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Output regulation of multi-input systems under packet dropout with application to trajectory tracking of cooperative robots*

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Abstract— **This paper addresses the output regulation problem for discrete-time linear systems over lossy actuating channels. In order to solve this problem, a co-design approach of controller, encoder and decoder is adopted, i.e., an encoder matrix and a decoder matrix are introduced at each end of the controller-actuator channels for taking full advantage of the communication resource. The solvability conditions are firstly derived to design the feedback gain such that the closed-loop system is mean-square (MS) stable. Then the regulation equations are deduced for designing the feedforward gain to drive the controlled output following the desired trajectory. Finally, a simulation is performed to implement the design method on cooperative robots, thereby showing the effectiveness of the proposed results.**

Index Terms— **Networked control systems; co-design; multiple channels; packet dropouts; cooperative robots.**

I. INTRODUCTION

Networked control systems (NCSs) are closed-loop systems where the plants, the sensors, the actuators and controllers are coordinated through a communication network. NCSs are more and more involved in control engineering such as sensor networks [1], multi-agent systems [2], grid system [3] etc, and have received increasing attention from both industry and academia in the recent years [4], [5]. Networked constraints such as packet dropouts [6], [7], time delays [8], [9], fading [10], [11], limited data rate [12], [13] and quantization [14], [15] are phenomena often encountered in NCSs. These factors can be source of instability or performance degradation of the whole and have received numerous research attention. To name a few, authors in [16] consider a stabilization problem with actuating channel subject to packet dropouts, and necessary and sufficient conditions are determined in terms of the packet dropouts probability and the spectral radius of the system matrix. [17] solves the state feedback stabilization problem for single-input systems with both packet dropout and logarithmic quantization. [18] considers the output feedback controller synthesis for systems over quantized lossy channel, and shows stabilization is indeed related to a tradeoff between robust stability and robust performance. In order to find the least channel capacity of achieving stabilization for multi-input NCSs, the technique of channel resource allocation is appealed to deal with the constraints as signal-to-noise ratio (SNR), logarithmic quantization [19] and fading [20]. When it comes to the case where sub-channels' capacities are assumed to be fixed as a priori, a coding and controller co-design method is put forward in [21] to quest the stabilizability condition for multi-input systems under signal-to-noise ratio constraint. Recently, the quadratic mean square stabilization problem for systems with a norm bounded nonlinearity and stochastic multiplicative noise is discussed and a set of observer-based stabilizing controllers for such systems is conducted by solving two algebraic Riccati equations(AREs) and an algebraic Riccati inequality in [22].

The output regulation or output tracking problem known as letting the controlled output of systems to track signals generated by an exogenous system typically a trajectory generator is one of essential issues in the control theory. The classical output regulation or output tracking problem has been thoroughly investigated for continuous-time linear systems [23], [24] and discrete-time linear systems [25] respectively. In the last dozen years, there have been some studies focusing on the output regulation problem in networked environments with transmission uncertainties. The output regulation problem for continuous linear time-delay systems is respectively studied in [26] by using the operator approach and in [27] by using the finite-dimensional linear state space techniques. Moreover, the output regulation problem is extended to a cooperative and distributed scheme with constraints as switching network topology [28]. Output regulation is alternatively tackled by using input/output weighting filters, and the classical internal model principle is extended to the co-called comprehensive admissibility [29].

Some motivations for our study are briefly highlighted here. First, analytical solution to controller synthesis of NCSs with multiple channels turns out to be an essential μ-synthesis problem [19], [18], which is an obstacle to solve the tracking problem and deserves further studying. Second, with the increasing numbers of applications of NCSs in mobile robots, sensor networks, multi-agent systems and so on, it is necessary to consider how to apply the proposed output regulation method on practical networked applications. Motivated by the above discussion, in this study the output regulation problem is considered for discrete-time linear systems via multiple channels with packet dropouts. Inspired by [21], a pair of encoder and decoder is introduced respectively in the transmitting side and the receiving side of the channels in order to take full advantage of the communication resource, thereby introducing more channels

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than control signals and giving more design freedom there. The mean square stabilization problem is firstly considered for the closed-loop system and sufficient conditions are therefore derived for the co-design of encoder, decoder and feedback gain. Then some regulation equations are given for designing the feedforward gain to drive the controlled output tracking the desired trajectory. Finally, an application of a platform with two cooperative mobile robots is presented as a study case to prove the effectiveness of the proposed control method.

Notation: The notation is fairly standard. The superscripts H and T are the conjugate transpose and transpose, respectively. In a symmetric matrix symbol $*$ denotes the symmetric terms. For a real square matrix P, $P \geq 0$ (respectively $P > 0$) means that P is symmetric positive semidefinite (respectively positive definite). The notation $\mathrm{E}\{\cdot\}$, and $\mathrm{Tr}\{\cdot\}$, $\rho\{\cdot\}$, denote the standard expectation operator, trace of a square matrix, and spectral radius, respectively. A state-space realization of a rational proper transfer function is denoted by

$$G(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] = C(zI - A)^{-1}B + D.$$

Given a transfer function $G(z) \in R\mathcal{H}_2$ with dimension $m \times m$, its $\mathcal{H}_2$ norm is defined as $\|G\|_2 = \sqrt{\frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{Tr}\left\{G(e^{j\omega})G^H(e^{j\omega})\right\} d\omega}$, and the mixed norm for $G(z)$ is defined as follows $\|G\|_{2,1} = \sqrt{\max\limits_{1\leq j\leq m} \sum_{i=1}^{m} \|G_{ij}(z)\|_2^2}$, where, $G_{ij}(z)$ denotes the element of $G(z)$ in i^{th} row and j^{th} column. The matrix 1-norm for $P \in \mathbb{C}^{m\times n}$ is defined as $\|P\|_1 = \max\limits_{1\leq j\leq m} \sum_{i=1}^{n} |P_{ij}|$.

