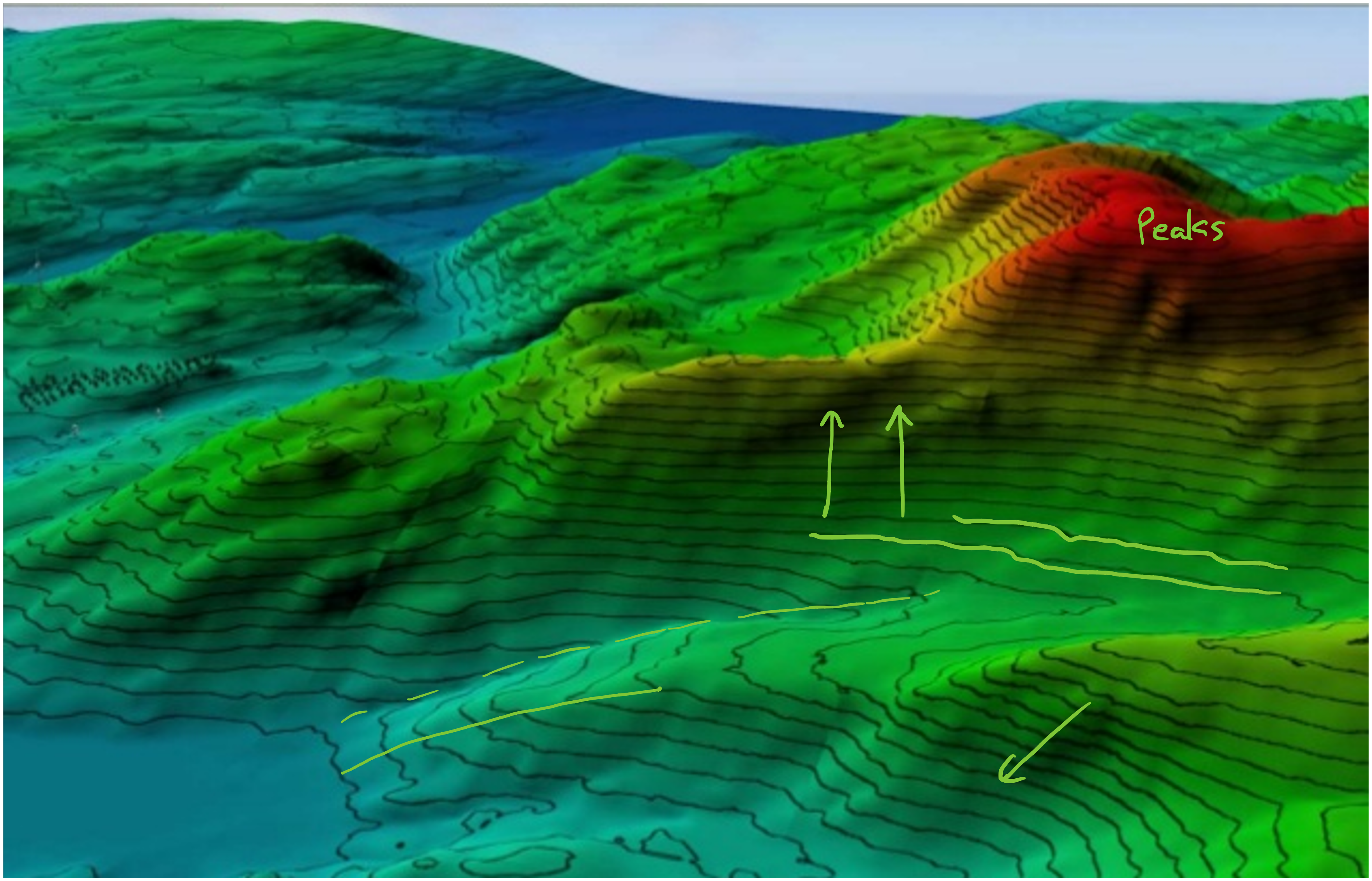

University of Colorado
Boulder

Deep Learning Applications for Computer Vision

Lecture 7: Gradients and Linear Filters

Terrain Concepts



Terrain Concepts

Basic notions:

