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Conversion from Full-Order Controllers to Observer-Structured Controllers*

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Abstract— **In this note, we compare the structures of Linear Time-Invariant (LTI) full-order controllers and LTI observer-structured controllers (whose structure is similar to that of observer-based controllers; however, they do not necessarily play as observers), and give an insight for the structural differences between those controllers. This insight consequently gives a parametrization of the state-space matrices of observer-structured controllers by using those of full-order controllers. This parametrization contains two free matrices, which can be used to loosely impose some structural constraints for observer-structured controllers in the conversion from full-order controllers to observer-structured ones. Several numerical examples are included to illustrate the usefulness of the two free matrices in our parametrization.**

I. INTRODUCTION

When designing Linear Time-Invariant (LTI) controllers for practical systems, engineers have a lot of options: Proportional-Integral-Derivative (PID) controller [2] is a good choice with respect to simplicity and intuitive understanding of gain values; state-feedback controller is a good choice to realize achievable best performance as long as all states are available; H_∞ and H_2 controllers [12] are also a good choice to satisfy design requirements given in frequency domain, etc. However, they also have their inherent drawbacks: PID controller has a strictly defined structural constraint, which usually fails to design the globally optimal PID controller gains and consequently leads to performance degradation; state-feedback controller cannot be implemented if some of the states are not available, *viz.*, state-feedback controller is not so practical in real world; H_∞ and H_2 controllers have the same orders as the generalized plant has, in other words, complicated controllers are obtained.

On the other hand, it is well-known that H_∞ and H_2 controllers are powerful and useful for controlling practical systems, because robust performance against parametric and/or nonparametric uncertainties can be easily incorporated into design specifications, and the controllers satisfying the design requirements are easily obtained by using convex optimization technique which is also known as Linear Matrix Inequalities (LMIs) [3] in control community. However, the properties that no structural constraints are imposed and the order of the controller is huge due to the incorporation of several weighting functions to satisfy design requirements in frequency domain prevent on-site engineers from understanding the meaning of the values of the state-space matrices.

One of the good compromise between simplicity and good control performance is observer-based controllers from the following aspects: i) The structure of observer-based controllers are simple, because they are composed of the state observer of the plant and the state-feedback controller with the estimated plant state, ii) it is therefore easy for everyone to understand the meaning of the values in the state-space matrices, i.e. state-feedback gain and observer gain, and iii) the controller has more freedom than PID controllers and this property has a possibility to achieve good control performance. However, observer-based controllers still have structural constraints as being composed of observers and state-feedback controllers. Thus, in general, the design problem of observer-based controllers is not given as a convex problem. (See [9]–[11] and the references therein.)

By considering the research background above, it is very helpful if there is a method to convert full-order controllers (designed by using H_∞ or H_2 control technique) to controllers whose structures are very similar to observer-based controllers. Hereafter, the controllers are referred as "observer-structured controllers" if they have the following property:

- The controllers have the structural constraints similarly to observer-based controllers.

However, note that the state of the controllers does not necessarily play as the estimated state of the plant. The rigorous definition of "observer-structured controllers" is given in the next section.

In this paper, we investigate the similarity and the difference between full-order controllers and observer-structured controllers, and give a parametrization for converting full-order controllers to observer-structured ones. We also show a useful application of the proposed parametrization with several numerical examples.

We summarize the objective of this note.

- Investigation of the similarity and difference of full-order controllers and observer-structured controllers with respect to the structure.
- Parametrization of the state-space matrices of observer-structured controllers by using those of full-order controllers.

The remainder of this note is as follows: In Section II, we define a plant to be controlled, a full-order controller and an observer-structured controller; In Section III, we then show our main results, i.e. the parametrization for the

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conversion from the full-order controller to the observer-structured controller, and show a useful application of the parametrization with several numerical examples; We finally give concluding remarks in Section IV.

We use the following notations in this paper: $\mathbf{I}_n$ denotes an $n \times n$-dimensional identity matrix, $\mathbf{0}$ denotes a zero matrix of appropriate dimensions, $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ respectively denote the sets of n-dimensional real vectors and $n \times m$-dimensional real matrices, and X^T for matrix X denotes the transpose of matrix X.

In this note, we consider continuous-time and discrete-time cases; however, for simplicity, we use a single notation for both cases. To this end, for the states of systems, let x represent the state, then $\delta[x]$ denotes $\frac{d}{dt}x(t)$ and $x(k+1)$ respectively in continuous-time case and discrete-time case.

