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An Optimal Displacement-Based Leader-Follower Formation Control for Multi-Agent Systems With Energy Consumption Constraints

Reza Babazadeh and Rastko Selmic *

Abstract— **In this paper, we propose an optimal control method for a displacement-based leader-follower formation control of multi-agent systems with energy consumption constraints. We developed a model for electric energy consumption of an agent and then formulate the formation control problem for a set of multi-agents in optimal control context. The proposed method minimizes a weighted cost function that includes formation and energy consumption for a given mission. To solve the formulated problem, we use State-Dependent Riccati Equation (SDRE) method. We choose the weighting matrices of the cost function to be dependent on the energy level of the agents, thus allowing for autonomous adjustment of the agents' trajectories that preserve the integrity of the overall formation in spite of energy levels. Simulation results illustrate the effectiveness of the proposed method in both two- and three-dimensional spaces.**

I. INTRODUCTION

Formation control has been an active research topic for more than two decades [1]. Results have shown that when multiple agents are assigned a certain task, the overall performance and efficiency is improved if agents form a certain geometric shape [1]. Various formation examples can be seen in nature, e.g. flocking birds, school of fishes, packs of wolves and other animals. The goal of formation control is to develop control methods for a group of agents to conduct a certain mission while maintaining the desired geometric characteristic (shape, distances, angles, etc.). In general, those methods can be categorized as position-based, distance-based and displacement-based formation control [2], [3].

A new formation control that is based on intra-agents bearings is proposed in [4] where the control algorithms preserve bearing formation regardless of distances. In [5], a distance-based formation control using angles between agents is proposed.

In a leader-follower formation there is one or more leaders that the rest of agents (followers) follow [6], [7], [8]. Behavioral approach defines a desired behavior such as obstacle or collision avoidance [9], [10]. Virtual structure approach assumes desired formation as a whole virtual rigid structure [11], [12]. There are also other classifications in the literature and can be found in review articles e.g. [1], [2].

There are several solutions proposed for formation control problems in the literature including optimal control methods. The formation control problem is formulated as a Linear Quadratic (LQ) optimal control problem in [13]. A Linear Quadratic Regulator (LQR)-based optimal controller has been proposed for the leader-follower formation in [14] and for formation reconfiguration and collision avoidance in [15]. Various cost/objective functions were proposed for formation control problems and solved in standard optimal control framework, e.g. [9], [10], [16], [17], [18]. A leader-follower formation control using SDRE control method was proposed in [8], [19].

The contribution of this paper is threefold. First, we develop a new dynamic model that includes agents' energy consumption. Second, we propose an SDRE-based sub-optimal leader-follower formation control for a set of agents that takes into account agents' energy levels. Based on SDRE method, the local asymptotic stability of the closed-loop system is guaranteed. And third, we extend a solution with guaranteed global asymptotic stability for the single-integrator scenario. All results are illustrated with detailed simulation results in both two and three-dimensional spaces.

A. Preliminaries and Notation

The following notation is used in this article. Let $\mathbb{R}$ indicate the real numbers set, $\mathbb{N}$ the natural numbers set, and $\mathbb{Z}$ the integer numbers. Let $\mathbb{R}^n$ indicate n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denote the set of $n \times m$ real matrices. Let I_n denote $n \times n$ identity matrix and $0_{n\times m}$ denote the $n\times m$ zero matrix. We use 1_n to indicate a column vector with n rows and all entries equal to one. Similarly, 0_n denotes a vector with n rows and all elements of zero. With $diag[a_1, a_2, ...]$ we denote a block diagonal matrix with matrices a_i on its diagonal. Let $\otimes$ denotes the Kronecker product. By $\|.\|$ we denote the Euclidean norm.