II. PROBLEM FORMULATION

Let's consider the control scheme depicted in Fig. 1. The modeling of the plant together with the analog-digital converter (sampler and zero-order holder) is assumed to be of the following form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), x(0) = x_0, \\ \quad z(k) = Cx(k) + Du(k), \\ \quad e(k) = z(k) - z_{ref}(k), k = 0, 1, \cdots. \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the plant state, $u(k) \in \mathbb{R}^m$ the control input, $z(k) \in \mathbb{R}^p$ the controlled output, $e(k) \in \mathbb{R}^p$ the tracking error, $z_{ref}(k) \in \mathbb{R}^p$ the reference signal to be tracked. It is assumed that the matrix pair (A, B) is stabilizable and the reference signal $z_{ref}(k)$ is assumed to be generated by the exosystem

$$\begin{cases} r(k+1) = A_r r(k), r(0) = r_0, \\ \quad z_{ref}(k) = C_r r(k), k = 0, 1, \cdots. \end{cases} \tag{2}$$

where $r(k) \in \mathbb{R}^q$ is the state of the exosystem.

In this paper, the following control law is considered

$$v(k) = K_{fb}x(k) + K_{ff}r(k), \tag{3}$$

where $v(k) \in \mathbb{R}^m$ is the control signal, the feedback gain

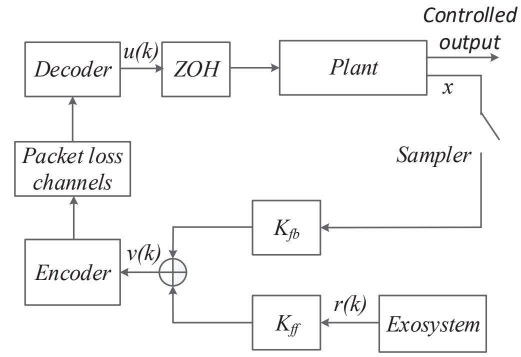

Fig. 1. Structural representation for the control problem under consideration

$K_{fb} \in \mathbb{R}^{m\times n}$ and the feedforward gain $K_{ff} \in \mathbb{R}^{m\times q}$ are constant matrices. Moreover, the controller interacts with the plant through l independent packet dropout channels. In this case, the number of channels is assumed to be more than the number of control signals, i. e., $l > m$. To specify the transmission process characterized by its input $v(k)$ and output $u(k)$, a set of Bernoulli variable $\alpha_i(k), i = 0, 1, \cdots, l$ is introduced to model the unreliable transmission process of each channel. At any time instant k, $\alpha_i(k)$ take value in $\{0, 1\}$. $\alpha_i(k) = 1$ indicates that the transmission succeeds, otherwise $\alpha_i(k) = 0$. The probability of successful transmission is given as $\mathrm{E}\{\alpha_i(k)\} = \beta_i$ with $\beta_i \neq 0$. Moreover, a pair of encoder and decoder is introduced on each side of the communication channels to give a degree of freedom for fully utilizing the resource of the l multiple channels. The encoder matrix is denoted by $\mathcal{T} \in \mathbb{R}^{l\times m}$ and the decoder matrix is denoted by $\mathcal{R} \in \mathbb{R}^{m\times l}$. As known, encoder and decoder are generally viewed as a pair of invertible operators, hence the design requirement on $\mathcal{T}$ and $\mathcal{R}$ is to satisfy the following constraint:

$$\mathcal{R}\Xi\mathcal{T} = I_m, \tag{4}$$

where $\Xi = \mathrm{diag}\left[\beta_1, \cdots, \beta_l\right]$. Considering the transmission process of the control signal $v(k)$ mentioned above, the control input $u(k)$ in (1) is clearly given as

$$u(k) = \mathcal{R}\alpha(k)\mathcal{T}v(k), \tag{5}$$

where $\alpha(k) = \mathrm{diag}\left[\alpha_1(k), \cdots, \alpha_l(k)\right]$.

Combining the plant (1), the exosystem (2) with the control input (5), the closed-loop system is obtained in the following form:

$$\begin{aligned} x(k+1) &= A_c x(k) + B_c r(k), \\ r(k+1) &= A_r r(k), \\ e(k) &= C_c x(k) + D_c r(k). \end{aligned} \tag{6}$$

where $A_c = A + B\mathcal{R}\alpha(k)\mathcal{T}K_{fb}$, $B_c = B\mathcal{R}\alpha(k)\mathcal{T}K_{ff}$, $C_c = C + D\mathcal{R}\alpha(k)\mathcal{T}K_{fb}$, $D_c = -C_r + D\mathcal{R}\alpha(k)\mathcal{T}K_{ff}$.

To describe the requirement on the closed-loop system (6), the following definitions are firstly introduced.

