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Parametrization of All Retrofit Controllers toward Open Control-Systems

Masaki Inoue[1], Takayuki Ishizaki[2], and Mizuki Suzumura[1]

***Abstract*— This paper is devoted to the foundation of decentralized design and implementation of multiple controllers. Motivated from the application to huge-scale systems, a localized-control problem, called retrofit control, is formulated: A local controller is designed for a local part of the huge-scale system such that the entire controlled system is stable independently of any change in the other remaining part. Then, the problem is solved, and all retrofit controllers are parametrized. Finally, design guidelines of the retrofit controller are illustrated.**

I. INTRODUCTION

The decentralized control for huge-scale systems has been studied in over past four decades, see e.g., [1], [2], [3], [4], [5], [6], [7], [8] and references therein. Even in recent years, with a growing interest in network systems, the decentralized or distributed control problem is still attractive, see e.g., [9], [10], [11], [12]. Most of the works focus on the controller structure such that its implementation can be decentralized. Although the convexity of the design problem is addressed [5], [7], [9], [10], [11], [12], less attention is paid to the decentralization of the controller design itself. In [13], the importance of the *decentralized design* (or said to be *distributed design*) is emphasized. This paper is devoted to the foundation of the *decentralized design and implementation* of the controllers.

In Fig. 1, a control problem of a huge-scale complex system is illustrated. To solve the problem, multiple controllers K_1, K_2, $\ldots$ are designed and implemented by participants $1, 2, \ldots$, respectively. The system P represents the plant to be controlled by the participant i and is composed of a baseline system and all other controllers K_j except K_i. The controller K_i is designed and implemented by the participant i with utilizing only the local model L_i independently of others K_j. In this sense, the design and implementation are decentralized. The decentralization can realize the *open control-system*: Anonymous participants design their own controller in order to achieve their own control objective. Then, each controller is connected to or disconnected from a baseline system independently of others.

There remains difficulty in the open control-system: If no rule or regulation is imposed on the participants, the entire

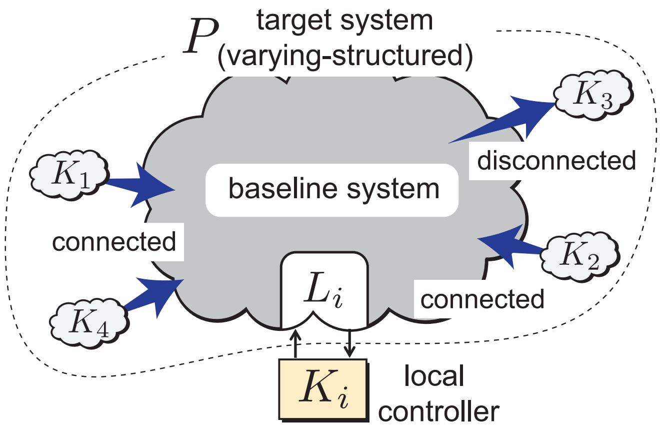

Fig. 1. Open control-system. Local-controller design from the view of a participant i is illustrated.

controlled system (P, K_i) can be easily destabilized. This is because that connecting or disconnecting a controller K_j is modeled as a structural variation in the plant P. Large variations in some feedback loop can destabilize the entire system. The rule and regulation must be imposed on the controller design and implementation such that the entire controlled system (P, K_i) is stable.

In this paper, a control problem, called *retrofit control*, is formulated with inheriting the concept proposed in [14], [15]: A local controller is designed for a local part of the huge-scale system such that the entire controlled system is stable independently of any change in the other remaining part. The problem is solved with the Youla parametrization (see e.g., [16]) applied to not only the controller but also the plant system. Then, the parametrization of all retrofit controllers is given with the Youla parameter of the controller that satisfies a particular equality constraint. Finally, the retrofit control problem is specialized, and the design guidelines of the controller are proposed.

Notation: Let

$$\bar{M} := \left[\begin{array}{c|c} \bar{M}_{11} & \bar{M}_{12} \\ \hline \bar{M}_{21} & \bar{M}_{22} \end{array}\right]$$

and $\bar{N}$ be transfer matrices. Then, the linear fractional transformations (LFTs) are defined as

$$\mathcal{F}_{\mathrm{l}}(\bar{M}, \bar{N}) := \bar{M}_{11} + \bar{M}_{12}\bar{N}(I - \bar{M}_{22}\bar{N})^{-1}\bar{M}_{21},$$
$$\mathcal{F}_{\mathrm{u}}(\bar{M}, \bar{N}) := \bar{M}_{22} + \bar{M}_{21}\bar{N}(I - \bar{M}_{11}\bar{N})^{-1}\bar{M}_{12},$$

which are called the lower and upper LFTs, respectively (see e.g., [17]).

