}\left[\frac{e^{-j\frac{\tau}{2}(\omega-\omega_c)} - e^{j\frac{\tau}{2}(\omega-\omega_c)}}{-j(\omega-\omega_c)}\right]$$

แปลงกลับเป็น sin

$$= \frac{V_o}{\omega+\omega_c} sin\left[\frac{\tau(\omega+\omega_c)}{2}\right] + \frac{V_o}{\omega-\omega_c} sin\left[\frac{\tau(\omega-\omega_c)}{2}\right]$$

$$= \frac{\frac{V_o\tau}{2}}{\frac{\tau}{2}(\omega+\omega_c)} sin\left[\frac{\tau}{2}(\omega+\omega_c)\right] + \frac{\frac{V_o\tau}{2}}{\frac{\tau}{2}(\omega-\omega_c)} sin\left[\frac{\tau}{2}(\omega-\omega_c)\right]$$

$$= \frac{V_o\tau}{2}\left[sinc\{(\omega+\omega_c)\frac{\tau}{2}\} + sinc\{(\omega-\omega_c)\frac{\tau}{2}\}\right]$$