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Robust observer–based control for TS Fuzzy models Application to vehicle lateral dynamics

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***Abstract*— This paper deals with the observer based robust control design problem for TS fuzzy systems with time-varying uncertain parameters. Sufficient design conditions of the fuzzy observer and the fuzzy controller are first proposed. Due to the use of Young relation, and the cone-complementary, the non-convex and Bilinear problems are then converted into convex optimization one with LMI constraints. Controller and observer gains are obtained under $\mathscr{L}_2$-bounded disturbance and $\mathscr{L}_2$-norm input constraint by set feasible solutions of LMI conditions. The effectiveness of the proposed fuzzy controller and fuzzy observer design methodology is finally demonstrated through numerical simulations on vehicle lateral dynamic model.**

***Keywords*—** Linear matrix inequality (LMI), observer-based control, Tagaki-Sugeno fuzzy model, uncertain systems, vehicle dynamics.

I. INTRODUCTION

Since the introduction the TS fuzzy models in 1985 to describe the nonlinear system behaviors [4], the fuzzy model based control has been successfully investigated due to the capacity of the TS fuzzy to approximate a large class of the complex nonlinear systems. The stability analysis and control design of this class of systems via LMI [3] are, generally presented as the convex optimization problems. The stabilizing output-feedback problem for the TS fuzzy systems with or without uncertainties have been investigated in [16], [17] and [18] and the design conditions are solved using two step algorithms. The observer based controller for linear systems have been studied in [10,11] using the Young's inequality to relax conditions, and a new two-step design technique [9] for a large uncertainty with more degree of freedom. For uncertain TS fuzzy systems, the observer based stabilizing control problem has been solved using LMI approach in [6,7]. However, the proposed single step algorithm requires to previously choose some tuning parameters. In this work, a unified observer based control design method for uncertain TS fuzzy systems with external disturbances and input constraints, is discussed. The set of design conditions are presented in LMI form with optimizing the tuning parameters. By expanding the Lyapunov stability theory and LMI approach, robust fuzzy observer-based controller for with external perturbations satisfied $\mathscr{L}_2$-norm and input constraints is parametrized in a set of LMI constraints. For the analysis and the design, the nonconvex optimization problem will be transformed into an optimization problem subject to LMI constraints due to the cone complementarity linearization - CCL method. Consequently, the less conservative results and better disturbance attenuation performance are derived.

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This paper is organized as follows: the problem statement and preliminaries are presented in Section 2. In Section 3, a summary of the available results in the literature for observer of TS fuzzy system, and the design of robust fuzzy observer based control via LMI approach are analysed. In Section 4, an application of the vehicle lateral dynamics control is presented. Finally, a conclusion is given in last Section.

Notations. Through this note, we use the following notations: symbol * is used as symmetric, and I is the identity matrix with appropriate dimension in block matrices. $P \succ 0$ that means P is symmetric positive definite matrix. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, the sum of transposition matrices $\mathscr{H}_e\{U\} = U + U^T$, and the block matrix:

$$X = \begin{bmatrix} X_{1,1} & \cdots \\ * & X_{i,i} \end{bmatrix}.$$

II. PROBLEM STATEMENT

Consider now the continuous-time TS model for nonlinear dynamical system in presence of disturbance, defined by the following set of equations:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(\alpha(t)) \left[(A_i + \Delta A_i(t))x(t) + B_i u(t) \right] + w(t) \\ y(t) &= (C + \Delta C(t))x(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, is the state vector, $y \in \mathbb{R}^p$ is the output measurement and $u \in \mathbb{R}^m$ is the control input vector, $w(t)$ is external disturbances assumed $\mathscr{L}_2$-Bounded. The nominal matrices $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, and $C_i \in \mathbb{R}^{p\times n}$ are constant of adequate dimensions. Firstly, we consider the following assumptions:

Assumption 1: Pairs (A_i, B_i) and (A_i, C) are respectively stabilizable and detectable.

Assumption 2: Uncertain matrices $\Delta A_i(t)$ and $\Delta C(t)$ are unknown representing time-varying parameter and satisfy:

$\Delta A_i(t) = M_1 F_i(t) N_1$, and $\Delta C(t) = M_2 F_{r+1}(t) N_2$,

with $F_k^T(t) F_k(t) \le I$, for $k = 1, 2, \ldots (r+1)$.

Assumption 3: $h_i(\alpha)$ do not depend on the state variables estimated by a fuzzy observer.

Lemma 1: [5] *$\forall\, \varepsilon$ positive constant, and the real matrices M, N, and $F \in \mathbb{R}$ with appropriate dimension, such that $F^T(t) F(t) \le I$ the following inequality holds:*

$$MFN + N^T F^T M^T \preceq \frac{1}{\varepsilon} MM^T + \varepsilon N^T N \tag{2}$$

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The structure of the observer-based controller we consider in this paper is under the form:

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{4} h_i(\alpha(t))\left(A_i\hat{x}(t) + B_i u(t) + L_i\left(y(t) - C\hat{x}(t)\right)\right)\\ u(t) &= -\sum_{j=1}^{4} h_j(\alpha(t))K_j\hat{x}(t)\end{aligned} \tag{3}$$

where $K_j \in \mathbb{R}^{m\times n}$ is the controller gains, and $L_i \in \mathbb{R}^{n\times p}$ is the observer gains to be determined. Denote $e(t) = x(t) - \hat{x}(t)$ error estimate and defining a vector $z(t) = [x(t) \quad e(t)]^T$ then the augmented system can be written as:

$$\begin{aligned}\begin{bmatrix}\dot{x}(t)\\ \dot{e}(t)\end{bmatrix} &= \sum_{i,j=1}^{r} h_i(\alpha)h_j(\alpha)\underbrace{\begin{bmatrix}A_i - B_iK_j + \Delta A_i & B_iK_j\\ \Delta A_i - L_i\Delta C & A_i - L_iC\end{bmatrix}}_{A_z}\underbrace{\begin{bmatrix}x(t)\\ e(t)\end{bmatrix}}_{z}\\ &+ \underbrace{\begin{bmatrix}I_n\\ I_n\end{bmatrix}}_{B_w}w(t)\end{aligned} \tag{4}$$

The robust H_∞ observer and controller gains of TS fuzzy parametric uncertain system (4) are calculated such that the following two requirements are satisfied:

- The closed loop system (4) is robustly asymptotically stable when $w(t) = 0$
- Letting $\gamma > 0$ be a given constant, under zero-initial conditions, the following H_∞ performance index holds:

$$\int_0^\infty z(t)^T z(t)\,dt \le \gamma^2 \int_0^\infty w(t)^T w(t)\,dt \tag{5}$$

III. Observer-Based Control Design

Theorem 1: For positive scalars ε_k and γ, $\lambda > 0$, system dynamics (4) is robustly asymptotically stable with the H_∞ *performance attenuation-γ* in (5), if there exist two positive definite matrices X, $P_2 \in \mathbb{R}^{n\times n}$, and matrices $Q_{1j} \in \mathbb{R}^{m\times n}$, $Q_{2i} \in \mathbb{R}^{n\times p}$ such that the following inequalities are satisfied.

$X \succ 0$, $P_2 \succ 0$, $\varepsilon_k > 0$, $k = 1,2,3,4$, and $i,j = 1,2, \ldots r$.

$$\begin{bmatrix}\Xi_{ij} & \Omega\\ * & \mathbb{C}\end{bmatrix} \prec 0 \tag{6}$$

where:

$$\Xi_{ij}(1,1) = XA_i^T + A_iX - Q_{1j}^TB_i^T - B_iQ_{1j},$$

$$\Xi_{ij}(1,2) = 0,$$

$$\Xi_{ij}(2,2) = A_i^TP_2 + P_2A_i - C^TQ_{2i}^T - Q_{2i}C + \gamma^{-1}I_n,$$

$$\Omega = \begin{bmatrix}I_n & X & XN_1^T & XN_2^T & BQ_{1j} & M_1 & 0 & 0 & 0\\ P_2 & 0 & 0 & 0 & 0 & 0 & P_2M_1 & Q_{2i}M_2 & I_n\end{bmatrix}$$

and $\mathbb{C} = diag(-\gamma I; -\gamma I; -(\varepsilon_1 + \varepsilon_2)^{-1} I; -\varepsilon_3^{-1}I; -\varepsilon_4^{-1}X;$
$-\varepsilon_1 I; -\varepsilon_2 I; -\varepsilon_3 I; -\varepsilon_4 X)$.

observer and controller gains are given by $K_j = Q_{1i}X^{-1}$ and $L_i = P_2^{-1}Q_{2i}$.

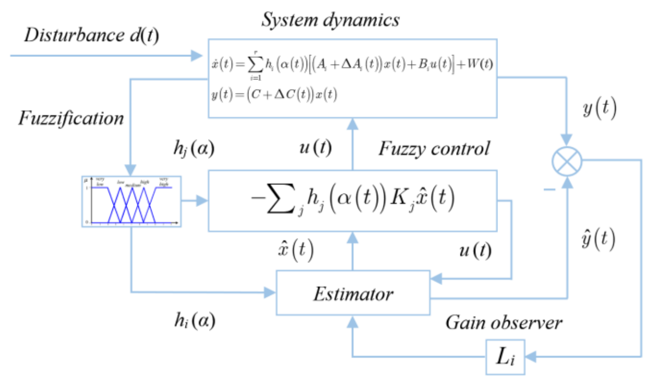

Figure. 1. *The block diagram of the system dynamic.*

Proof: Consider the Lyapunov function associated to dynamics (4):

$$V(z(t)) = z(t)^T P_z z(t) \tag{7}$$

with P_z is the *block diagonal* matrices of the positive definite matrices $P_i \in \mathbb{R}^{n\times n}$, i = 1, 2. Then the time derivate of $V(z(t))$ along the trajectories of (4) is given by:

$$\begin{aligned}\dot{V}(z(t)) &= z^T\sum_{ij} h_i(\alpha)h_j(\alpha)\left(A_z^TP_z + P_zA_z\right)z\\ &+ w^TB_w^TP_zz + z^TP_zB_ww\end{aligned} \tag{8}$$

With above analysis, to satisfy the attenuation level in (5) the following conditions must be satisfied:

$$\dot{V}(z(t),t) + \gamma^{-1}z(t)^Tz(t) - \gamma w(t)^Tw(t) < 0 \tag{9}$$

Which means:

$$\begin{aligned}z^T\left(\Xi_{ij}^0 + \Xi_{ij}^\Delta + \gamma^{-1}I\right)z + w(t)^TB_w^TP_zz(t)\\ + z(t)^TP_zB_ww(t) - \gamma w(t)^Tw(t) < 0\end{aligned} \tag{10}$$

where:

$$\Xi_{ij}^0 = \begin{bmatrix}\mathcal{H}_e(P_1(A_i - B_iK_j)) & P_1B_iK_j\\ * & \mathcal{H}_e(P_2(A_i - L_iC))\end{bmatrix},$$

$$\Xi_i^\Delta = \begin{bmatrix}\Delta A_i^TP_1 + P_1\Delta A_i & (\Delta A_i - L_i\Delta C)^TP_2\\ * & 0\end{bmatrix},$$

with $i, j = 1,2, \ldots r$.

