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Optimal Load Sharing for Serial Compressors via Modifier Adaptation

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Abstract— **This paper investigates the optimal load-sharing problem of serial gas compressors in the presence of plant-model mismatch. The problem is formulated as a static real-time optimization task that is solved via modifier adaptation for interconnected systems. The proposed approach guarantees optimal operation of the plant upon convergence. Furthermore, it is shown how the specific problem structure can be exploited during process operation for the efficient estimation of plant gradients with respect to local inputs. A simulated case study demonstrates the effectiveness of the proposed real-time optimization approach.**

Index Terms— **real-time optimization, modifier adaptation, gas compressors, optimal load sharing**

I. INTRODUCTION

Centrifugal gas compressors are energy-intensive units in many industrial processes [11]. They provide air for combustion, recirculate fluids through processes, and transport gas through pipes and pipelines. Hence, compressor operation with minimal energy consumption is of considerable interest [2, 13]. The present paper investigates the problem of optimal load sharing of gas compressors in the presence of plant-model mismatch. More specifically, we consider the problem of distributing a given load requirement among the available machines in a serial compressor station, when the compressor efficiency characteristics are not precisely known.

Current industrial practice uses the standard two-step procedure [2, 5, 13], namely, the identification of compressor maps via static nonlinear regression followed by the optimization of the load-sharing problem. However, it is well known that the two-step approach does not guarantee reaching optimal plant operation [3, 8]. Consequently, other RTO approaches, such as Modifier Adaptation (MA) [4, 7, 12], have been designed to enforce plant optimality upon convergence.

The most challenging part of MA lies in the fact that the gradients of the plant outputs with respect to the inputs must be available. Recently, a novel real-time optimization (RTO) formulation for interconnected systems that only requires *local* gradient information has been proposed [9]. This formulation is very useful for dealing with large-scale optimization problems for two main reasons: (i) the local input and output measurements can be used for local subsystem gradient correction, and (ii) the proposed problem can be solved in a distributed manner such that the subsystems need not share their models. Also, upon convergence, the scheme reaches optimal steady-state performance for the overall plant.

This paper discusses the implementation of a first-order MA scheme for interconnected systems that uses local input and output measurements of each subsystem [9]. The solution of the optimization scheme for interconnected systems is handled in centralized manner while we tackle the gradient estimation of subsystems' cost and constraints with respect to local inputs exploiting the interconnection structure. We address the key issue, namely, the gradient estimation with respect to the interconnection variables for the serial-compressor plant. We show how the structure of the serial-compressor problem can be exploited for the purpose of efficient gradient estimation. Furthermore, the complexity of this estimation is independent of the number of compressors. The real-time optimization algorithm works in synergy with a low-level controller that tracks the required discharge pressure.

The remainder of the paper is organized as follows. Section II revisits the MA scheme for interconnected systems, while Section III addresses the load-sharing optimization problem for a serial-compressor plant. A case study is presented in Section IV, and conclusions in Section V.

II. REAL-TIME OPTIMIZATION OF UNCERTAIN INTERCONNECTED SYSTEMS

An RTO problem for a system consisting of interconnected subsystems is typically expressed as the following NLP:

$$(u^\star, v_p^\star) = \underset{u, v_p}{\operatorname{argmin}} \sum_{i \in \mathcal{N}} \Phi_{p,i}(u_i, v_{p,i}) \tag{1a}$$

$$\text{s.t. } \forall i \in \mathcal{N} \quad y_{p,i} = F_{p,i}(u_i, v_{p,i}) \tag{1b}$$

$$v_p = H y_p \tag{1c}$$

$$G_{p,i}(u_i, v_{p,i}) \leq 0 \tag{1d}$$

$$u_i \in \mathcal{U}_i, \; v_{p,i} \in \mathcal{V}_i, \tag{1e}$$

where $u \in \mathbb{R}^{n_u}$, $v_p \in \mathbb{R}^{n_v}$ and $y_p \in \mathbb{R}^{n_y}$ are the feasible set of steady-state plant inputs, interconnection variables and outputs, respectively. The interconnection variables v_p connect the different subsystems, with H being *the interconnection matrix*. We consider that the plant consists of N interconnected subsystems, $i \in \mathcal{N} = \{1, \dots, N\}$. Subsystem i has the inputs $u_i \in \mathcal{U}_i$, with $\mathcal{U}_i = \{u_i \in \mathbb{R}^{n_{u_i}} : u_i^L \leq$

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$u_i \leq u_i^U\}$, the interconnection variables $v_{p,i} \in \mathcal{V}_i$, with $\mathcal{V}_i = \{v_{p,i} \in \mathbb{R}^{n_{v_i}} : v_i^L \leq v_{p,i} \leq v_i^U\}$, the outputs $y_{p,i} \in \mathbb{R}^{n_{y_i}}$, the cost $\Phi_{p,i} : \mathbb{R}^{n_{u_i}} \times \mathbb{R}^{n_{v_i}} \mapsto \mathbb{R}$ and the constraints $G_{p,i} : \mathbb{R}^{n_{u_i}} \times \mathbb{R}^{n_{v_i}} \mapsto \mathbb{R}^{n_{G_i}}$. In practice, the plant functions $\Phi_{p,i}$, $F_{p,i}$, $G_{p,i}$ are typically not known accurately, as only models of these functions are available.

