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Controller redesign for (nonlinear) systems with input saturation*

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Abstract— **In this paper we propose a method to redesign given controllers for systems with input saturation. In contrast to typical approaches, we do not saturate the output of the controller and we do not rely on anti-windup compensators. Instead, we reformulate required bounds on the input as constraints on the controller state and we formulate redesign objectives such that adjusted controllers satisfy the required constraints. Particularly, we focus on redesigning existing controllers in a way that the associated dissipation property is preserved, allowing for a straightforward adaptation of existing stability results. This makes the suggested method particularly suitable for nonlinear systems, where the overall stability often relies on the existence of certain dissipation properties. To demonstrate the proposed methodology, we redesign a popular distributed controller aiming at power sharing in electricity networks such that the new controller additionally satisfies important transient constraints.**

I. Introduction

In many applications the actuators have bounds on their inputs and the importance of studying the effect of possible input saturation has been widely recognized. Neglecting the effect of these bounds often results in a discrepancy between the system input and controller output, generally called windup, which can cause reduced closed-loop performance or even instability. To prevent these issues, various methods have been proposed to design so-called anti-windup compensators [1], [2], [3], [4]. In a typical design approach, first controllers are proposed without taking into account possible saturation effects, whereafter additional compensators are incorporated to avoid the adverse effects of possible windup. However, analyzing the stability of the obtained overall system is generally difficult and methods to incorporate the saturation effect are often tailored to specific applications. And although the effect of saturation is quite well understood for linear systems [5], it is still largely unresolved in case the control system is nonlinear, where the unconstrained controller design exploits system-theoretical properties such as passivity or input–to–stability. Incorporating the input saturation afterwards is challenging as it might destroy useful dissipation related properties [6], such that obtained stability results might become invalid. Indeed, we notice that satisfying input constraints is relatively straightforward, using e.g. saturation functions, while preserving the stability of the overall system is difficult.

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A. Contributions

Inspired by the previous observations, we propose a new method to redesign given controllers for systems with input saturation, such that desired input constraints are met and existing stability results can be more easily adapted. We summarize the main contributions of this work as follows:

1) We suggest that a viable alternative to saturating the input to the system is to constrain the states of the controller. Required modifications of unconstrained controllers are formulated in three redesign objectives. Particularly, we argue that it is essential that a dissipative controller results, after the redesign, in another dissipative controller with the same dissipation property, permitting straightforward adaptation of existing stability results.
2) In case the storage function associated to the constrained states is quadratic, we propose an explicit redesign of the controller dynamics and provide a new storage function that preserves the dissipation property.
3) Based on the obtained results, we redesign a popular distributed controller aiming at power sharing in electricity networks such that the new controller additionally satisfies important transient constraints.

B. Outline

Section II introduces systems with input saturation and discusses how bounded inputs can be obtained by properly constraining the controller states. In Section III we provide a general introduction to the proposed methodology to redesign existing control systems. Section IV makes the approach explicit for the case when the storage term, associated to the to be constrained state component, is quadratic. In Section V we demonstrate how the obtained results can be applied to redesign suggested distributed controllers in the literature that aim at power sharing in electricity networks. Finally, some conclusions and future directions are gathered in Section VI.

II. Systems with input saturation

The focus of this section is to define the constraints on the *input* to a dynamical system (input saturation) and how they can be casted to constraints on the *state* of an interconnected controller. To do so, we consider the following system with state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$ and output $y \in \mathbb{R}^m$:

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x), \end{aligned} \tag{1}$$

where $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^m$ are sufficiently smooth mappings. The desired input constraints are formulated in the following objective:

Objective 1 (Bounded system input) *Let $\mathcal{V}_u \subseteq \{1, \ldots, m\}$ the subset indicating the bounded input components u_i, i.e for $i \in \mathcal{V}_u \subseteq \{1, \ldots, m\}$, the input component u_i is required to be lower bounded and upper bounded, i.e. $u_i^{\min} < u_i < u_i^{\max}$, where $u_i^{\min} \in \mathbb{R}$ and $u_i^{\max} \in \mathbb{R}$ are constants.*

