

# Using Particle Swarm Optimization for Fuzzy Antecedent Parameter Identification in Active Suspension Control

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**Abstract**—This paper addresses fuzzy parameter identification by using Particle Swarm Optimization (PSO) techniques with an application to active suspension control. In particular, the target fuzzy controller is a zero-order Takagi-Sugeno system with a standard fuzzy partition (SFP) of its antecedent variables. The major contribution of this paper with respect to previous works is that learning of the fuzzy suspension control is not limited to the scale factors of the input-output variables. Thus, the proposed approach allows optimization of SFP triangular membership functions for the antecedents with a manageable dimension of the search space. This method has been successfully applied to control a quarter-car test rig.

## I. INTRODUCTION

Active and semi-active suspension systems represent a relevant control problem in the automotive field. These systems present inherent difficulties, such as their non-linearity and the conflict between comfort and driving response, which make them an interesting target problem for new control design strategies [1][2][3].

Fuzzy logic control allows expressing heuristic knowledge in the form of rules while offering appropriate features for complex systems [4][5]. However, the adjustment of fuzzy control parameters to implement these fuzzy rules is not trivial [6], so meta-heuristic optimization techniques such as simulated annealing [7], genetic algorithms [8] and particle swarm optimization (PSO) can be useful tools for fuzzy control tuning.

In this work, we focus on PSO, which is a bio-inspired optimization strategy that simulates the social behaviour of individuals in nature, like in banks of fish or bee swarms [9]. PSO can improve performance regarding convergence and computation time with respect to other stochastic methods [10]. This is why it has been applied to a variety of engineering problems, including proportional-integral-derivative (PID) parameter tuning for suspension systems [11].

Fuzzy parameter identification with PSO has been addressed in recent works. For instance, [12] uses PSO to tune a fuzzy controller with symmetric partitions that only requires three parameters. In [13], PSO is used for data clustering to obtain the antecedent parameters of a fuzzy Takagi-Sugeno (TS) [14] controller. Furthermore, online identification of scalar TS consequents has been addressed with hierarchical PSO [15].

Regarding active suspension, PSO optimization has been considered to tune PID control parameters [1]. Furthermore,

PSO has been proposed to optimize a prediction model used in combination with a fuzzy controller of the suspension system [16]. As for fuzzy control parameter adjustment, PSO has been applied to identify scale factors for a fuzzy system with predefined membership functions (MF) [17][10].

The major contribution of this paper is a novel application of PSO to optimize membership function parameters for fuzzy a fuzzy controller. In particular, we consider a zero order Takagi-Sugeno fuzzy inference systems where antecedents are defined as a standard fuzzy partition (SFP) [18]. The proposed fuzzy identification method has been applied to control a quarter-car active suspension system. Experimental results, performed with a laboratory test rig, indicate an improvement in control performance with respect to optimized scale factors.

The remaining of the paper is organized as follows. The next section reviews the definition of standard fuzzy partitions. Section III introduces the proposed fuzzy parameter identification method with PSO. Section IV offers the application of the fuzzy controller to a quarter-car suspension system. Section V discusses experimental results. The paper is closed with the conclusions.

## II. TAKAGI-SUGENO WITH STANDARD FUZZY PARTITIONS

This work proposes the adjustment of antecedent membership function parameters for a Takagi-Sugeno fuzzy inference system. Fig. 1 illustrates the basic structure of this fuzzy system for the case of two input variables. A SFP is considered for the definition of the membership functions of each input variable. In this work, it is considered that the system has  $N_v$  input variables ( $x_1, \dots, x_i, \dots, x_{N_v}$ ) with  $M$

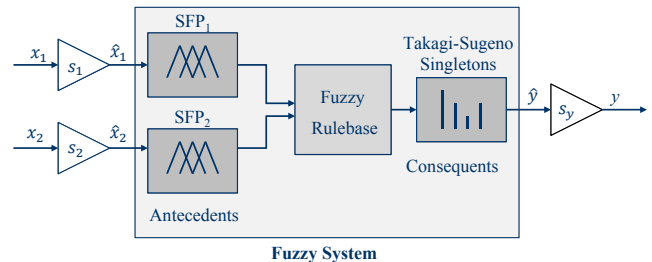


Fig. 1. Structure of a Takagi-Sugeno fuzzy systems with two SFP antecedents and scale factors.

partitions each. Moreover, it is assumed that the universe of discourse of the input and output variables is normalized in the  $[-1, 1]$  interval, which implies the use of scale factors for the inputs  $(s_1, \dots, s_i, \dots, s_{N_v})$  and the output  $s_y$  outside of the fuzzy system.

