Final_submission

March 27, 2023

0.1 1. Explore Data

```
[ ]: # get updated version here : https://drive.google.com/drive/folders/
     ↪1Pht5gprUm14P2RaIrVP2fOSpAF8U8cFt?usp=share_link
```

```
[ ]: #import libraries
     import numpy as np
     import pandas as pd
     # import cvxpy as cp
     from sklearn.cluster import KMeans
     import matplotlib.pyplot as plt
     from scipy.stats import shapiro, skew, kurtosis
     from scipy.cluster.hierarchy import linkage, dendrogram
     from scipy.cluster.hierarchy import fcluster
     from statsmodels.tsa.stattools import kpss
     import seaborn as sns
     import statsmodels.api as sm
     import statsmodels.formula.api as smf

     #settings
     pd.set_option('display.max_rows', 500)
     pd.set_option('display.max_columns', 500)
     plt.rcParams["figure.figsize"] = (12, 9)
     import warnings
     warnings.filterwarnings('ignore')
```

Let's dig the data!

```
[ ]: df_index = pd.read_csv("data_challenge_index_prices.csv")
     df_stock = pd.read_csv("data_challenge_stock_prices.csv")
```

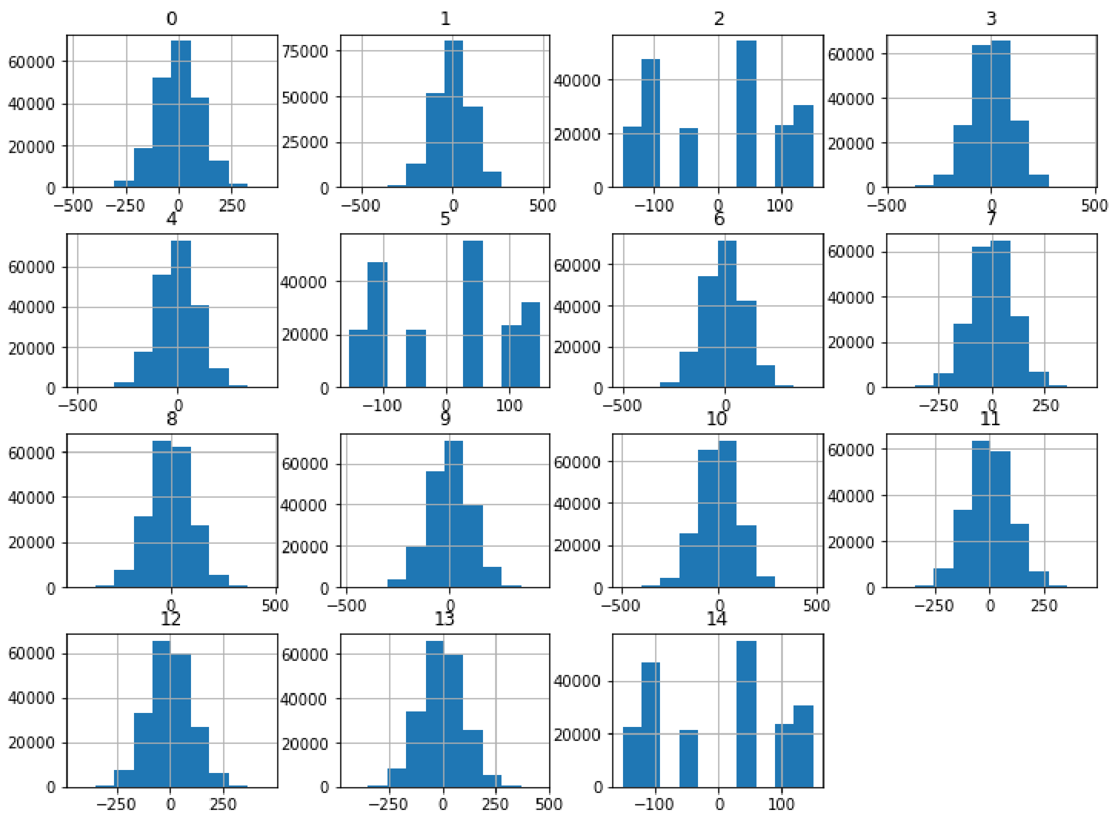
```
#Compute returns and stndardize them
returns_stock = df_stock.pct_change().fillna(0)
returns_stock = ((returns_stock - returns_stock.mean()) / returns_stock.std())*100
returns_index = df_index.pct_change().fillna(0)
returns_index = (((returns_index - returns_index.mean()) / returns_index.std())*100)
```

[ ]:
```
# returns_stock.info()
# returns_index.info()
```

All non - null Values Data type set to float We have a clear visual confirmation that data doesn't have outliers as all stock price lie within 0-250 range that is permissible, so we are not going to perform statistical tests for outlier detection, So we have a clean and nice ready to go data, is it really nice? We need to check some more things wrt to time series

As we know in Finance we depend heavily on Normal distribution, so it's important to look at returns distribution to infer our data better

[ ]:
```
index_corr = returns_index.corr()
returns_index.hist();
```



We have a culprits here lets catch them, we can see 2,5,14 are clearly a non normal distribution, lets call them **special indices** but we need to confirm this statistically and for that we will use Sharpio-wilk as our normality test.

[ ]:
```
# perform Shapiro-Wilk test for each stock and index

def normality_test(df,significance_level = 0.05):

    sw_results = df.apply(lambda x: shapiro(x))
    p_values = sw_results.apply(lambda x: x[1])

    # Get column indices where p-value is less than 0.05
    non_normal_columns = np.where(p_values < 0.05)[0]
    non_normal_columns = non_normal_columns.astype(int)

    return (non_normal_columns)
```

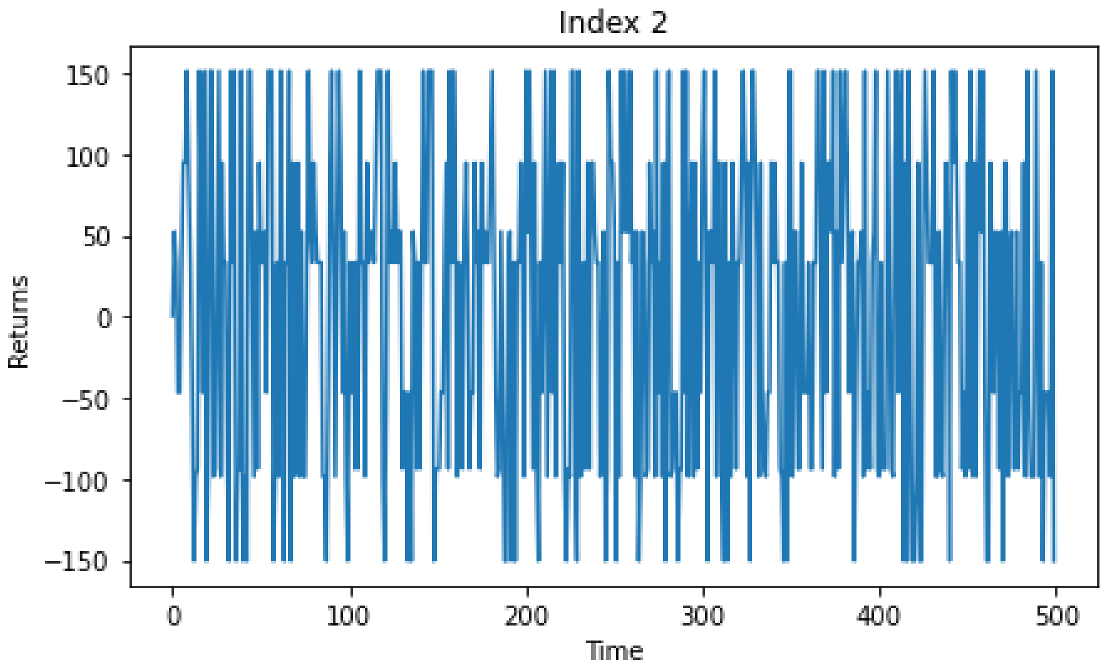
[ ]:
```
print(normality_test(returns_stock))
print(normality_test(returns_index))
```

```
[]
[ 1  2  3  5  9 10 11 12 13 14]
```

