2018 26th Mediterranean Conference on Control and Automation (MED)
Zadar, Croatia, June 19-22, 2018

PID Controller Design based on Generalized Stability Boundary Locus to Control Unstable Processes with Dead Time*

Serdal Atic, Ibrahim Kaya

***Abstract*— This paper proposes a method so that all PID controller tuning parameters, which are satisfying stability of any unstable time delay processes, can be calculated by forming the stability boundary loci. Processes having a higher order transfer function must first be modeled by an unstable first order plus dead time (UFOPDT) transfer function in order to apply the method. Later, UFOPDT process transfer function and the controller transfer function are converted into normalized forms to obtain the stability boundary locus in $\left(KK_c, KK_c(T/T_i)\right)$, $\left(KK_c, KK_c\left(T_d/T\right)\right)$ and $\left(KK_c(T/T_i), KK_c\left(T_d/T\right)\right)$ planes for PID controller design. PID controller parameter values achieving stability of the control system can be determined by the obtained stability boundary loci. The advantage of the method given in this study compared with previous studies in this subject is to remove the need of re-plotting the stability boundary locus as the process transfer function changes. That is, the approach results in somehow generalized stability boundary loci for unstable plus time delay processes under a PID controller. Application of the method has been clarified with examples.**

I. Introduction

Researchers have always been interested in PID controllers due to their simple structure and performing robustly for many industrial applications Being the most popular controller, they still attract researchers' attention. An excellent collection on the PID controller design methods can be found in [1], [2].

Most commonly used methods for determination of PID controllers are Ziegler and Nichols [3], Cohen and Coon [4] and Aström and Haggland [5] methods. Methods based on integral performance criteria [6] are among very standard approaches as well. Other methods being used for calculating PID controller tuning parameters are Internal Model Control (IMC) [7] and controller synthesis [8] methods.

After the study of Ho et al. [9]–[11], researchers has paid special interest to determination of all stabilizing PI and PID controllers. These studies led to achieve all integral and derivative gain values of a PID controller in the same plane for a fixed proportional gain value. Although, the approach suggested by Ho et al. [9]–[11] made calculation of all PI and PID controller tuning parameters available, its application takes time. Therefore, researchers have searched for developing different approaches. Munro and Söylemez [12] and Söylemez et al. [13] proposed a new method that provided a faster calculation of all PID controller tuning parameters. Shafiei and Shenton [14] and Huang and Wang [15] provided graphical solutions for determination of all stabilizing PID controller parameter values. Tan et al. [16] and Tan [17] suggested a new approach providing a faster calculation of all stabilizing PI or PID controller tuning parameters, based on stability boundary locus calculation. This approach has been used in different studies up to date. Zàvackà et al. [18] suggested a robust PI controller design for a continuous stirred tank reactor with multiple steady-states. Sandeep and Yogesh [19] gave design of a PID controller for an inverted pendulum. Deniz et al. [20] recommended an integer order approximation method based on stability boundary locus for fractional order derivative/integrator operators. All of the studies mentioned above can be used to find the stability boundary locus of only a specific plant transfer function.

In this study, the method suggested by Kaya and Atic [21] for obtaining all stabilizing PI controllers to control open loop stable time delay processes has been extended to all stabilizing PID controllers to control open loop unstable time delay processes. In this approach, modeling of higher order processes by a first order unstable plus dead time (UFOPDT) model is required. It is assumed that relay feedback identification method of Kaya and Atherton [22] can be used for this purpose. The relay feedback method gives exact solutions assuming that there are no measurement errors and disturbances entering the control system. Process transfer function model and the controller transfer function are first converted into normalized forms and then used to form stability boundary loci for obtaining all stabilizing PID controller tuning parameters for varying normalized dead time ratio, $\tau=\theta/T$. The advantage of the method is to eliminate the need of re-plotting the stability boundary locus whenever the transfer function changes so that calculation of all stabilizing PID controllers becomes easier.

The rest of paper is organized as follows. Next section gives the procedure for achieving the stability boundary locus in $\left(KK_c\left(T/T_i\right), KK_c\left(T_d/T\right)\right)$ plane for a fixed value of KK_c to obtain all stabilizing PID controllers. In Section 3, the application of method is illustrated with several examples. Conclusions are given in Section 4.

