Econometrics, second half, first problem set. Due 1 April, in class.

1. For practice using fminunc, estimate a Poisson model by ML using the 10 independent data points

y	0	0	0	1	1	1	2	2	2	3
x	-1	-1	1	0	-1	-1	1	1	2	2

For the Poisson model, the density $f_Y(y|x) = \frac{\exp(-\lambda)\lambda^y}{y!}$, $y = 0, 1, 2, ...$ To make the model depend on conditioning variables, use the parameterization $\lambda(x) = \exp(-\theta_1 - \theta_2 x)$. The example EstimatePoisson.m, in the notes, should be helpful

   (a) create a data file that contains these observations

   (b) find the log-likelihood function

   (c) write a Matlab function that computes the log-likelihood function, using the form obj=loglik(theta, data)

   (d) use fminunc to find the ML estimator. You need to use an anonymous function for this.

   (e) optional, but worth thinking about: can you propose a GMM estimator for this model? Hint: what is $E(Y|X = x)$ and/or $V(Y|X = x)$ for this model?

2. Consider the model

$$y_t = \alpha + \rho_1 y_{t-1} + \rho_2 y_{t-2} + \beta x_t + \epsilon_t$$

where ϵ_t is a $N(0, 1)$ white noise error. This is an autogressive model of order 2 (AR2) model, with an additional exogenous regressor. Suppose that data is generated from the AR2 model, but the econometrician mistakenly decides to estimate an AR1 model, $y_t = \alpha + \rho_1 y_{t-1} + \beta x_t + v_t$. This is a case of omitted relevant variables.

   (a) show that weak exogeneity fails for the AR1 model.

   (b) Consider IV estimation of the AR1 model, using lags of x_t as instruments. Is this a consistent estimator?

   (c) simulate data from the correct AR2 model, using $\alpha = 0$, $\rho_1 = 0.5$, $\rho_2 = 0.4$, $\beta = 2$, and $x_t \sim IIN(0, 1)$. Use a sample size of 30 observations.

      i. estimate the incorrectly specified AR1 model by OLS

      ii. estimate the correctly specified AR2 model by OLS

      iii. implement your proposed IV estimator of the AR1 model

      iv. embed the simulations and estimations in a loop, to do a Monte Carlo study using 1000 replications. Provide histograms for the distribtions of the estimators of the parameter ρ_1 for the 3 estimators.

(d) discuss all results thoroughly, focussing on bias and standard errors of the estimators of the autoregressive parameters

3. For the IV estimator of the AR1 model of the previous exercise, present the IV estimator in the form of a GMM estimator.

   (a) Define precisely what are the moment conditions $m_n(\theta)$ and the weight matrix W_n that define the estimator.

   (b) Give the form of the asymptotic distribution of the estimator, and explain clearly how to estimate all terms in the asymptotic distribution

   (c) Using the same simulated data as in 2ciii above, provide the GMM estimates and the estimated standard errors.

4. Estimate the investment equation of the Klein Model 1 using GMM. Be sure to get the latest version of the notes from the class web page, then see section 10.10 for the equation and see the examples at the end of section 14.10 for hints.

5. Verify the missing steps needed to show that $n \cdot m(\hat{\theta})'\hat{\Omega}^{-1}m(\hat{\theta})$ has a $\chi^2(g-K)$ distribution. That is, show that the big ugly matrix is idempotent and has trace equal to $g-K$.

6. The GMM estimator with an arbitrary weight matrix has the asymptotic distribution

$$\sqrt{n}\left(\hat{\theta}-\theta^0\right) \stackrel{d}{\to} N\left[0,\left(D_\infty W_\infty D_\infty'\right)^{-1} D_\infty W_\infty \Omega_\infty W_\infty D_\infty' \left(D_\infty W_\infty D_\infty'\right)^{-1}\right]$$

   Supposing that you compute a GMM estimator using an arbitrary weight matrix, so that this result applies. Carefully explain how you could test the hypothesis $H_0 : R\theta^0 = r$ versus $H_A : R\theta^0 \neq r$, where R is a given $q \times k$ matrix, and r is a given $q \times 1$ vector. I suggest that you use a Wald test. Explain exactly what is the test statistic, and how to compute every quantity that appears in the statistic.