System (1) is controlled, in feedback interconnection, by the controller

$$\begin{aligned} \dot{z} =&\ F(z, u_c) \\ y_c =&\ H(z), \end{aligned} \tag{2}$$

with state $z \in \mathbb{R}^{n_c}$, input $u_c \in \mathbb{R}^m$ and output $y_c \in \mathbb{R}^m$, where $F : \mathbb{R}^{n_c} \times \mathbb{R}^m \to \mathbb{R}^{n_c}$ and $H : \mathbb{R}^{n_c} \to \mathbb{R}^m$ are sufficiently smooth mappings. Specifically, we assume the negative feedback interconnection $u = -y_c$ and $u_c = y$, yielding (see also Figure 1) the closed loop system

$$\begin{aligned} \dot{x} =&\ f(x, -H(z)) \\ \dot{z} =&\ F(z, h(x)). \end{aligned} \tag{3}$$

We assume that there exists a suitable steady state solution to system (3).

Assumption 1 (Existence of steady state) *There exists a constant pair $(\overline{x}, \overline{z})$ satisfying the steady state equations*

$$\begin{aligned} \mathbf{0} =&\ f(\overline{x}, -H(\overline{z})) \\ \mathbf{0} =&\ F(\overline{z}, h(\overline{x})), \end{aligned} \tag{4}$$

where $u_i^{\min} < H_i(\overline{z}) < u_i^{\max}$ for all $i \in \mathcal{V}_u$.

Note that we have not made any assumption on the output $y_c = H(z)$ of controller (2), and that the output y_c might exceed required bounds on u. A common approach to maintain nevertheless desired bounds on the input to the system, is to employ saturation functions, by replacing for all $i \in \mathcal{V}_u$ the component u_i in (1) with $\mathrm{sat}_i(u_i)$, where the range of sat_i satisfies $\mathcal{R}(\mathrm{sat}_i) = [u_i^{\min}, u_i^{\max}]$. In this paper we propose another approach that offers a few advantages over the use of saturation functions and anti-windup compensators.

A. Constraints on the controller state

We now discuss how the desired bounds on the input u can be realized by satisfying appropriate bounds on some of the controller state components. To do so, we notice that the input constraints in Objective 1 are equivalent to constraints on the controller output $y_c = H(z)$, and consequently we require

$$u_i^{\min} \leq H_i(z) \leq u_i^{\max} \text{ for all } i \in \mathcal{V}_u. \tag{5}$$

Let $\mathcal{V}_z \subseteq \{1, \ldots, n_c\}$ the subset indicating the bounded controller components z_i. We aim at finding a subset $\mathcal{V}_z \subseteq \{1, \ldots, n_c\}$, lower bounds $z_i^{\min} \in \mathbb{R}$ and upper bounds

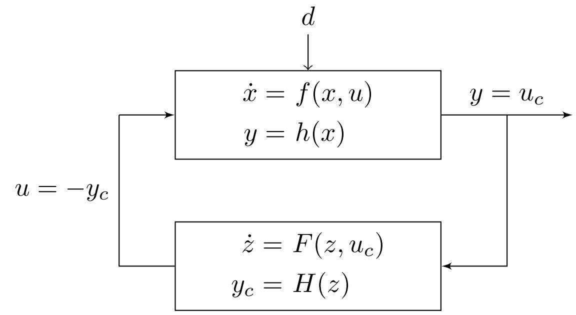

Fig. 1: System (1) in feedback interconnection with the controller (2), with d a disturbance.

$z_i^{\max} \in \mathbb{R}$ for all $i \in \mathcal{V}_z$, such that the following implication holds:

$$z_i^{\min} \leq z_i \leq z_i^{\max} \qquad \text{for all } i \in \mathcal{V}_z \tag{6}$$

$$\Downarrow$$

$$u_i^{\min} \leq H_i(z) \leq u_i^{\max} \qquad \text{for all } i \in \mathcal{V}_u. \tag{7}$$

According to (5), we define the set of feasible points z as

$$S_{H(z)} = \{z \in \mathbb{R}^{n_c} : u_i^{\min} \leq H_i(z) \leq u_i^{\max} \text{ for all } i \in \mathcal{V}_u\}. \tag{8}$$

Similarly, relation (6) defines the set of point z that satisfy the required component-wise constraints,

$$S_z = \{z \in \mathbb{R}^{n_c} : z_i^{\min} \leq z_i \leq z_i^{\max} \text{ for all } i \in \mathcal{V}_z\}. \tag{9}$$

Given Objective 1, we now assume that satisfying the input constraints can be achieved by appropriately chosen constraints on the controller state.

