$$\begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix}$$

1.1 ด้วยวิธี Jacobi Method โดยค่าเริ่มต้น x1=x2=x3=x4=0 โดยแสดงวิธีทำจำนวน 4 iterations (คำนวณมือ) และถ้ากำหนดค่า

$5x_1 + 2x_2 = 12 \quad ; \quad x_1^{k+1} = \dfrac{12 - 2x_2^k}{5}$ / $2x_2 + 5x_3 + 2x_4 = 14 \quad ; \quad x_3^{k+1} = \dfrac{14 - 2x_2^k - 2x_4^k}{5}$

$2x_1 + 5x_2 + 2x_3 = 17 \quad ; \quad x_2^{k+1} = \dfrac{17 - 2x_1^k - 2x_3^k}{5}$ / $2x_3 + 5x_4 = 7 \quad ; \quad x_4^{k+1} = \dfrac{7 - 2x_3^k}{5}$

Initial $x_1^1 = x_2^1 = x_3^1 = x_4^1 = 0$

Iteration 1 ; k = 1

$x_1^2 = \dfrac{12 - 2x_2^1}{5} = 2.4 \qquad \varepsilon = |2.4 - 0| = 2.4$

$x_2^2 = \dfrac{17 - 2x_1^1 - 2x_3^1}{5} = 3.4 \qquad \varepsilon = |3.4 - 0| = 3.4$

$x_3^2 = \dfrac{14 - 2x_2^1 - 2x_4^1}{5} = 2.8 \qquad \varepsilon = |2.8 - 0| = 2.8$

$x_4^2 = \dfrac{7 - 2x_3^1}{5} = 1.4 \qquad \varepsilon = |1.4 - 0| = 1.4$

Iteration 2 ; k = 2

$x_1^3 = \dfrac{12 - 2x_2^2}{5} = 1.44 \qquad \varepsilon = |1.44 - 2.4| = 0.96$

$x_2^3 = \dfrac{17 - 2x_1^2 - 2x_3^2}{5} = 1.32 \qquad \varepsilon = |1.32 - 3.4| = 2.08$

$x_3^3 = \dfrac{14 - 2x_2^2 - 2x_4^2}{5} = 0.88 \qquad \varepsilon = |0.88 - 2.8| = 1.92$

$x_4^3 = \dfrac{7 - 2x_3^2}{5} = 0.28 \qquad \varepsilon = |0.28 - 1.4| = 1.12$

Iteration 3 ; k = 3

$x_1^4 = \dfrac{12 - 2x_2^3}{5} = 1.824 \qquad \varepsilon = |1.824 - 1.44| = 0.384$

$x_2^4 = \dfrac{17 - 2x_1^3 - 2x_3^3}{5} = 2.472 \qquad \varepsilon = |2.472 - 1.32| = 1.152$

$x_3^4 = \dfrac{14 - 2x_2^3 - 2x_4^3}{5} = 2.16 \qquad \varepsilon = |2.16 - 0.88| = 1.28$

$x_4^4 = \dfrac{7 - 2x_3^3}{5} = 1.048 \qquad \varepsilon = |1.048 - 0.28| = 0.768$

Iteration 4 ; k = 4

$x_1^5 = \dfrac{12 - 2x_2^4}{5} = 1.4112 \qquad \varepsilon = |1.4112 - 1.824| = 0.4128$

$x_2^5 = \dfrac{17 - 2x_1^4 - 2x_3^4}{5} = 1.8064 \qquad \varepsilon = |1.8064 - 2.472| = 0.6656$

$x_3^5 = \dfrac{14 - 2x_2^4 - 2x_4^4}{5} = 1.392 \qquad \varepsilon = |1.392 - 2.16| = 0.768$

