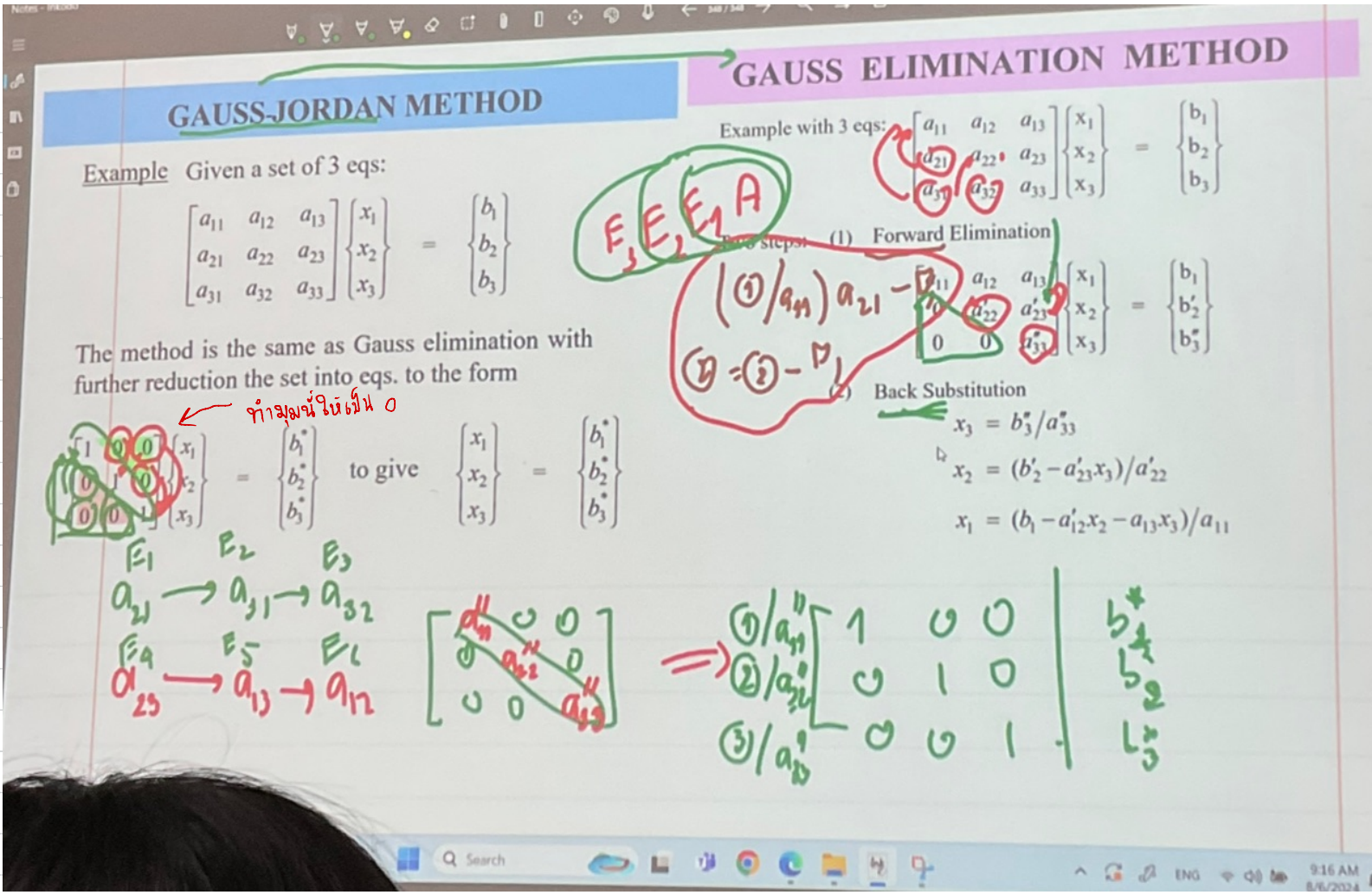
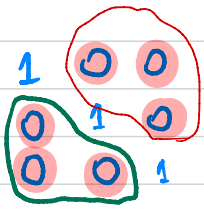




