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Integral action for uncertain switched affine systems with application to DC/DC converters

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Abstract— **The paper addresses the problem of designing a stabilizing control for switched affine systems in presence of a model uncertainties. We formulate the problem both in the case where the set of affine subsystems is finite and in the case where the set of affine subsystems is not finite but given by a convex polytope, i.e., the convex hull of finitely many affine subsystems. The main contribution of this work shows how to include in the design an integral action and how a switched control with a global asymptotic stability property can be deduced. It is proved that the design procedure ensures zero steady state error on the controlled output when the discrepancy between the model and the real system is bounded. Finally, a DC/DC Flyback converter is considered to illustrate the effectiveness of the proposed method. We also show that the proposed strategy allows to cancel the steady state error in mean value when the continuous time feedback is sampled.**

Index Terms— **Integral action, switched affine systems, DC-DC power converter.**

I. INTRODUCTION

A considerable interest has been devoted to switched systems by many researchers both for theoretical and practical reasons. These hybrid systems consists of continuous or discrete time dynamical subsystems and a switching rule that determines at each instant of time the active subsystem [13]. They are encountered in many applications such as embedded systems, automotive, aerospace, and many other fields. From a theoretical point view, the analysis and design problems are very challenging and many contributions have been proposed during the last decades (see, for instance, [14], [17] and references therein).

Here, we focus on a specific class of switched systems called switched affine systems. This class captures essential features of many applications including power systems and power electronics [1], [7], [15], [18]. It is also characterized by the fact that many challenging problems have not been solved yet mainly because of the affine nature of the linear subsystems which introduces additional difficulties in analysis and control design problems. Among these difficulties, the one related to equilibrium points is of particular interest. There are contributions dedicated to the design of stabilizing switching rules under the assumption that the equilibrium point is perfectly known in advance [6], [8], [10], [19], [12]. However, these approaches cannot be applied in the realistic case when a certain discrepancy in the modeling is considered.

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Only few papers have investigated the case of uncertain equilibrium points [16]. In [3], the authors proposed an adaptive control design that guarantees that the closed loop system has global convergence properties despite the fact that the equilibrium point is not perfectly known. In this work, the mismatch in the modeling which is investigated, recovers the case of uncertain equilibrium but not only and is more general. Assuming a bounded discrepancy in the modeling, an integral action is introduced in the design in order to guarantee global asymptotic convergence of the controlled output to its reference.

The paper is organized as follows. In the next section, we first recall the classical state feedback switching control design problem for switched affine systems and its solution in the case where the set of affine subsystems is finite. We also formulate the problem in the case where the set of affine subsystems is not finite. We assume that this set is given by a convex polytope, i.e., the convex hull of finitely many affine subsystems. Section III is dedicated to the main contribution of this paper. A methodology to include an integral action and design a global stabilizing control law, for both classes of systems considered in this paper, is provided. The result is not trivial since it implies to deal with *weak* Lyapunov functions. When a bounded discrepancy in the modeling occurs, it is proved that the integral action allows to cancel asymptotically the error in the controlled output, thanks to a local exponential stability property of the closed loop [2]. In section IV, the results are applied to a DC- DC power converter subject to unknown load and input variations and in case where the continuous time feedback law is sampled. As expected, the simulations show clearly that all these uncertainties and perturbations are rejected.

Notations: The set composed by the N first integers is denoted by $\mathbb{K} = \{1, \cdots, N\}$. The $(N-1)$-dimension simplex is denoted $\Lambda := \left\{\lambda \in \mathbb{R}^N \mid \forall i \in \mathbb{K},\ \lambda_i \geq 0,\ \sum_{i=1}^N \lambda_i = 1\right\}$. The convex combination of a set of matrices $\mathcal{A} = \{A_1, \cdots, A_N\}$ is denoted $A(\lambda) = \sum_{i=1}^N \lambda_i A_i$, with $\lambda \in \Lambda$. For a square symmetric matrix, $M \succ 0$ ($M \prec 0$) indicates that it is positive (negative) definite.

II. STATE FEEDBACK STABILIZATION

We consider the class of continuous−time switched affine systems given by:

$$\dot{x}(t) = A_\sigma x(t) + b_\sigma \tag{1}$$

where $x : \mathbb{R} \mapsto \mathbb{R}^n$ is the state and $\sigma : \mathbb{R}^n \mapsto \mathbb{K}$ refers to the state dependent switching law that selects at each time one of the N subsystems characterized by the pairs (A_i, b_i),

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$i \in \mathbb{K}$. Our main objective in this paper is the design of adaptive and global stabilizing control laws for this class of hybrid systems in case of parameter uncertainties.

We also consider the class of dynamical systems obtained by taking the convex combination of these N subsystems and which is given by:

$$\dot{x}(t) = A(\lambda)x(t) + b(\lambda) \qquad (2)$$

where the control λ takes its values in the whole simplex Λ.

