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Assignment No. 4

1 Camera Calibration I

Ans. (a) Given Projection Matrix equation, $p = MP$ → world point 3DH; p ↑ image point 2DH; M ↓ Projection matrix 3×4

Forward Projection:- Radical lens distortion and weak perspective camera will arise in forward projection. In lens distortion, there is a larger shrink away from center. It can be correlated by finding parameters.

$$p^i = \begin{bmatrix} 1/\lambda & & \\ & 1/\lambda & \\ & & 1 \end{bmatrix} K^* [R^* | T^*] P^i$$

where $\lambda = 1 + K_1 d + K_2 d^2$

In weak perspective camera, the depth variation in scene is small compared with distance from camera.

Calibration:- It is difficult to match the world points with image points & calculate correspondence between them.

The calibration depends on intrinsic & extrinsic parameters.

$$M = K^* [R^* | T^*]$$

$$K^* = \text{Intrinsic parameter} = \begin{bmatrix} \alpha_v & s & v_0 \\ 0 & \alpha_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Reconstruction:- The object point can be sometimes

difficult to distinguish in uniform region because in this approach we need to find the depth of each pixel from 3D point to 2D point.
Due to some feature, point could be hard to interpret leading to problem of ambiguity.

Forward projection is the easiest while reconstruction is difficult.

Ans (b): Necessary inputs for Camera Calibration are as follows:-
1. 3D world points (x, y, z)
2. Its image 2D corresponding points (x_i, y_i)

Ans: (c) Steps in non coplanar calibration algorithm:-
1. find projection matrix M.
2. Find parameters (intrinsic & extrinsic) i.e. (K^*, R^*, T^*)

In step one, we need to find the projection matrix M with given 2D image points & 3D world point.
Here, $\underset{\substack{\uparrow \\ (2DH)}}{p} = M \underset{\substack{\uparrow \\ (3DH)}}{P}$

M → 3x4 matrix
→ $K^* [R^* | T^*]$

Ans: (d) $P_i' = M P_i$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 18 \\ 14 \\ 7 \end{bmatrix} \Rightarrow \begin{bmatrix} 18/7 \\ 2 \end{bmatrix}$$

Ans. (e) $P_i' = M P_i$

Given world image coordinates,

$$\begin{pmatrix} 100 \\ 200 \\ 1 \end{pmatrix} = M \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$$

The first 2 rows of the unknown projection matrix M are as follows:-

$$\begin{bmatrix} \cancel{1 \; 2 \; 3 \; 1 \; 0 \; 0 \; 0 \; 0 \; -100 \; -200 \; -300 \; -100} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}_{12\times1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times1}$$

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 & 0 & 0 & -100 & -200 & -300 & -100 \\ 0 & 0 & 0 & 0 & 1 & 2 & 3 & 1 & -200 & -400 & -600 & -200 \end{bmatrix}}_{2\times12} \underbrace{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{12\times1} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{2\times1}$$

Ans. (f) Minimal 6 points are necessary to be able to find a unique solution for M.

In order to find matrix M, we consider

$$Ax = 0$$

where $A = 2n\times12$ matrix

$x = 12\times1$ matrix

$$\begin{bmatrix} l_1^T & 0 & -x_1 P_1^T \\ 0 & l_1^T & -y_1 P_1^T \\ \vdots & & \\ l_m^T & 0 & -x_n P_m^T \\ 0 & l_m^T & -y_n P_m^T \end{bmatrix}_{2m \times 12} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}_{12 \times 1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}_{2m \times 1}$$

To find the solution of $Ax = 0$, we use SVD using singular value decomposition,

$$A = VDV^T$$

Column of V belonging to zero singular value is the solution.

$$\therefore \hat{x} = \begin{bmatrix} \hat{m}_1 \\ \hat{m}_2 \\ \hat{m}_3 \end{bmatrix} \Longrightarrow \hat{M} = \begin{bmatrix} - \hat{m}_1^T - \\ - \hat{m}_2^T - \\ - \hat{m}_3^T - \end{bmatrix}$$

The solution is not unique if $\hat{A}x = 0$

$A(\delta \hat{x}) = 0 \Rightarrow \delta \hat{x}$ is a solution.

