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ABSTRACT

Black-Scholes model is a very famous model using to estimate option prices in stock market. The model is based on Brownian motion and Normal distribution as an evolutionary process of the underlying asset. However, the evolutionary process of the underlying asset is always in a changing environment that changing from "good" to "bad", "bad" to "normal", and so on. Therefore, the option values can be affected by this environment. Janssen (Janssen, Manca and E.Volpe, 2009) proposed a new model for Black-Scholes formula by assuming the environment as a Markov chain with the state space including "good", "normal", "bad", etc. (sometime called Markov Black-Scholes model in this paper). The most importance of this model is to estimate the parameters of the Markov chain which is the best appropriate to the environment that the underlying asset in. This issue still not be addressed reasonably up to now. On the other hand, Zucchini and Macdonald 2009 introduced Hidden Markov models for time series using R programming. Thus, we applied the Hidden Markov model using Normal distribution to the historical data of VN-Index to find out the Markov model that fits best the data. It is clear that Hidden Markov model can be used to estimate the parameters for the Markov Black-Scholes model. In this article, we present in detail how to use Hidden Markov model in Markov Black-Scholes model and illustrate the acquired results for daily VN-Index historical data from 2009 to 2011. This combination makes the parameters in the model have more significance for the reality.

Keywords: Option prices, Markov model, Hidden Markov model, VN-Index.

2010 Mathematics Subject Classification: 60H30, 60J22, 62M05, 62P20, 91G20, 91B25.

1 Introduction

To price and hedge derivative securities, it is important to get a good probability distribution of the underlying asset. The most famous and popular model with continuous time established is Black-Scholes model (Black and Scholes, 1973). It uses Normal distribution to fit the logarithm returns of underlying asset. The price process of the underlying asset is given by the geometric Brownian motion:

$$S_t = S_0.e^{(\mu-\frac{\sigma^2}{2})t+\sigma.B_t} \tag{1.1}$$

where $\{B_t, t \geq 0\}$ is a standard Brownian motion, i.e. B_t has the Normal distribution with the mean 0 and variance t. Nevertheless, the price process of the underlying evolves in a non-stable environment that can change from ”good” to ”bad”. Janssen (Janssen and Manca, 2009) proposed a model representing the environment as a Markov chain with various states (sometime we call Markov Black-Scholes model in this article). The parameters μ, σ then be governed by on the states i, j of the environment at the time t, t' respectively. Hence, the option value depends not only on σ_{ij} but also on transition probabilities of the Markov chain. However, there are the lack of appropriate methods to estimate the mentioned parameters reasonably and actually.

On the other hand, Hidden Markov Models (HMMs) is a widely tool to analyze and predict time series phenomena. HMMs has been used successfully to analyze various types of time series including DNA sequence analysis (Cheung, 2004), Speech Signal recognition (Xie, Andreae, Zhang and Warren, 2004), ECG analysis (Coast, Stern, Cano and Briller, 1990), e.t.c. In financial fields, an earlier study (Hassan and Nath, 2005), HMM has been used to generate one-day forecasts of stock prices in a novel way. Other study of Hassan (M.D.R, Nath and Kerley, 2007) has combined the HMM used in Hassan and Nath (2005) with an Artificial Neural Network (ANN) and a Genetic Algorithm (GA) to achieve better forecasts. We can refer a more recent study of Rafiul Hassan (Hassan, 2009) as the other combination of Hidden Markov Model and fuzzy model for stock market forecasting.

The best advantage of HMM is to find the Markov chain of the system's evolution which cannot be seen such that it is the best fit of the observation data. Therefore, this paper uses HMM to find the Markov chain of a financial time series together with some other method to estimate the parameters for the Black-Scholes model in Markov environment.

The empirical example for this article we use the historical data of VN-Index form `http://www.cophieu68.com` website and the inter-bank average interest rate data form the State Bank of Vietnam `http://www.sbv.gov.vn`.

