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Learning Observer-based Robust H_∞ Fault-Tolerant Control for Takagi-Sugeno descriptor systems with time-delay

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***Abstract*— This brief is concerned with the problem of learning observer-based robust H_∞ fault-tolerant Control (FTC) for Takagi-Sugeno (T-S) fuzzy descriptor systems with time-delay affected by both actuator faults and external disturbances. The proposed observer allows simultaneous estimation of system states and actuator faults. Based on the state estimation and the actuator fault reconstruction, a FTC is designed to maintain the performance of the faulty system. Using the H_∞ optimization technique, the analysis and design conditions of the fuzzy learning observer-based FTC are provided and then formulated into a set of delay-dependent linear matrix inequalities (LMIs) which can be solved in a single step. An example is finally presented to validate our findings.**

I. INTRODUCTION

Actuator faults tend to degrade the system performance or even cause some instability issues. The high requirements for safety and reliability of modern control system, have led to improve new methodologies for both fault reconstruction and FTC. Moreover, fault reconstruction has a vital role in FTC, which can stabilize a closed-loop faulty system and guarantee satisfactory performances. Few efforts have been considered in FTC and fault reconstruction areas for T-S fuzzy descriptor systems with time-delay [1], [2]. In [3] for example, authors have extended the adaptive observer proposed in [4] to deal with adaptive FTC for standard T-S fuzzy systems with time-delay. Moreover, it should be pointed that nonlinear T-S descriptor systems can be used to model many practical systems (see for instance [5], [6], [1]).

Therefore, for time-delayed T-S descriptor systems, the criteria of stability and stabilization may be characterized by two categories: delay-independent case [7], [8] and delay-dependent case [3], [9] which is known to provide less conservative results than delay-independent ones. In the survey paper [2], considerable efforts have been devoted to the issue of FTC and fault reconstruction using fuzzy learning observer for time-delayed T-S descriptor systems with actuator faults and external disturbances. However, design conditions of the learning observer and FTC are given as Bilinear Matrix Inequalities (BMIs) and solved not only by fixing some tuning parameters but also by using a two-step algorithm. Similar approaches have been proposed in [10] and [11], based on the fault information obtained by reduced-order observer and adaptive observer, respectively, FTC is proposed to compensate the actuator faults. Observer and controller are also presented separately and the corresponding LMI conditions can be solved only by using a two-step solving algorithm. Actually, the present framework contributes the previous works. Delay-dependent design conditions of time-delayed descriptor systems affected by actuator faults are given in terms of LMIs. Using the H_∞ optimization technique, the learning observer and the FTC gains are obtained by solving a set of LMIs in a single step.

Section 2 presents problem statements and some preliminaries necessary for further developments. Section 3 includes the main results. Delay-dependent sufficient conditions for the fuzzy learning observer-based robust H_∞ FTC are derived and expressed in terms of LMIs. A simulation example is given in section 4 to validate the developed approach. Finally, some conclusions are presented in section 5.

II. PROBLEM FORMULATION

Consider a T-S fuzzy descriptor system with state delay.
Plant Rule $i(i=1,2,..,r)$: If θ_1 is μ_{i1} and, $\cdots$, and θ_p is μ_{ip}, Then

$$\begin{aligned} E\dot{x}(t) &= A_i x(t) + A_{hi} x(t-h) + B_i(u(t)+f_a(t)) + Dd(t) \\ z(t) &= C_L x(t) \\ x(t) &= \varphi(t), \forall t \in [-\bar{h}, 0] \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $\mu_{ij}(i=1,\cdots,r, j=1,\cdots,p)$ represent the fuzzy sets which are characterized by the membership functions and depend on the premise variables $\theta_j(x(t))$ which are supposed to be measurable. r represents the number of If-Then rules and p is the total number of the premise variables. $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$, $z(t) \in \mathbb{R}^{n_z}$, $y(t) \in \mathbb{R}^{n_y}$ denote, respectively, the system state, the control input, the controlled output and the measured output. The actuator fault is represented by $f_a(t) \in \mathbb{R}^{n_u}$ and may be constant or time-varying function. $d(t) \in \mathbb{R}^{n_d}$ is the external disturbance. h represents the system delay satisfying $0 < h \leq \bar{h}$ and $\varphi(t)$ is a continuously differentiable real-valued initial function. Matrix $E \in \mathbb{R}^{n_x \times n_x}$ may be rank deficient, i.e., $rank(E) = q \leq n_x$.

A_i, A_{hi}, B_i, C_L, C and D are known constant matrices with appropriate dimensions. Matrices B_i and C are supposed to be of full column rank and of full row rank, respectively.

By fuzzy blending, the overall fuzzy system is inferred as follows :

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{r} \varrho_i(\theta(x(t)))[A_i x(t) + A_{hi} x(t-h) \\ &+ B_i(u(t)+f_a(t)) + Dd(t)] \\ z(t) &= C_L x(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2}$$

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in which $\theta(x(t)) = [\theta_1(x(t)), ..., \theta_p(x(t))]$

$$\varrho_i(\theta(x(t))) = \frac{\nu_i(\theta(x(t)))}{\sum_{i=1}^{r}\nu_i(\theta(x(t)))}; \nu_i(\theta(x(t))) = \prod_{j=1}^{p}\mu_{ij}(\theta_i(x(t)))$$

where $\mu_{ij}(\theta_i(x(t)))$ is the grade of membership of $\theta_i(x(t))$ in μ_{ij}. $\varrho_i(\theta(x(t)))$ is the weighting function which is in general nonlinear, $0 \le \varrho_i(\theta(x(t))) \le 1$ and $\sum_{i=1}^{r}\varrho_i(\theta(x(t))) = 1$. Then, for brevity ϱ_i is given to denote $\varrho_i(\theta(x(t)))$.

Before starting the main results, the following two assumptions and two lemmas are needed.

