2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Design and Verification of Autonomous Steering Control Based on Driver Modeling

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Abstract— **The paper proposes the design and verification of an autonomous steering control system. The method is based on the control-oriented modeling of the human driver, which results in the comfort of the passengers and provides predictability for the drivers of the conventional vehicles and the pedestrians. The optimization procedure is based on test measurements, which is able to guarantee the steering characteristics of the driver in the autonomous control system. The designed controller is based on the robust $\mathcal{H}_\infty$ method, which is extended with a prediction rule. The proposed autonomous steering functionality is demonstrated through real vehicle tests, which show that the control system approximates the steering behavior of the driver accurately.**

I. INTRODUCTION AND MOTIVATION

In the last decade the autonomous vehicle systems have had considerable impact on the research of vehicle control. Several intelligent sensors and actuators have been built into the vehicles and there are several possibilities to access information about the traffic environment, e.g. cloud technology. Although the focus of the research in the vehicle control is on the self-driving functionality, there is a growing interest in the behavior of the drivers. It indicates that the cruising and maneuvering of autonomous vehicles must approximate the style of the human driver, which is a significant challenge in the vehicle control design. It requires knowledge not only about the vehicles, but also the drivers, e.g. the different information sources, the human data processing, the actuation capabilities of the driver. Moreover, the driving behavior of the drivers can vary, depending on age, experience, mental state etc. These circumstances make the modeling of the driver behavior difficult. At the same time the autonomous vehicle must guarantee the comfort of the passengers and monitor and predict the drivers of the conventional vehicles and the pedestrians. Thus, the modeling of the human driving behavior is an important factor in the autonomous vehicle control design. The autonomous system should prevent the characteristics of the human driver, while the limitations must be eliminated.

There are several papers in which the human driving capabilities are examined. Modeling the human driver can be divided into three subtasks, such as the human sensory system, the human brain's processing and human actuation.

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The research was supported by the National Research, Development and Innovation Fund through the project "SEPPAC: Safety and Economic Platform for Partially Automated Commercial vehicles" (VKSZ 14-1-2015-0125). This paper was partially supported by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences.

The role of human sensory system in the driving task was reviewed by [1]. It was presented that the steering control task is the superposition of a target path and a disturbance rejection task, see [2]. During the driving task the driver looks ahead of the upcoming road and determines an optimal target path to follow. Simultaneously, the driver perceives the discrepancies from the previously planned trajectory and acts against them as a feedback controller. The driver uses various sensory systems to estimate the state of the vehicle and perceive its surrounding environment. Unquestionably the most dominant of them is the visual system, since this is the only one with which the driver can sense the future road geometry and the vehicle's relative motion to its surroundings.

Furthermore, drivers can control their vehicle using nothing else but their visual system [3]. One of the proposed mechanisms for human visual motion perception is the optic flow [4], which is being used in recent models, see [5]. An alternative method is calculating the time to collision by measuring the rates of change of vectors between the drivers and specific objects [6] [7]. Several different visual driver models exist. One of the earliest one is the well-known crossover model [8]. Later models usually also incorporate preview information in order to improve tracking performance [9] [10]. It is proved that human driving performance does not decrease significantly if only two, a closer and a further region, are visible [11]. Therefore many preview-predictive model use the visual cue from these points as input [12] [13] [14].

Steering is one of the most important components in the autonomous vehicle since it is able to guarantee lateral stability and trajectory tracking. Several papers deal with the design of intelligent steering systems, which must be precise and fast for safe cruising. The modeling and analysis of the electric power steering system are proposed by [15]. In that research the effects of the steering on vertical dynamics are in the focus. A fault-tolerant tracking control for four-wheel steering autonomous vehicles is presented by [16]. Generally, the control system requires signals about the road in order to monitor the road conditions. However, [17] proposes a control design based on a multiple model adaptive method, which does not require road information. A parameter adaptive steering control is proposed by [18]. A PID-based steering control is proposed by [19]. A nonlinear model predictive control is proposed by [20], [21]. Similarly, [22] presents a method based on a model predictive envelope of the steering control, in which the physical limits are considered.