$(\varepsilon_1) \xrightarrow{\text{then}} (\varepsilon_2) \xrightarrow{\text{then}} (\varepsilon_3)$

$a_{21} \rightarrow a_{31} \rightarrow a_{32}$

$a_{23} \rightarrow a_{13} \rightarrow a_{12}$

$\rightarrow \begin{bmatrix} a_{11}'' & 0 & 0 \\ 0 & a_{22}'' & 0 \\ 0 & 0 & a_{33}'' \end{bmatrix}$

Matrix inversion [ใช้ เกาส์จอร์แดน]

A^{-1} : A inverse I = เมทริกซ์เอกลักษณ์

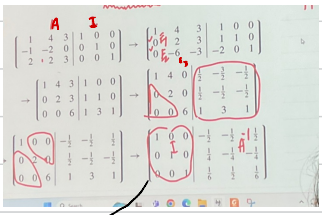
:

$$A^{-1}Ax = A^{-1}B$$

$$Ix = A^{-1}B$$

$$x = A^{-1}B$$

1. มี A , I
2. ผัน A ให้เป็น I



3. ได้ I

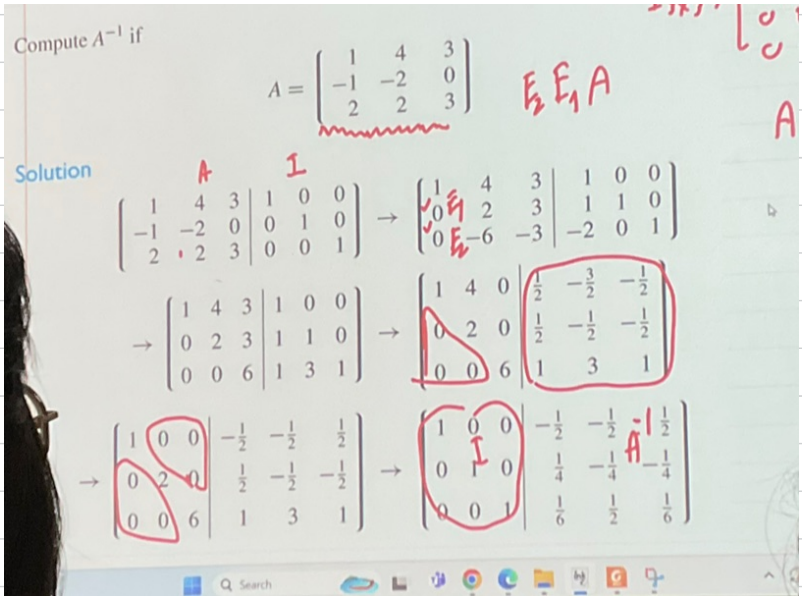
$E_n \cdots E_2 E_1 A = I$

คูณ A^{-1} ทั้งสองข้าง

$E_n \cdots E_2 E_1 \cdot A \cdot A^{-1} = I \cdot A^{-1}$

$A \cdot A^{-1} = I$

$E_n \cdots E_1 \cdot I = A^{-1}$



Ex

$$\begin{bmatrix} 4 & -4_{(4)} & 0 \\ -1_{(1)} & 4 & -2_{(3)} \\ 0 & -2_{(2)} & 4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix}$$

ทำ – เกาส์จอร์แดน (2 ทีพ)

– อินเวอร์ส

950

350

275

$$\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 400 \\ 400 \\ 400 \end{vmatrix}$$

#1 กำจัด → (−1)

#1.

$\frac{①}{4} = [1 \quad -1 \quad 0] \; [100]$

$\cdot -1 = [-1 \quad 1 \quad 0] \; [-100]$

② − ①' $= [-1 \quad 4 \quad -2] \; [400]$

① $[-1 \quad 1 \quad 0] \; [-100]$

$R_2' = [0 \quad 3 \quad -2] \; [500]$

$\frac{R_2 - R_1(-1)}{4} = \frac{(-1) - (4(-1))}{4}$

$$= \begin{bmatrix} 4 & -4 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 400 \\ 500 \\ 400 \end{bmatrix}$$

กำจัด #2 (−2)

#2.

