Output Robust Controller Design for Input-Saturated Robotic Boat with Disturbance Cancellation

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Abstract—In this paper a problem of saturated control for a robotic boat with unknown parameters and unmeasurable velocity and acceleration is addressed. The controller design is based on the output robust control approach consecutive compensator. It was augmented with an internal model, which allows to eliminate a static error and implement the anti-windup scheme to reduce overshoot of the output variable. As result, the regulator generating the bounded control signal and avoiding windup for the boat was obtained. The proof of closed-loop system stability is presented. The efficiency of the proposed algorithm was illustrated by series of experiments using the setup with robotic boat. The experimental results and comparison between three types of controllers (regular consecutive compensator, integral modification and one equipped with anti-windup) are presented.

I. INTRODUCTION

This study is focused on designing simple control algorithms which can be easily implemented for various technical systems with bounded inputs. Output control approaches are significantly useful when using of the sensors to measure output derivatives is complicated or even impossible due to design reasons. Robust controller development is important for real applications since precise plant parameters might be unknown.

Real technical systems have nonlinearity and some various constraints. In this study input saturation effects are considered. Control signals generated by regulators are saturated due to hardware constraints. Also integral term inside the controller may cause performance decrease, loss of stability, etc. This issue can be resolved with anti-windup techniques which prevents error accumulation.

The control algorithm proposed in the paper is based on the robust approach named the consecutive compensator (see [1],[2],[3]). The main advantage of this approach is simplicity of implementation to robotic systems in various cases of plant uncertainties and unavailability of output derivatives. It was applied to various robotic application: for the robotic model of surface vessel (see [4]), for the quadcopter model (see [5],[6]), for the mobile robot (see [7]). An adaptive modification of this approach with implementation to the surface vessel is presented in [8].

The robotic surface vessel is a good example of MIMO system with uncertainties for implementing robust controller with saturated output. In [9] and [10] described sophisticated structure of such systems since they are supposed to move

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in water. The control algorithms for multisinusoidal external disturbance rejection presented in [11], [12], [13]. Description of subsystems of the vessel and their interconnections as well as power plant simulator described in [14] and [15].

In this paper the consecutive compensator was augmented with integral term eliminate the static error, which may occur. At the same time the anti-windup scheme applied to decrease an overshoot value of the output variable under condition of bounded input always caused by hardware constraints. The proof of closed-loop system stability is presented. Efficiency of the proposed algorithm was demonstrated by the experimental approval using the robotic boat setup (used in [4]). Three series of experiments were carried out with different types of controllers for comparison. Regular regulator provides small control values for small errors, which are not sufficient to move the boat. The precision was achieved using the improved controller proposed in this paper. The experimental results and comparison are presented.

This paper is organized as follows. The problem addressed in the paper is mathematically formulated in Section II. Control design and stability analysis are provided in Section III. A robotic boat used to illustrate efficiency of the proposed approach is described in Section IV. Experimental results are given in Section V. Finally, the paper is summarized in Conclusions.

II. PROBLEM STATEMENT

Consider the LTI system

$$\dot{x} = Ax + bu + Rw,\tag{1}$$

$$y = c^T x, (2)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^1$ is the control signal, $w \in \mathbb{R}^{n_w}$ is the disturbance vector, $y \in \mathbb{R}^1$ is the measured output variable, A, R, b, c are matrices and vectors of corresponding dimensions.

The disturbances vector \boldsymbol{w} is considered as the state of the linear system

$$\dot{w} = S(\varrho)w,\tag{3}$$

where ϱ is the vector of parameters, S is the state matrix of corresponding dimension.

The control signal u satisfies the saturation condition

$$u = \operatorname{sat}(v) = \begin{cases} u_{max} & \text{if } v \ge u_{max}, \\ u & \text{if } u_{min} < v < u_{max}, \\ u_{min} & \text{if } v \le u_{min}, \end{cases}$$
(4)

 $u_{min}\ u_{max}$ are input saturation limits, v is the control signal generated by a nominal linear regulator.

Impose the following assumptions.

Assumption 1: The plant (1), (2) is minimum-phase (its zero-dynamics is stable).

Assumption 2: The relative degree of the plant model (1), (2) $\rho \geq 1$ is known.

Assumption 3: The vector ρ of the disturbances generator (3) is known.