$x_4^5 = \dfrac{7 - 2x_3^4}{5} = 0.536 \qquad \varepsilon = |0.536 - 1.048| = 0.512$

```
double fx1(dou x2){
    return (12 -(2 * x2 )/5 ;
}

double fx2(dou x1, dou x3){
    return (17 - (2 * x1) - (2 * x3))/5 ;
}

double fx3 (dou x2, dou x4){
    return (14 - (2*x2) - (2 * x4))/5;
}

double fx4(dou x3){
    return (7-(2*x3))/5;
}

int main(void){
    double ans[4];
    jacobi(0,0,0,0,ans);

}

double[] jacobi(dou x1, dou x2, dou x3, dou x4, dou ans[]){
    double newx1 = fx1(x2);
    double newx2 = fx2(x1, x3);
    double newx3 = fx3(x2, x4);
    double newx4 = fx4(x3);

    if(fabs(newx1 - x1) < 0.000001 && fabs(newx2 - x2) < 0.000001 &&
       fabs(newx4 - x4) < 0.000001 && fabs(newx3 - x3) < 0.000001 {
        ans = {newx1, newx2, newx3, newx4)
        return ans;
    else {
       return jacobi(newx1, newx2, newx3, newx4, ans);
    }
```

1.2 ด้วยวิธี Gauss-Seidel Iteration Method โดยค่าเริ่มต้น x1=x2=x3=x4=0 โดยแสดงวิธีทำจำนวน 4 iterations (คำนวณมือ)

Initial $x_1^1 = x_2^1 = x_3^1 = x_4^1 = 0$

Iteration 1 ; k = 1

$x_1^2 = \frac{12-2x_2^1}{5} = 2.4 \quad \varepsilon = |2.4 - 0| = 2.4$

$x_2^2 = \frac{17-2x_1^2-2x_3^1}{5} = 2.44 \quad \varepsilon = |2.44 - 0| = 2.44$

$x_3^2 = \frac{14-2x_2^2-2x_4^1}{5} = 1.824 \quad \varepsilon = |1.824 - 0| = 1.824$

$x_4^2 = \frac{7-2x_3^2}{5} = 0.6704 \quad \varepsilon = |0.6704 - 0| = 0.6704$

Iteration 2 ; k = 2

$x_1^3 = \frac{12-2x_2^2}{5} = 1.424 \quad \varepsilon = |1.424 - 2.4| = 0.976$

$x_2^3 = \frac{17-2x_1^3-2x_3^2}{5} = 2.1 \quad \varepsilon = |2.1 - 2.44| = 0.34$

$x_3^3 = \frac{14-2x_2^3-2x_4^2}{5} = 1.69 \quad \varepsilon = |1.69 - 1.824| = 0.132$

$x_4^3 = \frac{7-2x_3^3}{5} = 0.724 \quad \varepsilon = |0.724 - 0.6704| = 0.0536$

Iteration 3 ; k = 3

$x_1^4 = \frac{12-2x_2^3}{5} = 1.56 \quad \varepsilon = |1.56 - 1.424| = 0.136$

$x_2^4 = \frac{17-2x_1^4-2x_3^3}{5} = 2.1 \quad \varepsilon = |2.1 - 2.1| = 0$

$x_3^4 = \frac{14-2x_2^4-2x_4^3}{5} = 1.67 \quad \varepsilon = |1.67 - 1.69| = 0.02$

$x_4^4 = \frac{7-2x_3^4}{5} = 0.732 \quad \varepsilon = |0.732 - 0.724| = 0.008$

Iteration 4 ; k = 4

$x_1^5 = \frac{12-2x_2^4}{5} = 1.56 \quad \varepsilon = |1.56 - 1.56| = 0$

$x_2^5 = \frac{17-2x_1^5-2x_3^4}{5} = 2.108 \quad \varepsilon = |2.108 - 2.1| = 0.008$

$x_3^5 = \frac{14-2x_2^5-2x_4^4}{5} = 1.664 \quad \varepsilon = |1.664 - 1.67| = 0.006$

$x_4^5 = \frac{7-2x_3^5}{5} = 0.7344 \quad \varepsilon = |0.7344 - 0.732| = 0.0024$

$D_1 = |5| = 5$

$D_2 = \begin{vmatrix} 5 & 2 \\ 2 & 5 \end{vmatrix} = 21$

$D_3 = \begin{vmatrix} 5 & 2 & 0 \\ 2 & 5 & 2 \\ 0 & 2 & 5 \end{vmatrix} = 85$

$D_4 = \begin{vmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{vmatrix} = 341$

