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New LMI Conditions for Static Output Feedback Control of Continuous-Time Linear Systems with Parametric Uncertainties

Hassène Gritli and Safya Belghith

Abstract— **The design of static output feedback (SOF) is of fundamental importance in control theory. This paper studies the SOF control problem of continuous-time linear systems subject to norm-bounded parametric uncertainties. New simple sufficient linear matrix inequality (LMI) conditions with a line search over a scalar variable for designing robust SOF controllers are proposed. We show that the new design method gives less conservative results than those available in the literature by inserting an equality constraint. Numerical examples are given to show the validity and superiority of our proposed method.**

I. Introduction

Considerable efforts have been assigned to the design of static/dynamic output feedback controllers since in almost all practical control applications, a full access to the state of the physical system is not always possible [1], [2], [3], [4]. Moreover, the problem of robust stabilization of both linear and nonlinear systems subject to norm-bounded or polytopic parametric uncertainties has received a significant amount of attention to date [5]. Generally, robust observer-based controllers are very often used to stabilize uncertain linear and nonlinear systems. However, in contrast to the observer-based control scheme, the static output feedback (SOF) controllers are with simpler structures and more easily realized in practice compared to the observer-based controllers and also the dynamic output feedback controllers.

While a wide variety of problems related to the controller design can be recast as convex Linear Matrix Inequality (LMI), this is not the case for the design of the SOF controller. The most general characterization of the SOF design is Bilinear Matrix Inequalities (BMI). However, it is well known that solving a BMI is an NP-hard problem from computation complexity point-of-view, which is a drawback for numerical implementations [6]. For this reason, the SOF controller design is one of the most challenging open problems in control theory and practice. The SOF control problem of linear systems has been extensively studied and there are various approaches to deal with it, see the survey papers [7], [1] and references therein. In recent years, there are various numerical algorithms addressing the design problem of SOF controllers through various approaches where iterative algorithms based on LMI conditions are widely used. We refer our readers to [1], [2], [8], [9], [10], [11], [12], just to mention a few, for some other methods. In contrast to these approaches, various convex sufficient conditions for designing SOF stabilizing controllers are proposed in the literature. By inserting an linear matrix equality constraint depending on the Lyapunov matrix [13], sufficient LMI-based conditions for designing SOF controllers are given, see also [12] for other interesting approaches. In addition, in [13] which is the main motivation of this paper, it was assumed that the output matrix of the continuous-time linear system must be full row rank. However, due to the presence of the equality constraint and the rank condition, the problem becomes more conservative and hence obtaining a solution becomes more difficult and in some special cases, the problem is completely unrealizable.

Motivated by the above discussions, and in order to derive less conservative results than those obtained in [13], a new design methodology for the SOF controller synthesis of continuous-time linear systems under norm-bounded parametric uncertainties is proposed in this paper. In this work, only the state and output matrices are considered uncertain and then the input matrix is not uncertain. A new LMI condition is developed, where the output matrix is not required to be of full row rank. It is proved that the new method gives less conservative results that the method proposed in [13]. Our main idea lies in the use of a new technical Lemma that allows us to decouple the Lyapunov, the output and the input matrices from each other. This leads to a quite simple LMI condition that is numerically tractable with any LMI software. It is important to underline that the proposed LMI condition is derived and hence solved with a line search over a scalar variable, which is fixed a priori.

In this paper, we present first LMI conditions for the linear systems without uncertainties in order to make comparisons from conservatism point-of-view with the results in [13]. Moreover, we describe our new design methodology of the LMI conditions by considering parametric uncertainties. Our approach is applied to the numerical examples taken from [14] aiming at providing comparisons and to show the superiority of our proposed new design methodology.

II. Problem Statement and Background Results

A. Notations

The following notations will be used throughout this paper. In large matrix expressions, the symbol $(\star)$ replaces terms that are induced by symmetry. Moreover, $\mathcal{X}+(\star)=\mathcal{X}+\mathcal{X}^{\mathrm{T}}$. In addition, the null matrix and the identity matrix with appropriate dimensions are denoted by $\mathcal{O}$ and $\mathcal{I}$, respectively.

Hassène Gritli is with Institut Supérieur des Technologies de l'Information et de la Communication, Université de Carthage, 1164 Borj Cedria, Tunis, Tunisia. grhass@yahoo.fr

Hassène Gritli and Safya Belghith are with Laboratoire Robotique, Informatique et Systèmes Complexes (RISC-LR16ES07), Ecole Nationale d'Ingénieurs de Tunis, Université de Tunis El Manar, BP. 37, Le Belvédère, 1002 Tunis, Tunisia.

B. Technical Lemmas

Lemma 1: [15] For any matrices $\boldsymbol{\mathcal{M}} \in \mathbb{R}^{r\times s}$ and $\boldsymbol{\mathcal{N}} \in \mathbb{R}^{s\times r}$, and any matrix $\boldsymbol{\mathcal{F}}(t) \in \mathbb{R}^{s\times s}$ satisfying $\boldsymbol{\mathcal{F}}^{\mathrm{T}}(t)\boldsymbol{\mathcal{F}}(t) \leq \boldsymbol{\mathcal{I}}$, and a scalar $\epsilon > 0$, the following inequality holds:

$$\boldsymbol{\mathcal{M}}\boldsymbol{\mathcal{F}}(t)\boldsymbol{\mathcal{N}} + (\star) \leq \epsilon^{-1}\boldsymbol{\mathcal{M}}\boldsymbol{\mathcal{M}}^{\mathrm{T}} + \epsilon\boldsymbol{\mathcal{N}}^{\mathrm{T}}\boldsymbol{\mathcal{N}} \tag{1}$$

Lemma 2: For any square matrices $\boldsymbol{\Omega} = \boldsymbol{\Omega}^{\mathrm{T}}$, $\boldsymbol{\mathcal{G}} > 0$ (resp. $\boldsymbol{\mathcal{G}} < 0$) and $\boldsymbol{\mathcal{H}}$, the inequality

$$\boldsymbol{\Omega} + \boldsymbol{\mathcal{G}}\boldsymbol{\mathcal{H}} + \boldsymbol{\mathcal{H}}^{\mathrm{T}}\boldsymbol{\mathcal{G}} < 0 \tag{2}$$

is fulfilled if the following condition

$$\begin{bmatrix} \boldsymbol{\Omega} - \mu^{-1}\boldsymbol{\mathcal{G}} & (\star) \\ \boldsymbol{\mathcal{I}} + \mu\boldsymbol{\mathcal{H}} & \mu^{-1}\boldsymbol{\mathcal{G}} - 2\boldsymbol{\mathcal{I}} \end{bmatrix} < 0 \tag{3}$$

holds for some scalar $\mu > 0$ (resp. $\mu < 0$).

