

Theoretical and experimental research of the discrete output robust controller for uncertain plant

Dmitrii Dobriborsci¹, Alexey Margun¹ and Sergey Kolyubin¹

Abstract—The paper is devoted to the synthesis of discrete robust output controller for a class of parametric uncertain linear plants under external disturbances. Controller synthesis is based on consecutive compensator method. Proposed algorithm provides convergence of tracking error to the bounded area. Lyapunov function method is used for stability analysis. Performance of proposed controller is confirmed by experimental implementation for Ball and Plate system.

I. INTRODUCTION

One of the topical areas of the modern control study is the solution of problems of designing fairly simple algorithms for control plants with unknown parameters. By "simple algorithms" we mean a limited number of arithmetic operations in the structure and quantity of measured variables, a decrease of the controller dynamic order and a decrease in the number of configurable parameters. This area of research is relevant as well as for solutions for object models. When solving practical problems, it may turn out that a number of characteristics of a real object may be unknown in advance or change during its functioning. In adaptive control systems, the lack of a priori information is replenished in the process of its functioning on the basis of current data on the behavior of the object. These data are processed in real time and used to improve the quality of the control system. Of particular interest are the problems of control by a linear parametrically indeterminate object without measuring the derivatives of the controlled variable. Today, a lot of solutions are received both in the class of adaptive tasks and in the class of robust control problems. These methods serve to construct control systems with significant uncertainties in the parameters of the control plants and under conditions of its operation (environmental characteristics) available at the design stage or before the system is operational. We consider such control problems in which the dynamic properties of an object can vary widely in an unknown manner. As a rule, known algorithms are rather complicated to implement and, therefore, are not attractive for use due to the complexity and cumbersomeness of proposed controllers. In addition, not all of the existing schemes can solve the problem of synthesizing the control law to compensate external disturbances, which is a fundamental problem of the modern theory of automatic control systems.

R. Monopoli proposed an extended error scheme in 1974 [12]. The basic stages of the development of adaptive control

systems according to the Monopoly scheme consisted in the implementation of simple schemes for generating extended error signals, as well as in modifying the basic adaptive control schemes in order to guarantee the stability of the closed system. In the process of designing adaptive control systems based on an extended error, the structure of the regulated controller is formulated in accordance with the principle of direct compensation.

In [14] another way of controllers design for a linear system with respect to an output variable is proposed. This method considers the usage of iterative synthesis procedures, also known as "adaptive bypass of integrators". The main advantage of this method is the very small dimension of the controller. In addition, as merits, we should mention some very positive properties of a closed-loop system that can not be met when implementing the method of direct compensation. One of these properties is parametric robustness, as well as making it possible to use a priori information about the control object. Another property is the fact that the algorithm, which is synthesized on the basis of iterative methods of adaptive bypass of integrators, guarantees improved quality indicators of transient processes. However, the use of iterative synthesis methods also have negative properties. In particular, a fairly complex control law is attributed to shortcomings due to a very problematic synthesis procedure.

The main aspects of the synthesis of control algorithms using the hidden implementation of the reference model are proposed, as well as the parallel connection of the correcting loop (the so-called shunt) to the control plant. The proposed method provides reduction of the dynamic order by reducing the number of adjustable parameters and auxiliary filters. Let us check that for any object of a minimal phase with a scalar relative degree there is an included parallel compensator whose order is equal to, which is equivalent to transforming the object into a strictly minimal phase.

Analyzing the information considered previously we made the following conclusions. The method of controller design based on an extended error which is synthesized by direct compensation does not present difficulties in the construction and we obtain a rather simple controller structure. Also note that the use of this method ensures compliance with weak properties of the closed system. In contrast, the method based on the use of iterative synthesis procedures guarantees improved quality of transient processes, but in its turn has the most complex structure. The merits of this method also should be attributed to the presence of useful properties such as parametric robustness and the use of a priori information

*This work is supported by the Russian Science Foundation grant (project 17-79-20341).

