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A risk-constrained and energy efficient stochastic approach for autonomous overtaking

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***Abstract*— In this paper a control approach for safe and energy efficient autonomous overtaking is presented. The proposed method combines a stochastic approach based on the use of safety indicators as the Time headway (TH) and Time to collision (TTC) as boundary conditions and a cost index on fuel consumption for the trajectory of an Ego vehicle in a traffic environment. The approach has been evaluated in two typical overtaking scenarios: 1) braking of the leading vehicle and 2) entry of a third vehicle in the gap between the leading and the ego vehicle. The efficiency of TH and TTC- based stochastic approach have been compared for the same maximal risk threshold. Results show that both risk function are suitable to perform overtaking manoeuvres both with and without an additive fuel cost. Moreover the TTC–based stochastic approach shows a reduction of about 5% in fuel consumption with respect to the TH-based approach for both considered scenarios.**

I. INTRODUCTION

Advanced Driver Assistance Systems (ADAS), are attracting much attention both in the academia and industry for several reasons, among others as support for fast increasing elder drivers population, but also as a step in direction of true autonmous driving (see e.g. [1], [2], [3], [4] for some examples).

While ADAS typically address a single function at a time, e.g. parking or cruise control, more rather general case is the so called highway pilot, which combines the cruise control (CC) and autonomous overtaking. CC is widely used in several forms, on new vehicles increasingly in the form of adaptive cruise control (ACC) which combines speed tracking with a minimum speed.

Instead autonomous overtaking is not as advanced, mainly due to safety concerns. Indeed, ADAS are expected not only to fulfill their function, but also to make sure that the collision risk is sufficiently small. Not surprisingly, the problem of safety in overtaking has attracted much attention [3], [4], [5], in particular the problem of collision avoidance in normal traffic environment (see e.g. [6], [7]).

To achieve this function, first a sensible metric for the risk is needed. In this context, time-headway (TH) and time-to-collision (TTC) are frequently used as risk functions, where risk is defined by some authors [1], [8] as the probability of the violation of a given distance policy to obstacles, which usually is more conservative than collision risk to account for uncertainties. However, if the uncertainties can be somehow restricted by stochastic models, a stochastic, risk constreined optimal control approach as in [9] can provide better results.

However, safety and comfort are not the only possible benefits of ACC, also significant gains in terms of energy efficiency can be obtained. The question behind this paper is whether it is possible and sensible to extend the previous work to take in account energy efficiency, in our case in terms of fuel consumption. This paper concentrates on overtaking, as the case for the ACC is already solved, but of course in a final implementation energy optimization must be considered for all conditions.

The basic challenge during overtaking is that the classical vehicle following approach is not appropriate, as the reference car - the leading vehicles - typically changes over time. This aspect has been considered tackling autonomous overtaking in previous publications such as [4], [11], [5]. However, most of them suppose the exact knowledge of the surrounding vehicles along the prediction horizon, a rather optimistic assumption.

Using a stochastic approach, as in this paper, this problem can be relaxed. In addition, a mixed integer programming problem in the context of MPC can be used to take into account the switching nature of this problem, without the need to choose explicitly a reference vehicle. A maximum risk level is used to collision avoidance, while reducing fuel consumption.

The rest of the paper is organized as follows: the problem statement of ego car dynamics, risk functions and fuel consumption are given in section two. The control design is proposed in section three. A comparative study for two different scenario is demonstrated in section four to show the effectiveness of the proposed algorithm. Finally, the conclusions are given in section five.

II. PROBLEM STATEMENT

A. System dynamics

We assume the vehicle dynamic of the controlled vehicle (ego) to be described by the following simple linear model (see [10] for more details):

$$\mathbf{x}^{(E)}(k+1) = A\mathbf{x}^{(E)}(k) + B\mathbf{u}^{(E)}(k), \tag{1}$$

with the *state* $\mathbf{x}$ and the *control* $\mathbf{u}$ vectors are defined as:

$$\begin{aligned} \mathbf{x}^{(E)}(k) &= \left[y^{(E)}(k)\; v_y^{(E)}(k)\; x^{(E)}(k)\right]^T \\ \mathbf{u}^{(E)}(k) &= \left[a_y^{(E)}(k)\; v_x^{(E)}(k)\right]^T \end{aligned} \tag{2}$$

where: $y^{(E)}(k)$, $v_y^{(E)}(k)$, $a_y^{(E)}(k)$ indicate respectively the *longitudinal* position, speed and acceleration at time k and

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$x^{(E)}(k)$, $v_x^{(E)}(k)$ respectively the *lateral* positions and speeds at time k.

