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Discrete robust controller for Ball and Plate system

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Abstract— **In this paper we present recent results on designing the discrete output robust controller called "'consecutive compensator"' suitable for multiple-input multiple- output discrete systems with cross-line connections and uncounted plate dynamics. It is an extension of the previous results where the continuous version was considered for uncertain Single- input single-output plants. The proposed algorithm provides convergence of the tracking error to the bounded area. Lyapunov function method for stability analysis was used. The experimental validation was performed using well-known Ball and Plate System.**

I. INTRODUCTION

One of the actual research field of modern control becomes designing the discrete controllers for multiple-input multiple-output (MIMO) systems with uncertainties. This area of research is relevant, as well as for solutions for object models. This paper is devoted to development of simple output control algorithms, which could be used for different real technical systems. Output control method is used in cases when the direct measurements of the output derivatives (velocity or acceleration) are complex or it impossible due to various reasons. So, in this study the object parameters are unknown or can be changed during the process. That is why the robust controller design is an important area for engineers. Control approach proposed in this paper is based on the consecutive compensator introduced in [1] for linear plants, nonlinear plants [2], for a tracking problem with disturbance compensation [3], for time-delay systems [4],[5],[6]. The main advantage of this method is its simple engineering implementation in the cases of control object uncertainties and unavailability of output derivatives. It was applied for different robotic setups. For instance, for the quadcopter model [13], for the robotic surface vessel [14] Here we present its extension for MIMO discrete systems with cross-line connections. In the past papers authors presented an especially built parallel kinematics platform known as "ball-and-plate".In [15] we describe results on design and experimental validation of the parallel kinematics robotic platform for nonprehensile manipulation tasks. Such type of systems are widely used in flight simulators, automobile simulators, industrial automation such as fast sorting. Alternatively to [15] we replaced computer vision system to resistive touch screen. It provides faster and more accurate measurements. Software integration of the system was performed in MATLAB/Simulink. The platform was specially built to conduct the following experiments: object stabilization in the specific point on the plate, optimal point-to-point motions and desired trajectory tracking as well as object-to-plate contact model identification. The presented laboratory setup is suitable for research and education as well [16].

The paper is organized as follows: after the short introduction the controller synthesis is described. In the next part includes the proof based on the Lyapunov function method of the proposed algorithm. The third section represents the laboratory setup description and then we provide some experimental validation results. The paper finishes with a short discussion on proposed algorithm.

II. CONTROLLER SYNTHESIS

A. Problem Statement

Consider linear MIMO plant with input and output couplings. Each subsystem of plant is described by equation

$$\begin{aligned}&Q_i(p)y_i(t) = R_i(p)(u_i(t)+f_i(t))+\\&\sum_{i=1,i\neq j}^{N} c_{ij}(p)y_i(t)+\sum_{j=1,i\neq j}^{M}\gamma_{ij}(p)u_j(t),\\&i=\overline{1,N},\, j=\overline{1,M},\end{aligned} \tag{1}$$

where $Q_i(p)$ and $R_i(p)$ are linear differential operators of i-th subsystem with unknown coefficients, deg $Q_i = n_i$, deg $R_i = m_i$, $y_i(t)$ is a subsystem output, $u(t)$ is a subsystem control signal, $f_i(t)$ is a bounded external disturbances acting on i-th subsystem input, $\rho_i = n_i - m_i$ is a subsystem relative degree.

Reference model is described by set of linear differential equations

$$Q_{mi}(p)y_{mi}(t) = R_{mi}(p)r_i(t), \tag{2}$$

where $Q_{mi}(p)$ and $R_{mi}(p)$ are linear differential operators with known coefficients, $y_{mi}(t)$ are reference model outputs, $r_i(t)$ are piece-wise smooth inputs of reference model.

The control goal is to design controller which provides tracking of plant (1) outputs for the outputs of reference model (2) with pre-specified accuracy for finite time:

$$|y_i(t)-y_{mi}(t)| \leq \delta_i, \forall t > T, \tag{3}$$

where δ_i is a tracking accuracy, T is a transient time.

Let us introduce following assumptions

- Unknown coefficients of plant (1) belong to the known compact set Ξ.
- $R_i(\lambda)$ are Hurwitz polynomials, where λ is an imaginary unit.

