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# Robust Attitude Stabilization of a Quadrotor by Using LFT Linear-Parameter-Varying $H_\infty$

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***Abstract*–In this paper, robust stabilization of a quadrotor aircraft attitude is investigated. The exact nonlinear attitude of a quadrotor model is presented by Linear-Parameter-Varying system with Linear-Fractional-Transformation representation (LFT-based LPV), when we took the nonlinearities as varying parameters. The closed loop interconnection is established by taking into account the external disturbances, measurement noises and actuator dynamics. The synthesis of $H_\infty$ gain-scheduling controller is achieved by scaling small gain theorem when the controller is an LFT-based LPV too. The simulations show the efficiency and the robustness of the proposed controller against the disturbances, noises and parametric uncertainties.**

**Keywords:** Linear-Parameter-Varying systems; Quadrotor; Robust Control; Linear-Fractional-Transformation (LFT).

## I. Introduction

Unmanned Aerial Vehicles (UAV) control becomes a very interesting field due to its efficiency in surveillance and security applications because of their capability of navigation of small and hard areas. The quadrotor is one of popular UAV which has received an attraction from the control researchers because of its advantages; vertical take-off and landing, stationary flight and low speed flight which make it highly maneuverable. However, the quadrotor is an unstable, complex, nonlinear system; it has six degrees of freedom (6DOF); three for translation and three for rotation (attitude) [1]. The motion subsystem doesn't have direct inputs to the quadrotor, but virtual ones, associated with the attitude. Therefore, the attitude control is significantly interesting by researchers; large different linear and nonlinear control techniques and structures were implemented. A simple PD and LQR controllers were achieved in [2] but the linearization was required to implement these linear controllers, which has degraded the system performance. The high computational fuzzy-logic controller was overridden by using fuzzy-padé controller was proposed in [3], because it is a knowledge based controller and do not require the exact modeling of the system. An adaptive Backstepping controller has been proposed in [4], where the authors reduced the model of the system and then estimated the uncertainties and external disturbances, in this case, the stability was investigated by Lyapunov theory. The sliding mode was used in [5], [6] where the robustness against disturbances was analysis.

Robust control against both, uncertainties and external disturbances, were carried out; against actuator faults in [7], with present input time delay [8]. On other hand, the $H_\infty$ has been shown as a very efficient robust control approach, originally created for Linear-Time-Invariant (LTI) systems. For the quadrotor attitude control, we can find; a control strategy based on a black box model of quadrotor and the pitch angle control has been analyzed and controlled by $H_\infty$ [9]. Mixed $H_2/H_\infty$ control has been applied for attitude control in [10], but for linearized model, which degraded the performance of the quadrotor, the control algorithm is not applicable in the nonlinear range of angles; big angles.

An extension of $H_\infty$ to nonlinear systems can be used by two kinds of techniques; the first is the nonlinear $H_\infty$ method via game-theory when we replace the Riccati equations by Hamilton-Jacobi ones [11], the second is the use of Linear-Parameter-Varying (LPV) techniques, which is the aim of our work. The application of the first approach, game-theory, can be found in the aircraft field [12] where it is used to reject the disturbances for a nonlinear system. In [13] the authors designed a nonlinear $H_\infty$ controller with PID structure, applied to a robot manipulator. Also, the same technique can be found in attitude robust control of quadrotor [14], the work's authors conclude that the main difficulty they face is that there is no general method to solve Hamilton-Jacobi equations.

The second kind of approaches is based on the use of Linear-Parameter-Varying (LPV) characteristics, although the techniques have been created as an extension of LTI $H_\infty$ to Linear-Parameter-Varying (LPV) systems, but, the real-time measurement of states makes the use of them in nonlinear systems valuable. In this case, the parameters are functions of states or/and inputs, therefore, the nonlinear systems will be a quasi-LPV [15]. The characterizations and the handling with the quasi-LPV systems remain the same as LPV systems. Unlike the first technique of nonlinear $H_\infty$, game-theory, the use of LPV systems' characteristics to nonlinear systems has more than one approach, whether the representation or the controller synthesis. There is affine representation [16], and LFT representation which allows for an exact representation of nonlinear systems by the rational dependence of parameters [17], which have the advantage that no need to discrete the parameter range.