An MA scheme for the real-time optimization of interconnected systems has been proposed recently in [9]. This scheme, which uses a coordinator and measurements of the interconnection variables, guarantees plant optimality upon convergence, provided perfect knowledge of plant gradients is available. The overall optimization problem at the k^{th} RTO iteration reads:

$$\left(u_{k+1}^\star, v_{k+1}^\star\right) = \underset{u,v}{\operatorname{argmin}} \sum_{i \in \mathcal{N}} \left(\Phi_i(z_i) + \left(\lambda_k^{\Phi_i}\right)^{\mathrm{T}} (z_i - z_{p,i,k})\right) \tag{2a}$$

s.t. $\forall i \in \mathcal{N}$:

$$y_{m,i} = F_i(z_i) + \varepsilon_k^{F_i} + \left(\lambda_k^{F_i}\right)^{\mathrm{T}} (z_i - z_{p,i,k}) \tag{2b}$$

$$v = H y_m \tag{2c}$$

$$G_i(z_i, v_i) + \varepsilon_k^{G_i} + \left(\lambda_k^{G_i}\right)^{\mathrm{T}} (z_i - z_{p,i,k}) \leq 0 \tag{2d}$$

$$u_i \in \mathcal{U}_i, \;\; v_i \in \mathcal{V}_i, \tag{2e}$$

where $z_i^{\mathrm{T}} := [u_i^{\mathrm{T}}, v_i^{\mathrm{T}}]$ and $z_{p,i}^{\mathrm{T}} := [u_i^{\mathrm{T}}, v_{p,i}^{\mathrm{T}}]$, for all $i \in \mathcal{N}$. Furthermore, $v_i \in \mathbb{R}^{n_{v_i}}$ are the modeled interconnection variables. Similarly, the steady-state outputs F_i, the constraints G_i, and the cost functions Φ_i are the model counterparts of the plant functions defined in (1), while the modifiers are defined as:

$$\lambda_k^{\Phi_i} = \nabla_z \Phi_{p,i}\left(z_{p,i,k}\right) - \nabla_z \Phi\left(z_{p,i,k}\right) \tag{3a}$$

$$\varepsilon_k^{G_i} = G_{p,i}\left(z_{p,i,k}\right) - G_i\left(z_{p,i,k}\right) \tag{3b}$$

$$\lambda_k^{G_i} = \nabla_z G_{p,i}\left(z_{p,i,k}\right) - \nabla_z G_i\left(z_{p,i,k}\right) \tag{3c}$$

$$\varepsilon_k^{F_i} = F_{p,i}\left(z_{p,i,k}\right) - F_i\left(z_{p,i,k}\right) \tag{3d}$$

$$\lambda_k^{F_i} = \nabla_z F_{p,i}\left(z_{p,i,k}\right) - \nabla_z F_i\left(z_{p,i,k}\right). \tag{3e}$$

We remark that one can either solve (2) in a centralized or distributed fashion. Here we focus on the former, while exploiting the interconnection structure for gradient estimation. We refer to [9] for distributed solution approaches.

III. Optimal load sharing of serial compressors

We start this section by introducing the model of a single compressor. Then, the load-sharing optimization problem is formulated using the concept of MA for interconnected systems. Subsection III-D reviews material for the closed-loop implementation of MA, while Subsection III-E presents a way to efficiently estimate the plant gradients with respect to local inputs.

A. Compressor Model

A static compressor model, based on the work of [1], was presented in a previous study [10]. As depicted in Fig. 1, the main element is the centrifugal compressor that is surrounded by piping and valves. The main line consists of a suction side valve ($'in'$), a recycle valve line ($'rec'$) that connects the compressor outlet with the compressor inlet, and the discharge side valve ($'out'$).

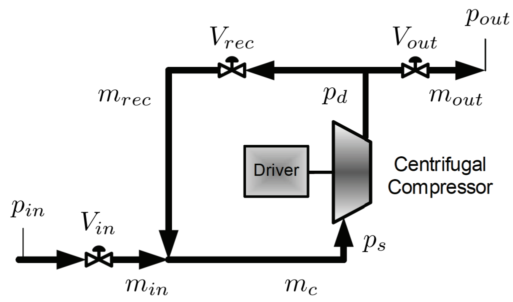

Fig. 1. Diagram of a single compressor.