Proof: Let $\boldsymbol{\mathcal{G}}$ be a symmetric matrix and μ a scalar such that $\mu\boldsymbol{\mathcal{G}} > 0$. It follows from the following condition:

$$\begin{aligned}\left(\mu^{-1}\boldsymbol{\mathcal{G}} - \boldsymbol{\mathcal{I}}\right)\left(\mu^{-1}\boldsymbol{\mathcal{G}}\right)^{-1}\left(\mu^{-1}\boldsymbol{\mathcal{G}} - \boldsymbol{\mathcal{I}}\right)^{\mathrm{T}} \\ = \left(\mu^{-1}\boldsymbol{\mathcal{G}}\right)^{-1} + \mu^{-1}\boldsymbol{\mathcal{G}} - 2\boldsymbol{\mathcal{I}} \geq 0\end{aligned} \tag{4}$$

that

$$-\mu\boldsymbol{\mathcal{G}}^{-1} \leq \mu^{-1}\boldsymbol{\mathcal{G}} - 2\boldsymbol{\mathcal{I}} \tag{5}$$

Therefore, based on (5), matrix inequality (3) becomes:

$$\begin{bmatrix} \boldsymbol{\Omega} - \mu^{-1}\boldsymbol{\mathcal{G}} & (\star) \\ \boldsymbol{\mathcal{I}} + \mu\boldsymbol{\mathcal{H}} & -\mu\boldsymbol{\mathcal{G}}^{-1} \end{bmatrix} < 0 \tag{6}$$

By applying the Schur complement and since $\mu^{-1}\boldsymbol{\mathcal{G}} > 0$, matrix inequality (6) is equivalent to:

$$\boldsymbol{\Omega} - \mu^{-1}\boldsymbol{\mathcal{G}} + (\boldsymbol{\mathcal{I}} + \mu\boldsymbol{\mathcal{H}})^{\mathrm{T}}\left(\mu\boldsymbol{\mathcal{G}}^{-1}\right)^{-1}(\boldsymbol{\mathcal{I}} + \mu\boldsymbol{\mathcal{H}}) < 0 \tag{7}$$

By developing the left-hand side quantity in (7), we obtain then:

$$\boldsymbol{\Omega} + \boldsymbol{\mathcal{G}}\boldsymbol{\mathcal{H}} + \boldsymbol{\mathcal{H}}^{\mathrm{T}}\boldsymbol{\mathcal{G}} + \mu\boldsymbol{\mathcal{H}}^{\mathrm{T}}\boldsymbol{\mathcal{G}}\boldsymbol{\mathcal{H}} < 0 \tag{8}$$

It is obvious from condition (8) that, for any scalar μ such that $\mu\boldsymbol{\mathcal{G}} > 0$, the condition (2) is satisfied. This completes the proof of Lemma 2. ■

C. Problem Statement

Consider the continuous-time uncertain linear systems described by the following equations:

$$\dot{\boldsymbol{x}} = (\boldsymbol{\mathcal{A}} + \boldsymbol{\Delta}\boldsymbol{\mathcal{A}}(t))\,\boldsymbol{x} + \boldsymbol{\mathcal{B}}\boldsymbol{u} \tag{9}$$

$$\boldsymbol{y} = (\boldsymbol{\mathcal{C}} + \boldsymbol{\Delta}\boldsymbol{\mathcal{C}}(t))\,\boldsymbol{x} \tag{10}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, $\boldsymbol{y} \in \mathbb{R}^p$ is the output measurement and $\boldsymbol{u} \in \mathbb{R}^m$ is the control input vector. $\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{B}}$ and $\boldsymbol{\mathcal{C}}$ are constant matrices of adequate dimensions.

First, we consider the following assumptions:

- the uncertain linear system (9)-(10) is stabilizable;
- there exist matrices $\boldsymbol{\mathcal{M}}_1$, $\boldsymbol{\mathcal{M}}_2$, $\boldsymbol{\mathcal{N}}_1$, $\boldsymbol{\mathcal{N}}_2$, $\boldsymbol{\mathcal{F}}_1(t)$ and $\boldsymbol{\mathcal{F}}_2(t)$ of appropriate dimensions so that:

$$\boldsymbol{\Delta}\boldsymbol{\mathcal{A}}(t) = \boldsymbol{\mathcal{M}}_1\boldsymbol{\mathcal{F}}_1(t)\boldsymbol{\mathcal{N}}_1 \tag{11a}$$

$$\boldsymbol{\Delta}\boldsymbol{\mathcal{C}}(t) = \boldsymbol{\mathcal{M}}_2\boldsymbol{\mathcal{F}}_2(t)\boldsymbol{\mathcal{N}}_2 \tag{11b}$$

where the unknown matrices $\boldsymbol{\mathcal{F}}_i(t)$, $i = 1, 2$, satisfy the following condition:

$$\boldsymbol{\mathcal{F}}_i^{\mathrm{T}}(t)\,\boldsymbol{\mathcal{F}}_i(t) \leq \boldsymbol{\mathcal{I}}, \quad \text{for } i = 1, 2, \quad \forall\, t \in \mathbb{R} \tag{12}$$

Our objective in this work is to design a linear static output feedback controller

$$\boldsymbol{u} = \boldsymbol{\mathcal{K}}\boldsymbol{y} \tag{13}$$

in order to robustly stabilize the linear system (9)-(10) under the norm-bounded parametric uncertainties satisfying condition (12). In (13), $\boldsymbol{\mathcal{K}} \in \mathbb{R}^{m\times p}$ is the feedback gain matrix to design.