II. PID Controller Design For Unstable Process

Consider single-input single-output control system depicted in Fig. 1.

*Research supported by DUBAP (MUHENDISLIK.15.013).

S. Atic is with Electricity and Energy Department, Vocational High School, Batman University, Batman, 72060, Turkey
(e-mail: serdal.atic@batman.edu.tr)
I. Kaya is with the Department of Electrical Electronics Engineering, Dicle University, Diyarbakir, 21280 Turkey
(Phone: 0412-241-1000; e-mail: ikaya@dicle.edu.tr)

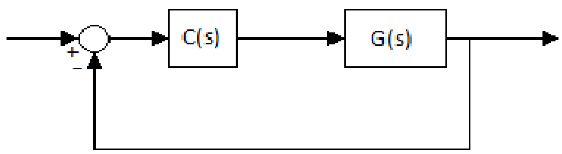

Figure 1. SISO Control System

In the figure, $C(s)$ and $G(s)$ are, respectively, the controller and the process transfer functions. Transfer function of the ideal PID controller is:

$$C(s) = K_c\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{1}$$

UFOPDT model of process transfer function is assumed to be given by:

$$G(s) = \frac{Ke^{-\theta s}}{Ts - 1} \tag{2}$$

By substituting $sT = \bar{s}$ into (1) and (2), the normalized controller and process transfer functions can be obtained as:

$$C(\bar{s}) = K_c\left(1 + \frac{T}{T_i\bar{s}} + \frac{\bar{s}T_d}{T}\right) \tag{3}$$

$$G(\bar{s}) = \frac{Ke^{-\frac{\theta}{T}\bar{s}}}{\bar{s} - 1} = \frac{Ke^{-\tau\bar{s}}}{\bar{s} - 1}. \tag{4}$$

Here, the aim is to calculate all controller parameter values in (1) satisfying the stability of the control system shown in Fig. 1. Closed-loop characteristic equation of the system is $1 + C(\bar{s})G(\bar{s})$. Hence, substituting $C(\bar{s})$ and $G(\bar{s})$, correspondingly, from (3) and (4), one can find that closed-loop characteristic polynomial is given by:

$$\begin{aligned}&\Delta(\bar{s}) = \\ &KK_cTT_i\bar{s}e^{-\tau\bar{s}} + KK_cT^2e^{-\tau\bar{s}} + KK_cT_iT_d\bar{s}^2e^{-\tau\bar{s}} + TT_i\bar{s}^2 - TT_i\bar{s}\end{aligned} \tag{5}$$

Decomposing the numerator and the denominator of (2) into its even and odd parts, and replacing $\bar{s} = j\bar{\omega}$ one can find the following:

$$G(j\bar{\omega}) = \frac{N_e(-\bar{\omega}^2) + j\bar{\omega}N_o(-\bar{\omega}^2)}{D_e(-\bar{\omega}^2) + j\bar{\omega}D_o(-\bar{\omega}^2)}. \tag{6}$$

Dropping the dash over ω for simplicity, the characteristic equation can be written as:

$$\begin{aligned}\Delta(j\omega) = {} & j\omega KK_cTT_i\cos(\omega\tau) \\ & + \omega KK_cTT_i\sin(\omega\tau) + KK_cT^2\cos(\omega\tau) \\ & - jKK_cT^2\sin(\omega\tau) - \omega^2KK_cT_iT_d\cos(\omega\tau) \\ & + j\omega^2KK_cT_iT_d\sin(\omega\tau) \\ & - \omega^2TT_i - j\omega TT_i \\ & = R_\Delta + jI_\Delta = 0.\end{aligned} \tag{7}$$

Equating the real and imaginary parts of the characteristic equation equal to zero, the following two equations are obtained:

$$KK_c\left[\omega\sin(\omega\tau)\right] + \frac{KK_cT}{T_i}\left[\cos(\omega\tau)\right] + \frac{KK_cT_d}{T}\left[-\omega^2\cos(\omega\tau)\right] = \omega^2 \tag{8}$$

$$KK_c\left[\omega\cos(\omega\tau)\right] + \frac{KK_cT}{T_i}\left[-\sin(\omega\tau)\right] + \frac{KK_cT_d}{T}\left[\omega^2\sin(\omega\tau)\right] = \omega. \tag{9}$$

