COMPARISON OF VALUE AT RISK MODELS AND FORECASTING REALIZED VOLATILITY BY USING INTRADAY DATA

An Empirical Study on American Stock Exchanges

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BY

MUHAMMET BOZKAYA

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Head of MSc Programme: LACOSTE, Vincent

Thesis Supervisor: SBRANA, Giacomo

Approval of the thesis:

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submitted by **MUHAMMET BOZKAYA** in partial fulfillment of the requirements for the degree of **Master of Science in Finance in Department of Finance , Neoma Business School** by,

Pascal Krupka

Director of Postgraduate Programs, **Neoma Business School**

Vincent Lacoste

Head of Programme, **Department of Finance**

Giacomo Sbarana

Thesis Supervisor, **Department of Information Systems, Supply Chain Management and Decisions**

NEOMA Business School
Campus de Rouen
1, rue du Maréchal Juin - BP 215
76825 Mont Saint Aignan Cedex
Tél : 02 32 82 57 00

DECLARATION OF ACADEMIC INTEGRITY

I, Mr., **Muhammet Bozkaya** student enrolled in the MSc in FINANCE certify that the data and information contained in the professional thesis entitled COMPARISON OF VALUE AT RISK MODELS AND FORECASTING REALIZED VOLATILITY BY USING INTRADAY DATA “An Empirical Study on American Stock Exchanges” have not been plagiarized.

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ÖZET

GÜN İÇİ VERİLER KULLANILARAK RİSKE MARUZ DEĞER MODELLERİ KARŞILAŞTIRMASI VE GERÇEKLEŞEN VOLATİLİTE TAHMİNLEMESİ

Amerikan Menkul Kıymet Borsaları Üzerine Bir Uygulama

BOZKAYA, Muhammet

Yüksek Lisans Finans Bölümü

Danışman: SBRANA, Giacomo

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Son yıllarda finansal piyasalarda artan belirsizlik ve karışıklık ile birlikte, finansal kuruluşlar için piyasalardaki beklenmeyen hareketlerin ölçülmesi büyük önem kazanmıştır. Piyasadaki bu hareketlenme ve volatilitelerin ölçülmesi için finansal kuruluşlar arasında en yaygın kullanılan yöntem (RMD) Riske Maruz Değer'dir. Fakat, tutarlı bir RMD hesaplanması ve tahminlenmesi için farklı yöntemler üzerine bir çok farklı soru ve görüş vardır. Hangi yöntemle RMD hesaplanması ve hangi yöntemle volatilite hesaplanması gerektiği olmak üzere iki temel soru vardır.

Bu çalışmanın amacı, yüksek frekans gün içi finansal verilerin volatilite ve RMD tahminleme yöntemlerine dahil edilmesi ve EWMA,GARCH(1,1) gibi volatilite hesaplamasında kullanılan yöntemlerin gerçekleşen volatilite tahmin gücünü araştırma ve analiz etmektir. Ayrıca, hem Hareketli Ortalama(MA), Üssel Ağırlıklandırılmış Hareketli Ortalama(EWMA), Genelleştirilmiş Otoregresif Koşullu Heteroskedastisite Yöntemi (GARCH) gibi parametrik ve Tarihsel Simülasyon gibi parametrik olmayan yöntemlerle RMD hesaplaması yapılmıştır. Ayrıca her bir yöntemin tutarlılığını ölçmek için geriye dönük testler yapılmıştır.

Anahtar Kelimeler: Gerçekleşen Volatilite, Riske Maruz Değer, Volatilite, Gün İçi Veri, EWMA, GARCH, MA, Tarihsel Simülasyon, Geriye Dönük Test, Risk Yönetimi, Yüksek Frekans Finansal Veri.

ABSTRACT

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BOZKAYA, Muhammet

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In recent years with increasing uncertainties and turmoil in financial markets, importance of measuring movements in financial markets has tremendously risen for financial institutions. The most widely used approach between financial institutions to measure these movements and volatilities in the market; is Value at Risk (VaR) approach. However, there are many questions and discussions on different methods to calculate and forecast a consistent VaR. There are two main questions which method to calculate VaR and which method to calculate Variance or Volatility that is the main step to calculate VaR.

The aim of this study is to introduce high frequency Intraday financial data into volatility and value at risk estimation methods and investigate and analyze realized volatility forecasting power of each main method that are used to calculate volatility such as EWMA,GARCH(1,1). Moreover, we calculate VaR with both parametric methods like Moving Average (MA), Exponentially Weighted Moving Average (EWMA), and Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) and a non-parametric method Historical Simulation. We also do Backtesting analysis to measure the accuracy of each method.

Keywords: Realized Volatility, Value at Risk, Volatility, Intraday Data, EWMA, GARCH, MA, Historical Simulation, Backtesting, risk management, high frequency financial data.

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Chapter 1

Introduction

In last decades, increasing globalization in international economies has affected financial markets by increasing competition, transaction volumes and diversity in financial products. This affect caused financial markets to be more complex and connected to each other. Therefore, because of this increasing complexity and connectivity, financial institutions have been more vulnerable to risk arising from price movements in financial markets. Because of this, institutions began to look for effective solutions and risk methodologies to define, measure, control and update their risks. Moreover, with recent financial crisis and turmoil in economies, financial authorities have brought certain rules and measures to force institutions to take precaution and improve their risk management systems.

With increasing importance of measuring and controlling market risk, several methodologies has been produced such as; sensitivity analysis and scenario analysis. After presentation of Value at Risk method, it has been the most widely used market risk method since it is easy to implement and interpret, at the end of analysis we get a single value in real currency amounts, that is why VaR is widely used and preferred. Value at Risk method is simply a method that uses historical data to calculate maximum level of loss in real currency amounts at a certain time, given a certain confidence level.

In this study, we use Intra-day 1-minute, 5-minute and 10-minute historical prices of three US stock exchanges: S&P500, DowJones and NASDAQ. We create a benchmark daily-realized volatility from 1-minute historical prices for 5-minute and 10-minute historical prices and then we calculate volatility directly from 5-minute and 10-minute historical prices by using each method to measure their forecasting power. Moreover, we analyze both parametric and non-parametric approaches to calculate Value at Risk for each type of historical prices, and then we do backtesting to measure the accuracy of each approach and we choose which approach and method is the best to calculate Value at risk for each type of historical prices.

In first chapter of this study, we discuss the general risk concept and different types of risks including market risk. In second chapter, we focus on concept of market risk and we give theoretical information on market risk and its measures. In third chapter, we focus on Value at Risk method, we give theoretical information and we explain the approaches and methods to calculate it. In fourth chapter we start to our analysis first, we give information about our data and our portfolio and then we create our benchmark minute-volatilities. Moreover, we continue our analysis by calculating Value at Risk with each approach and method and we do backtesting to compare results from each approach and method. In fifth and last chapter, we do our conclusion and summarize our results.

1. Definition of Risk and Types of Risk

1.1. What is Risk?

Risk is defined as the positive probability of ending up in a state of the world in which returns or consumption are low[1] (MALZ, 2011, pg. 50). What we understand from this definition risk is probability of having unexpected results or amounts. Risk is a certain situation when there is negative deviation from what people expect. We mentioned deviation as negative because if there would be only and only positive deviations which is a profit in economic terms, in that case rational people would not complain and there would not be a problem of risk.

1.2. Types of Risk

1.2.1. Credit Risk

Credit risk is the risk that the creditworthiness of the issuer of a debt obligation you own deteriorates[2] (MALZ, 2011, pg. 35). It may take different forms such as; Default Risk, which is the risk of debtor's being unable to pay the debt and its installments due in time. Counterparty Risk, which is the risk in case of CDSs or other advanced credit products, risk of third party to be unable to deliver a security or any financial product. Liquidity Risk is the risk that arises when debt issuer is not available to execute its own debts and create new funds to continue credit-creating cycle.

1.2.2. Market Risk

Market is the risk of having loss due to movement in market prices. It may cause from several issues:

Price Risk is the risk that the market price of a security goes the wrong way[3] (MALZ, 2011, pg. 35). In case of a portfolio price risk is more complicated, because in that case overall risk is not only arising from single price movements, but also arises from correlations between assets that form the portfolio. Execution Risk, is the risk that one cannot execute a trade quickly enough or skillfully enough to avoid a loss. An important example is stop-loss risk, the risk that you cannot exit a trade at the worst price you were willing to accept[4] (MALZ, 2011, pg. 35). Political Risk is the risk arises from political and regulatory situation in a country or region, for example, a regulatory change in market dynamics can have huge effects on market therefore prices in one's portfolio or a political up rise or unrest can have very big effects on market prices.

1.2.3. Operational Risk

Is the risk of loss that causes from disorders and failures in organizations. It is generally risk that left after market and credit risk. It may take several forms; Fraud risk is the risk of loss that arises from violation of organization system and regulation by its employees for example an accountant who transfers bank cash to his own account. Regulatory risk is the risk of loss that may arise from regulatory sanctions due to organization's violation of regulations. Reputational Risk is the risk of loss that arises from internal failures and breakdown that decrease an organization's reputation so its credibility and this causes eventually to a profit loss, for example an organization which losses the customer data because of a failure may loss reputation therefore customers and profit.

1.2.4. Model Risk

Model Risk is the risk of potential loss arising from incorrect models. It can take many forms; one source of model risk is correlation risk. The risk of applying the "wrong" return correlation arises frequently because correlation is hard to estimate[5] (MALZ, 2011, pg. 35).

Model risk may also cause from failures in data, or parameter estimation or risk estimation. For example in 2007 financial crisis there was an example to model risk, for subprime mortgages both credit rating agencies and most individual investors build their model on historical data but subprime mortgages loans were not available 2 decades ago.

Chapter 2

2. Concept of Market Risk and Methods for Measuring Market Risk

Why market risk is important? Why institutions need a market risk management? Market is risk is the most crucial and difficultly managed risk for institutions, because it is a risk that cannot be managed or controlled, since it does not arise from institutions itself. For example in case of credit risk, an institution has ability to decide to whom to issue debt but in case of market risk no tie in the hands of institutions. Only way to mitigate the market risk is to take precautions like hedging or doing diversification in portfolio allocation to mitigate effects of risk. Since it is that much important for institutions to measure and control the market risk, many methodologies have been found to calculate it in time.

With high advancements and increasing complexity in financial markets day by day, the importance of managing market risk has risen for institutions. Therefore, many institutions and academicians started to do research and suggest different techniques to calculate and predict the market risk. As a result, many approaches and models like Stress Testing, Scenario Analysis, Sensitivity Analysis, Option Pricing Models, Black&Scholes Model, Parametric Value at Risk, Non-Parametric Value at Risk and advance Simulation techniques have been created for controlling market risk.

2.1. General Methods for Measuring Risk

To obtain an efficient risk management, first and most important step is choosing the model or methodology for measuring risk, when we look at the history and literature we see that mainly several measures have been used such as Duration analysis, Gap analysis, Scenario analysis and Sensitivity analysis.

Duration analysis is one of the very basic and straightforward methods used to measure market risk. Duration is simply the life of a financial instrument; its calculation is a bit complex and depends on, maturity, coupon rate, yield and face value of the financial instrument. Duration is important to analyze how sensitive a financial instrument to interest rate changes, longer duration means higher interest rate risk. Because of this, institutions try to have their assets with shorter duration than those of liabilities.

Gap analysis is another straightforward measure of risk; it is mainly used to measure interest rate and liquidity risks in asset liability management. It is mainly the process of ordering all assets and liabilities according to their terms structures and then defining asset sensitive and liability sensitive portfolios.

Scenario and sensitivity analysis are computerized and programmed techniques to measure effects of changes in single components or a group of component together on portfolio. For example for sensitivity analysis a risk manager might ask what would be impact of a sudden 100 basis points decrease in interest rate on our portfolio. For scenario analysis, analysts mostly define a worst-case scenario and measure its effects on portfolio.

2.2. Basic Concepts in Measuring Risk

Since measuring risk is a matter of measuring movements in profit and loss due to market price changes, the basics concepts used in measuring risk are some statistical and economical concepts.

First step in measuring risk is calculating returns, to find profit or loss due to a price change we need to find what is the effect of that change that is why we calculate return, return is simply positive or negative outcome of a price change. The reason why we use return instead of prices is "normalization", probability distribution of prices are not normally distributed so to normalize our data to make easy comparisons we need to use returns which have a normal probability distribution. There are two main ways to calculate returns,

2.2.1. Return

Discretely Compounded Return;

Discrete or percentage returns are relative changes in prices from one period to another; it is simply difference between two prices of consecutive period denominated in older period in sequence. We can show it mathematically as below,

$$r_{\mathrm{t}} = \frac{P_{\mathrm{t}} - P_{\mathrm{t-1}}}{P_{\mathrm{t-1}}}$$

r refers to Return, t refers to current time, P refers to Price and t-1 refers to one period before current period t.

Continuously Compounded Return;

Continuously compounded or logarithmic return is simply calculated as difference between natural logarithm of two consecutive prices. Natural logarithm is inverse of exponential function and it is used for linearization purposes. Mathematically we can show continuously compounded return as follow,

$$r_{\mathrm{t}} = \ln\left(\frac{P_{\mathrm{t}}}{P_{\mathrm{t-1}}}\right) \quad = \ln P_{\mathrm{t}} - \ln P_{\mathrm{t-1}}$$

“ln” refers to Natural Logarithm.

Why we use Logarithmic return?

In our this study, we use logarithmic return, mainly because of several logical advantages it has, First advantage of logarithmic return is that it linearizes the returns over a time interval, because logarithmic return is not exponential, it is linear; sums of logarithmic returns over a time interval is equal to logarithmic return of that time interval. What we mean here is that logarithmic return is additive, let's think about discrete compounding in that case we compound mathematically,

$$(1 + x_1).(1 + x_2) \ldots \ldots \ldots .(1 + x_n)$$

As we can see here, there is a multiplication and it goes exponentially, so this creates a problem because it violates the normality assumption. On the other hand, for logarithmic return we know that;

$$\ln(1 + x_1) + \ln(1 + x_2) \ldots \ldots \ldots \ln(1 + x_n)$$

Therefore, we can easily see here that logarithmic return is not violating the normality assumption due to its additive property.[1]

Another advantage is approximation of log returns, by default $\boldsymbol{\ln(1 + x) \approx x \; given \; that \; x < 1}$.

2.2.2. Statistical Concepts

There are several statistical concepts that we use for measuring risk such as Mean, Variance, Standard Deviation, Covariance, Correlation, Probability distribution, Skewness, Kurtosis, Jarque-Bera, and Goodness of Fit.

Mean or Average is sum of all the elements in a population or sample divided by number of elements in population or sample. It is usually symbolized as $\boldsymbol{\bar{X}}$ and statistical formula for mean is:

$$\bar{X} = \frac{\sum_{i=1}^{n} Xi}{n}$$

[1] http://quantivity.wordpress.com/2011/02/21/why-log-returns/

X_i refers to single elements in sample or population, n refers to size of sample or population.

Variance is the sum of squared differences of each single element from their mean divided by number of elements in sample or population. It shows how each element is dispersed relative to mean, how away they are to mean. Statistically we can represent the variance as follow,

$$V(XY) = E[(X - E(X))^2]$$

$$V(XY) = E(X^2) - E(X)^2$$

It is symbolized in parameter notation as follow σ^2 and its formula is,

$$\sigma^2 = \frac{\sum_{i=1}^{N}(Xi - \bar{X})^2}{N}$$

Standard Deviation is simply square root of Variance and it measures variability of each single element from their mean. It is very similar to variance but we use it because in variance we squared the original data so in variance data types are not same, for example if single elements would be meter then variance would be meter square. That is why we take the square root of variance and find Standard Deviation. Standard Deviation is symbolized as σ and its statistical formula is,

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(Xi - \bar{X})^2}{N}}$$

Volatility is standard deviation of return derived from a single asset or portfolio at a certain time unit, for risk management purposes, we use daily volatility mainly. Symbol of volatility is same as for standard deviation and is σ**.** Volatility is most important component in measuring risk with Value at Risk, in most cases volatility is the only thing to calculate so we can easily say that volatility is strongest risk measure. After calculating daily volatility, then volatility can be calculated for other time units including annual. For example for market risk we generally need 10 days volatility, to get 10 days volatility from 1 day volatility we simply multiply square root of 10 with 1 day volatility,

$$\sigma Annual = \sqrt{252} \times \sigma 1Day$$

$$\sigma 10day = \sqrt{10} \times \sigma 1Day$$

Volatility Estimation Methods

Estimating a variance according to the formulae given by a model, using historical data, gives an observed variance that is "realized" by the process assumed in our model. However, this "realized variance" is still only ever an estimate. Sample estimates are always subject to sampling error, which means that their value depends on the sample data used[2] (Alexander, 2007, Pg. 2). As a result, we should say that even if we use historical real data different models could give different volatility estimates.[3]

There are three methods; we can use to estimate volatility,

- **Moving Average (MA) Model,**

It is in fact called Equally Weighted Moving Average model but not to confuse with EWMA model that we will explain later, we call it shortly MA. In moving average model, volatility for current day is estimated by using most fresh return from previous days. Volatility for day t is estimated by using return from en the end of day t-1. Moving

[2] Carol Alexander, November2, 2007, Reading University.
[3] For detailed information, please refer to article "Carol Alexander, November2, 2007, Reading University".

average model is weighting all historical return data with same average weight, for this model recent and closest data has same value as old data. Mathematically we can represent MA model as follow,

$$\sigma_t^2 = \frac{1}{N-1}\sum_{i=1}^{N}(r_{t-1} - \bar{r})^2$$

In above formula $\bar{r}$ is mean of returns, in risk management mean of returns are very close to zero and because of it, we assume its zero. Therefore, in above formula when we remove the mean we get,

$$\sigma_t^2 = \frac{1}{N}\sum_{i=1}^{N}(r_{t-1})^2$$

Therefore above formula is how Moving Average model estimates the volatility, of course please not that above we get σ_t^2 which is variance, therefore to get standard deviation we need to take the square root of σ_t^2,

$$\sigma_t = \sqrt{\sigma_t^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_{t-1})^2}$$

Above formula is how Moving Average model estimates the volatility. Another issue we need to know about moving average model is how it weights the historical returns, below we show the weighting scale of moving average model,

$$\sigma_t^2 = \frac{1}{N}\sum_{i=1}^{N}(r_{t-1})^2$$

$$= \frac{1}{N}\times(r_{t-1})^2 + \frac{1}{N}\times(r_{t-2})^2 + \frac{1}{N}\times(r_{t-3})^2 + \cdots + \frac{1}{N}\times(r_{t-N})^2$$

As we see moving average model gives equal weights to each return does not matter how recent they are. There are some pitfalls of moving average model, the biggest problem with this model is that it does kill the effects of huge jumps in the historical process, and it shares equally effects of huge jumps among all historical returns. However, this is not true, the day after a crisis, a shock or turmoil is more affected by this jump then a year later. In addition, it averages effect of even a small one time huge ump among all historical returns. That way, it increases the effect of a single jump.