¹Dmitrii Dobriborsci, Alexey Margun and Sergey Kolyubin are with Control Systems and Informatics Department, ITMO University, Kronverksky dmitrii.dobriborsci@corp.ifmo.ru

about the control object. Special attention should be paid to the shunt insertion method, which is characterized by a simple controller structure and a calculation that does not present difficulties, however, it was not designed in conditions of external disturbances.

$$\begin{cases} \dot{\varepsilon}(t) = A\varepsilon(t) + B(\beta e(t) + (\alpha + \beta)L\eta(t)) + Bh\psi(t) + \\ B_1\varphi(t), \\ \dot{\eta}(t) = L^T \dot{\varepsilon}(t) + \sigma\Gamma\eta(t) + \sigma h\Gamma\psi(t) \end{cases} \quad (13)$$

Statement

There exist $\alpha, \beta, \sigma, \bar{h} > 0$ such that for any $h < \bar{h}$ discrete observer (10) and control law (12) provide exponential convergence of tracking error $e(t)$ to the bounded area.

C. Proof of Statement

Consider Lyapunov function

$$V(t) = \varepsilon^T(t)P_1\varepsilon + \eta^T(t)P_2\eta(t), \quad (14)$$

where P_1, P_2 are solutions of Lyapunov equations $A^T P_1 + PA = -Q_1, \Gamma^T P_2 + P\Gamma = -Q_2$, Q_1 and Q_2 are positive defined symmetric matrices.

Obtain derivative of Lyapunov function (14) along trajectories (13)

$$\begin{aligned} \dot{V} = & -\varepsilon^T(Q_1 + 2\beta L^T B^T P_1)\varepsilon + 2\varepsilon^T P_1 B_1 \varphi + 2h\varepsilon^T P_1 B \psi + \\ & 2(\alpha + \beta)\varepsilon^T P_1 B L \eta - \eta^T(\sigma Q_2 - 2(\alpha + \beta)P_2 B L) + \\ & 2\sigma h\eta^T P_2 \Gamma \psi + 2\eta^T P_2 A \varepsilon - 2\beta\eta^T P_2 B L \varepsilon + 2h\eta^T P_2 B \psi + \\ & 2\eta^T P_2 B_1 \varphi. \end{aligned} \quad (15)$$

Bound terms of the right part of (15) by inequalities

$$\begin{aligned} 2\varepsilon^T P_1 B_1 \varphi & \leq v\varepsilon^T P_1 B_1 B_1^T P_1 \varepsilon + \frac{1}{v}\varphi^T \varphi \\ 2\varepsilon^T P_1 B \psi & \leq v\varepsilon^T P_1 B B^T P_1 \varepsilon + \frac{1}{v}\psi^T \psi, \\ 2\varepsilon^T P_1 B L \eta & \leq v\varepsilon^T P_1 B B^T P_1 \varepsilon + \frac{1}{v}\eta^T \eta, \\ 2\eta^T P_2 \Gamma \psi & \leq v\eta^T P_2 \Gamma \Gamma^T P_2 \eta + \frac{1}{v}\psi^T \psi, \\ 2\eta^T P_2 A \varepsilon & \leq \beta\eta^T P_2 A A^T P_2 \eta + \frac{1}{\beta}\varepsilon^T \varepsilon, \\ 2\eta^T P_2 B L \varepsilon & \leq v\eta^T P_2 B L L^T B^T P_2 \eta + \frac{1}{v}\varepsilon^T \varepsilon, \\ 2\eta^T P_2 B \psi & \leq v\eta^T P_2 B B^T P_2 \eta + \frac{1}{v}\psi^T \psi, \\ 2\eta^T P_2 B_1 \varphi & \leq v\eta^T P_2 B_1 B_1^T P_2 \eta + \frac{1}{v}\varphi^T \varphi. \end{aligned} \quad (16)$$

where v is a small positive number.