One can define the following equations:

$$\begin{aligned}Q(\omega) &= \omega\sin(\omega\tau), \\ R(\omega) &= \cos(\omega\tau), \\ F(\omega) &= -\omega^2\cos(\omega\tau), \\ X(\omega) &= \omega^2 + \omega^2\frac{KK_cT_d}{T}\cos(\omega\tau), \\ H(\omega) &= \omega^2 - \frac{KK_cT}{T_i}\cos(\omega\tau).\end{aligned} \tag{10}$$

and

$$\begin{aligned}S(\omega) &= \omega\cos(\omega\tau), \\ U(\omega) &= -\sin(\omega\tau), \\ B(\omega) &= \omega^2\sin(\omega\tau), \\ Y(\omega) &= \omega - \omega^2\frac{KK_cT_d}{T}\sin(\omega\tau), \\ N(\omega) &= \omega + \frac{KK_cT}{T_i}\sin(\omega\tau).\end{aligned} \tag{11}$$

Using the above definitions, one can rearrange (8) and (9) as follows:

$$\begin{aligned}KK_cQ(\omega) + KK_c\frac{T}{T_i}R(\omega) &= X(\omega), \\ KK_cS(\omega) + KK_c\frac{T}{T_i}U(\omega) &= Y(\omega).\end{aligned} \tag{12}$$

and

$$\begin{aligned}KK_cQ(\omega) + \frac{KK_cT_d}{T}F(\omega) &= H(\omega), \\ KK_cS(\omega) + \frac{KK_cT_d}{T}B(\omega) &= N(\omega).\end{aligned} \tag{13}$$

One can solve (12) and (13) to obtain the following expressions:

$$KK_c = \frac{X(\omega)U(\omega) - Y(\omega)R(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \tag{14}$$

$$KK_c\frac{T}{T_i} = \frac{Y(\omega)Q(\omega) - X(\omega)S(\omega)}{Q(\omega)U(\omega) - R(\omega)S(\omega)} \tag{15}$$

and

$$\frac{KK_cT_d}{T} = \frac{N(\omega)Q(\omega) - H(\omega)S(\omega)}{Q(\omega)B(\omega) - F(\omega)S(\omega)} \tag{16}$$

Substituting (10) and (11) into (14), (15) and (16), one can find the following equations:

$$KK_c = \omega\sin(\omega\tau) + \cos(\omega\tau) \tag{17}$$

$$KK_c \frac{T}{T_i} = -\omega \sin(\omega\tau) + \omega^2 \cos(\omega\tau) + \omega^2 \frac{KK_c T_d}{T} \qquad (18)$$

and

$$\frac{KK_c T_d}{T} = \omega^{-1} \sin(\omega\tau) - \cos(\omega\tau) + \omega^{-2} \frac{KK_c T}{T_i} \qquad (19)$$

Stability boundary loci in $(KK_c, KK_c(T/T_i))$ plane for the normalized dead time ratio of $\tau = 1$ and fixed $KK_c T_d / T$ values of 1 and 0.5 are drawn in Fig. 2, by using (17) and (18). Fig. 3 illustrates the stability boundary loci in $(KK_c, KK_c(T_d/T))$ plane for the normalized dead time ratio of $\tau = 1$ and fixed $KK_c(T/T_i)$ values of 1 and 0.5, by using (17) and (19).

