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Interferometry

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Inhaltsverzeichnis

1 Objective

In this experiment a Sagnac-interferometer is used in combination with a HeNe-laser is used to determine the refractive indices of glass and air. To achieve this the Interferometer is first adjustet and the visibility measured to guarantee a high quality in the interference pattern.

2 Theory

intererometry uses the effect of interference between lightwaves to make measurements for example regarding properties of certain materials like in this case the refractive indices of glass and air. To achieve interference several factors have to be taken into consideration, which will be discussed in the following chapter.

2.1 Coherence

When trying to observe interference between lightwaves it is important to consider, that interference occurs only with coherent light. This means, the different lightwaves have to have a constant phase relationship with each other to make interference possible. This property of light is called coherence and is distinguished between spatial and temporal coherence. Real lightsources emitt light in a certain spectrum and therefore constantly shift their phase, which means that a constant phase relation can only be achieved within a certain timeframe. Perfect monochromatic light theoretically is perfectly temporally coherent but can not be achieved in reality. Spatial coherence is achieved when wavefronts do not shift their phase difference due to spatial differences between the propagating wavefronts.

Coherence is a phenomenon that exists on a certain scale, meaning that light can not only be incoherent or perfectly coherent. This is quantified by the Degree of coherence:

$$\tilde{\gamma}_{12}(\tau) = \frac{\langle \tilde{E}_1(t+\tau)\tilde{E}_2^*(t)\rangle_T}{\sqrt{\langle|\tilde{E}_1|^2\rangle\langle|\tilde{E}_2|^2\rangle}} \tag{1}$$

with the electric field at two points E_1, E_2 and the relative offset τ. Here $\langle ...\rangle_T$ denotes the time average over an Intervall T. The degree of coherence takes up values between 1 and 0 with $\gamma_{12} = 1$ indicating perfect coherence.

2.2 Polarisation

Another important factor in interference of transversal waves like lightwaves is the polarisation of the wave. This describes the orientation of the plane in which the electric filed oscillates. The most important distinction in terms of polarisation is between linear and circluar/elliptical polarisation where the electric field vector is not polarised in a single plane but rotates. Linearly polarised light can be described by the polarization angle and generated from unpolarized light by using a polarisation filter, which only allows the component with the corresponding polarization angle to pass. Linearly polarized lightbeams can only produce interference if they have the same polarization angle. A

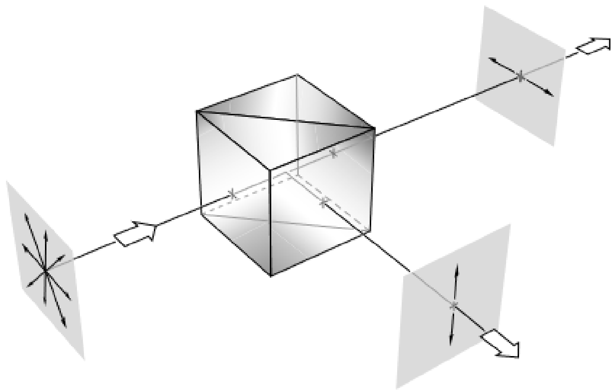

Abbildung 1: Depictioin of a Polarising beam-splitter cube splitting unpolarised light into the s-polarised and p-polarised components. [**Hecht_2018**]

polarizing beam-splitter cube (PBSC) can be used to split light with a certain polarization into two perpendicular components. The transmitted beam is then polarized parallel (p-polarization) and the reflected beam perpendicular (s-polarization) to the incident plane. The effect of a PBSC on unpolarized light is depicted in Figure 1.

2.3 Visibility

A metric to describe the quality of an interference pattern is the so called visibility:

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \tag{2}$$

which quantifies the contrast between the minimum Intensity I_{min} and maximum Intensity I_{max}. The intensity is proportional to the square of the electric field strength $I \propto E^2$.