Ans (g) To find the unknown parameters projection matrix M,

1. estimate M
2. Break M into R^*, K^* & T^*

$$M = \delta \hat{M}$$

$$\Rightarrow [K^* R^* \mid K^* T^*] = \delta \hat{M}$$

$$\hat{M} = \left[\begin{array}{c|c} \text{------ } q_1^T \text{ ----} & \\ \text{------ } q_2^T \text{ ----} & b \\ \text{------ } q_3^T \text{ ----} & \end{array}\right] \qquad K^* R^* = \delta \begin{bmatrix} \text{--} q_1^T \text{---} \\ \text{-} q_2^T \text{---} \\ \text{---} q_3^T \text{--} \end{bmatrix}$$

$$K^* T^* = \delta b$$

Ans. (h) Quality of Projection Matrix can be given as:-

$$\{p_i\}^n = \{P_i\}$$

Image points world points

For camera calibration, we calculate the estimate of parameters K^*, R^* & T^* ~~respectively~~ where,

$$M = K^*[R^* | T^*]$$

We need to compute error:

$$E(K^*, R^*, T^*) = \frac{1}{m}\sum_{i=1}^{n}\left(x_i - \frac{m_1^T P_i}{m_3^T P_i}\right)^2 + \left(y_i - \frac{m_2^T P_i}{m_3^T P_i}\right)^2$$

The lower the error is, the better the quality of fit.

Ans. (i) Principal of planar camera calibration:-

1. Estimate 2D homography (projective map) between calibration plane and image (for several images)
2. Estimate intrinsic parameters.
3. Compute extrinsic parameters for view of interest.

In non planar calibration, it uses 3D image as calibration target. So we need to find the pixel coordinates of all the corners and based on these values camera calibration is done while in planar, a single plane picture is used with different values.

Ans: (j) Homography (H)

→ Given as,

$$\underset{2DH}{p_i'} = \underset{3\times3}{M}\ \underset{2DH}{p_i^*}$$

or

$$\underset{2DH}{\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix}} = \underbrace{K^*\begin{bmatrix} r_1 & r_2 & T^* \end{bmatrix}}_{\substack{3\times3 \\ \text{homography}}} \underset{2DH}{\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}}$$

Projection matrix M

→ Given as

$$\underset{2DH}{p_i'} = \underset{3\times4}{M}\ \underset{3DH}{p_i}$$

or

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = K^*\begin{bmatrix} r_1 & r_2 & r_3 & T^* \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \\ 1 \end{bmatrix}$$

→ 2 equation for each point pair. need a minimum of 4 point pairs.	3 equation for each point pair and need a minimum of 6 point pairs.

→ Assumption used to make sure we deal with homography matrices..
To get a 2D projective ~~matrix~~ map we assume
$\{P_i\}_3 = 0 \iff P_i = (x_i, y_i, 0)$

2. Camera Calibration 2.

Ans: (a) $P_i' = M P_i$

where M = projection matrix

Given world image coordinates.

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = M \begin{pmatrix} 3 \\ 4 \\ 5 \\ 1 \end{pmatrix}$$

Estimating projection matrix; First 2 rows:-

$$\underbrace{\begin{bmatrix} 3 & 4 & 5 & 1 & 0 & 0 & 0 & 0 & -3 & -4 & -5 & -1 \\ 0 & 0 & 0 & 0 & 3 & 4 & 5 & 1 & -6 & -8 & -10 & -2 \end{bmatrix}}_{2\times12} \underbrace{\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}}_{12\times1} = \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{2\times1}$$

Ans: (b) Given Estimated Projection Matrix M,

$$M = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

$a_1 = [1 \quad 2 \quad 3]$

$a_2 = [2 \quad 3 \quad 4]$

$a_3 = [3 . 4 \quad 5]$

$b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$\therefore |S| = \dfrac{1}{|a_3|} = \dfrac{1}{\sqrt{9+16+25}} = \dfrac{1}{\sqrt{50}} = 0.141$