Taking into account (16) rewrite (15) in the form

$$\dot{V} \leq -\varepsilon^T R_1 \varepsilon - \eta^T R_2 \eta + \theta, \quad (17)$$

where R_1, R_2 are positive defined matrices due to the choose of α, β and σ . θ is a bounded function.

$$\begin{aligned} R_1 &= Q_1 + 2\beta L^T B^T P_1 - vP_1 B_1 B_1^T P_1 - hvP_1 B B^T P_1 - \\ & (\alpha + \beta)vP_1 B B^T P_1 + (\beta^{-1} + \beta v)I, \\ R_2 &= \sigma Q_2 - 2(\alpha + \beta)P_2 B L - v^{-1}I - 2\sigma hvP_2 \Gamma \Gamma^T P_2 - \\ & \beta P_2 A A^T P_2 - \beta vP_2 B L L^T B^T P_2 - hvP_2 B B^T P_2 - vP_2 B_1 B_1^T P_2, \\ \theta &= \sup\left(\frac{2+2\sigma h}{v}\varphi^T \varphi + \frac{2h}{v}\psi^T \psi\right). \end{aligned} \quad (18)$$

Remark 2. In perfect case the value of θ must equals to zero. So, here to reduce it we should choose the smallest possible value of h . The value of θ depends on the σ parameter as well. That's the problem of choosing compatible coefficients α and β . Possible solution: Implementing of adaptive control algorithms. Bound derivative of Lyapunov function by inequality

$$\begin{aligned} \dot{V} & \leq -\zeta V + \theta, \zeta = \frac{\lambda_{\min} R_1}{\lambda_{\max}(P_1)}, \\ V & \leq \left(V(0) - \frac{\theta}{\zeta}\right)e^{-\zeta t} + \frac{\theta}{\zeta}. \end{aligned} \quad (19)$$

Taking into account $\lambda_{\min}(P_1)e^2 \leq \lambda_{\min}(P_1)\varepsilon^2 \leq V$ we obtain

$$|e| \leq \sqrt{\frac{1}{\lambda_{\min}(P_1)} \left[\left(V(0) - \frac{\theta}{\zeta}\right)e^{-\zeta t} + \frac{\theta}{\zeta} \right]} \quad (20)$$

Thereby, tracking error exponentially converges to the area bounded by inequality

$$|y - y_m| = |e| \leq \sqrt{\frac{1}{\lambda_{\min}(P_1)} \frac{\theta}{\zeta}} \quad (21)$$

III. EXPERIMENTAL SETUP

We built a tiltable platform with two degrees of freedom consisting of the following components: resistive touch-screen, servo-drives, square plate and connecting links. The control system was implemented on the Arduino single-board computer. Resistive touch-screen determines the coordinates of the object.

The kinematic scheme of the developed system is presented in the Fig. ?? Each tilting axis is operated on by a servomotor SpringGRC SM-S2213m with JR connector.

The platform was specially built to conduct the following experiments: object stabilization in the specific point on the plate, its optimal point-to-point motions and desired motion trajectory tracking as well as dynamics object-to-plate contact model identification.

In contrast to previous papers [10], [11] here we detect the ball position using resistive touch-screen system. An analog resistive touch-screen consist of a glass or acrylic panel that is coated with electrically conductive and resistive layers made with indium tin oxide (ITO). These layers are separated by invisible spacers/ To generate a position, the user or an object must exceed the activation force of the screen, pressing the two layers together. This actions create an electrical connection between layers. However, both X and

Y axes cannot be read simultaneously, because the concept works by applying a voltage across one layer, and looking for the voltage that appears on the another. Therefore, the system operates by alternately applying a voltage to one layer and reading off of the another. The voltages are then read in by an analog-to-digital (A/D) converter to be used as a coordinate value. There are several types of the resistive touch-screens, e.g. 4-wire, 5-wire and 8-wire. These labels refer to number of wires between the screen and the controller. In the current study we chosen a 4-wire resistive touch AST-150C. It features the typical advantages of the resistive type such as operational stability, detection accuracy, ease of introducing, competitive cost and so on. It have great durability with up to as much as 10 million touches, due to its structure.

