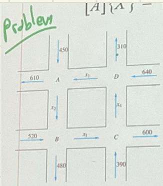
Solutions of Linear Algebra Equation : $[A]\{x\} = \{B\}$ $(Ax = B)$

$$\overset{A}{\begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}} \overset{x}{\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}} = \overset{B}{\begin{Bmatrix} 5 \\ 6 \end{Bmatrix}} \quad ; \quad \boxed{\begin{aligned} 2x_1 + 3x_2 &= 5 \\ 5x_1 + 6x_2 &= 6 \end{aligned}}$$

* สมการเกิดจากปัญหาในชีวิตประจำวัน

รถเข้า = รถออก



A : $450 + x_1 = 610 + x_2$

B : $520 + x_2 = 480 + x_3$

C : $x_3 + 390 = 600 + x_4$

D : $x_4 + 640 = x_1 + 310$

$\longrightarrow$

$x_1 - x_2 + 0x_3 + 0x_4 = 160$

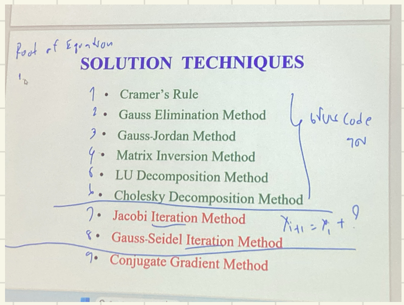
$0x_1 + x_2 - x_3 + 0x_4 = -40$

$0x_1 + 0x_2 + x_3 - x_4 = 210$

$-x_1 + 0x_2 + 0x_3 + x_4 = -330$

ตั้ง $Ax = B$ จากสมการ

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{Bmatrix} = \begin{Bmatrix} 160 \\ -40 \\ 210 \\ -330 \end{Bmatrix}$$



สอบ 1-8

Cramer's Rule (find x)

Cramer$_{2\times2}$; $x_i = \dfrac{\det[A]_i}{\det[A]}$, Given A, x, B to form $Ax = B$

where $[A]_i$ = matrix $[A]$ with column i replaced by $\{B\}$

ex. $\overset{A}{\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}} \overset{x}{\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}} \overset{=}{=} \overset{B}{\begin{Bmatrix} 4 \\ -1 \end{Bmatrix}}$

$x_1 + x_2 = 3$ (หาคำตอบไม่ได้)

$x_1 + x_2 = 4$

สมการไม่สอดคล้อง $\det A = 0$

$x_1 + x_2 = 3$ (สมการขนานกัน)

$2x_1 + 2x_2 = 6$

$\det A = 0$

ไม่เกิดจุดตัด

$$x_1 = \frac{\begin{vmatrix} 4 & 1 \\ -1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-4+1}{-2-1} = \frac{-3}{-3} = 1 \qquad x_2 = \frac{\begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}} = \frac{-2-4}{-2-1} = \frac{-6}{-3} = 2$$

Cramer$_{3\times3}$; ทำเหมือนเดิม det 3×3

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \end{vmatrix}}$$

Gauss Elimination

Example with 3 eqs:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

Two steps: (1) Forward Elimination

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b'_2 \\ b''_3 \end{Bmatrix}$$

(2) Back Substitution

แทนค่ากลับ

$$x_3 = b''_3 / a''_{33}$$
$$x_2 = (b'_2 - a'_{23}x_3)/a'_{22}$$
$$x_1 = (b_1 - a'_{12}x_2 - a_{13}x_3)/a_{11}$$

eliminate a_{21}, a_{31} reference to a_{11}

$$\begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \cancel{a_{21}} & a_{22} & a_{23} \\ \cancel{a_{31}} & \cancel{a_{32}} & a_{33} \end{bmatrix}$$

III

eliminate a_{32} reference to a_{22}

$$\begin{bmatrix} 3 & 4 & 1 \\ 2 & 6 & 4 \\ 5 & 2 & 7 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 5 \\ 10 \end{Bmatrix}$$

a_{21}

$$\begin{array}{c} ① \\ ② \\ ③ \end{array} \left[\begin{array}{ccc|c} 3 & 4 & 1 & 6 \\ 2 & 6 & 4 & 5 \\ 5 & 2 & 7 & 10 \end{array}\right]$$

$①/3$; $1 \quad 4/3 \quad 1/3 \mid 2$

$(①/3) \times 2$; $2 \quad 8/3 \quad 2/3 \mid 4 \rightarrow ♡_1$

$② \Leftarrow ② - ♡_1$

update $\longrightarrow$

a_{31}

$$\begin{array}{cccc} & a'_{22} & a'_{23} & b'_2 \end{array}$$
$$\left[\begin{array}{ccc|c} 3 & 4 & 1 & 6 \\ 0 & 6-\frac{8}{3} & 4-\frac{2}{3} & 1 \\ 5 & 2 & 7 & 10 \end{array}\right]$$

$①/3$; $1 \quad 4/3 \quad 1/3 \mid 2$

$(①/3) \times 5$; $5 \quad 20/3 \quad 5/3 \mid 10 \rightarrow ♡_2$

$③ \Leftarrow ③ - ♡_2$

update $\longrightarrow$

a_{32}

$$\left[\begin{array}{ccc|c} 3 & 4 & 1 & 6 \\ 0 & 6-\frac{8}{3} & 4-\frac{2}{3} & 1 \\ 0 & 2-\frac{20}{3} & 7-\frac{5}{3} & 0 \end{array}\right]$$

