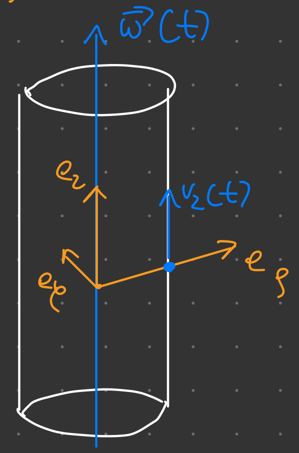
ZADANIE 4 SZYMON CEDROWSKI




Bierzemy układ walcowy u. w $t=0$ $\vec{r}=(R,0,0)$.

$$\dot{\varphi}(t) = \frac{A}{R}t^2, \quad \dot{z}(t) = Bt^2$$

$$\vec{v}(t) = \begin{bmatrix} \dot{\rho} \\ \rho\dot{\varphi} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ At^2 \\ 2Bt \end{bmatrix}$$

$$\vec{a}(t) = \begin{bmatrix} \ddot{\rho} - \rho\dot{\varphi}^2 \\ 2\dot{\rho}\dot{\varphi} + \rho\ddot{\varphi} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -\frac{A^2}{R}t^4 \\ 2At \\ 2B \end{bmatrix}$$

$$|\vec{v}| = \sqrt{A^2t^4 + 4B^2t^2} = t\sqrt{A^2t^2+4B^2}$$

$$ds = |\vec{v}|\,dt = \sqrt{A^2t^2+4B^2}\,t\,dt$$

$$\vec{t} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{t\sqrt{A^2t^2+4B^2}}\begin{bmatrix} 0 \\ At^2 \\ 2Bt \end{bmatrix} = \frac{1}{\sqrt{A^2t^2+4B^2}}\begin{bmatrix} 0 \\ At \\ 2B \end{bmatrix}$$

$$\frac{d\vec{t}}{ds} = \frac{d\vec{t}}{dt}\frac{dt}{ds} = \frac{1}{|\vec{v}|}\frac{d}{dt}\left(\frac{1}{\sqrt{A^2t^2+4B^2}}\begin{bmatrix} 0 \\ At \\ 2B \end{bmatrix}_{(\rho,\varphi,z)}\right)$$

$$= \frac{1}{|\vec{v}|}\frac{d}{dt}\left(\frac{1}{t\sqrt{A^2t^2+4B^2}}\right)\begin{bmatrix} 0 \\ At^2 \\ 2Bt \end{bmatrix} +$$

$$+ \frac{1}{|\vec{v}|}\frac{1}{t\sqrt{\phantom{x}}}\underbrace{\frac{d}{dt}\begin{bmatrix} 0 \\ At^2 \\ 2Bt \end{bmatrix}}_{\vec{a}} = -\frac{1}{|\vec{v}|}\frac{2(A^2t^2+2B^2)}{t^2(A^2t^2+4B^2)^{3/2}}\vec{v} + \frac{1}{|\vec{v}|^2}\vec{a}$$

$$= \frac{-2(A^2t^2+2B^2)}{t^2(A^2t^2+4B^2)^{3/2}}\vec{t} + \frac{1}{A^2t^4+4B^2t^2}\vec{a} = \kappa\vec{n}$$

Składowa styczna $\vec{a}$:

$$a_t = (\vec{a}|\vec{t}) = \frac{1}{\sqrt{A^2t^2+4B^2}}(2A^2t^2+4B^2)$$

$$a_n^2 = a^2 - a_t^2 = \frac{A^4}{R^2}t^8 + 4A^2t^2 + 4B^2 - \frac{(2A^2t^2+4B^2)^2}{A^2t^2+4B^2}$$