An important feature of this purely continuous and non linear class of dynamical systems is the fact that it allows to characterize the set of equilibrium points of the switched affine systems (1). Indeed, it has been shown in [6], [8], [9], [10] that the set of accessible equilibrium points can be defined as follows:

Definition 1: Let $\Lambda^{\mathcal{H}}$ be the subset of Λ such that $A(\lambda)$ is Hurwitz, that is:

$$\Lambda^{\mathcal{H}} := \{\lambda \in \Lambda : \ \exists P \succ 0, \ A^T(\lambda)P + PA(\lambda) \prec 0\}. \quad (3)$$

Definition 2: Let X^e be the set of the equilibrium points defined as:

$$X^e := \{x^e \in \mathbb{R}^n, \ x^e = -A(\lambda^e)^{-1}b(\lambda^e), \ \lambda^e \in \Lambda^{\mathcal{H}}\} \quad (4)$$

A classical approach for the stabilization of a given equilibrium point characterized by the pair (x^e, λ^e) is to use the following switched state feedback law [6][8]:

$$\sigma(x) \in \arg\min_{i \in \mathbb{K}} (x - x^e)^T P(A_i x + b_i), \qquad (5)$$

where $P \succ 0$ satisfies the Lyapunov equation $A(\lambda^e)^T P + PA(\lambda^e) + Q \prec 0$, for a given $Q \succ 0$.

Another interest of the class of dynamical systems given by (2) is related to the property of density of the trajectories of system (1) into trajectories generated by (2) which allows to use (2) instead of (1) for stability analysis and/or control design [5], [9], [11]. Indeed, this density property is useful to establish some bridges between control law dedicated to (1) and to the one dedicated to (2) and conversely. For example, by means of average value of a switched control law, one can obtain a control law for (2), while by Pulse-width Modulation technic (PWM) a switched based control can be deduced from a control λ.

Now recall that for a given (x^e, λ^e), system (2) can be rewritten as follows:

$$\dot{x} = A(\lambda^e)(x - x^e) + B(x)(\lambda - \lambda^e)_{[1,N-1]} \qquad (6)$$

where the column of matrix $B(x)$ are $B_i(x) = (A_i - A_N)x + (b_i - b_N)$, $i = 1 \cdots, N-1$ and the notation $v_{[k_1,k_2]}$ refers to the components between range k_1 and k_2 of a vector v.

Proposition 1: Let the dynamical system (2) and P obtained as in (5). The equilibrium point corresponding to the pair (x^e, λ^e) is globally asymptotically stable under the state feedback law:

$$\lambda_{[1,N-1]}(x, x^e) = \lambda^e_{[1,N-1]} - Ky(x, x^e), \qquad (7)$$

$$\lambda_N(x, x^e) = \lambda^e_N + \mathbb{1}^T Ky(x, x^e) \qquad (8)$$

with $y(x, x^e) = B^T(x)P(x - x^e)$, $B(x)$ the matrix whose columns are the $B_i(x) = (A_i - A_N)x + (b_i - b_N)$, $K = diag(k_1, \cdots, k_{N-1})$ with positive real numbers $k_i > 0$ and $\mathbb{1} = (1, \cdots, 1)$.

Proof: For a given (x^e, λ^e), and from (6), it is straightforward to show that the derivative of the Lyapunov function $V(x, x^e) = z^T Pz$ with $z = x - x^e \neq 0$ satisfies $\dot{V}(x, x^e) = -z^T Qz - 2y(x, x^e)^T Ky(x, x^e) < 0$ ■

As the control domain Λ is bounded, the embedded control of Proposition 1 must be saturated as follows:

Definition 3: For a given control of the form $\lambda = \lambda^e + \delta\lambda$ with $\lambda^e \in \Lambda$ and $\sum_{i\in\mathbb{K}} \delta\lambda_i = 0$, its saturation denoted by $sat(\lambda)$, is defined by its projection in the direction $\delta\lambda$ on Λ:

$$sat(\lambda) = Proj(\lambda; \delta\lambda) = \lambda^e + \alpha\delta\lambda \qquad (9)$$

where $\alpha = \max\{\alpha \in [0\ 1] : \lambda^e + \alpha\delta\lambda \in \Lambda\}$.

Lemma 1: The values of α are determined by the following relations: $\alpha = \min_{j\in\mathbb{K}} \alpha_j$ and

$$\alpha_j = \begin{cases} \min\left(1, \frac{1-\lambda^e_j}{\delta\lambda_j}\right) & \text{if } \delta\lambda_j > 0 \\ \min\left(1, \frac{-\lambda^e_j}{\delta\lambda_j}\right) & \text{if } \delta\lambda_j < 0 \\ 1 & \text{if } \delta\lambda_j = 0 \end{cases}$$

We are now in position to state the main result of this section.

