A Bayesian Approach to Predicting Football Match Outcomes Considering Time Effect Weight

Article · January 2015

DOI: 10.1007/978-3-319-12454-4_12

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Journal of the Operational Research Society (2009), 1–10 0160-5682/09

www.palgrave-journals.com/jors/

A Bayesian approach for predicting match outcomes: The 2006 (Association) Football World Cup

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In this paper we propose a Bayesian methodology for predicting match outcomes. The methodology is illustrated on the 2006 Soccer World Cup. As prior information, we make use of the specialists' opinions and the FIFA ratings. The method is applied to calculate the win, draw and loss probabilities at each match and also to simulate the whole competition in order to estimate classification probabilities in group stage and winning tournament chances for each team. The prediction capability of the proposed methodology is determined by the DeFinetti measure and by the percentage of correct forecasts.

Journal of the Operational Research Society advance online publication, 14 October 2009
doi:10.1057/jors.2009.127

Keywords: Bayesian inference; forecasting; simulation; Soccer World Cup; sports; statistics

Introduction

The World Cup tournament organized by FIFA takes place every 4 years and is the most important international soccer championship. This competition joins 32 teams from around the world and is composed of two stages: a group stage followed by a knockout stage.

In the group stage, teams compete within eight groups of four teams each. Each group plays a round-robin tournament and the top two teams from each group advance to the next stage. Points are assigned to each team within a group, in which a win counts for 3 points and a draw counts for 1. The teams are ranked on the following criteria in order: greatest number of points, greatest total goal difference, greatest number of goals scored. If teams remain level after applying these criteria, a mini-group is formed with these teams and the same criteria applied. If after this teams remain level, a draw of lots is held.

The knockout stage is a single-elimination tournament in which teams play each other in one-off matches, with extra time of 30 min (2 halves of 15 min each) and penalty shootouts used to decide the winners, if necessary.

Although several authors have considered soccer scores prediction applied to championship leagues (Keller, 1994; Lee, 1997; Everson and Goldsmith-Pinkham, 2008), few articles can be found concerning score predictions for the World Cup. This can be explained by the limited amount of valuable data related to international matches due to great changes in national squads in the large elapsed time between World Cups (4 years), and also due to the fact that few competitions join teams from different continents.

Dyte and Clarke (2000) presents a log-linear Poisson regression model for soccer match predictions applied to the 1998 World Cup tournament, which takes the FIFA ratings as covariates. In that paper, the authors give some results about the predictive power of the model and also present simulation results to estimate winning championship probabilities.

Taking a different approach, Brillinger (2008) proposed to model directly the win, draw and loss probabilities. In that paper, Brillinger employed a trinomial model and applied it to the Brazilian 2006 Series A championship to obtain a probability estimate of any particular team's being champion, fit the team's final points and to evaluate the chance of a team's being in the top four places.

Recently, Karlis and Ntzoufras (2009) have applied the Skellam's distribution to model the goal difference between home and away teams. The authors argue that this approach does not rely neither on independence nor on the marginal Poisson distribution assumptions for the number of goals scored by the teams. A Bayesian analysis for predicting match outcomes for the English Premiere League (2006–2007 season) is carried out using a log-linear link function and non-informative prior distributions for parameters.

Using a counting processes approach, Volf (2009) modelled the development of a match score as two interacting time-dependent random point processes. The interaction between teams are modelled via a semi-parametric multiplicative regression model of intensity. The author has applied this model to the analysis of the performance of the eight teams that reached the quarter-finals of 2006 World Cup.

In this article, we propose a model to forecast the 2006 World Cup similar to that of Dyte and Clarke (2000),

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but within a Bayesian framework that takes into account specialists' opinion as prior information and FIFA ratings as covariate. The incorporation of specialists' knowledge has the advantage of overcoming the little amount of information about teams and of updating team's strength as competition progresses.

Probabilistic model

In this section, we will be concerned about the score prediction of a match between teams A and B with respective FIFA ratings R_A and R_B. We shall assume X_{AB} and X_{BA}, the number of goals scored by team A and B, two independent random variables such that

$$X_{AB}|\lambda_A \sim \text{Poisson}\left(\lambda_A \frac{R_A}{R_B}\right), \tag{1}$$

$$X_{BA}|\lambda_B \sim \text{Poisson}\left(\lambda_B \frac{R_B}{R_A}\right), \tag{2}$$

where λ_A and λ_B can be interpreted as the mean number of goals teams A and B score against a team with the same ability.

Observe that ratings are used to quantify each team's ability. So, in our model the mean number of goals that team A scored against B is directly proportional to the rating of team A and inversely proportional to the rating of team B. This assumption is natural, because it is expected, for instance, that Brazil (rating 827) scores more goals against Togo (rating 569) than against France (rating 749).

At this point, it is worthwhile noting that Dyte and Clarke (2000) observe, in the light of their parameter estimates, that the fitted logarithm of the mean number of goals is approximately a linear function of the difference of teams' ratings. Taking a similar idea, the proposed model (1) and (2) states that the mean number of goals is proportional to the ratio of team's ratings with a proportionality constant for each team. This is an important feature that allows for teams' evolution to be incorporated into the model as will be clarified in the next sections.