- **Exponentially Weighted Moving Average (EWMA) Model**

This model estimates volatility by giving exponentially increasing weights to recent data. For this model, last available recent data has more information than old data to estimate the future data. It estimates volatility for tomorrow by using square of yesterday's volatility and square of yesterday returns, and both data have exponentially increasing weights. This model removes the pitfalls that we mentioned for moving average model, since it gives a higher weight to recent data, it does not share effects of a single huge jump among all historical returns. Therefore, effect of a single jump is not increased, instead decreased in time. Mathematically we can represent and prove EWMA model as below,

$$\sigma_t^2 = (1-\lambda)\times(r_{t-1})^2 + (1-\lambda)\lambda\times(r_{t-2})^2 + (1-\lambda)\lambda^2\times(r_{t-3})^2 + \cdots + (1-\lambda)\lambda^{N-1}\times(r_{t-N})^2$$

$$= (1-\lambda)\times(r_{t-1})^2 + \lambda[(1-\lambda)\times(r_{t-2})^2 + (1-\lambda)\lambda\times(r_{t-3})^2 + (1-\lambda)\lambda^2\times(r_{t-4})^2 \cdots + (1-\lambda)\lambda^{N-2}\times(r_{t-N})^2]$$

Above in the second equation the term, $[(1-\lambda)\times(r_{t-2})^2 + (1-\lambda)\lambda\times(r_{t-3})^2(1-\lambda)\lambda^2\times(r_{t-4})^2 \cdots + (1-\lambda)\lambda^{N-2}\times(r_{t-N})^2]$ is equal to σ_{t-1}^2 therefore we can rewrite the equation as,

$$\sigma_t^2 = (1-\lambda)(r_{t-1})^2 + \lambda(\sigma_{t-1})^2$$

To get the volatility we simply take the square root of above equation,

$$\sigma_t = \sqrt{\sigma_t^2} = \sqrt{(1-\lambda)(r_{t-1})^2 + \lambda\sigma_{t-1}^2}$$

σ_t = is volatility or standard deviation in time t.

r_{t-1} = is rate of return in time t-1 which just one period before t.

σ_{t-1} = is volatility or standard deviation in time t-1.

About λ, in above equation lambda is a value between zero and one. $0 < \lambda < 1$. It gives automatically higher weights to recent data. The higher λ, the lower weight of the recent return and vice versa. The higher the λ the higher weight for previous volatility and vice versa. In formula for EWMA there are two expressions one is $\lambda(\sigma_{t-1})^2$ which implies the continuity of volatility, it shows how todays volatility is dependent on yesterday's volatility. Higher λ means, volatility is more dependent to yesterday's volatility and less dependent on market events. Second expression is $(1-\lambda)(r_{t-1})^2$ it shows how today's volatility is related to yesterday's return or market events. A low λ gives more impression on recent market events and means today's volatility is more related to yesterday's market events.

As we can see here, two parameters or two weights are dependent on each other; if one increases; the other should decrease and if one decrease the other should increase. Because of this, different λ values may give different results, later in our analysis we will test EWMA model for different values of . Another question is how we decide which λ to use. In fact, there is no a clear answer to this question. In practice, a good λ may vary between 0.75 and 0.98. In our analysis, we will use 0.94 value for λ which is suggested by famous Riskmetrics model that was developed by J.P. Morgan.

- **Generalized AutoRegressive Conditional Heteroskedasticity GARCH (1, 1) Model**

GARCH (1, 1) model estimates volatility by using long run mean of variance, previous market returns and previous volatility. It uses long run mean of variance because of mean reversion purpose, mean reversion concept states that both high and low prices in the long run tend converge to long run mean.

GARCH model has been developed by Bollerslev in 1986. According to GARCH model in addition to error values of conditional variance, variance also depends on its own lagged values and it tends to converge to its long run historical average. We can represent GARCH model mathematically as follows,

$$\sigma_t^2 = \gamma V_l + \alpha(r_{t-1})^2 + \beta(\sigma_{t-1})^2$$

V_l = long term mean of variance

γ, α, β are parameters of model and $\gamma + \alpha + \beta = 1$.

We can rewrite above equation as below,

$$\sigma_t^2 = \omega + \alpha(r_{t-1})^2 + \beta(\sigma_{t-1})^2$$

$\omega = \gamma V_l$ after a few adjustments we can get this equation $V_l = \frac{\omega}{\gamma}$ since we know that $\gamma + \alpha + \beta = 1$ we can say $\gamma = 1 - \alpha - \beta$ therefore eventually we get $V_l = \frac{\omega}{1-\alpha-\beta}$

Please note that GARCH model is a specific kind of EWMA model when long run mean variance is zero.

In recent years with the development of GARCH model, there has been a wide range of choices for volatility estimation. Because of this diverse list of choices, the choice of model is very difficult to estimate an efficient volatility. Theoretically, GARCH model is most efficient one, because we know that in the long run variance tend to converge to its mean and GARCH model includes the long run mean variance. However, in practice EWMA is mostly preferred model, for example Riskmetrics uses EWMA as a standardized model for volatility estimation mainly because of easy use of model and easy calculation of model parameters.[4] For choice of model, we can follow a rule of thumb,

- Start with the GARCH Model
- Estimate the Parameters of GARCH Model
- If $\omega > 0$ continue with GARCH Model
- If $\omega < 0$ GARCH Model is not appropriate, use EWMA Model and estimate λ. (Jung-Hyun Ahn, Market Risk I: Volatility and Correlation, Page 18).

Covariance, in statistics covariance is a measure of dependence between two variable, and it measure how related two variable move together. In finance covariance it measures how average value of two financial assets move together, how they vary together. Covariance helps financial manager to decide which assets are similar and move together and which moves inverse. It is especially crucial to know in portfolio choice process. In risk

[4] Riskmetrics, Page 78.

management process, we need to consider covariance between assets when we have estimate the risk for a portfolio consisting of two or more assets.

Covariance can be negative or positive. A positive covariance between two assets mean if one asset has a positive return more than average therefore the other assets must have also a positive return over average. A negative covariance means assets move in inverse direction if one has positive return the other will definitely have negative return, in real life that kind of two assets are very attractive and very difficult to find for a portfolio manager because having such two assets in portfolio will make portfolio highly diversified and minimized market risk.

Mathematically representation of covariance for a two assets X and Y is as follow,

$$Cov_{XY} = E[(X - \bar{X})(Y - \bar{Y})]$$

Where E(X) is the expectation or mean of a variable,

$$Cov_{XY} = E(XY) - E(X)E(Y)$$

In risk management since mean of returns are very close to zero, we accept that E(X) is zero. Son that case from second formula above we get,

$$Cov_{XY} = E(XY)$$

If we use a more financial representation including asset returns, finally covariance would be,

$$Cov_{XY} = \frac{\sum_{i=1}^{N}(r_x - \bar{r}_x)(r_y - \bar{r}_y)}{N}$$

How covariance is measured? In practice for risk management purposes, again same methodologies for volatility estimation are used to estimate the covariance.

- **Moving Average (MA) Model**

Moving average model approach to covariance estimation same as in volatility estimation, mathematical representation is as below,

$$Cov_{XY} = \frac{1}{N}\sum_{i=1}^{N} r_{xt-1}\, r_{yt-1}$$

- **Exponentially Weighted Moving Average (EWMA) Model**

Exponentially weighted moving average model approach the covariance estimation same as before but here we use the lagged value of covariance not the variance, mathematical representation is as below,

$$Cov_{XY} = (1-\lambda) r_{xt-1} r_{yt-1} + \lambda Cov_{XYt-1}$$

- **Generalized AutoRegressive Conditional Heteroskedasticity GARCH (1, 1) Model**

Generalized AutoRegressive Conditional Hetereskedasticity model estimates covariance in the same as estimating volatility, mathematical representation is as below,

$$Cov_{XY} = \omega + \alpha r_{xt-1} r_{yt-1} + \beta Cov_{XYt-1}$$

Correlation is a measure how degree of dependence between two variables, it shows what percentage and direction two variable move together. For portfolios consisting of more than one asset it is necessary to know and estimate the coefficient of correlation otherwise we cannot estimate a portfolio variance. Correlation takes value between -1 and +1 and it's symbol is $\rho_{x,y}$.

$\boldsymbol{\rho}_{x,y}$ **= +1** in this case it means that assets x and y are **perfectly correlated**. They are moving exactly in same direction at same degrees. For example if against a positive shock return for asset x rise 10% then return for asset y will definitely have 10% rise too.

$\boldsymbol{\rho}_{x,y}$ **= -1** in this case it means asset x and y are **perfectly inversely correlated**. They are moving exactly in inverse directions but their degree of change is exactly same. For example against a positive shock for asset x if return of asset x increase 10% then return for asset y will definitely decrease 10%.

$\boldsymbol{\rho}_{x,y}$ **= 0** in this case it means that asset x and y are **independent** of each other and they completely move in random ways and degrees.

Mathematically we represent correlation as below,

$$Corr_{xy} = \rho_{xy} = \frac{Cov_{xy}}{\sqrt{\sigma_x{}^2 \times \sigma_y{}^2}}$$

$$Corr_{xy} = \rho_{xy} = \frac{Cov_{xy}}{\sigma_x \times \sigma_y}$$

There are some additional statistical concept that we use in risk analysis not directly in volatility estimation but during evaluating the model data and it's assumptions. This concepts tell us about distribution of returns are they normally distributed or not. We introduce these concepts shortly below,

Properties of Variance,

Since we completed our explanations for crucial statistical concepts that we will use to estimate market risk, now we can talk about some useful properties of variance concept that we will use in later calculations,

- $\sigma^2(aX + bY) = a^2\sigma_X{}^2 + b^2\sigma_Y{}^2 + 2ab \times Cov(X, Y)$

- $\sigma^2(aX + b) = a^2\sigma_X{}^2$

The first property above is used for calculating portfolio variance that is very crucial in risk estimation process.

Skewness is statistically a measure of asymmetry. It measures how and which way returns are distributed around mean. It is useful when we try to find out if our data is normally distributed or not. For normal distribution skewness equals to zero and returns are distributed around mean equally which means that 50% percent of returns lie below the mean and remaining 50% lie above the mean. Its mathematical representation is as follow,

$$SK = E\left[(\frac{X - E(X)}{\sigma})^3\right]$$

Kurtosis measures how peaked is the distribution. Kurtosis tells us about tails of distribution and it is very crucial during model evaluation, as we will see later in our analysis. In normal distribution, kurtosis equals to three therefore a kurtosis below or above three means that data is not normally distributed. Its mathematical representation is as follow,

$$Kurtosis = E\left[(\frac{X - E(X)}{\sigma})^4\right]$$

Jarque-Bera[5] is test of goodness of fit. It measures how well data fits in distribution. It uses skewness and kurtosis and tells us if our data is normally distributed or not. In our analysis, we accept that for normal distribution Jarqu-Bera equals to six. Therefore, a value above six will mean that data is not normally distributed. It has a formula as below,

$$JB = N \times \left[\frac{SK^2}{6} + \frac{(K-3)^2}{24}\right]$$

Where N is number of observations, SK is skewness and K is kurtosis.

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[5] For more information, about kurtosis, skewness and Jarque-bera please refer to Gerhard Bohm, Introduction to Statistics, Page 26.

Chapter 3

3. Value at Risk (VaR)

3.1. Definition of Value at Risk

In recent years, some of the world's leading financial institutions have suffered huge losses in the financial markets. One implication of this has been the increased monitoring of exposure to market risk.

Value at risk (VaR) is a method for calculating and controlling exposure to market risk. It measures the volatility of an institution's assets - the greater the volatility, the greater is the risk of a loss. We can define VaR as a single number which estimates the maximum expected loss of a portfolio over a given time horizon (the holding period) and at a given confidence level.[6]

3.2. Calculating Value at Risk

VaR calculates the expected maximum loss of a portfolio as a result of an adverse change in risk factors (for example, interest rates, exchange rates and stock prices). The VaR estimate is dependent some concepts such as, a specified holding period, confidence level, volatility and, usually, correlation among the variables.[7]

Broadly speaking, the calculation of VaR involves the following steps:

3.2.1. Step 1 – Determining Holding Period

Holding period is a concept related with underlying assets in portfolio and its symbol is t. For example for portfolios with liquid assets, value at risk for a short holding period is calculated such as 1 day. On the other hand, for portfolios consisting of illiquid assets a longer holding period must be used such as 10 days, and 10 day value a t risk should be estimated.

[6] Institution KnowHow
[7] Institution KnowHow

3.2.2. Step 2 –Confidence Level

Confidence level is degree determined according to desired level of guarantee or certainty. It depends how much certain and guarantee results an institution desires. If an institution wants a value at risk amount with 99 percent certainty that loss will not exceed this amount, then this institution should use 99% confidence level for VaR estimation. As confidence level increases, value at risk increases too. Most used confidence levels in real life are between 95% and 99%.

3.2.3. Step 3 – Create a Probability Distribution of Likely Returns

In vale at risk estimation, most used probability distribution is normal distribution because distribution of returns are not normally distributed but very close to normal distribution.

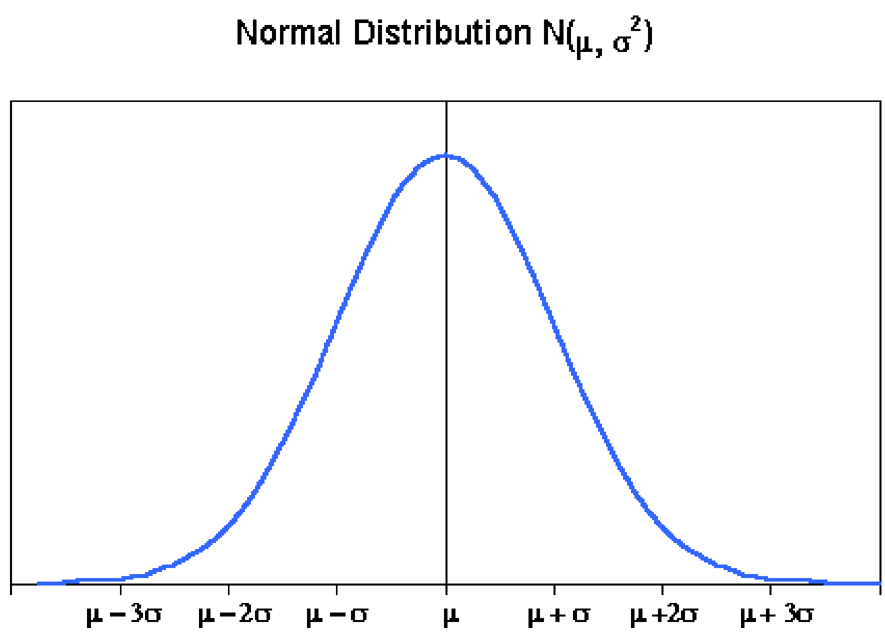


Figure 1 Normal Distribution (ALLAN M. MALZ Financial Risk Management)

3.2.4. Step 4 – Determine Correlations between Assets

Financial instruments are not generally independent of each other. Correlation measures the extent to which the value of one variable is related to the value of another variable. For example, the value of one currency may be correlated to movements in the value of another currency and the value of real estate stocks tends to be correlated with changes in interest rates. Correlation between assets impacts on the risk of a portfolio. Therefore, the degree of correlation between assets is a vital consideration for portfolio managers who wish to reduce risk through diversification.[8]

3.2.5. Step 5 - Calculate the Volatility of the Portfolio

The most crucial step in estimating value at risk is, calculating volatility shows the deviation or dispersion of returns. Volatility is simply calculated as standard deviation which is dispersion from the mean.

