Liquid Volume Estimation in a Glass Container

Vojtěch Vondráček

***Abstract*—This paper presents the design and implementation of a liquid volume estimation method for a glass container based on the analysis of sound frequencies emitted when the container is excited. The core methodology involves Short-time Fourier Transform (STFT) analysis to identify specific frequencies corresponding to different fluid volumes. A transfer characteristic between frequency and fluid volume is established, enabling real-time volume estimation using audio data captured from a microphone. The proposed solution is validated through several experiments, including a demonstration where distinct liquid levels were used to play a simple melody on a set of glass bottles. The results demonstrate the method's feasibility and highlight its potential for application in fluid diagnostics and acoustic-based sensing technologies.**

***Index Terms*—Liquid volume estimation; Acoustic-based volume estimation; FFT; STFT; Ovčáci, čtveráci; Bottle music**

I. Introduction

.

THE determination of fluid volume in container is a fundamental requirement in many fields and across various applications. Numerous traditional method exist, such as those based on weight measurement, the use of volumetric cylinders, or optical and pressure sensors. While these methods typically provide reliable results, they often require specialized sensors and do not support real-time measurements or non-invasive access to the medium [1].

This paper explores an alternative method for determining fluid volume. The described method estimates the volume based on the analysis of sound emitted by container after excitation, enabling non-invasive measurement using almost any microphone (e.g., from a computer, mobile phone, etc.).

The following section describes the fundamental principle on which the measurement process is based, supplemented by an analysis of vibrational modes in a glass container. The first part is subsequently followed by a description of selected containers, and then, of course, the procedure used to determine the volume.

VV
December 31, 2024

II. Fundamental principle

When a glass container is excited, mechanical vibrations are generated, which subsequently propagate and can be measured. These vibrations can be divided into two main components: vibrations of the container walls (glass) and standing air waves inside the container.

According to my experiments, the predominant type of vibration in the resulting sound depends on the method used to excite the vibrations. If vibrations are excited by blowing into the container, standing air waves above the liquid level dominate. Conversely, if vibrations are induced by tapping the container (in my case, using a wooden spoon), the resulting is vibrations of the container walls, influenced by the liquid level. A comparison of these two methods is shown in figures 1a and 1b. These figures also show that when vibrations are excited by blowing, the highest frequency is approximately 250 Hz, whereas after tapping, the dominant frequency is around 2800 Hz. Figure 1b also shows a slightly increased amplitude at the frequency of the standing waves, which is due to the fact that even when tapping the glass container, some standing waves are excited.

A. Expected Frequency for Vibrations Excited by Blowing

The approximate resonant frequency of vibrations induced by blowing can be estimated using the formula for the resonant frequency of standing waves in an ideal cylinder, as shown in [2], [3]. For a container with a neck diameter of d, an air column height of l, and a speed of sound of v_s, the frequency can be approximated as

$$f = \frac{v_s}{4(l + 0.3d)}. \tag{1}$$

B. Expected Frequency for Vibrations Excited by Tapping

I was unable to find an exact formula for determining the resonant frequency of the container influenced by the liquid level height. However, based on experiments and fundamental physical principles, it can be stated that the frequency response as a function of volume should have the opposite characteristic to the previous case. That is, the higher the liquid level (i.e., the smaller the air column), the lower the frequency. For an ideal cylinder, this relationship can be expressed as follows

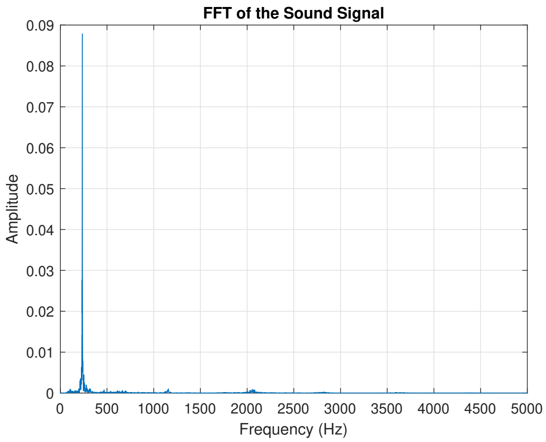
$$f \propto l, \tag{2}$$

where l is the height of the remaining air column.