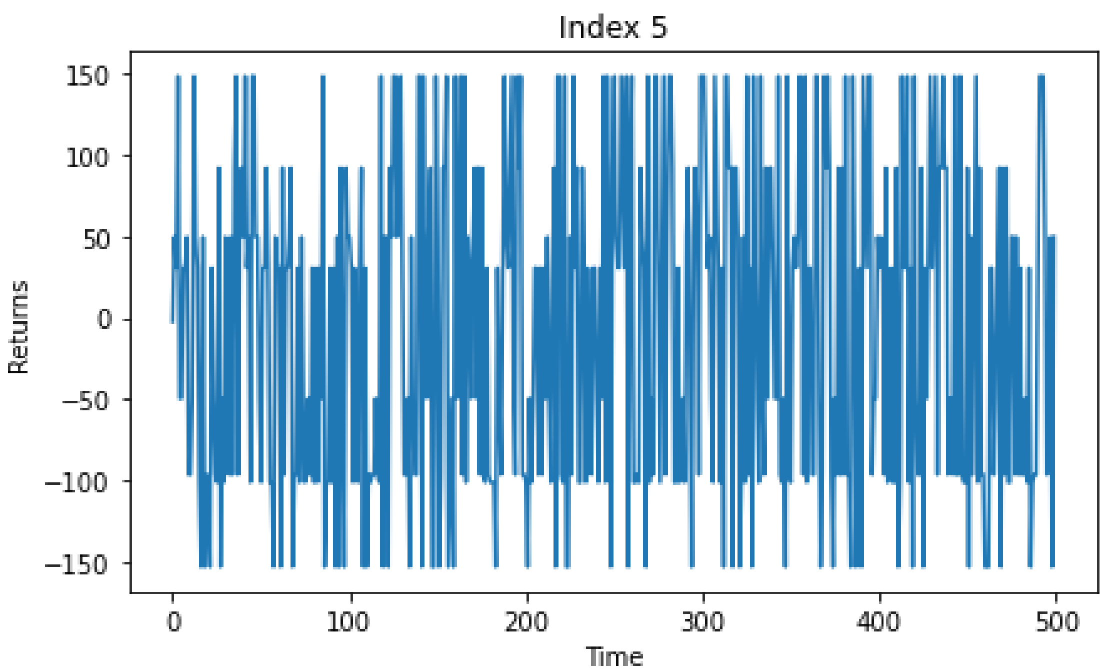
We see p-values less than threshold hence we reject the null hypothesis that data comes from normal distribution. Inferences : Stocks are normally distributed, Some indices are not

[ ]:
```
special_indexes = [2,5,14]

for i in special_indexes:
    data = returns_index.iloc[:,i]
    plt.rcParams["figure.figsize"] = (7, 4)
    fig, ax = plt.subplots()
    ax.plot(returns_index.iloc[:500,i])
    ax.set_xlabel("Time")
    ax.set_ylabel("Returns")
    ax.set_title(f"Index {i}")
    skewness = skew(data)
    kurt = kurtosis(data)
    print("Skewness: ", skewness)
    print("Kurtosis: ", kurt)
    plt.show()
```

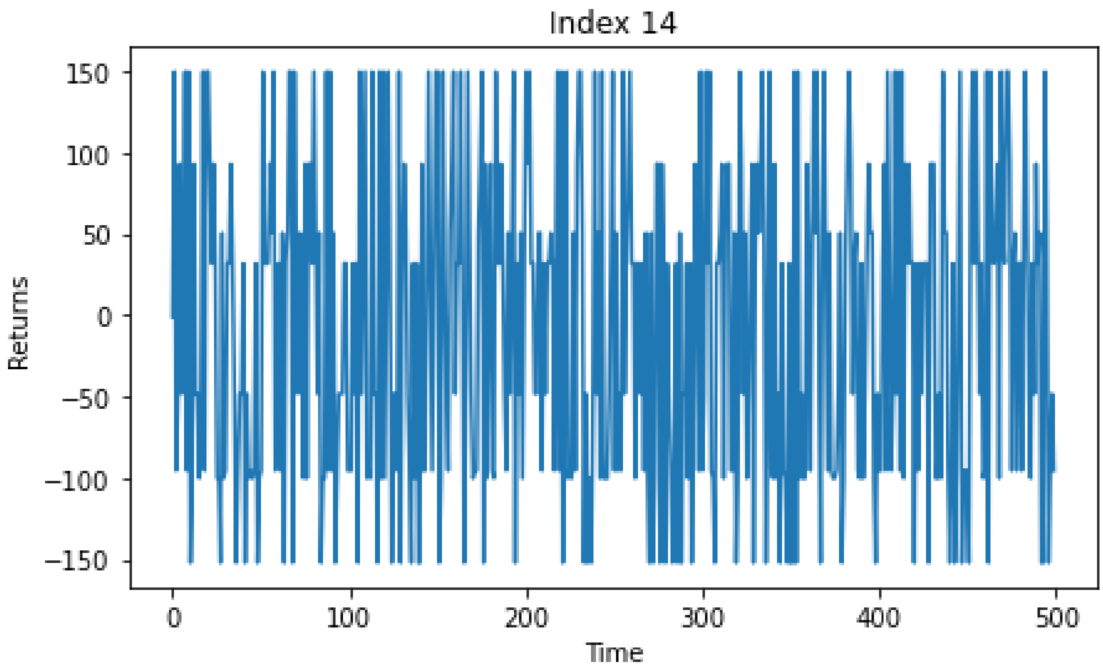
```
Skewness:  0.03362579367076073
Kurtosis:  -1.3149209268902733
```




```
Skewness:  0.007279169710851204
Kurtosis:  -1.3135858562573255
```



```
Skewness:  0.00903016153872343
Kurtosis:  -1.3144583103743086
```



Nice !!! These are indeed special indices, what can we do with this information? Looks like returns are periodic in nature for these indices...How can we use this data to exploit, Let's Try to Decompose this via Fast Fourier Transform FFT

[ ]:
```
def compute_dominant_freq(data):

    # Apply the Fourier transform to the detrended data
    fft = np.fft.fft(data)

    # Compute the frequency axis
    freq = np.fft.fftfreq(data.size)

    # Find the index of the maximum amplitude in the frequency spectrum
    max_idx = np.argmax(abs(fft))

    # Get the dominant frequency from the frequency axis
    dominant_freq = freq[max_idx]

    return dominant_freq
```

[ ]:
```
dominant_freq = compute_dominant_freq(returns_index.iloc[:,2].values)
dominant_freq
```