It is a fact that the quadrotor is an underactuated vehicle with non-holonomic constraints in its dynamics, but in this paper, we focus on the full actuated subsystem, the attitude. The LFT-LPV representation of the nonlinear attitude model has been achieved, and then, the LPV controller synthesis is done by scaling small gain theorem for LPV systems [18].

The paper structure is as follows; in the next section we show some preliminaries. In section III, we present the nonlinear model of quadrotor-attitude and its LFT-LPV

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representation. In section IV, we present the synthesis algorithm of the selected approach applied to quadrotor-attitude. Section V, state our simulation results and lastly a conclusion is exposed in section VI.

## II. PRELIMINARIES

In this section, we present a review about the LPV systems and the used representation of a nonlinear system by LFT representation. In addition, we present the chosen controller synthesis design.

The general state-space of a Linear-Parameter-Varying system is as shown in (1):

$$\begin{cases} \dot{x} = A(\rho(t))\,x + B(\rho(t))\,u \\ y = C(\rho(t))\,x + D(\rho(t))\,u \end{cases} \tag{1}$$

Where: A(.), B(.), C(.) and D(.) are a parameter depending matrices, and ρ(t) the vector of time-varying parameters which is unknown in advance but measurable in real time. The common representation of an LPV system is affine or polytopic representation [19], [20]. With any linearization method, the system is divided into a set of LTI subsystems. When we extract a quasi-LPV representation for a nonlinear system, the main disadvantage of this representation is that, no stability guarantee exists for the nonlinear system itself, but we can prove just the stability of each LTI subsystem, [21]. Therefore, we tend to LFT representation which achieves an exact representation of the nonlinear system.

### A. LFT-based LPV Representation:

In this paper, the LFT-LPV representation is achieved for both; the plant and the controller which is shown below by Fig.1.

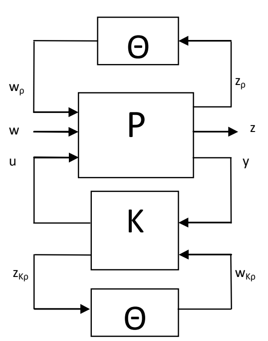


Figure 1. LFT-LPV representation of the closed loop plant-controller.

The LFT representation of LPV plants, as well as the quasi-LPV ones, can be presented by its state-space as (2):

$$\begin{aligned} \dot{x} &= A\,x + B_\rho\, w_\rho + B_1 w + B_2 u \\ z_\rho &= C_\rho x + D_{\rho\rho} w_\rho + D_{\rho 1} w + D_{\rho 2} u \\ z &= C_1 x + D_{1\rho} w_\rho + D_{11} w + D_{12} u \\ y &= C_2 x + D_{2\rho} w_\rho + D_{21} w + D_{22} u \end{aligned} \tag{2}$$

$$w_\rho = \Theta z_\rho$$

*Notation:* $x$ the vector of states, $u$ the input vector, $w$ the exogenous inputs; disturbances and noises, $y$ the measured output, $z$ the controlled output, $z_\rho$ and $w_\rho$ are inputs and outputs of the parameter block, $\Theta = diag(\rho_1 I_{r1}, \rho_2 I_{r2} \ldots \rho_k I_{rk})$.

Fig.1 shows that the controller has the same representation (LFT-LPV) and the same complexity likewise the plant; the same recurrence of each varying parameter; the same structure of the parameter block (*Θ*). Expression (3) shows the state-space of the LPV gain-scheduled robust controller.

$$\begin{aligned} \dot{x}_K &= A_K x_K + B_{K1} y + B_{K\rho} w_{K\rho} \\ u &= C_{K1} x_K + D_{K11} y + D_{K1\rho} w_{K\rho} \\ z_{K\rho} &= C_{K\rho} x_K + D_{K\rho 1} y + D_{K\rho\rho} w_{K\rho} \end{aligned} \tag{3}$$

$$w_{K\rho} = \Theta z_{K\rho}$$

*Notation*: $x_K$ the controller states vector, $y$ the measurement outputs from the plant, $u$ the controller outputs, $z_{K\rho}$ and $w_{K\rho}$ are inputs and outputs of the parameter block.