Theorem 1: Let the dynamical system (2). The equilibrium point corresponding to the pair (x^e, λ^e) is globally asymptotically stable under the saturated version of the state feedback control $\lambda(x)$ of Prop. 1:

$$sat(\lambda(x)) = \begin{cases} \lambda^e_{[1,N-1]} - \alpha Ky(x, x^e), \\ \lambda^e_N + \mathbb{1}^T \alpha Ky(x, x^e) \end{cases} \qquad (10)$$

with function $sat(\cdot)$ and number α defined by (9).

Proof: The proof is easily obtained from the expression of the derivative of V and noticing that the function sat does not modify the sign of the derivative by substituting the term $-2y(x, x^e)^T Ky(x, x^e)$ by $-2\alpha y(x, x^e)^T Ky(x, x^e)$. ■

A key assumption behind the state feedback control laws (5) and (10) is the knowledge of the pair (x^e, λ^e) which allows to determine the feedback law in real time. A question of practical interest which one may ask is whether these control laws can be used in the case where the affine system under interest is subject to parameter variations or uncertainties. We answer this question in the next section by considering an integral action.

III. INTEGRAL ACTION AND PERTURBATION REJECTION

A. Integral action

The design of the proposed control laws are based on the existence of a Lyapunov function and is related to the solvability of the Lyapunov equation given in (5).

In case of constant perturbations, uncertainties or even of a discrete time implementation of the control law, a steady state error may occurs on the control output $y = Cx$. So, one can have the temptation to add in the system an integral action in order to cancel this error. Obviously this cannot be done directly since a Lyapunov equation does not admit an unique solution when the matrix A admits the value 0 as eigenvalue. Nevertheless, this can be achieved indirectly by the following steps:

a) Add integral action by considering the augmented system:

$$\dot{z} = \tilde{A}_i z + \tilde{b}_i + \tilde{c}x^e \tag{11}$$

with $z = (x, I_y)$, where $I_y = \int C(x - x^e)$ (Assume $rank(C) = dim I_y = n_y < n$),

$$\tilde{A}_i = \begin{pmatrix} A_i & 0 \\ C & 0 \end{pmatrix}, \quad \tilde{b}_i = \begin{pmatrix} b_i \\ 0 \end{pmatrix}, \quad \tilde{c}_i = \begin{pmatrix} 0 \\ -C \end{pmatrix}.$$

b) For a given equilibrium (x^e, λ^e), determine a matrix T such that

$$D(\lambda^e) = T^{-1}\tilde{A}(\lambda^e)T = \begin{pmatrix} D_s & 0 \\ 0 & 0 \end{pmatrix}$$

where the n_s eigenvalues of matrix Ds all lie in the open left-half plane.

c) Solve the Lyapunov equation $P_s D_s + D_s^T P_s + Q_s = 0$ with $Q_s \succ 0$

d) For a given $\rho > 0$, define $P(\lambda^e)$ as

$$P = T^{-T} \begin{pmatrix} P_s & 0 \\ 0 & \rho Id \end{pmatrix} T^{-1}. \tag{12}$$

Then, P satisfies $PA(\lambda^e) + A(\lambda^e)^T P + Q = 0$ where $Q = T^{-T} \begin{pmatrix} Q_s & 0 \\ 0 & 0 \end{pmatrix} T^{-1} \geq 0$.

e) Define the weak Lyapunov function $V(z) = (z - z^e)^T P(z - z^e)$ with $z^e = (x^e, I_y^e)$ for any I_y^e.

Theorem 2: Consider an equilibrium point corresponding to a pair (x^e, λ^e) of (2). For the augmented system (11), define the weak Lyapunov function $V(z) = (z - z^e)^T P(z - z^e)$ with P satisfying (12) and form the following two switching laws:

$$\sigma(z) \in \arg\min_{i \in \mathbb{K}} (z - z^e)^T P(\tilde{A}_i z + \tilde{b}_i + \tilde{c}x^e), \tag{13}$$

or

$$sat(\lambda(z)) = \begin{cases} \lambda^e_{[1,N-1]} - \alpha K y(z, z^e), \\ \lambda^e_N + \mathbb{1}^T \alpha K y(z, z^e) \end{cases} \tag{14}$$

with $y(z) = \tilde{B}^T(z)P(z - z^e)$, with matrix $\tilde{B}(z)$ whose columns are $\tilde{B}_i(z) = (\tilde{A}_i - \tilde{A}_N)z + (\tilde{b}_i - \tilde{b}_N)$, $K = diag(k_1, \cdots, k_{N-1})$ with positive real numbers $k_i > 0$ and with function $sat(\cdot)$ defined by (9). Then, both switching law stabilize globally and asymptotically the state x on the equilibrium x^e. Moreover, if the pair $(\tilde{A}(\lambda^e), \tilde{B}(z^e))$ is controllable, the convergence rate is locally exponential.