$\therefore$ A is symmetric and positive definite

```
double fx1(dou x2){
    return (12 -(2 * x2 ))/5 ;
}

double fx2 (dou x1, dou x3)}
    return (17 - (2 * x1) - (2 * x3))/5;
}

double fx3 (dou x2, dou x4)}
    return (14 - (2*x2) - (2*x4))/5;
}

double fx4(dou x3)}
    return (7-(2*x3))/5;
}

int main (void)}
    double ans[4];
    jacobi(0,0,0,0,ans);

}

double[] jacobi (dou x1, dou x2, dou x3, dou x4, dou ans[])}
    double newx1 = fx1 (x2);
    double newx2 = fx2 (newx1, x3);
    double newx3 = fx3 (newx2, x4);
    double newx4 = fx4 (newx3);

    if(fabs(newx1 - x1) < 0.000001 && fabs(newx2 - x2) < 0.000001 &&
       fabs(newx4 - x4) < 0.000001 && fabs(newx3 - x3) < 0.000001 }
        ans = }newx1, newx2, newx3, newx4)
        return ans ;
    else }
        return jacobi (newx1, newx2, newx3, newx4, ans);
    }
```

1.3 ด้วยวิธี Conjugate Gradient Method โดยค่าเริ่มต้น x1=x2=x3=x4=0 โดยแสดงวิธีทำจำนวน 4 iterations (คำนวณมือ) และ

Iteration 1 ; $k = 0$

$$\{R\}^0 = [A]\{x\}^0 - \{B\} = \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix} = \begin{Bmatrix} -12 \\ -17 \\ -14 \\ -7 \end{Bmatrix}$$

$$\{D\}^0 = -\{R\}^0 = \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix}$$

$$\lambda_0 = -\frac{\lfloor D \rfloor^0 \{R\}^0}{\lfloor D \rfloor^0 [A] \{D\}^0} = -\frac{\begin{bmatrix} 12 & 17 & 14 & 7 \end{bmatrix} \begin{Bmatrix} -12 \\ -17 \\ -14 \\ -7 \end{Bmatrix}}{\begin{bmatrix} 12 & 17 & 14 & 7 \end{bmatrix} \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix}} = \frac{286}{5550}$$

$$\{x\}^1 = \{x\}^0 + \lambda_0 \{D\}^0 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \frac{286}{5550} \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix} = \begin{Bmatrix} 0.618 \\ 0.876 \\ 0.721 \\ 0.36 \end{Bmatrix}$$

$$\{R\}^1 = [A]\{x\}^1 - \{B\} = \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} 0.618 \\ 0.876 \\ 0.721 \\ 0.36 \end{Bmatrix} - \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix} = \begin{Bmatrix} -7.158 \\ -9.942 \\ -7.923 \\ -3.758 \end{Bmatrix}$$

$$\text{Error} = \sqrt{\lfloor R \rfloor^1 \{R\}^1} = \sqrt{\begin{Bmatrix} -7.158 & -9.942 & -7.923 & -3.758 \end{Bmatrix} \begin{Bmatrix} -7.158 \\ -9.942 \\ -7.923 \\ -3.758 \end{Bmatrix}} = 15.0657$$

Iteration 2 ; k = 1

$$\alpha_0 = \frac{\lfloor R \rfloor^1 [A] \{D\}^0}{\lfloor D \rfloor^0 [A] \{D\}^0} = \frac{\begin{Bmatrix} -7.158 \\ -9.942 \\ -7.923 \\ -3.758 \end{Bmatrix} \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix}}{\begin{bmatrix} 12 & 17 & 14 & 7 \end{bmatrix} \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix}} = \frac{-3206.574}{5550}$$

$$\{D\}^1 = -\{R\}^1 + \alpha_0 \{D\}^0 = -\begin{Bmatrix} -7.158 \\ -9.942 \\ -7.923 \\ -3.758 \end{Bmatrix} + \frac{-3206.574}{5550} \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix} = \begin{Bmatrix} 0.225 \\ 0.12 \\ -0.166 \\ -0.286 \end{Bmatrix}$$