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[1]Department of Applied Physics and Physico-Informatic, Faculty of Science and Technology, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama, Kanagawa, Japan. minoue@appi.keio.ac.jp, mantikann@z6.keio.jp

[2]Department of Systems and Control Engineering, Graduate School of Engineering, Tokyo Institute of Technology, 2-12-1-W8-1 Oh-Okayama, Meguro-ku, Tokyo, Japan. ishizaki@mei.titech.ac.jp

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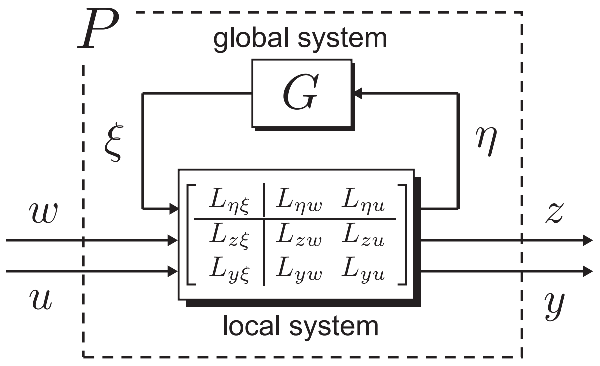

Fig. 2. Feedback system P is composed of the local system L and global system G.

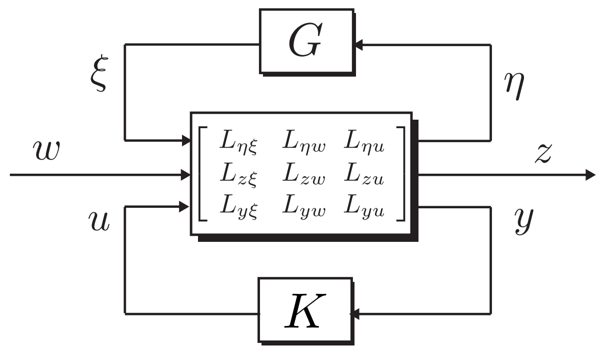

Fig. 3. Entire controlled system Σ_{all}.

II. PROBLEM FORMULATION

A. System Description

We consider two dynamical systems L and G which constitute a feedback system P as illustrated in Fig. 2. The system L is described by the following generalized plant

$$L: \begin{bmatrix} \eta \\ \hline z \\ y \end{bmatrix} = \bar{L} \begin{bmatrix} \xi \\ \hline w \\ u \end{bmatrix},$$

where $\eta \in \mathbb{R}^r$ and $\xi \in \mathbb{R}^s$ are the interaction signals, $z \in \mathbb{R}^p$ is the control output, $w \in \mathbb{R}^q$ is the disturbance input, $y \in \mathbb{R}^\ell$ is the measured output, $u \in \mathbb{R}^m$ is the control input, and $\bar{L}$ is the transfer matrix:

$$\bar{L} := \left[\begin{array}{c|cc} \bar{L}_{\eta\xi} & \bar{L}_{\eta w} & \bar{L}_{\eta u} \\ \hline \bar{L}_{z\xi} & \bar{L}_{zw} & \bar{L}_{zu} \\ \bar{L}_{y\xi} & \bar{L}_{yw} & \bar{L}_{yu} \end{array}\right].$$

The system L represents a *local* part in P and is a target system for a control problem. The system G represents the rest part of P and is described by

$$G: \ \xi = \bar{G}\eta,$$

where $\bar{G}$ is the transfer matrix of G. The system G is called the *global* system. Then, the plant system P is defined by the feedback connection of L and G:

$$P: \ \begin{bmatrix} z \\ y \end{bmatrix} = \mathcal{F}_{\mathrm{u}}(\bar{L}, \bar{G}) \begin{bmatrix} w \\ u \end{bmatrix}.$$

An assumption is imposed on G such that

$$\bar{G} \in \mathcal{S}_{\mathrm{g}} := \{\bar{G} \,|\, \mathcal{F}_{\mathrm{u}}(\bar{L}, \bar{G}) \in \mathcal{RH}_\infty\} \tag{1}$$

holds[1]. It should be emphasized here that we do not impose any limitation on the magnitude or phase of G, which is typically considered in standard robust control problems in e.g., [17]. Any structure and dimension can be considered in G as long as P is stable.

[1]For simplicity, it is implicitly assumed that any feedback system studied in this paper is well-posed.

B. Problem Formulation: Retrofit Control Problem

Let K be the controller described by

$$K: \ u = \bar{K}y,$$

where $\bar{K}$ is the transfer matrix of K. Then, the entire controlled system Σ_{all} is described by

$$\Sigma_{\mathrm{all}}: \ z = \bar{\Sigma}_{\mathrm{all}} w,$$

where $\bar{\Sigma}_{\mathrm{all}}$ is the transfer matrix:

$$\bar{\Sigma}_{\mathrm{all}} := \mathcal{F}_{\mathrm{l}}(\mathcal{F}_{\mathrm{u}}(\bar{L}, \bar{G}), \bar{K}). \tag{2}$$

The entire controlled system Σ_{all} is illustrated in Fig. 3.