3.2.6. Step 6 - Calculate the VaR Estimate

After completing all previous steps the last step is simply calculating the VaR. By using simple VaR formula;

$$VaR = \sqrt{t} \times P \times Z(1 - CL) \times \sigma_{\mathrm{p}}$$

$\sqrt{t}$ = Holding period

P = Portfolio value in currency amounts.

CL= Confidence level.

Z = Inverse of the normal distribution which simply helps us to find, how many standard deviations should we go from mean to include the mentioned confidence level.

σ_{p} = Portfolio Volatility

As we can see here, only crucial factor to estimate VaR is portfolio volatility. Therefore, the calculation of VaR is then really a question of finding the appropriate volatility of the

[8] Institution KnowHow

portfolio. Below we will discuss how to calculate VaR, what methodologies are used estimate VaR or in other words portfolio volatility.

3.3. Comments on Value at Risk Method

3.3.1. Advantages of Value at Risk Method

The most important advantage of value a t risk is its simplicity and easiness to apply. It is not very technical and in real money terms therefore, managers or other related people without technical knowledge can easily understand and interpret the risks.

Other benefits include:

- Value at risk does not need to focus on individual assets in portfolio; it can give a clear aggregate result for a portfolio.

3.3.2. Disadvantages of Value at Risk Method

The most important drawback of value at risk is its normal distribution assumption that all VaR methods use except historical simulation. However, in fact returns are not normally distributed and there a lot extreme events in market. These, extreme events cause model to underestimate or overestimate the risks sometimes.

3.4. Methods of Calculating Value at Risk

There are two different methods to estimate Value at Risk, one is parametric method, which is also called variance-covariance method, and second one is non-parametric method that consists of two simulation methods, historical simulation and Monte Carlo simulation. Both methods have advantages and disadvantages. Suitability of these methods are evaluated according to shape of distribution of returns and linear movements returns. We should note that outcome of value at risk estimations generated by each method usually vary, different methods generate different results.

In cases that future price movements can be explained by normal distribution parameters or price changes follow a normal distribution, price movements can be estimated by variance and covariance estimation therefore in these cases we can use variance covariance approach. In cases where price movements do not follow normal distribution, each expected change has different probability, and returns do not follow normal distribution because of this it is not very easy to calculate variance and covariance therefore in such cases it's better to use historical simulation.[9]

Another important indicator to determine value at risk calculating method is linear dependency of portfolio return to return on single assets in portfolio. Return on options and other derivative products are not linearly dependent on portfolio including them; therefore, it is not possible to use variance-covariance and historical simulation methods. For portfolios including derivative products, we use Monte Carlo simulation method.[10]

There are three popular approaches to calculating Value at Risk:

3.4.1. Non-Parametric Approach Monte Carlo Simulation

Monte Carlo Simulation is useful to VaR estimation for portfolios including options, since in our analysis, our portfolio does not include options we will not include this method in our analysis.

3.4.2. Non-Parametric Approach Historical Simulation

Historical simulation is a method for Value at Risk estimation, which does not make any assumption for distribution of asset returns and does not need any variance or covariance calculation. Therefore, historical simulation method can be used for linear or non-linear asset returns. In this method, scenarios are being created from historical data, and an ordered historical portfolio return series is created to find the lowest return at the confidence level. This method firstly takes historical returns of each asset in portfolio, and then multiplies each asset return by its weight in portfolio to find portfolio return. Then sorts the portfolio return series from lowest to highest and create percentile series for each

[9] Bolgün, Akçay, p. 396.
[10] Riskmetrics, p. 11.

return. After doing all, the percentile, which corresponds to confidence level, the return value corresponding to this percentile is Value at Risk.

For example, let us say that we have a portfolio of one million US dollar, consisting of three assets A, B and C and we have daily prices for last 100 days for all three assets. To estimate the value at risk at 99% confidence level first, we find daily returns for each asset, and then we multiply each asset by its weight in portfolio, considering each asset has equal weight we have, $Rp = \frac{1}{3} \times A + \frac{1}{3} \times B + \frac{1}{3} C$. Now we have the portfolio return as a ratio, to get portfolio return in dollar amounts we multiply Rp by portfolio amount .Now we have Rp$= Rp x 1000000 for each day. Finally, we just sort all the portfolio returns in dollars from lowest to the highest and we create a percentile for each day. The 99% percentiles is our value at risk at 99% confidence level. In other word, 99^{th} smallest return is our value at risk at 99% confidence level.

Comments on Historical Simulation

As we see historical simulation method, is very straightforward and easy to apply the only work matters is collecting data, rest is very quick. Historical simulation does not need any complex calculation or iteration, it is non-parametric therefore need to calculate any parameters therefore we have no risk of errors arising from incorrect calculations. Since, we do not do any assumption in the model for example there is no normality assumption as in case of variance-covariance method; there is no risk of bias arising from model assumptions. Historical simulation method is comprehensive, since it uses historical data it can be applies to and kind of financial instrument which can provide historical data.

Although there are certain advantages of historical simulation method, there are also some certain drawbacks of method. First and most important drawback is its reliance on past data. Model tries to estimate future movements from past movements but this has some problems. Past data may be from a highly volatile or highly stable period but future may not be. Past data may include some one time huge shocks that will not repeat in future, similarly, there might be some big shocks in future that past data does not include; all of those may result in under or over estimated value at risk. Another drawback is length of past data period, data from very short time period may not have meaningful information or data from long time period may include irrelevant old information. Besides, finding historical data for all types of asset may not be possible always.

3.4.3. Parametric of Variance-Covariance Approach

Variance-covariance approach is most popular and most widely used value at risk calculation method; it is also used by riskmetrics which was initially developed by JP Morgan. Parametric methods that used often in market risk estimation based on the assumption that returns from financial assets have a normal distribution. Besides, variance-covariance approach also assumes that portfolio risk has a linear relationship with single asset risk factors that follow a normal distribution. In parametric methods by using historical return series of financial assets, basic risk parameters like standard deviations and variances are calculated. Then by using these parameters, variance-covariance matrix is calculated and then finally value at risk for portfolio is estimated.

In parametric methods, biggest concern is calculation of variance covariance matrix or mainly volatility. Because of this, there are different methodologies used in parametric methods for calculation of volatility and variance covariance matrix. In our study, we will use three approaches that were explained before. We will try to estimate value at risk with variance-covariance approach by using MA, EWMA and GARCH (1, 1) approaches for calculating volatility and variance-covariance matrix.

Assumptions of Variance-Covariance Approach

Parametric or variance-covariance approach for estimating value at risk has following assumptions,

- The future distribution and movements on financial assets can be measured based on the distribution and movements on financial assets in past.
- All of assets returns are normally distributed.
- Assets have constant correlation between each other.

The biggest and most crucial assumption here is that returns of financial assets are normally distributed. Normal distribution assumption brings a lot ease into model, because normal distribution has only two parameters mean and standard deviation.

Therefore, if you estimate these two parameters, that means you have the value at risk. In practice mean of returns are accepted to be zero, because in short time periods mean of returns are very close to zero and very small compared to standard deviation. This mean after all, estimation of value at risk is only matter of calculating standard deviation or volatility. On the other hand, this assumption brings a lot problem into model, because as we will see later in our analysis returns of financial assets are not always normally distributed.

Normal Distribution

Before we go into detail and explanations of the variance covariance approach, we want to explain the main and most crucial assumption in the model.

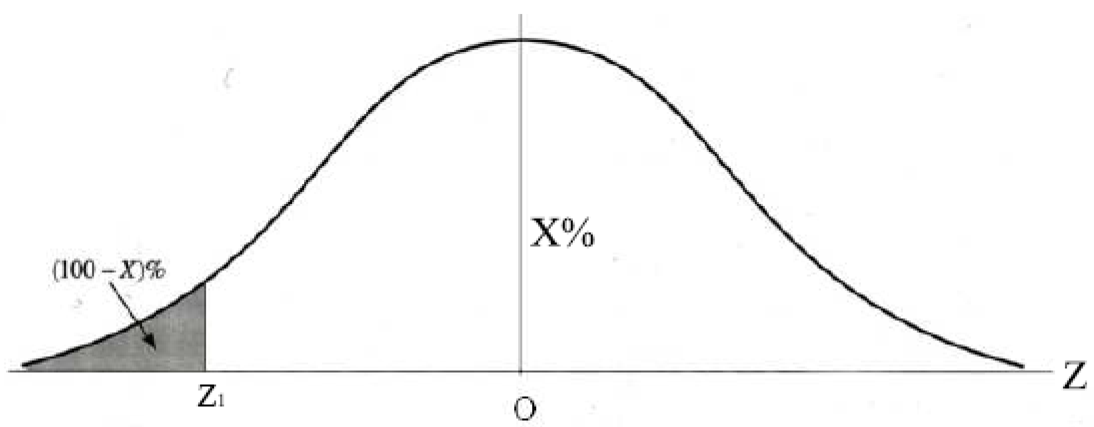


Figure 2 Standard Normal Distribution, Jung-Hyun Ahn, Market Risk II

Standard normal distribution has above figure and this representation $Z \sim N(0, 1^2)$.

Since we assume our data is normally distributed we need to change normal distribution into standard normal distribution to find how many standard deviations we need to go from mean to correspond to our value at risk at given confidence level. Let us call our normal distribution as X and confidence level as 99%. To get the normal X value for our confidence level we need to apply following formula,

Given $X \sim N(\mu, \sigma^2$

$$Z = \frac{X - \mu}{\sigma} \sim N(0, 1^2)$$

$$X = Z \times \sigma + \mu$$

Given $X \sim N(\mu, \sigma^2$ is one percentile smallest value of X , X(1%) or in other words value at risk at 99% confidence level,

$$X(1\%) = Z(1\%) \times \sigma \times \mu$$

We find appropriate Z value from predefined Z table or by simply using Microsoft Excel built in function NORMALSINV(1-%99).[11]

As we said before in risk management we assume that mean of returns are zero so we can remove the mean μ from above formula,

$$X(1\%) = Z(1\%) \times \sigma \approx -2.33 \times \sigma$$

[11] Jung Hyun Ahn, Market Risk II, Pg.14

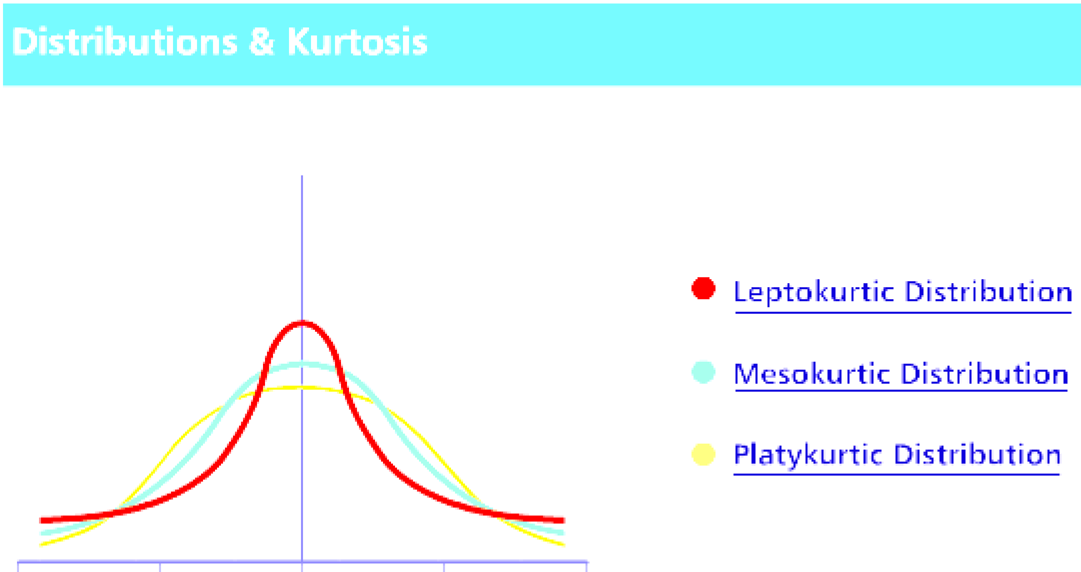


Figure 3 Kurtosis, Intuition Know-How

However, is normality assumption holds, are returns from financial assets really normally distributed? No! In practice, mostly returns of financial assets are not normally distributed, but they are at best most closely approximating to normal distribution. Normality assumption of variance-covariance approach brings a problem called "Fat Tails", as we explained before normal distribution has a kurtosis of three. However, in real world distribution of returns of financial assets have kurtosis highly above three, in other words they are leptokurtic therefore they create the fat tails problem, fat tails mean that there are some more values in the tail than normal distribution, this may cause an extreme value problem and therefore underestimation of value at risk.

Calculating Value at Risk with Variance-Covariance Approach

We can summarize variance-covariance approach for value at risk estimation in three steps,

- Decide and make assumption of distribution of returns
    - We assume normal distribution
- Calculate mean of returns
    - We assume that its zero.

- Calculate variance-covariance matrix
  - We will use MA, EWMA and GARCH (1, 1) methods to calculate variance covariance matrix.

Variance-covariance approach is most popular and most widely used value at risk calculation method; it is also used by riskmetrics, which was initially developed by JP Morgan. Parametric methods that used often in market risk estimation based on the assumption that returns from financial assets have a normal distribution. Besides, variance-covariance approach also assumes that portfolio risk has a linear relationship with single asset risk factors that follow a normal distribution. In variance-covariance method value at risk is calculated as below,

$$VaR = P \times \sigma \times Z(1 - CL) \times \sqrt{t}$$

t = Holding period

P = Portfolio value in currency amounts.

CL= Confidence level.

Z = Inverse of the normal distribution which simply helps us to find, how many standard deviations should we go from mean to include the mentioned confidence level.

σ = Volatility of single asset

Above formula gives value at risk for portfolios consisting of only one single financial asset. In case when there are two financial assets in portfolio, portfolio volatility is calculated by considering; weights of two assets in portfolio and coefficient of correlation for two assets as bellows,

Let us say that our portfolio consist of two asset X and Y,

α_X= Weight of asset X in portfolio

α_Y= Weight of asset Y in portfolio

$\alpha_X + \alpha_Y = 1$

σ_X= Volatility of asset X

σ_Y= Volatility of asset Y

σ_p = Volatility of portfolio

$\rho_{X,Y}$ = Coefficient of correlation between asset X and Y

$$\sigma_p = \sqrt{\alpha_X{}^2\sigma_X{}^2 + \alpha_Y{}^2\sigma_Y{}^2 + 2\alpha_X\alpha_Y\rho_{X,Y}\sigma_X\sigma_Y}$$

$$\sigma_p = \sqrt{\alpha_X{}^2\sigma_X{}^2 + \alpha_Y{}^2\sigma_Y{}^2 + 2\alpha_X\alpha_Y Cov(X,Y)}$$

After calculating portfolio volatility for portfolio with two assets with above formula, we can put the amount we found into the VaR formula, we get portfolio value at risk as below,

$$VaR_p = P \times \sigma_p \times Z(1 - CL) \times \sqrt{t}$$

In case portfolio consist of more than two financial assets, portfolio volatility or standard deviation is calculated by using matrixes, for this purpose variance-covariance matrix need to be calculated, variance-covariance matrix is calculated as below,

C = Variance-Covariance Matrix

σ_i = Volatility of i^{th} asset

ρ_{ij} = Coefficient of correlation between i^{th} and j^{th} asset

$\alpha_i =$ Weight of ith asset

$\alpha_j =$ Weight of jth asset

$$\sigma_p^2 = \sum_{i=1}^{n} \alpha_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \alpha_i \alpha_j Cov_{ij}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j \sigma_i \sigma_j \rho_{ij}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_i \alpha_j Cov_{ij}$$

$$C = VCM = \begin{matrix} \sigma_1 & Cov_{12} & Cov_{13} \\ Cov_{21} & \sigma_2 & Cov_{23} \\ Cov_{31} & Cov_{32} & \sigma_3 \end{matrix}$$

Standard deviation or volatility for portfolios consisting of more than two assets is calculated by using variance-covariance matrix as follow,

$C =$ Variance-Covariance Matrix

$\sigma_p =$ Volatility of portfolio

P = Portfolio value in currency amounts

$\alpha =$ Column vector consisting of individual weights of all assets in portfolio

$$\alpha = \begin{matrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{matrix}$$

$\alpha^T =$ Transpose of column vector of weights

$$\sigma_p = \sqrt{\alpha \times C \times \alpha^T}$$

After calculating portfolio standard deviation or volatility now we can estimate portfolio value at risk by putting portfolio volatility calculated by variance-covariance matrix, into value at risk formula as below,

$$VaR_p = P \times \sqrt{\alpha \times C \times \alpha^T} \times Z(1 - CL) \times \sqrt{t}$$

3.5. Comparison of Value at Risk Estimation Methods

Value at risk is very useful for estimation of market risk; in practice, it is most widely used method to estimate market risk. However, to estimate an efficient value at risk we need to decide which approach to estimate value at risk is best and most suitable to our portfolio.

In fact to answer which approach is the best is very difficult, there is no a single best. Institutions decide which model is best for them according to, portfolio structure, ease of application, cost of model, time consumption of model. For example sophisticated financial institutions mostly use simulation methods, because simulation methods are flexible data changes does not affect the model, and ever decreasing of cost of applying simulation models. On the other hand, smaller firms mostly prefer variance-covariance model, because mostly they do not have significant amount of options in their portfolio, and they easily outsource their risk estimations to sophisticated firms like Riskmetrics.