Assuming, for simplicity, a uniform sampling time T_s, the model in eq.(1) can be rewritten, in its discrete form, as:

$$A = \begin{bmatrix} 1 & T_s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ T_s & 0 \\ 0 & T_s \end{bmatrix}. \tag{3}$$

In order to ensure the intrinsic limits of the vehicle dynamics, we assume that the states and inputs are constrained in bounded set:

$$\mathbf{x}^{(E)}(k) \in \left[\underline{\mathbf{x}}^{(E)}, \overline{\mathbf{x}}^{(E)}\right], \quad \mathbf{u}^{(E)}(k) \in \left[\underline{\mathbf{u}}^{(E)}, \overline{\mathbf{u}}^{(E)}\right]$$

and these bounds are specified as

$$\underline{\mathbf{x}}^{(E)} = \begin{bmatrix} 0 & \underline{v}_y^{(E)} & l_1 \end{bmatrix}^T, \ \overline{\mathbf{x}}^{(E)} = \begin{bmatrix} +\infty & \overline{v}_y^{(E)} & l_2 \end{bmatrix}^T,$$
$$\underline{\mathbf{u}}^{(E)} = \begin{bmatrix} \underline{a}_y^{(E)} & \underline{v}_x^{(E)} \end{bmatrix}^T, \ \overline{\mathbf{u}}^{(E)} = \begin{bmatrix} \overline{a}_y^{(E)} & \overline{v}_x^{(E)} \end{bmatrix}^T,$$

where $\overline{q}$ and $\underline{q}$ denote respectively the upper and lower bounds of the quantity q (with $q \in \{v_x, v_y, a_y\}$) as well as l_1 and l_2 represent respectively the lower and upper bounds for the lateral position on the roadway. In addition, to avoid an unrealistic steering behavior, the lateral speed of the vehicle is constrained to its longitudinal speed trough the relation

$$v_x^{(E)}(k) \in \left[-\tan(\beta), \ \tan(\beta)\right] v_y^{(E)}(k), \tag{4}$$

to the maximal slip angle β. Finally, for simplicity, the roadway in this paper is assumed to be straight.

B. Risk constraints

The two considered risk functions based on Time Headway (TH) and time-to-collision (TTC) described below in the text lead to a probabilistic risk constraint:

$$\Pr\left(\mathrm{R} \le \mathrm{R}_{\max}\right) \ge 1-\alpha, \tag{5}$$

where R and $\mathrm{R}_{\max}$ represent respectively a chosen risk function and its maximum accepted value. The parameter α represents the admitted probability level for a chosen risk function R to exceed the risk threshold $\mathrm{R}_{\max}$.

Notice that it may seem surprising to have an accepted collision probability, but, in view of the unavoidable uncertainty in measurements and prediction models, there will always be a crash probability, even if very small. As shown in [12], it pays off even in term of final risk of constraint violation to consider explicitly this risk than to assume a correct information and use a deterministic, apparently risk-free approach.

C. Fuel efficiency

Energy efficiency can be improved by including it explicitly in the objective function. In our simulations we shall use a conventional drive-line (BMW 320d), so energy efficiency boils down to fuel-efficiency. The fuel consumption rate q_{f} is modeled here as a *static map* of the speed and acceleration of the vehicle $q_{\mathrm{f}} = \hat{q}_f(\mathbf{a}, \mathbf{v})$ and in turn, it can be expressed in terms of the state and input variables $q_{\mathrm{f}} = \hat{q}_f(\mathbf{x}, \mathbf{u})$. The gear-dependence of the fuel consumption cost is here for simplicity encoded in the function q_{f} (for further details about the gear dependence reference is made to [1]).

Within the MPC framework, the predicted vehicle's fuel consumption along an optimization horizon t_h is introduced as a cost function of the system dynamics as follow:

$$Q_{\mathrm{f},k} = T_s \sum_{i=0}^{h_{\mathrm{P}}-1} \hat{q}_{\mathrm{f}}\left(\mathbf{x}_{k+i|k}, \mathbf{u}_{k+i|k}\right) \tag{6}$$

where h_{P} is the number uniformly distributed evaluation instants along the horizon ($t_h = h_{\mathrm{P}} \cdot T_s$), $\mathbf{x}_{k+i|k}$ and $\mathbf{u}_{k+i|k}$ are the *i*- steps ahead prediction of the system-state and input at time k.