Proof: The derivative of V along the direction defined by λ^e, is given by:

$$\begin{aligned} \dot{V}(z; \lambda^e) &= 2(z - z^e)^T P(\tilde{A}(\lambda^e)z + \tilde{b}(\lambda^e) + \tilde{c}x^e) && (15) \\ &= 2(z - z^e)^T P \tilde{A}(\lambda^e)(z - z^e) && (16) \end{aligned}$$

The later term is obtained using the fact that, for any value I_y, the point $z^e = (x^e, I_y)$ is an equilibrium point of the system: $\dot{z} = \tilde{A}(\lambda^e)z + \tilde{b}(\lambda^e) + \tilde{c}x^e$. It follows using the definition of P, that:

$$\dot{V}(z; \lambda^e) = -(z - z^e)^T Q(z - z^e) \leq 0\,. \tag{17}$$

Now, note that, by definition of T and Q, the subspace $\mathcal{K}er\tilde{A}(\lambda^e) = \mathcal{K}erQ^{\frac{1}{2}}$ (and is generated by the $n + n_y - n_s$ last column vectors of T) which means that $Q^{\frac{1}{2}}v = 0$ if and only if $\tilde{A}(\lambda^e)v = 0$. So, from (17), $\dot{V}(z; \lambda^e) = 0$ if and only if $(z - z^e) \in \mathcal{K}er\tilde{A}(\lambda^e)$. As it is assumed that $A(\lambda^e)$ is Hurwitz, it follows that $\mathcal{K}erA(\lambda^e) = \{0\}$ and then $(z - z^e) \in \mathcal{K}er\tilde{A}(\lambda^e)$ implies

$$(z - z^e) = (x - x^e, I_y - I_y^e) = (0, I_y - I_y^e)\,.$$

This is obvious in view of the form of matrix $\tilde{A}(\lambda^e)$.

Now, applying a switching law of the type (13) or (14) yields to a derivative of V along the trajectory satisfying:

$$\dot{V}(z; u(z)) \leq -(z - z^e)^T Q(z - z^e) \leq 0 \tag{18}$$

with $u(z) = \sigma(z)$ or $sat(\lambda(z))$. The first inequality is justified by the following facts:

- when $u(z) = \sigma(z)$, the chosen direction corresponds to a steepest descend direction which is more decreasing than the direction corresponding to λ^e.
- when $u(z) = sat(\lambda(z))$, a simple calculus shows that:
  $$\dot{V}(z; u(z)) = \dot{V}(z; \lambda^e) - 2\alpha y(z, z^e)^T K y(z, z^e) \leq \dot{V}(z; \lambda^e)\,.$$

So, it can be concluded that both switching laws are globally asymptotically stable for the equilibrium x^e since $\dot{V}(z; u(z)) = 0$ if only if $x = x^e$.

Moreover, let us show that the convergence of x to x^e is locally exponential. In closed loop with $u(z) = sat(\lambda(z))$, the system evolves following the dynamic:

$$\dot{z} = (\tilde{A}(\lambda^e) - \alpha\tilde{B}(z)K\tilde{B}^T(z)P)(z - z^e)\,. \tag{19}$$

The linearization of the closed loop around the equilibrium z^e leads to:

$$\dot{\delta}z = A_\ell \delta z$$

with $A_\ell = \tilde{A}(\lambda^e) - \alpha\tilde{B}(z^e)K\tilde{B}^T(z^e)P$. It follows that:

$$\begin{aligned} PA_\ell + A_\ell^T P &= P\tilde{A}(\lambda^e) + \tilde{A}(\lambda^e)^T P - 2\alpha P\tilde{B}(z^e)K\tilde{B}^T(z^e)P \\ &= -Q - 2\alpha P\tilde{B}(z^e)K\tilde{B}^T(z^e)P\,. \end{aligned}$$

Suppose that

$$\mathcal{K}er(\tilde{B}^T(z^e)P) \cap \mathcal{K}erQ^{\frac{1}{2}} = \{0\}\,. \tag{20}$$

Then we obtain $PA_\ell + A_\ell^T P < 0$ which implies that the matrix A_ℓ is Hurwitz and the nonlinear system (19) is locally exponentially stable. Let us show that condition (20) is effectively verified. Since the pair $(\tilde{A}(\lambda^e), \tilde{B}(z^e))$ is controllable, when expressed in basis T, it takes the form

$$\left(\begin{pmatrix} D_s & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} \bar{B}_1 \\ \bar{B}_2 \end{pmatrix} \right)$$

with $\bar{B} = \begin{pmatrix} \bar{B}_1 \\ \bar{B}_2 \end{pmatrix} = T^{-1}\tilde{B}$ and it is still controllable. It can be noted that any row of $\bar{B}_2$ cannot be identically zeros (if not the previous pair would not be controllable). Thus, by following the decomposition of $\bar{B}$, the product $\bar{B}\bar{B}^T$ can be written as:

$$\bar{B}\bar{B}^T = \begin{pmatrix} \bar{B}_1\bar{B}_1^T & \bar{B}_1\bar{B}_2^T \\ \bar{B}_2\bar{B}_1^T & \bar{B}_2\bar{B}_2^T \end{pmatrix}$$

