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Spacecraft Collision Avoidance with Constrained Control via Discrete-Time Generating Functions

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Abstract— **This study presents sub-optimal collision-free transfers of spacecraft subject to constraints on control magnitude. In order to mitigate the difficulty in solving an optimal control problem considering directly inequality constraints, the penalty and barrier functions are incorporated into the cost function of optimal tracking problem. Then, the sub-optimal control law is derived by employing the discrete-time generating functions representing the canonical transformation in the discrete-time Hamilton-Jacobi theory. The proposed approach allows us to derive the control law as an algebraic form of the states of spacecraft, reference solution, and obstacles without any iterative process and initial guess. The numerical simulations validate the proposed approach by showing that spacecraft can reach the target point while avoiding obstacles with constrained control.**

I. Introduction

The topic of spacecraft collision avoidance has been actively studied with the advent of operations with multiple small spacecraft such as cube satellites at low cost. Considering that hardware capacity of small spacecraft is usually limited, and that rapid maneuver might be required for collision avoidance occasionally, the optimal collision-free control law under input constraints is necessary for operating spacecraft robustly and efficiently. Previous studies usually employ the receding horizon technique such as Model Predictive Control (MPC) based on the direct optimizations for deriving an optimal solution [1]-[5]. They can implement the constraints for both collision avoidance and limitation on control input directly, but generally require iterative process and initial guess. Some studies use the indirect shooting method for an optimal control problem while employing the penalty and barrier functions to mitigate the sensitivity in numerical calculations [6]-[7], but they also require iterative process and initial guess. There were other attempts to adopt an optimal control for collision avoidance problems by combination with an artificial potential function [8]-[10], but they do not consider the entire optimality including avoidance maneuvers and input constraints.

Recently, the sub-optimal approach employing the discrete-time generating function was developed for optimal tracking and collision-free transfers [11]-[13]. They can derive the control law as an explicit function of the current states of the reference solution and obstacles *without* repetitive calculations, but they did not consider input constraints. Expanding the study of [12]-[13], we incorporate the logarithmic barrier function into the cost function presented in [13] for implementing optimal collision-free transfers under constraints on control magnitude. Then, the alternative optimal control law for the cost function including the logarithmic barrier function is derived as a function of the costates of the discrete-time Hamiltonian system, and the discrete-time generating function is used to derive the costates as an explicit function of the current states of the reference solution and obstacles. The proposed approach allows us to derive the control law without any repetitive calculations while implementing the constraint on control magnitude.

II. Sub-optimal Collision Avoidance Control with Input Constraints

A. Problem Statement

We define the following optimal tracking problem for implementing collision-free transfers under control input constraints in discrete-time domain:

Problem 1: Minimize

$$J = \frac{1}{2}(\mathbf{x}_N - \mathbf{x}_r)^T Q(\mathbf{x}_N - \mathbf{x}_r) + \sum_{k=0}^{N-1}[\frac{1}{2}(\mathbf{x}_k - \mathbf{x}_r)^T Q(\mathbf{x}_k - \mathbf{x}_r) + \frac{1}{2}\mathbf{u}_k^T R\mathbf{u}_k + P] \tag{1}$$

subject to discrete-time nonlinear equations of motion in affine form and input constraints

$$\mathbf{x}_{k+1} = \mathbf{f}_d(\mathbf{x}_k, t_k) + \mathbf{g}_d(\mathbf{x}_k, t_k)\mathbf{u}_k, \quad \|\mathbf{u}_k\| < u_{\max} \tag{2}$$

where $\mathbf{x}_k \in R^{n\times 1}$ and $\mathbf{u}_k \in R^{m\times 1}$ are the states and control inputs of spacecraft, respectively. $\mathbf{f}_d$ and $\mathbf{g}_d$ represent a discrete-time nonlinear dynamical system with $\mathbf{u}_k$, and $u_{\max}$ is the upper bound on control magnitude. The subscripts k and N denote the k-th and final time step, respectively. $\mathbf{x}_r$ is the states of *reference solution* designated as an *optimal transfer not considering constraints on both states and control inputs*. $Q \in \mathbf{R}^{n\times n}$ and $R \in \mathbf{R}^{m\times m}$ are weighting matrices, and P is a penalty function for implementing collision avoidance indirectly. If the penalty function is properly designed to increase sharply as the spacecraft approaches obstacles, then the spacecraft in Problem 1 can avoid obst

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acles while tracking the reference solution [12]-[13].

