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A distributed disagreement-based protocol for synchronization of uncertain heterogeneous agents

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Abstract— **In networks with heterogeneous and uncertain agents, fixed-gain control can lead to synchronization only if the uncertainties are relatively small. If the uncertainties are larger, we need to develop adaptive-gain approaches to achieve synchronization. In this work we propose an adaptive synchronization protocol, in case of full-state measurement, for uncertain heterogeneous agents based on a distributed disagreement reasoning. Specifically, we first define unknown gains (feedback and coupling gains) that could lead all agents to a homogeneous behavior and thus synchronization: however, since these gains are unknown in view of the unknown dynamics, we design adaptive laws for these gains that lead the agents toward synchronization. The adaptive laws are driven by a disagreement error which is calculated among neighbors: a Lyapunov analysis is presented for showing convergence of the synchronization error to zero.**

Index Terms— **Adaptive synchronization, uncertain heterogeneous agents, distributed disagreement.**

I. Introduction

A wide range of multi-agent coordination missions, such as leader-following, formation keeping, and many more tasks, require the agents to achieve synchronization [1]. A very popular method to achieve synchronization is to formulate the problem as a cooperative output regulation problem. [2] shows that an internal model requirement is necessary and sufficient for synchronizability of a network to an autonomous exogenous system. This means that the well-known internal model principle [3] can be used to solve synchronization problems. Motivated by this result, synchronization protocols were designed for both linear [4] and nonlinear networks [5]. Most approaches to cooperative output regulation problem can be divided into two families: the internal model approach [6], and the feedforward approach [7]. The main advantages and disadvantages of these two families can be summarized as follows: the internal model approach works in a smaller number of cases (a transmission zero condition must be satisfied), but it is robust to system uncertainty; on the other hand, the feedforward method requires less assumptions, but it cannot handle system uncertainty. Recent advances in the cooperative output regulation problem include: removing the assumption that all systems know the matrix of the exosystem [8]; removing the need to exchange state information among the communication network so as to reduce the communication burden [9]; addressing the presence of switching communication topologies [10], [11]. Despite these advances in cooperative output regulation, the approach relies on fixed-gain control, and it can lead to synchronization only when uncertainties are relatively small.

Another interesting approach to deal with synchronization of heterogeneous agents is via homogenization [12], [13]. Being synchronization of homogeneous networks a well known result in literature, the idea behind homogenization is to steer the behavior of an heterogeneous network to that of an homogeneous one, by compensating or cancelling the heterogeneity.

Since the very beginning, researchers in synchronization have recognized the need for addressing *parameter uncertainties* in system matrices [14], from which a fruitful line of research on uncertain heterogeneous systems stemmed, aiming to solve the synchronization problem not only when agents differ from each other, but their matrices also lie in an uncertainty set: [4] addresses synchronization in heterogeneous groups of a class of linear dynamical agents (non-identical double-integrators and harmonic oscillators) that are coupled by diffusive links. It is shown that in the presence of parameter uncertainties arising from heterogeneity, structural requirements are needed for robust output synchronization.

As a result, we can summarize this overview of the state of the art by saying that there are mature synchronization approaches for heterogeneous systems in the presence of small uncertainty, but the study of adaptive synchronization approaches for heterogeneous systems in the presence of large uncertainty is not equally mature: to be more specific, only limited classes of uncertainty have been addressed via adaptive synchronization, namely unknown (but identical) control directions [15], unknown but homogeneous agents [16], [17], unknown leader parameters [18], unknown heterogeneous agents with passifiable dynamics [19], nonlinear systems in output feedback form with unknown (but identical) parameters [20], unknown agents in specific platooning protocols [21], unknown agents on acyclic networks [22], and agents in the Euler-Lagrange form [23], [24] or in the parametric strict-feedback form [25]. Having adaptive in place of fixed gains is particularly relevant because the restriction of having fixed-gain control implies that synchronization can be achieved only for small uncertainties [4].