and we have $\bar{B}_2\bar{B}_2^T > 0$ since the row of $\bar{B}_2$ are not identically zeros. Note that the expression in basis T of $Q + 2\alpha P\tilde{B}(z^e)K\tilde{B}^T(z^e)P$ takes the form:

$$\begin{pmatrix} Q_s & 0 \\ 0 & 0 \end{pmatrix} + 2\alpha K \begin{pmatrix} P_s & 0 \\ 0 & \rho \end{pmatrix}\begin{pmatrix} \bar{B}_1\bar{B}_1^T & \bar{B}_1\bar{B}_2^T \\ \bar{B}_2\bar{B}_1^T & \bar{B}_2\bar{B}_2^T \end{pmatrix}\begin{pmatrix} P_s & 0 \\ 0 & \rho \end{pmatrix}.$$

Thus, taking into account that $\bar{B}_2\bar{B}_2^T > 0$, it is clear that (20) is verified. Therefore, we conclude that exponential stability is achieved. Under the same hypothesis, the same result can be established if the switched law $u(x) = \sigma(z)$ is used instead of $u(z) = sat(\lambda(z))$. ■

B. Perturbation rejection

Suppose now that the real process is described by equations:

$$\dot{x} = \xi(x, \lambda) \tag{21}$$

$$y = \zeta(x, \lambda) \tag{22}$$

where the function ξ and ζ are assumed continuously differentiable but unknown. In view of Theorem 2, let $X \times Y \subset \mathbb{R}^n \times \mathbb{R}^{n_y}$ be some compact sets containing the nominal equilibrium $(x^e, 0, \lambda^e)$ and such that the set $X \times Y \times \Lambda$ is forward invariant for the extended system (11) in closed-loop with (13) or (14). Such compact sets exist in view of Theorem 2. We have then the following result.

Proposition 2: For any open neighborhood $\mathcal{N}_X$ of X, there exists a $\delta > 0$ such that if the discrepancies between systems satisfy the following bound:

$$\|A(\lambda)x + b(\lambda) - \xi(x, \lambda)\| + \|Cx - \zeta(x, \lambda)\| < \delta$$

for all $(x, \lambda) \in \mathcal{N}_X \times \Lambda$, then under feedback laws of Theorem 2, the real process (21) has equilibria and at any such point $C(x - x^e) = 0$.

Proof: As both proposed feedbacks in Theorem 2 stabilize globally and locally exponentially the augmented system (11) and as the perturbation is bounded on the whole state space Proposition 2 (and in particular Lemma 4) in [2] yields the result. ■

IV. Application to DC-DC power converters

The theoretical approach we propose is applied to a DC-DC power converter subject to parameter uncertainties as well as load and input variations. The so-called Flyback converter is depicted in Fig 1 and it is composed of passive components (a resistor R, an inductor L, a capacitor C), a transformer and two types of switches: a controlled switch (transistor S) and an uncontrolled switch (diode D). The transformer is also supposed ideal with a turns ratio of n.

For simplicity, we discard DCM (Discontinuous Conduction Mode) operating mode since it does not modify the stability property of our proposed control as proved in [4]. The Flyback converter in CCM (Continuous Conduction Mode) is a switched affine system given by:

$$\dot{x} = A_\sigma x + B_\sigma u, \tag{23}$$

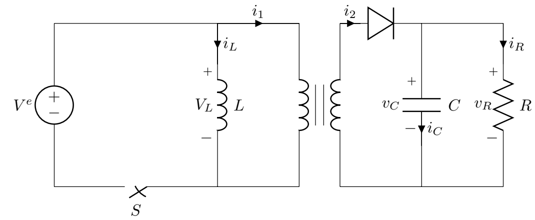

Fig. 1. A Flyback converter

where the state $x = (i_L, v_C)$ is composed by the current i_L that flows through the inductor L and the voltage v_C across the capacitor C, u is the input considered as fixed ($u(t) = V^e$) and cannot be used as a control variable. Therefore, denoting $b_\sigma = B_\sigma u$ allows to use the framework developed in this paper. The control is the switching rule $\sigma : \mathbb{R}^n \to \{1, 2\}$ which indicates the active Mode at each time instant. The matrices A_i and B_i, $i \in \{1, 2\}$ are given by:

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{RC} \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -\frac{n}{L} \\ \frac{n}{C} & \frac{-1}{RC} \end{bmatrix}, B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The control goal is to steer and maintain the output voltage $x_2 = v_C$ to a prescribed value. For simulation purposes, the following nominal values are chosen for the five parameters of the converter: $V_e = 28V$, $R = 75\Omega$, $L = 200\mu H$, $C = 2.6\mu F$ and $n = 2$ and we assume that the target point is $x_2^e = 15V$.