Definition 1: [10] The closed-loop system (6) is said to be mean square stable if for any bounded initial state, $\Upsilon(t) := \mathrm{E}\{x(t)x^T(t)\}$ is well defined for all t and converges to zero when $r(k) = 0, k = 0, 1, 2, \cdots$.

Definition 2: For any bounded initial state $x(0)$ and $r(0)$, the closed-loop system (6) is said to meet output regulation property if the tracking error in (6) satisfies

$$\mathrm{E}[\lim_{k\to\infty} e(k)] = \mathrm{E}[\lim_{k\to\infty} (C_c x(k) + D_c r(k))] = 0.$$

Now the problem under consideration is described as follows.

Problem 1: Find the control gains K_{fb}, K_{ff}, and a pair of encoder $\mathcal{T}$ and decoder $\mathcal{R}$ such that the closed-loop system (6) is MS stable with output regulation property.

III. Main Results

In this section, Problem 1 is solved. And some lemmas are therefore given in Appendix for the derivation of main results.

A. The MS stabilization

The solvability conditions to MS stability of the closed-loop system (6) are firstly provided. For the sake of simplicity, we assume without loss of generality that the matrix pair (A, B) is under the Wonham decomposition [31].

$$A = \begin{bmatrix} A_1 & A_{12} & \cdots & A_{1m} \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & A_{(m-1)m} \\ 0 & \cdots & 0 & A_m \end{bmatrix},$$

$$B = \begin{bmatrix} b_1 & b_{12} & \cdots & b_{1m} \\ 0 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & b_{(m-1)m} \\ 0 & \cdots & 0 & b_m \end{bmatrix},$$

where $A_i \in \mathbb{R}^{n_i \times n_i}$, $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $b_i, b_{ij} \in \mathbb{R}^{n_i}$, $\Sigma_{i=1}^m n_i = n$, each matrix pair (A_i, b_i) is stabilizable, and the symbol $*$ represents the entry irrelevant to the discussion.

Theorem 1: The closed-loop system (6) is MS stable if there exists a matrix $U \in \mathbb{R}^{l\times m}$ satisfying $U^{\mathrm{T}}U = I$, matrices $P_i > 0$, $W_i > 0$, Q_i and scalars $\gamma_i > 0$, $i = 1, 2, \cdots m$, such that the following conditions hold

$$\left\{U\left(\mathrm{diag}\left[\gamma_1, \cdots, \gamma_m\right]\right)U^{\mathrm{T}}\right\}_{ii} < \frac{1}{\beta_i^{-1} - 1}, \tag{7}$$

$$\mathrm{Tr}(W_i) < \gamma_i, \tag{8}$$

$$\begin{bmatrix} W_i & Q_i \\ * & P_i \end{bmatrix} > 0, \tag{9}$$

$$\begin{bmatrix} P_i & A_i P_i + b_i Q_i & b_i \\ * & P_i & 0 \\ * & * & I \end{bmatrix} > 0, \tag{10}$$

then the state feedback gain K_{fb} in (3), encoder $\mathcal{T}$ and decoder $\mathcal{R}$ in (5) for the Problem 1 are constructed respectively as

$$K_{fb} = \mathrm{diag}\left[Q_1 P_1^{-1}, \cdots, Q_m P_m^{-1}\right], \tag{11}$$

$$\mathcal{T} = \Xi^{-1/2} U D^{-1}, \mathcal{R} = D U^{\mathrm{T}} \Xi^{-1/2}, \tag{12}$$

with $\Xi = \mathrm{diag}\,[\beta_1, \cdots, \beta_l]$, $D = \mathrm{diag}[1, \varepsilon, \cdots, \varepsilon^{(m-1)}]$ and ε is a small positive number.

Proof: Note that for the Bernoulli process $\alpha_i(k)$ mentioned in (5) , $i = 1, 2, \cdots, l$, we have

$$\mathrm{E}\{\alpha_i(k)\} = \beta_i \neq 0, \quad \mathrm{E}\{(\alpha_i(k) - \beta_i)^2\} = \beta_i(1 - \beta_i).$$

Setting new random variables as $\phi_i(k) = (\alpha_i(k) - \beta_i)/\beta_i$ yields

$$\mathrm{E}\{\phi_i(k)\} = 0, \quad \mathrm{E}\{(\phi_i)^2\} = \beta_i^{-1} - 1,$$

$i = 1, 2, \cdots, l$. Then A_c in (6) can be rewritten as

$$\begin{aligned} A_c &= A + B\mathcal{R}\Xi\mathcal{T}K_{fb} + B\mathcal{R}\Xi\Phi(k)\mathcal{T}K_{fb} \\ &= A + BK_{fb} + B\mathcal{R}\Xi\Phi(k)\mathcal{T}K_{fb} \end{aligned}$$

where

$$\Xi = \mathrm{diag}\left[\beta_1, \cdots, \beta_l\right], \Phi(k) = \mathrm{diag}\left[\phi_1(k), \cdots, \phi_l(k)\right].$$

With the help of Lemma 3, it is easy to see that the system (6) is MS stable if there exists a feedback gain K_{fb}, an encoder/decoder pair $\mathcal{T}$ and $\mathcal{R}$ such that

$$\rho\{\tilde{T}\bar{\beta}^2\} < 1 \tag{13}$$

where

$$\tilde{T} = \begin{bmatrix} \|T_{1,1}(z)\|_2^2 & \cdots & \|T_{1,l}(z)\|_2^2 \\ \vdots & \ddots & \vdots \\ \|T_{l,1}(z)\|_2^2 & \cdots & \|T_{l,l}(z)\|_2^2 \end{bmatrix},$$

$$\bar{\beta} = \mathrm{diag}\left[(\beta_1^{-1} - 1)^{\frac{1}{2}}, \cdots, (\beta_l^{-1} - 1)^{\frac{1}{2}}\right],$$

$$T(z) = \mathcal{T}K_{fb}(zI - A - BK_{fb})^{-1}B\mathcal{R}\Xi,$$

$T_{i,j}(z)$ denotes the element of $T(z)$ in i^{th} row j^{th} column.