The dynamic compressor model can be written as:

$$\frac{dp_s}{dt} = C_1(m_{in} + m_{rec} - m_c) \tag{4a}$$

$$\frac{dp_d}{dt} = C_2(m_c - m_{rec} - m_{out}) \tag{4b}$$

$$\frac{dm_c}{dt} = C_3(p_s \Pi - p_d) \tag{4c}$$

$$\frac{d\omega}{dt} = C_4(\tau - \tau_{comp}) \tag{4d}$$

$$\frac{dm_{rec}}{dt} = C_5(m_{rec,ss} - m_{rec}), \tag{4e}$$

where m_c, m_{rec}, m_{in} and m_{out} denote the compressor, recycle, inlet, and outlet flows, respectively; p_s is the suction pressure, p_d the discharge pressure, Π the pressure ratio, ω the rotational speed of the shaft, τ the applied torque, while $C_i, i = 1, ..., 5$, are constant parameters. Furthermore, one can write:

$$m_{in} = k_{in} V_{in} \sqrt{|p_{in} - p_s|} \tag{5a}$$

$$m_{out} = k_{out} V_{out} \sqrt{|p_d - p_{out}|} \tag{5b}$$

$$m_{rec,ss} = k_{rec} V_{rec} \sqrt{|p_d - p_s|} \tag{5c}$$

$$\tau_{comp} = \sigma r \omega m_c, \tag{5d}$$

where k_{in}, k_{out}, and k_{rec} are the inlet, outlet, and recycle valve gains; p_{in} and p_{out} are the inlet and outlet pressures.

The pressure ratio Π is modeled as a polynomial function of ω and m_c. Likewise, the polytropic efficiency η_p is modeled as a polynomial function of ω and Π [2]:

$$\Pi = \alpha_1 + \alpha_2 \omega + \alpha_3 m_c + \alpha_4 \omega m_c + \alpha_5 \omega^2 + \alpha_6 m_c^2 \tag{6a}$$

$$\eta_p = \beta_1 + \beta_2 \omega + \beta_3 \Pi + \beta_4 \omega \Pi + \beta_5 \omega^2 + \beta_6 \Pi^2. \tag{6b}$$

These compressor maps are typically provided by the manufacturer or they can be identified based on historical data [2]. The shaft power is calculated as

$$\Phi = \frac{y_p}{\eta_p} m_c, \tag{7a}$$

where y_p is the polytropic head:

$$y_p = \frac{Z_{in} R T_{in}}{M_W} \frac{n_\nu}{n_\nu - 1} \left[\Pi^{\frac{n_\nu - 1}{n_\nu}} - 1\right]. \tag{7b}$$

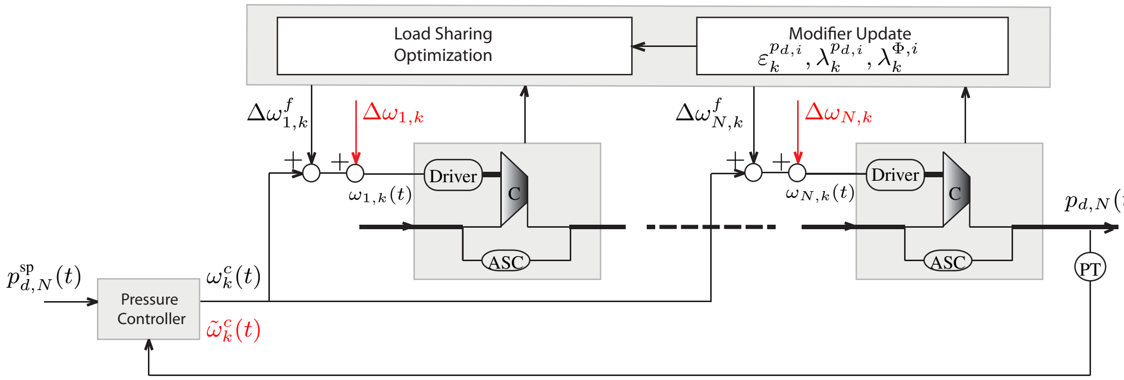

Fig. 2. Diagram of a serial-compressor configuration with pressure control. PT represents a pressure transmitter. ASC is an anti-surge controller. The control is executed in continuous time t, while the optimization part is executed only once the system has reached steady state. $\Delta\omega_{i,k}^f$ is the contribution of the optimization layer to the rotational speed $\omega_{i,k}$, while $\Delta\omega_{i,k}$ is the perturbation that is added to compute the experimental gradients. ω_k^c is the speed reached at the k^{th} RTO iteration without the perturbation $\Delta\omega_{i,k}$, while $\tilde{\omega}_k^c$ is the speed reached at steady state with the perturbations $\Delta\omega_{i,k}$ added. This study assumes $V_{rec,i} = 0$.

