

## แบบฝึกหัด 6 LEAST-SQUARES REGRESSION

### 1. LINEAR REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

1.1 จงใช้ LINEAR REGRESSION ในการคำนวณหา  $f(x) = a_0 + a_1x_1$  และค่า  $f(65)$

1.2 จงเขียน code

### 2. POLYNOMIAL REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

2.1 จงใช้ POLYNOMIAL REGRESSION order  $m = 2$  ในการสร้างสมการ  $f(x) = a_0 + a_1x + a_2x^2$

2.2 จงเขียน code

### 3. MULTIPLE LINEAR REGRESSION

$x_1$	$x_2$	$x_3$	Y
1	0	1	4
0	1	3	-5
2	4	1	-6
3	2	2	0
4	1	5	-1
2	3	3	-7
1	6	4	-20

3.1 จงใช้ MULTIPLE LINEAR REGRESSION ในการสร้างสมการ  $f(x) = a_0 + a_1x_1 + a_2x_2 + a_3x_3$

3.2 จงเขียน code

## 1. LINEAR REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

1.1 จงใช้ LINEAR REGRESSION ในการคำนวณหา  $f(x) = a_0 + a_1x_1$  และค่า  $f(65)$

n , q

x	y	$x^2$	xy
10	5	100	50
15	9	225	135
20	15	400	300

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

$$9a_0 + 375a_1 = 215$$

$$a_0 = \frac{215}{9} - \frac{375a_1}{9} \quad \text{--- (1)}$$

30	18	900	540
40	22	1600	880
50	30	2500	1500

$$\begin{bmatrix} 9 & 375 \\ 375 & 20625 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 215 \\ 11605 \end{bmatrix}$$

$$375a_0 + 20625a_1 = 11605$$

$$a_1 = \frac{11605}{20625} - \frac{375a_0}{20625} \quad \text{--- (2)}$$

60	35	3600	2100
70	38	4900	2660
80	43	6400	3440

(แทน (2) ใน (1) ;  $a_0 \cdot \frac{215}{9} - \frac{375}{9} \left( \frac{11605}{20625} - \frac{375a_0}{20625} \right) = \frac{215}{9} - \frac{211}{9} + \frac{25}{33}a_0$

$$\frac{8}{33}a_0 = \frac{4}{9} ; a_0 = 1.8333$$

$$\therefore a_1 = 0.529$$

$$f(65) = 1.8333 + 0.5293(65) = 36.24$$

## 2. POLYNOMIAL REGRESSION

x	10	15	20	30	40	50	60	70	80
f(x)	5	9	15	18	22	30	35	38	43

m (order) = 2

unknown = m+1 = 3 (3x3)

2.1 จงใช้ POLYNOMIAL REGRESSION order m = 2 ในการสร้างสมการ  $f(x) = a_0 + a_1x + a_2x^2$

x	y	$x^2$	$x^3$	$x^4$	xy	$x^2y$
10	5	100	1000	10000	50	500
15	9	225	3375	50625	135	2025
20	15	400	8000	160000	300	6000
30	18	900	27000	810000	540	16200
40	22	1600	64000	2560000	880	35200
50	30	2500	125000	6250000	1500	75000
60	35	3600	216000	12960000	2100	126000
70	38	4900	343000	24010000	2660	186200
80	43	6400	512000	40960000	3440	275200
$\Sigma$	375	215	20625	1299375	87775	722025

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix} \longrightarrow \begin{bmatrix} 9 & 375 & 20625 \\ 375 & 20625 & 1299375 \\ 20625 & 1299375 & 87770625 \end{bmatrix} \cdot \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 215 \\ 11575 \\ 722025 \end{bmatrix}$$

$$\left[ \begin{array}{ccc|c} 9 & 375 & 20625 & 212 \\ 375 & 20625 & 1299375 & 11575 \\ 20625 & 1299375 & 87770625 & 722025 \end{array} \right] \xrightarrow{P_1/9} \left[ \begin{array}{ccc|c} 1 & 41.6 & 2291.6 & 23.5 \\ 375 & 20625 & 1299375 & 11575 \\ 20625 & 1299375 & 87770625 & 722025 \end{array} \right] \begin{array}{l} P_2 - 375P_1 \\ P_3 - 20625P_1 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 41.6 & 2291.6 & 23.5 \\ 0 & 5000 & 440000 & 2741.6 \\ 0 & 440000 & 40505000 & 836191.6 \end{array} \right] \xrightarrow{P_2/5000} \left[ \begin{array}{ccc|c} 1 & 41.6 & 2291.6 & 23.5 \\ 0 & 1 & 88 & 0.5483 \\ 0 & 440000 & 40505000 & 836191.6 \end{array} \right] \begin{array}{l} P_1 - 41.6P_2 \\ P_3 - 440000P_2 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1375 & 0.7083 \\ 0 & 1 & 88 & 0.5483 \\ 0 & 0 & 1785000 & -5075 \end{array} \right] \xrightarrow{P_3/1785000} \left[ \begin{array}{ccc|c} 1 & 0 & -1375 & 0.7083 \\ 0 & 1 & 88 & 0.5483 \\ 0 & 0 & 1 & -0.00284313 \end{array} \right] \begin{array}{l} P_1 + 1375P_3 \\ P_2 - 88P_3 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -3.20098 \\ 0 & 1 & 0 & 0.7985294 \\ 0 & 0 & 1 & -0.00284313 \end{array} \right] \begin{array}{l} a_0 = -3.20098 \\ a_1 = 0.7985294 \\ a_2 = -0.00284313 \end{array}$$