It is worth mentioning that plotting stability boundary locus for $\omega \in [0, \omega_c]$ will be enough since the controller operates in this frequency range [17]. Here, ω_c is the critical frequency value where the Nyquist plot of a plant transfer function intersects the negative real axis, or open loop transfer function phase is equal to -180^o. Therefore, with the help of these graphs, one can find the following four linear equations by using $KK_c(T/T_i)$ and $KK_c(T_d/T)$ values corresponding to a constant KK_c value.

$$l_1: \quad KK_c(T_d/T) = 0.0348(KK_c(T/T)_i) + 0.978,$$
$$l_2: \quad KK_c(T_d/T) = 0,$$
$$l_3: \quad KK_c(T_d/T) = 0.6992(KK_c(T/T)_i) - 0.3557,$$
$$l_4: \quad KK_c(T_d/T) = 0.6993(KK_c(T/T)_i) - 0.3559. \qquad (20)$$

Using the above obtained linear equations, stability boundary locus in $(KK_c(T/T_i), KK_c(T_d/T))$ plane can be formed for the normalized dead time ratio $\tau = 0.5$ and fixed value of $KK_c = 1.5$. The result is depicted in Fig. 4.

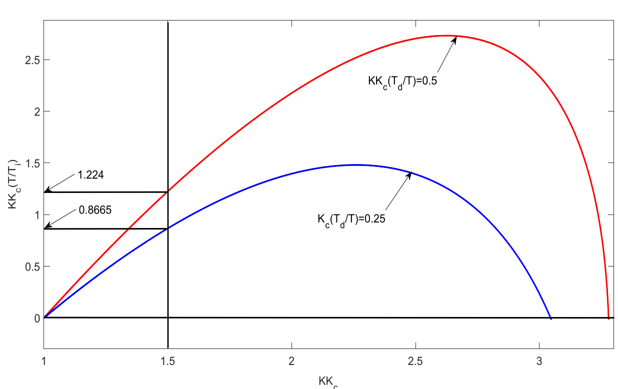

Figure 2. Stability boundary locus in $(KK_c, KK_c(T/T_i))$ plane for fixed values of $KK_c(T_d/T)$

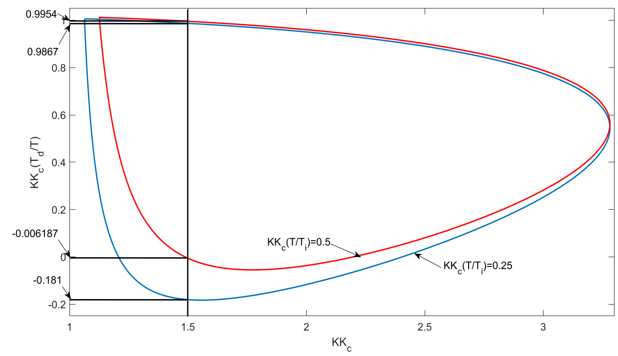

Figure 3. Stability boundary locus in $(KK_c, KK_c(T_d/T))$ plane for fixed values of $KK_c(T/T_i)$

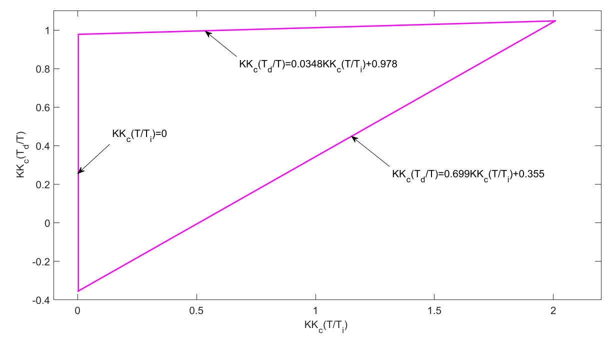

Figure 4. Stability region in $(KK_c(T/T_i), KK_c(T_d/T))$ plane for for fixed $KK_c = 1.5$ and $\tau = 0.5$.

Similar computations can be carried out for different normalized dead time ratios. Stability boundary loci corresponding to different dead time ratios are presented in Figs. 5, 6 and 7. The stability boundary loci given in Figs. 5, 6 and 7 can be considered as generalized, since, once the UFOPDT model is known, all stabilizing PID controller tuning parameters can be found from Fig. 5 for the fixed value of $KK_c = 1.5$ and varying values of normalized dead time. If it is required, stability boundary loci can be plotted for different normalized dead time and KK_c values. By this way, the approach can be made more generalized.