Thus, under the static output feedback controller $\boldsymbol{u}$ in (13), the uncertain linear system (9) becomes as follows:

$$\dot{\boldsymbol{x}} = (\boldsymbol{\mathcal{A}} + \boldsymbol{\Delta}\boldsymbol{\mathcal{A}}(t) + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}(\boldsymbol{\mathcal{C}} + \boldsymbol{\Delta}\boldsymbol{\mathcal{C}}(t)))\,\boldsymbol{x} \tag{14}$$

To derive stability condition of the closed-loop uncertain linear system (14), we define the following Lyapunov function candidate:

$$\mathcal{V}(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\mathcal{P}}\boldsymbol{x} \tag{15}$$

where $\boldsymbol{\mathcal{P}}$ is a symmetric positive-definite matrix to be determined next together with the feedback matrix $\boldsymbol{\mathcal{K}}$ using the LMI approach.

A sufficient condition on the stability of the closed-loop linear system (14) is that:

$$\begin{aligned}\dot{\mathcal{V}}(\boldsymbol{x}) =& 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}})\,\boldsymbol{x} + \\ & 2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\mathcal{P}}(\boldsymbol{\Delta}\boldsymbol{\mathcal{A}}(t) + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\Delta}\boldsymbol{\mathcal{C}}(t))\,\boldsymbol{x} < 0\end{aligned} \tag{16}$$

It is worth noticing that $\dot{\mathcal{V}}(\boldsymbol{x}) < 0$, $\forall\ \boldsymbol{x} \neq \boldsymbol{0}$, if the following matrix inequality holds:

$$\boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}}) + \boldsymbol{\mathcal{P}}(\boldsymbol{\Delta}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\Delta}\boldsymbol{\mathcal{C}}) + (\star) < 0, \quad \boldsymbol{\mathcal{P}} > 0 \tag{17}$$

We provide also another stability condition by multiplying (17) from both sides by $\boldsymbol{\mathcal{S}} = \boldsymbol{\mathcal{P}}^{-1}$. Then, we get

$$(\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}})\,\boldsymbol{\mathcal{S}} + (\boldsymbol{\Delta}\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\Delta}\boldsymbol{\mathcal{C}})\,\boldsymbol{\mathcal{S}} + (\star) < 0, \quad \boldsymbol{\mathcal{S}} > 0 \tag{18}$$

The uncertain continuous-time linear system (9)-(10) is stabilizable via the static output feedback (13) if there exist matrices $\boldsymbol{\mathcal{P}} > 0$ (resp. $\boldsymbol{\mathcal{S}} > 0$) and $\boldsymbol{\mathcal{K}}$, of compatible dimensions, such that condition (17) (resp. (18)) holds.

To synthesize traceable stability condition through constraint (17) or systematically constraint (18), we will consider first the uncertainty-free case, that is a linear system (9)-(10) without parametric uncertainties. As a result, the two conditions (17) and (18) will be recast, respectively, as:

$$\boldsymbol{\mathcal{P}}(\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}}) + (\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}})^{\mathrm{T}}\boldsymbol{\mathcal{P}} < 0, \quad \boldsymbol{\mathcal{P}} > 0 \tag{19}$$

$$(\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}})\,\boldsymbol{\mathcal{S}} + \boldsymbol{\mathcal{S}}(\boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}})^{\mathrm{T}} < 0, \quad \boldsymbol{\mathcal{S}} > 0 \tag{20}$$

Notice that these stability conditions (19) and (20) are well known in the literature for the static output feedback control, especially in [13] which is the main motivation of this paper. Actually, the main contribution of this paper consists of developing a new design methodology that we compare efficiently to the design methods provided in [13]. We stress that in [13], the design of the static output feedback was achieved by considering mainly two cases (among others): the uncertainty-free case and the polytopic-uncertainty case. The case of norm-bounded parametric uncertainties was not considered in [13]. To compare our results with those in [13], we will consider first the uncertainty-free case, and then conditions (19) and (20).

It is worth to note that the matrix inequality (19) (resp. (20)) is a BMI, which is not exploitable numerically to solve for matrices $\mathcal{P}$ (resp. $\mathcal{S}$) and $\mathcal{K}$. On the other hand, linearizing such BMI is a very difficult task because of the presence of the coupling term $\mathcal{PBKC}$ (resp. $\mathcal{BKCS}$). Many researchers in this field have attempted to solve this problem, but the resulting methods remain conservative because of the presence of some imposed conditions on the structure of the input and output matrices [13], [1], [12], [11], [7], [9]. In the following subsection, we recall some available results in the literature, and we describe, with details, especially the results of Crusius and Trofino in [13], which constitutes the main motivation of this paper.

D. Background Results

In order to give a comparison with the existing methods, the results in [13] are recalled. As discussed before, the problem of numerically solving the matrix inequality (19) (resp. (20)) for $\mathcal{P}$ (resp. $\mathcal{S}$) and $\mathcal{K}$, is in fact a very difficult one because it is not convex in general. Related to this nonconvex problem is the following convex one [13].

Theorem 1: ([13]) Let $\mathcal{P}$, $\mathcal{M}$ and $\mathcal{N}$ be solutions of the following $\mathcal{P}$-problem:

$$\mathcal{PA} + \mathcal{A}^{\mathrm{T}}\mathcal{P} + \mathcal{BNC} + \mathcal{C}^{\mathrm{T}}\mathcal{N}^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} < 0, \quad \mathcal{P} > 0 \tag{21}$$

$$\mathcal{BM} = \mathcal{PB} \tag{22}$$

Then, the static output feedback control (13) with

$$\mathcal{K} = \mathcal{M}^{-1}\mathcal{N} \tag{23}$$

stabilizes the certain linear system (9)-(10).

