2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Humans-in-the-loop: A Game-Theoretic Perspective on Adaptive Building Energy Systems

Annika Eichler, Georgios Darivianakis and John Lygeros

Abstract— **Efficient building energy management has attracted a great interest in diverse research areas due to significant potential energy savings. A remaining challenge is how to combine the efforts of engineers on improving the energy management system with approaches developed by social scientists to integrate the occupants actively in the energy management system. This paper proposes the formulation of a game between the building energy management system and occupants to agree on room temperature comfort bounds. Under mild assumptions on the cost functions of the occupants, we show that a generalized Nash equilibrium exists and it can be shown to equal the social optimum. The alternating direction method of multipliers is used to solve the resulting consensus optimization problem in a distributed way, with the building management system as coordinator. An advantage of the proposed method is that the building energy management system does not need to rely on an explicit model of the occupant behavior but due to the game theoretic approach, indirectly receives an adapted model at each iteration. An extensive numerical study demonstrates the efficacy of the proposed approach.**

I. Introduction

Recent studies have shown that buildings account for 20%-40% of the total energy consumption in developed countries [1]. The potential for energy savings in this area has motivated intensive research efforts in various different disciplines. While engineers aim to reduce the energy consumption by improving the building construction characteristics and developing sophisticated control strategies for the efficient operation of building devices [2], [3], social scientists seek energy savings by employing behavior-based approaches to improve the occupants' energy-use efficiency. Modern technological advances (e.g., low cost sensors, smart metering, wireless proximity communication, or mobile internet) together with the evolution of web-enabled technologies and the growing market of smart-phones, have enabled the deployment of modern participation schemes [4].

Despite considerable progress in both fields, the combination of engineers' efforts on improving the energy management system with approaches of social scientists so as to actively integrate the occupants in the energy management system remains a challenge [5]. The final goal of a combined integrated approach is to reduce the energy consumption while guaranteeing the human comfort; studies have shown that the occupants' performance decreases up to 10% at thermal discomfort [6]. One of the most widely used measures of thermal comfort is the predicted mean vote (PMV) [7] and, based on this, the predicted percentage dissatisfied index (PPD), which predicts the percentage of thermally dissatisfied people at each PMV value. This measure represents average values. To simulate the behavior of individuals detailed information and fine tuning is needed.

A first step towards bridging the approaches from control engineers and social scientists is discussed in [8], where a control scheme that encapsulates building occupant behavior into the building energy management system (BEMS) was developed. The control scheme proposed there allows direct online feedback of occupant behavior, which is then exploited by the BEMS to further reduce energy consumption. In particular, the occupants' willingness to tolerate deviations of predefined comfort bounds is exploited by treating the occupant behavior in the BEMS as a random variable, whose distribution is assumed to be static and determined through social studies based on historical survey data. These stochastic models are static models that are not adaptable to the current situation and the correlation with potential compensation, e.g., monetary reward, is missing.

To overcome these limitations, in this paper we propose a game theoretic approach, in which the BEMS actively interacts with the occupants to agree on the level of allowed comfort constraint deviations. We show that, under mild assumptions on the occupant behavior supported by the PPD and physiological studies [9], a generalized Nash equilibrium for the formulated game exists and coincides with the social optimum. This enables the use of well-established distributed algorithms, such as the alternating direction method of multipliers [10], to solve the game. The novelty of the proposed approach is that it does not rely on a model or distribution of the occupants, but the real occupants themselves are integrated in the control loop. An advantage of this formulation is that due to the receding horizon implementation of the game the BEMS can continuously adapt to changes in the occupants' thermal comfort perception.

Section II briefly reviews the problem setup introduced in [8]. The game formulation between the BEMS and the occupants is presented in Section III. In Section IV the alternating direction method of multipliers is introduced as a distributed algorithm to find a generalized Nash equilibrium for the game. Section V confirms the positive effect of occupants participation by an extensive numerical study.

Notation. The concatenation of vectors v_1, v_2 up to v_n is defined as $[v_i]_{i=1,\dots,n} = \begin{bmatrix} v_1^\top & v_2^\top & \dots & v_n^\top \end{bmatrix}^\top$. Dimensions of vectors and matrices will be clear from the context.

This research project is part of the Swiss Competence Center for Energy Research SCCER FEEB&D of the Swiss Innovation Agency Innosuisse.

The authors are with the Automatic Control Laboratory, Department of Electrical Engineering and Information Technology, ETH Zurich, 8092 Zurich, Switzerland. {eichlean, gdarivia, lygeros}@control.ee.ethz.ch

II. Problem Setup

1) Building Dynamics: We use linear dynamics to model the buildings thermal dynamics and the dynamics of devices for energy storage, generation or conversion like a photovoltaic (PV) array, battery, chiller or boiler that are controlled by the BEMS, as

$$x_{t+1} = Ax_t + Bu_t + C\xi_t \,. \tag{1}$$

Here x_t are the building states (e.g., room, wall and floor temperatures) and the states of the devices (e.g., state of charge of the battery), u_t are the inputs to building actuation systems (e.g., air handling units or radiators) and devices and ξ_t are the disturbances acting on building and devices (e.g., ambient temperature or solar radiation). The system matrices A, B, C describe the building and device characteristics and are of appropriate dimensions. The operational constraints (e.g., power limits of the building actuation system or charging limits of the battery) are modeled by

$$Ex_t + Fx_{t+1} + Gu_t + H\xi_t \leq h \,, \tag{2}$$

with matrices E, F, G, H and vector h of appropriate dimensions. The conversion of energy is guaranteed by

$$L_{\mathrm{p}} p_t + L_{\mathrm{u}} u_t = 0 \,, \tag{3}$$

where p_t refers to the energy purchased from the grid and L_{p} and L_{u} balance the in-feed of electricity, heating and cooling energy from the corresponding grids with the inputs to the building and the energy devices.