The aim of controller design and implementation is briefly mentioned here. The final goal of this study is to realize *open control-systems*: Anonymous participants design and implement their own controller to a baseline system independently each other. From the view of one participant, the target system to be controlled is uncertain and has variable structure, which depends on the controllers implemented by other participants. Controller design and implementation without any specific rule and regulation can easily destabilize the entire controlled system. The rule and regulation to guarantee the stability are studied in this paper by addressing the following control problem.

Problem 1: Retrofit control problem: Find K such that $\bar{\Sigma}_{\mathrm{all}} \in \mathcal{RH}_\infty$ holds for all $\bar{G} \in \mathcal{S}_{\mathrm{g}}$.

Notation 1: The set of all solutions to Problem 1, i.e., all *retrofit controllers*, is denoted by $\mathcal{S}_{\mathrm{RC}}$.

The retrofit or related control problems have been addressed in e.g., [18], [14], [15]. In [18], the problem is partially solved under the additional stability assumption on G with the *dual* Youla parametrization [19] applied to the plant system. In [14], [15], motivated from the application to the *open control-system*, the concept of the retrofit control is proposed. Then, the papers [14], [15] propose a systematic design procedure of a class of retrofit controllers that satisfies a specification of available measurement signals.

In this paper, we refine and mathematically formulate the retrofit control problem with inheriting the essence in the original concept [14], [15]. Then, we give the general solution to the problem without the specification of available measurement signals. The refined problem formulation and the general solution are the main contributions of this paper.

III. PARAMETRIZATION OF ALL RETROFIT CONTROLLERS

A. Preliminaries for Solution

To simplify Problem 1 and extract its essence, a technical assumption is imposed on the local system L.

Assumption 1: $\bar{L} \in \mathcal{RH}_\infty$ holds.

Under the assumption, we have the following two propositions: One shows a parametrization of the global system $\bar{G} \in \mathcal{S}_{\rm g}$, and the other provides candidates of the solution to Problem 1.

Proposition 1: Under Assumption 1, $\mathcal{S}_{\rm g}$ is expressed as

$$\mathcal{S}_{\rm g} = \{\, \bar{Q}_{\rm g}(I + \bar{L}_{\eta\xi}\bar{Q}_{\rm g})^{-1} \,|\, \bar{Q}_{\rm g} \in \mathcal{RH}_\infty \,\}. \tag{3}$$

In addition, the transfer matrix of P is given by

$$\mathcal{F}_{\rm u}(\bar{L}, \bar{G}) = \mathcal{F}_{\rm u}\left(\left[\begin{array}{c|cc} 0 & \bar{L}_{\eta w} & \bar{L}_{\eta u} \\ \hline \bar{L}_{z\xi} & \bar{L}_{zw} & \bar{L}_{zu} \\ \bar{L}_{y\xi} & \bar{L}_{yw} & \bar{L}_{yu} \end{array}\right], \bar{Q}_{\rm g}\right). \tag{4}$$

Proof: The expression

$$\bar{G} = \bar{Q}_{\rm g}(I + \bar{L}_{\eta\xi}\bar{Q}_{\rm g})^{-1} \tag{5}$$

is straightforwardly derived by the Youla parametrization (see e.g., [16]) of $\bar{G}$. Noting that (5) is equivalent to

$$\bar{Q}_{\rm g} = \bar{G}(I - \bar{L}_{\eta\xi}\bar{G})^{-1},$$

we have (4). ■

Proposition 2: Let

$$\mathcal{S}_{\rm k} := \{\, \bar{Q}_{\rm k}(I + \bar{L}_{yu}\bar{Q}_{\rm k})^{-1} \,|\, \bar{Q}_{\rm k} \in \mathcal{RH}_\infty \,\}.$$

Then, under Assumption 1, $\mathcal{S}_{\rm RC} \subseteq \mathcal{S}_{\rm k}$ holds.

Proof: From Assumption 1, we see that $0 \in \mathcal{S}_{\rm g}$ holds, i.e., $\bar{G} = 0$ is an element of $\mathcal{S}_{\rm g}$. Note here that $\mathcal{F}_{\rm l}(\mathcal{F}_{\rm u}(\bar{L}, 0), \bar{K})$ represents the transfer matrix of the entire controlled system $\Sigma_{\rm all}$ when $\bar{G} = 0$. It follows that for any $\bar{K} \in \mathcal{S}_{\rm RC}$,

$$\mathcal{F}_{\rm l}(\mathcal{F}_{\rm u}(\bar{L}, 0), \bar{K}) \in \mathcal{RH}_\infty \tag{6}$$

must hold. Then,

$$\mathcal{F}_{\rm l}(\mathcal{F}_{\rm u}(\bar{L}, 0), \bar{K}) = \mathcal{F}_{\rm l}\left(\begin{bmatrix} \bar{L}_{zw} & \bar{L}_{zu} \\ \bar{L}_{yw} & \bar{L}_{yu} \end{bmatrix}, \bar{K}\right) \in \mathcal{RH}_\infty$$

holds, where the feedback loop composed of $\bar{L}_{yu}$ and $\bar{K}$ is included. Then, the Youla parametrization shows that the set of all $\bar{K}$ satisfying (6) is given by $\mathcal{S}_{\rm k}$. In other words, $\mathcal{S}_{\rm RC} \subseteq \mathcal{S}_{\rm k}$ holds. ■

