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Shared-Control for the Lateral Motion of Vehicles

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***Abstract*— Car accidents caused by driver's fatigue, inattention, drowsiness and illness are one of the leading reasons for preventable fatalities and injuries on the road. This paper studies the assistant driving problem for the nonlinear lateral dynamic model of vehicles in the lane keeping case. By means of the developed controller the car is able to keep itself in a given lane even if the human driver does not drive properly. The shared-control algorithm is based on a hysteresis switch and the formal properties of the closed-loop system are established via a Lyapunov-like analysis. Simulation results are presented to show the effectiveness of the driving assistant system.**

NOMENCLATURE

β	ratio between the lateral speed and the longitudinal speed
$\dot{\psi}$	yaw rate [rad/s]
ψ_L	heading error, relative yaw angle [rad]
δ	actual steering angle [rad]
δ_d	steering angle at the column system [rad]
ρ	road curvature $[\mathrm{m}^{-1}]$
η_t	width of the tyre contact patch [m]
a_y	lateral acceleration of the car $[\mathrm{m} \times \mathrm{s}^{-2}]$
B_u	damping coefficient of the steering system $[\mathrm{Nm}/(\mathrm{rad} \times \mathrm{s})]$
$C_f(C_r)$	front (rear) tyre cornering stiffness [N/rad]
D_s	reduction ratio of the steering system, *i.e.* $D_s = \delta_d/\delta$
$F_f(F_r)$	front (rear) tyre lateral force [N]
I_z	moment of inertia of the car about the yaw-axis $[\mathrm{kg} \times \mathrm{m}^2]$
J_s	moment of inertia of the steering system $[\mathrm{kg} \times \mathrm{m}^2]$
k	sharing function
K_a	visual anticipatory control of the driver
K_c	proportional gain of the transfer function representing the compensatory steering control of the driver
$l_f(l_r)$	distances of the front (rear) tyres to the mass center of the vehicle [m]
m	mass of the car [kg]
$\mathcal{S}_s(\mathcal{S}_h, \mathcal{S}_d)$	the safe subset(the hysteresis subset, the dangerous subset)
τ	external torque input to the steering system [Nm]
$\tau_f(\tau_h)$	steering torque generated by the feedback controller (the human driver) [Nm]
$T_i(T_l)$	lag (lead) time constant of the transfer function representing the compensatory steering control of the driver
T_n	neuromuscular lag time constant of the driver
T_p	driver's preview time [s]
τ_s	self-aligning moment of the steering system [Nm]
v_x	longitudinal speed of the car [m/s]
v_y	lateral speed of the car [m/s]
y_L	lateral deviation of the car [m]

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I. INTRODUCTION

According to statistics provided by the World Health Organization around 1.25 million people are killed by road accidents per year, 31% of which are car occupants [1]. Even though progress has been made in improving the road safety legislation and in making vehicles safer, the data show that the pace is too slow. Moreover, [2] states that more than 40% of the fatalities caused by car accidents are due to improper turning of the steering wheel. Therefore, we need urgent actions in improving the road safety by developing a steering assistant system.

On the basis of increasing autonomy level the driving assistant systems can be classified into four categories: Electronic Stability Control (ESC), Vision Enhancement Systems (VES), Lane Departure Warning Systems (LDWS) and Lane Keeping Assistance Systems (LKAS). ESC is designed to brake one or more wheels to prevent skidding [3]. However, such a system is active only if the driver has lost control of the car and cannot be used to prevent car crashes. VES is particularly helpful in foggy, hazy and inclement weather when the vision is unclear [4]. Note that VES is only used to enhance driver's vision. In other words, the driver holds complete control authority on the vehicle and takes responsibility of driving the car. Compared with the previous two systems, drivers trust more on LDWS and LKAS [5]. LDWS can warn the driver if the vehicle is close to the boundary of the lane and starts to leave the lane, while LKAS is able to provide continuously physical help on the steering system. The cooperation mode LKAS is regarded as a better way to help the driver than simply giving warnings and attracting the driver's attentions [2].

Even though the longitudinal and the lateral dynamics of vehicles are coupled, it is often assumed that they are decoupled when the road curvature is small [6]. The longitudinal control of vehicles has been studied in [7], while this paper focuses on the lateral assistant control. The paper [8] has proposed an adaptive steering control law based on experimental data of human mechanical impedance properties measured in turning scenarios according to the steering angles and the steering torques, while the paper [9] has presented a robust lateral tyre force control based on a feed-forward compensator and a feedback compensator and the controller has been verified by tests on an electric vehicle. Model Predictive Control (MPC) has also been used to control the steering system in autonomous vehicles [10], [11]. Even though the MPC approach based on successive on-line linearization of a nonlinear model could significantly reduce the computational complexity, it is difficult to calculate the computational time in the worst case. The paper [12] have developed the steering assistant systems for the linear lateral dynamics, while this paper studies the steering assistant system for the nonlinear lateral dynamics.

