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3D autonomous underwater navigation using seabed acoustic sensing

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KEYWORDS

AUV; acoustic sensing; navigation.

ABSTRACT

Autonomous Underwater Vehicle (AUV) being a powerful tool for exploring and investigating ocean resources can be used in a large variety of oceanographic, industry and defense applications. Underwater navigation for AUV is still a challenging task and is one of the fundamental elements in modern robotics because the ability of AUV to correctly understand its position and attitude within the underwater environment is determinant for the success in the different applications. AUV navigation usually is based only on information obtained from Doppler Velocity Loggers, Inertial Navigation Systems, etc. due to the absence of an external reference sources. But this type of navigation is subjected to a continuously growing error due to the absence of the absolute position measurement (for example, received from GPS or GLONASS) which is typical for the majority of UAV applications. These measurements might be provided by observation of so-called feature points like in Unmanned Aerial Vehicles (UAV) case, but the big difference between acoustical and optical images makes it a rather difficult problem which solution needs detailed preliminary mapping of the operational seabed area. The new generation of acoustic imaging gives rise to the new approaches to AUV navigation based on the absolute velocity measurements. By analogy with the optical flow approach coming from the area of UAV the evolution of the seabed map produces the information related to the absolute motion of the AUV. The principal advantage of the proposed algorithm is that the fusion of the acoustic mapping and the Inertial Navigation System (INS) gives the absolute velocity of the vehicle with respect to the seabed. In some sense the suggested algorithm operates as multibeam Doppler Velocity Log (DVL), though in different way and in different environment. Even in the theory the DVL operates perfectly over the flat surface, but the suggested algorithm needs the presence of the relief and uses the evolution of the relief range obtained by sonar and measured from the ship to the seabed as a source of own speed.

I. INTRODUCTION

Modern sonars make it possible to obtain acoustic images of the seabed and the surrounding space, which basically is an additional means for solving navigational problems [1], [2]. Nowadays, it has been known to use sonar as a means of selflocalization the underwater vehicle in analogy with UAV, by comparing the recorded bottom relief with a preloaded map [2], as well as tracking means for mobile objects and inspecting underwater communication lines [5]. Sonars are undoubtedly useful for collisions with an obstacles avoiding and for detecting of chracteristic objects located on the seabed [3], [5], [6]. But its use as a measuring tool, which is necessary for navigation, is very problematic [7]. However, with a sufficient frame rate, which provides for example side-looking sonar, the sonar allows to obtain an analog of the video sequence used for UAV (unmanned aerial vehicle) navigation. The difficulty is that the distribution of the relief is uaually unknown, therefore at the first stage it is necessary to determine the characteristics of the relief, its slope. After that it is possible to determine the AUV shift using the evolution of the measured profile from frame to frame. Here, by analogy with the theory linking the movement of the UAV and the speed of the image shift in the focal plane, that is so-called Optical Flow, it is necessary to find the relations that allow to extract the speed of motion of the AUV from the sequence of acoustic images [11], [12], [13]. In the present paper, these relations are determined in a form that connects the rate of change of the measured relief range with the speed of AUV motion. The ability to perform measurements over a wide range of angles yields a system of equations giving the relations of the measured rates of range relief with the AUV velocities. Since the number of equations is equal to the number of measurements at different angles which is much larger than the number of estimated elements of motion (3 velocity components), it is possible to estimate fairly accurately the latter by the method of least squares, which is the basis for the proposed algorithm.

In this article we are going to demonstrate the new approach to the absolute velocity measurements based on the observation of the evolution of the acoustical images captured by sonar at relatively wide observation angle. In the next sections we demonstrate the idea for 2D and 3D motion of the AUV and then show the quality of estimation of the AUV position for perturbed motion on the basis of noisy relief range measurement. This approach in some sense is opposite to the used in multibeam Doppler Velocity Log, which operates perfectly over the flat seabed surface [10], though the suggested approach needs the presence of the relief captured by sonar as an acoustic image of the seabed surface. The evolution of this image serves as a source of the infromation about the vehicle own velocity with respect to grownd.

II. 2D AUV MOTION

For the sake of simplicity to demonstrate the idea we consider 2D motion of the AUV in the ahead and vertical directions. In this case AUV can move along x and z axes only (Fig.1). So the axis $0X$ is directed along the motion and $0Z$ in the vertical direction from the bottom to the surface. In the case of 3D motion one needs to add the angle transformation, but we omit this to make the idea clearer, however general formulas are in [9].