Remark 1: The expression (3) or (5) is utilized for simplifying the following development of equations. Proposition 2 implies that we can find the solution to Problem 1 only in $\mathcal{S}_{\rm k}$ without loss of generality.

B. Main Result

Now, we find the general solution to Problem 1 without restricting the problem such as $\bar{G} = 0$. As implied in Propositions 1 and 2, any $\bar{G} \in \mathcal{S}_{\rm g}$ and $\bar{K} \in \mathcal{S}_{\rm RC}$ are parametrized with $\bar{Q}_{\rm g}$ and $\bar{Q}_{\rm k}$, respectively. Then, the transfer matrix of the entire controlled system $\Sigma_{\rm all}$ is expressed as follows:

$$\begin{aligned} \bar{\Sigma}_{\rm all} &= \mathcal{F}_{\rm l}(\mathcal{F}_{\rm u}(\bar{L}, \bar{G}), \bar{K}) \\ &= \mathcal{F}_{\rm l}\left(\mathcal{F}_{\rm u}\left(\left[\begin{array}{c|cc} 0 & \bar{L}_{\eta w} & \bar{L}_{\eta u} \\ \hline \bar{L}_{z\xi} & \bar{L}_{zw} & \bar{L}_{zu} \\ \bar{L}_{y\xi} & \bar{L}_{yw} & 0 \end{array}\right], \bar{Q}_{\rm g}\right), \bar{Q}_{\rm k}\right) \\ &= \mathcal{F}_{\rm u}\left(\mathcal{F}_{\rm l}\left(\left[\begin{array}{cc|c} 0 & \bar{L}_{\eta w} & \bar{L}_{\eta u} \\ \bar{L}_{z\xi} & \bar{L}_{zw} & \bar{L}_{zu} \\ \hline \bar{L}_{y\xi} & \bar{L}_{yw} & 0 \end{array}\right], \bar{Q}_{\rm k}\right), \bar{Q}_{\rm g}\right) \\ &= (\bar{L}_{zw} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{yw}) \\ &\quad + (\bar{L}_{z\xi} + \bar{L}_{zu}\bar{Q}_k\bar{L}_{y\xi})\bar{Q}_{\rm g}(I - \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{y\xi}\bar{Q}_{\rm g})^{-1} \\ &\quad \cdot (\bar{L}_{\eta w} + \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{yw}). \end{aligned} \tag{7}$$

This equation development is graphically illustrated in Fig. 4. Note that the original expression of $\Sigma_{\rm all}$, where the parametrization of $\bar{G}$ and $\bar{K}$ is not applied, is illustrated in Fig. 3. By applying the parametrization, Fig. 3 is equivalently transformed into Fig. 4(a). Further applying the loop transformation to Fig. 4(a), the blocks $L_{\eta\xi}$ and L_{yu} in Fig. 4(a) are canceled out, and their corresponding feedback loops are erased. The resulting expression of $\Sigma_{\rm all}$ is illustrated in Fig. 4(b).

The essence for deriving the solution to Problem 1 is briefly mentioned here. Note that there remains a feedback loop in Fig. 4(b), which is characterized with the loop transfer matrix $\bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{y\xi}\bar{Q}_{\rm g}$. The loop also appears as $(I - \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{y\xi}\bar{Q}_{\rm g})^{-1}$ in the expression (7). The essence to guarantee the stability of $\Sigma_{\rm all}$ is cutting down the loop to transform the feedback-structured system to a cascade- and parallel-structured one. We can show that the cut down is also necessity for the stability of $\Sigma_{\rm all}$ for all $\bar{Q}_{\rm g} \in \mathcal{RH}_\infty$. Then, $\Sigma_{\rm all}$ is stable for all $\bar{Q}_{\rm g} \in \mathcal{RH}_\infty$ if and only if the loop transfer matrix $\bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{y\xi}\bar{Q}_{\rm g}$ is zero for all $\bar{Q}_{\rm g} \in \mathcal{RH}_\infty$. This is further equivalent to that

$$\bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{y\xi} \equiv 0 \tag{8}$$

holds. We have the following theorem.