$$f(65) = -3.20098 + 0.7985294(65) - 0.00284313(65)^2 = 36.69117$$

### 3. MULTIPLE LINEAR REGRESSION

$X_1$	$X_2$	$X_3$	$Y$
1	0	1	4
0	1	3	-5
2	4	1	-6
3	2	2	0
4	1	5	-1
2	3	3	-7
1	6	4	-20

$k = 3$

Unknown =  $k + 1 = 4$

3.1 จงใช้ MULTIPLE LINEAR REGRESSION ในการสร้างสมการ  $f(X) = a_0 + a_1X_1 + a_2X_2 + a_3X_3$

3.2 จงเขียน code

$X_{1i}$	$X_{2i}$	$X_{3i}$	$X_{1i}X_{1i}$	$X_{1i}X_{2i}$	$X_{1i}X_{3i}$	$X_{2i}X_{1i}$	$X_{2i}X_{3i}$	$X_{3i}X_{3i}$	$y$	$X_{1i}y_i$	$X_{2i}y_i$	$X_{3i}y_i$
1	0	1	1	0	1	0	0	1	4	4	0	4
0	1	3	0	0	0	1	3	9	-5	0	-5	-15
2	4	1	4	8	2	16	4	1	-6	-12	-24	-6
3	2	2	9	6	6	4	4	4	0	0	0	0
4	1	5	16	4	20	1	5	25	-1	-4	-1	-5
2	3	3	4	6	6	9	9	9	-7	-14	-21	-21
1	6	4	1	6	4	36	24	16	-20	-20	-120	-80
$\Sigma$ 13	17	19	35	30	39	67	49	65	-35	-46	-171	-123

$$\begin{bmatrix} n & \Sigma X_{1i} & \Sigma X_{2i} & \Sigma X_{3i} \\ \Sigma X_{1i} & \Sigma X_{1i}X_{1i} & \Sigma X_{1i}X_{2i} & \Sigma X_{1i}X_{3i} \\ \Sigma X_{2i} & \Sigma X_{1i}X_{2i} & \Sigma X_{2i}X_{2i} & \Sigma X_{2i}X_{3i} \\ \Sigma X_{3i} & \Sigma X_{1i}X_{3i} & \Sigma X_{2i}X_{3i} & \Sigma X_{3i}X_{3i} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma X_{1i}y_i \\ \Sigma X_{2i}y_i \\ \Sigma X_{3i}y_i \end{bmatrix} \rightarrow \begin{bmatrix} 7 & 13 & 17 & 19 \\ 13 & 35 & 30 & 39 \\ 17 & 30 & 67 & 49 \\ 19 & 39 & 49 & 65 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -35 \\ -46 \\ -171 \\ -123 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 13 & 17 & 19 & -35 \\ 13 & 35 & 30 & 39 & -46 \\ 17 & 30 & 67 & 49 & -171 \\ 19 & 39 & 49 & 65 & -123 \end{bmatrix} \xrightarrow{R_1/7} \begin{bmatrix} 1 & 1.857 & 2.428 & 2.714 & -5 \\ 13 & 35 & 30 & 39 & -46 \\ 17 & 30 & 67 & 49 & -171 \\ 19 & 39 & 49 & 65 & -123 \end{bmatrix} \xrightarrow{\begin{matrix} R_2 - 13R_1 \\ R_3 - 17R_1 \\ R_4 - 19R_1 \end{matrix}} \begin{bmatrix} 1 & 1.857 & 2.428 & 2.714 & -5 \\ 0 & 10.857 & -1.571 & 3.714 & 19 \\ 0 & -1.571 & 25.714 & 2.857 & -86 \\ 0 & 3.714 & 2.857 & 13.428 & -28 \end{bmatrix}$$