In this paper, we are primarily interested in the effect of the building occupants on the BEMS. In this context, we make the assumption that perfect forecasts of the weather disturbances are available. Although unrealistic in practice, this assumption is certainly not restrictive since the methods proposed here can easily be combined with stochastic and robust building control approaches, e.g., [3], that can efficiently handle the effect of disturbances in the system.

2) Comfort Constraints: Human thermal comfort depends on various different parameters such as room temperature, relative humidity, performed activity, clothing or relative air velocity and can be expressed by different measures like the PMV. As a first step, we assume here that room temperature is the only factor influencing the thermal comfort; extension to other factors such as humidity can be considered in a similar fashion. Current practice is to enforce thermal comfort by bounding the room temperature as

$$\underline{T}_{r,t} \leq x_{r,t} \leq \overline{T}_{r,t} \,, \quad \forall r \in \mathcal{R} \,. \tag{4}$$

Here $x_{r,t}$ is the entry of x_t for the temperature in room r with $r \in \mathcal{R} = \{1, \dots, R\}$, where R is the number of rooms in the building. The comfort bounds $\underline{T}_{r,t}$ and $\overline{T}_{r,t}$ are chosen according to standards, e.g. by the European Committee for Standardization in [11]. However, the average parameters considered in these standards obscure the variability in the preferences of the individual occupants. To account for this, we adapt these temperature comfort bounds as

$$\underline{T}_{r,t} - \underline{\delta}_{r,t} \leq x_{r,t} \leq \overline{T}_{r,t} + \overline{\delta}_{r,t} \,, \;\; \underline{\delta}_{r,t}, \overline{\delta}_{r,t} \geq 0 \,, \;\; \forall r \in \mathcal{R} \,, \tag{5}$$

where $\underline{\delta}_{r,t}$ and $\overline{\delta}_{r,t}$ are positive deviations below and above the fixed thermal comfort bounds.

III. Game Formulation

Assuming the existence of N occupants and an automated building energy management system, we now formulate a game of $N+1$ players, interpreted as a generalized Nash equilibrium problem (GNEP). In a GNEP of $N+1$ players, player $i = 0, \dots, N$ is assumed to have access to decision variables $\zeta_i \in \mathbb{R}^{n_i}$ that he uses to minimize his scalar cost function $\theta_i(\zeta_i, \zeta_{-i}) : \mathbb{R}^n \to \mathbb{R}$, where $n = \sum_{i=0}^{N} n_i$. As usual, the cost of player i depends on his own strategy $\zeta_i \in \mathbb{R}^{n_i}$ and on the other players' strategies $\zeta_{-i} = [\zeta_i]_{i \in \{0,\dots,N\} \setminus i}$. We assume that the agents actions are coupled through a constraint set $Q \subseteq \mathbb{R}^n$; if the actions of other agents ζ_{-i} are assumed fixed, this implies a constraint on the possible actions of agent i of the form $\zeta_i \in Q_i(\zeta_{-i}) \subseteq \mathbb{R}^{n_i}$. This leads to each player solving an optimization problem of the form

$$\begin{aligned} &\min_{\zeta_i} \, \theta_i(\zeta_i, \zeta_{-i}) \\ &\text{s.t. } \zeta_i \in Q_i(\zeta_{-i}) = \{\zeta_i \in \mathbb{R}^{n_i} \mid (\zeta_i, \zeta_{-i}) \in Q\} \,, \end{aligned} \tag{6}$$

for all $i = 0, \dots, N$. The solution of the GNEP (6) is called a generalized Nash equilibrium (GNE). This is a collection of strategies for the players, for which no player has any incentive to unilaterally change his strategy. As defined in [12], [13], game (6) admits a GNE $\zeta^* = [\zeta_i^*]_{i=0,\dots,N}$ if for each player $i = 0, \dots, N$, it holds that $\theta_i(\zeta_i^*, \zeta_{-i}) \leq \theta_i(\zeta_i, \zeta_{-i})$ for all $\zeta_i \in Q_i(\zeta_{-i})$.

In the game formulation considered here, the BEMS tries to minimize the energy costs with respect to the temperature deviations, while the occupants try to minimize their thermal discomfort with respect to the temperature deviations. It is intuitive that the smaller the deviation the larger the energy costs for the BEMS, but the smaller the occupants' thermal discomfort. The temperature deviation, for which the BEMS as well as the occupants have no incentive to deviate, defines this game's generalized Nash equilibrium.