Scanned with CamScanner

Finding parameters from M

$u_0 = |S|^2 a_1 \cdot a_3$

$= (0.141)^2 \cdot (3+8+15)$

$= 0.0196 \times 26$

$= 0.5096$

and,

$v_0 = |S|^2 a_2 \cdot a_3$

$= (0.141)^2 \cdot (6+12+20)$

$= 0.0196 \times 38$

$= 0.7448$

$(u_0, v_0) = (0.5096, 0.7448)$

Ans: (c) Given world image points and projection matrix M,

$$p_i = M P_i$$

$$\underset{3\times 1}{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}} = \underset{3\times 4}{\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}} \underset{4\times 1}{\begin{pmatrix} 3 \\ 4 \\ 5 \\ 1 \end{pmatrix}}$$

Projection error of matrix M,

$$E = \frac{1}{m}\left(\left\| x_i - \frac{m_1^T P_i}{m_3^T P_i} \right\|^2 + \left\| y_i - \frac{m_2^T P_i}{m_3^T P_i} \right\|^2 \right)$$

Here, $m_1^T P_i = 3+8+15+4 = 30$

$m_2^T P_i = 6+12+20+5 = 43$

$m_3^T P_i = 9+16+25+6 = 56$

$$E = \frac{1}{1} \times \left(\left| 1 - \frac{30}{56} \right|^2 + \left| 2 - \frac{43}{56} \right|^2 \right)$$

$$= \frac{1}{1} \times \left(\left| \frac{26}{56} \right|^2 + \left| \frac{69}{56} \right|^2 \right)$$

$$= (0.4643)^2 + (1.2321)^2$$
$$= 0.2155 + 1.518$$
$$= 1.733$$

Ans. (d) Given $R^* = I + Q$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^* = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

To obtain rotation & translation of camera with respect to world.

$$R = (R^*)^T$$

$$= \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = -RT^*$$

$$= -\begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4\times4} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}_{4\times1}$$

$$= \begin{bmatrix} -6 \\ -2 \\ -3 \\ -1 \end{bmatrix}$$

Ans. (c) $\underset{2DH}{P_i'} = \underset{3\times3}{H} \underset{2DH}{P_i^*}$

where H = homography 3x3 matrix.
Given world-image coordinates

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = H \begin{pmatrix} 3 \\ 4 \\ 0 \\ 1 \end{pmatrix}$$

Calculate $P_i^* = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$

Homography ~~calculation~~ matrix calculation:-

$$\underset{2m\times9}{\begin{bmatrix} P_i^{*T} & 0 & -x_i P_i^{*T} \\ 0 & P_i^{*T} & -y_i P_i^{*T} \\ & \vdots & \end{bmatrix}} \underset{9\times1}{\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}} = \underset{2m\times1}{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}$$

$$\underbrace{\begin{bmatrix} 3 & 4 & 1 & 0 & 0 & 0 & -3 & -4 & -1 \\ 0 & 0 & 0 & 3 & 4 & 1 & -6 & -8 & -2 \\ & & & & \vdots & & & & \end{bmatrix}}_{\text{homography matrix}} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

3. Multiple View geometry 1:-

Ans: (a)

Sparse	Dense
1. Find feature points eg. corners.	1. Compare all patches
2. Find local characteriza-tion (eg.. SIFT, AOG)	2. Instead of distance b/w feature vectors it measures correl-ion of SSD.
~~3. Find corresponding points having similar features.~~	~~3~~
3. Compute epipolar ~~gemotery~~ geometry using techniques like RANSAC algorithms.	3. Compute epipolar geometry derived from the set of sparse matches.
Advantages:- It handles large disparities Greedy vs optimal Assignment.	Apply regularization to reduce errors and find correspondence in difficult areas (eg. uniform) It will produce more points
Disadvantages:- Constraints for reducing # of candidates (eg. Search around current location)	→ Having dependency on absolute value.

Ans: (b) Normalized cross correlation (NCC):-

$$\psi(\omega_1, \omega_2) = \sum_i \frac{(\omega(x_i, y_i) - \mu_{\omega_1})(\omega_2(x_i, y_i) - \mu_{\omega_2})}{\sigma_{\omega_1} \sigma_{\omega_2}}$$

Normalized SSD:-

$$\psi(\omega_1, \omega_2) = \sum_i \left(\frac{(\omega_i(x_i, y_i) - \mu_{\omega_1})}{\sigma_{\omega_1}} - \frac{(\omega_2(x_i, y_i) - \mu_{\omega_2})}{\sigma_{\omega_2}} \right)^2$$

More the value of NCC, higher is the correlation between the point in two windows.
In SSD, the less is the distance between the points, the more correlated they are.