Assumption 2 (Redesign feasibility) *The set $S_{H(z)}$ given by (8) is non-empty. Furthermore, there exists a subset $\mathcal{V}_z \subseteq \{1, \ldots, n_c\}$, such that for all $i \in \mathcal{V}_z$, there exist scalars $z_i^{\min}$ and $z_i^{\max}$ satisfying $z_i^{\min} < \overline{z}_i < z_i^{\max}$, and the induced set S_z, given by (9), satisfies $S_z \subseteq S_{H(z)}$.*

Notice that Assumption 2 above imposes that the components $\overline{z}_i$, for all $i \in \mathcal{V}_z$, of the steady state $\overline{z}$ satisfying (4), are within the corresponding bounds $z_i^{\min}$ and $z_i^{\max}$.

Remark 1 (Determining $\mathcal{V}_z$, $z_i^{\min}$ and $z_i^{\max}$) *The focus of this work is not to fully characterize $\mathcal{V}_z$, $z_i^{\min}$ and $z_i^{\max}$, such that Assumption 2 holds. Indeed, we recognize that this characterization largely depends on the output mapping $H(z)$. However, obtaining this characterization is straightforward for the important case where*

$$H(z) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 1 & 0 & \ldots & 0 \end{bmatrix}}_{H_c \in \mathbb{R}^{m \times n_c}} z, \tag{10}$$

with $H_{c_{ij}} = 1$ if $i = j$ and 0 otherwise. It follows that in this case $\mathcal{V}_z = \{1, \ldots, m\}$ and that for all $i \in \mathcal{V}_z$ the lower and upper bounds are given by $z_i^{\min} = u_i^{\max}$ and $z_i^{\max} = u_i^{\min}$, respectively.

III. Controller redesign objectives

In this section we discuss how previously established desired bounds on some components of the controller state z lead to redesign objectives of given controllers (2). Additionally, we particularly pay attention to the preservation of the dissipation inequality associated to the controller dynamics, which is a major aspect where our proposed approach differs from existing results (see also Remark 2 below). To this end, we assume the existence of a suitable storage function [6].

Assumption 3 (Differential dissipation inequality) *There exists a positive definite, continuous differentiable storage function $V_1(z, \overline{z})$ satisfying the differential dissipation inequality*[1]

$$\frac{\partial V_1}{\partial z} F(z, u_c) \leq -W(z, \overline{z}, u_c, \overline{u}_c). \tag{11}$$

along the solutions to (2). Here, $W : \mathbb{R}^{n_c} \times \mathbb{R}^{n_c} \times \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}$ is a function, and we call W the dissipation property.

We continue with formulating three redesign objectives, where requirements on a function $G_i : \mathbb{R} \to \mathbb{R}$ are established, such that the redesigned dynamics of the controller state component z_i become

$$\dot{z}_i = G_i(F_i(z, u_c)), \tag{12}$$

having desirable properties that we will make explicit in the remainder of this section. The first design objective we formalize is the enforcement of the transient constraints, such that z_i remains for all $i \in \mathcal{V}_z$ between its lower bound $z_i^{\min} \in \mathbb{R}$ and its upper bound $z_i^{\max} \in \mathbb{R}$, when properly initialized.

Design objective 1 (Satisfying transient constraints) *Design smooth functions $G_i : \mathbb{R} \to \mathbb{R}$ such that for all $i \in \mathcal{V}_z$ the redesigned controller*

$$\dot{z}_i = G_i(F_i(z, u_c)), \tag{13}$$

ensure that $z_i(t) \in (z_i^{\min}, z_i^{\max})$ for all $t > 0$, if $z_i(0) \in (z_i^{\min}, z_i^{\max})$.