Let  $\mathbf{F}_j$  be the  $j$ -th MF defined over the universe of discourse of a given input variable  $x_i$ . Then, variable  $x_i$  has a SFP if, for all  $j$ ,  $\mathbf{F}_j$  is convex, normalized, and overlaps only with its neighboring MFs [18]:

$$\forall u \in U, \quad \sum_{\forall i} \mathbf{F}_j(u) = 1. \quad (1)$$

In the case of triangular MFs,  $\mathbf{F}_j$  is defined by the position of its three vertices, where the central one is the modal value  $f_j$ , as in the example shown in Fig. 2. The adoption of SFP reduces the total number of parameters required to tune the fuzzy MFs in the input variable. Thus, the  $M$  triangular MFs of a each input variable are defined by a vector of  $M - 2$  elements  $\mathbf{f}_i = [f_{i1} \dots f_{ij} \dots f_{i(M-2)}]$ . Note that the modal values  $f_{i0}$  y  $f_{i(M-1)}$  have not been included, as these correspond to both ends of the universe of discourse  $[-1, 1]$ .

### III. PSO IDENTIFICATION OF FUZZY PARAMETERS

PSO allows optimization of an objective function in an  $N$ -dimensional search space. To this end, it is necessary to define a set of  $n$  particles (i.e., candidate solutions) that will evolve in subsequent iterations of the algorithm outlined in Fig. 3. The PSO method admits non-derivable objective functions, has a relatively simple implementation, converges to reasonable solutions, and its random component is useful to avoid falling into local minima [9].

The evolution of the  $i$ -th particle depends on its position ( $p_i$ ) and velocity ( $v_i$ ) at each iteration ( $k$ ):

$$v_i(k+1) = wv_i(k) + c_1r_1(k)(L_i^{best} - p_i(k)) + c_2r_2(k)(G^{best} - p_i(k)) \quad (2)$$

$$p_i(k+1) = p_i(k) + v_i(k), \quad (3)$$

where  $w$  is the inertia weight for the particle swarm,  $c_1$  is a cognitive component that attracts the particles towards the best local position,  $c_2$  is a social component that drives the

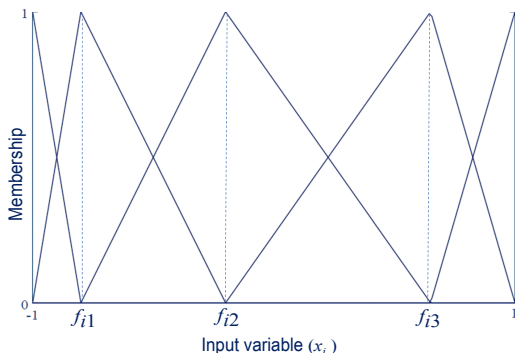


Fig. 2. Illustration of triangular fuzzy sets with SFP for  $M = 5$ .

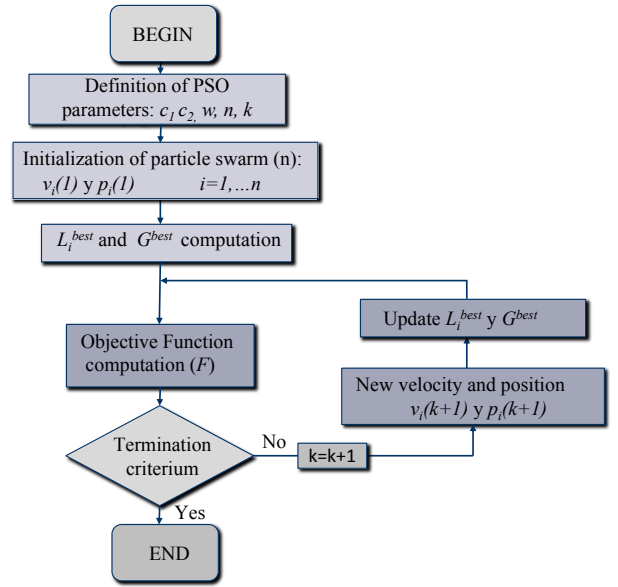


Fig. 3. Flow diagram for the PSO algorithm.