Here, Z_{in} is the inlet compressibility factor, T_{in} the suction temperature, M_W the molecular weight of the gas mixture, which is assumed to be constant, and n_ν the polytropic exponent.

B. Load-Sharing Optimization Problem

Consider a station with N compressors operating in a serial configuration as illustrated in Figure 2. The inputs are the setpoints for the rotational speed ω_i and the recycle valve opening $V_{rec,i}$, with $i \in \mathcal{N}$. Their optimal values can be obtained by minimizing the sum of the power consumption of all compressors. Here, we consider that the power consumption is minimized by keeping the inlet $V_{in,i}$ and outlet $V_{out,i}$ valves at constant values.

In the serial compressor system, the compressors are arranged such that the discharge tank of the i^{th} compressor feeds into the suction tank of the $(i+1)^{st}$ compressor. In fact, the discharge line and suction line of two neighboring compressors share the same valve, and their interconnection constraint can be expressed as $p_{d,i} = p_{in,i+1}$ with $m_{out,i} = m_{in,i+1}$. It is desired to enforce that the discharge pressure of the N^{th} compressor be equal to the pressure setpoint $p_{d,N}^{\text{sp}}$. The pressure upstream of the first compressor is considered to be at atmospheric pressure, that is, $p_{in,1} = p_{atm}$.

Mathematically, a suitable RTO problem can be stated as follows:

$$\min_{\{\omega_i, V_{rec,i}, p_{in,i}\}} \quad \sum_{i\in\mathcal{N}} \Phi_i \tag{8a}$$

$$\text{s.t.} \quad \text{steady-state equations}$$

$$p_{d,N} = p_{d,N}^{\text{sp}} \tag{8b}$$

$$p_{d,i} = p_{in,i+1} \quad i \in \{1, \dots, N-1\} \tag{8c}$$

$$m_{out,i} = m_{in,i+1} \quad i \in \{1, \dots, N-1\}\,, \tag{8d}$$

$$s_{0,i} - s_{1,i} m_{c,i} + \Pi_i \le 0, \quad i \in \mathcal{N} \tag{8e}$$

$$c_{0,i} + c_{1,i} m_{c,i} - \Pi_i \le 0, \quad i \in \mathcal{N} \tag{8f}$$

$$\omega_i \in \mathcal{W}_i,\ p_{in,i} \in \mathcal{P}_{in,i},,\ i \in \mathcal{N} \tag{8g}$$

$$m_{c,i} \in \mathcal{M}_i,\ V_{rec,i} \in \mathcal{V}_{rec,i},\ i \in \mathcal{N} \tag{8h}$$

where $s_{0,i}$ and $s_{1,i}$ are positive constants that define the surge constraints (8e). The violation of this constraint is prevented through an anti-surge controller (ASC). Furthermore, $c_{0,i}$ and $c_{1,i}$ define the choke constraints (8f). Box constraints on the operating speeds and pressures are included in (8g), and also for flows and valve openings in (8h).

C. MA Using Measurements of Interconnection Variables

For simplicity, we restrict our analysis to the case when all compressors operate far from the surge line, that is, without a need for anti-surge control. In other words, we assume $V_{rec,i} = 0$. Hence, all mass flows $m_{c,i}, \forall i \in \mathcal{N}$, are equal to the outlet flow

$$m_{out,N} = k_{out,N} V_{out,N} \sqrt{|p_{d,N} - p_{out,N}|}. \tag{9}$$

Consequently, if $p_{d,N}$, $p_{out,N}$ and $V_{out,N}$ do not change, the mass flows of all compressors will remain constant at the plant steady state. It follows from constant mass flow that the operating conditions of the i^{th} compressor are defined entirely by the local inputs

$$z_i = [\omega_i \quad p_{in,i}]^{\mathrm{T}}. \tag{10}$$

The interconnection variable $v_i = p_{in,i}$ is given by the interconnection constraint (8c), while the output of the i^{th} compressor is $y_i = p_{d,i}$. Hence, Problem (8) can be solved via the MA formulation for interconnected systems:

$$\left(\omega_{k+1}^\star, p_{in,k+1}^\star\right) = \operatorname*{argmin}_{\omega, p_{in}} \sum_{i\in\mathcal{N}} \left(\Phi_i(z_i) + \left(\lambda_k^{\Phi_i}\right)^{\mathrm{T}} (z_i - z_{p,i,k})\right) \tag{11a}$$