Theorem 2: ([13]) Let $\mathcal{S}$, $\mathcal{M}$ and $\mathcal{N}$ be solutions of the following $\mathcal{S}$-problem:

$$\mathcal{AS} + \mathcal{SA}^{\mathrm{T}} + \mathcal{BNC} + \mathcal{C}^{\mathrm{T}}\mathcal{N}^{\mathrm{T}}\mathcal{B}^{\mathrm{T}} < 0, \quad \mathcal{S} > 0 \tag{24}$$

$$\mathcal{MC} = \mathcal{CS} \tag{25}$$

Then, the static output feedback control (13) with

$$\mathcal{K} = \mathcal{NM}^{-1} \tag{26}$$

stabilizes the certain linear system (9)-(10).

Note that the equality constraint (22) (resp. (25)) is imposed between the input (resp. output) matrix and the Lyapunov matrix, which might lead to a strict constraint. Moreover, according to [13], it was assumed in Theorem 1 (resp. Theorem 2) that $\mathcal{B}$ (resp. $\mathcal{C}$) is full column (resp. row) rank. Moreover, notice that even if Theorem 1 (resp. Theorem 2) provides (if possible) solutions, it remains quite conservative because of the presence of the equality constraint (22) (resp. (25)).

III. Main Theoretical Results

In this section, we present our main contribution for the design of the static output feedback controller allowing the robust stabilization of the continuous-time linear system under parameter uncertainties (14). We develop first the main results in the uncertainty-free case. After that, we will provide the results in the presence of the uncertainties.

A. Systems Without Parametric Uncertainties

For simplicity of the presentation and to understand easily our proposed main results for the design of the static output feedback, we first consider linear systems without parameter uncertainties, i.e. $\boldsymbol{\Delta}\mathcal{A} = \mathcal{O}_{n\times n}$ and $\boldsymbol{\Delta}\mathcal{C} = \mathcal{O}_{p\times n}$. Our design methodology will be achieved mainly through Lemma 2.

Let us consider the closed-loop system (14) without uncertainties and its stability condition (19) or systematically (20), which are BMIs. As noted before, the problem comes from the presence of the coupling term $\mathcal{PBKC}$ in (19) or $\mathcal{BKCS}$ in (20). To solve such problem, we need to decouple the two matrices $\mathcal{P}$ and $\mathcal{K}$ (or $\mathcal{S}$ and $\mathcal{K}$) from each other. In the following, we present our main results for solving such problem in the uncertainty-free case.

Theorem 3: Let $\mathcal{P} > 0$ and $\mathcal{K}$ be two matrices of adequate dimensions and solutions of the following LMI for some fixed scalar $\mu > 0$:

$$\begin{bmatrix} \mathcal{PA} + \mathcal{A}^{\mathrm{T}}\mathcal{P} - \mu^{-1}\mathcal{P} & (\star) \\ \mathcal{I} + \mu\mathcal{BKC} & \mu^{-1}\mathcal{P} - 2\mathcal{I} \end{bmatrix} < 0 \tag{27}$$

Then, the static output feedback control law (13) stabilizes the linear system (9)-(10) without parametric uncertainties.

Proof: Let us consider Lemma 2. It is obvious that the matrix inequality (19) can be recast as the matrix inequality (2) with $\boldsymbol{\Omega} = \mathcal{PA} + \mathcal{A}^{\mathrm{T}}\mathcal{P}$, $\mathcal{G} = \mathcal{P}$ and $\mathcal{H} = \mathcal{BKC}$. Therefore, the BMI (19) is satisfied if the matrix inequality (27) holds. We emphasize that as $\mathcal{P} > 0$ (and then $\mathcal{G} > 0$), we must have $\mu > 0$. Obviously, the matrix inequality (27) is a LMI with respect to the unknown variables $\mathcal{P}$ and $\mathcal{K}$. This completes the proof of Theorem 3. ■

Theorem 4: Let $\mathcal{S} > 0$ and $\mathcal{K}$ be solutions of the following LMI for some fixed scalar $\mu > 0$:

$$\begin{bmatrix} \mathcal{AS} + \mathcal{SA}^{\mathrm{T}} - \mu^{-1}\mathcal{S} & \mathcal{I} + \mu\mathcal{BKC} \\ (\star) & \mu^{-1}\mathcal{S} - 2\mathcal{I} \end{bmatrix} < 0 \tag{28}$$

Then, the static output feedback control law (13) stabilizes the linear system (9)-(10) without parametric uncertainties.

Proof: Consider the matrix inequality (20) and the matrix inequality (2) with $\boldsymbol{\Omega} = \mathcal{AS} + \mathcal{SA}^{\mathrm{T}}$, $\mathcal{G} = \mathcal{S}$ and $\mathcal{H}^{\mathrm{T}} = \mathcal{BKC}$. Thus, the BMI (20) is fulfilled if the LMI (28) is feasible for some prescribed positive scalar μ. This ends the proof. ■

B. On the Necessary Conditions for the LMI Feasibility

This part is devoted to some remarks about the necessary conditions for the feasibility of the new designed LMI condition (27) (or systematically LMI (28)) for the computation of the static output feedback gain $\mathcal{K}$ in the uncertainty-free case. It should be noticed that the necessary conditions for the feasibility of the LMI (27) are:

$$\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{P}}-\mu^{-1}\boldsymbol{\mathcal{P}}<0 \tag{29}$$

$$\mu^{-1}\boldsymbol{\mathcal{P}}-2\boldsymbol{\mathcal{I}}<0 \tag{30}$$

From (29) and (30), we obtain hence the following condition:

$$\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{P}}<2\boldsymbol{\mathcal{I}} \tag{31}$$

which is feasible for (all) stable and unstable linear systems without uncertainties.