Defining $Q_{2i} = P_2L_i$, the uncertain matrices are deployed as follows:

$$\begin{aligned}\Xi_i^\Delta &= \begin{bmatrix}P_1M_1\\ 0\end{bmatrix}F_i\begin{bmatrix}N_1 & 0\end{bmatrix} + \begin{bmatrix}N_1 & 0\end{bmatrix}^TF_i^T\begin{bmatrix}P_1M_1\\ 0\end{bmatrix}^T\\ &+ \begin{bmatrix}0\\ P_2M_1\end{bmatrix}F_i\begin{bmatrix}N_1 & 0\end{bmatrix} + \begin{bmatrix}N_1 & 0\end{bmatrix}^TF_i^T\begin{bmatrix}0\\ P_2M_1\end{bmatrix}^T\end{aligned}$$

$$+\begin{bmatrix}0\\-Q_{2i}M_2\end{bmatrix}F_{r+1}\ N_2\quad 0\ +\ N_2\quad 0^{\ T}\ F_{r+1}^{T}\begin{bmatrix}0\\-Q_{2i}M_2\end{bmatrix}^{T} \tag{11}$$

According to *assumption* 2, and *lemma* 1 we have the inequality:

$$\begin{aligned}\Xi_i^{\Delta}\ \preceq\ &\varepsilon_1^{-1}\begin{bmatrix}P_1M_1\\0\end{bmatrix}\begin{bmatrix}P_1M_1\\0\end{bmatrix}^T+\varepsilon_2^{-1}\begin{bmatrix}0\\P_2M_1\end{bmatrix}\begin{bmatrix}0\\P_2M_1\end{bmatrix}^T\\&+\varepsilon_3^{-1}\begin{bmatrix}0\\-Q_{2i}M_2\end{bmatrix}\begin{bmatrix}0\\-Q_{2i}M_2\end{bmatrix}^T\\&+\ \varepsilon_1+\varepsilon_2\ \cdot\ N_1\quad 0^{\ T}\ N_1\quad 0+\varepsilon_3\cdot\ N_2\quad 0^{\ T}\ N_2\quad 0\end{aligned} \tag{12}$$

$\forall\ \varepsilon_k$ positive, $k = 1, 2, 3$.

From matrix inequalities (10) and (12), rearranging as:

$$\begin{bmatrix}\Xi_i^1+\Xi_{ij}^2 & P_zB_w\\ * & -\gamma I\end{bmatrix}\prec\ 0 \tag{13}$$

where:

$$\Xi_{ij}^2=\mathcal{H}_e\left\{\begin{bmatrix}P_1B_i\\0\end{bmatrix}\cdot\begin{bmatrix}-K_j & K_j\end{bmatrix}\right\} \tag{14}$$

$$\begin{aligned}\Xi_i^1\ \ 1{,}1\ &=\mathcal{H}_e\ \ P_1A_i\ \ +\ \ \varepsilon_1+\varepsilon_2\ \ N_1^TN_1+\varepsilon_3N_2^TN_2\\&\quad+\varepsilon_1^{-1}P_1M_1M_1^TP_1+\gamma^{-1}I,\\ \Xi_i^1\ \ 1{,}2\ &=0,\\ \Xi_i^1\ \ 2{,}2\ &=\mathcal{H}_e\ \ P_2A_i-Q_{2i}C\ \ +\varepsilon_2^{-1}P_2M_1M_1^TP_2\\&\quad+\varepsilon_3^{-1}Q_{2i}M_2M_2^TQ_{2i}^T+\gamma^{-1}I,\end{aligned}$$

Defining matrices $Q_{1j}=K_jX$, with $X\ =\ P_1^{-1}$ a symmetric definite positive, and multiplying both side of (13) with $diag\ \ X,I,I$, we get:

$$\begin{bmatrix}\Xi_i^3+\Xi_{ij}^4 & \Upsilon\\ * & -\gamma I\end{bmatrix}\prec\ 0 \tag{15}$$

where i, j = 1,2,3 4 and

$$\begin{aligned}\Xi_{ij}^3\ \ 1{,}1\ &=\mathcal{H}_e\ \ A_iX-B_iQ_{1j}\ \ +\ \ \varepsilon_1+\varepsilon_2\ \ XN_1^TN_1X\\&\quad+\varepsilon_3XN_2^TN_2X+\varepsilon_1^{-1}M_1M_1^T+\gamma^{-1}XX,\\ \Xi_{ij}^3\ \ 1{,}2\ &=0,\\ \Xi_{ij}^3\ \ 2{,}2\ &=\mathcal{H}_e\ \ P_2A_i-Q_{2i}C\ \ +\varepsilon_2^{-1}P_2M_1M_1^TP_2\\&\quad+\varepsilon_3^{-1}Q_{2i}M_2M_2^TQ_{2i}^T+\gamma^{-1}I,\\ \Xi_{ij}^4&=\begin{bmatrix}0 & B_iK_j\\ * & 0\end{bmatrix},\qquad\text{and }\ \Upsilon=\begin{bmatrix}I\\P_2\end{bmatrix}.\end{aligned}$$