$②/a'_{22}$

$(②/a'_{22}) \cdot a'_{32} \rightarrow ♡_3$

$③ \Leftarrow ③ - ♡_3$

update $\longrightarrow$

$$\begin{bmatrix} 3 & 4 & 1 & 6 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & 0 & a''_{33} & b''_3 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a'_{22} & a'_{23} \\ 0 & 0 & a''_{33} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b'_2 \\ b''_3 \end{bmatrix}$$

ได้จากย้ายข้าง
สมการ $Ax = B$

$$\begin{cases} x_3 = b''_3 / a''_{33} \\ x_2 = (b'_2 - a'_{23})x_3 / a'_{22} \\ x_1 = (b_1 - a'_{12}x_2 - a_{13}x_3)/a_{11} \end{cases}$$

$$\begin{matrix} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ a_{21} & a_{22} & a_{23} & a_{24} & b_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & b_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & b_4 \end{matrix}$$

1. $\cancel{a_{21}}$ $(①/a_{11})\cdot a_{21}$ — $\heartsuit_1$

$② \Leftarrow ② - \heartsuit_1$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ 0 & a'_{22} & a'_{23} & a'_{24} & b'_2 \\ a_{31} & a_{32} & a_{33} & a_{34} & b_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & b_4 \end{array}\right]$$

2 $\cancel{a_{31}}$ $(①/a_{11})\cdot a_{31}$ — $\heartsuit_2$

$③ \Leftarrow ③ - \heartsuit_2$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ 0 & a'_{22} & a'_{23} & a'_{24} & b'_2 \\ 0 & a'_{32} & a'_{33} & a'_{34} & b'_3 \\ a_{41} & a_{42} & a_{43} & a_{44} & b_4 \end{array}\right]$$

3 $\cancel{a_{41}}$ $(①/a_{11})\cdot a_{41}$ — $\heartsuit$

$④ \Leftarrow ④ - \heartsuit$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ 0 & a'_{22} & a'_{23} & a'_{24} & b'_2 \\ 0 & a'_{32} & a'_{33} & a'_{34} & b'_3 \\ 0 & a'_{42} & a'_{43} & a'_{44} & b'_4 \end{array}\right]$$

4. $\cancel{a'_{32}}$ $(②/a'_{22})\cdot a'_{32}$ — $\heartsuit_4$

$③ \Leftarrow ③ - \heartsuit_4$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ 0 & a'_{22} & a'_{23} & a'_{24} & b'_2 \\ 0 & 0 & a''_{33} & a''_{34} & b''_3 \\ 0 & a'_{42} & a'_{43} & a'_{44} & b'_4 \end{array}\right]$$

5. $\cancel{a'_{42}}$ $(②/a'_{22})\cdot a'_{42}$ — $\heartsuit_5$

$④ \Leftarrow ④ - \heartsuit_5$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ 0 & a'_{22} & a'_{23} & a'_{24} & b'_2 \\ 0 & 0 & a''_{33} & a''_{34} & b''_3 \\ 0 & 0 & a''_{43} & a''_{44} & b''_4 \end{array}\right]$$

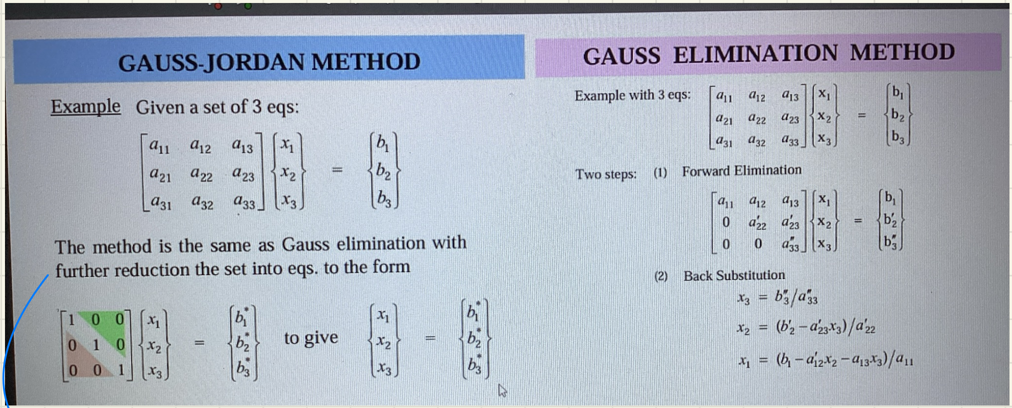
6. $\cancel{a''_{43}}$ $(③/a'_{33})\cdot a'_{43}$ — $\heartsuit_6$

$④ \Leftarrow ④ - \heartsuit_6$

$$\left[\begin{array}{cccc|c} a_{11} & a_{12} & a_{13} & a_{14} & b_1 \\ 0 & a'_{22} & a'_{23} & a'_{24} & b'_2 \\ 0 & 0 & a''_{33} & a''_{34} & b''_3 \\ 0 & 0 & 0 & a'''_{44} & b'''_4 \end{array}\right]$$