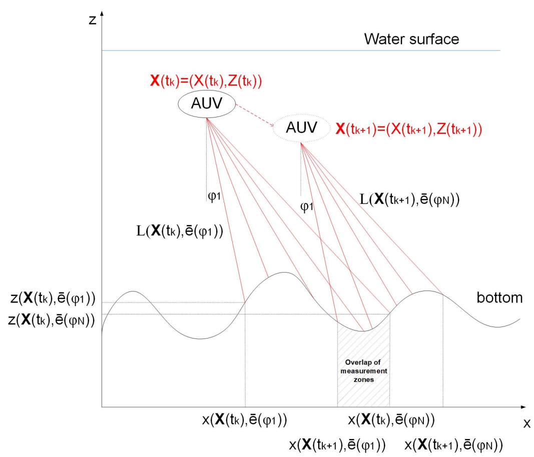

Fig. 1. Measurements model of AUV motion, the time instants t_k, t_{k+1} represent the successive range measurements

We consider the AUV motion in discrete time, so by vector

$$\mathbf{X}(t_k) = \begin{pmatrix} X(t_k) \\ Z(t_k) \end{pmatrix}.$$

At every time instant t_k AUV multibeam locator produces the set of N measurements $L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i))$, where ϕ_i is the angle between the beam and vertical axis, so the vector $\bar{\mathbf{e}}(\phi_i)$ defines the direction of the i-th acoustic beam of N:

$$\bar{\mathbf{e}}(\phi_i) = \begin{pmatrix} e_x(\phi_i) \\ e_z(\phi_i) \end{pmatrix} = \begin{pmatrix} \cos\phi_i \\ \sin\phi_i \end{pmatrix}.$$

Assume that the bottom profile is given by smooth function $z = z(x)$, later in Section V we consider the function of two variables (x, y). If at time t_k the AUV position is $\mathbf{X}(t_k)$ the measurement beam in the direction $\bar{\mathbf{e}}(\phi_i)$ reaches the bottom at the point $(x(\mathbf{X}(t_k), z(\mathbf{X}(t_k))$ satisfying following equation:

$$\begin{pmatrix} x(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i)) \\ z(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i)) \end{pmatrix} = \begin{pmatrix} X(t_k) \\ Z(t_k) \end{pmatrix}$$
$$+L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i)) \begin{pmatrix} e_x(\phi_i) \\ e_z(\phi_i) \end{pmatrix}.$$

We assume the depth profile is unknown and given by some smooth function $z = z(x)$, so the measured value $L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i))$ satisfies the following equation:

$$\begin{aligned} &Z(t_k) + e_z(\phi_i) L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i)) \\ &= z(X(t_k) + e_x(\phi_i) L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i))). \end{aligned} \tag{1}$$

Assuming rather frequent measurements (say $\Delta t = 0.1s$) with respect to the AUV motion (for example $3 - 5\frac{m}{s}$) and the smooth seabed relief, one can consider equation (1) as continuous one in t and use the differentiation of the measurements. Differentiating (1) equation in t we get:

$$\frac{dZ}{dt} + e_z \frac{dL}{dt} = \frac{\partial z}{\partial x} \left(\frac{dX}{dt} + e_x \frac{dL}{dt} \right). \tag{2}$$

In this equation elements $\frac{dX}{dt}$ and $\frac{dZ}{dt}$ define AUV velocities along axis x and z respectively. The problem is how to evaluate these elements. In the next section we provide the algorithm for $\frac{dX}{dt}$, $\frac{dZ}{dt}$ evaluation using equation (2). Equation (2) gives the relation between unknown velocities $\frac{dX}{dt}$, $\frac{dZ}{dt}$, the seabed profile $\frac{\partial z}{\partial x}$ and measured derivative $\frac{dL}{dt}$. So the first issue is to estimate the seabed profile $\frac{\partial z}{\partial x}$ and after that the velocities of the AUV. The multiple measurements of the range L under various directions of the acoustic beams $\bar{\mathbf{e}}$ permits to solve the first issue basing on the least square method. One needs to underline that this approach does not need the seabed profile map to be preliminary known which provides the real means for navigation in unknown environment.

