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MIMO Direct Adaptive Control with Relaxing Constrains on High-Frequency Matrix Gain Assumption*

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Abstract — **The problem of direct model reference adaptive control for multi input multi output (MIMO) uncertain plants with unknown high frequency gain matrix (HFGM) is addressed in the paper. Two schemes of adaptive control algorithms using augmented error estimators and relaxed assumption concerning HFGM are proposed. It is assumed that the minimum of modulus of HFGM determinant is known. The first scheme involves the resettable approximation of the inversion of HFGM estimate in control law. The second scheme is based on modification of conventional augmented error estimator by introduction of a switched projection mechanism. The proposed theoretical ideas are wrapped up by simulation examples.**

I. INTRODUCTION

Model reference adaptive control (MRAC) of single input single output systems is one of the most extensively studied areas of adaptive control theory, see e.g. [1, 2, 3, 4]. Conditions of MRAC problems and constraints on the plant are established and became traditional for a long time. One of these constraints concerns the high frequency gain defining the control direction and finally being responsible for stability of the closed-loop system. In the classical MRAC problem the sign of high frequency gain is assumed to be known what is conventional for the overwhelming majority of practical tasks.

A quite different problem arises from generalization of the problem for MIMO plants. The HFGM, say K_p, as is uncertain at least due to interactions between channels. Typically MRAC of MIMO systems uses assumption [3] about the knowledge of such nonsingular matrix Γ that

$$K_p\Gamma^T = \Gamma K_p^T \succ 0. \qquad (1)$$

This inequality called symmetry condition is highly restrictive and can be admitted in a quite narrow class of physical systems.

One of the first conjectures of relaxation of (1) was described in [1, 5] (the authors referred to work [6]) and implied the knowledge of symmetric matrix S such that

$$K_pS^T + SK_p^T = Q \succ 0. \qquad (2)$$

As it has been shown with counterexample in [7], condition (2) is not sufficient for MRAC.

However, in [8] authors proposed an alternative solution of MRAC problem based on immersion and invariance (I&I) control wherein condition (2) is sufficient.

In [7], [9], [10] MRAC problem is resolved with a weakened assumption (1) — all the leading minors of K_p are nonzero and have known signs. The proposed solution uses standard gradient adaptation algorithms applied to the parametrized plant. In [7] authors chose hierarchy control approach based on sliding mode control theory.

Alternative solution presented in [11], [12] considers assumption about the knowledge of upper bound on the norm of K_p. The authors apply the gradient adaptation algorithms based on standard MRAC parametrization and implement the control containing inversion $\hat{K}_p^{-1}$. The risk of singularity of $\hat{K}_p$ is avoided by means of hysteresis transformation or switching.

Research presented in [13] is motivated by ideas from [14] and relaxing of (1) for the case when K_p satisfies positive diagonal Jordan form condition and has real positive eigenvalues.

The contribution of the paper is to propose two solutions of MIMO MRAC problem with relaxed constraints. It is assumed that the lower bound of $\left|\det(K_p)\right|$ is known to prevent possible division by zero in the control laws. In authors' opinion this assumption potentially expands the practical meaning of MRAC problem for MIMO systems. The results presented in the paper are the logical continuation of SISO MRAC proposed by the authors in [15].

The remaining of the paper is organized as follows. Section 2 formulates the MIMO MRAC problem addressed in the paper. Section 3 presents two schemes of adaptive control with modified augmented error estimators. Section 4 contains simulation results and discussion.

II. PROBLEM STATEMENT

Consider the problem of relaxing the knowledge of the high frequency gain matrix in MRAC of MIMO linear time-invariant plant

$$y = W(p)\left[K_pu\right], \qquad (3)$$

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where $y \in \mathrm{R}^m$ is the plant output, $u \in \mathrm{R}^m$ is the plant input, $K_p \in \mathrm{R}^{m \times m}$ is the high frequency gain matrix whose parameters are unknown, $W(p)$ is the $m \times m$ transfer matrix with unknown parameters but known order, $p = d/dt$ is the differential operator. The MRAC objective is to design a control u ensuring the following limiting equality:

$$\lim_{t \to \infty} \left(y(t) - y_r(t) \right) \le \Delta, \quad (4)$$

where Δ is a constant depending on controller parameters, $y_r \in \mathrm{R}^m$ is the output of reference model

$$y_r = W_r(p)[r]$$

with stable $m \times m$ diagonal transfer matrix $W_r(p) = diag\left\{W_{ri}(p),\, i = \overline{1,m}\right\}$ of full rank and piecewise continuous uniformly bounded reference vector $r \in \mathrm{R}^m$.