However, in [13] (Theorem 1 and Theorem 2), the necessary condition for the feasibility of LMI (21) (resp. LMI (24)) is analytically more conservative because of the presence of the equality constraint (22) (resp. (25)). To illustrate this statement, we follow the strategy used in [14]. We consider first the particular class of linear systems with matrices $\boldsymbol{\mathcal{B}}=\begin{bmatrix}\boldsymbol{\mathcal{B}}_1\\ \boldsymbol{\mathcal{O}}\end{bmatrix}$ and $\boldsymbol{\mathcal{C}}=\begin{bmatrix}\boldsymbol{\mathcal{C}}_1 & \boldsymbol{\mathcal{C}}_2\end{bmatrix}$, where $\boldsymbol{\mathcal{B}}_1$, $\boldsymbol{\mathcal{C}}_1$ and $\boldsymbol{\mathcal{C}}_2$ are of appropriate dimension. Now, we write: $\boldsymbol{\mathcal{A}}=\begin{bmatrix}\boldsymbol{\mathcal{A}}_{11} & \boldsymbol{\mathcal{A}}_{12}\\ \boldsymbol{\mathcal{A}}_{21} & \boldsymbol{\mathcal{A}}_{22}\end{bmatrix}$, $\boldsymbol{\mathcal{P}}=\begin{bmatrix}\boldsymbol{\mathcal{P}}_{11} & \boldsymbol{\mathcal{P}}_{12}\\ \boldsymbol{\mathcal{P}}_{12}^{\mathrm{T}} & \boldsymbol{\mathcal{P}}_{22}\end{bmatrix}$ and $\boldsymbol{\mathcal{N}}=\begin{bmatrix}\boldsymbol{\mathcal{N}}_1 & \boldsymbol{\mathcal{N}}_2\end{bmatrix}$. Moreover, let $\boldsymbol{\mathcal{B}}^{\perp}$ be the kernel of $\boldsymbol{\mathcal{B}}$, i.e, $\boldsymbol{\mathcal{B}}^{\perp}\boldsymbol{\mathcal{B}}=\boldsymbol{\mathcal{O}}$. We take $\boldsymbol{\mathcal{B}}^{\perp}=\begin{bmatrix}\boldsymbol{\mathcal{O}} & \boldsymbol{\mathcal{B}}_2\end{bmatrix}$. The equality constraint (22), i.e. $\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{M}}=\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{B}}$, leads to $\boldsymbol{\mathcal{B}}^{\perp}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{B}}=\boldsymbol{\mathcal{B}}_2\boldsymbol{\mathcal{P}}_{12}^{\mathrm{T}}\boldsymbol{\mathcal{B}}_1=\boldsymbol{\mathcal{O}}$, and then $\boldsymbol{\mathcal{P}}_{12}=\boldsymbol{\mathcal{O}}$, which means that the Lyapunov matrix $\boldsymbol{\mathcal{P}}$ is diagonal.

On the other hand, after some manipulations, a necessary condition for the feasibility of the LMI (21) is:

$$\boldsymbol{\mathcal{P}}_{22}\boldsymbol{\mathcal{A}}_{22}+\boldsymbol{\mathcal{A}}_{22}^{\mathrm{T}}\boldsymbol{\mathcal{P}}_{22}<0 \tag{32}$$

which means that the matrix $\boldsymbol{\mathcal{A}}_{22}$ must be Schur stable.

Therefore, the necessary condition for the feasibility of the LMI (21) is the Schur stability of $\boldsymbol{\mathcal{A}}_{22}$ in the sense of (32). This restriction on the state matrix $\boldsymbol{\mathcal{A}}$ and the Lyapunov matrix $\boldsymbol{\mathcal{P}}$ shows analytically the superiority of our proposed design methodology of the static output feedback control at least for this particular class of linear systems.

The same feasibility problem occurs also for the LMI (24) with the equality constraint (25). To prove this, we take $\boldsymbol{\mathcal{B}}=\begin{bmatrix}\boldsymbol{\mathcal{B}}_1\\ \boldsymbol{\mathcal{B}}_2\end{bmatrix}$, $\boldsymbol{\mathcal{C}}=\begin{bmatrix}\boldsymbol{\mathcal{C}}_1 & \boldsymbol{\mathcal{O}}\end{bmatrix}$ and $\boldsymbol{\mathcal{C}}^{\perp}=\begin{bmatrix}\boldsymbol{\mathcal{O}}\\ \boldsymbol{\mathcal{C}}_2\end{bmatrix}$. Finally, we will found the condition: $\boldsymbol{\mathcal{A}}_{22}\boldsymbol{\mathcal{S}}_{22}+\boldsymbol{\mathcal{S}}_{22}\boldsymbol{\mathcal{A}}_{22}^{\mathrm{T}}<0$, which means that the matrix $\boldsymbol{\mathcal{A}}_{22}$ must be Schur stable also for this case.

C. Systems With Parametric Uncertainties

We take now into consideration the norm-bounded parametric uncertainties injected in the linear system (14) and the stability condition (17).

Theorem 5: The linear system (9)-(10) under the norm-bounded parametric uncertainties defined by (11)-(12), is robustly stable via the static output feedback control law (13), if, for a fixed scalar $\mu>0$, there exist matrices $\boldsymbol{\mathcal{P}}>0$ and $\boldsymbol{\mathcal{K}}$ of adequate dimensions and scalars ϵ_1 and ϵ_2 so that the LMI condition (33) (given at the top of the next page) is feasible.

