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Missile Guidance Based on Tracking of Predicted Target Trajectory

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***Abstract*—This study presents missile guidance methods based on path-following approach for the interception of a high-speed maneuvering target. In the problem of high-speed incoming target interception, it is usually preferred to form a stable head-on engagement geometry a few seconds before the end of engagement to allow enough time for improved hit-to-kill performance. One plausible approach in this regard is to let the missile first track the predicted target trajectory and then encounter the target while moving along the predicted path. To this end, two approaches based on path-following guidance are proposed; one is to modify the parameters of guidance law, and the other is to design the path to be followed by the missile. The effectiveness of the proposed methods are verified by comparing their performances with pure proportional navigation guidance.**

I. Introduction

Intercepting a high-speed target including hypersonic vehicles and ballistic missiles has been recognized as one of the most difficult tasks for anti-air missiles [1]. An interceptor should approach the target with maintaining the head-on geometric configuration because the closing speed is very high, and even a small misalignment of the collision course near the interception may result in a large miss-distance. To successfully intercept this type of target, advanced guidance strategy as well as precise target state estimation technique [2], [3] are required.

To deal with the problem, proportional navigation guidance (PNG) and nonlinear control schemes including sliding mode control [4] have been widely studied for decades. Most of which are designed based on line-of-sight (LOS) information. Despite the good performance, the LOS-based guidance laws usually need a large amount of corrective actions near the collision, because LOS changes rapidly near the target. Guidance laws utilizing other types of measurements such as cross-track error from target path or predicted miss distance, i.e., zero-effort-miss (ZEM), [5], [6], [7] avoid direct use of LOS rate information, thus improving the hit-to-kill probability. However, ZEM is usually obtained from the linearized kinematics and can be converted to LOS angle. In this respect, an alternative scheme that depends less on the LOS information is needed.

In this study, tracking the target path is considered. Trajectory of the target is represented as a parametric curve by considering the locus of predicted intercept point (PIP) as the future path of the target. A path-following guidance method for exact tracking [8] is modified to bring head-on intercept. In the first method, a look-ahead vector is modified so that accurate tracking as well as required approaching time before time-to-intereception can be achieved. In the other method, a reference path for the missile is generated to make the missile approach and then move along the predicted path of the target until the interception.

The remainder of the paper is organized as follows. Section II presents the problem formulation and the guidance law utilized for path-following. In Sec. III, two different methods to form head-on collision geometry ahead of the end-phase are proposed. Numerical simulation results are shown in Sec. IV, and concluding remarks are provided in Sec. V.

II. Problem Formulation

A. Engagement Kinematics

In this study, a planar engagement between a target T and an interceptor M is considered as shown in Fig. 1.

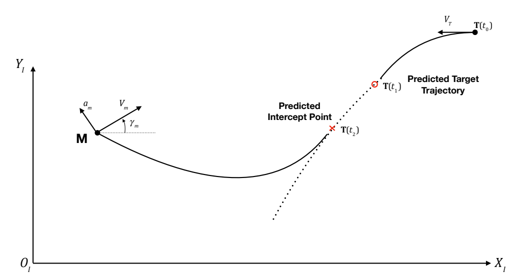

Fig. 1. Engagement Geometry in Planar Motion

In ballistic missile defense, the target is usually faster than the interceptor, and therefore head-on interception is required to compensate the speed disadvantage. In this situation, tracking of the target's predicted trajectory may lead to the head-on interception.

Equations of motion for the interceptor described as a two-dimensional point-mass model can be expressed as

$$\begin{aligned} \dot{x}_m &= V_m \cos\gamma_m \\ \dot{y}_m &= V_m \sin\gamma_m \\ \dot{\gamma}_m &= \frac{a_m}{V_m} \end{aligned} \tag{1}$$

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where (x_m, y_m) is the position of the missile, γ_m is the flight-path angle of the missile, V_m is the speed of the missile, and a_m is the acceleration of the missile.