A. Cost Function of the BEMS

The BEMS aims to minimize the costs of purchased energy from the grid over a given time horizon T with $\mathcal{T} = \{1, \dots, T\}$, while satisfying the dynamics of building and devices (1), operational and conversion constraints (2) and (3) and comfort constraints (5). Given the time-varying price of the energy purchased from the grid c_t and the initial states x_1 and s_1, the cost of operating the BEMS is given by the optimal solution of the following optimization problem

$$\min_{u,v,p,\underline{\delta},\overline{\delta}} \; \sum_{t \in \mathcal{T}} c_t^\top p_t \quad \text{s.t. } (1)-(3) \text{ and } (5) \quad \forall t \in \mathcal{T} \,. \tag{7}$$

The decision variables $(u, v, p, \underline{\delta}, \overline{\delta})$ are indexed by time, i.e., $u = [u_t]_{t \in \mathcal{T}}$ with v, p, $\underline{\delta}$, $\overline{\delta}$, $\underline{\epsilon}$ and $\overline{\epsilon}$ defined similarly. Here, $\underline{\delta}_t = [\underline{\delta}_{r,t}]_{r \in \mathcal{R}}$ and $\overline{\delta}_t = [\overline{\delta}_{r,t}]_{r \in \mathcal{R}}$ comprise the downward and upward temperature deviations desired by the BEMS for each room $r \in \mathcal{R}$ for time $t \in \mathcal{T}$, while $\underline{\epsilon}_t = [\underline{\epsilon}_{i,t}]_{i=1,\dots,N}$ and $\overline{\epsilon}_t = [\overline{\epsilon}_{i,t}]_{i=1,\dots,N}$ comprise the deviations each of the N occupants. Note that (x, s) are also functional variables of (7) but are completely determined by the decision variables $(u, v, p, \underline{\delta}, \overline{\delta})$.

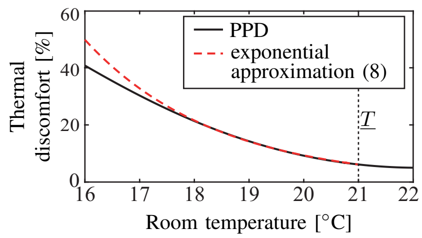

Fig. 1. Thermal discomfort measure PPD according to ISO 7730 [14] for average values and exponential approximation (8)

B. Cost Function of the Occupants

We propose the following cost function for the i-th occupant's thermal discomfort

$$\min_{\substack{\underline{\epsilon}_{i,t},\overline{\epsilon}_{i,t} \\ \forall t\in\mathcal{T}}} \sum_{t\in\mathcal{T}} \underline{\alpha}_{i,t}\exp(\underline{\beta}_{i,t}\underline{\epsilon}_{i,t}) + \sum_{t\in\mathcal{T}} \overline{\alpha}_{i,t}\exp(\overline{\beta}_{i,t}\overline{\epsilon}_{i,t}), \quad (8)$$

where $\underline{\alpha}_{i,t}$, $\overline{\alpha}_{i,t}$, $\underline{\beta}_{i,t}$, $\overline{\beta}_{i,t} > 0$ are occupant dependent model parameters. The choice of this exponential model for the thermal discomfort is motivated by two considerations:

(i) Physiological studies [9] have shown that humans weight cold exposure exponentially to a linear monetary reward. Although derived in studies under more extreme conditions than we will confront in a building, we assume here that this relationship holds also under moderate conditions.

(ii) The PPD predicts the percentage of thermally dissatisfied people at each PMV value. As verified by Fig. 1, in the temperature region of interest the PPD can be approximated well by an exponential curve, while for extreme temperature regions the PPD is quasiconvex and approaches to 100%, i.e., everybody feels uncomfortable. These temperature regions, however, will not be attained in this setting.

C. Coupling Constraints

To ensure consensus between BEMS and occupants we require the deviations for the BEMS and all occupants in each room to match. This is enforced through

$$g(\underline{\delta},\overline{\delta},\underline{\epsilon},\overline{\epsilon}) = \left[\left[\left[\begin{array}{c}\underline{\delta}_{r,t}-\underline{\epsilon}_{i,t}\\ \overline{\delta}_{r,t}-\overline{\epsilon}_{i,t}\end{array}\right]_{i\in\mathcal{N}_{r,t}}\right]_{r\in\mathcal{R}}\right]_{t\in\mathcal{T}}, \quad (9)$$

where $\mathcal{N}_{r,t}$ denotes the subset of occupants present in room r at time t. In addition to the perfect weather forecast, we make the assumption of a perfect room occupation plan here.