$$\xrightarrow{R_2/10.857} \begin{bmatrix} 1 & 1.857 & 2.428 & 2.714 & -5 \\ 0 & 1 & -0.148 & 0.342 & 1.75 \\ 0 & -1.571 & 25.714 & 2.857 & -86 \\ 0 & 3.714 & 2.857 & 13.428 & -28 \end{bmatrix} \xrightarrow{\begin{matrix} R_1 - 1.857R_2 \\ R_3 + 1.571R_2 \\ R_4 - 3.714R_2 \end{matrix}} \begin{bmatrix} 1 & 0 & 2.697 & 2.079 & -8.25 \\ 0 & 1 & -0.148 & 0.342 & 1.75 \\ 0 & 0 & 25.487 & 3.395 & -82.25 \\ 0 & 0 & 3.395 & 12.158 & -34.5 \end{bmatrix} \xrightarrow{R_3/25.487} \begin{bmatrix} 1 & 0 & 2.697 & 2.079 & -8.25 \\ 0 & 1 & -0.148 & 0.342 & 1.75 \\ 0 & 0 & 1 & 0.133 & -3.235 \\ 0 & 0 & 3.395 & 12.158 & -34.5 \end{bmatrix}$$

$$\begin{array}{c|c|c|c|c}
 1 & 0 & 2.697 & 2.079 & -8.25 \\
 0 & 1 & -0.148 & 0.342 & 1.75 \\
 0 & 0 & 1 & 0.133 & -3.266 \\
 0 & 0 & 3.395 & 12.158 & -34.5
 \end{array}
 \begin{array}{l}
 R_1 - 2.697R_3 \\
 R_2 + 0.148R_3 \\
 \hline
 R_4 - 3.395R_3
 \end{array}
 \rightarrow
 \begin{array}{c|c|c|c|c}
 1 & 0 & 0 & 1.719 & 0.56 \\
 0 & 1 & 0 & 0.361 & 1.277 \\
 0 & 0 & 1 & 0.133 & -3.266 \\
 0 & 0 & 0 & 11.7 & -23.411
 \end{array}
 \xrightarrow{R_4/11.7}
 \begin{array}{c|c|c|c|c}
 1 & 0 & 0 & 1.719 & 0.56 \\
 0 & 1 & 0 & 0.361 & 1.277 \\
 0 & 0 & 1 & 0.133 & -3.266 \\
 0 & 0 & 0 & 1 & -2
 \end{array}$$

$$\begin{array}{c|c|c|c|c}
 R_1 - 1.719R_4 \\
 R_2 - 0.361R_4 \\
 \hline
 R_3 - 0.133R_4
 \end{array}
 \rightarrow
 \begin{array}{c|c|c|c|c}
 1 & 0 & 0 & 0 & 4 \\
 0 & 1 & 0 & 0 & 2 \\
 0 & 0 & 1 & 0 & -3 \\
 0 & 0 & 0 & 1 & -2
 \end{array}
 \begin{array}{l}
 a_0 = 4 \\
 a_1 = 2 \\
 a_2 = -3 \\
 a_3 = -2
 \end{array}$$

$$f(x) = 4 + 2x_1 - 3x_2 - 2x_3$$

$$\begin{array}{c}
 0 \quad 1 \quad 2 \quad 3 \\
 0 \begin{bmatrix} n & \sum x_{ji} & \sum x_{2i} & \sum x_{3i} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_{1i}y_i \\ \sum x_{2i}y_i \\ \sum x_{3i}y_i \end{bmatrix} \\
 1 \begin{bmatrix} \sum x_{1i} & \sum x_{1i}x_{1i} & \sum x_{1i}x_{2i} & \sum x_{1i}x_{3i} \end{bmatrix} \\
 2 \begin{bmatrix} \sum x_{2i} & \sum x_{2i}x_{1i} & \sum x_{2i}x_{2i} & \sum x_{2i}x_{3i} \end{bmatrix} \\
 3 \begin{bmatrix} \sum x_{3i} & \sum x_{3i}x_{1i} & \sum x_{3i}x_{2i} & \sum x_{3i}x_{3i} \end{bmatrix}
 \end{array}$$

3. MULTIPLE LINEAR REGRESSION

0 1 2

X

0

1

2

3

4

5

6

	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	Y
0	1	0	1	4
1	0	1	3	-5
2	2	4	1	-6
3	3	2	2	0
4	4	1	5	-1
5	2	3	3	-7
6	1	6	4	-20

3.1 จงใช้ MULTIPLE LINEAR REGRESSION ในการสร้างสมการ  $f(x) = a_0 + a_1x_1 + a_2x_2 + a_3x_3$

3.2 จงเขียน code

$$\begin{array}{c}
 0 \quad 1 \quad 2 \quad 3 \\
 A = \begin{bmatrix} 7 & & & \\ & - & - & - \\ & - & - & - \\ & - & - & - \\ & - & - & - \end{bmatrix} \\
 B = \begin{bmatrix} -35 & & & \end{bmatrix}
 \end{array}$$

1