III. 2D AUV MOTION ESTIMATION

Algorithm in general:

1) Using successive measurements $L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i))$ and $L(\mathbf{X}(t_{k+1}), \bar{\mathbf{e}}(\phi_i))$ evaluating $\frac{dL}{dt}$.
2) Evaluating partial derivative $\frac{\partial z}{\partial x}$ for different directions, defined by angle ϕ_i:
   - Since we have the whole series of L measurements we can evaluate the partial derivative of measured value L with respect to e_x and e_z. The partial derivative for $\frac{\partial}{\partial e_x}$ for (1):
   $$\frac{\partial z}{\partial x} \left(L + e_x \frac{\partial L}{\partial e_x} \right) = \frac{\partial e_z}{\partial e_x} L + e_z \frac{\partial L}{\partial e_x}$$
   or introducing new symbols:
   $$\frac{\partial z}{\partial x} a = b.$$
   - $\frac{\partial}{\partial e_z}$ for (1):
   $$\frac{\partial z}{\partial x} \left(e_x \frac{\partial L}{\partial e_z} + \frac{\partial e_x}{\partial e_z} L \right) = e_z \frac{\partial L}{\partial e_z} + L$$

or introducing new symbols:

$$\frac{\partial z}{\partial x}c = d.$$

- We get a joint system for estimation of the seabed slope

$$\frac{\partial z}{\partial x}\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix}$$

and then by using least square method we get the estimate:

$$\frac{\partial z}{\partial x} \approx \frac{ab + cd}{a^2 + c^2}.$$

3) Evaluating $\frac{dX}{dt}$, $\frac{dZ}{dt}$:

- By substitution of the increments instead of derivatives in (2) we get the following equation:

$$\Delta Z(t_k) - \frac{\partial z}{\partial x}\Delta X(t_k) = -\Delta L(t_k)\left(e_z - \frac{\partial z}{\partial x}e_x\right)$$

- Introducing new symbols: $M = -\left(e_z - \frac{\partial z}{\partial x}e_x\right)$ we are estimating $\Delta Z(t_k), \Delta X(t_k)$ with the aid of least square method:

$$\sum_{i=1}^{N}\left(\Delta\hat{Z}(t_k) - \tfrac{\partial z}{\partial x}\Delta\hat{X}(t_k) - M\Delta L(t_k)\right)^2 \rightarrow \rightarrow \min_{\Delta\hat{X}(t_k), \Delta\hat{Z}(t_k)}$$

and getting the following estimates:

$$\Delta\hat{X}(t_k) = \frac{\sum_{i=1}^{N}\frac{\partial z}{\partial x}M\Delta L(t_k) - \sum_{i=1}^{N}\frac{\partial z}{\partial x}\sum_{i=1}^{N}M\Delta L(t_k)}{-\sum_{i=1}^{N}(\frac{\partial z}{\partial x})^2 + (\sum_{i=1}^{N}\frac{\partial z}{\partial x})^2}$$

$$\Delta\hat{Z}(t_k) = \frac{-\sum_{i=1}^{N}(\frac{\partial z}{\partial x})^2\sum_{i=1}^{N}M\Delta L(t_k) + \sum_{i=1}^{N}\frac{\partial z}{\partial x}\sum_{i=1}^{N}\frac{\partial z}{\partial x}M\Delta L(t_k)}{-\sum_{i=1}^{N}(\frac{\partial z}{\partial x})^2 + (\sum_{i=1}^{N}\frac{\partial z}{\partial x})^2}$$

Summing up the above, the estimations of the AUV velocities are the following. Here we use the approximate values of the derivatives and the relation $e_x^2 + e_z^2 = 1$.