The main contributions of the paper are stated as follows. The paper studies global asymptotic stabilization problem for a class of nonlinear systems and exploits the result to solve the lateral assistant control problem in the lane keeping case. A Lyapunov-like analysis is used to prove the stability properties of the closed-loop system with the given shared-control. With the proposed controller, the magnitude of the vehicle's lateral deviation from the center line of the lane is within a given and prespecified margin.

This paper is organized as follows. Section II describes the model we study and formulates the shared-control problem for the lateral dynamics in the trajectory tracking case (*i.e.* lane keeping case), while Section III presents a preliminary theorem which is essential to solve the shared-control problem. The solution to the shared-control problem is given in Section IV, in which formal properties of the closed-loop system are established. Simulation results are presented in Section V illustrating the effectiveness of the developed shared-control algorithm. Finally, Section VI provides conclusions and suggestions for future work.

II. SYSTEM MODELING, PROBLEM STATEMENT, DEFINITIONS AND ASSUMPTIONS

As detailed in [13], the lateral dynamics of the vehicle can be described as

$$\begin{aligned}
\dot{\beta} =& \frac{2C_f}{mv_x}\delta - \dot{\psi} - \frac{2C_f}{mv_x}\arctan(\beta + \frac{l_f\dot{\psi}}{v_x}) \\
& - \frac{2C_r}{mv_x}\arctan(\beta - \frac{l_r\dot{\psi}}{v_x}), \\
\ddot{\psi} =& \frac{2C_f l_f}{I_z}\delta - \frac{2C_f l_f}{I_z}\arctan(\beta + \frac{l_f\dot{\psi}}{v_x}) \\
& + \frac{2C_r l_r}{I_z}\arctan(\beta - \frac{l_f\dot{\psi}}{v_x}),
\end{aligned} \tag{1}$$

where the actual steering angle δ can be regarded as the input to the system. The description of all parameters is given at the beginning of the paper.

Throughout the paper we take the following assumption.

Assumption 1: We assume that the longitudinal speed is a positive constant, *i.e.* $v_x > 0$ and v_x is a constant.

In a real car the steering angle cannot be controlled directly. Instead, it is controlled through the steering system, the dynamics of which is given by the equation

$$J_s\ddot{\delta}_d + B_u\dot{\delta}_d = \tau - \tau_s, \tag{2}$$

where the self-aligning moment τ_s can be calculated as

$$\tau_s = -\frac{2C_f\eta_t}{R_s}\beta - \frac{2C_f l_f\eta_t}{R_s v_x}\dot{\psi} + \frac{2C_f\eta_t}{R_s^2}\delta_d. \tag{3}$$

In the lane keeping dynamics we introduce two additional variables to describe the relationship between the vehicle and the center line of the lane. These are the lateral deviation y_L and the yaw error ψ_L. Their dynamics can be described by the equations

$$\begin{aligned}
\dot{y}_L =& v_x\beta + T_p v_x\dot{\psi} + v_x\psi_L, \\
\dot{\psi}_L =& \dot{\psi} - v_x\rho,
\end{aligned} \tag{4}$$

where the definitions of the parameters and variables are given at the beginning of the paper.

Definition 1: We use the names *h-control* and *f-control*, denoted as τ_h and τ_f, to describe the steering torque generated by the human driver and the feedback controller, respectively. In addition, we use the name *sharing function*, denoted as k, to define how the control authority is shared between the human driver and the feedback controller, *i.e.*

$$\tau = k\tau_h + (1-k)\tau_f. \tag{5}$$

Note that $k(t) \in [0,1]$ for all $t \geq 0$.

The block diagram of the overall system is illustrated in Fig. 1, which shows that the value of the sharing function k is a feedback signal to the human driver, indicating how dangerous his/her performance is. This is helpful not only in improving the driver's driving skills but also in reducing driver's anger and irritation when he/she feels that the car does not respond to him/her completely.

Definition 2: In the lane keeping case we assume that $\mathcal{S}_a \in \mathbb{R}^6$ is a given set describing the feasible values of the state of the system (1)-(2)-(4) and is defined as

$$\mathcal{S}_a = \left\{(\beta, \dot{\psi}, \delta, \dot{\delta}, y_L, \psi_L) \in \mathbb{R}^6 \mid |y_L - y_{L_R}| < \sigma\right\}, \tag{6}$$

where y_{L_R} is the reference signal for y_L and σ is a strictly positive constant.

