2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Distributed Communication-Free Control of Multiple Robots for Circumnavigation of a Speedy Unpredictably Maneuvering Target*

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Abstract— **Multiple fully actuated mobile robots travel in a plane with upper-limited speeds and are driven by accelerations limited in magnitude. The robots should arrive at a pre-specified distance from an unpredictable speedy target, then maintain this distance and achieve an even self-distribution over the respective moving circle, along with a given angular velocity of rotation about the target. The robots do not communicate with anybody and are anonymous to one another; pre-assignment of different roles to various robots is impossible. Every robot measures only the relative position of the target and companion robots (within a finite range of "visibility" in the latter case) and has access to the angular velocity of its own pure rotation; access to its own linear velocity may also be needed in some cases. Necessary conditions for the solvability of the mission are disclosed and a distributed control strategy is proposed. Its global convergence and collision avoidance property are rigorously proved under slight and partly unavoidable enhancement of the just mentioned necessary conditions. The performance of the control law is illustrated by computer simulations.**

I. Introduction

Moving multiple robots into a desired stable geometric formation is a fundamental task in multi-agent robotics [1]. In the area of formation control, considerable attention has been paid to the problem of driving many mobile robots into a capturing formation around a target object, see e.g., [2]–[18]. This research is motivated by a variety of applications concerned with minimization of security risks, rescue operations, exploration, surveillance, and improvement of situational awareness in hazardous environments, deployment of mobile sensor/actuator networks [19], escorting and patrolling missions, troop hunting, moving large objects, etc. [2], [4]. The involved navigation task is to approach the target and then to surround and follow it, often at a pre-specified distance. Since the pursuers typically travel at higher speeds than the target, the result is moving around the target, which in turns motivates the term *circumnavigation*. Typically, the pursuers should also achieve an effective (e.g., equi-spaced) grasping configuration around the taget.

The gradient decent approach is used in [12] to achieve capturing of a moving target by planar fully actuated and kinematically driven robots, each with access to data on the angularly closest robots and target, which is assumed to be motionless until an affective distribution of the robots is determined. The control input is the rate at which the distance to the target and the polar angle relative to it should evolve over time; its implementation calls for access to the target's velocity vector. A strategy for cooperative hunting by means of Hilare-type robots is offered in [13], along with an evidence of the team's local stability in closed-loop; however, no rigorous convergence results are provided. Conditions under which a simple distributed linear control law drives a team of identical planar linear mobile agents with full actuation and observation into a target-centric formation are established in [15], provided that the target obeys a known and beneficial for the capturing task escaping rule,

In [5], the cyclic pursuit paradigm underlies a control law for capturing a moving target in 3D at a given altitude by kinematically driven identical vehicles. As in [12], the control input is the rate at which the distance to and polar angle relative to the target evolve over time. Basic findings of [5] are extended in [11] to stable under-actuated vehicles described by a common linear model and a steady target. A limitation of pursuit-based approaches is reliance on a fixed ring-like information-flow graph: the pursuer identifies its prey agent, sees it or/and communicates with it.

In [6], [8], [10], not necessarily a ring-like graph, a moving target, and robots with access to its velocity vector are studied. Control laws are given for fully actuated kinematically driven unicycles in 2D and a time-invariant graph [6], [8] and simple integrators in 3D and a time-varying graph [10]. Research similar to that in [6], [8] is reported in [14], [20] for the case of a time-varying graph and uncertainties in data on the target [14] or information exchange [20]. All these works assume that the information-flow graph does not depend on the team's motion and has special properties, unnecessary in general. However, this graph is often defined by the positions of the robots, and its traits are affected by the control law.

Such a case is studied in [17]: every of anonymous simple-integrator planar robots observes its angular predecessor and follower in the circular order around a moving target, irrespective of their distances from the robot at hand. However, visibility of an object typically depends on the distance to it for most real sensors, whereas the angular discrepancy with respect to a third party (target) position does not matter.

The unlimited range of visibility is assumed in all above papers. In [16], a more realistic case is handled where the robot measures the relative positions and orientations of the companions within the union of a disc sector with a disc of a smaller radius. For identical under-actuated unicycles, a distributed control law is proposed and shown to ensure local stability of the uniform formation around a steady target.

*This work was suported by the Russian Foundation for Basic Research under the grant 17-08-00715

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However, no rigorous results on global stability are provided.

The issue of collision avoidance is addressed in [9], [18], [21], [22] for target capturing missions. In [9], the control objective is a collective collision-free motion of fully actuated unicycles with all-to-all communication to a desired distance from a steady beacon, with their distribution around the beacon being out of control. The control law from [21] safely drives constant-speed planar simple-integrator robots with limited sensing radii into an uniform circular formation around a steady beacon. The main focus of [18] is in fact on a proximity of a steady beacon, where identical, unlabeled, and fully actuated unicycle-like robots acquire all-to-all visibility. Collision avoidance is guaranteed only if the robots start from distinct rays rooted at the beacon, the ultimately achieved evenly spaced circular formation is not burdened by the need to ensure a pre-specified speed of rotation about the beacon. This goal is achieved in [22] for non-identical and unlabeled unicycle-like fully-actuated robots with a finite control range and sector of vision, which have access only to the distance data about a beacon; collisions are excluded if the robots are initially far enough from one another.