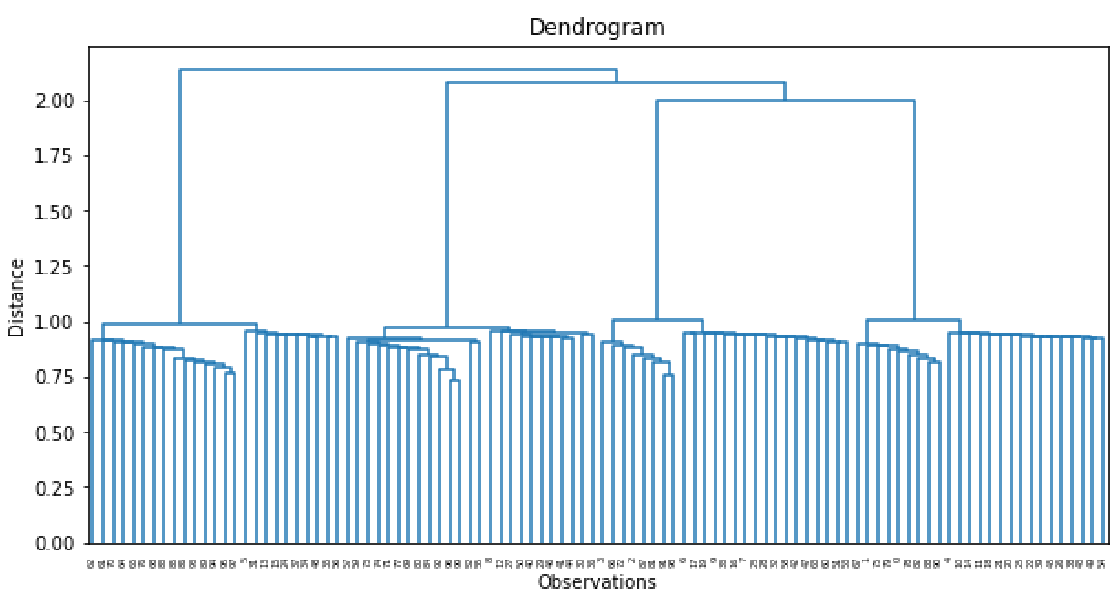
[ ]: 0.008475

1 2. Compute M

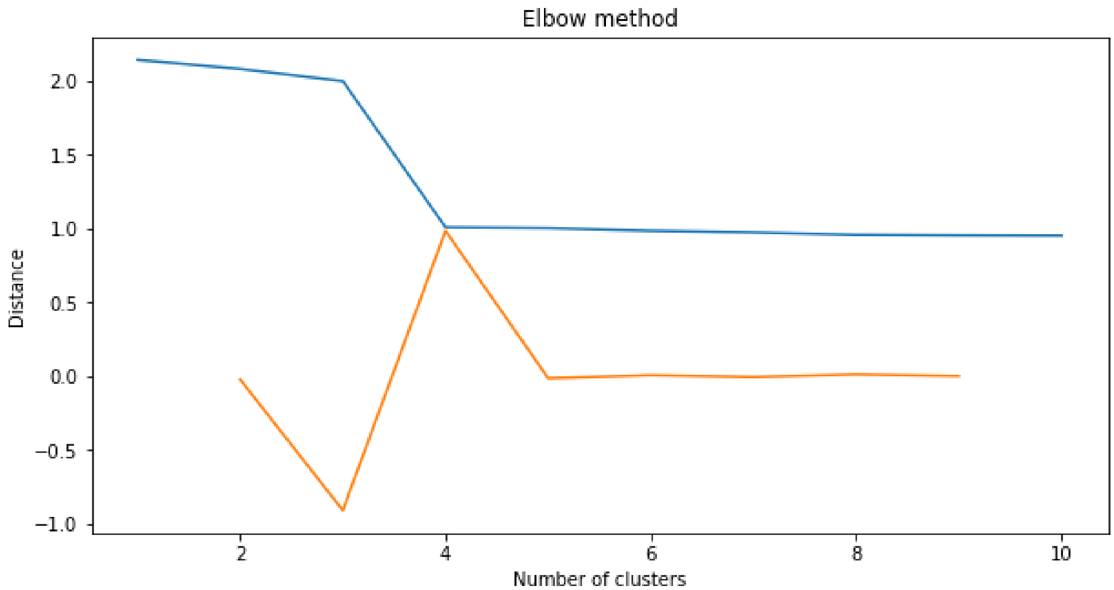
Now After exploring We move to compute M, So Here are the things to try to get No. of Sectors Out of these 100 stocks to explain Indexes: 1. Decompostion of Correlation Matrix of returns 2. Clustering on the basis of Correlation Matrix of returns 3. Denoise and do the FFT of every stock and get the clusters of frequency (To try)

1.0.1 2.1 Clustering on the basis of Correlation Matrix of return

[ ]:
```
# compute the correlation matrix
returns_corr = returns_stock.corr()

# Convert the correlation matrix to a distance matrix
dist_matrix = np.sqrt((1 - returns_corr.abs()).clip(0)) # Distance = sqrt(1 -␣
↪|correlation|)
dist_array = dist_matrix.values[np.triu_indices_from(dist_matrix, k=1)]

# Perform hierarchical clustering
Z = linkage(dist_array, method='ward')

# Plot the dendrogram
plt.figure(figsize=(10, 5))
dendrogram(Z, color_threshold=0)
plt.title("Dendrogram")
plt.xlabel("Observations")
plt.ylabel("Distance")
plt.show()
```



[ ]:
```python
# Determine the optimal number of clusters using the elbow method
last = Z[-10:, 2]
last_rev = last[::-1]
idxs = np.arange(1, len(last) + 1)
plt.figure(figsize=(10, 5))
plt.plot(idxs, last_rev)

# Compute the second derivative of the distances
acceleration = np.diff(last, 2)
acceleration_rev = acceleration[::-1]
plt.plot(idxs[:-2] + 1, acceleration_rev)
plt.title('Elbow method')
plt.xlabel('Number of clusters')
plt.ylabel('Distance')
plt.show()

# Based on the elbow plot, choose the optimal number of clusters
k = acceleration_rev.argmax() + 2
print("Optimal number of clusters:", k)

# Perform agglomerative hierarchical clustering with k clusters
clusters = fcluster(Z, k, criterion='maxclust')
returns_corr['cluster'] = clusters

# Group the stocks by sector based on the cluster labels
sectors = returns_corr.groupby('cluster').groups
for sector, stocks in sectors.items():
    # Convert the stock indexes from string to integer
    stocks = [int(stock) for stock in stocks]
    print(f"Sector {sector}: {stocks}")
```




```
Optimal number of clusters: 4
Sector 1: [5, 13, 15, 24, 31, 34, 35, 37, 48, 56, 61, 62, 64, 65, 68, 70, 76,
83, 85, 86, 89, 93, 94, 95, 97]
Sector 2: [8, 12, 27, 29, 30, 36, 40, 41, 44, 46, 50, 52, 55, 57, 59, 69, 71,
73, 74, 77, 80, 84, 92, 96, 99]
Sector 3: [2, 3, 6, 7, 9, 16, 17, 19, 23, 28, 32, 33, 42, 47, 51, 53, 58, 60,
63, 66, 72, 81, 87, 91, 98]
Sector 4: [0, 1, 4, 10, 11, 14, 18, 20, 21, 22, 25, 26, 38, 39, 43, 45, 49, 54,
67, 75, 78, 79, 82, 88, 90]
```

1.0.2 2.2 Decomposition of Correlation Matrix of return

[ ]:
```
eigenvalues, eigenvectors = np.linalg.eig(returns_corr)
M = np.sum(eigenvalues > 2)
print("Optimal number of clusters:", M)
```

```
Optimal number of clusters: 4
```

[ ]:
```
# Optimal = 4, suseptibility to threshold
```

[ ]:
```
kmeans = KMeans(n_clusters=4, random_state=0)
kmeans.fit(returns_corr)

# The predicted cluster labels for each stock
stock_clusters = kmeans.labels_
```