Below table summarizes the comparison of three methods,

	Variance-Covariance	Monte Carlo	Historical
Portfolio Valuation	Delta approximation (linear)	Full/Approximation	Full
Assumed Distribution of Returns	Normal distribution	Flexible (usually the normal distribution)	Actual
Requirement for Cashflow Mapping	Yes	No	No
Suitability for Options	No (unless holding period is short and portfolio has moderate options content)	Yes	Yes
Computation and Implementation	Relatively easy to compute and implement	Can be costly and slow depending on the number of simulation runs required	Easy to compute and implement
Accuracy of VAR Estimate	Depends on validity of normality assumption and degree of portfolio optionality	Generally the most reliable method provided sufficient simulation runs are carried out	Good, provided the historical data is a reliable guide to future market behavior

Figure 4-Comparison of VaR Models, Intuition Know-How

3.6. Methods to Support and Check Accuracy of Value at Risk Method

3.6.1. Backtesting

Basel Committee obliges institutions to test their Value at Risk models regularly in its famous Market Risk Amendment. In addition, Basel committee explains the standards of testing the models in Market Risk Amendments document. For testing purpose Backtesting method is suggested by Basel Committee. To compute and measure the accuracy of value at risk estimations, and to measure performance of value at risk models backtesting should be done. Backtesting is executed by comparison of estimated value at risk value for next day and actual value at risk value observed on next day.

The VaR value that estimated by value at risk method should be bigger than VaR value or profit and loss amount calculated by backtesting. If VaR value or actual profit and loss calculated on the actual day by backtesting is more than VaR value that value at risk method estimated on previous day we call it a violation to model and it deteriorate the model.

To clarify backtesting evaluates performance of the Value at Risk estimations by using a constant past data range. What backtesting does is simply estimate a VaR for today by using past data and then compare this estimated VaR with today's portfolio profit and loss. If portfolio loss today exceeds the VaR estimate then it is a violation to model. To run a backtesting analysis we need to choose an estimation window and a testing window for our portfolio. We need to have testing window bigger than estimation window. Below we explain main concepts and preparation of back testing analysis,

W_E = Estimation window the rolling but constant number of observations that we use to estimate value at risk from past data

W_T= Testing window the rolling but constant number of observations that we use to compare value at risk estimates

$T = W_E + W_T$ = Total number of observations

For example, let us say that we decided to test our VaR model for last 1500 days this is our testing window, and we decided to estimate VaR we use 500 day past and this is our estimation window. In total, we have 2000 observations for backtesting. It works like that; starting from 1st day to 500th day, we use the first estimation window to estimate a VaR for 501st day which is the first day of testing window. Then we start from 2nd day to 501st day we take second estimation window (which again consist of 500 observations), and we estimate a VaR for 502nd day which is second day of testing window. This process continues until we make VaR estimations for all 1500 observations in testing window. Then we start to compare, for example we compare VaR estimate on 501st day, in other words VaR estimate for first day of testing window and we compare it with portfolio profit and loss on 501st day. If portfolio loss exceeds the VaR estimate then it is counted as a violation to model.

Below in figure 5 we show how estimation window and testing windows roll together in a loop,

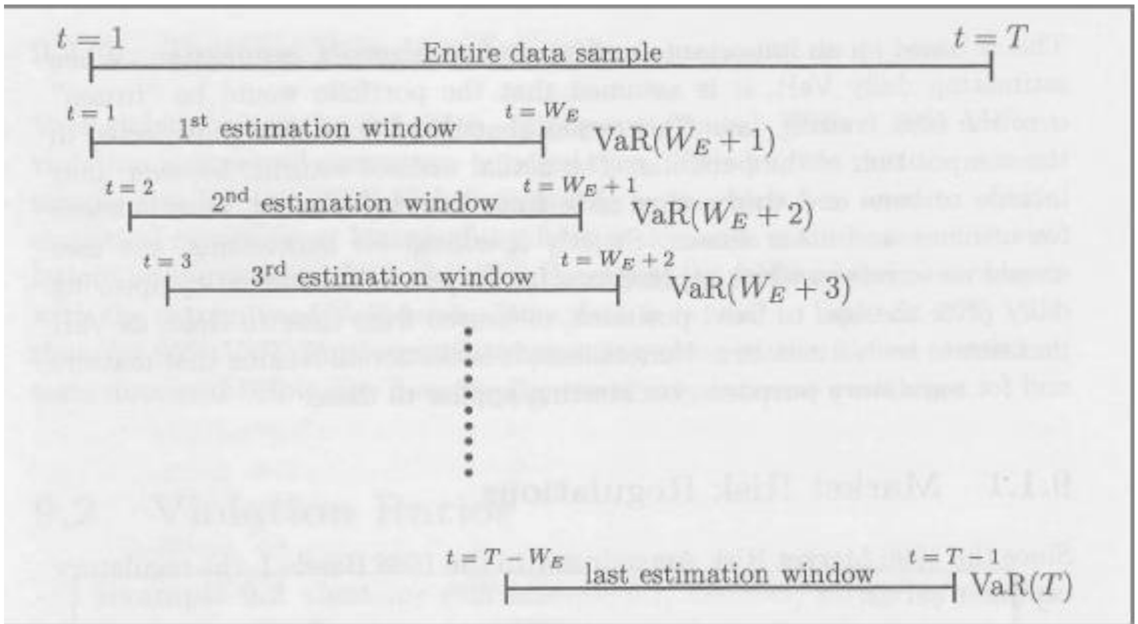


Figure 5, Backtesting Observations, Jung-Hyun Ahn Market Risk II: Value-at-Risk

How we decide if the VaR model is accurate?

After deciding to estimation, testing windows, estimating the VaR, and comparing it to actual portfolio profit and loss. How can we decide model is doing well or not, how many number of violations are acceptable? For purpose of decision making for model accuracy we use a ratio called Violation Ratio.

Violation Ratio

Violation ratio is simply ratio of number of total violations to number of expected violations; expected violation is number of acceptable violations we calculate the number of acceptable violations by simply multiplying number of observations in testing window with 1-confidence level. So in our case our expected violation will be 1500 x (1-0.99) = 15.

Mathematically we can represent Violation Ratio as below,

$$VR = \frac{V}{E(V)}$$

VR= Violation Ratio

V= Total number of violations

E (V) = Expected violation, number of acceptable violations

We interpret the result of Violation Ratio as below,

- VR > 1 means that model underestimates the market risk
- VR < 1 means that model overestimates the market risk
- VR = 1 means that model is accurate

Chapter 4

4. Intraday Value at Risk Analysis, VaR Model Comparisons and Realized Volatility Forecasting for a US Stock Exchange Portfolio

4.1. Scope and Objectives of the Study

In last several years world of finance have increased its speed day by day. Classical way of approaching to risk management is no more sufficient, daily analysis are nor more satisfying for brokers, traders and risk analysts. Now speed of trading and transactions are very high, high frequency financial instruments spread in financial world, many transactions start and end in less than a day. Therefore, risk managers, brokers or traders should have more information than daily basis; they should keep track of high frequency movements. For example, a trader may need risk value at risk of its portfolio at 13:55 afternoon.

Because of these rapidly changing needs, in this study we wanted to apply value at risk models to intraday financial data, and try to see their realized volatility forecasting power and to understand which model is more accurate on which data range. We aim to make optimal model suggestion for intraday value at risk users.

In this study, we choose a portfolio consisting of three US stock exchanges. We use intraday 1 minute, 5 minute and 10 minute prices to forecast realized volatility and estimate value at risk with both parametric and non-parametric approaches. After that, we do backtesting analysis to measure the accuracy of each model. For preparation of this study we followed a step by step process as below,

- We choose our portfolio
- Decide data type and period
- Decide which models and approaches will be used
- Define assumptions and rules for model usage
- Doing calculations and analysis

- Testing and controlling results

4.2. Portfolio Choice, Data Type and Period

In this empirical study, we constitute a portfolio consisting of three US stock market indexes: Standard & Poor's (S&P500), Dow Jones Industrial Average (DJI) and NASDAQ Composite Index (NASDAQ). Value of our portfolio is ten million US dollars, it consist of four million S&P500, three million DJI and three million NASDAQ shares.

Index	**S&P500**	**DJI**	**NASDAQ**	**Total**
Amount	4 Million	3 Million	3 Million	10 Million

Figure 6, Portfolio Allocation

For intraday value at risk analysis, we used 1 minute, 5 minute and 10 minute closing prices of three stock market indexes between 29/07/2013 and 31/08/2013 time period and we created our portfolio three times for each data type. We gathered data from Bloomberg financial database.

For backtesting analysis, we used our portfolio as estimation window, which means that for backtesting analysis our estimation window is 29/07/2013 and 31/08/2013 one month time period. For testing window, we choose following three months after estimation window. Therefore, our testing window is three months time period between 03/09/2013 and 27/11/2013.

In risk management, generally it is known that prices of financial assets are increasing exponentially, therefore for our portfolio rerun we calculated logarithmic or compounded returns. In excel we applied following formula and calculated returns for each closing price,

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

r refers to Return, t refers to current time, P refers to Price and t-1 refers to one period before current period t.

Below in table 2 we include a screen shot from our excel work to show how we calculate returns for our portfolio,

Clipboard | Font | Alignment | Number | Styles

J13 f_x =LN(D13/D12)

	A	B	C	D	E	F	I	J	K	L	M
9								Return			
10	Pricing Date	TIME	# of Data	S&P500	DJI	NASDAQ		S&P500	DJI	NASDAQ	Portfolio
11							Weight	0.4	0.3	0.3	
12	29-07-13	17:30	1	1688.62	15529.97	3606.99					
13	29-07-13	17:35	2	1688.26	15522.62	3610.74		-0.00021321	-0.00047339	0.00103911	844.29418859
14	29-07-13	17:40	3	1687.88	15524.43	3612.13		-0.00022511	0.00011660	0.00038489	604.02077085
15	29-07-13	17:45	4	1689.38	15536.64	3616.57		0.00088829	0.00078619	0.00122844	9597.06641214
16	29-07-13	17:50	5	1689.77	15538.96	3613.6		0.00023083	0.00014931	-0.00082156	-1093.42349298
17	29-07-13	17:55	6	1688.79	15532.76	3614.05		-0.00058013	-0.00039908	0.00012452	-3144.18007463
18	29-07-13	18:00	7	1689.12	15538.66	3612.84		0.00019539	0.00037977	-0.00033486	916.27751930
19	29-07-13	18:06	8	1688.38	15531.26	3612.18		[illegible]	[illegible]34	-0.00018270	-3729.90604622
20	29-07-13	18:13	9	1686.97	15520.04	3610.15		[illegible]	[illegible]68	-0.00056215	-7196.33753338
21	29-07-13	18:18	10	1686.78	15520.18	3607.26		[illegible]	[illegible]02	-0.00080084	-2825.99959810
22	29-07-13	18:23	11	1687.7	15528.32	3608.52		[illegible]	[illegible]34	0.00034923	4801.80352274
23	29-07-13	18:28	12	1687.72	15527.67	3609.78		0.00001185	-0.00004186	0.00034911	969.15989647
24	29-07-13	18:33	13	1687	15523.11	3607.48		-0.00042670	-0.00029371	-0.00063736	-4500.02862980
25	29-07-13	18:38	14	1686.39	15519.01	3605.79		-0.00036165	-0.00026416	-0.00046858	-3644.83018150
26	29-07-13	18:43	15	1686.11	15511.86	3608.11		-0.00016605	-0.00046083	0.00064320	-117.08181335
27	29-07-13	18:48	16	1687.07	15523.61	3607.87		0.00056920	0.00075720	-0.00006652	4348.82081259
28	29-07-13	18:53	17	1687.04	15522.19	3607.61		-0.00001778	-0.00009148	-0.00007207	-561.76495801

Formula for portfolio return
=(J13*J11+K13*K11+L13*

Figure 7, Excel Return Calculation for 5 Minute Data

4.3. Rules, Assumptions and Analysis of Data

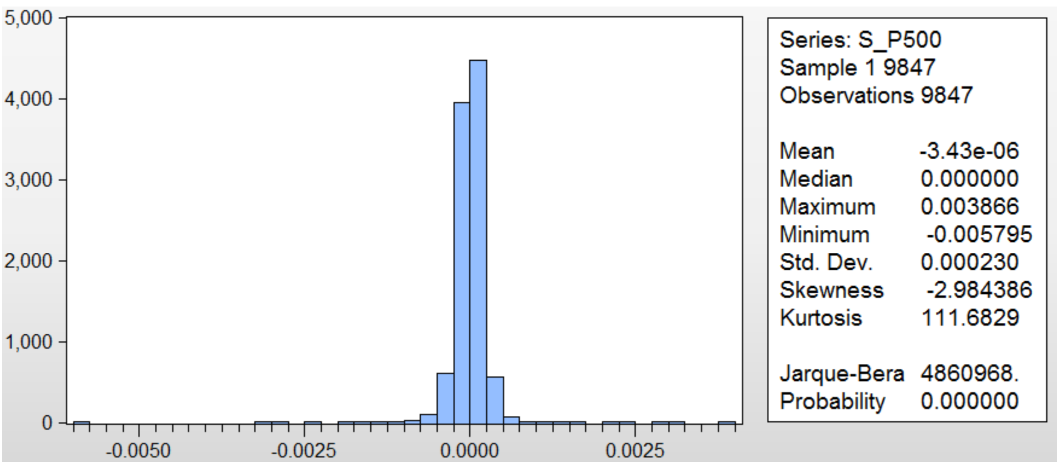
In our value at risk analysis, we mainly use 99% confidence level since it is the one mostly used in finance world and Riskmetrics mostly use it. For Exponentially Weighted Moving Average EWMA estimation of VaR, we used Lambda $\lambda = 0.94$ since it is used and recommended by Riskmetrics, but we will also use other possible Lambdas in our analysis of backtesting to see the effect of different Lambdas.

Biggest assumption in Value at Risk analysis is Normality assumption, which assumes that returns are normally distributed. We will test the validity of normality assumption by applying Jarque-Bera test and calculating skewness and kurtosis for returns. We also assume that mean of returns is equal to zero.

Below we will analyze each individual asset in portfolio and portfolio as a whole, we will calculate basic descriptive statistics by using 1-minute returns, to check validity of assumptions, and we will show graphs showing structure of data. Figures are prepared by using Eviews.

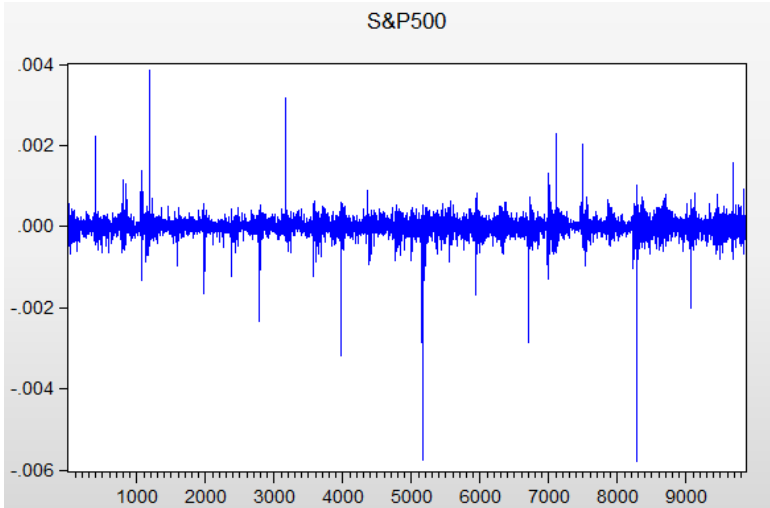
S&P500

Figure 8, S&P500 Descriptive Statistics and Histogram



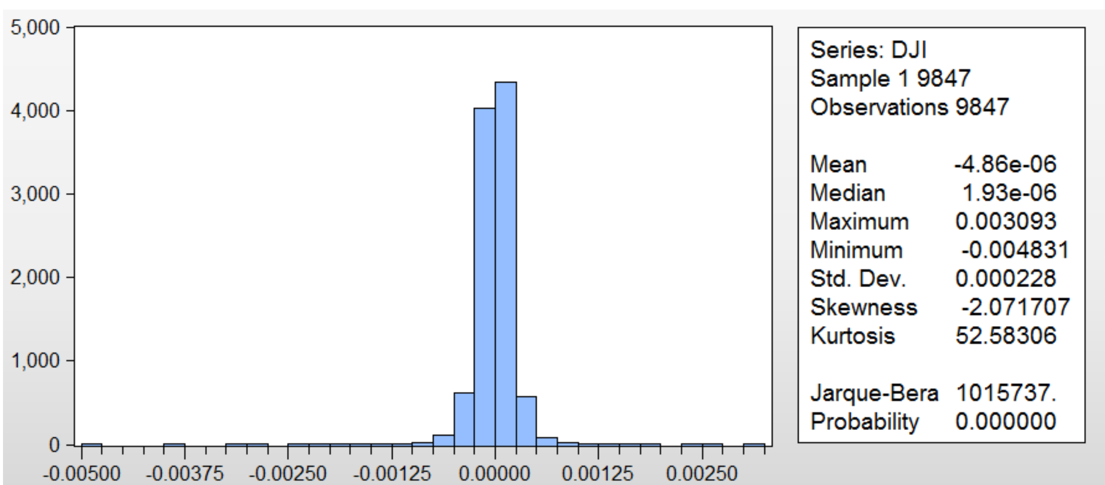
Below in Figure 8, we see that mean of S&P500 1-minute returns are very close to zero so we ca accept it is zero, that mean our assumption of zero mean is valid. For normality assumption, let's first look at Skewness it is almost -3 but in normal distribution it should zero. We see that kurtosis is over 100 this an extremely huge number compared to kurtosis 3 for normal distribution. Jarque-Bera is extremely high but in normal distribution it should be 6, if it is more than then we can easily say that distribution is not normally distribution. If we look at the histogram, we see the effect of high skewness and kurtosis and we see the fat tail problem. Below in figure 9, we plot the 1-Minute returns and we see that it is highly volatile with spikes.