We make the following assumptions:

A.1 $W(p)$ has stable zeros, controllable and full rank.

A.2 An upper bound $\bar{\nu}$ on the observability index ν of $W(p)$ is known.

A.3 Matrix given by $\lim_{p \to \infty} \xi_r(p) W_r(p)$, where $\xi_r(p)$ is the known left interactor matrix of $W(p)$, is finite and nonsingular.

A.4 A constant $\underline{k}_p \in \mathrm{R}_+$ verifying

$$\left|\det(K_p)\right| \ge \underline{k}_p \quad (5)$$

is known.

With regard to the assumptions, the following comments are made.

— Assumptions A.1, A.2 and A.3 are standard in MRAC of MIMO systems [1, 3].

Assumption A.4 is the significant relaxation of high frequency gain assumptions discussed in the introduction. The boundness away from zero of the determinant is a property of vast majority of physical systems and can be interpreted as the inevitable condition for control implementation without division by zero.

Relaxation of constraints on K_p according to assumption A.4 is the main *contribution* of this paper.

Remark 1. Generally, the value Δ is assumed nonzero. However, as it will be shown, for majority of scenarios the solutions proposed below drive control error $e = y - y_r$ to zero (i.e. $\Delta = 0$). Condition $\Delta \ne 0$ corresponds to particular case, when the estimate of K_p denoted as $\hat{K}_p$ stays inside the set defined by inequality $\left|\det(\hat{K}_p)\right| < \underline{k}_p$. This scenario can occur in Scheme 1 due to the lack of persistent excitation in reference r.

Instrumental for the development of MRAC is the *lemma* below, known as direct control model reference parametrization presented in [3, 4].

Lemma. Consider plant (3) and tracking error

$$e = y - y_r = y - W_r(p)[r]. \quad (6)$$

There exist matrices $\Theta_1, \Theta_2 \in \mathrm{R}^{m(\bar{\nu}-1) \times m}$ and $\Theta_3, \Theta_4 \in \mathrm{R}^{m \times m}$ forming matrix $\Theta = col\left(\Theta_1, \Theta_2, \Theta_3, \Theta_4\right)$ such that

$$e = y - y_r = W_r(p)\left[K_p u - \Theta^T \Phi\right] + \vartheta, \quad (7)$$

where $\vartheta \in \mathrm{R}^m$ is exponentially decaying term[1], $\Phi \in \mathrm{R}^{2m\bar{\nu} \times 1}$ is the matrix of measurable variables (regressor) given by

$$\Phi = col\left(\omega_1, \omega_2, y, r\right), \quad (8)$$

$$\omega_1 = \frac{A(p)}{\Lambda(p)}[u], \quad \omega_2 = \frac{A(p)}{\Lambda(p)}[y],$$

$$A(p) = col(I \cdot p^{\bar{\nu}-2}, I \cdot p^{\bar{\nu}-3}, \ldots, I \cdot p, I),$$

$\Lambda(p)$ is a monic Hurwitz polynomial of degree $\bar{\nu} - 1$.

The MRAC design based on certainty equivalence principle motivates the following choice of adjustable controller:

$$u = \hat{K}_p^{-1} \hat{\Theta}^T \Phi, \quad (9)$$

where $\hat{\Theta} \in \mathrm{R}^{2m\bar{\nu} \times m}$ is the matrix of adjustable parameters generated by an integral type adaptation algorithm

$$\dot{\hat{\Theta}} = F(\hat{\Theta}, e, \Phi),$$

$\hat{K}_p$ is the estimate of K_p.

III. Main Results

In this section we propose two schemes of adaptive estimation derived from parameterization (7) and implemented to avoid singularity in control (9).