Following the Young relation [1], for a positive scalar ε_4:

$$\begin{aligned}\Xi_{ij}^4&=\begin{bmatrix}0\\I\end{bmatrix}\begin{bmatrix}B_iK_j\\0\end{bmatrix}^T+\begin{bmatrix}B_iK_j\\0\end{bmatrix}\begin{bmatrix}0\\I\end{bmatrix}^T\ \preceq\ \varepsilon_4^{-1}\begin{bmatrix}0\\I\end{bmatrix}X^{-1}\begin{bmatrix}0\\I\end{bmatrix}^T\\&\qquad+\varepsilon_4\begin{bmatrix}B_iK_j\\0\end{bmatrix}X\begin{bmatrix}B_iK_j\\0\end{bmatrix}^T\\&=\varepsilon_4^{-1}\begin{bmatrix}0\\I\end{bmatrix}X^{-1}\begin{bmatrix}0\\I\end{bmatrix}^T+\varepsilon_4\begin{bmatrix}B_iQ_{1j}\\0\end{bmatrix}X^{-1}\begin{bmatrix}B_iQ_{1j}\\0\end{bmatrix}^T\end{aligned} \tag{16}$$

By incorporating inequalities (15) and (16), subsequently applying Schur complement then the conditions can be represented as LMIs (19) which complete the proof.

Lemma 2: [3] *Assume that the initial condition is known and there exist a positive scalar* $\theta\in[0,1]$. *Constraint* $\left\|u\ \ t\right\|_2<\mu$ *is enforced* $\forall\ t\in[0,\infty)$ *if the LMIs*:

$$\begin{bmatrix}\theta & x^T\ 0\\ * & X\end{bmatrix}\succeq 0 \tag{17}$$

$$\begin{bmatrix}X & Q_{1j}^T\\ * & \mu^2I\end{bmatrix}\succeq 0 \tag{18}$$

hold, where $X=P_1^{-1}\in\mathbb{R}^{n\times n}$ symmetric definite positive, and matrices $Q_{1j}\in\mathbb{R}^{m\times n}$, $Q_{1j}=K_jX$.

$$\left[\begin{array}{cc|ccccccccc}\Xi_{ij}(1,1) & \Xi_{ij}(1,2) & I & X & XN_1^T & XN_2^T & B_iQ_{1j} & M_1 & 0 & 0 & 0\\ * & \Xi_{ij}(2,2) & P_2 & 0 & 0 & 0 & 0 & 0 & P_2M_1 & Q_{2i}M_2 & I\\ \hline * & * & -\gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -\gamma I & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & \dfrac{-1}{\varepsilon_1+\varepsilon_2}I & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -\varepsilon_3^{-1}I & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & -\varepsilon_4^{-1}X & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & * & -\varepsilon_1I & 0 & 0 & 0\\ * & * & * & * & * & * & * & * & -\varepsilon_2I & 0 & 0\\ * & * & * & * & * & * & * & * & * & -\varepsilon_3I & 0\\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_4X\end{array}\right]\prec 0 \tag{19}$$

Remark. To avoid the nonlinearity term in condition (19), it is necessary to choose scalars $\varepsilon_k > 0$, $k = 1,2,3$, and 4. Besides, using the Young relative to handle with bilinear coupling in (14) related to control gain K_j that results an implicit conservatism condition. In such context, the cone-complementary linearization deployed with 2nd step design method in [14,16,18] is the reasonable approach. At this stage, we use the substitution as [14] gives a positive scalar $\varepsilon_5 > 0$ such that $\varepsilon_5 = \sqrt{\varepsilon_4}$, $V_1 = \varepsilon_5^{-1} X \varepsilon_5^{-1}$, and $V_2 = \varepsilon_5 X \varepsilon_5$ then according CCL method the nonconvex problems are transformed into the optimization problem as follows:

$$\text{Minimize } Tr\ \ \bar{E}E + XP_1 + 0.5\ \ \bar{V}_1 V_1 + \bar{V}_2 V_2 + \bar{\varepsilon}_5 X \bar{\varepsilon}_5 \bar{V}_1 + .. \\ + \varepsilon_5 X \varepsilon_5 \bar{V}_2 + \varepsilon_5 P_1 \varepsilon_5 V_1 + \bar{\varepsilon}_5 P_1 \bar{\varepsilon}_5 V_2 \quad (20)$$

subject to (17), (18), (19) and these inequalities

$$\begin{bmatrix} \bar{E} & I \\ * & E \end{bmatrix} \succeq 0, \begin{bmatrix} X & I_n \\ * & P_1 \end{bmatrix} \succeq 0, \begin{bmatrix} \bar{V}_i & I_n \\ * & V_i \end{bmatrix} \succeq 0, \begin{bmatrix} X & \varepsilon_5 I_n \\ * & \bar{V}_1 \end{bmatrix} \succeq 0,$$
$$\begin{bmatrix} X & \bar{\varepsilon}_5 I_n \\ * & \bar{V}_2 \end{bmatrix} \succeq 0, \begin{bmatrix} P_1 & \bar{\varepsilon}_5 I_n \\ * & V_1 \end{bmatrix} \succeq 0, \begin{bmatrix} P_1 & \varepsilon_5 I_n \\ * & V_2 \end{bmatrix} \succeq 0. \quad (21)$$

with setting $\bar{E} = E^{-1}$ and $E = diag\ \ \varepsilon_i\ \ , i = 1,2,..5$.