→ When we search entire image as search space, we can face a problem in uniform regions of image.
→ There could be multiple point in one image that matches the points in other image.
→ To reduce the search ~~people~~ space, we can apply constraints for reducing number of candidates.
→ For example, we can look close to the current location (neighbourhood)

Ans: (c) Consider axis aligned stereo:-

Depth (z-coordinate), $z = f \frac{t}{d}$ where focal length = f

$P_l = (100, 200)$

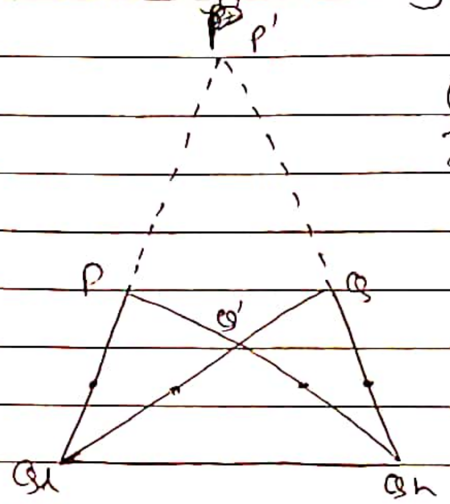
$P_r = (103, 200)$

T = baseline

$$z = \frac{100 \times 10}{(103-100)(200-200)}$$

$$= \frac{1000}{3} = 333.33$$

Ans. (d) The point of ambiguity owing to the ambiguous local appearances of image points is the one of the main causes making the stereo problem difficult. Under the point of ambiguity, local similarity measures are easy to be ambiguous and this results in false matches in ambiguous regions.



Correct :- P, Q
Incorrect :- P', Q'

Ans. (e) Expression for rotation and translation of the right camera wrt left camera:-

Rotation, $\boxed{R = R_l^T R_r}$

Translation, $\boxed{T = R_l^T (T_r - T_l)}$

Here, R_l, T_l = Rotation & translation of left camera wrt world.

and R_r, T_r = Rotation & translation of right camera wrt world.

4. Multiple View Geometry 2:-

Ans. (a) Given an axis aligned stereo system:-

$$\text{Depth}, z = \frac{f\,T}{d}$$

$$= \frac{10 \times 20}{30}$$

$$= \frac{20}{3} = 6.67$$

where, f = focal length.
T = baseline.
d = disparity

Ans:(b) Cross product as matrix multiplications:-

$$A \times B = [A]_\times B \equiv \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} B$$

skew-symmetric matrix

$$[A]_x = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix}$$

Hence if we multiply the above matrix by B we get the cross product A x B.

Verification:-

$A \times B = (1\,2\,3) \times (2\,3\,4) = (-1\,2\,-1)$

Using the matrix we obtained.

$$[A\times]B = \begin{bmatrix} 0 & -3 & 2 \\ 3 & 0 & -1 \\ -2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0-9+8 \\ 6+0-4 \\ -4+3+0 \end{bmatrix} = [-1\,2\,-1]$$

Ans: (c) Given fundamental matrix, $F = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$

Corresponding left & right points are (1,2) & (2,3)

Applying eight point's algorithm, value of

$P_r^T F P_l$

$= \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 11 & 17 & 23 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$= 11 + 34 + 23$

$= 68$

Ans: (d) Given left & right points (1,2) (2,3)

$$\underbrace{\begin{bmatrix} x_i x_i' & x_i y_i' & x_i & y_i x_i' & y_i y_i' & y_i & x_i' & y_i' & 1 \\ & & & \vdots & & & & & \\ & & & & & & & & \end{bmatrix}}_{n \times 9} \underbrace{\begin{bmatrix} f_{11} \\ f_{12} \\ \vdots \end{bmatrix}}_{9 \times 1} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{n \times 1}$$

$$\begin{bmatrix} 2 & 3 & 1 & 4 & 6 & 2 & 2 & 3 & 1 \\ & & & \vdots & & & & & \\ & & & \vdots & & & & & \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$