

Assignment No. 5

Q.1

$$\begin{aligned} \text{a. } & \forall x P(x) \Rightarrow Q(x) \\ & \forall x \neg P(x) \vee Q(x) \\ & \neg P(x) \vee Q(x) \end{aligned}$$

$$\begin{aligned} \text{b. } & \forall x \forall y P(x, y) \Rightarrow Q(x) \\ & \forall x \forall y \neg P(x, y) \vee Q(x) \\ & \neg P(x, y) \vee Q(x) \end{aligned}$$

$$\begin{aligned} \text{c. } & \exists x P(x) \wedge Q(x) \\ & P(A) \wedge Q(A) \end{aligned}$$

$$\begin{aligned} \text{d. } & \exists x \exists y P(x, y) \wedge Q(y, x) \\ & P(A, B) \wedge Q(B, A) \end{aligned}$$

$$\begin{aligned} \text{e. } & \exists x \forall y P(x, y) \\ & \forall y P(A, y) \\ & P(A, y) \end{aligned}$$

$$\begin{aligned} \text{f. } & \forall x \exists y P(x, y) \\ & \forall x P(x, F(x)) \\ & P(x, F(x)) \end{aligned}$$

$$\begin{aligned} \text{g. } & \forall x \forall y \exists z P(x, y, z) \\ & \forall x \forall y P(x, y, F(x, y)) \\ & P(x, y, F(x, y)) \end{aligned}$$

$$\begin{aligned} \text{h. } & \exists x \forall y \forall z P(x, y, z) \\ & \forall y \forall z P(A, y, z) \\ & P(A, y, z) \end{aligned}$$

i. $\forall x (\exists y P(x, y) \wedge Q(y)) \Rightarrow R(x)$
 $\forall x \neg (\exists y P(x, y) \wedge Q(y)) \vee R(x)$
 $\forall x (\forall y \neg P(x, y) \vee \neg Q(y)) \vee R(x)$
 $(\neg P(x, y) \vee \neg Q(y)) \vee R(x)$
 $\neg P(x, y) \vee \neg Q(y) \vee R(x)$

j. $\forall x (\forall y P(x, y) \Rightarrow Q(y)) \Rightarrow R(x)$
 $\forall x \neg (\forall y P(x, y) \Rightarrow Q(y)) \vee R(x)$
 $\forall x \neg (\forall y \neg P(x, y) \vee Q(y)) \vee R(x)$
 $\forall x (\exists y P(x, y) \wedge \neg Q(y)) \vee R(x)$
 $\forall x (P(x, F(x)) \wedge \neg Q(F(x))) \vee R(x)$
 $(P(x, F(x)) \wedge \neg Q(F(x))) \vee R(x)$
 $(P(x, F(x)) \vee R(x)) \wedge (\neg Q(F(x)) \vee R(x))$

Q.2.

a. $P(x)$

b. $P(A)$

Ans: x/A (x will be replaced by A)

g. $P(x, A) \vee Q(A, x)$

h. $P(B, y) \vee Q(y, B)$

Ans: $x/B, y/A$
(x will be replaced by B and y by A)

c. $P(x) \vee Q(x, A)$

d. $P(B) \vee Q(x, A)$

Ans: x/B
(x will be replaced by B)

i. $P(x) \vee Q(F(x))$

j. $P(A) \vee Q(F(A))$

Ans: x/A (x will be replaced by A)

e. $P(x) \vee Q(A, x)$

f. $P(x) \vee Q(A, B)$

Ans: x/B
(x will be replaced by B)

k. $P(x, A) \vee Q(F(x), x)$

l. $P(B, y) \vee Q(F(B), B)$

Ans: $x/B, y/A$
(x will be replaced by B and y by A)

m. $P(x, A) \vee Q(F(x), x)$

n. $P(B, y) \vee Q(F(A), A)$

Ans: Fails because substituting x/B won't satisfy the condition to unify the sentence.

o. $P(x, y) \vee Q(F(A), B)$

p. $P(x, y) \vee Q(x, y)$

Ans: $x/F(A), y/B$. (x will be replaced by function $F(A)$ and y by B)

q. $P(x, y) \vee Q(F(A), A)$

r. $P(x, y) \vee Q(x, y)$

Ans: $x/F(A), y/A$ (x will be replaced by function $F(A)$ and y by A)

s. $P(x, y) \vee Q(F(x), y)$

t. $P(z, y) \vee Q(z, y)$

Ans: Fails because substituting $x/F(x)$ won't satisfy the condition to unify the sentence.

Q.3

A	B	C	$P(A, B, C)$
T	T	T	0.014
T	T	F	0.126
T	F	T	0.012
T	F	F	0.048
F	T	T	0.392
F	T	F	0.168
F	F	T	0.144
F	F	F	0.096

$$\begin{aligned}
 a. P(A, C) &= P(A=T, C=T, B=T) + P(A=T, C=T, B=F) \\
 &= 0.014 + 0.012 \\
 &= 0.026
 \end{aligned}$$

Ans: $P(A, C) = 0.026$

$$\begin{aligned}
 b. P(C) &= P(C=T, A=F, B=F) + P(C=T, A=T, B=T) + \\
 &\quad P(C=T, A=F, B=T) + P(C=T, A=T, B=F) \\
 &= 0.144 + 0.014 + 0.392 + 0.012 \\
 &= 0.562 \quad \boxed{P(C) = 0.562}
 \end{aligned}$$

$$\begin{aligned}
 c. P(A|C) &= \frac{P(A, C)}{P(C)} \\
 &= \frac{0.026}{0.562} \\
 &= 0.046 \quad \boxed{P(A|C) = 0.046}
 \end{aligned}$$

$$\begin{aligned}
 d. P(A, B|C) &= P(A|C) * P(B|C) \\
 &= \frac{P(A, C)}{P(C)} * \frac{P(B, C)}{P(C)} \\
 &= \frac{0.026}{0.562} * \frac{0.014 + 0.392}{0.562} \\
 &= 0.046 * 0.722 = 0.033 \quad \boxed{P(A, B|C) = 0.033}
 \end{aligned}$$

$$\begin{aligned}
 e. P(B|A, C) &= \frac{P(B=T, A=T, C=T)}{P(A=T, C=T)} \\
 &= \frac{0.014}{0.026} \\
 &= 0.538
 \end{aligned}$$

$P(B|A, C) = 0.538$

Q. 4.

a. Variables are binary

$$1. P(x_2) = 1$$

$$2. P(x_n) = 1$$

$$3. P(x_2, x_3, \dots, x_n) = 2^{(n-1)} - 1$$

$$4. P(x_2 | x_3, x_4, \dots, x_n) = 2^{(n-2)}$$

$$5. P(x_2, x_3, \dots, x_{n-1} | x_n) = 2(2^{(n-2)} - 1)$$

b. Variables are non-binary

$$1. P(x_2) = 1$$

$$2. P(x_n) = n - 1$$

$$3. P(x_2, x_3, \dots, x_n) = \frac{n!}{2} - 1$$

$$4. P(x_2 | x_3, x_4, \dots, x_n) = \frac{n!}{2}$$

$$5. P(x_2, x_3, \dots, x_{n-1} | x_n) = n \times (n-2)! - 1$$