Meanwhile, among the survey of papers on target capturing navigation, the authors failed to come across one that tackles the issues of a bounded control input and collision avoidance in the case of a moving target. For such a target, most papers also assume either access to its velocity vector [5], [6], [8], [10], [12] or a certain level of predictability [15]. This paper is aimed at filling these gaps. Among the incentives, there is the fact that in many modern and emerging applications, the robots have to handle an unpredictable target with a comparable maneuverability and that the target may intentionally provoke collisions among the pursuers.

Specifically, we consider a speedy unpredictable target in 2D and a team of point-wise robots, each driven by the acceleration vector. Upper bounds are imposed on its magnitude and the speeds of the robots. Every robot has access to the angular velocity of its own pure rotation and to the vector of its own velocity in the local frame (the latter is not needed in some cases). It cannot assess the speeds of other objects and distinguish between the companions, does not communicate with them, but measures their relative positions within a finite range of visibility; the relative position of the target is also measured. The robots should approach and then circumnavigate the target in a given common direction with a given rotational speed while ultimately achieving an even self-distribution over the respective circle.

We first disclose conditions necessary for the mission to be feasible. Then we present a control strategy that solves the mission under a slight enhancement of these conditions and ensures no collisions among the robots and with the target. These are shown via a rigorous global convergence result and confirmed by computer simulation tests.

The body of the paper is organized as follows. Sect. II introduces the robot's model and problem setup. Sect. III offers necessary conditions for the mission feasibility and assumptions. The navigation law and main results are presented in Sects. IV and V, respectively. Section VI reports on computer simulation tests, Sect. VII offers brief conclusions. Due to the page limit, the proofs of the results stated in this paper will be given in its full version.

II. System description and problem setup

Every of N planar robots, labeled 0 through $N-1$, is driven by the acceleration vector limited in magnitude by $\overline{a}$. There also is an upper bound $\overline{v} > 0$ on the speed of the robot. Every of them has access to the angular velocity of its own pure rotation and to the vector of its own velocity (in its local frame).[1] It also measures the relative positions of the companions within a "visibility range" R_{viz}, but cannot distinguish between them or communicate with them.

The objective is to approach a speedy unpredictable target and then circumnavigate it at a pre-specified distance d_0 from it, in a given direction, and with a given rotational speed ω_0. An even self-distribution over the d_0-circle centered at the target should also be achieved. Every robot measures the relative position of the target but can sense neither the speed nor acceleration of the target and other robots.

The following nomenclature is adopted in the paper:

- $\boldsymbol{r}_i$, absolute position of the ith robot;
- $\boldsymbol{v}_i, \boldsymbol{a}_i$, vectors of its velocity and acceleration;
- $\boldsymbol{p}$, absolute position of the target (prey);
- $\boldsymbol{v}_p, \boldsymbol{a}_p$, vectors of its velocity and acceleration;
- $\|\cdot\|$, standard Euclidean norm;
- $\langle \cdot ; \cdot \rangle$, standard Euclidean inner product;
- $a_i := \|\boldsymbol{a}_i\|, v_i := \|\boldsymbol{v}_i\|, a_p := \|\boldsymbol{a}_p\|, v_p := \|\boldsymbol{v}_p\|$;
- d_i, distance between $\boldsymbol{r}_i$ and $\boldsymbol{p}$;
- $\mathcal{R}_i$, ray going from $\boldsymbol{p}$ to $\boldsymbol{r}_i$;
- φ_i, polar angle of $\mathcal{R}_i$ in the world frame;
- ϖ_i, angular velocity of its own pure rotation;
- $\boldsymbol{e}_i$, unit vector colinear with $\mathcal{R}_i$;
- $\boldsymbol{u}_i$, unit vector colinear with $\boldsymbol{v}_i$;
- α_i, angle from $\boldsymbol{e}_i$ to $\boldsymbol{v}_p$ (positive angles are counted counterclockwise);
- $\boldsymbol{w}^\perp$, vector $\boldsymbol{w}$ rotated through $+\pi/2$;
- $w_i^x := \langle \boldsymbol{w}_i ; \boldsymbol{e}_i \rangle, w_i^y := \langle \boldsymbol{w}_i ; \boldsymbol{e}_i^\perp \rangle$, where $w = v, a$;
- β_p, angle from $\boldsymbol{v}_p$ to $\boldsymbol{a}_p$;
- $\boldsymbol{F}_i$, a Cartesian frame rigidly attached to robot i;
- $\sigma = \pm 1$ gives the desired direction of rotation around the target (counterclockwise/clockwise);
- $\oplus$, addition modulo N.