particles towards the best global position in the swarm,  $L_i^{best}$  is the local best known position found by the  $i$ -th particle,  $G^{best}$  is the global best, and  $r_1$  and  $r_2$  are random numbers in  $[0, 1]$ . Evolution continues until a termination criterium is met, such as reaching a maximum number of iterations  $k_{max}$ .

In the proposed solution for antecedent parameter identification, a particle is defined as vector  $[\mathbf{f}_1 \dots \mathbf{f}_{N_v}]$ , where the modal values of every  $\mathbf{f}_i$  are sorted in ascending order. This definition determines the dimension of the search space as:

$$N = N_v \cdot (M - 2). \quad (4)$$

### IV. FUZZY CONTROL OF AN ACTIVE SUSPENSION SYSTEM

The structure of active suspension system for a quarter-car vehicle with fuzzy control is shown in Fig. 4. In this system,  $z_s$  and  $z_{us}$  represent the displacement of the sprung mass and the unsprung mass, respectively, in response to a road perturbation  $z_r$ . The characteristics of the suspension systems are defined by stiffness coefficients ( $k_s$  and  $k_{us}$ ) and damping coefficients ( $B_s$  y  $B_{us}$ ).

As for the fuzzy controller, its control objective is to minimize the sprung mass acceleration ( $A_s$ ). With this purpose, two input variables are defined:  $e$ , representing the deviation of  $z_s$  with respect to  $z_r$  and  $e_r$ , which corresponds to the derivative of  $e$ . The output of the fuzzy system is actuator force  $F_a$ . All input variables are normalized in  $[-1, 1]$  by scaling factors.

A set of heuristic rules that is generally adopted for this problem [19][20] is summarized in Table I. This table requires  $M = 7$  fuzzy partitions for each input variable as well as seven linguistic terms for the output variable, which correspond to seven equally spaced singleton values in the case of Takagi-Sugeno inference. The seven

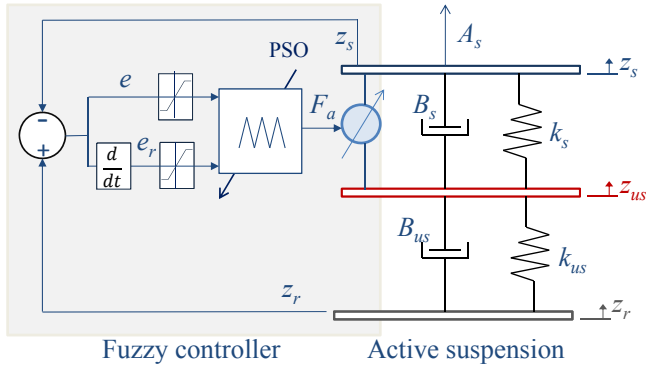


Fig. 4. Structure of the active suspension system for a quarter-car vehicle with fuzzy control.

TABLE I  
HEURISTIC RULES FOR ACTIVE SUSPENSION CONTROL.

		Error, $e$						
		PB	PM	PS	ZE	NS	NM	NB
Rate of error, $e_r$	PB	PB	PB	PB	PB	PM	PS	ZE
	PM	PB	PB	PB	PM	PS	ZE	NS
	PS	PB	PB	PM	PS	ZE	NS	NM
	ZE	PB	PM	PS	ZE	NS	NM	NB
	NS	PM	PS	ZE	NS	NM	NB	NB
	NM	PS	ZE	NS	NM	NB	NB	NB
	NB	ZE	NS	NM	NB	NB	NB	NB

linguistic terms  $NB, NM, NS, ZE, PS, PM, PB$  are distributed in ascending order within the universe of discourse of each input/output variable. Thus, (4) yields  $N = 10$  tuning parameters, which correspond to the modal values of  $NM, NS, ZE, PS, PM$  for input variables  $e$  and  $e_r$ .