[ ]:
```
# Map sector labels to stock names
sector_map = {}
for i in range(len(stock_clusters)):
    if stock_clusters[i] not in sector_map:
        sector_map[stock_clusters[i]] = []
    sector_map[stock_clusters[i]].append(i)

# Print out the mapping of sectors to stock names
for sector in sector_map:
    print("Sector {}: {}".format(sector, sector_map[sector]))
```

```
Sector 2: [0, 1, 4, 10, 11, 14, 18, 20, 21, 22, 25, 26, 38, 39, 43, 45, 49, 54,
67, 75, 78, 79, 82, 88, 90]
Sector 3: [2, 3, 6, 7, 9, 16, 17, 19, 23, 28, 32, 33, 42, 47, 51, 53, 58, 60,
63, 66, 72, 81, 87, 91, 98]
Sector 1: [5, 13, 15, 24, 31, 34, 35, 37, 48, 56, 61, 62, 64, 65, 68, 70, 76,
83, 85, 86, 89, 93, 94, 95, 97]
Sector 0: [8, 12, 27, 29, 30, 36, 40, 41, 44, 46, 50, 52, 55, 57, 59, 69, 71,
73, 74, 77, 80, 84, 92, 96, 99]
```

We see similar No. of clusters and the sectors being clusterd from 2 methods and we can be sure about M to be 4, so we have 4 sectors with 25 stocks in each

1.0.3 2.3 M via FFT idea explore

To apply FFt we need to be sure our data is stationary and since we are applying it on returns which is are stationary but for the sake of completeness let's check it statistically:

[ ]:
```
# Define a function to perform the KPSS test on a single column of data

def kpss_test(data):
    # KPSS test
    kpss_result = kpss(data)
    return pd.Series({
        "KPSS Statistic": kpss_result[0],
        "p-value": kpss_result[1],
        "Lags Used": kpss_result[2],
        "Critical Values": kpss_result[3]
    })

# Apply the KPSS test to each column of the DataFrame
results = returns_stock.iloc[:,12:30].apply(kpss_test)
results
```

[ ]:
```
                                                                  12  \
KPSS Statistic                                              0.065254
p-value                                                          0.1
Lags Used                                                         29
```

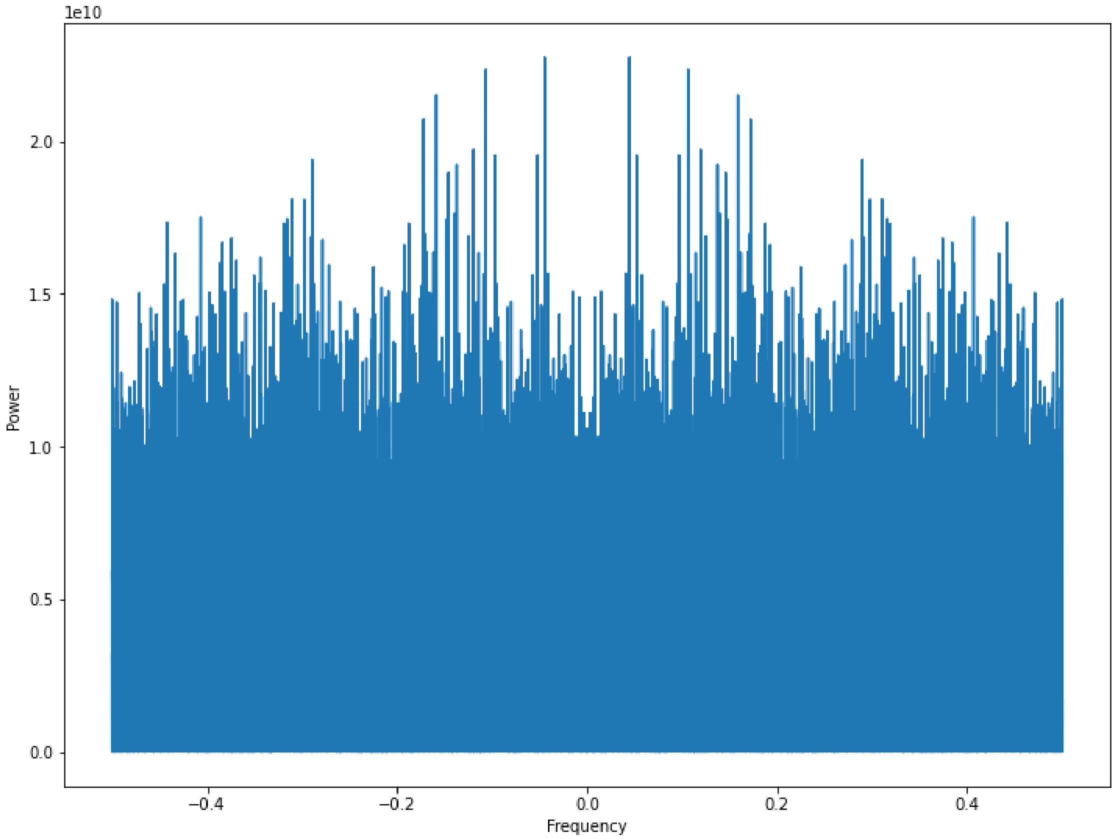
```
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 13  \
KPSS Statistic                                             0.117732
p-value                                                         0.1
Lags Used                                                         3
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 14  \
KPSS Statistic                                             0.459744
p-value                                                    0.051403
Lags Used                                                        25
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 15  \
KPSS Statistic                                             0.074597
p-value                                                         0.1
Lags Used                                                         4
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 16  \
KPSS Statistic                                             0.095594
p-value                                                         0.1
Lags Used                                                        19
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 17  \
KPSS Statistic                                             0.162247
p-value                                                         0.1
Lags Used                                                         9
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 18  \
KPSS Statistic                                               0.1284
p-value                                                         0.1
Lags Used                                                         2
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 19  \
KPSS Statistic                                             0.050397
p-value                                                         0.1
Lags Used                                                        14
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…

                                                                 20  \
KPSS Statistic                                             0.067251
p-value                                                         0.1
```

```
Lags Used                                                       26
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                21  \
KPSS Statistic                                            0.225199
p-value                                                        0.1
Lags Used                                                       30
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                22  \
KPSS Statistic                                            0.075188
p-value                                                        0.1
Lags Used                                                       11
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                23  \
KPSS Statistic                                            0.101921
p-value                                                        0.1
Lags Used                                                       13
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                24  \
KPSS Statistic                                            0.644177
p-value                                                    0.01862
Lags Used                                                       10
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                25  \
KPSS Statistic                                            0.475641
p-value                                                   0.047153
Lags Used                                                        2
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                26  \
KPSS Statistic                                            0.023388
p-value                                                        0.1
Lags Used                                                       31
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                27  \
KPSS Statistic                                            0.071768
p-value                                                        0.1
Lags Used                                                        9
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%...

                                                                28  \
KPSS Statistic                                            0.320473
```

```
p-value                                                              0.1
Lags Used                                                             12
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…


                                                                      29
KPSS Statistic                                                  0.083337
p-value                                                              0.1
Lags Used                                                             14
Critical Values  {'10%': 0.347, '5%': 0.463, '2.5%': 0.574, '1%…
```

The test is based on the null hypothesis that the time series is stationary, and the alternative hypothesis that the time series has a unit root and is non-stationary.

[ ]:
```
data = returns_stock["0"] #Try for 1st stock

# Apply the Fourier transform
fft = np.fft.fft(data)

# Compute the frequency axis
freq = np.fft.fftfreq(data.size)

# Compute the power spectrum
power_spectrum = np.abs(fft) ** 2

# Plot the power spectrum against the frequency axis
plt.plot(freq, power_spectrum)
plt.xlabel('Frequency')
plt.ylabel('Power')
plt.show()

# Identify the dominant frequencies
dominant_freqs = freq[np.argsort(power_spectrum)[::-1][:5]] # get the 5 highest␣
 ↪power frequencies
print('Dominant frequencies:', dominant_freqs)
```




```
Dominant frequencies: [-0.04474  0.04474  0.10635 -0.10635  0.15877]
```

We have captured 5 Dominant frequencies but We see a lot of Noise and instead we should take a range of frequency, the idea seems interesting but as stated in the problem in lieu of time we should move forward, I leave this idea to explore in future.