2 Markov Model in Finance

2.1 Markov Chain

Let us consider an economic or physical system S with m possible states, represented by the set I:

$$I = \{1, 2, \cdots, m\}. \tag{2.1}$$

Let the system S evolve randomly in discrete time $(t = 0, 1, 2, ..., n, ...)$, and let J_n be the random variable representing the state of the system S at time n.

Definition 2.1. The random sequence $(J_n, n \in \mathbb{N})$ is a Markov chain if and only if for all $j_0, j_1, \cdots, j_n \in I$:

$$\begin{aligned} &P(J_n = j_n | J_0 = j_0, ..., J_{n-1} = j_{n-1}) \\ &= P(J_n = j_n | J_{n-1} = j_{n-1}). \end{aligned} \tag{2.2}$$

(Provided this probability have meaning).

Definition 2.2. A Markov chain $(J_n, n \geq 0)$ is homogenous if and only if the probability (2.2) do not depend on n and non-homogenous in the other cases.

For the moment, we will only consider the homogenous case for which we write:

$$P(J_n = j | J_{n-1} = i) = p_{ij}, \tag{2.3}$$

and we introduce matrix $\mathbf{P}$ defined as

$$\mathbf{P} = [p_{ij}]. \tag{2.4}$$

Of course, the elements of matrix $\mathbf{P}$ have the following properties:

$$\begin{array}{lll} \text{(i)} & p_{ij} \geq 0, & \text{for all } i, j \in I \\ \text{(ii)} & \sum\limits_{j \in I} p_{ij} = 1, & \text{for all } i \in I. \end{array}$$

A matrix P satisfying these two conditions is called a *Markov matrix* or a *transition matrix*.
To fully define the evolution of a Markov chain, it is also necessary to fix an initial distribution for the state J_0, i.e. a vector

$$\mathbf{p} = (p_1, p_2, ..., p_m), \tag{2.5}$$

such that

$$\begin{gathered} p_i \geq 0, \forall i \in I \\ \sum_{i \in I} p_i = 1. \end{gathered}$$

For all i, p_i represents the initial probability of starting from i:

$$p_i = P(J_0 = i). \tag{2.6}$$

For the rest of this article, we will consider homogenous Markov chains characterized by the couple $(\mathbf{p}, \mathbf{P})$.
Now we introduce the *transition probabilities of order* n, $p_{ij}^{(n)}$, defined as

$$p_{ij}^{(n)} = P(J_{v+n} = j | J_v = i). \tag{2.7}$$

From the Markov property (2.2), by induction, it is clear that $\mathbf{P}^{(n)} = \left[p_{ij}^{(n)}\right] = \mathbf{P}^n$.
This means the transition probability matrix in n steps is equal to the n-th power of matrix $\mathbf{P}$.
For the marginal distributions related to J_n, we define for $i \in I$ and $n \geq 0$:

$$p_i(n) = P(J_n = i). \tag{2.8}$$

If let $\mathbf{p}(n) = (p_1(n), p_2(n), ..., p_m(n))$ then it can solve in matrix notation that $\mathbf{p}(n) = \mathbf{p}\mathbf{P}^n$.

Definition 2.3. A Markov matrix $\mathbf{P}$ is *regular* if there exists a positive integer k such that all elements of matrix $\mathbf{P}^k$ are strictly positive.

Definition 2.4. A Markov chain is said to be *Ergodic Markov chain* if from any state, it can reach any other state (unnecessary in one step), i.e $\exists k \in \mathbb{N} : p_{ij}^k > 0$ for all $i, j \in I$.

That implies that a regular Markov chain is also an Ergodic Markov chain.