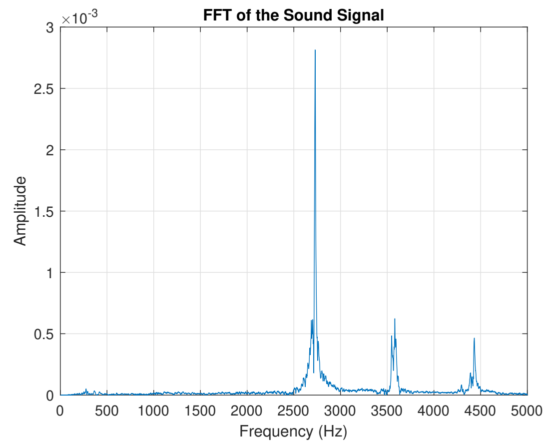
C. Choice of Vibration Excitation Method for the Container

Figure 1 shows that after blowing excitation, the frequency is approximately 300 Hz, which is problematic for several reasons. The main issue, in my view, is that it fully overlaps with the range of human speech, which averages between

Vojtěch Vondráček is with the Faculty of Electrical Engineering, Czech Technical University in Prague, Czech Republic (e-mail: vondrvo5@fel.cvut.cz).



(a) Excited by blowing - standing waves, container A



(b) Excited by tapping, container A

Fig. 1. Comparison of two methods of excitation

$78 - 307$ Hz [4], making this method highly susceptible to interference, which would be difficult to filter out.

The second method, where the container is excited by tapping, has a resonant frequency that is several times higher—approximately 10x. For this reason, I decided to use this method as it should be more robust, allow easier noise removal, and be more reliable overall[1].

III. Choise of Containers

I selected two types of containers to test the proposed methods. Both types are shown in Fig. 2. Type A containers are narrow-neck glass bottles originally used for Opočenské milk. The reason why there are five of them will be explained in section VII. Each bottle has an approximate capacity of 750 ml. The second type, labeled as B, is of unknown origin and has an approximate capacity of 350 ml.

IV. Determination of Reference Volume

In several parts of this project, it was necessary to determine the reference volume of liquid in the container. While multiple methods could be used for this purpose, I considered only two straightforward approaches.

The first method involves using a volumetric cylinder, but this approach has limitations, including reduced precision and practical challenges in measurement accessibility. Instead, I decided for a second method based on weight measurement. Specifically, the volume of the liquid V is calculated using the formula $V = (m_C - m_E)/\rho_W$, where m_C is the mass of the container with the liquid, m_E is the mass of the empty container, and ρ_W is the density of the liquid (in this case, water).

This method was chosen for its simplicity and its ability to provide accurate and rapid measurements. Throughout the project, only this method was used. A standard kitchen scale was used, and its uncertainty was considered negligible.

Fig. 2. Used bottles

V. Procedure

As mentioned previously, the resonant frequency produced by a container upon tapping depends on the volume of liquid inside. The primary task, therefore, is to identify the resonant frequency in the recorded signal and establish a transfer function between frequency and volume. The procedure is demonstrated using data collected from the Type A container, while the Type B container is referenced only for comparative purposes. Additionally, all subsequent measurements use water as the liquid.

A. *Processing the Recorded Signal*

Determining the frequency directly from the time-domain signal is challenging, as illustrated in Fig. 3. Therefore, I utilize the Fast Fourier Transform (FFT) to convert the signal into the frequency domain, as shown in Fig. 4. Next, irrelevant parts of the signal are filtered out, focusing solely on the resonant frequencies of the container, which eliminates most background noise and human speech. A band-pass filter is employed for this purpose. The processed signal is shown in Fig. 5a, and the same filtered signal, converted back into the time domain using the inverse FFT, is shown in Fig. 5b.

[1]From the comparison of the graphs in Figure 1, it might seem that when the container is excited by blowing, the signal at the resonant frequency has a higher amplitude than when excited by tapping. However, this is not the case, as the amplitudes are not scaled identically.

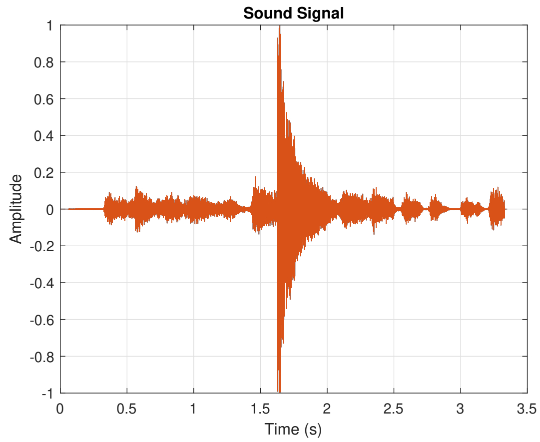


Fig. 3. Measured sound signal in time domain.