Given a small positive scalar $\delta > 0$, according to the feasible solution of the above minimization is smaller than $|16n + \delta|$, that could be say from *Theorem* 1 the close-loop system (4) is asymptotically stable with a prescribed robust H_∞ disturbance attenuation level-γ and $\mathscr{L}_2$-gain between the state estimate and observer-based control input strictly less than μ can be computed as $K_j = Q_{1j} X^{-1}$ and $L_i = P_2^{-1} Q_{2i}$.

The standard method of the iterative optimization as the following algorithm:

Algorithm 1.

Step 1: Choose the prerequisite parameters γ_0 and μ_0 such that conditions (6), (20) and (21) are feasible.

Step 2: Assigning *above solutions* be initial set $\bar{E}_0, E_0, X_0, P_{10}, \bar{V}_{10}, V_{10}, \bar{V}_{20}, V_{20}$, then let k = 0.

Step 3: Solve LMI problem given as:

$$\boldsymbol{min}\ \ Tr\ \ J_1 + J_2 + 0.5 J_3 + 0.5 J_4 \quad (22)$$

subject to (17), (18), (19), and (21).

with:

$$J_1 = E\bar{E}_k + \bar{E}E_k,$$
$$J_2 = XP_{1k} + X_k P_1,$$
$$J_3 = \ V_1 \bar{V}_{1k} + V_{1k} \bar{V}_1 + V_2 \bar{V}_{2k} + V_{2k} \bar{V}_2$$
$$J_4 = \ \bar{\varepsilon}_5 X_k \bar{\varepsilon}_{5k} \bar{V}_{1k} + \bar{\varepsilon}_{5k} X \bar{\varepsilon}_{5k} \bar{V}_{1k} + \bar{\varepsilon}_{5k} X_k \bar{\varepsilon}_5 \bar{V}_{1k} + \bar{\varepsilon}_{5k} X_k \bar{\varepsilon}_{5k} \bar{V}_1$$
$$+\ \varepsilon_5 X_k \varepsilon_{5k} \bar{V}_{2k} + \varepsilon_{5k} X \varepsilon_{5k} \bar{V}_{2k} + \varepsilon_{5k} X_k \varepsilon_5 \bar{V}_{2k} + \varepsilon_{5k} X_k \varepsilon_{5k} \bar{V}_2$$
$$+\ \varepsilon_5 P_{1k} \varepsilon_{5k} V_{1k} + \varepsilon_{5k} P_1 \varepsilon_{5k} V_{1k} + \varepsilon_{5k} P_{1k} \varepsilon_5 V_{1k} + \varepsilon_{5k} P_{1k} \varepsilon_{5k} V_1$$
$$+\ \bar{\varepsilon}_5 P_{1k} \bar{\varepsilon}_{5k} V_{2k} + \bar{\varepsilon}_{5k} P_1 \bar{\varepsilon}_{5k} V_{2k} + \bar{\varepsilon}_{5k} P_{1k} \bar{\varepsilon}_5 V_{2k} + \bar{\varepsilon}_{5k} P_{1k} \bar{\varepsilon}_{5k} V_2$$

Letting k-th set be the optimal solution of (22)

Step 4: Fix a positive scalar $\delta \in \mathbb{R}^+$, and a sufficient small tolerance ϵ , if

$$Tr\ \ J_1 + J_2 + 0.5 J_3 + 0.5 J_4\ \ - 16n < \delta, \quad (23)$$
$$or\ \left\| E_k - \bar{E}_k^{-1} \right\| < \epsilon,\ \left\| X_k - P_{1k}^{-1} \right\| < \epsilon,$$
$$and\ \left\| V_{ik} - \bar{V}_{ik}^{-1} \right\| < \epsilon$$

then decrease γ, set $\gamma_0 = \gamma_k$ and go back to Step 2. Otherwise, increase the count value-k by 1 and go to Step 3 *within* a specified iteration loop number other than, return $\gamma_{\min} = \gamma_k$ then exit.

IV. Application to Vehicle Lateral Dynamics

In this work, we consider the vehicle lateral dynamics obtained by considering the single-track model [21] as shown in *Fig.* 2. Where the longitudinal and lateral velocity are (u_x, u_y), respectively, and the yaw rate of vehicle frame $\dot{\psi}$. The lateral tire forces at the front wheel and the rear wheel are (F_{yf}, F_{yr}), and the tire slip angles (α_f, α_r). Then the dynamics of the vehicle systems is given:

$$\begin{cases} \dot{u}_y = \dfrac{2F_{yf} + 2F_{yr}}{m} - u_x \dot{\psi} \\ \ddot{\psi} = \dfrac{2F_{yf} - 2F_{yr}}{I_z} + M_z \end{cases} \quad (24)$$