Theorem 2.1. *Janssen and Manca 2006 Let* $\mathbf{P}$ *be the transition matrix of an Ergodic Markov chain, then the limit* $\lim_{n\to\infty} p_i(n) = \pi_i$ *exists regardless to the initial distribution* $\mathbf{p} = (p_1, p_2, ..., p_m)$ *and* $\{\pi_i\}$ *is the unique solution of the system:*

$$\begin{cases} \pi_j = \sum_{i\in I} \pi_i . p_{ij} \\ \sum_{i\in I} \pi_i = 1 \end{cases} . \tag{2.9}$$

2.2 Black-Scholes Model in Markov Environment

2.2.1 Black-Scholes Model

Black and Scholes used a continuous time model for the underlying asset introduced by Samuelson (1965). Let $\{S_t, t \geq 0\}$ represent the time evolution of the underlying asset. The basic assumptions is that the stochastic dynamic of S-process given by:

$$\begin{cases} dS_t & = \mu S_t dt + \sigma S_t dB_t \\ S_{t=0} & = S_0 \end{cases} , \tag{2.10}$$

where the process $\{B_t, t \in [0, T]\}$ is a standard Brownian process.
The solution of this stochastic differential equation is $S_t = S_0 . e^{(\mu - \frac{\sigma^2}{2})t + \sigma B_t}$. This means $\frac{S_t}{S_0}$ has lognormal distribution with parameters $\left((\mu - \frac{\sigma^2}{2})t; \sigma^2 t\right)$.
Now we consider a European call option of maturity T and exercise price K at every time t in $[0, T]$. Let r be the non-risky coinstantaneous interest rate and i be the non-risky annual rate we have $e^r = 1 + i$. We call the value of this call option under AOA (absence of opportunity arbitrage) condition as $C(S, t)$, Black and Scholes (1973) obtained the explicit form of $C(S, t)$ as below:

$$\begin{cases} C(S,t) & = S\Phi(d_1) - K.e^{-r(T-t)}\Phi(d_2) \\ d_1 & = \frac{1}{\sigma\sqrt{T-t}} \left[\log \frac{S}{K} + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right] \\ d_2 & = d_1 - \sigma\sqrt{T-t} \\ S & = S_t \end{cases} \tag{2.11}$$

where $\Phi(x)$ is the cumulative distribution function of standard normal distribution.

2.2.2 The Markov Extension of the Black-Scholes Model

Starting with considering a Black-Scholes model for the evolution of one asset having the known value $S(0) = S_0$ at time 0 and random value S_t at time t. The economic and financial environment is defined with random variables J_0, J_t representing the environmental states respectively at time 0 and time t. These random variables take their values in the state space $I = 1, 2, ..., m$, which defined on the probability space by:

$$P(J_0 = i) = p_i, \quad \forall i \in I,$$
$$P(J_t = j | J_0 = i) = p_{ij}, \quad \forall i, j \in I.$$

Furthermore, let us introduce the following functions $r_{J_0 J_t}, \sigma_{J_0 J_t}$ of J_0, J_t to be the random variables of non-risky interest rate and volatility of the asset respectively with state J_0 at time 0 and J_t at time t. So we have two matrices $\mathbf{r} = [r_{ij}], \sigma = [\sigma_{ij}]$. To use the classical notation in the Black and Scholes (1973), we can define the instantaneous interest rate intensity ρ_{ij} with $\rho_{ij} = \log(1 + r_{ij})$.

Theorem 2.2. *Janssen and Manca 2009 Under the assumption the Markov chain of matrix $\mathbf{P}$ of the environment process is Ergodic and given the initial environment state $i \in I$ and the environment state at time t $j \in I$, the non-risky rate is given by ρ_{ij} and the annual volatility by σ_{ij}, then we have the following results concerning the European call price at time 0 with exercise price K and maturity t:*
With knowledge of state $J_0 = i$ and $J_t = j$, the call value is given by

$$\begin{cases} C_{ij}(S_0, t) & = S_0 \Phi(d_{ij1}) - K.e^{-\rho_{ij}.t} \Phi(d_{ij2}) \\ d_{ij1} & = \frac{1}{\sigma_{ij}\sqrt{t}} \left[\log \frac{S_0}{K} + \left(\rho_{ij} + \frac{\sigma_{ij}^2}{2} \right) t \right]. \\ d_{ij2} & = d_{ij1} - \sigma_{ij}\sqrt{t} \end{cases} \tag{2.12}$$