Figure 9, S&P500 Plot of 1-Minute Return



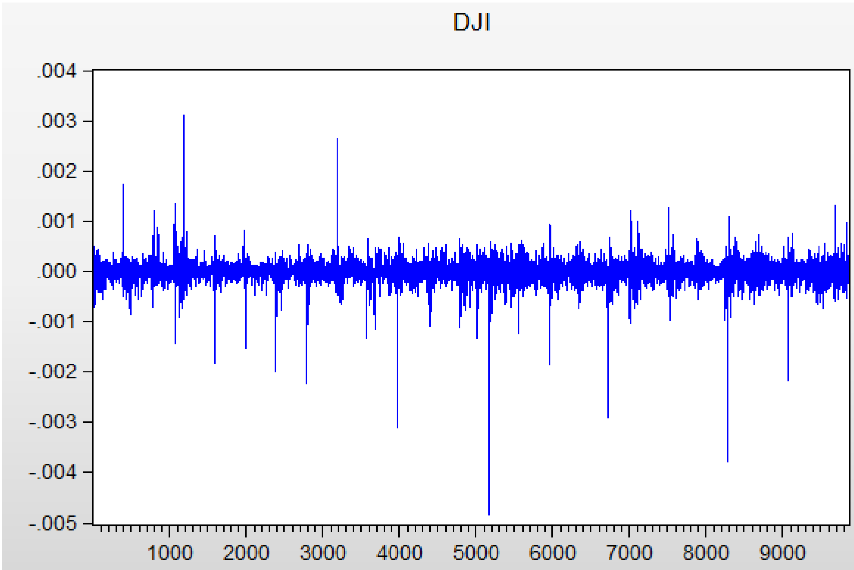
Dow Jones

Figure 10, Dow Jones Descriptive Statistics and Histogram



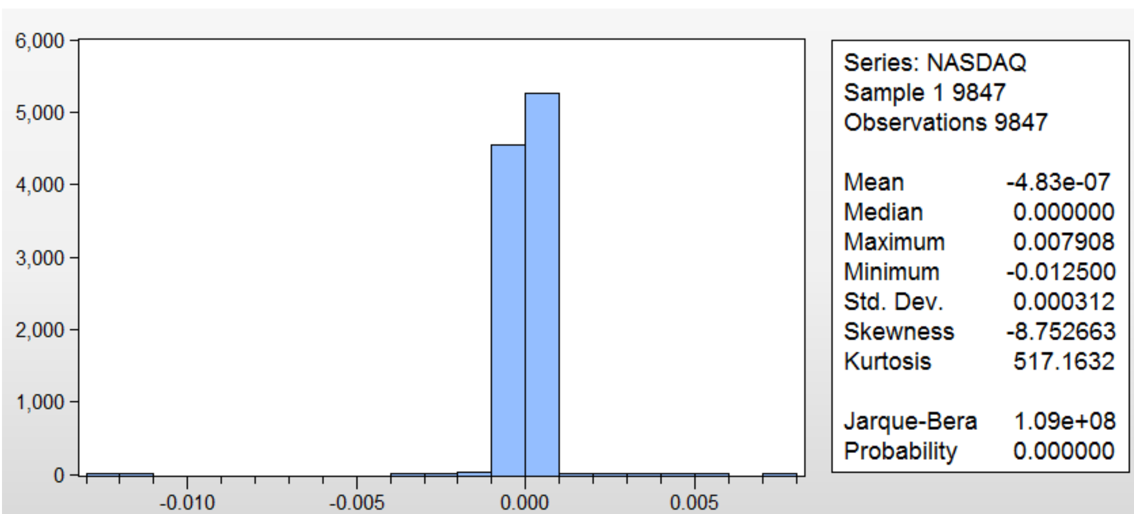
Above in Figure 10, we see that again mean is very close to zero therefore we accept it as zero and validate zero mean assumption. However, skewness, kurtosis and Jarque-Bera are again extremely higher than normal distribution; therefore say that returns are not normally distributed. By looking at histogram, we see that there are some extreme values at tails and there is a fat tail problem. Below in Figure 11 we can see the plot of 1-Minute returns, we see that returns are highly volatile with extreme values.

Figure 11, Dow Jones Plot 1-Minute Returns



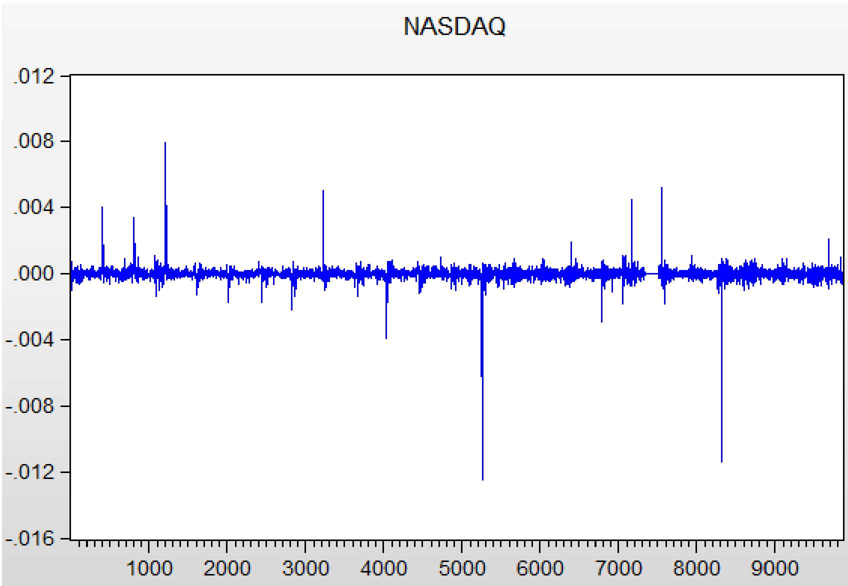
NASDAQ

Figure 12, NASDAQ Descriptive Statistics and Histogram



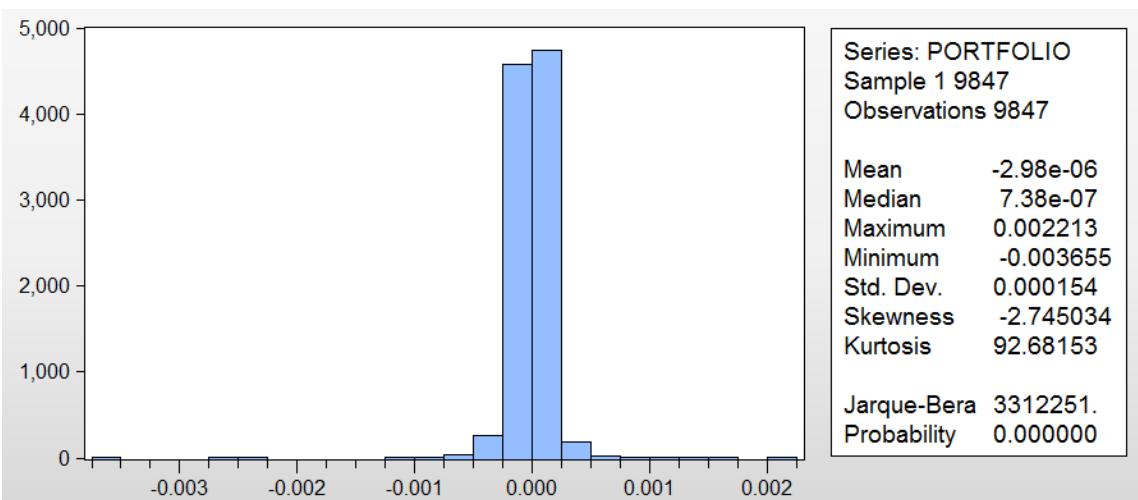
Above in Figure 12, we see that again mean is very close to zero therefore we accept it as zero and validate zero mean assumption. However, skewness, kurtosis and Jarque-Bera are again extremely higher than normal distribution; therefore say that returns are not normally distributed. By looking at histogram, we see that there are some extreme values at tails and there is a fat tail problem. Below in Figure 13 we can see the plot of 1-Minute returns, we see that returns are highly volatile with extreme values.

Figure 13, NASDAQ Plot of 1-Minute Returns



Portfolio

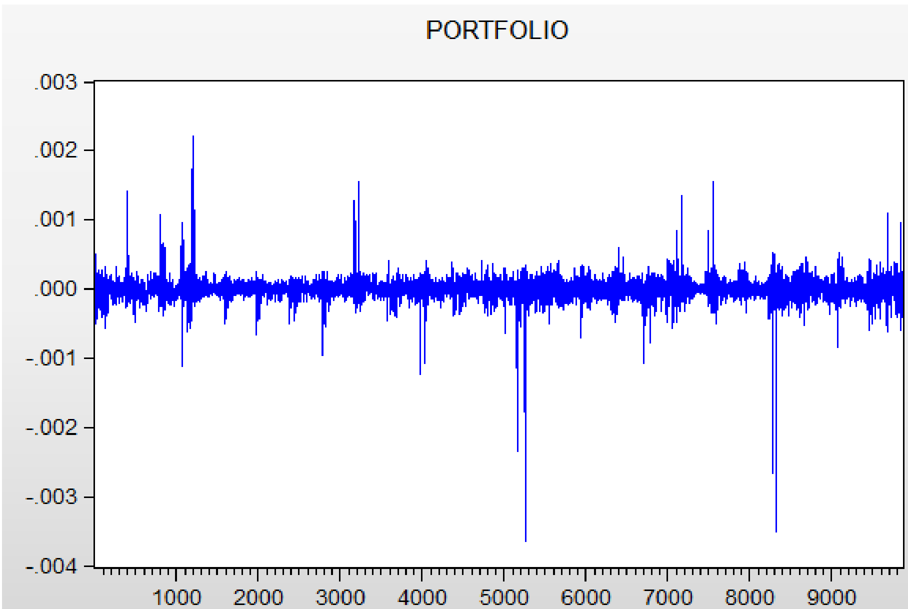
Figure 14, Portfolio Descriptive Statistics and Histogram



Above in Figure 14, we see descriptive statistics for our portfolio, again mean is close to zero, and other statistics clearly show that returns are not normally distributed. We see extreme values in tail and there is a fat tail problem. Therefore, our Value at Risk analysis maybe flawed, this is the biggest problem with value at risk estimation with variance-covariance approach.

In Figure 15, we see that our portfolio returns are highly volatile, with some extreme jumps.

Figure 15, Portfolio Plot of 1-Minute Returns



4.4. Value at Risk Estimation and Realized Volatility Forecasting

4.4.1. Forecasting Realized Volatility

In this part, we start to do our analysis; firstly, we will try to measure realized volatility forecasting power of our models. To do this we construct a benchmark 5-minute and 10-minute volatility from 1-minute volatility then we calculate realized volatility from 5-minute and 10-minute data itself. Then we compare realized volatilities with benchmark volatilities and we try to see forecasting power of our models, and accuracy of our data.

Realized Volatility

Realized volatility is the daily standard deviation of log returns of an underlying asset, index, instrument, security, or ETF, over a defined period, with an assumed mean of zero, no degrees of freedom, and a constant 252-day annualization factor.[12](Volatility Exchange). Mathematically we represent realized volatility as below,

$$\sigma = \sqrt{\frac{\sum_{t=1}^{n} R_t^2}{n}}$$

Where:

R= return on each single time period

t = is time period

n = is total number of time periods

Benchmark Volatilities

In our analysis for creating benchmark we use basic excel standard deviation function which includes mean of returns, but since mean is very small and close to zero, this approach is basically OK.

For forecasting realized volatility, we created a series of benchmark volatilities for 5-minute and 10-minute returns by using 1-minute returns. For example, to calculate benchmark volatility for 5-minute volatility; we took each first five rows of 1-minute returns and from these five rows we calculated benchmark 5 minute volatility with simple excel standard deviation function. Then, continued to this process in loop for all the 5-minute periods in our data. Then we used the same approach for 10-minute benchmark volatility. We took first 10 rows of 1-minute data then from these we calculated single 10-minute volatility. This approach is quite logical if we look closely at data. 1-minute return series already include all 5-minute and 10-minute returns. For example, starting from 12:01 minute data series continue with 12:02, 12:03, 12:04, 12:05, 12:06, 12:07, 12:08, 12:09, 12:10, as we can see ten 1-minute return series includes two 5-minute (12:05 and 12:10) and one 10-minute (12:10) returns. Therefore summing up volatility of single 1-minute

[12] http://www.volx.us/realizedvolatilitydefinition.htm

volatilities, we can create a benchmark for real volatilities from 5-minute and 10-minute returns itself. Below we represent a figure from our analysis to show how we calculated benchmark volatilities,

Figure 16, Benchmark Volatility Calculation

					1-Minute RETURNS					Benchmark Volatilties (5 Mins)		
Pricing Date	TIME	# of Data		S&P500	DJI	NASDAQ	Portfolio		S&P500	DJI	NASDAQ	Portfolio
			Weights	0.4	0.3	0.3						
29-07-13	17:31	1		-0.000367231	-0.000706626	0.000121978	-0.00032229		0.000358603	0.00046663	0.000226127	0.000337506
29-07-13	17:32	2		1.18483E-05	0.00012049	0.00014968	8.57903E-05		0.000187048	0.000140896	0.00038997	0.000143731
29-07-13	17:33	3		-0.0004207	-0.000473023	-9.7012E-05	-0.00033929		0.000205893	0.000201763	0.00037266	0.000102993
29-07-13	17:34	4		0.000118525	0.000235251	0.000440637	0.000250176		0.000352382			
29-07-13	17:35	5		0.000444343	0.000350518	0.000423826	0.00041004		0.000326			
29-07-13	17:36	6		0.000112536	0.000136566	0.000318444	0.000181517		0.000130034			
29-07-13	17:37	7		0.000106601	-2.64098E-05	0.000276825	0.000117765		0.000321094	0.000357849	0.000265641	0.000268807
29-07-13	17:38	8		-1.1844E-05	0.00018099	-0.00055927	-0.00011822		0.00022404	0.000304548	0.000150542	0.000182932
29-07-13	17:39	9		-0.000343537	-1.28807E-06	-3.3234E-05	-0.00014777		0.000215978	0.000187481	0.000102523	0.000145897
29-07-13	17:40	10		-8.88649E-05	-0.00017326	0.000382119	2.71117E-05		8.63478E-05	9.42192E-05	7.52365E-05	5.81909E-05
29-07-13	17:41	11		0.000272494	0.00028596	7.19771E-05	0.000216379		0.000238998	0.000249028	0.00019603	0.000156129
29-07-13	17:42	12		0.000254656	0.000312917	0.000351506	0.000301189		0.00033819	0.000276175	0.000328696	0.000207296
29-07-13	17:43	13		0.000254591	0.000182168	0.00039011	0.00027352		0.000233742	0.000315438	0.000308724	0.000156108
29-07-13	17:44	14		0.000295954	0.000194361	-0.00028773	9.03723E-05		0.000163502	0.000170929	9.51219E-05	0.000118438

Formula used here
=STDEV(OFFSET(H13,(ROW(M13)-ROW(M13))*5,0,5,1))

As we can see above screenshot which shows an example of how we calculated benchmark volatility for 5-minute volatility, we simply applied excel standard deviation function for each five 1-minute returns in a loop. For example, we calculated standard deviation of minutes 17:31, 17:32, 17:33, 17:34 and 17:35 and as a result, on left hand side in red box we obtain benchmark 5-minute volatility for 17:35.

Realized Volatilities

After calculating benchmark volatilities, now we can calculate realized volatilities for 5-minute and 10-minute returns itself. After calculating realized volatilities, we will compare the results with benchmark volatilities and see how good we forecast the volatility and how good our data is confident. We calculated realized volatility with EWMA and GARCH methods so we had a chance to compare which method is more convenient.