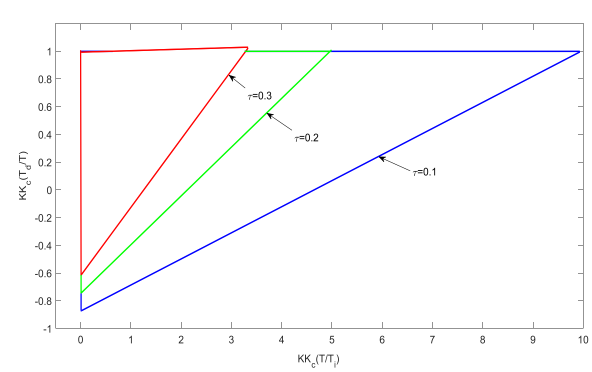

Figure 5. Stability regions in $(KK_c(T/T_i), KK_c(T_d/T))$ plane for different normalized dead time ratios and $KK_c = 1.5$

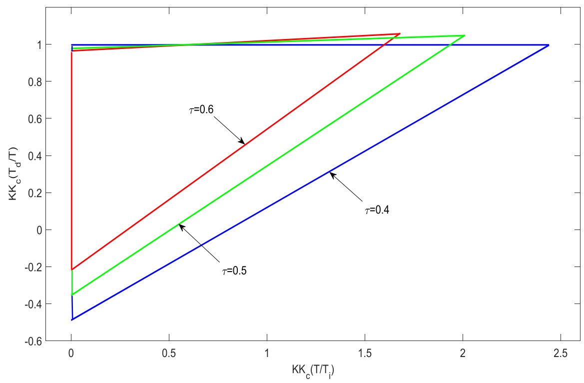

Figure 6. Stability regions in $(KK_c(T/T_i), KK_c(T_d/T))$ plane for different normalized dead time ratios and $KK_c = 1.5$

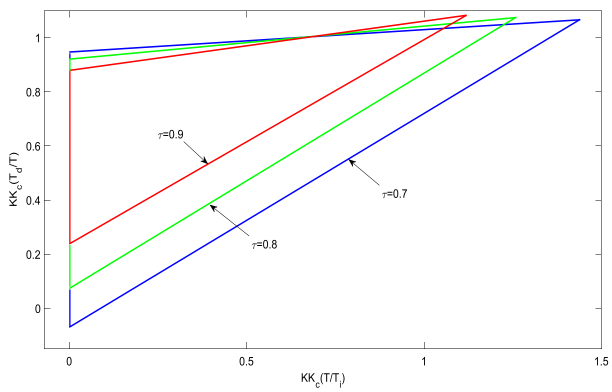

Figure 7. Stability regions in $(KK_c(T/T_i), KK_c(T_d/T))$ plane for different normalized dead time ratios and $KK_c = 1.5$

III. Examples

This section provides simulation examples to illustrate the use of proposed approach.

A. Example 1

Let's consider a process transfer function of $G(s) = e^{-0.2s}/(s-1)$. The normalized dead time value for this transfer function is $\tau = 0.2$. Since the actual system transfer function matches exactly the UFOPDT model transfer function, the relay feedback identification method of Kaya and Atherton [18] will give exact solutions for the UFOPDT model. In Fig. 5, the region remaining inside dead time ratio of $\tau = 0.2$ can be used to determine all stabilizing PID controller tuning parameters. Some points taken from the stability region corresponding $\tau = 0.2$ stability region and the resultant PID tuning parameters are summarized in Table I. Note that the controller gain $K_c = 1.5$ in all cases, as $K = 1$, $T = 1$ for this example. Fig. 8 shows the closed-loop step responses of the closed loop system for the determined PID controllers. The figure proves the validity of the obtained stability region.