With knowledge of state $J_0 = i$, the call value represented by $C_i(S_0, t)$ is given by

$$C_i(S_0, t) = \sum_{j \in I} \pi_j . C_{ij}(S_0, t). \tag{2.13}$$

With knowledge of state $J_t = j$, the call value represented by $C^j(S_0, t)$ is given by

$$C^j(S_0, t) = \sum_{i \in I} p_i . C_{ij}(S_0, t). \tag{2.14}$$

Without any environment knowledge, the call value represented by $C(S_0, t)$ is given by

$$C(S_0, t) = \sum_{i \in I} p_i . C_i(S_0, t). \tag{2.15}$$

or

$$C(S_0, t) = \sum_{j \in I} \pi_j . C^j(S_0, t). \tag{2.16}$$

The main problem for the Markov Black-Scholes model is how to estimate the matrix parameters $\mathbf{P}$, σ and $\mathbf{r}$. In this article, we propose a new approach for this issue using Hidden Markov model. This method allows creating an algorithm to calculate the call value based on the historical data as an input data.

3 Hidden Markov Model

3.1 Definitions

A hidden Markov model (HMM) is a statistical Markov model in which the system modeled is assumed to be a Markov process with unobserved (hidden) states. An HMM can be considered as the simplest dynamic Bayesian network. In a regular Markov model, the state is directly visible to the observer, and therefore the state transition probabilities are the only parameters. In a hidden Markov model, the state is not directly visible, but output, dependent on the state, is visible. Each state has a probability distribution over the possible output tokens. Therefore the sequence of tokens generated by an HMM gives some information about the sequence of states. Figure 1 illustrates how a hidden Markov model works, where:

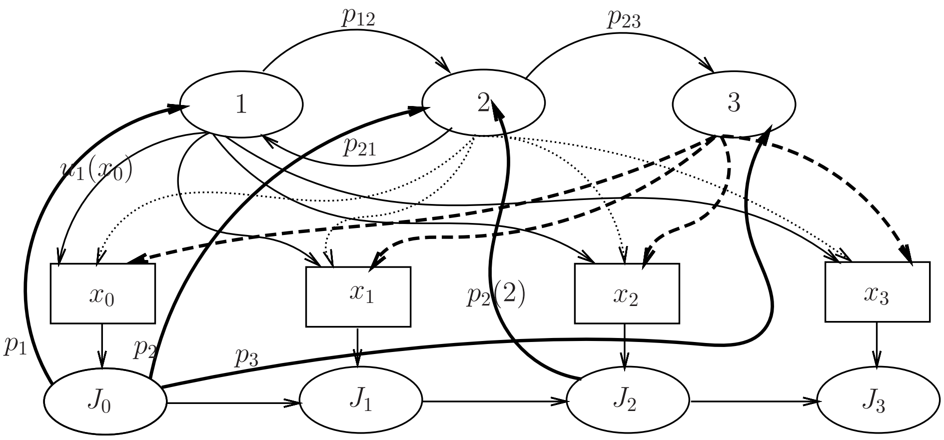

Figure 1: Illustration of hidden Markov models

- J_i - States,
- p_{ij} - states transition probabilities,
- x_t - possible observations,
- $u_i(x_t)$ - output probabilities.