Theorem 1: Under Assumption 1, $\mathcal{S}_{\rm RC}$ is given by

$$\mathcal{S}_{\rm RC} = \left\{\bar{Q}_{\rm k}(I + \bar{L}_{yu}\bar{Q}_{\rm k})^{-1} \,|\, (8),\, \bar{Q}_{\rm k} \in \mathcal{RH}_\infty\right\}.$$

Proof: In the proof, we let

$$\mathcal{S}_{\rm Thm} := \left\{\bar{Q}_{\rm k}(I + \bar{L}_{yu}\bar{Q}_{\rm k})^{-1} \,|\, (8),\, \bar{Q}_{\rm k} \in \mathcal{RH}_\infty\right\}.$$

Then, the proof to show $\mathcal{S}_{\rm RC} = \mathcal{S}_{\rm Thm}$ is composed of two steps: showing i) $\mathcal{S}_{\rm Thm} \subseteq \mathcal{S}_{\rm RC}$ holds and ii) conversely $\mathcal{S}_{\rm Thm} \subseteq \mathcal{S}_{\rm RC}$ holds.

i) In this step, we suppose $\bar{K} \in \mathcal{S}_{\rm Thm}$ to conclude $\bar{K} \in \mathcal{S}_{\rm RC}$. The controller $\bar{K}$ is parametrized as

$$\bar{K} = \bar{Q}_{\rm k}(I + \bar{L}_{yu}\bar{Q}_{\rm k})^{-1}, \tag{9}$$

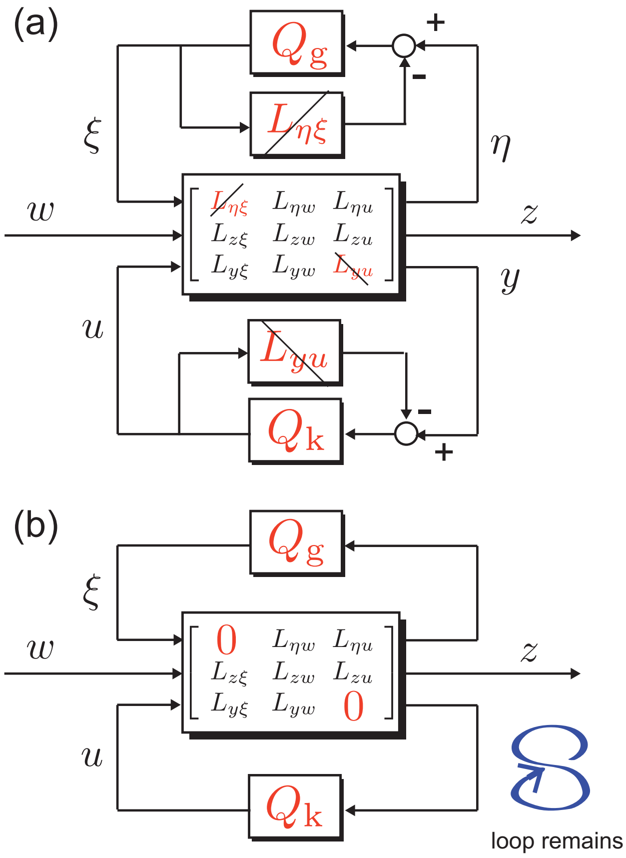

Fig. 4. Equivalent expression of entire controlled system $\Sigma_{\rm all}$. Through the parameterization of $\bar{G}$ and $\bar{K}$ as illustrated in Fig. (a), $\Sigma_{\rm all}$ is expressed by the block diagram illustrated in Fig. (b). There remains a feedback loop in Fig. (b), which is characterized with the loop transfer matrix $\bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{y\xi}\bar{Q}_{\rm g}$.

where (8) holds. Then, the expression (7) is further reduced to

$$\begin{aligned}\bar{\Sigma}_{\rm all} = &(\bar{L}_{zw} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{yw})\\ &+ (\bar{L}_{z\xi} + \bar{L}_{zu}\bar{Q}_{k}\bar{L}_{y\xi})\bar{Q}_{\rm g}(\bar{L}_{\eta w} + \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{yw}).\end{aligned} \tag{10}$$

From $\bar{L}$, $\bar{Q}_{\rm g}$, $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$, it follows that $\bar{\Sigma}_{\rm all} \in \mathcal{RH}_\infty$ holds. In other words, any controller $\bar{K} \in \mathcal{S}_{\rm Thm}$ is also in $\mathcal{S}_{\rm RC}$.

ii) Suppose that $\bar{K} \in \mathcal{S}_{\rm RC}$ holds. From Proposition 2, $\bar{K} \in \mathcal{S}_{\rm k}$ also holds, i.e., $\bar{K}$ is expressed as (9) without loss of generality. We further suppose that (8) does not hold to show a contradiction. Then,

$$\bar{L}_{\eta u}(j\omega_0)\bar{Q}_{\rm k}(j\omega_0)\bar{L}_{y\xi}(j\omega_0) \neq 0$$

holds at some frequency point $\omega_0 \in \mathbb{R} \cup \{\infty\}$. It follows that there exists $\bar{Q}_{\rm g} \in \mathcal{RH}_\infty$ such that

$$\det(I - \bar{L}_{\eta u}(j\omega_0)\bar{Q}_{\rm k}(j\omega_0)\bar{L}_{y\xi}(j\omega_0)\bar{Q}_{\rm g}(j\omega_0)) = 0$$

holds. This means that $\Sigma_{\rm all}$ is ill-posed or unstable. This contradicts $\bar{K} \in \mathcal{S}_{\rm RC}$, and therefore (8) must hold.