$$\text{s.t.} \quad \text{steady-state model equations}$$

$$p_{d,m,i} := p_{d,i} + \varepsilon_k^{p_{d,i}} + \left(\lambda_k^{p_{d,i}}\right)^{\mathrm{T}} (z_{p,i} - z_{p,i,k})\,, i \in \mathcal{N}$$

$$p_{in,i+1} = p_{d,m,i} \quad i \in \{1, \dots, N-1\} \tag{11b}$$

$$p_{d,m,N} = p_{d,N}^{\text{sp}} \tag{11c}$$

$$\text{inequality constraints (8e)-(8g)}$$

The modifiers are computed as:

$$\varepsilon_k^{p_{d,i}} = p_{d,p,i,k} - p_{d,i,k} \tag{11d}$$

$$\lambda_k^{p_{d,i}} = \nabla_{z_i} p_{d,p,i}(z_{p,i,k}) - \nabla_{z_i} p_{d,i}(z_{p,i,k}) \tag{11e}$$

$$\lambda_k^{\Phi_i} = \nabla_{z_i} \Phi_{p,i}(z_{p,i,k}) - \nabla_{z_i} \Phi_i(z_{p,i,k}). \tag{11f}$$

Here, $\nabla_{z_i} \Phi_{p,i}(z_{p,i,k})$ and $\nabla_{z_i} p_{d,p,i}(z_{p,i,k})$ are the estimated values of the derivatives of the plant cost $\Phi_{p,i}$ and plant discharge pressure $p_{d,p,i}$ with respect to z_i, respectively, evaluated at $z_{p,i,k}$. Note that (i) the gradient modifiers $\lambda_k^{\Phi_i}$ and $\lambda_k^{p_{d,i}}$ are vectors, and (ii) a single perturbation of the inputs ω_i is required to estimate the plant derivatives $\nabla_{z_i} \Phi_{p,i}(z_{p,i,k})$ and $\nabla_{z_i} p_{d,p,i}(z_{p,i,k})$ for the i^{th} compressor. Furthermore, it is possible to perturb all the speeds simultaneously in such a way that the disturbance of the outlet pressure setpoint $p_{d,N}^{\text{sp}}$ is negligible, as will be explained in the next subsection.

D. MA for Controlled Compressors in Series

As shown in Figure 2, the discharge pressure $p_{d,N}$ is regulated around the varying station demands $p_{d,N}^{\text{sp}}$. In conventional load sharing, each compressor receives the same speed setpoint $\omega_{p,k}^{\text{c}}(t)$, thereby achieving an "equal-load" distribution among the interconnected compressors. Load-sharing optimization comes as an additional layer that generates asymmetries in the load distribution based on the current efficiencies of the various units.

Consider the steady state reached at the k^{th} RTO iteration, with $\omega_{p,k}^{\text{c}}$ the speed generated by the pressure controller and $\omega_{p,i,k}$ the speed setpoint applied to the i^{th} compressor. The RTO algorithm suggests applying next $\omega_{i,k+1}^{\text{f}}$, which is the filtered value of the optimal speed computed from Problem (11), that is,

$$\omega_{i,k+1}^{\text{f}} = \mathsf{K}\omega_{i,k+1}^{\star} + (\mathbf{I} - \mathsf{K})\omega_{p,i,k}, \tag{12}$$

which means that the contribution of the optimization to the next RTO iteration is:

$$\Delta\omega_{i,k+1}^{\text{f}} := \omega_{i,k+1}^{\text{f}} - \omega_{p,k}^{\text{c}}. \tag{13}$$

This closed-loop implementation maintains the attractive MA property of converging to a Karush-Kuhn-Tucker (KKT) point for the plant. Before stating this as a Proposition, we will discuss the required assumptions.

Assumption 1 (Differentiable Problem Data): The functions $\Phi_{p,i}(z_i)$, $p_{d,p,i}(z_i)$, $\Phi_i(z_i)$ and $p_{d,i}(z_i)$, $\forall i \in \mathcal{N}$, are continuously differentiable for the plant and the model. Also, $p_{d,i}(z_i)$ is measured without noise, and the derivatives $\nabla_{z_i} \Phi_{p,i}$ and $\nabla_{z_i} p_{d,p,i}$ are perfectly estimated. □

Proposition 1 (Optimality upon Convergence): Consider the serial-compressor station under pressure control with the MA scheme (11)–(13), and let Assumption 1 hold. Then, if the MA scheme converges, the converged point $\omega_\infty = \lim_{k\to\infty} \omega_k$ is a KKT point for the plant.