D. Game Between BEMS and Occupants

To ease notation, we denote the BEMS as player 0 with its strategy $\zeta_0 = [u^\top v^\top p^\top \underline{\delta}^\top \overline{\delta}^\top]^\top$. Accordingly, we introduce the strategy of the i-th occupant with $i = 1,\dots,N$ as $\zeta_i = [\underline{\epsilon}_{i,t}, \overline{\epsilon}_{i,t}]_{t\in\mathcal{T}}^\top$. With these we can formulate the game between the BEMS and the occupants as a special case of (6)

$$\begin{aligned}&\min_{\zeta_i} \theta_i(\zeta_i) && (10)\\ &\text{s.t. } \zeta_i \in Q_i(\zeta_{-i}) = \{\zeta_i \in \overline{Q}_i \,:\, g(\zeta_i,\zeta_{-i}) = 0\},\end{aligned}$$

for all $i = 0,\dots,N$. The cost function $\theta_0(\zeta_0)$ and the local constraint set $\overline{Q}_0$ of the BEMS are given by (7). The thermal discomfort of the occupants in (8) defines the cost function of the occupants $\theta_i(\zeta_i)$ for $i = 1,\dots,N$. Their strategies are not locally constrained, thus $\overline{Q}_i = \mathbb{R}^{2T}$ for $i = 1,\dots,N$. The joint constraint is $g(\zeta_i,\zeta_{-i}) = g(\underline{\delta},\overline{\delta},\underline{\epsilon},\overline{\epsilon})$, as defined in (9). The constraint set Q_i of each player is the intersection of its local constraints $\overline{Q}_i$ and the joint constraint $g(\zeta_i,\zeta_{-i}) = 0$.

The game is to be solved in a receding horizon manner, i.e., at each time step a new game is formulated to decide on the BEMS operation. This allows the BEMS to adapt to the current mood of the occupants, which may change over time. Note that this would not have been possible if a static model for the occupants were to be used as in [8].

GNEPs are in general very difficult to solve and uniqueness of the solution is not guaranteed unless strong assumptions like uniform convexity of the cost functions hold. However, the game between BEMS and occupants in (10) has a special structure that can be exploited.

Definition 1 (Jointly convex GNEP [12])**.** *A GNEP is jointly convex if for every player i with $i = 0,\dots,N$*

(i) *the cost function $\theta_i(\zeta_i,\zeta_{-i})$ is convex in his strategy ζ_i,*
(ii) *his constraint set $Q_i(\zeta_{-i})$ is convex and bounded given ζ_{-i}, and*
(iii) *the coupling constraint set Q is closed and convex.*

Proposition 1. *The game between BEMS and the occupants, as described in (10), is a jointly convex GNEP.*

Proof. The cost functions of BEMS and occupants are convex, their constraint sets are closed and bounded and (9) is linear, thus, (i), (ii) and (iii) are fulfilled. ∎

Joint convexity of the game can be exploited to solve the GNEP using variational inequalities.

Definition 2 (Variational inequality [15])**.** *A vector $\zeta' \in \mathbb{R}^n$ is a solution of the variational inequality $\mathrm{VI}(Q,\Theta)$ for a continuous mapping $\Theta : \mathbb{R}^n \to \mathbb{R}^n$ and a set $Q \subseteq \mathbb{R}^n$ if $(\zeta-\zeta')^\top\Theta(\zeta') \geq 0$ for all $\zeta \in Q$.*

Noting that all the cost functions are differentiable, let us define the following variational inequality

$$\begin{aligned}\mathrm{VI}(Q,\Theta) \;\text{ with }\; Q = \{\zeta_i \in \bar{Q}_i,\ \forall i = 0,\dots,N \,:\, g(\zeta) = 0\},\\ \Theta = [\nabla_{\zeta_i}\theta_i(\zeta_i)]_{i=0,\dots,N}\,, \quad \zeta = [\zeta_i]_{i=0,\dots,N}\,. \quad (11)\end{aligned}$$

As shown in [16], assuming that generalized Slaters' constrained qualification holds, ζ' is a solution of the $\mathrm{VI}(Q,\Theta)$ defined in (11) if and only if it satisfies the KKT system

$$\begin{aligned}0 \in \nabla_{\zeta_i}\theta_i(\zeta_i') + {\lambda'}^\top\nabla_{\zeta_i}g(\zeta') + \mathcal{C}(\zeta_i',\overline{Q}_i)\,, \quad (12)\\ \forall i = 0,\dots,N\,,\end{aligned}$$

with $\mathcal{C}(\zeta_i',\overline{Q}_i)$ being the normal cone to $\overline{Q}_i$ at ζ_i', in other words ζ' is a solution of the GNEP if and only if there exist multipliers $\lambda_i' \in \mathbb{R}^m$ such that (12) holds. Moreover, ζ^* is as a solution of the jointly convex GNEP in (10) if and only if there exist multipliers $\lambda_i^* \in \mathbb{R}^m$ such that

$$\begin{aligned}0 \in \nabla_{\zeta_i}\theta_i(\zeta_i^*,\zeta_{-i}^*) + {\lambda_i^*}^\top\nabla_{\zeta_i}g(\zeta^*) + \mathcal{C}(\zeta_i^*,\overline{Q}_i)\,, \quad (13)\\ 0 \leq \lambda_i^* \perp g(\zeta^*) \leq 0\,, \quad \forall i = 0,\dots,N\,.\end{aligned}$$