Robot i is modeled as a double integrator

$$\ddot{\boldsymbol{r}}_i = \boldsymbol{a}_i, \quad \|\boldsymbol{a}_i\| \le \overline{a}, \quad \boldsymbol{r}_i(0) = \boldsymbol{r}_i^0, \quad \boldsymbol{v}_i(0) = \boldsymbol{v}_i^0, \qquad (1)$$

where $\boldsymbol{a}_i$ is the control input and $\|\boldsymbol{v}_i^0\| \le \overline{v}$. We assume that (1) is in effect if the speed $v_i \le \overline{v}$, and controller design is constrained to meet this bound. This permits us not to discuss what may occur above the bound or under a trespass attempt.

The control objective is in fact described in the following.

Definition 2.1: The robots *achieve regular circumnavigation of the target* if the following statements hold for any i, possibly after a proper re-enumeration of the robots:

i) $d_i \to d_0$ as $t \to \infty$;

[1] The latter measurement is not needed in some cases, see Remark 4.1.

ii) The angle from $\mathcal{R}_i$ to $\mathcal{R}_{i\oplus 1}$ goes to $2\pi/N$ as $t \to \infty$;
iii) Its interior does not contain robots for large t's;
iv) The angular velocity $\dot{\varphi}_i(t)$ goes to $\sigma\omega_0$ as $t \to \infty$.

This should be achieved without any data exchange among the robots in a fully decentralized and distributed fashion.

III. Assumptions and Necessary Conditions for the Mission Feasibility

To avoid setting our assumptions high above the necessary level, we first disclose the conditions that are necessary for the control objective to be attainable. The assumptions of our theoretical analysis will be slight enhancements of these.

Less knowledge about the target typically entails stronger demands to the robots. To address this knowledge, we define a *scenario* (of the target behavior) as the set $\mathfrak{S}$ of all feasible (subject to what is known) combinations $\mathfrak{s} = (a_p, \beta_p, v_p, t)$. The two simplest examples are as follows: **1.** Steady target: $\mathfrak{S} = \{(0, \beta_p, 0, t) : \beta_p \in \mathbb{R}\}$; **2.** Only upper bounds on a_p and v_p are known: $\mathfrak{S} = \{(a_p, \beta_p, v_p, t) : a_p \leq \overline{a}_p, \beta_p \in \mathbb{R}, v_p \leq \overline{v}_p\}$.

We start with a technical assumption about $\mathfrak{S}$.

Assumption 3.1: The scenario $\mathfrak{S}$ is invariant to the time shifts and the substitution $\beta_p \mapsto \beta_p \pm \pi$; the following function of $v_p \in \mathfrak{V} := \{v_p : \mathfrak{S}(v_p) \neq \emptyset\}$ and $\alpha \in \mathbb{R}$ is continuous

$$\xi(\alpha, v_p) := \sup_{(a_p,\beta_p)\in\mathfrak{S}(v_p)} \left(a_p |\sin(\alpha + \beta_p)|\right), \quad \text{where}$$
$$\mathfrak{S}(v_p) := \{(a_p, \beta_p) : (a_p, \beta_p, v_p) \in \mathfrak{S}\}. \tag{2}$$

Thanks to the first claim, t can be dropped in $\mathfrak{s}$. Assumption 3.1 clearly holds for the scenarios 1, 2.

Lemma 3.1: Suppose that irrespective of the angle relative to the target, robot i can maintain the distance d_0 to it and the angular speed $|\omega_i| = \omega_0$ of rotation around it. Then

$$\overline{v} \geq v_p + d_0\omega_0. \tag{3}$$

To achieve even local closed-loop stability, some controllability of the regulated outputs d_i, φ_i is classically needed. Since their degree is 2, local controllability means that irrespective of the target's behavior, any combination of signs in $\ddot{d}_i$ and $\ddot{\varphi}_i$ can be ensured by a proper choice of a feasible acceleration $\boldsymbol{a}_i$. We require this only on the targeted surface $d_i = d_0, \dot{d}_i = 0, \sigma\omega_i > 0$ of the state space of the robot. More precisely, we require this only in its part where $v_i < \overline{v}$. In the limit case $v_i = \overline{v}$, the need to respect the speed bound $v_i \leq \overline{v}$ reduces the set of feasible accelerations from the disc $\{\boldsymbol{a}_i : \|\boldsymbol{a}_i\| \leq \overline{a}\}$ to its half-disc given by $\langle \boldsymbol{a}_i; \boldsymbol{v}_i \rangle \leq 0$. So four "degrees-of-freedom" in manipulation of $\mathbf{sgn}\,\ddot{d}_i$ and $\mathbf{sgn}\,\ddot{\varphi}_i$ (any combination) become unrealistic. We sacrifice one "degree" and request only the sign $\mathbf{sgn}\,\ddot{\varphi}_i$ that is associated with respecting the speed bound.

