JOURNAL OF INDUSTRIAL AND
MANAGEMENT OPTIMIZATION
Volume **16**, Number **4**, July **2020**

doi:10.3934/jimo.2019021

pp. **1635–1654**

MODEL SELECTION BASED ON VALUE-AT-RISK BACKTESTING APPROACH FOR GARCH-TYPE MODELS

Hao-Zhe Tay

Institute of Mathematical Sciences
Faculty of Science, University of Malaya
Kuala Lumpur, Malaysia

Kok-Haur Ng*[1], You-Beng Koh[2] and Kooi-Huat Ng[3]

[1,2]Institute of Mathematical Sciences
Faculty of Science, University of Malaya
Kuala Lumpur, Malaysia
[3]Department of Mathematical and Actuarial Sciences
Lee Kong Chian Faculty of Engineering and Science, Universiti Tunku Abdul Rahman
Malaysia

(Communicated by Hailiang Yang)

Abstract. This paper aims to investigate the efficiency of the value-at-risk (VaR) backtests in the model selection from different types of generalised autoregressive conditional heteroskedasticity (GARCH) models with skewed and non-skewed innovation distributions. Extensive simulation is carried out to compare the model selection based on VaR backtests and Akaike Information Criteria (AIC). When the model is given but the innovation distribution is one of the six selected distributions which may be skewed or non-skewed, the simulation results show that both AIC and the VaR backtests succeed in selecting the correct innovation distribution from the set of six distributions under consideration. This indicates that both AIC and the VaR backtests are able to distinguish between skewed and non-skewed distributions when the innovation distribution is misspecified. Using an empirical data from NASDAQ index, we observe that the selected combination of model and innovation distribution based on the smallest AIC does not agree with that selected by using the in-sample VaR backtests. Examination of confidence limits for VaR and the expected shortfall forecasts under various loss functions provides evidence that the selected combination of model and innovation distribution using the VaR backtests tends to possess smaller mean absolute percentage error and logarithmic loss.

1. **Introduction.** Volatility is a statistical measure of the dispersion of returns for a particular asset index. This measure often relates to the uncertainty or the risk of the asset index. Since the stock market crash of 1987, stock price volatility has captured the eyes of both academic researchers as well as the regulators' attentions [29]. Therefore, there has been considerable search for appropriate volatility forecasting and risk measurement methodologies that have extensively been studied and

2010 *Mathematics Subject Classification.* Primary: 62P05; Secondary: 91B84, 37M10, 62M10.
Key words and phrases. Model selection, backtest, value-at-risk, GARCH model, skewed distribution.

* Corresponding author: Ng, Kok-Haur.

reviewed by many academicians and practitioners in the past two decades. Some of the noteworthy characteristics that are commonly observed in asset returns are the persistence volatility clustering, time-varying volatility, leptokurtic and leverage effects.

In the past, various time series volatility models were proposed and used to formulate the volatility forecast. Autoregressive conditional heteroskedasticity (ARCH) model was one of the popular models, and yet, being extensively enhanced in the late nineties [15]. [15] proposed to use ARCH of order q to model time varying variance of inflation data. One of the novel features of ARCH model in the volatility modelling is its conditional variance of the model written as a function of the past squared returns, which are capable of capturing the volatility clustering property that exists in numerous financial time series. Generalised conditional autoregressive heteroskedasticity (GARCH) model proposed by [7], which is based on an infinite ARCH specification, allows one to reduce the number of estimated parameters from infinity to a finite number [1]. Even though the ARCH and GARCH models are capable in detecting volatility clustering, however, these two models fail to capture some of the common characteristics such as leverage effect that may occur in financial data. To address this issue, different nonlinear and asymmetric extension models have been proposed. [31] proposed the exponential GARCH model, which specifies conditional variance in logarithmic form and also includes the additional terms for capturing the leverage effect. On the other hand, various types of models that allow for asymmetric dependencies were introduced such as Glosten-Jagannathan-Runkle GARCH (GJRGARCH) by [20], Threshold GARCH (TGARCH) by [33] , Asymmetry Power ARCH (APARCH) by [14], nonlinear asymmetric GARCH (NAGARCH) by [16], Quadratic GARCH by [34], Fractional Integrated GARCH by [5], Bilinear GARCH (BLGARCH) by [38], Threshold BLGARCH by [12] and etc.

In addition to the choices of appropriate GARCH-Type models, attention has also been centralised on the specification of innovation distribution. Traditionally, stock market returns are modelled based on the normality assumption. However, this distributional assumption does not embrace the stylised characteristics of financial time series such as heavy tailed, leptokurtic and asymmetry [25]. Thus, a number of papers focusing on different forms of distribution assumptions were proposed to discern the respective characteristics. Among them were skewed normal distribution [3], student-t distribution [8], generalised error distribution [31], and skewed student-t distribution with an additional skew parameter proposed by [18], which was later extended to the GARCH framework by [24]. [4] considered the symmetric/asymmetric GARCH models with the skewed generalised error and skewed student-t distributions in modelling the interest rate volatility. The asymmetric exponential power distribution [42] and asymmetric student-t distribution [41] were also used for the modelling of the five popular commodities [30]. [27] modelled crude oil returns using NAGARCH model with skewed distributions that provide improvements in terms of its forecasting ability.

Value-at-risk (VaR) is one of the major concern and popular risk measures for quantifying market risks. Most of the practitioners and investors refer it as the maximal loss of financial position at a given time period and probability (see [39]), which are served as guideline for the regulatory committee to set their margin requirements. To evaluate the accuracy of the estimation of VaR, some backtests such as Kupiec test [23], conditional coverage Christoffersen test [13], and dynamic quantile test [17] were performed. [19] showed that the Kupiec test results on the estimation of in-sample VaR for returns using APARCH model based on skewed student-t distribution agreed with the chosen confidence level. Most studies have noted that generalising the non-skewed innovation distribution of GARCH-Type models to skewed family distribution will significantly improve the accuracy of VaR forecast [2, 10, 19, 22]. Meanwhile, [37] asserted that the best selected model based on the smallest AIC does not always give an satisfactory backtests' results for estimating VaR using GARCH-Type models in empirical application. However, to date, there is lack of simulation study on the misspecified distributional assumption on the GARCH-Type models for VaR forecast. As discussed by [27], though under the same volatility model specification, different distribution assumptions may cause a great discrepancy of VaR values.

Hence the aim of this paper is threefold. First, we investigate the model selection capability when estimating the in-sample long and short positions VaR of returns. The accuracy of VaR is further tested using the backtests. A number of GARCH-Type models with known skewed and non-skewed, as well as misspecified innovation distributions, are then compared and reported. Secondly, we examine the choices of model based on the smallest Akaike Information Criteria (AIC) and the significance of all in-sample VaR backtests using an empirical study of NASDAQ index. Lastly, we present the forecasts of the returns, the VaR and the expected shortfall (ES) of the returns on the basis of the best fitted models. Meanwhile, the predictive performances of these models are evaluated using various loss functions, and VaR backtests are carried out to assess the comparative forecasting accuracy across the models.

This paper is organised as follows. Section 2 discusses the general description on several GARCH-Type models and the related distributions that employed in volatility modelling. The simulation study is then implemented in Section 3 to investigate the GARCH-Type models with the misspecified innovation distributions. Section 4 describes the empirical data set with their corresponding summary statistics. The best fitted models using model selection criteria, including AIC and VaR as well as its backtests results, are elaborated and compared. It is then followed by the comparison of its forecasting performance via the models. Finally, the concluding remarks are presented in Section 5.

2. Model specification and distributional assumption.

Conditional heteroskedasticity models are well-known tools that are frequently applied in modelling and forecasting the volatility of the financial returns. This section reviews various types of GARCH models used in this study and different types of distributions for innovation associated with the criteria for selecting the best fitted models.

Consider the closing price of a stock, P_t at time t, where $t = 1, 2, \ldots, n$. The log return of the stock is defined as $r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$ which can be decomposed into two components: the mean component μ_t and the error component (or shock) a_t, i.e.,

$$r_t = \mu_t + a_t = \mu_t + \sigma_t \varepsilon_t, \tag{1}$$

where ε_t is independent and identically distributed (i.i.d) innovation with mean zero and unit variance, and σ_t is the conditional standard deviation of the returns.

2.1. GARCH-Type models.

The conditional variance for the ARCH of order q model proposed by [15] is

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2, \tag{2}$$

where $\alpha_0 > 0$ and $\alpha_i \geq 0$ for $i = 1, 2, \ldots, q$. It can be observed from (2) that large past squared shocks $\left\{a_{t-i}^2\right\}_{i=1}^q$ imply a large value of conditional variance, σ_t^2.