Trajectory of the target should be precisely estimated, which includes the information of position, velocity, and time-to-go profile. In general, the motion of a ballistic missile is mainly governed by the gravitational force, and therefore the predicted trajectory can be parametrized as $\mathbf{T} = \mathbf{P}(t)$ and $\mathbf{V} = \dot{\mathbf{P}}(t)$. For example, the gravitational motion of the target can be expressed as

$$\begin{aligned} \mathbf{P}(t) &= \mathbf{T}_0 + \mathbf{V}_{T0}t + 0.5\mathbf{g}t^2 \\ \dot{\mathbf{P}}(t) &= \mathbf{V}_{T0} + \mathbf{g}t \end{aligned} \tag{2}$$

where $\mathbf{T}_0$ and $\mathbf{V}_{T0}$ are the initial position and velocity vectors obtained from radar, and $\mathbf{g}$ is the gravitational acceleration vector. Because the target moves along the trajectory, the time-to-go and PIP can be estimated from the trajectory.

In this study, design objective can be summarized as follows,

- Generate appropriate trajectory or guidance command for the missile to reach and move along the target path.
- For head-on interception, the missile should reach the trajectory ahead of the PIP, i.e., $t_s \leq t_{go}$ where t_s is a time incident at reaching point, and t_{go} is the time to intercept.

B. Guidance Law

Nonlinear differential geometry based path-following guidance law (NDGPFG) [8] considered in this study can be represented as

$$\mathbf{a}_c = k\left(\mathbf{V}_m \times \hat{\mathbf{L}}\right) \times \mathbf{V}_m \tag{3}$$

where $\hat{\mathbf{L}}$ indicates the look-ahead vector, and $\mathbf{V}_m = [V_m \cos\gamma_m, V_m \sin\gamma_m]^T$ is the velocity vector of the missile. Performance of the guidance law may differ depending on the choice of gain k and look-ahead vector $\hat{\mathbf{L}}$. For example, the look-ahead vector can be represented as

$$\hat{\mathbf{L}} = \cos\theta_L \hat{\mathbf{d}} + \sin\theta_L \hat{\mathbf{T}}_p \tag{4}$$

where $\mathbf{d} = \mathbf{P}_c - \mathbf{M} + d_{\text{shift}}\hat{\boldsymbol{\kappa}}_p$ is the shifted error vector, $\mathbf{P}_c$ is the closest point on the path, $\mathbf{M}$ is the position vector of the missile, $\hat{\mathbf{T}}_p$ and $\hat{\boldsymbol{\kappa}}_p$ is the unit tangent and unit curvature vectors at the closest point, and $\hat{(\cdot)}$ notation is defined to indicate the unit vector in the direction of a given vector. For exact tracking of a specified path, the look-ahead angle θ_L and the shift distance d_{shift} can be designed as

$$\begin{aligned} \theta_L(\|\mathbf{d}\|) &= \cos^{-1}\left((1-\epsilon)\text{sat}\left(\frac{\|\mathbf{d}\|}{\delta_{BL}}\right)\right) \\ d_{\text{shift}} &= \frac{\kappa_p}{k}\frac{\delta_{BL}}{1-\epsilon} \end{aligned} \tag{5}$$

Using Eq. (5), transient performance characteristics can be analysed. By approximating the desired path around the closest point of each instance as the osculating circle with small cross-track error $e_N = ||\mathbf{P}_c - \mathbf{M}||$, the tracking error dynamics near the path can be linearized as follows,

$$\ddot{e}_N + 2\zeta\omega_N\dot{e}_N + \omega_N^2 e_N = 0 \tag{6}$$

where the natural frequency and damping ratio can be expressed as

$$\begin{aligned} \omega_N &= V_m\sqrt{\frac{k(1-\epsilon)}{\delta_{BL}} + \kappa_p^2} \\ \zeta &= \sqrt{\frac{k^2 - \kappa_p^2}{\frac{k(1-\epsilon)}{\delta_{BL}} + {\kappa_p}^2}} \end{aligned} \tag{7}$$