Realized Volatility by EWMA

To calculate realized volatility for 5-minute and 10-minute volatilities with EWMA, we applied standard EWMA formula in excel. EWMA formula is as follow,

$$\sigma_t = \sqrt{(1-\lambda)(r_{t-1})^2 + \lambda\sigma_{t-1}{}^2}$$

For each asset and portfolio, we applied above formula to get individual realized volatilities. Since we do not have previous period variance for first period of each return series, therefore as first period of variance series we take the variance of all population and put it as first period variance. Below we represent a screen shot showing how we calculated realized volatility by EWMA,

Figure 17, Realized Volatility EWMA

O14 =M11*O13+(1-M11)*H13^2

A	B	C	F	G	H	I	J	K	L	M	N	O	P	Q	R
						RETURN				Lambda		EWMA VARIANCEs			
Pricing Date	TIME	# of Data	NASDAQ												
			0.3	Weights	S&P500	DJI	NASDAQ	Portfolio		0.94		S&P500	DJI	NASDAQ	Portfolio
29-07-13	17:30	1	3606.99												
29-07-13	17:35	2	3610.74	0	-0.00021321	-0.00047339	0.00103911	0.00008443				0.000000412	0.000000405	0.000000597	0.000000181
29-07-13	17:40	3	3612.13		-0.00022511	0.00011660	0.00038489	0.00006040				0.000000390	0.000000394	0.000000626	0.000000171
29-07-13	17:45	4	3616.57		0.00088829	0.00078619	0.00122844	0.00095971				0.000000369	0.000000371	[illegible]	[illegible]
29-07-13	17:50	5	3613.6		0.00023083	0.00014931	-0.00082156	-0.00010934				0.000000395	0.000000386	[illegible]	[illegible]
29-07-13	17:55	6	3614.05		-0.00058013	-0.00039908	0.00012452	-0.00031442				0.000000374	0.000000364	0.000000654	0.000000195
29-07-13	18:00	7	3612.84		0.00019539	0.00037977	-0.00033486	0.00009163				0.000000372	0.000000352	0.000000615	0.000000189
29-07-13	18:06	8	3612.18		-0.00043819	-0.00047634	-0.00018270	-0.00037299				0.000000352	0.000000340	0.000000585	0.000000178
29-07-13	18:13	9	3610.15		-0.00083547	-0.00072268	-0.00056215	-0.00071963				0.000000342	0.000000333	0.000000552	0.000000176
29-07-13	18:18	10	3607.26		-0.00011263	0.00000902	-0.00080084	-0.00028260				0.000000364	0.000000344	0.000000538	0.000000196
29-07-13	18:23	11	3608.52		0.00054527	0.00052434	0.00034923	0.00048018				0.000000342	0.000000323	0.000000544	0.000000189
29-07-13	18:28	12	3609.78		0.00001185	-0.00004186	0.00034911	0.00009692				0.000000340	0.000000321	0.000000519	0.000000192
29-07-13	18:33	13	3607.48		-0.00042670	-0.00029371	-0.00063736	-0.00045000				0.000000319	0.000000301	0.000000495	0.000000181
29-07-13	18:38	14	3605.79		-0.00036165	-0.00026416	-0.00046858	-0.00036448				0.000000311	0.000000289	0.000000490	0.000000182
29-07-13	18:43	15	3608.11		-0.00016605	-0.00046083	0.00064320	-0.00001171				0.000000300	0.000000275	0.000000473	0.000000179
29-07-13	18:48	16	3607.87		0.00056920	0.00075720	-0.00006652	0.00043488				0.000000284	0.000000272	0.000000470	0.000000169
29-07-13	18:53	17	3607.61		-0.00001778	-0.00009148	-0.00007207	-0.00005618				0.000000286	0.000000290	0.000000442	0.000000170
29-07-13	18:58	18	3609.13		0.00034966	0.00034461	0.00042124	0.00036962				0.000000269	0.000000273	0.000000416	0.000000160
29-07-13	19:03	19	3608.85		-0.00013630	-0.00011207	-0.00007758	-0.00011141				0.000000260	0.000000264	0.000000401	0.000000158
29-07-13	19:08	20	3604.16		-0.00126309	-0.00113166	-0.00130043	-0.00123486				0.000000246	0.000000249	0.000000378	0.000000150
29-07-13	19:13	21	3600.78		-0.00053418	-0.00042954	-0.00093825	-0.00062401				0.000000327	0.000000310	0.000000456	0.000000232
29-07-13	19:18	22	3599.1		-0.00048695	-0.00035938	-0.00046667	-0.00044260				0.000000324	0.000000303	0.000000482	0.000000242

=VAR(H13:H1981)

Realized Volatility by GARCH

In GARCH method since parameter estimation is alone a very detailed work and can be a topic to another study, we did not do parameter estimation in our study, and therefore we could not apply GARCH (1, 1) formula that we represented before. Therefore, we estimated realized volatilities with GARCH method by using Eviews statistical software, which does parameter estimation and formula application automatically once you provide data. We will represent GARCH realized volatilities in comparisons part.

Comparisons

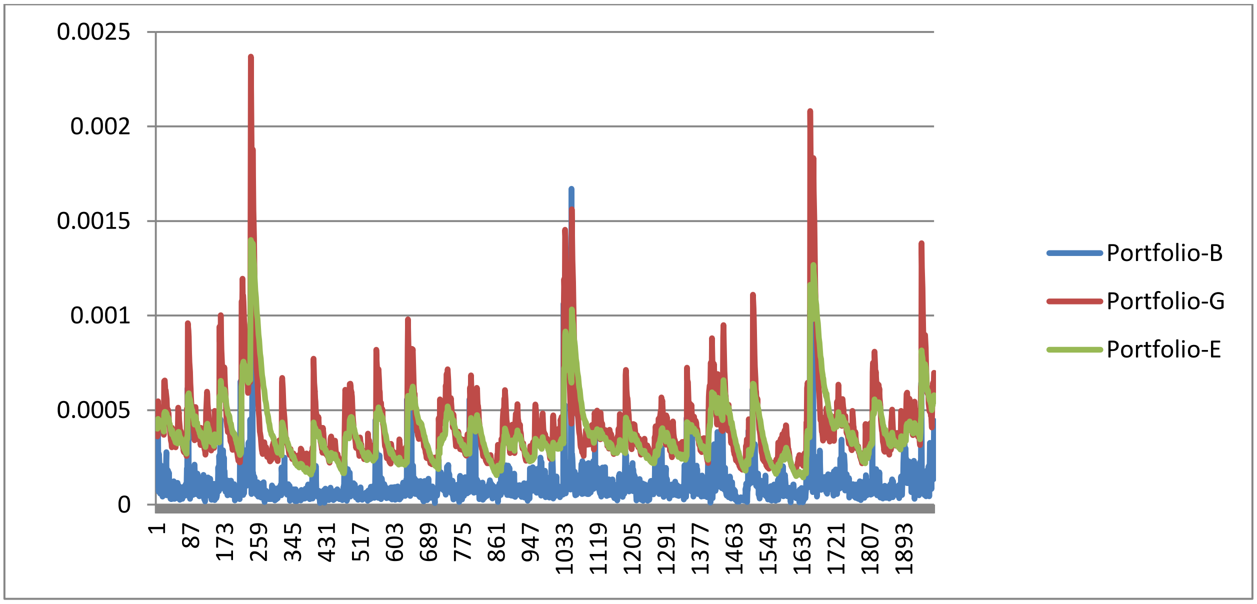
After doing benchmark volatility calculation and realized volatility estimation now we can compare our results and comment on outcome. Below we represent a comparison of benchmark volatility with EWMA and GARCH 5-minute volatility.

Figure 18, Comparisons of Volatility

	Benchmark Volatilties (5 Mins)			5 min Volatilities GARCH				5 min Volatilities EWMA			
S&P500-B	DJI-B	NASDAQ-B	Portfolio-B	S&P500-G	DJI-G	NASDAQ-G	Portfolio-G	S&P500-E	DJI-E	NASDAQ-E	Portfolio-E
0.00035860	0.00046663	0.00022613	0.00033751	0.00047600	0.00049282	0.00082515	0.00042459	0.00064159	0.00063639	0.00077285	0.00042591
0.00018705	0.00014090	0.00038997	0.00014373	0.00046960	0.00051939	0.00083977	0.00039126	0.00062423	0.00062781	0.00079135	0.00041345
0.00020589	0.00020176	0.00037266	0.00010299	0.00046529	0.00051998	0.00081651	0.00036098	0.00060772	0.00060935	0.00077302	0.00040113
0.00035238	0.00031963	0.00054808	0.00038725	0.00057335	0.00057327	0.00084882	0.00054645	0.00062810	0.00062138	0.00080762	0.00045444
0.00032600	0.00032195	0.00015345	0.00020505	0.00055075	0.00056265	0.00084415	0.00050010	0.00061159	0.00060356	0.00080846	0.00044141
0.00013003	0.00008834	0.00011737	0.00006839	0.00057212	0.00056602	0.00081514	0.00047794	0.00060974	0.00059329	0.00078442	0.00043483
0.00032109	0.00035785	0.00026564	0.00026881	0.00054758	0.00056734	0.00079407	0.00043866	0.00059310	0.00058268	0.00076494	0.00042218
0.00022404	0.00030455	0.00015054	0.00018293	0.00054949	0.00057526	0.00077293	0.00043568	0.00058497	0.00057686	0.00074298	0.00041939
0.00021598	0.00018748	0.00010252	0.00014590	0.00061866	0.00060509	0.00076662	0.00051460	0.00060294	0.00058663	0.00073339	0.00044318
0.00008635	0.00009422	0.00007524	0.00005819	0.00058319	0.00058596	0.00077469	0.00048632	0.00058522	0.00056876	0.00073761	0.00043522
0.00023900	0.00024903	0.00019603	0.00015613	0.00059209	0.00059335	0.00076009	0.00049418	0.00058290	0.00056620	0.00072024	0.00043805
0.00033819	0.00027617	0.00032870	0.00020730	0.00055898	0.00057678	0.00074774	0.00045325	0.00056515	0.00054904	0.00070351	0.00042537
0.00023374	0.00031544	0.00030872	0.00015611	0.00055719	0.00057079	0.00074938	0.00046182	0.00055781	0.00053716	0.00069972	0.00042689
0.00016350	0.00017093	0.00009512	0.00011844	0.00054850	0.00056474	0.00074286	0.00045328	0.00054803	0.00052480	0.00068805	0.00042340
0.00019654	0.00023956	0.00022806	0.00012413	0.00052635	0.00057208	0.00074565	0.00041485	0.00053289	0.00052118	0.00068544	0.00041051
0.00011188	0.00008701	0.00017092	0.00004353	0.00055170	0.00060679	0.00073051	0.00042837	0.00053514	0.00053827	0.00066476	0.00041202
0.00020318	0.00016583	0.00021818	0.00013794	0.00052496	0.00058799	0.00071772	0.00039356	0.00051885	0.00052235	0.00064475	0.00039970
0.00006673	0.00007259	0.00006497	0.00006258	0.00052122	0.00058222	0.00071455	0.00039856	0.00051028	0.00051342	0.00063356	0.00039796

If we examine the above data, we see that benchmark volatilities and realized volatilities are very close to each other. Data is obtained from a very volatile period, which is the period after FEDs announcement of gradual termination of Quantitive Easing policy. Because of this there are some times huge differences between benchmark and realized volatilities. Overall, we can say our benchmark volatility creates a good forecast of realized volatility; especially we can improve accuracy of results if we apply this method for a period of no extreme events.

Figure 19, Graph Volatility Comparisons



Above graph represents plot of 5-minute benchmark, EWMA and GARCH volatilities, we can clearly see all three volatilities move exactly together, and EWMA and GARCH volatilities are very close, although benchmark volatility is little away to realized volatilities still it is a good forecast. Besides, we can easily see EWMA volatilities are closer to benchmark more than that of GARCH.

4.4.2. Value at Risk Estimation

For Value at Risk estimation, we used Parametric-Variance Covariance approach and non-parametric Historical Simulation method. At variance-covariance approach, we used MA, EWMA and GARCH methods to calculate variance covariance matrix. We used our 10 million US dollar Portfolio that consists of 4 million S&P500, 3 million Dow Jones and 3 million NASDAQ shares.

VaR with Historical Simulation

To run a historical simulation analysis all we need to know is portfolio return series and confidence level, which is 99% in our case. Since historical simulation, approach is non-parametric approach it is quiet straight forward to apply. First, we estimated value at risk only for 5 minute and 10 minute returns; therefore, we used periodic logarithmic return series between 29-07-13 and 30-08-13 and we calculated portfolio return by weighting individual periodic returns with their portfolio weight. After calculating minute portfolio returns we start our scenario. Below we represent a screen shot of our analysis in excel to show how we calculated VaR with historical simulation for 5-minute returns,

Figure 20, Historical Simulation

Portfolio	Portfolio Return		Percentile
	Simulation	Rank	
844.294189	-44553.72543	1	0.0507872%
604.020771	-34214.93018	2	0.1015744%
9597.066412	-33520.64111	3	0.1523616%
-1093.423493	-23777.81181	4	0.2031488%
-3144.180075	-22728.40407	5	0.2539360%
916.277519	-22269.85461	6	0.3047232%
-3729.906046	-16073.49257	7	0.3555104%
-7196.337533	-15766.55266	8	0.4062976%
-2825.999598	-15766.52671	9	0.4570848%
4801.803523	-14563.87482	10	0.5078720%
969.159896	-13964.03259	11	0.5586592%
-4500.028630	-13722.67951	12	0.6094464%
-3644.830182	-13233.46997	13	0.6602336%
-117.081813	-13022.40411	14	0.7110208%
4348.820813	-12348.64418	15	0.7618080%
-561.764958	-12332.25177	16	0.8125952%
3696.209232	-12307.40254	17	0.8633824%
-1114.128160	-11867.68405	18	0.9141696%
-12348.644182	-11632.64922	19	0.9649568%
-6240.086550	-11219.96715	20	1.0157440%
-4425.961476	-10810.95761	21	1.0665312%

Once we obtain portfolio return series, then we take all portfolio return and sort them from smallest to largest in excel, after that we create rank for this return series, we simply say the smallest is 1 and second smallest is 2 and it goes like that till end of series. Then we divide each rank by total number of observations to get percentile for each return, for example for 5-minute return total number of observations 1969 therefore, fore first return we divide its rank 1 to 1969 and we get 0.0507872% percentile. After getting cumulative percentiles for all series, no all we need to do is to find the return value corresponding to our confidence level, since our confidence level is 99% we need to find value at risk the lefts out of our confidence level which is 1-99% = 1%. Therefore, value at risk at 99% confidence level is return value that corresponds to 1% percentile. However, sometimes you may not see exactly 1% percentile in percentile series, as you can see in Figure 20, in this case we need do linear interpolation to find exact percentile. After following all above process for both 5-minute and 10-minute return series, we get results as below,

Figure 21, VaR Historical Simulation

	Value at Risk- Historical Simulation	
	5-Minute	**10-Minute**
Confidence Level	99%	99%
Portfolio Value	10000000	10000000
Value at Risk	-11347.89859$	-18721.68353$

Therefore according to Historical Simulation approach our portfolios 5-munite VaR is -11347.89859$ which means with 99% confidence level our loss will not exceed -11347.89859$. Same comment is valid for 10-minute VaR which is-18721.68353$. That means with 99% confidence level our portfolio loss will not exceed -18721.68353$.

VaR with Variance Covariance Approach

Under variance–covariance approach, we are going to estimate value at risk for 5-minute and 10-minute returns by using MA, EWMA and GARCH methods. Our assumptions are zero mean and normality assumption that assumes returns have normal distribution. We will use 99% confidence level. We applied following value at risk and variance-covariance matrix formulas in each method,

Portfolio volatility with variance-covariance matrix = $\sigma_p = \sqrt{\alpha \times C \times \alpha^T}$

Value at Risk $= VaR_p = P \times \sqrt{\alpha \times C \times \alpha^T} \times Z(1 - CL) \times \sqrt{t}$

Moving Average (MA) Method

To calculate variance-covariance matrix with MA we do not go deep into parameter estimation we simply use excel data analysis tool to estimate variance-covariance matrix. We simply give single asset returns as input to data analysis tool and we get variance-covariance matrix. After getting variance covariance matrix, we apply the above formula for portfolio volatility with variance-covariance matrix and we multiply transpose of weights vector by variance-covariance matrix then again we multiply it by weights vector at the end we get a single portfolio variance. To get portfolio volatility we simply take square root of portfolio variance. After obtaining portfolio volatility, we start to apply value at risk formula given above. First, we multiply portfolio volatility with portfolio value, which is 10 million, and we get portfolio volatility in dollars. Then we calculate Z value corresponding to our percentile which is what percentage lefts out of confidence level, therefore we calculate Z (1-99%) = Z (1%), to get Z value we use excel function "NORM.S.INV". After finding Z value, we multiply it by portfolio volatility in dollars and we have value at risk. Below, we represent a screen shot showing how we followed this process for 5-minute returns,

Figure 22, VaR MA

	Variance-Covariance Matrix			
	S&P500	DJI	NASDAQ	
S&P500	0.0000004114	0.0000000990	-0.0000000008	
DJI	0.0000000990	0.0000004048	0.0000000096	
NASDAQ	-0.0000000008	0.0000000096	0.0000005970	
Weights Vector	0.4			
	0.3			
	0.3			
Portfolio Variance	0.00000018131			Formula here =MMULT(MMULT(TRANSPOSE(N30:N32),N26:P28),N30:N32)
Portfolio Volatility	0.0004258			
Portfolio size	10000000			
Portfolio Volatility ($)	4258.001839			
Z(1%)	-2.326347874			=NORM.S.INV(1-B4)
99% VaR	-9905.593527			

After completing above process for both 5-minute and 10-minute returns we get following results,

Figure 23, VaR Results MA

	Value at Risk- MA	
	5-Minute	**10-Minute**
Confidence Level	99%	99%
Portfolio Value	10000000	10000000
Value at Risk	-9905.593527$	-14839.67657$

As we analyzed before asset returns are not normally distributed and kurtosis for our returns are very high, that means our distribution is fat tailed. Fat tailed means that distribution has much more extreme events in tail than normal distribution. This means that some models can miss some of the extreme events in distribution therefore, underestimate the value at risk. Since MA method weights all past returns with equal weights it divides the effect of extreme values, and sometimes it may underestimate the value at risk. To improve our value at risk estimation we can use some more specific methods like EWMA and GARCH methods.