[7] introduced a more general class of ARCH process, which is the GARCH model that allows for more flexible lag structure. The GARCH model appears to be a simple generalisation of the ARCH model, that is, a low order GARCH may capture the properties similar to high order ARCH model. Thus, the conditional variance of the GARCH (p, q) model at time t can be expressed as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{3}$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1, 2, \ldots, q$ and $\beta_j \geq 0$ for $j = 1, 2, \ldots, p$, in which the sufficient condition for ensuring the stationarity of the variance is $\left(\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j\right) < 1$.

Note that GARCH (p, q) model will reduce to ARCH (q) model when $\beta_j = 0$ for all j. Meanwhile, a special case of GARCH, which is the integrated generalised autoregressive conditional heteroskedasticity model (IGARCH), is restricted to the case where $\left(\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j\right) = 1$.

To account for the asymmetry of the returns time series, various asymmetric GARCH-Type models were proposed. [20] introduced the GJRGARCH (p, q) model by imposing dummy variable that may effectively identify the asymmetry effect of the data. The GJRGARCH (p, q) model is therefore

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i a_{t-i}^2 + \gamma_i \mathbb{I}_{t-i} a_{t-i}^2\right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{4}$$

where γ_i is the asymmetric component and the dummy variable $\mathbb{I}_{t-i} = 1$ if $a_{t-i} < 0$ and $\mathbb{I}_{t-i} = 0$ if $a_{t-i} > 0$ for $i = 1, 2, \ldots q$. GJRGARCH (p, q) model with positive shock a_{t-i} will lead to a lower volatility and negative shock will lead to a higher volatility when $\gamma_i > 0$.

Another example of asymmetry GARCH model is TGARCH model proposed by [33]. Rather than modelling on the conditional variance, TGARCH (p, q) model seeks to express the conditional standard deviation of the data in the following way, that is

$$\sigma_t = \alpha_0 + \sum_{i=1}^{q} \left(\alpha_i^+ a_{t-i}^+ - \alpha_i^- a_{t-i}^-\right) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}, \tag{5}$$

where $a_t^+ = \max\left(a_t, 0\right)$ and $a_t^- = \min\left(a_t, 0\right)$. Note that $\alpha_0 > 0$, $\alpha_i^+, \alpha_i^- \geq 0$ and $\beta_j \geq 0$ for $i = 1, 2, \ldots, q$, and $j = 1, 2, \ldots, p$, in order to ensure the positivity of the conditional standard deviation. If $\alpha_i^+ < \alpha_i^-$, then the negative shock would indicate a higher conditional standard deviation (as well as the conditional variance). This result coincides with the leverage effect mentioned by [6] .

Meanwhile, bilinear GARCH model proposed by [38] allows one to capture the asymmetry property in the time series data by means of the interaction between the past shocks and volatilities, and the BLGARCH (p, q) model is defined as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{p^*} \zeta_k a_{t-k} \sigma_{t-k}, \tag{6}$$

where α_i, β_j and ζ_k are the parameters to be estimated and $p^* = \min(p, q)$. Here, ζ_k is the asymmetry component in the model and the leverage effect is accounted by the interaction between the past shocks and volatilities. Notice that for the BLGARCH $(1,1)$ model, $4\alpha_1\beta_1 > \zeta_1$ is the necessary and sufficient condition for the positivity of σ_t^2 [38].

2.2. **Distributions of innovation.** By convention, financial time series models generally assume that the logarithmic of asset returns (refer to returns hereafter) would follow normal distribution. However, many empirical studies show that financial returns are usually non-normal, and demonstrating asymmetric, fat-tail and leptokurtic properties (see [11, 28]). In this study, we consider six different popular innovation distributions, which are normal distribution (NORMD), student-t distribution (STD), generalised error distribution (GED), skewed normal distribution (SNORMD), skewed student-t distribution (SSTD) and skewed generalised error distribution (SGED). The probability density function (pdf) for each standardised distribution is given as below:

1. Normal distribution (NORMD)

$$f(\varepsilon_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\varepsilon_t^2}{2}}, \quad -\infty < \varepsilon_t < \infty. \tag{7}$$

2. Student-t distribution (STD)

$$f(\varepsilon_t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{\varepsilon_t^2}{v-2}\right)^{-\frac{v+1}{2}}, \quad -\infty < \varepsilon_t < \infty, \quad v > 2, \tag{8}$$

where $\Gamma(\cdot)$ is the gamma function.

3. Generalised error distribution (GED)

$$f(\varepsilon_t) = \frac{v \exp\left[-\frac{1}{2}\left|\frac{\varepsilon_t}{\lambda}\right|^v\right]}{\lambda 2^{1+\frac{1}{v}}\Gamma\left(\frac{1}{v}\right)}, \quad -\infty < \varepsilon_t < \infty, \quad v > 0, \tag{9}$$

where $\lambda = \left[\frac{2^{-\frac{2}{v}}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right]^{\frac{1}{2}}$.

4. Skewed normal distribution (SNORMD)

$$f(\varepsilon_t|\xi) = \begin{cases} \frac{2}{\xi+\frac{1}{\xi}} s \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{s\varepsilon_t+m}{\xi}\right)^2}\right], & \varepsilon_t \geq -\frac{m}{s}, \\ \frac{2}{\xi+\frac{1}{\xi}} s \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\xi[s\varepsilon_t+m])^2}\right], & \varepsilon_t < -\frac{m}{s}, \end{cases} \tag{10}$$

where $m = \frac{2}{\sqrt{2\pi}}\left(\xi - \frac{1}{\xi}\right)$ and $s = \left(1 - \frac{2}{\pi}\right)\left(\xi^2 + \frac{1}{\xi^2}\right) + \frac{4}{\pi} - 1$.

5. Skewed student-t distribution (SSTD)

$$f(\varepsilon_t|\xi) = \begin{cases} \frac{2}{\xi+\frac{1}{\xi}} s \left[\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{\left(\frac{s\varepsilon_t+m}{\xi}\right)^2}{v-2}\right)^{-\frac{v+1}{2}}\right], & \varepsilon_t \geq -\frac{m}{s}, \\ \frac{2}{\xi+\frac{1}{\xi}} s \left[\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{\pi(v-2)}\Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{(\xi[s\varepsilon_t+m])^2}{v-2}\right)^{-\frac{v+1}{2}}\right], & \varepsilon_t < -\frac{m}{s}, \end{cases} \tag{11}$$

where $m = M_1\left(\xi - \frac{1}{\xi}\right), s = (1 - M_1)\left(\xi^2 + \frac{1}{\xi^2}\right) + 2M_1^2 - 1$ and $M_1 = \frac{2\sqrt{v-2}\Gamma\left(\frac{v+1}{2}\right)}{(v-1)\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}$.

6. Skewed generalised error distribution (SGED)

$$f\left(\varepsilon_t|\xi\right)=\begin{cases}\frac{2}{\xi+\frac{1}{\xi}}s\left[\frac{v\exp\left[-\frac{1}{2}\left|\frac{s\varepsilon_t+m}{\lambda\xi}\right|^v\right]}{\lambda 2^{1+\frac{1}{v}}\Gamma\left(\frac{1}{v}\right)}\right], & \varepsilon_t\geq-\frac{m}{s},\\ \frac{2}{\xi+\frac{1}{\xi}}s\left[\frac{v\exp\left[-\frac{1}{2}\left|\frac{\xi\left[s\varepsilon_t+m\right]}{\lambda}\right|^v\right]}{\lambda 2^{1+\frac{1}{v}}\Gamma\left(\frac{1}{v}\right)}\right], & \varepsilon_t<-\frac{m}{s},\end{cases}\tag{12}$$

where $\lambda=\left[\frac{2^{-\frac{2}{v}}\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}\right]^{\frac{1}{2}}$, $m=M_1\left(\xi-\frac{1}{\xi}\right)$, $s=\left(1-M_1\right)\left(\xi^2+\frac{1}{\xi^2}\right)+2M_1^2-1$ and $M_1=2^{\frac{1}{v}}\lambda\left(\frac{\Gamma\left(\frac{2}{v}\right)}{\Gamma\left(\frac{1}{v}\right)}\right)$.

The construction of these skewed distributions was explicitly described by [18] and [24].

3. **Model selection criteria.** All GARCH-Type models will be fitted by employing the maximum likelihood method. To select the best fitted models for the financial data set used, two approaches which are based on the smallest AIC and the agreement of VaR backtests, will be considered.