Note 1. *Despite the good tracking performance, the guidance law itself does not impose a time constraint. To achieve the "On track" condition within a specified time $t_s < t_{PIP}$, the following condition should be satisfied*

$$t_s = \int_{l(\mathbf{M}_0)}^{l(\mathbf{P}_s)} \frac{dL(l)}{V_m} < \int_{l(\mathbf{M}_0)}^{l(\mathbf{P}_{PIP})} \frac{dL(l)}{V_m} \tag{8}$$

where $\mathbf{P}_s$ is the reaching point at the specified t_s, $\int dL(l)$ is the length of the missile path which can be determined by a guidance law, and $l(\mathbf{P})$ denotes the curvilinear coordinate of a point $\mathbf{P}$ on the curve parametrized in l. Right hand side of Eq. (8) *is equal to time-to-intercept, i.e., $\int_{l(\mathbf{M}_0)}^{l(\mathbf{P}_{PIP})} \frac{dL(l)}{V_m} = t_{P_{PIP}}$, and therefore Eq.* (8) *addresses the time condition $t_s < t_{PIP}$.*

III. Guidance Laws Based on Trajectory Tracking

In this study, two approaches are proposed to accomplish the design objective. The method presented in Sec. III-A is based on the modification of the look-ahead vector in the guidance law to make the missile move toward a certain stationary point before it transits smoothly to perform pure path-following. In Sec. III-B, another method based on specifying the reference path for the missile to follow is presented.

A. Path-Following of Target Trajectory Considering Approach Time

The concept of the proposed guidance law is illustrated in Fig. 2. When a missile is far from a target trajectory, look-ahead vector aims at the approaching point $\mathbf{P}$. As the missile gets close to the approach point, it comes inside the boundary layer region specified by $||\mathbf{e}|| < \delta_{BL}$, and exact tracking of the path can be performed using the guidance law of Eq. (3).

1) Modification of the look-ahead vector: In this section, the look-ahead vector is modified to reflect the time constraint. Let us define a cross track error as a relative position from a closest point $\mathbf{P}_c$ to $\mathbf{M}$ as

$$\mathbf{e} = \mathbf{P}_c - \mathbf{M} \tag{9}$$

Suppose that the missile is away from the target trajectory such that $||\mathbf{e}|| > \delta_{BL}$. A possible choice is to pursue a point $\mathbf{P}$ onto

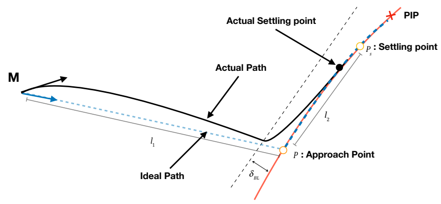


Fig. 2. Modification of the Look-ahead Angle

the trajectory until it gets close to the path. The look-ahead vector can be expressed as

$$\hat{\mathbf{L}}_{\text{outer}} = \frac{\mathbf{P} - \mathbf{M}(t)}{||\mathbf{P} - \mathbf{M}(t)||} \tag{10}$$

where $\mathbf{P}$ is The approach point to be determined, and $\mathbf{M}(t)$ is the position vector of the missile at time t.

Now, let us choose an approach point $\mathbf{P}$ satisfying the approaching condition. Assuming that the trajectory of the target is given with predicted intercept time, a settling point $\mathbf{P}_s$ can be selected as a point on the predicted trajectory of the target preceded by the predicted intercept point $\mathbf{P}_{PIP}$. Given a fixed $\mathbf{P}_s$, a candidate $\mathbf{P}$ is the point on the path such that the arc-length between the points $\mathbf{P}$ and $\mathbf{P}_s$ satisfies the following inequality.