Proof: It has been shown in [10] for the case of parallel compressors that the plant input adaptation scheme (12)–(13) guarantees $\omega_{p,\infty} = \omega_\infty^\star$ upon convergence. The same reasoning holds for the case of serial compressors considered here. Moreover, since $\omega_\infty^\star$ corresponds to a KKT point of Problem (11), MA ensures that $\omega_\infty^\star$ corresponds to a KKT point of the plant (see Theorem 1 in [7]). ■

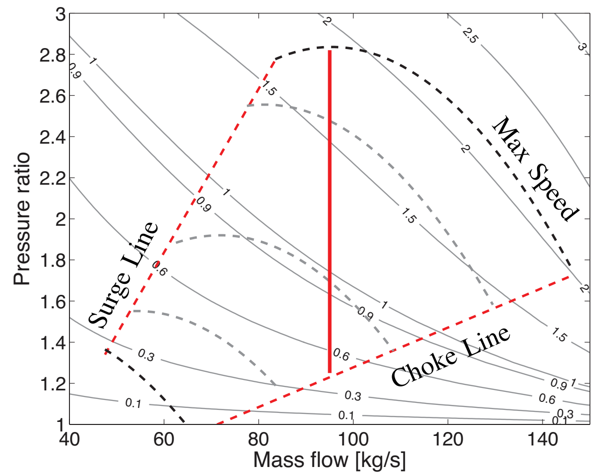

Fig. 3. Compressor operating constraints and cost contour curves (gray line). Upper and lower bound on speed (dashed black), operating points for three constant speeds (dashed gray), choke and surge line (dashed red) and operating points for a constant mass flow (red line)

E. Gradient Estimation Exploiting the Problem Structure

The most challenging part of MA consists in estimating the plant gradients defined in (11e)-(11f). In general, steady-state perturbation methods require at least $(n_u + 1)$ steady-state operating points to be available, which means that one has to wait for steady state each time the inputs are changed. It turns out that MA for interconnected systems can help since the input dimension of each compressor is $n_{z_i} = 2$. The price for being able to reduce the dimensionality of the input space via MA for interconnected systems is that the plant gradients with respect to the interconnection variables be available.

We will next see that this is not an obstacle in the case of the load-sharing problem (11) since the plant structure can be exploited for the efficient estimation of the plant derivatives $\nabla_{z_i} \Phi_{p,i}$ and $\nabla_{z_i} p_{d,p,i}$. In fact, it is possible to perturb the speeds of all compressors at the same time to estimate the required derivatives. This has the advantage that the complete gradients can be estimated using only two steady-state operating points, regardless of the number of compressors. These steady states correspond to the current RTO operating point $\omega_{p,k}$ and the perturbed operating point $\tilde{\omega}_{p,k}$. The perturbed point is obtained by adding the perturbations $\Delta\omega_{i,k}$ to each compressor (see Figure 2):

$$\tilde{\omega}_{p,i,k} = \tilde{\omega}_{p,k}^{\text{c}} + \Delta\omega_{i,k}^{\text{f}} + \Delta\omega_{i,k}, \tag{14a}$$

$$\omega_{p,i,k} = \omega_{p,k}^{\text{c}} + \Delta\omega_{i,k}^{\text{f}}, \quad \forall i \in \mathcal{N}, \tag{14b}$$

where $\tilde{\omega}_{p,k}^{\text{c}}$ is the controller speed setpoint reached at steady state for the perturbed operating point.

Next, we need to estimate the cost gradient $\Phi_{p,i}$ in the directions of ω_i and $p_{in,i}$. Figure 3 indicates that the compressor can only operate along vertical line, that is, in the subspace of ω_i and $p_{in,i}$ that corresponds to constant mass flow. Hence, we can estimate the plant gradients only along the direction denoted $U_{i,1}^r = \tilde{z}_{p,i,k} - z_{p,i,k}$, where $\tilde{z}_{p,i,k} = [\tilde{\omega}_{p,i,k} \quad \tilde{p}_{in,p,i,k}]^{\mathrm{T}}$ and $\tilde{p}_{in,p,i,k}$ is the measured inlet pressure at $\tilde{\omega}_{p,i,k}$. For this, we will use the fact that the cost does not change for constant mass flow and constant speed, that is, in the direction $U_{i,2}^r = [0 \quad 1]^{\mathrm{T}}$. As a result, we obtain the plant cost gradient estimate as

$$\nabla_{z_i} \Phi_{p,i}^{\mathrm{T}} = \begin{bmatrix} \Phi_{p,i}(\tilde{z}_{p,i,k}) - \Phi_{p,i}(z_{p,i,k}) \\ 0 \end{bmatrix}^{\mathrm{T}} U_{i,1}^{-1}, \qquad (15)$$

where $U_{i,1} = [U_{i,1}^r \quad U_{i,2}^r]$.