$$\lambda_1 = -\frac{\lfloor D \rfloor^1 \{R\}^1}{\lfloor D \rfloor^1 [A] \{D\}^1} = -\frac{\begin{bmatrix} 0.225 & 0.12 & -0.166 & -0.286 \end{bmatrix} \begin{Bmatrix} -7.158 \\ -9.942 \\ -7.923 \\ -3.758 \end{Bmatrix}}{\begin{bmatrix} 0.225 & 0.12 & -0.166 & -0.286 \end{bmatrix} \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} 0.225 \\ 0.12 \\ -0.166 \\ -0.286 \end{Bmatrix}} = \frac{0.413584}{1.090109}$$

$$\{x\}^2 = \{x\}^1 + \lambda_1 \{D\}^1 = \begin{Bmatrix} 0.618 \\ 0.876 \\ 0.721 \\ 0.36 \end{Bmatrix} + \frac{0.413584}{1.090109} \begin{Bmatrix} 0.225 \\ 0.12 \\ -0.166 \\ -0.286 \end{Bmatrix} = \begin{Bmatrix} 0.7034 \\ 0.9215 \\ 0.658 \\ 0.2515 \end{Bmatrix}$$

$$\{R\}^2 = [A]\{x\}^2 - \{B\} = \begin{bmatrix} 5 & 2 & 0 & 0 \\ 2 & 5 & 2 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 2 & 5 \end{bmatrix} \begin{Bmatrix} 0.7034 \\ 0.9215 \\ 0.658 \\ 0.2515 \end{Bmatrix} - \begin{Bmatrix} 12 \\ 17 \\ 14 \\ 7 \end{Bmatrix} = \begin{Bmatrix} -6.64 \\ -9.6697 \\ -8.364 \\ -4.4265 \end{Bmatrix}$$

$$\text{Error} = \sqrt{\lfloor R \rfloor \{R\}} = \sqrt{\{-6.64 \;\; -9.6697 \;\; -8.364 \;\; -4.4265\} \begin{Bmatrix} -6.64 \\ -9.6697 \\ -8.364 \\ -4.4265 \end{Bmatrix}} = 15.071$$

2. กำหนดให้ $A = \begin{bmatrix}2 & 5\\ 5 & 1\end{bmatrix}$, $B = \begin{bmatrix}3\\ 2\end{bmatrix}$ และ

$$f(x_1, x_2) = \frac{1}{2}\underset{(1\times2)}{\lfloor X \rfloor}\underset{(2\times2)}{[A]}\underset{(2\times1)}{\{X\}} - \underset{(1\times2)}{\lfloor B \rfloor}\underset{(2\times1)}{\{X\}}$$

จงหาร $f(x_1, x_2)$ พร้อมวาดกราฟ contour

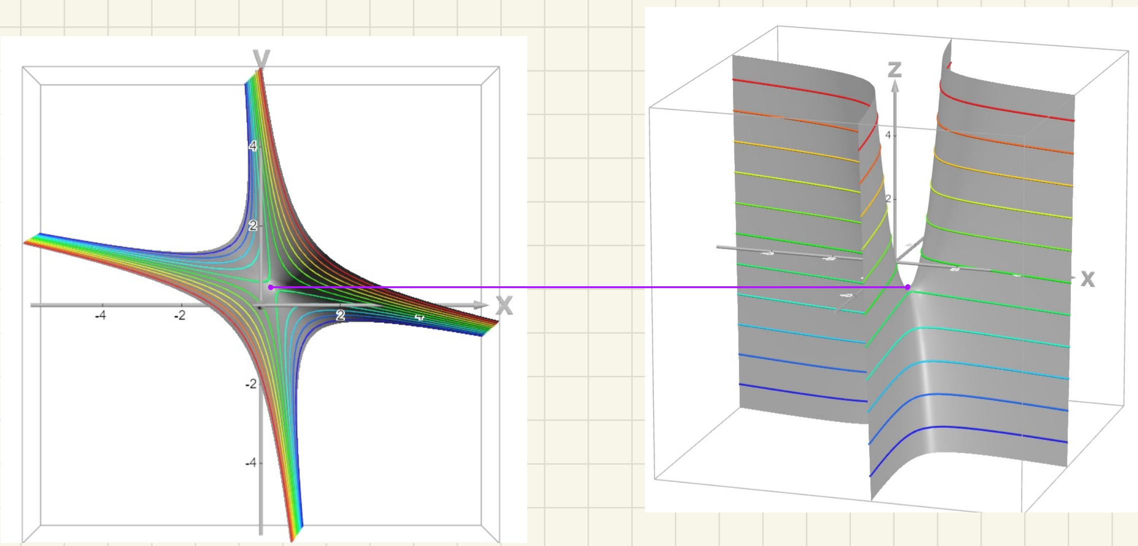
$$f(x_1,x_2) = \frac{1}{2}\begin{bmatrix}x_1 & x_2\end{bmatrix}\begin{bmatrix}2 & 5\\ 5 & 1\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix} - \begin{bmatrix}3 & 2\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}2x_1 + 5x_2 & 5x_1 + x_2\end{bmatrix}\begin{Bmatrix}x_1\\ x_2\end{Bmatrix} - (3x_1 + 2x_2)$$