More precisely the intensity of two interfering lightbeams can be described as:

$$I \propto \langle |E_1 \cos(wt) + E_2 \cos(wt + \delta)|^2 \rangle$$

Assuming a phase difference of $\delta_{max} = n2\pi$ for complete constructive interference at the maximum and $\delta_{min} = (2n + 1)\pi$ with $n\epsilon N^0$ for destructive interference at the minimum and consequently using $\cos(wt + \delta_{max}) = \cos(wt)$ and $\cos(wt + \delta_{min}) = -\cos(wt)$ the corresponding intensities can be calculated:

$$I_{max/min} \propto \langle \cos^2(wt)(E_1 \pm E_2)^2 \rangle$$
$$\iff I_{max/min} \propto E_1^2 \pm 2E_1E_2 + E_2^2$$

The strengths of the electric fields E_1 and E_2 is determined by the polarisation angle Φ of the initial light beam with the electric field strength E_0 before beeing split by the beam-splitter cube:

$$\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} = E_0 \begin{pmatrix} \cos(\Phi) \\ \sin(\Phi) \end{pmatrix}$$

Inserting this into the previous equation yields:

$$I_{max/min} \propto E_0^2(1 \pm 2\sin(\Phi)\cos(\Phi)) \tag{3}$$

When inserting the results for the Intensities into the visibility it can be expressed depending only on the polarization Angle:

$$V(\Phi) = 2\sin(\Phi)\cos(\Phi) \tag{4}$$

2.4 Refractive indicex of glass

Using a glass holder holding two glass plates placed in the interferometer the refractive index of the glass can be determined based on the number of observed maxima or minima:

$$M = \frac{\Delta\phi}{2\pi}$$

with the phase shift $\Delta\phi$ depending on the rotation angle of the glass plate Θ and the refractive index of the glass n_{glass}:

$$\Delta\phi = \frac{2\pi}{\lambda_{vac}} D(\frac{n_{glass} - 1}{2n_{glass}} \Theta^2 + O(\Theta^4)) \tag{5}$$

With the vacuum-wavelength of light λ_{vac} and the thickness of the glass plate D. Because the plates in the glass holder are already rotated by an angle of $\pm 10°$ the formula for the phase shift has to be slightly changed and results in a linear relationship between the number of Maxima and the rotation angle:

$$M = \frac{D}{\lambda_{vac}} \frac{n_{glass} - 1}{2n_{glass}}((\Theta + \frac{\pi}{18})^2 - (\Theta - \frac{\pi}{18})^2) = \frac{D}{\lambda_{vac}} \frac{n_{glass} - 1}{n_{glass}} 2\Theta \frac{\pi}{18} \tag{6}$$

$$n = \frac{1}{1 - \frac{M\lambda_{vac}}{D2\Theta\frac{\pi}{18}}} \tag{7}$$

2.5 Refractive index of air

The refractive index of air can be determined analogously by placing a gas cell of Length L in the beam. This again results in a phase shift of

$$\Delta\Phi = \frac{2\pi}{\lambda_{vac}}(n_{air} - 1)L$$

corresponding to a refractive index of:

$$n_{air} = \frac{M\lambda_{vac}}{L} + 1 \tag{8}$$

Alternatively the Refractive index of air can also be determined using the Lorentz-Lorentz-Law. Applying a Taylor series by $n = 1$ a linear dependency can be approximated:

$$\frac{A \cdot p}{R \cdot T} = \frac{n_{air}^2 - 1}{n_{air}^2 + 1} \approx \frac{3}{2}(n-1) \tag{9}$$

which describes the relationship between refractive index, pressure p and temparature T. In this case A denotes the molrefraction value and R the universal gas constant. The refraction index can therefore be determined as

$$n \approx \frac{3}{2}\frac{A \cdot p}{R \cdot T} + 1 \tag{10}$$

3 Measurement

For this experiment a Sagnac-Interferometer is chosen because of its high resistance against external disturbances. A schematic picture of the interferomter used can be seen in Figure 2. The light for the interferometer is generated by a HeNe-Laser emitting linearly polarised light tilted by 45° from the vertical with a wavelength of 632,990 nm.

3.1 Adjustment

To be able to properly use the interferometer for measurements it first has to be adjustet. For this purpose adjustement plates are used to stepwise adjust the interferometer.

First the mirror $M1$ is adjusted so that the laser beam hits the center of Mirror $M2$. This mirror can then be positioned in a way, that the beam transmitted by the PBSC passes through the center of the adjustment plates in positions 1 and 2. Afterwards the reflected beam is adjusted to pass the center of adjustment plates in positions 8 and 9 by moving the PBSC itself. Then the mirrors Ma and MC are adjusted using plates in positions 5 and 6. In a last step the mirror Mb can be adjusted by using plates in the positions 7 and 4. With all adjustement steps finished a polarization filter has to be introduced to the interfering beams to synchronize their polarization angle. The resulting picture should then still show no interference fringes if the interferometer is perferctly adjusted. By moving the mirror $M2$ away from the adjusted central position the two overlapping beams can be separated so that each beam can be manipulated individualy.