In this work we propose an adaptive synchronization

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protocol, in case of full-state measurement, for uncertain heterogeneous agents based on adaptive homogenization reasoning. In particular, we first define unknown gains (feedback and coupling gains) that could lead all agents to a homogeneous behavior and thus synchronization: however, since these gains are unknown in view of the unknown dynamics, we design adaptive laws for these gains that lead the agents toward synchronization. The adaptive laws are driven by a disagreement error which is calculated among neighbors: thus, the algorithm is fully distributed. A Lyapunov analysis is presented for showing the stability of the proposed protocol.

The rest of the paper is organized as follows: in Section II we give the problem formulation; in Section III the distributed disagreement-based protocol is presented; a numerical example is provided in Section IV, while Section V concludes the work.

Notation: The notation in this paper is standard. The transpose of a matrix or of a vector is indicated with X^T and x^T respectively. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathscr{L}_2$ class ($x \in \mathscr{L}_2$), if $\int_0^t \|x(\tau)\|^2 \, \mathrm{d}\tau < \infty$, $\forall t \geq 0$. A vector signal $x \in \mathbb{R}^n$ is said to belong to $\mathscr{L}_\infty$ class ($x \in \mathscr{L}_\infty$), if $\max_{t\geq 0} \|x(t)\| < \infty$, $\forall t \geq 0$. A time-invariant undirected communication graph of order N is completely defined by the pair $\mathscr{G} = (\mathscr{V}, \mathscr{E})$, where $\mathscr{V} = \{1, \ldots, N\}$ is a finite nonempty set of nodes, and $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is a set of corresponding non-ordered pair of nodes, called edges. The adjacency matrix $\mathscr{A} = [a_{ij}]$ of an unweighted undirected graph is defined as $a_{ii} = 0$ and $a_{ij} = a_{ji} = 1$ if $(i, j) \in \mathscr{E}$, where $i \neq j$. The Laplacian matrix of the unweighted graph is defined as $\mathscr{L} = [l_{ij}]$, where $l_{ii} = \sum_j a_{ij}$ and $l_{ij} = -a_{ij}$, if $i \neq j$. An undirected graph $\mathscr{G}$ is said to be connected if, taken any arbitrary pair of nodes (i, j) where $i, j \in \mathscr{V}$, there is a path that leads from i to j. In this work we indicate with N the number of nodes (or agents) in the network, while n represents the order of the agents in the network.

II. Problem formulation

A network of heterogeneous agents with unknown dynamics is considered in this work.

$$\dot{x}_i = A_i x_i + b_i u_i, \qquad i \in \mathscr{V} \triangleq \{1, 2, \ldots, N\} \tag{1}$$

where $x_i \in \mathbb{R}^{n\times 1}$ is the state and $u_i \in \mathbb{R}$ is the input. Time index t is usually omitted when obvious. The matrices A_i and b_i are unknown matrices of appropriate dimensions, with possibly $A_i \neq A_j$ and $b_i \neq b_j$, $i \neq j$ (uncertain heterogeneous agents). For simplicity, we deal with the single-input case (for extension to multiple-input the tools in [26] should be used).

The following connectivity assumption is made.

Assumption 1: The graph $\mathscr{G}$ of the network is undirected and connected.

The following problem is considered in this work:

Problem 1: [Adaptive state synchronization] Consider a network of uncertain heterogeneous agents (1) satisfying Assumption 1. Find a distributed state-feedback strategy (i.e. exploiting only state measurements from neighbors) for the control input u_i such that, without any knowledge of the entries of A_i and b_i, the network state synchronizes to the same behavior, i.e. $x_i - x_j \to 0$, $\forall i, j$.

III. Adaptive state synchronization

Two results are now given which are instrumental to solving Problem 1. Synchronization is favored in the presence of a homogeneous structure [27]: here we define the desired structure.

Proposition 1: [State homogenization via reference model] Consider the following reference model

$$\dot{x}_m = A_0 x_m + b_0 u \tag{2}$$

with $x_m \in \mathbb{R}^{n\times 1}$. If there exist a family of vectors $k_i^* \in \mathbb{R}^{n\times 1}$ and a family of scalars l_i^* (with known sign) such that the following matching conditions are satisfied

$$\begin{cases} A_i + b_i k_i^{*T} = A_0 \\ \qquad l_i^* b_i = b_0 \end{cases} \tag{3}$$

then, there exists an ideal controller

$$u_i^* = k_i^{*T} x_i + l_i^* f^T \sum_{j=1}^{N} a_{ij}(x_i - x_j) \tag{4}$$

with $f \in \mathbb{R}^{n\times 1}$ to be designed, which leads to the following dynamics

$$\dot{x}_i = A_0 x_i + b_0 f^T \sum_{j=1}^{N} a_{ij}(x_i - x_j), \quad i \in \mathscr{V}. \tag{5}$$

Proof: The proof directly follows from applying the control input (4) to agent (1), and using (3).