II. PRELIMINARIES

A. Plant System

Let us consider the following LTI plant:

$$G : \begin{bmatrix} \delta[x] \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}, \quad (1)$$

where $x \in \mathbb{R}^n$, $z \in \mathbb{R}^{n_z}$, $y \in \mathbb{R}^{n_y}$, $w \in \mathbb{R}^{n_w}$ and $u \in \mathbb{R}^{n_u}$ respectively denote the state, the performance output, the measurement output, the external input and the control input. All the matrices in (1) are supposed to be constant and to have compatible dimensions.

B. Full-Order Controller

For the LTI plant G, let us suppose that a full-order controller with *no* direct term, i.e. a strictly proper full-order controller, has already designed by some methods, e.g. H_∞ control, H_2 control, etc., and its state-space representation is also supposed to be given as follows:

$$C_{FO} : \begin{bmatrix} \delta[x_c] \\ u \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ C_c & \mathbf{0} \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix}, \quad (2)$$

where $x_c \in \mathbb{R}^n$ denotes the state, and matrices $A_c \in \mathbb{R}^{n \times n}$, $B_c \in \mathbb{R}^{n \times n_y}$ and $C_c \in \mathbb{R}^{n_u \times n}$ are the state-space matrices.

Then, the closed-loop system G_{cl}^{FO} comprising G and C_{FO} is straightforwardly obtained.

$$G_{cl}^{FO} : \begin{bmatrix} \delta[x_{cl}] \\ z \end{bmatrix} = \begin{bmatrix} A_{cl}^{FO} & B_{cl}^{FO} \\ C_{cl}^{FO} & D_{cl}^{FO} \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix}, \quad (3)$$

where $x_{cl} = \begin{bmatrix} x^T & x_c^T \end{bmatrix}^T$ denotes the state, and the matrices A_{cl}^{FO}, etc. are given as follows:

$$\left[\begin{array}{c|c} A_{cl}^{FO} & B_{cl}^{FO} \\ \hline C_{cl}^{FO} & D_{cl}^{FO} \end{array}\right] = \left[\begin{array}{cc|c} A & B_2 C_c & B_1 \\ B_c C_2 & A_c + B_c D_{22} C_c & B_c D_{21} \\ \hline C_1 & D_{12} C_c & D_{11} \end{array}\right]. \quad (4)$$

Remark 1: In H_∞ control, if the methods in [5], [6] are used, then, in general, controllers have direct terms from y to u, because the existence of H_∞ controllers is parametrized with the feasibility of two LMIs which are derived by eliminating the matrices corresponding the state-space matrices of H_∞ controllers. However, if the methods in [4], [8] are used, then it is always possible to obtain controllers without direct terms, i.e. strictly proper controllers, by setting the matrix corresponding to the direct term to be a zero matrix.

In continuous-time H_2 control, D_{11} in (1) should be a zero matrix for proper H_2 norm definition. In addition to this assumption, strictly proper controllers should be designed for proper H_2 norm definition if $D_{12} \neq \mathbf{0}$ and $D_{21} \neq \mathbf{0}$ hold.

From the discussion above, considering only a strictly proper controller C_{FO} in (2) does not severely lose generality. □

C. Observer-Structured Controller

For the LTI plant G, we first define "observer-structured controller" as follows:

$$C_{OS} : \begin{cases} \delta[\hat{x}] &= A_o \hat{x} + B_o u - L(y - C_o \hat{x} - D_o u) \\ u &= K \hat{x} \end{cases}, \quad (5)$$

where $\hat{x} \in \mathbb{R}^n$ denotes the state of the observer, matrices $K \in \mathbb{R}^{n_u \times n}$ denotes the state-feedback gain using $\hat{x}$, and $L \in \mathbb{R}^{n \times n_y}$ denotes the observer gain. In addition to those gains, C_{OS} has additional matrices to be designed; that is, matrices $A_o \in \mathbb{R}^{n \times n}$, $B_o \in \mathbb{R}^{n \times n_u}$, $C_o \in \mathbb{R}^{n_y \times n}$ and $D_o \in \mathbb{R}^{n_y \times n_u}$ are also set as design matrices.

