Chapter 1: Vonguulian Education System

Pointing Out the Inadequacies

To Vonguul, one of the main problems we noticed with most things is the dependency upon financial resources that a business requires. When you make something into a business there are rules that must be adhered to, to claim the title of a successful business. One of those things being the need to secure “repeat customers,” the need to establish quotas and the lack of empathy for the collective. With the education system of today, humanity has advertised that children are the future and the importance of their development, but the practices applied do not address these issues as problems but as milestones and objectives to reach. Teachers are not compensated for the level of importance they serve in human society, nor are they provided with the tools or training to effectively tutor their students. Humanity has been funneling most of their resources into military advancements, entertainment and distractions designed for the masses, at least from a public relations standpoint.

Vonguul is not here to change those that do not want to change, we are here to provide another option to those that desire it. With Vonguulian Education, the focus is on the students learning the information that humanity has accumulated. It begins with the fundamentals of any topic and then we teach the applied variant of those fundamentals, so that every student is aware of the potential possibilities that the knowledge allows them. Too many times we have heard over the years, “when are we ever going to use this information?” Vonguul aims to answer that question before it is a thought in a student’s mind. It is understood that this would involve an entire curriculum change and/or revamping if we assumed the full position of correction. Vonguul does want to correct but our aim is targeted at creation. We desire to create a form of education that is not a carbon copy of the current public education. There is importance in both the physical and the metaphysical knowledge an individual can obtain, Vonguul will implement both.

A Sample of Vonguulian Math

Before we begin it must be understood that Language Arts are imperative to being able to comprehend the information provided. Though we do have some material for Language it is mostly for advanced techniques. So, with that you may have our disclaimer for Vonguulian Math: This information has been formed under the assumption that the student or teacher is fluent in the English Language and is only unaware of Math itself. Yes, we do desire to have it translated into other languages as well but as with the writing of this book we do not have access to that luxury.

Numbers are symbols that, for the sake of Math, hold quantitative value. These numbers are aligned to specific positions to help signify different quantitative values.

Number Symbols – In English

1 – One	4 – Four	7 – Seven	0 – Zero
2 – Two	5 – Five	8 – Eight	
3 – Three	6 – Six	9 – Nine	

Number Positions

7	6	5	4	3	2	1
1,	0	0	0,	0	0	0

Each vertical segment is referred to as columns.

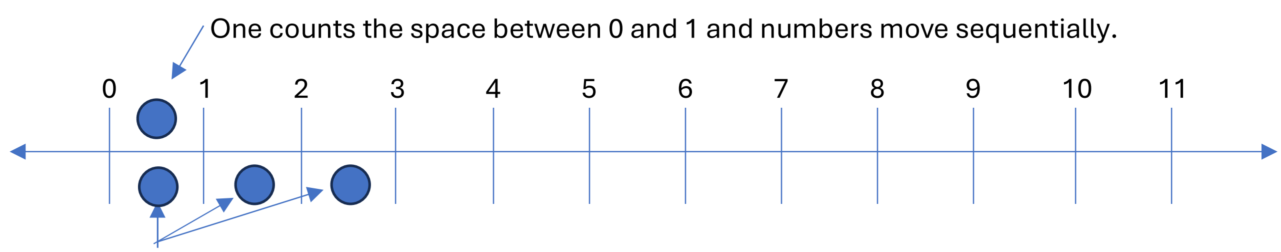
1 = Ones Column
2 = Tens Column
3 = Hundreds Column
4 = Thousands Column
5 = Ten Thousands Column
6 = Hundred Thousands Column
7 = Millions Column
8, 9, 10, etc. = So Forth and So On

The columns are used throughout math and are ever present. The information placed in this book is done in such a way as to start building a solid foundation and each of these blocks of information will assist in the building of an easily understood final product.

Addition

Addition is when we take two or more numbers and combine them to form a new number. What is important to take note of is the logic behind the "combining and forming" of these numbers.

To understand the logic, let us look at the equation 1 + 3 and we can use a number line:



So, if we were to add 1 space and 3 spaces, we would get 4 spaces. This is the logic behind Addition, every number occurs in a sequence and when adding we are combining numbers using the logic shown with the number line. It may also help to understand what a count is. A count is making something match the quantitative value of another thing, and it is just more widely accepted to use numbers. Instead of saying "spaces" like in the previous example we would just understand it as the "numbers" instead of "spaces."

It can be beneficial to look at the numbers as if they followed a particular order or pattern. Numbers in the "Ones" column start with 0 and are considered single digit numbers.

Observe the following:
0 → 1 → 2 → 3 → 4 → 5 → 6 → 7 → 8 → 9

From 0 to 9 you would remain in the "Ones" column, moving into the "Tens" column we would begin with 10. This is where positioning becomes apparent, and you can see a pattern start to form.

Observe the following:
10 → 11 → 12 → 13 → 14 → 15 → 16 → 17 → 18 → 19
20 → 21 → 22 → 23 → 24 → 25 → 26 → 27 → 28 → 29
30 → 31 → 32 → 33 → 34 → 35 → 36 → 37 → 38 → 39
40 → 41 → 42 → 43 → 44 → 45 → 46 → 47 → 48 → 49
50 → 51 → 52 → 53 → 54 → 55 → 56 → 57 → 58 → 59
60 → 61 → 62 → 63 → 64 → 65 → 66 → 67 → 68 → 69
70 → 71 → 72 → 73 → 74 → 75 → 76 → 77 → 78 → 79
80 → 81 → 82 → 83 → 84 → 85 → 86 → 87 → 88 → 89
90 → 91 → 92 → 93 → 94 → 95 → 96 → 97 → 98 → 99

From 10 to 99 you would have both the "Ones" column as well as the "Tens" column and it would follow the same pattern into the next column which is for the "Hundreds" place, and the column after that. Once the pattern is understood the rest becomes simpler to understand even if it can be a bit tedious.

We have our building blocks, now you can try putting them together to begin adding like a veteran mathematician. Now you may be able to add single digit numbers into larger single digit numbers but run into issues when attempting to add single digit numbers that equate to double digit numbers. It is important to remember to place each number into the proper column. It can help to view each "Number" in the "Ones" column as a "Number" with a 0 (zero) preceding it.

Observe the following:

00	01	02	03	04	05	06	07	08	09
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49

50 etc. ...

Notice how each number in the “Ones” column has a zero in the “Tens” column as a placeholder. Now let us try to take what we have learned here and apply it.