The second design objective is to guarantee that the redesigned dynamics do not alter the steady state and only constrain the transient behaviour.

Design objective 2 (Unaltered steady state) *Given $0 = F_i(\overline{z}, \overline{u}_c)$, with $\overline{z}_i \in (z_i^{\min}, z_i^{\max})$ for all $i \in \mathcal{V}_z$. Design smooth functions $G_i : \mathbb{R} \to \mathbb{R}$ such that for all $i \in \mathcal{V}_z$ the redesigned dynamics satisfy*

$$0 = G_i(F_i(\overline{z}, \overline{u}_c)). \tag{14}$$

[1]Note that $\frac{\partial V_1}{\partial \overline{z}} F(\overline{z}, \overline{u}_c) = 0$.

The third objective is to preserve the dissipation inequality associated to the controller dynamics.

Design objective 3 (Preserving the dissipation property) *Given a positive definite, continuously differentiable storage function $V_1(z, \overline{z})$ that satisfies*

$$\begin{aligned} &\frac{\partial V_1}{\partial z_i} F_i(z, u_c) \leq -W_i(z, \overline{z}, u_c, \overline{u}_c) \quad i \in \mathcal{V}_z \\ &\frac{\partial V_1}{\partial z_j} F_j(z, u_c) \leq -W_j(z, \overline{z}, u_c, \overline{u}_c) \quad j \in \{1, \ldots, n_c\} \backslash \mathcal{V}_z, \end{aligned} \tag{15}$$

and possesses a (local) minimum at $z = \overline{z}$. Here,

$$\sum_{i \in \mathcal{V}_z} W_i + \sum_{j \in \{1,\ldots,n_c\} \backslash \mathcal{V}_z} W_j = W. \tag{16}$$

Design for all $i \in \mathcal{V}_z$ smooth functions $G_i : \mathbb{R} \to \mathbb{R}$ such that there exists a (possibly different) continuously differentiable storage function $V_2(z, \overline{z})$ that satisfies

$$\begin{aligned} &\frac{\partial V_2}{\partial z_i} G_i(F_i(z, u_c)) \leq -W_i(z, \overline{z}, u_c, \overline{u}_c) \quad i \in \mathcal{V}_z \\ &\frac{\partial V_2}{\partial z_j} F_j(z, u_c) \leq -W_j(z, \overline{z}, u_c, \overline{u}_c) \quad j \in \{1, \ldots, n_c\} \backslash \mathcal{V}_z, \end{aligned} \tag{17}$$

and also possesses a (local) minimum at $z = \overline{z}$.

Note that the dissipative terms W_i at the right hand sides of (17) are identical to the dissipative terms W_i at the right hand sides of (15).

Remark 2 (Preserving the dissipation property) *We like to stress the importance of preserving the dissipative term $W(z, \overline{z}, u_c, \overline{u}_c)$ in (11). Indeed, many useful system theoretical properties such as (incremental), (equilibrium independent) passivity [7], [8] or input–state–stability (ISS) [9], rely on the particular expression of W. Therefore, we believe that preserving the dissipative term (11) permits a more straightforward incorporation of the redesigned controllers in the existing stability analysis established with the original controllers (2).*

In the next section we provide for a particular case an explicit design of G_i and provide the associated storage function $V_2(z, \overline{z})$.

IV. An explicit redesign

In this section we provide the explicit design of function G_i and storage function $V_2(z, \overline{z})$ in case the, to be constrained, state components $z_i, i \in \mathcal{V}_z$ appear as $\alpha_i(z_i - \overline{z}_i)^2$ in the storage function $V_1(z, \overline{z})$ associated to the unmodified system (2).