A. Simulation results

1) Embedded versus switched control: Before considering integral action, let us make a comparison between the embedded based control (10) and the switched based control (5). To determine the Lyapunov function $V(x, x^e) = (x - x^e)^T P(x - x^e)$ for all admissible x^e, we solve the following LMI :

$$PA_i + A_i^T P + \alpha_i P \leq 0, \; i = 1, 2 \,.$$

With $\alpha_1 = 772$, $\alpha_2 = 0$, the obtained result is provided by

$$P = \begin{pmatrix} 3.2170 & -0.0032 \\ -0.0032 & 0.0418 \end{pmatrix}.$$

For the embedded control, the matrix K is reduced here to one parameter k_1 and can be used to meet some performance requirements. Fig. 2 shows the start-up transient for three values of this parameter $k_1 = 3.10^{-7},\ 1.10^{-6},\ 7.10^{-6}$ and also the start-up transient corresponding to the switched case. For these simulations, the embedded control is applied using Pulse-width modulation with a chosen cutting frequency equals to $f_s = 10\ MHz$. The switched control law is sampled with a period $T_e = \frac{1}{2f_s}$ in order to reach for both laws, the same number of switchings.

Not surprisingly, it can be observed that the switched based control corresponding to a steepest descent strategy is not the best choice to reduce the settling time as a better transient can be achieved by an appropriate setting of parameter k_1. It can be noticed that a large value of k_i leads to a saturation of the control thus recovering the switched case, while a very small value corresponds to an open loop control $\lambda(t) \approx \lambda^e$ for all t. Obviously, the later is not suitable due to large oscillations in transient of the natural response of the converter (not shown in the Fig. 2).

In the sequel and in order to show some comparisons, the best controller is retained that is the embedded control with $k_1 = 3.10^{-7}$.

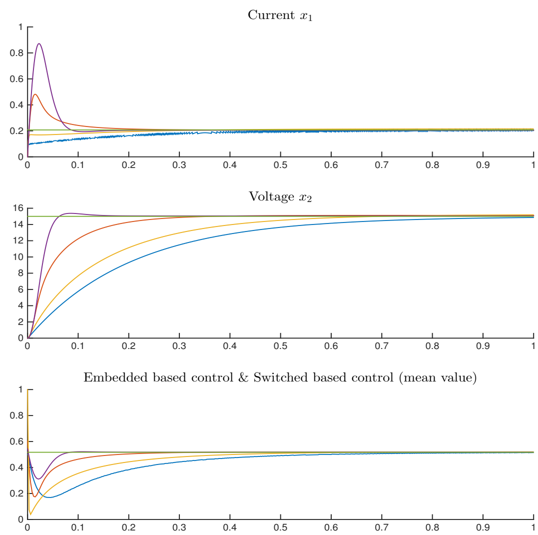

Fig. 2. Start-up transient: Plot 1 shows the current x_1, Plot 2 shows the voltage x_2. Plot 3 shows the control (in average for the switched based control case). Switched based control (Blue); Embedded based control (Purple, $k_1 = 3.10^{-7}$)(Red, $k_1 = 1.10^{-6}$)(Yellow, $k_1 = 7.10^{-6}$). Time is given in (ms).

2) Uncertain parameters: As a simple computation shows the equilibrium points x^e associated to a control value $\lambda^e = (\lambda_1^e, \lambda_2^e) \in \Lambda$. It is determined by :

$$x^e = \begin{pmatrix} \frac{\lambda_1^e V_e}{R(n\lambda_2^e)^2} \\ \frac{\lambda_1^e V_e}{n\lambda_2^e} \end{pmatrix}$$

and it is directly related to the values of the load R and of the voltage input V_e. Thus, the two above control laws presented in the previous subsection are clearly sensitive to these parameters. Therefore, in order to show the effectiveness of the proposed design in case of parameter uncertainties, we assume that R and V_e may change and we do not measure these parameters. We use as a scenario a piecewise constant function with variations between 50Ω and 100Ω for the load, and variations between $20V$ and $40V$ for the input. First, let us design a controller including an integral action. For the fixed target value $x_2^e = 15v$, we consider the augmented state $z = (x, I_y)$ with $I_y = \int(x_2 - x_2^e)dt$. So, by following the proposed methodology of Section III.A with $Q_s = \begin{pmatrix} 400 & 0 \\ 0 & 1 \end{pmatrix}$ and $\rho = 6000$ we get:

$$P = \begin{pmatrix} 3.3227 & -0.0001 & 1.2167 \\ -0.0001 & 0.0400 & -0.0000 \\ 1.2167 & -0.0000 & 5.9619 \end{pmatrix}$$

and we finally consider the embedded control (14): $\lambda_1(z) = sat(\lambda_1^e - Ky(z, z^e)) \in [0\ 1]$ with $K = 3.10^{-7}$. Note that $\lambda_2 = 1 - \lambda_1$.