	Tens	Ones
	1	1
+	0	8
	1	9

	Tens	Ones
	1	
	1	6
+	0	7
	2	3

6 + 7 = 1 | 3

Tens	Ones

1 + 1 = 2

Subtraction

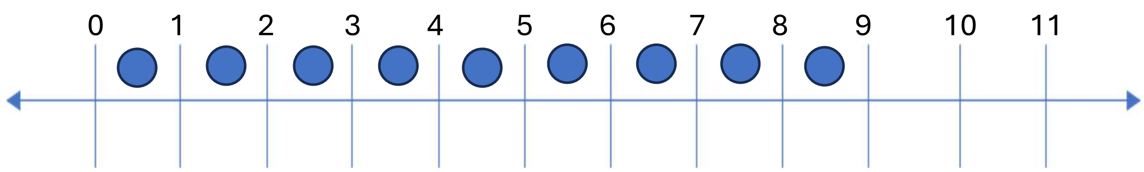
Now these principles can be used by anyone to perform the task of addition. Next, we are going to explain subtraction. In “Addition” we worked our way along the number line and with subtraction we will be going the other way but still along the number line.

Starting immediately with an example.

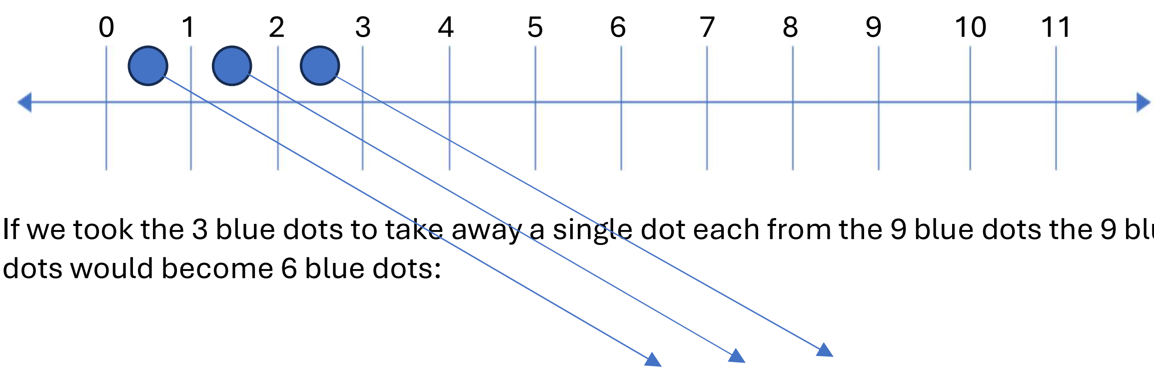
	Tens	Ones
	0	9
-	0	3
	0	6

The concept of subtraction is like addition but instead of going up in quantity, you will be going down in quantity. The given equation asks us to subtract 3 from 9. Let us visualize the equation with the number line.

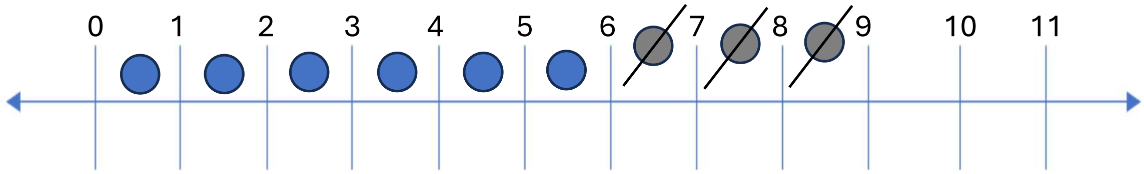
Here we have 9 blue dots:



Here we have 3 blue dots:

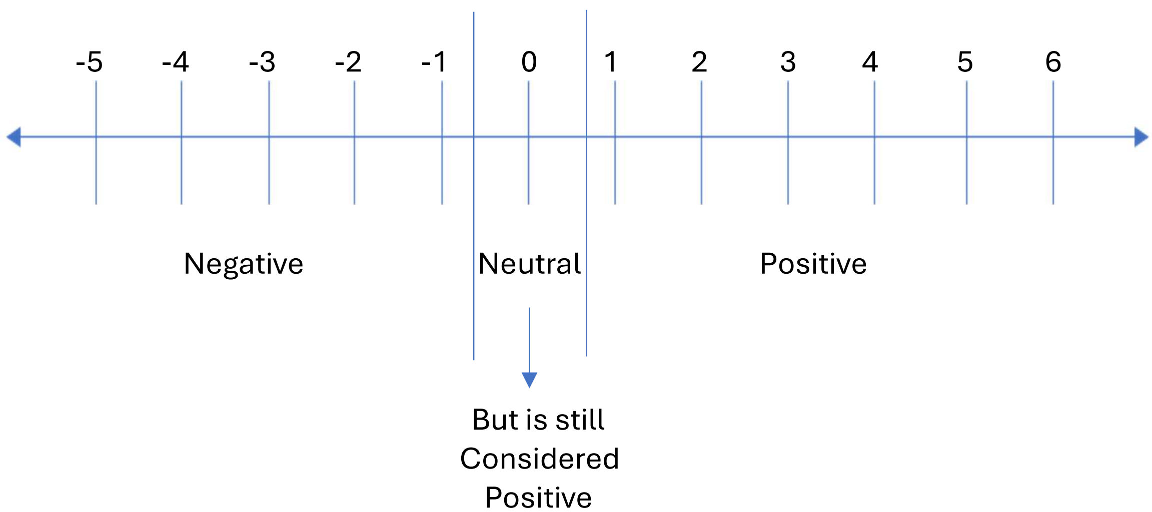


If we took the 3 blue dots to take away a single dot each from the 9 blue dots the 9 blue dots would become 6 blue dots:



Some have explained subtraction as taking away, which also fits the description of what is happening in the subtraction equation, you are taking 3 (dots) away from the 9 (dots). However similar subtraction may be to addition there are some key differences that must be discussed. First, let us go over what we already know so far. We know what numbers are, the logic behind the sequence of said numbers and the positions these numbers can be found in, as well as their pattern. What you need to be introduced to with the lesson regarding subtraction would be positive and negative numbers, how positive and negative numbers manifest in real life, the symbols for positive and negative, and the importance of the order of the equation.

Let us look at our trustworthy number line once again and let us see if there are any differences.