Remark 2: In conventional observer-based controllers (e.g. [2]), matrices A_o, B_o, C_o and D_o are respectively set as A, B_2, C_2 and D_{22}. However, observer-structured controller C_{OS}, they have no need to be identical to A, B_2, C_2 and D_{22} respectively, *viz.*, they can be obtained by optimizing control performance. Thus, in other words, C_{OS} has a very similar structure as observer-based controllers; however, the state $\hat{x}$ in C_{OS} does not necessarily play as the estimated state of the plant G due to the fact that matrices A_o, B_o, C_o and D_o are not identical to A, B_2, C_2 and D_{22} respectively. By considering that the design of those matrices has an additional freedom, controller C_{OS} is not referred to as observer-based controller, but as *observer-structured controller*. □

Then, the closed-loop system G_{cl}^{OS} comprising G and C_{OS} is given as follows:

$$G_{cl}^{OS} : \begin{bmatrix} \delta[x_{cl}] \\ z \end{bmatrix} = \begin{bmatrix} A_{cl}^{OS} & B_{cl}^{OS} \\ C_{cl}^{OS} & D_{cl}^{OS} \end{bmatrix} \begin{bmatrix} x_{cl} \\ w \end{bmatrix}, \quad (6)$$

where $x_{cl} = \begin{bmatrix} x^T & \hat{x}^T \end{bmatrix}^T$ denotes the state, and the matrices A_{cl}^{OS}, etc. are given as follows:

$$\left[\begin{array}{c|c} A_{cl}^{OS} & B_{cl}^{OS} \\ \hline C_{cl}^{OS} & D_{cl}^{OS} \end{array}\right] = \left[\begin{array}{cc|c} A & B_2 K & B_1 \\ -LC_2 & \begin{pmatrix} A_o + B_o K + L C_o \\ -L(D_{22} - D_o)K \end{pmatrix} & -LD_{21} \\ \hline C_1 & D_{12} K & D_{11} \end{array}\right]. \quad (7)$$

III. MAIN RESULTS

We first give a parametrization of the state-space matrices of observer-structured controller C_{OS} using those of full-order controller C_{FO} by comparing the structures of the closed-loop systems in (4) and (7). Then, a useful application of our derived parametrization is shown with several numerical examples.

A. Parametrization of Matrices in C_{OS} by Structural Comparison

Comparison between the state-space matrices of G_{cl}^{FO} in (4) and G_{cl}^{OS} in (7) reveals that the two systems are equivalent if the following relations hold.

- $K = C_c$
- $-L = B_c$
- $A_o + B_o K + LC_o - L(D_{22} - D_o)K = A_c + B_c D_{22} C_c$

The last equation is equivalently written as follows under the supposition that the first and the second relations are satisfied.

$$\begin{bmatrix} \mathbf{I}_n & -B_c \end{bmatrix} \underbrace{\begin{bmatrix} A_o & B_o \\ C_o & D_o \end{bmatrix}}_{X_o} \begin{bmatrix} \mathbf{I}_n \\ C_c \end{bmatrix} = A_c \tag{8}$$

Regarding the constraint (8), one of the solutions for X_o, i.e. the additional design matrices, i.e. A_o, etc., is parametrized as follows:

$$X_o = \begin{bmatrix} A_c & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} B_c \\ \mathbf{I}_{n_y} \end{bmatrix} F + G \begin{bmatrix} -C_c & \mathbf{I}_{n_u} \end{bmatrix},$$

where matrices $F = [F_1 \; F_2] \in \mathbb{R}^{n_y \times (n+n_u)}$ and $G = \begin{bmatrix} G_1^T \; G_2^T \end{bmatrix}^T \in \mathbb{R}^{(n+n_y) \times n_u}$ are free matrices. By noting the following two equalities, it is straightforwardly confirmed that X_o in the above satisfies the constraint (8):

$$\begin{bmatrix} \mathbf{I}_n & -B_c \end{bmatrix} \begin{bmatrix} B_c \\ \mathbf{I}_{n_y} \end{bmatrix} = \mathbf{0},$$
$$\begin{bmatrix} -C_c & \mathbf{I}_{n_u} \end{bmatrix} \begin{bmatrix} \mathbf{I}_n \\ C_c \end{bmatrix} = \mathbf{0}.$$

