2018 26th Mediterranean Conference on Control and Automation (MED)
Zadar, Croatia, June 19-22, 2018

Reduced-order Multiple Observer Design for Takagi-Sugeno Models with Unknown Inputs

Mihai Lungu, Olfa Boubaker, and Romulus Lungu

***Abstract*— The paper presents the design of a new reduced-order multiple observer (ROMO) for state estimation of unknown input Takagi-Sugeno (T-S) models; this is only the second reduced-order multiple observer ever designed for such systems. The proposed observer is obtained by applying the classical methods of reduced-order observers' design for linear multivariable systems with unknown inputs to the case of T-S models. The originality of the designed observer compared to the only existing one is related to: 1) no existence conditions are required, 2) no use of the pole placement method leading to sensitivity problems is necessary and, 3) the solution is designed using available commercial LMI tools. The related algorithm of the proposed ROMO is structured into five steps; its efficiency is proved by using a concrete example of a light aircraft lateral-directional motion.**

Keywords — multiple observer; Takagi-Sugeno model; state estimation; unknown input.

I. Introduction

The unknown input observers are designed for systems affected by both known and unknown inputs; noises, disturbances' uncertainty measurements, sensors and actuators' fault or other disturbances can influence any process' behavior [1]; therefore, to minimize the negative effects of disturbances in controlled systems, the state and disturbances' estimation is a necessity. It is obvious that designing observers for nonlinear systems is much more difficult than those for linear systems for which the control theory is more mature. One interesting solution to overcome difficulties related to the design and implementation of the nonlinear observers is the usage of Takagi-Sugeno (T-S) multiple models [2]; the sum of some linear models characterizing the system's behavior in a specific operating regime represents a multiple model. The principle of the T-S multiple model approach is based on the reduction of the nonlinear system's complexity by the decomposition of its operating space in a finite number of operating zones [3]; one obtains local models which are linear, affine, and time invariant due to some linearization assumptions. The relative contribution of each sub-model is quantified by a designed weighting function.

Recently, the problem of state reconstruction via multiple observers has received considerable attention and different algorithms are designed [1-8]. For example, in the paper [6] where the objective is to construct the state variables of a perturbed nonlinear system, a Lyapunov based sliding mode multiple observer is presented as a solution to the complex problem. The proof of the observer stability was ensured using suitable estimation gains computed via linear matrix inequality (LMI) tools. In [7], an observer to estimate both the states and the parameters is proposed for a Takagi-Sugeno model of heat-exchanger-zones whose model depends on the unknown parameters; by means of some sector extreme values and a sector nonlinearity approach, the authors completed the rewriting of the unknown parameters; then, an L_2 formulation is used for canceling the estimation error associated to the weighting functions. The paper [8] provides the multiple actuator faults' estimation by an observer designed for T-S multiple models; one modifies the discontinuous observer term (causing the sliding motion) such that the output errors of the system are weighted by means of appropriate gains, the distinguish between dynamic and static relations in the Takagi-Sugeno descriptor model being the main innovation.

The multiple observers have been initially used for failure detection [9] and switching control [10]. Later, in the research area of multiple models' estimation, in paper [11], the state estimation for a unknown input nonlinear T-S multiple model is achieved by an approach converting the unknown outputs into "pseudo" unknown inputs, not taken into consideration in the observer's design algorithm. The design of multiple observers for nonlinear systems in multiple model has been achieved in [12] using the sliding mode technique and an innovative LMI based condition. In [13], for descriptor systems, the authors design a robust unknown input observer by using the LMI theory, while in [14] a mathematical transformation is the key for the design of a multiple observer associated to a class of uncertain nonlinear systems written under the form of Takagi-Sugeno multiple models; this way, the unknown outputs of the system become unknown inputs. In [15] an output feedback controller for time-varying state-delay T-S fuzzy systems is obtained; the conservatism of the precedent works was reduced in paper [16] by a method which relaxes the existence conditions of the approach and reduces the number of LMIs that have to be solved to make the method viable. Also, many papers applied the T-S fuzzy models for networked nonlinear systems and fuzzy stochastic systems [17].

All the above presented observers are full-order order multiple observers (FOMOs); in the literature, the only designed and software implemented reduced-order multiple observer (ROMO) for Takagi-Sugeno systems with unknown inputs has been introduced in [18]; therefore, we can remark that the design of such observers is still an open research area.

This work was supported by a grant of the Ministry of National Education and Scientific Research, RDI Programme for Space Technology and Advanced Research - STAR, project number 181/20.07.2017.

Mihai Lungu is with University of Craiova, Faculty of Electrical Engineering, Craiova, Romania (e-mail: Lma1312@yahoo.com).
Olfa Boubaker is with National Institute of Applied Sciences and Technology, Tunis, Tunisia (e-mail: olfa_insat@yahoo.com).
Romulus Lungu is with University of Craiova, Faculty of Electrical Engineering, Craiova, Romania (e-mail: romulus_lungu@yahoo.com).