The following result, taken from [28], allows us to design f to achieve synchronization for the homogeneous dynamics in (5).

Proposition 2: [Homogeneous network state synchronization] The homogeneous network (5) synchronizes if

$$A_0 + \lambda_i b_0 f^T \ \text{ is Hurwitz}, \quad \forall i \in \mathscr{V}/\{1\} \tag{6}$$

where λ_i's, $i \in \mathscr{V}/\{1\}$, are the non-zero eigenvalues of the Laplacian.

Proof: The overall homogeneous network (5) can be written in the more compact form

$$\dot{x} = (I_N \otimes A_0 + \mathscr{L} \otimes b_0 f^T)x \tag{7}$$

where $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$. Let us now define the synchronization error as

$$e_i = \sum_{j=1}^{N} a_{ij}(x_i - x_j), \qquad e = [e_1^T, e_2^T, \ldots, e_N^T]^T \tag{8}$$

where the error for the overall network can be written as $e = (\mathscr{L} \otimes I_n)x$. Since the graph is undirected and connected, we know, e.g. from [28], that there exists a unitary matrix $\mathscr{U} = [\frac{1}{\sqrt{N}} \mathbf{1}_N \ \mathscr{U}_2]$ with $\mathscr{U}_2 \in \mathbb{R}^{N\times(N-1)}$ such that $\mathscr{U}^T \mathscr{L} \mathscr{U} = \mathrm{diag}(0, \lambda_2, \ldots, \lambda_N) \triangleq \mathrm{diag}(0, \bar{\Lambda}) \triangleq \Lambda$. This can be used to define $\bar{e} = (\mathscr{U} \otimes I_n)e$. Moreover let $\bar{e} = [\bar{e}_1^T, \bar{e}_2^T, \ldots, \bar{e}_N^T]^T$ and $\tilde{e} = [\bar{e}_2^T, \ldots, \bar{e}_N^T]^T$. It is easily checked that

$$\begin{aligned}\bar{e}_1 &= \left(\frac{1}{\sqrt{N}}\mathbf{1}_N \otimes I_n\right) e \\ &= \left(\frac{1}{\sqrt{N}}\mathbf{1}_N \otimes I_n\right)(\mathscr{L} \otimes I_n)x = \mathbf{0}_{(N\times n)\times 1}.\end{aligned}$$

We can now write the overall error dynamics as:

$$\begin{aligned}\dot{e} &= (\mathscr{L} \otimes I_n)\dot{x} \\ &= (\mathscr{L} \otimes I_n)[(I_N \otimes A_0)x + (I_N \otimes b_0 f^T)e] \\ &= (\mathscr{L} \otimes I_n)(I_N \otimes A_0)x + (\mathscr{L} \otimes I_n)(I_N \otimes b_0 f^T)e \\ &= [(I_N \otimes A_0) + (\mathscr{L} \otimes b_0 f^T)]e.\end{aligned} \tag{9}$$

Consider the Lyapunov function candidate:

$$V_1 = e^T(I_N \otimes P)e \tag{10}$$

where $P \in \mathbb{R}^{n\times n}$ is a symmetric positive definite matrix. Then we have

$$\begin{aligned}\dot{V}_1 &= 2e^T(I_N \otimes P)[(I_N \otimes A_0) + (\mathscr{L} \otimes b_0 f^T)]e \\ &= 2e^T(I_N \otimes PA_0 + \mathscr{L} \otimes Pb_0 f^T)e \\ &= 2\bar{e}^T(I_N \otimes PA_0 + \Lambda \otimes Pb_0 f^T)\bar{e} \\ &= 2\tilde{e}^T(I_{N-1} \otimes PA_0 + \bar{\Lambda} \otimes Pb_0 f^T)\tilde{e} \\ &= \sum_{i=2}^{N} \bar{e}_i^T[P(A_0 + \lambda_i b_0 f^T) + (A_0 + \lambda_i b_0 f^T)^T P]\bar{e}_i\end{aligned} \tag{11}$$

which is negative definite if

$$P(A_0 + \lambda_i b_0 f^T) + (A_0 + \lambda_i b_0 f^T)^T P < \mathbf{0},\ \forall i \in \mathscr{V}/\{1\} \tag{12}$$

which completes the proof.