### B. LPV Controller Synthesis:

In fact, the controller synthesis approach is a direct application of the work of [18], scaling small-gain theorem. The objective is to minimize the gain between the exogenous inputs and the so-called outputs, with next theorem:

*Theorem 1*: (scaling small-gain theorem [18]): there exists a lower LFT-LPV controller (3) if there is a scaling matrix $L$ verified (4).

$$\left\| \begin{pmatrix} L^{1/2} & 0 \\ 0 & I \end{pmatrix} . F_l(P_a, K) . \begin{pmatrix} L^{-1/2} & 0 \\ 0 & I \end{pmatrix} \right\|_\infty \le \gamma \tag{4}$$

Where: $F_l(P_a,K)$ the LFT of closed loop plant-controller and $\gamma$ is the specified performance.

The approach used the Linear-Matrices-Inequalities (LMI), basically created for the LTI systems [22], but extended to LPV systems [18] when we augment $B_1$, $C_1$ and $D_{11}$ from (2) by including the parameter matrices:

$B_1 = (B_\rho, B_1)$, $C_1 = \begin{pmatrix} C_\rho \\ C_1 \end{pmatrix}$, and $D_{11} = \begin{pmatrix} D_{\rho\rho} & D_{\rho 1} \\ D_{1\rho} & D_{11} \end{pmatrix}$.

There is a gain-scheduled controller with structure (3), if there are checked matrices; R, S, $L_3$ and $J_3$ solved (5)-(8).

$$N_R^T \begin{pmatrix} AR + RA^T & RC_1^T & B_1 \\ C_1 R & -\gamma \begin{pmatrix} J_3 & 0 \\ 0 & I \end{pmatrix} & D_{11} \\ B_1^T & D_{11}^T & -\gamma \begin{pmatrix} L_3 & 0 \\ 0 & I \end{pmatrix} \end{pmatrix} N_R < 0 \tag{5}$$

$$N_S^T \begin{pmatrix} A^T S + SA & SB_1 & C_1^T \\ B_1^T S & -\gamma \begin{pmatrix} L_3 & 0 \\ 0 & I \end{pmatrix} & D_{11}^T \\ C_1 & D_{11} & -\gamma \begin{pmatrix} J_3 & 0 \\ 0 & I \end{pmatrix} \end{pmatrix} N_S < 0 \tag{6}$$

$$\begin{pmatrix} R & I \\ I & S \end{pmatrix} \ge 0 \tag{7}$$

$$\begin{pmatrix} L_3 & I \\ I & J_3 \end{pmatrix} \ge 0 \tag{8}$$

The theorem was proofed in [22] for LTI systems and adopted for LPV systems in [18]. We assume a given solution of R, S, $L_3$ and $J_3$ of (5)-(8); in last we can get the controller's matrices with structure (9) and the solution of the LMI is shown in (10).

$$\Omega = \begin{pmatrix} A_K & B_{K1} & B_{K\rho} \\ C_{K1} & D_{K11} & D_{K1\rho} \\ C_{K\rho} & D_{K\rho 1} & D_{K\rho\rho} \end{pmatrix} \tag{9}$$

$$\Psi + \begin{pmatrix} X_{cl} & 0 \\ 0 & I \end{pmatrix} P^T \Omega Q + Q^T \Omega^T P \begin{pmatrix} X_{cl} & 0 \\ 0 & I \end{pmatrix} < 0 \tag{10}$$

Where:

$$\Psi = \begin{pmatrix} A_0^T X_{cl} + X_{cl} A_0 & X_{cl} B_0 & C_0^T \\ B_0^T X_{cl} & -\gamma \mathcal{L} & \mathcal{D}_{11}^T \\ C_0 & \mathcal{D}_{11} & -\gamma \mathcal{J} \end{pmatrix} \qquad (11)$$

And:

$$P = (\mathcal{B}^T, 0, \mathcal{D}_{12}^T),\ Q = (\mathcal{C}, \mathcal{D}_{21}, 0) \qquad (12)$$

## III. Quasi-LPV Modeling of Quadrotor-Attitude

The mathematical model of the quadrotor was generated by both of techniques; Euler-Newton [23] and Euler-Lagrange [14]. Fig.2 presents a general view of the quadrotor, which has four rotors, and where each two rotors turn on the same direction. We can find two kind of configurations, the cross (×) configuration and the plus (+) configuration, that is chosen in this paper.