Assumption 4: Input saturation limits u_{min} and u_{max} satisfy

$$|u_{min}| = |u_{max}|,$$

$$u_{min} + u_{max} = 0.$$

Assumption 5: The disturbance is bounded $w \in \mathcal{L}_{\infty}$ and the nominal control signal u_0 necessary to its compensation at steady state satisfies

$$u_{min} \le |u_0| \le u_{max}$$
.

The purpose of this paper is to design the control law uusing only measurements of the output variable y such, that

$$\lim_{t \to \infty} y(t) = 0$$

under action of external disturbances with compensation of the integral windup taking into account the input saturation condition (4).

III. CONTROL DESIGN

Let us rewrite the model of the plant (1), (2) in the state space with replacing the variables to extract the zero dynamics

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ b_2 \end{bmatrix} u + \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} w, \quad (5)$$

$$y = \begin{bmatrix} 0 & c_2^T \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \tag{6}$$

where A_{11} is Hurwitz under Assumptions 1, vectors b_2 and c_2 are equal $b_2^T = \begin{bmatrix} 0 & \cdots & 0 & b_0 \end{bmatrix}$, $c_2^T = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$. Choose the control law of the form

$$v = -\kappa (c_q^T \xi + y) - \gamma \eta, \tag{7}$$

$$\dot{\xi} = A_a \xi + b_a y,\tag{8}$$

$$\dot{\eta} = \kappa(c_q^T \xi + y) + \nu \varkappa(v), \tag{9}$$

$$\varkappa(v) = v - \operatorname{sat}(v),\tag{10}$$

where $\varkappa(v)$ is nonlinear anti-windup correction signal, $\kappa >$

where
$$\varkappa(\sigma)$$
 is nonlinear anti-whittip confection signal, $\kappa > 0$, $\gamma > 0$, $\nu > 0$; matrix A_q and vectors b_q , c_q such that
$$A_q = \begin{bmatrix} -q'_\rho \sigma & 1 & 0 & \cdots & 0 \\ -q'_{\rho-1} \sigma^2 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ -q'_2 \sigma^{\rho-1} & 0 & 0 & \cdots & 1 \\ -q'_1 \sigma^{\rho} & 0 & 0 & \cdots & 0 \end{bmatrix}, b_q = \begin{bmatrix} q'_\rho \sigma \\ q'_{\rho-1} \sigma^2 \\ \vdots \\ q'_2 \sigma^{\rho-1} \\ q'_1 \sigma^{\rho} \end{bmatrix}, c_q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q'_{\rho-1} \\ q_1 \end{bmatrix}, c_q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_{\rho-1} \\ q_{\rho} \end{bmatrix}$$
where $\beta_\kappa(s) = \kappa b_q(s)(s+\gamma)b(s)$ and $\frac{b_q(s)}{a_q(s)} = c_q^T(sI - A_q)^{-1}b_q + 1$.
Assume that $\varpi = 0$. To prove the absolute stability of the system it is necessary to show the strictly positive realness of the transfer function
$$W(s) = 1 + W_{\ell}(s) = 0$$

where $\sigma > 0$, q'_i $(i = \overline{1, \rho})$ are chosen from the Hurwitz condition of the system (8), q_i ($i = \overline{1, \rho}$) are the coefficients of an arbitrary Hurwitz polynomial of the form q(s) = $q_{\rho}s^{\rho-1} + \cdots + q_2s + q_1.$

Combining (5), (6) with the control law (7)-(10) yields the closed-loop plant in the equation (11).

Introduce the new variable $\chi = z_2 - \xi$ and rewrite the closed-loop plant (11) in the form (12) where

$$I_0 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

And finally introduce the new variable ζ $(b_2^T b_2)^{-1} b_2^T z_2 + \eta, \ \overline{b_2}^T = \begin{bmatrix} 0 & \dots & 0 & \frac{1}{b_0} \end{bmatrix} \text{ with } b_0 \neq 0 \text{ and }$ rewrite model (12) in the form (13).

Thus, by virtue of the choice of the parameters γ , κ , σ (sufficient large $\kappa \geq \kappa_0, \sigma \geq \sigma_0$ and with Assumption 1) in system (13) it is possible to achieve the Hurwitz property of all diagonal block matrix elements A, which implies that the whole matrix is Hurwitz.