3.2 Measuring the visibility

To measure the visibility a glass holder with two tilted glass plates at an adjustable angle is introduced to the beam. Each plate has a thickness of $D = 1\,\text{mm}$. The polarisation

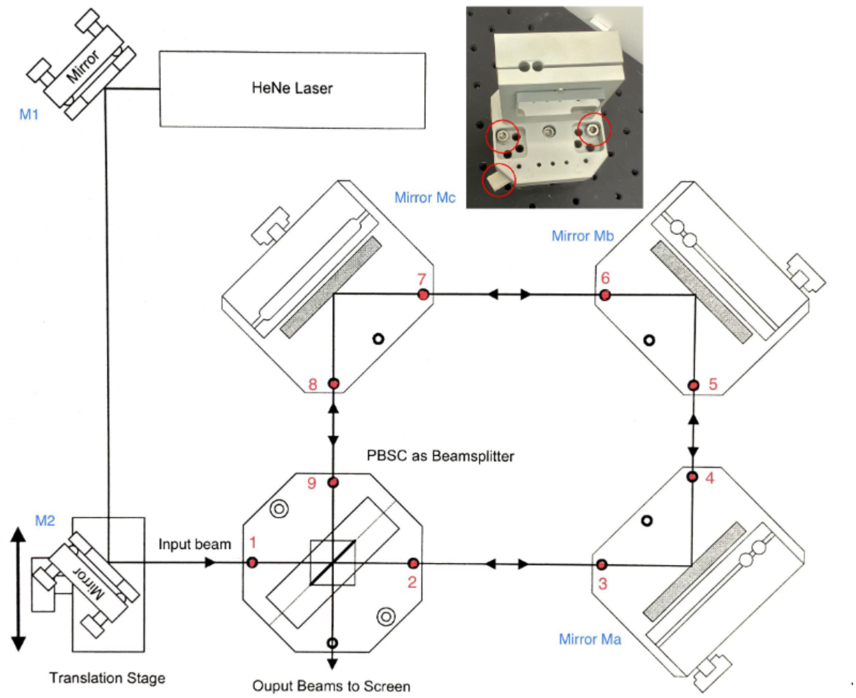

Abbildung 2: Schematic picture of the Sagnac-Interferometer used for the purpose of this experiment. [**V64**]

angle of the input beam is also additionally controlled by a polarisation filter in front of the PBSC. By utilizing the angle of the glass plates the interference pattern can now be changed to either show a minimum or a maximum. The intensity is measured using the output voltage of a photo-diode. The polarisation of the input beam is then increased in 10° steps from 0° to 180° and for each angle the intensity at the minimum and the maximum are recorded. This measurement is repeated three times.

3.3 Refractive index of glass

After the visibility is measured in the previous step the polarisation angle of the input beam is set to guarantee the maximal visibility. For the measurements regarding the refractive indices the difference voltage method is used with two diodes. The angle of the glass plates is then increased from 0° to 10° and an automatic counter in combination with an oscilloscope is used to measure the number of Maxima. This procedure is repeated ten times.

3.4 Refractive index of air

Instead of the glass holder a gas cell with a length of $L = (100{,}0 \pm 0{,}1)\,\mathrm{mm}$ is then introduced to the beam. The cell can be avacuated using a pump and then slowly filled with air again. The number of Maxima is recorded in steps of 50 mbar. This measurement is repeated five times.

4 Analysis

4.1 Kontrast

In order to study the usage of interferrometrie to determine refraction indices, first of all the kontrast of the used Sagnac Interferometer was calculated. The kontrast was calculated with equation (2) and the values of table 1. Table 1 also contains the calculated kontrast values. The maximum contrast K = 0.92 was measured at $\varPhi = 130°$ Therefore the polarization filter was set to $\varPhi = 130°$ for the following measurements. In addition, a fit of the form

$$K = A \cdot |\cos(\varPhi)\sin(\varPhi)| \tag{11}$$

is performed with the mean values of the measured values. This can be seen for the determined value of $A = 1.76 \pm 0.07$ in graph 3.