$$= \frac{1}{2}(2x_1^2 + 5x_1x_2 + 5x_1x_2 + x_2^2) - 3x_1 - 2x_2$$

$$= \frac{1}{2}(2x_1^2 + 10x_1x_2 + x_2^2) - 3x_1 - 2x_2$$

$$= x_1^2 + 5x_1x_2 + \frac{x_2^2}{2} - 3x_1 - 2x_2$$



CONJUGATE GRADIENT PROCEDURE

Start from the quadratic function for the system of n eqs.,

$$f(x_1, x_2, \ldots, x_n) = \frac{1}{2} \underset{(1\times n)}{\lfloor X \rfloor} \underset{(n\times n)}{[A]} \underset{(n\times 1)}{\{X\}} - \underset{(1\times n)}{\lfloor B \rfloor} \underset{(n\times 1)}{\{X\}}$$

So that their first derivatives wrt. $x_1, x_2, \ldots, x_n$ are determined and set to zero.

$$\frac{\partial f}{\partial \{X\}} = [A]\{X\} - \{B\} = 0$$

yielding the original system of n eqs.

The conjugate gradient method is an iterative technique, a set of initial guess values in $\{X\}$ is needed. The new $\{X\}$ at $(k+1)^{th}$ iteration is obtained from,

$$\{X\}^{k+1} = \{X\}^k + \lambda_k \{D\}^k$$

where

$\{D\}^k$ = search direction vector

λ_k = step size of $\{D\}^k$

$$\{D\}^{k+1} = -\{R\}^{k+1} + \alpha_k \{D\}^k$$

จงพิสูจน์ว่า

$$\lambda_k = -\frac{\lfloor D \rfloor^k \{R\}^k}{\lfloor D \rfloor^k [A] \{D\}^k}$$

$$\alpha_k = \frac{\lfloor R \rfloor^{k+1} [A] \{D\}^k}{\lfloor D \rfloor^k [A] \{D\}^k}$$

find λ_k

$$f(x) = \frac{1}{2}\lfloor X \rfloor [A]\{X\} - \lfloor B \rfloor \{X\}$$

จาก $\{X\}^{k+1} = \{X\}^k + \lambda_k \{D\}^k$

$$f(x^{k+1}) = \frac{1}{2}\cdot\lfloor \{X\}^k + \lambda_k\{D\}^k \rfloor \cdot [A] \cdot \{\{X\}^k + \lambda_k \{D\}^k\} - \lfloor B \rfloor \cdot \{\{X\}^k + \lambda_k\{D\}^k\}$$

$$= \frac{1}{2}\left(\lfloor \{X\}^k + \lambda_k\{D\}^k \rfloor \cdot (A\{X\}^k + A\lambda_k\{D\}^k)\right) - \lfloor B \rfloor\{X\}^k - \lfloor B \rfloor \lambda_k \{D\}^k$$

$$= \frac{1}{2}\left(\lfloor \{X\}^k \rfloor A\{X\}^k + 2\lambda_k \lfloor \{X\}^k \rfloor [A]\{D\}^k + \lambda_k^2 \lfloor \{D\}^k \rfloor A \{D\}^k\right) - \lfloor B \rfloor\{X\}^k - \lfloor B \rfloor \lambda_k \{D\}^k$$

$$\frac{d}{d\lambda_k} f(x^{k+1}) = \lfloor \{X\}^k \rfloor [A]\{D\}^k + \lambda_k \lfloor \{D\}^k \rfloor A\{D\}^k - \lfloor B \rfloor \{D\}^k$$