By using identification and linearization of the cornering forces [7,8] the following TS fuzzy model is obtained:

$$\begin{bmatrix} \dot{u}_y \\ \ddot{\psi} \end{bmatrix} = \sum_{i=1}^{2} h_i\ \ \alpha_f\ \ t\ \ \ A_i + \Delta A_i\ \ t\ \ \begin{bmatrix} u_y \\ \dot{\psi} \end{bmatrix} \\ + \sum_{i=1}^{2} h_i\ \ \alpha_f\ \ t\ \ B_i \begin{bmatrix} \delta \\ M_z \end{bmatrix} + w(t) \quad (25)$$
$$y\ \ t\ \ =\ \ C + \Delta C\ \ t\ \ \begin{bmatrix} u_y \\ \dot{\psi} \end{bmatrix}$$

With the full description of the parameters using in the vehicle system (25) provided in *Table*. *I*, and the parameters of the membership functions [7], the state space *subsystem* can be represented as:

$$A_i = \begin{bmatrix} -\dfrac{2C_{fi} + 2C_{ri}}{mu_x} & \dfrac{-2C_{fi} l_f + 2C_{ri} l_r}{mu_x} - u_x \\ \dfrac{-2C_{fi} l_f + 2C_{ri} l_r}{I_z u_x} & -\dfrac{2C_{fi} l_f^2 + 2C_{ri} l_r^2}{I_z u_x} \end{bmatrix},$$

$$B_i = \begin{bmatrix} \dfrac{2C_{fi}}{m} & 0 \\ \dfrac{2C_{fi} l_f}{I_z} & \dfrac{1}{M_z} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The uncertainty matrices for the above system are taken

$M_1 = [0.1\ \ 0.1\ ]^T$, $M_2 = [0.1\ \ 0.1]^T$,

$N_1 = \rho_1\, [0.1\ \ 0.1]$, $N_2 = \rho_2\, [0.1\ \ 0.1]^T$.

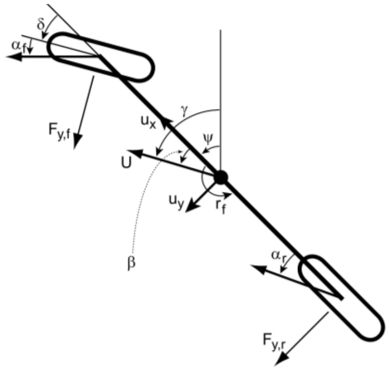

Figure. 2. [21] *The single-track model.*

Starting from the initial conditions $x\ 0\ =\ 0.5\quad -0.3\ ^{T}$ and $\hat{x}\ 0\ =\ 0.05\quad 0.02\ ^{T}$ the numerical simulations have been carried out at constant speed 72 km.h^{-1} through the wind-affected area treated as disturbances. In the supposition of neglect the effect of lift force, the wind velocity is given as [24]:

$$v_{wind}^{*} = -v_{windX}\sin\psi + v_{windy}\cos\psi \qquad (26)$$

The side force effect and yawing moment of crosswind as shown in *Fig.* 3 is expressed as follows:

$$F_{windY} = \text{sign}\ v_{wind}^{*}\ C_{airS}A_S\frac{\rho}{2}\ u_y - v_{wind}^{*}\ ^{2} \qquad (27)$$

$$M_{wind\psi} = -\text{sign}\ v_{wind}^{*}\ C_{air\psi}A_S l_w\frac{\rho}{2}\ u_y - v_{wind}^{*}\ ^{2} \qquad (28)$$

with the aerodynamic parameters for a car as seen in [23] is descripted in *Table. II.*

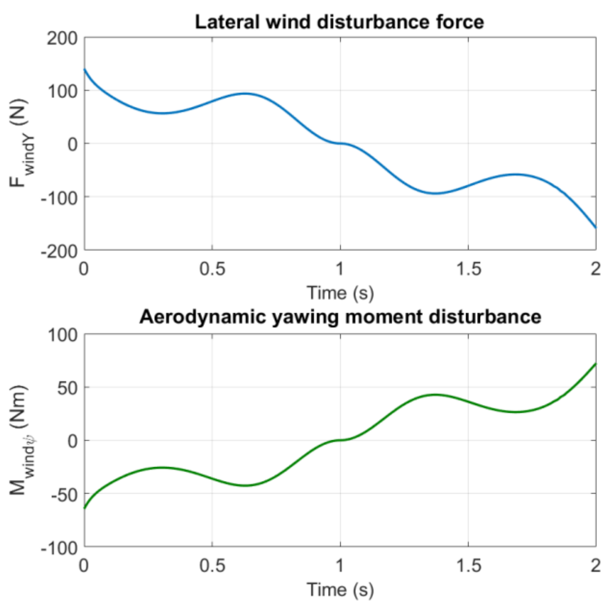

Figure. 3. *The crosswind disturbance.*

For $\rho_1 = \rho_2 = 100$, by using toolbox [25] and optimization algorithm 1 to solve the convex problem for Theorem 1, with the H_∞ attenuation level $\gamma = 1.382$ and input constraint $\mu_1 = 0.12$ (for the steering angle), $\mu_2 = 10^3$ (for yawing moment) a feasible set of variables are obtained as: $\varepsilon_i = \{0.28; 0.20; 0.21; 0.29\}$, respectively, where $i = 1, 2, 3, 5$, and the LMI variables:

$$W = \begin{bmatrix} 4.4817 & 0.7326 \\ 0.7326 & 0.4030 \end{bmatrix},\quad P_2 = \begin{bmatrix} 282.4548 & -243.4555 \\ -243.4555 & 259.3303 \end{bmatrix}.$$