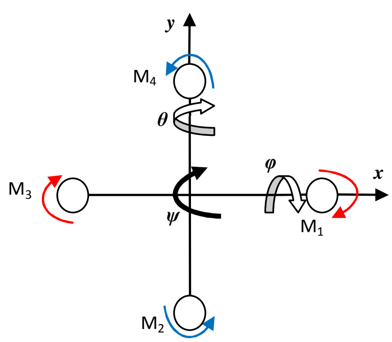


Figure 2. The geometric structure of the quadrotor

In this paper, we focus on the attitude subsystem, where the model has three degrees of freedom; roll, pitch and yaw; Six states; which are three angles $[\varphi\ \theta\ \psi]$ with their derivatives$[\dot{\varphi}\ \dot{\theta}\ \dot{\psi}]$, and three inputs (13):

$$\begin{cases} \ddot{\varphi} = \frac{I_y - I_z}{I_x}\dot{\theta}\dot{\psi} - \frac{J_r \Omega_r}{I_x}\dot{\theta} + \frac{l}{I_x} U_\varphi \\ \ddot{\theta} = \frac{I_z - I_x}{I_y}\dot{\varphi}\dot{\psi} + \frac{J_r \Omega_r}{I_y}\dot{\varphi} + \frac{l}{I_y} U_\theta \\ \ddot{\psi} = \frac{I_x - I_y}{I_z}\dot{\varphi}\dot{\theta} + \frac{1}{I_z} U_\psi \end{cases} \qquad (13)$$

Where :

$$\Omega_r = \omega_1 - \omega_2 + \omega_3 - \omega_4 \qquad (14)$$

And:

$$\begin{cases} U_\varphi = b.(\omega_4^2 - \omega_2^2) \\ U_\theta = b.(\omega_3^2 - \omega_1^2) \\ U_\psi = d.(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \end{cases} \qquad (15)$$

Table I. shows the model parameters definition from selected project, OS4 [23].

TABLE I. OS4 PARAMETERS

| Parameter | Name | Value | Unit |
|---|---|---|---|
| $l$ | Arm length | 0.23 | m |
| $b$ | Thrust coefficient | $3.13\ 10^{-5}$ | $N.s^2$ |
| $d$ | Drag coefficient | $7.5\ 10^{-7}$ | $N.m.s^2$ |
| $I_x$, $I_y$ | Inertia on $x$ and $y$ axis | $7.5\ 10^{-3}$ | $Kg.m^2$ |
| $I_z$ | Inertia on $z$ axis | $1.3\ 10^{-2}$ | $Kg.m^2$ |
| $J_r$ | Rotor inertia | $6\ 10^{-5}$ | $Kg.m^2$ |
| $\omega_i$ | Rotor speed | [0, 400] | $rad.s^{-1}$ |

Due to the fact, that the selected model is symmetric when ($I_x$=$I_y$), the yaw model can be linearized and will be ( $\ddot{\psi} = \frac{1}{I_z} U_\psi$ ). We get two subsystems; the nonlinear one, roll-pitch ($\varphi$, $\theta$) subsystem and the linear one, yaw ($\psi$) subsystem.

We don't need a complex approach for the yaw subsystem; therefore we have controlled it by an LTI robust $H_\infty$. What is next is the modeling of roll-pitch subsystem as a quasi-LPV system.