D. Surrounding traffic

We recall that in the MPC framework, not only the actual, but also the future state of the system and constraints are needed. In particular, a control formulation based on constraints upon TH and TTC indexes will involve necessary a prediction of the surrounding traffic participants future position (see (25) and (33) in the next sections).

In this work specifically, the prediction of the future *longitudinal* position (named $y^{(S)}(k+i+1|k), i = 0, \ldots, h_{\mathrm{P}}$ in the next sections) is provided by a *stochastic model* of the future longitudinal speed $v_y^{(S)}(k+i+1|k)$ along the optimization horizon. A stochastic model of the *lateral* position $x^{(S)}(k+i+1|k), i = 0, \ldots, h_{\mathrm{P}}$ is additionally presented. In this article we used a stochastic model based on a graphical model denominated *Bayesian Network*(BN) developed in [13] for *lane change* prediction. A BN is a *directed acyclic graph*(see Fig.1) in which nodes represent random variables and arcs represent *conditional dependence* assumptions. A

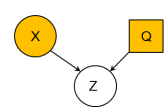

Fig. 1. Sketch of a Bayesian network describing the dependence of a continuous random variable Z with continuous-valued parent X and discrete-valued parent Q.

special class of Bayesian network called *conditional linear Gaussian* model (CG) for modeling the future longitudinal velocity and lateral position of the surrounding vehicles has been chosen in this case. A detailed explanation of this modeling method can be found in [14]. In this case (CG), it is assumed that the future velocity $v_y^{(S)}$ lateral position $x^{(S)}$ can be described using a Gaussian distribution Z_i for each sampling instance $i = 0, \ldots, h_P - 1$ i.e.

$$Z_i \sim \mathcal{N}\left\{\mu_{Z_i} \sigma_{Z_i}\right\} \tag{7}$$

where $\mathcal{N}\left\{\mu_{Z_i} \sigma_{Z_i}\right\}$ denotes a 1-dimensional Gaussian distribution with mean μ_{Y_i} and standard deviation σ_{Z_i}. Moreover the dependent random vector Z (each one the Z_i defined above) is assumed to have the following conditional distribution with respect to its *continuous* parents-nodes X and a *discrete* parent Q:

$$p(Z|X, Q=j) = c|\Sigma_j|^{-\frac{1}{2}} e^{-\frac{1}{2}\left(Z-B_jX-\mu_i\right)^T \Sigma_j^{-1}\left(Z-B_jX-\mu_j\right)} \tag{8}$$

where $c = (2\pi)^{-d/2}$ is a normalization constant, d is the dimension of Z, Σ_j its covariance matrix and B_j is the regression matrix of Z when the discrete random variable $Q = j$. The parameter matrix B_j estimated by means of a *maximum likelihood* algorithm. The prediction of both $v_y^{(S)}(k+i+1)$ and $x^{(S)}(k+i+1)$ is modeled by the introduction of a dependence from their two previous states and from the actual lane position $m^{(S)}(k+i+1) \in \{\text{left}, \text{current lane}, \text{right}\}$, this last depending in turn from the previous value of the indicator signal $I^{(S)}(k+i) \in \{\text{left}, \text{off}, \text{right}\}$. The compete structure of this Bayesian network is sketched in Fig.2

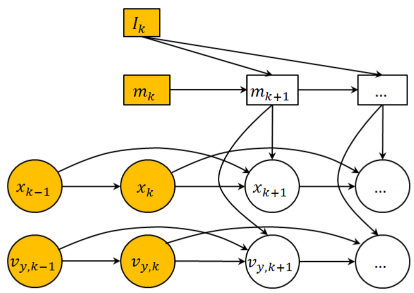

Fig. 2. The *lane-change* Bayesian network.

III. Control formulations

In this section we will expose the calculations that allow to formulate the *time headway* and the *time-to-collision* as risk-constraints for a scenario involving overtaking.

A. TH-induced formulation

Time headway TH is defined as the time that passes between the front of the preceding vehicle passing a point on the road and the front of the following one passing the same point [15].

$$\mathrm{TH} = \frac{y_1 - y_2}{v_2}. \tag{9}$$

where y_1, y_2, v_2 denote respectively the longitudinal position of the two vehicles and the longitudinal speed of the following vehicle. As a smaller TH means a larger risk, it is usual to use its inverse as the TH induced risk function :

$$\mathrm{R} := \hat{\mathrm{R}}(\mathrm{TH}) = \frac{1}{\mathrm{TH}} = \frac{v_2}{y_1 - y_2} \tag{10}$$