Using ROMOs instead of FOMOs decreases certainly the number of sensors and guarantees a lot of other advantages related to general usage of reduced-order observers. The drawbacks of the only existing ROMO (the one from [18]) are: 1) the large number of existence conditions; 2) the usage of pole placement method leading to the undesired phenomena so called eigenvalues' sensitivity; 3) high convergence time for the estimation error. It will be shown that the ROMO designed in this paper has no existence conditions, does not use the pole placement technique, and the problem of ROMO's design for T-S unknown input systems can be reduced to a standard one (the unknown inputs are not part of the observer's equations). The observer proposed in this paper is obtained by applying the classical methods of design reduced-order observers for linear multi-variable systems with unknown inputs to the case of T-S models.

The paper is organized as follows: the problem statement, the stability analysis, and the design methodology of the new ROMO are presented in the second section; in the next section, the design procedure is summarized into an algorithm for software implementation, while in section IV the new observer is validated for the case of a light aircraft lateral-directional motion. Some final remarks are provided in section V.

Notations: Throughout the paper, the transpose of the matrix X is denoted with X^{T}; for an easy notation, the time variable is omitted.

II. Design of the New Reduced-order Multiple Observer

Due to their simplicity, the vast majority of nonlinear systems can be represented by the well known T-S models. Any nonlinear system can be then put under the form of T-S model by using an interpolation function between some linear sub-models and their associated activation functions. These are associated to each sub-model.

A. Problem Statement

Consider the T-S model described by [1, 19]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left[A_i x(t) + B_i u(t) + D v(t)\right], \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$ is the state vector, $u(t) \in \mathcal{R}^m$ – the known inputs' vector, $v(t) \in \mathcal{R}^q$ – vector of unknown inputs, $y(t) \in \mathcal{R}^p$ – the outputs vector, while C, D, A_i, and $B_i, i = \overline{1,M}$ are constant matrices of appropriate dimensions assumed to be known. The activation functions $\mu_i(\xi(t)), i = \overline{1,M}$, fulfil the conditions: $\sum_{i=1}^{M} \mu_i(\xi(t)) = 1, 0 \le \mu_i(\xi(t)) \le 1, (\forall)\, i = \overline{1,M}$; the decision vector $\xi(t)$ is influenced by the inputs and other variables. There is a close connection between M (the number of local models), the activation functions' structure, the original nonlinear system's complexity, and on the modeling accuracy.

Without loss of the generality, the following assumptions are considered for system (1): **A1)** $\operatorname{rank}(C) = p, \operatorname{rank}(D) = q$; **A2)** the pairs (C, A_i) are observable; **A3)** $n > r > p \ge q$; **A4)** $n = p + q$. A particular form for the matrix C can be chosen as $C = [C_1 \;\; 0_{p\times q}]$, where $C_1 \in \mathcal{R}^{p\times p}$ is a full rank matrix. As long as the matrix C is full row rank (condition A1), the system's output will always can be written as $y(t) = [C_1 \;\; 0]x(t)$ by means of an orthogonal transformation.

The problem to be solved in this paper is the estimation of the system state vector $x(t)$ by designing an r-order ROMO which generates the estimation of the vector:

$$z(t) = Nx(t), \tag{2}$$

where $N \in \mathcal{R}^{r\times n}$ and $z(t) \in \mathcal{R}^r$ is the vector enclosing a part of the system's states or combinations of them. Let us denote the system's estimated vector with $\hat{x}(t)$ and the estimation vector of $z(t)$ with $\hat{z}(t)$. The dynamics of the new ROMO is:

$$\begin{cases} L\dot{\hat{z}}(t) = \sum_{i=1}^{M} \mu_i(\xi(t))\left[F_i \hat{z}(t) + G_i u(t) + H_i y(t)\right], \\ \hat{x}(t) = P\hat{z}(t) + Qy(t), \end{cases} \tag{3}$$

where the constant matrices $F_i \in \mathcal{R}^{r\times r}, G_i \in \mathcal{R}^{r\times m}, H_i \in \mathcal{R}^{r\times p}$, $P \in \mathcal{R}^{n\times r}, Q \in \mathcal{R}^{n\times p}$, $N \in \mathcal{R}^{r\times n}$, and $L \in \mathcal{R}^{r\times r}$ will be computed such that when $t \to \infty$, $\hat{z}(t)$ and $\hat{x}(t)$ converge to $z(t)$ and $x(t)$, respectively.

B. Stability Analysis

Consider the observer error vector given by $e(t) = \hat{z}(t) - z(t)$; it is easy to prove that:

$$\begin{aligned} L\dot{e}(t) = &\sum_{i=1}^{M} \mu_i(\xi(t))F_i e + \sum_{i=1}^{M} \mu_i(\xi(t))(F_i N - LNA_i + H_i C)x(t) + \\ &+ \sum_{i=1}^{M} \mu_i(\xi(t))(G_i - LNB_i)u(t) - \sum_{i=1}^{M} \mu_i(\xi(t))LNDv(t). \end{aligned} \tag{4}$$

If $LND = 0, F_i N - LNA_i + H_i C = 0, G_i = LNB_i$, and the matrix L is non-singular, the error dynamics is given by $\dot{e}(t) = \widetilde{F}e(t)$, where $\widetilde{F} = \sum_{i=1}^{M} \mu_i(\xi) L^{-1} F_i$; it corresponds to a stable dynamics if the matrix $\widetilde{F}$ is Hurwitz. Calculating the state estimation error $e_x(t) = \hat{x}(t) - x(t)$, we obtain: $e_x(t) = Pe + (QC + PN - I_n)x(t)$; if $QC + PN - I_n = 0$ and $e(t) \to 0$, it results: $e_x(t) \to 0$.