### A. Parameters selection

The LPV representation of a nonlinear system is not unique; the parameter selection has an influence on synthesis complexity, by a look at (13), if the goal is to decouple the yaw from roll-pitch subsystem. We have chosen ($\rho_1 = \dot{\psi}$), and ($\rho_2 = \Omega_r$) as varying parameters, then we have created the LFT-LPV representation of the roll-pitch subsystem and we have got a state-space as shown in (16) and Fig.3:

$$\begin{bmatrix} \dot{x} \\ z_\rho \\ y \end{bmatrix} = \begin{bmatrix} \mathcal{A} & \mathcal{B}_\rho & \mathcal{B}_1 \\ \mathcal{C}_\rho & \mathcal{D}_{\rho\rho} & \mathcal{D}_{\rho 1} \\ \mathcal{C}_1 & \mathcal{D}_{1\rho} & \mathcal{D}_{11} \end{bmatrix} . \begin{bmatrix} x \\ w_\rho \\ u \end{bmatrix} \qquad (16)$$

$$w_\rho = \Theta\, z_\rho$$

Where: $\mathcal{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $\mathcal{B}_\rho = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{I_y - I_z}{I_x} & -\frac{J_r}{I_x} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{I_z - I_x}{I_y} & \frac{J_r}{I_y} \end{bmatrix}$,

$$\mathcal{B}_1 = \begin{bmatrix} 0 & 0 \\ \frac{l}{I_x} & 0 \\ 0 & 0 \\ 0 & \frac{l}{I_y} \end{bmatrix}, \mathcal{C}_\rho = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathcal{C}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\mathcal{D}_{\rho\rho} = 0_{4\times 4}, \quad \mathcal{D}_{\rho 1} = 0_{4\times 2}, \mathcal{D}_{1\rho} = 0_{2\times 4}, \mathcal{D}_{11} = 0_{2\times 2}$$

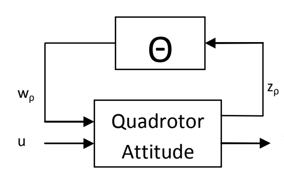


Figure 3. LFT-LPV scheme of the quadrotor-attitude

We have 4 states $[\varphi\ \dot{\varphi}\ \theta\ \dot{\theta}]$, 2 inputs ($U_\varphi$, $U_\theta$) and 2 varying parameters by two recurrences of each; $\Theta = diag\ (\rho_1, \rho_2, \rho_1, \rho_2)$. About the yaw, it is a simple LTI system with 2 states and just one input. Here, we have the LFT-LPV representation of the quadrotor attitude, but we have not generated yet the closed loop system. In the next section, the closed loop is obtained and then the synthesis of the gain-scheduled $H_\infty$ controller is done.

## IV. Controllers Synthesis

After getting the model, we need to establish the interconnection between the plant and the different inputs; reference signals, disturbances and noises. In addition, the weighting functions whose present the desired performance, are ordered as usual, by the error and the input signals.

Actually, the quadrotor is controlled by the rotors' speed ($\omega_1 \dots \omega_4$), but to simplify the plant understanding, the rotors' speed are coupled to get a roll control, pitch control and yaw control (15). In fact, the rotor doesn't have a real-time response; the rotor dynamic in the selected project [23] is as follows:

$$D(s) = \frac{\omega_i}{\omega_{di}} = \frac{0.936}{0.178\, s+1} \tag{17}$$

Where: $\omega_i$ and $\omega_{di}$ are the actual and desired speed of $i^{th}$ rotor, respectively. We have generated inputs from the controller; desired inputs ($U_d$) and actual inputs ($U_a$) which directly applied to the model after passing on the actuators, by combining this with (15) and (17):

$$U_a = D^2(s).U_d \tag{18}$$

Where: Eq. (18) presents the actuators dynamic for each input ($U_\varphi$, $U_\theta$, $U_\psi$); pitch, roll and yaw controls.

In this paper, we consider three types of external inputs; the reference which presents the desired angles, the input disturbances and the sensor noises (Fig.4).

Due to the division of the attitude model into two different subsystems; an LTI (yaw subsystem) and an LPV (roll-pitch subsystem), the controller of each subsystem has the same nature as the plant.

### A. Linear-Time-Invariant $H_\infty$

The synthesis of the LTI $H_\infty$ controller for the yaw subsystem in this work is achieved by solving the Riccati-Equations and loop shaping [24], [25].

