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Distributed leader-tracking for autonomous connected vehicles in presence of input time-varying delay

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Abstract— **In this paper, the leader tracking problem for a platoon of connected vehicles in presence of homogeneous time-varying Vehicle-to-Vehicle communication delays is addressed. To this aim, the platoon is recast as a network of multi-agent systems and consensus is achieved by leveraging a delayed distributed strategy that complements the standard linear diffusive control protocol with additional distributed integral and derivative actions. The asymptotic stability of the closed-loop delayed system is hence analytically proven by exploiting the Lyapunov-Krasovskii theory. Stability conditions are expressed as a set of Linear Matrix Inequalities, whose solution allows the proper tuning of proportional, derivative and integral gains such as to counteract the presence of the time-varying input delay. An exemplar tracking maneuver is considered for evaluating the performance of a connected vehicles fleet and the numerical results confirm the effectiveness of the theoretical derivation.**

I. INTRODUCTION

Platooning of connected vehicles has been attracting an increasing attention in the automotive field due to its potential benefits in terms of the driving safety, traffic congestion and fuel consumption [1]. The aim of the platoon control is to ensure that all vehicles within the fleet move following a desired speed profile, given by the first vehicle of the fleet (called leader) or by an external infrastructure, while they simultaneously keep desired inter-vehicular distances. The platoon problem mainly deals with the longitudinal dynamics of the vehicles and with the design of a hierarchical control architecture, composed by an upper level and lower level controller [2]. This work focuses on the design of the upper level strategy for determining the desired acceleration profile that has to be imposed to the autonomous vehicles within a platoon in order to reach a common leader velocity and desired inter-vehicular distances [2]. Earlier platoon controllers, such as classical Adaptive Cruise Control (ACC) strategies (see [2] and references therein), exploit only information coming from on-board sensors (radar, lidar, etc.) of each vehicle to measure its distance and velocity w.r.t. its preceding vehicle. Nowadays, the use of wireless communication technologies, on-board integrated, provide a major level of connectivity among vehicles (Vehicle-to-Vehicle (V2V)) and/or infrastructures (Vehicle-to-Infrastructure (V2I)). In so doing, each connected vehicle is capable of obtaining and exploiting information that is beyond the line of sight of its on-board sensors. Hence, the control strategies (often referred as Cooperative Adaptive Cruise Control (CACC) in technical literature) have to be capable of leveraging these information varieties for improving the driving behavior of platoons of vehicles (see [3] and references therein). However, since vehicles share information through a non ideal V2V wireless network, communication impairments are unavoidable. Indeed, control strategies have to cope with delays in the shared information before their effective deployment in a real smart road scenario since the control action is computed on the basis of outdated information [4]. In this technological framework different control approaches, such as decentralized diffusive state-feedback control, distributed Model Predictive Control (MPC) and sliding mode strategies have been recently proposed to solve the leader tracking problem under the main restrictive usual assumption that the communication among vehicles is ideal and reliable (see for example [5], [6], [7], [8] and references therein). Consensus-based strategies have been also proposed as an alternative for explicitly coping with communication delays (e.g., see [1], [9], [10] and references therein). However, the approaches usually neglect the tracking of a leader that moves with a time-varying velocity profile and instead address the convergence and robustness of platoons of autonomous vehicles traveling with a common constant speed. Moreover, the exploitation of Multi-Agent Systems (MASs) for the distributed platoon control is restricted to full state-feedback schemes based on diffusive coupling.

Since accurate full-state on-line measurements could not be available in practice, in this paper we propose an output-feedback distributed control for platooning able to cope with time-varying communication delays, where the classical diffusive consensus protocol is augmented with additional distributed both integral and derivative actions. The proposed distributed control strategy is on-board computed by only using information shared through the wireless communication. Hence, it relies on delayed state information as received by neighbors through the V2V links affected by the time-varying delay, whose current value depends on the actual condition of the communication network. Note that the exploitation of distributed integral and derivative actions has been only very recently proposed in the wide technical literature on MASs for increasing the dynamic performance achievable with the simple proportional action [11]. However, the approaches are usually designed under the restrictive main assumption of a perfect communication among agents (e.g., see [12] and