Next, we will show that under the conditions (7)-(10), (13) holds with K_{fb}, $\mathcal{T}$, $\mathcal{R}$ given in (11) and (12).

Since the matrix pair (A, B) is under the Wonham decomposition. $T(z)$ can be rewritten as

$$T(z) = \Xi^{-1/2} U \bar{K}_{fb}(zI - \bar{A} - \bar{B}\bar{K}_{fb})^{-1}\bar{B}U^{\mathrm{T}}\Xi^{1/2}$$

where

$$\bar{A} = Q^{-1}AQ = \begin{bmatrix} A_1 & o(\varepsilon) & \cdots & o(\varepsilon) \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & o(\varepsilon) \\ 0 & \cdots & 0 & A_m \end{bmatrix},$$

$$\bar{B} = Q^{-1}BD = \begin{bmatrix} b_1 & o(\varepsilon) & \cdots & o(\varepsilon) \\ 0 & b_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & o(\varepsilon) \\ 0 & \cdots & 0 & b_m \end{bmatrix},$$

$$\begin{aligned} &\bar{K}_{fb} = D^{-1}K_{fb}Q, \quad D = \mathrm{diag}[1, \varepsilon, \cdots, \varepsilon^{(m-1)}], \\ &Q = \mathrm{diag}[1, \varepsilon, \cdots, \varepsilon^{(m-1)}], \end{aligned}$$

and $o(\varepsilon)/\varepsilon$ approaches to a finite constant as $\varepsilon \to 0$.

Moreover, it is observed by Lemma 4 that

$$\rho\{\tilde{T}\bar{\beta}^2\} = \inf_{\mathcal{D}} \left\|\mathcal{D}^{-1}T(z)\bar{\beta}\mathcal{D}\right\|_{2,1}^2 .$$

Next, we will show

$$\rho\{\tilde{T}\bar{\beta}^2\} \leq \left\|\mathcal{D}^{-1}T(z)\bar{\beta}\mathcal{D}\right\|_{2,1}^2 < 1$$

with $\mathcal{D}$ being selected as $\mathcal{D} = \Xi^{-1/2}$.

By the definition, $\left\|\Xi^{1/2}T(z)\bar{\beta}\Xi^{-1/2}\right\|_{2,1}^2$ can be rewritten as

$$\begin{aligned}
&\left\|\Xi^{1/2}T(z)\bar{\beta}\Xi^{-1/2}\right\|_{2,1}^2 \\
&= \max_i \tfrac{1}{2\pi}\int_{-\pi}^{\pi}\left\{\bar{\beta}\Xi^{-1/2}T^H(e^{jw})\Xi_l T(e^{jw})\Xi_l{}^{-1/2}\bar{\beta}\right\}_{ii} dw \\
&\leq \max_i \left\{\bar{\beta}U\left(\mathrm{diag}\left[\|T_1(z)\|_2^2, \cdots, \|T_m(z)\|_2^2\right]\right)U^{\mathrm{T}}\bar{\beta}\right\}_{ii} \\
&+ o(\varepsilon, z).
\end{aligned}$$

where $T_i(z) = f_i(zI - A_i - b_i f_i)b_i$. With the help of lemma 1 in [34], Eq. (8)-(10) indicate that $\|T_i(z)\|_2^2 < \gamma_i$. Therefore, it can be concluded by (7) that

$$\begin{aligned}
&\left\|\Xi^{1/2}T(z)\bar{\beta}\Xi^{-1/2}\right\|_{2,1}^2 \\
&< \max_i \left\{\bar{\beta}U\left(\mathrm{diag}\left[\gamma_1, \cdots, \gamma_m\right]\right)U^{\mathrm{T}}\bar{\beta}\right\}_{ii} + o(\varepsilon, z), \\
&< 1 + o(\varepsilon, z).
\end{aligned}$$

When ε is sufficiently small, it is clear that

$$\rho\{\tilde{T}\bar{\beta}^2\} \leq \left\|\Xi^{1/2}T(z)\bar{\beta}\Xi^{-1/2}\right\|_{2,1}^2 < 1.$$

Consequently, (13) holds.