Notice that the TH induced risk function in (10) depends on which vehicle precedes the other one. However, during overtaking the relative position will change, so that it is more appropriate to consider an ego vehicle (E) and a surrounding car (S) instead of a preceding and follower. As only two cases can occur, the above definition can be re-formulated for a general time instant k as:

$$R = \begin{cases} \dfrac{v_y^{(E)}(k)}{y^{(S)}(k+1) - y^{(E)}(k+1)}, \; \textit{if} \text{ the car (E) follows the car (S)} \\ \dfrac{v_y^{(S)}(k)}{y^{(E)}(k+1) - y^{(S)}(k+1)}, \; \textit{if} \text{ the car (S) follows the car (E)} \end{cases} \tag{11}$$

In order to avoid collisions between vehicles, a maximal risk level Rmax=1/THmin will be used:

- if at time $k+i+1$ the ego vehicle (E) follows the surrounding vehicle (S), then

$$y^{(S)}(k+i+1|k) - y^{(E)}(k+i+1|k) \geq \mathrm{THmin}\; v_y^{(E)}(k+i|k), \tag{12}$$

- otherwise if at time $k+i+1$ the ego vehicle (E) precedes the other one, then

$$y^{(E)}(k+i+1|k) - y^{(S)}(k+i+1|k) \geq \mathrm{THmin}\; v_y^{(S)}(k+i|k). \tag{13}$$

This implies a speed-depending minimum distance between these two vehicles. To prevent this distance to become too small, an additional constraint is used:

$$x^{(E)}(k+i|k) \notin \left[x^{(S)}(k+i|k) - l, x^{(S)}(k+i|k) + l\right] \tag{14}$$

where l is a suitable minimum distance. The predicted values $y^{(S)}(k+i+1|k), v_y^{(S)}(k+i|k), x^{(S)}(k+i|k)$ in the constraints (12), (13) and (14), are obtained along the prediction horizon using the prediction model described in sect.II-D. Notice that the stochastic nature of the prediction implies that no absolute guarantee of collision avoidance is possible, but only an arbitrarily small one. However, as shown in [1], the stochastic approach does not perform worse in terms of constraint satisfaction than a deterministic one, as all models are approximations of the actual traffic situation.

The probability of infringement of the constraints can be bounded by Chebyshev's inequality as:

$$\Pr(k_1 < X < k_2) \geq 4\frac{(k_2 - \mu)(\mu - k_1) - \delta^2}{(k_2 - k_1)^2}, \tag{15}$$

where X denotes a real random variable, $\mu = \mathrm{E}(X)$, $\delta = \sqrt{\mathrm{Var}(X)}$ and $k_1 + k_2 = 2\mu$. Accordingly, if $k_1 = \mu - \varepsilon$ and $k_2 = \mu + \varepsilon$ for $\varepsilon > 0$, then we have:

$$\Pr(\mu - \varepsilon < X < \mu + \varepsilon) \geq 1 - \frac{\delta^2}{\varepsilon^2}. \tag{16}$$

Hereafter the level of uncertainty α defined in (5) will be named α_1 for the longitudinal direction and α_2 for the transversal (lateral) direction.

By applying (15) to predict the lateral position of vehicle (S) at time $k+i$ predicted at time k with $1 > \alpha_2 > 0$, one obtains:

$$\begin{aligned} &\Pr(\underline{x}^{(S)}(k+i|k) < x^{(S)}(k+i|k) < \overline{x}^{(S)}(k+i|k)) \geq 1 - \alpha_2 \\ &\underline{x}^{(S)}(k+i|k) = \mathrm{E}(x^{(S)}(k+i|k)) - \frac{\sqrt{\mathrm{Var}(x^{(S)}(k+i|k))}}{\sqrt{\alpha_2}} \\ &\overline{x}^{(S)}(k+i|k) = \mathrm{E}(x^{(S)}(k+i|k)) + \frac{\sqrt{\mathrm{Var}(x^{(S)}(k+i|k))}}{\sqrt{\alpha_2}}. \end{aligned} \tag{17}$$