Remark 1: Since A_i, b_i are unknown, the ideal control (4) *cannot be implemented to solve Problem 1. Therefore, some adaptation mechanisms must be devised to estimate the unknown ideal gains in Proposition 1, by exploiting only measurements from neighbors.*

The following state synchronizing protocol is proposed

$$u_i(t) = k_i^T(t)x_i + l_i(t)f^T \sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) \tag{13}$$

where k_i and l_i are the (time-dependent) estimates of k_i^* and l_i^*, respectively. The following synchronization result holds.

Theorem 1: Under Assumption 1, the uncertain heterogeneous network (1), controlled using the synchronizing protocol (13) and the following adaptive laws

$$\begin{aligned}\dot{k}_i^T &= -sgn(l_i^*)\gamma\left(\sum_{j=1}^{N} a_{ij}(e_i - e_j)\right)^T Pb_0 x_i^T \\ \dot{l}_i &= -sgn(l_i^*)\gamma\left(\sum_{j=1}^{N} a_{ij}(e_i - e_j)\right)^T Pb_0 f^T e_i\end{aligned} \tag{14}$$

with adaptive gain $\gamma > 0$, reaches synchronization provided that the matrix P and the vector f are chosen such that condition (12) holds.

Proof: The closed-loop network formed by (1) and (13) is given by

$$\dot{x}_i = (A_i + b_i k_i^T)x_i + l_i b_i f^T \sum_{j=1}^{N} a_{ij}(x_i - x_j) \tag{15}$$

which can be rewritten as a function of the estimation errors,

$$\dot{x}_i = (A_0 + b_i\tilde{k}_i^T(t))x_i + (b_0 + \tilde{l}_i(t)b_i)f^T \sum_{j=1}^{N} a_{ij}(x_i - x_j) \tag{16}$$

where $\tilde{k}_i(t) = k_i(t) - k_i^*$ and $\tilde{l}_i(t) = l_i(t) - l_i^*$. By defining for compactness

$$\begin{aligned}B_k(t) &= \mathrm{diag}(b_1\tilde{k}_1^T(t), \ldots, b_N\tilde{k}_N^T(t)) \\ B_l(t) &= \mathrm{diag}(\tilde{l}_1(t)b_1 f^T, \ldots, \tilde{l}_N(t)b_N f^T)\end{aligned} \tag{17}$$

the closed-loop for the overall network can be written as

$$\dot{x} = (I_N \otimes A_0 + B_k(t))x + (I_N \otimes b_0 f^T + B_l(t))e. \tag{18}$$

Recalling that the synchronization error is $e = (\mathscr{L} \otimes I_n)x$, the error dynamics are

$$\begin{aligned}\dot{e} = &[(I_N \otimes A_0) + (\mathscr{L} \otimes b_0 f^T)]e + \\ &+ (\mathscr{L} \otimes I_n)(B_k(t)x + B_l(t)e).\end{aligned} \tag{19}$$

The adaptive laws (14) arise from considering the Lyapunov function candidate $V = V_1 + V_2 + V_3$, where V_1 is (10), and

$$V_2 = \sum_{i=1}^{N} \frac{\tilde{k}_i^T(t)\gamma^{-1}\tilde{k}_i(t)}{|l_i^*|} \qquad V_3 = \sum_{i=1}^{N} \frac{\tilde{l}_i(t)\gamma^{-1}\tilde{l}_i^T(t)}{|l_i^*|} \tag{20}$$