Note that the task of predicting the number of goals team A scores is entirely analogous to that for team B. So, in the next subsections we just show the derivations for team A.

Prior distribution

In this paper, we decided to ask specialists for a guess about the match's scores, instead of asking them directly for information about the parameters. This procedure of prior elicitation is easier to apply, since not all specialists have statistical knowledge and a plausible guess for the parameter is difficult. Assuming that the specialists' opinions are independent, and following a Poisson distribution, we shall obtain the prior distribution for the parameters using a procedure analogous to the *Power Prior* method (Ibrahim and Chen, 2000) with the historical data replaced by the specialists' opinion.

Previous to the knowledge of specialists' opinions, we assume total absence of information, which will be expressed by the Jeffreys prior (non-informative) for the Poisson model given by

$$\pi_0(\lambda_A) \propto \lambda_A^{-\frac{1}{2}} \tag{3}$$

$$\pi_0(\lambda_B) \propto \lambda_B^{-\frac{1}{2}}. \tag{4}$$

Now, considering the specialists' opinions (about the number of goals A scores against a team OA) $\tilde{x}^i_{A,OA}$, $i = 1, \ldots, s$, a random sample of a Poisson distribution with parameter $\lambda_A \frac{R_A}{R_{OA}}$, the power prior of λ_A is expressed as

$$\pi(\lambda_A|\mathscr{D}_0) \propto \pi_0(\lambda_A)\left\{\prod_{i=1}^{s} \exp\left(-\frac{\lambda_A R_A}{R_{OA}}\right)\left[\frac{\lambda_A R_A}{R_{OA}}\right]^{\tilde{x}^i_{A,OA}}\right\}^{a_0}$$
$$\propto \exp\left(-\frac{a_0 s \lambda_A R_A}{R_{OA}}\right)\lambda_A^{a_0\sum_{i=1}^{s}\tilde{x}^i_{A,OA}-\frac{1}{2}}, \tag{5}$$

where $0 \leqslant a_0 \leqslant 1$ and $\mathscr{D}_0$ denotes all the specialists' opinions.

So, denoting $\sum_{i=1}^{s}\tilde{x}^i_{A,OA}$ by $\tilde{x}_{A,OA}$, it follows from (5) that the power prior distribution of λ_A when $a_0 > 0$ is given by

$$\lambda_A|\mathscr{D}_0 \sim \text{Gamma}\left(a_0\tilde{x}_{A,OA} + \frac{1}{2}, a_0 s\frac{R_A}{R_{OA}}\right), \tag{6}$$

and is given by the Jeffreys prior (3) when $a_0 = 0$.

Analogously for team **B**, λ_B has power prior given by

$$\lambda_B|\mathscr{D}_0 \sim \text{Gamma}\left(a_0\tilde{x}_{B,OB} + \frac{1}{2}, a_0 s\frac{R_B}{R_{OB}}\right), \tag{7}$$

when $a_0 > 0$ and the Jeffreys prior (4) when $a_0 = 0$.

The distributions (6) and (7) will be used as prior distributions for the parameters λ_A and λ_B.

Posterior and predictive distributions

In this section, we want to predict the number of goals team A scores against team B, using all the available information (hereafter denoted by $\mathscr{D}$). This information is originated from two sources: the specialists' opinions and the scores of matches already played. So, we may be in two distinct situations: (i) we do have the specialists' opinions but no matches have been played; (ii) we have both the specialists' opinions and the scores of matches played.

In situation (i), we do not have observed data, only the specialist opinions. So, from the model given in (1) and the prior distribution in (6), it follows from Result 2 that the prior predictive distribution of X_{AB} is

$$X_{AB} \sim NB\left(a_0\tilde{x}_{A,OA} + \frac{1}{2}, \left[1 + \frac{R_{OA}}{R_B a_0 s}\right]^{-1}\right), \tag{8}$$

where NB denotes the negative binomial distribution.

Analogously for team B, from model (2) and the prior distribution (7), it follows from Result 2 that the prior predictive distribution of X_{BA} is

$$X_{BA} \sim NB\left(a_0\bar{x}_{B,OB} + \frac{1}{2}, \left[1 + \frac{R_{OB}}{R_A a_0 s}\right]^{-1}\right). \tag{9}$$

In situation (ii), assume that team A has played k matches against the opponent teams $C_1, \ldots, C_k$ and that, given λ_A, $X_{A,C_1}, \ldots, X_{A,C_k}$ are independent Poisson distributed random variables with parameters $\lambda_A \frac{R_A}{R_{C_1}}, \ldots, \lambda_A \frac{R_A}{R_{C_k}}$. Hence, it follows from the model (1) that the likelihood for these data is given by

$$\begin{aligned} P[X_{A,C_1} = x_A^1, \ldots, X_{A,C_k} = x_A^k | \lambda_A] &= \prod_{i=1}^{k} \frac{e^{-\lambda_A \frac{R_A}{R_{C_i}}} (\lambda_A \frac{R_A}{R_{C_i}})^{x_A^i}}{(x_A^i)!} \\ &\propto \exp\left\{-\lambda_A \sum_{i=1}^{k} \frac{R_A}{R_{C_i}}\right\} \lambda_A^{\sum_{i=1}^{k} x_A^i}, \end{aligned} \tag{10}$$

where $x_A^i = 0, 1, \ldots$ with $i = 1, \ldots, k$.