Hence, in a hidden Markov model, there are a state space $I = \{1, 2, ..., m\}$, a transition matrix for the Markov chain $\mathbf{P} = [p_{ij}]$, a sequence of possible observations $\{x_0, x_1, ..., x_n\}$ and m probability distributions $u_i(x)$ implying the probability of observing x in state i (i.e. $u_i(x) = P(X_t = x|C_t = i)$). The distributions $u_i(x)$ can be chosen such as Normal, Poisson and so on depending on the characters of the observation data $\{x_0, x_1, ..., x_n\}$.
As in Markov chain theory (se Section 2.1), we are known $p_i = P(J_0 = i)$ and $p_i(t) = P(J_t = i)$ for $t = 0, 1, 2, ..., n$ and $i \in I$. For descrete valued observations x_t, we have:

$$\begin{aligned} P(X_t = x) &= \sum_{i \in I} P(J_t = i).P(X_t = x|J_t = i) \\ &= \sum_{i \in I} p_i(t) u_i(x). \end{aligned} \tag{3.1}$$

Notice that $\mathbf{p}(t) = \mathbf{p}.\mathbf{P}^t$, hence that

$$P(X_t = x) = \mathbf{p}.\mathbf{P}^t \mathbf{U}(x) \mathbf{1}', \tag{3.2}$$

in which, $\mathbf{p}(t)$ is the row vector of elements $p_i(t)$ and $\mathbf{1}'$ is the column vector of elements 1. $\mathbf{U}(x)$ is the diagonal matrix with i-th diagonal element $u_i(x)$. Thus, we have the likelihood function as below

$$L_n = \mathbf{p}\mathbf{U}(x_1)\mathbf{P}\mathbf{U}(x_2)...\mathbf{P}\mathbf{U}(x_n)\mathbf{1}'. \tag{3.3}$$

3.2 Three Basic Problems of HMMs

We know that, in Janssen's model, the most important issue is to find a Markov chain which is the most appropriate for the financial environment based on historical data. Fortunately, solving three following problems of HMMs will address this issue:

- Given observations $\{x_0, x_1, ..., x_n\}$ and model $\lambda = (\mathbf{P}, \mathbf{U}, \mathbf{p})$, efficiently compute $P(x_0, x_1, ..., x_n|\lambda)$:
  - Hidden states complicate the evaluation.
  - Given two models λ_1 and λ_2, this can be used to choose the better one.

- Given observations $\{x_0, x_1, ..., x_n\}$ and model $\lambda = (\mathbf{P}, \mathbf{U}, \mathbf{p})$, find the optimal state sequence $\{j_0, j_1, ..., j_n\}$:
  - Optimality criterion has to be decided (e.g. maximum likelihood).
  - "Explanation" for the data.

- Given observations $\{x_0, x_1, ..., x_n\}$, estimate model parameters $\lambda = (\mathbf{P}, \mathbf{U}, \mathbf{p})$ that maximize the probability $P(x_0, x_1, ..., x_n|\lambda)$.

According to Zucchini and Macdonald 2009, each of the problems has own solving algorithm. To address the first problem, they use an algorithm called Forward-Backward algorithm. The Forward-Backward algorithm based on a given HMM model, it means given the initial parameters including transition matrix $\mathbf{P}$ and the parameters of output probabilities $u_i(x)$, to calculate the likelihood $P(x_0, x_1, ..., x_n|\lambda)$ using given observations $\{x_0, x_1, ..., x_n\}$.
However, the initial parameters normally are not good, i.e. not fit best for the observed data. Therefore, the most importance is to estimate the new parameters, hence having a new model, which fits best the data, i.e. maximizing the likelihood, so the third problem can be solved. To address this issue, EM algorithm has been used to find the parameters as well as the number of states of the model that maximize the likelihood. The EM algorithm implements by replacing the initial parameters by the new one which are obtained from maximal log-likelihood method. After the number of training times, the best model will be found.
To solve the second problem, it means to find the optimal state sequence $j_0, j_1, ..., j_n$, which maximizes the likelihood, Viterbi algorithm is used. After this algorithm, there are some forecast distributions can be made from the best model.