Proof: To demonstrate the result in Theorem 5, we consider as previously Lemma 2. The stability condition (17) can be rewritten under the form of inequality (2) by taking $\boldsymbol{\Omega}=\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{P}}+\boldsymbol{\mathcal{P}}\boldsymbol{\Delta}\boldsymbol{\mathcal{A}}+\boldsymbol{\Delta}\boldsymbol{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{P}}$, $\boldsymbol{\mathcal{G}}=\boldsymbol{\mathcal{P}}$ and $\boldsymbol{\mathcal{H}}=\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}}+\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\Delta}\boldsymbol{\mathcal{C}}$. Therefore, relying on the matrix inequality (3), we obtain the following result:

$$\begin{bmatrix}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{P}}\boldsymbol{\Delta}\boldsymbol{\mathcal{A}}+(\star)-\mu^{-1}\boldsymbol{\mathcal{P}} & (\star)\\ \boldsymbol{\mathcal{I}}+\mu\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\left(\boldsymbol{\mathcal{C}}+\boldsymbol{\Delta}\boldsymbol{\mathcal{C}}\right) & \mu^{-1}\boldsymbol{\mathcal{P}}-2\boldsymbol{\mathcal{I}}\end{bmatrix}<0 \tag{34}$$

Based on relations in (11), the matrix inequality (34) can be rewritten like so:

$$\begin{aligned}&\begin{bmatrix}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{P}}-\mu^{-1}\boldsymbol{\mathcal{P}} & (\star)\\ \boldsymbol{\mathcal{I}}+\mu\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}} & \mu^{-1}\boldsymbol{\mathcal{P}}-2\boldsymbol{\mathcal{I}}\end{bmatrix}\\ &+\begin{bmatrix}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{M}}_1\\ \boldsymbol{\mathcal{O}}\end{bmatrix}\boldsymbol{\mathcal{F}}_1(t)\begin{bmatrix}\boldsymbol{\mathcal{N}}_1 & \boldsymbol{\mathcal{O}}\end{bmatrix}+(\star)\\ &+\mu\begin{bmatrix}\boldsymbol{\mathcal{O}}\\ \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{M}}_2\end{bmatrix}\boldsymbol{\mathcal{F}}_2(t)\begin{bmatrix}\boldsymbol{\mathcal{N}}_2 & \boldsymbol{\mathcal{O}}\end{bmatrix}+(\star)<0\end{aligned} \tag{35}$$

Relying on Lemma 1 and by taking into account (12), it follows that:

$$\begin{aligned}&\begin{bmatrix}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{M}}_1\\ \boldsymbol{\mathcal{O}}\end{bmatrix}\boldsymbol{\mathcal{F}}_1(t)\begin{bmatrix}\boldsymbol{\mathcal{N}}_1 & \boldsymbol{\mathcal{O}}\end{bmatrix}+(\star)\leq\\ &\epsilon_1^{-1}\begin{bmatrix}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{M}}_1\\ \boldsymbol{\mathcal{O}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{M}}_1\\ \boldsymbol{\mathcal{O}}\end{bmatrix}^{\mathrm{T}}+\epsilon_1\begin{bmatrix}\boldsymbol{\mathcal{N}}_1^{\mathrm{T}}\\ \boldsymbol{\mathcal{O}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\mathcal{N}}_1^{\mathrm{T}}\\ \boldsymbol{\mathcal{O}}\end{bmatrix}^{\mathrm{T}}\end{aligned} \tag{36}$$

$$\begin{aligned}&\mu\begin{bmatrix}\boldsymbol{\mathcal{O}}\\ \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{M}}_2\end{bmatrix}\boldsymbol{\mathcal{F}}_2(t)\begin{bmatrix}\boldsymbol{\mathcal{N}}_2 & \boldsymbol{\mathcal{O}}\end{bmatrix}+(\star)\leq\\ &\epsilon_2^{-1}\begin{bmatrix}\boldsymbol{\mathcal{O}}\\ \mu\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{M}}_2\end{bmatrix}\begin{bmatrix}\boldsymbol{\mathcal{O}}\\ \mu\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{M}}_2\end{bmatrix}^{\mathrm{T}}\\ &+\epsilon_2\begin{bmatrix}\boldsymbol{\mathcal{N}}_2^{\mathrm{T}}\\ \boldsymbol{\mathcal{O}}\end{bmatrix}\begin{bmatrix}\boldsymbol{\mathcal{N}}_2^{\mathrm{T}}\\ \boldsymbol{\mathcal{O}}\end{bmatrix}^{\mathrm{T}}\end{aligned} \tag{37}$$

Hence, by substituting constraints (36) and (37) in the matrix inequality (35) and by applying the Schur Lemma, inequality (35) is equivalent to the LMI (33). This completes the proof of Theorem 5. ■

As in the previous section, we develop now a LMI condition based on constraint (18). As a result, we state the following new Theorem.

Theorem 6: If, for some fixed constants $\mu>0$ and $\epsilon_2>0$, there exist matrices $\boldsymbol{\mathcal{S}}>0$ and $\boldsymbol{\mathcal{K}}$ and a scalar $\epsilon_1>0$ solutions of the LMI (38) (given in the next page), then the uncertain linear system (9)-(10) is robustly stable via the static output feedback control law (13).

Proof: Let us consider condition (18). To apply Lemma 2 as previously, we take $\boldsymbol{\Omega}=\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{S}}+\boldsymbol{\Delta}\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{S}}+\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\Delta}\boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{S}}+(\star)$, $\boldsymbol{\mathcal{G}}=\boldsymbol{\mathcal{S}}$ and $\boldsymbol{\mathcal{H}}^{T}=\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}}$. Moreover, via Lemma 1, it is easy to show that:

$$\boldsymbol{\Delta}\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{S}}+(\star)\leq\epsilon_1\boldsymbol{\mathcal{M}}_1\boldsymbol{\mathcal{M}}_1^{\mathrm{T}}+\epsilon_1^{-1}\boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{N}}_1^{\mathrm{T}}\boldsymbol{\mathcal{N}}_1\boldsymbol{\mathcal{S}} \tag{39}$$

$$\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\Delta}\boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{S}}+(\star)\leq\epsilon_2\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{M}}_2\boldsymbol{\mathcal{M}}_2^{\mathrm{T}}\boldsymbol{\mathcal{K}}^{\mathrm{T}}\boldsymbol{\mathcal{B}}^{\mathrm{T}}+\epsilon_2^{-1}\boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{N}}_2^{\mathrm{T}}\boldsymbol{\mathcal{N}}_2\boldsymbol{\mathcal{S}} \tag{40}$$