Lemma 3.2: Suppose that robot i is locally controllable in the described sense. Then the following inequalities hold

$$\overline{a}^2 \geq \eta^2 + \xi^2, \quad \forall v_p \in \mathfrak{V}, \alpha \in \mathbb{R}, \tag{4}$$
$$\sqrt{1 - (v_p/\overline{v})^2 \cos^2\alpha} \geq \eta/\overline{a}, \quad \forall v_p \in \mathfrak{V}, \alpha \in \mathbb{R}, \tag{5}$$

where $\xi = \xi(\alpha, v_p)$ is given by (2) and

$$\eta = \eta(\alpha, v_p) := \xi(\alpha + \pi/2, v_p) + \left(\sqrt{\overline{v}^2 - v_p^2\cos^2\alpha} + v_p|\sin\alpha|\right)^2 / d_0. \tag{6}$$

We assume that the necessary conditions from Lemmas 3.1 and 3.2 hold with $>$ put in place of $\geq$ everywhere and that these $>$'s do not regress to $\geq$ as $t \to \infty$.

Assumption 3.2: There exist $\Delta_v, \Delta_a, \Delta > 0$ such that (3) and (4), (5) hold with $\overline{v} := \overline{v} - \Delta_v$ and $\overline{a} := \overline{a} - \Delta_a$, respectively, even if the left hand side of (5) is reduced by $\Delta < (2\overline{v})^{-1}\sqrt{2\overline{v}\Delta_v - \Delta_v^2}$.

By continuity, this remains true if $\overline{v}$ and $\Delta_v, \Delta_a, \Delta$ are slightly reduced. We assume that this is carried out. So $\overline{v}$ is a feasible (factually sub-critical) speed from now on.

Assumption 3.3: Under the even distribution over the d_0-circle centered at $\boldsymbol{p}$, any robot "sees" at least one partner:

$$2d_0\sin(\pi/N) < R_{\text{viz}}. \tag{7}$$

Here the distance between two adjacent points of the even distribution is on the left. Brutal violation of (7) ($<\mapsto>$) means that the feedback from the inter-robot distance is unavailable if the even distribution is nearly achieved. Then the control objective is clearly infeasible.

IV. Navigation Law

All robots are driven by a common navigation law, which individually operates on any of them. The proposed law is hybrid with three discrete modes: $\mathfrak{C}$ (search of controllability), $\mathfrak{A}$ (approaching the target), and $\mathfrak{M}$ (main mode).

The initial mode is $\mathfrak{C}$, where the goal is to drive the robot at a posture with $\dot{d}_i \approx 0$. This brings the local controllability that is addressed in Lemma 3.2 and needs $\dot{d}_i = 0$ or a like.

For mode $\mathfrak{A}$, the main goal is to drive d_i in a proximity of the desired value d_0. A secondary objective is to ensure that the robot rotates around the target in the desired direction σ.

Whereas modes $\mathfrak{C}$ and $\mathfrak{A}$ carry out preparatory operations, the main control objective is to be achieved in mode $\mathfrak{M}$.

Now we introduce the regulation rules for every mode.

Mode $\mathfrak{C}$: The acceleration $\boldsymbol{a}_i$ of robot i is maximal in magnitude, normal to its velocity, and is given by $\boldsymbol{a}_i = \bar{a} \cdot \boldsymbol{u}_i^{\perp}$.

Mode $\mathfrak{A}$: The control input $\boldsymbol{a}_i$ is given by

$$\boldsymbol{a}_i = a_i^x \boldsymbol{e}_i + a_i^y \boldsymbol{e}_i^{\perp}, \tag{8}$$
$$a_i^x := -\bar{a}_x(\alpha_i, v_p)\mathbf{sgn}\{\dot{d}_i + \mu\chi[d_i - d_0]\}, \tag{9}$$
$$a_i^y := -\bar{a}_y(\alpha_i, v_p) \cdot \mathbf{sgn}\left[\dot{\varphi}_i - \sigma c_\omega/d_i\right]. \tag{10}$$

In (8), the first and second addend are commissioned to regulate the distance d_i and angle φ_i, respectively. In (9) and (10), the maps $\overline{a}_x(\cdot), \overline{a}_y(\cdot), \chi(\cdot)$ and parameters $\mu, c_\omega > 0$ are to be chosen by the designer of the controller.

In (10), $\dot{\varphi}_i = \dot{\psi}_i + \varpi_i$, where ψ_i is the measured polar angle of $\mathcal{R}_i$ in the frame $\boldsymbol{F}_i$ and ϖ_i the measured angular velocity at which this frame rotates. Thus $\dot{\varphi}_i$ is accessible by the robot.

Any method to evaluate $\dot{d}_i, \dot{\psi}_i$ is acceptable; there is numerical differentiation of the readings d_i, ψ_i among the options. The angle α_i and speed v_p are computable from the

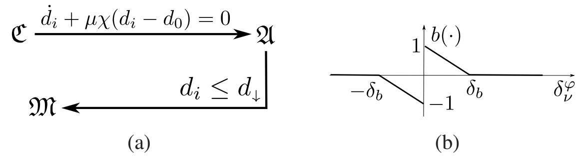

Fig. 1. (a) Switching logic; (b) Function $b(\cdot)$.