แทนค่าย้อนกลับ

Gauss Jordan



After Gauss Eliminate

$$\left[\begin{array}{ccc|c} a_{11} & \cancel{a_{12}} & \cancel{a_{13}} & b_1 \\ 0 & a'_{22} & \cancel{a'_{23}} & b'_2 \\ 0 & 0 & a''_{33} & b''_3 \end{array}\right] \xrightarrow[\text{(Gauss Jordan)}]{\text{eliminate } a_{12},\, a_{13},\, a'_{23}} \left[\begin{array}{ccc|c} a''_{11} & 0 & 0 & b''_1 \\ 0 & a''_{22} & 0 & b''_2 \\ 0 & 0 & a''_{33} & b''_3 \end{array}\right] \longrightarrow \begin{array}{c} ①/a''_{11} \\ ②/a''_{22} \\ ③/a''_{33} \end{array} \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & b^*_1 \\ 0 & 1 & 0 & b^*_2 \\ 0 & 0 & 1 & b^*_3 \end{array}\right]$$

$$\left.\begin{array}{l} (①/a_{11})\,a_{21} - ♡ \\ ② \Leftarrow ② - ♡ \end{array}\right\} E$$

① $$E_1 \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right]$$

② $$E_2E_1 \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & a'_{32} & a'_{33} & b'_3 \end{array}\right]$$

③ $$E_3E_2E_1 \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a'_{22} & a'_{23} & b'_2 \\ 0 & 0 & a''_{33} & b''_3 \end{array}\right]$$

④ $$E_4E_3E_2E_1 \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a''_{22} & 0 & b''_2 \\ 0 & 0 & a''_{33} & b''_3 \end{array}\right]$$

⑤ $$E_5E_4E_3E_2E_1 \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a'_{11} & a'_{12} & 0 & b'_1 \\ 0 & a''_{22} & 0 & b''_2 \\ 0 & 0 & a''_{33} & b''_3 \end{array}\right]$$

⑥ $$E_6E_5E_4E_3E_2E_1 \left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ a_{21} & a_{22} & a_{23} & b_2 \\ a_{31} & a_{32} & a_{33} & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a''_{11} & 0 & 0 & b''_1 \\ 0 & a''_{22} & 0 & b''_2 \\ 0 & 0 & a''_{33} & b''_3 \end{array}\right]$$

⑦ $$\begin{array}{c} ①/a''_{11} \\ ②/a''_{22} \\ ③/a''_{33} \end{array} \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & b^*_1 \\ 0 & 1 & 0 & b^*_2 \\ 0 & 0 & 1 & b^*_3 \end{array}\right]$$

$$E_n \dots E_1 A = I \; ; \quad I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_{2\times2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I_{n\times n} = \begin{cases} 1 & \text{เมื่อ } i = j \\ 0 & \text{เมื่อ } i \neq j \end{cases}$$

Matrix Inversion

$Ax = B \quad ; \quad x_1, x_2, x_3 = ?$

เอา A^{-1} คูณทั้งสองสมการ

$A^{-1}Ax = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$\cancel{A^{-1} = \frac{1}{\det A} \cdot \text{adj } A}$

เราจะไม่หา A^{-1} ด้วยวิธีนี้แต่จะใช้ $E_n \dots E_1 A = I$ และเอา A^{-1} คูณทั้งสองสมการ

$E_n \dots E_1 A A^{-1} = I A^{-1}$

$\downarrow$

* $E_n \dots E_1 I = A^{-1}$

MATRIX INVERSION

Given $[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ Want $[A]^{-1} = ?$

IDEA: Since $\underset{(3\times3)}{[A]} \; \underset{(3\times3)}{[A]^{-1}} = \underset{(3\times3)}{[I]}$

EXAMPLE 4 Compute A^{-1} if

$$A = \begin{bmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$$

Solution

$$\begin{array}{cc} A & I \\ \end{array} \qquad\qquad \begin{array}{cc} E_2E_1A & I' \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ \cancel{-1} & -2 & 0 & 0 & 1 & 0 \\ \cancel{2} & 2 & 3 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ 0 & 2 & 3 & 1 & 1 & 0 \\ 0 & \cancel{-6} & -3 & -2 & 0 & 1 \end{array}\right]$$

$$\begin{array}{cc} E_3E_2E_1A & I'' \end{array} \qquad\qquad \begin{array}{cc} E_5 \dots E_1 A & I''' \end{array}$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 4 & \cancel{3} & 1 & 0 & 0 \\ 0 & 2 & \cancel{3} & 1 & 1 & 0 \\ 0 & 0 & 6 & 1 & 3 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & \cancel{4} & 0 & \frac{1}{2} & -\frac{3}{2} & -\frac{1}{2} \\ 0 & 2 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 6 & 1 & 3 & 1 \end{array}\right]$$

$$\begin{array}{cc} E_6 \dots E_1 A & I'''' \end{array} \qquad\qquad \begin{array}{cc} I & A^{-1} \end{array}$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 2 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 6 & 1 & 3 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{6} & \frac{1}{2} & \frac{1}{6} \end{array}\right]$$

Check : $A \cdot A^{-1} = I$

$$\begin{bmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} -1/2 & -1/2 & 1/2 \\ 1/4 & -1/4 & -1/4 \\ 1/6 & 1/2 & 1/6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore \quad X = A^{-1}B$