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references therein) or of a constant delay (e.g., see [13] and references therein). An alternative attempt to cope with time-varying delay has been instead very recently proposed in [14] where nevertheless the integral action is neglected and the analytically derivation is explicitly tailored to deal only with the derivative action. The stability of the distributed control algorithm has been analytically proven through Lyapunov-Krasovskii theory [15]. The derived condition, expressed as Linear Matrix Inequalities (LMIs), allows the proper tuning of the control gains. Numerical results confirm the theoretical derivation and the effectiveness of the control approach in ensuring the leader-tracking. It is worth here to mention that the ability to achieve good leader-tracking performance is fundamental in vehicular networks both during normal operation - where deceleration or acceleration maneuvers must be safely executed (e.g. in the occurrence of sudden traffic) to avoid that any vehicle in formation falls too far behind the vehicle ahead - and during join or emergency braking maneuvers to prevent collisions (e.g., see [16] where the problem as been adressed under the main assumption of full state feedback). Finally, the paper is organized as follows. In Section II the notation and the mathematical background are introduced. In Section III the problem statement and the control approach are presented. In Section IV the stability analysis is carried out and the stability conditions are derived. Numerical results are presented in Section V while the conclusions are given in Section VI.

II. MATHEMATICAL PRELIMINARIES

A set of connected vehicles can be modeled as a directed graph, defined as $\mathscr{G}_N = (\mathscr{V}_N, \mathscr{E}_N)$, where $\mathscr{V}_N = 1,2,...,N$ is the set of vehicles, while $\mathscr{E}_N \subseteq \mathscr{V}_N \times \mathscr{V}_N$ is the set of edges that mimic the communication links. Hence, the communication structure can be described by the adjacency matrix $\mathscr{A}$ and the in-degree matrix $\mathscr{D}$. The adjacency matrix $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is such that $a_{ij} = 1$ if there is a link from vehicle j to vehicle i and $a_{ij} = 0$ otherwise. The in-degree matrix $\mathscr{D} = \mathrm{diag}\{d_1, d_2, ..., d_N\}$ is a diagonal matrix whose element, i.e. $d_i = \sum_{j=1}^{N} a_{ij}$, indicates the number of vehicles that communicate with vehicle i. In so doing the Laplacian matrix is defined as $\mathscr{L} = \mathscr{D} - \mathscr{A}$ [7]. In the rest of the paper, we consider N vehicles plus a leader agent, labeled with 0. Therefore we use an augmented directed graph $\mathscr{G}_{N+1}$ to model the resulting network topology. Moreover, we define a pinning matrix [8], [17] $\mathscr{P} = \mathrm{diag}\{p_1, p_2, ..., p_N\}$, such that $p_i = 1$ when the leader information is directly available for the $i-th$ vehicle, 0 otherwise. It means that if the i vehicle directly obtains the leader information, it is called pinned.

Moreover, in what follows a useful Theorem dealing with stability in delayed systems is also briefly introduced.

Consider the following time-delay system:

$$\begin{aligned} \dot{x}(t) =& f(t, x_t), \;\; t \geq t_0 \\ x(t_0 + s) =& \phi(s), \; s \in [-h, 0] \end{aligned}$$

where $h > 0$ is the delay and $\phi \in C([-h,0], R^n)$ is the functional of initial conditions. The state of the system $x_t \in C([-h,0], R^n)$ is defined as $x_t(\theta) = x(t+\theta)$ and $x_t(t_0, \phi)$ denotes the state value at time t with initial condition $x_{t0} = \phi$. It holds:

Theorem 1 (Lyapunov-Krasovskii Stability Theorem, [15]): Suppose $f : R_{\geq 0} \times C([-h,0], R^n) \to R^n$ that maps $R_{\geq 0}\times$ (bounded sets of $C([-h,0], R^n)$) into a bounded sets of R^n, and that $u, v, w : R_{\geq 0} \to R_{\geq 0}$ are continuous non-decreasing functions, $u(s)$ and $v(s)$ are positive for $s > 0$ and $u(0) = v(0) = 0$. Assume further that there exists a continuous differentiable functional $V : R \times C([-h, 0, R^n]) \to R$ such that: $u(\|\phi(0)\|) \leq V(t, \phi) \leq v(\|\phi\|_c)$, and $\dot{V}(t, \phi) \leq -w(\|\phi(0)\|)$, then the trivial solution of the time delay system is uniformly stable and if $w(s) > 0$ for $s > 0$, then the trivial solution is uniformly asymptotically stable.

III. PROBLEM STATEMENT AND SYSTEM DYNAMICS

Consider a platoon of N vehicles plus a leader, moving on a single lane, that is organized as string with vehicles following one another along a straight line. In the scenario under investigation, each vehicle within the platoon measures its absolute position and speed [18], and shares this information with all the other vehicles in its communication range through a V2V communication protocol [19]. The reference behavior for each vehicle is imposed by the leader, i.e. the first vehicle of the fleet, that is labeled with index 0.