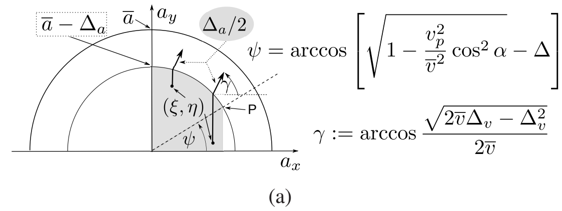

Fig. 2. Construction of $(\overline{a}_x, \overline{a}_y)$

equations $v_p \cos\alpha_i = v_i^x - \dot d_i$ and $v_p \sin\alpha_i = v_i^y - d_i \dot\varphi_i$, where v_i^x and v_i^y are measured by the robot due to our assumptions.

Remark 4.1: The reason for $\overline{a}_d, \overline{a}_\varphi$ to depend on α_i, v_p is that our design is under the conditions that are close to be necessary for controlability. Their appropriate enhancement enables one to pick $\overline{a}_d$ and $\overline{a}_\varphi$ constant (details will be given in the full version of the paper). This fully overcomes the need for computation of v_p, α_i and so for access to v_i^x, v_i^y, i.e., to the robot's own velocity in the local reference frame.

The parameter μ and map $\chi(\cdot)$ from (9) are also used to trigger the switch $\mathfrak{C} \to \mathfrak{A}$ according to the switching logic from Fig. 1(a), where $d_\downarrow > d_0$ is one more design parameter.

Mode $\mathfrak{M}$ uses extra parameters $\delta_b, \varkappa_b, \Delta^\varphi > 0$.

Definition 4.1: An *essential neighbor* of robot i is robot j within the visibility range R_{viz} of i for which $d_j < d_\downarrow$ and the angular distance $\rho_{i\to j|\sigma} \ge 0$ from i to j computed in direction of σ is nonzero but less than Δ^φ.

Thus any robot i can identify all its essential neighbors.

Formulas (8), (9) are in use for $\mathfrak{M}$; (10) is replaced by

$$a_i^y := -\bar{a}_y(\alpha_i, v_p) \cdot \mathbf{sgn}\,\Sigma_i - \varsigma_i, \tag{11}$$

$$\Sigma_i := \dot\varphi_i - \sigma\Xi(\rho_i), \quad \varsigma_i := \varkappa_b d_i \sum_\nu b\left(\delta_\nu^\varphi\right), \tag{12}$$

where $\delta_\nu^\varphi := \varphi_{j_{\nu+1}} - \varphi_{j_\nu}$. The map $\Xi(\cdot)$ is designer chosen, $b(\cdot)$ is given by Fig. 1(b), and $\rho_i := \min_j \rho_{i\to j|\sigma} > 0$, where $\min_j$ is over all essential neighbors j of i; if they do not exist, $\rho_i := \Delta^\varphi$. Robot i and these j's are ordered so that their polar angles $\varphi_{j_0} := \varphi_i, \varphi_{j_1}, \varphi_{j_2}, \ldots, \varphi_{j_s}$ ascend in direction of σ. In (12), the sum is from $\nu = 0$ to the first zero addend, and any sum over the empty set is defined to be 0.

In (11), the first addend is viewed as the main part of the control input, the second addend adds a "braking" correction to it. Formula (12) sets a "braking" effort ς_i for robot i so that it exceeds the similar effort of any close robot j in front.

Before the above control law is put in use, a preliminary maneuver should be carried out: the robots should be brought far enough from one another and the target, they should be accelerated to the maximal speed. We omit discussion of related and fairly trivial implementation issues and merely state the targeted result of this maneuver as our last assumption.

Assumption 4.1: At the start of the mission, the robots are deployed so that for Δ_v, Δ_a from Assumption 3.2,

$$v_i(0) = \overline{v}, \quad d_i(0) > \left[\frac{3\pi + 2}{\overline{a}} + \frac{4}{\Delta_*}\right]\overline{v}^2 + d_\downarrow,$$
$$|\boldsymbol{r}_i(0) - \boldsymbol{r}_j(0)| > [3\pi/\overline{a} + 2/\Delta_*]\,\overline{v}, \tag{13}$$

where $\Delta_* := \Delta_a/(4\overline{v})\sqrt{3\overline{v}^2 + (\overline{v} - \Delta_v)^2}$.

For the proposed discontinuous control law, the solutions of the closed-loop system are meant in Filippov's sense [23].

V. Main results

The first result shows that the navigation law is intrinsically capable of solving the mission under proper tuning.

Theorem 5.1: Suppose that Assumptions 3.1—3.3 and 4.1 hold. Then the proposed navigation law can be tuned so that the speed and acceleration bounds are always respected $v_i \le \overline{v}, a_i \le \overline{a}$ by all robots, the robots achieve regular circumnavigation of the target, as is described in Definition 2.1, and do not collide with one another and the target.

The reminder of the section discusses controller tuning based on $\Delta_v, \Delta_a, \Delta$ from Assumption 3.2 and Δ_* from (13).