4 Markov Financial Model Using Hidden Markov Model

Since Markov Black-Scholes model and HMM model presented above, we find out that it can use Hidden Markov model for the historical data to find the Markov process that maximizes the likelihood and its parameters. Then we use these parameters for Markov Black-Scholes model in calculating the call value of European options.

4.1 The Operation of The Model

The operation for the model can be illustrated as Figure 2. In detail, the Hidden Markov model processes as below:

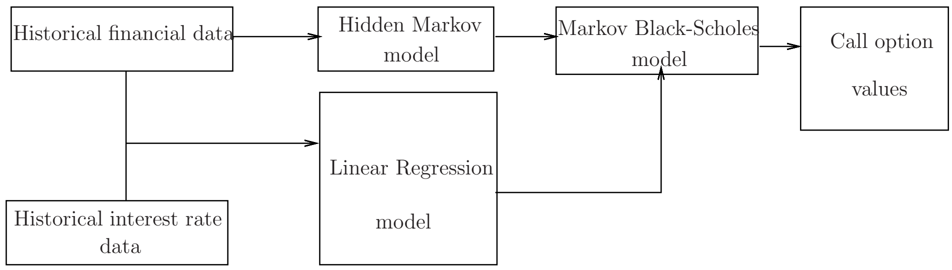

Figure 2: The peration of Markov financial model using Hidden Markov model

- Input historical data.
- Initialize the initial parameters including the number of states m, initial distribution $\mathbf{p} = \{p_1, p_2, ..., p_m\}$, transition matrix $\mathbf{P} = [p_{ij}]$ and the $\{\mu_i, \sigma_i\}_{i=1}^{m}$ of m Normal distributions $u_i(x)$.
- Perform Forward-Backward algorithm.
- Train the model by EM algorithm.
- Using AIC and BIC criteria to choose the best value of m (see Zucchini and Macdonald 2009).
- Output the new parameters fitting best the data including transition matrix $\mathbf{P} = [p_{ij}]$ and the $\{\mu_i, \sigma_i\}_{i=1}^{m}$.
- Estimate the limit distribution $\pi_, i = 1, ..., m$ of the new Markov chain.
- Using Viterbi algorithm to find the state sequence $\{j_0, j_1, ..., j_n\}$ corresponding to observation sequence $\{x_0, x_1, ..., x_n\}$ with the best HMM model found above.

To take the parameters of this Markov model into the Markov Black-Scholes model, we not only need transition matrix $\mathbf{P}$, limit distribution π_i and initial distribution p_i but also volatility matrix $\sigma = [\sigma_{ij}]$ and non-risky interest rate matrix $\rho = \rho_{ij}$, which are not obtained from HMM process. For the matrix , this article propose a brand new method. From the state sequence $\{j_0, j_1, ..., j_n\}$ is identified, we divide the historical data $\{x_0, x_1, ..., x_n\}$ into m groups $G_1, G_2, ..., G_m$ corresponding to m states of the Markov chain. For σ_{ij}, we calculate the standard deviation of the set $G_i \cup G_j$. Therefore, we obtain matrix $\sigma = [\sigma_{ij}]$.

For the matrix ρ, we know that the interest rate $\mathbf{r}$ is quite stable over time. To estimate r_{ij}, we propose using linear regression model. First, using linear regression model for $\{r_0, r_1, ..., r_n\}$ and $\{x_0, x_1, ..., x_n\}$ to estimate the parameters β_0, β_1 in the relationship $\mathbf{r} = \beta_1 \mathbf{x} + \beta_0 +$. Then calculate $r_i = \beta_1 \mu_i + \beta_0$ for all $i = 1, ..., m$. Finally, we estimate $r_{ij} = r_{ji} = \frac{r_i + r_j}{2}$. Hence we have the matrix $[\rho_{ij}]$.

For the initial distribution $\mathbf{p}$, because the time t=0 can be taken any time in the future, we suppose $p_i = \pi_i$ for all $i = 1, ..., m$.

