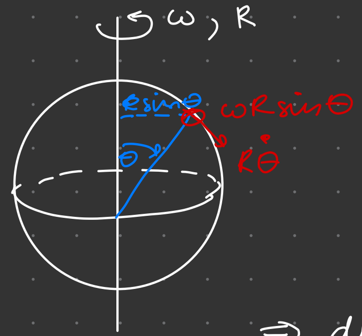
(1)



Założenie: $(\omega R \sin\theta)^2 + R^2\dot\theta^2 = R^2\omega^2$

$$\Rightarrow \omega^2 \sin^2\theta + \dot\theta^2 = \omega^2$$

$$\dot\theta^2 = \omega^2\cos^2\theta \quad \wedge \quad \dot\theta > 0$$

$\Rightarrow$ dla $\theta \in [0, \frac{\pi}{2}]$: $\dot\theta = \omega \cos\theta$

$$\int \frac{d\theta}{\cos\theta} = \int \omega\, dt \Rightarrow \int \frac{d\theta}{\cos\theta} = \left| \begin{array}{l} t = \cos\theta \\ dt = -\sin\theta\, d\theta \\ \quad = -\sqrt{1-t^2}\, d\theta \end{array} \right| = -\int \frac{dt}{t\sqrt{1-t^2}}$$

$$= \left| \begin{array}{l} s = \sqrt{1-t^2} \\ ds = -\frac{t}{\sqrt{1-t^2}}\, dt \end{array} \right| = \int \frac{ds}{t^2} = \int \frac{ds}{1-s^2} = \tanh^{-1}(s) + C$$

$$= \tanh^{-1}(\sin\theta) + C = \omega t \quad , (t=0, \theta=0) \Rightarrow C = 0$$

$$\sin\theta = \tanh(\omega t) \Rightarrow \theta(t) = \sin^{-1}(\tanh(\omega t))$$

Zauważmy, że $\lim_{x \to 1} \tanh^{-1}(x) = +\infty \Rightarrow$ mrówka dotrze do równika po $T \to +\infty$.

Krzywizna: $\vec t = \frac{\vec v}{|\vec v|}$, $ds = |\vec v| dt = \omega R\, dt$

w ukł. sferycznym: $\vec v = (0, R\omega\cos\theta, R\omega\sin\theta)$, $|\vec v| = \omega R$

$$\Rightarrow \vec t = (0, \cos\theta, \sin\theta)$$

$$\frac{d\vec t}{ds} = \frac{1}{\omega R}\frac{d}{dt}(0, \cos\theta, \sin\theta)_{sph.} = \frac{1}{\omega R}\Big(-R\dot\theta^2 - R\sin^2\theta\,\omega^2,$$
$$, R\ddot\theta - R\sin\theta\cos\theta\,\omega^2, 2R\cos\theta\,\omega\dot\theta\Big) =$$

$$= \frac{1}{\omega R}\Big(-R\omega^2\cos^2\theta - R\omega^2\sin^2\theta, -\omega^2 R\sin\theta\cos\theta - R\omega^2\sin\theta\cos\theta,$$
$$, 2R\omega^2\cos^2\theta\Big)$$

$$= (-\omega, -2\omega\sin\theta\cos\theta, 2\omega\cos^2\theta)$$

$$\left|\frac{d\vec t}{ds}\right|^2 = \omega^2 + 4\omega^2\sin^2\theta\cos^2\theta + 4\omega^2\cos^2\theta =$$
$$= \omega^2[1 + \cos^2\theta(4\cos^2\theta + 4\sin^2\theta)]$$
$$= \omega^2(1 + 4\cos^2\theta)$$

$$\frac{d\vec t}{ds} = \kappa \vec n = \kappa \frac{d\vec t}{ds} : \left|\frac{d\vec t}{ds}\right| \Rightarrow \kappa(\theta) = \left|\frac{d\vec t}{ds}\right| = \omega\sqrt{1 + 4\cos^2\theta}$$

$$\rho(0) = \frac{1}{\omega\sqrt5}$$

② $\rho(\varphi) = A(\sin\varphi + \cos\varphi)$, $\varphi(t) = \omega t$, $z(t) = 0$