Temporarily, assume that w=0 and write the closed-loop system (13) in the compact form of the plant (1),(2) with the control law (7)-(10)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}\boldsymbol{\varkappa}(v),\tag{14}$$

$$v = \mathbf{c}^T \mathbf{x},\tag{15}$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ \xi \\ \eta \end{bmatrix}, \ \mathbf{A} = \begin{bmatrix} A - \kappa b c^T & -\kappa b c_q^T & -\gamma b \\ b_q c^T & A_q & 0 \\ \kappa c^T & \kappa c_q^T & 0 \end{bmatrix},$$
$$\mathbf{b} = \begin{bmatrix} -b \\ 0 \\ \nu \end{bmatrix}, \ \mathbf{c} = \begin{bmatrix} -\kappa c \\ -\kappa c_q \\ -\gamma \end{bmatrix}.$$

Statement 1: A closed-loop system, consisting of the linear part (14)–(15) and memory-less function $\varkappa(v)$, is absolutely stable under all initial conditions.

Proof: According Popov criteria [16] a closed-loop system, consisting of the linear part (14)-(15) and memoryless functions $\varkappa(v)$, is absolutely stable in case there exist the constant $\varpi \geq 0$ such that $(1 + \lambda_i \varpi) \neq 0$ for each eigenvalue λ_i of matrix **A** as well as the transfer function $W(s) = 1 + (1 + s\varpi)W_{\ell}(s)$ is strictly positive real where $W_{\ell}(s) = \mathbf{c}^{T}(sI - \mathbf{A})^{-1}\mathbf{b}$ is transfer function of linear part of the system (14)-(15) defined as follows

$$W_{\ell}(s) = \frac{\beta_{\kappa}(s) - \gamma \nu a_q(s) a(s)}{s a_q(s) a(s) + \beta_{\kappa}(s)}.$$
 (16)

$$W(s) = 1 + W_{\ell}(s) = \frac{2\kappa b_{q}(s)(s+\gamma)b(s) + (s-\gamma\nu)a_{q}(s)a(s)}{sa_{q}(s)a(s) + \kappa b_{q}(s)(s+\gamma)b(s)}.$$
 (17)

It is known that with the given Assumption 1 there is a constant κ^* such that when $\kappa \geq \kappa^*$ then the polynomials of the numerator and denominator of the transfer function

$$\begin{bmatrix}
\dot{z}_{1} \\
\dot{z}_{2} \\
\dot{\xi} \\
\dot{\eta}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & 0 & 0 \\
A_{21} & A_{22} - \kappa b_{2} c_{2}^{T} & -\kappa b_{2} c_{q}^{T} & -b_{2} \gamma \\
0 & b_{q} c_{2}^{T} & A_{q} & 0 \\
0 & \kappa c_{2}^{T} & \kappa c_{q}^{T} & 0
\end{bmatrix} \begin{bmatrix}
z_{1} \\
z_{2} \\
\xi \\
\eta
\end{bmatrix} + \begin{bmatrix}
0 \\
-b_{2} \\
0 \\
\nu
\end{bmatrix} \varkappa + \begin{bmatrix}
R_{1} \\
R_{2} \\
0 \\
0
\end{bmatrix} w.$$

$$\begin{bmatrix}
\dot{z}_{1} \\
\dot{z}_{2} \\
\dot{\chi} \\
\dot{\eta}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & 0 & 0 \\
A_{21} & A_{22} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) & \kappa b_{2} c_{q}^{T} & -b_{2} \gamma \\
A_{21} & A_{22} - I_{0} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) & \kappa b_{2} c_{q}^{T} & -b_{2} \gamma \\
0 & \kappa (c_{q}^{T} + c_{2}^{T}) & -\kappa c_{q}^{T} & 0
\end{bmatrix} \begin{bmatrix}
z_{1} \\
z_{2} \\
\chi \\
\eta
\end{bmatrix} + \begin{bmatrix}
0 \\
-b_{2} \\
\chi \\
\eta
\end{bmatrix} \varkappa + \begin{bmatrix}
R_{1} \\
R_{2} \\
R_{2} \\
\nu
\end{bmatrix} w.$$