Then we have

$$\begin{aligned}\dot{V}_1 = &2e^T(I_N \otimes P)[(I_N \otimes A_0) + (\mathscr{L} \otimes b_0 f^T)]e + \\ &+ 2e^T(I_N \otimes P)[(\mathscr{L} \otimes I_n)(B_k(t)x + B_l(t)e)]\end{aligned} \tag{21}$$

and following the same procedure as in (11):

$$\begin{aligned}\dot{V}_1 = &\sum_{i=2}^{N} \bar{e}_i^T[P(A_0 + \lambda_i b_0 f^T) + (A_0 + \lambda_i b_0 f^T)^T P]\bar{e}_i + \\ &+ 2\sum_{i=1}^{N} \tilde{k}_i^T(t)x_i b_i^T P\left(\sum_{j=1}^{N} a_{ij}(e_i - e_j)\right) + \\ &+ 2\sum_{i=1}^{N} \tilde{l}_i(t)e_i^T f b_i^T P\left(\sum_{j=1}^{N} a_{ij}(e_i - e_j)\right)\end{aligned} \tag{22}$$

Moreover, by using (14) we have:

$$\begin{aligned}\dot{V}_2 &= -2\sum_{i=1}^{N} \frac{sgn(l_i^*)\gamma^{-1}}{|l_i^*|}\tilde{k}_i(t)x_i b_0^T P\left(\sum_{j=1}^{N} a_{ij}(e_i - e_j)\right) \\ \dot{V}_3 &= -2\sum_{i=1}^{N} \frac{sgn(l_i^*)\gamma^{-1}}{|l_i^*|}\tilde{l}_i(t)e_i^T f b_0^T P\left(\sum_{j=1}^{N} a_{ij}(e_i - e_j)\right)\end{aligned} \tag{23}$$

leading to:

$$\begin{aligned}\dot{V} &= \dot{V}_1+\dot{V}_2+\dot{V}_3 \\ &= \sum_{i=2}^{N} \bar{e}_i^T[P(A_0+\lambda_i b_0 f^T)+(A_0+\lambda_i b_0 f^T)^T P]\bar{e}_i\end{aligned} \quad (24)$$

which is negative semi-definite provided that condition (12) holds. Using standard Lyapunov arguments we can prove boundedness of all closed-loop signals and convergence of e to 0. In fact, since $V>0$ and $\dot{V}\le 0$, it follows that $V(t)$ has a limit, i.e.,

$$\lim_{t\to\infty} V(e(t),\tilde{\Omega}(t)) = V_\infty < \infty \quad (25)$$

where we have collected all parametric errors in $\tilde{\Omega}$. The finite limit implies V, e, $\tilde{\Omega}\in\mathscr{L}_\infty$. In addition, by integrating $\dot{V}$ it follows that for some $Q>0$

$$\int_0^\infty e^T(\tau)Qe(\tau)\,\mathrm{d}\tau \le V(e(0),\tilde{\Omega}(0)) - V_\infty \quad (26)$$

from which we establish that $e\in\mathscr{L}_2$. Finally, since $\dot{V}$ is uniformly continuous in time (this is satisfied because $\ddot{V}$ is finite), the Barbalat's lemma implies $\dot{V}\to 0$ as $t\to\infty$ and hence $e\to 0$, from which we derive $x_i\to x_j$, $\forall i,j$. This concludes the proof.

Remark 2: In order to implement (14)*, and in particular the term* $\sum_{j=1}^N a_{ij}(e_i-e_j)$*, it is required to communicate the variable* e_i *among neighbors (extra local information). Communication of extra auxiliary variables is often at the core of many synchronization protocols: for example, synchronization based on distributed observer [11], [8] requires communication of auxiliary variables representing the observer states. Adaptive synchronization protocols based on distributed observer have been adopted in literature for the so-called Euler-Lagrange (heterogeneous uncertain) agents [23], [24]. Now, comparing these approaches with* (14)*, we see that the proposed disagreement-based protocol is essentially simpler, because it does not require to construct in a distributed manner the observer variables.*