This paper presents the results concerning the design of autonomous steering functions, in which the characteristics of the human driver is incorporated. The driver is modeled in a control-oriented form, considering the sensory and the brain process systems. As a novelty of the paper, the robust $\mathcal{H}_\infty$ controller structure is built-in the driver model. An advantage of the method that it is able to reject the disturbances of the measured signals, which can be also performed by the human driver, e.g. the noises in the visual and vestibular systems. The contribution of the paper is the formulation of the optimization method, with which the characteristics of the human driver is incorporated in the design of the autonomous steering control. The control system is verified through real vehicle tests, which demonstrates the efficiency of the proposed method.

The paper is organized as follows. Section II presents the modeling of the driver's sensors, such as the visual channel and the vestibular system. The processing of this information using the $\mathcal{H}_\infty$-based modeling and a prediction rule are proposed in Section III. The verification of the autonomous steering system compared to the driver's steering is demonstrated in Section IV. Finally, the paper is concluded in Section V.

II. Sensing the environment and the vehicle motion

The human driver senses the environment and the vehicle-driver motion through several information channels. The most important is the visual channel, but the vestibular system (otolith signals, semicircular canals), and the somatosensory system (muscle spindles, Golgi tendon organs) also have roles in the sensing of the vehicle-driver motion and the environment. Thus, the human perception incorporates several different signals in an integrated way, which results in a comprehensive evaluation of the current dynamics of the vehicle [1].

As an assumption, in the following model the visual channel and the information from the semicircular canals are considered. Since the visual and the vestibular systems are the most important primary information channels [3], the elimination of the further channels has a low impact on the results. Thus, the following signals are used in the model:

- the current lateral error between the reference trajectory and the position of the vehicle $e_{y,a}$,
- the current angle error between the heading angle of the trajectory and the yaw angle of the vehicle $e_{\psi,a}$,
- the lateral error between the reference trajectory and the predicted position of the vehicle $e_{p,i}$, where i is the number of the prediction points.

The lateral error signals are sensed through the visual channels, while the yaw error is related to the visual and the semicircular systems. In the control-oriented modeling of the driver the human sensors are handled as filters, thresholds, limiters and delays [3]. Moreover, there are also sensor noises in the system [1].

The sensing of the lateral error and the semicircular system are formulated as transfer functions, see Figure 1. The measuring system contains the dynamics of the sensor G_{sens} and a time delay G_{delay}. Moreover, the noise w on the measurement is also considered. The dynamics of the sensor

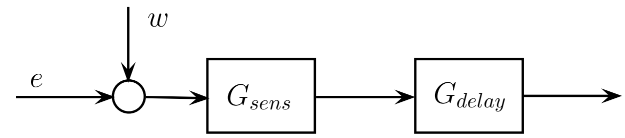

Fig. 1. Human driver sensor modeling

is formulated typically in the form

$$G_{sens} = \frac{\alpha_2 s^2 + \alpha_1 s + \alpha_0}{T_2 s^2 + T_1 s + 1}, \quad (1)$$

where α_i and T_i are the parameters related to the visual and the semicircular systems. Furthermore, the time delay can be transformed into a transfer function using the Padé approximation

$$G_{delay} = e^{-s\tau} \approx \frac{1 - k_1 s + k_2 s^2 - \ldots \pm k_n s^n}{1 + k_1 s + k_2 s^2 + \ldots + k_n s^n}, \quad (2)$$

where τ is the delay of the sensor and k_i are the coefficients in the approximation [23]. As an example, the time delay of the semicircular sensory system is $\tau_{y,a} = 5 \ldots 440 ms$, while the sensor dynamics has a second-order transfer function [1]. The Bode function of the exact transfer function of the sensor $G_{sens} \cdot G_{delay}$ and its approximation are illustrated in Figure 2. It can be seen that a fifth-order Padé approximation results in an accurate transfer function for the sensory system.