TABLE I. SIMULATION VEHICLE PARAMETERS

Parameters	Descriptions	Values
m	Total vehicle mass (kg)	1832
υ	Vehicle speed (m s^{-1})	20
I_z	Yaw moment of inertia at center of gravity (CG) (kg m^2)	2988
l_r	Distance from CG to rear axe (m)	1.77
l_f	Distance from CG to front axle (m)	1.18
M_z	Driving/Braking force control (Nm)	
δ	Front steering angle (rad)	

TABLE II. SIMULATION AERODYNAMIC PARAMETERS

Parameters	Descriptions	Values
C_{airS}	Side force coefficient	
$C_{air\psi}$	Yawing moment coefficient	
ρ	Air density (kg m^{-3})	1.292
l_w	Distance from CG to the aerodynamic center (m)	0.295
υ_{windX}	The longitudinal wind velocity (m s^{-1})	$22\sin(2\pi t)$
υ_{windY}	The lateral wind velocity (m s^{-1})	$10\sin(2\pi t)$

The evolutions of states and estimation errors are demonstrated in below fig.

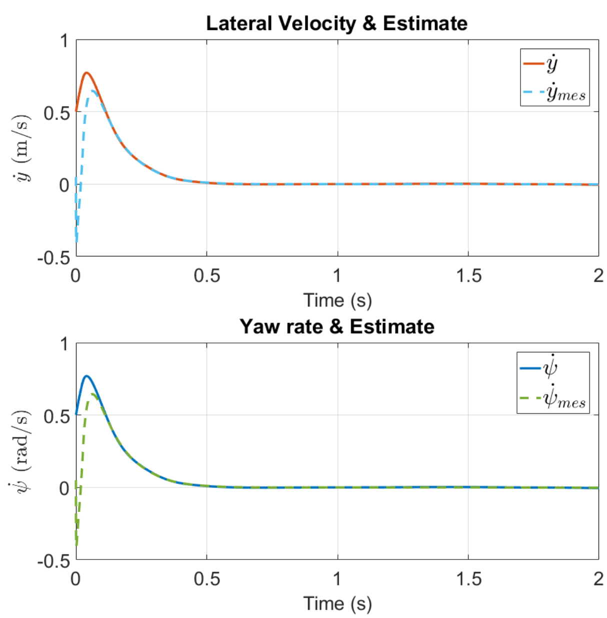

Figure. 4. *Convergence of states and estimation errors under parametric uncertainties and disturbance with the H_∞ attenuation level $\gamma_{min} = 0.271$*

The simulation results show an asymptotic of the lateral velocity/yaw rate state estimates under the influence of disturbance and uncertain parameters to the origins which demonstrates the good estimation of the proposed methodology. The performance obtained at the optimal disturbance attenuation level $\gamma_{min} = 0.271$, respectively, the signal input control (the steering angle and the yawing moment control) in the following figure:

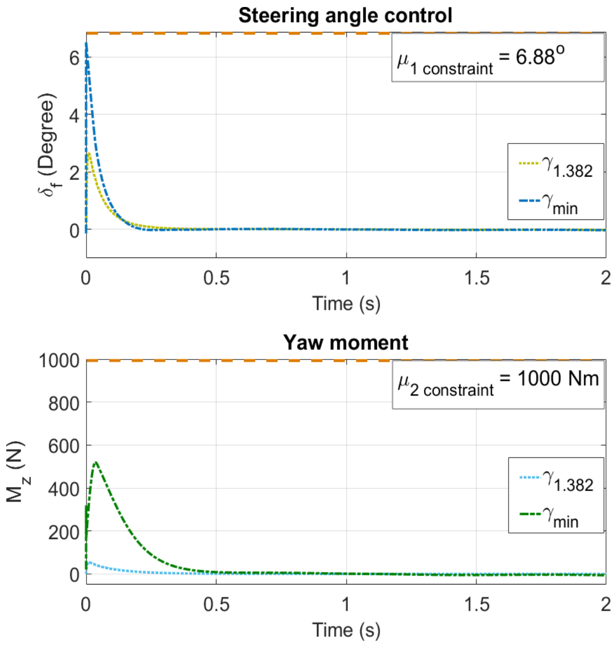

Figure. 5. *The input signal controls under constraint at the H_∞ attenuation level γ_{min}* and $\gamma = 1.382$.

Under $\mathscr{L}_2$ constraint, thank to LMI toolbox [25], the fuzzy observer-based control feedback gains are given:

$$K_1 = 10^3 \times \begin{bmatrix} -0.0000 & 0.0001 \\ -1.0407 & 2.2006 \end{bmatrix},$$

$$K_2 = 10^3 \times \begin{bmatrix} -0.0000 & 0.0002 \\ -0.9315 & 1.5955 \end{bmatrix}.$$

And the following LMI-observer gains:

$$L_1 = \begin{bmatrix} 136.42 & -135.82 \\ -396.17 & 396.82 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 227.85 & -227.83 \\ -696.11 & 696.12 \end{bmatrix}.$$

V. Conclusion

In this paper, we have investigated the problem of the observer-based robust control for TS fuzzy with time-varying norm-bounded parameter uncertainties, external disturbances and input constraint. Based on the Lyapunov function, we present an observer based control design method via set of linear matrix inequalities (LMIs) that has been attained by solving the cone-complementary linearization problem through state transformation. Finally, illustrative simulations are given to show the high performances of the proposed design technique.