1.0.4 2.4 Can Volatility help?

The Idea was to explore this idea we see Volatility correlation when market is in fear or we say correlation increases as vol increase so this may create unnecessary noise in the data, as we want pure correlation between stocks among sector but when vol increases for the whole market every stock becomes correlated and it may harm data points

[ ]:
```
volatility_index = returns_index.rolling(window=30).std().fillna(0)
volatility_stock = returns_stock.rolling(window=30).std().fillna(0)
```

[ ]:
```
for i in range(100):
    corr = np.corrcoef((returns_stock.iloc[:,i]), volatility_stock.iloc[:
 ↪,i])[0, 1]
    if corr >0.1:
        print(corr)
```

No Output, as We have correlation, around 0.002, but as we know correlation takes into account the sign and -ve returns are harming the potential results, we can go to other robust correlation methods. Here I am using Absolute values of returns, which is not so correct but on large dataset it won't harm

```
[ ]: corr_list = []
     for i in range(100):
         corr = np.corrcoef(abs(returns_stock.iloc[:,i]), volatility_stock.iloc[:
      ↪,i])[0, 1]
         if corr > 0.1:
             corr_list.append(corr)
     print(len(corr_list))
```

```
100
```

Now the idea is **whenever index were very volatile we drop that day to compute correlation matrix and hence we avoid the days when all sectors were correlated**

```
[ ]: cutoff = volatility_index.quantile(0.20, axis=0)
     sums = (volatility_index >= cutoff).sum(axis=1)
     top_20_perc = volatility_index.index[sums > (volatility_index.shape[1] / 3)]
```

```
[ ]: excluded = returns_stock.drop(top_20_perc)

     # Compute the correlation matrix
     new_corr_matrix = excluded.corr()
```

Not including further computations here as it will be a mess, **but the No. of clusters didnot changed nor did the classifications**, possible reason is the dataset is huge so in the long run the effect of such events fades away!

2 3. Get the Functional form

```
[ ]: Sector_0 = [5, 13, 15, 24, 31, 34, 35, 37, 48, 56, 61, 62, 64, 65, 68, 70, 76,␣
      ↪83, 85, 86, 89, 93, 94, 95, 97]
     Sector_1 = [8, 12, 27, 29, 30, 36, 40, 41, 44, 46, 50, 52, 55, 57, 59, 69, 71,␣
      ↪73, 74, 77, 80, 84, 92, 96, 99]
     Sector_2 = [2, 3, 6, 7, 9, 16, 17, 19, 23, 28, 32, 33, 42, 47, 51, 53, 58, 60,␣
      ↪63, 66, 72, 81, 87, 91, 98]
     Sector_3 = [0, 1, 4, 10, 11, 14, 18, 20, 21, 22, 25, 26, 38, 39, 43, 45, 49,␣
      ↪54, 67, 75, 78, 79, 82, 88, 90]
     sectors = [Sector_0,Sector_1,Sector_2,Sector_3]

     sector_names = ['Sector_0', 'Sector_1', 'Sector_2', 'Sector_3']
```

```
[ ]: # OLS Regression

     def check_covariance(sector,index,returns_stock,returns_index):
```

```
    train_x =  returns_stock.iloc[5000:100000, sector ]
    train_y  = returns_index.iloc[5000:100000, index]

    test_x = returns_stock.iloc[100000:200000, sector ]
    test_y = returns_index.iloc[100000:200000, index]

    ols_final = sm.OLS(train_y, sm.add_constant(train_x)).fit()

    # Compute test R^2 and test mean squared error
    ols_pred = ols_final.predict(sm.add_constant(test_x))
    ols_pred = pd.DataFrame(ols_pred, columns=["ols_p"])
    ols_actual = test_y

    ols_rss = np.sum(np.power(ols_pred.ols_p - ols_actual, 2))
    ols_tss = np.sum(np.power(ols_actual - np.mean(ols_actual), 2))
    ols_rsq = 1 - (ols_rss / ols_tss)
#     print("\n OLS_R^2", ols_rsq)

    ols_MSE = np.sqrt(ols_rss / len(test_y))
#     print(" OLS_SME", ols_MSE)

    x = ols_final.predict(sm.add_constant(test_x))
    y = test_y

    corr = np.corrcoef(x, y)[0, 1]


    return corr, x
```

[ ]:
```
corr_dict = {}

sector_dict = {
    'Sector_0': Sector_0,
    'Sector_1': Sector_1,
    'Sector_2': Sector_2,
    'Sector_3': Sector_3,
    'Sector_4': Sector_4,
    'Sector_5': Sector_5,
    'Sector_6': Sector_6,
    'Sector_7': Sector_7
}

for sector_index in r_index:
    if sector_index:
        sector_name = sector_index[0]
        index = sector_index[1]
```

```
        sector_list = sector_dict[sector_name]
        corr = check_covariance(sector_list, index, returns_stock,␣
 ↪returns_index)
        corr_dict[tuple(sector_index)] = corr

for key, value in corr_dict.items():
    print(key, value)
```

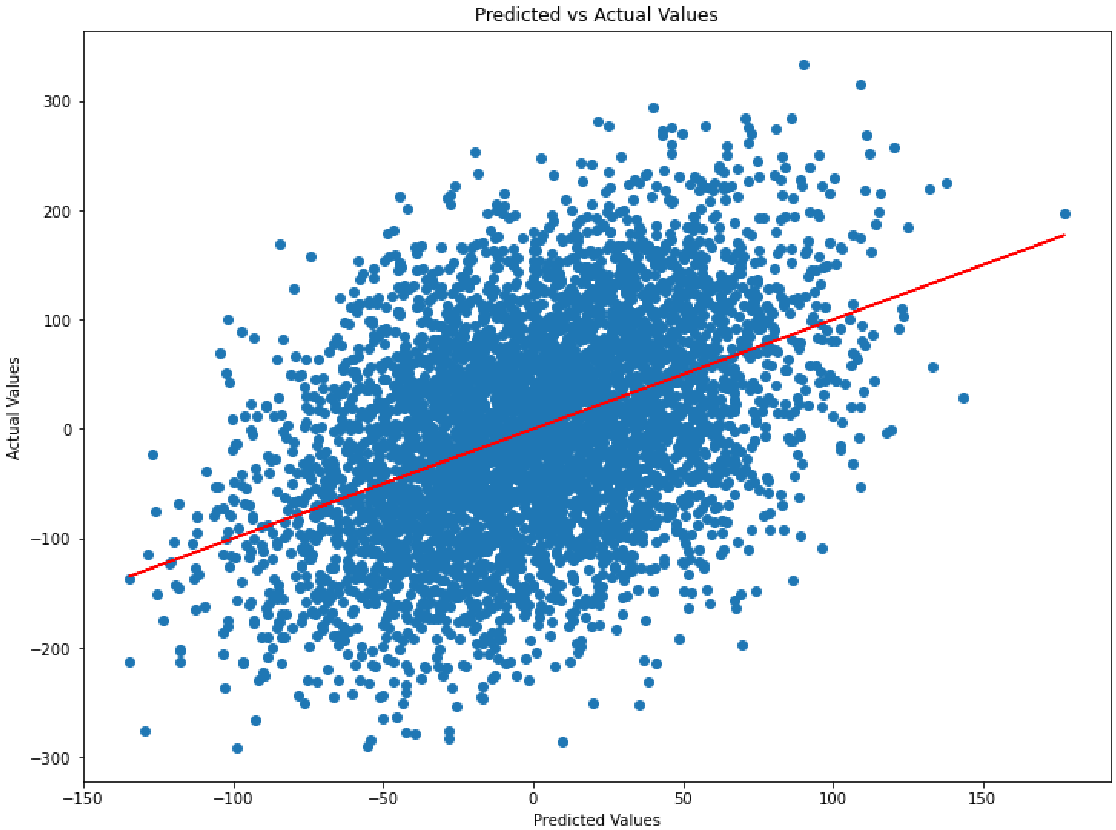
```
('Sector_7', 0) 0.40204417297927175
('Sector_4', 1) 0.30425278442714737
('Sector_7', 2) 0.20083661354964677
('Sector_5', 3) 0.2868239099593076
('Sector_2', 4) 0.40991735139618285
('Sector_1', 5) 0.20174049593765148
('Sector_5', 6) 0.43429976064863024
('Sector_0', 7) 0.250788462759403
('Sector_5', 8) 0.3220364392607265
('Sector_2', 9) 0.30951645306415565
('Sector_4', 10) 0.30720682859196574
('Sector_6', 14) 0.20563694917113223
```