II. PROBLEM FORMULATION

We consider a set of N agents described by

$$\dot{x_i} = f_i(x_i, u_i) \tag{1a}$$

$$y_i = h_i(x_i, u_i), \tag{1b}$$

where $x_i \in \mathbb{R}^q$, $u_i \in \mathbb{R}^p$, and $y_i \in \mathbb{R}^r$ denote the state, input, and output vectors of the agent $i \in \{1, ..., N\}$, q, p, r are dimensions of state, input, and output spaces respectively, and $f_i : \mathbb{R}^{q\times p} \rightarrow \mathbb{R}^q$, $h_i : \mathbb{R}^{q\times p} \rightarrow \mathbb{R}^r$. Define the augmented output vector of swarm $\mathbf{y} = [y_1^T, ..., y_N^T]^T \in \mathbb{R}^{rN}$, a desired formation for the set of agents can be described by M-constraints as

$$F(\mathbf{y}) = F(\mathbf{y}^*), \tag{2}$$

where $F : \mathbb{R}^{rN} \rightarrow \mathbb{R}^M$, for some given $\mathbf{y}^* \in \mathbb{R}^{rN}$. A formation control problem for the set of agents, described

R. Babazadeh and R. Selmic are with Department of Electrical and Computer Engineering, Concordia University, Montreal, QC, Canada. r.babaza@encs.concordia.ca, rastko.selmic@concordia.ca
* Corresponding author

by equations (1), is to design a control law u_i such that the set

$$E_{\mathbf{y}^*} = \Big\{[x_1^T, ..., x_N^T]^T : F(\mathbf{y}) = F(\mathbf{y}^*)\Big\}, \quad (3)$$

becomes asymptotically stable [1].

A. Agent Modeling

There are several dynamical models of agents in the literature (see [1] for the survey of models). In this paper we model agents as a point mass using single-integrator and double-integrator models, [1], [3].

1) Single-Integrator Model: The agent model is given by

$$\dot{p_i} = u_i, \quad (4)$$

where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ denote the position and control input of agent i in an n-dimensional space, $n \in \{2, 3\}$, with respect to a global coordinate system. Forming the state vector for a set of N agents as $\mathbf{x}_s = [p_1^T, ..., p_N^T]^T$, and input vector as $\mathbf{u} = [u_1^T, ..., u_N^T]^T$, we can write the state-space form for a set of N agents modeled by single-integrator dynamics as

$$\dot{\mathbf{x}}_s = \hat{B}\mathbf{u}, \quad (5)$$

where

$$\hat{B} = I_N \otimes I_n. \quad (6)$$

2) Double-Integrator Model: The agents are modeled by

$$\dot{p_i} = v_i \quad (7a)$$

$$\dot{v_i} = u_i, \quad (7b)$$

where $v_i \in \mathbb{R}^n$ denotes the velocity of agent i with respect to a global coordinate system and the control input is the acceleration of the agent.

Defining the state vector as $\mathbf{x}_d = [p_1^T, ..., p_N^T, v_1^T, ..., v_N^T]^T$, and input vector as $\mathbf{u} = [u_1^T, ..., u_N^T]^T$, the state space form for a set of N agents modeled by the double-integrator model is then given by

$$\dot{\mathbf{x}}_d = \tilde{A}\mathbf{x}_d + \tilde{B}\mathbf{u}, \quad (8)$$

with

$$\tilde{A} = \begin{bmatrix} 0_{N\times N} & I_N \\ 0_{N\times N} & 0_{N\times N} \end{bmatrix} \otimes I_n \quad (9a)$$

$$\tilde{B} = \begin{bmatrix} 0_{N\times N} \\ I_N \end{bmatrix} \otimes I_n, \quad (9b)$$

where $\mathbf{x}_d \in \mathbb{R}^{2nN}$ and $\mathbf{u} \in \mathbb{R}^{nN}$ are respectively the aggregate state and control input of all agents.