In general, Problem 1 is not easy to be solved directly without iterative process and initial guesses because of the inequality constraint on the control magnitude. To mitigate the difficulty in the procedure for solving Problem 1, we employ the logarithmic barrier function instead of the inequality constraint, which is similar to the technique presented in [6]-[7]. The problem employing the barrier function is stated as follows.

Problem 2: Minimize

$$\begin{aligned} J = \frac{1}{2}(\mathbf{x}_N - \mathbf{x}_r)^T Q(\mathbf{x}_N - \mathbf{x}_r) + \sum_{k=0}^{N-1}[\frac{1}{2}(\mathbf{x}_k - \mathbf{x}_r)^T Q(\mathbf{x}_k - \mathbf{x}_r) \\ + \frac{1}{2}\mathbf{u}_k^T R\mathbf{u}_k + P - \varepsilon \log(u_{\max} - \|\mathbf{u}_k\|)] \end{aligned} \tag{3}$$

subject to discrete-time nonlinear equations of motion in affine form

$$\mathbf{x}_{k+1} = \mathbf{f}_d(\mathbf{x}_k, t_k) + \mathbf{g}_d(\mathbf{x}_k, t_k)\mathbf{u}_k \tag{4}$$

where $\varepsilon > 0$ is a sufficiently small constant. This problem formulation allows us to implement the inequality constraint on the control magnitude indirectly because the barrier function, $-\varepsilon \log(u_{\max} - \|\mathbf{u}_k\|)$, diverges to $+\infty$ as the control magnitude approaches $u_{\max}$.

The optimal control law of Problem 2 is derived by the Pontryagin's Principle as follows.

Proposition 1: Suppose R is the identity matrix and P does not depend on $\mathbf{u}_k$. Then, the optimal control law $\mathbf{u}_k$ for Problem 2 is

$$\begin{aligned} \mathbf{u}_k &= -u_{opt}\frac{\mathbf{p}_{k+1}}{\|\mathbf{p}_{k+1}\|} \\ u_{opt} &= \frac{2(u_{\max}\|\mathbf{p}_{k+1}\| - \varepsilon)}{u_{\max} + \|\mathbf{p}_{k+1}\| + \sqrt{(u_{\max} - \|\mathbf{p}_{k+1}\|)^2 + 4\varepsilon}} \\ &\mathit{if}\ \ \|\mathbf{p}_{k+1}\| \le \frac{\varepsilon}{u_{\max}},\ \ \mathit{then}\ \ u_{opt} = 0 \end{aligned} \tag{5}$$

where $\mathbf{p}_{k+1} = \mathbf{g}_d(\mathbf{x}_k, t_k)^T \boldsymbol{\lambda}_{k+1}$ and $\boldsymbol{\lambda}_k$ is the costates at the k-th time step in the discrete-time Hamiltonian system.

Proof: The right discrete-time Hamiltonian for Problem 2 is defined as follows:

$$\begin{aligned} H_d(\mathbf{x}_k, \boldsymbol{\lambda}_{k+1}, \mathbf{u}_k, t_k) = \frac{1}{2}(\mathbf{x}_k - \mathbf{x}_r)^T Q(\mathbf{x}_k - \mathbf{x}_r) + \frac{1}{2}\|\mathbf{u}_k\|^2 \\ + P - \varepsilon \log(u_{\max} - \|\mathbf{u}_k\|) \\ + \boldsymbol{\lambda}_{k+1}^T \mathbf{f}_d(\mathbf{x}_k, t_k) + \mathbf{p}_{k+1}^T \mathbf{u}_k \end{aligned} \tag{6}$$

H_d satisfies the following equation by the Cauchy-Schwarz inequality [7].