In order to find other existence conditions of the ROMO, we consider the Lyapunov function $V(e) = e^{\mathrm{T}}Re$, where R is a symmetrical and positive-defined matrix; it results: $\dot{V}(e) = \sum_{i=1}^{M} \mu_i(\xi)\left\{e^{\mathrm{T}}\left[(L^{-1}F_i)^{\mathrm{T}} R + R(L^{-1}F_i)\right]e\right\}$. The convergence of the ROMO is achieved if the nonlinear matrix inequality:

$$(L^{-1}F_i)^{\mathrm{T}} R + R(L^{-1}F_i) < 0 \tag{5}$$

is satisfied (constraint of the observer (3)) for any $i = \overline{1,M}$.

Having in mind that for the ROMO algorithm, the design of the matrices F_i will be achieved before solving the above matrix inequalities, the inequalities (5) are then LMIs and can be easily solved without any problem by using a commercial software. The convergence of the new ROMO can be then synthesized into the following theorem:

Theorem 1: Consider the T-S model (1) under the assumptions A1-A4. For any initial conditions $x(0)$, $z(0)$, $v(t)$ and input vector $u(t)$, $\lim_{t\to\infty} e(t)=0$ and $\lim_{t\to\infty} e_x(t)=0$ if the matrix L is non-singular and the following conditions are satisfied: 1) the matrix $\widetilde{F}$ is Hurwitz, 2) the LMIs (5) are verified, and

$$\begin{aligned} &LND=0,\\ &F_iN-LNA_i+H_iC=0,\\ &G_i=LNB_i,\\ &QC+PN=I_n. \end{aligned} \tag{6}$$

C. Design of the New Observer

To determine the unknown matrices of the new ROMO, one judiciously chooses the matrix N and partitionate the matrices $L, A_i, D, F_i, H_i, Q, P$ as following:

$$\begin{aligned} &N=\begin{bmatrix} N_1 & N_2\\ N_3 & N_4\end{bmatrix}=\begin{bmatrix} 0_{p\times q} & I_{p\times p}\\ 0_{(r-p)\times q} & 0_{(r-p)\times p}\end{bmatrix}, A_i=\begin{bmatrix} A_1^i & A_2^i\\ A_3^i & A_4^i\end{bmatrix},\\ &L=\begin{bmatrix} L_1 & L_2\\ L_3 & L_4\end{bmatrix}=\begin{bmatrix}\widetilde{L}_1 & \widetilde{L}_2\end{bmatrix}, \widetilde{L}_1=\begin{bmatrix} L_1\\ L_3\end{bmatrix}, \widetilde{L}_2=\begin{bmatrix} L_2\\ L_4\end{bmatrix},\\ &D=\begin{bmatrix} D_1\\ D_2\end{bmatrix}, F_i=\begin{bmatrix} F_1^i & F_2^i\\ F_3^i & F_4^i\end{bmatrix}=\begin{bmatrix}\overline{F}_1^i & \overline{F}_2^i\end{bmatrix}, \overline{F}_1^i=\begin{bmatrix} F_1^i\\ F_3^i\end{bmatrix},\\ &\overline{F}_2^i=\begin{bmatrix} F_2^i\\ F_4^i\end{bmatrix}, H_i=\begin{bmatrix} H_1^i\\ H_2^i\end{bmatrix}, Q=\begin{bmatrix} Q_1\\ Q_2\end{bmatrix}, P=\begin{bmatrix} P_1 & P_2\\ P_3 & P_4\end{bmatrix}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} &A_1^i\in\mathcal{R}^{q\times q}, A_2^i\in\mathcal{R}^{q\times p}, A_3^i\in\mathcal{R}^{p\times q}, A_4^i\in\mathcal{R}^{p\times p},\\ &L_1\in\mathcal{R}^{q\times p}, L_2\in\mathcal{R}^{q\times(r-p)}, L_3\in\mathcal{R}^{(r-q)\times p}, L_4\in\mathcal{R}^{(r-q)\times(r-p)},\\ &\widetilde{L}_1\in\mathcal{R}^{r\times p}, \widetilde{L}_2\in\mathcal{R}^{r\times(r-p)}, D_1\in\mathcal{R}^{q\times q}, D_2\in\mathcal{R}^{p\times q},\\ &F_1^i\in\mathcal{R}^{q\times p}, F_2^i\in\mathcal{R}^{q\times(r-p)}, F_3^i\in\mathcal{R}^{(r-q)\times p}, F_4^i\in\mathcal{R}^{(r-q)\times(r-p)},\\ &\overline{F}_1^i\in\mathcal{R}^{r\times p}, \overline{F}_2^i\in\mathcal{R}^{r\times(r-p)}, H_1^i\in\mathcal{R}^{q\times p}, H_2^i\in\mathcal{R}^{(r-q)\times p},\\ &P_1\in\mathcal{R}^{q\times p}, P_2\in\mathcal{R}^{q\times(r-p)}, P_3\in\mathcal{R}^{p\times p}, P_4\in\mathcal{R}^{p\times(r-p)},\\ &Q_1\in\mathcal{R}^{q\times p}, Q_2\in\mathcal{R}^{p\times p}. \end{aligned} \tag{8}$$

The following theorem descibes the design of the new reduced-order multiple observer.