$$\vec{a} = [\ddot{\rho} - \rho\dot{\varphi}^2, 2\dot{\rho}\dot{\varphi} + \rho\ddot{\varphi}, 0]$$

$$\ddot{\rho} = A\frac{d^2}{dt}(\cos\omega t + \sin\omega t) = A\omega\frac{d}{dt}(-\sin\omega t + \cos\omega t)$$

$$= A\omega^2(-\cos\omega t - \sin\omega t) = -A\omega^2(\cos\varphi + \sin\varphi)$$

$$\ddot{\varphi} = \frac{d}{dt}(\omega) = 0$$

$$\vec{v} = [A\omega(\cos\varphi - \sin\varphi), A\omega(\sin\varphi + \cos\varphi), 0]$$

$$\vec{a} = [-2A\omega^2(\cos\varphi + \sin\varphi), 2\omega^2 A(\cos\varphi - \sin\varphi), 0]$$

$$|\vec{v}|^2 = (A\omega)^2(1 + 1 + 0)^{1/2} \Rightarrow |\vec{v}| = \sqrt{2}A\omega$$

$$\Rightarrow \vec{t} = \frac{1}{\sqrt{2}}[\cos\varphi - \sin\varphi, \sin\varphi + \cos\varphi, 0]$$

$$ds = \sqrt{2}A\omega\, dt$$

$$\frac{d\vec{t}}{ds} = \frac{d\vec{t}}{dt}\cdot\frac{1}{v} = \frac{d\frac{\vec{v}}{v}}{dt}\cdot\frac{1}{v} \overset{v=\text{konst}}{=} \frac{1}{v^2}\frac{d\vec{v}}{dt} = \frac{1}{v^2}\vec{a}$$

$$= \frac{2A\omega^2}{2A^2\omega^2}[-(\cos\varphi + \sin\varphi), \cos\varphi - \sin\varphi, 0]$$

$$= \frac{1}{A}[-\cos\varphi - \sin\varphi, \cos\varphi - \sin\varphi, 0] = \frac{\sqrt{2}}{A}\vec{n}$$

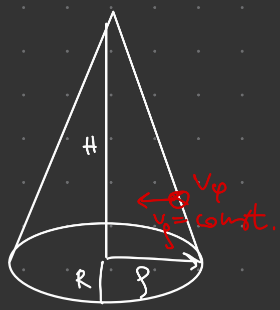
$$\vec{n} = \frac{1}{\sqrt{2}}[-\cos\varphi - \sin\varphi, \cos\varphi - \sin\varphi, 0]$$

$$a_t = (\vec{a}\,|\,\vec{t}) = 0 \Rightarrow a_n = a = 2\sqrt{2}A\omega^2$$

$$R = \frac{A}{\sqrt{2}} = \frac{A\sqrt{2}}{2}$$

$$s = \int_{t_1}^{t_2} ds = \int_0^{3\pi/4\omega} \sqrt{2}A\omega\, dt = \sqrt{2}A\omega\frac{3\pi}{4\omega} = \frac{3\sqrt{2}\pi A}{4}$$

③



$$v_0^2 = \rho(t)^2\dot\varphi^2 + \frac{R^2}{T^2} + \dot z^2$$

$$|v_\rho|t = R-\rho(t) \Rightarrow \rho(t) = R - \frac{Rt}{T} = \frac{R}{T}(T-t)$$

$$\Rightarrow z(T) = H$$

$$v_0^2 = \frac{R^2}{T^2}\left(\dot\varphi^2(T-t)^2+1\right) + \dot z^2 \qquad \frac{H}{R} = \frac{H-z}{\rho}$$

$$v_0^2 = \rho^2\dot\rho^2\left(\frac{d\varphi}{d\rho}\right)^2 + \frac{R^2}{T^2} + \dot\rho^2\left(\frac{dz}{d\rho}\right)^2, \quad \begin{cases} z(\rho) = H\left(1-\frac{\rho}{R}\right) \\ z(t) = H\left(1-\frac{T-t}{T}\right) = H\cdot\frac{t}{T} \end{cases}$$

$$\dot\rho = -\frac{R}{T}, \quad \frac{dz}{d\rho} = -\frac{H}{R}$$

$$\Rightarrow v_0^2 = \rho^2\frac{R^2}{T^2}\left(\frac{d\varphi}{d\rho}\right)^2 + \frac{R^2}{T^2} + \frac{H^2}{T^2}$$

$$\frac{T^2v_0^2 - R^2 - H^2}{R^2} = \rho^2\left(\frac{d\varphi}{d\rho}\right)^2$$