So, from the likelihood (10) and the prior distribution (6), it follows that the posterior distributions of parameter λ_A is

$$\lambda_A|\mathscr{D} \sim \text{Gamma}\left(a_0\tilde{x}_{A,OA} + \sum_{i=1}^{k} x_A^i + \frac{1}{2}, \sum_{i=1}^{k} \frac{R_A}{R_{C_i}} + \frac{a_0 s R_A}{R_{OA}}\right). \tag{11}$$

So, from the model (1), the posterior (11) and Result 2, the posterior predictive distribution of X_{AB} is

$$X_{AB}|\mathscr{D} \sim NB\left(a_0\tilde{x}_{A,OA} + \sum_{i=1}^{k} x_A^i + \frac{1}{2}, \frac{\sum_{i=1}^{k} \frac{1}{R_{C_i}} + \frac{a_0 s}{R_{OA}}}{\sum_{i=1}^{k} \frac{1}{R_{C_i}} + \frac{a_0 s}{R_{OA}} + \frac{1}{R_B}}\right). \tag{12}$$

Analogously for team B, the posterior distribution of λ_B is

$$\lambda_B|\mathscr{D} \sim \text{Gamma}\left(a_0\tilde{x}_{B,OB} + \sum_{i=1}^{k} x_A^i + \frac{1}{2}, \sum_{i=1}^{k} \frac{R_B}{R_{D_i}} + \frac{a_0 s R_B}{R_{OB}}\right), \tag{13}$$

where $D_i, i = 1, \ldots, k$, are the opponent teams faced by B.

Hence, from the model (2), the posterior (13) and Result 2, the posterior predictive distribution of X_{BA} is

$$X_{BA}|\mathscr{D} \sim NB\left(a_0\tilde{x}_{B,OB} + \sum_{i=1}^{k} x_B^i + \frac{1}{2}, \frac{\sum_{i=1}^{k} \frac{1}{R_{D_i}} + \frac{a_0 s}{R_{OB}}}{\sum_{i=1}^{k} \frac{1}{R_{D_i}} + \frac{a_0 s}{R_{OB}} + \frac{1}{R_A}}\right). \tag{14}$$

Methods

In this section, we shall consider the competition divided into seven rounds, where the first three rounds are in the group stage and the last four in the knockout stage. Just before each round, 10 specialists give their opinions about the scores of all matches in that round. To account for the mean specialists' opinion, we have chosen $a_0 = 1/10$, in the sense that, if one observation equal to the mean of the specialists' opinion is taken from the sampling distribution, then under the non-informative Jeffreys priors the posterior distribution is the same as the power prior distributions (6) and (7). It is important to note that is not allowed for the specialists to discuss about their guesses in order to make the guesses as much independent as possible. Also, the opinions only refer to one round at a time and they may be influenced by the outcomes of matches already played.

Predictions for single matches

For a given match played by teams A and B, we calculate the probabilities of win (P_W), draw (P_D) and loss (P_L) of team A from the predictive distributions, using the following formulas

$$P_W = P(X_{AB} > X_{BA}) = \sum_{i=1}^{\infty} \sum_{j=0}^{i-1} P(X_{AB} = i) P(X_{BA} = j), \tag{15}$$

$$P_D = P(X_{AB} = X_{BA}) = \sum_{i=0}^{\infty} P(X_{AB} = i) P(X_{BA} = i), \tag{16}$$

$$P_L = P(X_{AB} < X_{BA}) = \sum_{j=1}^{\infty} \sum_{i=0}^{j-1} P(X_{AB} = i) P(X_{BA} = j). \tag{17}$$

It is useful to consider the set of all possible forecasts given by the simplex set

$$S = \{(P_W, P_D, P_L) \in [0, 1]^3 : P_W + P_D + P_L = 1\}.$$

Observe that the vertices $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ of S represent the outcomes win, draw and loss, respectively. Thus,

Table 1 Probability distribution of scores for Germany *verus* Poland

		Poland					*Total*
		0	1	2	3	4	
Germany	0	0.053	0.013	0.004	0.001	0.000	0.071
	1	0.115	0.028	0.008	0.002	0.001	0.154
	2	0.144	0.035	0.010	0.003	0.001	0.193
	3	0.137	0.033	0.009	0.003	0.001	0.183
	4	0.109	0.026	0.007	0.002	0.001	0.145
	5	0.076	0.018	0.005	0.002	0.000	0.102
	6	0.049	0.012	0.003	0.001	0.000	0.065
	7	0.029	0.007	0.002	0.001	0.000	0.039
	8	0.016	0.004	0.001	0.000	0.000	0.022
	9	0.009	0.002	0.001	0.000	0.000	0.012
	10	0.005	0.001	0.000	0.000	0.000	0.006
	Total	0.743	0.179	0.051	0.015	0.005	

