COMP 3761 Assignment 2

Due Date: Wednesday **May 21, 2014** at 6:30pm

1. Given the following recurrence relations:

   1.1. $x(n) = 3\,x(n-1)$ for $n > 1$, $x(1) = 4$

   1.2. $y(n) = y(n/2) + n$ for $n > 1$, $y(1) = 1$, assume $n = 2^k$, where k is any positive integer.

   (a) Mathematically solve each of the above recurrence relations. Express each function as an exact function of n.   [4]

   (b) For each of the given recurrence relations, implement a recursive function in Java to compute the value of $x(n)$ or $y(n)$, for any integer $n \geq 1$.

      Write a test program that reads a positive integer n from the standard input, and outputs the corresponding values of $x(n)$ and $y(n)$.   [4]

   (c) Theoretically analyse the time efficiency for each of the recursive algorithms you implemented in (b).   [4]

      - what is the basic operation?
      - Set up a recurrence relation for the function's basic operation count and solve it. Express the number of basic operations as an exact function of input size n.
      - What is the worst-case time efficiency class of the algorithm?

2. Consider the following recursive functions:

   Suppose $a[0..n-1]$ is an array with $n \geq 1$ integers. $a[l..r]$ represents a sub-array with indexes starting at l and ending at r, where $0 \leq l \leq r \leq n-1$, initially $l = 0, \quad r = n-1$.

   2.1.
   ```
   int foo(int n, int a[l..r])
   {
       if( n == 1 )
           return a[l];

       int result1 = foo(n-1, a[l..r-1] );
       int result2 = foo(n-1, a[l+1..r] );

       return result1 + result2;
   }
   ```

   2.2.
   ```
   int bar( int n, int a[l..r] )
   {
       if( n == 1 )
           return 1;

       int mid = n/2;
   ```

```
        int result = bar(n/2, a[l..mid] );
        result += bar(n/2, a[mid+1..r] );

        for( int i = 0; i < n; ++i )
            result += a[i];

        return result;
    }
```

For each of the given recursive functions,

(a) Set up a recurrence relation for the number of times the algorithm's basic operation is executed for the input size n.   [2]

(b) Mathematically solve the recurrence relations for the basic operation count. Express the number of basic operations as an exact formula of n.   [4]

(c) What is the time efficiency class of each algorithm? You must show your work to support your answer.   [2]