Example $\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix}$ จงหา x_1, x_2, x_3 โดยใช้ 1. Gauss Jordan 2. Matrix Inversion

$(x_1 = 450,\ x_2 = 350,\ x_3 = 275)$

1. Gauss Jordan

$$\left[\begin{array}{ccc|c} 4 & -4 & 0 & 400 \\ -1 & 4 & -2 & 400 \\ 0 & -2 & 4 & 400 \end{array}\right] \xrightarrow[② - ♡]{(①/4)(-1) - ♡} \left[\begin{array}{ccc|c} 4 & -4 & 0 & 400 \\ 0 & 3 & -2 & 500 \\ 0 & -2 & 4 & 400 \end{array}\right] \xrightarrow[③ - ♡]{(②/3)(-2) - ♡} \left[\begin{array}{ccc|c} 4 & -4 & 0 & 400 \\ 0 & 3 & -2 & 500 \\ 0 & 0 & 8/3 & \frac{2200}{3} \end{array}\right] \xrightarrow[② - ♡]{(③/\frac{8}{3})(-2) - ♡} \left[\begin{array}{ccc|c} 4 & -4 & 0 & 400 \\ 0 & 3 & 0 & 1050 \\ 0 & 0 & 8/3 & \frac{2200}{3} \end{array}\right]$$

$$\xrightarrow[① - ♡]{(②/3)(-4) - ♡} \left[\begin{array}{ccc|c} 4 & 0 & 0 & 1800 \\ 0 & 3 & 0 & 1050 \\ 0 & 0 & 8/3 & \frac{2200}{3} \end{array}\right] \xrightarrow[③/\frac{8}{3}]{①/4,\ ②/3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 450 \\ 0 & 1 & 0 & 350 \\ 0 & 0 & 1 & 275 \end{array}\right]$$

$\therefore\ x_1 = 450,\ x_2 = 350,\ x_3 = 275$ #

2. Matrix Inversion

$$\left[\begin{array}{ccc|ccc} 4 & -4 & 0 & 1 & 0 & 0 \\ -1 & 4 & -2 & 0 & 1 & 0 \\ 0 & -2 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[② - ♡]{(①/4)(-1) - ♡} \left[\begin{array}{ccc|ccc} 4 & -4 & 0 & 1 & 0 & 0 \\ 0 & 3 & -2 & 1/4 & 1 & 0 \\ 0 & -2 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[③ - ♡]{(②/3)(-2) - ♡} \left[\begin{array}{ccc|ccc} 4 & -4 & 0 & 1 & 0 & 0 \\ 0 & 3 & -2 & 1/4 & 1 & 0 \\ 0 & 0 & 8/3 & 1/6 & 2/3 & 1 \end{array}\right]$$

$$\xrightarrow[② - ♡]{(③/\frac{8}{3})(-2) - ♡} \left[\begin{array}{ccc|ccc} 4 & -4 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & 3/8 & 3/2 & 3/4 \\ 0 & 0 & 8/3 & 1/6 & 2/3 & 1 \end{array}\right] \xrightarrow[① - ♡]{(②/3)(-4) - ♡} \left[\begin{array}{ccc|ccc} 4 & 0 & 0 & 3/2 & 2 & 1 \\ 0 & 3 & 0 & 3/8 & 3/2 & 3/4 \\ 0 & 0 & 8/3 & 1/6 & 2/3 & 1 \end{array}\right] \xrightarrow[③/\frac{8}{3}]{①/4,\ ②/3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/8 & 1/2 & 1/4 \\ 0 & 1 & 0 & 1/8 & 1/2 & 1/4 \\ 0 & 0 & 1 & 1/16 & 1/4 & 3/8 \end{array}\right]$$

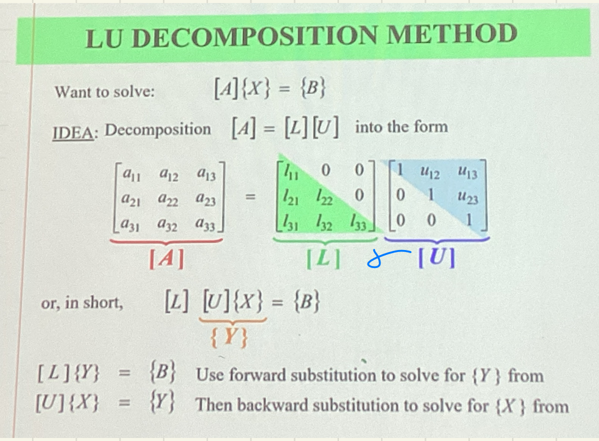
Check $AA^{-1} = I$

$$\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 3/8 & 1/2 & 1/4 \\ 1/8 & 1/2 & 1/4 \\ 1/16 & 1/4 & 3/8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore\ x = A^{-1}B = \begin{bmatrix} 3/8 & 1/2 & 1/4 \\ 1/8 & 1/2 & 1/4 \\ 1/16 & 1/4 & 3/8 \end{bmatrix} \begin{bmatrix} 400 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 450 \\ 350 \\ 275 \end{bmatrix}$$