The used weighting functions are; $W_{e\psi} = \frac{s+1}{s+0.2}$ for the error criteria and $W_{u\psi} = 10^{-2}\frac{s^2+15\, s+1}{5\, s^2+5\, s+1}$ for the control criteria.

These functions present the desired behavior in closed-loop, which aims in our case to have; a peak sensitivity equal to 1, a bandwidth limited to 1 rad/s and an allowable steady-state error equal to 0.2. The high frequency gain is limited to $10^{-2}$ by the controller bandwidth (7.5 rad/s) [26].

As a result, we have got an LTI $H_\infty$ with performance criteria $\gamma_\psi = 1.0008$.

### B. Linear-Parameter-Varying $H_\infty$

The LPV $H_\infty$ controller is designed by the scaling small gain theorem in terms of LMI convex optimization [18]. The advantage of the LFT-LPV controller synthesis algorithm in this paper is that there is no need to take into account the bounds of varying-parameters' derivative, the gotten performance criteria, $\gamma$, is the induced $L_2$ norm when no imposed limits to speed changing of the parameter in the time. Table II explains the range of parameters of the roll-pitch subsystem.

TABLE II. VARYING-PARAMETERS RANGE

| Parameter | description | Range |
|---|---|---|
| $\rho_1$ | $\dot{\psi}$ | *[-2, 2] rad.s$^{-1}$* |
| $\rho_2$ | $\Omega_r$ | *[-300, 300] rad.s$^{-1}$* |

The weighting functions are as well as the yaw subsystem, one of the error tracking; $W_{e\varphi,\theta} = \frac{s+1}{s+0.2} * I_2$ and one of the input criteria; $W_{u\varphi,\theta} = 10^{-2}\frac{s^2+25\, s+1}{5\, s^2+5\, s+1} * I_2$.

Here, the weighting functions present the desired behavior in closed-loop as that in the yaw subsystem, with a high frequency gain limited to $10^{-2}$ by the controller bandwidth (12.5 rad/s) [26].

The roll-pitch subsystem is a MIMO system, so we present either their $W_e$ and $W_u$ as a diagonal matrices, where $I_2$ is the identity matrix of size 2.

The algorithm of controller's design is detailed in sec. II.B. As a result, we have got an LPV $H_\infty$ with performance criteria $\gamma_{\varphi,\theta} = 2.89$. The control algorithm of LFT-LPV systems [18], at first, doesn't impose a variation's rate of the parameters with time. Secondly, it is just an upper bound on the induced $L_2$ norm [27], subsequently, if the disturbances and noise inputs have an induced $L_2$ norms limited by *1*, the $L_2$ norm is no greater than *2.89*. For any parameters' path ($\dot{\psi}$, $\Omega_r$) such as mentioned in Table II.

In order to combine the two different controllers, we assume that the LFT-LPV controller (for the roll-pitch subsystem) has a state-space as (3). Let's call this controller $K_{LPV} = \begin{bmatrix} A_{K1} & B_{K1} & B_{K\rho} \\ C_{K1} & D_{K11} & D_{K1\rho} \\ C_{K\rho} & D_{K\rho1} & D_{K\rho\rho} \end{bmatrix}$ which has a lower LFT as shown in Fig.4.

We call the LTI controller (for the yaw subsystem) $K_{LTI} = \begin{bmatrix} A_{K2} & B_{K2} \\ C_{K2} & D_{K2} \end{bmatrix}$, the combined controller for the whole system is presented by its matrices;

$$A_K = \begin{bmatrix} A_{K1} & \mathbb{0} \\ \mathbb{0} & A_{K2} \end{bmatrix}, B_K = \begin{bmatrix} B_{K1} & \mathbb{0} & B_{K\rho} \\ \mathbb{0} & B_{K2} & \mathbb{0} \end{bmatrix}, C_K = \begin{bmatrix} C_{K1} & \mathbb{0} \\ \mathbb{0} & C_{K2} \\ C_{K\rho} & \mathbb{0} \end{bmatrix},$$

$$D_K = \begin{bmatrix} D_{K11} & \mathbb{0} & D_{K1\rho} \\ \mathbb{0} & D_{K2} & \mathbb{0} \\ D_{K\rho1} & \mathbb{0} & D_{K\rho\rho} \end{bmatrix}.$$

