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Constraints Tightening Approach Towards Model Predictive Control Based Rendezvous and Docking with Uncooperative Targets

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Abstract— **This paper presents a guidance & control strategy and a case study for rendezvous and docking with an uncooperative target based on Model Predictive Control (MPC). It is shown that linear invariant constrained MPC is not able to stay in the centre of the line of sight constraint while approaching a target. Then, online constraint tightening framework is proposed to mimic straight line approach in the relative frame while orbiting around the tumbling target. Our formulation also includes quadratic input constraints to address the physical limitations of a thrust configuration along a single axis. The concept of the tightening constraints framework is eventually validated through simulations to achieve docking conditions while staying in the centre of the line of sight cone.**

I. INTRODUCTION

Spacecraft rendezvous and docking (RVD) capabilities are crucial for a number of astronautic missions. The capability of two or more spacecrafts to mechanically dock through spaceflight manoeuvers enabled historic missions such as Manned Lunar Mission, International Space Station in-orbit assembly, etc. However, these missions were mainly assisted by on-board crew or ground stations. Today, the main focuses are set towards fully autonomous sample & return; collection of the sample of Martian soil inserted by *Mars Ascent Vehicle*, and small asteroid explorations [1]. Moreover, It is envisioned that autonomous RVD missions will enable in-orbit maintenance, assembly, refuelling and capture missions in the future [2], [3].

The upcoming RVD missions are inherently concerned with the constraints of the system. Firstly, due to performance limits of the actuators, i.e thrusters or reaction wheels, it is necessary to define the control input constraints. In addition, line-of-sight (LOS) is the constraint which limits the path to be followed to a conical or tetrahedral safe region along the way of the docking port. For a tumbling target, time variant LOS constraints must be included in the system while considering the relative motion dynamics. The traditional open-loop maneuver planning and ad hoc error correction techniques do not explicity handle the aforementioned constraints [4].

This paper describes the design and implementation of a MPC paradigm to address the LOS issue in the RVD problem, as presented thoroughly in [5]. Particularly, MPC system is designed and employed in the final phase of RVD and capture mission where the chaser is assumed to be in the vicinity of the tumbling target, at a few kilometers or hundreds of meters depending on the orbit semi-major axis, and target is flying in a circular orbit. In the formulation proposed here; first, quadratically constrained optimization problem is defined to account for a single axis thruster configuration of chaser satellite, and together with time variant constraints to predict the future orientation of the target satellite in the presence of 3-axes tumbling motion. The main contribution of this paper is the integration of constraint tightening to mimic a Straight Line Approach (SLA), while the chaser follows a pseudo orbit around the target. SLA mandates the chaser to continuously follow the instantaneous docking axis, denoted by the y-axis in our work, and therefore is a safe and practical approach, especially for uncooperative targets [4]. Given the background, the tightening concept ensures safety of the mission, simplifies weight tuning effort and allows the navigation sensors (i.e. LIDAR) to provide measurements all the time.

A number of MPC strategies have been proposed to deal with RVD problem. [6] and [7] specify LOS constraint with the slopes of the tetrahedral cone and docking port dimensions in the target's body frame. [7] is further extended to an elliptic orbit in [8]. For simplicity, [9] formulates LOS cone in 2D (i.e local-vertical/ local-horizontal (LVLH) frame) but further investigates target's docking port by including the distance of the target's centre (origin) with the docking port (or the platform) and docking tolerance. These LOS constraints consist of user defined, linear invariant inequalities.

This paper further develops the LOS constraint formulation given in [7] by imposing online constraint tightening approach. The proposed framework provides optimal inputs and the resulting predicted states generate a pseudo orbit around the target satellite allowing the chaser to achieve docking conditions in terms of relative attitude and distance while minimizing the error along in-track axis as the target tumbles. In addition to [10]–[12], our formulation includes quadratic input constraints to account for the physical limitations of a single axis thruster configuration.

MPC is an attractive framework for guidance and control for rapidly changing environments, and when uncertainties are present. The proposals and applications of MPC in space systems are very diverse; formation flying [13], actuator control [14], Mars Sample Return [10], rover steering [15], powered descent [16] etc., suggesting that MPC is a powerful candidate to deal with RVD problem as it can compensate modelling uncertainties, enables superior precision with feedback control and explicitly handles constraints present in

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the system, while providing guidance and control alongside the relative motion dynamic models which can be used for prediction [2], [3].