What we notice is index 0,4,6 have high adjusted R^2 you notice these were the special indices, and what we also saw was 0,4,6 are the indexes which follow normal distribution suggesting others will fit probably a non-linear curve better

With this simple OLS model we see which sector may be able to explain the indices better, or which indices can have functional form for a particular sector. **We notice our special Indices are not having good R squared as we saw from their distribution was bimodal they may follow a quadratic fit!**

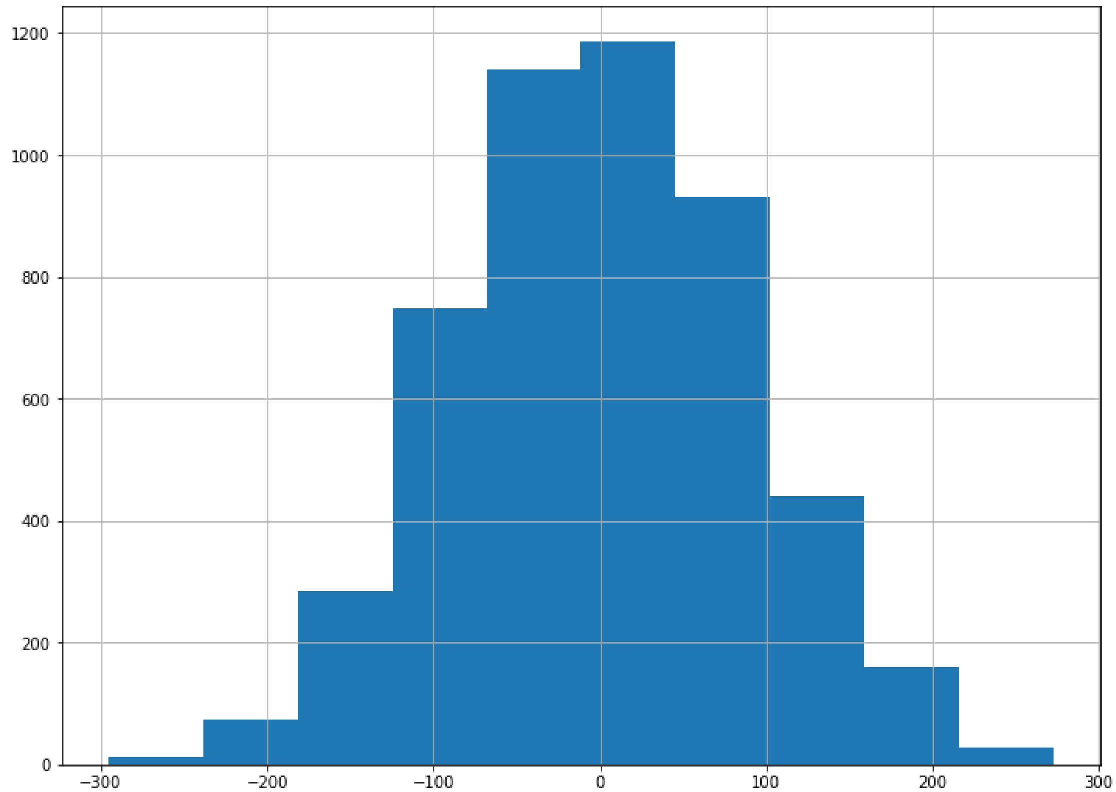
Now that we have best explaining sector for each index lets see if we were able to do a good job

[ ]:
```
# scatter plot of predicted vs actual values
plt.scatter(x, y)

# add a line showing the relationship between predicted and actual values
plt.plot(x, x, color='red')

# set axis labels and title
plt.xlabel('Predicted Values')
plt.ylabel('Actual Values')
plt.title('Predicted vs Actual Values')
plt.show()
```



[ ]:
```
(y-x).hist();
```

[ ]: <AxesSubplot:>



Histogram of residuals shows normal distribution indicates that We were able to use all the information from the data! for Index 0, but what about special indexes? but for index 0 we conclude the outlier detection models won't improve the r squared let's see

```
[ ]: #OLS:

train_x =  returns_stock.iloc[5000:20000, Sector_5 ]
train_y  = returns_index.iloc[5000:20000, 6]

test_x = returns_stock.iloc[:5000, Sector_5 ]
test_y = returns_index.iloc[:5000, 6]

# OLS Regression
ols_final = sm.OLS(train_y, sm.add_constant(train_x)).fit()

# Compute test R^2 and test mean squared error
ols_pred = ols_final.predict(sm.add_constant(test_x))
ols_pred = pd.DataFrame(ols_pred, columns=["ols_p"])
ols_actual = test_y

ols_rss = np.sum(np.power(ols_pred.ols_p - ols_actual, 2))
```

```python
ols_tss = np.sum(np.power(ols_actual - np.mean(ols_actual), 2))
ols_rsq = 1 - (ols_rss / ols_tss)
print("\n OLS_R^2", ols_rsq)

ols_MSE = np.sqrt(ols_rss / len(test_x))
print(" OLS_SME", ols_MSE)

# Ridge Regression
# generate a sequence of lambdas to try
lambdas = [np.power(10, i) for i in np.arange(4, -4, -0.1)]
alphas = lambdas

# Scale
# train_x_scale = scale(train_x) #In case you want to scale the variables.

# Use 10-fold Cross Validation to find optimal lambda
ridge_cv = RidgeCV(alphas=alphas, cv=10, scoring="neg_mean_squared_error")
ridge_cv.fit(train_x, train_y)

# Build final ridge regression model
ridge_final = Ridge(alpha=ridge_cv.alpha_, fit_intercept=True)
ridge_final.fit(train_x, train_y)

# R squared formula and mean squared error
ridge_pred = ridge_final.predict(test_x)
ridge_actual = test_y
ridge_rss = np.sum(np.power(ridge_pred - ridge_actual, 2))
ridge_tss = np.sum(np.power(ridge_actual - np.mean(ridge_actual), 2))
ridge_rsq = 1 - ridge_rss / ridge_tss
print("\n Ridge_R^2", ridge_rsq)

ridge_MSE = np.sqrt(ridge_rss / len(test_x))
print("Ridge_SME", ridge_MSE)
# LASSO Regression

# generate a sequence of lambdas to try
lambdas = [np.power(10, i) for i in np.arange(6, -6, -0.1)]

# Compile model
lasso_cv = LassoCV(cv=10, alphas=lambdas)
lasso_cv.fit(train_x, train_y)  # Fit Model

# Build final LASSO regression model
lasso_final = Lasso(alpha=lasso_cv.alpha_, fit_intercept=True)
lasso_final.fit(train_x, train_y)
```

```
# R squared formula and mean squared error
lasso_pred = lasso_final.predict(test_x)
lasso_actual = test_y
lasso_rss = np.sum(np.power(lasso_pred - lasso_actual, 2))
lasso_tss = np.sum(np.power(lasso_actual - np.mean(lasso_actual), 2))
lasso_rsq = 1 - lasso_rss / lasso_tss
print("\n LASSO_R^2: ", lasso_rsq)

lasso_MSE = np.sqrt(lasso_rss / len(test_x))
print("LASSO_SME: ", lasso_MSE)
```

```
 OLS_R^2 0.18854365722305466
 OLS_SME 87.40656291834547

 Ridge_R^2 0.1885435568035969
Ridge_SME 87.40656832671985

 LASSO_R^2:  0.1885428705718849
LASSO_SME:  87.40660528566376
```