Scheme 1. To derive new MRAC we firstly rewrite control (9) with approximation of matrix inversion:

$$\hat{K}_p^{-1} \approx \hat{K}_p \left(\sigma I_{m \times m} + \hat{K}_p^2\right)^{-1},$$

$$u = \hat{K}_p \left(\sigma I_{m \times m} + \hat{K}_p^2\right)^{-1} \hat{\Theta}^T \Phi, \quad (10)$$

where $I_{m \times m}$ is the identity matrix, σ is a positive small gain. Then we represent each row of (7) as

$$e_i = -\Theta_i^T W_{ri}(p)[\Phi] + K_{pi}^T W_{ri}(p)[u], \; i = \overline{1,m}, \quad (11)$$

[1] Variable vector ϑ will be omitted, since it does not influent on stability of the closed-loop system and final result of tracking.

where Θ_i^T is the i th row of Θ^T, K_{pi}^T is the i th row of K_p, e_i is the i th element of e. Then introduce augmented errors

$$\tilde{e}_i = e_i + \hat{\Theta}_i^T W_{ri}(p)[\Phi] - \hat{K}_{pi}^T W_{ri}(p)[u], \qquad (12)$$

where $\hat{K}_{pi}^T$ are the estimates of K_{pi}^T, and after substitution of (11) get:

$$\tilde{e}_i = -\tilde{\Theta}_i^T W_{ri}(p)[\Phi] + \tilde{K}_{pi}^T W_{ri}(p)[u]. \qquad (13)$$

Expressions (13) represent static error models and offer unnormalized version of adaptation algorithms generating rows $\hat{\Theta}_i^T$ and $\hat{K}_{pi}^T$ [1, 3] [2]:

$$\dot{\hat{\Theta}}_i^T = -\gamma_{1i}\tilde{e}_i W_{ri}(p)\left[\Phi^T\right],$$
$$\dot{\hat{K}}_{pi}^T = \gamma_{2i}\tilde{e}_i W_{ri}(p)\left[u^T\right],$$

where γ_{1i}, γ_{2i}, $i = \overline{1,m}$ are positive gains. Combination of all the rows in the obtained equations gives the matrix form of the adaptation algorithm

$$\dot{\hat{\Theta}}^T = -\Gamma_1 E \Phi_f, \qquad (14)$$

$$\dot{\hat{K}}_p = \Gamma_2 E u_f \qquad (15)$$

with filtered regressors

$$\Phi_f = \begin{bmatrix} W_{r1}(p)\left[\Phi^T\right] \\ W_{r2}(p)\left[\Phi^T\right] \\ \vdots \\ W_{rm}(p)\left[\Phi^T\right] \end{bmatrix}, \quad u_f = \begin{bmatrix} W_{r1}(p)\left[u^T\right] \\ W_{r2}(p)\left[u^T\right] \\ \vdots \\ W_{rm}(p)\left[u^T\right] \end{bmatrix},$$

$E = diag\left\{\tilde{e}_i, i = \overline{1,m}\right\}$, $\Gamma_1 = diag\left\{\gamma_{1i}, i = \overline{1,m}\right\}$, $\Gamma_2 = diag\left\{\gamma_{2i}, i = \overline{1,m}\right\}$ and matrix $\hat{K}_p$ representing estimate of K_p.

According to the properties of the algorithm, $\|\tilde{\Theta}\|$ and $\|\tilde{K}_p\|$ are nonincreasing functions, $\|E\|$ vanishes asymptotically fast and independently from the control structure [1, 3, 16 ,18]. In this regard, implementable form of control (10) converges to an approximation of "ideal" control

$$u = K_{pss}\left(\sigma I_{m\times m} + K_{pss}^2\right)^{-1} \Theta_{ss}^T \Phi,$$

where subscript ss denotes steady state constant value. Substitution of this approximation into (7) gives equality for generally nonzero steady state error:

$$e_{ss} = -W_r(p)\left[\sigma I_{m\times m}\left(\sigma I_{m\times m} + K_{pss}^2\right)^{-1} \Theta_{ss}^T \Phi\right]. \qquad (16)$$

It is easy to see that, if parameter σ is zero, the control provides zero steady state control error. This fact motivates to design a condition for timely resetting of σ that provides complete compensation of plant uncertainties.

By applying condition (5) we introduce the following resetting mechanism for σ:

$$\sigma(t_+) = \begin{cases} \sigma(t_-) & if \ \left|\det(\hat{K}_p)\right| < \underline{k}_p, \\ 0 & if \ \left|\det(\hat{K}_p)\right| \geq \underline{k}_p. \end{cases} \qquad (17)$$

Thus, the first scheme of adaptive control consists of adjustable controller (9), resetting mechanism (17) and adaptation algorithms (14), (15).