Proposition 2 ([12], [16])**.** *For the jointly convex GNEP (10) where all θ_i are continuously differentiable, every solution ζ' of the variational inequality $\mathrm{VI}(Q,\Theta)$ as defined in (11)* *is a solution ζ^* of the GNEP, i.e., $\zeta' = \zeta^*$, and it holds that $\lambda' = \lambda_0^* = \lambda_1^* = \ldots = \lambda_N^*$, with multipliers λ_i^* satisfying (12) and λ' satisfying (13).*

Note that, according to Proposition 2, every solution of the $\mathrm{VI}(Q, \Theta)$ in (11) is also a solution of the GNEP (10), but not vice versa. These GNEP solutions are referred to as variational solutions. The proposition states furthermore that for the variational solutions the multipliers λ_i^* for all players are equal. If we interpret the multiplier λ^* as the unit of price paid by the BEMS to the occupants for a temperature deviation, then we are interested in solutions where the prices for deviations that fulfill the coupling constraint $g(\zeta) = 0$, match for occupants and BEMS, i.e., the variational solutions. The unitary prices for different occupants and different rooms may however differ due to different occupant preferences, since for every occupant there is a row in $g(\cdot)$ corresponding to an entry of the multiplier λ.

Finally, we note that the cost function of each player is independent of the other players' strategies, i.e., $\theta_i(\zeta_i, \zeta_{-i}) = \theta_i(\zeta_i)$. This allows us to establish the following proposition as it can be found similarly for potential games [13].

Proposition 3. *The variational solutions of the VI in (11) coincide with the global solutions of*

$$\min_{\zeta} \ \theta(\zeta)\,, \quad \text{s.t. } \zeta \in Q\,, \quad \text{with} \quad \theta(\zeta) = \sum_{i=0}^{N} \theta_i(\zeta_i)\,. \quad (14)$$

Proof. As given in [15], ζ^* is a global minimum to the (14) if and only if ζ^* is a solution to the $\mathrm{VI}(Q, \nabla\theta)$. The solution to $\mathrm{VI}(Q, \nabla\theta)$ coincides in turn with the solutions of (11) since $\nabla_{\zeta_i}\theta(\zeta) = \nabla_{\zeta_i}\theta_i(\zeta_i)$. The result follows. ■

Summarizing, for the game in (10) every GNE, which is also a variational solution, attains the social optimum, i.e., is an optimizer of problem (14).

Remark 1. *For the game between BEMS and occupants, the global optimization problem in (14) can be reformulated to admit a strictly convex cost function, therefore guaranteeing the uniqueness of the variational solution.*

Under Proposition 3 the global optimization problem (14) can be solved to find a GNE. In practice, this is not possible due to privacy concerns of the occupants. But, with the decoupled nature of (14) a distributed optimization algorithm can be used to alleviate this difficulty.

IV. Solving the GNEP using ADMM

Any distributed optimization algorithm can be chosen to solve the GNEP, which ensures the cost functions and the constraints of the BEMS and the occupants to be kept private. Due to its simplicity and convergence guarantees, we consider the alternating direction method of multipliers (ADMM) [10]. ADMM is an iterative algorithm. For each iteration k the BEMS solves the local optimization problem

$$\zeta_0^{k+1} = \operatorname*{argmin}_{\zeta_0} \ \theta_0(\zeta_0) + \left(\sum_{i=1}^{N} \lambda_i^k S_{r,i}(\zeta_0)\right) + \frac{\rho}{2}\left\|g(\zeta_0, \zeta_{-0}^k)\right\|_2^2$$
$$\text{s.t. } \zeta_0 \in \overline{Q}_0\,. \quad (15)$$

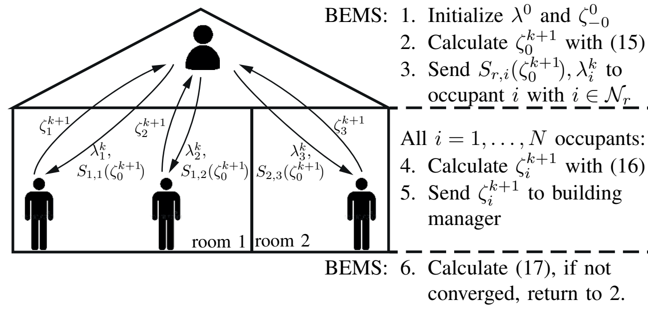

Fig. 2. ADMM algorithm to solve the GNEP between BEMS and occupants

Here $S_{r,i}(\zeta_0)$ is a linear mapping resulting in $S_{r,i}(\zeta_0) = [\underline{\delta}_{r,t} \ \overline{\delta}_{r,t}]_{t\in\mathcal{T}}^\top$ if r is the room, where the i-th occupant is in, i.e., $i \in \mathcal{N}_r$. Then the BEMS communicates the current temperature deviations $S_{r,i}(\zeta_0^{k+1})$ to all occupants i with $i \in \mathcal{N}_r$. Each occupant with $i = 1, \ldots, N$ solves