Theorem 2: Consider the T-S model (1) under the assumptions A1-A4; the ROMO (3) is convergent if, for any $i=\overline{1,M}$, there exists a symmetrical and positive-defined matrix R verifying the matrix inequality (5), if the matrix L is full rank, the matrix $\widetilde{F}$ is Hurwitz, and the matrices L, G_i, F_i, H_i, Q, and P are given by:

$$\begin{aligned} &L=\begin{bmatrix}\widetilde{L}_1 & \widetilde{L}_2\end{bmatrix}, \widetilde{L}_1=I_{r\times p}-\overline{D}_2(C_1D_2)^+C_1,\\ &G_i=\begin{bmatrix}0_{r\times q} & \widetilde{L}_1\end{bmatrix}B_i, H_i=\widetilde{L}_1A_3^i\widetilde{C}_1^+,\\ &F_i=\begin{bmatrix}\overline{F}_1^i & \overline{F}_2^i\end{bmatrix}=\begin{bmatrix}\widetilde{L}_1\left(A_4^i-A_3^i\widetilde{C}_1^+\widetilde{C}_2\right) & \overline{F}_2^i\end{bmatrix},\\ &Q=\begin{bmatrix}\widetilde{C}_1^+\\ I_p-\widetilde{C}_1\left(C_1\widetilde{C}_1\right)^+C_1\end{bmatrix},\\ &P=\begin{bmatrix}-\widetilde{C}_1^+\widetilde{C}_2 & 0_{q\times(r-p)}\\ I_p-\widetilde{C}_2+\widetilde{C}_1\left(C_1\widetilde{C}_1\right)^+C_1\widetilde{C}_2 & 0_{p\times(r-p)}\end{bmatrix}, \end{aligned} \tag{9}$$

where matrices $\widetilde{L}_2\in\mathcal{R}^{r\times(r-p)}$ and $\overline{F}_2^i\in\mathcal{R}^{r\times(r-p)}$ are randomly chosen, $\widetilde{C}_1\in\mathcal{R}^{p\times q}$ and $\widetilde{C}_2\in\mathcal{R}^{p\times p}$ are sub-matrices of the matrix C, and $\overline{D}_2=I_{r\times p}D_2$. □

Proof: Using equations (7) and (8), the first equation (6) becomes: $\widetilde{L}_1D_2=0_{r\times q}$; to compute the matrix $\widetilde{L}_1$ from this equation, one chooses this matrix of the following form: $\widetilde{L}_1=I_{r\times p}+T_1C_1$, with $I_{r\times p}$ – the identity matrix with r lines and p columns and $T_1\in\mathcal{R}^{r\times p}$ – an unknown matrix to be calculated. With $\overline{D}_2=I_{r\times p}D_2$, one obtains: $T_1=-\overline{D}_2(C_1D_2)^+$, where $(C_1D_2)^+$ is the generalized pseudo-inverse of (C_1D_2), given by [20, 21]: $(C_1D_2)^+=\left[(C_1D_2)^{\mathrm{T}}(C_1D_2)\right]^{-1}(C_1D_2)^{\mathrm{T}}$. The generalized pseudo-inverse of (C_1D_2) can be obtained if and only if (C_1D_2) is full column rank, i.e. $\operatorname{rank}(C_1D_2)=q$; having in mind the assuption A1 and that C_1 is a full rank matrix, it is easy to show that $\operatorname{rank}(C_1D_2)=q$; thus, one obtains $\widetilde{L}_1=I_{r\times p}-\overline{D}_2(C_1D_2)^+C_1$; the matrix $\widetilde{L}_2\in\mathcal{R}^{r\times(r-p)}$ is chosen randomly; the first expression (9) can be obtained. Using now this expression and the third equation (6), we obtain the expressions of the matrices G_i.