Assumption 4 (Separate quadratic storage term) *The storage function $V_1(z, \overline{z})$, associated to the original controller (2), is of the form*

$$V_1(z, \overline{z}) = \tilde{V}(z_{-\mathcal{V}_z}, \overline{z}_{-\mathcal{V}_z}) + \sum_{i \in \mathcal{V}_z} \alpha_i (z_i - \overline{z}_i)^2, \tag{18}$$

where $z_{-\mathcal{V}_z}$ denote all state components z_j with $j \in \{1,\ldots,n_c\}\backslash\mathcal{V}_z$, and $\alpha_i \in \mathbb{R}_{>0}$ is a positive constant for all $i \in \mathcal{V}_z$.

We remark that Assumption 4 is not very restrictive, since quadratic storage terms are widely used to show stability. We now propose an explicit design of G_i that ensures satisfying the transient constraint, resulting in the redesigned dynamics of state component $z_i, i \in \mathcal{V}_z$;

$$\dot{z}_i = \underbrace{(z_i^{\max} - z_i)(z_i - z_i^{\min})}_{G_i} F_i(z, u_c). \tag{19}$$

The following two result are immediate and cover the first two design objectives proposed in the previous section:

Lemma 1 (Transient constraints) *Consider (19). If $z_i(0) \in (z_i^{\min}, z_i^{\max})$, then $z_i(t) \in (z_i^{\min}, z_i^{\max})$ for all $t > 0$.*

Lemma 2 (Steady state) *If $0 = F_i(\overline{z}, \overline{u}_c)$, then also*

$$0 = (z_i^{\max} - \overline{z}_i)(\overline{z}_i - z_i^{\min})F_i(\overline{z}, \overline{u}_c). \tag{20}$$

To show that also the third design objective is met, we consider the storage function

$$V_2(z, \overline{z}) = \tilde{V}(z_{-\mathcal{V}_z}, \overline{z}_{-\mathcal{V}_z}) + \sum_{i\in\mathcal{V}_z} Z_i(z_i, \overline{z}_i), \tag{21}$$

where $\tilde{V}(z_{-\mathcal{V}_z}, \overline{z}_{-\mathcal{V}_z})$ is given in (18) and $Z_i(z_i, \overline{z}_i)$ replaces the quadratic term in $(z_i - \overline{z}_i)$ appearing in (18), and is given by

$$\begin{aligned} Z_i(z_i, \overline{z}_i) = \\ 2\alpha\Bigg(&\frac{-z_i^{\max}\ln(z_i^{\max} - z_i) + z_i^{\min}\ln(z_i - z_i^{\min})}{z_i^{\max} - z_i^{\min}} \\ &- \frac{-\overline{z}_i\ln(z_i^{\max} - z_i) + \overline{z}_i\ln(z_i - z_i^{\min})}{z_i^{\max} - z_i^{\min}}\Bigg). \end{aligned} \tag{22}$$

We now derive a few important properties of $Z_i(z_i, \overline{z}_i)$.

Lemma 3 (Strict convexity and minimum at $z_i = \overline{z}_i$) *Function $Z_i(z_i, \overline{z}_i)$, given by (28), is within the domain $z_i \in (z_i^{\min}, z_i^{max})$, strictly convex and attains a strict minimum at $z_i = \overline{z}_i$.*

Next, we show that $Z_i(z_i, \overline{z}_i)$ has a similar property to the so-called 'radially unboundedness' [10].

Lemma 4 (Asymptotic behaviour of $Z_i(z_i, \overline{z}_i)$) *The function $Z_i(z_i, \overline{z}_i)$ satisfies*

$$\begin{aligned} \lim_{z_i \downarrow z_i^{\min}} Z_i(z_i, \overline{z}_i) = \infty \\ \lim_{z_i \uparrow z_i^{\max}} Z_i(z_i, \overline{z}_i) = \infty \end{aligned} \tag{23}$$

The following theorem shows, together with Lemma 3, that also the third design objective is met:

Theorem 1 (Dissipativity preservation) *Given $V_1(z, \overline{z})$ in (18) and $V_2(z, \overline{z})$ in (21), it holds that*

$$\frac{\partial V_1(z, \overline{z})}{\partial z} F(z, u_c) = \frac{\partial V_2(z, \overline{z})}{\partial z} \tilde{G}(F(z, u_c)), \tag{24}$$

where $\tilde{G}_i(F_i(z, u_c)) = (z_i^{\max} - z_i)(z_i - z_i^{\min})F_i(z, u_c)$ if $i \in \mathcal{V}_z$ and $\tilde{G}_i(F_i(z, u_c)) = F_i(z, u_c)$ otherwise.