We thus give a parametrization of the state-space matrices of observer-structured controller C_{OS} in (5) by using the state-space matrices of full-order controller C_{FO} in (2) as follows:

$$\begin{cases} K = C_c \\ L = -B_c \\ X_o = \begin{bmatrix} A_c & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} B_c \\ \mathbf{I}_{n_y} \end{bmatrix} F + G \begin{bmatrix} -C_c & \mathbf{I}_{n_u} \end{bmatrix} \end{cases} \tag{9}$$

Substituting the matrices in (9) into C_{OS} gives the following state-space representation:

$$\begin{cases} \delta[\hat{x}] &= (A_c + B_c F_1 - G_1 C_c)\hat{x} + (G_1 + B_c F_2) C_c \hat{x} \\ & \quad + B_c \left[y - (F_1 - G_2 C_c)\hat{x} - (F_2 + G_2) C_c \hat{x} \right] \\ u &= C_c \hat{x} \end{cases}$$

Then, the following representation is straightforwardly derived from the above.

$$\begin{cases} \delta[\hat{x}] &= A_c \hat{x} + B_c y \\ u &= C_c \hat{x} \end{cases}$$

Obviously, this is just itself of C_{FO}. Thus, full-order controller C_{FO} can be always expressed as observer-structured controller C_{OS} with two arbitrarily chosen matrices F and G.

B. Application of Derived Parametrization

One useful application of the free matrices in (9) is to *loosely* impose structural constraints on matrices A_o, etc. It is not always possible to *rigorously* impose structural constraints on matrices A_o, etc., because, as in (9), X_o has only two free matrices with being multiplied by *a priori* designed matrices B_c and C_c. However, with help of some numerical optimization methods, structural constraints on the matrices of observer-structured controller C_{OS} can be imposed *loosely* by using the two free matrices F and G.

Several numerical examples are shown in this subsection.

1) Example 1: Let us consider the following continuous-time G given in [14].

$$\left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array}\right] = \left[\begin{array}{cccc|ccc|c} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \tag{10}$$

For this plant, we use `hinflmi` command in Matlab®with `option = [0,0,1e-5,1]` to design a full-order H_∞ controller and obtain C_{FO} with the following state-space matrices with optimal H_∞ norm being given as 1.1094.

$$\begin{aligned} A_c &= \begin{bmatrix} 615.78 & -6106.5 & 617.74 & -3.2279e+6 \\ 32.061 & -316.04 & 30.531 & -1.6695e+5 \\ 448.17 & -4437.7 & 449.1 & -2.3464e+6 \\ 98.294 & -286.61 & -81.772 & -99052 \end{bmatrix} \\ B_c &= \begin{bmatrix} 0.16803 \\ -0.95136 \\ -0.12412 \\ -442.04 \end{bmatrix} \\ C_c &= \begin{bmatrix} -1231.2 & 12202 & -1234.8 & 6.45e+6 \end{bmatrix} \end{aligned}$$

From (9), we set K and L as C_c and $-B_c$ respectively, and optimize the following cost function using two free matrices $F \in \mathbb{R}^{1\times 5}$ and $G \in \mathbb{R}^{5\times 1}$ via SeDuMi [13] with YALMIP [7]:

$$\begin{aligned} f_{cost} &= \textstyle\sum_{i=1}^{5} |B_2(i,1) - B_o(i,1)| \\ & \quad + \textstyle\sum_{j=1}^{5} |C_2(1,j) - C_o(1,j)| \\ & \quad + |D_{22} - D_o|. \end{aligned} \tag{11}$$

The optimized F and G are obtained as follows:

$$\left[\begin{array}{c|c} F^T & G \end{array}\right] = \left[\begin{array}{c|c} 9.5064e-10 & -1.1348e-13 \\ 1 & 7.1326e-13 \\ 9.5339e-10 & 1 \\ -4.98e-6 & 3.3638e-10 \\ 7.6099e-13 & -7.721e-13 \end{array}\right],$$

and consequently obtained matrices, A_o, etc. are given as (12) at the top of the next page. Obviously, $B_o \approx B_2$, $C_o \approx$

$$\left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}\right] = \left[\begin{array}{cccc|c} 615.78 & -6106.3 & 617.74 & -3.2279e+6 & 1.4387e-14 \\ 32.061 & -316.99 & 30.531 & -1.6695e+5 & -1.0713e-14 \\ 1679.4 & -16640 & 1683.9 & -8.7964e+6 & 1 \\ 98.294 & -728.65 & -81.772 & -99052 & -2.2748e-15 \\ \hline -2.5908e-15 & 1 & 6.395e-15 & -2.0752e-14 & -1.1117e-14 \end{array}\right] \tag{12}$$

C_2 and $D_o \approx D_{22}$ hold with very small errors; however, it is also obvious that A_o is totally different from A.