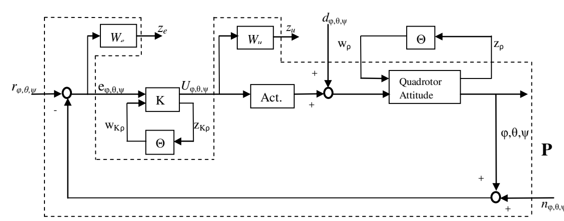


Figure 4. Closed-loop plant-controller

## V. SIMULATION RESULTS

The simulation results are presented in this section. We have to notice that we have applied the obtained controller to the nonlinear quadrotor-attitude itself (13); the initial angles are $[\varphi(0), \theta(0), \psi(0)] = \left[\frac{\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{3}\right]$ *(rad)*, as mentioned previously, where the exogenous inputs are; references, input-disturbances and noises.

Fig.5 presents a comparison between two simulation trajectories; the first is without disturbances, neither noises, the second is with both of them.

The input-disturbances are supposed to be as an impulses with amplitude of *0.1 N.m* ; $d_\varphi$ in [4, 4.07] *s*, $d_\theta$ in [6, 6.07] *s* and *0.03 N.m* $d_\psi$ in [7, 7.1] *s*, while the applied noises are permanent random signals by a range of [-0.5, 0.5] *rad.*

It could be clear that the responses with the presence of disturbances and noises have a good performance, and the disturbances are smoothly rejected. The system keeps the stability on the presence of disturbances and the responses stabilize after 3 *s* for all angles, wherever presented overshoot is about 35% in roll response, 44 % in pitch response and null in yaw response. Peak responses after the stability don't exceed *0.2 rad* for all angles.

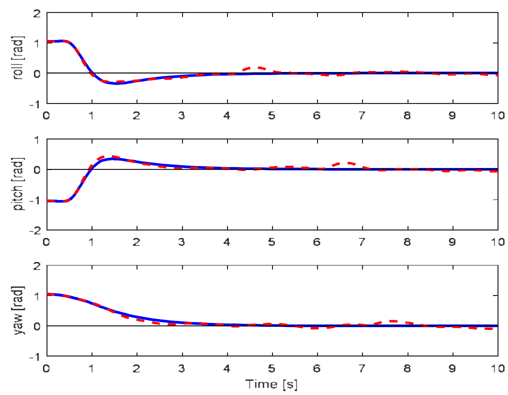


Figure 5. The angles responses, with applied disturbances and noises (dashed line), without them (solid line).

The next simulation proposes to discuss the robustness against uncertainties. We suppose that there is an uncertainty of inertias' identification ($I_x$, $I_y$, $I_z$ and $J_r$) of ($w_\Delta$ *=0.5)*, by considering three different models; nominal parameters, and nominal ± 50%. In Fig.6 we show these three trajectories.

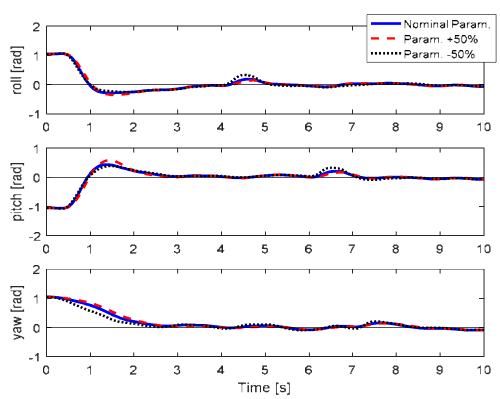


Figure 6. The angles responses with presence of uncertainties

We can notice that the proposed controller keeps the stability against the uncertainties on quadrotor inertias and rotor inertia by $w_\Delta$*=0.5* of weight moreover the disturbances and noises.