$\frac{②}{3} = [0 \quad 1 \quad -\frac{2}{3}] \; [\frac{500}{3}]$

$♡ = \cdot -2 = [0 \quad -2 \quad \frac{4}{3}] \; [-\frac{1000}{3}]$

③ $[0 \quad -2 \quad -4] \; [400]$ −

♡ $[0 \quad -2 \quad \frac{4}{3}] \; [-\frac{1000}{3}]$

③ $= [0 \quad 0 \quad -\frac{16}{3}] \; [\frac{2200}{3}]$ ← ③'

$$\begin{bmatrix} 4 & -4 & 0 \\ 0 & 3 & -2 \\ 0 & 0 & -\frac{16}{3} \end{bmatrix} \begin{bmatrix} 400 \\ 500 \\ \frac{2200}{3} \end{bmatrix}$$

#3, กำจัด

$\frac{③}{-\frac{16}{3}} = [0 \quad 0 \quad \frac{-\frac{16}{3}}{-\frac{16}{3}}] \left[\frac{\frac{2200}{3}}{-\frac{16}{3}}\right]$

$= [0 \quad 0 \quad 1] \; [-\frac{275}{2}]$

$♡ = \cdot -2 = [0 \quad 0 \quad -2] \; [275]$

② $[0 \quad 3 \quad -2] \; [500]$ −

♡ $[0 \quad 0 \quad -2] \; [275]$

② $[0 \quad 3 \quad 0] \; [225]$ ← ②''

$$\begin{bmatrix} 4 & -4 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -\frac{16}{3} \end{bmatrix} \begin{bmatrix} 400 \\ 225 \\ \frac{2200}{3} \end{bmatrix}$$

#4 ก่อน

$$\frac{②}{3} = [0 \quad 1 \quad 0] \; [75]$$

$$② = \cdot -4 = [0 \quad -4 \quad 0] \; [-300]$$

$$① \; [4 \quad -4 \quad 0] \; [400]$$
$$- \; ② \; [0 \quad -4 \quad 0] \; [-300]$$
$$① \; [4 \quad 0 \quad 0] \; [700]$$

$$\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -\frac{16}{3} \end{bmatrix} \begin{bmatrix} 700 \\ 225 \\ \frac{2200}{3} \end{bmatrix}$$

$$\frac{①}{4}, \; \frac{②}{3}, \; \frac{③}{-\frac{16}{3}} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 175 \\ 75 \\ -\frac{275}{2} \end{bmatrix}$$

$$0x + 0y + 1z = -\frac{275}{2} \qquad 0x + 1y + 0z = 75 \qquad 1x + 0y + 0z = 175$$

$$z = -\frac{275}{2} \qquad y = 75 \qquad x = 175$$

Matrix inversion [ใช้ เกาส์จอร์แดน]

A^{-1} : A inverse I = เมทริกซ์เอกลักษณ์

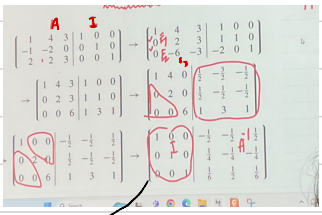
:

$$A^{-1}Ax = A^{-1}B$$

$$Ix = A^{-1}B$$

$$x = A^{-1}B$$

1. มี A , I
2. ผัน A ให้เป็น I

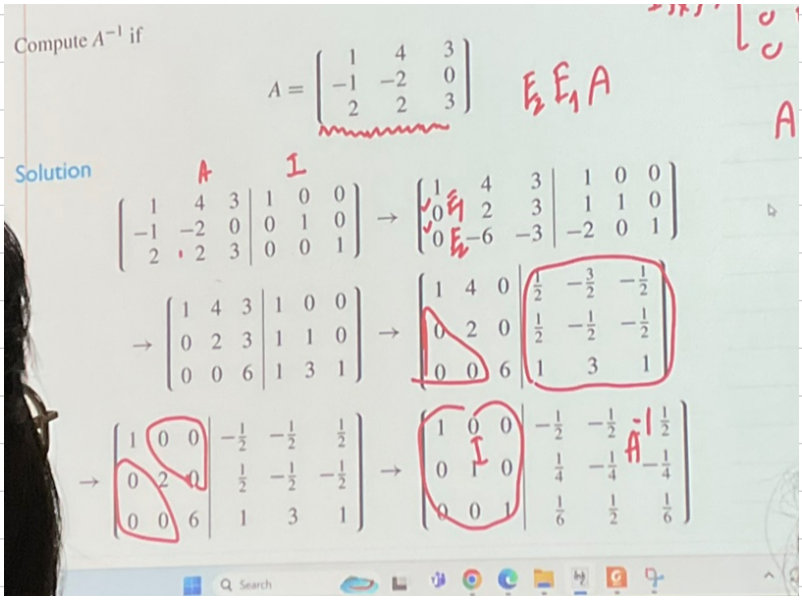


3. ได้ I

$E_n \cdots E_2 E_1 \; A = I$

$E_n \cdots E_2 E_1 \cdot A \cdot A^{-1} = I \cdot A^{-1}$ (คูณ A^{-1} ทั้งสองข้าง)

$E_n \cdots E_2 E_1 \cdot I = A^{-1}$



Ex

$$\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix}$$

A

$$= \begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A I

ดัน A ให้เป็น I

$$\begin{bmatrix} 4 & -4 & 0 \\ (-1) & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#1 $♡ = \frac{①}{4} = [1 \quad -1 \quad 0] \quad [\tfrac{1}{4} \quad 0 \quad 0]$

$② + ♡$: $[-1 \quad 4 \quad -2] + [0 \quad 1 \quad 0]$ +

$(1 \quad -1 \quad 0) \quad [\tfrac{1}{4} \quad 0 \quad 0]$

$② = [0 \quad 3 \quad -2] \quad [\tfrac{1}{4} \quad 1 \quad 0]$

$$= \begin{bmatrix} 4 & -4 & 0 \\ 0 & 3 & -2 \\ 0 & (-2) & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(×2 beside the circled −2)

#2 $♡ = \frac{②}{3} = [0 \quad 1 \quad -\tfrac{2}{3}] \quad [\tfrac{1}{12} \quad \tfrac{1}{3} \quad 0]$

$♡ \times 2 = [0 \quad 2 \quad \tfrac{4}{3}] \quad [\tfrac{1}{6} \quad \tfrac{2}{3} \quad 0]$

$3 + ♡$: $[0 \quad -2 \quad 4] \quad [0 \quad 0 \quad 1]$ +

$(0 \quad 2 \quad \tfrac{4}{3}) \quad (\tfrac{1}{6} \quad \tfrac{2}{3} \quad 0)$

$③ = [0 \quad 0 \quad \tfrac{16}{3}] \quad [\tfrac{1}{6} \quad \tfrac{2}{3} \quad 1]$

$$\begin{bmatrix} 4 & -4 & 0 \\ 0 & 3 & (-2) \\ 0 & 0 & \frac{16}{3} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{1}{6} & \frac{2}{3} & 1 \end{bmatrix}$$

#3 $\frac{③}{\frac{16}{3}} = [0 \quad 0 \quad 1] \quad [\tfrac{1}{32} \quad \tfrac{1}{8} \quad \tfrac{3}{16}]$

↓ ×2

$♡ \quad [0 \quad 0 \quad 2] \quad [\tfrac{1}{16} \quad \tfrac{1}{4} \quad \tfrac{3}{8}]$

$2 + ♡$: $[0 \quad 3 \quad -2] \quad [\tfrac{1}{4} \quad 1 \quad 0]$ +

$[0 \quad 0 \quad 2] \quad [\tfrac{1}{16} \quad \tfrac{1}{4} \quad \tfrac{3}{8}]$