B. Agent Energy Model

In general, energy consumption of agents with electric energy sources is a function of several parameters such as speed of electric motors, aerodynamic shape of an agent, agent's weight, electric efficiency, external disturbances such as wind speed, and more. The energy level of an agent i can be modeled as

$$\dot{l_i}(t) = \mathcal{L}_i(v_i, u_i), \quad (10)$$

where $l_i(t) \in \mathbb{R}_+$ is the energy level at time t, $v_i \in \mathbb{R}^q$ and $u_i \in \mathbb{R}^p$ are velocity and control inputs respectively, and $\mathcal{L}_i$ is a nonlinear mapping: $\mathbb{R}^{q\times p} \to \mathbb{R}$. In modeling of energy consumption we focus here on ground-based agents where the energy consumption depends on traveled distance. We assume that agents use no energy when they are stationary (no hovering modeled here). For simplicity, we assume that the gear ratio between electric motor and tires is one. Suppose $l_{0i} \in \mathbb{R}_+$ is an initial energy level of an agent i at initial time t_0, expressed as a percentage of maximum energy storage capacity (current energy level over maximum energy that can be stored) and α_i is the normalized energy consumption rate per traveled distance unit. Then $l_i(t)$ can be described by

$$l_i(t) = l_i(t_0) - \alpha_i \int_{t_0}^{t} \|\dot{p_i}\| dt. \quad (11)$$

The total consumed energy of agent i can be expressed as

$$l_i(t_0) - l_i(t_f) = \alpha_i \int_{t_0}^{t_f} \|v_i\| dt, \quad (12)$$

where t_f is the final time. Consequently, from the equation (11) the agent energy dynamics can be written as

$$\dot{l_i}(t) = -\alpha_i \|v_i\|, \quad (13)$$

with the initial condition $l_i(t_0) = l_{0i}$.

Remark 1: The experimental results shown in [20], [21] justify use of our proposed model (13) for energy dynamics of mobile robots.

C. Augmented Agent Model with Energy State

Adding the energy dynamics as an extra state to the agent model results in an augmented dynamic model. For the single-integrator model, the augmented state vector is $\mathbf{x}_{si} = [p_i^T, l_i]^T$ where energy dynamic is given by (13). Similarly, for the double-integrator model, the augmented state vector is $\mathbf{x}_{di} = [p_i^T, v_i^T, l_i]^T$.

III. OPTIMAL FORMATION CONTROL FOR AGENTS WITH ENERGY CONSTRAINTS

Suppose a nonlinear in the state but affine in control system is given by

$$\dot{\mathbf{x}} = f(\mathbf{x}) + B(\mathbf{x})\mathbf{u}, \quad (14)$$

where $\mathbf{x} \in \mathbb{R}^q$ and $\mathbf{u} \in \mathbb{R}^p$ are the state and input vectors of the system respectively. Note that $f(\mathbf{x}) : \mathbb{R}^q \to \mathbb{R}^q$ is a nonlinear mapping and $B(\mathbf{x}) \in \mathbb{R}^{q\times p}$ is a matrix-valued, state-dependent input matrix. Under the assumption that $f(\mathbf{x}) \in C^1$, we can write (14) in a linear-like form

$$\dot{\mathbf{x}} = A(\mathbf{x})\mathbf{x} + B(\mathbf{x})\mathbf{u}, \quad (15)$$

where $A : \mathbb{R}^q \to \mathbb{R}^{q\times q}$ is a matrix valued function and is non-unique for non-scalar systems [22].

Remark 2: There are several proposed methods for finding $A(\mathbf{x})$ in the literature. Note that non-uniqueness of $A(\mathbf{x})$ results in extra degrees of freedom in control design procedure. The reader may refer to [22], [23], [24] for more details.

Suppose an LQR-like state-dependent cost function is given by

$$J(\mathbf{x}_0, \mathbf{u}) = \frac{1}{2}\int_{t_0}^{\infty}(\mathbf{x}(t)^T Q(\mathbf{x})\mathbf{x}(t) + \mathbf{u}(t)^T R(\mathbf{x})\mathbf{u}(t))dt, \tag{16}$$

where $\mathbf{x}_0$ is the initial state of the system at the initial time t_0. Also, $Q: \mathbb{R}^q \rightarrow \mathbb{R}^{q\times q}$ is a non-negative, symmetric matrix-valued function and $R: \mathbb{R}^q \rightarrow \mathbb{R}^{p\times p}$ is a symmetric positive definite matrix-valued function. The objective of the optimal control problem is to find a control $\mathbf{u}$ for the system (15) such that it minimizes the cost function (16) while stabilizing the system. The desired optimal control will be in the state feedback form of $\mathbf{u}(\mathbf{x}) = -k(\mathbf{x})\mathbf{x}(t)$, with $k(\mathbf{x})$ being the optimal feedback gain.