■

B. The output regulation under MS stability

In this section, a sufficient condition is given for designing the feedforward gain K_{ff} such that the closed-loop system (6) is MS stable and has output regulation property with the K_{fb}, $\mathcal{R}$ and $\mathcal{T}$ obtained in the section III-A

Theorem 2: The closed-loop system (6) has output regulation property if it is MS stable and there exists matrix X and Y such that the following Sylvester-type equation holds:

$$\begin{aligned}
XA_r &= AX + BY, \\
0 &= CX + DY - C_r.
\end{aligned} \tag{14}$$

Moreover, the gain K_{ff} is given as:

$$K_{ff} = Y - K_{fb}X. \tag{15}$$

Proof: Substitute Eq. (15) into Eq. (14), then

$$\begin{aligned}
XA_r &= (A + BK_{fb})X + BK_{ff}, \\
0 &= (C + DK_{fb})X + DK_{ff} - C_r.
\end{aligned} \tag{16}$$

By defining $\bar{x}(k) = x(k) - Xr(k)$, for the closed-loop system (6) we get

$$\begin{aligned}
&x(k+1) - Xr(k+1) = A_c x(k) + B_c r(k) - Xr(k+1) \\
&= (A + BK_{fb})x(k) + BK_{ff}r(k) + B\mathcal{R}\Xi\Phi(k)\mathcal{T}K_{fb}x(k) \\
&+ B\mathcal{R}\Xi\Phi(k)\mathcal{T}K_{ff}r(k) - XA_r r(k)
\end{aligned}$$

Substituting (15) and (16) into the above formula yields

$$\begin{aligned}
\bar{x}(k+1) &= x(k+1) - Xr(k+1) \\
&= (A + BK_{fb})(x(k) - Xr(k)) \\
&+ B\mathcal{R}\Xi\Phi(k)\mathcal{T}K_{fb}x(k) + B\mathcal{R}\Xi\Phi(k)\mathcal{T}K_{ff}r(k) \\
&= (A + B\mathcal{R}\alpha(k)\mathcal{T}K_{fb})\bar{x}(k) \\
&+ (B\mathcal{R}\Xi\Phi(k)\mathcal{T}Y)r(k).
\end{aligned}$$

By the MS stability of the closed-loop system (6), $\lim_{k\to\infty}\mathrm{E}[\bar{x}(k)] = 0$ is guaranteed with $\mathcal{R}$, $\mathcal{T}$, K_{fb} given in (11) and (12). Then by (16),

$$\begin{aligned}
&\mathrm{E}[\lim_{k\to\infty} e(k)] \\
&= [\lim_{k\to\infty}(C_c x(k) + D_c r(k))] \\
&= \mathrm{E}[C_c\bar{x}(k)] + (C + DK_{fb}X + C_r + DK_{ff})r(k) \\
&= 0
\end{aligned}$$

holds.

■

Remark 1: As shown in eq. (3), final expression of the control signal is

$$v(k) = K_{fb}x(k) + K_{ff}r(k),$$

that can be rewrite thanks to eq. (15) as

$$\begin{aligned}
v(k) &= K_{fb}x(k) + (Y - K_{fb}X)r(k), \\
&= K_{fb}(x(k) + Xr(k)) + Yr(k), \\
&= K_{fb}(x(k) - x_{ref}(k)) + u_{ref}
\end{aligned}$$

with x_{ref} and u_{ref} the state and input reference signals generated by gains X and Y solutions of eq. (14).

IV. Application on a cooperative control platform

A. Platform description

As depicted in Fig.2, the control platform consists of two cooperative mobile robots, one camera and a supervisor. A Matlab simulation is considered for this platform. The objective of this platform is to drive two tank-like mobile robots that need to cooperate in order, for example, to manipulate a load too cumbersome and heavy to be manipulated by only one of the robots. There is no physical link between the two robots but they are required to keep a given distance. It is also assumed that the two robots cannot communicate between themselves. The supervisor can measure the absolute position and velocity of each robot thanks to cameras and communicate with each robot by a wireless mean. With the cameras, it is assumed in a realistic way that the measurement signals are available without any disturbance or delays. The control signals computed by the supervisor have to be send to each robot through a wireless protocol (*e.g.* a Zigbee MIMO protocol). Thus here communication disturbances (packet dropouts) must be considered.

B. Control objectives

The control objectives can be formulated as follows:

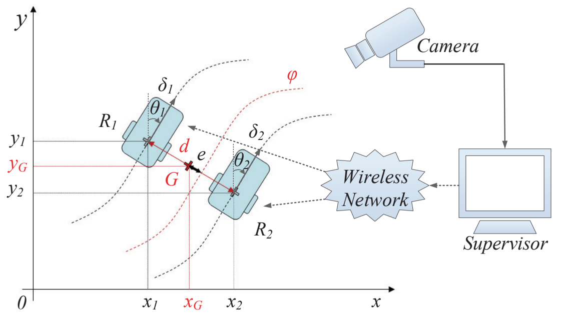

Fig. 2. Setup of the control system with two cooperative tank-like robots

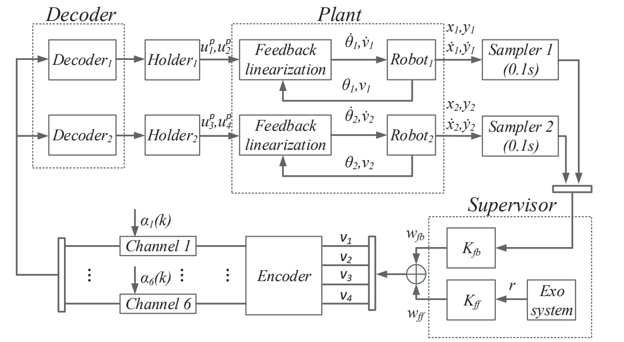

Fig. 3. Control implementation of the control platform with two robots

- The barycenter $G(x_G; y_G)$ of the two robots must be driven along a pre-defined trajectory.
- A constant interdistance between the two robots must be ensured. The interdistance d is defined here as the euclidian norm between centers of $robot_1$ $(x_1; y_1)$ and $robot_2$ $(x_2; y_2)$ in the frame $(0; x; y)$ (see Fig. 2).