The structure of this paper is as follows. Section II presents the relative motion dynamic model to be employed by the chaser satellite for the RVD problem. Next, MPC design is introduced with problem formulation, constraints and objective function. In section IV, results are given for different scenarios and the conclusions are provided in Section V.

II. Relative Motion Dynamics

To be able to mathematically explain relative motion of two satellites, Hill-Clohessy-Wiltshere (HCW) equations were employed [17]. One must notice that HCW equations are restricted to the cases where the initial relative distance is low (not more than few kilometers) and target satellite is flying in a circular orbit. Therefore, chaser is assumed to be in the vicinity of a tumbling target where relative distance is less than 200m while flying in a circular orbit. Finally, the *time to dock* is as short as 1-3 minutes, disturbances in the space (i.e. atmospheric drag, gravity gradient, solar radiation pressure, etc.) are neglected at this stage. Having said that, a linearization of the HCW equations can be made for circular orbits since the relative distance $\delta\vec{r}$ is small compared to the orbit radius R: $\delta\vec{r} \ll R$ [4] and model can be written in continous time state-space form in the following way;

$$\dot{X} \ = \ AX \ + \ BU, \tag{1}$$

where,

$$A \ = \ \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix}, \ B \ = \ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and, $X \ \in \mathbb{R}^6$ and $U \ \in \mathbb{R}^3$ are the state and control vectors respectively. Furthermore,

$$X \ = \ [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^\top, \ U \ = \ [u_x, \ u_y, \ u_z]^\top, \tag{2}$$

where, $x, y,$ and z are the relative position coordinates of the chaser and $u_x,\ u_y,\ u_z$ are the inputs in three different directions in LVLH frame of reference fixed on the centre of gravity of the target spacecraft. The most commonly used coordinate frame, LVLH, rotates with the chief orbit where x direction is in the local vertical or in other words radial position, y axis is aligned with the motion of the satellite and z axis completes the frame with the right hand rule. Eventually, the angular rate of the target spacecraft through its orbit is given by $n \ = \ \sqrt{\frac{\mu}{R^3}}$ where R is the radius of circular orbit, μ is the gravitational constant of the Earth, $\mu \ = \ 398600.4 \ km^3/s^2$. and, F_x, F_y, F_z are the external force components (i.e. input Forces) acting on the chaser vehicle. One can note from A matrix in continuous time state-space model that there is a strong coupling in the orbital plane x and y whereas out-of-plane (z) is decoupled. Now, the model in (1) can be discretized with a sampling period of ΔT in order to be used as a prediction model:

$$X(k+1) \ = \ A_d X(k) \ + \ B_d U(k); \tag{3}$$

III. Model Predictive Control Formulation

A. *Prediction of States*

Given the state at time k and input signals, we can now calculate the state at time $k+j$ as;

$$X(k+j) \ = \ A_d^j X(k) \ + \ \sum_{i=0}^{j-1} A_d^{j-i-1} B_d U(k+i) \tag{4}$$

If we now define $X(k)$ and $U(k)$ in general form as a set of $N_p \cdot n_x$ states and, $N_p \cdot n_u$ input signals starting at time k, where N_p is the prediction horizon, n_x is the number of state variables and n_u is the number of control inputs. The states and input values on this prediction horizon can be merged into:

$$X_s(k) = \begin{bmatrix} X(k+1) \\ X(k+2) \\ \vdots \\ X(k+N_p) \end{bmatrix}, \ U_s(k) = \begin{bmatrix} U(k) \\ U(k+1) \\ \vdots \\ U(k+N_p-1) \end{bmatrix} \tag{5}$$

Finally,

$$X_s(k) = \begin{bmatrix} A_d X(k) \ + \ B_d U(k) \\ A_d^2 X(k) \ + \sum_{i=0}^{1} A_d^{1-i} (B_d U(k+i)) \\ \vdots \\ A_d^{N_p} X(k) \ + \sum_{i=0}^{N_p-1} A_d^{N_p-1-i} (B_d U(k+i)) \end{bmatrix} \tag{6}$$