$$\begin{bmatrix}
\dot{z}_{1} \\
\dot{z}_{2} \\
\dot{z}_{2} \\
\dot{\chi}
\end{bmatrix} = \begin{bmatrix}
A_{11} & 0 & A_{12} & 0 \\
\bar{b}_{2}^{T} A_{21} & -\gamma & \bar{b}_{2}^{T} (A_{22} + \gamma I) & 0 \\
A_{21} & -b_{2} \gamma & A_{22} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) + \gamma b_{2} \bar{b}_{2}^{T} & \kappa b_{2} c_{q}^{T} \\
A_{21} & -b_{2} \gamma & A_{22} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) + \gamma b_{2} \bar{b}_{2}^{T} & \kappa b_{2} c_{q}^{T} \\
A_{21} & -b_{2} \gamma & A_{22} - I_{0} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) + \gamma b_{2} \bar{b}_{2}^{T} & \kappa b_{2} c_{q}^{T} \\
A_{21} & -b_{2} \gamma & A_{22} - I_{0} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) + \gamma b_{2} \bar{b}_{2}^{T} & A_{q} + \kappa b_{2} c_{q}^{T} \\
A_{q} + \kappa b_{2} c_{q}^{T}
\end{bmatrix} \begin{bmatrix}
z_{1} \\
\zeta \\
z_{2} \\
\chi
\end{bmatrix} + \begin{bmatrix}
0 \\
(\nu - 1) \\
-b_{2} \\
-b_{2}
\end{bmatrix} \varkappa + \begin{bmatrix}
R_{1} \\
\bar{b}_{2}^{T} R_{2} \\
R_{2}
\end{bmatrix} w.$$

$$(12)$$

$$\begin{vmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{\chi} \\ \dot{\eta} \end{vmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} - \kappa b_2 (c_q^T + c_2^T) & \kappa b_2 c_q^T & -b_2 \gamma \\ A_{21} & A_{22} - I_0 - \kappa b_2 (c_q^T + c_2^T) & A_q + \kappa b_2 c_q^T & -b_2 \gamma \\ 0 & \kappa (c_q^T + c_2^T) & -\kappa c_q^T & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \chi \\ \eta \end{bmatrix} + \begin{bmatrix} 0 \\ -b_2 \\ -b_2 \\ \nu \end{bmatrix} \varkappa + \begin{bmatrix} R_1 \\ R_2 \\ R_2 \\ 0 \end{bmatrix} w.$$
 (12)

$$\begin{bmatrix}
\dot{z}_{1} \\
\dot{\zeta} \\
\dot{z}_{2} \\
\dot{\chi}
\end{bmatrix} = \begin{bmatrix}
A_{11} & 0 & A_{12} & 0 \\
\bar{b}_{2}^{T} A_{21} & -\gamma & \bar{b}_{2}^{T} (A_{22} + \gamma I) & 0 \\
A_{21} & -b_{2}\gamma & A_{22} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) + \gamma b_{2} \bar{b}_{2}^{T} & \kappa b_{2} c_{q}^{T} \\
A_{21} & -b_{2}\gamma & A_{22} - I_{0} - \kappa b_{2} (c_{q}^{T} + c_{2}^{T}) + \gamma b_{2} \bar{b}_{2}^{T} & A_{q} + \kappa b_{2} c_{q}^{T}
\end{bmatrix} \begin{bmatrix}
z_{1} \\
\zeta \\
z_{2} \\
\chi
\end{bmatrix} + \begin{bmatrix}
0 \\
(\nu - 1) \\
-b_{2} \\
-b_{2}
\end{bmatrix} \varkappa + \begin{bmatrix}
R_{1} \\
\bar{b}_{2}^{T} R_{2} \\
R_{2} \\
R_{2}
\end{bmatrix} w. (13)$$

(17) are Hurwitz and the relative degree of which is equal to zero at the same time. In this case its strictly positive realness and the absolute stability of the system (14), (15) are ensured according to the Popov criterion [16].

Consider $w \neq 0$ and analyze the value of the steady error. Obviously $v = \operatorname{sat}(v)$ in the steady state that provides us that $\varkappa(v) = v - \operatorname{sat}(v) = 0$. In this case it is possible to use corresponding Sylvester equation for the model (11)

$$\begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_\xi \\ \Sigma \end{bmatrix} \! S \! = \! \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} - \kappa b_2 c_2^T & -\kappa b_2 c_q^T & -b_2 \gamma \\ 0 & b_q c_2^T & A_q & 0 \\ 0 & \kappa c_2^T & \kappa c_q^T & 0 \end{bmatrix} \! \begin{bmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_\xi \\ \Sigma \end{bmatrix} \! + \! \begin{bmatrix} R_1 \\ R_2 \\ 0 \\ 0 \end{bmatrix},$$

where we pay attention to the fourth line $\Sigma S = \kappa (c_2^T \Pi_2 +$ $c_a^T \Pi_{\mathcal{E}}$) from which it can be seen that in the steady state

$$c_2^T \Pi_2 + c_q^T \Pi_{\xi} = 0. {18}$$

Let us find the relation between $c_2^T \Pi_2$ and $c_q^T \Pi_{\xi}$. Consider the auxiliary variable z_0 :

$$z_0 = y + c_q^T \xi, \tag{19}$$

The steady-state value of which with accordance to the (18) is equal to zero.