We make the choice to compare in the simulation the embedded based control design of the previous subsection and the control law chosen in this section including the integral action. Moreover in order to meet some performance in the rejection of perturbations, we consider also the adaptive embedded control proposed in [3] that allows to estimate the unknown parameters R and V_e but in this paper, we also add in the design an integral action.

The results is depicted in Fig.3. We start the scenario of the simulation with the nominal values (ie $V_e = u = 28V$ and $R = 75\Omega$) and a cutting frequency fixed to $f_s = 1\ MHz$. The variations of R and V_e are introduced after $1\ ms$.

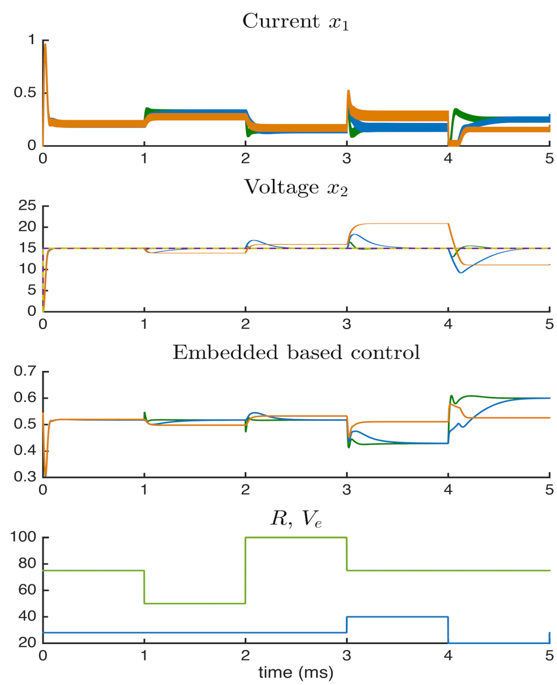

Fig. 3. Load and line transient: Plot 1 shows respectively the current x_1: (Orange) without integral action, (Blue) with integral action, (Green) adaptive embedded control including integral action. Plot 2 shows the voltage x_2 using the same convention, the reference is $x_2^e = 15v$. Plot 3 shows the control u using the same convention. Finally, Plot 4 shows load R and input V_e variations. Time is given in (ms) and $f_s = 1MHz$.

During the first $1ms$, we can see that each control steers the state to the reference x_2^e when all parameters are known. It is observed that once the load and input voltage variations begin, both control laws remain stable. However, a huge steady state error appears on the voltage x_2 (Orange) when the design does not include an integral action. As expected, the embedded control with integral action (Blue) rejects the perturbation induced by the parameter variations and allows the controlled output x_2 to follows its reference. Finally, performances (Green) are improved using both an integral action and an adaptive scheme as proposed in [3]. In this latter case, the load variations are particularly nicely rejected (quasi invisible on the plot) and the effects of input variations are seriously limited in comparison with the other controls.

3) Influence of the cutting frequency: The control values of a power converter is fundamentally of discrete nature (On, Off). Thus, practical implementations of a control law lead to use sampling or PWM technique. As proved in [10], [9] when a continuous time stabilizing state feedback control for an affine system is applied with a sampled period T_s or using PWM technique, a steady state error appears. More precisely, an Input to State Stability (ISS) property established in these papers implies that the state of the system is ultimately bounded in a set whose size is depending of the period T_s. In other words, state x satisfies a relation of the type: $\lim_{t\to\infty} \|x(t)\| \leq \gamma(T_s)$ with γ a positive and monotonically increasing function satisfying $\gamma(0) = 0$.

One more advantage in the inclusion of an integral action is illustrated on Figure 4 which shows the voltage x_2 in steady state for the embedded control with and without an integral action and for different values of the chosen cutting frequency $f_s = 0.5,\ 1,\ 10\ MHz$. As it can be observed on Figure 4, the steady state error increases with $T_s = 1/f_s$ when no integral action is considered (Plot 2) while integral action allows to cancel the mean value of the tracking error (Plot 1).

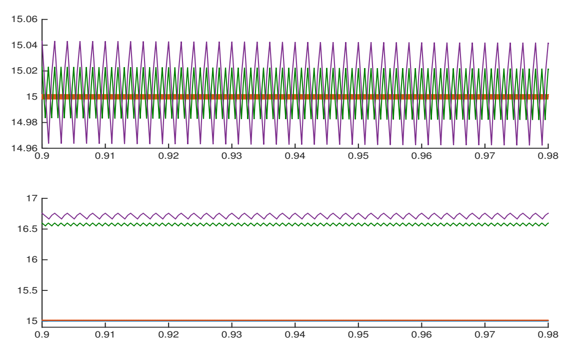

Fig. 4. Steady state voltage x_2 (when $x_2^e = 15v$) for different cutting frequencies (PWM) $f_s = 0.5,\ 1,\ 10\ MHz$ (Purple, Green and Red respectively): Plot 1 shows the voltage for the embedded control with integral action. Plot 2 without integral action. Time is given in (ms).