TABLE I. Some calculated tuning parameters for example 1

case	*Selected points*		*Calculated tuning parameters*	
	$KK_c\left(\frac{T}{T_i}\right)$	$KK_c\left(\frac{T_d}{T}\right)$	T_i	T_d
a	0.5	0.8	3	0.533
b	1	0.6	1.5	0.4
c	1.5	0.5	1	0.333
d	2	0.4	0.75	0.266
e	3	0.7	0.5	0.466
f	3.5	0.9	0.428	0.6

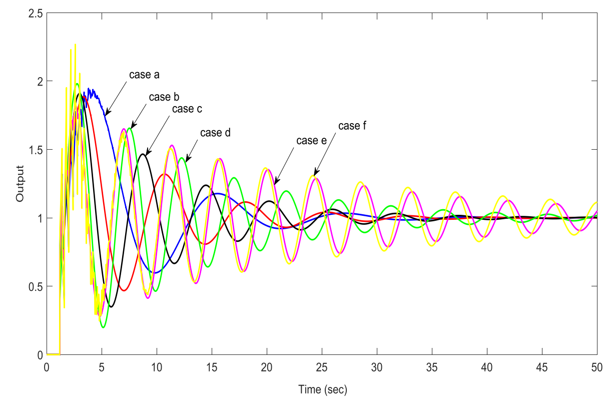

Figure 8. Closed-loop step responses for determined PID controllers for example1

B. Example 2

In this example, let's take a higher order process transfer function given by $G(s) = e^{-0.5s}/(2s-1)(0.5s+1)$. This process transfer function is modeled as UFOPDT model of $G_m(s) = e^{-0.9184s}/(2.6956s-1)$ by using relay feedback identification method of Kaya and Atherton [18]. Obtained UFOPDT model transfer function has the normalized dead time ratio of $\tau = 0.34$. Before determining all stabilizing PID controller parameters for this example, it would be appropriate to show the matching between the stability boundary locus of the actual process transfer function and the stability boundary locus of UFOPDT model transfer function. The result is presented in Fig. 9, illustrating a mismatch between the stability boundary locus obtained by the actual process transfer function and the stability boundary locus obtained by the UFOPDT model transfer function. To overcome this problem, the stability region corresponding to a smaller dead time ratio than dead time ratio of the UFOPDT model, which is 0.34 for this example, must be used. It has been observed through extensive trials, adding 0.2 to dead time ratio of the UFOPDT model, usually, results in the stability region that matches with the stability region of actual plant transfer function. So, the stability region obtained in Fig. 5 for the dead time ratio of $\tau = 0.5$, which is the closest value to the normalized dead time ratio of the UFOPDT model transfer function (0.34) plus 0.2, is used to determine all stabilizing PID controller tuning parameters. Table II summarizes the results for this example. In this example, the controller gain $K_c = 1.5$ in all cases, and $K = 1$, $T = 2.6956$. Fig. 10 demonstrates closed-loop responses to a unit step input for the determined PID controller parameter values. Responses in Fig. 10 validate the proposed approach to calculate all stabilizing PID controllers.

TABLE II. SOME CALCULATED TUNING PARAMETERS FOR EXAMPLE 2

case	*Selected points*		*Calculated tuning parameters*	
	$KK_c\left(\frac{T}{T_i}\right)$	$KK_c\left(\frac{T_d}{T}\right)$	T_i	T_d
a	0.1	-0.1	40.434	-0.179
b	0.2	0.1	20.217	0.179
c	0.6	0.5	6.739	0.898
d	0.7	0.6	5.776	1.078
e	1	0.7	4.043	1.257
f	1.4	0.8	2.888	1.437

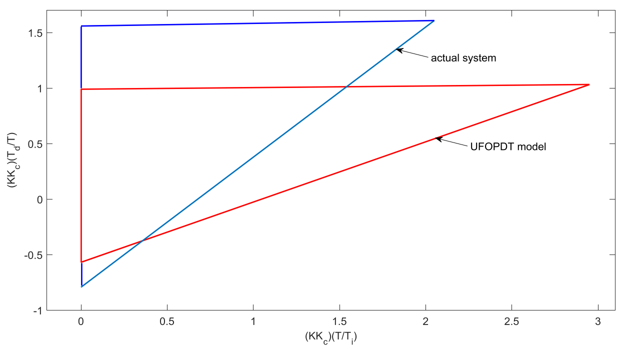

Figure 9. Stability regions for actual system and UFOPDT model transfer function of example