For solving the second equation (6), we partitionate the matrix C as: $C=\begin{bmatrix}\widetilde{C}_1 & \widetilde{C}_2\end{bmatrix}$, with $\widetilde{C}_1\in\mathcal{R}^{p\times q}, \widetilde{C}_2\in\mathcal{R}^{p\times p}$ and we randomly choose the matrices F_2^i and F_4^i; as a consequence, the matrix $\overline{F}_2^i=\begin{bmatrix}F_2^i\\ F_4^i\end{bmatrix}$ is random. The second equation (6) leads the following system:

$$\begin{cases} -L_1A_3^i+H_1^i\widetilde{C}_1=0_{q\times q},\\ F_1^i-L_1A_4^i+H_1^i\widetilde{C}_2=0_{q\times p},\\ -L_3A_3^i+H_2^i\widetilde{C}_1=0_{(r-q)\times q},\\ F_3^i-L_3A_4^i+H_2^i\widetilde{C}_2=0_{(r-q)\times p}; \end{cases} \tag{10}$$

solving the above system with respect to $F_1^i, F_3^i, H_1^i, H_2^i$, we get: $F_1^i=L_1\left(A_4^i-A_3^i\widetilde{C}_1^+\widetilde{C}_2\right), F_3^i=L_3\left(A_4^i-A_3^i\widetilde{C}_1^+\widetilde{C}_2\right), H_1^i=L_1A_3^i\widetilde{C}_1^+$, $H_2^i=L_3A_3^i\widetilde{C}_1^+$; having in mind the forms of matrices $\widetilde{L}_1$ and $\overline{F}_1^i$, the fourth and the fifth expressions (9) can be deduced.

To solve the last equation (6) with respect to the matrices Q and P, one uses the partitioning of these matrices as in (7) and

(8); choosing $P_2 = 0_{q\times(r-p)}$ and $P_4 = 0_{p\times(r-p)}$, the last equation (6) is transformed into the following system:

$$\begin{cases} Q_1\widetilde{C}_1 = I_q, \\ Q_1\widetilde{C}_2 + P_1 = 0_{q\times p}, \\ Q_2\widetilde{C}_1 = 0_{p\times q}, \\ Q_2\widetilde{C}_2 + P_3 = I_p; \end{cases} \tag{11}$$

from the first two equations (11), we obtain: $Q_1 = \widetilde{C}_1^+$ and $P_1 = -Q_1\widetilde{C}_2 = -\widetilde{C}_1^+\widetilde{C}_2$. The matrix Q_2 is calculated from the third equation (11) as follows: one writes $Q_2 = I_p + T_2C_1$, where $T_2 \in \mathcal{R}^{p\times p}$ – an unknown matrix to be calculated; with this, one succesively gets: $T_2 = -\widetilde{C}_1\left(C_1\widetilde{C}_1\right)^+$ and $Q_2 = I_p - -\widetilde{C}_1\left(C_1\widetilde{C}_1\right)^+C_1$. From the fourth equation (11), we obtain: $P_3 = I_p - \widetilde{C}_2 + \widetilde{C}_1\left(C_1\widetilde{C}_1\right)^+C_1\widetilde{C}_2$; by concatenation of the matrices Q_1, Q_2, P_1, P_3, $P_2 = 0_{q\times(r-p)}$, and $P_4 = 0_{p\times(r-p)}$, the last two expressions (9) are obtained. Now, the proof of the Theorem 2 is complete. ■

Remark 1: The matrices of the observer, obtained with (9), should satisfy three constraints: **C1)** there exists a symmetrical and positive-defined matrix R verifying the matrix inequality (5); **C2)** the matrix L is non-singular; **C3)** the matrix $\widetilde{F} = \sum_{i=1}^{M}\mu_i(\xi)L^{-1}F_i$ is Hurwitz. If the constraints are satisfied, the matrices of the new ROMO have been obtained properly; otherwise, other random matrices $\widetilde{L}_2$ and $\widetilde{F}_2^i$ are chosen and the matrices of the observer are again calculated with (9) until the constraints C1-C3 are fulfilled. In the new algorithm software implementation, the fulfillment of the three constraints is made in a "while" loop.

III. Algorithm and Block Diagram for Software Implementation

The ROMO's design procedure can be summarized now into the following algorithm:

Step 1: The matrices $A_i, B_i, i = \overline{1,M}$, C, D, and the vector of unknown inputs (v) are introduced; the order of the observer (r) is chosen. The fulfillment of the four assumptions (A1-A4) is checked; the expressions of the decision vector $\xi(t)$ and of the activation functions $\mu_i(\xi(t)), i = \overline{1,M}$ are established.

Step 2: The matrix N of the form (7) is chosen; the matrices L, D, F_i, A_i, H_i, Q, and P are partitioned as in (7) and (8). The matrices L_2 and L_4, and thus the matrix $\widetilde{L}_2$, are randomly chosen; the matrices L and G_i are obtained with Eqs. (9).

Step 3: The matrices F_2^i, F_4^i, and thus the matrix $\widetilde{F}_2^i$, are randomly chosen; by solving the system (10) with respect to $F_1^i, F_3^i, H_1^i, H_2^i$, these four matrices are obtained; the matrices F_i and H_i are then calculated with (9). Matrices Q and P are computed by solving the system (11).

Step 4: The matrix $\widetilde{F}$ as well as its eigenvalues are calculated; for any $i = \overline{1,M}$, one checks the fulfillment of the conditions C1-C3. If these constraints are satisfied, the matrices of the new ROMO have been obtained properly; otherwise, we return to step 2 and repeat the steps 2-4 in a "while" loop until all the conditions are met.

Step 5: We design the ROMO described by (3) for the estimation of the state vector x. For the validation of the observer, the T-S model (1) is also used.

The block diagram associated to the ensemble T-S multiple model – new ROMO is given in Fig. 1.