$$l_2 = \int_{l(\mathbf{P}_s)}^{l(\mathbf{P})} dL(l) \geq V_m T_s \tag{11}$$

where $L = \int dL(l)$ is the arc-length of the target's path, and $l(\mathbf{P})$ denotes a parameter corresponding to a point $\mathbf{P}$, T_s is the settling time that will be determined later. For a given reference path and boundary points $\mathbf{M}$ and $\mathbf{P}_s$, it is required to find an optimal $\mathbf{P}^*$ satisfying the inequality constraint of Eq. (11) as

$$\mathbf{P}^* = \arg\min \left[||\mathbf{P} - \mathbf{M}|| + \int_{l(\mathbf{P}_s)}^{l(\mathbf{P})} dL \right] \tag{12}$$

If the desired path is parametrized, the minimization problem Eq. (12) can be expressed as

$$\begin{aligned} &\text{minimize } J(l) = ||\mathbf{P}(l) - \mathbf{M}|| + \int_{l_s}^{l} ||\mathbf{P}'(l)||dl \\ &\text{subject to } l \geq l_{\text{min}} \end{aligned} \tag{13}$$

where $l_{\text{min}} > l_s$ satisfies

$$\int_{l_s}^{l_{\text{min}}} ||\mathbf{P}'(l)||dl = V_m T_s \tag{14}$$

If the desired path is convex (Parabola, circular path), the following condition is satisfied.

Lemma 1. *If the parametrized path is given, the optimal point* $\mathbf{P}^*$ *is determined by* $\mathbf{P}(l_{min})$.

Proof. At any point l satisfying $l \geq l_{\text{min}} \geq l_s$, J can be expressed as

$$\begin{aligned} J(l) &= ||\mathbf{P}(l) - \mathbf{M}|| + \int_{l_s}^{l} ||\mathbf{P}'(l)||dl \\ &= ||\mathbf{P}(l) - \mathbf{M}|| \\ &+ \int_{l_{min}}^{l} ||\mathbf{P}'(l)||dl + \int_{l_s}^{l_{min}} ||\mathbf{P}'(l)||dl \end{aligned} \tag{15}$$

For a set of end points $[l_1, l_2]$, the arc-length is always longer than the straight-line distance, i.e.,

$$||\mathbf{P}(l_2) - \mathbf{P}(l_1)|| \leq \int_{l_1}^{l_2} ||\mathbf{P}'(l)||dl \tag{16}$$

By the triangular inequality, we have

$$\begin{aligned} ||\mathbf{P}(l_1) - \mathbf{M}|| &\leq ||\mathbf{P}(l_2) - \mathbf{P}(l_1)|| + ||\mathbf{P}(l_2) - \mathbf{M}|| \\ &\leq \int_{l_1}^{l_2} ||\mathbf{P}'(l)||dl + ||\mathbf{P}(l_2) - \mathbf{M}|| \end{aligned} \tag{17}$$

Substituting Eqs. (16) and (17) into Eq. (15), $J(l) \leq J(l_{min})$. Therefore, J is minimal at $l = l_{min}$, and minimum-length path is always determined as $\mathbf{P}^*(l) = \mathbf{P}(l_{min})$. □

When the optimal $\mathbf{P}^*$ is obtained for given $\mathbf{P}_s$, the missile should reach the path $\mathbf{P}_s$ ahead of the target. Note that the missile has a constant speed during the engagement, the total flight time should satisfy the following condition.

$$\frac{J^*}{V_m} \leq t_{go} - \epsilon_1 \tag{18}$$

where ϵ_1 is positive for transition time in actual trajectory. Considering that the desired path can be approximated as an osculating circle for small $||\mathbf{e}||$, the tracking motion near the path behaves like a second-order system expressed as Eq. (6). The settling time of the convergence to the target trajectory can be obtained as

$$T_s = \frac{c}{\zeta\omega_N} = \frac{c}{V_m\sqrt{\frac{k(1-\epsilon)}{\delta_{BL}} + \kappa_p^2}\sqrt{\frac{k^2-\kappa_p^2}{\frac{k(1-\epsilon)}{\delta_{BL}} + {\kappa_p}^2}}} \tag{19}$$

where $c = 3.9$ for 2% settling time. If the condition holds for $\mathbf{P}^*$, the approach point can be determined. Otherwise, the approach point $\mathbf{P}$ is corrected, and the procedure is repeated by changing the settling point $\mathbf{P}_s$ until the condition satisfies. Algorithm 1 summarizes the procedure of determination of the approach point $\mathbf{P}$.