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B. Control Law

In [1] continuous control law is proposed for SISO plants. This result is extended for MIMO systems class with use of decentralized control in . Decentralized controller takes the form

$$u_i = -(\alpha_i + \beta_i) D_i(p) \hat{e}_i(t), \tag{4}$$

where α_i and β_i are positive constants chosen by designer, $\hat{e}_i(t)$ is an estimate of tracking error $e_i(t) = y_i(t) - y_{mi}(t)$, $D_i(\lambda)$ are Hurwitz polynomials of degrees $\rho_i - 1$ such that $Q_i(\lambda) + \alpha_i R_i(\lambda) D_i(\lambda)$ are Hurwitz polynomials. Last condition always can be satisfied by choose of big enough α_i.

It is necessary to know $\rho_i - 2$ derivatives of tracking errors for implementation of control law (4). Thus, introduce observers for its estimation:

$$\begin{cases} \dot{\xi}_i(t) = \sigma_i \Gamma_i \xi_i(t) + \sigma_i G_i e_i(t), \\ \hat{e}_i(t) = L_i \xi_i(t), \end{cases} \tag{5}$$

where $\xi_i(t) \in \mathbb{R}^{\rho_i - 1}$ is an observer state vector, $\Gamma_i = \begin{pmatrix} 0 & & I_{\rho_i-2} \\ -c_1 & \dots & -c_{\rho_i-1} \end{pmatrix}$ is Hurwitz matrix, $G = [0 \quad 0 \quad c_1]^T$, I_{ρ_i-2} is a unit matrix of $\rho_i - 2$ order, $L = [1 \quad 0 \quad \dots \quad 0]$, $\sigma_i > \alpha_i + \beta_i$.

Choose observation errors as follows:

$$\eta_i(t) = L_i^T e_i(t) - \xi_i(t). \tag{6}$$

Taking into account (5), calculate derivative of (6):

$$\dot{\eta}_i(t) = \sigma_i \Gamma_i \eta_i(t) + L_i^T \dot{e}_i(t). \tag{7}$$

According to [1], [9], MIMO closed loop subsystems consists of plant (1), control law (4) and observer (5) takes the form:

$$\begin{cases} \dot{\varepsilon}_i(t) = A_i \varepsilon_i(t) + B_i(-\beta_i e_i + (\alpha_i + \beta_i)(e_i(t) - \hat{e}_i(t)) + \\ B_{1i}\varphi_i(t) + (\alpha_i + \beta_i) \sum\limits_{j=1, i\neq j}^{M} U_{ij}(e_i(t) - \hat{e}_i(t)) + \\ \sum\limits_{i=1, i\neq j}^{N} \varepsilon_i(t) + \sum\limits_{i=1, i\neq j}^{N} D_{ij} \varepsilon_i(t), \\ \dot{\eta}_i(t) = \sigma_i \Gamma_i \eta_i(t) + L_i^T \dot{e}_i(t), \end{cases} \tag{8}$$

where $\varphi_i(t) = R_i(p) f_i(t) - \frac{Q_i(p)}{R_i(p) D_i(p)} y_{mi} + \sum_{i=1, i\neq j}^{N} c_{ij}(p) y_{mi}(t)$ are bounded disturbance functions.

Rewrite closed-loop system in matrix form for brevity

$$\begin{cases} \dot{\varepsilon} = A\varepsilon + B(-\beta e(t) + (\alpha + \beta)(e(t) - \hat{e}(t))) + B_1 \varphi(t) \\ + W\varepsilon(t) + U(e(t) - \hat{e}(t)) + Y\varepsilon(t) \\ \dot{\eta}(t) = \sigma\Gamma\eta(t) + L^T \dot{e}(t) \\ e(t) = L\varepsilon(t), \end{cases} \tag{9}$$

where $\varepsilon(t) = col\{\varepsilon_i(t)\}$, $e(t) = col\{e_i(t)\}$, $\eta_i(t) = col\{\eta(t)\}$, $A = diag\{A_i\}$, $B = diag\{B_i\}$, $\alpha = diag\{\alpha\}$, $\beta = diag\{\beta_i\}$ etc.