$$V_x = \frac{dX}{dt} \simeq \frac{\sum_{i=1}^{N}\frac{\partial z}{\partial x}M\frac{dL}{dt} - \sum_{i=1}^{N}\frac{\partial z}{\partial x}\sum_{i=1}^{N}M\frac{dL}{dt}}{-\sum_{i=1}^{N}(\frac{\partial z}{\partial x})^2 + (\sum_{i=1}^{N}\frac{\partial z}{\partial x})^2},$$

$$V_z = \frac{dZ}{dt} \simeq \frac{-\sum_{i=1}^{N}(\frac{\partial z}{\partial x})^2\sum_{i=1}^{N}M\frac{dL}{dt} + \sum_{i=1}^{N}\frac{\partial z}{\partial x}\sum_{i=1}^{N}\frac{\partial z}{\partial x}M\frac{dL}{dt}}{-\sum_{i=1}^{N}(\frac{\partial z}{\partial x})^2 + (\sum_{i=1}^{N}\frac{\partial z}{\partial x})^2}, \quad (3)$$

where

$$M = \tfrac{\partial z}{\partial x}e_x - e_z,$$

$$\frac{dL}{dt} \simeq \frac{L(\mathbf{X}(t_{k+1}), \bar{\mathbf{e}}(\phi_i)) - L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i))}{\Delta t},$$

$$\frac{\partial z}{\partial x} \simeq \frac{\left(L + e_x\frac{\partial L}{\partial e_x}\right)\left(-\frac{e_x}{e_z}L + e_z\frac{\partial L}{\partial e_x}\right) + \left(e_x\frac{\partial L}{\partial e_z} - \frac{e_z}{e_x}L\right)\left(e_z\frac{\partial L}{\partial e_z} + L\right)}{\left(L + e_x\frac{\partial L}{\partial e_x}\right)^2 + \left(e_x\frac{\partial L}{\partial e_z} - \frac{e_z}{e_x}L\right)^2},$$

$$\frac{\partial L}{\partial e_x} \simeq \frac{L(\mathbf{X}(t_k), e_x(\phi_{i+1})) - L(\mathbf{X}(t_k), e_x(\phi_i))}{e_x(\phi_{i+1}) - e_x(\phi_i)},$$

$$\frac{\partial L}{\partial e_z} \simeq \frac{L(\mathbf{X}(t_k), e_z(\phi_{i+1})) - L(\mathbf{X}(t_k), e_z(\phi_i))}{e_z(\phi_{i+1}) - e_z(\phi_i)}.$$

IV. Modelling

In order to evaluate the quality of the estimation we use the simulation of the moving AUV. In modelling the following conditions were used.

- AUV moving from point $(X_0, Z_0) = (1, -10)$ to the (X_T, Z_T) where we assume the motion along X axis with constant velocity $0.3\frac{m}{s}$ and the motion along axis Z with harmonically changing velocity with the amplitude $0.2\frac{m}{s}$. Both motions were perturbed by white noise with intensities, approximately $\sigma(a_x) = 0.06\frac{m}{s^2}$ and $\sigma(a_z) = 0.02\frac{m}{s^2}$, respectively. The accuracy of the range measurement assumed $0.1m$, which corresponds to the sonar's accuracy for the depth of the order $10m$.
- Number of measurements made in each point of trajectory is $N = 80$
- Direction of the i-th measurement beam is $\phi_i = 0.0285(i+1)\frac{\pi}{10}$
- Unknown seabed profile is modeled in the following form $z(x) = -20 + 0.001x^2 - 0.3\sin(2.5x)$

For each number of measurements N we performed the Monte-Carlo modelling with $k = 100$ samples. Fig. 2 shows the real AUV trajectory (black) and the estimated AUV trajectory (blue). As one can see proposed algorithm shows good quality of estimation. In the next table which illustrates the quality of estimation we use the following values:

- $E_X(t)$ is the error of the X coordinate estimate at time instant t;
- $E_Z(t)$ is the error of the X coordinate estimate at time instant t;
- $SD_X(t)$ is the standard deviation of the X coordinate estimate at time instant t;
- $SD_Z(t)$ is the standard deviation of the Z coordinate estimate at time instant t.

Fig. 3 shows the AUV moving under the seabed and performing range measurements in the beginning of its mission and in the end. The black is the real AUV trajectory and the red is the estimate. Bold are the points where the acoustic beams meet the seabed.