The aim is to ensure that each vehicle tracks the leading dynamic behavior while preserving a pre-fixed inter-vehicular spacing policy without any centralized control approach. In a control design perspective, the $i-th$ vehicle motion is described by its longitudinal dynamics [2]. According to the technical literature, exploiting nonlinear compensation, the i-th vehicle dynamics can be approximated with the following dynamical system according to the classical modelling approach proposed in [20]:

$$\begin{aligned} &\dot{p}_i(t) = v_i(t) \\ &\dot{v}_i(t) = a_i(t) \\ &\dot{a}_i(t) = -\frac{1}{T} a_i(t) + \frac{1}{T} u_i(t), \end{aligned} \tag{1}$$

where $p_i(t)$ $[m]$, $v_i(t)$ $[m/s]$ and $a_i(t)$ $[m/s^2]$, $(i = 1, ..., N)$ are the $i-th$ absolute vehicle position, velocity and acceleration, respectively, while T $[s]$ is the powertrain time constant. $u_i(t)$ is the control input that provides the desired acceleration to be imposed to $i-th$ vehicle that is on-board computed by handling both local measurements and networks information. The reference leading dynamics are instead described as a third order linear autonomous system, whose state variables are labelled with index 0. The leader tracking problem for platooning can now be expressed as the following second-order consensus problem:

$$\begin{cases} p_i(t) = p_{i-1}(t) + d_{i,i-1}, \\ v_i(t) = v_0(t), \end{cases} \tag{2}$$

$\forall i = 1, 2, \ldots, N$, being $d_{i,i-1}$ a positive constant representing the desired spacing policy between vehicle i and vehicle $i-1$ [6].

To solve this problem here we propose the following delayed distributed protocols leveraging proportional, integral and derivative actions:

$$
\begin{aligned}
u_i(t-\tau(t)) = & -K_p \sum_{j=0}^{N} a_{ij}\Big(p_i(t-\tau(t)) - p_j(t-\tau(t)) - d_{ij}\Big) \\
& - K_d \sum_{j=0}^{N} a_{ij}\Big(v_i(t-\tau(t)) - v_j(t-\tau(t))\Big) \\
& - K_i \sum_{j=0}^{N} a_{ij}\Big(\int_0^{t-\tau(t)} (p_i(s) - p_j(s) - d_{ij})ds\Big)
\end{aligned} \tag{3}
$$

where K_p, K_d and K_i are proportional, derivative and integral gains, respectively; a_{ij} models the network topology emerging from the presence/absence of communication link between the $i-th$ and $j-th$ vehicle; $\tau(t)$ is a time-varying homogeneous communication delay arising from the non-ideal communication network; d_{ij} is the desired distance between vehicle i and vehicle j. Note that the homogeneous delay is here considered as the summation of computation time and execution time. Moreover it is assumed to be bounded, i.e. $0 \le \tau(t) < \tau^*$ (e.g., see [21] and references therein). Indeed, as in reality, vehicles share information through a wireless communication channel and, therefore, it happens that information can be delivered with time-varying delay whose current value depends on the network conditions [22]. These considerations lead to the need of running the controller on the basis of outdated information.

A. Closed-Loop Vehicular Network

To prove the ability of vehicle dynamics (1) in tracking the leader motion under the action of the proposed control protocol (3), we define the error of the $i-th$ vehicle with respect to the leader as

$$
e_i(t) = \begin{bmatrix} \bar{p}_i(t) \\ \bar{v}_i(t) \\ \bar{a}_i(t) \end{bmatrix} = \begin{bmatrix} p_i(t) - p_0(t) - d_{i,0} \\ v_i(t) - v_0(t) \\ a_i(t) - a_0(t) \end{bmatrix}, \tag{4}
$$

where $d_{i,0}$ [m] is the inter-vehicular distance between the i-th vehicle and the leader. Given the definition in (4) and taking into account both i-th agent dynamics in (1) and the leading dynamics, the following dynamic error model for the i-th vehicle ($i = 1, 2\cdots, N$) can be written as:

$$
\begin{aligned}
\dot{\bar{p}}_i &= \bar{v}_i(t) \\
\dot{\bar{v}}_i &= \bar{a}_i(t) \\
\dot{\bar{a}}_i &= -\frac{1}{T}\bar{a}_i(t) + \frac{1}{T}u_i(t-\tau(t)).
\end{aligned} \tag{5}
$$