Similarly, for the longitudinal speed of the surrounding vehicle (S), the prediction is

$$\begin{aligned} &\Pr(\underline{v}_y^{(S)}(k+i|k) < v_y^{(S)}(k+i|k) < \overline{v}_y^{(S)}(k+i|k)) \geq 1 - \alpha_1 \\ &\underline{v}_y^{(S)}(k+i|k) = \mu_y^{(S)}(k+i|k) - \delta_y^{(S)}(k+i|k)/\sqrt{\alpha_1} \\ &\overline{v}_y^{(S)}(k+i|k) = \mu_y^{(S)}(k+i|k) + \delta_y^{(S)}(k+i|k)/\sqrt{\alpha_1}. \end{aligned} \tag{18}$$

where $\mu_y^{(S)}(k+i|k)$, $\delta_y^{(S)}(k+i|k)$ denote the mean value and the standard deviation of the longitudinal speed of the surrounding vehicle (S) at time $k+i$, predicted at time k. In addition, $1 > \alpha_1 > 0$,. Based on the predicted longitudinal speed of the surrounding vehicle (S), the longitudinal position will be:

$$y^{(S)}(k+1|k) \in y^{(S)}(k) \oplus \left[T_s \underline{v}_y^{(S)}(k|k),\ T_s \overline{v}_y^{(S)}(k|k)\right],$$

$$\begin{aligned}
&Y^{(S)}(k+i+1|k) = \left[\underline{y}^{(S)}(k+i+1|k),\ \overline{y}^{(S)}(k+i+1|k)\right] \\
&\underline{y}^{(S)}(k+i+1|k) = y^{(S)}(k) + \sum_{j=0}^{i} T_s \underline{v}_y^{(S)}(k+j|k) \\
&\overline{y}^{(S)}(k+i+1|k) = y^{(S)}(k) + \sum_{j=0}^{i} T_s \overline{v}_y^{(S)}(k+j|k).
\end{aligned} \tag{19}$$

Also the set $Y_{\text{THmin}}^{(S)}(k+i+1|k)$ is defined as below:

$$\begin{aligned}
Y_{\text{THmin}}^{(S)}(k+i+1|k) = {}& Y^{(S)}(k+i+1|k) \oplus \\
& \text{THmin}\left[\underline{v}_y^{(S)}(k+i|k), \overline{v}_y^{(S)}(k+i|k)\right].
\end{aligned}$$

As the time headway constraints (12) and (13) require the information of $y^{(S)}(k+i+1|k)$ and $v_y^{(S)}(k+i|k)$, applying the Chebyshev inequality (15), one can prove that

$$\Pr\left(y^{(S)}(k+i|k) \in Y^{(S)}(k+i|k)\right) \geq 1-\alpha_1 \tag{20}$$

$$\begin{aligned}
\Pr\Big(&y^{(S)}(k+i|k) + \text{THmin} v_y^{(S)}(k+i-1|k) \\
&\in Y_{\text{THmin}}^{(S)}(k+i|k)\Big) \geq 1-\alpha_1.
\end{aligned} \tag{21}$$

Since the TH constraints (12) and (13) hold for any value of $y^{(S)}(k+i+1|k)$ and $v_y^{(S)}(k+i|k)$, so do they for the worst cases. More precisely, if $y^{(S)}(k+i+1|k) \in Y^{(S)}(k+i+1|k)$, then constraint (12) holds true for the lowest value of $y^{(S)}(k+i+1|k)$ in $Y^{(S)}(k+i+1|k)$, moreover constraint (13) holds true for the largest value of $y^{(S)}(k+i+1|k) + \text{TH} v_y^{(S)}(k+i|k)$ in $Y_{\text{THmin}}^{(S)}(k+i+1|k)$.

Adding the term for the fuel efficiency leads to following optimization problem:

$$\min_{\mathbf{u}^{(E)}(k+i|k)} \ \big\{Q_{\text{f},k}+ \tag{22}$$

$$\begin{aligned}
&\sum_{i=0}^{N} \left(\mathbf{u}^{(E)}(k+i|k)\right)^T R \mathbf{u}^{(E)}(k+i|k) + \\
&\left(\mathbf{x}^{(E)}(k+i|k) - \mathbf{x}_{\text{ref}}^{(E)}\right)^T Q \left(\mathbf{x}^{(E)}(k+i|k) - \mathbf{x}_{\text{ref}}^{(E)}\right) + \\
&\left(\Delta\mathbf{u}^{(E)}(k+i|k)\right)^T R_{\Delta u} \Delta\mathbf{u}^{(E)}(k+i|k)\Big\}
\end{aligned} \tag{23}$$

s.t.