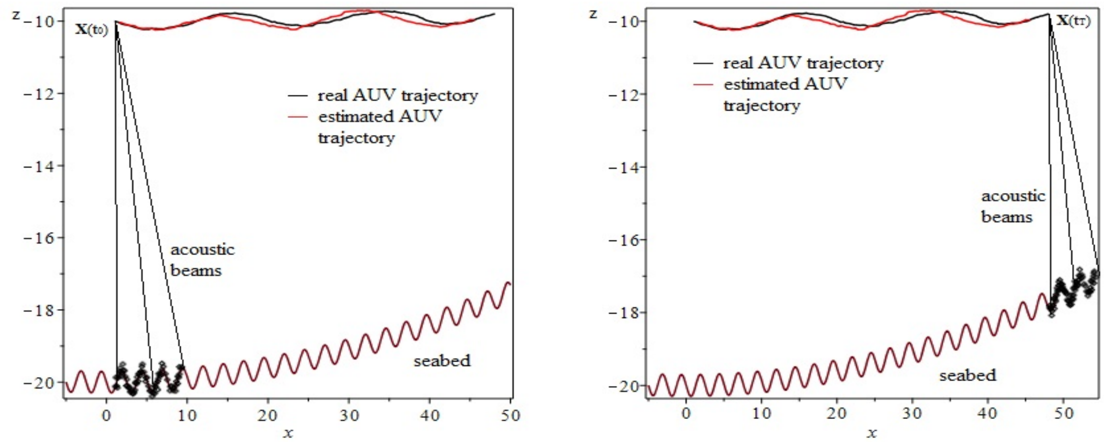

Fig. 3. AUV moving under the seabed and performing range measurements, on the left - at the time instant t_0, on the right - at time instant t_T. Bold are the points of the seabed used for acoustic sensing.

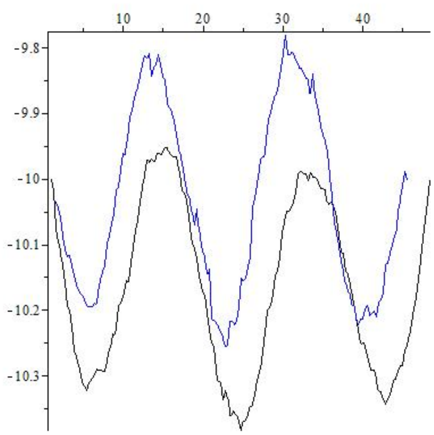

Fig. 2. Tracking of the AUV motion

t	$E_X(t)$	$E_Z(t)$	$SD_X(t)$	$SD_Z(t)$
20	-0.1636	0.0554	0.2157	0.07396
40	-0.6625	0.08098	0.6861	0.1058
80	0.08098	0.14268	1.5632	0.1851
160	-3.3115	-0.14459	3.3371	0.2721

V. 3D AUV MOTION

Here is the 3D model of AUV motion under the water surface. We assume that at time t_k AUV echo locator produces a set of range measurements $L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i))$ were (ϕ_i, θ_i) is the direction of the i-th beam supposing that AUV uses set $i = 1..N$ of such beams.

Here we solving the problem of AUV velocities evaluation analogous to the 2D case. Assume AUV motion is described by vector

$$\mathbf{X}(t_k) = \begin{pmatrix} X(t_k) \\ Y(t_k) \\ Z(t_k) \end{pmatrix}.$$

Vector $\bar{\mathbf{e}}(\phi_i, \theta_i)$ defines the direction of the i-th acoustic beam of N:

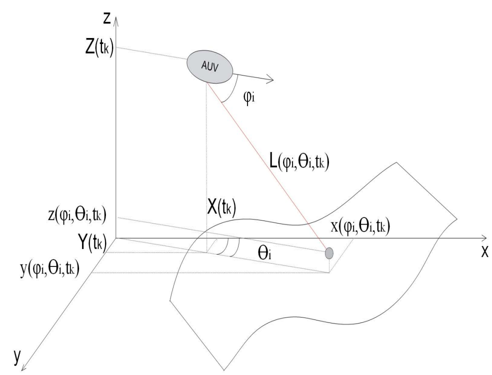

Fig. 4. 3D model of AUV motion

$$\bar{\mathbf{e}}(\phi_i, \theta_i) = \begin{pmatrix} e_x(\phi_i, \theta_i) \\ e_y(\phi_i, \theta_i) \\ e_z(\phi_i, \theta_i) \end{pmatrix} = \begin{pmatrix} \sin\phi_i \cos\theta_i \\ \sin\phi_i \sin\theta_i \\ \cos\phi_i \end{pmatrix}.$$