This process is a self-contained process so it is easy for one who would like to create a software for the model. Also, because the input is historical data, so it is convenient to update the data frequently. Therefore, the model becomes more and more accurate.

4.2 An Empirical Result in VN-Index Data

For the historical data of daily VN-Index (Vietnam Stock Market) from 2009 to 2011, after training by Hidden Markov model, we find out that the model with 4 states is the best fit for the data (using AIC and BIC criteria) and the obtained parameters as Table 1.

Table 1: Four obtained states of Normal distributions.

Parameterss	State 1	State 2	State 3	State 4
μ	453.9839	484.6801	505.9007	530.8300
σ	10.685743	7.152381	6.421777	13.074597

This result indicates that the 4-state Hidden Markov model is the best for VN-Index data in which each of the states corresponding to a Normal distribution Normal(μ_i, σ_i) for $i = 1, ..., 4$. Thus, predicting the changes of the stock market can help investers to make their decisions accurately.
And the transition matrix $\mathbf{P}$

$$\mathbf{P} = \begin{pmatrix} 0.9717 & 0.0283 & 0.0000 & 0.0000 \\ 0.0927 & 0.8106 & 0.0804 & 0.0163 \\ 0.0000 & 0.0748 & 0.8624 & 0.0628 \\ 0.0000 & 0.0000 & 0.0818 & 0.9182 \end{pmatrix} \tag{4.1}$$

This matrix is regular so the limit distribution is calculated as

$$\pi = (0.4845107, 0.1479942, 0.1912153, 0.1762798) \tag{4.2}$$

By using the method estimating the matrix $\sigma = [\sigma_{ij}]$ mentioned above, we obtain:

$$\sigma = \begin{pmatrix} 0.01369208 & 0.01399554 & 0.01357780 & 0.01701312 \\ 0.01392719 & 0.01416523 & 0.01323580 & 0.01727688 \\ 0.01353990 & 0.01314098 & 0.01152992 & 0.01355887 \\ 0.01563711 & 0.01559066 & 0.01279659 & 0.01467313 \end{pmatrix}$$

Now we are going to use linear regression model to find the linear relationship between daily interest rate $\mathbf{r}$ and VN-Index value $\mathbf{x}$, we have:

$$\mathbf{r} = -0.331327.10^{-7}\mathbf{x} + 0.333333.10^{-3} + \epsilon. \tag{4.3}$$

So we have $\{r_i\}$ equal

$$\begin{pmatrix} 0.0003182913 & 0.0003172742 & 0.0003165711 & 0.0003157452 \end{pmatrix}$$

Therefore, matrix $[r_{ij}]$ can be calculated:

$$\mathbf{r} = \begin{pmatrix} 0.0003182913 & 0.0003177828 & 0.0003174312 & 0.0003170182 \\ 0.0003177828 & 0.0003172742 & 0.0003169227 & 0.0003165097 \\ 0.0003174312 & 0.0003169227 & 0.0003165711 & 0.0003161582 \\ 0.0003170182 & 0.0003165097 & 0.0003161582 & 0.0003157452 \end{pmatrix}$$

From the relationship $\rho_{ij} = \log(1 + r_{ij})$, we have matrix $[\rho_{ij}]$:

$$[\rho_{ij}] = \begin{pmatrix} 0.0003182406 & 0.0003177323 & 0.0003173808 & 0.0003169680 \\ 0.0003177323 & 0.0003172239 & 0.0003168725 & 0.0003164596 \\ 0.0003173808 & 0.0003168725 & 0.0003165210 & 0.0003161082 \\ 0.0003169680 & 0.0003164596 & 0.0003161082 & 0.0003156953 \end{pmatrix}$$

Now applying to the Markov Black-Scholes model for $S(0) = 400$, $K = 450$, $t = 3$ months, we have $C_{ij}(S_0, t)$ (i.e. have knowledge of state i and state j), shown in Table 2. With knowledge

Table 2: Call option values with knowledge of state i and j.