$x_1 = 450,\ x_2 = 350,\ x_3 = 275$ #

LU Decomposition Method



LU DECOMPOSITION METHOD

Want to solve: $[A]\{X\} = \{B\}$

IDEA: Decomposition $[A] = [L][U]$ into the form

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{[A]} = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}}_{[U]}$$

or, in short, $[L]\underbrace{[U]\{X\}}_{\{Y\}} = \{B\}$

$[L]\{Y\} = \{B\}$ Use forward substitution to solve for $\{Y\}$ from

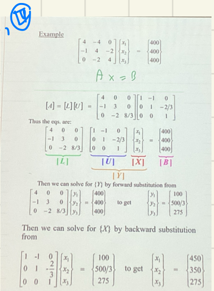
$[U]\{X\} = \{Y\}$ Then backward substitution to solve for $\{X\}$ from

$$A x = B$$

Lower $[L]$ Upper $[U]$ โดยใช้การคูณ matrix

$$\begin{array}{c} A \\ \begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix} \end{array} = \begin{array}{c} L \\ \begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 8/3 \end{bmatrix} \end{array} \cdot \begin{array}{c} u \\ \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

II, III



Example

$$\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix}$$

$A x = B$

$$[A] = [L][U] = \begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 8/3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus the eqs. are:

$$\underbrace{\begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 8/3 \end{bmatrix}}_{[L]} \underbrace{\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}}_{[U]} \underbrace{\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}}_{[X]}}_{[Y]} = \underbrace{\begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix}}_{[B]}$$

Then we can solve for $\{Y\}$ by forward substitution from

$$\begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 8/3 \end{bmatrix}\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix} \quad \text{to get} \quad \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} 100 \\ 500/3 \\ 275 \end{Bmatrix}$$

Then we can solve for $\{X\}$ by backward substitution from

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 100 \\ 500/3 \\ 275 \end{Bmatrix} \quad \text{to get} \quad \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 450 \\ 350 \\ 275 \end{Bmatrix}$$

Step I ; Decompose A to LU ($L\widetilde{Ux} = B$) ได้ $Ux = Y$ } forward

Step II ; $LY = B$ แก้สมการหาค่า Y โดยใช้คุณสมบัติการคูณ

Step III ; หาค่า x จาก $Ux = Y$ } backward

① หา L, U

$$\begin{array}{c} a \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \end{array} = \begin{array}{c} l \\ \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \end{array} \cdot \begin{array}{c} u \\ \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

} ต้องการ 9 สมการ

$$\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix}$$

	สูตร	แทนค่า
1. r_1c_1 ; $a_{11} = l_{11}\cdot 1 + 0\cdot 0 + 0\cdot 0$	$l_{11} = a_{11}$	$l_{11} = 4$
2. r_1c_2 ; $a_{12} = l_{11}\cdot u_{12} + 0\cdot 1 + 0\cdot 0$	$u_{12} = a_{12}/l_{11}$	$u_{12} = -4/4 = -1$
3. r_1c_3 ; $a_{13} = l_{11}\cdot u_{13} + 0\cdot u_{23} + 0\cdot 1$	$u_{13} = a_{13}/l_{11}$	$u_{13} = 0/4 = 0$
4. r_2c_1 ; $a_{21} = l_{21}\cdot 1 + l_{22}\cdot 0 + 0\cdot 0$	$l_{21} = a_{21}$	$l_{21} = -1$
5. r_2c_2 ; $a_{22} = l_{21}\cdot u_{12} + l_{22}\cdot 1 + 0\cdot 0$	$l_{22} = a_{22} - l_{21}\cdot u_{12}$	$l_{22} = 4 - (-1\cdot -1) = 3$
6. r_2c_3 ; $a_{23} = l_{21}\cdot u_{13} + l_{22}\cdot u_{23} + 0\cdot 1$	$u_{23} = \dfrac{a_{23} - l_{21}\cdot u_{13}}{l_{22}}$	$u_{23} = \dfrac{-2-(-1\cdot 0)}{3} = -2/3$
7. r_3c_1 ; $a_{31} = l_{31}\cdot 1 + l_{32}\cdot 0 + l_{33}\cdot 0$	$l_{31} = a_{31}$	$l_{31} = 0$
8. r_3c_2 ; $a_{32} = l_{31}\cdot u_{12} + l_{32}\cdot 1 + l_{33}\cdot 0$	$l_{32} = a_{32} - l_{31}\cdot u_{12}$	$l_{32} = -2 - (0\cdot -1) = -2$
9. r_3c_3 ; $a_{33} = l_{31}\cdot u_{13} + l_{32}\cdot u_{23} + l_{33}\cdot 1$	$l_{33} = a_{33} - (l_{31}\cdot u_{13}) - (l_{32}\cdot u_{23})$	$l_{33} = 4 - (0\cdot 0) - (-2\cdot -\frac{2}{3}) = 8/3$

Cholesky Decomposition Method

CHOLESKY DECOMPOSITION METHOD

IDEA : Same as LU decomposition for solving

$$[A]\{X\} = \{B\}$$

$Ax = [$

$L \; L^T$

When $[A]$ is **symmetric matrix** such that

$$[A] = [L][L]^T$$

$a_{ij} = a_{ji}$

or, in detail,

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}}_{[A]} = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}}_{[L]} \underbrace{\begin{bmatrix} l_{11} & l_{21} & l_{21} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}}_{[L]^T}$$