The first approach is grounded on the smallest AIC value of the model. This method has been extensively adopted (see [21, 37]). Hence, it will be used to obtain the best fitted model for the data set in which the AIC value for each model is computed by

$$\mathrm{AIC}=2k-2\mathrm{LLH},\tag{13}$$

where k is the number of estimated parameters and LLH is the logarithmic likelihood value.

The second approach is based on the accuracy of the estimation of VaR. Value-at-risk at α level for a time series is defined as the corresponding empirical quantile at $\alpha\%$ [19]. For the long and short trading positions, the VaR values in time t are determined by

$$\text{Long Position: }\mathrm{VaR}^l_{t,\alpha}=\mu_t+q_\alpha\sigma_t,\tag{14}$$

$$\text{Short Position: }\mathrm{VaR}^s_{t,\alpha}=\mu_t+q_{1-\alpha}\sigma_t,\tag{15}$$

in which case, μ_t and σ_t denote the conditional mean and the conditional standard deviation at time t, respectively, while q_α and $q_{1-\alpha}$ are the corresponding critical values of α and $(1-\alpha)$ quantiles from the empirical distribution.

To evaluate the accuracy of the estimation of VaR, several VaR backtests are carried out including unconditional coverage Kupiec (UCK) test [23], conditional coverage Christoffersen (CCC) test [13], and dynamic quantile (DQ) test [17]. These VaR backtests are used to compare the forecasted losses from a fitted model with the actual calculated losses realised at the ends of a fixed time horizon. As indicated by [40], the backtests results provide information on whether the VaR is underestimated or the losses are greater than the original expected VaR value.

In order to select the best fitted GARCH-Type model based on the backtesting for VaR approach, we first calculate the number of actual exceedances from the VaR values, in which the number of actual exceedances for the long position is the total number of observed return at time t that is smaller than $\mathrm{VaR}^l_{t,\alpha}$ for $t=1,2,\ldots,n$. Analogously, the total number of actual exceedances for short position is equal to the number of observed return at time t that is greater than $\mathrm{VaR}^s_{t,\alpha}$ for $t=1,2,\ldots,n$. The best fitted model is then selected based on the minimum absolute difference between the actual and the expected exceedances. Note that, the expected exceedance is $n\alpha$ with n is the sample size. We would expect the poor performance of the model for both long and short trading positions on that returns if the model was inadequate in explaining the data in terms of its VaR measure.

4. **Simulation study.** Oftentimes, in application of real data set, it is impossible to realise the true underlying innovation distribution for the model, therefore the misspecification of innovation distribution is of highly concerned. In this study, although AIC measure is traditionally found to be instrumental in fitting the best model [21], the in-sample VaR backtests approach in selecting the model is considered as an alternative method.

To commence, we shall generate an innovation series $\{\varepsilon_t\}$ of length 2,500 under each of true innovation distributions. Setting initial value $\sigma_1 = 0.5$, the simulated returns can be calculated using $r_t = a_t = \sigma_t \varepsilon_t$, where σ_t is computed based on GARCH $(1, 1)$ model in (3) with the mean component $\mu_t = 0$ for simplicity. Note that the first 500 realisations are discarded from the simulated r_t in the process for the reason to avoid the dependency on initial values when simulating the data series. Leaving us with a series of length $n = 2,000$, the simulated r_t will be first fitted with GARCH $(1, 1)$ model under the six different types of distributions. The AIC values are then computed together with various VaR backtests performed on the series. For each of the GARCH-Type models, the simulation process is repeated for $N = 100$ runs with the six distributions as indicated in Section 2.

Tables 1 to 4 show the simulation results for the GARCH $(1, 1)$, GJRGARCH $(1, 1)$, TGARCH $(1, 1)$ and BLGARCH $(1, 1)$ models, respectively, with the average of AIC values, and the total number (out of 100) of non-rejection in null hypothesis of the model being 'correct' with $\alpha = 0.05$ using various VaR backtests.

From Table 1, we observe that when the data are generated based on GARCH $(1, 1)$ model with the true innovation distribution, the fitted GARCH model with its corresponding true innovation distribution always gives the smallest average AIC value. These findings are observed for other GARCH-Type models under studied as shown in Tables 2 to 4. It can be seen that the AIC approach tends to fit the model best for the true distribution in terms of its precision and consistency as anticipated by [21].

Also from Table 1, when the data are generated from the GARCH $(1, 1)$ model with true non-skewed innovation distributions, majority of in-sample VaR backtests results will comply with the non-rejection null hypothesis decision regardless of whether the fitted model is from skewed or non-skewed innovation distribution. In contrast, when the generated data come from the model with skewed distributions (SNORMD, SSTD and SGED), the VaR backtests results based on the fitted model with non-skewed distributions (NORM, STD GED) show about 40% to 100% (out of 100) of rejections of null hypothesis at $\alpha = 0.05$ level. However, the fitted model with skewed distributions (such as SSTD and SGED) apparently produce at least 81% of non-rejections of null hypothesis implying that the backtests are capable in distinguishing whether the data are stemming from non-skewed or skewed distribution. The similar patterns are observed for the GJRGARCH $(1, 1)$, TGARCH $(1, 1)$ and BLGARCH $(1, 1)$ models as shown in Tables 2 to 4.

5. Application.

5.1. Data description.
Data excerpted from the NASDAQ composite index for daily adjusted closing price of the index were used in this study dating back from

Table 1. Average values of AIC and the number of non-rejections in null hypothesis at 5% significance level for various backtests out of 100 runs for the fitted GARCH $(1, 1)$ model with parameters $\alpha_0 = 0.2, \alpha_1 = 0.1$ and $\beta_1 = 0.6$.

		Fitted Model with Related Innovation Distribution					
True Distribution	Measure	NORMD	STD ($v = 5$)	GED ($v = 1.5$)	SNORMD ($\xi = 2.5$)	SSTD ($\xi = 2.5$)	SGED ($v, \xi = 1.5, 2.5$)
NORMD	AIC	**4839.88**	4842.51	4841.12	4840.79	4843.42	4842.05
	UCK-long	95	95	95	97	97	98
	UCK-short	99	98	99	99	99	99
	CCC-long	96	96	96	99	98	98
	CCC-short	97	97	97	99	99	99
	DQ-long	95	94	95	93	93	94
	DQ-short	97	96	97	98	98	97
STD	AIC	4794.80	**4615.59**	4639.27	4794.10	4616.55	4639.77
	UCK-long	75	99	83	80	98	87
	UCK-short	70	94	83	74	98	83
	CCC-long	86	96	89	86	97	89
	CCC-short	78	95	85	84	97	89
	DQ-long	97	100	98	99	100	98
	DQ-short	88	88	89	90	91	90
GED	AIC	4826.53	4795.85	**4790.65**	4827.24	4796.84	4791.49
	UCK-long	96	92	97	100	95	100
	UCK-short	99	94	98	98	96	100
	CCC-long	93	89	97	96	95	97
	CCC-short	98	96	98	99	99	98
	DQ-long	93	89	93	94	91	96
	DQ-short	97	95	97	98	96	97
SNORMD	AIC	4820.81	4809.73	4820.76	**4397.94**	4400.95	4398.99
	UCK-long	0	0	0	99	99	100
	UCK-short	1	0	1	93	91	95
	CCC-long	0	0	0	98	98	99
	CCC-short	2	1	2	93	92	95
	DQ-long	0	0	0	99	99	99
	DQ-short	5	4	6	97	96	96
SSTD	AIC	4784.12	4446.89	4559.11	4071.87	**3865.32**	3889.15
	UCK-long	0	0	0	4	100	100
	UCK-short	26	0	8	47	99	81
	CCC-long	0	0	0	11	99	98
	CCC-short	38	0	17	55	96	87
	DQ-long	0	0	0	45	96	97
	DQ-short	60	3	35	72	98	97
SGED	AIC	4804.63	4727.06	4770.63	4224.61	4189.49	**4183.04**
	UCK-long	0	0	0	70	100	100
	UCK-short	0	0	0	44	86	95
	CCC-long	0	0	0	84	99	99
	CCC-short	1	0	1	52	84	96
	DQ-long	0	0	0	97	97	96
	DQ-short	3	0	3	59	92	97

January 1, 1984 to June 30, 2006. A total of 5677 composite index observations were retrieved from Yahoo Finance website (Source: https://finance.yahoo.com/) with the returns of the composite index is evaluated via $r_t = 100 \times \log\left(\frac{P_t}{P_{t-1}}\right)$.

Figure 1 presents the time series plots for the closing prices and the returns of NASDAQ index. It is found that the NASDAQ index shows an overall upward trend from 1984 to 2000, followed by the decreasing in trend for about two years after year 2000, and tends to rise again after the overall declining of another two years. The volatility of the prices can also be observed from the fluctuation of the returns. From Figure 1(b), the volatility of returns for NASDAQ index are changing over the time indicating that the GARCH-Type models might be useful in explaining the data.