Next, we consider the following cost function instead of f_{cost} in (11):

$$\begin{aligned} f_{cost} \;=\; & \textstyle\sum_{j=1}^{5}\sum_{i=1}^{5}|A(i,j)-A_o(i,j)| \\ & +\textstyle\sum_{i=1}^{5}|B_2(i,1)-B_o(i,1)| \\ & +\textstyle\sum_{j=1}^{5}|C_2(1,j)-C_o(1,j)| \\ & +|D_{22}-D_o|. \end{aligned} \tag{13}$$

The optimized F and G are obtained as follows:

$$\left[\begin{array}{c|c} F^T & G \end{array}\right] = \left[\begin{array}{c|c} 0.17733 & -0.50046 \\ -0.22222 & -0.025885 \\ -0.22789 & -0.36379 \\ 6.4597e-7 & -0.015357 \\ -3.4741e-5 & 9.3505e-1 \end{array}\right],$$

and consequently obtained matrices, A_o, etc. are given as (14) at the top of the next page. Obviously, $D_o \approx D_{22}$ holds with the maximum error $3.4741e-5$; however, neither $B_o \approx B_2$ nor $C_o \approx C_2$ hold. Regarding A_o, some elements are very close to the corresponding elements of A. For example, the lower-right 2×2 block is almost a zero matrix, and $(2,4)$, $(4,1)$ and $(4,2)$ elements are identical to the corresponding elements in A. However, we cannot obtain state-space matrices which are closer to those of the plant than those in (14).

2) Example 2: Let us consider an example which represents the nominal linearized model of a VTOL aircraft. The state-space matrices, which are borrowed from [15], are given in (15) at the top of the next page.

For this plant, we use `hinflmi` command in Matlab® with `option = [0,0,1e-5,1]` to design a full-order H_∞ controller and obtain C_{FO} with the following state-space matrices with optimal H_∞ norm being given as 10.059.

$$A_c = \begin{bmatrix} -3249.9 & -604.63 & -72.412 & -90.878 \\ -457.48 & -91.018 & -19.715 & 64.256 \\ 4470.9 & 839.01 & 111.89 & 35.768 \\ 7633.5 & 1434.7 & 197.42 & -7.4461 \end{bmatrix}$$

$$B_c = \begin{bmatrix} 3381.9 \\ 482.54 \\ -4670.5 \\ -7978.3 \end{bmatrix}$$

$$C_c = \begin{bmatrix} -0.33691 & 0.97546 & 1.605 & -16.222 \\ 1.3762 & 0.26955 & -0.80659 & 2.1398 \end{bmatrix}$$

We first set the following cost function to obtain optimal free matrices:

$$\begin{aligned} f_{cost} \;=\; & \textstyle\sum_{j=1}^{4}\sum_{i=1}^{4}|A_o(i,j)| \\ & +\textstyle\sum_{j=1}^{2}\sum_{i=1}^{4}|B_o(i,j)| \\ & +\textstyle\sum_{j=1}^{4}|C_o(1,j)| \\ & +\textstyle\sum_{j=1}^{2}|D_o(1,j)|. \end{aligned} \tag{16}$$

The optimized F and G are obtained as follows:

$$F = \begin{bmatrix} 0.98626 \\ 0.18857 \\ 0.011697 \\ 7.3724e-8 \\ -0.0028603 \\ -0.022117 \end{bmatrix}^T,$$

$$G = \begin{bmatrix} 14.753 & 69.369 \\ -2.2678 & 12.837 \\ -15.685 & -102.19 \\ -22.817 & -176.46 \\ 0.047876 & 0.36295 \end{bmatrix}.$$

and consequently obtained matrices, A_o, etc. are given as (17) at the top of the next page. It is inferred that the degree of the controller might be reduced from the fact that several elements of A_o, B_o and C_o are very close to zeros, in particular, the fourth column of A_o and the fourth element of C_o.