$$\begin{aligned}
&\mathbf{x}^{(E)}(k+i+1|k) = A\mathbf{x}^{(E)}(k+i|k) + B\mathbf{u}^{(E)}(k+i|k) \\
&\mathbf{x}^{(E)}(k+i|k) \in \left[\underline{\mathbf{x}}^{(E)}, \overline{\mathbf{x}}^{(E)}\right], \mathbf{u}^{(E)}(k+i|k) \in \left[\underline{\mathbf{u}}^{(E)}, \overline{\mathbf{u}}^{(E)}\right] \\
&v_x^{(E)}(k+i|k) \in [-\tan(\beta),\ \tan(\beta)]\, v_y^{(E)}(k+i|k)
\end{aligned} \tag{24}$$

$$\begin{aligned}
-y^{(E)}(k+i|k) &\leq -\overline{y}^{(S)}(k+i|k) + M\gamma_1 \\
&\quad - \text{THmin}\, \overline{v}_y^{(S)}(k+i-1|k) \\
y^{(E)}(k+i|k) &\leq \ \ \underline{y}^{(S)}(k+i|k) + M\gamma_2 \\
&\quad - \text{THmin}\, v_y^{(E)}(k+i-1|k) \\
-x^{(E)}(k+i|k) &\leq -\overline{x}^{(S)}(k+i|k) - l + M\gamma_3 \\
x^{(E)}(k+i|k) &\leq \ \ \underline{x}^{(S)}(k+i|k) - l + M\gamma_4 \\
\gamma_j \in \{0,1\} &\text{ for all } j = 1\ldots 4,\ \sum_{j=1}^{4}\gamma_j \leq 3.
\end{aligned} \tag{25}$$

where $\mathbf{x}_{\text{ref}}^{(E)}$ denotes the reference trajectories. M is a suitable positive constant, and $\gamma_j \in \{0,1\}$ for $j = 1,...,4$ are binary. The purpose of the controller is to track the longitudinal speed and lateral position references which are: $\mathbf{x}_{\text{ref}}^{(E)} = \left[0\ v_{y_{\text{ref}}}\ x_{\text{ref}}\right]^T$,. in the equation (23), $Q, R, R_{\Delta u}$ are the positive semidefinite matrices that they are chosen to cancel $y^{(E)}(k+i|k)$ in the cost function (23).

B. TTC-induced formulation

Time-to-collistion, TTC, is defined as the time required for two vehicles to collide if they continue at their present speeds and on the same path [16].

$$\text{TTC} = \frac{y_1 - y_2 - L}{v_2 - v_1} \quad \text{for} \quad v_2 > v_1, \tag{26}$$

The TTC induced risk function is defined:

$$\text{R} := \hat{\text{R}}(\text{TTC}) = \frac{v_2 - v_1}{y_1 - y_2 - L} \quad \text{for} \quad v_2 > v_1, \tag{27}$$

where y_1, y_2 denote the longitudinal positions of the leading and following vehicles; v_1, v_2 denote the longitudinal speeds of the leading and following vehicles, respectively; and L is the length of the leading car. Following the same considerations as in sect.III-A, (27) can be generalized for the overtaking case at time $k+1$ as:

$$R = \begin{cases} \frac{v_y^{(S)}(k) - v_y^{(E)}(k)}{y^{(E)}(k+1) - y^{(S)}(k+1) - L}, if \text{ the car (E) procedes the car (S)} \\ \frac{v_y^{(E)}(k) - v_y^{(S)}(k)}{y^{(S)}(k+1) - y^{(E)}(k+1) - L}, if \text{ the car (S) procedes the car (E)} \end{cases} \tag{28}$$

In order to ensure collision avoidance, $\text{R}_{\max} = 1/\text{TTCmin}.$, where $\text{R}_{\max}$ is a maximum risk level.

This leads to following conditions

- if the ego car (E) precedes the surrounding car (S) at time $k+i+1$, then

$$\begin{aligned}
&y^{(E)}(k+i+1|k) - y^{(S)}(k+i+1|k) - L \\
&\quad \geq \text{TTCmin}\left(v_y^{(S)}(k+i|k) - v_y^{(E)}(k+i|k)\right),
\end{aligned} \tag{29}$$

- otherwise if the ego car (E) follows the surrounding car (S) at time $k+i+1$, then

$$\begin{aligned}
&y^{(S)}(k+i+1|k) - y^{(E)}(k+i+1|k) - L \\
&\quad \geq \text{TTCmin}\left(v_y^{(E)}(k+i|k) - v_y^{(S)}(k+i|k)\right).
\end{aligned} \tag{30}$$

The same considerations about the stochastic nature of the prediction apply here as for the TH case. For ease of presentation, we define the following set:

$$Y^{(S)}_{\text{TTCmin}}(k+i+1|k) = Y^{(S)}(k+i+1|k) \oplus \text{TTCmin}\left[\underline{v}_y^{(S)}(k+i|k),\, \overline{v}_y^{(S)}(k+i|k)\right], \tag{31}$$

whose parameters are defined in (18)–(19). Following the same argument used in Subsection III, one can also bound the probability as follows:

$$\Pr\Big(y^{(S)}(k+i|k) + \text{TTC}v_y^{(S)}(k+i-1|k) \in Y^{(S)}_{\text{TTC}}(k+i|k)\Big) \geq 1-\alpha_1. \tag{32}$$