$$\begin{aligned} H_d(\mathbf{x}_k, \boldsymbol{\lambda}_{k+1}, \mathbf{u}_k, t_k) \ge \frac{1}{2}(\mathbf{x}_k - \mathbf{x}_r)^T Q(\mathbf{x}_k - \mathbf{x}_r) + \frac{1}{2}\|\mathbf{u}_k\|^2 \\ + P - \varepsilon \log(u_{\max} - \|\mathbf{u}_k\|) \\ + \boldsymbol{\lambda}_{k+1}^T \mathbf{f}_d(\mathbf{x}_k, t_k) - \|\mathbf{p}_{k+1}\|\|\mathbf{u}_k\| \end{aligned} \tag{7}$$

The equality is satisfied when

$$\mathbf{u}_k = -\|\mathbf{u}_k\|\frac{\mathbf{p}_{k+1}}{\|\mathbf{p}_{k+1}\|} \tag{8}$$

H_d is minimized when Eq. (8) is satisfied and $\|\mathbf{u}_k\|$ is an optimal value u_{opt}. u_{opt} can be obtained by solving the following equation from the partial derivative of the right part of Eq. (7) by $\|\mathbf{u}_k\|$.

$$\|\mathbf{u}_k\| + \varepsilon\frac{1}{u_{\max} - \|\mathbf{u}_k\|} - \|\mathbf{p}_{k+1}\| = 0 \tag{9}$$

The two roots of Eq. (9) are obtained as follows:

$$\|\mathbf{u}_k\| = \frac{u_{\max} + \|\mathbf{p}_{k+1}\| + \sqrt{(u_{\max} - \|\mathbf{p}_{k+1}\|)^2 + 4\varepsilon}}{2} \tag{10}$$

$$\|\mathbf{u}_k\| = \frac{u_{\max} + \|\mathbf{p}_{k+1}\| - \sqrt{(u_{\max} - \|\mathbf{p}_{k+1}\|)^2 + 4\varepsilon}}{2} \tag{11}$$

If $\|\mathbf{u}_k\|$ satisfies Eq. (10), then $\|\mathbf{u}_k\| \ge u_{\max}$ because $(u_{\max} - \|\mathbf{p}_{k+1}\|)^2 + 4\varepsilon > (u_{\max} - \|\mathbf{p}_{k+1}\|)^2$. $\|\mathbf{u}_k\| < u_{\max}$ only satisfies Eq. (11). Thus, u_{opt} is obtained from Eq. (11), which can be transformed into Eq. (5) by simple algebraic manipulations. This completes the proof. ■

B. *Derivation of Control Law via Discrete-Time Generating Functions*

The optimal control law presented in Eq. (5) can be expressed as an explicit function of the states of spacecraft, obstacles, and reference solution by employing the discrete-time generating functions [12]-[13]. The procedure obtaining the generating function is based on Taylor expansion of the Hamiltonian. Thus, we reformulate Problem 2 as Problem 3 in the local frame of the relative states $\mathbf{z}_k$ for precisely approximating the penalty and barrier functions with a relatively low-order expansion about the simple zero nominal states [12]-[13].

Problem 3: Minimize

$$\begin{aligned} J = \frac{1}{2}(\mathbf{z}_N - \mathbf{z}_r)^T Q(\mathbf{z}_N - \mathbf{z}_r) + \sum_{k=0}^{N-1}[\frac{1}{2}(\mathbf{z}_k - \mathbf{z}_r)^T Q(\mathbf{z}_k - \mathbf{z}_r) \\ + \frac{1}{2}\mathbf{u}_k^T \mathbf{u}_k + P - \varepsilon \log(u_{\max} - \|\mathbf{u}_k\|)] \end{aligned} \tag{12}$$

subject to discrete-time nonlinear equations of motion in affine form:

$$\mathbf{z}_{k+1} = \mathbf{f}_d(\mathbf{z}_k + \mathbf{x}_c, t_k) + \mathbf{g}_d(\mathbf{z}_k + \mathbf{x}_c, t_k)\mathbf{u}_k - \mathbf{x}_c \tag{13}$$

Here, $\mathbf{x}_c = [\mathbf{r}_c, \mathbf{0}]^T$ where $\mathbf{r}_c$ is an arbitrary current position of spacecraft. The relative states $\mathbf{z}_k$ are defined as $\mathbf{z}_k = \mathbf{x}_k - \mathbf{x}_c$ such that the origin of the local frame is consistently placed at the current position of spacecraft and that the orientation of the local frame is the same as that of the frame of states $\mathbf{x}_k$. $\mathbf{z}_r$ is the states of the reference solution presented in the local frame. *R* is set as an identity matrix.