On the other hand, the objective function for PSO has been defined as follows:

$$F_{obj} = \int E^2 + z_s^{max}, \quad (5)$$

where  $E$  is the mean squared error and  $z_s^{max}$  represents the maximum displacement for the suspended mass of the system.

## V. EXPERIMENTAL RESULTS

### A. Quarter-Car Test Rig

The laboratory plant considered in the experiments has been a quarter-car test rig by Quanser [21], which can be seen in Fig. 5. This system corresponds to the basic active suspension presented in Fig. 4. This plant consists of three steel plates: the top plate, which includes an accelerometer, simulates the sprung mass of the vehicle, the intermediate plate corresponds to the unsprung mass, and the bottom plate simulates the road. The positions of the sprung and unsprung masses are obtained from angular encoders. The system has two motors: the first is used to generate road disturbances and the second is the actuator for active suspension. The connection with the control computer is done via a data

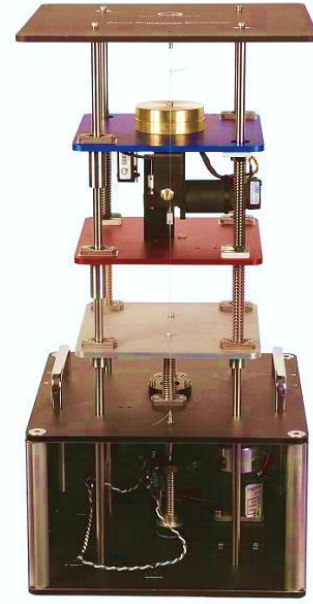


Fig. 5. Quarter-car test rig by Quanser [21]. Blue, red and grey plates simulate the sprung mass, the unsprung mass and the road, respectively.

TABLE II  
SUSPENSION SYSTEM PARAMETERS [21].

Parameter	Value
$K_s$	2000 N/m
$B_s$	2500 Ns/m
$K_{us}$	10 N/m
$B_{us}$	30 Ns/m

TABLE III  
PSO ALGORITHM PARAMETERS.

Parameter	Value
$n$	20
$k_{max}$	110
$N$	10
$w$	0.9
$c_2$	2.2
$c_1$	1.8

acquisition card. The parameters of this quarter-car test rig are summarized in Table II.

### B. Experiments

This section offers experimental results from the application of the fuzzy controller with PSO tuning of antecedent MFs with the Quanser suspension test rig. In this experiments, two different fuzzy controllers have been compared:

- 1) Scale factors only. A TS fuzzy controller where antecedents have a uniform fuzzy partition. In this controller, input and output scale factors have been optimized with PSO, as proposed in [22]. The resulting scale factors are:  $s_1 = 93.456$ ,  $s_2 = 30.604$  y  $s_y = 15.541$ .
- 2) MF optimization. Using the same scale factors as above, the approach presented in this paper has been

used to optimize the modal values of SFP triangular MFs.

The PSO algorithm is initialized with  $n$  particles which that are initially distributed randomly in the search space. The rest of the parameters in (2) have been determined considering values from previous works [22] as well as by empirical experimentation. An appropriate set of parameters has been set to obtain good convergence performance. Table III summarizes PSO parameters.

PSO optimization has been performed offline simulating the response of each candidate solution by means of a Matlab/Simulink model of the plant [21]. The response to two road perturbations was considered to evaluate each candidate solution: a road pot at  $t = 2s$  and a bump at  $t = 4s$ , which were implemented by a square signal with a period of 4s and an amplitude of  $\pm 0.01m$ . Furthermore, the response to road noise has also been evaluated by generating a white noise road perturbation corresponding to a road class roughness-C in terms of power spectral density [20]. The optimized fuzzy partitions produced by the PSO algorithm are shown in Fig. 6.

The sprung mass acceleration performance of the fuzzy control with the actual plant is shown in Fig. 7. The response to  $z_r$  perturbations corresponding to a pot and a bump indicates that the optimized MF parameters provide a smoother response than the scale-only optimization. Furthermore, the maximum acceleration and the settling time are also reduced.