References

[1] S. Boyd, L. Ghaoui, E Feron, V. Balakrishnan, "Linear matrix inequalities in system and control theory," vol. 15 of Studies in Applied Mathematics. Philadelphia: SIAM, 1994.

[2] L. El Ghaoui, F. Oustry, and M. A. Rami, "A cone complementarity linearization algorithm for static output-feedback and related problems," IEEE Trans. Autom. Control, vol. 42, no. 8, pp. 1171–1176, Aug. 1997.

[3] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," IEEE Trans. Syst., Man, Cybern., vol. SMC-15, no. 1, pp. 116–132, Feb. 1985.

[4] K. Tanaka and H. O. Wang, Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach. New York: Wiley, 2001.

[5] C.-H. Lien, "Robust observer-based control of systems with state perturbations via LMI approach," IEEE Trans. Autom. Control, vol. 49, no. 8, pp. 1365–1370, Aug. 2004.

[6] M. Oudghiri, M. Chadli and A. El Hajjaji, "One-Step procedure for robust output H ∞ fuzzy control, " Mediterranean Conference on Control & Automation, Athens, 2007, pp. 1–6.

[7] H. Dahmani, M. Chadli, A. Rabhi, and A. El Hajjaji, "Detection of impending vehicle rollover with road bank angle consideration using a robust fuzzy observer," International Journal of Vehicle Mechanics and Mobility. vol. 12, no. 1, pp. 93–101, Feb. 2015.

[8] H. Dahmani, O. Pagès, and A. El Hajjaji, "Observer-Based State Feedback Control for Vehicle Chassis Stability in Critical Situations," IEEE Trans. Autom. Control, vol.24, no. 2, pp. 636–643, Mar. 2016.

[9] A. Zemouche, R. Rajamani, H. Kheloufi, and F. Bedouhene, "Robust observer-based stabilization of Lipschitz nonlinear uncertain systems via LMIs - discussions and new design procedure," Int. J. Robust Nonlinear Control, vol. 27, pp. 1915–1939, 2017.

[10] H. Kheloufi, A. Zemouchek, B. Boulkroune, and M. Boutayeb, " A new observer-based stabilization method for linear systems with uncertain parameters ", Control Conference (ECC) 2013 European, pp. 1120–1125, 2013.

[11] H. Kheloufi, A. Zemouche, F. Bedouhene, and M. Boutayeb, "On LMI conditions to design observer-based controllers for linear systems with parameter uncertainties," Automatica, vol. 49, iss. 12, pp. 3700–3704, 2013.

[12] Akitaka Nishio, et. al., "Development of Vehicle Stability Control System Based on Vehicle Sideslip Angle Estimation," SAE Paper No. 2001–01–0137.

[13] A. Nishio et al., "Development of vehicle stability control system based on vehicle sideslip angle estimation," 2001, sAE Paper No. 2001–01–0137.

[14] S. Ahmada, M. Rehana, and K. S. Hong, "Observer-based robust control of one-sided Lipschitz nonlinear systems," ISA Transactions, vol.65, pp. 230–240, Nov. 2016.

[15] P. Bergsten, R. Palm, "Thau-Luenberger observers for TS fuzzy systems, " IEEE International Conference on Fuzzy Systems, no. 2, 2000.

[16] W.-J. Chang and C.-C. Sun, "Robust static output feedback H_∞ control for uncertain fuzzy systems," Fuzzy Sets Syst., vol. 273, pp. 87–104, 2015.

[17] X. Chang and G. Yang, "New results on output feedback H_∞ control for linear discrete-time systems," IEEE Transactions on Automatic Control, vol. 59, no. 5, 2014.

[18] M. Nachidi, A. Benzaouia, F. Tadeo, and M. Ait Rami, "LMI-Based Approach for Output-Feedback Stabilization for Discrete-Time Takagi–Sugeno Systems," IEEE Trans. Fuzzy Syst., vol. 16, no. 5, pp. 1188–1196, Oct. 2008.

[19] G. Zong, L. Hou, H. Yang, "Further Results Concerning Delay-Dependent H_∞ Control for Uncertain Discrete-Time Systems with Time-Varying Delay," Math Probl Eng., vol. 24, no. 732181, 2009.

[20] L. Hou, G. Zong, Y. Wu, "Robust exponential l_2 - l_∞ control for discrete-time switched system: A cone complement linearization method, " Optim. Control Appl. Meth., vol. 33, pp. 631-653, 2012.

[21] J. Ryu and J. C. Gerdes, "Estimation of vehicle roll and road bank angle," in Proc. Amer. Control Conf., Boston, MA, USA, Jun./Jul. 2004, pp. 2110–2115.

[22] J. Guldner, S. Tan, S. Patwardhan, "Analysis of Automatic Steering Control for Highway Vehicles with Look-down Lateral Reference Systems," Vehicle System Dynamics, vol. 26, no. 4, pp. 243–269, 1996.

[23] C.J. Baker, "Measures to control vehicle movement at exposed sites during windy periods," J. Wind Eng. Ind. Aerodyn., vol. 25, pp. 151–161, 1987

[24] U. Kiencke, L. Nielsen, "Automotive Control Systems For Engine, Driveline, and Vehicle," Springer, 2005.

[25] Yalmip (2017), website. URL: https://yalmip.github.io/. SeDuMi (v1.3), website. URL: http://sedumi.ie.lehigh.edu/