- Uphill / downhill
- Contour lines (curves of constant elevation)
- Steepness of slope
- Peaks/Valleys (local extrema) : max , min

More mathematical notions:

- Tangent Plane
- Normal vectors
- Curvature

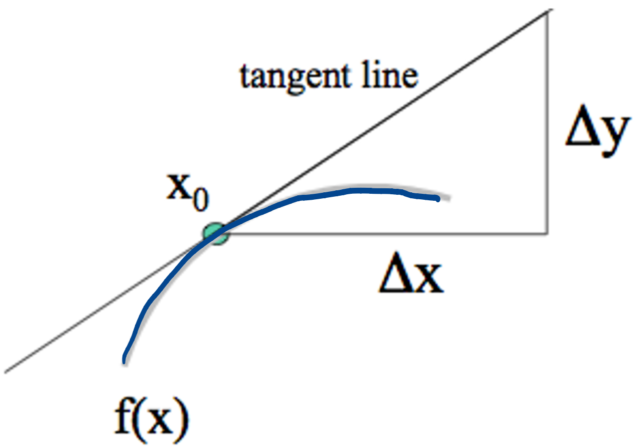
Gradient vectors (vectors of partial derivatives) will help us define/compute all of these.

Gradients – 1D

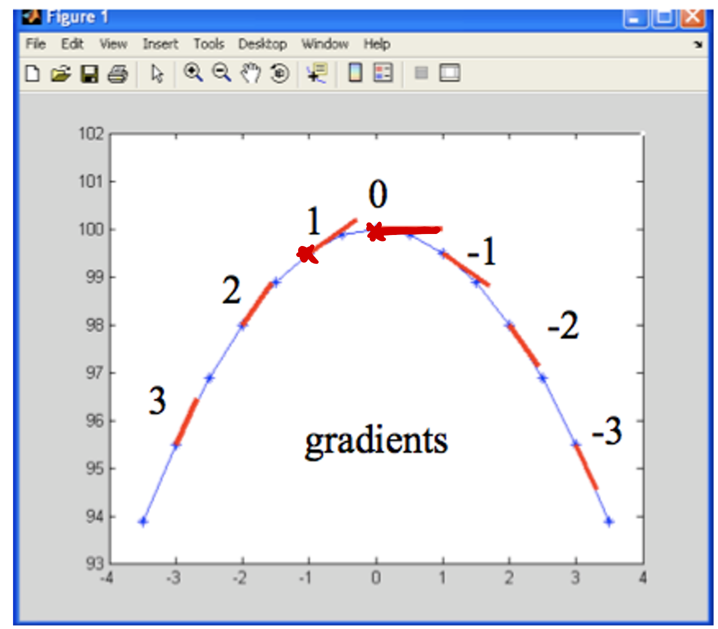
Consider function $f(x) = 100 - 0.5 * x^2$

Gradient: $\frac{d\,f(x)}{dx} = (-0.5) * 2 * x = -x$

Geometric interpretation: the gradient at x_0 is the slope of tangent line to the curve at point x_0