Table 5 shows the summary statistics of returns for NASDAQ index. The minimum values represent the maximum loss (gain) that the investors will bear (obtain)

Table 2. Average values of AIC and the number of non-rejections in null hypothesis at 5% significance level for various backtests out of 100 runs for the fitted GJRGARCH (1, 1) model with parameters $\alpha_0 = 0.2, \alpha_1 = 0.1, \gamma_1 = 0.2$ and $\beta_1 = 0.6$.

		Fitted Model with Related Innovation Distribution					
True Distribution	Measure	NORMD	STD ($v = 5$)	GED ($v = 1.5$)	SNORMD ($\xi = 2.5$)	SSTD ($\xi = 2.5$)	SGED ($v, \xi = 1.5, 2.5$)
NORMD	AIC	**5480.34**	5482.90	5481.25	5481.46	5484.05	5482.36
	UCK-long	99	99	99	100	100	100
	UCK-short	99	99	100	99	99	99
	CCC-long	99	99	99	99	99	99
	CCC-short	99	99	99	99	99	99
	DQ-long	95	96	96	95	96	95
	DQ-short	99	99	99	97	97	98
STD	AIC	5387.99	**5201.69**	5226.05	5386.65	5202.52	5226.47
	UCK-long	73	99	77	76	98	79
	UCK-short	72	97	79	76	98	82
	CCC-long	79	98	83	80	98	85
	CCC-short	78	98	87	81	97	92
	DQ-long	94	95	95	94	95	97
	DQ-short	96	98	98	96	99	98
GED	AIC	5123.43	5093.04	**5087.91**	5124.13	5094.03	5088.75
	UCK-long	96	91	97	100	95	100
	UCK-short	99	95	99	99	98	100
	CCC-long	95	92	94	96	94	98
	CCC-short	97	97	99	99	98	99
	DQ-long	94	90	93	94	92	97
	DQ-short	98	97	100	98	99	97
SNORMD	AIC	5352.98	5341.66	5352.87	**4928.49**	4931.46	4929.62
	UCK-long	0	0	0	100	100	100
	UCK-short	0	0	0	97	96	99
	CCC-long	0	0	0	99	97	97
	CCC-short	0	0	0	95	96	98
	DQ-long	0	0	0	98	96	97
	DQ-short	5	3	5	98	98	97
SSTD	AIC	5159.29	4834.74	4945.27	4466.38	**4261.63**	4285.05
	UCK-long	0	0	0	5	100	99
	UCK-short	18	0	7	50	98	86
	CCC-long	0	0	0	9	96	95
	CCC-short	31	0	14	63	95	88
	DQ-long	0	0	0	52	99	97
	DQ-short	47	1	32	74	93	95
SGED	AIC	5270.80	5194.20	5237.44	4691.87	4656.93	**4650.46**
	UCK-long	0	0	0	75	100	100
	UCK-short	0	0	0	44	86	96
	CCC-long	0	0	0	82	99	100
	CCC-short	1	0	1	54	83	97
	DQ-long	0	0	0	97	98	99
	DQ-short	2	0	2	64	91	96

if they hold for the long (short) position on the commodities for the specified period of time. Similarly, the maximum values indicate the maximum gain (loss) on the commodities if they hold for the long (short) position. From the perspective of long position, the maximum loss of NASDAQ index is -12.04%, while the maximum gain is 13.25%. The mean of the returns is 0.036% indicating that the NASDAQ index has a slightly positive return on the whole, with the standard deviation of the returns of 1.38%. It also noted that the NASDAQ index returns are negatively skewed with skewness of -0.234. The excess kurtosis for the NASDAQ index return is 8.771, which may mean that the returns might have a thicker tail on both positive and negative sides in comparing to normal distribution. Jarque-Bera statistic appears to reject null hypothesis of normality assumption at 1% significance level, suggesting that the returns might follow fat-tail distribution. Furthermore, Ljung Box statistic on the returns has the p-value lesser than 0.05 that might support the

Table 3. Average values of AIC and the number of non-rejections in null hypothesis at 5% significance level for various backtests out of 100 runs for the fitted TGARCH $(1,1)$ model with parameters $\alpha_0 = 0.2, \alpha_1^+ = 0.1, \alpha_1^- = 0.3$ and $\beta_1 = 0.6$.

		Fitted Model with Related Innovation Distribution					
True Distribution	Measure	NORMD	STD ($v = 5$)	GED ($v = 1.5$)	SNORMD ($\xi = 2.5$)	SSTD ($\xi = 2.5$)	SGED ($v, \xi = 1.5, 2.5$)
NORMD	AIC	**4848.75**	4851.33	4849.66	4849.86	4852.48	4850.76
	UCK-long	99	99	99	100	100	100
	UCK-short	99	98	100	98	98	98
	CCC-long	99	99	99	99	99	99
	CCC-short	99	99	99	99	99	99
	DQ-long	95	95	95	94	94	94
	DQ-short	97	98	96	96	96	95
STD	AIC	4661.30	**4475.00**	4499.38	4660.01	4475.84	4499.81
	UCK-long	73	98	78	75	98	81
	UCK-short	74	97	81	72	98	82
	CCC-long	75	97	87	77	98	87
	CCC-short	77	98	85	80	99	89
	DQ-long	94	93	94	93	94	96
	DQ-short	94	97	96	93	98	97
GED	AIC	4743.45	4712.98	**4707.84**	4744.16	4713.97	4780.68
	UCK-long	98	91	98	99	95	100
	UCK-short	100	97	100	99	97	100
	CCC-long	97	89	97	97	94	98
	CCC-short	97	96	98	99	97	99
	DQ-long	93	90	93	94	91	95
	DQ-short	97	95	97	97	97	97
SNORMD	AIC	4308.67	4297.26	4308.55	**3884.16**	3887.12	3885.27
	UCK-long	0	0	0	100	100	100
	UCK-short	0	0	0	97	98	100
	CCC-long	0	0	0	100	98	98
	CCC-short	0	0	0	95	95	98
	DQ-long	0	0	0	97	98	97
	DQ-short	4	4	6	95	98	96
SSTD	AIC	4650.27	4326.84	4437.44	3958.36	**3753.78**	3777.25
	UCK-long	0	0	0	1	100	100
	UCK-short	21	0	8	49	98	87
	CCC-long	0	0	0	8	97	96
	CCC-short	33	0	14	61	95	88
	DQ-long	0	0	0	49	98	99
	DQ-short	47	1	28	70	93	93
SGED	AIC	4812.02	4735.54	4778.74	4233.28	4198.40	**4191.91**
	UCK-long	0	0	0	73	100	100
	UCK-short	0	0	0	45	85	96
	CCC-long	0	0	0	82	98	98
	CCC-short	0	0	0	52	94	94
	DQ-long	0	0	0	98	98	98
	DQ-short	2	0	2	63	89	97

presence of serial correlation in the returns. Meanwhile, the result of Lagrange Multiplier (LM) test reveals the existence of ARCH effect in the returns data leading us to propose that the GARCH-Type model might be useful in explaining the data. For graphical displays, the histogram and QQ-plot of the returns for NASDAQ index are depicted in Figure 2.

5.2. **Model fitting and model selection.** This section intends to determine the most appropriate model for the returns data from NASDAQ index on the basis of its modelling performance from the corresponding models. Initially, the sample period is divided into two sub-periods, with that the first portion of sample period of length 5551 is used for modelling while the second portion of length 125 is retained for validating purpose among the models.

To proceed, the returns of NASDAQ index will first be fitted to the four GARCH-Type models [GARCH $(1,1)$, GJRGARCH $(1,1)$, TGARCH $(1,1)$ and BLGARCH

TABLE 4. Average values of AIC and the number of non-rejections in null hypothesis at 5% significance level for various backtests out of 100 runs for the fitted BLGARCH $(1,1)$ model with parameters $\alpha_0 = 0.2, \alpha_1 = 0.1$, $\beta_1 = 0.6$ and $\gamma_1 = 0.2$.