The penalty function for Problem 3 is designed as follows [13]:

$$P = \sum_{i=1}^{s}\left[\frac{\alpha_i}{\left\|\mathbf{r}_{oz,i}\right\|^4} + \frac{\beta_i}{1+\exp(\kappa \mathbf{h}_i^T \mathbf{r}_z)}\exp(-\eta \left\|\mathbf{r}_{oz,i}\right\|^2)\right] \tag{14}$$

Here, *s* is the number of obstacles, and $\mathbf{r}_z$ is the position vector in the local frame. $\mathbf{r}_{o,i}$ and $\mathbf{v}_{o,i}$ are the relative position and velocity vectors of the *i*-th obstacle from the spacecraft in the local frame, respectively, and $\mathbf{r}_{oz,i} = \mathbf{r}_{o,i} - \mathbf{r}_z$. α_i, β_i, κ and η are positive design parameters. $\mathbf{h}_i$ is defined as

$$\mathbf{h}_i = \frac{(\mathbf{v}_{o,i} \times \mathbf{r}_{o,i}) \times \mathbf{r}_{o,i}}{\left\|(\mathbf{v}_{o,i} \times \mathbf{r}_{o,i}) \times \mathbf{r}_{o,i}\right\|} \tag{15}$$

such that $\mathbf{h}_i^T \mathbf{r}_z = 0$ is the equation of the plane containing $\mathbf{r}_{o,i}$ and is perpendicular to the plane including both $\mathbf{r}_{o,i}$ and $\mathbf{v}_{o,i}$. In two-dimensional motion, Eq. (15) is reduced to

$$\mathbf{h}_i = \frac{sign(\dot{x}_{o,i} y_{o,i} - y_{o,i}\dot{x}_{o,i})}{\left\|\mathbf{r}_{o,i}\right\|}[-y_{o,i}, x_{o,i}]^T \tag{16}$$

such that $\mathbf{h}_i^T \mathbf{r}_z = 0$ is the equation of the line including $\mathbf{r}_{o,i}$.

The first term which is the reciprocal of $\left\|\mathbf{r}_{oz,i}\right\|^4$ indirectly implements the forbidden region around obstacles to avoid collision. The second term is the guidance term for cooperatively avoiding the obstacles in advance by considering the relative velocity between the spacecraft and obstacles. Because $\mathbf{h}_i$ always satisfies $\mathbf{h}_i^T \mathbf{v}_{o,i} \le 0$, the second term decreases as the spacecraft is guided to the side not including $\mathbf{v}_{o,i}$. α_i and β_i are designed to selectively/conditionally activate the first and second terms, respectively [13].

$$\alpha_i = \alpha_{0,i}[1 - \exp(-\gamma_i \left\|\mathbf{r}_f\right\|)] \tag{17}$$

$$\beta_i = \begin{cases} \beta_{0,i} & if \;\; \mathbf{r}_{o,i}{}^T \mathbf{r}_f \ge 0 \; and \; \left\|\mathbf{r}_{o,i}\right\| < \left\|\mathbf{r}_f\right\| \\ 0 & otherwise \end{cases} \tag{18}$$

Here, $\alpha_{0,i}$, $\beta_{0,i}$, and γ_i are positive constants, and $\mathbf{r}_f$ is the final desired position vector of spacecraft in the local frame. The first term becomes deactivated as the spacecraft approaches $\mathbf{r}_f$, and the second term is only activated when the obstacles exist between the final desired and current position of spacecraft.