$$\zeta_i^{k+1} = \operatorname*{argmin}_{\zeta_i} \ \theta_i(\zeta_i) - \lambda_i^k \zeta_i + \frac{\rho}{2}\left\|g_i(\zeta_i, S_{r,i}(\zeta_0^{k+1}))\right\|_2^2\,,$$
$$\text{s.t. } \zeta_i \in \overline{Q}_i\,, \quad (16)$$

with the constraint $g_i(\zeta_i, S_{r,i}(\zeta_0)) = S_{r,i}(\zeta_0) - \zeta_i$ being the respective lines of g as defined in (9). After solving (16), the occupants communicate the resulting temperature deviations ζ_i^{k+1} back to the BEMS. Finally, an update of the dual variables $\lambda = [\lambda_i]_{i=1,\ldots,N}$ is performed by the BEMS as

$$\lambda^{k+1} = \lambda^k + \rho g(\zeta_0^{k+1}, \zeta_{-0}^{k+1}) \quad \text{with} \quad \rho > 0\,, \quad (17)$$

and the updated multipliers are also broadcast to the occupants. The GNEP is solved if at iteration k the joint constraint $g(\zeta_0^k, \zeta_{-0}^k) = 0$ is fulfilled. Here the ADMM formulation in [10, 3] is used, which is applicable since the constraints can be considered by indicator functions. The splitting is performed with respect to the BEMS and all occupants. However, the update step of all occupants completely decouples for individual occupants, leading to (16) that does not depend on other occupants' temperature deviations. Thus, (16) can be performed in parallel for all occupants. Due to convexity of the cost functions of BEMS $\theta_0(\zeta_0)$ and occupants $\theta_i(\zeta_i)$ and the indicator function, ADMM is guaranteed to converge to the global solution [10, 3.2.1]. The implementation of the ADMM algorithm is visualized in Fig. 2.

The ADMM formulation allows one to interpret λ_i as the price of a unit temperature deviation, that is paid from the building to the i-th occupant for an agreed deviation ζ_i. With this, in (16) a linear monetary reward is added to the occupants' cost function. Thus, (16) represents the dependency resulting from the physiological studies [9].

It is likely that the prices for different occupants in one room differ for the same deviation given different cost functions. With the distributed algorithm in (15)–(17), no information of other occupants or their corresponding multiplier need to be communicated among the occupants.

Remark 2. *Although we model the occupants here with the cost function (8), this is not crucial for solving the game. The occupants can be seen as a black box, to which the BEMS* *gives a price and gets a temperature deviation back. As long as reactions of the occupants are governed by a convex cost function the game will converge to the global optimum.*

V. Numerical Results

A. Problem Formulation

1) Building Model: We consider the same setup as in [8] with a one zone building of $2{\times}8\,\mathrm{m}^2$ ground area. The building is equipped with a boiler, a chiller, a heat pump, a lead-acid battery and a south oriented PV installation with maximum capacity of $2\,\mathrm{kW}$ for boiler and chiller, $1\,\mathrm{kW}$ for the heat pump $5\,\mathrm{kW}$ for the battery and $4.1\,\mathrm{kW}$ for the PV installation. The coefficients of performance are 0.9, 0.7 and 3 for boiler, chiller and heat pump, respectively. The price of electricity from the grid follows a day-night tariff of 0.145 CHF/$\mathrm{kW\,h}$ between 5am and 11pm and 0.097 CHF/$\mathrm{kW\,h}$ otherwise. The building is assumed to be located in Zurich, Switzerland, and simulations are performed using weather data from January 2007.

2) Comfort Constraints: Since only winter days are considered, only deviations of the lower bound are studied. The fixed comfort bounds are set to $\underline{T} = 21\,°\mathrm{C}$ and $\overline{T} = 25\,°\mathrm{C}$ for day hours between 6am-11pm, as proposed by [11].

3) Occupants: The building is assumed to be occupied by 5 occupants. Their cost functions are modeled by (8) with $\underline{\alpha}_{i,t}$ taking values between 0.032 and 0.079 CHF and $\underline{\beta}_{i,t}$ taking values between 0.06 and $0.11°\mathrm{C}^{-1}$, differing among the occupants, but not in time. These parameter values were chosen to show a good fit with the survey data used in [8]. It is conceivable that the ADMM implementation is done on the occupants' behalf by a smart-phone program with individual settings (e.g., the α's and β's), where personal preferences can be set to steer the game in the desired direction.

B. Numerical Results

The optimization problem is solved in a receding horizon fashion for ten consecutive weeks starting from January the 8th 2007 with a prediction horizon set to 8 hours and a sampling period of one hour.

The minimum, maximum and mean values of the ten weeks with occupant participation denoted as 'Occ' are compared to fixed bounds in Table I. The occupant participation has positive effect on the overall energy consumption and on the cost function of the occupants, while it has almost no effect on the cost to be paid by the BEMS. This is achieved by reducing the lower temperature bound on average from $21\,°\mathrm{C}$ to $20.41\,°\mathrm{C}$. Note that for both the BEMS and the occupants the values in Table I include the monetary exchange among them, i.e., the results of (15) and (16).