Function $\chi(\cdot)$ is piece-wise smooth and such that

$$\chi(0) = 0, \quad \chi(z) < 0\ \forall z < 0, \quad \chi(z) > 0\ \forall z > 0,$$
$$\overline{\chi} := \sup_{z\in\mathbb{R}} |\chi(z)| < \infty, \quad \overline{\chi}' := \sup_{z\in\mathbb{R}} |\chi'(z\pm)| < \infty.$$

Examples are given by the linear function with saturation and by $\chi(z) = a\arctan(z/b), a, b > 0$.

Maps $\overline{a}_x$, $\overline{a}_y$ in (9)–(11) are subjected to the following constraints, which must hold for any α, v_p and where ξ, η, and Δ_* are taken from (2), (6) and (13), respectively:

$$\overline{a}_x \ge \eta(\alpha, v_p) + \Delta_a/(4\overline{v})\sqrt{2\overline{v}\Delta_v - \Delta_v^2} > 0, \tag{14}$$

$$\overline{a}_y \ge \xi(\alpha, v_p) + \Delta_*, \quad \overline{a}_x^2 + (\overline{a}_y + \Delta_a/2)^2 \le \overline{a}^2, \tag{15}$$

$$\frac{\overline{a}_y}{\overline{a}_x} \ge \frac{v_p|\cos\alpha|}{\sqrt{\overline{v}^2 - v_p^2\cos^2\alpha}} + \Delta. \tag{16}$$

Lemma 5.1: There exist continuous functions $\overline{a}_x = \overline{a}_x(\alpha, v_p)$ and $\overline{a}_y = \overline{a}_y(\alpha, v_p)$ for which (14)—(16) do hold. The required point $(\overline{a}_x, \overline{a}_y)$ can be obtained by the shift of the point (η, ξ) as is described in Fig. 2.

In Fig. 2, (η, ξ) lies in the grey domain thanks to (4), (5).

Parameter μ. By Assumption 3.2, the unknown speed

$$v_p \le \overline{v}_p := \overline{v} - \Delta_v^\omega, \quad \text{where} \quad \Delta_v^\omega := \Delta_v + d_0\omega_0 \tag{17}$$

and $\le$ holds for all $v_p \in \mathfrak{V}$. Parameter μ is chosen so that

$$\mu\overline{\chi} \le \Delta_v/2, \quad \Delta_* d_0/[2(\overline{v} + \overline{v}_p)], \tag{18}$$

$$\kappa(\mu, v_p) := \frac{\mu\overline{\chi}(\overline{v}_p + \mu\overline{\chi})}{\sqrt{\overline{v}^2 - (\overline{v}_p + \mu\overline{\chi})^2}} < \frac{\Delta_v}{2}, \tag{19}$$

$$2\frac{\overline{v} + \overline{v}_p}{d_0}\kappa + \mu^2\overline{\chi}\,\overline{\chi}' < \frac{\sqrt{2\overline{v}\Delta_v - \Delta_v^2}}{4\overline{v}}\Delta_a, \tag{20}$$

$$\frac{\mu\overline{\chi}\,\overline{v}^2}{\left[\overline{v}^2 - (\overline{v}_p + \mu\overline{\chi})^2\right]^{3/2}} < \Delta. \tag{21}$$

These are evidently met by any sufficiently small μ.

Parameters $\boldsymbol{\Delta^{\varphi}, \delta_b}$ (tacitly used in (12)) are such that

$$2\pi/N < \Delta^{\varphi} < 2\arcsin\left[R_{\text{viz}}/(2d_0)\right], \quad \delta_b < \Delta^{\varphi}/N. \qquad (22)$$

This is feasible thanks to (7).

Function $\boldsymbol{\Xi(\rho)}$ of $\rho \in [0, 2\pi]$ is smooth and such that

$$\Xi(0) \geq 0, \quad \Xi[2\pi/N] = \omega_0, \quad \min_{\rho\in[0,2\pi]} \Xi'(\rho) > 0, \qquad (23)$$

$$\overline{\omega} < \omega_0 + \Delta_v/(2d_0), \quad 4d_0\overline{\omega}\,\overline{\omega}' < \Delta_*, \qquad (24)$$

$$\text{where} \quad \overline{\omega} := \Xi(2\pi), \quad \overline{\omega}' := \max_{\rho\in[0,2\pi]} \Xi'(\rho).$$

These do hold for any linear ascending function with sufficiently small slope and $\Xi(2\pi/N) = \omega_0$.

Parameter $\boldsymbol{\varkappa_b}$ from (12) is chosen so that

$$0 < \varkappa_b < (2N)^{-1} \min\{\Delta_*/d_0; \Delta_a\}. \qquad (25)$$

This is satisfied by all small enough $\varkappa_b > 0$.