a method used to measure the goodness of a prediction is to calculate the DeFinnetti distance (DeFinetti, 1972), which is the Euclidean distance between the point correspondent to the outcome and that one correspondent to the prediction, for example, if a prediction is $(0.3, 0.6, 0.1)$ and the outcome is a draw (0, 1, 0), then the DeFinetti distance is $(0.3-0)^2+(0.6-1)^2+(0.1-0)^2=0.26$. Also, we can associate to a set of predictions the average of its DeFinetti distances, known as the DeFinetti measure. So, we shall consider the best method among some prediction methods the one with the least DeFinetti measure. Furthermore, the DeFinetti value 2/3 can be considered as a reference for comparison, because an equiprobable predictor that assigns equal probability for all outcomes ($P_W=P_D=P_L=1/3$) has 2/3 as its DeFinetti measure.

Another standard way of measuring the goodness of a prediction method is calculating the percentage of correct forecasts, in which a forecast ($P_W,\ P_D,\ P_L$) shall be considered correct if the outcome with the greatest probability coincides with the observed outcome.

Predictions for the whole tournament

Besides the prediction of outcomes for a single match, we also use the predictive distributions to perform a simulation of the whole competition. The purpose of this simulation is to estimate some probabilities that are difficult to obtain, such as the probability that a given team wins the tournament, reaches the final, qualifies to the knockout stage, etc.

Just before each round, we display the estimated probabilities of classification within group F and the winning tournament probabilities obtained by a simulation of 10 000 tournaments.

Results

Single match prediction

In this section, we first consider the match Germany *versus* Poland played at the second round in group A. The mean specialists' opinion for this match was 2 for Germany and 0.3 for Poland. Also, we note a very high skill difference between the two teams given by their ranking positions, 14th and 25th. The previous round results for this group were Germany 4 – Costa Rica 2, and Ecuador 2 – Poland 0, which indicate a high goal rate for Germany.

From the predictive distributions (12) and (14), we constructed Table 1, which shows the probability distribution of scores for this match. Also, from equations (15), (16) and (17) we obtain the following probabilities

Germany win	87.07%
Poland win	9.43%
Draw	3.50%

The observed result was Germany 1 – Poland 0, which is one of the most plausible results according to Table 1 (11.5%)

Other observed results for this group, with probabilities attached to the outcomes, DeFinnetti measures and correct prediction indicator, are shown in Table 2. The mean DeFinneti measure for this group is 0.423 and the correct prediction proportion is 83.33%.

Classification probabilities in group stage

In this section, we illustrate how the simulation is applied to obtain the classification probabilities in group F. Tables 3, 4 and 5 display for each team the probabilities of reaching any of the four group positions (the classification probabilities) and the probability of qualifying for the knockout stage (the qualifying probability) prior to rounds 1, 2 and 3, respectively.

From Table 3, we see clearly that, with no matches played, Brazil's squad is the favourite team in this group, which can be explained by the highest FIFA ranking and the mean opinion about the Brazil *versus* Croatia match (2.4 *versus* 0.7). Also, the Australia squad has the lowest qualification probability, which can be justified by the lowest FIFA ranking (26th) in this group.

Table 2 Probability predictions for outcomes in Group A

Round	*Team*	*Observed score*	*Team*	*W*	*D*	*L*	*DeFinetti*	*Correct*
1	Germany	4-2	Costa Rica	0.701	0.173	0.126	0.135	Yes
1	Poland	0-2	Ecuador	0.326	0.208	0.466	0.434	Yes
2	Germany	1-0	Poland	0.871	0.094	0.035	0.027	Yes
2	Ecuador	3-0	Costa Rica	0.455	0.174	0.371	0.464	Yes
3	Costa Rica	1-2	Poland	0.504	0.307	0.189	1.007	No
3	Ecuador	0-3	Germany	0.371	0.177	0.453	0.468	Yes

Table 3 Classification probabilities before first round for Group F

	1st Place	*2nd Place*	*3rd Place*	*4th Place*	*Qualif. Prob.*
Australia	0.0587	0.1789	0.3010	0.4614	0.2376
Japan	0.1881	0.3440	0.2761	0.1918	0.5321
Brazil	0.6392	0.2226	0.0959	0.0423	0.8618
Croatia	0.1140	0.2545	0.3270	0.3045	0.3685

Table 4 Classification probabilities before second round for Group F

	1st Place	*2nd Place*	*3rd Place*	*4th Place*	*Qualif. Prob.*
Australia	0.4532	0.3416	0.1698	0.0354	0.7948
Japan	0.0156	0.1048	0.2776	0.602	0.1204
Brazil	0.4793	0.3545	0.1385	0.0277	0.8338
Croatia	0.0519	0.1991	0.4141	0.3349	0.2510

Table 5 Classification probabilities before third round for Group F

	1st Place	*2nd Place*	*3rd Place*	*4th Place*	*Qualif. Prob.*
Australia	0.0385	0.8421	0.1105	0.0089	0.8806
Japan	0	0.0038	0.3769	0.6193	0.0038
Brazil	0.9615	0.0385	0	0	1
Croatia	0	0.1156	0.5126	0.3718	0.1156

Table 6 Final standings for Group F

Team	*Points*	*W*	*D*	*L*	*GD*	*GF*	*GA*
Brazil	9	3	0	0	6	7	1
Australia	4	1	1	1	0	5	5
Croatia	2	0	2	1	−1	2	3
Japan	1	0	1	2	−5	2	7

In Table 4, we see a great change in the classification probabilities, due to the unexpected win of Australia against Japan (3-1) in the first round. Observe that the Australia qualification probability increased from 24 to 79%. The Brazil team confirmed the favouritism and defeated Croatia (1-0), and maintained approximately the same qualification probability, but with smaller first place probability (48%).