The generalized form can be written as;

$$X_s(k) \ = \ \Phi X(k) \ + \ G_U U_s(k) \tag{7}$$

where,

$$\Phi = \begin{bmatrix} A_d \\ A_d^2 \\ \vdots \\ A_d^{N_p} \end{bmatrix}, \ G_U = \begin{bmatrix} B_d & 0 & \cdots & 0 \\ A_d B_d & B_d & & \vdots \\ \vdots & & \ddots & \vdots \\ A_d^{N_p-1} B_d & A_d^{N_p-2} B_d & \cdots & B_d \end{bmatrix}$$

One should note that full state feedback assumption is made in this paper.

B. *Constraints in the System*

The requirements in rendezvous and docking missions pose control problems with imposed pointwise-in-time and terminal constraints on both state and control variables [9]. The main ones are Line of Sight (LOS), Field of View (FOV), plume impingement, obstacle avoidance and physical constraints. In this paper, we formulate LOS and physical constraints as follows. First, during the proximity maneuvering, it is necessary for chaser satellite to stay inside the LOS region. This region can be represented with a polyhedron and can be in general written as;

$$\mathscr{P} \ = \ \{x \ : \ c_\eta^T x \leq d_\eta\} \tag{8}$$

Inequality constraints are defined as;
$y \geq c_x(x - x_0),\ y \geq -c_z(z + z_0),\ y_B \geq -c_x(x + x_0),$
$y \geq c_z(z - z_0)$ where c_x and c_z are the slopes of the tetrahedral cone (i.e if $c_x = 1$, slope angle is 45 deg), x_0 and z_0 are the docking port 2D dimensions and, $y \geq 0$ ensures that chaser satellite does not crash the docking port. Finally, $x,\ y,\ \&\ z$ are the body fixed frame coordinates and therefore, LOS constraint can be written in the following inequality form [6], [7]:

$$A_{LOS}X(k) \leq b_{LOS} \tag{9}$$

where,

$$A_{LOS} = \begin{bmatrix} 0 & -1 & 0 \\ c_x & -1 & 0 \\ -c_x & -1 & 0 \\ 0 & -1 & c_z \\ 0 & -1 & -c_z \end{bmatrix},\ b_{LOS} = \begin{bmatrix} 0 \\ c_x x_0 \\ c_x x_0 \\ c_z z_0 \\ c_z z_0 \end{bmatrix} \tag{10}$$

and X_k is the part of the state vector that represents the relative position of the chaser satellite in target's body frame. The given inequality constraints in Eq. (9) are valid for stable or non-rotating target case where LVHL frame is identical to body fixed frame. However, for a tumbling target, transformation of A_{LOS} and b_{LOS} dynamic constraints are required to predict future feasible region in order to generate set of inputs that optimizes the RVD problem while respecting the LOS constraints. To this end, LOS constraint matrix can be updated in LVLH frame by using projective geometry in the following form;

$$A_{LVLH_{rot}} = A_{LOS}R_{Rot} \tag{11}$$

Where for a simple scenario, assuming target only rotates around it's z axis, R_{Rot} can be obtained provided that perfect knowledge of rotation rate is available;

$$R_{Rot-z} = \begin{bmatrix} cos(\alpha) & -sin(\alpha) & 0 \\ sin(\alpha) & cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

Rotations will be named as yaw, pitch and roll where rotation matrices in counter-clockwise direction and the other rotation matrices can be obtained by multiplication of the given three matrices.

C. Physical Constraint

The most commonly employed physical constraint is the following linear inequality constraints which are valid in the presence of the multiple axis thrusters on-board to fire in every direction;

$$U_{min,i} \ \leq\ U(k)\ \leq U_{max,i},\ i\ \in\ \{x,y,z\} \tag{13}$$

However, for a single axis thruster configuration, physical limits on the net thrust can be written in the form;

$$U_x^2\ +\ U_y^2\ +\ U_z^2\ \leq\ U_{max}^2 \tag{14}$$

which results in Quadratically Constrained Quadratic Program(QCQP). In the given inequality physical constraints, U_{max} is characterized by the chaser's thruster performance; F (Newton), mass; m (kg), sampling time; $T_s(s)$ in the following form;

