ZADANIE 1

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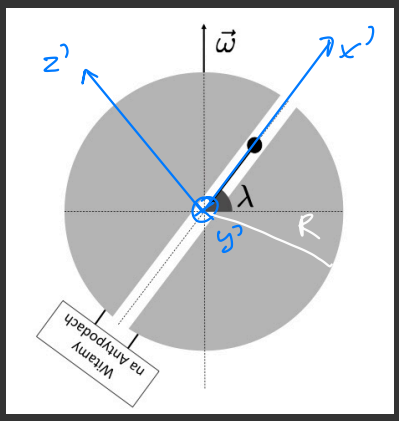

$$\vec{\omega} = \begin{bmatrix} \omega \sin\lambda \\ 0 \\ \omega \cos\lambda \end{bmatrix}_{u'} \qquad x' := r \Rightarrow \vec{r}' = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}_{u'}$$

$$\frac{GM(r)}{r^2} = g, \quad \frac{M(r)}{M(R)} = \left(\frac{r}{R}\right)^3 \Rightarrow \frac{g(r)}{g(R)} = \frac{r^3}{R^3}\cdot\frac{R^2}{r^2}$$

$$\Rightarrow g(r) = \frac{g}{R} r$$

$u' = (x', y', z')$
$u = (x, y, z)$

$$\Rightarrow m\vec{a} = \vec{F} = \begin{bmatrix} -r/R\, g \\ F_y \\ F_z \end{bmatrix}_{u'}$$

składowe siły reakcji

grawitacja na odcinku $x' > 0$. (na $x' < 0$ jest symetrycznie)

$$\vec{\omega} \times \vec{v}' = \begin{bmatrix} 0 \\ \dot{r}\omega\cos\lambda \\ 0 \end{bmatrix}_{u'}, \quad \vec{\omega}\times(\vec{\omega}\times\vec{r}') = \begin{vmatrix} e_{x'} & e_{y'} & e_{z'} \\ \omega\sin\lambda & 0 & \omega\cos\lambda \\ 0 & r\omega\cos\lambda & 0 \end{vmatrix} =$$

$$= \begin{bmatrix} -r\omega^2\cos^2\lambda \\ 0 \\ r\omega^2\sin\lambda\cos\lambda \end{bmatrix}_{u'}$$

$$\Rightarrow \vec{a}' = \vec{a} - 2\vec{\omega}\times\vec{v}' - \vec{\omega}\times(\vec{\omega}\times\vec{r}') \qquad \left(\begin{array}{l} v_{tr} = 0 \ \wedge \\ d\omega/dt = 0 \end{array}\right)$$

$$\begin{bmatrix} \ddot{r} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{m}\begin{bmatrix} -\frac{r}{R}g \\ F_y \\ F_z \end{bmatrix} - 2\omega\begin{bmatrix} 0 \\ \dot{r}\cos\lambda \\ 0 \end{bmatrix} - r\omega^2\begin{bmatrix} -\cos^2\lambda \\ 0 \\ \sin\lambda\cos\lambda \end{bmatrix}$$

$$\Rightarrow \begin{cases} F_y = 2m\omega\dot{r}\cos\lambda \\ F_z = mr\omega^2\sin\lambda\cos\lambda \end{cases} \wedge \ddot{r} = -\frac{r}{mR}g + r\omega^2\cos^2\lambda$$

$$\ddot{r} + \left(\frac{g}{mR} - \omega^2\cos^2\lambda\right) r = 0 \Rightarrow \Omega^2 = \frac{g}{mR} - \omega^2\cos^2\lambda$$

$$\Rightarrow r(t) = A\cos(\Omega t + \varphi) \qquad r(0) = R \ \wedge \ \dot{r}(0) = 0$$

$$\Rightarrow \boxed{r(t) = R\cos(\Omega t)}, \quad x' > 0$$

$$\Rightarrow \forall x' \quad r(t) = R\,|\cos(\Omega t)|$$

(symetria ruchu na obu półkulach)

Czas półobrotu to:

$$\boxed{T_A = \frac{\pi}{\Omega}}$$

Siła reakcji wyraźna w układzie $\mathcal{U}'$ to:

$$\vec{F}_R = \begin{bmatrix} 0 \\ 2m\omega v \cos\lambda \\ m r \omega^2 \sin\lambda \cos\lambda \end{bmatrix}_{\mathcal{U}'} = \begin{bmatrix} 0 \\ -2mR\omega\Omega \sin(\Omega t)\cos\lambda \\ mR\omega^2 \cos(\Omega t)\cos\lambda \sin\lambda \end{bmatrix}_{\mathcal{U}'}$$

I teraz liczenie.

$$\Omega = \sqrt{\frac{9{,}81}{6{,}37\cdot 10^6} - \left(\frac{2\pi}{24\cdot 3600}\right)^2 \cos^2(52)} \approx 1{,}24\cdot 10^{-3}\ s^{-1}$$

$$\Rightarrow T_A = \frac{\pi}{\Omega} \approx 42{,}2\ \text{min}$$

$$\vec{F}_R(t=0) = \begin{bmatrix} 0 \\ 0 \\ 0{,}016\,N \end{bmatrix}_{\mathcal{U}'} ; \quad \vec{F}_R\left(t=\frac{T_A}{2}\right) = \begin{bmatrix} 0 \\ -0{,}707\,N \\ 0 \end{bmatrix}_{\mathcal{U}'}$$