Moreover, by defining the following vectors as $\bar{p}(t) = [\bar{p}_1(t), \bar{p}_2(t), \dots, \bar{p}_N(t)]^T$; $\bar{v}(t) = [\bar{v}_1(t), \bar{v}_2(t), \dots, \bar{v}_N(t)]^T$; $\bar{a}(t) = [\bar{a}_1(t), \bar{a}_2(t), \dots, \bar{a}_N(t)]^T$; $\bar{u}(t) = [u_1(t), u_2(t), \dots, u_N(t)]^T$; $\bar{z}(t) = [\bar{z}_1(t), \bar{z}_2(t), \dots, \bar{z}_N(t)]^T$ being $\bar{z}_i(t) = \int_0^t \bar{p}_i(s)ds$, the control input vector can be recast as

$$
\begin{aligned}
\bar{u}(t-\tau(t)) = & -K_p\mathscr{H}\bar{p}(t-\tau(t)) - K_d\mathscr{H}\bar{v}(t-\tau(t)) + \\
& -K_i\mathscr{H}\bar{z}(t-\tau(t)),
\end{aligned} \tag{6}
$$

with $\mathscr{H} = \mathscr{L} + \mathscr{P}$, being $\mathscr{L}$ and $\mathscr{P}$ the Laplacian and the pinning matrix, respectively, as defined in Section II.
Now, naming $\bar{x}(t) = [\bar{p}(t), \bar{v}(t), \bar{a}(t), \bar{z}(t)]^T \in \mathbb{R}^{4N}$ and considering equations (5) and (6), the delayed closed-loop error systems can be derived as

$$
\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_\tau\bar{x}(t-\tau(t)), \tag{7}
$$

where

$$
\begin{aligned}
\bar{A} &= \begin{bmatrix} 0_{N\times N} & I_{N\times N} & 0_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & 0_{N\times N} & I_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & 0_{N\times N} & -\bar{T} & 0_{N\times N} \\ I_{N\times N} & 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \end{bmatrix}, \\
\bar{A}_\tau &= \begin{bmatrix} 0_{N\times N} & 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \\ 0_{N\times N} & 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \\ \bar{A}_\tau(3,1) & \bar{A}_\tau(3,2) & 0_{N\times N} & \bar{A}_\tau(3,4) \\ 0_{N\times N} & 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \end{bmatrix},
\end{aligned} \tag{8}
$$

being $0_{N\times N}$ and $I_{N\times N}$ N-order null and identity matrix, respectively and

- $\bar{T} = diag\left\{\frac{1}{T}, \cdots, \frac{1}{T}\right\} \in \mathbb{R}^{N\times N}$,
- $\bar{A}_\tau(3,1) = -\bar{T}K_p\mathscr{H} \in \mathbb{R}^{N\times N}$,
- $\bar{A}_\tau(3,2) = -\bar{T}K_d\mathscr{H} \in \mathbb{R}^{N\times N}$,
- $\bar{A}_\tau(3,4) = -\bar{T}K_i\mathscr{H} \in \mathbb{R}^{N\times N}$.

IV. Stability analysis

Given the closed-loop delayed system in (7), the leader tracking in presence of communication delay is proven here under the common assumption that delays are bounded, i.e. $\tau(t) \in [0, \tau^*]$, $\dot{\tau}(t) \in [0, \mu[\ \forall t$ and $\mu < 1$ [23][15]. The stability criterion, used to ensure leader tracking, is expressed as a delay-dependent LMI criterion that allows the proper tuning of the control gains in (3), by exploiting a Lyapunov-Krasovskii approach, according to the following Theorem.

Theorem 2: Consider the closed-loop system in (7), with matrices defined in (8), under the action of the proposed control strategy in (3). Assume the time varying communication delays $\tau(t)$ to be bounded. If there exist positive definite matrices $X \in \mathbb{R}^{4N\times 4N}$, $G \in \mathbb{R}^{4N\times 4N}$, $M \in \mathbb{R}^{4N\times 4N}$, $Q \in \mathbb{R}^{4N\times 4N}$, $W \in \mathbb{R}^{4N\times 4N}$, $Z \in \mathbb{R}^{4N\times 4N}$ and $F \in \mathbb{R}^{4N\times 4N}$ such that the following LMI holds:

$$
\begin{bmatrix} \bar{A}X + X^T\bar{A}^T - X - X^T + N & \bar{A}_\tau + I + M & \tau^* I & \tau^* X^T\bar{A}^T \\ \star & -Q(1-\mu) - W - W^T & -\tau^* W & \tau^*\bar{A}_\tau^T \\ \star & \star & -\tau^* Z & 0 \\ \star & \star & \star & -\tau^* F \end{bmatrix} < 0, \tag{9}
$$

then the leader tracking problem in (2) is asymptotically achieved, i.e.