Let obtain discrete realization of controller for its implementation on digital computing devices. Rewrite tracking error as $e(t) = e(t_k) + h\psi(\xi(t_k), t_k)$, where $e(t_k)$ is a discrete value of tracking error $e(t)$ on k-th step, $h = t_k - t_{k-1}$ is a sampling time, $\psi(\xi(t_k), t_k)$ is a Lipschitzian function because of plant is linear and disturbance is a piece-wise smooth. Discrete version of observer takes the form

$$\begin{cases} \xi_1(k+1) = \xi_1(k) + h\sigma\xi_2(k), \\ \xi_2(k+1) = \xi_2(k) + h\sigma\xi_3(k), \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ \xi_{\rho-1}(k+1) = \sigma(-c_1\xi_1(k) - c_2\xi_2(k) - \dots \\ + c_{\rho-1} e(k)). \end{cases} \tag{10}$$

From lemma [7] it follows existence of $\bar{h} > 0$ such, that $\forall h < \bar{h}, \lim_{t\to\infty} ||\xi(t) - \xi(t_k)|| < C$, where $C > 0$.

Dynamics of discrete observer error is described by equations

$$\begin{aligned} \eta(t) &= L^T e(t) - \xi(k) + h\psi(t), \\ \dot{\eta}(t) &= L^T \dot{e}(t) + \sigma\Gamma\eta(t) + \sigma\Gamma h\psi(t). \end{aligned} \tag{11}$$

Remark 1.So, here to reduce disturbances we have to choose h such small as possible. (see statement below) Change derivatives in (4) to the right hand differences:

$$u(t_k) = -(\alpha + \beta)(d_1\xi_1(t_k) + d_2\sigma\xi_2(t_k) + \dots). \tag{12}$$

Closed loop system takes the form

$$\begin{cases} \dot{\varepsilon}(t) = A\varepsilon(t) + B(\beta e(t) + (\alpha + \beta)L\eta(t)) + Bh\psi(t) + \\ B_1\varphi(t) + W\varepsilon(t) + UL\eta(t) + Y\varepsilon(t), \\ \dot{\eta}(t) = L^T \dot{e}(t) + \sigma\Gamma\eta(t) + \sigma h\Gamma\psi(t) \end{cases} \tag{13}$$

Statement

There exist $\alpha_i, \beta_i, \sigma_i, \bar{h} > 0$ and Hurwitz polynomials $D_i(\lambda)$ such that for any $h < \bar{h}$ discrete observer (10) and control law (12) provide exponential convergence of tracking error $e(t)$ to the bounded area.

C. Proof of Statement

Consider Lyapunov function

$$V(t) = \varepsilon^T(t) P \varepsilon + \eta^T(t) H \eta(t), \tag{14}$$

where P_1, P_2 are solutions of Lyapunov equations $A^T P_1 + PA = -Q_1$, $\Gamma^T P_2 + P\Gamma = -Q_2$, Q_1 and Q_2 are positive defined symmetric matrices.

Obtain derivative of Lyapunov function (14) along trajectories (13)

$$
\begin{aligned}
&\dot{V} = -\varepsilon^T(Q_1+2\beta L^T B^T P)\varepsilon + 2\beta\varepsilon^T PBL\eta \\
&+2\varepsilon^T P(U+(\alpha+\beta)B)L\eta + 2h\varepsilon^T PB\psi + \\
&2\varepsilon^T PB_1\varphi + 2\varepsilon^T P(W+Y)\varepsilon - \sigma\eta^T Q_2\eta + \\
&2\eta^T HL^T L(A+Y+W)\varepsilon - 2\beta\eta^T HL^T LBL\eta + \\
&2(\alpha+\beta)\eta^T HL^T LBL\eta + 2h\eta^T HL^T LB\psi \\
&2\eta^T HL^T LB_1\varphi + 2\eta^T HL^T L(W+Y)\varepsilon + 2\eta^T HL^T LUL\eta + \\
&2\sigma H\eta^T P\Gamma\psi.
\end{aligned}
\tag{15}
$$