$$U_{max} = \frac{F}{m}T_s. \tag{15}$$

D. Constraint Tightening

In order to represent the safe approach corridor, LOS constraints are formulated in Eqn. (9). It is desirable to increase LOS cone region as it enlarges the feasible initial condition sets. However, as a result of the optimization problem, chaser might stay close to constraints. In this case, it is worth reminding that the docking port of the chaser spacecraft must be aligned with the counterpart of the target for a successful docking. By following then trajectories close to wide constraints, the chaser may fail to approach while being aligned with the instantaneous docking axis. On the contrary, if a narrow LOS constraint is formulated, the chaser would be restricted to move almost only along the y-axis in the body frame of the target thereby the initial feasible start region would dramatically decrease. To this end, a quadratically constrained LOS corridor could be more beneficial than linear LOS constraints. As can be seen in the below **Convex Set** definition,

$$\mathscr{X} \text{ is convex } \Leftrightarrow\ \lambda x + (1-\lambda)y \in \mathscr{X}, \forall\lambda \in [0,1], \forall x,y \in \mathscr{X},$$

mentioned quadratic LOS constraint is not convex because a set $\mathscr{X}$ is convex if and only if for any pair of points x and y in $\mathscr{X}$ and, for all **convex combination** of x and y lies in $\mathscr{X}$ [18]. Considering the mentioned motivation and concerns, this paper proposes online LOS constraint tightening because, tightening constraints would mimic quadratic LOS constraints but allows user to formulate the problem in convex manner and enforces chaser to approach the target along y-axis while orbiting around it. Tightening and rotating LOS constraints can be written in the following general form: For every sampling instant; calculate future LOS cone slopes; c_x & c_z over the prediction horizon:

$$\begin{aligned} c_x(i+1|k) &= c_x(i|k) + T_s\tau_x,\ i = 0,...,N-1 \\ c_z(i+1|k) &= c_z(i|k) + T_s\tau_z,\ i = 0,...,N-1 \end{aligned} \tag{16}$$

where τ_x & τ_z are the tightening rates obtained by batch simulations; $c_x(0|k)$ is the user defined, initial slope of the cone between x and y axes, $c_x(N|k)$ is the future tightened slope at the end of the prediction horizon.Next, update current A_{LOS}, b_{LOS}, R_{Rot} Eq. (10) and (12) with obtained c_x & c_z values from Eq. (16). Subsequently, by updating A_{LOS} & b_{LOS}, Eqn (10) becomes;

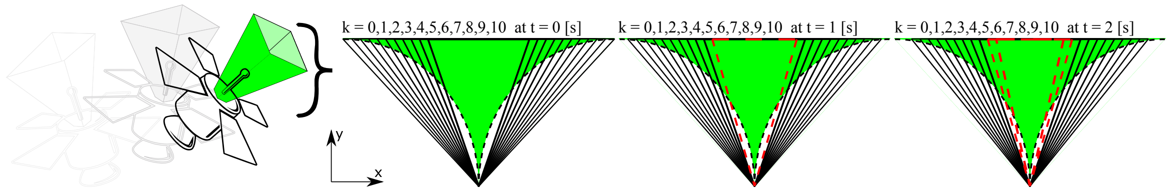

Fig. 1. Interpretation of Tightening Constraints Concept for Np = 10 for 3 Consecutive Steps

$$A_{LOS}(i|k) = \begin{bmatrix} 0 & -1 & 0 \\ c_x(i|k) & -1 & 0 \\ -c_x(i|k) & -1 & 0 \\ 0 & -1 & c_z(i|k) \\ 0 & -1 & -c_z(i|k) \end{bmatrix},$$

$$b_{LOS}(i|k) = \begin{bmatrix} 0 \\ c_x(i|k)x_0 \\ c_x(i|k)x_0 \\ c_z(i|k)z_0 \\ c_z(i|k)z_0 \end{bmatrix}, \ i = 0, ..., N. \quad (17)$$

Finally, calculate future A_{LOS} over prediction horizon mentioned in Eq. (11);

$$A_{LVLH_{rot}}(i+1|k) = A_{LOS}(i|k)R_{Rot}(i|k), \ i = 0, ..., N \quad (18)$$

The proposed framework can be summarized in the algorithm 1 and the interpretation of tightening constraints concept over prediction horizon for 3 consecutive time steps is depicted in Fig. 1. One must note that tightening rate is updated regarding to the actuator limits, initial line of sight cone angle, docking port dimensions, rotation angular rate of the target and initial distance from the target in LVLH frame.