Definition 3: For any given road curvature $\rho(t)$ the reference trajectory $(\beta_R, \dot{\psi}_R, \delta_R, y_{L_R}, \psi_{L_R})$ is said to be "feasible" if

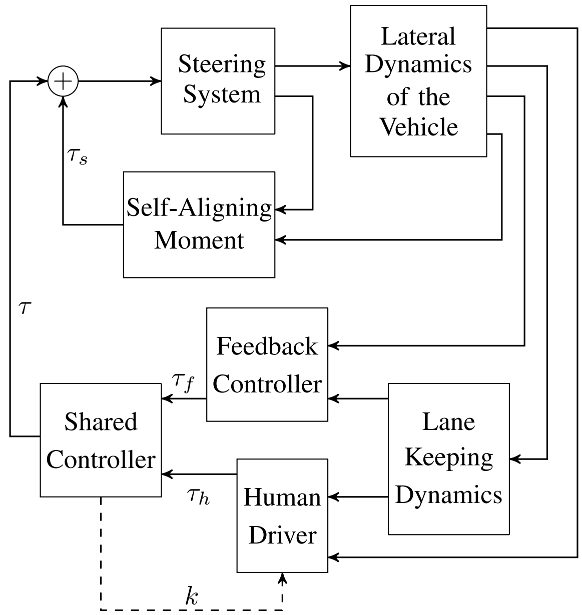

Fig. 1: Block Diagram of the Overall System

and only if there exists a τ_R such that the following equations

$$\begin{aligned}
\dot{\beta}_R =& \frac{2C_f}{mv_x}\delta_R - \dot{\psi}_R - \frac{2C_f}{mv_x}\arctan(\beta_R + \frac{l_f\dot{\psi}_R}{v_x}) \\
& - \frac{2C_r}{mv_x}\arctan(\beta_R - \frac{l_r\dot{\psi}_R}{v_x}), \\
\ddot{\psi}_R =& \frac{2C_f l_f}{I_z}\delta_R - \frac{2C_f l_f}{I_z}\arctan(\beta_R + \frac{l_f\dot{\psi}_R}{v_x}) \\
& + \frac{2C_r l_r}{I_z}\arctan(\beta_R - \frac{l_f\dot{\psi}_R}{v_x}), \\
\ddot{\delta}_R =& \frac{\tau_R - \tau_s - B_u R_s \dot{\delta}_R}{J_s R_s}, \\
\dot{y}_{L_R} =& v_x\beta_R + T_p v_x \dot{\psi}_R + v_x \psi_{L_R}, \\
\dot{\psi}_{L_R} =& \dot{\psi}_R - v_x \rho,
\end{aligned} \tag{7}$$

hold, where

$$\tau_s = -\frac{2C_f\eta_t}{R_s}\beta_R - \frac{2C_f l_f \eta_t}{R_s v_x}\dot{\psi}_R + \frac{2C_f\eta_t}{R_s}\delta_R,$$

and $(\beta_R(t), \dot{\psi}_R(t), \delta_R(t), \dot{\delta}_R(t), y_{L_R}(t), \psi_{L_R}(t)) \in \mathcal{S}_a$ for all $t \geq 0$.

Assumption 2: We assume that $\beta, \dot{\psi}, v_x, l_f, l_r, \beta_R$ and $\dot{\psi}_R$ are such that the inequalities

$$\begin{aligned}
(\beta_R + \frac{l_f\dot{\psi}_R}{v_x})(\beta + \frac{l_f\dot{\psi}}{v_x}) + 1 > 0 \\
(\beta_R - \frac{l_r\dot{\psi}_R}{v_x})(\beta - \frac{l_r\dot{\psi}}{v_x}) + 1 > 0
\end{aligned}$$

hold.

Note that Assumption 2 indicates that

$$\begin{aligned}
&\arctan(\beta + \frac{l_f\dot{\psi}}{v_x}) - \arctan(\beta_R + \frac{l_f\dot{\psi}_R}{v_x}) \\
&= \arctan\frac{\beta + \frac{l_f\dot{\psi}}{v_x} - \beta_R - \frac{l_f\dot{\psi}_R}{v_x}}{1 + (\beta_R + \frac{l_f\dot{\psi}_R}{v_x})(\beta + \frac{l_f\dot{\psi}}{v_x})}, \\
&\arctan(\beta - \frac{l_r\dot{\psi}}{v_x}) - \arctan(\beta_R - \frac{l_r\dot{\psi}_R}{v_x}) \\
&= \arctan\frac{\beta - \frac{l_r\dot{\psi}}{v_x} - \beta_R + \frac{l_r\dot{\psi}_r}{v_x}}{1 + (\beta_R - \frac{l_r\dot{\psi}_R}{v_x})(\beta - \frac{l_r\dot{\psi}}{v_x})},
\end{aligned}$$

and all arguments are bounded.