Bound terms of the right part of (15) by inequalities

$$
\begin{aligned}
&2\beta\varepsilon^T PBL\eta \leq \beta\upsilon\varepsilon^T PBB^T P + \frac{1}{\upsilon}\eta^T\eta, \\
&2\varepsilon^T P(U+(\alpha+\beta)B)L\eta \leq \upsilon\varepsilon^T P(U+(\alpha+\beta)B)\times \\
&(U+(\alpha+\beta)B)^T P\varepsilon + \frac{1}{\upsilon}\eta^T\eta, \\
&2h\varepsilon^T PB\psi \leq \upsilon h\varepsilon^T PBB^T P\varepsilon + \frac{h}{\upsilon}\psi^T\psi, \\
&2\varepsilon^T PB_1\varphi \leq \upsilon\varepsilon^T PB_1B_1^T P\varepsilon + \frac{1}{\upsilon}\varphi^T\varphi, \\
&2\eta^T HL^T L(A+Y+W)\varepsilon \leq \upsilon\varepsilon^T(A+Y+W)^T L^T LH\times \\
&HL^T L(A+Y+W)\eta + \frac{1}{\upsilon}\eta^T\eta, \\
&2h\eta^T HL^T LB\psi \leq \upsilon h\eta^T HL^T LH\eta + \frac{h}{\upsilon}\psi^T\psi, \\
&2\eta^T HL^T LB_1\varphi \leq \upsilon\eta^T HL^T LB_1H\eta + \frac{1}{\upsilon}\varphi^T\varphi, \\
&2\eta^T HL^T L(W+Y)\varepsilon \leq \upsilon\varepsilon^T(W+Y)^T L^T LH\times \\
&H(W+Y)\varepsilon\frac{1}{\upsilon}\eta^T\eta, \\
&2\sigma h\eta^T P\Gamma\psi \leq \sigma h\upsilon\eta^T P\Gamma\Gamma^T P\eta + \frac{\sigma h}{\upsilon}\psi^T\psi,
\end{aligned}
\tag{16}
$$

where υ is a small positive number.

Taking into account (16) rewrite (15) in the form

$$\dot{V} \leq -\varepsilon^T R_1\varepsilon - \eta^T R_2\eta + \theta, \tag{17}$$

where R_1, R_2 are positive defined matrices due to the choose of α, β and σ. θ is a bounded function.

$$
\begin{aligned}
&R_1 = Q_1 + \beta\upsilon PBB^T P - \upsilon P(U+\alpha+\beta B)(U+\alpha+\beta B)^T P - \\
&\upsilon hPBB^T P - \upsilon PB_1^T B_1 P - \upsilon(A+Y+W)^T L^T LHH(A+Y+W) \\
&-\upsilon(W+Y)^T L^T LHH(W+Y) - 2P(W+Y), \\
&R_2 = \sigma Q_2 + 2\beta HL^T LBL - 2(\alpha+\beta)HL^T LBL - 2HL^T LUL - \\
&\frac{4}{\upsilon}I - \sigma h\upsilon P\Gamma\Gamma^T P, \\
&\theta = sup(\frac{(2+\sigma)h}{\upsilon}\psi^T\psi + \frac{2}{\upsilon}\varphi^T\varphi.
\end{aligned}
\tag{18}
$$

Remark 2. In perfect case the value of θ must equals to zero.So, here to reduce it we should choose the smallest possible value of h.The value of θ depends on the σ parameter as well. That's the problem of choosing compatible coefficients α and β. Possible solution: Implementing adaptive control algorithms. Bound derivative of Lyapunov function by inequality

$$
\begin{aligned}
&\dot{V} \leq -\zeta V + \theta, \zeta = \frac{\lambda_{min} R_1}{\lambda_{max}(P_1)}, \\
&V \leq \left(V(0) - \frac{\theta}{\zeta}\right)e^{-\zeta t} - \frac{\theta}{\zeta}.
\end{aligned}
\tag{19}
$$

Taking into account $\lambda_{min}(P_1)e^2 \leq \lambda_{min}(P_1)\varepsilon^2\varepsilon \leq V$ we obtain

$$|e| \leq \sqrt{\frac{1}{\lambda_{min}(P_1)}\left[\left(V(0) - \frac{\theta}{\zeta}\right)e^{-\zeta t} + \frac{\theta}{\zeta}\right]} \tag{20}$$

Thereby, tracking error exponentially converges to the area bounded by inequality

$$|y - y_m| = |e| \leq \sqrt{\frac{1}{\lambda_{min}(P_1)}\frac{\theta}{\zeta}} \tag{21}$$

III. EXPERIMENTAL SETUP

We built a tiltable platform with two degrees of freedom consisting of the following components: resistive touch-screen, servo-drives, square plate and connecting links. The control system was implemented on the Arduino single-board computer. Resistive touch-screen determines the coordinates of the object.