For solution feasibility of the SDRE control method, the following conditions are required.

- Condition 1: The function $f(\mathbf{x})$ is continuously differentiable $f(\mathbf{x}) \in C^1$ and $B(\mathbf{x})$ is a matrix valued function such that $B(\mathbf{x}) \in C^0$.
- Condition 2: The origin $\mathbf{x} = 0$ is an equilibrium of the system with zero input such that $f(0) = 0$.
- Condition 3: The pair $\{A(\mathbf{x}), B(\mathbf{x})\}$ is point-wise stabilizable and pair $\{C(\mathbf{x}), A(\mathbf{x})\}$ is point-wise detectable in linear sense for all x, where $C^T(\mathbf{x})C(\mathbf{x}) = Q(\mathbf{x})$.

The following lemma can be found in [22], [25]:

Lemma 1 ([25]): Assume that the nonlinear system described by (14) meets the Conditions 1-3. Then there exists a sub-optimal control law in the form of a state feedback

$$\mathbf{u}(\mathbf{x}) = -k(\mathbf{x})\mathbf{x}(t). \tag{17}$$

The feedback gain $k(\mathbf{x})$ is given by

$$k(\mathbf{x}) = R^{-1}(\mathbf{x})B^T(\mathbf{x})S(\mathbf{x}), \tag{18}$$

where $S(\mathbf{x})$ is a unique, symmetric, positive-definite solution of the following corresponding state-dependent Riccati equation

$$\begin{aligned} Q(\mathbf{x}) + A^T(\mathbf{x})S(\mathbf{x}) &+ S(\mathbf{x})A(\mathbf{x}) \\ &- S(\mathbf{x})B(\mathbf{x})R^{-1}(\mathbf{x})B^T(\mathbf{x})S(\mathbf{x}) = 0. \end{aligned} \tag{19}$$

The control law (17) minimizes the cost function (16) asymptotically and guarantees local asymptotic stability of the closed-loop system.

Remark 3: Substituting control law (17) in the system (15), the closed-loop dynamic is then in the form $\dot{\mathbf{x}} = A_{CL}(\mathbf{x})\mathbf{x}$,

$$A_{CL}(\mathbf{x}) = A(\mathbf{x}) - B(\mathbf{x})R^{-1}(\mathbf{x})B^T(\mathbf{x})S(\mathbf{x}), \tag{20}$$

where $A_{CL}(\mathbf{x})$ is a closed-loop SDC matrix [26].

Lemma 2 ([27]): Assume that the system (14) satisfies Conditions 1-3 and $A_{CL}(\mathbf{x})$ is symmetric for all x, then SDRE solution is globally asymptotically stable.

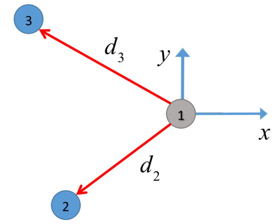

Fig. 1. Displacement-based formation for $N = 3$ agents.

A. Cost Function for Optimal Formation Control

In the leader-follower formation control, two main objectives are tracking performance and preserving the desired formation. Other objectives such as energy consumption, collision avoidance, obstacle avoidance, time of operation, control effort, etc. can also be included in the cost function. Here, we formulate the problem such that the obtained optimal control law preserves formation and satisfies required tracking performance.

Selection of Weighting Matrices: Although constant weighting matrices are widely used, our aim is to relate the weighting matrix of the cost function to energy levels of each agent and thus include the energy model into the control algorithm. We propose that the control weighting factor R_i (an element on the i-th row, i-th column of diagonal weighting matrix R) is a function of the energy level of each agent

$$R_i(t) = \frac{1}{l_i(t)}. \tag{21}$$

The weighting factor (21) is a time-varying and state (energy) dependent, resulting in an optimal controller that is state-dependent with continuous tuning throughout the state trajectory.