Discuss the properties of the proposed scheme.

1. Since the algorithm uses augmented error scheme with stability properties proved via the swapping lemma [1-3], it ensures objective equality (4) with $\Delta = 0$ all signals bounded at least when the value $\left|\det(\hat{K}_p)\right|$ is out of the "strip" defined by inequality $\left|\det(\hat{K}_p)\right| < \underline{k}_p$.

2. If condition inequality $\left|\det(\hat{K}_p)\right| < \underline{k}_p$ holds for arbitrary long time (see Remark 1), control error stays bounded together with all the other signal and approaches e_{ss}, while time tends to infinity. Modulus of e_{ss} can be made arbitrary small by decrease of parameter σ.

Scheme 2. To implement the second scheme we rewrite (7) in the following form:

$$e = W_r(p)\left[K_p u - K_p \Psi^T \Phi\right], \qquad (18)$$

where $\Psi^T = K_p^{-1}\Theta^T$ is the new $m \times 2m\bar{\nu}$ matrix of unknown parameters. Application of certainty equivalence principle to expression (18) gives the adjustable control

$$u = \hat{\Psi}^T \Phi, \qquad (19)$$

where $\hat{\Psi}$ is the estimate of Ψ given by

$$\hat{\Psi}^T = \hat{K}_p^{-1}\hat{\Theta}^T.$$

Note that control (19) is free from division by zero and does not require modifications.

To implement an adaptation algorithms generating $\hat{\Psi}$ and $\hat{K}_p$ we use algorithms (14), (15) derived from parameterization (18) or (7). By taking into account that

$$\dot{\hat{\Theta}}^T = \hat{K}_p \dot{\hat{\Psi}}^T + \dot{\hat{K}}_p \hat{\Psi}^T$$

and applying (14) and (15) we derive adaptation algorithm generating $\hat{\Psi}$:

$$\dot{\hat{\Psi}}^T = -\hat{K}_p^{-1} E\left\{\Gamma_1 \Phi_f + \Gamma_2 u_f \hat{\Psi}^T\right\}, \qquad (20)$$

[2] In the theoretical part of the paper we skip the stability issue for normalized versions of adaptation algorithms (14), (15) and forthcoming algorithms (23), (24) and refer the interested reader to [18]. In simulation example we insert normalization factors for rigorous presentation of results.

where $E = diag\left\{\tilde{e}_i, i = \overline{1,m}\right\}$ contains the augmented errors (12) rewritten in terms of new estimate $\hat{\Psi}$:

$$\tilde{e}_i = e_i + \hat{K}_{pi}^T \hat{\Psi}^T W_{ri}(p)[\Phi] - \hat{K}_{pi}^T W_{ri}(p)[u].$$

Assuming temporally that

$$\left|\det(\hat{K}_p)\right| \geq \underline{k}_p , \qquad (21)$$

and using relation of inversion of $\hat{K}_p$ to its adjugate

$$\hat{K}_p^{-1} = adj\left\{\hat{K}_p\right\} / \hat{d} , \qquad (22)$$

where $\hat{d} = det\left\{\hat{K}_p\right\}$, we get the implementable form of adaptation algorithm (20):

$$\dot{\hat{\Psi}}^T = -\frac{adj\left\{\hat{K}_p\right\}}{\hat{d}} E\left\{\Gamma_1 \Phi_f + \Gamma_2 u_f \hat{\Psi}^T\right\}. \qquad (23)$$

To enforce the condition (21) in (23) we propose the determinant estimator based on Jacobi's formula

$$\dot{\hat{d}} = tr\left\{adj\left\{\hat{K}_p\right\}\dot{\hat{K}}_p\right\}, \qquad (24)$$

where $\operatorname{tr}\{\cdot\}$ is the matrix trace, together with switching projection mechanism

$$\hat{d}(t_+) = \begin{cases} \hat{d}(t_-) & if \;\; \left|\hat{d}\right| > \underline{k}_p \; or \left(\dot{\hat{d}} = 0 \;\; and \; \left|\hat{d}\right| = \underline{k}_p\right), \\ \underline{k}_p & if \;\; \hat{d} = -\underline{k}_p \;\; and \;\; \dot{\hat{d}} > 0, \\ -\underline{k}_p & if \;\; \hat{d} = \underline{k}_p \;\; and \;\; \dot{\hat{d}} < 0. \end{cases} \qquad (25)$$

The conditions of the projection provide the "jump" of estimate $\hat{d}$ over zero and permit to avoid division by zero in (23).