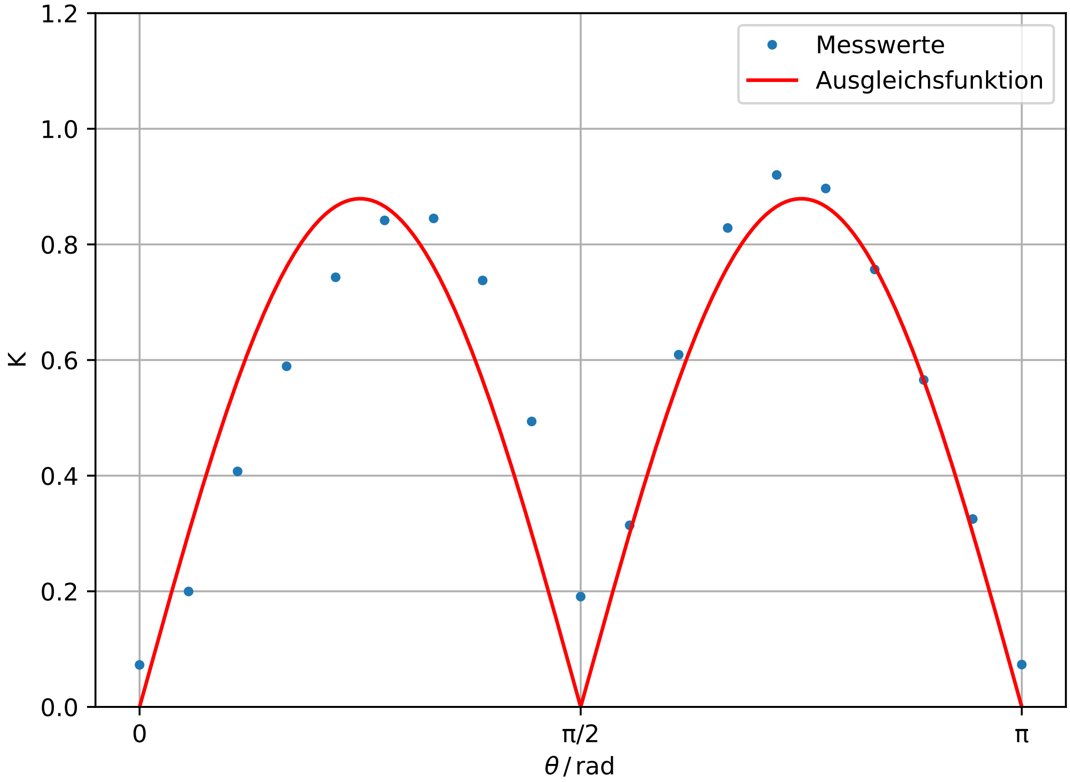

Abbildung 3: The measured values of the contrast according to equation (2) and the fit calculation according to equation (13).

Tabelle 1: Recorded measured values for contrast measurement, as well as the respective contrast value.

$\Phi/°$	$U_{\text{min},1}$/V	$U_{\text{max},1}$/V	K_1	$U_{\text{min},2}$/V	$U_{\text{max},2}$/V	K_2	$U_{\text{min},3}$/V	$U_{\text{max},3}$/V	K_3
0	1.73	1.98	0.06	1.8	2.1	0.07	1.76	2.04	0.07
10	1.21	1.8	0.19	1.16	1.78	0.21	1.24	1.83	0.19
20	0.69	1.65	0.41	0.7	1.63	0.39	0.69	1.66	0.41
30	0.36	1.44	0.60	0.39	1.46	0.57	0.38	1.47	0.58
40	0.21	1.48	0.75	0.23	1.47	0.72	0.21	1.46	0.74
50	0.14	1.7	0.84	0.15	1.64	0.83	0.14	1.66	0.84
60	0.17	1.98	0.84	0.16	1.93	0.84	0.16	1.92	0.84
70	0.37	2.35	0.72	0.32	2.15	0.74	0.33	2.25	0.74
80	0.85	2.36	0.47	0.7	2.22	0.52	0.79	2.31	0.49
90	1.64	2.36	0.18	1.48	2.28	0.21	1.62	2.33	0.17
100	1.43	2.93	0.34	1.46	2.68	0.29	1.49	2.79	0.30
110	1.08	4.13	0.58	0.92	3.78	0.60	0.91	4.06	0.63
120	0.52	5.06	0.81	0.49	5.06	0.82	0.44	5.36	0.84
130	0.24	5.53	0.91	0.25	5.56	0.91	0.21	5.76	0.92
140	0.34	5.92	0.89	0.33	5.91	0.89	0.3	5.99	0.90
150	0.73	5.26	0.75	0.77	5.39	0.75	0.73	5.45	0.76
160	1.16	4.13	0.56	1.22	4.25	0.55	1.16	4.38	0.58
170	1.63	3.18	0.32	1.67	3.19	0.31	1.62	3.29	0.34
180	1.86	2.08	0.05	1.85	2.18	0.08	1.79	2.11	0.08

4.2 Refraction index of glas

To determine the refractive index of glass, the number of intensity maxima M was recorded. The refractive index was determined using the equation (9). The thickness of the plates is $D = 1$ mm, the wavelength of the laser $\lambda_{\text{vac}} = 632.990$ nm. The measured values, as well as the refractive index determined in each case, can be found in Table 2. On average, the refractive index determined for glass is

$$n_{\text{Glas}} = 1.64 \pm 0.13.$$

Tabelle 2: AMeasured values to determine the refractive index of glass and the determined refractive index.