Accordingly, the control formulation for autonomous overtaking by means of the time-to-collision induced risk function is in the form of (23)–(25), except the collision avoidance formulation (25) is replaced with

$$\begin{aligned}
-y^{(E)}(k+i|k) \leq\ & \text{TTCmin}v_y^{(E)}(k+i-1|k) + M\gamma_1 \\
& -L - \overline{y}^{(S)}(k+i|k) - \text{TTC}\overline{v}_y^{(S)}(k+i-1|k) \\
y^{(E)}(k+i|k) \leq & -\text{TTCmin}v_y^{(E)}(k+i-1|k) + M\gamma_2 \\
& -L + \underline{y}^{(S)}(k+i|k) + \text{TTC}\underline{v}_y^{(S)}(k+i-1|k) \\
-x^{(E)}(k+i|k) \leq & -\overline{x}^{(S)}(k+i|k) - l + M\gamma_3 \\
x^{(E)}(k+i|k) \leq\ & \underline{x}^{(S)}(k+i|k) - l + M\gamma_4 \\
\gamma_j \in \{0,1\}\ & \text{ for all } j = 1\ldots 4,\ \sum_{j=1}^{4}\gamma_j \leq 3.
\end{aligned} \tag{33}$$

IV. Results

The evaluation of the proposed methods has been done using a model of a BMW320d in the high fidelity simulator Carmaker. The parameters of the ego car and road, also, the design controller parameters can be found in the table I.

Two different scenarios were considered:

A. Scenario1

In this scenario, a traffic environment with the ego car (E) and one surrounding vehicle (V_1) ahead (see Figure 3). In this scenario, THmin=TTCmin=3s were used. Fig.4 shows the

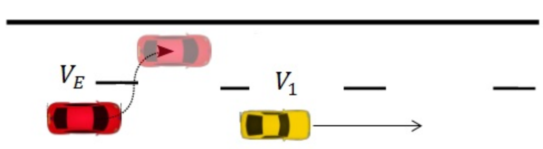

Fig. 3. Sketch of Scenario1

lateral position, longitudinal speed and acceleration for ego car with the induced risk functions during the autonomous overtaking. Both TH and TTC algorithms work but the overtaking with TH keeps a larger safety distance than TTC.

TABLE I

Parameters of Controllers

Parameter	Value
Lane width	$w_l = 3.75\,\text{m}$
Length of Leading car	$w_L = 4.3\,\text{m}$
Simulation time	$T_{\text{Sim}} = 70\,\text{s}$
Sampling time	$T_s = 0.1\,\text{s}$
Longitudinal speed constraints	$\underline{v}_y^{(E)} = 10\,[km.h^{-1}]$, $\overline{v}_y^{(E)} = 160\,[km.h^{-1}]$
Longitudinal acceleration constraints	$\underline{a}_y^{(E)} = -4\,[m^2.s^{-1}]$, $\overline{a}_y^{(E)} = 1\,[m^2.s^{-1}]$
Maximal slip angle	$\beta = 10\,[deg]$
The probability α	$\alpha_1 = 0.1,\ \alpha_2 = 0.2$
Desired lateral position	$x_{ref} = 0[m]$
Desired longitudinal speed	$v_{yref} = 110[km/h]$
Prediction horizon for fuel Consumption	$h_{\text{P}} = 4\,\text{s}$
Prediction horizon for overtaking	$N = 30\,\text{s}$
Considered gears	$g_k = [4,5,6,7]$
Semi Positive Matrices	$Q = 100 I_{2\times 2}$, $R = 0.01 I_{2\times 2}$, $R_{\Delta u} = 0.01 I_{2\times 2}$

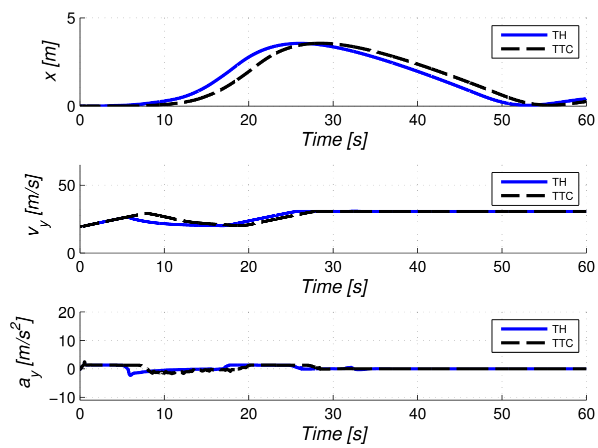

Fig. 4. The lateral position, the longitudinal velocity and acceleration with both the induced risk functions TH and TTC for Scenario1