		Fitted Model with Related Innovation Distribution					
True Distribution	Measure	NORMD	STD ($v = 5$)	GED ($v = 1.5$)	SNORMD ($\xi = 2.5$)	SSTD ($\xi = 2.5$)	SGED ($v, \xi = 1.5, 2.5$)
NORMD	AIC	**4775.04**	4777.6	4775.96	4776.16	4778.76	4777.07
	UCK-long	99	99	99	99	99	99
	UCK-short	100	100	100	99	99	99
	CCC-long	99	99	99	99	100	99
	CCC-short	99	99	99	99	99	99
	DQ-long	97	98	97	98	97	97
	DQ-short	96	97	96	97	97	97
STD	AIC	4773.34	**4587.04**	4611.49	4772.06	4587.89	4611.93
	UCK-long	75	98	77	75	100	78
	UCK-short	72	97	78	74	97	77
	CCC-long	81	99	86	83	98	89
	CCC-short	78	96	86	83	98	85
	DQ-long	97	97	98	94	96	96
	DQ-short	93	97	95	93	97	94
GED	AIC	4765.93	4735.55	**4730.36**	4766.63	4736.53	4731.20
	UCK-long	97	92	97	100	94	100
	UCK-short	99	95	97	99	95	100
	CCC-long	95	92	96	97	96	98
	CCC-short	97	97	99	98	98	100
	DQ-long	93	89	93	91	91	93
	DQ-short	98	98	96	100	98	98
SNORMD	AIC	4718.43	4707.04	4718.33	**4293.51**	4296.50	4294.63
	UCK-long	0	0	0	100	100	100
	UCK-short	0	0	0	97	94	100
	CCC-long	0	0	0	99	99	99
	CCC-short	0	0	0	96	94	98
	DQ-long	0	0	0	99	99	99
	DQ-short	4	3	5	97	98	98
SSTD	AIC	4663.37	4336.06	4446.73	3968.73	**3762.91**	3786.37
	UCK-long	0	0	0	4	100	100
	UCK-short	19	0	8	51	96	85
	CCC-long	0	0	0	9	97	96
	CCC-short	28	0	15	63	95	88
	DQ-long	0	0	0	43	97	97
	DQ-short	43	1	29	69	92	95
SGED	AIC	4692.89	4616.31	4659.56	4113.75	4078.82	**4072.33**
	UCK-long	0	0	0	72	100	100
	UCK-short	0	0	0	45	83	93
	CCC-long	0	0	0	83	100	100
	CCC-short	0	0	0	53	84	94
	DQ-long	0	0	0	96	98	98
	DQ-short	2	0	2	63	93	97

$(1,1)]$ under the six different types of distributions. Hence, there will be 24 models and the parameter estimates are computed by employing the ML method. Consequently, the best fitted models are determined based on the smallest in-sample AIC and VaR backtests values.

Table 6 gives the AIC values for various GARCH-Type models for NASDAQ index returns. It can be observed that the best fitted model for NASDAQ index return is TGARCH $(1,1)$ under the skewed student-t distribution. Contrasting the same fitted model, the GARCH model has the overall largest AIC value as compared to other models which might indicate that the asymmetric component is essential in modelling the returns of NASDAQ index.

Turning to Table 7 that shows the p-values for various VaR backtests results for the NASDAQ index returns. With different α level under various models, it can be seen that most of the tests have the p-values smaller than 0.05 when the

Table 5. Summary statistics of returns for NASDAQ index.

Minimum	1st Quartile	Median	Mean	3rd Quartile	Maximum
-12.04323	-0.50381	0.10549	0.03624	0.65116	13.25464
Standard deviation	Skewness	Excess Kurtosis	Jarque-Bera	$Q(10)$	ARCH(10)
1.376815	-0.2339677	8.771298	18264^{***} $(< 2.2e-16)$	34.191^{***} (0.0001714)	1137.7^{***} $(< 2.2e-16)$

Notes: $Q(10)$ is the Ljung and Box statistics of order 10 on the returns. ARCH(10) is the Lagrange Multiplier (LM) test of orders 10 (Engle, 1982). P-values of the statistics are reported in the parentheses.
*Denote rejection of the null hypothesis at the 10% significance level.
**Denote rejection of the null hypothesis at the 5% significance level.
***Denote rejection of the null hypothesis at the 1% significance level.

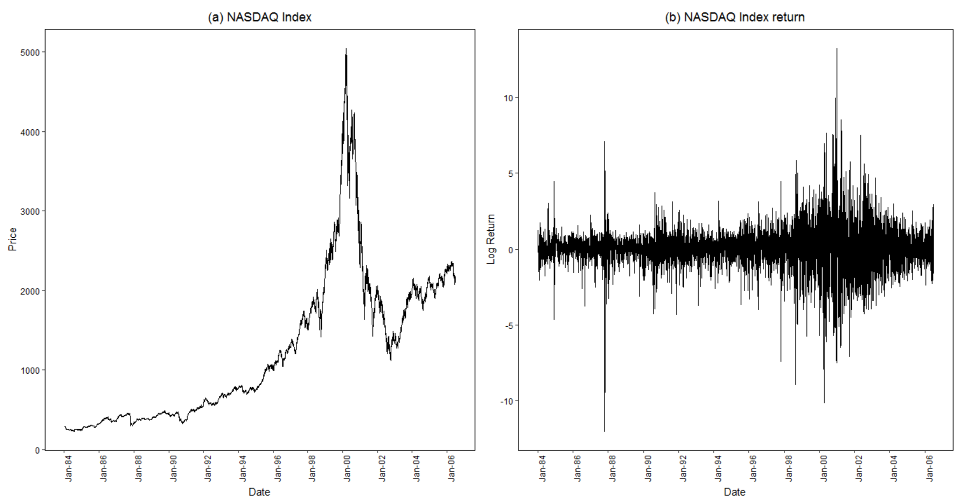

Figure 1. Time series plots for the prices of NASDAQ index and returns of NASDAQ index.

data are fitted under the non-skewed distributions. This might further suggest that the returns data appear to come from the skewed distribution. Out of these models, we observe that the selected models based on backtests results that are nonsignificant (boldface) are TGARCH $(1, 1)$ model with SNORM distribution, and BLGARCH $(1, 1)$ model with SNORM, SSTD and SGED distributions. To select the best fitted model, based on the difference between the expected and the actual exceedances with $\alpha = 0.05$, the VaR measures (number of exceedances) tabulated in Table 8 reveal that the models from the non-skewed distributions are incapable of capturing the large positive and negative returns. This might cause the models to overestimate or underestimate the variance of the returns. In light of the VaR backtests, the BLGARCH $(1, 1)$ model with the skewed student-t distribution is found to be the best fitted model.

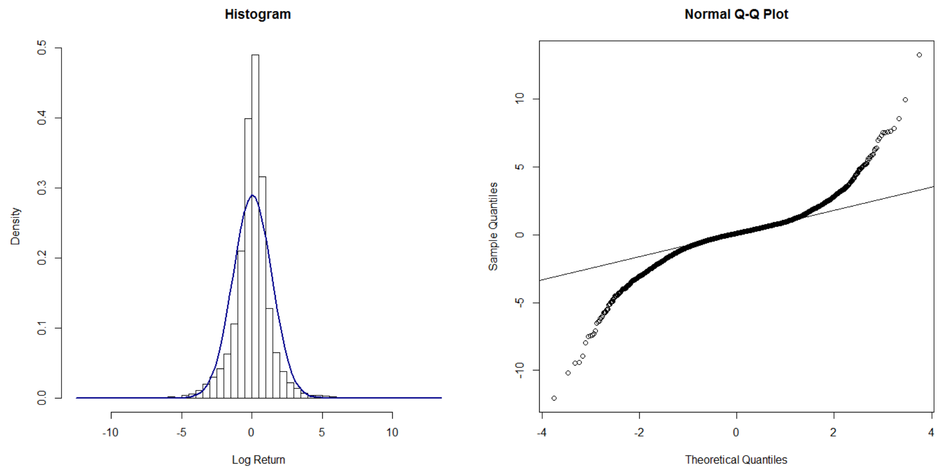

FIGURE 2. Histogram and QQ-plot for the returns of NASDAQ index (Solid curve is the normal distribution curve with corresponding mean and standard deviation).

TABLE 6. AIC values of various GARCH-Type models for the returns of NASDAQ index.

	NORMD	STD	GED	SNORMD	SSTD	SGED
GARCH $(1,1)$	15814.45	15559.17	15602.50	15720.81	15509.92	15545.09
GJRGARCH $(1,1)$	15762.01	15534.35	15574.46	15671.88	15482.61	15513.74
TGARCH $(1,1)$	15737.59	15506.22	15552.22	15641.02	**15452.94**	15487.44
BLGARCH $(1,1)$	15734.02	15512.67	15554.82	15641.02	15461.47	15492.78

The estimated parameters given in Table 9 for the two selected models show that all the parameter estimates are significant at 1% significance level. Overall, the values of α_1^+ are lower than α_1^- for the fitted TGARCH $(1,1)$ model indicating that the future prices are prone to be having negative shocks rather than the positive shocks. Moreover, the negative value of ζ_1 for the BLGARCH $(1,1)$ model would also suggest that the volatility tends to increase when there are in negative shocks. Accordingly, we might come to a conclusion that NASDAQ index has a leverage effect of negative shocks.