Thus, using these constraints (39) and (40), and by Schur complement, we obtain consequently LMI (38). ■

$$
\begin{bmatrix}
\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{A}}^{\mathrm{T}}\boldsymbol{\mathcal{P}}-\mu^{-1}\boldsymbol{\mathcal{P}}+\epsilon_1\boldsymbol{\mathcal{N}}_1^{\mathrm{T}}\boldsymbol{\mathcal{N}}_1+\epsilon_2\boldsymbol{\mathcal{N}}_2^{\mathrm{T}}\boldsymbol{\mathcal{N}}_2 & (\star) & \boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{M}}_1 & \boldsymbol{\mathcal{O}} \\
\boldsymbol{\mathcal{I}}+\mu\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}} & \mu^{-1}\boldsymbol{\mathcal{P}}-2\boldsymbol{\mathcal{I}} & \boldsymbol{\mathcal{O}} & \mu\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{M}}_2 \\
(\star) & (\star) & -\epsilon_1\boldsymbol{\mathcal{I}} & \boldsymbol{\mathcal{O}} \\
(\star) & (\star) & (\star) & -\epsilon_2\boldsymbol{\mathcal{I}}
\end{bmatrix} < 0 \tag{33}
$$

$$
\begin{bmatrix}
\boldsymbol{\mathcal{A}}\boldsymbol{\mathcal{S}}+\boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{A}}^{\mathrm{T}}-\mu^{-1}\boldsymbol{\mathcal{S}}+\epsilon_1\boldsymbol{\mathcal{M}}_1\boldsymbol{\mathcal{M}}_1^{\mathrm{T}} & \boldsymbol{\mathcal{I}}+\mu\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{C}} & \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{N}}_1^{\mathrm{T}} & \boldsymbol{\mathcal{S}}\boldsymbol{\mathcal{N}}_2^{\mathrm{T}} & \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{K}}\boldsymbol{\mathcal{M}}_2 \\
(\star) & \mu^{-1}\boldsymbol{\mathcal{S}}-2\boldsymbol{\mathcal{I}} & \boldsymbol{\mathcal{O}} & \boldsymbol{\mathcal{O}} & \boldsymbol{\mathcal{O}} \\
(\star) & (\star) & -\epsilon_1\boldsymbol{\mathcal{I}} & \boldsymbol{\mathcal{O}} & \boldsymbol{\mathcal{O}} \\
(\star) & (\star) & (\star) & -\epsilon_2\boldsymbol{\mathcal{I}} & \boldsymbol{\mathcal{O}} \\
(\star) & (\star) & (\star) & (\star) & -\epsilon_2^{-1}\boldsymbol{\mathcal{I}}
\end{bmatrix} < 0 \tag{38}
$$

Remark 1: It is worth to note that the matrix inequality (33) is a LMI if the scalar variable μ is fixed a priori. Similarly, inequality (38) is a LMI if μ and ϵ_2 are also fixed a priori. In order to overcome the difficulty of the choice of μ in (33) and μ and ϵ_2 in (38), we proceed as in [14] by using the gridding method. We can use also the logarithmic scale method as in [12]. In the present work, we adopted the first method.

Remark 2: It is worth to mention that the two obtained conditions (36) and (37) and also conditions (39) and (40) introduce some conservativeness in the developed LMI in Theorem 5 and that in Theorem 6.

In order to show the superiority of our new designed LMI (33) compared to the results in [13] by computing the tolerated maximum bounds of the parametric uncertainties, we provide the LMI-based conditions according to constraints (21)-(22) by taking into account the norm-bounded uncertainties. It is worth to recall that only the problem of polytopic uncertainties was considered in [13]. Then, relying on the previous results, it is straightforward to demonstrate that matrix inequality (21) under parametric uncertainties becomes as follows:

$$
\begin{bmatrix}
\begin{array}{c}\boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{A}}+\boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{N}}\boldsymbol{\mathcal{C}}+(\star)+\\ \epsilon_1\boldsymbol{\mathcal{N}}_1^{\mathrm{T}}\boldsymbol{\mathcal{N}}_1+\epsilon_2\boldsymbol{\mathcal{N}}_2^{\mathrm{T}}\boldsymbol{\mathcal{N}}_2\end{array} & \boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{M}}_1 & \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{N}}\boldsymbol{\mathcal{M}}_2 \\
(\star) & -\epsilon_1\boldsymbol{\mathcal{I}} & \boldsymbol{\mathcal{O}} \\
(\star) & (\star) & -\epsilon_2\boldsymbol{\mathcal{I}}
\end{bmatrix} < 0 \tag{41}
$$

It is possible to show also the superiority of our LMI (38) compared to the problem (24)-(25) by considering uncertainties. However, the obtained matrix inequality (from (24)) is complex enough to be solve numerically together with the already existing equality constraint (25). The establishment of this result is omitted here. Therefore, in the next section, we will compare only the LMI (33) with the LMI (41) combined with the equality constraint (22). We will also compare the conservatism of the LMI (33) with that in (38).

IV. Numerical Results and Conservatism Evaluation

In the sequel, we give two simple numerical examples in order to show the superiority of our design method of the SOF controller.

A. Example 1: On the Necessary Conditions

Consider the following example without uncertainties [14]:

$$
\boldsymbol{\mathcal{A}} = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}, \quad \boldsymbol{\mathcal{B}} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\mathcal{C}} = \begin{bmatrix} 1 & 0 \end{bmatrix}
$$

As demonstrated in the previous section, the design methodology of LMI-based stability conditions presented in [13] (Theorem 1 and Theorem 2) does not work on this simple example. Indeed, to satisfy the equality constraint (22), we should have $\boldsymbol{\mathcal{P}}_{22}\boldsymbol{\mathcal{A}}_{22} + \boldsymbol{\mathcal{A}}_{22}^{\mathrm{T}}\boldsymbol{\mathcal{P}}_{22} = 6\boldsymbol{\mathcal{P}}_{22} < 0$, which contractions the definition of $\boldsymbol{\mathcal{P}}_{22} > 0$ since $\boldsymbol{\mathcal{P}} > 0$. The same conclusion for the equality constraint (25). Therefore, Theorem 1 and Theorem 2 do not provide us any solution.