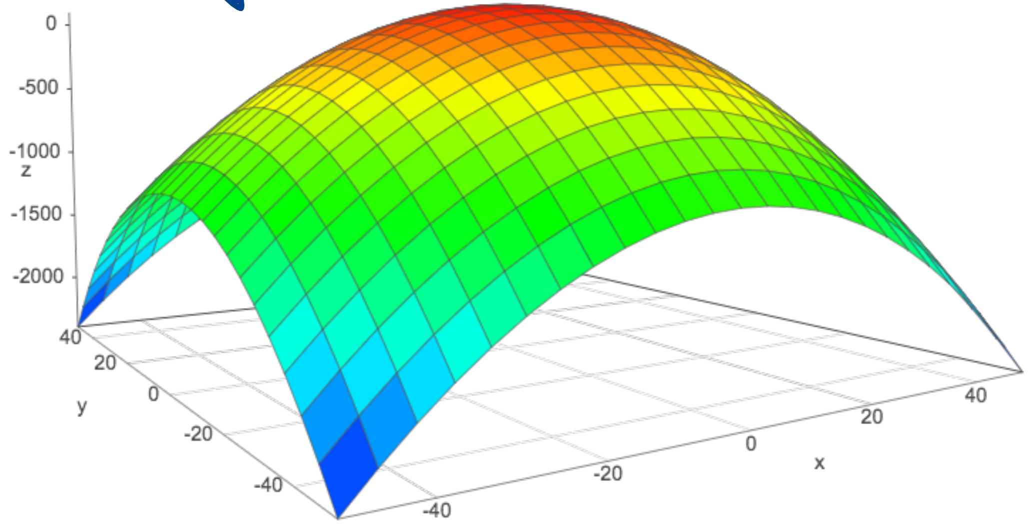
$$\text{slope} = \Delta y / \Delta x = df(x)/dx \Big|_{x_0}$$