Threshold $\boldsymbol{d_{\downarrow}}$ from Fig. 1(a) is chosen so that

$$\overline{\omega}(d_{\downarrow} - d_0) < \Delta_v/2 - d_0[\overline{\omega} - \omega_0],$$

$$d_{\downarrow} > d_0,\; 4d_{\downarrow}\overline{\omega}\,\overline{\omega}' < \Delta_*,\; \Upsilon(d_{\downarrow} - d_0) < \varkappa_b < \frac{\Delta_*}{2Nd_{\downarrow}},$$

$$\text{where} \quad \Upsilon := (2\overline{a} - \Delta_* + 2\overline{\omega}\mu\overline{\chi})/d_0^2 + 2\overline{\omega}\mu\overline{\chi}'/d_0,$$

$$2d_{\downarrow}\sin\frac{\Delta^{\varphi}}{2}, \sqrt{(d_{\downarrow} - d_0)^2 + 4d_{\downarrow}d_0\sin^2\frac{\Delta^{\varphi}}{2}} < R_{\text{viz}}.$$

Due to (22), (24), (25), these relations hold as $d_{\downarrow} \to d_0+$ and so are satisfied by all $d_{\downarrow} > d_0, d_{\downarrow} \approx d_0$.

Parameter $\boldsymbol{c_{\omega}}$ from (10) is chosen to be $c_{\omega} := d_{\downarrow}\Xi(0)$.

Theorem 5.2: Suppose that Assumptions 3.1—3.3 and 4.1 hold and the parameters of the control law are chosen subject to the recommendations stated in this section. Then the conclusion of Theorem 5.1 is true.

VI. Results of Computer Simulation Tests

The numerical values of the parameters used in the tests are as follows: $N = 5$, $\overline{a} = 1.2m/s^2$, $\overline{v} = 5.0m/s$, $d_0 = 30.0m$, $\omega_0 = 0.08rad/s$, $d_{\downarrow} = 40m$, $\delta_b = 0.2rad$, $\mu = 0.01s^{-1}$, $\varkappa_b = 0.002s^{-2}$, $R_{\text{viz}} = 40m$, $\Delta^{\varphi} = 1.3rad$, $\Xi(\rho) = 0.05 + 0.15\rho/(2\pi)$, $\overline{a}_x \equiv 1.0m/sec^2$, $\overline{a}_y \equiv 0.2m/sec^2$, $\tau = 1.0s$, $\chi(d) = 80\arctan\frac{d}{10}$, where τ is the control update period. Since $\overline{a}_x, \overline{a}_y$ are constant, access by every robot to its own velocity is not needed; see Remark 4.1. The distance measurements were corrupted by zero-mean Gaussian noises with the standard deviation $\approx 0.01m$. The simplest two-point quotient was used to assess time-derivatives. Multimedia of extended versions of all tests are available at `goo.gl/rVvF1v`.

In Figs. 3—5, the target $\boldsymbol{p}$ is shown as a blue dot, the blue circle is the locus of points at the distance d_0 from $\boldsymbol{p}$, the robots are depicted as red triangles. Recent porions of their paths are shown in pink, the remainder may be erased.

In Fig. 3, the target slowly traces an ellipsoidal path with a constant speed. Two (*distant*) robots are initially far from $\boldsymbol{p}$ and are not in the frame of Fig. 3(a), which shows the initial deployment of only the three other, *close* robots. In Figs. 3(b) and (c), the first and all of them, respectively, arrive at the distance d_0 to $\boldsymbol{p}$. One of the distant robots is already in the frame at the second moment. In Fig. 3(d), it reaches the distance d_0 to $\boldsymbol{p}$ and makes the robots' distribution rather uneven. As is shown in Fig. 3(e), this is promptly fixed, whereas the last distant robot gets in the frame. Later on, it arrives at the distance d_0 to $\boldsymbol{p}$ and an even distribution of five robots is achieved. Afterwards, this distance, distribution, and the requested angular speed of rotation around $\boldsymbol{p}$ are maintained. Fig. 3(f) corresponds to an occasional moment from this part of the experiment. Details of convergence

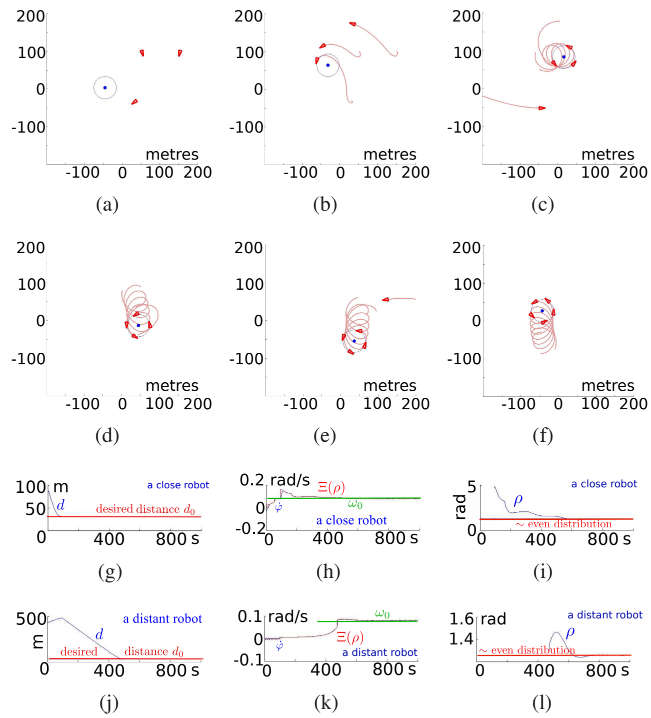

Fig. 3. Circumnavigation of a regularly moving target

(given in Figs. 3(g–i) for a close robot and in Figs. 3(j–l) for the "laggard" one) provide an extra evidence that the control law solves the mission despite target's motion.