The fuzzy controller with MF optimization also achieves improved results in the response of the sprung mass displacement, which is shown in Fig. 8. The figure indicates a

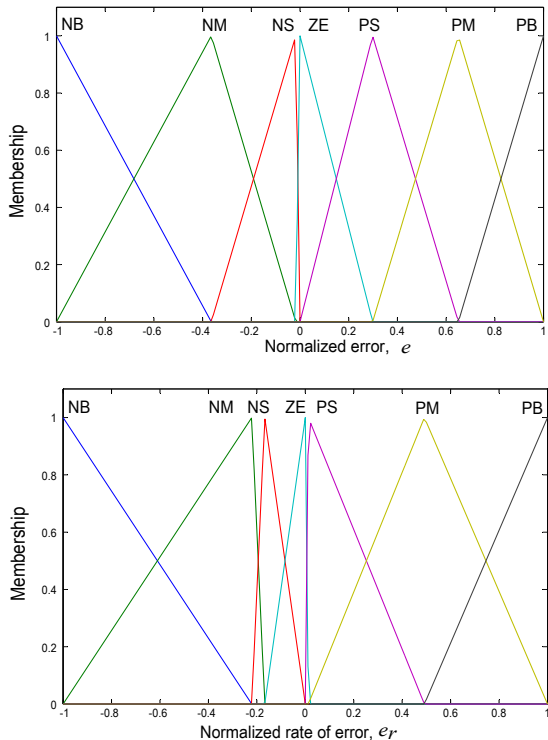


Fig. 6. Optimized fuzzy partitions for  $e$  (top) y  $e_r$  (bottom).

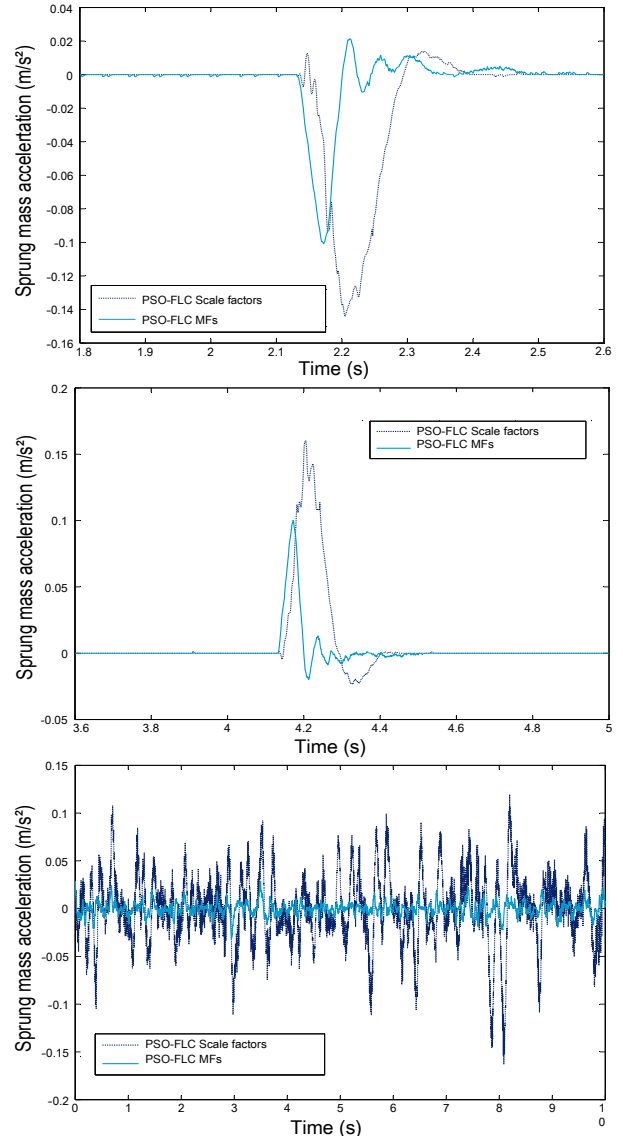


Fig. 7. Response of the sprung mass acceleration  $A_c$  for pot (top), bump (center) and noise (bottom) perturbations.

reduction in the peak value and overshoot of the sprung mass position for pot and bump road perturbations. The figure also shows differences between the stationaries of the two fuzzy controllers, which can be explained by some mechanical play in the test rig.