Here we derive the equations of motion for experimental setup Ball and Plate to design the robust output controller. In this part we introduce the following assumptions:

- There is no slipping for ball.
- The ball is completely symmetric and homogeneous.
- Friction forces are neglected.
- The ball and plate are in contact all time.

The angles of servo arms θ_x, θ_y are assumed to be the inputs, while the ball position on x, y axis are assumed to be the output. Here we derive dynamical equations of ball-on-plate system, by the help of Lagrangian [5], [6], [7]. The following mathematical equations are based on . So the nonlinear differential equations for the ball and plate system are presented below.

$$\left(m + \frac{I}{r^2}\right)\ddot{x} - m(x\ddot{\alpha}^2 + y\dot{\alpha}\dot{\beta}) + mg\sin\alpha = 0 \quad (22a)$$

$$\left(m + \frac{I}{r^2}\right)\ddot{y} - m(y\dot{\beta}^2 + x\dot{\alpha}\dot{\beta}) + mg\sin\beta = 0 \quad (22b)$$

where m, r, I are mass of the ball, radius of the ball, mass moment of inertia x, y are position of axis, $\alpha, \beta, \dot{\alpha}, \dot{\beta}$ are inclination angles of the plate, angular velocity of the plate,

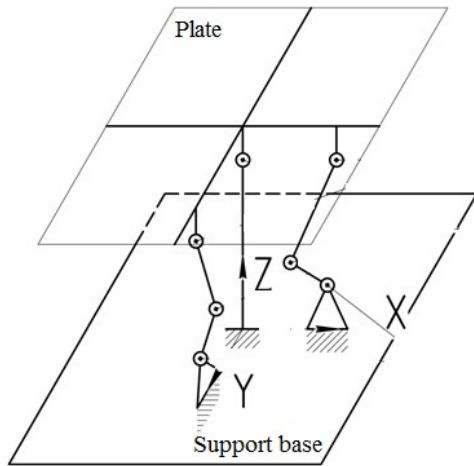


Fig. 1. Kinematic scheme of the mechanism

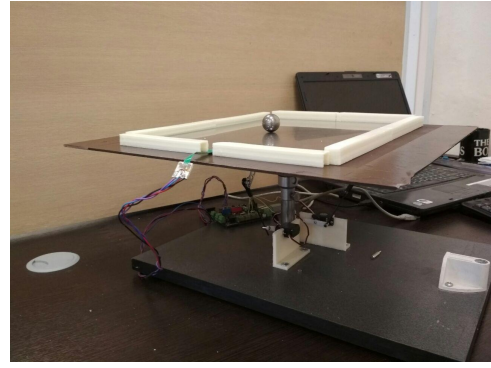


Fig. 2. Parallel kinematics robot Ball and Plate

g, L, d are gravitation, plate side length, length between the joint and the center of the gear, respectively.

The relations between inclination angles of the plate α, β and θ_x, θ_y are the following

$$\sin\alpha = \frac{2\sin\theta_x d}{L} \quad (23a)$$

$$\sin\beta = \frac{2\sin\theta_y d}{L} \quad (23b)$$

In the case of a slow rate of change for the plate angles equations (22a), (22b) can be linearized

$$\left(m + \frac{I}{r^2}\right)\ddot{x} - \frac{2mgd}{L}\theta_x = 0 \quad (24a)$$

$$\left(m + \frac{I}{r^2}\right)\ddot{y} - \frac{2mgd}{L}\theta_y = 0 \quad (24b)$$

The equations (24a), (24b) are equivalent because of the symmetry of the plate.

With the Laplace transformation, the following transfer functions can be obtained

$$P_x(s) = \frac{x}{\theta_x} = -\frac{2mgdr^2}{L(mr^2 + I)s^2} \quad (25a)$$

$$P_y(s) = \frac{x}{\theta_y} = -\frac{2mgdr^2}{L(mr^2 + I)s^2} \quad (25b)$$

where s is the Laplace operator. The control algorithm was first verified using the simulation model in Matlab/Simulink. Model parameters are assumed unknown.