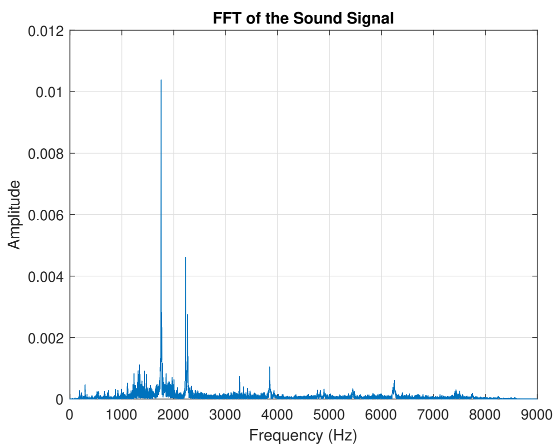
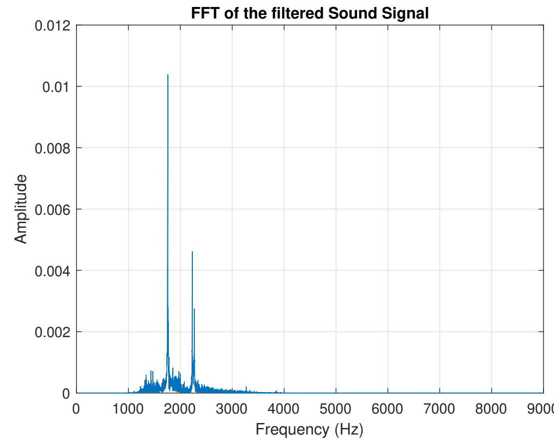
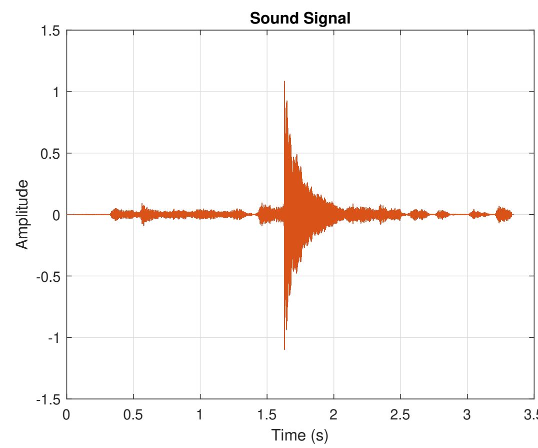


Fig. 4. Measured sound signal in frequency domain.



(a) Filtered sound signal in frequency domain



(b) Filtered sound signal in time domain

Fig. 5. Filtered sound signal

B. Identifying the Resonant Frequency

I chose a relatively simple yet quite effective approach to identify the resonant frequency: selecting the frequency with the highest amplitude in the filtered spectrum. This method is not entirely robust against noise; however, since the analysis operates at relatively high frequencies, no issues were encountered under normal conditions.

C. Transfer Function

Subsequently, resonant frequencies were determined for various liquid volumes. These frequency-volume pairs were then fitted using a least-squares polynomial regression to obtain the transfer function. The polynomial order was selected to minimize the relative error between the fitted curve and the measured data[2].

The resulting transfer functions are provided below, along with plots showing the fitted polynomial and the relative error. For the Type A container, the volume as a function of frequency is expressed as:

$$V_A(f_A) = \alpha_1 f_A^5 + \alpha_2 f_A^4 + \alpha_3 f_A^3 + \alpha_4 f_A^2 + \alpha_5 f_A + \alpha_6, \quad (3)$$

[2]The measured data were split into training and testing datasets. The transfer function was fitted on the training set and later validated on the testing set.

where the coefficients are:

$$\begin{aligned}
\alpha_1 &= -8.10005546311933 \cdot 10^{-13}, \\
\alpha_2 &= 8.71690921938181 \cdot 10^{-9}, \\
\alpha_3 &= -3.79652972989008 \cdot 10^{-5}, \\
\alpha_4 &= 8.37905216263537 \cdot 10^{-2}, \\
\alpha_5 &= -94.0829661873384, \\
\alpha_6 &= 43545.2170235549.
\end{aligned}$$

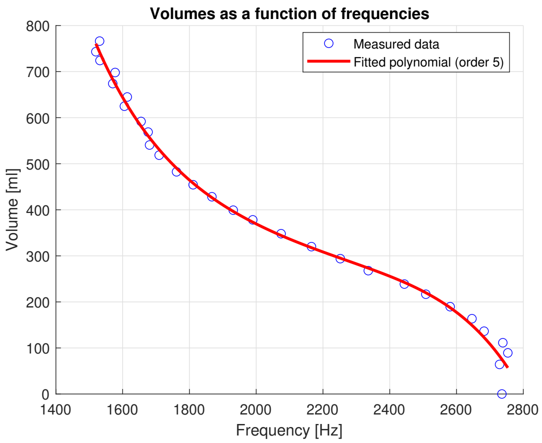
The corresponding transfer characteristic is shown in Fig. 6. Similarly, for the Type B container, the volume as a function of frequency is given by:

$$V_B(f_B) = \gamma_1 f_B^5 + \gamma_2 f_B^4 + \gamma_3 f_B^3 + \gamma_4 f_B^2 + \gamma_5 f_B + \gamma_6, \quad (4)$$

