Assignment 6 solution

1.a

P(X2)P(X3)P(X4)P(X5|X2,X3)P(X6|X3,X4)P(X7|X5)P(X8|X3,X5,X6)P(X9|X5,X7,X8)

1.b

i) 9! – 1 = 362879

ii) 1 + 2+ 3 + 4 * 2 * 3 + 5 * 3 * 4 + 6 * 5 + 7 * 3 * 5 * 6 + 8 * 5 * 7 * 8= 2990

1.c

i) T ii) F iii) T iv) F v) F

2. a

Eliminate D, C, A

B	P(B)
T	0.1*0.4 + 0.8*0.6 = 0.52
F	0.9*0.4 + 0.2*0.6 = 0.48

2.b

Eliminate D, B

C	P(C|A=T)	P(C|A=T) after normalization
T	0.4*(0.7*0.1 + 0.4*0.9) = 0.172	0.172/0.4 = 0.43
F	0.4*(0.3*0.1 + 0.6*0.9) = 0.228	0.228/0.4 = 0.57

2.c

Eliminate B, A

A	B	P(A,B|C=T,D=F)	P(A,B|C=T,D=F) after normalization
T	T	0.4*0.1*0.7*0.18=0.00504	0.00504/0.10008 = 0.051
T	F	0.4*0.9*0.4*0.18=0.02592	0.02592/0.10008 = 0.259
F	T	0.6*0.8*0.7*0.18=0.06048	0.06048/0.10008 = 0.604
F	F	0.6*0.2*0.4*0.18=0.00864	0.00864/0.10008 = 0.086

3. a

Y	P(Y) = P(X) * P(Y|X)
T	0.4*0.2+0.6*0.7=0.5
F	0.4*0.8+0.6*0.3=0.5

Action	EU
a	0.5 * 800 + 0.5 * 200 = 500
~a	0.5 * 400 + 0.5 * 1000 = 700

MEU = 700, take action ~a.

3.b

1) Z = T

Y	P(Y|Z=T) = P(X) * P(Y|X) * P(Z=T|Y) = P(Y) * P(Z=T|Y)	P(Y|Z=T) after normalization
T	0.5*0.9=0.45	0.45/0.55 = 0.82
F	0.5*0.2=0.10	0.10/0.55 = 0.18

Action	EU
a	0.82 * 800 + 0.18 * 200 = 692
~a	0.82 * 400 + 0.18 * 1000 = 508

MEU|(Z=T) = 692

2) Z = F

Y	P(Y|Z=F) = P(X) * P(Y|X) * P(Z=F|Y) = P(Y) * P(Z=F|Y)	P(Y|Z=F) after normalization
T	0.5*0.1=0.05	0.05/0.45 = 0.11
F	0.5*0.8=0.40	0.40/0.45 = 0.89

Action	EU
a	0.11 * 800 + 0.89 * 200 = 266
~a	0.11 * 400 + 0.89 * 1000 = 934

MEU|(Z=F) = 934

3) VOI(Z)

Z	P(Z) = P(Y)P(Z|Y)
T	0.5*0.9 + 0.5*0.2 = 0.55
F	0.5*0.1 + 0.5*0.8 = 0.45

VOI(Z) = P(Z=T)*(MEU|Z=T) + P(Z=F)*(MEU|Z=F) – MEU

= 0.55*692 + 0.45*934 - 700

= 100.9

3.c

1) X = T

Action	EU
a	0.2 * 800 + 0.8 * 200 = 320
~a	0.2 * 400 + 0.8 * 1000 = 880

MEU|(X=T) = 880

2) X = F

Action	EU
a	0.7 * 800 + 0.3 * 200 = 620
~a	0.7 * 400 + 0.3 * 1000 = 580

MEU|(X=F) = 620

3) VOI(x)

VOI(X) = P(X=T)*(MEU|X=T) + P(X=F)*(MEU|X=F) – MEU

= 0.4*880 + 0.6*620 – 700

= 24

3.d

1) X = T, Z = T

Y	P(Y|X=T, Z=T) = P(X=T) * P(Y|X=T) * P(Y|Z=T)	P(Y|X=T, Z=T) after normalization
T	0.4*0.2*0.9 = 0.072	0.072/0.136 = 0.53
F	0.4*0.8*0.2 = 0.064	0.064/0.136 = 0.47

Action	EU
a	0.53 * 800 + 0.47 * 200 = 518
~a	0.53 * 400 + 0.47 * 1000 = 682

MEU|(X=T, Z=T) = 682

2) X = F, Z = T

Y	P(Y|X=F, Z=T) = P(X=F) * P(Y|X=F) * P(Y|Z=T)	P(Y|X=F, Z=T) after normalization
T	0.6*0.7*0.9 = 0.378	0.378/0.414 = 0.91
F	0.6*0.3*0.2 = 0.036	0.036/0.414 = 0.09

Action	EU
a	0.91 * 800 + 0.09 * 200 = 746
~a	0.91 * 400 + 0.09 * 1000 = 454

MEU|(X=F, Z=T) = 746

3) VOI(X|Z=T)

X	P(X|Z=T) = P(X) * P(Y|X) * P(Z=T|Y)	P(X|Z=T) after normalization
T	0.4*0.2*0.9 + 0.4*0.8*0.2 = 0.136	0.136/0.55 = 0.25
F	0.6*0.7*0.9 + 0.6*0.3*0.2 = 0.414	0.414/0.55 = 0.75

VOI(X|Z=T) = P(X=T|Z=T)*(MEU|X=T,Z=T) + P(X=F|Z=T)*(MEU|X=F,Z=T) – (MEU|Z=T)

= 0.25*682 + 0.75*746 – 692

= 38