Remark 3: It can be noted from the adaptive protocol (13) *that the vectors* k_i *act as feedback gains, while the scalars* l_i *act as coupling gains. The proposed protocol can therefore adapt both the feedback and the coupling gains. Actually,* (13) *can be considered as a node-based protocol (because* l_i *is unique for each node): it is possible to modify* (13) *to be edge-based as follows*

$$u_i(t) = k_i^T(t)x_i + f^T\sum_{j=1}^N l_{ij}(t)a_{ij}(x_i(t)-x_j(t)) \quad (27)$$

and the corresponding adaptation laws would become

$$\begin{aligned}\dot{k}_i^T &= -sgn(l_i^*)\gamma\left(\sum_{j=1}^N a_{ij}(e_i-e_j)\right)^T Pb_0 x_i^T \\ \dot{l}_{ij} &= -sgn(l_i^*)\gamma\left(\sum_{j=1}^N a_{ij}(e_i-e_j)\right)^T Pb_0 f^T(x_i(t)-x_j(t))\end{aligned} \quad (28)$$

where l_{ij} *would be adapted on each edge separately.*

Remark 4: We want to emphasize that all the results and theorems in this work refer to leaderless synchronization, where the state to which the agents will synchronize are in general a priori unknown. Please note that, since the reference model (2) *simply guarentees the existence of ideal synchronizing gains and does not play the role of a leader, we have proven that the synchronization error* (8) *converges to zero, but the coherent state is in general a priori unknown. On the other hand, the addition of a leader in the network, having dynamics* (2)*, leads, via the proposed protocols, to convergence to the a priori known state of the leader. This will be shown in Sect. V. This will be shown in Sect. IV. Alternatively, adaptive leader-follower synchronizing protocols have been proposed in [29].*

IV. Numerical example

Simulations using controller (14) are carried out in the following. We consider the weighted graph shown in Figure 1, where agent 0 acts as a leader node with no adaptive law.

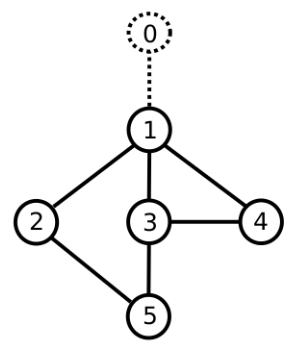

Fig. 1. The undirected communication graph.

The uncertain heterogeneous agents (1) are taken as second-order systems having transfer function numerator $n_1 s+n_2$ and denominator $s^2+d_1 s+d_2$. The parameters and initial conditions for each agent are reported in Table I (note that all agents are in state-space controllable canonical form). Recall that the agent parameters are unknown to the designer, i.e. the values in Table I are not used for control design (the values are used only for simulation).

TABLE I
Parameters and initial conditions for the agents

	d_1	d_2	n_1	n_2	$x(0)$
agent #1	1	2	1	1.5	$[0\ 1]^T$
agent #2	0.75	2.5	0.5	1	$[0\ 2]^T$
agent #3	1.25	2	1.25	1	$[0\ 3]^T$
agent #4	0.5	1	0.75	0.75	$[0\ 4]^T$
agent #5	0.75	1	1.5	2	$[0\ 5]^T$

The reference model is chosen as an harmonic oscillator

$$\dot{x}_m = \underbrace{\begin{bmatrix} 0 & 1 \\ -(0.7^2) & 0 \end{bmatrix}}_{A_0} x_m + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{b_0} u\,, \qquad x_m(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \quad (29)$$

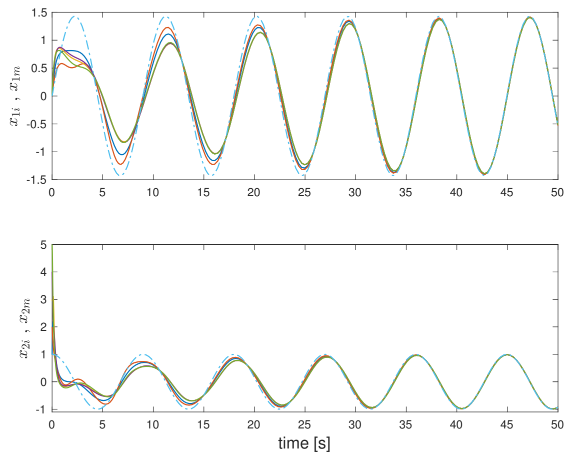

Fig. 2. Synchronization of the states of each agent i to the leader reference state using (14).