At time t_k AUV position is $\mathbf{X}(t_k)$ so the measurement signal in direction $\bar{\mathbf{e}}(\phi_i)$ reaches the bottom at the point:

$$\begin{pmatrix} x(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i)) \\ y(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i)) \\ z(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i)) \end{pmatrix} = \begin{pmatrix} X(t_k) \\ Y(t_k) \\ Z(t_k) \end{pmatrix}$$

$$+L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i)) \begin{pmatrix} e_x(\phi_i, \theta_i) \\ e_y(\phi_i, \theta_i) \\ e_z(\phi_i, \theta_i) \end{pmatrix}.$$

Assuming the depth profile is unknown and given by some smooth function $z = z(x, y)$ writing:

$$Z(t_k) + e_z(\phi_i, \theta_i) L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i))$$

$$= z(X(t_k) + e_x(\phi_i, \theta_i) L(\mathbf{X}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i)), \quad (4)$$

$$Y(t_k) + e_y(\phi_i, \theta_i) L(\mathbf{Y}(t_k), \bar{\mathbf{e}}(\phi_i, \theta_i))).$$

Differentiating this equation by t we get:

$$\frac{dZ}{dt}+e_z\frac{dL}{dt}=\frac{\partial z}{\partial x}\left(\frac{dX}{dt}+e_x\frac{dL}{dt}\right)+\frac{\partial z}{\partial y}\left(\frac{dY}{dt}+e_y\frac{dL}{dt}\right). \tag{5}$$

In this equation elements $\frac{dX}{dt}$, $\frac{dY}{dt}$ and $\frac{dZ}{dt}$ define AUV velocities along axis x, y and z respectively. In next section we evaluating these velocities analogous to 2D case.

VI. 3D AUV MOTION ESTIMATION

In general the algorithm is similar to 2D case:

1) $\frac{dL}{dt}$ evaluated using successive measurements $L(\mathbf{X}(t_k),\bar{\mathbf{e}}(\phi_i,\theta_i))$ and $L(\mathbf{X}(t_{k+1}),\bar{\mathbf{e}}(\phi_i,\theta_i))$.
2) Partial derivatives $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ are evaluated by the following steps:
   - Taking partial derivative $\frac{\partial}{\partial e_x}$ for (4):

$$\frac{\partial z}{\partial x}\left(L+e_x\frac{\partial L}{\partial e_x}\right)+\frac{\partial z}{\partial y}\left(L\frac{\partial e_y}{\partial e_x}+e_y\frac{\partial L}{\partial e_x}\right)$$
$$=\frac{\partial e_z}{\partial e_x}L+e_z\frac{\partial L}{\partial e_x} \tag{6}$$

   - Then taking partial derivative $\frac{\partial}{\partial e_y}$ for (4):

$$\frac{\partial z}{\partial x}\left(L\frac{\partial e_x}{\partial e_y}+e_x\frac{\partial L}{\partial e_y}\right)+\frac{\partial z}{\partial y}\left(L+e_y\frac{\partial L}{\partial e_y}\right)$$
$$=\frac{\partial e_z}{\partial e_y}L+e_z\frac{\partial L}{\partial e_y}$$

   - And finally taking partial derivative $\frac{\partial}{\partial e_z}$ for (4):

$$\frac{\partial z}{\partial x}\left(e_x\frac{\partial L}{\partial e_z}+\frac{\partial e_x}{\partial e_z}L\right)$$
$$+\frac{\partial z}{\partial y}\left(e_y\frac{\partial L}{\partial e_z}+\frac{\partial e_y}{\partial e_z}L\right)=e_z\frac{\partial L}{\partial e_z}+L$$

   - Introducing new symbols getting the following system:

$$\begin{pmatrix} a & b \\ f & g \\ q & w \end{pmatrix}\begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix}=\begin{pmatrix} c \\ h \\ p \end{pmatrix}.$$

   or

$$A\begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix}=B.$$

   -

$$A^T=\begin{pmatrix} a & f & q \\ b & g & w \end{pmatrix}$$

   so

$$A^TA=\begin{pmatrix} a^2+f^2+q^2 & ab+fg+qw \\ ab+fg+qw & b^2+g^2+w^2 \end{pmatrix}$$

   and

$$[A^TA]^{-1}=\frac{1}{\det[A^TA]}\times$$
$$\begin{pmatrix} b^2+g^2+w^2 & -(ab+fg+qw) \\ -(ab+fg+qw) & a^2+f^2+q^2 \end{pmatrix}$$

   and

$$A^TB=\begin{pmatrix} ac+fh+qp \\ bc+gh+wp \end{pmatrix}$$

   and finally

$$\begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix}=[A^TA]^{-1}[A^TB]$$