The properties of the adaptive estimator proposed are formulated and proved in the following theorem.

Proposition. Then the normalized version of estimator (23), (24) together with projection mechanism (25) guarantees the boundness of all the signals in closed-loop control system and asymptotical convergence of error e to zero.

Proof. As it is known and proved in literature (see [1-3], [16]), the estimator (14), (15) provides the boundness of all the signals in the MRAC system and asymptotical decaying of error e. An additional point to emphasize is that $\left\|\dot{\hat{\Theta}}\right\|$, $\left\|\dot{\hat{K}}_p\right\|$ tend to zero asymptotically fast and $\left\|\tilde{\Theta}\right\|$, $\left\|\tilde{K}_p\right\|$ are bounded.

It is followed from convergence properties of $\hat{K}_p$ that $\hat{d}$ is also bounded and bounded away from zero by conditions (25).

Thus, algorithm (23) derived from (14), (15) together with algorithm (15) and (24) guarantees the boundness of all signal in the closed-loop system and asymptotical convergence of error e to zero. □

Remark 2. Proposed adaptive estimator (14), (15) or (23), (24), (15) can suffer from chattering phenomena due to fast changing conditions in resetting mechanism (17) or switching mechanism (25). Indeed, nothing prevents the function $\left|\det(\hat{K}_p)\right|$ from changing its derivative sign after every switch. However, we propose basic adaptation algorithms that are not free from this problem, but reveal the main ideas of adaptive control of the plants with unknown control direction. To avoid chattering phenomena we refer an interested reader to, for example, DREM algorithm proposed in [17] and applied for MRAC of SISO plant in [15]. This algorithm provides monotonic convergence of the estimates and can guarantee chattering free convergence of corresponding DREM estimators in alliance with resetting (17) or switching mechanism (25). □

IV. SIMULATION RESULTS

As a simulation example, let us consider 2×2 unstable plant

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{9p-1}{p^2-p+4} & \dfrac{2p+2}{p^2-p+4} \\ \dfrac{16p+86}{p^2-p+4} & \dfrac{-2p+28}{p^2-p+4} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

with poles $p_{1,2}$=0.5 ± 1.94j, known upper bound of observability index $\bar{\nu} = 2$ and unknown high frequency gain matrix

$$K_p = \begin{bmatrix} 9 & 2 \\ 16 & -2 \end{bmatrix}.$$

The parameters of the plant transfer matrix are unknown.

Threshold parameter $\underline{k}_p = 2$ is known a priori.

Reference model is presented by 2×2 second order system

$$\begin{bmatrix} y_{m1} \\ y_{m2} \end{bmatrix} = \begin{bmatrix} W_{r1}(p) & 0 \\ 0 & W_{r2}(p) \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} \dfrac{3}{p+3} & 0 \\ 0 & \dfrac{2}{p+2} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix},$$

where $r_1(t) = 40 sign(sin 5t)$+50, $r_2(t) = 50\text{sign}(\cos t)$+30 .

Two proposed controllers are applied to the plant.

1. Adjustable controller (10) based on estimator (14), (15) with resetting mechanism (17).

The controller is presented by equality

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \hat{K}_p \left(\sigma I_{m\times m} + \hat{K}_p^2\right)^{-1} \hat{\Theta}^T \Phi \qquad (28)$$

with matrices of adjustable parameters

$$\hat{\Theta}^T = \begin{bmatrix} \hat{\Theta}_1^T \\ \hat{\Theta}_2^T \end{bmatrix} = \begin{bmatrix} \hat{\Theta}_{11} & \hat{\Theta}_{21} & \cdots & \hat{\Theta}_{81} \\ \hat{\Theta}_{12} & \hat{\Theta}_{22} & \cdots & \hat{\Theta}_{82} \end{bmatrix},$$

$$\hat{K}_p = \begin{bmatrix} \hat{K}_{p1}^T \\ \hat{K}_{p2}^T \end{bmatrix} = \begin{bmatrix} \hat{K}_{p11} & \hat{K}_{p12} \\ \hat{K}_{p21} & \hat{K}_{p22} \end{bmatrix} \tag{29}$$

and vector

$$\Phi = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{21} & \omega_{22} & y_1 & y_2 & r_1 & r_2 \end{bmatrix}^T \tag{30}$$

generated by filters

$$\omega_{11} = \frac{10}{p+10}[u_1], \quad \omega_{12} = \frac{10}{p+10}[u_2],$$
$$\omega_{21} = \frac{10}{p+10}[y_1], \quad \omega_{22} = \frac{10}{p+10}[y_2].$$