B. Displacement-Based Optimal Leader-Follower Formation Control

Consider a formation of N agents where the first agent is the leader and others are followers. The leader is assigned a reference trajectory $p^\star$ to follow also it can sense the followers' relative position.

1) Single-Integrator Model: We define the desired formation in a displacement-based form where desired followers' positions are vectors with regard to the leader's local coordinate frame. The formation vector in n-dimensional space, with the first agent being the leader, is given by

$$\hat{\mathbf{d}}_1 = [0_n^T, d_2^T, ..., d_N^T]^T, \tag{22}$$

where

$$d_i = p_i^\star - p_1^\star, \qquad i \in \{2, ..., N\}. \tag{23}$$

Note that $p_i^\star$ is the desired position of the agent i in the global coordinate frame. Fig. 1 shows the desired formation for $N = 3$ agents.

We propose a cost function that includes three parts related to tracking, formation stability, and energy consumption:

$$J = J_{tr} + J_{fm} + J_{en}. \tag{24}$$

The cost function components are given by

$$J_{tr} = \frac{1}{2}\int_{t_0}^{\infty}(p_1 - p^\star)^T Q_1 (p_1 - p^\star)dt, \quad (25)$$

$$J_{fm} = \frac{1}{2}\sum_{i=2}^{N}\int_{t_0}^{\infty}(p_i - p_1 - d_i)^T Q_i (p_i - p_1 - d_i)dt, \quad (26)$$

$$J_{en} = \frac{1}{2}\sum_{i=1}^{N}\int_{t_0}^{\infty}(\alpha_i u_i)^T R_i (\alpha_i u_i)dt. \quad (27)$$

Let define the error vector as

$$\hat{\mathbf{e}}_1 = [e_1^T, e_2^T, ..., e_N^T]^T, \quad (28)$$

where e_1 is the leader's reference tracking error given by

$$e_1 = p_1 - p^\star. \quad (29)$$

The formation errors are

$$e_i = p_i - p_1 - d_i, \qquad i \in \{2, ..., N\}. \quad (30)$$

The error dynamics is then given by

$$\dot{e_1} = \dot{p_1} = u_1 \quad (31a)$$

$$\dot{e_i} = \dot{p_i} - \dot{p_1} = u_i - u_1. \quad (31b)$$

The cost function (24) can now be written in the quadratic form

$$\begin{aligned} J^\star = min\frac{1}{2}\int_{t_0}^{\infty}&(\hat{\mathbf{e}}_1^T Q_s(\mathbf{x})\hat{\mathbf{e}}_1 + \mathbf{u}^T R_s(\mathbf{x})\mathbf{u})dt \\ s.t.& \\ &\dot{\hat{\mathbf{e}}}_1 = \hat{B}_1\mathbf{u} \\ &Q_s(\mathbf{x}) = diag[Q_1, ..., Q_N] \geq 0 \\ &R_s(\mathbf{x}) = diag[R_1, ..., R_N] > 0, \end{aligned} \quad (32)$$

where

$$\hat{B}_1 = \begin{bmatrix} 1 & 0_{N-1}^T \\ -1_{N-1} & I_{N-1} \end{bmatrix} \otimes I_n. \quad (33)$$