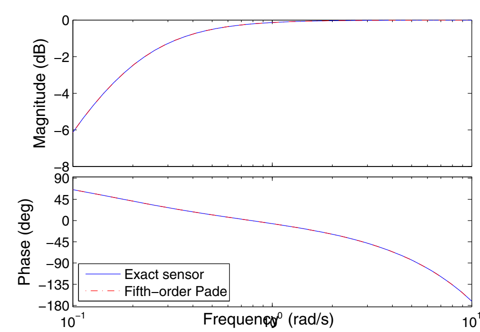

Fig. 2. Approximation of the sensor dynamics

In the prediction algorithm two look-ahead points are applied. Based on the results of scenarios with real vehicle tests, the first point from the current position of the driver is $T_{p,1} = 0.53s$ ahead, while the second point is $T_{p,2} = 0.93s$, see [11].

III. The human process concerning trajectory tracking

The visual and motion signals are processed through the brain of the driver. Since the aim of the paper is to formulate a control-oriented driver model, in the following a simplified method for the process of the sensed signals is proposed.

Through the process the trajectory tracking of the vehicle must be performed, which also considers the prediction of the vehicle motion.

The steering angle δ is divided into two parts, such as

$$\delta = \delta_a + \delta_p \tag{3}$$

where δ_a is the steering angle, which represents the input, generated from the current information $e_{y,a}$ and $e_{\psi,a}$. The steering angle δ_p is computed from the predicted errors $e_{p,i}$. In the following the design of δ_a and δ_p is detailed.

A. Design of the current steering component δ_a

The design of δ_a is based on the non-holonomic model of the vehicle. The equations of the lateral vehicle motion is formulated as

$$\dot{y} = v \sin\psi \tag{4a}$$
$$\dot{\psi} = \frac{v}{L}\tan\delta_a \tag{4b}$$

where v is the longitudinal velocity, L is the distance between the vehicle axles, ψ is the yaw angle and y is the lateral vehicle motion. During the control design the assumptions $\tan\delta_a \approx \delta_a$, $\sin\psi \approx \psi$ for small angles are considered. The equations (4) can be transformed into a state-space representation, where the state vector is $x = \begin{bmatrix} y & \psi \end{bmatrix}^T$, the control input is $u = \delta_a$. The state equation is written as

$$\dot{x} = A(\rho)x + Bu, \tag{5}$$

where the matrices contain the parameters:

$$A(\rho) = \begin{bmatrix} 0 & v \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ v/L \end{bmatrix}.$$

The role of the steering angle δ_a is to guarantee the tracking of the vehicle. It results in the performance

$$z_1 = y - y_{ref}, \qquad |z_1| \to min. \tag{6}$$

The appropriate trajectory tracking also requires the minimization of the heading error

$$z_2 = \psi - \psi_{ref}, \qquad |z_2| \to min. \tag{7}$$

Moreover, the driver has a limited capability on the turning of the steering wheel. Thus, the minimization of the control input is also considered as

$$z_3 = \delta_a, \qquad |z_3| \to min. \tag{8}$$

The defined three performances are formed in a vector $z = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}^T$. Thus, the state-space representation (5) can be extended with z and the measured signals $y_m = \begin{bmatrix} \psi & y \end{bmatrix}^T$:

$$\dot{x} = Ax + Bu \tag{9a}$$
$$z = C_1 x + D_{11} w + D_{12} u \tag{9b}$$
$$y_m = C_2 x \tag{9c}$$

where $C_{1,2}, D_{11,12}$ are matrices of the performances and the measurements. Moreover, $w = \begin{bmatrix} y_{ref} & \psi_{ref} \end{bmatrix}^T$ incorporates in the reference signals.

The performances of the system (9) are guaranteed by robust $\mathcal{H}_\infty$ control strategy, in which the robustness against the sensor noises are considered. The control design is based on a weighting strategy, which is formulated through a closed-loop interconnection structure, see Figure 3.

