Question 1.

a. $5n^3+2n^2+3n = O(n^3)$

According to the definition, $f(n) \le c \cdot O(g(n))$ for all $n > n_0$.

By setting $n_0 = 3$ and $C = 6$ works because

$$5 \cdot 6^3 + 2 \cdot 6^2 + 3 \cdot 6 < 6 \cdot 3^3$$

So, $5n^3+2n^2+3n = O(n^3)$

b. $\sqrt{7n^2+2n-8} = \theta(n)$

According to the definition of $\theta(n)$, ~~f(n)=~~ C

$C_1\theta(n) \le f(n) \le C_2\theta(n)$ for all $n > n_0$.

By setting $C_1 = 1$, $C_2 = 2$, and $n_0 = 1$ works because

$$1 \le 7+2-8 \le 2,$$

So $\sqrt{7n^2+2n-8} = \theta(n)$

c. If $d(n) = O(f(n))$, then there exists C_1 ($C_1 > 1$) for that $d(n) \le C_1 \cdot f(n)$. The same, if $e(n) = O(g(n))$, there exists C_2 ($C_2 > 0$) for that $e(n) \le C_2 \cdot g(n)$.

Therefore, $C_1 \cdot C_2 \cdot f(n) \cdot g(n) \ge d(n)e(n)$ must be true, which shows that $d(n)e(n) = O(f(n)g(n))$.

Question 2

def 1 = $\theta(n^2)$

def 2 = $\theta(n)$

def 3 = ~~$\theta(\log n)$~~ $\theta(\sqrt{n})$

def 4 = $\theta(n)$