Assumption 1: [11] System (E, A_i, C) is observable $\forall\, i \in [1, \cdots, r]$, i.e.,

$$rank \begin{bmatrix} E \\ C \end{bmatrix} = n_x \quad (3)$$

$$rank \begin{bmatrix} sE - A_i \\ C \end{bmatrix} = n_x, \forall\, s \in \boldsymbol{C}_+, \forall\, i = [1, \cdots, r] \quad (4)$$

Assumption 2: [2] Fault $f_a(t)$ satisfies $\|\tilde{f}_a(t)\|_\infty \le k$ where vector $\tilde{f}_a(t) := f_a(t) - K_1 f_a(t-\tau)$ and k is a small positive constant.

Remark 1: Assumption 1 assure the impulse observability of the the triple matrix $(E, A_i, C), \forall i \in [1, \cdots, r]$ and

$$rank \begin{bmatrix} E \\ C \end{bmatrix} = n_x$$

implies that there exists a full-row rank matrix

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} = \begin{bmatrix} E \\ C \end{bmatrix}^\dagger \quad (5)$$

such that

$$\begin{bmatrix} T_1 & T_2 \end{bmatrix} \begin{bmatrix} E \\ C \end{bmatrix} = I_{n_x} \quad (6)$$

It should be noted that the designed observer (9) require necessary conditions introduced in Assumptions 1.

Remark 2: Referring to [2], we note that learning interval τ can be regulated to guarantee fault reconstruction accuracy. For time-varying fault, it is selected to be sufficiently small.

Lemma 1: [3] For any vectors x and y and any symmetric positive-definite matrix Θ for a given scalar $\lambda > 0$, the following inequality holds:

$$2x^\intercal y \le \frac{1}{\lambda}x^\intercal \Theta x + \lambda y^\intercal \Theta^{-1} y\ ,\ x, y \in \mathbb{R}^n \quad (7)$$

Lemma 2: [1] Given a negative definite matrix $\Upsilon < 0$. Consider a matrix Y of appropriate dimension such that $Y^T \Upsilon Y < 0$, then $\exists\ \lambda > 0$ such that

$$Y^T \Upsilon Y \le -\lambda(Y + Y^T) - \lambda^2 \Upsilon^{-1} \quad (8)$$

III. Main results

A. Fuzzy Learning Observer-based Fault-Tolerant Controller Design

In order to achieve a simultaneous estimation of descriptor system states and reconstruction of actuator faults in system (2), the following fuzzy learning observer is given:

$$\begin{cases} \dot{w}(t) = \sum_{i=1}^{r}\varrho_i[T_1 A_i \hat{x}(t) + T_1 A_{hi}\hat{x}(t-h) + T_1 B_i(u(t) \\ \qquad + \hat{f}_a(t)) + L_{1i}(y(t) - \hat{y}(t)) + L_{2i}(y(t-h) - \hat{y}(t-h))] \\ \hat{x}(t) = w(t) + T_2 y(t) \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{f}_a(t) = K_1 \hat{f}_a(t-\tau) + \sum_{i=1}^{r}\varrho_i K_{2i}(y(t) - \hat{y}(t)) \end{cases} \quad (9)$$

and the fault tolerant control is:

$$u(t) = -\sum_{i=1}^{r}\varrho_i K_{3i}\hat{x}(t) - \hat{f}_a(t) \quad (10)$$

where $w(t) \in \mathbb{R}^{n_x}$ is the observer state, $\hat{x}(t) \in \mathbb{R}^{n_x}$ and $\hat{x}(t-h) \in \mathbb{R}^{n_x}$ are the estimated state vector at the sampling time t and $t-h$, respectively. $\hat{y}(t) \in \mathbb{R}^{n_y}$ and $\hat{y}(t-h) \in \mathbb{R}^{n_y}$ are the estimated output vectors at t and $t-h$, respectively. $\hat{f}_a(t) \in \mathbb{R}^{n_u}$ represents the fault reconstruction signal, which is updated by both its previous information at $t - \tau$ and current output estimation error. Parameter τ is the learning interval which can be adjusted to guarantee the accuracy of fault reconstruction [2]. $K_1 = diag(\sigma_1, \cdots, \sigma_{n_u})$ is a diagonal matrix with $\sigma_i \in (0, 1]$. T_1, T_2, L_{1i}, L_{2i},K_{2i} and K_{3i} are gain matrices with appropriate dimensions to be computed.

Under Remark 1, there exist nonsingular matrices $T_1 \in \mathbb{R}^{n_x \times n_x}$ and $T_2 \in \mathbb{R}^{n_x \times n_y}$ such that

$$T_1 E + T_2 C = I_{n_x} \quad (11)$$

Denote the state estimation error by $e_x(t) = x(t) - \hat{x}(t)$, the output estimation error by $e_y(t) = y(t) - \hat{y}(t)$, and the fault reconstruction error by $e_f(t) = f_a(t) - \hat{f}_a(t)$.

By using relation (11) and by considering (2) and (9), both estimation error dynamic $e_x(t)$ and output estimation error $e_y(t)$ can further be written as

$$\begin{aligned} \dot{e}_x(t) &= \sum_{i=1}^{r}\varrho_i[(T_1 A_i - L_{1i}C)e_x(t) + (T_1 A_{hi} - L_{2i}C) \\ &\times \ e_x(t-h) + T_1 B_i e_f(t) + T_1 D d(t)] \quad (12) \\ e_y(t) &= Ce_x(t) \quad (13) \end{aligned}$$

Under Assumption 2, one can obtain

$$e_f(t) = K_1 e_f(t-\tau) - \sum_{i=1}^{r}\varrho_i K_{2i} C e_x(t) + \tilde{f}_a(t) \quad (14)$$

The closed-loop T-S descriptor system is described by the following equation (obtained by applying FTC law (10) to system (2)).

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\varrho_i\varrho_j[(A_i - B_i K_{3j})x(t) + A_{hi}x(t-h) \\ &+ \ B_i K_{3j} e_x(t) + B_i e_f(t) + Dd(t)] \quad (15) \end{aligned}$$

B. Stability and Stabilization Analysis

Sufficient conditions of the proposed fuzzy learning observer-based robust H_∞ fault tolerant control are presented in delay-dependent results. These results are given in a set of strict LMIs.