Consider the equation $\xi(s) = (sI - A_q)^{-1}(b_q y(s) + \xi(0))$ where $\xi(0)$ is the initial condition.

Rewrite (19)

$$z_0(s) = y(s) + c_q^T (sI - A_q)^{-1} (b_q y(s) + \xi_1(0) + \xi_2(0))$$

= $(c_q^T (sI - A_q)^{-1} b_q + 1) y(s) + \varepsilon(s),$

where $\varepsilon(s)=c_q^T(sI-A_q)^{-1}(\xi_1(0)+\xi_2(0))$ corresponds to an exponentially fading function ε .

If c_q^T is chosen in such a way that the numerator of the transfer function $(c_q^T(sI-A_q)^{-1}b_q+1)$ is Hurwitz and the relative degree is equal to zero then from

$$y(s) = (c_q^T(sI - A_q)^{-1}b_q + 1)^{-1}(z_0(s) - \varepsilon(s))$$

find that the steady-state error y and $c_q^T \xi$ converges to zero.

IV. SETUP DESCRIPTION

The detailed description of the robotic boat setup used for experimental study is represented at the [4]. It is designed to test the control systems inside the experimental basin. The robotic boat is a scale 1:32 model of the real trawler ship and equiped with the same thruster configuration. The boat contains the main engine, two tunnel thrusters on the bow and stern and servo drive for heading control. Its dimensions are $(0.432 \times 0.096 \times 0.052)$ m. The experimental basin represents workspace for the boat with dimensions are (1.50 \times 1.10 \times 0.1) m. The digital camera attached to the basin with the tripod and connected to the computer.

Position and orientation of the boat as any rigid body in a plane can be specified by three numbers. Those are the linear coordinates, say, x and y and angle ψ , which usually called heading in marine navigation and control systems. Since the boat has the three controlled actuators (the main engine and two thrusters) and three output variables $(x(t), y(t), \psi(t))$, it should be mathematically described by the MIMO model of the form

$$x(t) = F(P_e, P_b, P_s, \alpha_e), \tag{20}$$

$$y(t) = G(P_e, P_b, P_s, \alpha_e), \tag{21}$$

$$\psi(t) = H(P_e, P_b, P_s, \alpha_e), \tag{22}$$

where P_e , P_b and P_s are the inputs of the main engine, bow thruster, stern thruster respectively, α_e is the value of steering.

To simplify control strategy perform decomposition of the model (20) as follows

$$P_x(t) = P_e(t), (23)$$

$$P_y(t) = P_b(t) + P_s(t), (24)$$

$$M_{\psi}(t) = -\alpha_e(t)P_e(t)L_e(t) + P_bL_b + P_sL_s,$$
 (25)

where $P_x(t)$, $P_y(t)$ and $M_{\psi}(t)$ are the generalized forces and moment (see Fig. 1), L_e , L_b and L_s are distances from the canter of mass to the main engine, bow and stern respectively.

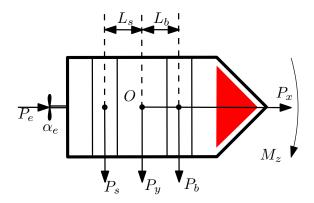


Fig. 1. Actuator configuration of the robotic boat

The robotic boat represents technical realization of the closed-loop system with the computer vision used for feedback. The output variables of the computer vision system are the linear coordinates x(t), y(t) with respect to, say, the absolute coordinate system (O, X, Y) and heading angle $\psi(t)$. These signals are used to calculate the local coordinates $\overline{x}(t)$ and $\overline{y}(t)$ of the system $(\overline{O}, \overline{X}, \overline{Y})$ attached to the boat and error $e_{\psi}(t)$ as follows

$$\begin{bmatrix} \overline{x}(t) \\ \overline{y}(t) \end{bmatrix} = \begin{bmatrix} \cos(\psi) & \sin(\psi) \\ -\sin(\psi) & \cos(\psi) \end{bmatrix} \begin{bmatrix} x^* - x(t) \\ y^* - y(t) \end{bmatrix}, \quad (26)$$

$$e_{\psi} = \psi^* - \psi(t), \quad (27)$$

where x^* , y^* and ψ^* are the desired coordinates and heading. These obtained signals can be used in a controller designed on the computer. In turn, its outputs correspond to $P_x(t)$, $P_{\nu}(t)$ and $M_{\psi}(t)$. Call them virtual control inputs. Note, that they should be saturated with the limits [-127; 127] due to the hardware constraints.