$② = [0 \quad 3 \quad 0] \quad [\tfrac{5}{16} \quad \tfrac{5}{4} \quad \tfrac{3}{8}]$

$$\left[\begin{array}{ccc} 4 & -4 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & \frac{16}{3} \end{array}\right] \quad \left[\begin{array}{ccc} 1 & 0 & 0 \\ \frac{5}{16} & \frac{5}{4} & \frac{3}{8} \\ \frac{1}{6} & \frac{2}{3} & 1 \end{array}\right]$$

#4 $\heartsuit = \frac{②}{3} = [\,0 \;\; 1 \;\; 0\,] \quad \left[\begin{array}{ccc} \frac{5}{48} & \frac{5}{12} & \frac{1}{8} \end{array}\right]$

$\heartsuit \cdot 4 = [\,0 \;\; 4 \;\; 0\,] \quad \left[\begin{array}{ccc} \frac{5}{12} & \frac{5}{3} & \frac{1}{2} \end{array}\right]$

① + $[\,4 \;\; -4 \;\; 0\,] \quad [\,1 \;\; 0 \;\; 0\,]$

$\heartsuit \; [\,0 \;\; 4 \;\; 0\,] \quad \left[\begin{array}{ccc} \frac{5}{12} & \frac{5}{3} & \frac{1}{2} \end{array}\right]$

① : $[\,4 \;\; 0 \;\; 0\,] \quad \left[\begin{array}{ccc} \frac{17}{12} & \frac{5}{3} & \frac{1}{2} \end{array}\right]$

$$\left[\begin{array}{ccc} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & \frac{16}{3} \end{array}\right] \quad \left[\begin{array}{ccc} \frac{17}{12} & \frac{5}{3} & \frac{1}{2} \\ \frac{5}{16} & \frac{5}{4} & \frac{3}{8} \\ \frac{1}{6} & \frac{2}{3} & 1 \end{array}\right]$$

$\frac{①}{4} = [\,1 \;\; 0 \;\; 0\,] \quad \left[\begin{array}{ccc} \frac{17}{48} & \frac{5}{12} & \frac{1}{8} \end{array}\right]$

$\frac{②}{3} = [\,0 \;\; 1 \;\; 0\,] \quad \left[\begin{array}{ccc} \frac{5}{48} & \frac{5}{12} & \frac{1}{8} \end{array}\right]$

$\frac{③}{\frac{16}{3}} \quad [\,0 \;\; 0 \;\; 1\,] \quad \left[\begin{array}{ccc} \frac{1}{32} & \frac{1}{8} & \frac{3}{16} \end{array}\right]$

$$= \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc} \frac{17}{48} & \frac{5}{12} & \frac{1}{8} \\ \frac{5}{48} & \frac{5}{12} & \frac{1}{8} \\ \frac{1}{32} & \frac{1}{8} & \frac{3}{16} \end{array}\right]$$

$I \qquad A^{-1}$

$E_n \cdots E_2 E_1 \cdot A = I$

$E_n \cdots E_2 E_1 \cdot A \cdot A^{-1} = I \cdot A^{-1}$ ← คูณ A^{-1} ทั้งสองข้าง

$E_n \cdots E_2 E_1 \cdot I = A^{-1}$

$= E_n \cdots E_2 E_1 \cdot A = I$

$E_n \cdots E_2 E_1 \cdot I = A^{-1}$

ε_1 ε_2 ε_3

a_{21} a_{31} a_{32}

ε_4 ε_5 ε_6

a_{23} a_{31} a_{12}

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} \frac{17}{96} & \frac{5}{12} & \frac{1}{8} \\ \frac{5}{48} & \frac{5}{12} & \frac{1}{8} \\ \frac{1}{32} & \frac{1}{8} & \frac{3}{16} \end{bmatrix}$$

I $\quad A^{-1}$

$$\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix}$$

A

$E_n \cdots E_2 E_1 \; A = I$

$E_n \cdots E_2 E_1 \cdot A \cdot A^{-1} = I \cdot A^{-1}$ ← คูณ A^{-1} ทั้งสองข้าง

I

$E_n \cdots E_1 \cdot I = A^{-1}$

$= \quad E_n \cdots E_2 E_1 \cdot A = I$

$E_n \cdots E_2 E_1 \cdot I = A^{-1}$