Theorem 1: Consider system (15) with Assumptions $1-2$, if there exist matrix $P_1 > 0$ and symmetric positive-definite matrices $Q_1, R_1, P_2, Q_2, R_2, Q_3, M_1$ and M_2 as well as K_{2i} and K_{3i} with appropriate dimensions such that $\forall i \in [1, \cdots, r]$ the following conditions hold:

$$E^T P_1 = P_1^T E \geq 0 \tag{16}$$

$$(T_1 B_i)^T P_2 = \beta_0 K_{2i} C \tag{17}$$

$$\beta_0 \beta_1 K_1^T K_1 - Q_3 \leq 0 \tag{18}$$

$$\Phi_{ij} + \Phi_{ji} < 0, \quad i, j = 1, 2, \cdots, r, i \leq j \tag{19}$$

then the fuzzy learning observer proposed in (9) can asymptotically estimate both descriptor system states and actuator faults and the FTC defined in (10) can realize the stability of the closed-loop faulty system (15) with the prescribed H_∞ performance.
with $\beta_0 = (1+\varepsilon_0)\lambda_{max}(Q_3)$ and $\beta_1 = 1+\varepsilon_1$; $\varepsilon_0 \geq 0$ and $\varepsilon_1 > 0$.
where

$$\Phi_{ij} = \begin{bmatrix} \varphi_{ij}^{11} & \varphi_i^{12} & P_1^T B_i K_{3j} & 0 & P_1^T B_i & P_1^T D & 0 & \varphi_{ij}^{17} & C_L^T \\ * & \varphi^{22} & 0 & 0 & 0 & 0 & 0 & A_{hi}^T P_1 & 0 \\ * & * & \varphi_i^{33} & \varphi_i^{34} & 0 & P_2 T_1 D & \varphi_i^{37} & K_{3j}^T B_i^T P_1 & 0 \\ * & * & * & \varphi^{44} & 0 & 0 & \varphi_i^{47} & 0 & 0 \\ * & * & * & * & -\epsilon_0 Q_3 & 0 & B_i^T T_1^T P_2 & B_i^T P_1 & 0 \\ * & * & * & * & * & -\gamma^2 M_2 & D^T T_1^T P_2 & D^T P_1 & 0 \\ * & * & * & * & * & * & -P_2(h^2 R_2)^{-1} P_2 & 0 & 0 \\ * & * & * & * & * & * & -P_1^T (h^2 R_1)^{-1} P_1 & 0 & \\ * & * & * & * & * & * & * & * & -M_1^{-1} \end{bmatrix} < 0 \tag{20}$$

in which

$$\begin{aligned} \varphi_{ij}^{11} &= sym(P_1^T A_i - P_1^T B_i K_{3j}) + Q_1 - E^T R_1 E \\ \varphi_i^{12} &= P_1^T A_{hi} + E^T R_1 E \\ \varphi_{ij}^{17} &= (A_i - B_i K_{3j})^T P_1 \\ \varphi^{22} &= -(Q_1 + E^T R_1 E) \\ \varphi_i^{33} &= sym(P_2 T_1 A_i - P_2 L_{1i} C) + Q_2 - R_2 \\ \varphi_i^{34} &= P_2 (T_1 A_{hi} - L_{2i} C) + R_2 \\ \varphi_i^{37} &= (T_1 A_i - L_{1i} C)^T P_2 \\ \varphi^{44} &= -(Q_2 + R_2) \\ \varphi_i^{47} &= (T_1 A_{hi} - L_{2i} C)^T P_2 \end{aligned}$$

Proof: Let's consider the Lyapunov-Krasovskii functional as follows:

$$\begin{aligned} V(t) &= (Ex(t))^T P_1 x(t) + \int_{t-h}^{t} x^T(s) Q_1 x(s)\,\mathrm{d}s + e_x^T(t) P_2 e_x(t) \\ &+ \int_{t-h}^{t} e_x^T(s) Q_2 e_x(s)\,\mathrm{d}s + \int_{t-\tau}^{t} e_f^T(s) Q_3 e_f(s)\,\mathrm{d}s \\ &+ h \int_{-h}^{0} \int_{t+\theta}^{t} (E\dot{x}(s))^T R_1 (E\dot{x}(s))\,\mathrm{d}s\,\mathrm{d}\theta \\ &+ h \int_{-h}^{0} \int_{t+\theta}^{t} \dot{e}_x^T(s) R_2 \dot{e}_x(s)\,\mathrm{d}s\,\mathrm{d}\theta \end{aligned} \tag{21}$$

The time derivative of $V(t)$ along the trajectory is:

$$\begin{aligned} \dot{V}(t) &\leq (E\dot{x}(t))^T P_1 x(t) + (Ex(t))^T P_1 \dot{x}(t) + x^T(t) Q_1 x(t) \\ &- x^T(t-h) Q_1 x(t-h) + 2\dot{e}_x^T(t) P_2 e_x(t) + e_x^T(t) Q_2 e_x(t) \\ &- e_x^T(t-h) Q_2 e_x(t-h) - \varepsilon_0 e_f^T(t) Q_3 e_f(t) + \beta_0 e_f^T(t) e_f(t) \\ &- e_f^T(t-\tau) Q_3 e_f(t-\tau) + h^2[(E\dot{x}(t))^T R_1 (E\dot{x}(s))] \\ &- h \int_{t-h}^{t} (E\dot{x}(s))^T R_1 (E\dot{x}(s))\,\mathrm{d}s \ + h^2[\dot{e}_x^T(s) R_2 \dot{e}_x(s)] \\ &- h \int_{t-h}^{t} \dot{e}_x^T(s) R_2 \dot{e}_x(s)\,\mathrm{d}s \end{aligned} \tag{22}$$

where $\beta_0 = (1+\varepsilon_0)\lambda_{max}(Q_3)$ with $\varepsilon_0 \geq 0$.