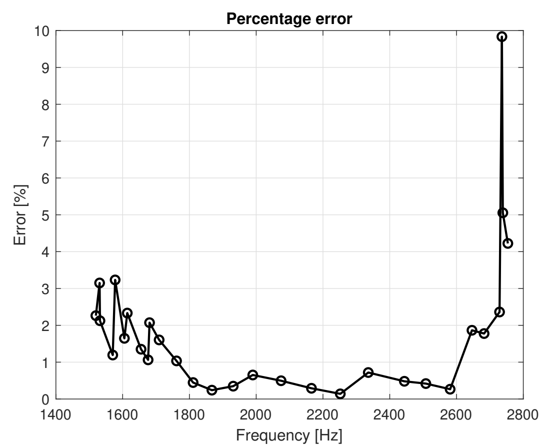
with the coefficients:

$$\begin{aligned}
\gamma_1 &= -2.87505647667480 \cdot 10^{-13}, \\
\gamma_2 &= 3.96064793875682 \cdot 10^{-9}, \\
\gamma_3 &= -2.17163742258517 \cdot 10^{-5}, \\
\gamma_4 &= 0.0593222651201244, \\
\gamma_5 &= -81.0059373477689, \\
\gamma_6 &= 44649.6799524653.
\end{aligned}$$

The corresponding transfer characteristic for the Type B container is shown in Fig. 7.

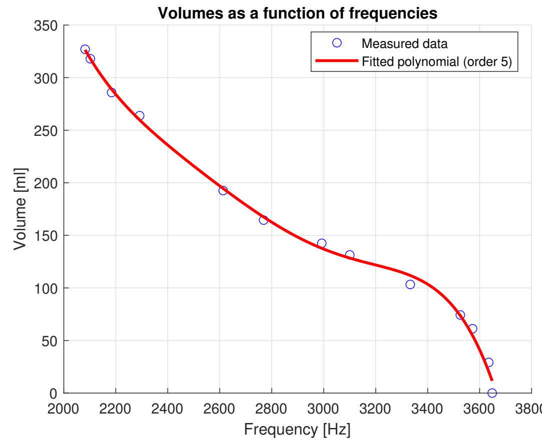


(a) Dependence of volume on frequency

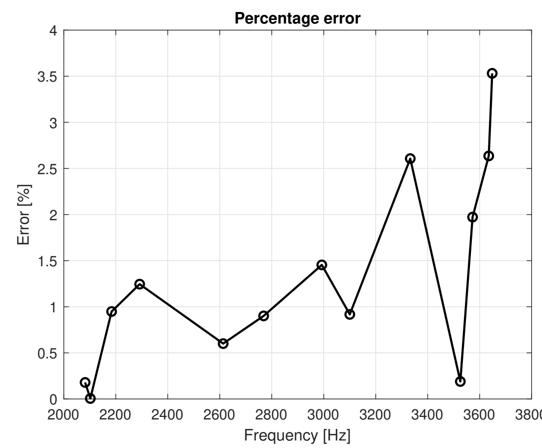


(b) Relative error between fitted curve and measured values

Fig. 6. Dependence of volume on frequency, Container A



(a) Dependence of volume on frequency



(b) Relative error between fitted curve and measured values

Fig. 7. [Dependence of volume on frequency, container B

D. Comparison

A comparison between the measured volumes and the volumes derived from the transfer function is shown in Fig. 6b for the Type A container and Fig. 7b for the Type B container. Both types exhibit the largest errors when the liquid volume is low. For the Type A container, the error in this range reaches up to 10%, while for the Type B container, it is around 3.5%. However, across the rest of the range, the error remains below 3%, often staying under 1%.

VI. Real-Time Volume Estimation Algorithm

This section describes a real-time liquid volume estimation in a container. Unlike offline processing, where the Fast Fourier Transform (FFT) can be used to analyze the entire signal, real-time processing requires techniques capable of temporal resolution. For this purpose, the Short-Time Fourier Transform (STFT) was used, which segments the signal into overlapping time windows, performing a DFT on each segment[3]. To reduce spectral leakage at the window edges, a Hamming window was applied.