$$
\lim_{t\to\infty} \bar{x}(t) = 0.
$$

Proof: In order to provide the stability conditions for the closed-loop system (7), we consider the following Lyapunov-Krasovskii functional:

$$
V(\bar{x}(t)) = V_1(\bar{x}(t)) + V_2(\bar{x}(t)) + V_3(\bar{x}(t)), \tag{10}
$$

where

$$V_1(\bar{x}(t)) = \bar{x}^T(t)P\bar{x}(t), \tag{11}$$

$$V_2(\bar{x}(t)) = \int_{t-\tau(t)}^{t} \bar{x}^T(s)Q\bar{x}(s)ds, \tag{12}$$

$$V_3(\bar{x}(t)) = \int_{-\tau^*}^{0}\int_{t+\theta}^{t} \dot{\bar{x}}^T(s)Z\dot{\bar{x}}(s)dsd\theta, \tag{13}$$

being $P \in \mathbb{R}^{4N\times 4N}$, $Q \in \mathbb{R}^{4N\times 4N}$ and $Z \in \mathbb{R}^{4N\times 4N}$ symmetric and positive definite matrices.
Differentiating $V_1(\bar{x}(t))$ in (11) along the trajectories of the closed-loop system (7), we have

$$\begin{aligned}\dot{V}_1(\bar{x}(t) &= \dot{\bar{x}}^T(t)P\bar{x}(t) + \bar{x}^T(t)P\dot{\bar{x}}(t) = \\ &= 2\bar{x}^T(t)P\Big[\bar{A}\bar{x}(t) + \bar{A}_\tau \bar{x}(t-\tau(t))\Big].\end{aligned} \tag{14}$$

By using the Newton-Liebnitz formula [15], i.e. $\bar{x}(t-\tau(t)) = \bar{x}(t) - \int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds$, we can re-write (14) as

$$\dot{V}_1(\bar{x}(t)) = 2\bar{x}^T(t)P\Big(\bar{A}+\bar{A}_\tau\Big)\bar{x}(t) - 2\bar{x}^T(t)P\bar{A}_\tau\int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds. \tag{15}$$

Introducing the slack variables matrices Y and $W \in \mathbb{R}^{4N\times 4N}$, after some algebraic manipulation expression (15) can be written as

$$\begin{aligned}&\dot{V}_1(\bar{x}(t)) = \\ &= 2\bar{x}^T(t)P\Big(\bar{A}+\bar{A}_\tau\Big)\bar{x}(t) + 2\bar{x}^T(t)\Big(Y - P\bar{A}_\tau\Big)\int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds + \\ &+ 2\bar{x}^T(t-\tau(t))W\int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds - \Big[2\bar{x}^T(t)Y\int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds + \\ &+ 2\bar{x}^T(t-\tau(t))W\int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds\Big].\end{aligned} \tag{16}$$

Exploiting again the Newton-Liebnitz formula for the first and second integral term in (16) and considering the boundedness condition for the time-varying delay $\tau(t)$ [23], expression (16) can be recast as

$$\begin{aligned}\dot{V}_1(\bar{x}(t)) \leq \frac{1}{\tau^*}\int_{t-\tau^*}^{t}&\Big[2\bar{x}^T(t)\Big(P\bar{A}+Y\Big)\bar{x}(t) + \\ &+ 2\bar{x}^T(t)\Big(P\bar{A}_\tau - Y + W^T\Big)\bar{x}(t-\tau(t)) + \\ &- 2\bar{x}^T(t-\tau(t))W\bar{x}(t-\tau(t)) - 2\tau^*\bar{x}^T(t)Y\dot{\bar{x}}(s) + \\ &- 2\tau^*\bar{x}^T(t-\tau(t))W\dot{\bar{x}}(s)\Big]ds.\end{aligned} \tag{17}$$