IV. EXPERIMENTAL RESULTS

The experiment validation was performed on the developed 2 DOF Ball and Plate laboratory setup. The objective is to stable the ball in user-dened coordinates. As discussed the ball and plate system can well be approximated by two linear decoupled systems. Therefore, this system with two inputs and two outputs can be treated as two decoupled single input single output systems, this means that controllers designing can be realized independently. For the controller design we make no further assumptions about the model. We assume that the parameters of the system are unknown. So, for the

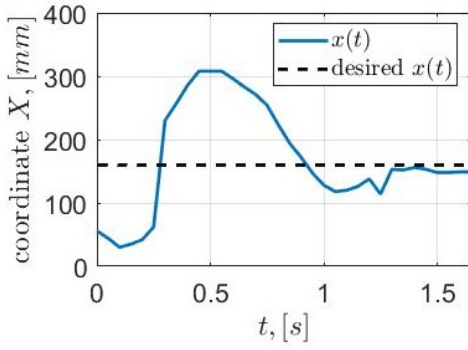


Fig. 3. Ball position in X-axis

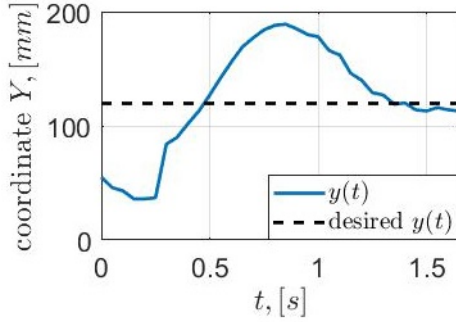


Fig. 4. Ball position in Y-axis

considerable setup the relative degree is $\rho = 2$. The graphical interpretation of the control goal is presented in fig.3. In this case we have got the control law and observer presented below. Discrete version of observer for experimental setup takes the form

$$\begin{cases} \xi_1(t_{k+1}) = \xi_1(t_k) + h\sigma\xi_2(t_k), \\ \xi_2(t_{k+1}) = \xi_2(t_k) + h\sigma\xi_3(t_k), \\ \xi_3(t_{k+1}) = \sigma h(-c_1\xi_1(t_k) - c_2\xi_2(t_k) - c_3\xi_3(t_k) + c\hat{e}(k)), \\ \hat{e}(t_k) = \xi_1(t_k). \end{cases} \quad (26)$$

The controller is as following:

$$\theta_x(t_k) = -(\alpha + \beta)(d_1\xi_1(t_k) + \sigma d_2\xi_2(t_k) + \sigma^2 d_3\xi_3(t_k)). \quad (27)$$

For Y direction the control law is similar, because of the symmetry of the plate. The experimental confirmation of obtained theoretical results can be seen on Fig.3 - Fig.6.

V. CONCLUSIONS

In this paper a robust controller design for uncertain plant and its implementation are presented. Proposed controller provides convergence of the tracking error to the bounded area. During the experiment implementation the problem of ball stabilization on the plate using output robust controller is solved. Future research will be devoted to other dynamic