We finish this section by providing a simple example illustrating the obtained results, whereafter we apply the developed redesign methodology to a more extensive case in the next section.

Example 1 (Integral control) *Consider an integral controller of the form*

$$\begin{aligned} \dot{z} &= u_c \\ y_c &= z, \end{aligned} \tag{25}$$

with state $z \in \mathbb{R}$, input $u_c \in \mathbb{R}$ and output $y_c \in \mathbb{R}$. System (25) is passive, as can be established by considering the storage function[2] $V_1(z) = \frac{1}{2}z^2$, that satisfies

$$\dot{V}_1 = u_c y_c, \tag{26}$$

along the solutions to (25). To prevent an overshoot of z we apply the proposed redesing in this section, yielding

$$\begin{aligned} \dot{z} &= (z^{\max} - z)(z - z^{\min})u_c \\ y_c &= z, \end{aligned} \tag{27}$$

where $z^{\min}$ and $z^{\max}$ are desired lower and upper bounds respectively. System (27) is also passive, as can be established by considering the storage function

$$V_2(z) = \frac{-z^{\max}\ln(z^{\max} - z) + z^{\min}\ln(z - z^{\min})}{z^{\max} - z^{\min}}, \tag{28}$$

that satisfies

$$\dot{V}_2 = u_c y_c, \tag{29}$$

along the solutions to (27).

V. Power sharing in electricity networks

In this section we apply the obtained results to redesign distributed controllers that have been proposed previously in the literature to obtain *optimal* power sharing in electricity networks, such as high voltage transmission networks or microgrids. Particularly, we focus on distributed averaging integral (DAI) controllers considered in e.g. [11], [12], [13], [14]. These type of controllers aim at regulating an interconnected electricity network, while solving at steady state an optimization problem of the form

$$\begin{aligned} &\min_{y_c} C(y_c) \\ &\text{s.t.} \quad 0 = \sum_{i\in\mathcal{V}_g} y_{ci} - \sum_{i\in\mathcal{V}_l} P_{li}, \end{aligned} \tag{30}$$

with $C(y_c) = \sum_{i\in\mathcal{V}_g} C_i(y_{ci})$. Here, $y_{ci} \in \mathbb{R}$ is the controlled power generation at node $i \in \mathcal{V}_g$ in the network and $P_{li} \in \mathbb{R}$ is the uncontrollable power load at node $i \in \mathcal{V}_l$. Associated to every generation y_{ci} is a strictly convex cost

[2] We take $\overline{z} = \overline{u}_c = 0$.

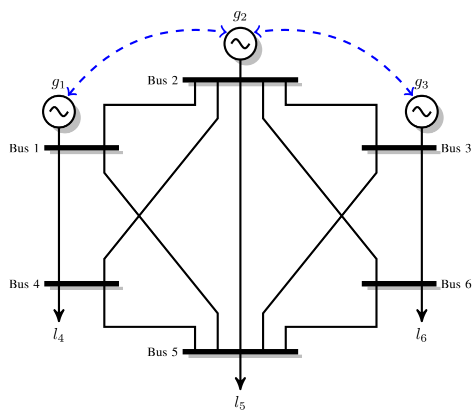

Fig. 2: Diagram for a 6–bus power network, consisting of 3 generator and 3 load buses. The communication links, employed by the distributed controllers, are represented by the dashed lines.

function $C_i(y_{ci}) : \mathbb{R} \to \mathbb{R}$, and optimization problem (30) aims at minimizing the total generation costs while satisfying the power balance constraint $\sum_{i\in\mathcal{V}_g} y_{ci} = \sum_{i\in\mathcal{V}_l} P_{li}$. We refer to the references above for a more detailed discussion on the electricity network, its control objectives and the controller design. We merely state a typical DAI controller in an ad-hoc fashion for the case $C_i(y_{ci})$ is linear-quadratic, i.e. of the form $C_i(y_{ci}) = \frac{1}{2}Q_i y_{ci}^2 + R_i y_{ci} + S_i$, such that the total costs can be expressed as $C(y_c) = \frac{1}{2}y_c^T Q y_c + R^T y_c + \mathbb{1}^T S$.