For the estimation of $\nabla_{z_i} p_{d,p,i}$ in (11e), we consider the discharge pressure in (4c) at steady state, $p_{d,p,i} = \Pi_{p,i} p_{s,p,i}$, and write

$$\nabla_{z_i} p_{d,p,i} = \nabla_{z_i} \Pi_{p,i} p_{s,p,i} + \Pi_{p,i} \nabla_{z_i} p_{s,p,i}, \qquad (16a)$$

where, similarly to the cost in (15), we estimate $\nabla_{z_i} \Pi_{p,i}$ as

$$\nabla_{z_i} \Pi_{p,i}^{\mathrm{T}} = \begin{bmatrix} \Pi_{p,i}(\tilde{z}_{p,i,k}) - \Pi_{p,i}(z_{p,i,k}) \\ 0 \end{bmatrix}^{\mathrm{T}} U_{i,1}^{-1}. \qquad (16b)$$

For the estimate $\nabla_{z_i} p_{s,p,i}$, we use the fact that the suction pressure does not change for the constant mass flow and constant inlet pressure according to (5a), which corresponds to the direction $U_{i,3}^r = [1 \quad 0]^{\mathrm{T}}$. The gradient estimate of the suction pressure is obtained as

$$\nabla_{z_i} p_{s,p,i}^{\mathrm{T}} = \begin{bmatrix} p_{s,p,i}(\tilde{z}_{p,i,k}) - p_{s,p,i}(z_{p,i,k}) \\ 0 \end{bmatrix}^{\mathrm{T}} U_{i,2}^{-1}, \qquad (16c)$$

where $U_{i,2} = [U_{i,1}^r \quad U_{i,3}^r]$.

The perturbations $\Delta\omega_{i,k}$ should be selected such that (i) the perturbation of $p_{d,N}$ is negligible, and (ii) the gradients are estimated with sufficient accuracy. With respect to (i), the prediction of the perturbed point $\tilde{\omega}_{i,k}$ and the perturbation level $h_{i,k}$ is obtained by solving Problem (11) subject to the additional constraints:

$$\tilde{\omega}_{i,k} = \omega_{p,i,k} + (-1)^i \, h_{i,k} \qquad (17a)$$

$$h_{i,k} \geq h_i^{\mathrm{L}}, \quad \forall i \in \mathcal{N}. \qquad (17b)$$

The solution defines a set of compressors being perturbed in one direction and the complementary set in the opposite direction such that the effect on $p_{d,N}$ is negligible. Hence, the perturbations $\Delta\omega_{i,k} = (-1)^i \, h_{i,k}$ are used in (14a). With respect to (ii), the error in the derivative estimates due to measurement noise will be large if the step $|\tilde{\omega}_{p,i,k} - \omega_{p,i,k}|$ is too small [6], while, if it is too large, the error due to the finite-difference approximation of the gradient will be too large. According to [6], h_i^{L} is obtained such that the gradient error is minimized. Namely, we take $h_i^{\mathrm{L}} = \sqrt{\frac{\delta_i}{d_i}}$, where δ_i is the noise level and d_i is the eigenvalue of the Hessian of $\Phi_i(\cdot)$ with largest absolute value, respectively.

IV. SIMULATION RESULTS

In this section, the MA scheme described in Section III-C is tested in simulation. We consider the scenario of a gas compressor station consisting of three compressors in series. Based on experimental data from real compressors, the measurement noises for the power consumption and the pressures are selected as Gaussian noises with standard deviations of 0.33%. Once the controlled plant satisfies near steady-state conditions, cost and constraint measurements are taken and averaged over a moving time window of 10 sec. The perturbations $\Delta\omega_{i,k}$ used to estimate the gradients are computed according to the procedure discussed in III-E. Instead of exciting at every RTO iteration, the perturbations are stopped and the gradient modifiers no longer updated once the cost improvement becomes negligible. For instance, at the k^{th} RTO iteration, excitation is not carried out if, for a given $\varepsilon > 0$, the following condition is met:

$$\frac{\left| \sum_{i=1}^{3} \Phi_{p,i}(z_{p,i,k}) - \sum_{i=1}^{3} \Phi_{p,i}(z_{p,i,k-1}) \right|}{\sum_{i=1}^{3} \Phi_{p,i}(z_{p,i,k-1})} \leq \varepsilon. \qquad (18)$$

Gradient estimation is restarted every time the pressure setpoint is changed. In order to obtain smooth transitions between steady states, the changes in both the pressure setpoint and the feedforward contributions to the speeds are implemented using ramps. The filter (12) is applied with $\mathrm{K} = 0.95\mathbf{I}$.

Plant-model mismatch is introduced by using compressor maps (6) that are different for the plant and the model. The plant efficiency maps A, B and C shown in Figure 4 are used in (7a) to compute the plant cost functions $\Phi_{p,i}$. The model cost functions Φ_i are obtained using the model efficiency map shown in Figure 4. The model assumes the same map for all compressors. It follows that the model optimum corresponds to equal-load distribution.