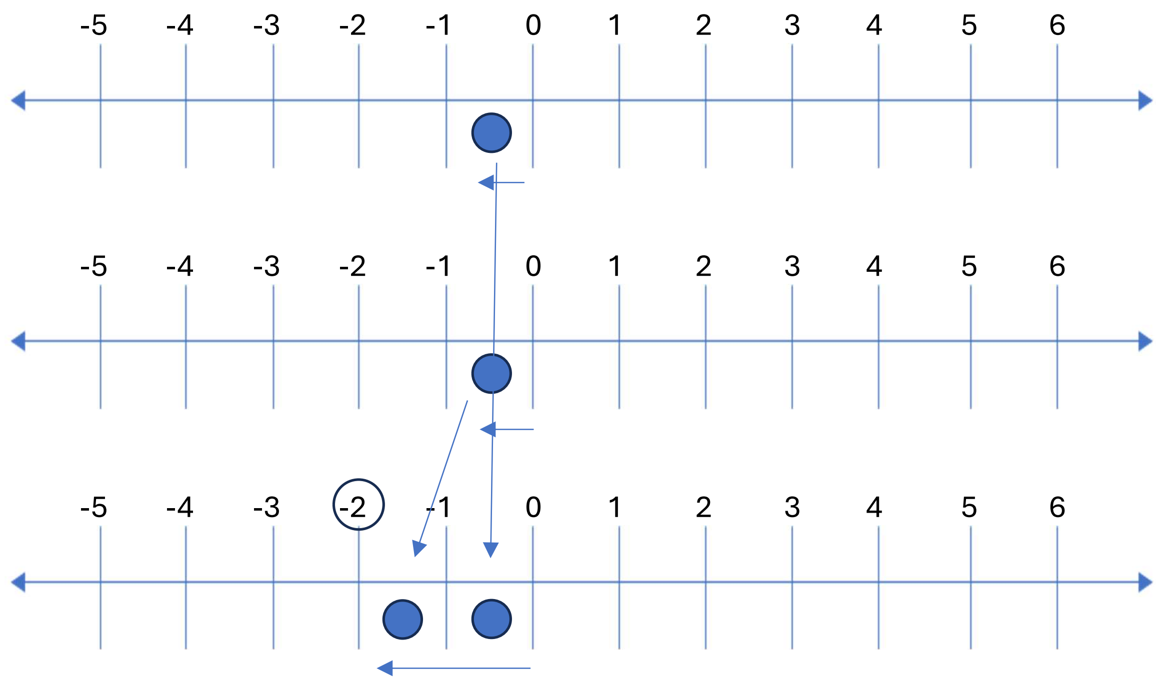


In math, understanding the types of numbers you are dealing with becomes more important the further you progress. A number being considered positive or negative tells us more details about the type of number we are dealing with. Positive and Negative are commonly referred to as "charge(s)" and is even relevant to other studies that utilize math mechanics, which will be more noticeable with Multiplication and helps to simplify some parts of Division. Either way it is another building block to our foundation for mathematics but for now, with addition and subtraction we would simply traverse the number as usual. Let us look at a few equations to see this in action:

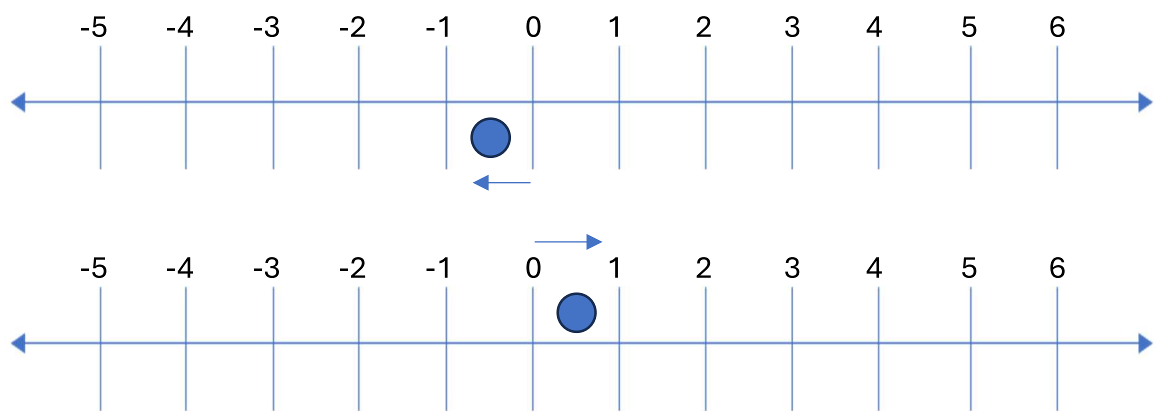
$$\begin{array}{r} -1 \\ + \quad -1 \\ \hline -2 \end{array} \qquad \begin{array}{r} +1 \\ + \quad -1 \\ \hline 0 \end{array} \qquad \begin{array}{r} +1 \\ + \quad +1 \\ \hline 2 \end{array} \qquad \begin{array}{r} -1 \\ - \quad -1 \\ \hline 0 \end{array}$$

Vonguulian Math is meant to always be logical.

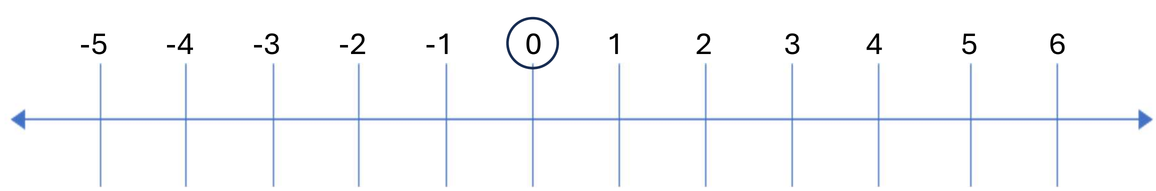
Starting with the first equation, we have 2 negative numbers, -1 and -1, being added together and we look at what this looks like on our number line we have the following:



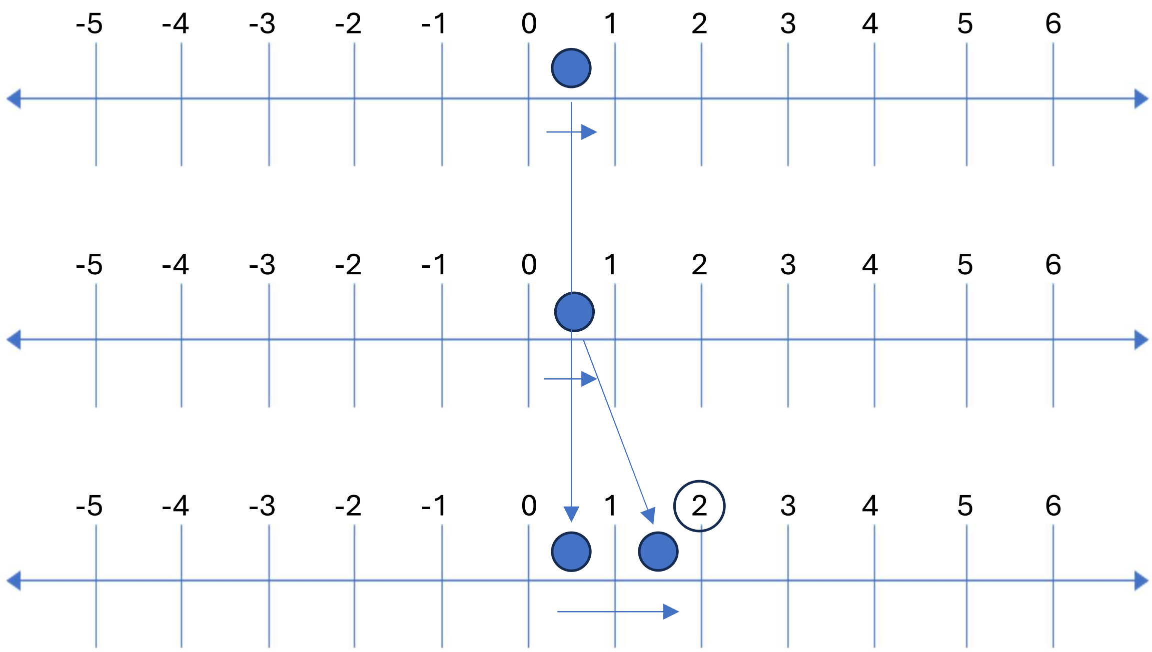
The second equation has one negative number, -1, one positive number, 1, and it wants us to add them together. Observe the following:



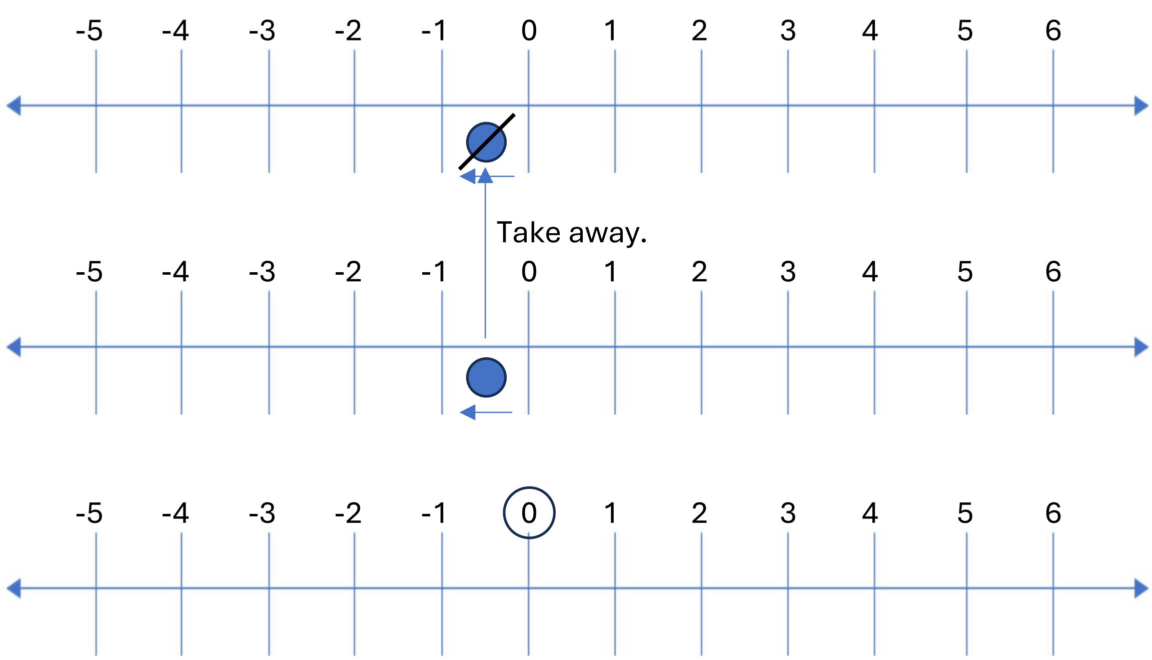
If we start from 0 go left once for the negative and right once for the positive, we return to 0.



The third equation is standard addition that we are familiar with, but we will still explain it as follows:



The fourth equation consists of two negative numbers (-1, and -1), and it wants us to take one away from the other using subtraction. This could become tricky for some, but we can still utilize the number line to understand it. Observe the following:



This last equation only seems more difficult because it is a different operation from the previous equations but, this also leads us into a useful pneumonic device that can be used to make the equation easier. Observe the following diagram:

++ = +
-- = +
+- = -
-+ = -

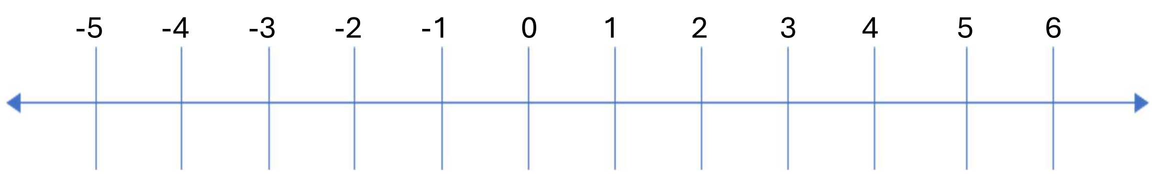
Now let us look at the equations again:

$$\begin{array}{rr} & -1 \\ + & -1 \\ \hline & -2 \end{array} \qquad \begin{array}{rr} & +1 \\ + & -1 \\ \hline & 0 \end{array} \qquad \begin{array}{rr} & +1 \\ + & +1 \\ \hline & 2 \end{array} \qquad \begin{array}{rr} & -1 \\ - & -1 \\ \hline & 0 \end{array}$$

What do we notice about the equations given the information we have from the diagram? That we can see similar patterns in +-, -+, ++, and --. Since we know what those patterns equate to let us apply them to our equations and see if the equations become easier to understand. Observe the following:

$$\begin{array}{rr} & -1 \\ - & 1 \\ \hline & -2 \end{array} \qquad \begin{array}{rr} & +1 \\ - & 1 \\ \hline & 0 \end{array} \qquad \begin{array}{rr} & +1 \\ + & 1 \\ \hline & 2 \end{array} \qquad \begin{array}{rr} & -1 \\ + & 1 \\ \hline & 0 \end{array}$$

This should make it easier to utilize the number line to solve the equations, so I want you to use the following number line to try them yourself.