Controller 1 (DAI)

$$\begin{aligned} \dot{\theta} &= -Q\mathcal{L}\big(Q\theta + R\big) + u_c \\ y_c &= \theta, \end{aligned} \tag{31}$$

where $\mathcal{L}$ is the Laplacian matrix associated to the topology of the connected communication network. Without the proposed redesign, controller (31) has been shown to be incrementally passive with respect to its steady state solution, which turned out to be essential to the overall stability of the network [11]. However, since the controllers adjust the generation, it is of importance that they respect capacity restrictions of the power generation units [15] i.e. it is required that $y_{ci}(t) \in [y_{ci}^{\min}, y_{ci}^{\max}]$ for all $i \in \mathcal{V}_g$ and for all $t \geq 0$. Following the suggested modifications in (19), the newly obtained constrained DAI (C–DAI) controller is given by

Controller 2 (C–DAI)

$$\begin{aligned} \dot{\theta} &= \mathrm{diag}(\theta - \theta^{\min})\mathrm{diag}(\theta^{\max} - \theta)\big(-Q\mathcal{L}\big(Q\theta + R\big) + u_c\big) \\ y_c &= \theta. \end{aligned} \tag{32}$$

Using the results from Section IV, also (32) can be shown to be incrementally passive with respect to its steady state solution, permitting a straightforward adaptation of the stability results given in e.g. [11] and [16]. The corresponding analysis is omitted here. Instead, we show the effectiveness of the redesigned controller, by comparing controllers (31) and (32) when interconnected with an academic example of a high voltage power network, shown in Figure 2. The case study is taken from [16], and we refer to that reference for additional details and used numerical values. Following [17], generator bus $i \in \mathcal{V}_g$ is modelled as

	State variables
δ_i	Voltage angle
ω_{gi}	Frequency deviation at the generator bus
ω_{li}	Frequency deviation at the load bus
	Parameters
M_i	Moment of inertia
D_{gi}	Damping constant of the generator
D_{li}	Damping constant of the load
B_{ij}	Susceptance of the transmission line
V_i	Voltage
	Controllable input
y_{ci}	Power generation
	Uncontrollable disturbance
P_{li}	Unknown constant power demand

Table 1: Description the variables and parameters appearing in the power network model.

$$\begin{aligned} \dot{\delta}_i &= \omega_{gi} \\ M_i\dot{\omega}_{gi} &= -D_{gi}\omega_{gi} - \sum_{j\in\mathcal{N}_i} V_iV_jB_{ij}\sin(\delta_i - \delta_j) + y_{ci}, \end{aligned}$$

where $\mathcal{N}_i$ is the set of buses connected to bus i. The uncontrollable loads are assumed to consist of a constant and a frequency dependent component. We model a load bus for $i \in \mathcal{V}_l$ therefore as

$$\begin{aligned} \dot{\delta}_i &= \omega_{li} \\ 0 &= -D_{li}\omega_{li} - \sum_{j\in\mathcal{N}_i} V_iV_jB_{ij}\sin(\delta_i - \delta_j) - P_{li}. \end{aligned}$$

An overview of the used symbols is provided in Table 1. The system is initially at steady state with loads P_{l1}, P_{l2} and P_{l3} being 0.01, 0.20 and 0.18 pu respectively. After 5 seconds the loads are increased to 1.15, 1.25 and 1.21 pu, respectively. In (31) and in (32), we take $Q = \mathbb{I}_3$, $R = \mathbf{0}$ and $u_c = -100\omega_g$. The maximum power the generators can safely provide is 1.4 pu, i.e. we require that $y_{ci}^{\min} = 0 \leq y_{ci} \leq 1.4 = y_{ci}^{\max}$, for all $i \in \{1,2,3\}$. From Figure 3, where the response with controller (31) is depicted, we see that the power output clearly exceeds 1.4 pu at all generators. On the other hand, from Figure 4, where the response with the redesigned controller (31) is depicted, we can see how the output never exceeds 1.4 pu. Note that both controllers achieve frequency regulation, i.e. $\lim_{t\to\infty 0} \omega_g(t) = \mathbf{0}$ and that the steady state power generation is identical.