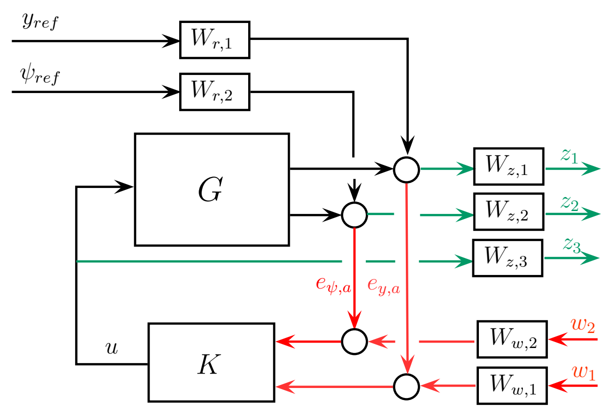

Fig. 3. Closed-loop interconnection

The interconnection structure contains several weighting functions, whose roles are to guarantee the trade-off between the performances and to scale the signals. The weights $W_{w,1}, W_{w,2}$ are related to the sensor characteristics on the actual lateral error $e_{y,a} = y - y_{ref}$ and yaw error $e_{\psi,a} = \psi - \psi_{ref}$ measurement. These weights are related to the sensor dynamics, as detailed in Section II. The role of $W_{r,1}, W_{r,2}$ is to scale the reference signals. These weights are formed as constant values. $W_{z,1}$, $W_{z,2}$ and $W_{z,3}$ are the weights for the performances, which represent the minimization of z_1, z_2 and z_3, see (6)-(8). These are chosen in the form

$$W_{z,i} = A_i \frac{\epsilon_{2,i}s^2 + \epsilon_{1,i}s + 1}{T_{2,i}s^2 + T_{1,i}s + 1}, \tag{10}$$

where $A_i, \epsilon_{1,i}, \epsilon_{2,i}, T_{1,i}$ and $T_{2,i}$ are design parameters.

The objective of $\mathcal{H}_\infty$ control is to minimize the inf-norm of the transfer function $T_{z_\infty w}$. More precisely, the problem can be stated as follows [24], [25]. The LMI (Linear Matrix Inequality) problem of $\mathcal{H}_\infty$ performance is formulated as: the closed-loop RMS gain from w to z_∞ does not exceed γ if and only if there exists a symmetric and positive definite matrix X_∞ such that

$$\begin{bmatrix} A_{cl}X_\infty + X_\infty A_{cl}^T & X_\infty B_{cl} & C_{cl}^T \\ B_{cl}^T X_\infty & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \tag{11}$$

with $\gamma > 0$.

For the plant G find an admissible control K which satisfies the following design criteria:

- the closed-loop system must be asymptotically stable,
- the closed-loop transfer function from w_1, w_2 to z_1, z_2, z_3 satisfies the constraint:

$$\left\| T_{z_\infty F_{d,fb}}(s) \right\|_\infty < \gamma, \tag{12}$$

for a given real positive value γ, where $T_{z_\infty F_{d,fb}}(s)$ is cosensitivity function.

The result of the LMI problem (11) is used to calculate the controller for the computation of the steering angle δ_a.

B. Design of the predicted steering component δ_p

The design of δ_p results from the error of the predicted road section.

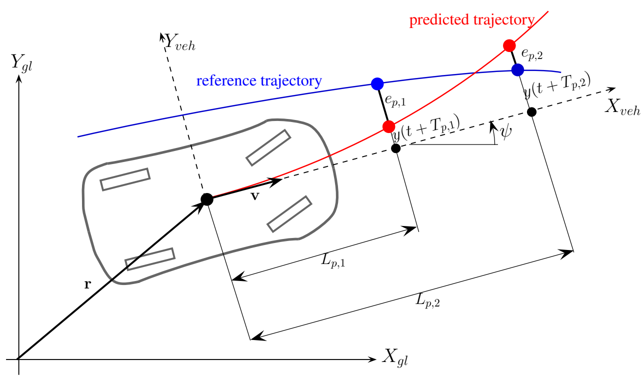

Fig. 4. Prediction of the oncoming trajectory

The prediction of the lateral error in $T_{p,i}$ requires the prediction of the motion, see Figure 4. The position of the vehicle in the coordinate system of the vehicle at the time $t+T_{p,i}$ is computed using the Taylor series technique. It can result in adequate approximation of the trajectory for $n=2$, such as