When the cross track error is small such that $||\mathbf{e}|| \leq \delta_{BL}$, the look-ahead vector is chosen as Eqs. (4) and (5). For smooth transition, the look-ahead vector is considered as

$$\hat{\mathbf{L}} = \alpha(||\mathbf{e}||)\hat{\mathbf{L}}_{\text{outer}} + (1 - \alpha(||\mathbf{e}||))\hat{\mathbf{L}}_{\text{inner}} \tag{20}$$

2) Guaranteed Reaching Time Before Interecption: In this section, reaching time to the target path before interception is analyzed. As shown in Fig. 2, the actual path is slightly different from the ideal path, and the settling point can be dif-

Algorithm 1 Determination of Approach Point $\mathbf{P}$ (Parameterized Curve)

procedure DETERMINATION OF $\mathbf{P}$
  Set desired t_{PIP} and $\mathbf{P}_{PIP} = \mathbf{P}(t_{PIP})$ from the data of the target trajectory.
  Set settling parameter $s_s = t_{PIP} + \epsilon_1$
  while (1) **do**
    Obtain s_1 such that

$$\int_{s_1}^{s_s} ||\mathbf{P}'(s)||ds = V_m T_s$$

    Obtain $\mathbf{P}^* = \mathbf{P}(s_1)$.
    Calculate J such that

$$J = ||\mathbf{P}(s_1) - \mathbf{M}|| + \int_{s_1}^{s_s} ||\mathbf{P}'(s)||ds$$

    if $\frac{l^*}{V_m} \leq t_{PIP} - \epsilon$ **then**
      Return $\mathbf{P}^*$
    end if
    $t_{PIP} = t_{PIP} + \Delta t_{PIP}$
  end while
end procedure

ferent. To guarantee the reaching time, the actual reaching time should be always smaller than time to intercept $t_{r,actual} \leq t_{go}$.

Theorem 1. *Suppose that the transient time due to initial heading error and cross-track error is bounded by $t_t \leq \epsilon_1$, the reaching time is always smaller than time to intercept, that is*

$$t_{r,actual} \leq t_{go} \tag{21}$$

Proof. In ideal case, the reaching time to the target trajectory can be obtained as

$$t_r = \frac{J^*}{V_m} = \frac{||\mathbf{M}_0 - \mathbf{P}^*||}{V_m} + T_s \tag{22}$$

Using the bounded transient time $t_t \leq \epsilon$, the actual reaching time is bounded by

$$t_{r,actual} \leq \frac{||\mathbf{M}_0 - \mathbf{P}^*||}{V_m} + T_s + \epsilon_1 \tag{23}$$

Since the $\mathbf{P}^*$ is selected satisfying $\frac{J^*}{V_m} \leq t_{go} - \epsilon_1$, the reaching time is always smaller than the time to intercept. □

Remark 1. *Unlike existing NDGPFG(Nonlinear Differential Geometric Path-Following Guidance) method, the proposed guidance law can achieve required reaching time before interception as well as the "On track" condition.*

B. Trajectory Generation Considering Approach Time

In this section, a path-following problem is addressed by generating a trajectory for the missile. In this approach, as shown in Fig. 3, trajectory for the missile is generated to approach the target path by considering various constraints, and the missile follows the designed path instead of the target path. After approaching the target path, the missile follows the target path and moves along the path.