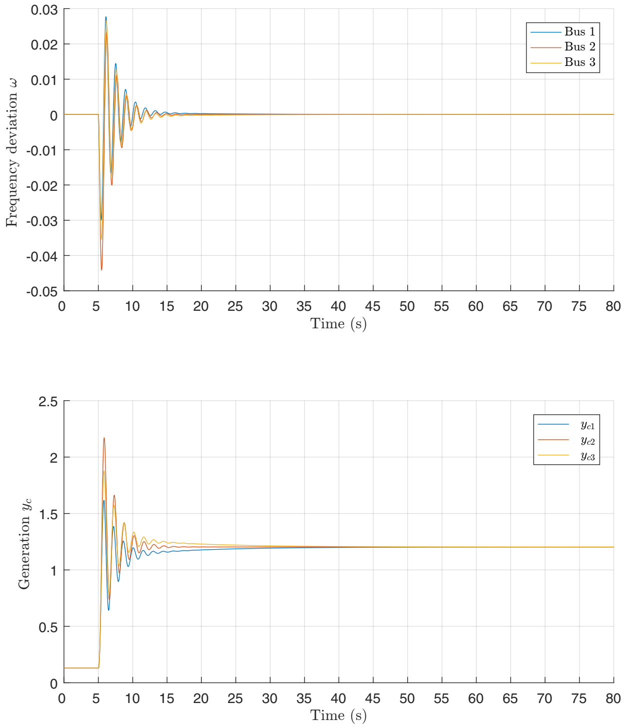

Fig. 3: Time evolution of the frequency deviation ω_g and power generation y_c when controller (31) is used.

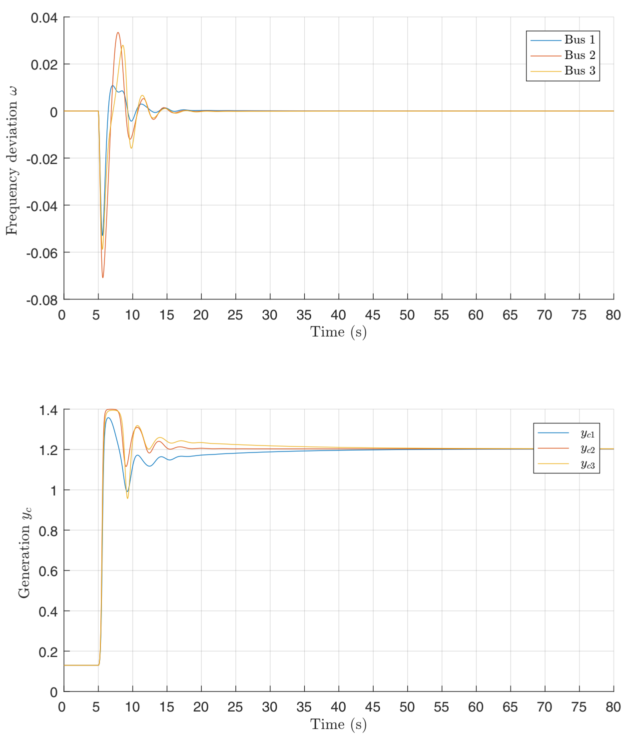

Fig. 4: Time evolution of the frequency deviation ω_g and power generation y_c when controller (32) is used.

VI. Conclusions and Future Research

We proposed a method to redesign given controllers for systems with input saturation. Instead of constraining the input, we argue that a viable alternative is to constrain the controller state. Particularly, we focus on redesigning existing controllers in a way that the associated dissipation property is preserved, allowing for a straightforward adaptation of existing stability results. To demonstrate the proposed methodology, we redesign a popular distributed controller aiming at power sharing in electricity networks. A case study indicates the effectiveness of the proposed solution.

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