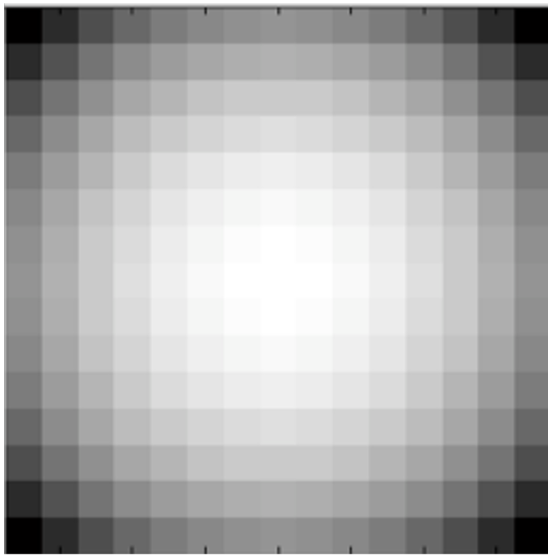


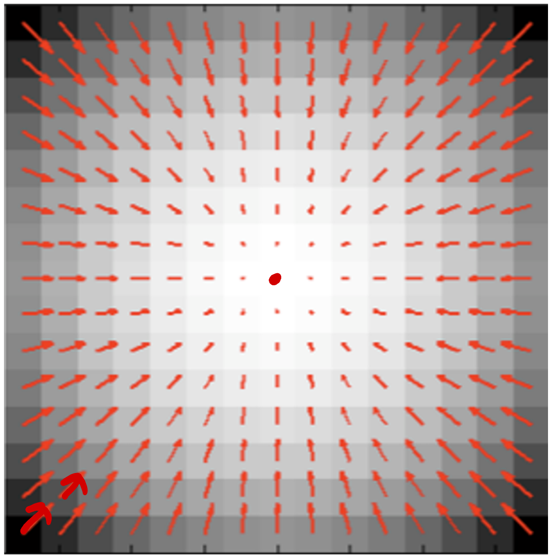
Gradients – 2D

col

row





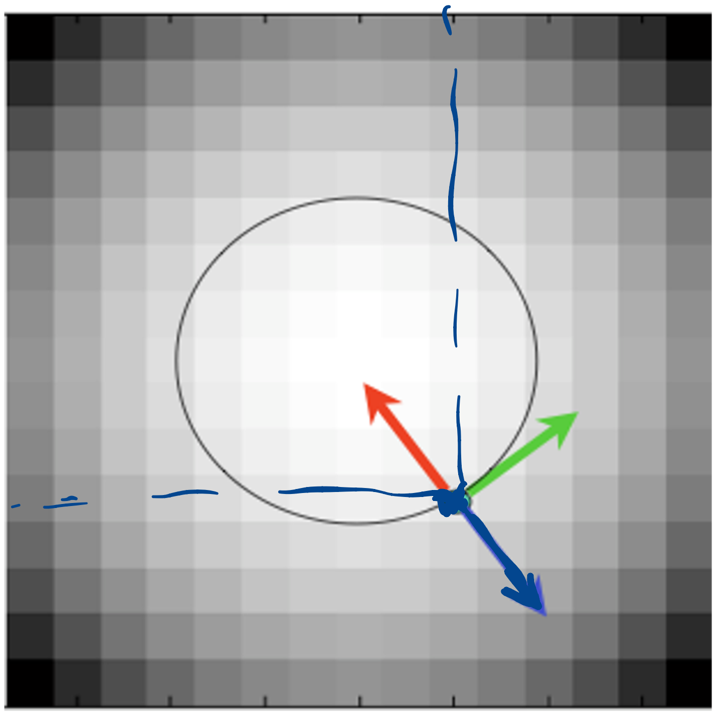


2D Gradients

$$f(x,y) = 100 - 0.5 * x^2 - 0.5 * y^2$$

x component → $$\frac{d\,f(x,y)}{dx} = (-0.5) * 2 * x = -x$$

y component → $$\frac{d\,f(x,y)}{dy} = (-0.5) * 2 * y = -y$$



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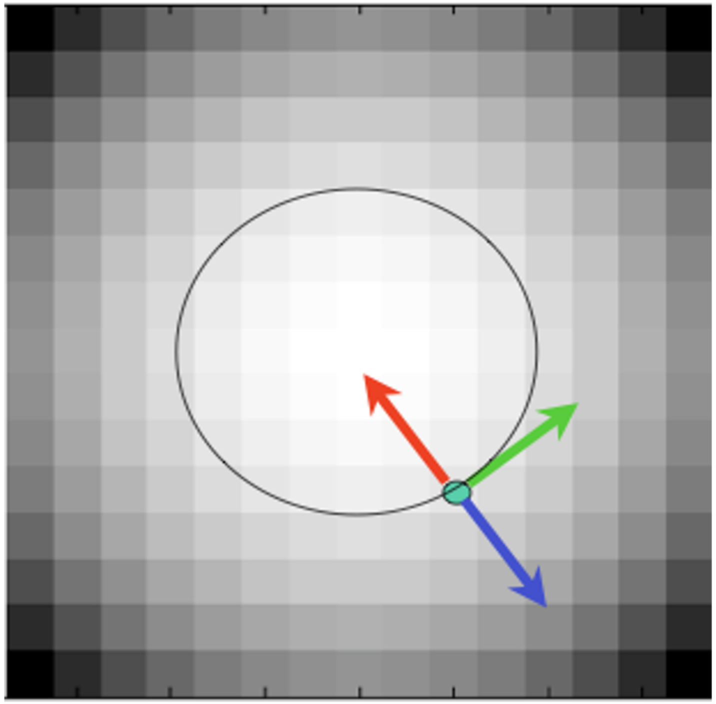


Let g=[g_x,g_y] be the gradient vector at point/pixel (x_0,y_0)

Vector **g** points uphill
(direction of steepest ascent)

Vector **-g** points downhill
(direction of steepest descent)