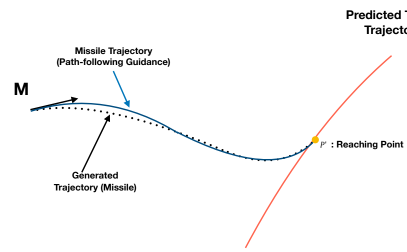


Fig. 3. Concept of the trajectory generation

1) Trajectory Generation: There have been lots of methods for trajectory generation which include trajectory optimization, parametric curve, etc. In this study, missile path is generated using polynomials, which can be represented as

$$\mathbf{p}(l) = \sum_{i=0}^{N} \mathbf{a}_i l^i \tag{24}$$

where $l \in [0, 1]$ is the normalized parameter, and $\mathbf{a}_i$ is the polynomial coefficient vector to be determined. For head-on interception, following boundary condition can be considered.
Initial Condition: $\mathbf{p}(0) = \mathbf{M}_0$, (Optional: $\mathbf{p}'(0) \propto \hat{\mathbf{V}}_m(0)$)
Terminal Condition: $\mathbf{p}(1) = \mathbf{P}_f$, (Optional: $\mathbf{p}'(1) \propto -\hat{\mathbf{V}}_T(\mathbf{P}_f)$)
where $\mathbf{P}_f$ is the reaching point, which is not specified but lies in the target trajectory. Therefore, the problem can be addressed as

$$t_s = \int_{M_0}^{P'} \frac{dL}{V_m} < t_{go} = t(\mathbf{P}_{PIP}) - t(\mathbf{P}_0) \tag{25}$$

Note 2. *Depending on the degree of polynomial, the generated trajectory has different properties. For example, cubic spline curve can achieve four boundary conditions (two initial condition, and two terminal condition).*

Note 3. *Higher degree curves are computationally expensive and may suffer from Gibbs phenomenon. Only the arc-length of a general polynomial curve can be calculated numerically.*

To generate the path of the missile, cubic polynomials are considered in this study, which can be expressed as

$$\begin{aligned} \mathbf{p}(l) &= \mathbf{a}_0 + \mathbf{a}_1 l + \mathbf{a}_2 l^2 + \mathbf{a}_3 l^3 \\ \mathbf{p}'(l) &= \mathbf{a}_1 + 2\mathbf{a}_2 l + 3\mathbf{a}_3 l^2 \end{aligned} \tag{26}$$

To impose boundary condition, set the terminal point lying on the target path is fixed. Then, we have

$$\begin{aligned} \mathbf{a}_0 &= \mathbf{M}_0, \mathbf{a}_1 = K_1 \hat{\mathbf{V}}_m(0) \\ \mathbf{p}(1) &= \mathbf{M}_0 + K_1 \hat{\mathbf{V}}_0 + \mathbf{a}_2 + \mathbf{a}_3 = \mathbf{P}_f \\ \mathbf{p}'(1) &= K_1 \hat{\mathbf{V}}_0 + 2\mathbf{a}_2 + 3\mathbf{a}_3 = K_2 \hat{\mathbf{V}}_f \end{aligned} \tag{27}$$

where $K_1 = ||\frac{d\mathbf{p}}{dl}||_{l=0}$ and $K_2 = ||\frac{d\mathbf{p}}{dl}||_{l=1}$ are scaling factors to be determined. Regarding a constant transition rate at the boundary points, i.e., $||\frac{d\mathbf{p}}{dl}|| = ||\mathbf{P}_f - \mathbf{P}_0|| = K$, the coefficients can be obtained as

$$\begin{bmatrix}\mathbf{a}_2\\ \mathbf{a}_3\end{bmatrix} = \begin{bmatrix} I_2 & I_2 \\ 2I_2 & 3I_3\end{bmatrix}^{-1} \begin{bmatrix}(\mathbf{P}_f - \mathbf{M}_0) - K\hat{\mathbf{V}}_0 \\ K(\hat{\mathbf{V}}_f - \hat{\mathbf{V}}_0)\end{bmatrix} \tag{28}$$