5.3. **Out-of-sample forecast.** In time series analysis, the out-of-sample performance of the model might be crucial to academicians and practitioners. This is due to that the market players are more concerned about how well the models are capable in capturing the future trend of market price, rather than the "information" about the historical price and their performance as time goes on. A number of studies have investigated this question, for instance, [36] stated that there might be several plausible models in the forecasting process. While [21] argued that the best fitted model (based on AIC), which is not necessary statistically different from the best forecasted model, might be employed in volatility forecasting. To warrant a better model, we would contradistinguish across different types of GARCH models on the basis of AIC and in-sample VaR model selection criterion integrating the

Table 7. P-values for different backtests for the returns of NASDAQ index with varying α levels under various models.

DIST	Model	GARCH (1, 1)			GJRGARCH (1, 1)			TGARCH (1, 1)			BLGARCH (1, 1)		
	Measure	5%	2.5%	1%	5%	2.5%	1%	5%	2.5%	1%	5%	2.5%	1%
NORM	UCK-long	0.008	0.000	0.000	0.065	0.001	0.000	0.074	0.000	0.000	0.172	0.004	0.001
	UCK-short	0.000	0.003	0.028	0.000	0.004	0.028	0.000	0.004	0.079	0.000	0.006	0.057
	CCC-long	0.027	0.000	0.000	0.163	0.004	0.000	0.197	0.000	0.000	0.386	0.008	0.004
	CCC-short	0.000	0.006	0.051	0.001	0.014	0.051	0.000	0.008	0.137	0.001	0.011	0.101
	DQ-long	0.018	0.000	0.000	0.078	0.002	0.000	0.130	0.000	0.000	0.129	0.008	0.000
	DQ-short	0.000	0.121	0.360	0.019	0.175	0.056	0.010	0.062	0.495	0.049	0.071	0.612
STD	UCK-long	0.000	0.001	0.045	0.001	0.001	0.264	0.001	0.001	0.133	0.003	0.002	0.133
	UCK-short	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
	CCC-long	0.000	0.001	0.081	0.003	0.002	0.513	0.003	0.001	0.011	0.012	0.004	0.317
	CCC-short	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.002	0.000	0.000
	DQ-long	0.000	0.000	0.000	0.004	0.000	0.157	0.000	0.000	0.000	0.016	0.001	0.084
	DQ-short	0.000	0.001	0.000	0.027	0.002	0.000	0.006	0.000	0.001	0.035	0.005	0.002
GED	UCK-long	0.001	0.023	0.264	0.032	0.019	0.390	0.032	0.036	0.390	0.043	0.015	0.323
	UCK-short	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	CCC-long	0.005	0.033	0.251	0.085	0.057	0.650	0.095	0.044	0.298	0.105	0.024	0.582
	CCC-short	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	DQ-long	0.003	0.000	0.003	0.076	0.019	0.240	0.016	0.009	0.053	0.085	0.017	0.186
	DQ-short	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002	0.000	0.000
SNORM	UCK-long	0.563	0.125	0.045	0.641	0.263	0.390	**0.597**	**0.595**	**0.550**	**0.334**	**0.341**	**0.550**
	UCK-short	0.159	0.782	0.080	0.474	0.341	0.013	**0.554**	**0.263**	**0.080**	**0.779**	**0.229**	**0.104**
	CCC-long	0.715	0.237	0.081	0.779	0.532	0.650	**0.613**	**0.272**	**0.329**	**0.559**	**0.570**	**0.769**
	CCC-short	0.363	0.486	0.040	0.772	0.249	0.030	**0.839**	**0.227**	**0.213**	**0.881**	**0.214**	**0.145**
	DQ-long	0.150	0.017	0.001	0.431	0.162	0.247	**0.085**	**0.141**	**0.060**	**0.397**	**0.102**	**0.260**
	DQ-short	0.539	0.870	0.044	0.930	0.310	0.045	**0.629**	**0.248**	**0.297**	**0.962**	**0.311**	**0.289**
SSTD	UCK-long	0.213	0.744	0.300	0.832	0.849	0.079	0.605	0.811	0.300	**0.832**	**0.878**	**0.079**
	UCK-short	0.554	0.348	0.057	0.738	0.744	0.143	0.785	0.617	0.057	**0.785**	**0.849**	**0.143**
	CCC-long	0.403	0.364	0.116	0.736	0.958	0.137	0.825	0.389	0.116	**0.584**	**0.412**	**0.137**
	CCC-short	0.839	0.540	0.021	0.934	0.630	0.055	0.741	0.555	0.101	**0.947**	**0.754**	**0.233**
	DQ-long	0.003	0.007	0.022	0.233	0.050	0.345	0.089	0.014	0.168	**0.307**	**0.051**	**0.336**
	DQ-short	0.227	0.897	0.082	0.715	0.765	0.030	0.267	0.241	0.140	**0.807**	**0.865**	**0.692**
SGED	UCK-long	0.832	0.229	0.537	0.436	0.348	0.040	0.436	0.196	0.186	**0.334**	**0.141**	**0.057**
	UCK-short	0.097	0.082	0.028	0.474	0.265	0.107	0.400	0.196	0.186	**0.597**	**0.500**	**0.143**
	CCC-long	0.736	0.234	0.200	0.588	0.540	0.073	0.588	0.097	0.072	**0.616**	**0.332**	**0.101**
	CCC-short	0.247	0.152	0.009	0.725	0.269	0.041	0.643	0.337	0.293	**0.830**	**0.472**	**0.233**
	DQ-long	0.017	0.056	0.037	0.311	0.175	0.293	0.043	0.009	0.127	**0.408**	**0.342**	**0.315**
	DQ-short	0.436	0.723	0.042	0.923	0.538	0.021	0.617	0.518	0.351	**0.868**	**0.536**	**0.074**

rolling window technique with window size $(M-H)$ in the forecasting procedure, where M represents the total number of observations. This technique will reserve $H = 125$ observation points for forecasting evaluation purpose at the end of the process. It is started from the first $(M-H)$ observations which denote the in-sample period, and model will be fitted based on these $(M-H)$ observations along with the one-step ahead forecasting value evaluated. The in-sample period is then rolled by adding the next observation and dropping the oldest observation, and continuing for H times until each one-step forecasting is computed. In this case, in-sample period size is fixed so that the estimation of the forecasted values will not be overlapping.

To quantify the forecasting performance of the respective GARCH-Type models, the following six loss functions are considered:

1. Mean absolute error (MAE)

$$\mathrm{MAE} = \frac{1}{H}\sum_{t=1}^{H}\left|\sigma_t^2 - \hat{\sigma}_t^2\right|, \tag{16}$$

2. Mean square error (MSE)

$$\mathrm{MSE} = \frac{1}{H}\sum_{t=1}^{H}\left(\sigma_t^2 - \hat{\sigma}_t^2\right)^2, \tag{17}$$

TABLE 8. VaR measures for the returns of NASDAQ index under various models (in-sample).

Model	GARCH $(1,1)$		GJRGARCH $(1,1)$		TGARCH $(1,1)$		BLGARCH $(1,1)$	
$\alpha\%$	5% Long	5% Short	5% Long	5% Short	5% Long	5% Short	5% Long	5% Short
NORM	322	204	308	218	307	217	300	220
Diff	44	74	30	60	29	61	22	58
STD	346	207	334	220	335	214	327	223
Diff	68	71	56	58	57	64	49	55
GED	332	193	313	197	313	193	311	203
Diff	54	85	35	81	35	85	33	75
SNORM	287	255	270	266	269	268	262	273
Diff	9	23	8	12	9	10	16	5
SSTD	298	268	281	283	286	282	281	282
Diff	20	10	3	5	8	4	3	4
SGED	281	251	265	266	265	264	262	269
Diff	3	27	13	12	13	14	16	9

Notes: Expected exceed $= 5551 \times 0.05 \approx 278$ and Diff $= |\text{actual exceed} - \text{expected exceed}|$.

TABLE 9. Parameter estimates based on AIC and VaR backtests criteria for the selected GARCH-Type models using the returns of NASDAQ index.