An important point for the uncertainties, in case that the inertias change during the system working for any reasons, the system starts with nominal parameters and then we applied uncertainties of +50% about inertias of the quadrotor and -50% for rotors' inertias in instant *5 s*. Fig.7 presents the angles responses with the presence of uncertainties in the middle of working, the robustness is still guaranteed, as well as the previous simulation. Fig.8 presents a zoom of the errors in steady-state, after the instant *2 s*; We can notice that the errors have never increased than *0.2 rad*, in roll dynamic and yaw dynamic, but the error of pitch dynamic reaches *-0.25 rad.* Fig.9 presents the varying-parameter trajectories; $\rho_1=\dot{\psi}$ and $\rho_2=\Omega_r$ under their lower and upper bounds, we can notice that with selected initial conditions and simulated disturbances and noises, the varying-parameters still in the ranges that we imposed them in the controller's synthesis.

At last Fig.10 presents the control signals, it is remarkable that the control signals, torques, don't cost big energy; under *0.5 N.m* for roll and pitch controls, and under *0.05 N.m* for yaw control.

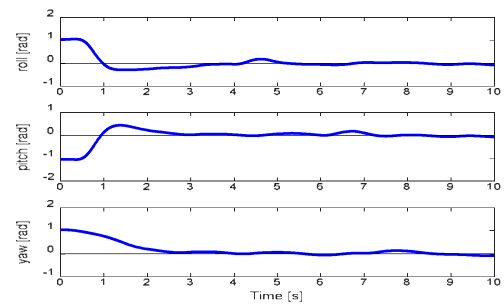


Figure 7. The angles responses, in presence of uncertainties at *5 s*.

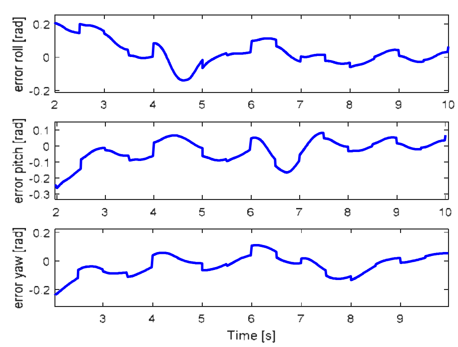


Figure 8. Zoom in error signals

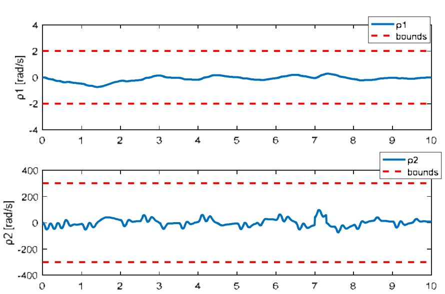


Figure 9. Trajectories of varying parameters

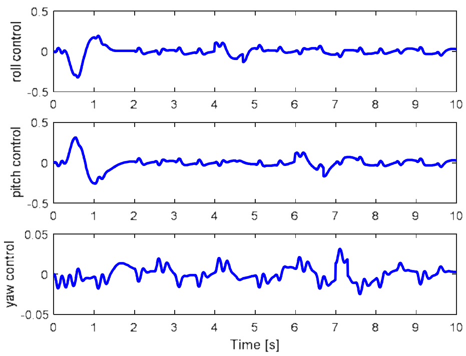


Figure 10. Inputs signals

## VI. Conclusion

In this work, we have proposed to control the full actuated subsystem of quadrotor's attitude by using a robust LPV technique; where the model is not linearized or limited in its range of running. However, we have presented it as a quasi-LPV system, when we have chosen the nonlinearities as varying-parameters and present them by LFT structure. The controller's design is achieved by LPV $H_\infty$, this approach was applied in roll-pitch subsystem when the yaw subsystem settled for LTI $H_\infty$. The system's performance showed clearly that with the proposed controller, the robustness is guaranteed against the disturbances, noises and the parametric uncertainties. The chosen synthesis approach is the scaling small gain theorem, because of its advantage which doesn't impose to know the rate of the varying-parameters. In addition, the input signals have low costs with proposed technique. As a future work, the use of this technique for the whole system, with the motion, can be an attractive challenge for the automatic control researchers.

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