Note that in the optimal control problem (32) the error dynamic is linear and only the weighting matrices of cost function are state-dependent. The notation $Q_s(\mathbf{x})$ and $R_s(\mathbf{x})$ are used for showing state-dependency. As a result, by proper selection of $Q_s(\mathbf{x})$ we can ensure satisfaction of Conditions1-3 of Lemma 1. Then based on Lemma 1, we conclude that the sub-optimal state-dependent feedback control law that minimizes the cost function in (32) and locally asymptotically stabilize the closed-loop system is

$$\mathbf{u}^\star = -k\hat{\mathbf{e}}_1, \quad (34)$$

where k is an sub-optimal gain given by

$$k = R_s(\mathbf{x})^{-1}\hat{B}_1^T S, \quad (35)$$

and S is the positive definite solution of the following state-dependent Riccati equation

$$S\hat{B}_1 R_s(\mathbf{x})^{-1}\hat{B}_1^T S = Q_s(\mathbf{x}). \quad (36)$$

Theorem 3.1: Select the state weighting matrix as

$$Q_s = K \times I, \quad (37)$$

where K is a positive scalar. Then, for a set of agents described by the single-integrator model, the proposed control law (34) results in global asymptotic stability of the closed-loop system.

Proof: Substituting (34) in error dynamic, the closed-loop error dynamics of the system is

$$\dot{\mathbf{e}}_1 = A_{CS}\mathbf{e}_1, \quad (38)$$

where

$$A_{CS} = -\hat{B}_1 R_s(\mathbf{x})^{-1}\hat{B}_1^T S. \quad (39)$$

Rearranging equation (36) yields

$$A_{CS} = -S^{-1}Q_s. \quad (40)$$

Since S as a solution of Riccati equation is symmetric, by selecting Q_s as proposed in (37), the closed-loop SDC matrix A_{CS} will be symmetric for all x. According to Lemma 2, this implies global asymptotic stability of the closed-loop system. ■

Theorem 3.1 provides a sufficient condition for the global stability of the closed-loop system in case of the single-integrator dynamics.

Remark 4: Choosing constant weighting matrix, the optimal control problem (32) reduces to standard LQR that guarantees global asymptotic stability of the closed-loop system.

2) Double-Integrator Model: In case of a double-integrator model, we define the state vector as $\mathbf{x}_d = [p_1^T, ..., p_N^T, v_1^T, ..., v_N^T]^T$. Similarly to (22), N agents in a formation can be defined by a constant formation vector

$$\hat{\mathbf{d}}_2 = [0_n^T, d_2^T, ..., d_N^T, 0_{nN}^T]^T, \quad (41)$$

where d_i for $i \in \{2, ..., N\}$ is given by (23). Defining the error vector we have

$$\hat{\mathbf{e}}_2 = [e_1^T, ..., e_N^T, v_1^T, v_2^T - v_1^T, ..., v_N^T - v_1^T]^T, \quad (42)$$

where components e_i are defined in (29) and (30). Derivatives are then given by

$$\dot{e_1} = \dot{p_1} = v_1 \quad (43a)$$

$$\dot{e_i} = \dot{p_i} - \dot{p_1} = v_i - v_1 \quad (43b)$$

$$\dot{v}_i - \dot{v}_1 = u_i - u_1. \quad (43c)$$

Then we can rewrite the system equations as

$$\dot{\hat{\mathbf{e}}}_2 = \hat{A}_2\hat{\mathbf{e}}_2 + \hat{B}_2\mathbf{u}, \quad (44)$$

with

$$\hat{A}_2 = \begin{bmatrix} 0_{N\times N} & I_N \\ 0_{N\times N} & 0_{N\times N} \end{bmatrix} \otimes I_n \quad (45a)$$

$$\hat{B}_2 = \begin{bmatrix} 0_{N\times N} \\ \hat{I} \end{bmatrix} \otimes I_n, \quad (45b)$$

and

$$\hat{I} = \begin{bmatrix} 1 & 0_{N-1}^T \\ -1_{N-1} & I_{N-1} \end{bmatrix}. \tag{46}$$

Now we can formulate an optimal control problem for the double-integrator model as

$$\begin{aligned} J^\star = min\frac{1}{2}\int_{t_0}^{\infty} & (\hat{\mathbf{e}}_2^T Q_d(\mathbf{x})\hat{\mathbf{e}}_2 + \mathbf{u}^T R_d(\mathbf{x})\mathbf{u})dt \\ s.t. & \\ & \dot{\hat{\mathbf{e}}}_2 = \hat{A}_2\hat{\mathbf{e}}_2 + \hat{B}_2\mathbf{u} \\ & Q_d(\mathbf{x}) = diag[Q_1, ..., Q_{2N}] \geq 0 \\ & R_d(\mathbf{x}) = diag[R_1, ..., R_N] > 0. \end{aligned} \tag{47}$$