After the second round, Brazil's squad confirms its participation in the next stage with a win against Australia (2-0), and has a great chance (96%) of reaching the first place in the group. With the draw between Japan and Croatia (0-0), Australia's team increased even more the qualification probability (88%) to the next stage, but has little chance of reaching the first place (4%). In spite of the high favouritism of Brazil and Australia, Japan and Croatia still have chances to qualify for the knockout stage.

After the third round, Brazil confirmed the predictions and defeated Japan (4-1). Australia also confirmed the qualification for the next stage with a draw against Croatia (2-2). Table 6 displays the final standings for group F.

Tournament simulation

In this section, we have performed a simulation study in order to estimate the winning tournament probabilities. Just prior to each of the seven rounds, one simulation of 10,000 tournament replicas has been carried out and the percentage of tournament replica wins is calculated for each team. Tables 7, 8, 9 and 10 display, respectively, the latter percentages for the a_0 values 0.01, 0.1, 0.5 and 0.9, which represent different weights given to the specialists' opinion in the prior distribution. In the referred tables, the rows has been ordered according to the world cup final standings, with the first position team placed at the first row and so on.

From the presented results, we see a high variability in the estimated probabilities as the competition progresses. For the first three rounds (group stage), the most remarkable impact of the a_0 value in the estimated probabilities is seen for Brazil and Germany teams, with the former with a positive impact and the latter a negative one. Besides this, the major influence of the a_0 weight is observed when there is no matches played (before 1st round column).

Also, from the obtained results, it is remarkable that both tournament finalists (Italy and France) could not be considered favourite for winning the competition at any of the first six rounds despite of the a_0 value. Before reaching the semi-finals, these two teams' winning probabilities remained low and did not change substantially for any of the a_0 values. This feature may be explained in part by the low FIFA rating, the lack of confidence of specialists in these teams and the small number of goals made before semi-finals stage.

Table 7 Percentage of tournament wins for each team ($a_0 = 0.01$)

Team (Ranking)	*Before 1st round*	*Before 2nd round*	*Before 3rd round*	*Before round of 16*	*Before QF*	*Before SF*	*Before Final*
Italy (11th)	5.51	2.52	1.55	5.56	8.09	27.10	51.96
France (8th)	1.53	0.01	0.01	0.72	6.15	22.61	48.04
Germany (14th)	6.68	24.57	10.76	16.09	19.82	32.78	
Portugal (7th)	3.42	0.37	1.70	5.45	9.13	17.51	
Brazil (1st)	6.58	1.05	3.80	16.92	30.05		
Argentina (9th)	6.60	3.73	32.05	18.79	19.63		
England (10th)	6.19	0.54	0.48	2.77	4.56		
Ukraine (28th)	0.33	0.00	1.08	3.24	2.57		
Spain (5th)	5.59	20.40	29.25	17.70			
Switzerland (24th)	1.10	0.00	0.20	1.53			
Netherlands (3rd)	3.11	0.44	2.44	1.24			
Ecuador (25th)	2.45	2.43	7.56	3.32			
Ghana (30th)	0.36	0.00	0.21	1.19			
Sweden (12th)	2.48	0.00	0.11	0.61			
Mexico (4th)	8.85	13.47	1.53	1.61			
Australia (26th)	3.10	5.69	1.92	3.26			
Korea Republic (19th)	3.49	1.35	1.38				
Paraguay (22nd)	2.09	0.00	0.00				
Cote d'Ivoire (21st)	2.91	0.36	0.00				
Czech Republic (2nd)	9.96	17.35	2.25				
Poland (19th)	0.11	0.00	0.00				
Croatia (16th)	0.86	0.01	0.00				
Angola (31st)	0.31	0.00	0.00				
Tunisia (15th)	3.04	1.82	1.59				
USA (5th)	0.82	0.00	0.07				
Iran (16th)	2.48	0.18	0.00				
Trinidad and Tobago (29th)	0.85	0.01	0.00				
Saudi Arabia (23rd)	2.10	1.64	0.05				
Japan (13th)	2.21	0.13	0.01				
Togo (32nd)	1.86	0.07	0.00				
Costa Rica (18th)	2.36	1.86	0.00				
Serbia and Montenegro (27th)	0.67	0.00	0.00				

Table 8 Percentage of tournament wins for each team ($a_0 = 0.1$)