Exponentially Weighted Moving Average (EWMA) Method

For EWMA method, we used Lambda parameter of 0.94, which is used by Riskmetrics. EWMA method gives more weight and pressure on recent data therefore it does not share and decrease effect of extreme events. Besides, portfolio volatility with variance-covariance matrix formula we will use following EWMA formula for estimating individual variances and covariances,

$$\sigma_t^2 = (1-\lambda)(r_{t-1})^2 + \lambda\sigma_{t-1}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots..1$$

$$Cov_t = (1-\lambda)r_{At-1}r_{Bt-1} + \lambda Cov_{t-1}^2 \quad \ldots\ldots\ldots..2$$

Once we have return series start to process, first we calculate variance series for all periods and all assets, to do this we apply formula 1 above, after getting variance series we apply formula 2 above to get the covariance series for all periods and assets. After calculating variance and covariance series, we create variance-covariance matrix manually, for corresponding variances in matrix, we take last period variance from variance series and for corresponding covariances, we take last period covariance from covariance series, this way we obtain variance-covariance matrix. Below we represent a screen shot from our analysis to show how we get step-by-step variance covariance matrix for 5-minute returns,

Figure 24, EWMA Process

	RETURN			Lambda		VARIANCEs				COVARIANCEs	
S&P500	DJI	NASDAQ	Portfolio	0.94	S&P500	DJI	NASDAQ	Portfolio	S&P500/DJI	S&P500/NQ	DJI/NQ
-0.00021321	-0.00047339	0.00103911	0.00008443		0.000000412	0.000000405	0.000000597	0.000000181	0.000000099	-0.000000001	0.000000010
-0.00022511	0.00011660	0.00038489	0.00006040		0.000000390	0.000000394	0.000000626	0.000000171	0.000000099	-0.000000014	-0.000000020
0.00088829	0.00078619	0.00122844	0.00095971		0.000000369	0.000000371	0.000000598	0.000000161	0.000000092	-0.000000018	-0.000000017
0.00023083	0.00014931	-0.00082156	-0.00010934		0.000000395	0.000000386	0.000000652	0.000000207	0.000000128	0.000000048	0.000000042
-0.00058013	-0.00039908	0.00012452	-0.00031442		0.000000374	0.000000364	0.000000654	0.000000195	0.000000122	0.000000034	0.000000033
0.00019539	0.00037977	-0.00033486	0.00009163		0.000000372	0.000000352	0.000000615	0.000000189	0.000000129	0.000000028	0.000000028
-0.00043819	-0.00047634	-0.00018270	-0.00037299		0.000000352	0.000000340	0.000000585	0.000000178	0.000000126	0.000000022	0.000000018
-0.00083547	-0.00072268	-0.00056215	-0.00071963		0.000000342	0.000000333	0.000000552	0.000000176	0.000000131	0.000000025	0.000000022
-0.00011263	0.00000902	-0.00080084	-0.00028260		0.000000364	0.000000344	0.000000538	0.000000196	0.000000159	0.000000052	0.000000045
0.00054527	0.00052434	0.00034923	0.00048018		0.000000342	0.000000323	0.000000544	0.000000189	0.000000149	0.000000054	0.000000042
0.00001185	-0.00004186	0.00034911	0.00009692		0.000000340	0.000000321	0.000000519	0.000000192	0.000000158	0.000000063	0.000000051
-0.00042670	-0.00029371	-0.00063736	-0.00045000		0.000000319	0.000000301	0.000000495	0.000000181	0.000000148	0.000000059	0.000000047
-0.00036165	-0.00026416	-0.00046858	-0.00036448		0.000000311	0.000000289	0.000000490	0.000000182	0.000000147	0.000000072	0.000000055

After getting variance-covariance matrix rest of the process is same as for MA approach. We show process in below screen shot,

Figure 25, VaR EWMA

	Variance-Covariance Matrix		
	S&P500	DJI	NASDAQ
S&P500	0.000000378	0.000000409	0.000000287
DJI	0.000000409	0.000000393	0.000000279
NASDAQ	0.000000287	0.000000279	0.000000277
Weights Vector	0.4		
	0.3		
	0.3		
Portfolio Variance	0.0000003383		
Portfolio Volatility	0.000581623		
Portfolio size	10000000		
Portfolio Volatility ($)	5816.228972		
Z(1%)	-2.326347874		
99% VaR	-13530.5719		

For EWMA approach to see the effect of change in parameter, we estimated value at risk also for lambda = 0.955 and 0. 96, after completing all above process for both 5-minute and 10-minute returns we get following results,

	Value at Risk- EWMA	
	5-Minute	**10-Minute**
Confidence Level	99%	99%
Portfolio Value	10000000	10000000
Value at Risk $\lambda = 0.94$	-13530.5719$	-20297.7032$
Value at Risk $\lambda = 0.955$	-13373.09645$	-19784.2329$
Value at Risk $\lambda = 0.96$	-13288.6755$	-19532.5123$

As we can see EWMA has higher estimate than MA, which means it may decrease possibility of underestimating value at risk, we will discuss this in detail later. As lambda is getting

Generalized AutoRegressive Conditional Heteroskedasticity GARCH Method

To estimate variance-covariance matrix and calculate portfolio volatility to estimate value at risk, we also used GARCH (1, 1) method. As we explained in detailed before, GARCH is a parametric method and has following formula and parameters,

$$\sigma_t^2 = \gamma V_l + \alpha (r_{t-1})^2 + \beta (\sigma_{t-1})^2$$

However, since estimation of GARCH parameters is a very advance, highly mathematical issue and might be subject to separate study; in our study we did not estimate GARCH parameters. Instead, we used Eviews statistical software, which estimate all GARCH parameters. Therefore, to calculate variance-covariance matrix and estimate value at risk with GARCH method we followed these steps. First, we submitted our 5 –minute and 10-minute returns in Eviews as foreign work file, then first we created single equations for each asset and portfolio return series and we created series of GARCH Variance, this way we obtained all individual variances for both single assets and portfolio. Then, to calculate covariances with GARCH, we created equations in Eviews for each combination of two assets and we calculated covariance series. After getting variance and covariance series, we followed the same way as for EWMA method and we chose last period variances and covariances to create variance-covariance matrix. After calculating variance-covariance matrix, we followed same procedure as we explained for other methods. Below we have a screen shot of our work to show how we completed value at risk estimation with GARCH method,

Figure 26, VaR with GARCH

	Variance-Covariance Matrix(Eviews)		
	S&P500	DJI	NASDAQ
S&P500	0.00000053696	0.000000133	0.000000587
DJI	0.000000133	0.00000047283	0.000000465
NASDAQ	0.000000587	0.000000465	0.00000055282
Weights Vector	0.4		
	0.3		
	0.3		
Portfolio Variance	0.0000004347		
Portfolio Volatility	0.000659305		
Portfolio size	10000000		
Portfolio Volatility ($)	6593.047627		
Z(1%)	-2.326347874		
99% VaR	-15337.72233		

After completing all of above process for both 5-minute and 10-minute returns, we get following value at risk values with GARCH method,

	Value at Risk- GARCH	
	5-Minute	**10-Minute**
Confidence Level	99%	99%
Portfolio Value	10000000	10000000
Value at Risk	-15337.72233$	-22337.43183$

Comparison of VaR Estimations of All Methods

Below table compares value at risk estimations from each method used above,

	Value at Risk Comparisons	
	5-Minute VaR	**10-Minute VaR**
Historical Simulation	-11347.89859$	-18721.68353$
MA	-9905.593527$	-14839.67657$
EWMA $\lambda = 0.94$	-13530.5719$	-20297.7032$
EWMA $\lambda = 0.955$	-13373.09645$	-19784.2329$
EWMA $\lambda = 0.96$	-13288.6755$	-19532.5123$
GARCH (1, 1)	-15337.72233$	-22337.43183$

When we look at the comparisons table, we see that MA method has the lowest estimation, which makes it most vulnerable to bias, or fat tail effect. Besides, we see that EWMA and GARCH results are very similar and close to each other, because they are both parametric method and both of them gives more weight to recent data.

4.5. Backtesting

After estimating value at risk with each model, now we will do backtesting analysis to measure accuracy of each model. Backtesting evaluates performance of the Value at Risk estimations by using a constant past data range. What backtesting does is simply estimate a VaR for today by using past data and then compare this estimated VaR with today's portfolio profit and loss. If portfolio loss today exceeds the VaR estimate then it is a violation to model. To run a backtesting analysis we need to choose an estimation window and a testing window for our portfolio. We need to have testing window bigger than estimation window. Below we explain main concepts and preparation of back testing analysis,

W_E = Estimation window the rolling but constant number of observations that we use to estimate value at risk from past data

W_T= Testing window the rolling but constant number of observations that we use to compare value at risk estimates

T= $W_E + W_T =$ Total number of observations

Violation Ratio

Violation ratio is simply ratio of number of total violations to number of expected violations; expected violation is number of acceptable violations we calculate the number of acceptable violations by simply multiplying number of observations in testing window with 1-confidence level.

Mathematically we can represent Violation Ratio as below,

$$VR = \frac{V}{E(V)}$$

VR= Violation Ratio

V= Total number of violations

E (V) = Expected violation, number of acceptable violations

We interpret the result of Violation Ratio as below,

- VR > 1 means that model underestimates the market risk
- VR < 1 means that model overestimates the market risk
- VR = 1 means that model is accurate

In our analysis both for 5-minute and 10-minute returns, we used 1-month estimation window and 3 months testing window. Estimation window is from 29-07-13 to 30-08-13 and testing window we took 3 following months therefore, testing window is from 03-09-13 to 27-11-13. Below table summarizes the estimation and testing windows for both data types,

	5-Minute	**10-Minute**
Estimation Window	1969	984
Testing Window	4924	2462
Total	6893	3446

We did backtesting analysis for Historical Simulation, MA and EWMA methods by using Excel-VBA user defined sub procedures. We did not do backtesting for GARCH because we can do not estimate parameters for model in this study and Excel does not have any function to estimate its parameters. To do backtesting we prepared VBA sub procedures that, starts from first day of estimation window and calculates value at risk in a loop for all estimation windows equal to number of observations in testing window. For example, for 5-minute returns, our VBA procedure starts from 29-07-13 and takes first 1969 rows to calculate a value at risk value for first row of testing window that is 03-09-13. Then, sub procedure starts from second row and again takes 1969 rows to estimate value at risk for second row of testing window.

After calculating value at risk for entire testing window, now our sub procedure compares each single value at risk with profit and loss of portfolio on that time period. For example, it compares value at risk calculated for first row of testing window, with portfolio profit and loss on this minute. After, comparing if portfolio loss is higher than value at risk estimation then we take it as a violation to model. Then, our sub procedure counts number of violations and calculates violation ratio for each method.

Backtesting Results

We did back testing analysis for Historical Simulation, MA and EWMA methods. For, EWMA method we calculated value at risk for different lambda λ values, results are summarized in below table,

5-Minute Returns	V	E(V)	VR
HS	47	49	0.959183673
MA	57	49	1.163265
EWMA $\lambda = 0.94$	76	49	1.55102
EWMA $\lambda = 0.955$	75	49	1.530612
EWMA $\lambda = 0.98$	71	49	1.44898
EWMA $\lambda = 0.999$	57	49	1.163265

10-Minute Returns	V	E(V)	VR

HS	16	25	0.64
MA	25	25	1.0
EWMA $\lambda = 0.94$	47	25	1.88
EWMA $\lambda = 0.955$	40	25	1.6
EWMA $\lambda = 0.98$	40	25	1.6
EWMA $\lambda = 0.999$	26	25	1.04

Above results show that, Historical Simulation method overestimates the value at risk especially for 10-minute returns violation ration is very low. Main reason for that is our value is from a highly volatile and uncertain period. EWMA method underestimates the value at risk, especially at $\lambda = 0.94$ violation ratio is far above 1, but as we increase value of lambda model gets better and violation ration converges to 1, at $\lambda = 0.999$ violation ratio is very low and model is much more accurate than in $\lambda = 0.94$. MA approach has the best value at risk estimation for both 5-minute and 10-minute returns. Violation ratio is almost 1 for 5-minute and exactly 1 for 10-minute.

CHAPTER 5

5. Conclusion

In last decades, increasing globalization in international economies has affected financial markets by increasing competition, transaction volumes and diversity in financial products. This affect caused financial markets to be more complex and connected to each other. Therefore, because of this increasing complexity and connectivity, financial institutions have been more vulnerable to risk arising from price movements in financial markets. Because of this, institutions began to look for effective solutions and risk methodologies to define, measure, control and update their risks. Moreover, with recent financial crisis and turmoil in economies, financial authorities have brought certain rules and measures to force institutions to take precaution and improve their risk management systems.

With increasing importance of measuring and controlling market risk, several methodologies has been produced such as; sensitivity analysis and scenario analysis. After presentation of Value at Risk method, it has been the most widely used market risk method since it is easy to implement and interpret, at the end of analysis we get a single value in real currency amounts, that is why VaR is widely used and preferred. Value at Risk method is simply a method that uses historical data to calculate maximum level of loss in real currency amounts at a certain time, given a certain confidence level.

In last several years world of finance have increased its speed day by day. Classical way of approaching to risk management is no more sufficient, daily analysis are nor more satisfying for brokers, traders and risk analysts. Now speed of trading and transactions are very high, high frequency financial instruments spread in financial world, many transactions start and end in less than a day. Therefore, risk managers, brokers or traders should have more information than daily basis; they should keep track of high frequency movements. For example, a trader may need risk value at risk of its portfolio at 13:55 afternoon.

Because of these rapidly changing needs, in this study we wanted to apply value at risk models to intraday financial data, and try to see their realized volatility forecasting power and to understand which model is more accurate on which data range. We aim to make optimal model suggestion for intraday value at risk users.

In this study, we choose a portfolio consisting of three US stock exchanges. We use intraday 1 minute, 5 minute and 10 minute prices to forecast realized volatility and estimate value at risk with both parametric and non-parametric approaches. After that, we do backtesting analysis to measure the accuracy of each model.

After doing realized volatility forecasting we conclude that methodology we used is accurate to forecast realized volatility although there is a bit gap between benchmark volatility and realized volatility. This gap is mainly because of volatile time period that we collected data from. We also conclude that, EWMA method has results more closely to benchmark volatility than GARCH method has.

After successfully estimating value at risk for all methods and doing backtesting analysis we conclude that for both 5-minute and 10-minute returns the best method to estimate value at risk is Moving Average method. Although, before we mentioned that Moving Average method is always open to fat tail problem and in most cases; it might be missing some extreme cases, here in our analysis of intraday value at risk it generated the best risk estimation. Because our data is from a highly volatile period with extreme spikes, moving average model helps to smooth the effects of spikes and generates an overall risk estimation.

Overall, we conclude that for estimating value at risk for intraday prices, best method is Moving Average method. However, we must note that this study is a very specific case, data is taken from a very volatile period, portfolio consist of only equity indexes therefore portfolio has only one type of underlying asset. Results may change if these conditions are changed, and completely different method can prove better.