Analogously, by differentiating $V_2(\bar{x}(t))$ in (12) after some algebraic manipulation we have

$$\begin{aligned}\dot{V}_2(\bar{x}(t)) = \frac{1}{\tau^*}\int_{t-\tau^*}^{t}&\Big[\bar{x}^T(t)Q\bar{x}(t) + \\ &- \bar{x}(t-\tau(t))^T Q\bar{x}(t-\tau(t))(1-\mu)\Big]ds.\end{aligned} \tag{18}$$

Finally, differentiating $V_3(\bar{x}(t))$ in (13), we obtain

$$\dot{V}_3(\bar{x}(t)) = \int_{t-\tau^*}^{t}\Big[\dot{\bar{x}}^T(t)Z\dot{\bar{x}}(t) - \dot{\bar{x}}^T(s)Z\dot{\bar{x}}(s)\Big]ds. \tag{19}$$

Taking into account the closed-loop system (7), we can now re-write (19) as

$$\begin{aligned}\dot{V}_3(\bar{x}(t)) &= \frac{1}{\tau^*}\int_{t-\tau^*}^{t}\Big[\tau^*\bar{x}^T(t)\bar{A}^T Z\bar{A}\bar{x}(t) + \\ &+ \tau^*\bar{x}^T(t)\bar{A}^T Z\bar{A}_\tau\bar{x}(t-\tau(t)) + \tau^*\bar{x}^T(t-\tau(t))\bar{A}_\tau^T Z\bar{A}\bar{x}(t) + \\ &+ \tau^*\bar{x}^T(t-\tau(t))\bar{A}_\tau^T Z\bar{A}_\tau\bar{x}(t-\tau(t)) - \tau^*\dot{\bar{x}}^T(s)Z\dot{\bar{x}}(s)\Big]ds.\end{aligned} \tag{20}$$

By summing (17), (18), (20), we obtain:

$$\begin{aligned}\dot{V}(\bar{x}(t)) \leq \frac{1}{\tau^*}\int_{t-\tau^*}^{t}&\Big[2\bar{x}^T(t)\Big(P\bar{A}+Y\Big)\bar{x}(t) + 2\bar{x}^T(t)\Big(P\bar{A}_\tau + \\ &- Y + W^T\Big)\bar{x}(t-\tau(t)) - 2\bar{x}^T(t-\tau(t))W\bar{x}(t-\tau(t)) + \\ &- 2\tau^*\bar{x}^T(t)Y\dot{\bar{x}}(s) - 2\tau^*\bar{x}^T(t-\tau(t))W\dot{\bar{x}}(s) + \bar{x}^T(t)Q\bar{x}(t) + \\ &- \bar{x}^T(t-\tau(t))Q\bar{x}(t-\tau(t))(1-\mu) + \tau^*\bar{x}^T(t)\bar{A}^T Z\bar{A}\bar{x}(t) + \\ &+ \tau^*\bar{x}^T(t)\bar{A}^T Z\bar{A}_\tau\bar{x}(t-\tau(t)) + \tau^*\bar{x}^T(t-\tau(t))\bar{A}_\tau^T Z\bar{A}\bar{x}(t) \\ &+ \tau^*\bar{x}^T(t-\tau(t))\bar{A}_\tau^T Z\bar{A}_\tau\bar{x}(t-\tau(t)) - \tau^*\dot{\bar{x}}^T(s)Z\dot{\bar{x}}(s)\Big]ds.\end{aligned} \tag{21}$$

Defining now the following augmented state vector $q(t,s) = [\bar{x}(t)\ \bar{x}(t-\tau(t))\ \dot{\bar{x}}(s)]^T \in \mathbb{R}^{\upsilon}$ ($\upsilon = 12N$) [24], inequality (21) can be re-written in a more compact form as

$$\dot{V}(\bar{x}(t)) < \frac{1}{\tau^*}\int_{t-\tau^*}^{t} q^T(t,s)\,\Lambda\, q(t,s)ds, \tag{22}$$

where

$$\Lambda = \begin{bmatrix}\Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \star & \Lambda_{22} & \Lambda_{23} \\ \star & \star & \Lambda_{33}\end{bmatrix} \in \mathbb{R}^{\upsilon\times\upsilon}, \tag{23}$$