[ ]:
```
OLS_df = pd.DataFrame(ols_final.summary2().tables[1]["Coef."]).
 ↪rename(columns={"Coef.": "OLS"})
OLS_df.index = ["Intercept",3, 6, 7, 9, 16, 17, 19, 23, 28, 32, 33, 42, 47, 51,␣
 ↪53, 58, 60, 63]

Ridge_df = pd.DataFrame(
    np.insert(ridge_final.coef_, 0, ridge_final.intercept_),
    index= ["Intercept",3, 6, 7, 9, 16, 17, 19, 23, 28, 32, 33, 42, 47, 51, 53,␣
 ↪58, 60, 63],
    columns=["Ridge"],)

Lasso_df = pd.DataFrame(
    np.insert(lasso_final.coef_, 0, lasso_final.intercept_),
    index=["Intercept",3, 6, 7, 9, 16, 17, 19, 23, 28, 32, 33, 42, 47, 51, 53,␣
 ↪58, 60, 63],
    columns=["Lasso"],
)

df = OLS_df.merge(Ridge_df, left_index=True, right_index=True)
df = df.merge(Lasso_df, left_index=True, right_index=True)

df.append(
    pd.DataFrame(
        {
            "OLS":   [ols_rsq, ols_MSE],
            "Ridge": [ridge_rsq, ridge_MSE],
```

```
            "Lasso": [lasso_rsq, lasso_MSE],
        },
        index=["R sq", "Mean Sq. Err"],
    ),
    ignore_index=False,
)
```

[ ]:

	OLS	Ridge	Lasso
Intercept	-0.996119	-0.996115	-0.996063
3	0.072563	0.072562	0.072553
6	0.047825	0.047825	0.047816
7	0.051135	0.051135	0.051133
9	0.071319	0.071315	0.071285
16	0.031397	0.031399	0.031397
17	0.067067	0.067065	0.067048
19	0.076927	0.076920	0.076871
23	0.057811	0.057809	0.057783
28	0.055770	0.055769	0.055759
32	0.041866	0.041867	0.041866
33	0.074638	0.074630	0.074567
42	0.042412	0.042413	0.042412
47	0.059430	0.059429	0.059417
51	0.060064	0.060063	0.060052
53	0.076216	0.076213	0.076193
58	0.074409	0.074401	0.074344
60	0.069084	0.069082	0.069064
63	0.064605	0.064604	0.064586
R sq	0.188544	0.188544	0.188543
Mean Sq. Err	87.406563	87.406568	87.406605

So we have all the coefficients and an Intercept for our functional form for Index 7 explained by Sector 5, and similarly we store for all of them and their predictions now!

[ ]:
```
predictions.to_csv("predictions_OLS.csv")
```

[ ]:
```
df_predictions = pd.read_csv("predictions_OLS.csv", index_col= 'Unnamed: 0')
df_predictions
```

[ ]:

	0	1	2	3	4	5 \
100001	14.599583	-2.868377	-11.218019	-10.659914	49.083123	-9.721550
100002	9.429706	-35.651850	12.513619	24.191307	-4.739676	20.456032
100003	63.614170	-31.694544	-49.280298	-34.347178	-12.419088	-10.662190
100004	45.224822	-6.025127	-26.456619	-58.200514	70.550851	-7.282417
100005	43.966904	57.498722	-13.948657	-35.609277	-60.097190	-55.778331
…	…	…	…	…	…	…
199995	52.849782	18.790479	1.358003	71.044418	-9.458841	-25.852707
199996	32.271568	-4.176340	-27.472346	66.621942	23.872060	-35.353183

```
199997 -18.895914 -23.932701  -0.135992   9.752798 -38.035320  -3.321031
199998  18.455228 -14.580303  -2.328382 -23.805353 -39.529998  21.130877
199999  14.200704 -15.960729   2.297821  -1.112284  18.581018  -0.301367

                6          7          8          9         10         11  \
100001   8.634447   5.381625   0.491627  -6.123677 -52.280460   6.136776
100002 -37.912675   2.538937 -24.327310   3.325710  10.458176 -1.031368
100003 -29.694853  40.746073 -12.206510  22.031224  46.142682 -2.707917
100004  60.489385  33.130847  37.745020  19.134000  33.286838   2.263658
100005  19.142217  16.796243  12.348202  79.731543 -17.329954   1.494280
…             …          …          …          …          …         …
199995 -46.854888  28.124784 -28.705011 -21.680873   3.157729 -1.524393
199996 -37.089869  22.809914 -24.286472  49.458189  39.101951 -0.023425
199997 -36.094673  -9.354447 -28.284815  72.338899   8.050806   4.150536
199998 -25.746564  13.658382 -11.886517  26.156238 -46.271405   1.784112
199999  57.916202  12.845423  41.554940 -48.683396  -6.331413 -2.602827

               14
100001 -50.880467
100002   0.108155
100003   6.093103
100004 -24.189271
100005  -7.897371
…             …
199995  31.790769
199996 -11.698417
199997   9.338779
199998  15.003824
199999  19.504221

[99999 rows x 13 columns]
```

[ ]:
```python
corr_index = df_predictions.corr()
corr_index
```

[ ]:
```
           0         1         2         3         4         5         6  \
0   1.000000  0.168604 -0.646803 -0.100723  0.201042 -0.129139  0.189878
1   0.168604  1.000000 -0.095339 -0.101530  0.180111 -0.639282  0.195175
2  -0.646803 -0.095339  1.000000  0.060571 -0.117424  0.071206 -0.091976
3  -0.100723 -0.101530  0.060571  1.000000 -0.082980  0.077914 -0.479793
4   0.201042  0.180111 -0.117424 -0.082980  1.000000 -0.135318  0.173270
5  -0.129139 -0.639282  0.071206  0.077914 -0.135318  1.000000 -0.142724
6   0.189878  0.195175 -0.091976 -0.479793  0.173270 -0.142724  1.000000
7   0.981538  0.169192 -0.605527 -0.091527  0.199075 -0.131094  0.190376
8   0.185517  0.192679 -0.089878 -0.481038  0.174610 -0.140630  0.983335
9  -0.114321 -0.121983  0.071032  0.060119 -0.484733  0.092329 -0.110824
10 -0.058610 -0.468122  0.033385  0.042092 -0.061847  0.119316 -0.086869
```

```
11  0.057869  0.047617 -0.033446 -0.023067  0.275722 -0.036387  0.050152
14 -0.125274 -0.108240  0.069682  0.036310 -0.664185  0.080764 -0.114421

           7         8         9        10        11        14
0   0.981538  0.185517 -0.114321 -0.058610  0.057869 -0.125274
1   0.169192  0.192679 -0.121983 -0.468122  0.047617 -0.108240
2  -0.605527 -0.089878  0.071032  0.033385 -0.033446  0.069682
3  -0.091527 -0.481038  0.060119  0.042092 -0.023067  0.036310
4   0.199075  0.174610 -0.484733 -0.061847  0.275722 -0.664185
5  -0.131094 -0.140630  0.092329  0.119316 -0.036387  0.080764
6   0.190376  0.983335 -0.110824 -0.086869  0.050152 -0.114421
7   1.000000  0.185892 -0.114207 -0.059675  0.056856 -0.123086
8   0.185892  1.000000 -0.109910 -0.087189  0.049711 -0.115790
9  -0.114207 -0.109910  1.000000  0.051124 -0.061687  0.131941
10 -0.059675 -0.087189  0.051124  1.000000 -0.012146  0.032760
11  0.056856  0.049711 -0.061687 -0.012146  1.000000 -0.444017
14 -0.123086 -0.115790  0.131941  0.032760 -0.444017  1.000000
```

2.1 Let's try NN to see weather we have better fits in higher dimension

Idea is if we are able to achieve better fits for some indexes we will try to fit non-linear forms to that particular index wherever the **OLS performed bad and NN was better**

Also on observation I found if we give volatility of stocks as a feature along with returns we can achieve a better fits, but we will loose explainibility, But still the improvement was significant so I went with Volatility as a feature too!