From i) and ii), we show that $\mathcal{S}_{\rm RC} = \mathcal{S}_{\rm Thm}$ holds. ■

Remark 2: Although it is not explicitly stated, Theorem 1 provides the *parametrization of all retrofit controllers*. Any retrofit controller is expressed with the parameter $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$ satisfying (8). Note that the parametrized controller (9) includes $\bar{L}_{yu}$, which is the *local* plant model. Therefore, the retrofit control can be called the *localized*-internal model control after the standard internal model control, where the controller includes the *entire* plant model (see e.g., [20], [21]). Furthermore, the parametrization itself can be called the *localized* version of the Youla parametrization [22], [23].

Remark 3: A perspective of the retrofit controller to the *open control-systems* is mentioned here. The constraint (8) is defined as a rule required for every controller-designer. Suppose that each local controller is designed by the choice of $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$ such that (8) holds. Then, the entire controlled system is stable independently of the number and design aim of the other connected controllers. This is because that connecting the retrofit controller does not generate any additional feedback loop, which may destabilize the entire controlled system.

IV. DESIGN PROBLEMS OF RETROFIT CONTROLLERS

A. *Special Retrofit Controller*

The design rule (8) is more deeply studied in this section. To this end, we suppose that

$$\bar{Q}_{\rm k}\bar{L}_{y\xi} \equiv 0, \tag{11}$$

or otherwise

$$\bar{L}_{\eta u}\bar{Q}_{\rm k} \equiv 0. \tag{12}$$

Then, letting

$$\begin{aligned}\mathcal{S}_{\rm Cor1} &:= \left\{ \bar{Q}_{\rm k}(I + \bar{L}_{yu}\bar{Q}_{\rm k})^{-1} \,|\, (11),\, \bar{Q}_{\rm k} \in \mathcal{RH}_\infty \right\},\\ \mathcal{S}_{\rm Cor2} &:= \left\{ \bar{Q}_{\rm k}(I + \bar{L}_{yu}\bar{Q}_{\rm k})^{-1} \,|\, (12),\, \bar{Q}_{\rm k} \in \mathcal{RH}_\infty \right\},\end{aligned}$$

we have

$$\mathcal{S}_{\rm Cor1} \subseteq \mathcal{S}_{\rm RC}, \quad \mathcal{S}_{\rm Cor2} \subseteq \mathcal{S}_{\rm RC}.$$

In other words, any $\bar{K} \in \mathcal{S}_{\rm Cor1} \cup \mathcal{S}_{\rm Cor2}$ is a (special) retrofit controller.

Now, recall that $\bar{\Sigma}_{\rm all}$ is expressed by (10) when a retrofit controller $\bar{K} \in \mathcal{S}_{\rm RC}$ is applied. We further suppose that $\bar{K} \in \mathcal{S}_{\rm Cor1} \cup \mathcal{S}_{\rm Cor2}$ holds. Then, the expression (10) is reduced to

$$\begin{aligned}\bar{\Sigma}_{\rm all} = &(\bar{L}_{zw} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{yw})\\ &+ \bar{L}_{z\xi}\bar{Q}_{\rm g}(\bar{L}_{\eta w} + \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{yw})\end{aligned} \tag{13}$$

for $\bar{K} \in \mathcal{S}_{\rm Cor1}$ or

$$\begin{aligned}\bar{\Sigma}_{\rm all} = &(\bar{L}_{zw} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{yw})\\ &+ (\bar{L}_{z\xi} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{y\xi})\bar{Q}_{\rm g}\bar{L}_{\eta w},\end{aligned} \tag{14}$$

for $\bar{K} \in \mathcal{S}_{\rm Cor2}$. We analyze the structure of the expressions to propose design guidelines of $\bar{Q}_{\rm k}$.

From the expression (13) or (14), we easily see that any $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$ guarantees the *qualitative* stability in $\Sigma_{\rm all}$. On the other hand, we cannot obtain any design guideline of $\bar{Q}_{\rm k}$

that *quantitatively* improves the performance of $\Sigma_{\rm all}$. This is because that it is implicitly assumed in the retrofit control problem that $\bar{Q}_{\rm g}$ is unavailable for the design. In this section, the retrofit control problem is more *specified* than Problem 1, where z and w are *general* control output and general disturbance input, respectively. Then, for specified control aim or target systems, design guidelines of $\bar{Q}_{\rm k}$ are proposed.