In Fig. 3 the profiles of the room temperature evolution and the energy consumption from the grid are shown, comparing occupants participation with fixed bounds. We emphasize that using fixed bounds the BEMS makes use of the reduced electricity prices in the early morning to preheat the building. In contrast, with occupants participation it is economically more reasonable not to preheat but paying more to the occupants in the morning for a larger deviation. This behavior is advantageous from an energy efficiency point of view, since due to heat losses of the building the preheating energy can not be fully exploited. In average, the ADMM algorithm with $\rho = 2$ converges for one sample time within 720 iterations. Implemented in Matlab on a computer equipped with 8 GB RAM and a 2.9 GHz processor, one iteration is solved in average in 8.9 sec (8.88 sec for the BEMS and in average 0.02 sec for one occupant). This results in a calculation time per iteration of approximately 1.8 hours, which is clearly not feasible in real time for a sampling time of 1 hour.

1) Reducing the Horizon of Occupants Participation: One possibility to significantly reduce the number of iterations is to reduce the number of variables to agree on. Therefore, for the horizon of T, only for $t \in \{1, \ldots, T_a\}$ with $T_a < T$ adaptable comfort bounds as in (5) are considered. For the remaining time steps $t \in \{T_a + 1, \ldots, T\}$ the fixed bounds in (4) are implemented. In addition to reducing the number of variables and thus the number of iterations, this strategy offers one way to address the issue that the occupants might be unsure about predicting their desired temperature deviations in distant future. The drawback is that smaller energy savings can be expected compared to adaptable bounds for the whole horizon, since the BEMS plans in each sample time to be able to reach the static bounds after $T_a + 1$ time steps and thus reacts more conservative. In Table I the results for the extreme case with $T_a = 1$ are shown. As expected the costs for the BEMS and the occupants' cost function are almost unchanged, while more energy is consumed. But compared to the solution with fixed bounds there is still an improvement of 24% in energy consumption. For the reduced horizon the ADMM algorithm converges in average within 22 iterations for one time instance. Compared to the average time of 1.8 hours per iteration for a full adaptable horizon, with one step of adaptation one iteration takes in average 3.2 minutes with 8.88 sec per iteration (8.83 sec for the BEMS and 0.02 sec for one occupant), which is feasible in real time. Further parameters of influence on the convergence speed are the initial values as well as ρ, which were not in the

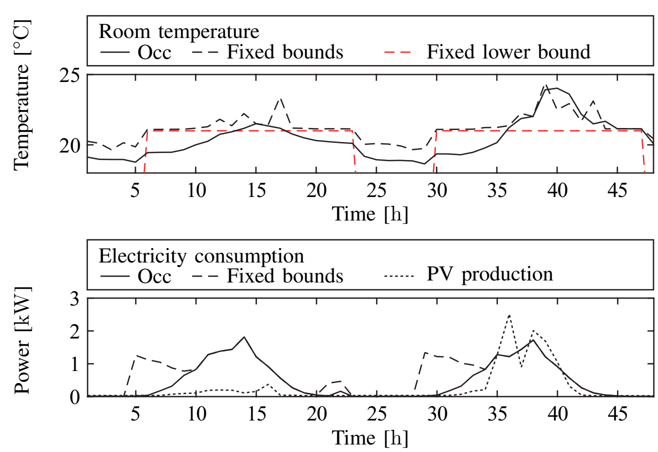

Fig. 3. Room temperature and electricity consumption from the grid. Comparison of 'Occ' (occupants participation) with fixed bounds for two typical days.

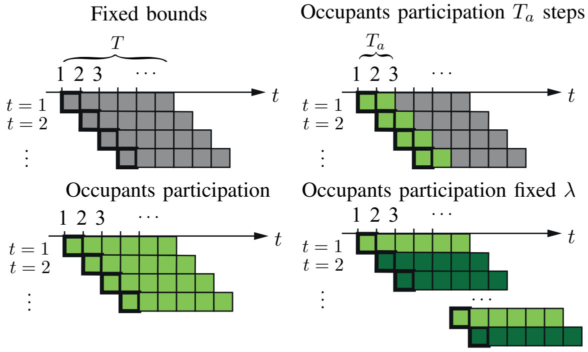

Fig. 4. The four different versions of implementation, with fixed bounds in gray, flexible bounds in green and flexible bounds but prefixed λ in dark green. The number of boxes vertically represents the optimization horizon, while only the current strategy in bold is implemented.

TABLE I

COMPARISON OF COST AND ELECTRICITY CONSUMPTION

	Minimum	Mean	Maximum
Electricity consumption (kW)			
Fixed bounds	74.50	116.42	207.16
Occ	71.95	75.15	78.07
Occ fixed λ	71.95	74.67	77.45
Occ 1 step	74.27	88.76	94.88
Cost function of occupants			
Fixed bounds	72.65	72.65	72.65
Occ	67.45	71.72	72.64
Occ fixed λ	72.33	72.87	76.07
Occ 1 step	68.52	72.02	72.64
Cost of BEMS (CHF)			
Fixed bounds	10.71	16.01	28.83
Occ	10.68	16.53	34.79
	(10.42+0.26)*	(10.88+5.65)*	(11.26 + 23.53)*
Occ fixed λ	10.52	15.17	26.90
	(10.42+0.10)*	(10.81+4.36)*	(10.92+15.98)*
Occ 1 step	10.82	16.10	31.37
	(10.64+0.18)*	(12.16+3.94)*	(12.18+19.19)*

*(grid+occupants)

focus of this paper. A comparison of the different strategies is visualized in Fig. 4.