[ ]:
```
import tensorflow as tf
import numpy as np
from tensorflow.keras import regularizers

index_sector = [3,0,3,2,1,0,2,3,2,1,0,3,1,2,1]
sectors = [Sector_0,Sector_1,Sector_2,Sector_3]


for i in range(1,15):

        s = index_sector[i]
        sector = sectors[s]

        train_x = pd.concat([returns_stock.iloc[50:50000, sector],␣
 ↪volatility_stock.iloc[50:50000, sector]], axis=1)
        train_y = returns_index.iloc[50:50000, i]

        # Define the neural network architecture
        model = tf.keras.Sequential([
            tf.keras.layers.Dense(64, activation='relu', input_shape=(train_x.
 ↪shape[1],),kernel_regularizer=regularizers.l2(0.01)),
```

```
            tf.keras.layers.Dropout(0.2),
            tf.keras.layers.Dense(32,␣
↪activation='relu',kernel_regularizer=regularizers.l2(0.01)),
            tf.keras.layers.Dropout(0.2),
            tf.keras.layers.Dense(1)
        ])

        # Compile the model
        model.compile(optimizer='adam', loss='mean_absolute_error')

        # Train the model
        model.fit(train_x, train_y, epochs=100, batch_size=32, verbose=0)

        # Make predictions on new data
        test_x = pd.concat([returns_stock.iloc[50000:190000, sector],␣
↪volatility_stock.iloc[50000:190000, sector]], axis=1)
        test_y = returns_index.iloc[50000:190000, i]
        prediction = model.predict(test_x)

        # Append predictions to list
        predictions_df2.append(prediction.flatten())

        corr = np.corrcoef(prediction.flatten(), test_y)[0, 1]

        print("index", i)
        print("Sector", s)
        print("corr", corr)

        # Add corr value to corr_dict with sector s as key
        if s not in corr_dict2:
            corr_dict2[s] = []
        corr_dict2[s].append(corr)


# Create dataframe from predictions list
predictions_df = pd.DataFrame(predictions_df2).transpose()
```

So we indeed see better fits for indexes that were not ablle to fit via OLS specially for our special indexes the correlation reached to as good as 60%

[ ]:
```
predictions_df.to_csv("predictions_using_vol.csv_01345678910111121314",␣
↪index=False)
```

While training I lost the index 2 so i decided to leave him behind as I didn't have time to do it again

[ ]:
```
predictions_df
```

```
[ ]:                0          1          2          3           4          5  \
0        17.594763  -5.907107 -41.906338 -31.463125   63.326897  46.584667
1        12.247170 -52.665287   4.143923 -42.779728  126.282486 -36.071136
2        46.177990  13.995529  85.398790  45.671730   10.235013  12.134693
3       -16.572327  -9.301745   4.143923   8.495736  121.733116 -44.318356
4       -10.954910 -34.153374   6.393514 -10.273191  125.194040 -93.087320
…              …          …          …          …           …          …
139995   -4.900393  25.528957   9.251000  -6.012039   38.088190 -53.646770
139996   46.831745  -5.095889   6.647972  32.627070   39.171944 -17.196499
139997   38.983707  91.675160  63.899853  45.233627   33.059624  18.747885
139998  -42.178562  18.908297   5.268075 -81.588326    5.752670 -21.569012
139999   50.115322  -9.587955  95.820380   7.427365  -12.094053  32.542150

                6          7          8           9         10         11  \
0        39.679550   0.086075  10.045840    7.147789  -9.416501  10.561757
1        -9.598078  -4.595795   7.403167    6.773143 -33.981880  15.748678
2        14.021536   9.902845  29.352910   34.357900  27.667934  53.057940
3       -63.041490  -1.520072 -11.133469    7.340301 -30.576363 -52.403263
4        23.836360 -68.631030   6.233618    7.072279 -35.366510 -45.121735
…              …          …          …           …          …          …
139995   22.751050 -31.991814  -2.113419   20.685860 -34.199657 -38.219707
139996  -20.002010  -6.774857  30.088495 -138.241800  16.342909  11.404474
139997  -16.684101  -5.815474 -41.439890 -127.795210  23.659883  18.779894
139998  -27.561820 -41.875580   6.749308    6.008812  23.403837  56.787205
139999   57.881330 -14.308416  16.358028   10.395222  34.054146  -1.540191

               12          13
0        17.711887 -118.660600
1         7.835031  119.730040
2       -36.433144   38.070065
3        17.191221   51.472550
4        59.949900   43.690270
…              …           …
139995   29.161823  -71.722626
139996    2.914329   48.649580
139997  -13.325199   31.962614
139998  -30.369127  151.525910
139999   13.080715   32.240562

[140000 rows x 14 columns]
```

2.2 5. Trading Strategy Using Portfolio Optimization

Now that we have Cov of Indexes and Predictions we will use a technique that optimizes a portfolio using the **Mean-Variance optimization model with a zero-sum constraint**. It takes as input the expected returns, covariance matrix, target volatility, and risk-free rate of the assets in the portfolio, and returns the optimal weights for each asset.

```
[ ]: def optimize_portfolio(expected_returns, cov_matrix, target_volatility,␣
      ↪risk_free_rate):

         n_assets = len(cov_matrix)

         # Define optimization variables
         w = cp.Variable(n_assets)

         # Define objective function
         objective = cp.Maximize(cp.sum(w.T @ expected_returns) - risk_free_rate)

         # Define constraints
         constraints = [cp.sum(w) == 0,
                        cp.quad_form(w, cov_matrix) <= target_volatility**2]

         # Solve the optimization problem
         problem = cp.Problem(objective, constraints)
         problem.solve()

         # Retrieve the optimal weights
         optimal_weights = w.value

         return optimal_weights

     # Test the function
     expected_returns = df_predictions.iloc[:50000].values
     cov_matrix = corr_index.values

     # Set the target volatility and risk-free rate
     target_volatility = 0.1
     risk_free_rate = 0.03

     # Optimize the portfolio
     optimal_weights = optimize_portfolio(expected_returns, cov_matrix,␣
      ↪target_volatility, risk_free_rate)

     print(optimal_weights)
```

Assumptions:

The returns on assets are normally distributed. (For Predictions from NN Yes they are!) The returns on assets are linearly related to each other. The expected returns and covariance matrix used in the optimization are accurate and reliable.

```
[ ]: expected_returns = df_predictions.iloc[:50000,]
     cov_matrix = corr_index

     # Set the target volatility and risk-free rate
```

```
target_volatility = 0.1
risk_free_rate = 0.03

# Optimize the portfolio
optimal_weights = optimize_portfolio(expected_returns, cov_matrix,␣
 ↪target_volatility, risk_free_rate)

print(optimal_weights)
```

2.2.1 Did not get enough time to train and Submit the Results but Will do it in Future!! Had a great time exploring the Quant-Research type of job!