The kinematic scheme of the developed system is presented in the Fig.1 Each tilting axis is operated on by a servomotor Hitec HS-8330SH with JR connector.

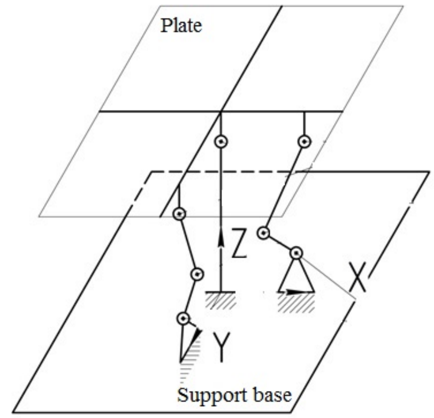

Fig. 1. Kinematic scheme of the mechanism

TABLE I
SERVO CHARACTERISTICS

Working Voltage	6-7.4 V
Working current	0.1-0.8 A
Torque	10.0 kg.cm; 13.0 kg.cm
Average speed	0.07-0.08 s/60 degrees
Weight	66 g

The platform was specially built to conduct the following experiments: object stabilization in the specific point on

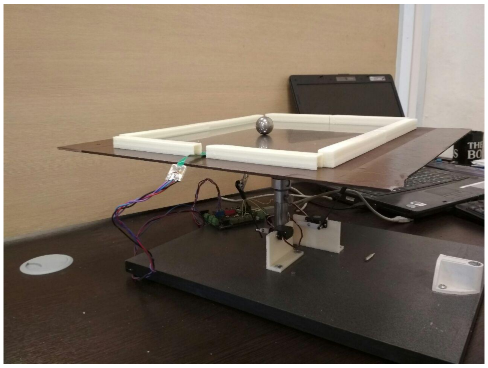

Fig. 2. Parallel kinematics robot Ball and Plate

the plate, its optimal point-to-point motions and desired motion trajectory tracking as well as dynamics object-to-plate contact model identification.
In contrast to previous papers [15], [16] here we detect the ball position using resistive touch-screen system. An analog resistive touch-screen consist of a glass or acrylic panel that is coated with electrically conductive and resistive layers made with indium tin oxide (ITO). These layers are separated by invisible spacers/ To generate a position, the user or an object must exceed the activation force of the screen, pressing the two layers together. This actions create an electrical connection between layers. However, both X and Y axes cannot be read simultaneously, because the concept works by applying a voltage across one layer, and looking for the voltage that appears on the another. Therefore, the system operates by alternately applying a voltage to one layer and reading off of the another. The voltages are then read in by an analog-to-digital (A/D) converter to be used as a coordinate value. There are several types of the resistive touch-screens, e.g. 4-wire, 5-wire and 8-wire. These labels refer to number of wires between the screen and the controller. In the current study we chosen a 4-wire resistive touch AST-150C. It features the typical advantages of the resistive type such as operational stability, detection accuracy, ease of introducing, competitive cost and so on. It have great durability with up to as much as 10 million touches, due to its structure. At the same time, measurement it's just a part of the system. To convert voltages generated by a touchscreen in to *mm* scale, and due to channel bounds and other digital devices a number of measures decrease. Nevertheless, it is enough for control ball on the plate.

IV. Mathematical model of the Ball-and-Plate system

Here we derive the equations of motion for experimental setup Ball on the Plate system by the help of Lagrangian. In this part we introduce the following assumptions:

- There is no slipping for ball.
- The ball is completely symmetric and homogeneous.
- Friction forces are neglected.
- The ball and plate are in contact all time.