Using the proposed MA scheme, the time profiles of the speeds applied to the three compressors are shown in Figure 5 for both equal-load distribution and optimal plant operation. The plant discharge pressure is compared to its setpoint for three different setpoint values in Figure 6. Note

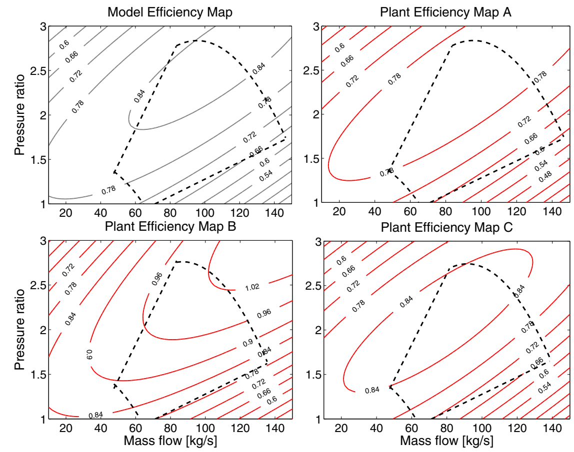

Fig. 4. Comparison between the model efficiency map (grey) and the plant efficiency maps (red) for two different compressors. The maps are shown as contour plots, whereas the dashed black lines represent the operating range of the gas compressor.

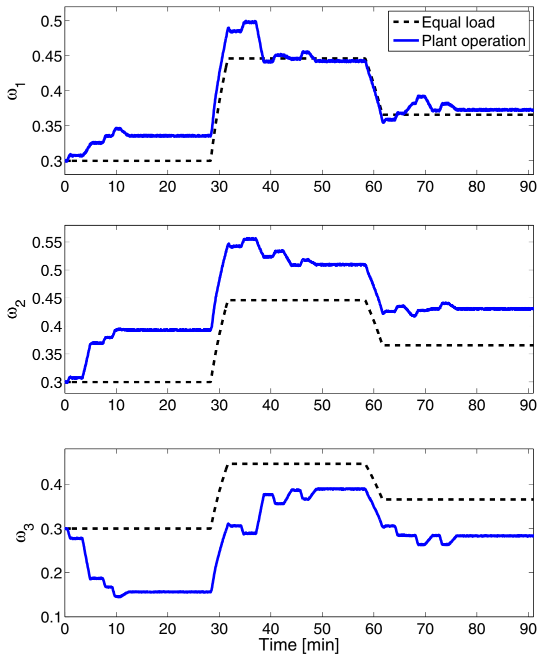

Fig. 5. Convergence of the normalized speed to the plant optimum.

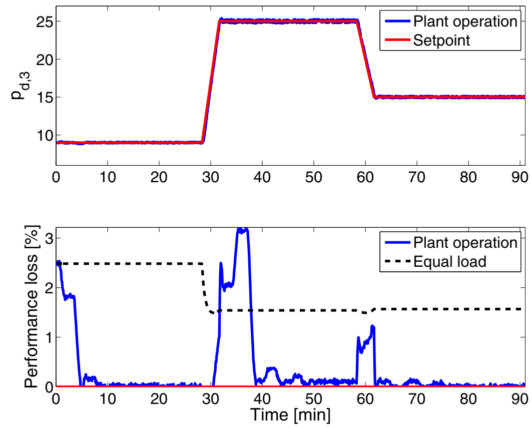

Fig. 6. Normalized discharge pressure of the station (top). Power loss using MA compared to equal-load distribution (bottom).

that the discharge pressure satisfies the desired setpoint even when the speeds are being perturbed for the purpose of gradient estimation. Using $\varepsilon = 10^{-3}$ in (18), the perturbations are stopped after only 2-3 RTO iterations. As can be seen in Figure 6, the improvement in power consumption is up to 2.5%, which is economically significant. Similarly, the instantaneous optimality loss after each pressure setpoint change is nearly suppressed after the first RTO iteration.

V. CONCLUSIONS

This paper has investigated the use of MA for load-sharing optimization in gas compression stations consisting of several compressors in series. The results show that *optimal operation* of the plant can be obtained after a few RTO iterations without having to update the model parameters or the compressor maps. We show that, by *using only local subsystem derivatives*, the algorithm is capable of quickly converging to the plant optimum. In fact, each subsystem relies on the estimation of the local power consumption and pressure derivatives with respect to its own inputs. An interesting feature that makes MA particularly well suited for this problem is that it is possible to excite all compressor speeds simultaneously without significantly perturbing the pressure setpoint. The simulation case study shows that the potential saving is up to 2.5% for the considered operating conditions.

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