B. Scenario2

In this traffic scenario two surrounding vehicles (V_1), (V_2) are in front of the ego car (E). The first vehicle (V_1) starts in the right lane and the second vehicle (V_2) starts in the left lane behind vehicle (V_1) (Figure5). THmin=TTCmin=3.5s were used.

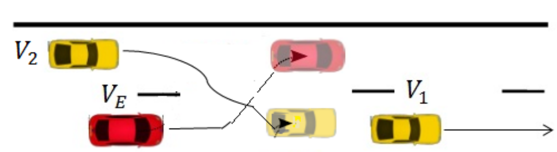

Fig. 5. Sketch of Scenario2

The lateral position, longitudinal speed and acceleration for ego car with both the induced risk functions are shown in the Figure 6. In addition, the lateral position of this scenario with three vehicles (ego and two surrounding) can be seen in the Figure 7.

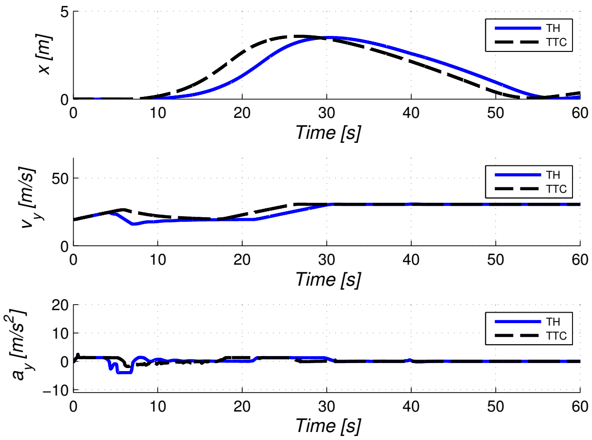

Fig. 6. The lateral position, the longitudinal velocity and acceleration with both the induced risk functions TH and TTC for Scenario2

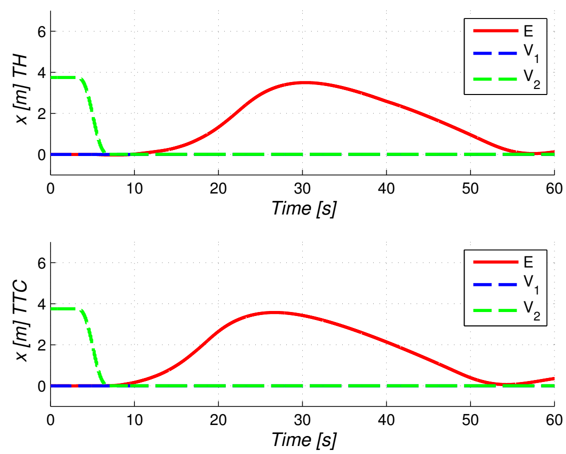

Fig. 7. The lateral position with both the induced risk functions TH and TTC with three vehicle in Scenario2

From the figures 6 and 7, it is demonstrated that both the TH and TTC work, however for the time to collision based algorithm the distance between the ego car (E) and vehicle (V_2) is very small before the overtaking process starts, *i.e.* the ego vehicle comes nearer to the preceding vehicle before starting the overtaking maneuver.

The fuel consumption values for both scenarios during the overtaking are shown in the table II.

V. Conclusions

The question behind this work was whether it was possible to combine energy optimization and risk based speed control even for more complex operations than standard leader-follower setups. As the simulations and the values in table II show, it is both possible and sensible. Interestingly, the TTC-based approach, which is more consistent with the manual driving, leads to better results than the TH-based approach.

TABLE II
Comparison of Fuel Consumption

Controller	Scenario	Δq_f [gr] (TH)	Δq_f [gr] (TTC)
MPC with Fuel cost Function	1	75.02	70.02
	2	75.25	71.24
MPC without Fuel cost Function	1	79.88	71.58
	2	79.85	74.39

Of course, overtaking does not happen continuously, so the total consumption of a trip will not be reduced by the same extent as for overtaking alone, but this is a clear hint that explicit consideration of fuel efficiency should not be pursued only for simple manouevers but also for general ones.

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