By replacing $\mathbf{u}_k$ as a function of $\boldsymbol{\lambda}_k$ by Eq. (5), the right discrete-time Hamiltonian and associated Hamiltonian system for Problem 3 can be stated as follows:

$$\begin{aligned} H_d(\mathbf{z}_k, \boldsymbol{\lambda}_{k+1}, t_k) = & \frac{1}{2}(\mathbf{z}_k - \mathbf{z}_r)^T Q(\mathbf{z}_k - \mathbf{z}_r) + Ru_{opt}^2 + P \\ & - \varepsilon \log(u_{\max} - u_{opt}) + \boldsymbol{\lambda}_{k+1}^T \mathbf{f}_d(\mathbf{z}_k + \mathbf{x}_c, t_k) \\ & - \boldsymbol{\lambda}_{k+1}^T \mathbf{x}_c - u_{opt}\left\|\mathbf{g}_d(\mathbf{x}_k, t_k)^T \boldsymbol{\lambda}_{k+1}\right\| \end{aligned} \tag{19}$$

$$\mathbf{z}_{k+1} = \frac{\partial H_d(\mathbf{z}_k, \boldsymbol{\lambda}_{k+1}, t_k)}{\partial \boldsymbol{\lambda}_{k+1}} \quad \boldsymbol{\lambda}_k = \frac{\partial H_d(\mathbf{z}_k, \boldsymbol{\lambda}_{k+1}, t_k)}{\partial \mathbf{z}_k} \tag{20}$$

In the frame work of the discrete-time Hamilton-Jacobi theory, the generating functions can be used to represent the canonical transformation between $(\mathbf{z}_k, \boldsymbol{\lambda}_k)$ and $(\mathbf{z}_N, \boldsymbol{\lambda}_N)$. It allows us to define the second-type generating function, F_2, which satisfies the following discrete-time Hamilton-Jacobi equation and associated relationships [14].

$$F_2(\mathbf{z}_{k-1}, \boldsymbol{\lambda}_N) = F_2(\mathbf{z}_k, \boldsymbol{\lambda}_N) - \boldsymbol{\lambda}_k{}^T \mathbf{z}_k + H_d(\mathbf{z}_{k-1}, \boldsymbol{\lambda}_k, t_k) \tag{21}$$

$$\boldsymbol{\lambda}_k = \frac{\partial F_2(\mathbf{z}_k, \boldsymbol{\lambda}_N)}{\partial \mathbf{z}_k}, \quad \mathbf{z}_N = \frac{\partial F_2(\mathbf{z}_k, \boldsymbol{\lambda}_N)}{\partial \boldsymbol{\lambda}_N} \tag{22}$$

After assigning the time-varying terms such as $\mathbf{x}_c$, $\mathbf{z}_r$, $\mathbf{r}_{o,i}$, $\mathbf{r}_f$, α_i, β_i, and $\mathbf{h}_i$ to independent variables, F_2 is obtained by solving the discrete-time Hamilton-Jacobi equation for a receding horizon arbitrary defined as N^* based on Taylor expansion [12]-[13]. Then, F_2 at the initial time step of N^* is used to derive $\mathbf{p}_{k+1}$ as follows:

$$\mathbf{p}_{k+1} = \mathbf{g}_d(\mathbf{z}_k + \mathbf{x}_c, t_k)^T \frac{\partial F_2(\mathbf{z}_{k+1}, \boldsymbol{\lambda}_{N^*})}{\partial \mathbf{z}_{k+1}} \tag{23}$$

The sub-optimal control law is finally obtained by applying Eq. (23), the transversality condition, and the linear state transformation between $\mathbf{z}_{k+1}$ and $\mathbf{z}_k$ to Eq. (5) [13]-[14]. Because the obtained F_2 is an explicit function of the time-varying terms, so is the proposed control law. Note that the process for deriving F_2 does *not require any iterative process and initial guess*, and that *the proposed control law is an algebraic equation* of the states of spacecraft, reference solution, and obstacles. These characteristics allow us to implement collision-free transfers under the constraint on control magnitude with relatively lower computational burden. As the algebraic evaluation is quite straightforward, this approach can be considered as a candidate for real-time sub-optimal feedback control.