B. Design Rule 1 for Retrofit Control

In this subsection, we consider that $\bar{K} \in \mathcal{S}_{\rm Cor1}$ holds, i.e., the constraint (11) holds. This constraint means that the control input u is not affected by the interaction signal ξ. An illustrative example of the control aim and corresponding retrofit controller is given as follows.

Example: Let us consider that the interaction signal ξ is measurable in addition to some other measurable signal y'. The signal ξ is also available for the controller design and implementation as in the setting of e.g., [14], [15]. Then, a part of the local model $\bar{L}$ is described by

$$\left[\begin{array}{c|c|c} \bar{L}_{y\xi} & \bar{L}_{yw} & \bar{L}_{yu} \end{array}\right] = \left[\begin{array}{c|c|c} \bar{L}'_{y\xi} & \bar{L}'_{yw} & \bar{L}'_{yu} \\ I & 0 & 0 \end{array}\right].$$

We let

$$\bar{Q}_{\rm k} = \bar{Q}_{\rm ex} \left[\begin{array}{cc} I & -\bar{L}'_{y\xi} \end{array}\right]$$

with $\bar{Q}_{\rm ex} \in \mathcal{RH}_\infty$. Then, (11) holds, and consequently the designed controller is a retrofit controller. As implied in this example, (11) requires that the measured output y includes sufficient information on ξ.

In addition to the constraint (11), the control aim is more specified than the general problem stated in Problem 1. To this end, suppose that there exist $\bar{M}_\eta, \bar{M}_z, \bar{L}'_{zw}, \bar{L}'_{zu} \in \mathcal{RH}_\infty$ such that

$$\begin{aligned} \bar{L}_{\eta w} &= \bar{M}_\eta \bar{L}'_{zw}, \quad \bar{L}_{\eta u} = \bar{M}_\eta \bar{L}'_{zu}, \\ \bar{L}_{zw} &= \bar{M}_z \bar{L}'_{zw}, \quad \bar{L}_{zu} = \bar{M}_z \bar{L}'_{zu} \end{aligned}$$

hold, i.e., the generalized plant in $\bar{L}$ is expressed as follows:

$$\bar{L} = \left[\begin{array}{c|cc} \begin{array}{c} \bar{L}_{\eta\xi} \\ \hline \bar{L}_{z\xi} \end{array} & \multicolumn{2}{c}{\left[\begin{array}{c} \bar{M}_\eta \\ \bar{M}_z \end{array}\right] \left[\begin{array}{cc} \bar{L}'_{zw} & \bar{L}'_{zu} \end{array}\right]} \\ \hline \bar{L}_{y\xi} & \bar{L}_{yw} & \multicolumn{1}{|c}{\bar{L}_{yu}} \end{array}\right]. \tag{15}$$

This (15) holds for $\bar{M}_z = \bar{M}_\eta = I$, for example, when $z = \eta$ holds, i.e., the control aim is to suppress the interaction signal η. The aim is said to be *localization* of the disturbance effect. A more generalized localization problem is addressed in e.g., [24], [25]. More general control problems can be expressed with (15), such as the multi-objective control for the localization with small input amplification.

For (15), the expression (13) is further reduced to

$$\bar{\Sigma}_{\rm all} = \bar{\Sigma}_{\rm global1} \bar{\Sigma}_{\rm local1}, \tag{16}$$

where

$$\begin{aligned} \bar{\Sigma}_{\rm global1} &:= \bar{M}_z + \bar{L}_{z\xi} \bar{Q}_{\rm g} \bar{M}_\eta, \\ \bar{\Sigma}_{\rm local1} &:= \bar{L}'_{zw} + \bar{L}'_{zu} \bar{Q}_{\rm k} \bar{L}_{yw}. \end{aligned}$$

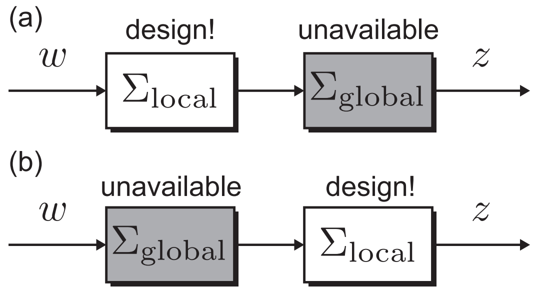

Fig. 5. Cascade-structured expression of the entire controlled system.

We see that $\bar{\Sigma}_{\rm all}$ has the *cascade-structure* of $\bar{\Sigma}_{\rm global1}$ and $\bar{\Sigma}_{\rm local1}$, which is illustrated in Fig. 5(a).

The discussion in this subsection is summarized in the following corollary and remark.

Corollary 1: Consider that $\bar{L}$ is given by (15). Then, if $\bar{K} \in \mathcal{S}_{\rm Cor1}$, $\bar{\Sigma}_{\rm all}$ is expressed by (16).