$$\begin{aligned} y(t+T_{p,i}) &= \sum_{n=0}^{\infty} \frac{y^n(t)}{n!} T_{p,i}^n \approx \\ &\approx y(t) + \dot{y}(t)T_{p,i} + \frac{1}{2}\ddot{y}(t)T_{p,i}^2 \end{aligned} \tag{13}$$

Considering that the longitudinal velocity is constant, the prediction horizon for $T_{p,i}$ is computed as $L_{p,i} = vT_{p,i}$. Thus, the position vector of the vehicle at the time $t+T_{p,i}$ is

$$\mathbf{r_{veh}}(t+T_{p,i}) = \begin{bmatrix} L_{p,i} \\ y(t+T_{p,i}) \end{bmatrix} \tag{14}$$

The computation of the predicted position of the vehicle in the global coordinate system $\mathbf{r_{gl}}$ requires the rotation of the vector $\mathbf{r_{veh}}(t+T_{p,i})$ with the current yaw angle $\psi(t)$ and its addition to the current position $\mathbf{r}(t)$. Thus, the resulting position is

$$\mathbf{r_{gl}}(t+T_{p,i}) = \mathbf{r}(t) + \begin{bmatrix} L_{p,i}\cos\psi(t) - y(t+T_{p,i})\sin\psi(t) \\ L_{p,i}\sin\psi(t) + y(t+T_{p,i})\cos\psi(t) \end{bmatrix} \tag{15}$$

The errors $e_{p,1}, e_{p,2}$ result in the difference between the reference position $\mathbf{r_{ref}}(t+T_{p,i}) = \begin{bmatrix} x_{ref}(t+T_{p,i}) & y_{ref}(t+T_{p,i}) \end{bmatrix}^T$ and the predicted positions $\mathbf{r_{gl}}(t+T_{p,i})$. The reference positions are defined as the intersection of the reference trajectory and the perpendicular on the vehicle direction x, see Figure 4. The errors are computed using (15) as the length of the resulting vector difference as

$$\mathbf{e_i} = \mathbf{r_{ref}}(t+T_{p,i}) - \mathbf{r_{gl}}(t+T_{p,i}) \tag{16a}$$

$$e_{p,i} = \left|\mathbf{e_i}\right| \tag{16b}$$

The predicted steering angle δ_p component is computed as the weighted sum of the errors using (16b), such as

$$\delta_p = K_{p,1}e_{p,1}\cdot sign(z_{p,1}) + K_{p,2}e_{p,2}\cdot sign(z_{p,2}) \tag{17}$$

where $K_{p,1}$ and $K_{p,2}$ are the weights and $sign(z_{p,i})$ represents the direction of the lateral error $e_{p,i}$. The value $z_{p,i}$ is the coefficient of the vertical component, resulting in the cross product $\mathbf{v}\times\mathbf{e_i}$, where $\mathbf{v}$ is the vector of the longitudinal velocity.

C. Determination of controller parameters

In the control design several tuning parameters are used, e.g., in the weighting of the $\mathcal{H}_\infty$ controller and in the computation of δ_p. Some of these parameters are selected based on physical measurements, such as the scaling weights of the noises $W_{w,1}, W_{w,2}$. Some parameters are selected intuitively, which characterizes the required operation of the entire system. In an autonomous steering control, which approximates the steering behavior of the driver, the intuitively selected parameters should represent the driver's intentions. In the following, the determination of the most important controller parameters is presented.