Now, let us try a bit of Applied Math. In Applied Math we use Math the world around us, and Positive Numbers are the predominantly used numbers, but Negative Numbers play just as big a role in society as Positive Numbers. This is just a bit of Applied Math so we will not be going into all the terminologies associated with the Fundamentals, but you can see the effect of Negative Numbers when looking at terms such as, Debt, Balance, or Owe in Finance.

Math is a vast subject, but it does not have to be complicated. There are a multitude of Mathematical Symbols that have meanings, we will give you the symbols we have already been using and it will be updated as we progress throughout the lessons.

Current Mathematical Symbols

+	= denotes Addition	meaning to Go up the Number line
-	= denotes Subtraction	meaning to Go down the Number line
=	= denotes Equal	meaning to Make both sides value the same

Now, these are the visible symbols for the Math we have gone over as of this point. There are so many words and terminologies in Mathematics that it has its own language that needs to be understood and deciphered into a readable language. We do not teach the language completely in the Fundamentals, but we do make sure it is known by the students to prime their mind to receive this information later in the teaching process and we do that quite frequently. As we progress more symbols will be added to our Mathematical Symbols list which will be used in future lessons.

In math there is a specific order in which an equation can be broken down or simplified into multiple smaller equations or expressions. The process of simplifying an equation uses what we call "operations," to determine the "specific order" we mentioned earlier. The Order of Operations is as follows:

Parenthesis	- Solve all equations encased in "()" Parenthesis,	First.
Exponents	- Resolve or Simplify any Exponents,	Second.
Multiplication Or Division	- Solve any Multiplication OR Division expressions,	Third.
Addition Or Subtraction	- Solve any Addition OR Subtraction expressions,	Fourth.

*Notice third and fourth have an "OR" placed in the instructions. This means that the order in which they are to be solved is interchangeable with each other.
* There is a helpful pneumonic device to help you remember the Order of Operations. Either PEMDAS as a single word that is used like an acronym or the phrase, "Please Excuse My Dear Aunt Sally" with the first letter of each word denoting a correlation to PEMDAS.

For now, let us look at an equation and dissect it, using what we know already. Observe the following:

Math Symbols

- We know what each symbol instructs us to do.

1 + 2 + 3 – 4 – 1 + 5 =?

-This is a variable and is typically a letter but we use the question mark here

Overall, the equation consists of Math Symbols and Numbers

Numbers

- We know the value of each Number

This equation may appear quite strange, but we will be using it to see how a longer equation can be broken down into smaller expressions. Instead of one big equation let us break it down into 3 smaller expressions:

1 + 2 = 3
3 – 4 = -1
-1 + 5 = 4

Which would leave us with the following equation:

$3 - 1 + 4 \xrightarrow{\text{Further simplified to}} 2 + 4 \quad = 6$

But this also is not the only way to solve the equation, you could go from left to right, and do one expression at a time. Observe the following:

1 + 2 + 3 – 4 – 1 + 5 =?

1 + 2 = 3

3 + 3 = 6

6 – 4 = 2

2 – 1 = 1

1 + 5 = 6

Notice how the math symbols before the number are carried over and used in the next equation dictating the operation.

Now the only reason you can go left to right in this equation without an issue is because you only have addition and subtraction micro expressions, but if we were to introduce Multiplication or Exponents into the mix, we would have to follow the PEMDAS Order of Operations. When looking at addition and subtraction equations like this it might seem easy to confuse them with positive and negative charges, especially since they also share symbols, "+" and "- ", so let us try to make sure we do not have that issue in the future. Allow me to give this rule of thumb, all numbers are positively charged until indicated otherwise. These charges are evident in an equation just like the above diagram, but here is another example:

Negative Charge Symbol

Positive Charge Symbol normally omitted.

$^{-}6 \quad + \quad {}^{+}6 \quad =$

Number

Addition Symbol

Equal Symbol

If we were to split these numbers apart, they take on the charge of the symbol to its' left and would just be written as -6 + 6 =

Multiplication

Interesting enough Math gets easier before it gets complicated the more you learn, so go at your own pace and make sure you gain a full understanding.

Following along with the Order of Operations we will now be discussing Multiplication. This includes the logic behind Multiplication, how to use multiplication, and when to multiply. First let us add some symbols to our Math Symbols List specifically for Multiplication:

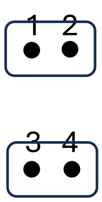
Mathematical Symbols List (continued)

x	= denotes Multiplication	meaning to Multiply to go up the Number Line
●	= denotes Multiplication	meaning to Multiply to go up the Number Line
*	= denotes Multiplication	meaning to Multiply to go up the Number Line
()	= denotes Multiplication	meaning to Multiply to go up the Number Line

Let us begin with the logic behind multiplication. The logic is to go up the Number Line using a different process we call multiplying. Observe the following:

$$2 \quad x \quad 2 \quad =?$$

When reading this equation, it can be read as "2 multiplied by 2 equals what?" but it helps to read it another way, "2 times 2 equals what?". Neither is incorrect, it just helps explain what is happening.



This is one instance of "2" particularly, 2 black dots. To multiply this instance by 2 we would add another instance of 2 black dots and counting the total number of dots we get 4 black dots.

2 is a placeholder variable for something that is quantified as 2 in value. It could be 2 black dots, 2 cars, 2 jobs, 2 anything. So, Multiplication is trying to figure out the total quantity of the "placeholder variable"(2) in "how many"(2) "times"(x). Let us label the previous equation:

"Placeholder Variable" "How Many Times?"

$$2 \quad x \quad 2 \quad =?$$ Equates to what quantified number.