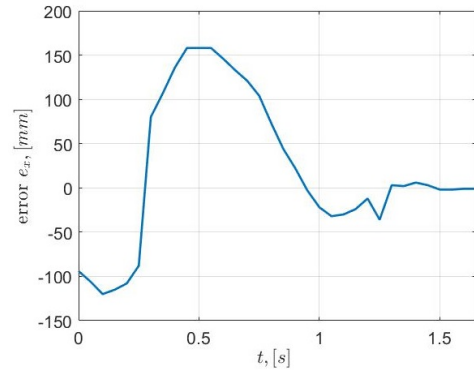


Fig. 5. Tracking error in X-axis

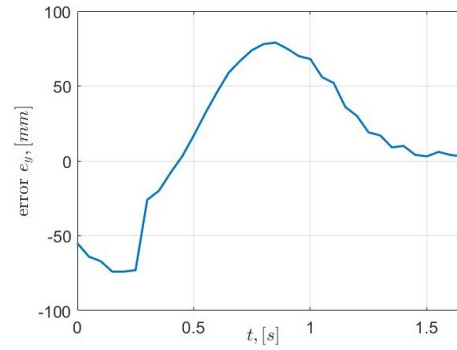


Fig. 6. Tracking error Y-axis

manipulation examples (such as trajectory control, ball-to-plate contact model identification) and implement various modern control methods such as described in [8], [9]. Proposed setup and algorithms can be effectively used for energy-efficient control and biomechanics, for instance, in reliability technologies.

REFERENCES

- [1] A.A. Bobtsov, Robust output-control for a linear system with uncertain coefficients, Automation and Remote Control, vol. 63 (11), 2002, pp. 1794–1802.
- [2] I.B. Furtat, A.L. Fradkov, D. Liberzon, Compensation of disturbances for MIMO systems with quantized output, Automatica, vol. 60, 2015, pp. 239–244.
- [3] A.A. Margun, A.A. Bobtsov and I.B. Furtat, Algorithm to control linear plants with measurable quantized output, Automation and Remote Control, vol. 78(5), 2017, pp. 826–835.
- [4] A. Margun and I. Furtat, Robust control of linear MIMO systems in conditions of parametric uncertainties, external disturbances and signal quantization, 20th International Conference on Methods and Models in Automation and Robotics, 2015, pp.341–346.
- [5] X. Fan, N. Zhang, and S. Teng, Trajectory planning and tracking of ball and plate system using hierarchical fuzzy control scheme, Fuzzy Sets and Systems, vol. 44, no. 2, pp. 297–312. 2004.
- [6] K. kyu Lee, G. Btz, and D. Wollherr, Basketball robot: Ball-on-plate with pure haptic information, IEEE International Conference on Robotics and Automation. 2008.
- [7] D.Yuan and Z. Zhang, Modelling and control scheme of the ball-plate trajectory-tracking pneumatic system with a touchscreen and a rotary cylinder, Control Theory and Application, IET, vol. 4, no. 4. pp. 573–589. 2010.

- [8] Pyrkin, A.A., Bobtsov, A.A., Kolyubin, S.A., Borisov, O.I., Gromov, V.S., Output controller for quadcopters based on mathematical model decomposition, 22nd Mediterranean Conference on Control and Automation, pp. 1281-1286. 2014.
- [9] Bobtsov, A.A., Pyrkin, A.A., Kolyubin, S.A., Simple output feedback adaptive control based on passification principle, International Journal of Adaptive Control and Signal Processing, vol. 28, issue 7-8. 2014.
- [10] Dobriborsci, D., Kolyubin, S., Design and control of parallel kinematics platform for nonprehensile manipulation, 2017 IEEE International Workshop of Electronics, Control, Measurement, Signals and their Application to Mechatronics, ECMSM 2017.
- [11] Dobriborsci, D., Kapitonov, A., Nikolaev, N. Application of the Stewart platform for studying in control theory, Proceedings of the International Conference on Information and Digital Technologies, IDT 2017, pp.95-99. 2017.
- [12] Monopoli R.V. Model reference adaptive control with an augmented signal, IEEE Trans. On Automatic Control. 1974. V. 19, No 5. P. 474-484.
- [13] Morse A.S. High order parameter tuners for adaptive control of nonlinear systems, Systems, Models and Feedback: Theory and Applications / Eds A. Isidori, T.J. Tarn. Birkhauser. 1992.
- [14] Kanellakopoulos I., Kokotovic P.V., Morse A.S. Systematic design of adaptive controllers for feedback linearizable systems // IEEE Trans. On Automatic Control. 1991. V. 36. P. 1241-1253.