V. Conclusion

A switched based control design including an integral action for uncertain switched affine systems has been proposed in this paper. A global and asymptotical stability is guaranteed for both derived switching laws from the design. Under the assumption that the discrepancy in the modeling is bounded, the use of an integral action allows to guarantee the existence of an equilibrium for which the reference of the controlled output is always achieved. The results have been applied to a Dc/Dc Flyback converter subject to unknown input and load variations. Simulations clearly show the interest of an integral action to reject these parameter variations. Moreover and in order to improve the dynamic performances, the proposed design has been applied to the state-dependent adaptive switching law proposed in [3]. Finally, when the control law is sampled, it has been shown that there is also a benefit to include this integral action to cancel the steady state error in mean value.

References

[1] Carolina Albea, Germain Garcia, and Luca Zaccarian. Hybrid dynamic modeling and control of switched affine systems: application to dc-dc converters. In *2015 54th IEEE Conference on Decision and Control (CDC)*, pages 2264–2269. IEEE, 2015.

[2] Daniele Astolfi and Laurent Praly. Integral action in output feedback for multi-input multi-output nonlinear systems. *IEEE Transactions on Automatic Control*, 62(4):1559–1574, 2017.

[3] G. Beneux, P. Riedinger, J. Daafouz, and L. Grimaud-Salmon. Robust stabilization of switched affine systems with unknown parameters and its application to dc/dc flyback converters. In *Proceedings of the 2017 IEEE American Control Conference*, May 2017.

[4] G. Beneux, P. Riedinger, J. Daafouz, and L. Grimaud-Salmon. Stabilisation of power converters with uncertain equilibrium: An adaptive switched approach with guarantee of stability in continuous and discontinuous conduction modes. *Proceedings of the IFAC World Congress 2017, Toulouse, France*, 2017.

[5] S. C. Bengea and R. A. Decarlo. Optimal control of switching systems. *Automatica*, 41(1):11 – 27, 2005.

[6] P. Bolzern and W. Spinelli. Quadratic stabilization of a switched affine system about a nonequilibrium point. In *American Control Conference, 2004. Proceedings of the 2004*, volume 5, pages 3890–3895 vol.5, June 2004.

[7] Jean Buisson, Pierre-Yves Richard, and Hervé Cormerais. *On the Stabilisation of Switching Electrical Power Converters*, pages 184–197. Springer Berlin Heidelberg, Berlin, Heidelberg, 2005.

[8] G.S. Deaecto, J.C. Geromel, F.S. Garcia, and J.A. Pomilio. Switched affine systems control design with application to dc−dc converters. *Control Theory & Applications, IET*, 4(7):1201–1210, 2010.

[9] Pascal Hauroigne, Pierre Riedinger, and Claude Iung. Switched affine systems using sampled-data controllers: robust and guaranteed stabilization. *IEEE Transactions on Automatic Control*, 56(12):2929–2935, 2011.

[10] L. Hetel and E. Fridman. Robust sampled data control of switched affine systems. *IEEE Transactions on Automatic Control*, 58(11):2922–2928, Nov 2013.

[11] Brian Ingalls, Eduardo Sontag, and Yuan Wang. An infinite-time relaxation theorem for differential inclusions. *Proceedings of the American Mathematical Society*, 131(2):487–499, 2003.

[12] Zohra Kader, Christophe Fiter, Laurentiu Hetel, and Lotfi Belkoura. Non-quadratic stabilization of switched affine systems. In *20th IFAC World Congress, Toulouse, France*, 2017.

[13] Daniel Liberzon. *Switching in systems and control*. Systems & Control: Foundations and Applications. Springer, 2003.

[14] H. Lin and P. J. Antsaklis. Stability and stabilizability of switched linear systems: A survey of recent results. *IEEE Transactions on Automatic Control*, 54(2):308–322, Feb 2009.

[15] Rami Naim, George Weiss, and Shmuel Ben-Yaakov. H_∞ control applied to boost power converters. *IEEE Transactions on Power Electronics*, 12(4):677–683, 1997.

[16] G. A. Senger and A. Trofino. Switching rule design for affine switched systems with guaranteed cost and uncertain equilibrium condition. *IEEE Transactions on Automatic Control*, 61(7):1925–1930, July 2016.

[17] R. Shorten, F. Wirth, O. Mason, K. Wulff, and C. King. Stability criteria for switched and hybrid systems. *Invited Paper SIAM Rev.*, 49(4):545–592, 2007.

[18] Hebertt Sira-Ramirez and Romeo Ortega. Passivity-based controllers for the stabilization of dc-to-dc power converters. In *Decision and Control, 1995., Proceedings of the 34th IEEE Conference on*, volume 4, pages 3471–3476. IEEE, 1995.

[19] T. A. F. Theunisse, J. Chai, R. G. Sanfelice, and W. P. M. H. Heemels. Robust global stabilization of the dc-dc boost converter via hybrid control. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 62(4):1052–1061, April 2015.