Algorithm:
Initialize model parameters in Eq. (16)
Start maneuver
$k := 0$
while $k \leqslant k_{max}$ **do**
    $\mathscr{A}\{1\} := A_{LOS}(0|k)$
    Measure current states $X(k)$ at k^{th} time
    for $i = 1$ *to* N **do**
        Compute c_x & c_z using (16)
        Tighten A_{LOS} and b_{LOS} using (17)
        Compute $A_{LVLH_{rot}}(i|k)$ using (18)
        $N :=$ prediction horizon
        Store future LOS constraints in a cell array;
        $\mathscr{A}\{i+1\} := A$
    end
    Find U by solving optimization problem in Eq. (19) subject to (20)
    $U(i+1|k) := U\{1\}$
    Apply $U(i+1|k)$ to the system.
    Run the model from k to $k+1$
    Update: $A_{LOS}(0|k)$ & $b_{LOS}(0|k)$
    Update: $k := k+1$
end

Algorithm 1: Constraint Tightening MPC Simulation Procedure

E. Cost Function

Now, an autonomous docking MPC can be designed. We employ linearized & time invariant relative dynamics motion model (i.e., HCW equations), linear equality and inequality constraints (i.e., Input+Line of Sight), and quadratic costs on the states and control inputs. Therefore, MPC can be formulated as a QP problem and MPC solves at every time step k the following baseline optimization problem.

$$\begin{aligned} \textbf{minimize} \quad & \sum_{i=0}^{N-1} [X(i|k)^T QX(i|k)] \\ & + \sum_{i=0}^{N-1} [U(i|k)^T RU(i|k)] \\ & + X(N|k)^T Q_p X(N|k) \end{aligned} \quad (19)$$

$$\begin{aligned} \textbf{subject to} \quad X(i+1|k) &= A_d X(i|k) + B_d U(i|k) \\ X(0|k) &= X(k) \\ U_x(i|k)^2 + U_y(i|k)^2 + U_z(i|k)^2 &\leq U_{max}^2, \quad i = 0, ..., N_c \\ A_{LOS}(i|k)X(i|k) &\leq b_{LOS}(i|k), \ i = 0, ..., N \end{aligned} \quad (20)$$

The problem is subject to dynamic equality, state and input inequality constraints where, N & N_c are the prediction and control horizons respectively. In the cost function, the cost of states and inputs is minimized because docking port is the origin where no state or input is needed (consistency of cost function). In the cost function, Q is positive-semidefinite weight matrix and penalizes the state values, R is positive-semidefinite weight matrix and penalizes the control effort together with Q_p being the solution of the Algebraic Riccati Equation that ensures local stability by penalizing the terminal cost of the MPC problem [5].

IV. Simulated Scenarios

In this section, we evaluate the performance of MPC for 2 different cases; stable target and rotating target. To formulate the optimization problem defined in Eq. (19), we used CVX, an open source Matlab based modeling language for convex optimization and MPC [19].

One can notice that in order to simplfy the figures, results are presented in Radial and In-track directions only. MPC prediction horizon was varied between 10 to 50 in order to avoid shortsighted MPC design and unnecessary computation load. Hence, it was our concern to select the horizon (Np) as short as possible while still ensuring feasibility. Np is selected 30, state penalty is $Q = I$, input penalty is $R = 1e5I$ and terminal penalty is the solution of Algebraic Riccati Equation. It is assumed that single axis thruster can generate

TABLE I
MPC SPECIFICATIONS

Parameter	Case-1.a	Case-1.b	Case-2
X0	[-140 180 0 -1 0 0]	[120 150 0 0 0 0]	[10 50 10 0 0 0]
Q	I_{6x6}	diag([1000 1 1 1 1 1])	I_{6x6}
R	1e5 I_{3x3}	1e5 I_{3x3}	1e5 I_{3x3}
Qp	Solution of	Algebraic Riccati	Equation
Ts	1[s]	1[s]	1[s]
Horizon	30	30	30