The coefficients in $[L]$ matrix can be determined as follows:

$$Ax = B$$

$$A \to L L^T$$

Step I $\quad L \underbrace{L^T x}_{Y} = B$

Step II $\quad LY = B$

Step III $\quad L^T x = B$

$$\overset{a}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}} = \overset{l}{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}} \cdot \overset{l^t}{\begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}}$$

ต้องการ 6 สมการ

$$\begin{bmatrix} 4 & -4 & 0 \\ -1 & 4 & -2 \\ 0 & -2 & 4 \end{bmatrix}$$

	สูตร	แทนค่า
1. $r_1 c_1$; $a_{11} = l_{11}^2$	$l_{11} = \sqrt{a_{11}}$	
2. $r_1 c_2$; $a_{12} = l_{11} \cdot l_{21} + 0 \cdot l_{22} + 0 \cdot 0$	$l_{21} = a_{12}/l_{11}$	
3. $r_1 c_3$; $a_{13} = l_{11} \cdot l_{31} + 0 \cdot l_{32} + 0 \cdot l_{33}$	$l_{31} = a_{13}/l_{11}$	
4. $r_2 c_2$; $a_{22} = l_{21}^2 + l_{22}^2 + 0 \cdot 0$	$l_{22} = \sqrt{a_{22} - l_{21}^2}$	
5. $r_2 c_3$; $a_{23} = l_{21} \cdot l_{31} + l_{22} \cdot l_{32} + 0 \cdot l_{33}$	$l_{32} = \dfrac{a_{23} - (l_{21} \cdot l_{31})}{l_{22}}$	
6. $r_3 c_3$; $a_{33} = l_{31}^2 + l_{32}^2 + l_{33}^2$	$l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2}$	

LU Decompose example

$$-2x_1 + 3x_2 + x_3 = 9$$
$$3x_1 + 4x_2 - 5x_3 = 0$$
$$x_1 - 2x_2 + x_3 = -4$$

$$\begin{bmatrix} -2 & 3 & 1 \\ 3 & 4 & -5 \\ 1 & -2 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 9 \\ 0 \\ -4 \end{bmatrix}$$

$$a = l \cdot u$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \cdot \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

00	01	02		a_{11}	a_{12}	a_{13}
10	11	12		a_{21}	a_{22}	a_{23}
20	21	22		a_{31}	a_{32}	a_{33}

	สูตร	แทนค่า
1. r_1c_1 ; $a_{11} = l_{11}\cdot 1 + 0\cdot 0 + 0\cdot 0$	$l_{11} = a_{11}$	$l_{11} = -2$
2. r_1c_2 ; $a_{12} = l_{11}\cdot u_{12} + 0\cdot 1 + 0\cdot 0$	$u_{12} = a_{12}/l_{11}$	$u_{12} = 3/-2$
3. r_1c_3 ; $a_{13} = l_{11}\cdot u_{13} + 0\cdot u_{23} + 0\cdot 1$	$u_{13} = a_{13}/l_{11}$	$u_{13} = \frac{1}{-2} = -\frac{1}{2}$
4. r_2c_1 ; $a_{21} = l_{21}\cdot 1 + l_{22}\cdot 0 + 0\cdot 0$	$l_{21} = a_{21}$	$l_{21} = 3$
5. r_2c_2 ; $a_{22} = l_{21}\cdot u_{12} + l_{22}\cdot 1 + 0\cdot 0$	$l_{22} = a_{22} - l_{21}\cdot u_{12}$	$l_{22} = 4 - (3\cdot\frac{3}{-2}) = 17/2$
6. r_2c_3 ; $a_{23} = l_{21}\cdot u_{13} + l_{22}\cdot u_{23} + 0\cdot 1$	$u_{23} = \frac{a_{23} - l_{21}\cdot u_{13}}{l_{22}}$	$u_{23} = \frac{-5-(3\cdot -\frac{1}{2})}{17/2} = -7/17$
7. r_3c_1 ; $a_{31} = l_{31}\cdot 1 + l_{32}\cdot 0 + l_{33}\cdot 0$	$l_{31} = a_{31}$	$l_{31} = 1$
8. r_3c_2 ; $a_{32} = l_{31}\cdot u_{12} + l_{32}\cdot 1 + l_{33}\cdot 0$	$l_{32} = a_{32} - l_{31}\cdot u_{12}$	$l_{32} = -2 - (1\cdot -\frac{3}{2}) = -1/2$
9. r_3c_3 ; $a_{33} = l_{31}\cdot u_{13} + l_{32}\cdot u_{23} + l_{33}\cdot 1$	$l_{33} = a_{33} - (l_{31}\cdot u_{13}) - (l_{32}\cdot u_{23})$	$l_{33} = 1-(1\cdot -\frac{1}{2}) - (-\frac{1}{2}\cdot -\frac{7}{17}) = \frac{22}{17}$

$$A = LU = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 17/2 & 0 \\ 1 & -1/2 & 22/17 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3/2 & -1/2 \\ 0 & 1 & -7/17 \\ 0 & 0 & 1 \end{bmatrix}$$