Otherwise, our LMI (27) in Theorem 3 provides the following SOF controller gain for a fixed scalar $\mu = 0.1$: $\boldsymbol{\mathcal{K}} = \begin{bmatrix} -5 & 0 \end{bmatrix}^{\mathrm{T}}$. It is worth to note that the LMI (27) was found to be feasible for all $\mu \leq \mu_\star^1 = 0.4157$.

Moreover, using our second designed LMI (28) in Theorem 4, and for $\mu = 0.1$, we obtained the same previous controller gain $\boldsymbol{\mathcal{K}}$. However, the LMI is feasible for all $\mu \leq \mu_\star^2 = 0.4134$. Accordingly, these results show the superiority of our proposed design methodology compared to that in [13].

B. Example 2: Conservatism Evaluation

We take the same example in [14]. The system has the following parameters:

$$
\boldsymbol{\mathcal{A}} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 1 & -2 & -5 \end{bmatrix}, \boldsymbol{\Delta\mathcal{A}} = \begin{bmatrix} 0 & 0 & a(t) \\ 0 & b(t) & 0 \\ c(t) & 0 & 0 \end{bmatrix},
$$

$$
\boldsymbol{\mathcal{B}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \boldsymbol{\mathcal{C}} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \boldsymbol{\Delta\mathcal{C}} = \begin{bmatrix} 0 & d(t) & 0 \end{bmatrix},
$$

where

$$
a(t) \leq \alpha \qquad b(t) \leq \beta \qquad c(t) \leq \gamma \qquad d(t) \leq \delta
$$

The uncertainties can be rewritten under the form (11) with:

$$
\boldsymbol{\mathcal{F}}_1(t) = \begin{bmatrix} \frac{a(t)}{\alpha} & 0 & 0 \\ 0 & \frac{b(t)}{\beta} & 0 \\ 0 & 0 & \frac{c(t)}{\gamma} \end{bmatrix}, \boldsymbol{\mathcal{N}}_1 = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & \beta & 0 \\ \gamma & 0 & 0 \end{bmatrix},
$$

$\boldsymbol{\mathcal{F}}_2(t) = \frac{d(t)}{\delta}$, $\boldsymbol{\mathcal{N}}_2 = \begin{bmatrix} 0 & \delta & 0 \end{bmatrix}$, $\boldsymbol{\mathcal{M}}_1 = \boldsymbol{\mathcal{I}}$ and $\boldsymbol{\mathcal{M}}_2 = 1$.

As in [14], we assume that $\alpha = \beta = \gamma = \delta$. In Table I, we test the feasibility of the LMI-based problem established in [13] augmented with the parametric uncertainties, i.e. the matrix inequality (41) with the equality (22), and that of our designed LMIs (33) and (38). To do that, we search for the maximum value of α, $\alpha_{\max}$, tolerated by each method. From this Table, the superiority of our design approach of the SOF controller to that in [13] (LMI (41) and equality constraint (22)) is quite clear. Our design methodology via LMI (38) provides solutions for all $\alpha \leq 0.5662$ with fixed parameters $\mu = 0.00054$ and $\epsilon_2 = 0.0001$. Moreover, our LMI (33) provides solutions for all $\alpha \leq 0.7668$ by taking $\mu = 0.2150$, while the design method using LMI (41) and equality constraint (22) fails for $\alpha > 0.4146$.

TABLE I

SUPERIORITY OF OUR PROPOSED LMIs FOR THE SOF DESIGN

Method	LMI (41) and (22)	LMI (38)	LMI (33)
		$\mu = 0.00054$ $\epsilon_2 = 0.0001$	$\mu = 0.2150$
$\alpha_{\max}$	0.4146	0.5662	0.7668

C. Simulation

Here we consider the same previous example. Our objective in this part is to observe the temporal evolution of the states, i.e. x_1, x_2 and x_3, of the uncertain linear system under the SOF controller $\boldsymbol{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^{\mathrm{T}}$. We used here the results obtained from LMI (33). In the simulation of the system under question, we fixed $\alpha = \beta = \gamma = \delta = \alpha_{\max} = 0.7668$. The SOF controller gain is $\boldsymbol{\mathcal{K}} = \begin{bmatrix} -4.0984 & -0.0257 \end{bmatrix}^{\mathrm{T}}$. The simulation results are given in Figures 1 and 2.

V. CONCLUSION

In this paper, a linear matrix inequality approach to design static output feedback controllers for continuous-time linear systems under norm-bounded parametric uncertainties is addressed. We have shown that our new design methodology leads to less restrictive LMI conditions. A comparison study of our results with respect to those given in [13] shows the superiority of our synthesis method of the robust SOF controller.

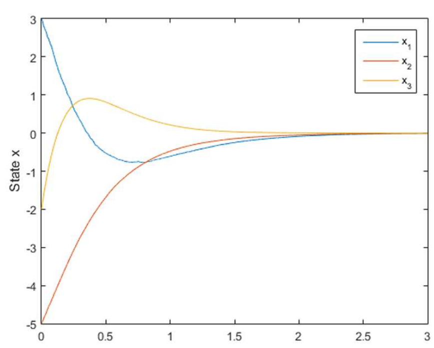

Fig. 1. Temporal evolution of the states x_1, x_2 and x_3 in the presence of norm-bounded parametric uncertainties.

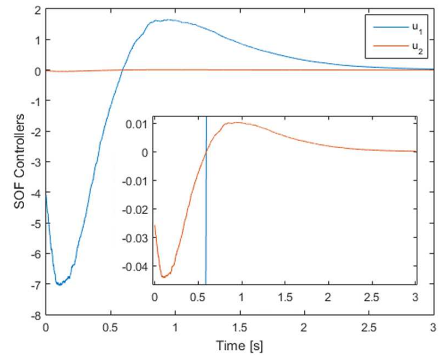

Fig. 2. Evolution of the SOF controller, i.e. u_1 and u_2.

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