The most effective weights to improve the proposed $\mathcal{H}_\infty$ controller are the performance weights $W_{z,i}$. These weights determine the balance between the performances (6)-(8). The weights are selected in the form (10), which contains the gain A_i and the frequency characteristics. Generally, the weights have high values at low frequency domain and low values at high frequency domain. The low frequency domain is linked to the steady state value of the lateral error and the yaw error. In the high frequency domain the disturbance rejection is important, disturbances have significant effects. Therefore, the frequency dependence of $W_{z,i}$ can be selected intuitively, while the gains A_i are selected with an optimization procedure, in which the steering intention of the driver is considered.

Moreover, further effective parameters to influence the operation of the entire control system are the weights $K_{p,i}$ and the prediction times $T_{p,i}$. The role of these parameters is to characterize the prediction in the driving behavior, e.g. the human adaptation to the forthcoming road curvature. Since the prediction is an important factor in the human steering, the parameters $K_{p,1}, K_{p,2}$ and $T_{p,1}, T_{p,2}$ are selected using an optimization process together with A_i.

Since the parameters $A_i, K_{p,i}, T_{p,i}$ defines the steering of the human driver in the autonomous control system, several measurements on human steering maneuvers must be performed. These measurements are significant in the the autonomous steering control. Using a test vehicle the longitudinal velocity v, the position of the vehicle (e.g. GPS coordinates), the yaw angle ψ and the steering angle δ are logged. Furthermore, it is required to receive information of the reference of the trajectory (GPS measurements or map database). This information is used to compute both y_{ref} and ψ_{ref}.

After the data collection the velocity is handled as a parameter of the vehicle model (5) and the references are considered as the input signals of the controller. The signals v, y_{ref}, ψ_{ref} results in the lateral position $\hat{y}$, the yaw angle $\hat{\psi}$ and the steering angle $\hat{\delta}$. Since the aim is to approximate the steering behavior of the driver, it is required to find the controller parameters $A_i, K_{p,i}, T_{p,i}$, with which the errors $|y - \hat{y}|$, $|\psi - \hat{\psi}|$ and $|\delta - \hat{\delta}|$ are minimized simultaneously. Thus, the following optimization problem is formulated:

$$\min_{A_i, K_{p,i}, T_{p,i}} \left(|y - \hat{y}| + |\psi - \hat{\psi}| + |\delta - \hat{\delta}| \right) \tag{18}$$

such that $A_i, K_{p,i}, T_{p,i} \in \mathcal{P}$, where $\mathcal{P}$ represents the set of the possible controller parameters. Thus, the role of $\mathcal{P}$ is to avoid the unstable controllers and the negative prediction times. In the optimization algorithm several methods can be selected, e.g., [26], [27]. The solution of the optimization task (18) leads to the controller parameters.

IV. Demonstration of the steering control on test vehicle

In this section the proposed driver steering controller is demonstrated on an electric real vehicle. The test scenario has been performed in a small test course in the surrounding of Győr, Hungary, see Figure 5. In the test the vehicle is controlled by a driver, and several signals are measured. The signals are the steering wheel angle, velocity and acceleration using the vehicle's own sensors and the position of the vehicle using GPS with 100 Hz frequency. Through the collection of the coordinates the Real-Time Kinetic (RTK) correction of the GPS data is used, which resulted in 0.008 m precision. The logged GPS data are transformed to a local coordinate system North-East-Down (NED) with m dimensions. The measured data have been synchronized and marked with timestamps.

The verification is shown in Figure 6. In the illustration the *Route* represents the middle of the lane in the test course, while the *Driver* shows the path, which has been previously performed by the driver. The figure shows that the path of the driver deviates from the middle of the lane. The driver starts with a cornering maneuver ahead of the bend and follows a cruising with high radius. The cornering maneuver is zoomed in Figure 7.