By considering (16) and substituting (12), (14) and (15) into equation (22), we can obtain:

$$\begin{aligned} \dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} \varrho_i \varrho_j [& x(t)^T [sym(P_1^T (A_i - B_i K_{3j})) + Q_1] x(t) \\ &+ 2x(t)^T P_1^T A_{hi} x(t-h) + 2x(t)^T P_1^T B_i K_{3j} e_x(t) \\ &+ 2x(t)^T P_1^T B_i e_f(t) + 2x(t)^T P_1^T D d(t) - x^T(t-h) \\ & Q_1 x(t-h) + e_x^T(t)[sym(P_2(T_1 A_i - L_{1i} C)) + Q_2] e_x(t) \\ &+ 2e_x^T(t) P_2 (T_1 A_{hi} - L_{2i} C) e_x(t-h) - e_x^T(t-h) Q_2 e_x(t-h) \\ &+ 2e_x^T(t)[P_2 T_1 B_i - \beta_0 (K_{2i} C)^T] K_1 e_f(t-\tau) + \beta_0 \tilde{f}_a^T(t) \tilde{f}_a(t) \\ &+ 2e_x(t) P_2 T_1 D d(t) - \varepsilon_0 e_f^T(t) Q_3 e_f(t) - e_x^T(t)[2P_2 T_1 B_i \\ &- \beta_0 (K_{2i} C)^T] K_{2i} C e_x(t) + 2\beta_0 e_f^T(t-\tau) K_1^T \tilde{f}_a(t) \\ &+ \beta_0 e_f^T(t-\tau) K_1^T K_1 e_f(t-\tau) + 2e_x^T(t)[P_2 T_1 B_i \\ &- \beta_0 (K_{2i} C)^T] \tilde{f}_a(t) - e_f^T(t-\tau) Q_3 e_f(t-\tau) \\ &+ h^2[(E\dot{x}(t))^T R_1 (E\dot{x}(s))] - h \int_{t-h}^{t} (E\dot{x}(s))^T R_1 (E\dot{x}(s))\,\mathrm{d}s \\ &+ h^2[\dot{e}_x^T(s) R_2 \dot{e}_x(s)] - h \int_{t-h}^{t} \dot{e}_x^T(s) R_2 \dot{e}_x(s)\,\mathrm{d}s\ ] \end{aligned} \tag{23}$$

If condition (17) holds, one obtains

$$\begin{aligned} &-e_x^T(t)[2P_2 T_1 B_i - \beta_0 (K_{2i} C)^T] K_{2i} C e_x(t) \\ &\quad = -\beta_0 e_x^T(t) (K_{2i} C)^T K_{2i} C e_x(t) \leq 0 \end{aligned} \tag{24}$$

And by using Lemma 1 we have:

$$2e_f^T(t-\tau) K_1^T \tilde{f}_a(t) \leqslant \varepsilon_1 e_f^T(t-\tau) K_1^T K_1 e_f(t-\tau) + \frac{1}{\varepsilon_1} \tilde{f}_a^T(t) \tilde{f}_a(t) \tag{25}$$

The above inequality can be simplified as:

$$\begin{aligned} \dot{V}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} \varrho_i \varrho_j [& x(t)^T [sym(P_1^T (A_i - B_i K_{3j})) + Q_1] x(t) \\ &+ 2x(t)^T P_1^T A_{hi} x(t-h) + 2x(t)^T P_1^T B_i K_{3j} e_x(t) \\ &+ 2x(t)^T P_1^T B_i e_f(t) + 2x(t)^T P_1^T D d(t) - x^T(t-h) Q_1 x(t-h) \\ &+ e_x^T(t)[sym(P_2(T_1 A_i - L_{1i} C)) + Q_2] e_x(t) + \frac{\beta_0 \beta_1}{\beta_1 - 1} \tilde{f}_a^T(t) \tilde{f}_a(t) \\ &+ 2e_x^T(t) P_2 (T_1 A_{hi} - L_{2i} C) e_x(t-h) - e_x^T(t-h) Q_2 e_x(t-h) \\ &- \varepsilon_0 e_f^T(t) Q_3 e_f(t) + e_f^T(t-\tau)[\beta_0 \beta_1 K_1^T K_1 - Q_3] e_f(t-\tau) \\ &+ h^2[(E\dot{x}(t))^T R_1 (E\dot{x}(s))] - h \int_{t-h}^{t} (E\dot{x}(s))^T R_1 (E\dot{x}(s))\,\mathrm{d}s \\ &+ h^2[\dot{e}_x^T(s) R_2 \dot{e}_x(s)] - h \int_{t-h}^{t} \dot{e}_x^T(s) R_2 \dot{e}_x(s)\,\mathrm{d}s\ ] \end{aligned} \tag{26}$$

Now, to minimize the effect of the perturbation on the controlled output in the H_∞ sense, we establish

$$J(t) = \dot{V}(t) + z^T(t) M_1 z(t) - \gamma^2 d^T(t) M_2 d(t), \tag{27}$$

It proceeds that

$$\dot{V}(t)+z^T(t)M_1z(t)-\gamma^2d^T(t)M_2d(t)<0, \quad (28)$$

Considering Jessen's inequality [12], one can obtain

$$-h\int_{t-h}^{t}(E\dot{x}(s))^TR_1(E\dot{x}(s))\,\mathrm{d}s \le \begin{bmatrix} Ex(t) \\ Ex(t-h) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ * & -R_1 \end{bmatrix} \begin{bmatrix} Ex(t) \\ Ex(t-h) \end{bmatrix}$$

$$\le \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix}^T \begin{bmatrix} -E^TR_1E & E^TR_1E \\ * & -E^TR_1E \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix} \quad (29)$$

$$-h\int_{t-h}^{t}\dot{e}_x^T(s)R_2\dot{e}_x(s)\,\mathrm{d}s \le \begin{bmatrix} e_x(t) \\ e_x(t-h) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ * & -R_2 \end{bmatrix} \begin{bmatrix} e_x(t) \\ e_x(t-h) \end{bmatrix} \quad (30)$$