Parameter is defined as $\sigma = 0.001$ and switched off according to (17) with threshold $\underline{k}_p = 2$. Matrices $\hat{\Theta}$ and $\hat{K}_p$ are calculated by estimators with involved normalization factors:

$$\dot{\hat{\Theta}}^T = -\frac{\Gamma_1}{1+\Phi^T\Phi} E\Phi_f, \quad \hat{\Theta}^T(0) = O_{2\times 8},$$
$$\dot{\hat{K}}_p = \frac{\Gamma_2}{1+\Phi^T\Phi} E u_f, \quad \hat{K}_p(0) = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}, \tag{31}$$

where

$$\Phi_f = \begin{bmatrix} \frac{3}{p+3}\left[\Phi^T\right] \\ \frac{2}{p+2}\left[\Phi^T\right] \end{bmatrix}, \quad u_f = \begin{bmatrix} \frac{3}{p+3}\left[u^T\right] \\ \frac{2}{p+2}\left[u^T\right] \end{bmatrix},$$

$\Gamma_1 = 10 I_{2\times 2}$, $\Gamma_2 = 200 I_{2\times 2}$ are the adaptation gains, $E = diag\{\tilde{e}_1, \tilde{e}_2\}$ is augmented error matrix with

$$\tilde{e}_1 = y_1 - y_{m1} + \hat{\Theta}_1^T \frac{3}{p+3}[\Phi] - \hat{K}_{p1}^T \frac{3}{p+3}[u],$$
$$\tilde{e}_2 = y_2 - y_{m2} + \hat{\Theta}_2^T \frac{2}{p+2}[\Phi] - \hat{K}_{p2}^T \frac{2}{p+2}[u].$$

Note, that the signs of the determinants $\hat{K}_p(0)$ and K_p are different.

Simulation results are presented in Fig.1 and illustrate the boundness of all the signals and convergence of errors e_1, e_2. It is shown that approximation of inversion $\hat{K}_p^{-1}$ presented in (28) prevents the division by zero and removes aggressive transient behavior.

2. Adjustable controller (19) based on estimator (23), (24), (15) with projection mechanism (25).

The controller is presented by equality

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \hat{\Psi}^T \Phi$$

with matrices (29) and

$$\hat{\Psi}^T = \begin{bmatrix} \hat{\Psi}_{11} & \hat{\Psi}_{21} & \cdots & \hat{\Psi}_{81} \\ \hat{\Psi}_{12} & \hat{\Psi}_{22} & \cdots & \hat{\Psi}_{82} \end{bmatrix},$$

vector (30), Matrices $\hat{K}_p$, $\hat{\Psi}^T$ and function $\hat{d}$ are generated by (31),

$$\dot{\hat{\Psi}}^T = -\frac{adj\left\{\hat{K}_p\right\}}{\hat{d}} \frac{1}{1+\Phi^T\Phi} E\left\{\Gamma_1\Phi_f + \Gamma_2 u_f \hat{\Psi}^T\right\},$$
$$\hat{\Psi}(0) = O_{8\times 2},$$
$$\dot{\hat{d}} = tr\left\{adj\left\{\hat{K}_p\right\}\dot{\hat{K}}_p\right\}, \ \hat{d}(0) = 21$$

with adaptation gains $\Gamma_1 = 10 I_{2\times 2}$, $\Gamma_2 = 200 I_{2\times 2}$ and augmented error matrix $E = diag\{\tilde{e}_1, \tilde{e}_2\}$, where

$$\tilde{e}_1 = y_1 - y_{m1} + \hat{K}_{p1}^T \hat{\Psi}^T \frac{3}{p+3}[\Phi] - \hat{K}_{p1}^T \frac{3}{p+3}[u],$$
$$\tilde{e}_2 = y_2 - y_{m2} + \hat{K}_{p2}^T \hat{\Psi}^T \frac{2}{p+2}[\Phi] - \hat{K}_{p2}^T \frac{2}{p+2}[u].$$

Projection mechanism (25) is implemented with threshold $\underline{k}_p = 2$.