2) Reducing the Instances of Playing the Game: It is unrealistic in practice that the iterative negotiation about the prices takes place every sample time. One possibility to overcome this issue is to perform the negotiation only occasionally to obtain representative prices, which can then be used in open-loop. If the BEMS is aware of significant changes of the occupants or the environment (e.g., seasonal changes), by repeating the game, an update of the prices can be received. This strategy is referred as 'Occ fixed λ'. Here for the first of the ten weeks the negotiation is performed as before. Then for each hour average prices over the first week are considered and no negotiation is performed. Instead the BEMS solves the optimization problem (15), assuming that the coupling constraint is fulfilled, in a receding horizon fashion with these fixed prices. As shown in Table I, this open-loop strategy leads to only minimum changes of the overall performance. But it is clear that the performance of the open-loop strategy strongly depends on the the variation within the weeks.

VI. Conclusion

This works presents a methodology to integrate the building occupant behavior into the building energy management system. This is achieved by formulating a generalized Nash equilibrium game between the occupants and the BEMS. The occupants' thermal comfort in return of a monetary reward is exploited therein to further reduce the energy impact of the building. Under mild assumptions on convexity of the occupants' cost functions, the existence of an equilibrium, which coincides with the social optimum, is shown. A distributed implementation of the proposed method is possible using the ADMM algorithm. A numerical study conducted shows the merits of the proposed approach which is capable of significantly reducing the energy footprint of the building while keeping occupants' satisfaction levels high.

References

[1] L. Pérez-Lombard, J. Ortiz, and C. Pout, "A review on buildings energy consumption information," *Energy Build.*, vol. 40, pp. 394–398, 2008.

[2] P. B. Luh, "Building Energy Management: Integrated Control of Active and Passive Heating, Cooling, Lighting, Shading, and Ventilation Systems," *IEEE Trans. Autom. Sci. Eng.*, vol. 10, no. 3, pp. 588–602, 2013.

[3] G. Darivianakis, A. Georghiou, R. S. Smith, and J. Lygeros, "The Power of Diversity: Data-Driven Robust Predictive Control for Energy Efficient Buildings and Districts," *IEEE Trans. Control Syst. Technol.*, vol. PP, no. 99, pp. 1–14, 2017.

[4] V. L. Chen, M. A. Delmas, and W. J. Kaiser, "Real-time, appliance-level electricity use feedback system: How to engage users?" *Energy Build.*, vol. 70, pp. 455–462, 2014.

[5] R. Gulbinas, A. Khosrowpour, and J. Taylor, "Segmentation and Classification of Commercial Building Occupants by Energy-Use Efficiency and Predictability," *IEEE Trans. Smart Grid*, vol. 6, no. 3, pp. 1414–1424, 2015.

[6] L. Lan, P. Wargocki, and Z. Lian, "Quantitative measurement of productivity loss due to thermal discomfort," *Energy Build.*, vol. 43, no. 5, pp. 1057–1062, 2011.

[7] P. O. Fanger, *Thermal comfort : analysis and applications in environmental engineering*. Danish Technical Press, 1970.

[8] A. Eichler, G. Darivianakis, and J. Lygeros, "Humans in the Loop: A Stochastic Predictive Approach to Building Energy Management in the Presence of Unpredictable Users," in *IFAC Proc. Vol.*, 2017, pp. 15 036–15 041.

[9] M. Cabanac and K. Johnson, "Human Thermoregulatory Behavior During a Conflict Between Cold Discomfort and Money," *Physiol. Behav.*, vol. 30, pp. 145–150, 1983.

[10] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers," *Found. Trends® Mach. Learn.*, vol. 3, no. 1, pp. 1–122, 2010.

[11] European Committee for Standardization, "CEN EN 15251:2007: Indoor environmental input parameters for design and assessment of energy performance of buildings addressing indoor air quality, thermal environment, lighting and acoustics," Tech. Rep., 2007.

[12] F. Facchinei and C. Kanzow, "Generalized Nash equilibrium problems," *4or*, vol. 5, no. 3, pp. 173–210, 2007.

[13] F. Facchinei, V. Piccialli, and M. Sciandrone, "Decomposition algorithms for generalized potential games," *Comput. Optim. Appl.*, vol. 50, no. 2, pp. 237–262, 2011.

[14] International Organization for Standardization, "ISO 7730: Moderate thermal enviroments - Determination of the PMV and PPD indices for specification of the conditions for thermal comfort," 1994.

[15] F. Facchinei and J. S. Pang, *Finite-Dimensional Variational Inequalities and Complementarity Problems*, 1st ed. Springer-Verlag, 2003.

[16] G. Scutari, D. P. Palomar, F. Facchinei, and J.-S. Pang, "Monotone Games for Cognitive Radio Systems," in *Distrib. Decis. Control*, 2011, pp. 83–112.