	AIC	VaR
Model	TGARCH-SSTD	BLGARCH-SSTD
μ	0.043^{***} [0.008]	0.041^{***} [0.011]
ϕ	0.147^{***} [0.014]	0.153^{***} [0.014]
α_0	0.011^{***} [0.002]	0.012^{***} [0.003]
α_1	-	0.111^{***} [0.012]
α_1^+	0.067^{***} [0.009]	-
α_1^-	0.134^{***} [0.013]	-
β_1	0.913^{***} [0.009]	0.887^{***} [0.011]
ζ_1	-	-0.079^{***} [0.013]
v	8.043^{***} [0.787]	8.175^{***} [0.813]
ξ	0.866^{***} [0.017]	0.869^{***} [0.017]

Notes: the numbers in square brackets are the standard error of the estimates.
*Denote rejection of the null hypothesis at the 10% significance level.
**Denote rejection of the null hypothesis at the 5% significance level.
***Denote rejection of the null hypothesis at the 1% significance level.

3. Mean absolute percentage error (MAPE)

$$\text{MAE} = \frac{1}{H}\sum_{t=1}^{H}\left|\frac{\sigma_t^2 - \hat{\sigma}_t^2}{\sigma_t^2} \times 100\%\right|, \tag{18}$$

4. Heteroskedasticity-adjusted MAE (HMAE)

$$\text{HMAE} = \frac{1}{H}\sum_{t=1}^{H}\left|1 - \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right|, \tag{19}$$

5. Heteroskedasticity-adjusted MSE (HMSE), [9]

$$\text{HMSE} = \frac{1}{H}\sum_{t=1}^{H}\left(1 - \frac{\sigma_t^2}{\hat{\sigma}_t^2}\right)^2, \tag{20}$$

6. Logarithmic loss (LL) function, [32]

$$\text{LL} = \frac{1}{H}\sum_{t=1}^{H}\left[\ln\left(\frac{\sigma_t^2}{\hat{\sigma}_t^2}\right)\right]^2, \tag{21}$$

where H is the number of forecasted data points, σ_t^2 and $\hat{\sigma}_t^2$ are the 'actual' volatility and the forecasted variance at time t, respectively. We adopt the approach discussed in [26] that uses the squared error term as a proxy for the 'actual' variance.

Note that the MAE and MSE will penalise the errors of opposite signs symmetrically while the remaining loss functions will penalise the errors in forecasting volatility asymmetrically. MAPE will penalise inaccurate forecast more heavily when σ_t^2 is low. Also, the LL will penalise the under-prediction volatility more severely, and more weight is given to the over-prediction volatility when using the HMSE and HMAE. Concurrently, the out-of-sample one-step-ahead VaR prediction values are computed in order to perform the UCK, CCC and DQ tests for out-of-sample VaR prediction.

Another measure, which is called the expected shortfall (ES), is defined as the expected loss conditioned on the loss being greater than VaR [35]. The expected shortfall at α level for the long and short positions are calculated via

$$\text{ES}_{\text{long}} = \mathbb{E}\left[r_t | r_t < \text{VaR}_{t,\alpha}^{l}\right], \tag{22}$$

$$\text{ES}_{\text{short}} = \mathbb{E}\left[r_t | r_t > \text{VaR}_{t,\alpha}^{s}\right]. \tag{23}$$

Table 10 displays the out-of-sample volatility forecasting evaluated under the six criteria of loss function, the model based on AIC tends to have better accuracy using MAE, MSE, HMAE and HMSE. Meanwhile, the BLGARCH model under the selection of in-sample VaR is superior to TGARCH model for its MAPE and LL implying that there is no single best fitted model that can outperform other models associating with all the loss functions utilised. We are in the view that the two approaches considered in this study might have their own strengths in forecasting of the returns. Moreover, Table 11 shows the various tests on the out-of-sample VaR, and both models seem to not to reject the null hypothesis for all tests in terms of their long and short position which might suggest that both models are adequate in explaining the returns. For the purpose of illustration, Figures 3 and 4 present the VaR and ES plots for NASDAQ index returns for the models selected on the basis of AIC and VaR measures.

Table 10. Out-of-sample volatility forecasting evaluated under the six criteria of loss functions for returns of NASDAQ index.

	MAE	MSE	MAPE	HMAE	HMSE	LL
TGARCH $(1,1)$-SSTD	**0.9109**	**1.9433**	874.463	**1.0313**	**2.5931**	6.9049
BLGARCH $(1,1)$-SSTD	0.9151	1.9831	**513.787**	1.0589	2.8460	**6.4131**

Table 11. Out-of-sample VaR for various tests of the returns of NASDAQ index.

$\alpha\%$	5%		2.5%		1%	
Position	Long	Short	Long	Short	Long	Short
TGARCH-SSTD						
UCK-Test	0.9178	0.7625	0.9425	0.3224	0.8159	0.5351
CCC-Test	0.7330	0.6881	0.9259	0.5184	0.9653	0.8117
DQ-Test	0.8268	0.9681	0.9968	0.8134	0.9960	0.9913
BLGARCH -SSTD						
UCK-Test	0.9176	0.9176	0.9425	0.3224	0.5352	0.5352
CCC-Test	0.7330	0.7721	0.9259	0.5184	0.7985	0.8117
DQ-Test	0.8843	0.9857	0.9935	0.8551	0.8960	0.9918

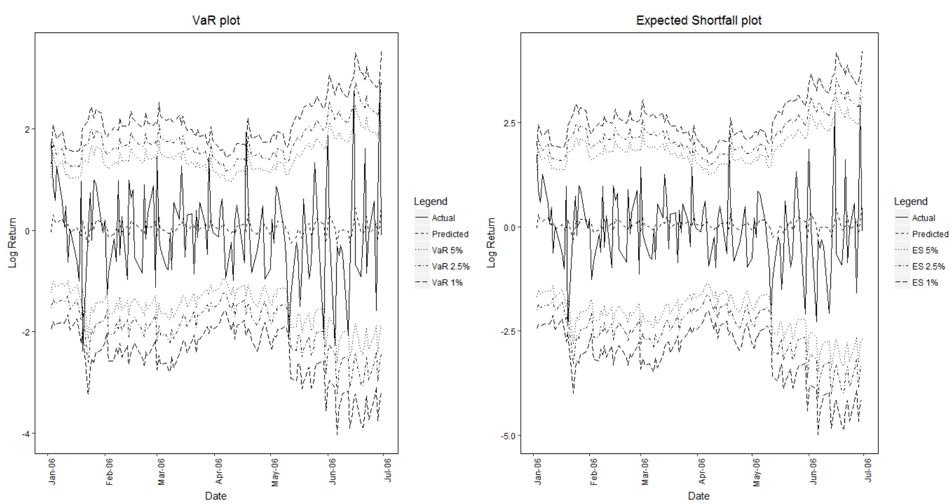

Figure 3. Out-of-sample VaR and the expected shortfall plots for the returns of NASDAQ index under TGARCH $(1,1)$-SSTD (AIC best model).

6. **Conclusion.** To sum up, in this research, we observe that the GARCH-Type models with the existence of innovation misspecification will greatly affect the model selection process. When the innovation of returns follows skewed distribution, inappropriate use of likelihood function of non-skewed distribution will lead to the

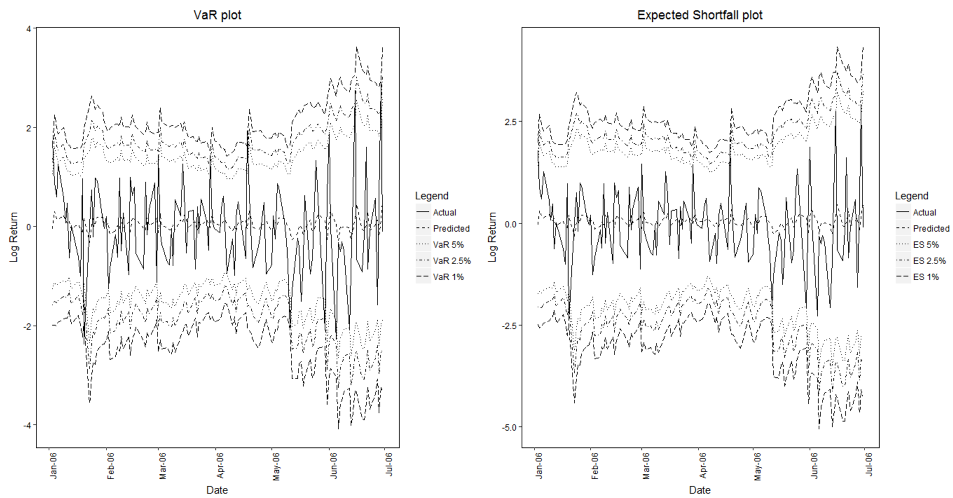

FIGURE 4. Out-of-sample VaR and the expected shortfall plots for the returns of NASDAQ index under BLGARCH $(1,1)$-SSTD (VaR best model).

significant indication of all VaR backtests that will eventually impacting the accuracy of VaR measurement. We have modelled the NASDAQ index returns through various GARCH-Type models under the six different potential distributions. There are two benchmarks utilised to determine the best fitted models for both the AIC and in-sample VaR backtests approaches, in which AIC measure is traditionally deemed to be more appropriate in fitting the model. The results appear to indicate that the TGARCH $(1,1)$ model with skewed student-t distribution is more appropriate based on AIC, while the BLGARCH $(1,1)$ model with skewed student-t distribution is the best fitted model when using in-sample VaR backtests.