Simulation results are presented in Fig.2. As in the first scheme, the results demonstrate the boundness of all signals in the closed-loop system and asymptotical convergence of tracking errors to zero. Singularity of the estimate of high frequency gain matrix $\hat{K}_p$ is prevented due to the switching projection mechanism.

V. Conclusion

In this paper we proposed less restrictive solutions to the problem of MRAC for MIMO parametrically uncertain linear plants. The solutions permit to relax the constraints on high frequency gain matrix K_p and require only the knowledge of lower bound of $\left|\det(K_p)\right|$.

Asymptotical and boundness properties of the system are established in theory and verified via simulation.

The future development of the problem will be focused on the following issues:

1. It is seen from simulation results that the transient performance of system is deteriorated what is due to the lack of persistent excitation in heavily filtered signal r and plant overparameterization. Therefore, application of algorithm with fast parametric convergence is preferable, however requires essential research.
2. Design of so called switching free modifications of the algorithms, which do require conditions (17) and (25) and, hence do not suffer from possible chattering phenomena. The basic ideas can be found in [19].
3. Relaxation of assumption A.4 to the full rank condition $\det(K_p) \neq 0$.

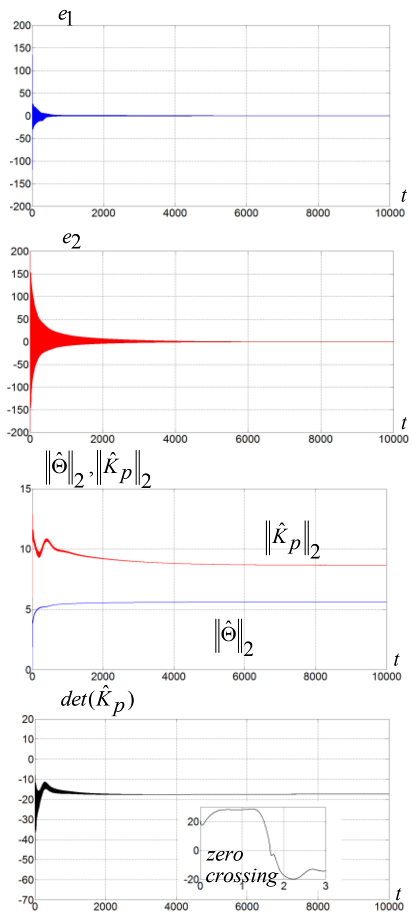

Fig.1. Transients in MIMO adaptive tracking system closed by estimator (14), (15) with resetting mechanism (17)

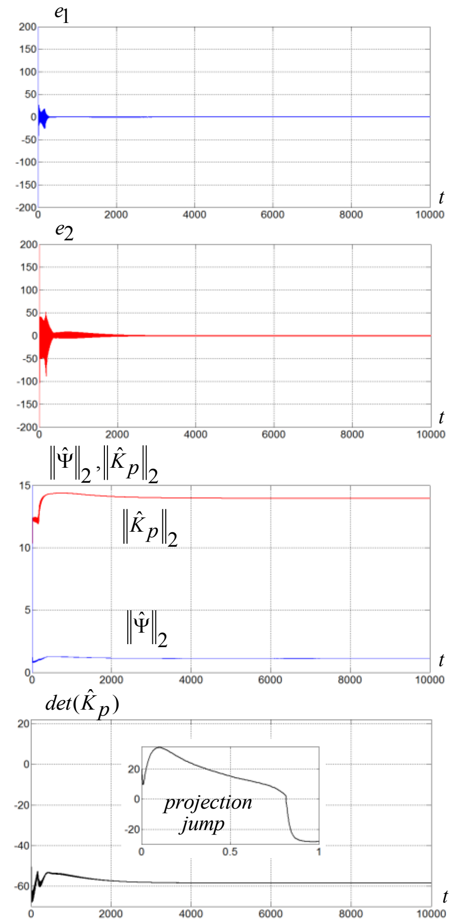

Fig.2. Transients in MIMO adaptive tracking system closed by estimator (23), (24), (15) with projection mechanism (25)

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