The processing workflow consists of the following steps:

1) **Audio Capture:** Continuous sound is recorded via a microphone in real-time.
2) **Preprocessing:** The recorded signal is filtered using a bandpass filter to isolate frequencies relevant to the container's resonances, as described in Section V-A.
3) **Resonant Frequency Identification:** For each time window, the frequency with the highest amplitude is identified from the filtered spectrum. If this amplitude exceeds a predefined threshold specific to the container and noise, then the frequency is assumed to correspond to the container's resonance.
4) **Volume Estimation:** Using the identified frequency, the volume of liquid in the container is estimated via the transfer function in equations 3 or 4.
5) **Visualization:** The estimated volume is displayed in real-time within a simple graphical user interface.

[3]https://www.mathworks.com/help/signal/ref/stft.html

A. Experiment

This algorithm, although not entirely robust against strong noise at some frequencies, performed reliably in controlled tests. The higher frequencies associated with tapping-induced vibrations significantly reduce susceptibility to common noise sources, such as speech. One of the experiments conducted can be found in the following online video at YouTube (https://youtu.be/X6KOk0fO4MQ). In the experiment, it can be seen that the estimation error is relatively small for larger volumes but increases for smaller ones. For instance, in the recorded experiment, for a volume of 600 ml, the relative error is 0.8%, while at 50 ml, the error rises to nearly 2%. This observation aligns with the graph 6b. The code in MATLAB can be found in the git repository (see appendix A).

VII. Ovčáci, čtveráci

To further demonstrate the versatility of the proposed frequency-to-volume characteristic, I attempted to play the Czech folk song Ovčáci, čtveráci on a set of milk bottles. The song was chosen due to its simplicity, requiring only five distinct musical tones.

First, it was necessary to determine the specific water volumes for each bottle to produce the desired frequencies corresponding to the notes. This involved utilizing the transfer function derived earlier (see Fig. 6).

The more challenging part of the task was collecting enough milk bottles to assemble the five required for the song. I took me nearly two months to collect them all, but shortly before Christmas I succeeded.

Finally, with the calculated water levels and all five bottles prepared, I was ready to play the melody. A recording of the performance is available online at YouTube (https://www.youtube.com/shorts/pi8bOXCfTYk) as well as in the git repository (see apendix A).

VIII. Conclusion

This paper presents an approach for estimating liquid volume in glass containers using the analysis of resonant frequencies produced by tapping the container. The relationship between resonant frequency and liquid volume was established through transfer functions, enabling accurate and non-invasive volume estimation. Experimental results validate the feasibility of this method, demonstrating reliable performance across a range of experimental conditions.

Key advantages of the proposed method include its simplicity, real-time applicability, and minimal equipment requirements, as it relies only on an audio recording device and basic signal processing. However, certain limitations of this method were identified. The method is not robust to significant noise across some frequencies and is also not particularly suitable for containers with highly irregular shapes.

Future work could focus on improving noise resistance and exploring additional applications, such as liquid diagnostics or acoustic-based quality control. Furthermore, the method could be used by creative bands seeking for new interesting instruments.

In summary, this work presents a simple and cost-effective solution for liquid volume estimation with broad potential for scientific, industrial, and creative applications.

Appendix A
Supplementary Materials

1. Source code repository:
https://github.com/Vondras1/Acoustic-Estimation-of-Volume.git
2. Live volume estimation video:
https://youtu.be/X6KOk0fO4MQ
3. Czech folk song Ovčáci, Čtveráci played on bottles:
https://www.youtube.com/shorts/pi8bOXCfTYk

References

[1] A. T. Pavel Ripka, *Modern Sensors Handbook*. ISTE Ltd, 2007.

[2] Z. Wei, L. Taifu, Z. Yuantao, Y. Jun, and H. Gang, "Research on liquid level detection based on acoustic field characteristic of the oil wells," in *2015 IEEE 10th Conference on Industrial Electronics and Applications (ICIEA)*, 2015, pp. 202–207.

[3] W. Zhou, J. Liu, and L. Gan, "Dynamic liquid level detection method based on resonant frequency difference for oil wells," *Turkish Journal of Electrical Engineering and Computer Sciences*, vol. 26, no. 6, p. Article 15, 2018. [Online]. Available: https://journals.tubitak.gov.tr/elektrik/vol26/iss6/15

[4] K. Pisanski, A. Groyecka-Bernard, and P. Sorokowski, "Human voice pitch measures are robust across a variety of speech recordings: methodological and theoretical implications," *Biological Letters*, vol. 17, no. 9, p. 20210356, September 2021.