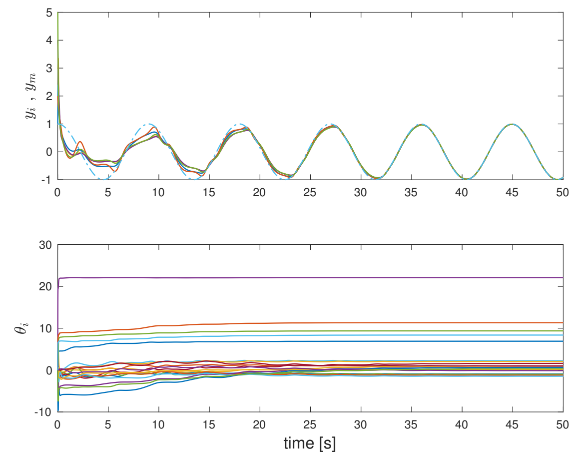

Fig. 4. Synchronization of the outputs of each agent i to the leader reference output, and adaptive gains.

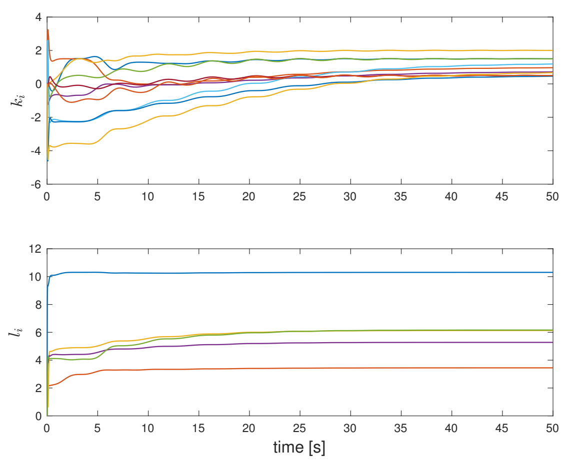

Fig. 3. Adaptive gains resulting from (14).

The used f vector and P matrix that satisfy condition (12) are

$$P = \begin{bmatrix} 1.4082 & 0.2671 \\ 0.2671 & 0.5551 \end{bmatrix}, \qquad f^T = \begin{bmatrix} -1 & -1 \end{bmatrix}. \tag{30}$$

Finally, the adaptive gain is taken $\gamma = 10$ and all estimated control gains k_i, l_i, are initialized to 0. The resulting adaptive state synchronization is shown in Figure 2 with adaptive gains shown in Figure 3.

Also, the simulations of the output feedback version of the proposed protocol are given without discussing the methodology, due to page constraints. The output is considered to be the second state of both the agents and the harmonic oscillator. The resulting adaptive output synchronization is shown in Figure 4 together with the adaptive gains.

V. Conclusions

In this work we have proposed an adaptive synchronization protocol for uncertain heterogeneous agents based on a distributed disagreement reasoning: in particular, we first defined unknown gains (both feedback and coupling gains) that lead all agents to a homogeneous behavior and thus synchronization: however, since these gains are unknown in view of the unknown dynamics, we have designed adaptive laws for these gains that lead the agents toward synchronization. The adaptive laws are driven by a disagreement error which is calculated among neighbors: thus, the algorithm is fully distributed. A Lyapunov analysis is presented for showing stability. Having adaptive in place of fixed gains is particularly relevant because the restriction of having fixed-gain control implies that synchronization can be achieved only for small uncertainties.

Future work could go in the following directions. First, it is important to investigate how to extend this protocol to nonlinear agents: a first step in this direction can be found in the companion paper [30], where the synchronization over a network of uncertain heterogeneous Kuramoto-like agents was studied. Then, the output synchronization protocol, tested only in simulations, has to be formalized and generalized to relative degree greater than one. This should be possible by using SPR filters in the spirit of [31, Sect. 6.4]. Another relevant topic would be to study the effect of delays in the calculations of the distributed disagreement. This potentially could lead to bounded synchronization errors using tools as in [32], [33]. Finally, the extension to switching topology and other network-induced constraints is relevant and can be achieved using adaptive switched tools [34], [35].

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