Instance of 2 "things" 2 Times

As we have stated before, Math is its own language, and we would need to decipher the information that that language is trying to communicate. So, when we read the equation with an understanding of the fundamentals, we know that 2 instances of 2 things totals an amount equal to 4 of those things. Or:

$$2 \quad x \quad 2 \quad =4$$

Let us try another equation:

$$2 \quad x \quad 4 \quad =?$$

Using the example of dots again:

1 2 3 4

[• •] X 4 = [• •] [• •]

One instance of 2 4 times equates to 8 black dots.

[• •] [• •]

5 6 7 8

Or if you are not a fan of dots:

$2 \text{ x } 4 = \underset{1}{2} + \underset{2}{2} + \underset{3}{2} + \underset{4}{2}$

Even $24 \text{ x } 3 = \underset{1}{24} + \underset{2}{24} + \underset{3}{24}$

It helps to think of a multiplication expression as, one “Number,” “Other Number” “Times.”

6 x 4 =?

So, One Number, 6, Other Number, 4, times

= 6 + 6 + 6 + 6

= 12 + 12

= 24

As you can see in the multiple examples, Multiplication is just another way of adding to or increasing the value of something. This is another reason you would need to resolve multiplication expressions before addition and subtraction. Multiplication combined with charges is another point that we should also take the time to mention. To do this best, we will use a modified equation. Observe the following:

Modified Equations	Key
+ x + = +	+ = Positively Charged Number
- x - = +	- = Negatively Charged Number
- x + = -	x = Multiplication Symbol
+ x - = -	= = Equates to

Looking at the Modified Equations we can spot a pattern:
- If 2 charges are the same the result will be a positive number
- If 2 charges are different the result will be a negative number

To show those Equations with the Number and the Charges it will look like the following:

+1 x +1 = +1
-1 x -1 = +1
-1 x +1 = -1
+1 x -1 = -1

What is important to take note of here, is how different charges interact with another charge.

Typically, at this point in Vonguulian Math I would experience some amounts of skepticism about Vonguul's methods or approach. To this, I would like to remind everyone that Vonguulian Education is divided into 2 parts, Fundamentals and Applied. The Fundamentals are there as a solid foundation for all Math, which also allows the freedom for an individual to create their own path or follow a well-known path and remain accurate. As Vonguulians we understand that everything is information, so for us to teach others, we must do it with that understanding as well, because of this, we present the student with all the tools and materials we had and have available and show them how we made our conclusions.

So, yes there are methods of processing math that are considered efficient to some, but we prefer to allow the student the opportunity to discover what works best for them. As well as being capable of comparing and competing with the status quo. This is the Vonguulian Way, and we have our reasons which have been found to be justifiable.

Division

Following Multiplication we have Division. As usual we will be discussing the logic behind our topic, how to use division, and when to divide.

Mathematical Symbols List (continued)

÷	= denotes Division	meaning to Divide to go down the Number Line
/	= denotes Division	meaning to Divide to go down the Number Line
$n \overline{)\, n}$	= denotes Division	meaning to Divide to go down the Number Line
$\frac{n}{n}$	= denotes Division	meaning to Divide to go down the Number Line

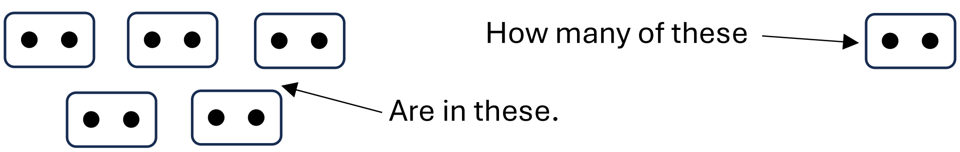
The “÷” symbol is the most common symbol used to denote Division, the “/” symbol is mostly utilized in computer sciences, “$n\overline{)\,n}$” is more of a format for an equation such as $2\overline{)\,4}$, and finally we have “$\frac{n}{n}$” which is more commonly known by the term “fraction” but also works as a format that denotes Division. *Also take notice of how the “n’s” in that format resembles the first symbol, “÷.”

Now that we know what symbols denote to Division let us look at the logic behind it. You may think that Division should be like Subtraction the same way Multiplication was like Addition, but it would be more accurate to say that Division is the opposite of Multiplication. The point of Division is to find out “How many” “Instances” of a set “Number,” can be found in “Another Number.” To demonstrate this let us look at an expression that tells us to use Division:

$$10 \div 2 = ?$$

In English, this is read as follows, “10 divided by 2 equates to what?”

To solve this equation, we need to find out how many instances of “2 things” can be found in “10 of the same things.” Using the dot example let us visualize this equation:



The answer would be 5 because we can count 5 instances of 2 dots within the 10 dots. So very simply 10 ÷ 2 = 5, and this is the logic behind Division, “How many instances of 1 number can we find in another number”. Now, even though we have introduced you to the logic behind Division we have only Divided using one format or used only one of the Division symbols. Normally, this information would be expanded upon in the Applied portion of the educational system, when we discuss formulas and formats, but we also want to make sure you can identify a Division expression when spotted. Let us look at an example:

We are already familiar with

9 ÷ 3 =?

Let us use a different format for this same expression,

$3\overline{)\,9}$

If we were to use our previous method of dividing, the dot method, we would get an answer of 9 divided by 3 equals 3, and the logic doesn’t change however to have to use the dot method for larger number can be space consuming and tedious, so this new format allows us to continue using smaller numbers to divide larger numbers. For example:

$36 \div 3 = ?$

It can be a task to draw 36 dots and circle 3 dots until we have a resulting number of instances. The new format helps to address this issue.

$$\begin{array}{r} ? \\ 3\overline{)36} \end{array}$$

Now all we must do is find out how many times 3 goes into "3" and "6" separately to get our definitive answer. 3 goes into 3, 1 time, and into 6, 2 times. So, our answer should be "12". This is a quick example of utilizing formats to your advantage, and inside of the Applied sector of the curriculum we will go into depth on how to use different formats to deduce expressions of various levels of difficulty. We eventually find ourselves looking at Fractions which can be converted into Decimals and now we start to see how vast Math can be. A fraction is another format for an equation, and it is also equal to a Decimal Number which is another type of number, so we can infer that these equations in math can have more than one correct answer.