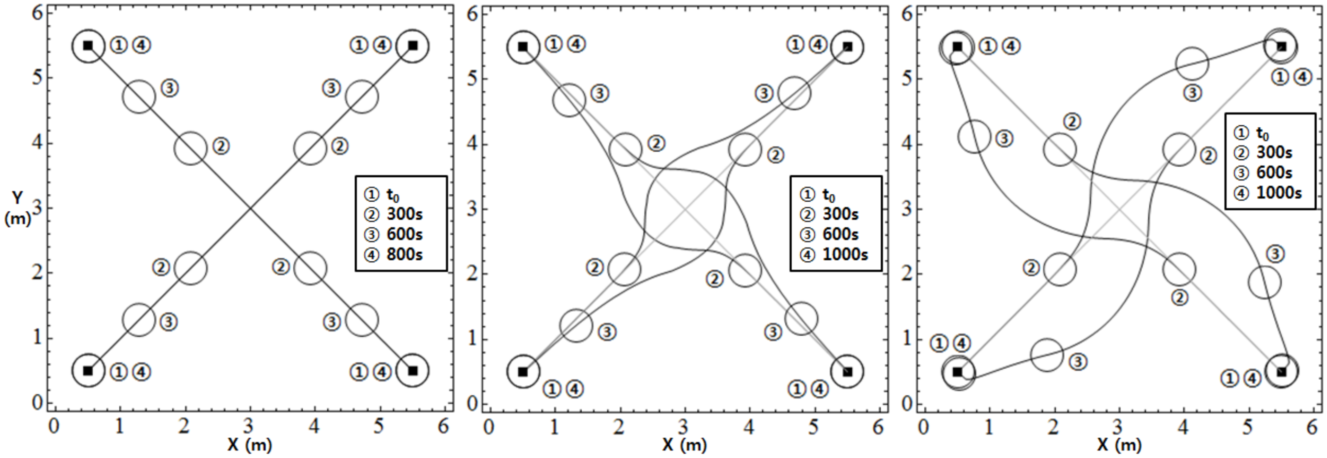

Figure 1. Resultant trajectories of four spacecraft (left: Case 1, middle: Case 2, right: Case 3)

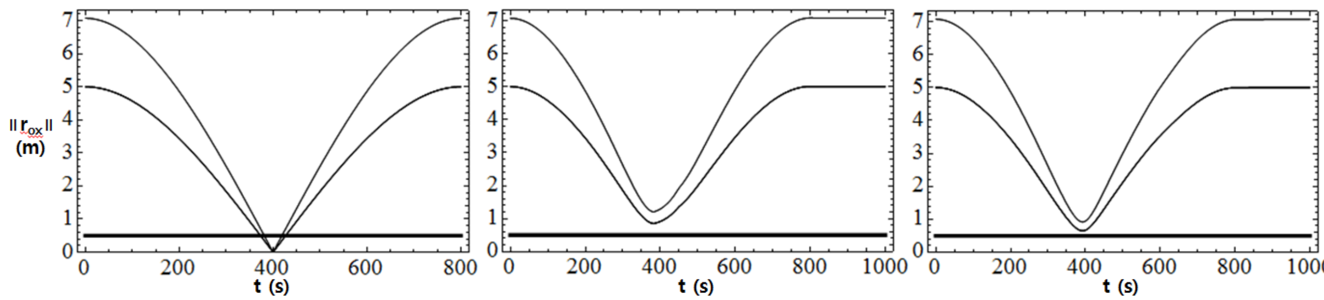

Figure 2. Relative distances between spacecraft (left: Case 1, middle: Case 2, right: Case 3)

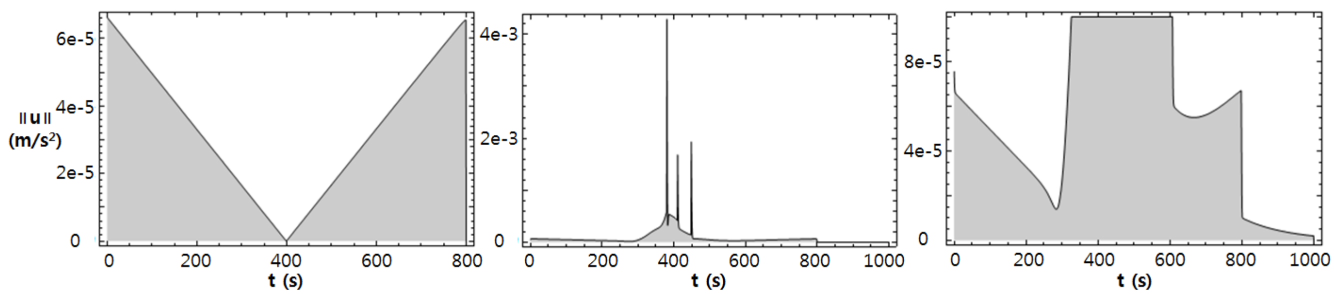

Figure 3. Control magnitude histories (left: Case 1, middle: Case 2, right: Case 3)