จาก $\{R\}^k = [A]\{X\}^k - \{B\}$

$[A]\{X\}^k = \{R\}^k - \{B\}$

$[A]\lfloor \{X\}^k \rfloor = \lfloor \{R\}^k - \{B\} \rfloor$

$$\frac{d}{d\lambda_k} f(x^{k+1}) = \lfloor \{R\}^k - \{B\} \rfloor \{D\}^k + \lambda_k \lfloor \{D\}^k \rfloor A\{D\}^k - \lfloor B \rfloor \{D\}^k$$

$$0 = \lfloor R \rfloor^k \{D\}^k - \lfloor B \rfloor\{D\}^k + \lambda_k \lfloor \{D\}^k \rfloor A\{D\}^k - \lfloor B \rfloor\{D\}^k$$

$$0 = -\lfloor B \rfloor \{D\}^k \left(\lfloor R \rfloor^k \{D\}^k + \lambda_k \lfloor \{D\}^k \rfloor A \{D\}^k\right)$$

$$0 = \lfloor R \rfloor^k \{D\}^k + \lambda_k \lfloor \{D\}^k \rfloor A\{D\}^k$$

$$\lambda_k = -\frac{\lfloor R \rfloor^k \{D\}^k}{\lfloor D \rfloor^k A \{D\}^k} \quad \#$$

find α_k

$$f(x) = \frac{1}{2}\lfloor x \rfloor [A]\{x\} - \lfloor B \rfloor \{x\}$$

$$\{x\}^{k+1} = \{x\}^k + \alpha_k \{D\}^k$$

$$f(\{x\}^{k+1}) = f(\{x\}^k + \alpha_k \{D\}^k)$$

$$f(\{x\}^k + \alpha_k\{D\}^k) = \frac{1}{2}\lfloor \{x\}^k + \alpha_k\{D\}^k \rfloor([A]\{x\}^k + [A]\alpha_k\{D\}^k) - \lfloor B \rfloor(\{x\}^k + \alpha_k\{D\}^k)$$

$$= \frac{1}{2}\left(\lfloor x \rfloor^k[A]\{x\}^k + 2\alpha_k\lfloor x \rfloor^k[A]\{D\}^k + \alpha_k^2\lfloor D \rfloor^k A\{D\}^k\right) - \lfloor B \rfloor\{x\}^k - \lfloor B \rfloor\alpha_k\{D\}^k$$

$$\frac{d}{d\alpha_k} f(x_{k+1}) = \lfloor X \rfloor^k[A]\{D\}^k + \alpha_k\lfloor D \rfloor^k A\{D\}^k - \lfloor B \rfloor\{D\}^k = 0$$

จาก $\{R\}^k = [A]\{x\}^k - \{B\}$

$[A]\{x\}^k = \{R\}^k + \{B\}$

$[A]\lfloor x \rfloor^k = \lfloor R \rfloor^k + \lfloor B \rfloor$

$\longrightarrow (\lfloor R \rfloor^k + \lfloor B \rfloor)\{D\}^k + \alpha_k\lfloor D \rfloor^k A\{D\}^k - \lfloor B \rfloor\{D\}^k = 0$

$\lfloor R \rfloor^k\{D\}^k \cancel{+ \lfloor B \rfloor\{D\}^k} + \alpha_k\lfloor D \rfloor^k A\{D\}^k \cancel{- \lfloor B \rfloor\{D\}^k} = 0$

$$\lfloor R \rfloor^k\{D\}^k + \alpha_k\lfloor D \rfloor^k A\{D\}^k = 0 \quad — \quad \alpha_k = \left| -\frac{\lfloor R \rfloor^k\{D\}^k}{\lfloor D \rfloor^k A\{D\}^k} \right|$$