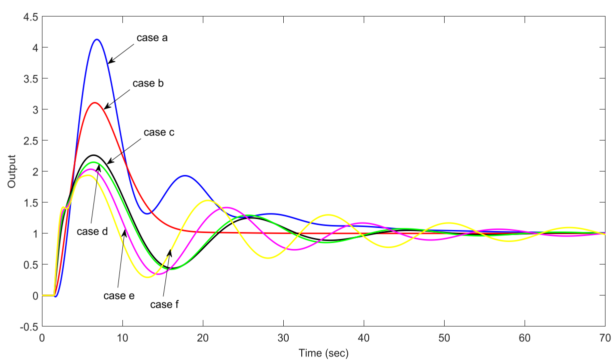

Figure 10. Closed loop step responses for determined PID controller parameter values for example 2.

C. Example 3

In this example, another high order transfer function of $G(s) = e^{-0.2s} / (0.1s-1)(s+1)^3$ is considered. The UFOPDT model was obtained as $G_m(s) = e^{-2.8376s} / (15.4099s-1)$ using the relay feedback identification method of Kaya and Atherton [18]. The UFOPDT model has the normalized dead time ratio of $\tau = 0.18414$. Adding 0.2 to this ratio value, the stability region corresponding to $\tau = 0.4$ in Fig. 6 has been used to determine all stabilizing PID controllers. The points and corresponding PID controller parameters taken from the inside of stability region corresponding to the normalized dead time of $\tau = 0.4$ are given in Table III. Since $K = 1$ and $T = 15.4099$ for this example, hence the controller gain is $K_c = 1.5$ in all cases. Fig. 11 illustrates closed-loop step responses for designed PID controllers. The validity of the proposed approach has been confirmed once again.

TABLE III. SOME CALCULATED TUNING PARAMETERS FOR EXAMPLE 3

case	*Selected points*		*Calculated tuning parameters*	
	$KK_c\left(\frac{T}{T_i}\right)$	$KK_c\left(\frac{T_d}{T}\right)$	T_i	T_d
a	0.2	0.9	115.57	9.245
b	0.5	0.8	46.228	8.218
c	1	0.5	23.114	5.136
d	1.5	0.4	15.409	4.109
e	2	0.65	11.557	6.677
f	2.5	0.85	9.2456	8.732

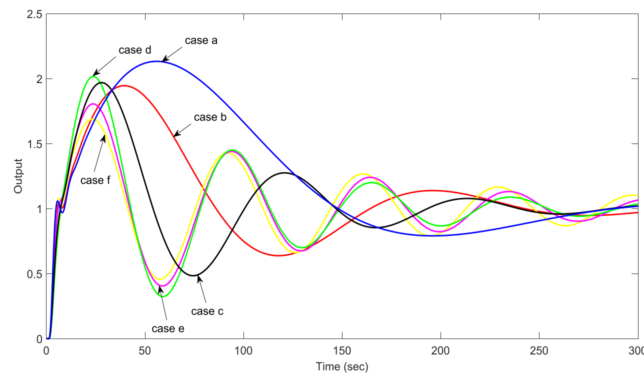

Figure 11. Closed-loop step responses for determined PID controller parameter values for example

IV. CONCLUSION

In this study, a generalized method has been given for determining all stabilizing PID controllers for stability of unstable processes plus dead time. In order to implement the method, the UFOPDT model of the actual process transfer function has to be obtained. If the actual process and the UFOPDT model transfer functions matches exactly, then obtained stability regions will give exact solutions. If the actual process transfer function is a high order one, then there exists a mismatch between the stability regions obtained from the actual and model transfer functions. In this case, it has been shown that adding 0.2 to the dead time ratio of the UFOPDT model and using this resultant dead time ratio gives the stability region for all stabilizing PID controllers. The proposed approach removes the necessity of redrawing the stability boundary locus each time as the process transfer function changes when the actual and model transfer functions matches exactly. However, more research must be carried out when there is a mismatch between the actual and model transfer functions.

ACKNOWLEDGMENT

This work is supported by Dicle University Coordinatorship of Scientific Research Projects (DUBAP) under Grant

no. MUHENDISLIK.15.013.

REFERENCES

[1] Aström, K. J. and Hagglund, T. (2001). The future of PID control. Control Engineering Practice, 9, 1163-1175.