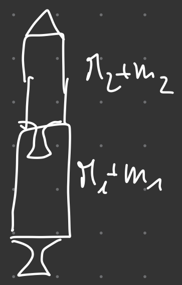
$$\frac{\sqrt{v_0^2T^2 - R^2 - H^2}}{\frac{R^2}{T}(T-t)} = \frac{1}{|\dot\rho|}\left|\frac{d\varphi}{dt}\right| \Rightarrow \left|\frac{d\varphi}{dt}\right| = \frac{\overbrace{\sqrt{v_0^2T^2 - R^2 - H^2}}^{A}}{R(T-t)}$$

$$\Rightarrow \vec v = \left(-\frac{R}{T}, \frac{\rho}{R}\frac{A}{T-t}, H\frac{t}{T}\right)$$

$$|\vec v| = v_0 \Rightarrow \vec t = \frac{1}{v_0}\vec v, \quad ds = v_0\,dt$$

$$\frac{d\vec t}{ds} = \frac{1}{v_0^2}\vec a = \frac{1}{v_0^2}\left(-\rho\frac{A^2}{R^2(T-t)^2}, -\frac{2R}{T}\frac{A}{R(T-t)} + \rho \text{ ---- }\right)$$

① $g\downarrow$ $\beta = \frac{dm}{dt} < 0$, $w = const$



$$\frac{dv}{dt} = \frac{w}{m}\frac{dm}{dt} - g$$

$$\frac{dv}{dm}\cdot\frac{dm}{dt} = \frac{w}{m}\frac{dm}{dt} - g$$

$$\frac{dv}{dm} = \frac{w}{m} - \frac{g}{\beta}$$

M_i – pusty człon
m_i – masa paliwa

$\Rightarrow v = w\log m - \frac{gm}{\beta} + C$; $C = -w\log(M_{tot}) + \frac{gM_{tot}}{\beta}$

$\Rightarrow v = w\log\frac{m}{M_{tot}} + \frac{g}{\beta}(M_{tot}-m)$

$$v_{1k} = w\log\frac{M_1+M_2+m_2}{M_1+M_2+m_1+m_2} + \frac{g}{\beta}m_1$$

Teraz, $v_{1k} = w\log(M_2+m_2) - \frac{g(M_2+m_2)}{\beta} + C$

$\Rightarrow v - v_{1k} = w\log\frac{m}{M_2+m_2} + \frac{g}{\beta}(M_2+m_2-m)$

$$v_{2k} - v_{1k} = w\log\frac{M_2}{M_2+m_2} + \frac{g}{\beta}m_2$$

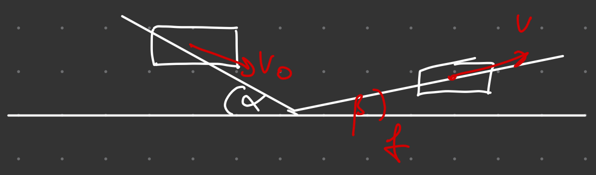
$\Rightarrow v_{2k} = w\log\frac{M_2(M_1+M_2+m_2)}{(M_2+m_2)(M_1+M_2+m_1+m_2)} + \frac{g}{\beta}(m_1+m_2)$

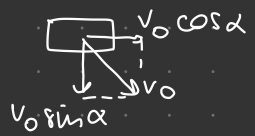
Wynik jest ciut inny ale równoważny. U nas $\vec{w} = w\hat{e}_x$