$$\begin{cases} x_G = (x_1 + x_2)/2, \\ y_G = (y_1 + y_2)/2, \\ \quad d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}. \end{cases} \tag{17}$$

Feet with the linear context, (17) can be reformulated as the 4 following signals to be controlled

$$\begin{cases} x_G = (x_1 + x_2)/2, \\ y_G = (y_1 + y_2)/2, \\ \Delta_x = x_1 - x_2, \\ \Delta_y = y_1 - y_2. \end{cases} \tag{18}$$

The reference trajectory for the gravity center of the two robots can be generated in several ways; directly by a human user thanks to a joystick, by using some robotic trajectory generation algorithms, etc. According to the controlled output defined in eq. (18) and notations in Fig. 2, the trajectory generator provide a vector signal $ref(k) \in \mathbb{R}^4$ such as

$$ref(k) = \begin{cases} x_G^{ref}(k), \\ y_G^{ref}(k), \\ \Delta_x^{ref}(k), \\ \Delta_y^{ref}(k). \end{cases} \tag{19}$$

C. Networked controller architecture

The whole control implementation is depicted in Fig.3. The two robots with embedded feedback linearization control are set as the plant(see hereafter). The absolute position and velocity of each robot is measured by a camera and sent to the supervisor with a sampling period tuned at 0.1s. Both the control law (based on a 2 degrees of freedom controller) and the trajectory generation are computed in the supervisor. Concretely, the controller is based on a feedback static gain K_{fb} and a feedforward static gains K_{ff} generating the control signals v_1, v_2, v_3 and v_4 to drive the two robots.

The wireless communication network is based on MIMO channels; the four control signals are firstly encoded and then transmitted via $l = 6$ independent channels with packet dropouts. After transmission the signals are decoded and used by the two robots. The encoder (static matrix $\mathcal{T} \in \mathbb{R}^{6\times 4}$) can be implemented directly in the supervisor, or locally in the hardware transmitter. The decoder algorithm (static matrix $\mathcal{R} \in \mathbb{R}^{4\times 6}$) can be implemented separately in the two robots following eq. (20).

$$R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}, R_1 \in \mathbb{R}^{2\times 6}, R_2 \in \mathbb{R}^{2\times 6}. \tag{20}$$

D. Robots modeling and local control law

Each robot can be modeled by a classic kinematic unicycle model

$$\begin{cases} \dot{x}_i = \delta_i \cos(\theta_i), \\ \dot{y}_i = \delta_i \sin(\theta_i), \\ \dot{\theta}_i = \eta_i, \\ \dot{\delta}_i = \gamma_i. \end{cases} \tag{21}$$

where $(x_i; y_i)$ are the position of the $robot_i$, δ_i and θ_i its velocity and its angular orientation, $(\dot{x}_i; \dot{y}_i)$ its accelerated velocity on (O_x) and (O_y) axle in the frame (O, x, y), $\dot{\theta}_i$ its angular velocity, η_i, γ_i the plant input of the $robot_i$, $i = 1, 2$. In each robot a classical linearizing feedback control law is implemented [30], leading to a new input-output mapping (see Fig. 2) based on two decoupled integrator chains.

$$Rob_i^{lin} = \begin{cases} x_i/a_i^x = 1/s^2, \\ y_i/a_i^y = 1/s^2. \end{cases} \tag{22}$$

The two new control inputs for each robots $(a_i^x; a_i^y)$ are homogeneous to the acceleration of the robot. Finally, without taking into consideration the decoder R_i embedded in each robot, the plant model, after exact discretization at $T_{samp} = 0.1s$, is given by

$$\dot{x}_p = A_p x_p + B_p u_p \tag{23}$$

with

$$
\begin{aligned}
x_p^T &= [x_1\ \dot{x}_1\ y_1\ \dot{y}_1\ x_2\ \dot{x}_2\ y_2\ \dot{y}_2]^T,\\
u_p^T &= \left[u_p^1\ u_p^2\ u_p^3\ u_p^4\right]^T = [a_1^x\ a_1^y\ a_2^x\ a_2^y]^T,\\
A_p &= \mathrm{diag}\,[A_1,\cdots,A_4], \quad B_p = \mathrm{diag}\,[b_1,\cdots,b_4]\\
A_i &= \begin{bmatrix}1 & 0.1\\ 0 & 1\end{bmatrix}, \quad b_i = \begin{bmatrix}0.005\\ 0.100\end{bmatrix}, \quad i = 1,2,3,4.
\end{aligned}
$$

Such sampling period T_{samp} leads to a good compromise between the dynamic of the robots, and the capacities of the supervisor (video tracking algorithm in particular) and of the wireless protocol (e.g. zigbee or Bluetooth).

E. Control problem formulation

Considering the control objectives formulated before, the regulation problem with internal mean-square stability as to be defined in the same way than in section 3.