Since the error dynamic in (47) is linear, proper choice of $Q_d(\mathbf{x})$ guarantees feasibility of SDRE solution. Then, sub-optimal control law is given by Lemma 1 as

$$\mathbf{u}^\star = -k_2\hat{\mathbf{e}}_2, \tag{48}$$

yields in local asymptotic stability of the closed-loop system. Note that the sub-optimal gain k_2 is given by

$$k_2 = R_d(\mathbf{x})^{-1}\hat{B}_2^T S, \tag{49}$$

where S is the positive definite solution of the following SDRE

$$Q_d(\mathbf{x}) + \hat{A}_2^T S + S\hat{A}_2 - S\hat{B}_2 R_d(\mathbf{x})^{-1}\hat{B}_2^T S = 0. \tag{50}$$

Note that according to equation (13) the energy usage can be modeled using agents' velocity. For the double-integrator agent model, the velocity appears in the state vector and the energy weighting factors can be adjusted in matrix $Q_d(\mathbf{x})$.

Remark 5: Selecting constant weighting matrix $Q_d(\mathbf{x})$ and $R_d(\mathbf{x})$, results into a standard LQR problem that guarantees global asymptotic stability of the closed-loop system.

IV. SIMULATION RESULTS AND DISCUSSION

Here we present results of simulations based on the proposed control methods. For simplicity, we choose $\alpha_i = 1$ in all simulations.

Fig. 2 (top) shows the result of the sub-optimal formation control law formulated in (34) for a set of $N = 5$ agents that are modeled by a single-integrator in 3-D. We first simulated the scenario where all followers have fully charged batteries (all $l_{0i} = 1$) and we selected $Q = R(0) = I_n$ (top). Note that with $R(0)$ we indicated the initial value of weighting matrix. In case when one follower (agent #3 on the right side) has less initial energy than other agents $l_{03} = 0.1$, we selected $R_3(0) = 10$ (21). The result is shown in Fig. 2 (bottom).

Fig. 3 shows the results of proposed optimal formation control law (48) for a set of agents modeled by the double-integrator model. All followers have initially fully charged batteries ($l_{0i} = 1$), and we selected Q and $R(0)$ equal to identity matrices, Fig. 3 (top). Then we consider the case where one of the followers (agent #3 on the left side) has lower initial energy charged than others ($l_{03} = 0.01$) with selected $R_3(0) = 100$, Fig. 3 (bottom). Note that as a result

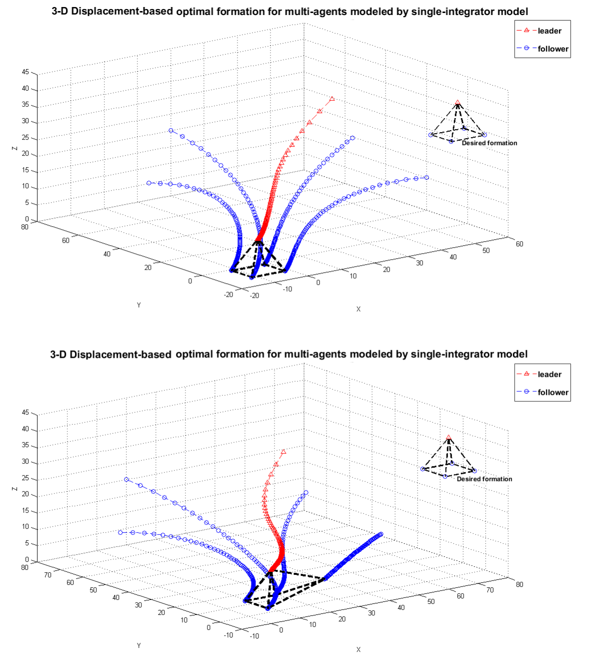

Fig. 2. Leader-follower formation control for $N = 5$ agents modeled by a single-integrator in 3-dimensional space; all followers have full initial energy level, $Q = R(0)$ (top); the agent #3 (on the right side) has less initial energy, $R_3(0) = 10$ (bottom).