Noting the extended state vector as follows:

$$\chi(t)=\begin{bmatrix} x^{\intercal}(t) & x^{\intercal}(t-h) & e_x^T(t) & e_x^T(t-h) & e_f^T(t) & d^T(t) \end{bmatrix}^T \quad (31)$$

And by considering condition (18), it follows that

$$J(t)\le \chi(t)^T\Phi_{ij}^{11}\chi(t)+h^2[(E\dot{x}(t))^TR_1(E\dot{x}(t))]+h^2[\dot{e}_x^T(t)R_2\dot{e}_x(t)]+\delta \quad (32)$$

where

$$\Phi_{ij}^{11}=\begin{bmatrix} \varphi_{ij}^{11} & \varphi_i^{12} & P_1^TB_iK_{3j} & 0 & P_1^TB_i & P_1^TD \\ * & \varphi^{22} & 0 & 0 & 0 & 0 \\ * & * & \varphi_i^{33} & \varphi_i^{34} & 0 & P_2T_1D \\ * & * & * & \varphi^{44} & 0 & 0 \\ * & * & * & * & -\epsilon_0Q_3 & 0 \\ * & * & * & * & * & -\gamma^2M_2 \end{bmatrix} \quad (33)$$

$$\Phi_{ij}=\sum_{i=1}^{r}\sum_{j=1}^{r}\varrho_i\varrho_j\begin{bmatrix} \Phi_{ij}^{11} & \Phi_i^{12} & \Phi_{ij}^{13} & \Phi_i^{14} \\ * & -\frac{1}{h^2}\tilde{R}_2^{-1} & 0 & 0 \\ * & * & -\frac{1}{h^2}\tilde{R}_1^{-1} & 0 \\ * & * & * & -M_1^{-1} \end{bmatrix} \quad (34)$$

in which

$$(\Phi_i^{12})^T=\begin{bmatrix} 0 & 0 & (\varphi_i^{37})^T & (\varphi_i^{47})^T & P_2(T_1B_i) & P_2(T_1D) \end{bmatrix}$$

$$(\Phi_{ij}^{13})^T=\begin{bmatrix} (\varphi_{ij}^{17})^T & P_1^TA_{hi} & P_1^TB_iK_{3j} & 0 & P_1^TB_i & P_1^TD \end{bmatrix}$$

$$(\Phi_i^{14})^T=\begin{bmatrix} C_L & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{R}_2=(P_2^{-1}R_2P_2^{-1})\ ,\ \widetilde{R}_1=(P_1^{-1}R_1P_1^{-T})$$

If condition (19) is satisfied, we can note that:

$$\frac{1}{2}(\Phi_{ij}+\Phi_{ji})<0, \quad i,j=1,2,\cdots,r, i\le j$$

Applying the Schur complement, the above inequality is equivalent to $\chi(t)^T\Phi_{ij}^{11}\chi(t)+h^2[(E\dot{x}(t))^TR_1(E\dot{x}(t))]+h^2[\dot{e}_x^T(t)R_2\dot{e}_x(t)]<0$.
Therefore, $\dot{V}(t)\le-\zeta\|\chi(t)\|^2+\delta$ with $\zeta=\lambda_{min}(-\Phi_{ij})$ and $\delta=\frac{\beta_0\beta_1}{\beta_1-1}k^2$.
It follows that $\dot{V}(t)<0$ for $\zeta\|\chi(t)\|^2>\delta$, and based on the Lyapunov stability theory, $\chi(t)$ will converge to a small set $\Psi=\{\chi(t)/\|\chi(t)\|^2\le\frac{\delta}{\zeta}\}$; thus $\chi(t)$ is uniformly bounded.
It follows that

$$\dot{V}(t)+z^T(t)M_1z(t)-\gamma_1^2d^T(t)M_2d(t)\le 0, \quad for \quad \zeta\|\chi(t)\|^2>\delta \quad (35)$$

when $d(t)=0$, (35) means $\dot{V}(t)\le 0$ for $\zeta\|\chi(t)\|^2>\delta$ and under the Lyapunov stability theory, $\chi(t)$ will converge to a small set $\Psi=\{\chi(t)/\|\chi(t)\|^2\le\frac{\delta}{\zeta}\}$; thus $\chi(t)$ is uniformly bounded in the case of $d(t)=0$.
when $d(t)\neq 0$, integrating both sides of (35) with respect to t over time period $[0\ \infty]$ yields

$$\begin{aligned} &V(\infty)-V(0)+\int_0^\infty z^T(s)M_1z(s)\,\mathrm{d}s \\ &-\gamma^2\int_0^\infty d^T(s)M_2d(s)\,\mathrm{d}s\le 0, \quad for \quad \zeta\|\chi(t)\|^2>\delta \end{aligned} \quad (36)$$

As $V(\infty)\ge 0$, and with zero initial condition $V(0)=0$, one obtains

$$\begin{aligned} &\int_0^\infty z^T(s)M_1z(s)\,\mathrm{d}s\le\gamma^2\int_0^\infty d^T(s)M_2d(s)\,\mathrm{d}s, \\ &for \quad \zeta\|\chi(t)\|^2>\delta. \end{aligned} \quad (37)$$

therefore, $J(t)<0$ for $\zeta\|\chi(t)\|^2>\delta$
This completes the proof. ■

Notice: Result on Theorem 1 is nonlinear matrix inequalities, which seems as a drawback. So by making further transformations, the LMI conditions presented in the following Theorem could be solved efficiently using LMI toolbox of MATLAB.