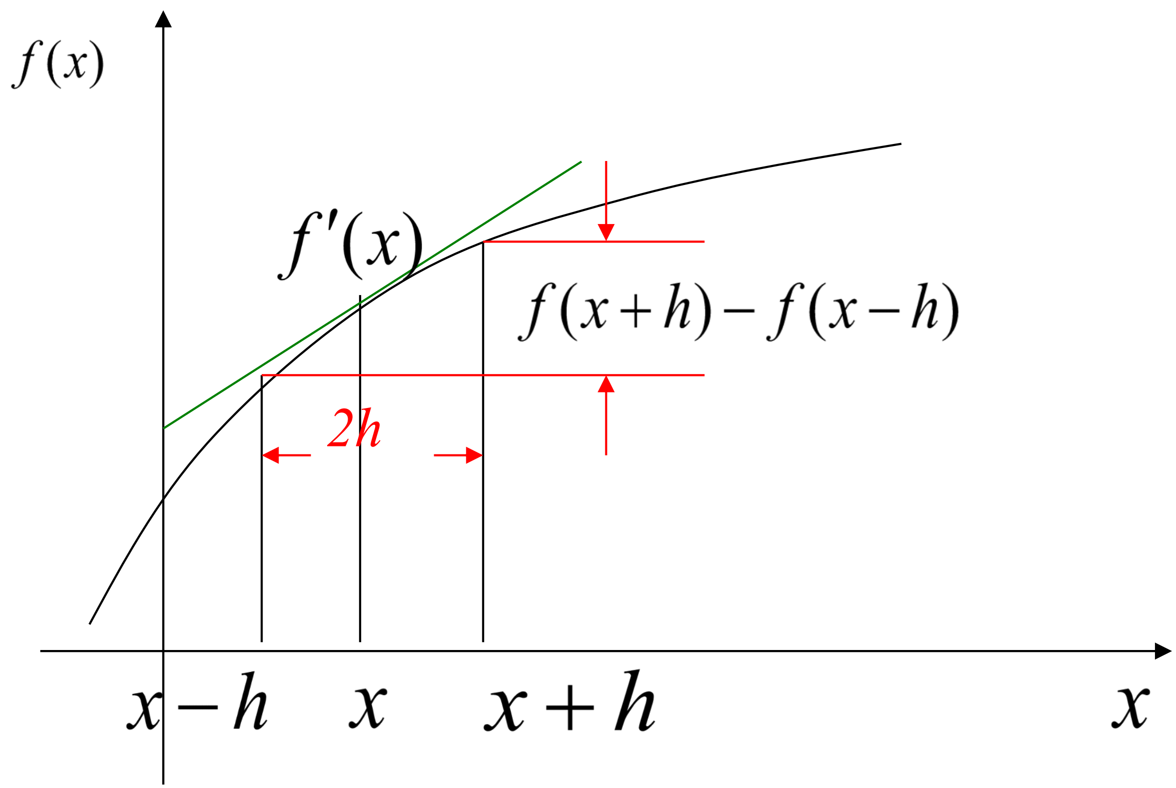
Vector [gy, -gx] is perpendicular to **g**
(direction of constant elevation)

Central Difference Formula for $f'(x)$

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

h - small

Geometrically



Introducing: Spatial Image Gradients

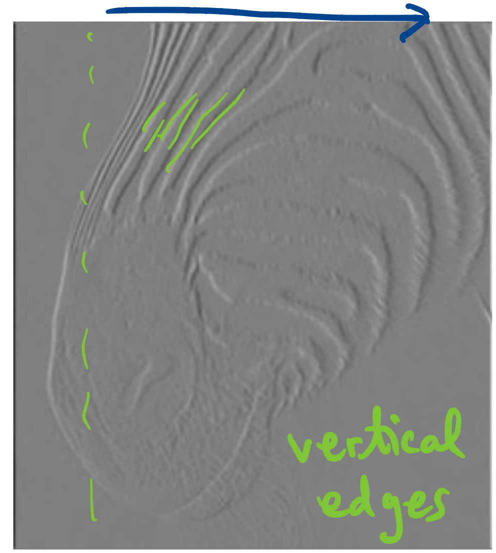
I

$I(x, y)$

$\boxed{h = 1}$

$$I_x = \frac{I(x+\overset{h}{\overset{\downarrow}{1}}, y) - I(x-\overset{h}{\overset{\downarrow}{1}}, y)}{2}$$

Partial derivative wrt x



$$I_y = \frac{I(x, y+\overset{h}{\overset{\downarrow}{1}}) - I(x, y-\overset{h}{\overset{\downarrow}{1}})}{2}$$

Partial derivative wrt y

Example:

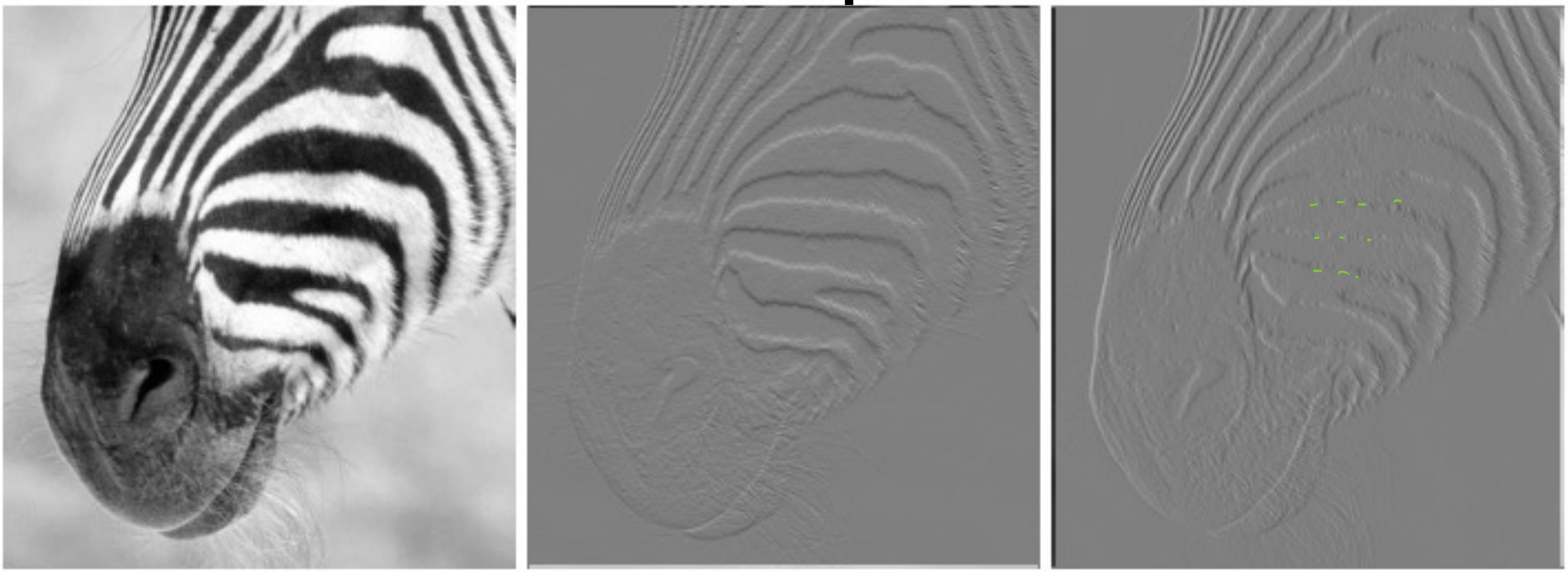
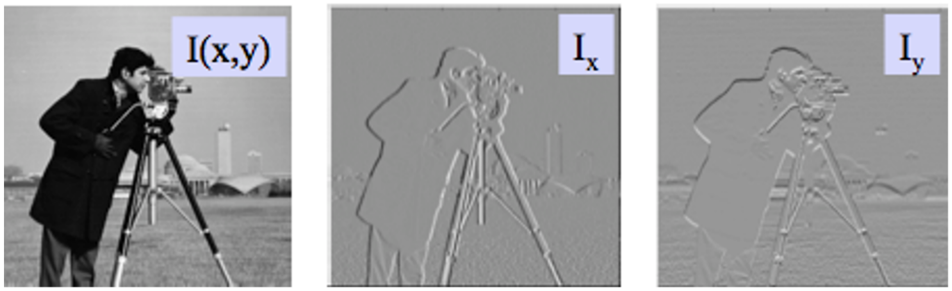


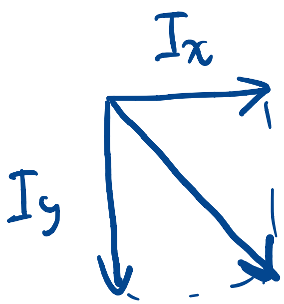
Figure 4.4 … shows estimates of derivatives obtained by finite differences. The image at the left shows a detail from a picture of a zebra. The center image shows the *partial derivative in the y-direction*—which responds strongly to horizontal stripes and weakly to vertical stripes—and the right image shows the *partial derivative in the x-direction*—which responds strongly to vertical stripes and weakly to horizontal stripes.