In Fig. 4, the target tries to break free out of encirclement by means of irregular moves, abrupt acceleration, deceleration, and turning. The target does not yet pose challenges exceeding the capacity of the robots: the necessary conditions from Lemmas 3.1 and 3.2 are met. Figs. 4(a–f) exhibit the situations at various successive moments within this hunting operation. Figs. 4(g–l) illustrate that despite the target's attempts, it does not succeed to escape from encircling or at least to visibly worsen the performance of the robotic team.

In Fig. 5, the escaping attempts of the target become sharper so that the necessary conditions from Lemmas 3.1 and 3.2 are temporarily violated from time to time. However, Fig. 5 shows that the robots are still able to carry out the circumnavigation mission, though the accuracy of achieving an even distribution and desired angular speed of rotation around the target degrades whereas that of maintaining the required distance to the target still seems to be very good.

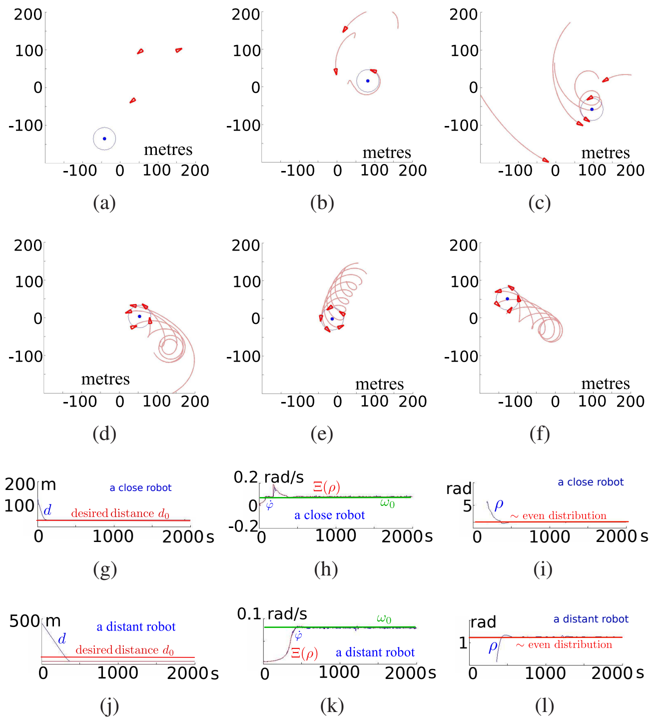

Fig. 4. Circumnavigation of an irregularly moving target

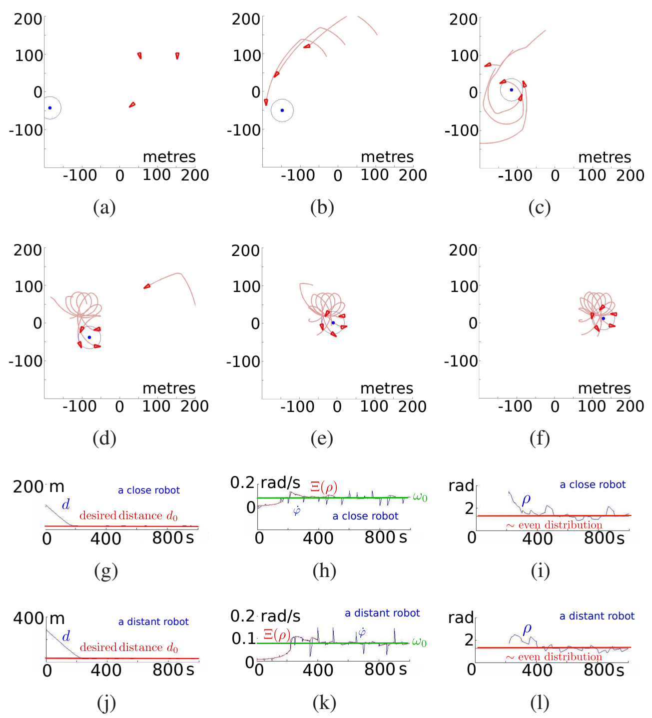

Fig. 5. Circumnavigation of an irregularly maneuvering and speedy target

VII. Conclusions

The paper proposed a distributed control law that drives a team of acceleration- and speed-limited robots into an even encircling formation around an unpredictably maneuvering target. The global convergence of the control law is rigorously justified and confirmed via computer simulation.

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