with

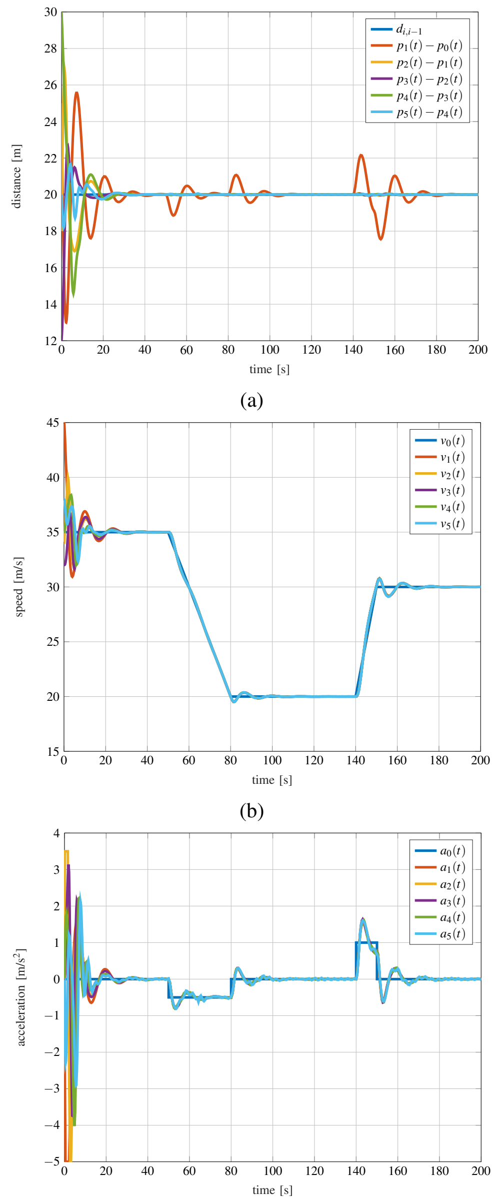
$$\begin{aligned}&\Lambda_{11} = P\bar{A} + \bar{A}^T P + Y + Y^T + \tau^*\bar{A}^T Z\bar{A} + Q, \\ &\Lambda_{12} = P\bar{A}_\tau - Y + W^T + \tau^*\bar{A}^T Z\bar{A}_\tau, \\ &\Lambda_{13} = -\tau^* Y,\ \ \Lambda_{22} = \tau^*\bar{A}_\tau^T Z\bar{A}_\tau - Q(1-\mu) - W - W^T, \\ &\Lambda_{23} = -\tau^* W,\ \ \Lambda_{33} = -\tau^* Z.\end{aligned}$$

The asymptotic stability of the delayed closed-loop system (7) is achieved if the nonlinear matrix inequality $\Lambda < 0$ is feasible. For dealing with it, we use the Schur Complement and other transformations in order to derive LMIs conditions to be satisfied. In particular, we apply Schur Complement (see lemma in [25]) on matrix (23) obtaining

$$\Lambda' = \begin{bmatrix} P\bar{A}+\bar{A}^T P+Y+Y^T+Q & P\bar{A}_\tau - Y + W^T & -\tau^* Y & \tau^*\bar{A}^T Z \\ \star & -Q(1-\mu)-W-W^T & -\tau^* W & \tau^*\bar{A}_\tau^T Z \\ \star & \star & -\tau^* Z & 0 \\ \star & \star & \star & -\tau^* Z\end{bmatrix}. \tag{24}$$

Then, matrix in (24) is pre and post-multiplied for the matrix $\Upsilon = diag\{X, I, I, Z^{-1}\} \in \mathbb{R}^{(\upsilon+4N)\times(\upsilon+4N)}$ (being $X \in \mathbb{R}^{4N\times 4N}$ a symmetric and positive definite matrix). Finally, by defining $X = P^{-1}$, $Y = -P$, $N = X^T Q X$, $M = X^T W^T$ and $F = Z^{-1}$, we get the LMIs in (9). In so doing the statement is proven. ■

Remark 1: Note that the LMI in (9) can be numerically verified by using, for example, the interior-point method [26] implemented in the *Yalmip*© Toolbox.



(a) (b) (c)

Fig. 1: Leader tracking maneuver under the action of the proportional, integral and derivative control in (3): a) time history of the inter-vehicular distance $p_i(t) - p_{i-1}(t)$ ($i = 1,2,3,4,5$); b) time history of the vehicle speed $v_i(t)$ ($i = 0,1,2,3,4,5$); c) time history of the vehicle acceleration $a_i(t)$ ($i = 0,1,2,3,4,5$).

Remark 2: Matrix F has been exploited to replace the reverse term Z^{-1} that would introduce nonlinearities into the optimization problem, whose solution provides the stabilizing control gains regions.