Functions of Gradients

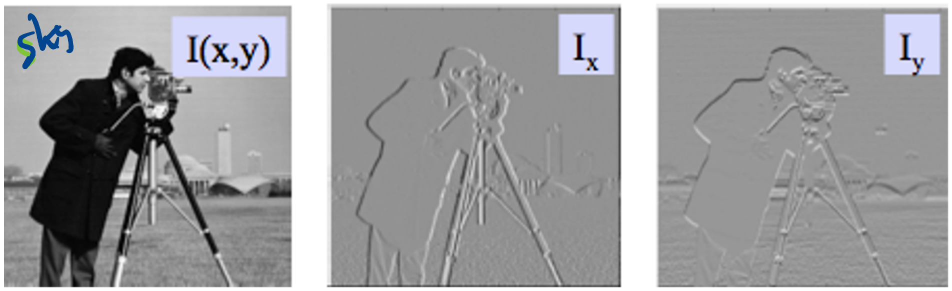


Magnitude of gradient

sqrt ($I_x^2 + I_y^2$)



Functions of Gradients



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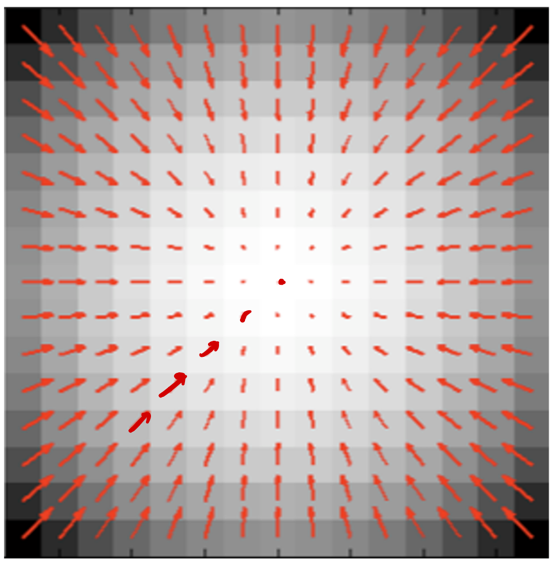


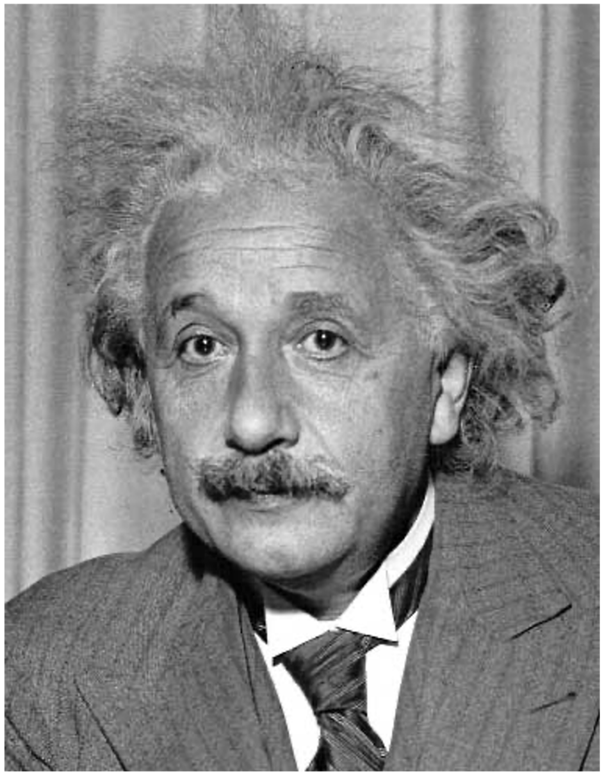
Gradients as linear operators

Gradients are an example of **linear operators**, i.e. the value at a pixel is computed as a linear combination of values of neighboring pixels.