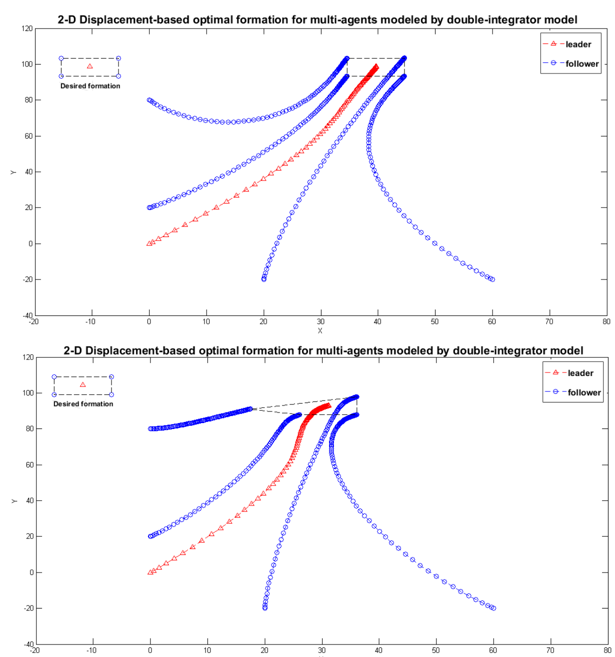

Fig. 3. Leader-follower formation control for $N = 5$ agents modeled by a double-integrator model in 2-D; all followers has same initial energy level, Q and $R(0)$ equal to identity matrix (top); the follower #3 has a very low initial energy, $R_3(0) = 100$ (bottom).

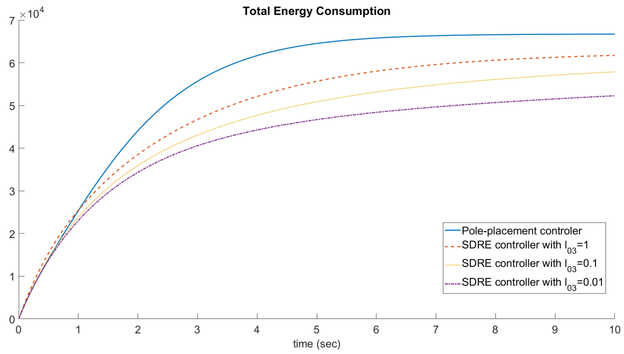

Fig. 4. Comparison of the total energy consumption for the formation governed by various controllers. The formation consists of $N = 5$ agents modeled by the single-integrator model.

of the control algorithm, the agents adjust their paths in order to preserve formation with regard to the "weak" agent.

In comparison with linear pole-placement controller, the proposed method shows significant reduction in energy consumption. Fig. 4 shows that SDRE controller saves more than 7.47 percent energy usage in comparison with a pole-placement controller with all poles placed at $s = -1$. The saved energy rate reaches 22 percent in the case of selecting $l_{03} = 0.01$ as the initial energy for agent #3.

V. CONCLUSIONS

In this paper, an optimal leader-follower formation control problem is considered. After introducing agent models, we proposed a model for agents' energy consumption. Involving energy dynamics in the agent model results in augmented state space model. We developed displacement-based, sub-optimal, leader-follower control scheme for a set of agents which asymptotically minimizes energy usage while satisfying tracking and formation performances. We also proposed a solution which results in the global asymptotic stability of the closed-loop system in the case of single-integrator dynamics. The simulation results show the effectiveness of proposed solution and reveal an interesting behavior of the group (swarm) when some agents become "weak" while maintaining the desired formation. Simulation results show significant reduction in the energy consumption.

VI. ACKNOWLEDGMENTS

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