V. EXPERIMENTAL RESULTS

Using model decomposition method described in (23), (24), (25) apply the proposed output robust control algorithm to solve station keeping problem for robotic boat.

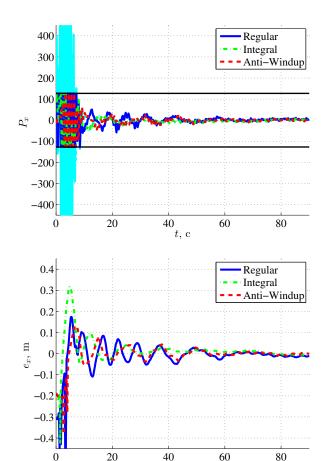
The control law is (7)–(10), where matrix A_q and vectors

$$A_q = \begin{bmatrix} -q_2'\sigma & 1 \\ -q_1'\sigma^2 & 0 \end{bmatrix}, \quad b_q = \begin{bmatrix} q_2'\sigma \\ q_1'\sigma^2 \end{bmatrix}, \quad c_q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix},$$

where $\sigma > 0$.

Three series of experiments was carried out. The first one is "Robuts output control": set $\gamma = 0$ and, as a result, exclude the internal model (9). The second is "Robuts output control with internal model": set $\gamma \neq 0$, $\nu = 0$ and, as a result, exclude anti-windup scheme (10). And the last is "Robuts output control with internal model and AW-compensation": set $\gamma \neq 0$, $\nu = 1$ (this is the final regulator).

The results of the experiments are presented in the Figs. 2, 3 and 4. The purpose of the experiments was to achieve the desired point which was provided by three desired coordinates. The control law for the main engine and two tunnel thrusters was used. The cyan curves illustrates the control



Plots of the control $P_x(t)$ and the error $e_x(t)$ signals of the closed-loop system along x-axis with various controllers

t. c

40

0

provided by unbounded regular consecutive compensator (robust output control). The blue curves in the figures is assigned to regular consecutive compensator with saturated output of the controller, the green one corresponds to the control with integral model and the last red curves refers to compensator equipped with the anti-windup scheme.

VI. CONCLUSIONS

This practical study is devoted to development of simple control algorithms, which can be useful in stabilization tasks of various applications. The proposed in this paper robust output controller with anti-windup compensation has shown the satisfactory experimental results at the robotic boat setup. This approach is applicable for plants with bounded input and uncertain parameters. Its useful feature is a possibility to specify limits of the generated control signal with reduced undesirable overshoot of the output variable.

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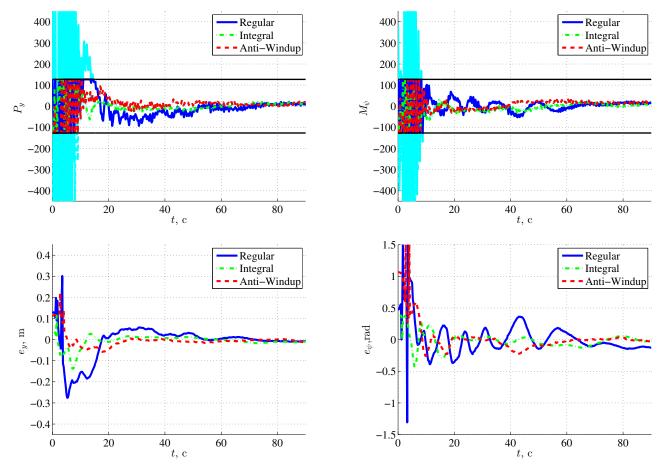


Fig. 3. Plots of the control $P_y(t)$ and the error $e_y(t)$ signals of the closed-loop system along y-axis with various controllers

Fig. 4. Plots of the control $M_{\psi}(t)$ and the error $e_{\psi}(t)$ signals of the closed-loop system along ψ angle with various controllers

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