TABLE 1. Minumim relative distance and costs of each case

Cases	Min. Distance Between Spacecraft (m)	$\sum_{k=0}^{N-1}\frac{1}{2}\mathbf{u}_k^T\mathbf{u}_k$	$\sum_{k=0}^{N}\frac{1}{2}(\mathbf{x}_k-\mathbf{x}_r)^T Q(\mathbf{x}_k-\mathbf{x}_r)$
Case 1	2.3000e-3	5.8759e-7	3.3552e-5
Case 2	8.5016e-1	2.1389e-5	4.2715e-1
Case 3	6.5131e-1	2.0840e-6	2.2806

III. Simulation Results

The proposed control law is validated via planar collision-free transfers of four spacecraft subject to double integrator dynamics. With the receding horizon set as two steps, the second order discrete-time generating function is obtained and used for deriving the control law. Then, the control law is applied for each spacecraft to switch their positions diagonally. Three cases are presented for comparative analyses:

Case 1: Unconstrained optimal transfers

Case 2: Collision-free transfers without input constraint

Case 3: Collision-free transfers with input constraint

Case 1 is the optimal transfers for 800 seconds *NOT considering both collision avoidance and input constraint*. Case 2 is the collision-free transfers without the upper bound of control magnitude, which is the same as in our previous study [13]. Case 3 uses the proposed control law with $u_{\max}=10^{-4}$ m/s^2. The reference solution for optimal tracking in Cases 2-3 is defined as the solution of Case 1 until 800 seconds, and is defined as the final states of Case 1 after 800 seconds. The desired minimum distance between each spacecraft is given as 0.5 m in all cases. The weighting matrices and design parameters are set as follows:

$$Q = \begin{bmatrix} 2\times10^{-2}\times I_{2\times2} & 0_{2\times2} \\ 0_{2\times2} & I_{2\times2} \end{bmatrix},\ R = I_{2\times2},$$
$$\alpha_0 = 7.5\times10^{-4},\ \beta_0 = 1\times10^{-2},\ \gamma = 6\times10^{-1} \tag{24}$$
$$\eta = 1,\ \kappa = 10,\ \varepsilon = 1\times10^{-20}$$

Figure 1 shows the trajectories of spacecraft in each case. The forbidden regions around the spacecraft at particular moments are described as solid circles of which the radii are a half of the desired minimum distance. Figure 2 presents the relative distance between spacecraft in each case. As shown in Figures 1-2, the spacecraft can avoid each other by the proposed approach, even though the resultant trajectories are different from that of Case 2. Figure 3 shows the resultant control magnitude histories of one spacecraft in each case. Case 2 shows that three extreme peaks appear when the guidance terms for avoiding the other three spacecraft are deactivated, as the cost function varies. By contrast, the control magnitude is obtained as a lower value than $u_{\max}$ by the proposed control law in Case 3. These results clearly show that both collision avoidance and constraints on control magnitude can be successfully accomplished by the proposed control law. Table 3 again states that the proposed control law increases the relative distances bigger than 0.5 m. Case 3 shows lower control effort than that of Case 2 because $u_{\max}$ is quite low when compared with the control magnitude history of Case 2. Note that the range of control magnitude in Case 2 for 300-500 seconds is about five times larger than $u_{\max}$ even if the extreme peaks are not considered. Although the tracking errors increase due to the lower control efforts in Case 3, the spacecraft reach the final desired positions.

IV. CONCLUSION

We proposed sub-optimal feedback control for collision-free transfers of spacecraft under constraints on control magnitude. After incorporating the penalty and barrier functions into the cost function for optimal tracking, the control law is derived as an explicit function of the states of spacecraft, reference solution, and obstacles by employing the discrete-time generating function. The proposed approach is suitable for small spacecraft of which the hardware has limited performance, because it does not require any iterative process and initial guess when deriving and applying the control law. It is expected that the proposed approach is applicable to realistic cases in space missions or hardware simulators on the ground. We will compare our approach with other approaches such as MPC and analytic approaches with artificial potential functions.

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