Team (Ranking)	*Before 1st round*	*Before 2nd round*	*Before 3rd round*	*Before round of 16*	*Before QF*	*Before SF*	*Before Final*
Italy (11th)	6.34	3.94	2.04	5.75	8.12	25.55	50.28
France (8th)	1.79	0.02	0.12	1.15	7.08	21.82	49.72
Germany (14th)	8.69	19.57	7.41	16.66	20.72	30.76	
Portugal (7th)	6.55	0.88	2.86	5.50	9.67	21.87	
Brazil (1st)	11.90	2.51	8.12	18.46	28.86		
Argentina (9th)	6.50	3.97	31.43	21.53	18.78		
England (10th)	6.19	2.36	0.44	3.02	4.61		
Ukraine (28th)	0.38	0	2.39	2.92	2.16		
Spain (5th)	4.98	19.31	30.60	14.40			
Switzerland (24th)	0.33	0.07	0.35	1.87			
Netherlands (3rd)	2.61	1.16	1.74	1.28			
Ecuador (25th)	2.11	4.05	5.03	2.61			
Ghana (30th)	0.03	0	0.31	1.26			
Sweden (12th)	4.17	0.05	0.07	0.61			
Mexico (4th)	12.26	15.75	1.40	1.23			
Australia (26th)	0.45	1.91	1.75	1.75			
Korea Republic (19th)	2.09	1.98	0.93				
Paraguay (22nd)	0.69	0.02	0.00				
Cote d'Ivoire (21st)	1.06	0.73	0.00				
Czech Republic (2nd)	15.02	19.3	2.11				

Table 8 (*continued*)

Team (Ranking)	*Before 1st round*	*Before 2nd round*	*Before 3rd round*	*Before round of 16*	*Before QF*	*Before SF*	*Before Final*
Poland (19th)	0.03	0	0.00				
Croatia (16th)	0.54	0.09	0.02				
Angola (31st)	0.06	0	0.00				
Tunisia (15th)	2.13	0.86	0.86				
USA (5th)	0.29	0	0.02				
Iran (16th)	0.34	0.1	0.00				
Trinidad and Tobago (29th)	0.01	0.01	0.00				
Saudi Arabia (23rd)	0.58	0.43	0.00				
Japan (13th)	1.52	0.04	0.00				
Togo (32nd)	0.10	0	0.00				
Costa Rica (18th)	0.11	0.89	0.00				
Serbia and Montenegro (27th)	0.15	0	0.00				

Table 9 Percentage of tournament wins for each team ($a_0 = 0.5$)

Team (Ranking)	*Before 1st round*	*Before 2nd round*	*Before 3rd round*	*Before round of 16*	*Before QF*	*Before SF*	*Before Final*
Italy (11th)	6.34	4.74	2.23	6.65	8.35	22.09	43.98
France (8th)	1.96	0.09	0.41	2.82	9.04	17.77	56.02
Germany (14th)	7.35	11.47	4.31	16.60	21.56	25.89	
Portugal (7th)	8.92	1.40	5.25	4.73	12.92	34.25	
Brazil (1st)	16.64	5.63	18.28	20.84	26.20		
Argentina (9th)	4.86	3.76	27.30	24.70	16.17		
England (10th)	5.77	4.55	0.45	4.20	4.71		
Ukraine (28th)	0.81	0.03	3.61	2.29	1.05		
Spain (5th)	4.06	14.12	29.81	8.78			
Switzerland (24th)	0.11	0.17	0.54	2.52			
Netherlands (3rd)	2.36	3.00	1.14	1.44			
Ecuador (25th)	1.55	5.72	2.27	1.29			
Ghana (30th)	0.02	0.00	0.39	1.55			
Sweden (12th)	4.94	0.26	0.16	0.82			
Mexico (4th)	13.49	16.75	0.94	0.60			
Australia (26th)	0.05	0.13	1.37	0.17			
Korea Republic (19th)	1.59	2.21	0.37				
Paraguay (22nd)	0.34	0.10	0.00				
Cote d'Ivoire (21st)	0.50	2.59	0.00				
Czech Republic (2nd)	14.94	20.89	0.86				
Poland (19th)	0.01	0.00	0.00				
Croatia (16th)	0.33	1.34	0.20				
Angola (31st)	0.01	0.01	0.00				
Tunisia (15th)	1.60	0.27	0.10				
USA (5th)	0.08	0.00	0.00				
Iran (16th)	0.06	0.02	0.00				
Trinidad and Tobago (29th)	0.00	0.00	0.01				
Saudi Arabia (23rd)	0.15	0.08	0.00				
Japan (13th)	1.09	0.02	0.00				
Togo (32nd)	0.00	0.00	0.00				
Costa Rica (18th)	0.01	0.64	0.00				
Serbia and Montenegro (27th)	0.06	0.01	0.00				

Prior to the semi-final matches (Germany *versus* Italy and Portugal *versus* France), we observe that as the a_0 value increases, the winning probabilities for Portugal increases, and concomitantly these probabilities decrease for Italy, France and Germany teams. This is surely due to the mean specialists' opinion about the match Portugal *versus* France (1.5 *versus* 0.5), which in turn can be explained by Portugal's good performance in past matches and its victorious coach Luiz Felipe Scolari, who won the previous 2002 World Cup for the Brazilian team.