จาก $LUx = B$ ให้ $Ux = Y$; $LY = B$

$$\begin{bmatrix} -2 & 0 & 0 \\ 3 & 17/2 & 0 \\ 1 & -1/2 & 22/17 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ -4 \end{bmatrix}$$

$$\left.\begin{aligned} -2y_1 &= 9 \\ 3y_1 + \frac{17y_2}{2} &= 0 \\ y_1 - \frac{y_2}{2} + \frac{22y_3}{17} &= -4 \end{aligned}\right\}$$

$$y_1 = \frac{-9}{2}$$

$$y_2 = \frac{(-3y_1\cdot 2)}{17} = 27/17$$

$$y_3 = \frac{\left(-4 - y_1 + \frac{y_2}{2}\right)\cdot 17}{22} = 1$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -9/2 \\ 27/17 \\ 1 \end{bmatrix}$$

แทน Y ใน $Ux = Y$

$$\begin{bmatrix} 1 & 3/2 & -1/2 \\ 0 & 1 & -7/17 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} -9/2 \\ 27/17 \\ 1 \end{bmatrix}$$

$$\begin{aligned} x_1 - \frac{3x_2}{2} - \frac{x_3}{2} &= -\frac{9}{2} \\ x_2 - \frac{7x_3}{17} &= 27/17 \\ x_3 &= 1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} x_3 &= 1 \\ x_2 &= \frac{27}{17} + \frac{7}{17} = 2 \\ x_1 &= -\frac{9}{2} + \frac{6}{2} + \frac{1}{2} = -1 \end{aligned}$$

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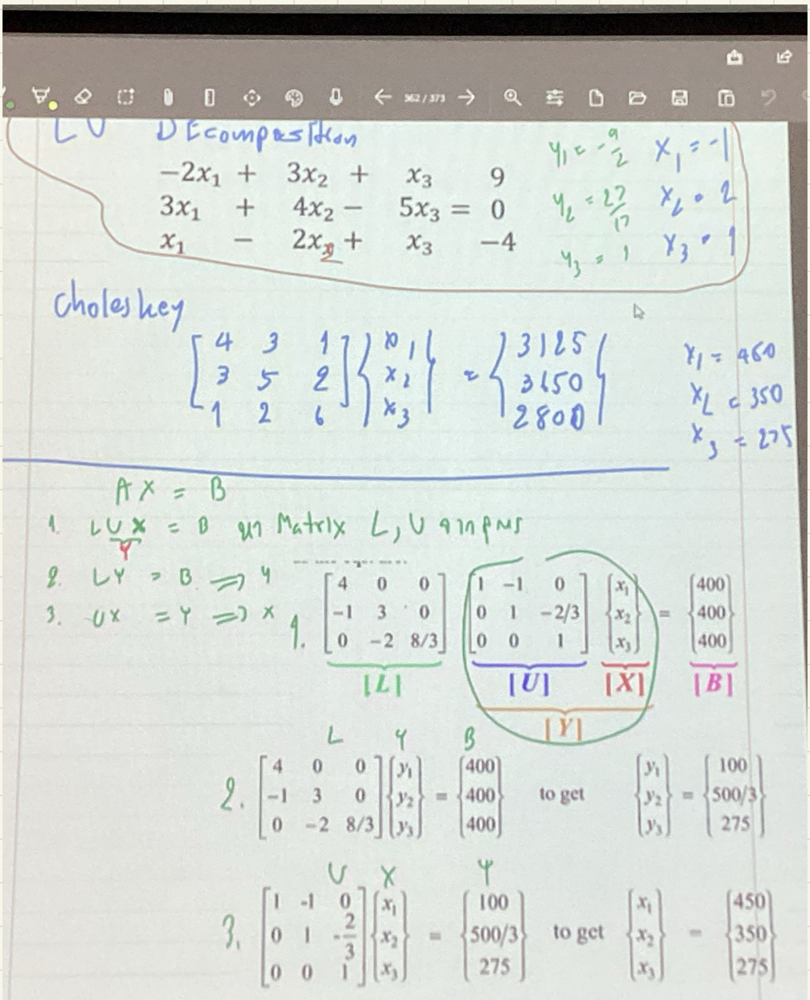
LU Decomposition

$$\begin{aligned} -2x_1 + 3x_2 + x_3 &\quad 9 \\ 3x_1 + 4x_2 - 5x_3 &= 0 \\ x_1 - 2x_2 + x_3 &\quad -4 \end{aligned}$$

$y_1 = -\frac{9}{2}$ $x_1 = -1$

$y_2 = \frac{27}{17}$ $x_2 = 2$

$y_3 = 1$ $x_3 = 1$

Cholesky

$$\begin{bmatrix} 4 & 3 & 1 \\ 3 & 5 & 2 \\ 1 & 2 & 6 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 3125 \\ 3150 \\ 2800 \end{Bmatrix}$$

$x_1 = 460$, $x_2 = 350$, $x_3 = 275$

$AX = B$

1. $LUX = B$ หา Matrix L, U ออกมา
2. $LY = B \Rightarrow Y$
3. $UX = Y \Rightarrow X$

1. $$\underbrace{\begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 8/3 \end{bmatrix}}_{[L]} \underbrace{\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{bmatrix}}_{[U]} \underbrace{\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}}_{[X]}}_{[Y]} = \underbrace{\begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix}}_{[B]}$$