Fractions and Decimals

We are still within the Section for Division, but we must go over Fractions and Decimals in greater detail to avoid confusion at the finish line. To explain, fractions are portions of "Whole Numbers." All the Numbers we have discussed or used in the past would be considered "Whole Numbers." A fraction is a portion of a Whole Number, and decimals are the same as fractions just expressed differently. Let us look at some examples:

Whole Number	=	6
6 as a Fraction	=	$\frac{6}{1}$
6 as a Decimal	=	6.0

When we are talking about Fractions, there are 3 types of them:

- Regular Fractions

- Improper Fractions

- Mixed Fractions

And every fraction has a decimal equivalent.

Starting with Regular Fractions let us look at the anatomy of the following fraction:

Numerator

$$\frac{2}{3}$$

Can be read as "2 over 3" or "two-thirds" In a proper fraction, the numerator is always less than the denominator.

Denominator

What is being communicated with this fraction is that it has the value of 2 out of 3 parts of a single Whole Number.

For example, let us say our Whole Number is 6 and you were tasked with finding two-thirds of its value, the answer would be 4 because if you broke 6 into 3 equal parts each part would have 2 pieces in them. 2 of those parts would equate to 4 pieces in total.

Next, we will look at Improper Fractions, and its anatomy:

*Notice that the numerator is greater in value than the denominator, but it is still read the same. Eleven-thirds or eleven over three.

$$\frac{11}{3}$$

This Improper Fraction is attempting to communicate that it has the value of 11 portions of the 3 required to make a Whole Number. Since there are more than enough portions to create a whole it is considered improper, and it is typically requested to convert it into a Mixed Number Fraction. If we did the mathematic operation denoted, which is Division still, we would be left with a Mixed Number Fraction.

That Mixed Number Fraction being:

*Notice the addition of a Whole Number and the numerator and denominator resembling a regular fraction.

Numerator

Whole Number $\rightarrow 3\frac{2}{3}$

Denominator

This Mixed Number Fraction is trying to communicate a similar result to the Improper Fraction it was derived from. The difference is that we simplify or resolve the Improper Fraction to see how many "Wholes" can be counted with the provided portions and in this one we see that there are 3 Wholes and 2 out of 3 portions that would equate to another whole. The way this type of fraction is read is, "Three and two-thirds" or "Three and two over three.” The “and” is particularly important to notice as well because it translates into what we call a “Decimal Point.” This Decimal Point is what makes a Number a Decimal. If we were to convert the above Mixed Number Fraction into a Decimal it would be 3.66 repeating. “Repeating” is a new term that is to also be associated with Decimal Numbers, and it is termed like this because of what happens during the resolving part of the expression. Let us look at just a Regular Fraction and attempt to resolve it:

$$\frac{2}{3} = 3\overline{)2}$$

It is not possible for “3” to go into “2” any number of times, but if we wanted to solve this equation, we would have to add a Decimal Point.

What times 3
Equal 0? → 0.

$$3\overline{)2.0}$$

We add a 0, 2 becomes 20, and our new question becomes, how many times does “3” go into “20” without going over. Which the answer would be 6 times, and 3 times 6 equates to 18.

$$\begin{array}{r} 0.6 \\ 3\overline{)2.0} \\ -18 \\ \hline 2 \end{array}$$

From here we would continue to add zeroes and we would continue to get a remainder of “2” as it is an infinite loop. Every 0 we add after the Decimal Point denotes to the placement of the corresponding value after the Decimal Point above it. When we encounter loops like this instead of continuously writing “6” we would just explain that it is repeating, and the process of Division used is referred to as “Long Division”.

Returning to Division

So, let us recap Fractions and Decimals before returning to finish our section on Division. There are 3 types of Fractions, Regular Fractions, Improper Fractions, and Mixed Number Fractions. Each of these fractions are communicating similar information that can be converted into an equation that can result in a Decimal Number. Decimal Numbers are denoted by having a Decimal Point, and because of this we know that in Math, equations can have multiple correct answers which is why it is important to know what we are solving for.

Great, now try the next few questions on your own:

1. 9/3
2. 12÷4
3. $\frac{6}{12}$
4. $8\overline{)12}$

I want you to translate them into the English language, write them as a different variant of the same expression, and finally solve the expression using Division. It is ok to get these answers wrong, what is important is that you try. We will go over each equation separately and breakdown how to solve it.

1. With the first equation we have, 9/3.
    a. In English this reads as, "Nine divided by three."
        i. The "/" directly translates to "divided by."
    b. A different variant of this expression could be, $\frac{9}{3}$, 9÷3, $3\overline{)9}$, or even simply "3".
    c. Using Division to resolve this equation would leave us with "3" as the answer.
2. Next, we have 12 ÷ 4.
    a. In English this reads like the previous question as, "Twelve divided by four."
        i. The "÷" symbol also directly translates to "divided by."
    b. The different variants look like, $\frac{12}{4}$, 12/4, $4\overline{)12}$, or "3".
    c. Using Division to resolve we get "3" again as the answer.
3. Third, we see $\frac{6}{12}$.
    a. In English this can be read as, "Six over twelve" or "Six-twelfths."
        i. Notice how we say the number of the numerator, but we add the 'ths' to the numerator, this is because it denotes to a Decimal Number.
    b. Variants appear as, 6/12, 6 ÷ 12, $12\overline{)6}$, or the Decimal Number "0.5".
    c. Using Division to resolve we get "0.5."
4. Finally, we have $8\overline{)12}$.
    a. In English this reads as "Twelve divided by eight"
        i. Unlike in previous formats notice how this equation appears to be read backwards, starting the right and going to the left.

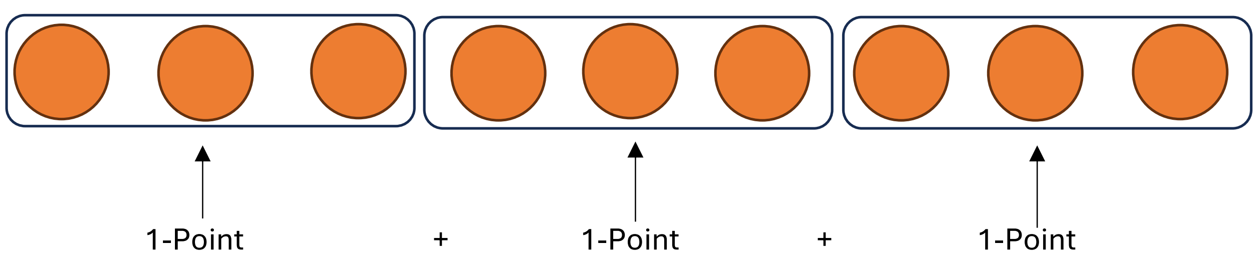
b. Variants of this expression appear as, 12/8, the Improper Fraction $\frac{12}{8}$, the Mixed Number Fraction(simplified) $1\frac{1}{2}$, the Mixed Number Fraction(not simplified) $1\frac{4}{8}$, or the Decimal Number 1.5.

c. Using Division to solve this equation we get “1.5.”