Table 10 Percentage of tournament wins for each team ($a_0 = 0.9$)

Team (Ranking)	*Before 1st round*	*Before 2nd round*	*Before 3rd round*	*Before round of 16*	*Before QF*	*Before SF*	*Before Final*
Italy (11th)	5.90	4.77	1.99	6.58	8.37	19.99	42.06
France (8th)	1.79	0.06	0.67	3.34	9.91	16.55	57.94
Germany (14th)	7.27	9.72	3.11	17.82	21.57	22.37	
Portugal (7th)	9.90	1.45	5.80	3.94	14.34	41.09	
Brazil (1st)	17.81	6.36	22.69	20.98	24.69		
Argentina (9th)	4.34	3.50	26.25	25.81	14.79		
England (10th)	5.84	5.65	0.39	5.19	5.52		
Ukraine (28th)	0.82	0.04	4.12	1.86	0.81		
Spain (5th)	3.33	13.67	28.35	6.75			
Switzerland (24th)	0.10	0.23	0.38	2.85			
Netherlands (3rd)	1.92	3.20	0.99	1.24			
Ecuador (25th)	1.33	6.24	1.50	0.95			
Ghana (30th)	0.01	0.00	0.28	1.50			
Sweden (12th)	5.22	0.27	0.22	0.73			
Mexico (4th)	13.89	16.77	0.84	0.38			
Australia (26th)	0.01	0.02	1.14	0.08			
Korea Republic (19th)	1.19	2.02	0.33				
Paraguay (22nd)	0.26	0.08	0.00				
Cote d'Ivoire (21st)	0.46	2.75	0.00				
Czech Republic (2nd)	16.07	20.44	0.62				
Poland (19th)	0.04	0.00	0.00				
Croatia (16th)	0.23	2.12	0.22				
Angola (31st)	0.01	0.00	0.00				
Tunisia (15th)	1.13	0.13	0.09				
USA (5th)	0.02	0.00	0.00				
Iran (16th)	0.04	0.00	0.00				
Trinidad and Tobago (29th)	0.00	0.00	0.01				
Saudi Arabia (23rd)	0.15	0.05	0.00				
Japan (13th)	0.84	0.07	0.01				
Togo (32nd)	0.01	0.00	0.00				
Costa Rica (18th)	0.02	0.39	0.00				
Serbia and Montenegro (27th)	0.05	0.00	0.00				

Just before the final match (Italy *versus* France), the mean specialists' opinion was 0.5 against 0, which explains the fact that France's winning probabilities increases with the increasing of the a_0 value. For instance, when a_0 is equal to 0.01, France's winning chance is 48.04% and, on the other hand, when a_0 is equal to 0.9, this chance increases to 57.94%. The actual match sore after the regular 90 min was levelled at 1-1. After 30 min extra time no goals were scored, forcing teams to follow to a penalty shootout, which Italy won 5-3.

Final remarks

Several factors that might influence a soccer match are difficult to model such as the team adaptation, physical conditioning, the attack and defence strength, tactic disciplines, team psychological conditions, goalkeeper and players of line with technical level above average, soccer fans, coach, referee etc. So, in this paper a Bayesian methodology for predicting match outcomes is proposed and illustrated for the 2006 Soccer World Cup. We considered the specialists' opinions and the FIFA ratings as prior information, taking into account some of these factors, which could be very difficult using standard methods like regression analysis (Lee, 1997; Dyte and Clarke, 2000; Everson and Goldsmith-Pinkham, 2008).

As explained in the 'Probabilistic model' section, the proposed model is very close to that of Dyte and Clarke (2000) and can be considered as an extension of that model, as updating the priors at each round can capture team's evolution, moral increase, important player suspensions or injuries and so on. These updating feature was not allowed for in Dyte and Clark's model, and those authors suggested that a method for updating the teams' ratings should be investigated. Although taking a different route (updating the relative team's strength), we think that the aim of their suggestion has been reached.

The proposed method is applied to calculate the win, draw and loss probabilities at each match and also to simulate the whole competition in order to estimate classification probabilities in the group stage and winning tournament chances for each team. The prediction capability of the proposed methodology can be stated when comparing it to the equiprobable predictor, which has $2/3$ as its DeFinetti measure. We can observe that, for the proposed method, in 62.5% of matches the De-Finetti measure is less than $2/3$, and 57.81% of matches were correctly predicted, which corresponds to a 73.45% increase in relation to the expected proportion of the equiprobable predictor (33.33%).

Furthermore, if we consider 64 independent guesses with success probability equal to $1/3$, in 95% of times, we will observe the number of correct guesses less than 28, which is much smaller than the 40 correct guesses provided by our method.