2) Correction of reaching point: The generated path should be appropriately corrected by considering the transverse time in order to achieve a head-on condition. Using the curve Eq. (26), the corresponding arc length can be obtained as

$$\begin{aligned} L &= \int dL = \int_0^1 \sqrt{\mathbf{p}'^T\mathbf{p}'}dl \\ &= \int_0^1 \sqrt{b_0 + b_1 l + b_2 l^2 + b_3 l^3 + b_4 l^4}dl \end{aligned} \tag{29}$$

where the corresponding coefficients are

$$\begin{aligned} b_0 &= \mathbf{a}_1^T\mathbf{a}_1 \\ b_1 &= 4\mathbf{a}_1^T\mathbf{a}_2 \\ b_2 &= 6\mathbf{a}_1^T\mathbf{a}_3 + 4\mathbf{a}_2^T\mathbf{a}_2 \\ b_3 &= 12\mathbf{a}_2^T\mathbf{a}_3 \\ b_4 &= 9\mathbf{a}_3^T\mathbf{a}_3 \end{aligned} \tag{30}$$

The proposed method is summarized in Algorithm 2. Obtained from Eq. (29), the path can be determined if $\frac{L}{V_m} < t_{go}$. Otherwise, the reaching point $\mathbf{P}_f$ is corrected until the condition is satisfied.

Algorithm 2 Path Generation

procedure PATH GENERATION
  Calculate t_{go} from the data of the target trajectory, and set corresponding $\mathbf{P}_f(t_{go})$
  while (1) **do**
    Find polynomial coefficients $\mathbf{a}_i$ for $i = 0, 1, 2, 3$
    Calculate L as
    $$L = \int_0^1 \sqrt{b_0 + b_1 l + b_2 l^2 + b_3 l^3 + b_4 l^4}dl$$
    if $\frac{L}{V_m} \leq t_{go}$ **then**
      Return $\mathbf{p}(l) = \sum_{i=0}^N \mathbf{a}_i l^i$
    end if
    $t_{go} = t_{go} + \Delta t_{go}$
    Choose $\mathbf{P}_f(t_{go})$
  end while
end procedure

IV. Numerical Simulation

To demonstrate the proposed methods, numerical simulation is performed. For simulation scenario, ballistic target interception is considered, in which the engagement occurs at hight altitude ($\mathbf{M}(0) = [40, 100]^T$km, $\mathbf{T}_0 = [120, 120]^T$km), and a missile is required to intercept a fast ballistic target $V_m(0) = 1,000$m/s $< V_t(0) = 1,500$m/s. For the motion of the target, gravitational motion is considered. The predicted position of the target can be expressed as

$$\mathbf{P}(T_{go}) = \mathbf{P}(0) - V_T(0)\mathbf{e_1}T_{go} + \mathbf{g}T_{go}^2 \tag{31}$$

where $\mathbf{P}(0) = [120, 120]^T$km, $\mathbf{e_1} = [1, 0]^T$, and $\mathbf{g} = [0, -9.81]^T$m/s2. Other parameters are summarized in Table IV.

TABLE I
INITIAL CONDITION AND SIMULATION PARAMETERS

Description	Symbol	Value
Initial Flight-path Angle	$\gamma_m(0)$	0
Guidance Feedback Gain	k	0.01
	ϵ	10^{-3}
Settling Time Parameter	T_s	10sec
Approximated Transient Bound	ϵ_1	3sec

A. Path-Following of Target Trajectory Considering Approach Time

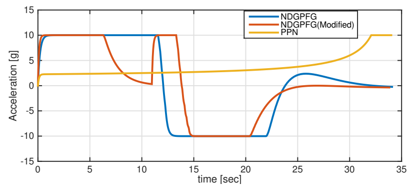
Figures 4 and 5 show the simulation using NDGPFG with look-ahead modification. Unlike PNG, the path-following guidance laws render the missile to converge to the target path using more acceleration in the early phase of the interception. Consequently, the missiles can regulate the cross track errors fast, and stable acceleration command is generated near the homing phase. Compared to the original NDGPFG, the proposed method improves transient characteristics by the tendency of pursuing a designed approaching point.