Two autonomous steering controllers are tested through the vehicle: the K_1 is a pure $\mathcal{H}_\infty$ controller, while the K_2 controller is constructed through the proposed control method and parameter optimization process. The purpose of K_1 is to verify that the $\mathcal{H}_\infty$ controller is able to guarantee the tracking of the course, i.e., the *Route* is approximated. However, the obtained path significantly differs from the driver's scenario, see the difference between K_1 path and the *Driver* path. In the controller K_2 the driver's characteristics are also built. Consequently, the deviation of the K_2 path

Fig. 5. Course of the vehicle near Győr, Hungary

from the *Driver* is significantly smaller. The prediction of K_2 is close to the human prediction. The resulted prediction times are $T_{p,1} = 0.62s$ and $T_{p,2} = 0.89s$. These values are close to the results of the publication [11].

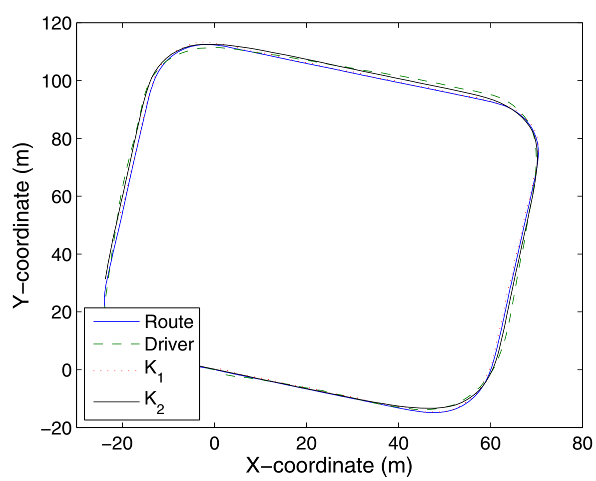

Fig. 6. Verification of the autonomous control system

Using K_2 controller the tracking of the predefined path is guaranteed. However, the overshooting features during the transient sections are not considered. A more precise tracking of the driver's path can be reached by applying a more complex performance weighing in the $\mathcal{H}_\infty$ control design and in the control rule of the prediction. However, the increase in the complexity of the weights might lead to difficulties in the implementation of the real-time controller. In the following step of the research the appropriate structure of the controller should be selected and a balance between the appropriate tracking and the implementation requirements.

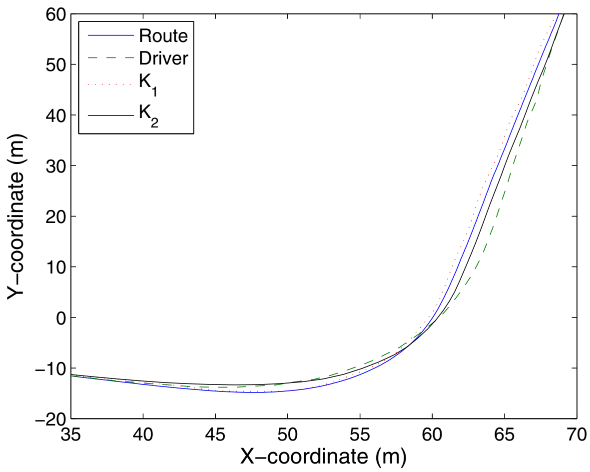

Fig. 7. Results in a bend of the test course

V. Conclusions

The paper has focused on the modeling of the human driver behavior during steering. Modeling the human driver can be divided into three subtasks, i.e., the human sensory system, the human brain's processing and human actuation. The human process for the determination of steering angle is modeled as an $\mathcal{H}_\infty$ controller, which is able to reject the disturbances and guarantee the path tracking performance. In the tracking problem the driver capability is also considered by using a parameter-dependent weighting function. Moreover, a prediction with two look-ahead points in the control rule is proposed. An optimization procedure, which results in a human-like autonomous steering control system, is also proposed. The autonomous steering control is verified through measurement on a test vehicle. The results show that the proposed method approximates the steering behavior of the driver adequately.

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