The angles of servo arms θ_x, θ_y are assumed to be the inputs, while the ball position on x, y axis are assumed to be the output. Here we derive dynamical equations of ball-on-plate system, by the help of Lagrangian [10], [11], [12]. The Euler-Lagrange equation of ball-plate system are as following:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \tag{22}$$

Where q_i stands for i-direction coordinate, T is kinetic energy of the system, V is potential energy of the system and Q is composite force.
The system has 4 degrees of freedom: two in ball motion and two in inclination of the plate. By assuming the generalized coordinates of system to be x_b, y_b position of the ball in each direction and α, β the inclinations of the plate.The kinetic energy of the ball consists of its both rotational with respect to its center of mass and translational energy:

$$T_b = \frac{1}{2}m_b(\dot{x_b}^2 + \dot{y_b}^2) + \frac{1}{2}I_b(\omega_x^2 + \omega_x^2) \tag{23}$$

where m_b, I_b are mass of the ball, moment of inertia of the ball; $\dot{x_b}, \dot{y_b}$ are translational velocities along x, y- axes. The relations between translational velocities and rotational velocities are the following:

$$\dot{x_b} = r\omega_y, \dot{y_b} = r\omega_x \tag{24}$$

where r_b is the radius of the ball. Taking into account (24) and (23) we get

$$\begin{aligned} T_b = & \frac{1}{2}\left[m_b(\dot{x_b}^2 + \dot{y_b}^2) + \frac{I_b}{r_b^2}(\dot{x_b}^2 + \dot{y_b}^2)\right] = \\ & = \frac{1}{2}\left(m_b + \frac{I_b}{r_b^2}\right)\left(\dot{x_b}^2 + \dot{y_b}^2\right) \end{aligned} \tag{25}$$

The kinetic energy of the plate, considering the fact that ball as a point mass which is positioned in (x_b, y_b) involves its rotational energy with respect to its center of mass

$$\begin{aligned} T_p = & \frac{1}{2}(I_p + I_b)\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + \frac{1}{2}m_b\left(x_b\dot{\alpha}^2 + y_b\dot{\beta}\right)^2 \\ = & \frac{1}{2}(I_p + I_b)\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + \frac{1}{2}m_b\left(x_b^2\dot{\alpha}^2 + x_b\dot{\alpha}y_b\dot{\beta} + y_b^2\dot{\beta}^2\right) \end{aligned} \tag{26}$$

where α, β, $\dot{\alpha}, \dot{\beta}$ are inclination angles of the plate, angular velocity of the plate. Here we can obtain the kinetic energy of the whole system:

$$\begin{aligned} & T = T_b + T_p \\ = & \frac{1}{2}\left(m_b + \frac{I_b}{r_b^2}\right)\left(\dot{x_b}^2 + \dot{y_b}^2\right) + \frac{1}{2}(I_p + I_b)\left(\dot{\alpha}^2 + \dot{\beta}^2\right) \\ & + \frac{1}{2}m_b\left(x_b^2\dot{\alpha}^2 + x_b\dot{\alpha}y_b\dot{\beta} + y_b^2\dot{\beta}^2\right) \end{aligned} \tag{27}$$

The potential energy of the ball relative (the relative potential) to horizontal plane in the center of the inclined plate can be calculated as:

$$V_b = m_b g h = m_b g(x_b sin\alpha + y_b sin\beta) \tag{28}$$

By the Lagrangian we can derive the equation of the system

$$L = T_b + T_p - V_b \tag{29}$$

Using L to derive system's equations we obtain

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{x}_b - m\left(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}\right) + m_b g sin\alpha = 0 \tag{30a}$$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{y}_b - m\left(y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta}\right) + m_b g sin\beta = 0 \tag{30b}$$

$$\begin{aligned}\tau_x = \left(I_b + I_p + m_b x_b^2\right)\ddot{\alpha} + 2m_b x_b \dot{x}_b \dot{\alpha} \\ + m_b x_b y_b \ddot{\beta} + m_b \dot{x}_b y_b \dot{\beta} + m_b x_b \dot{y}_b \dot{\beta} + m_b g x_b cos\alpha\end{aligned} \tag{30c}$$

$$\begin{aligned}\tau_y = \left(I_b + I_p + m_b y_b^2\right)\ddot{\beta} + 2m_b y_b \dot{y}_b \dot{\beta} \\ + m_b y_b x_b \ddot{\alpha} + m_b \dot{x}_b y_b \dot{\alpha} + m_b x_b \dot{y}_b \dot{\alpha} + m_b g y_b cos\beta\end{aligned} \tag{30d}$$