In our analysis, we have considered a fixed value for the weight parameter a_0. This choice has been made in order to obtain known prior distributions. In 'Tournament simulation' section, we have discussed the influence of different choices for the a_0 parameter in the estimated winning tournament probabilities. Thus, an important improvement of this study would be implementing a full hierarchical structure specifying a parametric distribution for the parameter a_0, for example a beta distribution, as suggested in Ibrahim and Chen (2000). Another possible extension would be allowing for distinct values for the a_0 parameter, one for each expert, which has also been suggested by Ibrahim and Chen (2000) to deal with multiple historical data sets. This feature can accommodate different beliefs about experts' prediction ability and may be updated according to the observed data.

Further research can also be made by considering different elicitation procedures. The prior elicitation technique used here is easy to apply and originates known prior distributions. Although this feature is mathematically convenient, other prior elicitation techniques may be investigated, such as the predictive elicitation. That technique makes use of a conjugated family of prior distributions (gama distribution in our case) and the hyperparameter choice is based on the predictive prior distribution. Percy (2002) gives a good description of this elicitation technique and presents illustrations in the reliability context.

Acknowledgements—The authors are grateful to the referees who pointed out many interesting issues which have enabled a substantial improvement of this paper. This work has received financial support from CNPq and CAPES.

References

Brillinger DR (2008). Modelling game outcomes of the Brazilian 2006 series a championship as ordinal-valued. *Braz J Probab Stat* **22**: 89–104.
DeFinetti B (1972). *Probability, Induction and Statistics*. John Wiley: London.
Dyte D and Clarke SR (2000). A ratings based Poisson model for World Cup soccer simulation. *J Opl Res Soc* **51**: 993–998.
Everson P and Goldsmith-Pinkham P (2008). Composite Poisson models for goal scoring. *J Quant Anal Sports* **4**.
Ibrahim JG and Chen MH (2000). Power prior distributions for regression models. *Stat Sci* **15**: 46–60.
Karlis D and Ntzoufras I (2009). Bayesian modelling of football outcomes: using the Skellam's distribution for the goal difference. *IMA J Mngt Math* **20**: 133–145.
Keller JB (1994). A characterization of the Poisson distribution and the probability of winning a game. *Am Stat* **48**: 294–298.
Lee A (1997). Modeling scores in the Premier League: Is Manchester United really the best? *Chance* **10**: 15–19.
Percy DF (2002). Bayesian enhanced strategic decision making for reliability. *Eur J Opl Res* **139**: 133–145.
Volf P (2009). A random point process model for the score in sport matches. *IMA J Mngt Math* **20**: 121–131.

Appendix A: Useful results

Result 1 If $X|\lambda \sim$ Poisson $(\lambda c), c > 0$, and λ follows a Jeffreys prior given by $\pi(\lambda) \propto \lambda^{-\frac{1}{2}}$, then $\lambda|X = x \sim$ Gamma $(x + \frac{1}{2}, c)$.

Proof The result follows immediately from

$$\pi(\lambda|x) \propto P\{X = x|\lambda\}\pi(\lambda)$$
$$\propto e^{-\lambda c}\lambda^{x+\frac{1}{2}-1}. \qquad \square$$

Result 2 If $X|\lambda \sim$ Poisson $(\lambda c), c > 0$, and $\lambda \sim$ Gamma (α, β), then the marginal distribution of X is a negative binomial distribution with parameters α and $\frac{\beta}{\beta+c}$.

Proof From the hypothesis above we have

$$P\{X = x|\lambda\} = \frac{e^{-\lambda c}(\lambda c)^x}{x!}, \qquad x = 0, 1, \ldots,$$

$$\pi(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}, \qquad \lambda > 0$$

Hence, the marginal distribution of X is given by

$$\begin{aligned} P\{X = x\} &= \int_0^\infty P\{X = x|\lambda\}(\pi\lambda)\,d\lambda \\ &= \int_0^\infty \frac{e^{-\lambda c}(\lambda c)^x}{x!}\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}\,d\lambda \\ &= \frac{c^x\beta^\alpha}{x!\Gamma(\alpha)}\int_0^\infty e^{-\lambda(c+\beta)}\lambda^{\alpha+x-1}\,d\lambda \\ &= \frac{\Gamma(\alpha+x)}{x!\Gamma(\alpha)}\left(\frac{c}{c+\beta}\right)^x\left(\frac{\beta}{c+\beta}\right)^a \qquad \square \end{aligned}$$

Appendix B: FIFA ratings

See Table B1.

Table B1 FIFA ratings prior to 2006 World Cup

Number	*Team*	*FIFA ratings*
1	Germany	696
2	Costa Rica	683
3	Poland	677
4	Ecuador	631
5	England	741
6	Paraguay	653
7	Trinidad and Tobago	604
8	Sweden	709
9	Argentina	746
10	Cote d'Ivoire	669
11	Serbia and Montenegro	610
12	Netherlands	768
13	Mexico	758
14	Iran	686
15	Angola	581
16	Portugal	750
17	Australia	612
18	Japan	705
19	USA	756
20	Czech Republic	772
21	Italy	728
22	Ghana	600
23	Korea Republic	677
24	Togo	569
25	France	749
26	Switzerland	648
27	Brazil	827
28	Croatia	686
29	Spain	756
30	Ukraine	609
31	Tunisia	693
32	Saudi Arabia	651

Received February 2009;
accepted August 2009 after one revision