B. Performance of Trajectory Generation Considering Approach Time

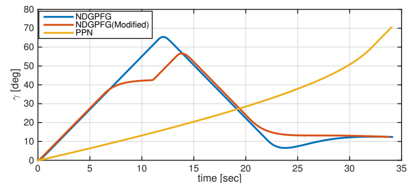
The proposed trajectory generation approach is compared with Pure PNG(PPN). Figures 6 and 7 show the simulation results. In the simulation result, reaching point $\mathbf{P}_f$ is obtained ahead of the actual PIP, $\mathbf{P}_{PIP}$. Followed by the path, the missile reaches the reaching point at $t = 25$s before the interception $t = 33$s and succeed in head-on interception. Owing to the generated path, the guidance law uses small guidance command than the existing method while achieving the interception.

V. Conclusion

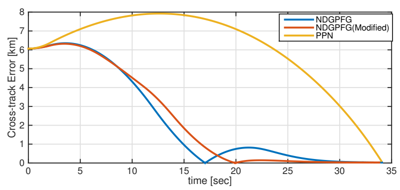
Missile guidance methods were proposed based on tracking of predicted target trajectory. Two different methods are proposed in that the approach of missile to the target path can be achieved either by modifying a guidance law considering approach point or by generating a proper path of the missile. Both methods regulated cross track error from the target trajectory fast while approaching to the target path prior to target interception.



(a) Acceleration

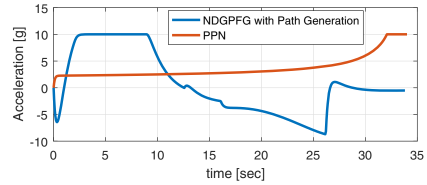


(b) Flight Path Angle

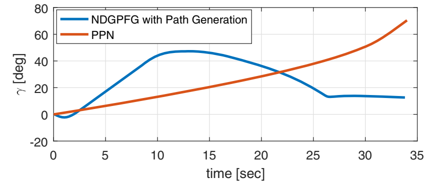


(c) Cross-track Error

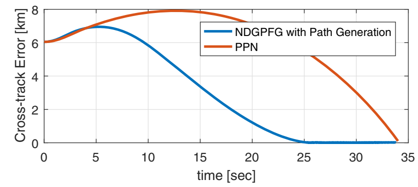
Fig. 4. Simulation Result for Path-Following Considering Approach Time



(a) Acceleration



(b) Flight Path Angle



(c) Cross-track Error

Fig. 6. Simulation Result for Path Generation Considering Approach Time

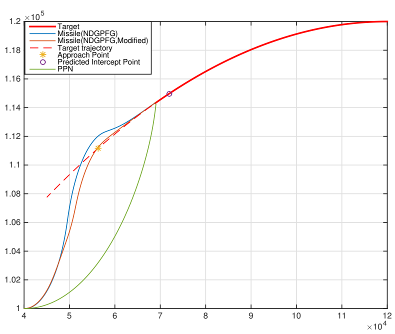


Fig. 5. Trajectory of Simulation Result for Path-Following Considering Approach Time

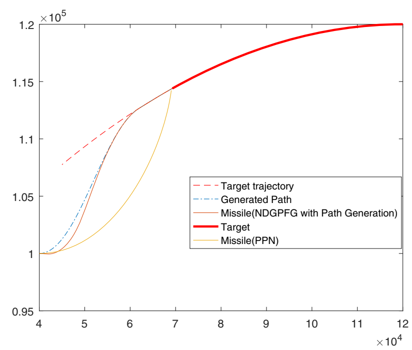


Fig. 7. Trajectory of Simulation Result for Path Generation Considering Approach Time

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