TABLE II
SCENARIO SPECIFICATIONS

Parameter	Case 1.a, 1.b, 2
F_{max}	50 [N]
Chaser Mass	500 [kg]
Altitude	590 [km]
Slope of LOS Cone	1
Tightening Rate	0.025 [s^{-1}]
Target Rotation Rate	2 [deg/s]
Thruster Configuration	single

50 [N] thrust and the chaser satellite's mass is 500 [kg]. Eventually, for 1 [s] sampling time, considering the relation given in Eq. (15), physical limit becomes $u_{max} = 0.1[m/s^2]$. Moreover, the docking port dimensions are assumed to be 20x20 cm and the initial cone slopes, c_x and c_z, are selected 0.5. Also, satellites fly in a circular orbit with an altitude of 590km.

A. *Case Studies*

Before presenting the simulation results, assumptions in the paper can be summarized as;

1) Target is flying in a circular orbit
2) Model uncertainties and orbital disturbances are neglected for these short-time manoeuvers.
3) Perfect knowledge of the states is available.
4) Chaser is assumed to orient itself accordingly to obtain single axis thrust vector.
5) No obstacle avoidance during the rendezvous

Two different cases are studied. *Case 1*: This case requires a maneuver along In-track axis. Number of initial conditions were simulated and two of them are given in Fig. (2). The chaser with initial coordinates conditions are [-140 180 0] m named $Case1.a$ and $Case1.b$ with [120 150 0] m initial start to approach a stable target. Details are given in Table I and II. This case can be considered as a $V-Bar$ approach. LOS constraints are imposed once the maneuver starts. Both approaches, with tightening and standard (no-tightening) LOS constraints succesfully proceeds towards the docking port. Yet, in $Case1.a$, it is seen that standard MPC stays close to constraints whereas tightening formulation enforces chaser to follow a trajectory close to y-axis.

Secondly, in $Case1.b$ it is seen that even with an aggressive controller design, tightening constrained formulation diverges from the origin as it generates set of inputs as a result of the optimization problem defined in Eq. (19) while respecting the time-variant constraints. Further simulations prove that the mentioned behaviour provides additional safety to chaser satellite for more complex scenarios, i.e. 3-Axes rotating targets. Tightening constrained and standard LOS constrained are depicted with red stars and blue diamonds respectively. As seen in Fig. 2, formulations with and without tightening both achieve docking conditions at the end of the maneuvers. However, with tightening linear constraints, we are now able to mimic non-convex quadratic line of sight constraint, thereby the trajectories differ. *Case 2*: In this scenario, a target which is potentially tumbling around any axis is studied. As an example, a target with 10m radial and 50m in-track separation from the chaser where initial condition [10 50 0]m is taken into account. Unlike the first case, target is assumed to be rotating around It's z axis with 2 deg/s rotation rate. It is no longer possible to approach the target along y axis in LVLH frame. Therefore, chaser tends to follow a path similar to a spiral as depicted in Fig. 4 for LVLH frame, while the main advantage of the proposed frome work; mimic of straight line approach is obtained in the target body frame as illustrated in Fig. 3..