3) Evaluating $\frac{dX}{dt}$, $\frac{dY}{dt}$, $\frac{dZ}{dt}$:
   - Rewriting (6) by representing the derivative via increments:

$$\Delta Z(t_k)-\frac{\partial z}{\partial x}\Delta X(t_k)-\frac{\partial z}{\partial y}\Delta Y(t_k)$$
$$=-\Delta L(t_k)\left(e_z-\frac{\partial z}{\partial x}e_x-\frac{\partial z}{\partial y}e_y\right)$$

   - Introducing new symbols: $M=-\left(e_z-\frac{\partial z}{\partial x}e_x-\frac{\partial z}{\partial y}e_y\right)$ we are estimating $\Delta X(t_k),\Delta Y(t_k),\Delta Z(t_k)$ with the aid of least square method:

$$\sum_{i=1}^{N}\Big(\Delta\hat{Z}(t_k)-\tfrac{\partial z}{\partial x}\Delta\hat{X}(t_k)$$
$$-\tfrac{\partial z}{\partial y}\Delta\hat{Y}(t_k)-M\Delta L(t_k)\Big)^2\to$$
$$\to\min_{\Delta\hat{X}(t_k),\Delta\hat{Y}(t_k),\Delta\hat{Z}(t_k)}$$

   Standard calculations give the following equation for the estimates vector

$$\Delta\hat{\mathbf{X}}(t_k)=\begin{pmatrix} \Delta\hat{Z}(t_k) \\ \Delta\hat{X}(t_k) \\ \Delta\hat{Y}(t_k) \end{pmatrix}$$

$$\tilde{\mathbf{B}}\Delta\hat{\mathbf{X}}(t_k)=\tilde{\mathbf{M}},$$

where

$$\tilde{\mathbf{B}} = \begin{pmatrix} \sum 1 & \sum \frac{\partial z}{\partial x} & \sum \frac{\partial z}{\partial y} \\ \sum \frac{\partial z}{\partial x} & \sum (\frac{\partial z}{\partial x})^2 & \sum \frac{\partial z}{\partial x}\frac{\partial z}{\partial y} \\ \sum \frac{\partial z}{\partial y} & \sum \frac{\partial z}{\partial x}\frac{\partial z}{\partial y} & \sum (\frac{\partial z}{\partial y})^2 \end{pmatrix},$$

and

$$\tilde{\mathbf{M}} = \begin{pmatrix} \sum M\Delta L(t_k) \\ \sum M\Delta L(t_k)\frac{\partial z}{\partial x} \\ \sum M\Delta L(t_k)\frac{\partial z}{\partial y} \end{pmatrix},$$

where symbol $\sum$ stands for $\sum\limits_{i=1}^{N_\phi}\sum\limits_{k=1}^{N_\theta}$. Calculations are analogous to the 2D case. We do not give here the final formulas for estimates in symbolic form due to their cumbersomeness.

VII. Conclusion

In this work we obtained the relations that allow to extract the speed of the AUV motion from the sequence of acoustic images. These relations are determined in a form that gives a relation between the rate of change of the measured range with the speed of AUV motion. We derived a system of equations relating the measured rates of range relief with the AUV velocities. It makes possible to estimate the AUV velocity components by using the least squares method. The proposed algorithm was examined in 2D when AUV can move only straight ahead and in vertical directions and extended this approach to the 3D case. Modeling for a 2D case shows good quality of estimation. In next works we will be perform the modelling for a 3D case and integration with INS. One should underline that navigation based on velocities measurements only, inevitably leads to the drift, which must be compensated by observation and bearing of some objects or parts of the seabed whose positions are more or less precisely known. This techniques have been tested already in UAV navigation [11] and highly likely should be effective in underwater applications.

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VIII. Author Biographies

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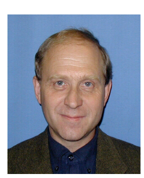

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