1) Reference model: As said in section IV-B, the reference trajectory $z_{ref}(k)$ can be generated in several way. Its model (2) must be representative of the dynamic characteristics of this exogenous signal all the while remaining simple. Here, as a first example, the reference trajectory $z_{ref}(k)$ for the gravity center G is a circle with radius 10 meters, and the robots should travel around this trajectory with a fixed interdistance of $2m$, at a fixed velocity of $1m/s$. The travel period is thus around $62.8s$. The reference signal $z_{ref}(k)$ is defined as

$$
z_{ref}(k) = C_r r(k) = \begin{cases} x_G^{ref}(k) = 10sin(0.1\pi k/20),\\ y_G^{ref}(k) = 10cos(0.1\pi k/20),\\ \Delta_x^{ref}(k) = 2sin(0.1\pi k/20),\\ \Delta_y^{ref}(k) = 2cos(0.1\pi k/20). \end{cases} \tag{24}
$$

The 4 reference signals are clearly coupled, thus the exosystem model (2) is a two order model (sinusoidal model) tuned, considering a sampling time of 0.1s, as

$$
A_r = \begin{bmatrix}0.99995 & -0.00800\\ 0.01250 & 0.99995\end{bmatrix}, \quad C_r = \begin{bmatrix}0 & 10 & 0 & 2\\ 8 & 0 & -1.6 & 0\end{bmatrix}^T.
$$

Remark 2: In this paper the trajectory is simple (clockwise circle), and the reference model (2) used to design the control law and the one simulated in the supervisor is assumed to be the same. More complex trajectory can be considered, typically with non-constant longitudinal velocity or distance between the two robots. It asks for a new exosystem model (2), with other physical meaning for the state vector $r(k)$.

2) Plant model: The standard model defined in (1) is linked to discretize plant model (23), and the matrices for the output regulation problem are defined as

$$
C = \begin{bmatrix}0.5 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0\\ 0 & 0 & 0.5 & 0 & 0 & 0 & 0.5 & 0\\ 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0\end{bmatrix}, D = 0.
$$

3) Network model: For the communication channels, it is assumed that there are $l = 6$ channels with parameters $\beta_i = 0.7, i = 1,2,\cdots,6$.

F. Simulation results

For the MS stabilization, the gain K_{fb} is firstly obtained for the supervisor as:

$$
K_{fb} = \mathrm{diag}\,[f_1,\cdots,f_4]
$$

with

$$
f_i = \begin{bmatrix}-0.0466 & -0.2498\end{bmatrix}, \quad i = 1,2,3,4.
$$

Meanwhile the encoder and decoder are designed as:

$$
T = \begin{bmatrix}-0.4829 & -0.2364 & -0.7822 & 0.1364\\ -0.3647 & -0.5124 & 0.0890 & -0.7756\\ -0.6221 & 0.0583 & 0.6858 & -0.2370\\ -0.4862 & 0.4948 & 0.3104 & 0.5298\\ -0.4787 & 0.6678 & -0.4920 & -0.3451\\ -0.4581 & -0.6451 & 0.0068 & 0.5938\end{bmatrix},
$$

$$
R = \begin{bmatrix}R_1\\ R_2\end{bmatrix},
$$

with

$$
R_1 = \begin{bmatrix}-0.4829 & -0.2364\\ -0.3647 & -0.5124\\ -0.6221 & 0.0583\\ -0.4862 & 0.4948\\ -0.4787 & 0.6678\\ -0.3647 & -0.6451\end{bmatrix}^T,
$$

$$
R_2 = \begin{bmatrix}-0.7822 & 0.1364\\ 0.0890 & -0.7756\\ 0.6858 & -0.2370\\ 0.3104 & 0.5298\\ -0.4920 & -0.3451\\ 0.0068 & 0.5938\end{bmatrix}^T.
$$

Then by solving the Sylvester-type equation (14), the gain K_{ff} is obtained as:

$$
K_{ff} = \begin{bmatrix}0.2742 & 0.3221\\ 0.4026 & -0.2194\\ 0.2244 & 0.2635\\ 0.3294 & -0.1795\end{bmatrix}.
$$

The Fig.4 shows the tracking performance of the robots under the proposed control strategy. The evolution of the tracking error and the distance between the two robots is given in Fig.5.

Remark 3: In fact, Theorem 1 ensures MS stability, but does not consider any performance requirements and the transient performance of the mobile robots as depicted in Fig.4 and Fig.5 may be improved. Our current work is studying this point to provide performance result within this stochastic framework.

In response to this, the following condition is added when designing K_{fb} for the cooperative control platform by (7)-(10).

$$
\begin{bmatrix} -P_i & \gamma(A_iP_i + b_iQ_i)^T\\ * & -P_i\end{bmatrix} < 0.
$$