The equations (30a)-(30b) show the relation between balls state and plates state that is plates inclination. The equations (30c)-(30d) show the effect of external torque on the ball-on-plate system. It is obvious that a deviation along one axis leads to a deviation in another direction. This effect is called "cross-line connections". Designed in Section II controller provides convergence of the tracking error to the bounded area and it is applicable in conditions of the influence of unaccounted dynamics, which appears in the links of the mechanism.

V. EXPERIMENTAL RESULTS

The experiment validation was performed on the developed 2 DOF Ball and Plate laboratory setup. The objective is realize trajectory control (follow circle). Model parameters are assumed unknown. Sample time $h = 0.02s$. The reference signal for each axis is as follows:

$$\begin{aligned} x^* = 60sin(t) + 100 \\ y^* = 60sin(t + \pi/2) + 100 \end{aligned} \tag{31}$$

The results can be seen Fig. 3- 4

VI. CONCLUSIONS

In this paper the robust output controller for discrete MIMO systems with cross-line connections was presented. The statement proof is performed using Lyapunov function method. The proposed algorithm provides convergence of the tracking error to the bounded area. Lyapunov function method for stability analysis was used. The experimental validation was performed using well-known Ball and Plate System.

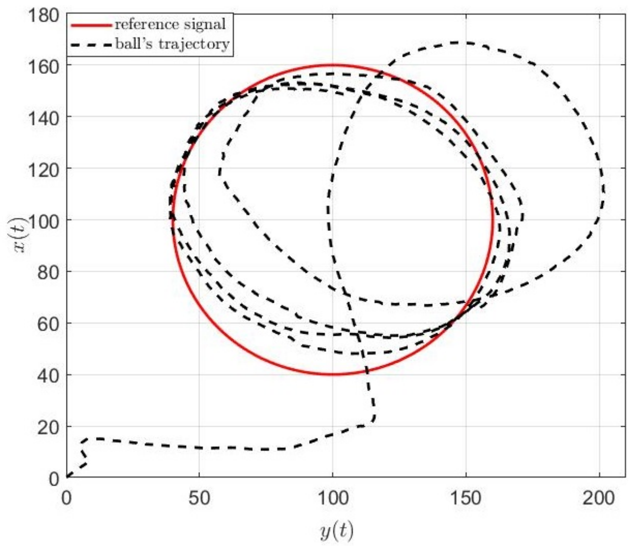

Fig. 3. Trajectory tracking: σ = 0.18, κ = 0.09

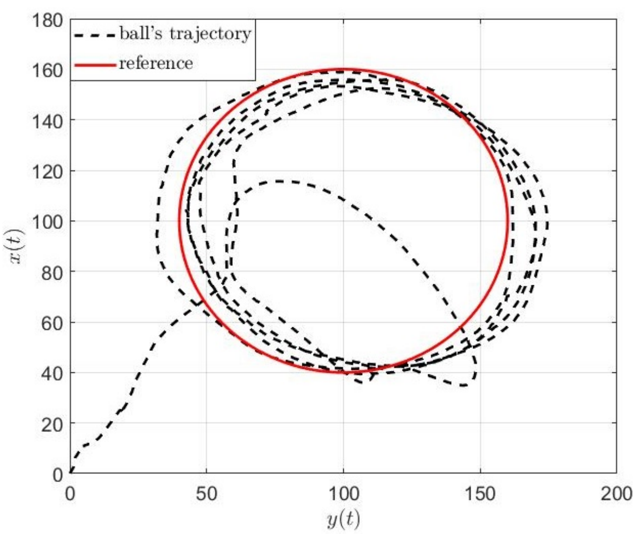

Fig. 4. Trajectory tracking: σ = 0.25, κ = 0.13

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