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APPENDIX

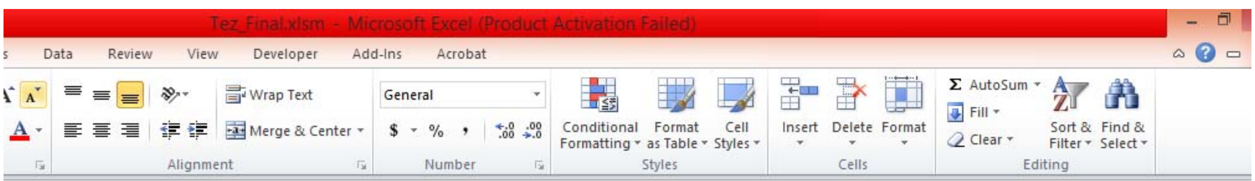
APPENDIX Print Screen of Work Done Step by Step

A portfolio of 10m $ consisting of 4m S&P500, 3m DowJones and 3m Nasdaq inde

Confidence level:	99.00%
Portfolio:	10000000
# of Assets:	3

			DATA					RETURN		
Pricing Date	TIME	# of Data	S&P500	DJI	NASDAQ		S&P500	DJI	NASDAQ	Portfolio
						Weights	0.4	0.3	0.3	
29-07-13	17:30	0	1688.62	15529.97	3606.99					
29-07-13	17:31	1	1688	15519	3607.43		-0.000367231	-0.000706626	0.000121978	-0.00032229
29-07-13	17:32	2	1688.02	15520.87	3607.97		1.18483E-05	0.00012049	0.00014968	8.57903E-05
29-07-13	17:33	3	1687.31	15513.53	3607.62		-0.0004207	-0.000473023	-9.7012E-05	-0.00033929
29-07-13	17:34	4	1687.51	15517.18	3609.21		0.000118525	0.000235251	0.000440637	0.000250176
29-07-13	17:35	5	1688.26	15522.62	3610.74		0.000444343	0.000350518	0.000423826	0.00041004
29-07-13	17:36	6	1688.45	15524.74	3611.89		0.000112536	0.000136566	0.000318444	0.000181517
29-07-13	17:37	7	1688.63	15524.33	3612.89		0.000106601	-2.64098E-05	0.000276825	0.000117765
29-07-13	17:38	8	1688.61	15527.14	3610.87		-1.1844E-05	0.00018099	-0.00055927	-0.00011822
29-07-13	17:39	9	1688.03	15527.12	3610.75		-0.000343537	-1.28807E-06	-3.3234E-05	-0.00014777
29-07-13	17:40	10	1687.88	15524.43	3612.13		-8.88649E-05	-0.00017326	0.000382119	2.71117E-05
29-07-13	17:41	11	1688.34	15528.87	3612.39		0.000272494	0.00028596	7.19771E-05	0.000216379
29-07-13	17:42	12	1688.77	15533.73	3613.66		0.000254656	0.000312917	0.000351506	0.000301189
29-07-13	17:43	13	1689.2	15536.56	3615.07		0.000254591	0.000182168	0.00039011	0.00027352

Volatility Comparisons | 1min | 5minDATA | 5min-HistSim | 5min-MA | 5min-EWMA | 5minGARCH(1,1) | 10minDATA



M	N	O	P	Q	R	S	T	U	V	W
	Realized Volatilties (5 Mins)							Realized Volatilties (10 Mins)		
S&P500	DJI	NASDAQ	Portfolio				S&P500	DJI	NASDAQ	Portfolio
0.000358603	0.00046663	0.000226127	0.000337506				0.000269639	0.000330856	0.000308336	0.000244571
0.000187048	0.000140896	0.00038997	0.000143731				0.00028077	0.000260781	0.000491858	0.000289935
0.000205893	0.000201763	0.00037266	0.000102993				0.000247853	0.000237222	0.000137595	0.000150325
0.000352382	0.000319635	0.00054808	0.000387248				0.000264358	0.000314341	0.000207448	0.000219824
0.000326	0.000321945	0.000153451	0.000205052				0.000169867	0.00015006	0.000147931	0.000132024
0.000130034	8.83382E-05	0.000117367	6.83871E-05				0.000279921	0.000249331	0.000275518	0.000182362
0.000321094	0.000357849	0.000265641	0.000268807				0.000191282	0.000240079	0.000245185	0.000135824
0.00022404	0.000304548	0.000150542	0.000182932				0.000162971	0.000192025	0.000190004	0.000101831
0.000215978	0.000187481	0.000102523	0.000145897				0.000151496	0.00012993	0.000160615	0.000112996
8.63478E-05	9.42192E-05	7.52365E-05	5.81909E-05				0.000169994	0.000179653	0.000174595	0.000154321
0.000238998	0.000249028	0.00019603	0.000156129				0.00018637	0.000190782	0.000270806	0.000172474
0.00033819	0.000276175	0.000328696	0.000207296				0.00013506	0.000114503	0.000309298	0.000160217
0.000233742	0.000315438	0.000308724	0.000156108				0.000220563	0.000198041	0.000205105	0.000197883

5min-HistSim | 5min-MA | 5min-EWMA | 5minGARCH(1,1) | 10minDATA

Portfolio Return		Percentile
Simulation	Rank	
-44553.72543	1	0.0507872%
-34214.93018	2	0.1015744%
-33520.64111	3	0.1523616%
-23777.81181	4	0.2031488%
-22728.40407	5	0.2539360%
-22269.85461	6	0.3047232%
-16073.49257	7	0.3555104%
-15766.55266	8	0.4062976%
-15766.52671	9	0.4570848%
-14563.87482	10	0.5078720%
-13964.03259	11	0.5586592%
-13722.67951	12	0.6094464%
-13233.46997	13	0.6602336%
-13022.40411	14	0.7110208%
-12348.64418	15	0.7618080%
-12332.25177	16	0.8125952%
-12307.40254	17	0.8633824%
-11867.68405	18	0.9141696%
-11632.64922	19	0.9649568%
-11219.96715	20	1.0157440%
-10810.95761	21	1.0665312%

Simulation	
RETURNS	
Mean	-151.4381016
Standard Error	95.98277408
Median	6.040586196
Mode	#N/A
Standard Deviation	4259.083512
Sample Variance	18139792.36
Kurtosis	24.14147835
Skewness	-0.042149196
Range	95398.80142
Minimum	-44553.72543
Maximum	50845.07599
Sum	-298181.622
Count	1969
Jarqu-Bera	36180.375
Interpolation	-11347.89859

inGARCH(1,1) | 10minDATA

Pricing Date	TIME	# of Data	Portfolio
29-07-13	18:33	13	-4500.0286298
29-07-13	18:38	14	-3644.8301815
29-07-13	18:43	15	-117.0818134
29-07-13	18:48	16	4348.8208126
29-07-13	18:53	17	-561.7649580
29-07-13	18:58	18	3696.2092319
29-07-13	19:03	19	-1114.1281601
29-07-13	19:08	20	-12348.6441824
29-07-13	19:13	21	-6240.0865497
29-07-13	19:18	22	-4425.9614764
29-07-13	19:23	23	-3251.9639677
29-07-13	19:28	24	-6178.5537019
29-07-13	19:33	25	3668.3605584
29-07-13	19:38	26	-4342.7808926
29-07-13	19:43	27	1309.3737636
29-07-13	19:48	28	832.2531013
29-07-13	19:53	29	5874.5207049
29-07-13	19:58	30	-3180.5077261
29-07-13	20:03	31	4459.7966619
29-07-13	20:08	32	658.4857763
29-07-13	20:13	33	-3116.5925216
29-07-13	20:18	34	2427.7511361
29-07-13	20:23	35	806.9034360

	Variance-Covariance Matrix		
	S&P500	DJI	NASDAQ
S&P500	0.0000004114	0.0000000990	-0.0000000008
DJI	0.0000000990	0.0000004048	0.0000000096
NASDAQ	-0.0000000008	0.0000000096	0.0000005970
Weights Vector	0.4		
	0.3		
	0.3		
Portfolio Variance	0.00000018131	Formula here =MMULT(MMULT(TRANSPOSE(N30:N32),N26:P28	
Portfolio Volatility	0.0004258		
Portfolio size	10000000		
Portfolio Volatility ($)	4258.001839		
Z(1%)	-2.326347874	=NORM.S.INV(1-B4)	
99% VaR	-9905.593527		
10-day 99% VaR	-31324.23712		

Volatility Comparisons | 1min | 5minDATA | 5min-HistSim | 5min-MA | 5min-EWMA | 5minGARCH(1,1) | 10minDATA

eady

Confidence level:	99.00%	
Portfolio:	10000000	
# of Assets:	3	

				DATA				RETURN		
Pricing Date	TIME	# of Data	S&P500	DJI	NASDAQ		S&P500	DJI	NASDAQ	Portfolio
						Weights	0.4	0.3	0.3	
30-08-13	21:43	972	1633.38	14808.84	3594.51		-0.000324428	1.28303E-05	-0.000795341	-0.00036
30-08-13	21:53	973	1634.46	14818.11	3597.23		0.000660987	0.000625782	0.000756423	0.000679
30-08-13	22:03	974	1631.26	14788.25	3589.99		-0.001959752	-0.002017135	-0.002014688	-0.00199
30-08-13	22:13	975	1631.75	14792.31	3592.55		0.000300336	0.000274505	0.00071284	0.000416
30-08-13	22:23	976	1632.55	14800.64	3592.39		0.000490151	0.000562972	-4.45376E-05	0.000352
30-08-13	22:33	977	1633.7	14812.73	3592.63		0.000704171	0.000816523	6.68057E-05	0.000547
30-08-13	22:43	978	1634.17	14817.46	3593.71		0.000287649	0.000319269	0.00030057	0.000301
30-08-13	22:53	979	1634.48	14821.16	3594.99		0.000189681	0.000249674	0.000356114	0.000258
30-08-13	23:03	980	1634.04	14813.96	3593.33		-0.000269235	-0.00048591	-0.00046186	-0.00039
30-08-13	23:13	981	1634.37	14817.09	3593.27		0.000201933	0.000211265	-1.66977E-05	0.000139
30-08-13	23:23	982	1634.9	14821.55	3595.45		0.000324231	0.000300958	0.000606506	0.000402
30-08-13	23:33	983	1633.02	14804.69	3591.73		-0.001150579	-0.00113818	-0.001035177	-0.00111
30-08-13	23:43	984	1630.83	14783.76	3587.53		-0.001341974	-0.001414741	-0.001170037	-0.00131
30-08-13	23:53	985	1628.99	14770.72	3584.7		-0.001128897	-0.000882438	-0.000789155	-0.00095

1min | 5minDATA | 5min-HistSim | 5min-MA | 5min-EWMA | 5minGARCH(1,1) | **10minDATA** | 10min-HistSim | 10mi

Confidence level:	99.00%	
Portfolio:	10000000	
# of Assets:	3	

Pricing Date	TIME	# of Data	NASDAQ	Portfolio	Portfolio Return		Percentile		
			0.3		Simulation	Rank			
29-07-13	17:30	1							
29-07-13	17:40	2	0.00142400	1448.31495945	-49267.43412	1	0.10%	*Column1*	
29-07-13	17:50	3	0.00040688	8503.64291915	-38036.40728	2	0.20%		
29-07-13	18:00	4	-0.00021034	-2227.90255533	-37675.82080	3	0.30%	Mean	-317.8626591
29-07-13	18:13	5	-0.00074484	-10926.24357961	-30687.93092	4	0.41%	Standard Error	203.4571791
29-07-13	18:23	6	-0.00045161	1975.80392464	-27140.39875	5	0.51%	Median	-117.3790081
29-07-13	18:33	7	-0.00028825	-3530.86873333	-23104.58228	6	0.61%	Mode	#N/A
29-07-13	18:43	8	0.00017462	-3761.91199485	-20326.90106	7	0.71%	Standard Deviation	6382.202326
29-07-13	18:53	9	-0.00013859	3787.05585458	-19934.47702	8	0.81%	Sample Variance	40732506.53
29-07-13	19:03	10	0.00034366	2582.08107183	-19419.68569	9	0.91%	Kurtosis	11.71049058
29-07-13	19:13	11	-0.00223867	-18588.73073217	-18588.73073	10	1.02%	Skewness	-0.347153126
29-07-13	19:23	12	-0.00115041	-7677.92544411	-18424.91228	11	1.12%	Range	96969.18803
29-07-13	19:33	13	-0.00038377	-2510.19314350	-17768.58072	12	1.22%	Minimum	-49267.43412
29-07-13	19:43	14	-0.00032270	-3033.40712899	-17561.33335	13	1.32%	Maximum	47701.75391

1min | 5minDATA | 5min-HistSim | 5min-MA | 5min-EWMA | 5minGARCH(1,1) | 10minDATA | **10min-HistSim** | 10mi

Confidence level:	99.00%
Portfolio:	10000000
# of Assets:	3

RETURN

Pricing Date	TIME	# of Data	NASDAQ					
			0.3	Weights	S&P500	DJI	NASDAQ	Portfolio
29-07-13	18:33	7	3607.48		-0.00041485	-0.00033557	-0.00028825	-0.00035309
29-07-13	18:43	8	3608.11		-0.00052770	-0.00072499	0.00017462	-0.00037619
29-07-13	18:53	9	3607.61		0.00055141	0.00066572	-0.00013859	0.00037871
29-07-13	19:03	10	3608.85		0.00021337	0.00023254	0.00034366	0.00025821
29-07-13	19:13	11	3600.78		-0.00179728	-0.00156120	-0.00223867	-0.00185887
29-07-13	19:23	12	3596.64		-0.00066516	-0.00052202	-0.00115041	-0.00076779
29-07-13	19:33	13	3595.26		-0.00025549	-0.00011231	-0.00038377	-0.00025102
29-07-13	19:43	14	3594.1		-0.00026744	-0.00033184	-0.00032270	-0.00030334
29-07-13	19:53	15	3596.15		0.00076054	0.00065132	0.00057022	0.00067068
29-07-13	20:03	16	3596.54		0.00014848	0.00012002	0.00010844	0.00012793
29-07-13	20:13	17	3596.36		-0.00037420	-0.00027038	-0.00005005	-0.00024581
29-07-13	20:23	18	3596.57		0.00027324	0.00065551	0.00005839	0.00032347
29-07-13	20:33	19	3596.84		0.00010690	0.00008578	0.00007507	0.00009101
29-07-13	20:43	20	3597.81		0.00011876	-0.00006256	0.00026964	0.00010963

Here Correlations and Covariances

COVARIANCES

	S&P500	*DJI*	*NASDAQ*
S&P500	0.0000009223		
DJI	0.0000002609	0.0000008868	
NASDAQ	0.0000000135	0.0000000337	0.0000011957

Variance-Covariance Matrix

	S&P500	DJI	NASDAQ
S&P500	0.0000009223	0.0000002609	0.0000000135
DJI	0.0000002609	0.0000008868	0.0000000337
NASDAQ	0.0000000135	0.0000000337	0.0000011957

Weights Vector	0.4
	0.3

5min-HistSim / 5min-MA / 5min-EWMA / 5minGARCH(1,1) / 10minDATA / 10min-HistSim / **10min-MA** / 10min-EWMA

onfidence level:	99.00%
ortfolio:	10000000
of Assets:	3

COVARIANCEs

ricing Date	TIME	# of Data					
			NASDAQ	Porfolio	S&P500/DJI	S&P500/NQ	DJI/NQ
29-07-13	19:13	11	0.00000082	0.00000034	0.000000325	0.000000040	0.000000041
29-07-13	19:23	12	0.00000107	0.00000053	0.000000474	0.000000279	0.000000249
29-07-13	19:33	13	0.00000108	0.00000053	0.000000466	0.000000308	0.000000270
29-07-13	19:43	14	0.00000103	0.00000051	0.000000440	0.000000295	0.000000256
29-07-13	19:53	15	0.00000097	0.00000048	0.000000419	0.000000283	0.000000247
29-07-13	20:03	16	0.00000093	0.00000048	0.000000424	0.000000292	0.000000255
29-07-13	20:13	17	0.00000088	0.00000045	0.000000399	0.000000275	0.000000240
29-07-13	20:23	18	0.00000082	0.00000043	0.000000381	0.000000260	0.000000227
29-07-13	20:33	19	0.00000078	0.00000041	0.000000369	0.000000245	0.000000215
29-07-13	20:43	20	0.00000073	0.00000038	0.000000348	0.000000231	0.000000203
29-07-13	20:53	21	0.00000069	0.00000036	0.000000326	0.000000219	0.000000190
29-07-13	21:03	22	0.00000069	0.00000036	0.000000325	0.000000232	0.000000204
29-07-13	21:13	23	0.00000064	0.00000034	0.000000305	0.000000218	0.000000191
29-07-13	21:23	24	0.00000061	0.00000032	0.000000287	0.000000205	0.000000179

Portfolio Variance	7.6128E-07
Portfolio Volatility	0.000872514
Portfolio size	10000000
Portfolio Volatility (€)	8725.13669
Z(1%)	-2.32634787
1-day 99% VaR	-20297.7032
10-day 99% VaR	-64186.9734

Confidence level:	99.00%	
Portfolio:	10000000	
# of Assets:	3	

COVARIANCEs

Pricing Date	TIME	# of Data	NASDAQ	Portfolio		S&P500/DJI	S&P500/NQ	DJI/NQ
29-07-13	17:40	2	0.00000092	0.00000038		0.000000146	0.000000712	0.000000601
29-07-13	17:50	3	0.00000113	0.00000031		0.000000139	0.000000674	0.000000612
29-07-13	18:00	4	0.00000107	0.00000043		0.000000231	0.000000913	0.000000772
29-07-13	18:13	5	0.00000102	0.00000035		0.000000255	0.000000792	0.000000685
29-07-13	18:23	6	0.00000104	0.00000058		0.000000273	0.000001076	0.000000928
29-07-13	18:33	7	0.00000101	0.00000044		0.000000190	0.000000892	0.000000832
29-07-13	18:43	8	0.00000099	0.00000037		0.000000161	0.000000786	0.000000742
29-07-13	18:53	9	0.00000096	0.00000033		0.000000128	0.000000745	0.000000774
29-07-13	19:03	10	0.00000095	0.00000031		0.000000111	0.000000727	0.000000776
29-07-13	19:13	11	0.00000094	0.00000027		0.000000101	0.000000660	0.000000699
29-07-13	19:23	12	0.00000146	0.00000109		0.000000357	0.000001419	0.000001145
29-07-13	19:33	13	0.00000141	0.00000089		0.000000284	0.000001168	0.000000955
29-07-13	19:43	14	0.00000125	0.00000064		0.000000214	0.000000923	0.000000792
29-07-13	19:53	15	0.00000114	0.00000049		0.000000157	0.000000780	0.000000719

Variance-Covariance Matrix(Eviews)

	S&P500	DJI	NASDAQ
S&P500	0.00000111658	0.000000178	0.000001191
DJI	0.000000178	0.00000114239	0.000001151
NASDAQ	0.000001191	0.000001151	0.00000116318
Weights Vector	0.4		

5min-EWMA | 5minGARCH(1,1) | 10minDATA | 10min-HistSim | 10min-MA | 10min-EWMA | 10minGARCH(1,1)