V. Numerical Validation

In this section we validate the effectiveness of the distributed control strategy in (3) and disclose its effectiveness in guaranteeing the leader tracking. Specifically, an exemplar platoon of five vehicles plus a leader connected through a Leader-Predecessor-Follower (L-P-F) [27] topology is considered. Note that, although many different communication topologies may arise for platooning, according to the V2V paradigm, the appraised L-P-F structure has been chosen as an exemplar case since it is very common in the technical literature (see [27] and references therein). According to definitions in II, the L-P-F topology is described by the following Laplacian and Pinning matrix:

$$\mathscr{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \quad \mathscr{P} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

To verify the LMI criterion (9), that allows the proper tuning of the control gains, we exploit the Yalmip© Toolbox [28]. The values of the control gains, resulting from the tuning procedure, are $K_p = 0.3623$, $K_d = 0.9679$ and $K_i = 0.1484$. Numerical simulations have been instead performed by exploiting the MATLAB© platform. In the simulation scenario communication delay has been emulated as a time-varying function whose maximum value, i.e. $\tau^* = 0.1$ $[s]$, is set above the typical value observed in practice for IEEE 802.11p vehicular networks (which is of the order of few hundredths of a second, i.e. 10^{-2} $[s]$ [21]). For testing the ability in tracking the leader motion, here we show an exemplar result related to the case of a leader traveling with an initial velocity of 35 $[m/s]$ that at time $t =$ 50 $[s]$ begins to decelerate with a constant deceleration of -0.5 $[m/s^2]$ until it reaches a constant velocity of 20 $[m/s]$. Then, at time $t =$ 140 $[s]$ it starts to accelerate with an acceleration of 1 $[m/s^2]$ until it achieves a final constant velocity equal to 30 $[m/s]$. According to the theoretical derivation in Section IV, results reported in Figs. 1 and 2 reveal the ability of the proposed distributed controller in guaranteeing that the formation correctly tracks the leader behavior (see Figs 1b and 1c) while preserving the desired inter-vehicular distance $d_{i,i-1} = 20$ $[m]$ ($\forall i = 1,\dots,5$) (see Figure 1a). Note that, as depicted in Figure 2, just some small bounded errors can be observed in correspondence of the sharp changing in the leader velocity profile. Indeed, position errors are equal at most at 2.65 $[m]$ while velocity errors are equal at most at 0.95 $[m/s]$ (see as Figs 2a and 2b).

VI. Conclusions

To solve the leader tracking problem for a platoon of connected vehicles, a novel distributed proportional, integral and derivative control strategy accounting for communication time-varying delays was proposed. The stability conditions of the control algorithm, derived through the Lyapunov-Krasovksii approach, were expressed as an LMI problem allowing the proper tuning of the control gains. Finally, numerical simulations disclosed the effectiveness of the strategy and the results confirmed that all vehicles track the leader motion while maintaining the desired inter-vehicular distance.

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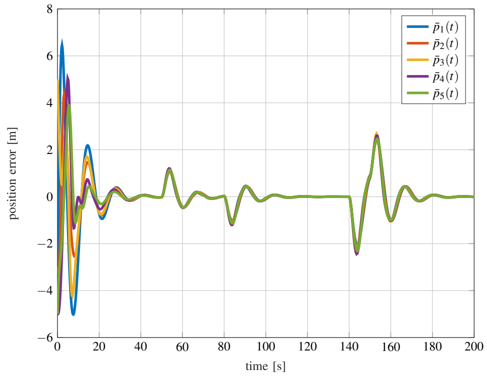
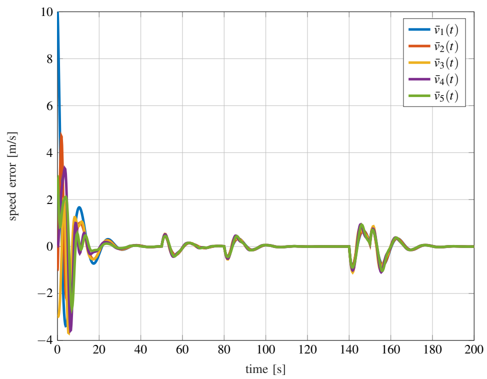


(a)



(b)

Fig. 2: Leader tracking maneuver under the action of the proportional, integral and derivative control in (3): a) time history of the position error $\bar{p}_i(t)$, computed as $p_i(t) - p_0(t) - d_{i,0}$ $(i = 1,2,3,4,5)$; b) time history of the speed error $\bar{v}_i(t)$, computed as $v_i(t) - v_0(t)$ $(i = 1,2,3,4,5)$.

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