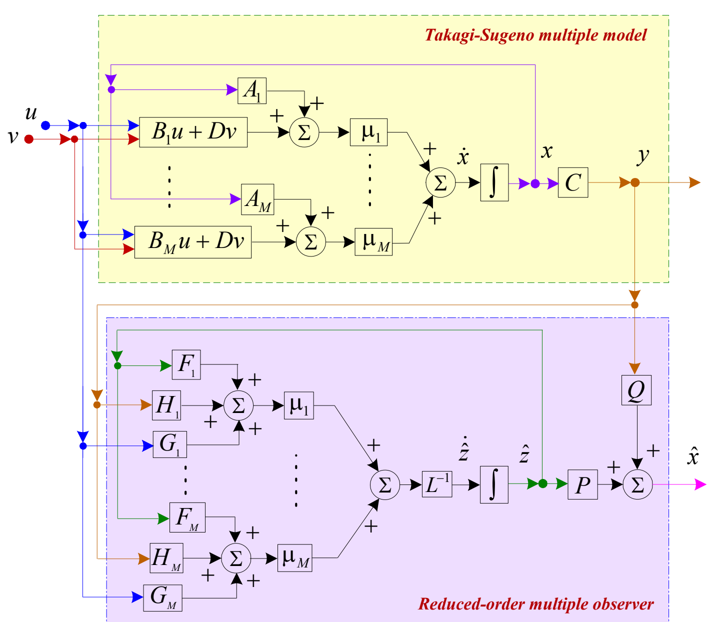

Fig. 1. Block diagram of the ensemble T-S multiple model – new ROMO

Remark 2: The new ROMO has four non-restrictive assumptions and no existence conditions (constraints) due to the "while" loop considered between the steps 2-4. Thus, another important advantage of the new ROMO is the lack of apriori restrictions on the class of T-S multiple models that can be considered, i.e. less restrictive applicability. □

IV. Numerical Simulation Results

To validate the ROMO algorithm, a T-S model of an unstable Charlie type light aircraft lateral-directional motion (M=2) is considered for simulation. The related matrices described by the form (1) are given by [22, 23]:

$$A_1 = \begin{bmatrix} -0.3 & 0 & -33 & 9.81 & 0 & -5.4 & 0 \\ 0.1 & -8.3 & 3.75 & 0 & 0 & 0 & -28.6 \\ 0.37 & 0 & -0.64 & 0 & 0 & -9.5 & 0 \\ 0 & 1 & 0.01 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0.001 & 0 & 0.001 & 0 \\ 0 & 0 & -0.01 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0.01 & 0 & -0.001 & 0 & -5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.28 & 0 & -33 & 9.81 & 0 & -5.2 & 0 \\ 0.1 & -7.6 & 3.95 & 0 & 0 & 0 & -27.6 \\ 0.34 & 0 & -0.59 & 0 & 0 & -9.6 & 0 \\ 0 & 1 & 0.01 & 0 & 0.01 & 0 & 0 \\ 0 & 0 & 1 & 0.002 & 0 & 0.001 & 0 \\ 0 & 0 & -0.02 & 0 & 0 & -9.9 & 0 \\ 0 & 0 & 0.02 & 0 & -0.001 & 0 & -4.95 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.012 \\ 10 & 0 \\ 0 & 10 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -0.009 \\ 9.8 & 0 \\ 0 & 9.8 \end{bmatrix},$$

$$C^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, D = \begin{bmatrix} 0.5 & 0.2 & -0.2 \\ 0.1 & 0.1 & -0.1 \\ 0.5 & -0.1 & -0.2 \\ 1 & -0.4 & -0.2 \\ -0.1 & 0.1 & 0.2 \\ 2 & 1 & 1 \\ 0.2 & 0.1 & 0.1 \end{bmatrix}.$$

The state of the system is $x = \begin{bmatrix} V_y & \omega_x & \omega_z & \varphi & \psi & \delta_r & \delta_a \end{bmatrix}^{\mathrm{T}}$, with V_y – the aircraft lateral velocity, ω_x – the roll angular rate, ω_z – the yaw angular rate, φ – the roll angle of aircraft, ψ – the yaw angle of aircraft, δ_r – the rudder deflection, and δ_a – the deflection of ailerons; the two components of the system input vector $u = \begin{bmatrix} \delta_{r_c} & \delta_{a_c} \end{bmatrix}^{\mathrm{T}}$ are the commands of the rudder and ailerons, respectively [22, 23]. For the above given matrices, one obtains: n=7, m=2, q=3, and p=4. From the form of matrix N, one can remark that the last p states of the system $(\varphi, \psi, \delta_r, \delta_a)$ are estimated.