Durchgang	M	n_{Glas}
1	38	1.652
2	38	1.652
3	38	1.652
4	37	1.625
5	38	1.652
6	38	1.652
7	37	1.625
8	38	1.652
9	38	1.652
10	37	1.625

4.3 Refraction index of air

To determine the refractive index of air, the measured values were recorded as described in chapter Abschnitt 3 and determined using equation (10). The length of the gas chamber is L = (100.0 ± 0.1) mm and the temperature T = 20.6 °C . The recorded values as well as the calculated reflection indices are shown in Table 3.

Tabelle 3: Measured values recorded to determine the refractive index of air next to the refractive index calculated according to equation (10). Here, M_i denotes the number of interference minima or maxima that have passed up to that point, where i indicates the passage.

p/mbar	M_1	n_1	M_2	n_2	M_3	n_3	M_4	n_4
50	2	1,000 012 66	2	1,000 012 66	2	1,000 012 66	3	1,000 018 99
100	4	1,000 025 32	4	1,000 025 32	4	1,000 025 32	5	1,000 031 65
150	7	1,000 044 31	6	1,000 037 98	6	1,000 037 98	7	1,000 044 31
200	9	1,000 056 97	8	1,000 050 64	8	1,000 050 64	9	1,000 056 97
250	11	1,000 069 63	10	1,000 063 30	10	1,000 063 30	11	1,000 069 63
300	13	1,000 082 29	12	1,000 075 96	12	1,000 075 96	13	1,000 082 29
350	15	1,000 094 95	15	1,000 094 95	15	1,000 094 95	15	1,000 094 95
400	17	1,000 107 61	17	1,000 107 61	17	1,000 107 61	18	1,000 113 94
450	20	1,000 126 60	19	1,000 120 27	19	1,000 120 27	20	1,000 126 60
500	22	1,000 139 26	21	1,000 132 93	21	1,000 132 93	22	1,000 139 26
550	24	1,000 151 92	23	1,000 145 59	23	1,000 145 59	24	1,000 151 92
600	26	1,000 164 58	25	1,000 158 25	25	1,000 158 25	26	1,000 164 58
650	28	1,000 177 24	27	1,000 170 91	27	1,000 170 91	28	1,000 177 24
700	30	1,000 189 90	30	1,000 189 90	29	1,000 183 57	30	1,000 189 90
750	32	1,000 202 56	31	1,000 196 23	32	1,000 202 56	33	1,000 208 89
800	35	1,000 221 55	34	1,000 215 22	34	1,000 215 22	35	1,000 221 55
850	37	1,000 234 21	36	1,000 227 88	36	1,000 227 88	37	1,000 234 21
900	39	1,000 246 87	38	1,000 240 54	38	1,000 240 54	39	1,000 246 87
950	41	1,000 259 53	40	1,000 253 20	40	1,000 253 20	41	1,000 259 53

4.4 Lorentz-Lorenz law

Since the refractive index also dependents on temperature and pressure according to the Lorentz-Lorenz law, an fit calculation is carried out according to equation 12 . The fit has the form

$$n = \frac{a}{TR} \cdot p + b \tag{12}$$

The temperate is T = 20.6 °C and R describes the universal gas constant. This results in the in table 4 shown values for a and b.

Tabelle 4: The results of the fit for the variables a and b for each run.