$\Rightarrow w<0 \wedge \log()<0$

$\Rightarrow w\log() \geq 0$

④





$\frac{dW}{dt} = P = Fv = N v_0\cos\alpha$

$dW = fNv_0\cos\alpha\,dt$

$N = \frac{dp}{dt}$

$\Rightarrow dW = f\,dp\,v_0\cos\alpha$

$$\left\{\frac{\cancel{m}v_0^2}{2} = \frac{\cancel{m}v^2}{2} + 2v_0\sin\alpha\, f\cancel{m}\, v_0\cos\alpha\right.$$

$$v_0^2 = v^2 + 4fv_0^2\sin\alpha\cos\alpha$$

$$v^2 = v_0^2(1-4f\sin\alpha\cos\alpha)$$

$$v = v_0\sqrt{1-4f\sin\alpha\cos\alpha}$$

$$dW = N f\, v_x(t)\, dt$$

$$N = \frac{dp_y}{dt} \qquad = \underbrace{dp_y}_{2 v_y m} f\, v_x$$

$$a = -\frac{\mu N(t)}{m}$$

$$v\cos\beta = v_0\cos\alpha - \frac{\mu}{m}\int N(t)\,dt = v_0\cos\alpha - \frac{\mu}{m}\int_{-v_0\sin\alpha}^{(*)\, v_0 \sin\alpha} dp_y$$

$$= v_0\cos\alpha - 2\mu v_0 \sin\alpha$$

$$v\sin\beta = v_0\sin\alpha$$

(*) $p_y(t)$ jest równoważna

$$\cot\beta = \cot\alpha - 2\mu$$

$$W = \int \mu N(t)\, dx = \int \mu N(t)\, \underset{v\cos\beta}{v_x(t)}\, dt$$

$$= \int \mu\, dp_y\, v_x = \mu m \int_{v_0\cos\alpha}^{v\cos\beta} v_x\, dv_x = \mu m \frac{v_x^2}{2}\Bigg|_{v_0\cos\alpha}^{v_0\cos\alpha - 2 v_0 \sin\alpha}$$

$$= \frac{\mu m}{2}\left(v_0^2\cos^2\alpha + 4\mu^2 v_0^2 \sin^2\alpha - 4 v_0^2 \mu \sin\alpha \cos\right.$$

$$- v_0$$

$$\int v_x\, dv_y$$

$$v_0^2 \cos^2\alpha$$

$$v_y +$$

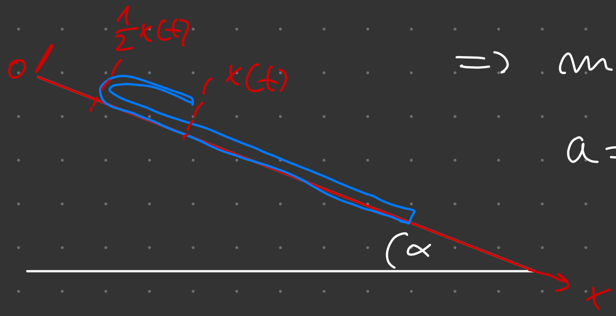
$$\frac{v_y^2}{2}\Bigg|_{-v_0\cos\alpha}^{v_0\cos\alpha}$$

$$v_x(t) = v_{0x} - f\int dv_y$$

$$f m \int \left(v_{0x} - f\int dv_y \right) dv_y$$

$$= f m \left[v_{0x} \int dv_y - f \int\!\!\int dv_y\, dv_y \right]$$

3



$$\Rightarrow m(t) = \frac{M}{L}\,\frac{x(t)}{2}$$

$$a = \text{const.} \Rightarrow x(t) = \frac{at^2}{2}$$

$$\Rightarrow m(t) = \frac{Ma}{4L}t^2$$

Pęd górnej części dywanu: $p = m(t)\cdot v(t) = \frac{Mat^2}{4L}\cdot at = \frac{Ma^2t^3}{4L}$

$$F_x = m(t)\, g \sin\alpha = \frac{dp}{dt} = \frac{3}{4}\,\frac{Ma^2t^2}{L}$$

$$\Rightarrow \frac{\cancel{M}a\cancel{t^2}}{\cancel{4L}}\, g \sin\alpha = \frac{3}{\cancel{4}}\,\frac{\cancel{M}a^{\cancel{2}}\cancel{t^2}}{\cancel{L}} \Rightarrow a = \frac{1}{3}\, g \sin\alpha \quad //$$