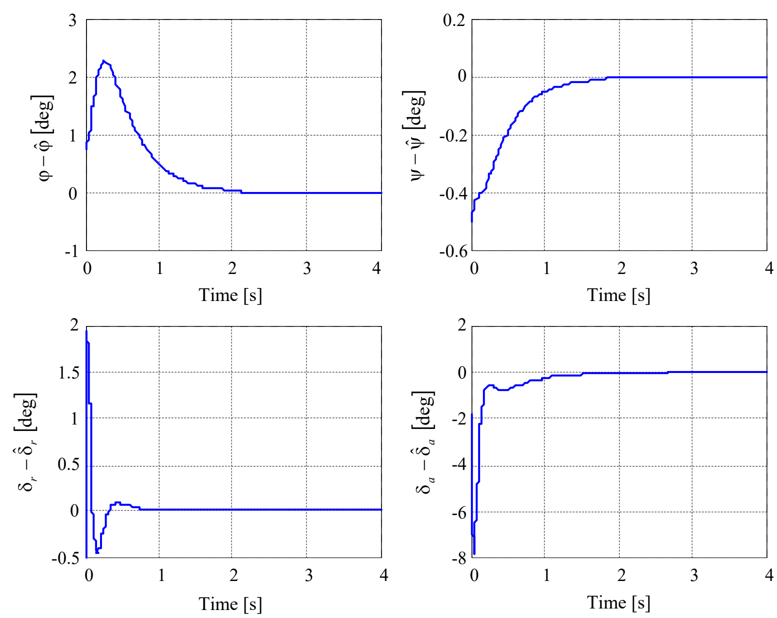

Fig. 2. The estimation errors of the new reduced-order multiple observer

The disturbances (wind shears, atmospheric turbulences, or sensors' errors) often influence the flight of aircraft; in the design of observers for such dynamics, these disturbances are unknown inputs [24]; observers for systems with unknown inputs estimate the unknown inputs, but, more important, these should estimate the system's states; in this paper, the unknown input vector $v(t)$ has been randomly chosen. The decision and activation functions have been chosen a following: $\xi(t) = u(t)$, $\mu_1(\xi(t)) = 0.4(1 - \tanh(\xi(t))), \mu_2(\xi(t)) = 1 - \mu_1(\xi(t))$, where the input has the form $u = -\overline{K}\hat{x}$; the gain matrix $\overline{K}$ is provided by a flight controller for the aircraft stabilization during its lateraldirection motion. There are diverse methods for the obtaining of this matrix; for example, the ALGLX optimal algorithm borrowed from [22] or the approaches presented in [25, 26] provides similar good results. One software implemented the dynamics of the light aircraft lateral-directional motion in Matlab environment as well as the algorithm associated to the new FOMO. In Fig. 2 one presents the time histories of the estimation errors, i.e. the four non-null components of the vector $e(t)=z(t)-\hat{z}(t)$. One remarks the cancel of the estimation errors, hence the effectiveness of the developed ROMO design algorithm; this is equivalent with the convergences $\hat{z}(t)\to z(t)$ and $\hat{x}(t)\to x(t)$.

Remark 3: The state estimation for any nonlinear system with unknown inputs can be achieved with the new designed ROMO if the system is brought to the form associated to the Takagi-Sugeno multiple model. Also, the new design procedure is effective because it can be carried out using commercially available software and because it provides all the degrees of design freedom which can be further utilized to achieve some additional system specifications. □

Remark 4: One can make a brief comparison between the proposed reduced-order multiple observer in this paper and the only existing ROMO in the literature, i.e. the one given in [18]. The drawbacks of this last could be summarized as follows: 1) the observer designed in [18] has 4 existence conditions, while the one designed in this paper has no existence conditions; 2) the ROMO from [18] uses the pole placement (eigenstructure assignment) method which can lead to the unwanted phenomena called eigenvalues' sensitivity. □

V. CONCLUSION

This paper proposes a new design of a ROMO for Takagi-Sugeno models with unknown inputs. The observer has been obtained by applying the classical methods of reduced-order observers' design for linear multivariable systems with unknown inputs to the case of T-S models. The primary advantage of the proposed observer is the decrease of the number of necessary sensors. Other advantages are the lack of existence conditions and the avoidance of pole placement usage which leads to easy implementation high scale systems. An algorithm summarizing all the steps of the design approach has been obtained, software implemented and validated for a light unstable aircraft lateral-directional motion. In future works, the estimation of the unknown inputs by extended robust ROMOs will be considered.

REFERENCES

[1] K. Bouassem, J. Soulami, A. El Assoudi, and E. El Yaagoubi, "Unknown Input Observer Design for a Class of Takagi-Sugeno Descriptor Systems," *Nonlinear Analysis and Differential Equations*, vol. 4, 2016, pp. 477-492.

[2] O. Boubaker and R. Iriarte, *The Inverted Pendulum in Control Theory and Robotics: From Theory to New Innovations*. Institution of Engineering and Technology (IET), Stevenage, United Kingdom, 2017.

[3] L. Xiaohang, F. Zhu, A. Chakrabarty, and S.H. Żak, "Nonfragile fault-tolerant fuzzy observer-based controller design for nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 24, 2016, pp. 1679-1689.

[4] M. Lungu,R. Lungu, R., and O. Preotu, "Estimation of Aircraft State during Landing by means of Multiple Observers," *23th International Conference on Systems, Signals and Image Processing*, 23-25 May 2016, Bratislava, Slovakia.