In addition, the forecasting performance of both selected models obtained via AIC and VaR backtests were also contrasted. Various measures based on different loss function are evaluated. Results tend to suggest that the model selected using VaR backtests consistently offer smaller MAPE and LL as compared to the model based on AIC. Various tests including Kupiec, Christoffersen and DQ tests were also applied in order to investigate their respective out-of-sample VaR forecasting performance. It turn out that both models selected on the basis of the two approaches perform emulously and might be a suggestive of their usefulness with their own strengths in forecasting the returns.

Acknowledgments. The authors thank the referees and editor for their useful comments and invaluable suggestions which have improved the quality of this version of the paper. This work is partially supported by the Special Research Fund Assistance with grant no: BKS076-2017 of the University of Malaya.

REFERENCES

[1] D. Alberg, H. Shalit and R. Yosef, Estimating stock market volatility using asymmetric GARCH models, *Applied Financial Economics*, **18** (2008), 1201–1208.

[2] T. Angelidis, A. Benos and S. Degiannakis, The use of GARCH models in VaR estimation, *Statistical Methodology*, **1** (2004), 105–128.

[3] A. Azzalini, A class of distributions which includes the normal ones, *Scandinavian Journal of Statistics*, **12** (1985), 171–178.

[4] T. G. Bali, Modeling the dynamics of interest rate volatility with skew fat-tailed distributions, *Annals of Operations Research*, **151** (2007), 151–178.

[5] R. Ballie, T. Bollerslev and H. Mikkelsen, Fractionally integrated generalized autoregressive conditional heteroskedasticity, *Jouranl of Econometrics*, **74** (1996), 3–30.

[6] F. Black, Studies of stock price volatility changes, *In: Proceedings of the 1976 Meeting of the Business and Economic Statistics Section, American Statistical Association, Washington DC*, (1976), 177–181.

[7] T. Bollerslev, Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics*, **31** (1986), 307–327.

[8] T. Bollerslev, A conditionally heteroskedastic time series model for speculative prices and rates of return, *Review of Economics and Statistics*, **69** (1987), 542–547.

[9] T. Bollerslev and E. Ghysels, Periodic autoregressive conditional heteroscedasticity, *Journal of Business and Economic Statisticss*, **14** (1996), 139–151.

[10] M. Braione and N. K. Scholtes, Forecasting value-at-risk under different distributional assumptions, *Econometrics*, **4** (2016), 1–27.

[11] J. Y. Campbell and L. Hentschel, No news is good news: An asymmetric model of changing volatility in stock returns, *Journal of Financial Economics*, **31** (1992), 281–318.

[12] M. S. Choi, J. A. Park and S. J. Hwang, Asymmetric GARCH processes featuring both threshold effect and bilinear structure, *Statistics and Probability Letters*, **82** (2012), 419–426.

[13] P. F. Christoffersen, Evaluating interval forecasts, *International Economic Review*, **39** (1998), 841–862.

[14] Z. Ding, C. W. Granger and R. F. Engle, A long memory property of stock market returns and a new model, *Journal of Empirical Finance*, **1** (1993), 83–106.

[15] R. F. Engle, Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation, *Econometrica*, **50** (1982), 987–1007.

[16] R. F. Engle and V. K. Ng, Measuring and testing the impact of new on volatility, *Journal of Finance*, **48** (1993), 1749–1778.

[17] R. F. Engle and S. Manganelli, CAViaR: Conditional Autoregressive value at risk by regression quantiles, *Journal of Business and Economic Statistics*, **22** (2004), 367–381.

[18] C. Fernández and M. F. Steel, On Bayesian modelling of fat tails and skewness, *Journal of the American Statistical Association*, **93** (1998), 359–371.

[19] P. Giot and S. Laurent, Value-at-risk for long and short trading positions, *Journal of Applied Econometrics*, **18** (2003), 641–663.

[20] L. R. Glosten, R. Jagannathan and D. E. Runkle, On the relation between the expected value and the volatility of the nominal excess return on stocks, *Journal of Finance*, **12** (1993), 1779–1801.

[21] C. Kosapattarapim, Y. X. Lin and M. McCrae, Evaluating the volatility forecasting performance of best fitting GARCH models in emerging Asian stock markets, *International Journal of Mathematics and Statistics*, **12** (2012), 1–15.

[22] K. Kuester, S. Mittnik and M. S. Paolella, Value-at-risk prediction: A comparison of alternative strategies, *Journal of Financial Econometrics*, **4** (2006), 53–89.

[23] P. H. Kupiec, Techniques for verifying the accuracy of risk measurement models, *Journal of Derivatives*, **3** (1995), 73–84.

[24] P. Lambert and S. Laurent, Modelling financial time series using GARCH-Type models and a skew Student distribution for the innovations, Discussion Paper No.0125. Institute de Statisque, Universite Catholique de Louvain, Louvain-la-Neuve, Belgium.

[25] H. C. Liu and J. C. Hung, Forecasting S&P-100 stock index volatility: The role of volatility asymmetry and distributional assumption in GARCH models, *Expert Systems with Applications*, **37** (2010), 4928–4934.

[26] J. A. Lopez, Evaluating the predictive accuracy of volatility models, *Journal of Forecasting*, **20** (2001), 87–109.

[27] Y. Lyu, P. Wang, Y. Wei and R. Ke, Forecasting the VaR of crude oil market: Do alternative distribution help?, *Energy Economics*, **66** (2017), 523–534.

[28] B. Mandelbrot, The variation of certain speculative prices, *Journal of Business*, **36** (1963), 394–419.

[29] D. McMillan, S. Alan and A. Owain, Forecasting UK stock market volatility, *Applied Financial Economics*, **10** (2000), 435–448.

[30] S. Nadarajah, E. Afuecheta and S. Chan, GARCH modeling of five popular commodities, *Empirical Economics*, **48** (2015), 1691–1712.
[31] D. B. Nelson, Conditional heteroskedasticity in asset returns: A new approach, *Econometrica*, **59** (1991), 347–370.
[32] A. R. Pagan and G. W. Schwert, Alternative models for conditional stock volatility, *Journal of Econometrics*, **45** (1990), 267–90.
[33] R. Rabemananjara and J. M. Zakoian, Threshold ARCH models and asymmetries in volatility, *Journal of Applied Econometrics*, **8** (1993), 31–49.
[34] E. Sentana, Quadratic ARCH models, *Review of Economic Studies*, **62** (1995), 639–661.
[35] O. Scaillet, Nonparametric estimation of conditional expected shortfall, *Insurance and Risk Management Journal*, **74** (2005), 639–660.
[36] A. Shamiri and Z. Isa, Modeling and Forecasting Volatility of the Malaysian Stock Markets, *Journal of Mathematics and Statistics*, **5** (2009), 234–240.
[37] J. C. Smolović, M. Lipovina-Božvić, and S. Vujošević, GARCH models in value at risk estimation: Empirical evidence from the Montenegrin stock exchange, *Economic Research-Ekonomska Istraživanja*, **30** (2017), 477–498.
[38] G. Storti and C. Vitale, BL-GARCH models and asymmetries in volatility, *Statistical Methods and Applications*, **12** (2003), 19–39.
[39] R. S. Tsay, *Analysis of Financial Time Series*, 3^{rd} edition, John Wiley & Sons, Hoboken, New Jersey, 2010.
[40] Y. Zhang and S. Nadarajah, A review of backtesting for value at risk, *Communications in Statistics-Theory and Methods*, **47** (2018), 3616–3639.
[41] D. Zhu and J. W. Galbraith, A generalized asymmetric student-t distribution with application to financial econometrics, *Journal of Econometrics*, **157** (2010), 297–305.
[42] D. Zhu and V. Zinde-Walsh, Properties and estimation of asymmetric exponential power distribution, *Journal of Econometrics*, **148** (2009), 86–99.

Received August 2018; revised October 2018.

E-mail address: tayhaozhe@siswa.um.edu.my
E-mail address: kokhaur@um.edu.my
E-mail address: kohyoubeng@um.edu.my
E-mail address: khng@utar.edu.my