Theorem 2: Consider system (15) with Assumptions $1-2$. Given a real scalar parameter $\gamma>0$ and tuning parameters $\mu,\lambda_1,\lambda_2,\lambda_3>0$, if there exist positive-definite matrices X_1, $\widetilde{Q}_1$ and $\widetilde{R}_1$ and symmetric positive-definite matrices X_2,Q_2,R_2,Q_3,M and M_2 as well as $K_{2i},K_{3i},Y_{1i},Y_{2i}$ and W_i with appropriate dimensions such that $\forall i\in[1,\cdots,r]$ the following conditions hold:

$$EX_1=X_1^TE^T\ge 0 \quad (38)$$

$$\begin{bmatrix} \mu I_m & (T_1B_i)^TX_2-\beta_0K_{2i}C \\ * & \mu I_n \end{bmatrix}>0, \quad i=1,2,\cdots,r \quad (39)$$

$$\beta_0\beta_1K_1^TK_1-Q_3\le 0 \quad (40)$$

$$\Omega_{ij}+\Omega_{ji}<0, \quad i,j=1,2,\cdots,r, i\le j \quad (41)$$

then the fuzzy learning observer proposed in (9) can asymptotically estimate the descriptor system states and reconstruct the actuator faults. As well as the FTC designed in (10) guarantee an asymptotic stability of the closed-loop system. Then the gains of the fuzzy learning observer and FTC are respectively derived from $L_{1i}=P_2^{-1}Y_{1i}$, $L_{2i}=P_2^{-1}Y_{2i}$ and $K_{3j}=W_jX_1^{-1}$.
where

$$\Omega_{ij}=\begin{bmatrix} \Omega_{ij}^{11} & \Omega_{ij}^{12} & \Omega_{ij}^{13} & 0 \\ * & \Omega^{22} & \Omega_{ij}^{23} & -\lambda_3I \\ * & * & \Omega^{33} & 0 \\ * & * & * & \Omega_i^{44} \end{bmatrix} \quad (42)$$

in which

$$\Omega_{ij}^{11}=\begin{bmatrix} \Theta_{ij}^{11} & A_{hi}X_1+E\tilde{R}_1E^T \\ * & -(\tilde{Q}_1+E\tilde{R}_1E^T) \end{bmatrix};\ \Omega_{ij}^{12}=\begin{bmatrix} B_iW_j & 0 & B_i & D & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Omega_{ij}^{13}=\begin{bmatrix} \Theta_{ij}^{13} & X_1^TC_L^T \\ (A_{hi}X_1)^T & 0 \end{bmatrix}\ \Omega^{22}=\begin{bmatrix} -\lambda_3(X_1+X_1^T) & 0 \\ * & -2\lambda_3I \end{bmatrix}$$

$$\Omega_{ij}^{23}=\begin{bmatrix} (B_iW_j)^T & 0 \\ 0 & 0 \\ B_i^T & 0 \\ D^T & 0 \\ 0 & 0 \end{bmatrix};\ \Omega^{33}=\begin{bmatrix} -\lambda_2(X_1+X_1^T)+\lambda_2^2h^2\tilde{R}_1 & 0 \\ * & -M \end{bmatrix}$$

$$\Omega_i^{44}=\begin{bmatrix} \Theta_i^{41} & \Theta_i^{42} & 0 & X_2T_1D & (X_2T_1A_i-Y_{1i}C)^T \\ * & \varphi^{44} & 0 & 0 & (X_2T_1A_{hi}-Y_{2i}C)^T \\ * & * & -\varepsilon_0Q_3 & 0 & (X_2T_1B_i)^T \\ * & * & * & -\gamma^2M_2 & (X_2T_1D)^T \\ * & * & * & * & -2\lambda_1X_2+\lambda_1^2h^2R_2 \end{bmatrix}$$

$$\begin{aligned}\Theta_{ij}^{11} &= sym(A_iX_1 - B_iW_j) + \widetilde{Q}_1 - E\widetilde{R}_1E^T\\ \Theta_{ij}^{13} &= (A_iX_1 - B_iW_j)^T\\ \Theta_i^{41} &= sym(X_2T_1A_i - Y_{1i}C) + Q_2 - R_2\\ \Theta_i^{42} &= X_2T_1A_{hi} - Y_{2i}C + R_2\end{aligned}$$

Proof: Inequality (20) can be written in this form

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{11} & \Pi_{ij}^{12} & \Pi_{ij}^{13}\\ * & \Pi_i^{22} & \Pi_{ij}^{23}\\ * & * & \Pi^{33}\end{bmatrix} < 0 \tag{43}$$

where

$$\Pi_{ij}^{11} = \begin{bmatrix} \varphi_{ij}^{11} & \varphi_i^{12}\\ * & \varphi^{22}\end{bmatrix}, \Pi_{ij}^{13} = \begin{bmatrix} \varphi_{ij}^{17} & C_L^T\\ A_{hi}^TP_1 & 0\end{bmatrix}$$

$$\Pi_{ij}^{12} = \begin{bmatrix} P_1^TB_iK_{3j} & 0 & P_1^TB_i & P_1^TD & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Pi_i^{22} = \begin{bmatrix} \varphi_i^{33} & \varphi_i^{34} & 0 & P_2T_1D & \varphi_i^{36}\\ * & \varphi^{44} & 0 & 0 & \varphi_i^{46}\\ * & * & -\varepsilon_0Q_3 & 0 & (T_1B_i)^TP_2\\ * & * & * & -\gamma^2M_2 & (T_1D)^TP_2\\ * & * & * & * & -P_2(h^2R_2)^{-1}P_2\end{bmatrix}$$

$$\Pi_{ij}^{23} = \begin{bmatrix} (B_iK_{3j})^TP_1 & 0\\ 0 & 0\\ B_i^TP_1 & 0\\ D^TP_1 & 0\\ 0 & 0\end{bmatrix}, \Pi^{33} = \begin{bmatrix} -P_1^T(h^2R_1)^{-1}P_1 & 0\\ * & -M_1^{-1}\end{bmatrix}$$