[5] H. Li, C. Wu, S. Yin, and H.K. Lam, "Observer-Based Fuzzy Control for Nonlinear Networked Systems Under Unmeasurable Premise Variables," *IEEE Transactions on Fuzzy Systems*, vol. 24, 2016, pp. 1233-1245.

[6] K. Hfaiedh, K. Dahech, Y. Massaoudi, and T. Damak, "Sliding mode multiple observer for perturbed output nonlinear systems," *16th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering*, 2015, pp. 740-745.

[7] K. Srinivasarengan, J. Ragot, D. Maquin, and C. Aubrun, "Nonlinear joint state-parameter observer for VAV damper position estimation," *3rd Conference on Control and Fault-Tolerant Systems*, 2016.

[8] H. Schulte and F. Poschke, "Estimation of Multiple Faults in Hydrostatic Wind Turbines using Takagi-Sugeno Sliding Mode Observer with Weighted Switching Action," *IFAC-PapersOnLine*, vol. 49, 2016, pp. 194-199.

[9] R.S. Mangoubi, *Robust Estimation and Failure Detection: A Concise Treatment*, Springer-Verlag Publisher, London, 1998.

[10] A. Morse, "Supervisory Control of Families of Linear Set-Point Controllers - Part 1: Exact Matching," *IEEE Transactions of Automatic Control*, vol. AC-41, 1996, pp.1413-1431.

[11] N. Bouguila, W. Jamel, A. Khedher, K. Othman, "Multiple observer design for a non-linear Takagi-Sugeno system submitted to unknown inputs and outputs," *IET Signal Processing*, vol. 7, no. 8, 2013, pp. 635-645.

[12] T.M. Guerra, R. Márquez, A. Kruszewski, M. Bernal, "H_∞ LMI-Based Observer Design for Nonlinear Systems via Takagi-Sugeno Models with Unmeasured Premise Variables," *IEEE Transactions on Fuzzy Systems*, vol. PP, no. 99, 2017.

[13] M. Sharifuddin, C. Goutam, B. Kingshook, "LMI approach to robust unknown input observer design for continuous systems with noise and uncertainties," *International Journal of Control, Automation, and Systems*, vol. 8, no. 2, 2010, pp. 210-219.

[14] W. Jamel, N. Bouguila, K. Ben Othman, "Observer design for nonlinear systems represented by Takagi-Sugeno models," *WSEAS Transactions on Systems*, vol. 9, no. 7, 2010, pp. 804-813.

[15] X. Su, P. Shi, L. Wu, and Y.D. Song, "A novel control design on discrete-time Takagi–Sugeno fuzzy systems with time-varying delays," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 4, 2013, pp. 655-671.

[16] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "Design of Observers for Takagi-Sugeno Systems with Immeasurable Premise Variables: an L_2 Approach," *17th World Congress of the International Federation of Automatic Control*, Seoul, Korea, July 6-11, 2008.

[17] X. Su, P. Shi, L. Wu, S.K. Nguang, "Induced L_2 filtering of fuzzy stochastic systems with time-varying delays," *IEEE Transactions on Cybernetics*, vol. 43, no. 4, 2013, pp. 1251-1264.

[18] M. Lungu and R. Lungu, "Reduced-Order Multiple Observer for Aircraft State Estimation during Landing," *Applied Mechanics and Materials*, vol. 841, 2016, pp. 253-259.

[19] D. Lee, "Relaxed LMI conditions for local stability and local stabilization of continuous-time Takagi–Sugeno fuzzy systems," *IEEE Transactions on Cybernetics*, vol. 44, 2014, pp. 394-405.

[20] V. Estrada-Manzo, Z. Lendek, and T. Guerra, "Unknown input estimation for nonlinear descriptor systems via LMIs and Takagi-Sugeno models," *54th IEEE Annual Conference on Decision and Control (CDC)*, 2015, pp. 6349-6354.

[21] M. Lungu and R. Lungu, "Design of Full-order Observers for Systems with Unknown Inputs by using the Eigenstructure Assignment," *Asian Journal of Control*, vol. 16, nr. 5, 2014, pp. 1470-1481.

[22] M. Lungu and R. Lungu, *Estimarea stării aparatelor de zbor (State estimation of the flying objects)*, Sitech Publisher, Craiova, 2014.

[23] R. Lungu and M. Lungu, "Automatic Control of Aircraft in Lateral-Directional Plane During Landing," *Asian Journal of Control*, vol. 18, no. 3, 2016, pp. 433-446.

[24] R. Lungu and M. Lungu, "Automatic Landing Control using H-inf Control and Dynamic Inversion," *Proceedings of the Institution of Mechanical Engineers Part G - Journal of Aerospace Engineering*, vol. 228, no. 14, 2014, pp. 2612-2626.

[25] O. Boubaker, "Gain scheduling control: an LMI approach," *International Review of Electrical Engineering*, vol. 3, 2008, pp. 378-385.

[26] M. Lungu, R. Lungu, and C. Rotaru, "New Systems for Identification, Estimation and Adaptive Control of the Aircrafts' Movements," *Studies in Informatics and Control*, vol. 20, no. 3, 2011, pp. 273-284