$i \to j$	$C_{ij}(S_0, t)$	$i \to j$	$C_{ij}(S_0, t)$
1→1	9.203245	2→1	9.519700
1→2	9.613652	2→2	9.842121
1→3	9.038489	2→3	8.570589
1→4	13.895776	2→4	14.275534
3→1	8.986982	4→1	11.908051
3→2	8.443215	4→2	11.835932
3→3	6.342079	4→3	7.976246
3→4	8.999373	4→4	10.531372

of state i without knowledge of state j, the results are shown in Table 3. With knowledge of

Table 3: Call option values without knowledge of state j

$i \to ?$	$C_i(S_0, t)$
1 → ?	10.059677
2 → ?	10.224289
3 → ?	8.402946
4 → ?	10.902876

state j without knowledge of state i, the results are shown in Table 4.

Table 4: Call option values without knowledge of state i

$? \to j$	$C^j(S_0, t)$
? → 1	9.685528
? → 2	9.815402
? → 3	8.266396
? → 4	12.422634

Finally, without knowledge of any state, we have

$$C(S_0, t) = 9.915886.$$

From these results, we find that the call value of the European option in the case of unknown any information about sates i and j has a significant difference from the cases of having knowledge about either state i or state j. However, this small difference plays a big role in forecasting option prices.

5 Conclusion

This article provides a new approach to find out the parameters for the Markov Black-Scholes model. With this method, the parameters are found automatically in a self-contained process. Especially, this method provides a high rationality for the real evolution of the financial time series. For its advantages, this new model can be become a helpful and effective tool for financial professors or everyone who would like to analyze and forecast option values.
In the near future, the model can be developed for other distribution that is better than Normal distribution such as Meixner distribution or other option pricing formulas.
Throughout this paper we use R software for computations (the software can be found at `http://www.r-project.org/`). This software is a very strong tool for programing and calculating.

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References

Black, F. and Scholes, M. 1973. The pricing of options and corporate liabilities, *Journal of Political Economy* **81**: 637–659.

Cheung, L. W. K. 2004. Use of runs statistics for pattern recognition in genomic dna sequences, *Journal of Computational Biology* **11**(1): 107–124.

Coast, D. A., Stern, R., Cano, G. and Briller, S. 1990. An approach to cardiac arhythmia analysis using hidden markov models, *IEEE Transactions on Biomedical Engineering* **37**(9): 826–836.

Hassan, M. 2009. A combination of hidden markov model and fuzzy model for stock market forecasting, *Elsevier* **72**: 3439–3446.

Hassan, M. and Nath, B. 2005. Stock market forecasting using hidden markov model: a new approach, *Proceedings of 5th international conference on intelligent system design and application, ISDA 2005, Wroclaw, Poland*, pp. 192–196.

Janssen, J. and Manca, R. 2006. *Applied semi-Markov Processes*, Springer Verlag, New York.

Janssen, J. and Manca, R. 2009. *Semi-Markov Risk Models for Finance, Insurance and Reliability*, Springer Verlag, New York.

Janssen, J., Manca, R. and E.Volpe 2009. *Mathematical Finance: Deterministic and Stochastic Models*, ISTE Ltd, London, UK.

M.D.R, H., Nath, B. and Kerley, M. 2007. A fusion model of hmm, ann and ga for stock market forecasting, *Expert Systems with Applications* **33**(1): 171–180.

Samuelson, P. 1965. Rational theory of warrant pricing, *Industrial Management Review* **6**: 13–31.

Xie, H., Andreae, P., Zhang, M. and Warren, P. 2004. Learning models for english speech recognition, *Proceedings of the 27th conference on Australasian computer science, ACSC 2004, Darlinghurst, Australia*, pp. 323–329.

Zucchini, W. and Macdonald, I. L. 2009. *Hidden Markov Models for Time Series: An Introduction Using R*, Chapman and Hall, New York.