Then, we set the following cost function to obtain optimal free matrices:

$$\begin{aligned} f_{cost} \;=\; & \textstyle\sum_{i=1}^{3}|A_o(i,4)|+\sum_{j=1}^{3}|A_o(4,j)|+|A_o(4,4)| \\ & +\textstyle\sum_{j=1}^{2}|B_o(4,j)|+|C_o(1,4)|. \end{aligned} \tag{18}$$

Then, the optimized F and G are obtained as follows:

$$F = \begin{bmatrix} 0.95694 \\ 0.17954 \\ 0.02424 \\ 0.00395 \\ 0.00030 \\ -3.9034e-05 \end{bmatrix}^T,$$

$$G = \begin{bmatrix} 4.6968 & -0.61651 \\ -4.0089 & 0.52881 \\ -1.0485 & 0.13831 \\ 2.3622 & -0.31142 \\ -0.00023952 & 3.1643e-05 \end{bmatrix},$$

and consequently obtained matrices, A_o, etc. are given in (19) at the top of the next page. It is confirmed that the elements in the fourth column and the fourth row of A_o and in the fourth column of C_o are very close to zeros, and the fourth row of B_o is indeed zero. The minimum singular value of A_o in (19) is $1.8951e-19$. Thus, by considering that

$$\left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}\right] = \left[\begin{array}{cccc|c} -0.37164 & 0.021301 & -0.25021 & -1.7906e-5 & -0.50046 \\ 0.02152 & 0.013431 & -1.2141 & 1 & -0.025852 \\ 0.22957 & 1.281 & -0.076206 & -1.2968e-5 & -0.36379 \\ 1 & -1 & -4.1146e-9 & -5.4488e-7 & 5.2351e-8 \\ \hline 0.17733 & -0.22222 & -0.22789 & 4.2868e-8 & -3.4741e-5 \end{array}\right] \tag{14}$$

$$\left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array}\right] = \left[\begin{array}{cccc|cccc|cc} -0.0366 & 0.0271 & 0.0188 & -0.4555 & 1 & 0 & 0 & 0 & 0.4422 & 0.1761 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 & 0 & 1 & 0 & 0 & 3.5446 & -7.5922 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 & 0 & 0 & 1 & 0 & -5.5200 & 4.4900 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{15}$$

$$\left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}\right] = \left[\begin{array}{cccc|cc} -5.0436 & -0.00017085 & -0.57865 & -6.4152e-5 & 5.0794 & -5.4273 \\ 0.0001514 & -1.2732 & -0.075967 & -3.6014e-5 & -3.648 & 2.1649 \\ 0.00059464 & 1.1414 & 0.00058154 & 6.8364e-5 & -2.326 & 1.1039 \\ -0.0004733 & -0.00070402 & -1.6101 & -6.7404e-5 & 0.0030788 & -0.0027244 \\ \hline 0.50291 & 0.044038 & 0.2276 & 2.4559e-5 & 0.045016 & 0.34083 \end{array}\right] \tag{17}$$

$$\left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}\right] = \left[\begin{array}{cccc|cc} -11.276 & -1.8641 & 1.5229 & -6.5503e-14 & 5.6981 & -0.74852 \\ 2.2039 & -0.61428 & -1.1578 & 6.1728e-14 & -3.8661 & 0.50997 \\ 1.0462 & 1.4543 & 0.47619 & 1.1213e-14 & -2.4313 & 0.32061 \\ 3.0098e-13 & 1.7888e-14 & 3.2752e-15 & -2.1205e-14 & 0 & 0 \\ \hline 0.95681 & 0.17977 & 0.024648 & 6.7356e-18 & 5.6559e-05 & -7.3907e-06 \end{array}\right] \tag{19}$$

observer gain $L(=-B_c)$ and state-feedback gain $K(=C_c)$ both have moderate values (not too large), it is inferred that the order of the controller can be reduced by one.

IV. CONCLUSIONS

In this note, we make a comparison between full-order controllers and observer-structured controllers, which has a very similar structure as observer-based controllers however the state does not always play as the estimated state of the plant, from the viewpoint of the structure of closed-loop systems. This comparison gives a parametrization of the state-space matrices of observer-structured controllers by using those of full-order controllers with two free matrices which can be arbitrarily chosen. By appropriately setting the two free matrices, some structural constraints can be *loosely* imposed for observer-structured controllers. This property has been illustrated by several numerical examples.

The connection between our results and the results in [1] is one of the future research topics to be addressed.

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