Now some of you may be asking “How” and this section is for you:

9/3

Using an example let us visualize this equation.
Let us play a game.
The objective of the game is to get as many points as possible.
For every 3 shots you take you get 1 point.
You have a total of 9 chances to take a shot.
If you take every shot, how many points would you have?



1 + 1 + 1 = 3-Points

$12 \div 4$

This would be solved the same way as the previous example.
It would be a total of 12 shots and 4 shots get you 1 point.
The Answer to both equations is “3”.

$\frac{6}{12}$

There are multiple ways to solve this expression as shown by the multiple answers given above. You can reformat and solve or simplify and solve by division. I recommend using the reformat to long division and solve option for these types of fraction expressions.

The long division format is shown below, let us go step by step to find out how to best utilize this format.

Question 1: How many times does 12 go into 6?
Answer 1: 0 times

	x 0.5	We need more numbers so we must add a Decimal Point.
	$12\overline{)6.0}$	Now that we have a Decimal Point, we can now add a 0.
12 x 0 = 0	-0	We bring that added 0 down to make the 6 a 60.
	6 0	12 times 5 equals 60, so 12 goes into 60, 5 times.
	- 6 0	Add our 5 up top and subtract 60 from 60
	0	The answer to this equation is 0.5.

Question 2: How many times does 12 go into 60?
Answer 2: 5 times
Question 3: What is 12 times 5?
Answer 3: 60

Our final question was already in long division format, so all we need to do is use what we just learned to solve the equation.

Question 1: How many times does 8 go into 12?
Answer 1: 1 time.

	1	
	$8\overline{)12}$	and
over	- 8	
	4	

From here we can choose to either continue to solve for a decimal number or we could write it as is, which would be, $1\frac{4}{8}$ and could be further simplified to $1\frac{1}{2}$. For sake of completion let us continue to solve for a Decimal Number.

$$
\begin{array}{r l}
1.5 & \text{We add our Decimal Point.} \\
8\overline{)12.0} & \text{Add our zero.} \\
-\ 8 & \text{Bring it down to turn the 4 into a 40.} \\
\hline
4\,0 & \text{And ask our next question.} \\
-\ 4\,0 & \text{Subtract 40 from 40} \\
\hline
0 & \text{When we get a 0 down here, our solving is complete.}
\end{array}
$$

Next Question: How many times can 8 go into 40?
Next Answer: 5 times.
Next Question 2: What is 8 times 5?
Next Answer 2: 40.

Our Decimal Number answer for this equation is "1.5."

Now, we have a better understanding of Division. That includes not the logic behind it but also the logic behind fractions and decimals. We know when to use division and we can now progress into the final stages of this portion of the Math Fundamentals. We will not be able to go into all of them in this book without it turning into a book that's "for educational purposes only." We want you to see a comprehensive example of what Vonguul offers, without trying to fit the universe into a booklet.

Exponents and Parenthesis

Exponents are quite simple to understand at this point in our journey. Exponents are like special equations, which ask you to multiply in a distinct way. First, let us look at the anatomy of an Exponent:

When we see an Exponent, it is coupled with a Number or Variable representing a Number, and it is raised slightly but noticeably higher than the Number or Variable.

This is the exponent.

$$n^2$$

What this expression is trying to communicate to us is the following equation:

$$n \times n$$

If the exponent were a 4 instead of a 2 it would communicate this instead:

$$n \times n \times n \times n$$

Whatever value the exponent has, is the number of times it wants you to multiply it by itself. This can be useful for expressing large numbers and advanced computing calculations. Before we go into Parenthesis, let us try a few Exponent problems:

1. 2^2
2. 4^3
3. 3^4

I am going to show how each expression is to be read:

1. 2 to the second power
2. 4 to the third power
3. 3 to the fourth power

Notice that we call the Exponent a "nth power."

What each expression translates to as an equation:

1. 2 x 2 =?
2. 4 x 4 x 4 =?
3. 3 x 3 x 3 x 3 =?

Now the answer for each equation:

1. 4
2. 64
3. 81

Did you get the right answers?

Now that we have explained Exponents and how they work, we can finally tackle Parenthesis. Now you may recall from an earlier section that we saw Parenthesis in a bit of action, we at least know that they can denote multiplication but more importantly it is the First Order of Operations. Anything we see inside of Parenthesis' we must give it our attention first. So, if we see Parenthesis' Inside of Parenthesis' we solve the innermost Parenthesis' before continuing to the subsequent Parenthesis. I say this now because it will be seen in more complex mathematic equations especially since the sections after this are Algebra and Geometry, in the Vonguulian Curriculum. Let us look at an equation that incorporates what we have learned in this Foundational Block of the Vonguulian Curriculum.

$$(4 \text{ x } 6 + 8) - (32 / 4^2) + 12 =?$$

Let us refer to our Order of Operations for instructions to solve this equation.

Parenthesis	- Solve all equations encased in "()" Parenthesis,	First.
Exponents	- Resolve or Simplify any Exponents,	Second.
Multiplication Or Division	- Solve any Multiplication OR Division expressions,	Third.
Addition Or Subtraction	- Solve any Addition OR Subtraction expressions,	Fourth.

$$(4 \text{ x } 6 + 8) - (32 / 4^2) + 12 =?$$
$$(4 \text{ x } 6 + 8) - (32 / (4 \text{ x } 4)) + 12 =?$$
$$(4 \text{ x } 6 + 8) - (32 / 16) + 12 =?$$
$$(24 + 8) - (2) + 12 =?$$
$$32 - 2 + 12 =?$$
$$30 + 12 =?$$
$$42 =?$$

From top to bottom we followed the Order of Operations and have come to a result of "42". We had two sets of Parenthesis and one of them had an Exponent, so we resolved the Exponent. Afterwards we solved the Multiplication and Division problems within Parenthesis. Completed the Addition inside the last Parenthesis and resolved the simple equation to a resulting 42. This concludes the Math portion of the Book of Vonguul.