Messung	$a/\left(10^{-2}\mathrm{m}^3/\mathrm{mol}\right)$	b
1	0.00066776 ±0.00000394	1.00000056 ±0.00000092
2	0.00065800 ±0.00000397	0.99999789 ±0.00000093
3	0.00065854 ±0.00000364	0.99999778 ±0.00000085
4	0.00066017 ±0.00000370	1.00000344 ±0.00000086

The fits and thus the calculated refractive indices are shown in Figure 4 . The average values for the variables are:

$$a = 0.0006611 \pm 0.0000019 \frac{m^3}{\mathrm{mol}}$$
$$b = 0.9999999 \pm 0.0000004.$$

According to the Loretz-Lorenz law, this results in the following refractive index in a normal atmosphere with T = 15 °C and p = 1013 hPa:

$$n = 1.0002795 \pm 0.0000009 \tag{13}$$

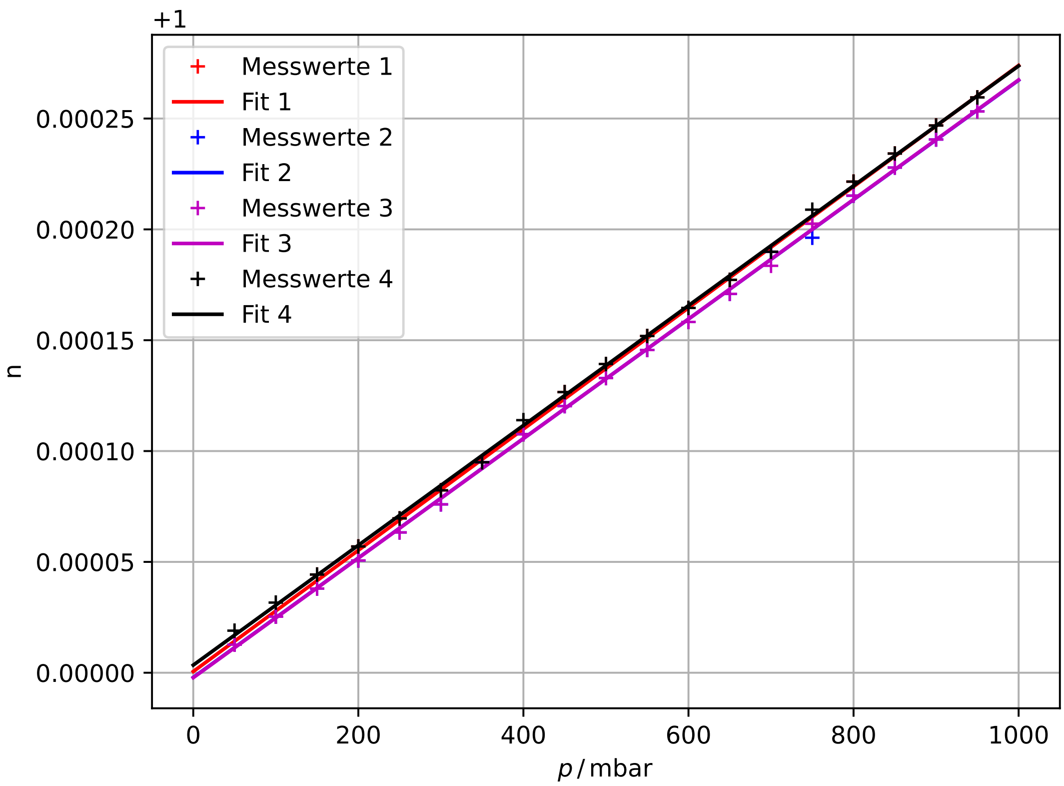

Abbildung 4: The calculated refractive indices n for air and the fit.

5 Diskussion

The measured values and the results obtained are in line with the expectations. An overview of the determined and theoretical values, as well as the respective deviation, can be found in table Tabelle 5. The determined contrasts follow the expected distribution and no major deviations are recognizable. Nevertheless, the contrast is not perfect. This could be due to the imperfect alignment of the Sagnac-interferometer, which prevents the intensity from dropping to zero at a minimum. As expected, the extremes can also be found at multiples of 45°.

For the refractive index of glass, $n_{\text{glass}} = 1.64 \pm 0.13$ was determined. The theoretical value is $n_{\text{glass, theo}} = 1.45$ [**Brechungsindex**]. The determined value therefore has a deviation of 13,10 %. One reason for this slightly higher deviation may be due to the different measurement method which seems to be less precise.

The theoretical value for air is $n_{\text{air, theo}} = 1.000292$ [**Brechungsindex**]. Averaged over all measurement series, the refractive index of air in a standard atmosphere is $n_{\text{air}} = 1.0002795 \pm 0.0000009$, which represents a deviation of $\ll 1\%$ from the theoretical value.

Tabelle 5: The refractive indices determined for glass and air compared to the respective theoretical values.

	n_{Glas}	n_{Luft}
Theorie	1,45	1,000292
Versuch	1.64 $\pm$0.13	1.0002795 $\pm$0.0000009
Abweichung	13.10 %	0.0012 %