Consider the following symmetric matrix :

$$\mathfrak{X} = \begin{bmatrix} \mathfrak{X}_{11} & 0 & 0\\ 0 & \mathfrak{X}_{22} & 0\\ 0 & 0 & \mathfrak{X}_{33}\end{bmatrix}$$

where $\mathfrak{X}_{11} = diag(P_1^{-T}, P_1^{-T})$, $\mathfrak{X}_{22} = diag(P_1^{-T}, I, I, I, I)$ and $\mathfrak{X}_{33} = diag(P_1^{-T}, I)$

Pre- and post-multiplying inequality (43) by $\mathfrak{X}$, one can obtain:

$$\begin{bmatrix} \mathfrak{X}_{11}\Pi_{ij}^{11}\mathfrak{X}_{11}^T & \mathfrak{X}_{11}\Pi_{ij}^{12}\mathfrak{X}_{22}^T & \mathfrak{X}_{11}\Pi_{ij}^{13}\mathfrak{X}_{33}^T\\ * & \mathfrak{X}_{22}\Pi_i^{22}\mathfrak{X}_{22}^T & \mathfrak{X}_{22}\Pi_{ij}^{23}\mathfrak{X}_{33}^T\\ * & * & \mathfrak{X}_{33}\Pi^{33}\mathfrak{X}_{33}^T\end{bmatrix} < 0 \tag{44}$$

Since Lemma 2 holds true, then we have

$$-P_2(h^2R_2)^{-1}P_2 \leq -2\lambda_1P_2 + \lambda_1^2h^2R_2 \tag{45}$$

$$P_1^{-T}(-P_1^T(h^2R_1)^{-1}P_1)P_1^{-1} \leq -\lambda_2(P_1^{-T} + P_1^{-1}) + \lambda_2^2h^2\widetilde{R}_1 \tag{46}$$

$$\mathfrak{X}_{22}\Pi_i^{22}\mathfrak{X}_{22}^T \leq -\lambda_3(\mathfrak{X}_{22} + \mathfrak{X}_{22}^T) - \lambda_3^2(\Pi_i^{22})^{-1} \tag{47}$$

Using Schur complement, (44) can further be written as

$$\begin{bmatrix} \mathfrak{X}_{11}\Pi_{ij}^{11}\mathfrak{X}_{11}^T & \mathfrak{X}_{11}\Pi_{ij}^{12}\mathfrak{X}_{22}^T & \mathfrak{X}_{11}\Pi_{ij}^{13}\mathfrak{X}_{33}^T & 0\\ * & -\lambda_3(\mathfrak{X}_{22} + \mathfrak{X}_{22}^T) & \mathfrak{X}_{22}\Pi_{ij}^{23}\mathfrak{X}_{33}^T & \lambda_3I\\ * & * & \mathfrak{X}_{33}\Pi^{33}\mathfrak{X}_{33}^T & 0\\ * & * & * & \Pi_i^{22}\end{bmatrix} < 0 \tag{48}$$

Letting $X_1 = P_1^{-1}$, $X_2 = P_2$, $\widetilde{R}_1 = P_1^{-1}R_1P_1^{-T}$, $\widetilde{Q}_1 = P_1^{-T}Q_1P_1^{-1}$, $Y_{1i} = P_2L_{1i}$, $Y_{2i} = P_2L_{2i}$, $W_i = K_{3i}P_1^{-1}$ and $M = M_1^{-1}$ inequality (42) can immediately be obtained. Making further transformation to (17), we can find the problem presented in (39). This completes the proof. ■

Remark 3: The proposed approach improves the existing results in the literature, at least, on the following points. First, the performances and stability of the closed-loop system are maintained, even in the presence of actuator faults and external disturbances. Second, although the problems of fault estimation/reconstruction and FTC have been investigated in [2], but the two-step design approach proposed in this reference appears as an inconvenient. This brief overcomes this drawback by using a one step solving algorithm.

IV. NUMERICAL EXAMPLE

In what follows, to illustrate the validity of our findings, we consider the following T-S fuzzy descriptor system with time-delay proposed in [13] and [14].

$$\begin{cases} E\dot{x}(t) = \sum_{i=1}^{3} h_i[A_ix(t) + A_{hi}x(t-h) + B_iu(t) + Dd(t)]\\ z(t) = C_Lx(t)\\ y(t) = Cx(t)\end{cases} \tag{49}$$

where

$$E = \begin{bmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix};\ A_1 = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ c & -e(\alpha^2+2) & a-1\end{bmatrix};$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ c & 0 & -a-1-a\alpha^2\end{bmatrix};\ A_3 = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ c & 0 & a-1\end{bmatrix};$$

$$A_{hi} = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ a_h & 0 & 0\end{bmatrix}; B_i = \begin{bmatrix} b\\ 0\\ b\end{bmatrix}; D = \begin{bmatrix} d\\ 0\\ d\end{bmatrix};$$

$$C = \begin{bmatrix} 0 & 1 & 1\\ 1 & 0 & 0\end{bmatrix}; C_L = \begin{bmatrix} 1 & 0 & 1\end{bmatrix}, i = 1,2,3$$

The membership functions are designed as follows:

$$h_1(t) = \frac{x_2^2(t)}{\alpha^2+2}, h_2(t) = \frac{1+cos(x_1(t))}{\alpha^2+2},$$

$$h_3(t) = \frac{\alpha^2 - x_2^2(t) + 1 - cos(x_1(t))}{\alpha^2+2}$$

where

$$-\sqrt{\alpha^2+2} \leq x_2 \leq \sqrt{\alpha^2+2}$$

For simulation purposes, we set : $\alpha = 2; a = b = c = d = e = 1; a_h = 0.8$.

T_1 and T_2 can be computed by solving (5)

$$T_1 = \begin{bmatrix} 0.5 & 0 & 0\\ 0 & 1 & 0\\ 0 & -1 & 0\end{bmatrix}; T_2 = \begin{bmatrix} 0 & 0.5\\ 0 & 0\\ 1 & 0\end{bmatrix}$$

By choosing the tuning parameter values as follows: $\lambda_1 = 3$, $\lambda_2 = 4$, $\lambda_3 = 3$, $h = 0.5$, $\gamma = 0.8367$, $\mu = 10^{-5}$, $\tau = 0.01$. Then by choosing $Q_3 = 12$, $\varepsilon_0 = 1$, $\varepsilon_1 = 0.009$ and $K_1 = 0.7039$ to satisfy (40) in Theorem 2 .