It is seen in Fig 4 that enforcing tightening constraints

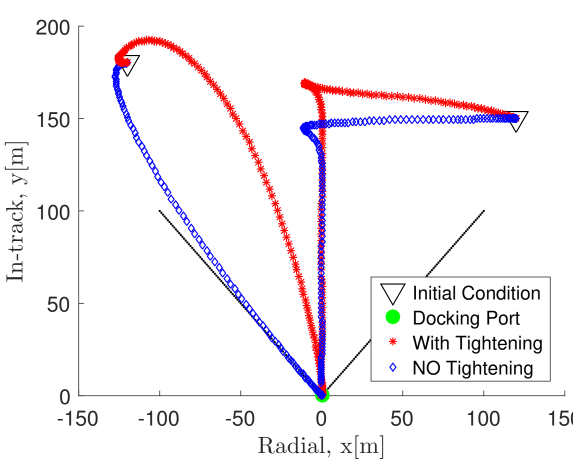

Fig. 2. Case 1a & 1b. Trajectory history of the chaser in LVLH frame

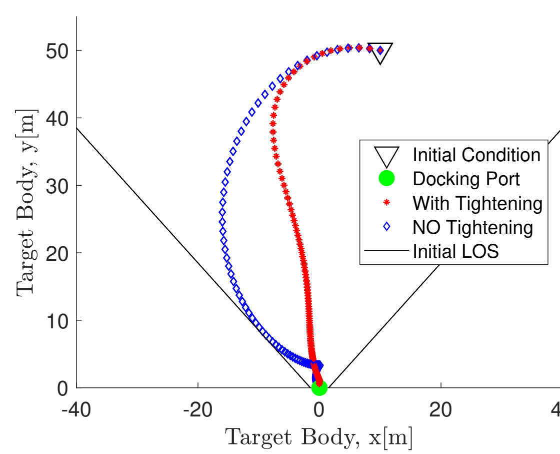

Fig. 3. Case 2.Trajectory history of the chaser in target's body fixed frame

enlarges the trajectory that takes more time steps to dock. Moreover, as seen in Fig. 5, tightening constrained MPC

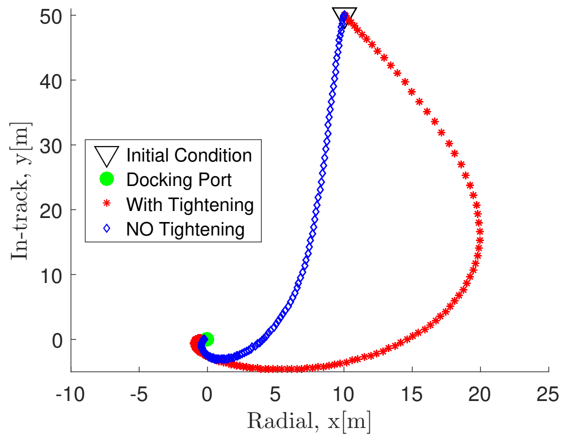

Fig. 4. Case 2. Trajectory history of the chaser in LVLH frame

design tends to fully utilize thrusters more frequently which results in more input use; therefore, fuel expenditure. Nevertheless, for such short period of time, performance can be evaluated with safety and trajectory rather than input cost. Therefore, with constraint tightening framework towards rendezvoud and docking problem, chaser is now able to follow a quasi-straight line approach while orbiting around the rotating target.

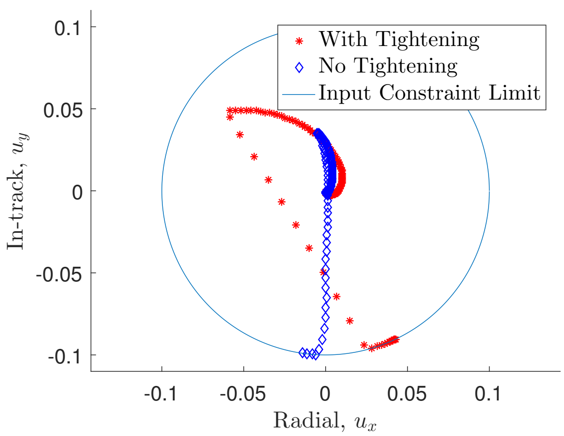

Fig. 5. Case 2. Input history comparison of chaser for single-axes thruster configuration

V. CONCLUSIONS

In this paper we presented a MPC framework to address rendezvous and docking problem by employing HCW relative motion dynamic model with tightening LOS constraints and quadratic input constraints. Starting with a feasible set as large as possible, tightening constraints would mimic quadratic LOS constraints but allows user to formulate the problem in a convex manner, and enforces the chaser to approach the target along y-axis while orbiting around it. guidance and control algorithm maintain the target docking

Throughout this paper, the relative states of the target is considered to be known. This hypothesis lies on the fact that port within the relative sensors Field-of-Views(FoV). In addition, vehicles must converge to the desired states without colliding with each other; repelling each other, causing contamination and degradation of surfaces such as solar arrays, due to the direction of thrusters. Hence, in a future step, FoV and plume impingement constraints will be formulated to include linear time variant dynamic model to achieve RVD conditions with a target flying in an elliptic orbit.

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