2. $$\begin{bmatrix} 4 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 8/3 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} 400 \\ 400 \\ 400 \end{Bmatrix} \text{ to get } \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} 100 \\ 500/3 \\ 275 \end{Bmatrix}$$

3. $$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 100 \\ 500/3 \\ 275 \end{Bmatrix} \text{ to get } \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 450 \\ 350 \\ 275 \end{Bmatrix}$$

Choleskey

$$\begin{bmatrix} 4 & 3 & 1 \\ 3 & 5 & 2 \\ 1 & 2 & 6 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 3125 \\ 3650 \\ 2800 \end{bmatrix}$$

$$\underset{a}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}} = \underset{l}{\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}} \cdot \underset{l^t}{\begin{bmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{bmatrix}}$$

ต้องการ 6 สมการ

1. r_1c_1 ; $a_{11} = l_{11}^2$
2. r_1c_2 ; $a_{12} = l_{11} \cdot l_{21} + 0 \cdot l_{22} + 0 \cdot 0$
3. r_1c_3 ; $a_{13} = l_{11} \cdot l_{31} + 0 \cdot l_{32} + 0 \cdot l_{33}$
4. r_2c_2 ; $a_{22} = l_{21}^2 + l_{22}^2 + 0 \cdot 0$
5. r_2c_3 ; $a_{23} = l_{21} \cdot l_{31} + l_{22} \cdot l_{32} + 0 \cdot l_{33}$
6. r_3c_3 ; $a_{33} = l_{31}^2 + l_{32}^2 + l_{33}^2$

สูตร

$l_{11} = \sqrt{a_{11}}$

$l_{21} = a_{12}/l_{11}$

$l_{31} = a_{13}/l_{11}$

$l_{22} = \sqrt{a_{22} - l_{21}^2}$

$l_{32} = \dfrac{a_{23} - (l_{21} \cdot l_{31})}{l_{22}}$

$l_{33} = \sqrt{a_{33} - l_{31}^2 - l_{32}^2}$

แทนค่า

$l_{11} = 2$

$l_{21} = 3/2$

$l_{31} = 1/2$

$l_{22} = \sqrt{5 - \left(\frac{3}{2}\right)^2} = \frac{\sqrt{11}}{2}$

$l_{32} = \dfrac{2 - \left(\frac{3}{2} \cdot \frac{1}{2}\right)}{\frac{\sqrt{11}}{2}} = \frac{5\sqrt{11}}{22}$

$l_{33} = \sqrt{6 - \frac{1}{4} - \left(\frac{5\sqrt{11}}{22}\right)^2} = \frac{\sqrt{627}}{11}$

$$A = LL^t = \begin{bmatrix} 2 & 0 & 0 \\ 3/2 & \sqrt{11}/2 & 0 \\ 1/2 & \frac{5\sqrt{11}}{22} & \frac{\sqrt{627}}{11} \end{bmatrix} \cdot \begin{bmatrix} 2 & 3/2 & 1/2 \\ 0 & \sqrt{11}/2 & \frac{5\sqrt{11}}{22} \\ 0 & 0 & \frac{\sqrt{627}}{11} \end{bmatrix}$$

จาก $LL^t x = B$ ได้ $L^t x = Y$

หา Y จาก $LY = B$

$$\begin{bmatrix} 2 & 0 & 0 \\ 3/2 & \sqrt{11}/2 & 0 \\ 1/2 & \frac{5\sqrt{11}}{22} & \frac{\sqrt{627}}{11} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{bmatrix} 3125 \\ 3650 \\ 2800 \end{bmatrix}$$

$2y_1 = 3125 \longrightarrow y_1 = \frac{3125}{2}$

$\frac{3}{2}y_1 + \frac{\sqrt{11}\,y_2}{2} = 3650 \longrightarrow y_2 = \dfrac{\left(3650 - \frac{3}{2} \cdot \frac{3125}{2}\right) \times 2}{\sqrt{11}} = \dfrac{475\sqrt{11}}{2}$

$\frac{1}{2}y_1 + \frac{5\sqrt{11}}{22}y_2 + \frac{\sqrt{627}}{11}y_3 = 2800$

$\hookrightarrow y_3 = \dfrac{\left(2800 - \frac{1}{2}\left(\frac{3125}{2}\right) - \frac{5\sqrt{11}}{22}\left(\frac{475\sqrt{11}}{2}\right)\right) 11}{\sqrt{627}}$

$= 25\sqrt{627}$

line y $)_{26}$ $\quad L^t x = y$

$$\begin{bmatrix} 2 & 3/2 & 1/2 \\ 0 & \sqrt{11}/2 & \frac{5\sqrt{11}}{22} \\ 0 & 0 & \frac{\sqrt{627}}{11} \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{bmatrix} 3125/2 \\ \frac{475\sqrt{11}}{2} \\ 25\sqrt{627} \end{bmatrix}$$

$$2x_1 + \frac{3}{2}x_2 + \frac{1}{2}x_3 = \frac{3125}{2}$$

$$\frac{\sqrt{11}}{2}x_2 + \frac{5\sqrt{11}}{22}x_3 = \frac{475\sqrt{11}}{2}$$

$$\frac{\sqrt{627}}{11}x_3 = 25\sqrt{627}$$

$$x_3 = 275$$

$$x_2 = 350$$

$$x_1 = 450$$

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