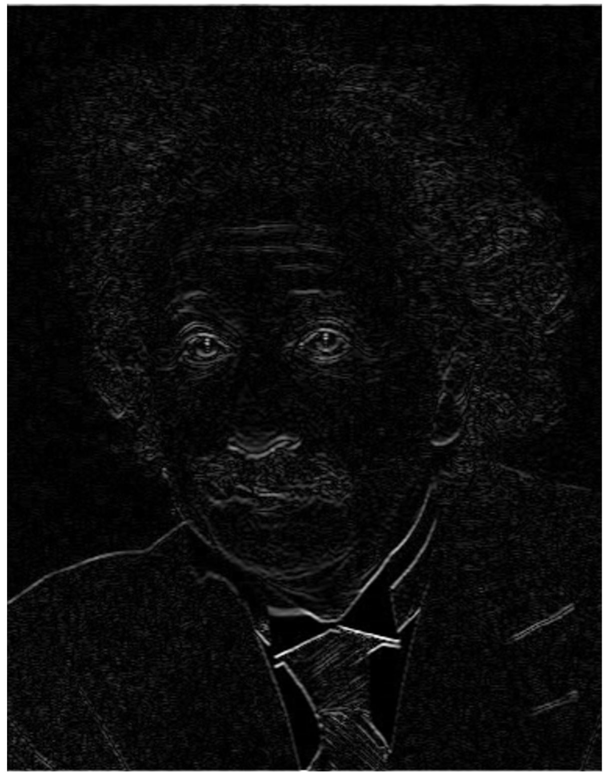
$$\begin{array}{c|c} & x \rightarrow \\ \hline y \downarrow & \begin{bmatrix} 0 & 0 & 0 \\ -1/2 & \boxed{0} & 1/2 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c|c} & x \\ \hline y & \begin{bmatrix} 0 & -1/2 & 0 \\ 0 & 0 & 0 \\ 0 & 1/2 & 0 \end{bmatrix} \end{array}$$

$$I_x = \frac{I(x+1, y) - I(x-1, y)}{2} \qquad I_y = \frac{I(x, y+1) - I(x, y-1)}{2}$$

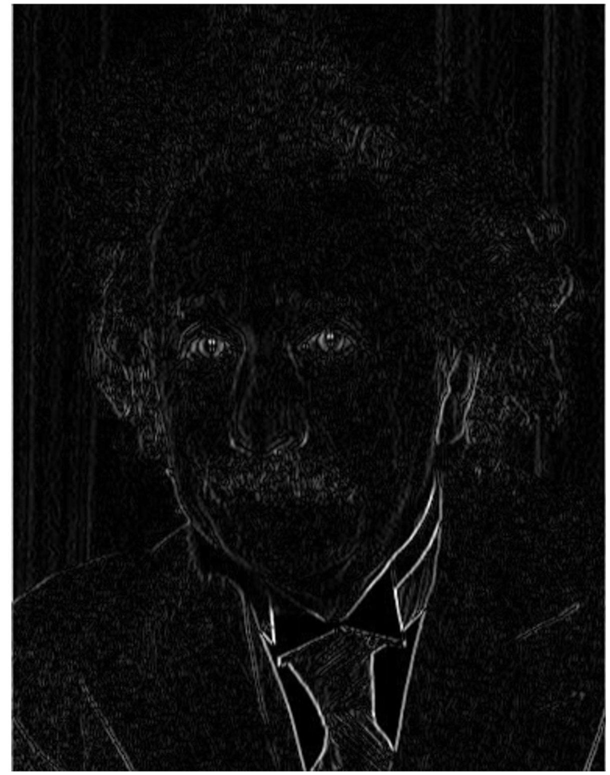
Examples – Sobel operator



original image



horizontal edges



vertical edges

Sobel
Prewitt
Roberts

1	2	1
0	$\boxed{0}$	0
-1	-2	-1

1	0	-1
2	0	-2
1	0	-1