Theorems 1 and 2 in [2] provide infeasible solutions. Now by applying Theorem 2 to solve the correspondence LMIs, we obtain a set of feasible solutions with the following observer and controller gains:

$$L_{11} = \begin{bmatrix} 0.2590 & 2.3929\\ 0.8401 & -0.0003\\ -0.1614 & 0.0003\end{bmatrix}, L_{12} = \begin{bmatrix} 0.2543 & 2.3914\\ 0.8383 & -0.0003\\ -0.1607 & 0.0003\end{bmatrix},$$

$$L_{13} = \begin{bmatrix} 0.2667 & 2.3919\\ 0.8386 & -0.0004\\ -0.1638 & 0.0003\end{bmatrix}, L_{21} = \begin{bmatrix} 0.0036 & 0.0079\\ 0.4547 & 0\\ 0.4619 & 0\end{bmatrix},$$

$$L_{22} = \begin{bmatrix} 0.0007 & 0.0066\\ 0.4565 & 0\\ 0.4606 & -0.0001\end{bmatrix}, L_{23} = \begin{bmatrix} 0.0011 & 0.0072\\ 0.4583 & -0.0001\\ 0.4668 & 0\end{bmatrix}$$

$$\begin{aligned} K_{31} &= \begin{bmatrix} 1.4544 & 0.6183 & 2.0590\end{bmatrix},\\ K_{32} &= \begin{bmatrix} 1.4301 & 2.8198 & -0.3953\end{bmatrix},\\ K_{33} &= \begin{bmatrix} 1.4563 & 3.5803 & 2.0371\end{bmatrix}\end{aligned}$$

M_1 and M_2 are given by : $M_1 = 1.1638.10^{-5}$ and $M_2 = 6.8622.10^4$.

We assume that the external disturbance is : $d(t) = 0.1(sin(x_1(t)) - x_1(t))$.

Considering the time-varying fault as follows :

$$f_a(t) = \begin{cases} 0 & t \leq 17\\ 0.05 + 0.1sin(0.2\pi(t-17)) & 17 < t \leq 30\end{cases} \tag{50}$$

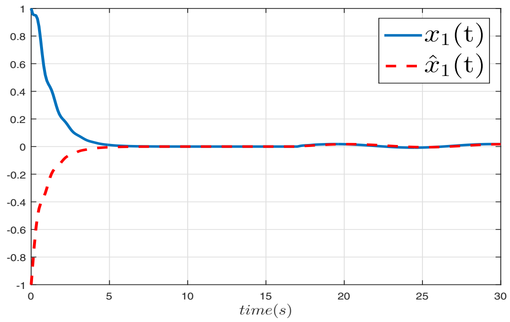

Fig. 1. System state $x_1(t)$ and its estimation under FTC law

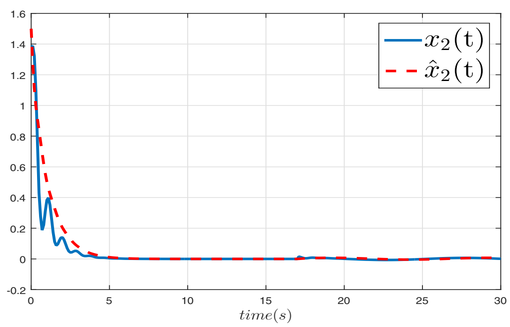

Fig. 2. System state $x_2(t)$ and its estimation under FTC law

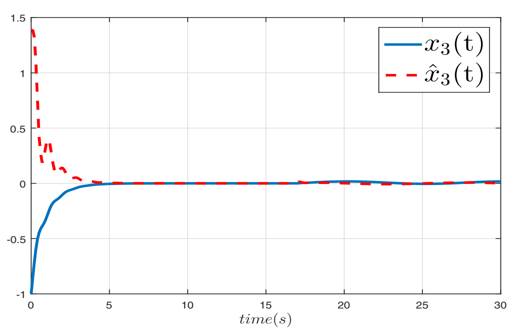

Fig. 3. System state $x_3(t)$ and its estimation under FTC law

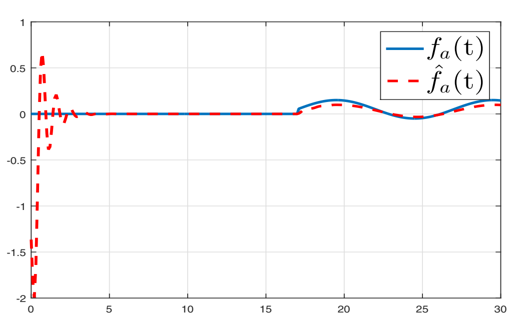

Fig. 4. Actuator fault f_a(t) and its estimated $\hat{f}_a$(t)

The simulation results are obtained after computing the observer and controller gains. In the simulation, initial states are set as $x_0 = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$ and $w_0 = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix}^T$. Results of the simulation are shown in Figs. 1-4. It can be seen that the fuzzy learning observer proposed in this work can estimate the actuator fault as well as the system states satisfactorily despite the existence of external disturbances. Moreover, it can be noticed that the learning observer-based robust H_∞ FTC can stabilize the closed-loop system in the presence of actuator faults and external disturbances.

V. Conclusion

In this paper, a fuzzy learning observer-based robust H_∞ fault tolerant control for T-S descriptor systems with time delay and external disturbances has been developed. The proposed design allows a simultaneous estimation of system states and time-varying actuator faults. The delay-dependent stability and stabilization conditions in presence of actuator faults and external disturbances are formulated in LMI terms that can be solved in only one step using LMI toolbox of MATLAB. The effectiveness of our results has been validated by considering a numerical example.

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