Remark 4: The cascade-structure in (16) provides us a design guideline of $\bar{Q}_{\rm k}$. Note here that if $\bar{Q}_{\rm k} = 0$, i.e., no controller is implemented to the plant P, $\bar{\Sigma}_{\rm all}$ is reduced to $\bar{\Sigma}_{\rm global1} \bar{L}'_{zw}$. Let the control aim be to reduce $\|\bar{\Sigma}_{\rm global1} \bar{\Sigma}_{\rm local1}\|$ for some norm compared with $\|\bar{\Sigma}_{\rm global1} \bar{L}'_{zw}\|$. An approach is to find $\bar{Q}_{\rm k}$ that minimizes $\|\bar{\Sigma}_{\rm local1}\|$. In the approach, the model information on $\bar{\Sigma}_{\rm global1}$ is not required.

C. Design Rule 2 for Retrofit Control

A controller and problem setting studied in this subsection are *dual* of those in Subsection IV-B. We consider that $\bar{K} \in \mathcal{S}_{\rm Cor2}$ holds, i.e., the constraint (12) holds in the designed controller. This constraint means that the control input u does not affect to the interaction signal η.

In a similar manner to the problem setting in Subsection IV-B, the control aim is more specified than Problem 1. We, suppose that there exist $\bar{L}'_{zw}$, $\bar{L}'_{yw}$, $\bar{M}_\xi$, $\bar{M}_w \in \mathcal{RH}_\infty$ such that

$$\begin{aligned} \bar{L}_{z\xi} &= \bar{L}'_{zw} \bar{M}_\xi, \quad \bar{L}_{zw} = \bar{L}'_{zw} \bar{M}_w, \\ \bar{L}_{y\xi} &= \bar{L}'_{yw} \bar{M}_\xi, \quad \bar{L}_{yw} = \bar{L}'_{yw} \bar{M}_w \end{aligned}$$

hold, i.e., $\bar{L}$ is expressed as follows:

$$\bar{L} = \left[\begin{array}{cc|c} \bar{L}_{\eta\xi} & \multicolumn{1}{|c|}{\bar{L}_{\eta w}} & \bar{L}_{\eta u} \\ \hline \multicolumn{2}{c|}{\left[\begin{array}{c} \bar{L}'_{zw} \\ \bar{L}'_{yw} \end{array}\right] \left[\begin{array}{cc} \bar{M}_\xi & \bar{M}_w \end{array}\right]} & \begin{array}{c} \bar{L}_{zu} \\ \hline \bar{L}_{yu} \end{array} \end{array}\right]. \tag{17}$$

For example, this (17) holds for $\bar{M}_\xi = \bar{M}_w = I$ when the disturbance input w is injected to the local system L through the same input port as the interaction signal ξ.

For (17), the expression (14) is further reduced to

$$\bar{\Sigma}_{\rm all} = \bar{\Sigma}_{\rm local2} \bar{\Sigma}_{\rm global2}, \tag{18}$$

where

$$\begin{aligned} \bar{\Sigma}_{\rm local2} &:= \bar{L}'_{zw} + \bar{L}_{zu} \bar{Q}_{\rm k} \bar{L}'_{yw}, \\ \bar{\Sigma}_{\rm global2} &:= \bar{M}_w + \bar{M}_\xi \bar{Q}_{\rm g} \bar{L}_{\eta w}. \end{aligned}$$

We see that $\bar{\Sigma}_{\rm all}$ has the *cascade-structure* as illustrated in Fig. 5(b).

The following corollary holds.

Corollary 2: Consider that $\bar{L}$ is given by (17). Then, if $\bar{K} \in \mathcal{S}_{\rm Cor2}$, $\bar{\Sigma}_{\rm all}$ is expressed by (18).

From the expression (18), we have a design guideline of $\bar{Q}_{\rm k}$ even if the model information on $\bar{\Sigma}_{\rm global2}$ is unavailable. An approach is to find $\bar{Q}_{\rm k}$ that minimizes $\|\bar{\Sigma}_{\rm local2}\|$ under the constraint (12).

V. CONCLUDING REMARKS

The problem of *retrofit control* was formulated with inheriting and generalizing the concept proposed in [14], [15]. By applying the Youla parametrization to the target system and controller, we reduced the problem into a parameter design which cut off a remaining feedback loop. Then, all retrofit controllers were characterized with the stable parameters satisfying the constraint (8) as stated in Remark 2. Design guidelines of the parameters were also proposed for special but practical control problems.

The proposed parametrization has many applications. For example, the retrofit control is applied to the decentralized design and implementation of multiple controllers for frequency regulation [26]. Another example is *data-driven* retrofit control of general systems. The internal local model $\bar{L}$ in the parametrized controller is replaced by plant data. The controller parameter $\bar{Q}_{\rm k}$ is determined *directly* from the data. We need to study the connection of the data-driven retrofit control to other control methods such as the *plug-and-play* control [27], [28].

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