[2] Ziegler, J.G. and Nichols, N. B. (1942). Optimum settings for automatic controllers. Transactions of the ASME, 64, 759–768.

[3] Cohen, G.H. and Coon, G.A. (1953). Theoretical considerations of retarded control. Transactions of ASME, 75, 827-834.

[4] Åström, K. J. and Hägglund, T. (1984). Automatic tuning of simple regulators with specifications on phase and amplitude margins. Automatica, 20, 645–651.

[5] Zhuang, M. and Atherton, D. P. (1993). Automatic tuning of optimum PID controllers. IEE Proceedings-D: Control Theory Applications, 140, 216-224.

[6] Morari, M. and Zafiriou, E. (1989). Robust process control, Englewood Cliffs: Prentice-Hall.

[7] Smith, C.A. and Corripio, A.B. (1997). Principles and Practice of Automatic Process Control, John Wiley & Sons, New York.

[8] Ho M.T., Datta A., and Bhattacharyya S.P. (1996). A new approach to feedback stabilization. Proceedings of the 35th CDC, 4, 4643–4648.

[9] Ho M.T., Datta A., and Bhattacharyya S.P. (1997a). A linear programming characterization of all stabilizing PID controllers. Proceedings of American Control Conference, 6, 3922-3928.

[10] Ho M.T., Datta A., and Bhattacharyya S.P. (1997b). A new approach to feedback design Part I: Generalized interlacing and proportional control, Department of Electrical Engineering, Texas A&M University, College Station, TX, Tech. Report TAMU-ECE97-001-A.

[11] Ho M.T., Datta A., and Bhattacharyya S.P. (1997c). A new approach to feedback design Part II: PI and PID controllers, Dept. of Electrical Eng., Texas A& M Univ., College Station, TX, Tech. Report TAMU-ECE97-001-B.

[12] Munro, N. and Söylemez, M.T. (2000) Fast calculation of stabilizing PID controllers for uncertain parameter systems. Proceedings of Symposium on Robust Control, Prague.

[13] Söylemez, M.T., Munro, N., and Baki, H. (2003) Fast calculation of stabilizing PID controllers. Automatica, 39, 121- 126.

[14] Shafiei, Z. and Shenton, A. T. (1997). Frequency domain design of PID controllers for stable and unstable systems with time delay. Automatica, 33, 2223- 2232.

[15] Huang. Y. J. and Wang, Y. J. (2000). Robust PID tuning strategy for uncertain plants based on the Kharitonov theorem. ISA Transactions, 39, 419-43 1.N. Kawasaki, "Parametric study of thermal and chemical nonequilibrium nozzle flow," M.S. thesis, Dept. Electron. Eng., Osaka Univ., Osaka, Japan, 1993.

[16] Tan, N., Kaya, I., and Atherton, D. P. (2003). Computation of stabilizing Pl and PID controllers. Proceedings of the IEEE International Conference on the Control Applications (CCA2003), Istanbul, Turkey. 2003.

[17] Tan, N. (2005). Computation of stabilizing PI and PID controllers for processes with time delay. ISA Transactions, 44, 213-223.

[18] Zàvackà, J., Bakošovà, M. and Matejičkovà, K. (2013). Robust PI controller design for a continuous stirred tank reactor with multiple steady-states. International Conference on Process Control (PC), 468-473.

[19] Sandeep, D.H. and Yogesh, V.H. (2014). Design of PID controller for inverted pendulum using stability boundary locus. Annual IEEE India Conference (INDICON).

[20] Deniz F.N., Alagoz B.B., Tan, N. and Atherton D.P. (2016). An integer order approximation method based on stability boundary locus for fractional order derivative/integrator operators. ISA Transactions, 62, 154-163.

[21] Kaya, I. and Atiç, S. (2016). PI Controller Design based on Generalized Stability Boundary Locus. International Conference on System Theory, Control and Computing (ICSTCC), 20, 24–28

[22] Kaya, I. and Atherton, D.P. (2001). Parameter estimation from relay autotuning with asymmetric limit cycle data. Journal of Process Control, 11, 429–439.