The response to road noise in  $z_r$  is also presented in Figs. 7-8. In this case, the controller with MF optimization achieves an improvement of about 20% in both acceleration and position of the sprung mass with respect to the scale-only controller. All in all, experiments confirm that the control performance can be improved by adjusting the antecedent fuzzy partitions with PSO optimization.

## VI. CONCLUSIONS

Experimental identification of fuzzy controllers can be limited by the dimension of the search space required to define a fuzzy system. In this work, we have proposed

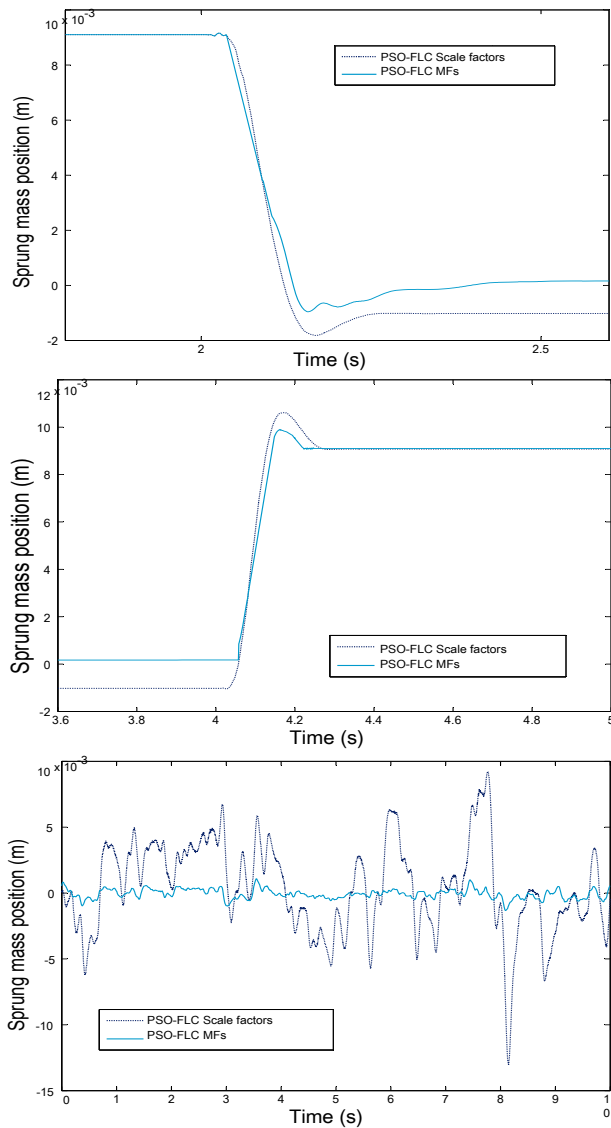


Fig. 8. Response of the sprung mass displacement  $z_s$  for pot (top), bump (center) and noise (bottom) perturbations.

the identification of fuzzy parameters by Particle Swarm Optimization (PSO) techniques, which has been applied to tune the fuzzy controller of an active suspension system. In particular, a zero-order Takagi-Sugeno system with standard fuzzy partition (SFP) of the antecedents has been considered. The SFP of triangular sets achieves a manageable set of tuning parameters, which allows that the membership functions can be optimized using meta-heuristic optimization techniques like PSO.

The proposed approach can be applied within a given universe of discourse. The paper has assumed a normalized universe of discourse for all variables, as well as given scale factors for all inputs and the output. The identification of scale factors with PSO optimization has been proposed in previous works. Thus, the method can be used as part of a two-step strategy in order to identify membership function parameters after scale factors have been determined.

The paper has offered experiments from a laboratory quarter-car test rig, where a fuzzy controller with two inputs and one output has been evaluated for three kinds of typical disturbances. In the experiments, the controller with optimized input partitions has been compared with an equivalent system with a uniform antecedent partition, where only the scaling factors had been optimized. Results indicate a noticeable improvement in performance when antecedent partitions are identified.

In future work, it will be interesting to evaluate the performance of alternative heuristic-based optimization techniques, like genetic algorithms, to find SFP antecedent parameters in fuzzy controllers. Furthermore, the possibility of adding the Takagi-Sugeno consequents to the optimization problem deserves additional research effort.

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