That is asking the system (23) to satisfy the exponentially

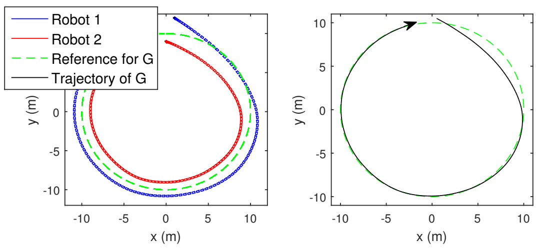

Fig. 4. Tracking performance of the two cooperative mobile tank-like robots

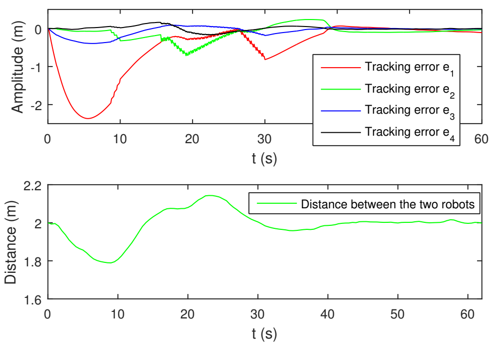

Fig. 5. Tracking error and the distance between the two cooperative robots

mean-square stable condition [32]

$$\mathrm{E}[\lim_{k\to\infty} \gamma^k x_p(k)] = 0.$$

for any bounded initial state $x_p(0)$, where $\gamma > 1$ is a given constant. The corresponding tracking performance, tracking error and distance evolution are shown in the Fig.6 and Fig.7.

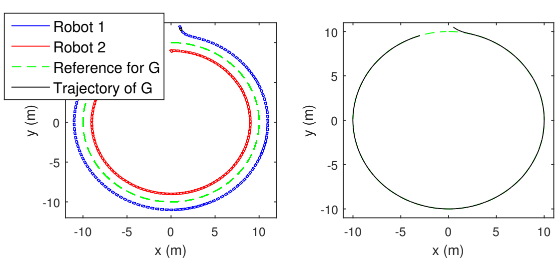

Fig. 6. Tracking performance of the two cooperative robots (improved)

V. CONCLUSIONS

In this paper, we have studied the output regulation of multi-input systems with packet dropouts. By utilizing the co-design method of encoder, decoder and controller, we have presented both the feedback gain and the feedforward gain to solve this problem. As an application, we have adopted the proposed method to solve the trajectory tracking problem of two cooperative robots. Future work will focus on the improvement of the tracking performance.

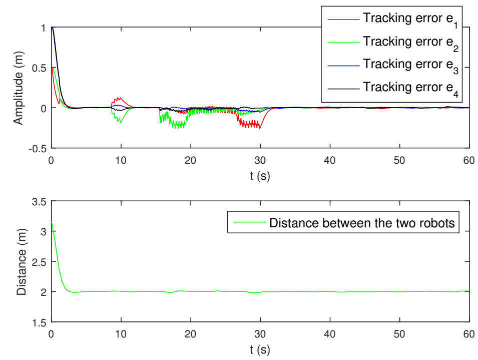

Fig. 7. Tracking error and the distance between the two robots (improved)

VI. APPENDIX

Lemma 3: [10] Given an internally stable system G with dimension $p \times p$ and a structured random process

$$\Delta(k) = \mathrm{diag}\left[\Delta_1(k), \cdots, \Delta_p(k)\right]$$

such that $\mathrm{E}\{\Delta_i(k)\} = 0$, $\mathrm{E}\{\Delta_i^2(k)\} = \tau_i^2$, $i = 1, 2, \cdots, p$, then the feedback interconnection in Fig. 8 is MS stable if and only if the following condition holds

$$\rho\{\tilde{G}\Phi\} < 1$$

where

$$\tilde{G} = \begin{bmatrix} \|G_{11}\|_2^2 & \cdots & \|G_{1p}\|_2^2 \\ \vdots & \ddots & \vdots \\ \|G_{p1}\|_2^2 & \cdots & \|G_{pp}\|_2^2 \end{bmatrix}, \Phi = \mathrm{diag}\left[\tau_1^2, \cdots, \tau_p^2\right].$$

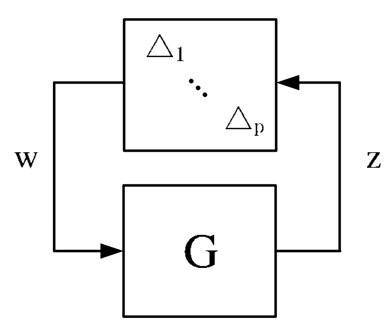

Fig. 8. Feedback interconnection of systems with stochastic perturbation

Lemma 4: Given an internally stable system G with dimension $p \times p$, there holds

$$\rho\{\tilde{G}\} = \inf_{\mathcal{D}} \|\mathcal{D}^{-1} G \mathcal{D}\|_{2,1}^2,$$

where $\mathcal{D}$ is a diagonal matrix with all diagonal elements being positive, and

$$\tilde{G} = \begin{bmatrix} \|G_{11}\|_2^2 & \cdots & \|G_{1p}\|_2^2 \\ \vdots & \ddots & \vdots \\ \|G_{p1}\|_2^2 & \cdots & \|G_{pp}\|_2^2 \end{bmatrix}.$$

Proof: As represented by Lemma 2.2 in [33], For nonnegative matrix M, there holds

$$\rho\{M\} = \inf_{\mathcal{D}} \|\mathcal{D}^{-1} M \mathcal{D}\|_p$$

for $1 \leq p \leq \infty$. Thus, there holds

$$\rho\{\tilde{G}\} = \inf_{\mathcal{D}} \|\mathcal{D}^{-1} \tilde{G} \mathcal{D}\|_1,$$

and by the definition of $\|\cdot\|_{2,1}$, it is observed that

$$\rho\{\tilde{G}\} = \inf_{\mathcal{D}} \|\mathcal{D}^{-1} G \mathcal{D}\|_{2,1}^2.$$

■

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