The shared-control problem for the lateral dynamics in the trajectory tracking case can then be formulated as follows.

Given the system (1)-(2)-(4) and a feasible reference trajectory $(\beta_r(t), \dot{\psi}_R(t), \delta_R(t), y_{L_R}(t), \psi_{L_R}(t))$, together with a reference torque $\tau_R(t)$ and the road curvature $\rho(t)$, find (if possible)

- a feedback controller τ_f;
- a sharing function k;
- a safe subset $\mathcal{R}_s \subset \mathcal{S}_a$;

such that closed-loop system (1)-(2)-(4)-(5) has the following properties.

P1) The state constraints are satisfied, *i.e.* $(\beta(t), \dot{\psi}(t), \delta(t), \dot{\delta}(t), y_L(t), \psi_L(t)) \in \mathcal{S}_a$ for all $t \geq 0$.

P2) The control effort is bounded, *i.e.* $\exists \mathcal{B} > 0$ such that $|\tau_f(t)| < \mathcal{B}$ for all $t \geq 0$.

P3) The tracking error converges to zero, *i.e.*

$$\lim_{t\to\infty}(\beta(t), \dot{\psi}(t), \delta(t), y_L(t), \psi_L(t)) - (\beta_R(t), \dot{\psi}_R(t), \delta_R(t), y_{L_R}(t), \psi_{L_R}(t)) = 0.$$

P4) If the human driver drives safely then he/she has complete control authority on the vehicle, *i.e.* $\tau = \tau_h$ if $(\beta, \dot{\psi}, \delta, \dot{\delta}, y_L, \psi_L) \in \mathcal{R}_s$.

III. PRELIMINARY RESULTS

This section provides some preliminary results which are essential to solve the shared-control problem stated in Section II.

Theorem 1: Consider a system, the dynamics of which can be described by the equations

$$\begin{aligned}
\dot{x} =& F(x) + B\xi, \\
\dot{\xi} =& g(x, \xi) + hu, \\
\dot{z} =& f(z)\xi,
\end{aligned} \tag{8}$$

where $x(t) \in \mathbb{R}^n$, $\xi(t) \in \mathbb{R}$ and $z(t) \in \mathbb{R}$ are the states and $u(t) \in \mathbb{R}$ is the control input. In addition, $B \in \mathbb{R}^n$ and $h \in \mathbb{R}$ are a constant vector and a constant scalar, respectively, while $F : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^{n+1} \to \mathbb{R}$ and $f : \mathbb{R} \to \mathbb{R}$ are continuous mappings. Assume that

A1) $f(z) \neq 0$, for all $z \in \mathbb{R}$;
A2) $h \neq 0$;
A3) $g(0,0) = 0$;
A4) $x = 0$ is an equilibrium point of the system $\dot{x} = F(x)$ and it is globally asymptotically stable.

Then there exists a state-feedback control law $u(x, \xi, z)$ such that the zero equilibrium of the closed-loop system is globally asymptotically stable. One such a choice is given by

$$u(x,\xi,z) = -\frac{\left(\frac{\partial L(x)}{\partial x}\right)^T B + g(x,\xi) + zf(z) + \alpha\xi}{h}, \quad (9)$$

for any $\alpha > 0$, where $L(x)$ is a Lyapunov function for the subsystem $\dot{x} = F(x)$.

IV. SHARED-CONTROL FOR THE LATERAL MOTION OF VEHICLES

This section provides a solution to the shared-control problem stated in Section II and such that the state constraint $|y_{L_e}(t)| = |y_L(t) - y_{L_R}(t)| < \sigma$ is satisfied for all $t \geq 0$ and for any given constant σ.

A. Feedback Control Design for the Lateral Dynamics of Vehicles

In this subsection a design for the feedback controller is given such that the closed-loop system (1)-(2)-(4)-(5) with $k = 0$ is able to track a given feasible reference and the constraints on the system is always satisfied.

In the beginning we design a controller for (1)-(2)-(4)-(5) with $k = 0$ and $\rho = 0$, in which case $\beta_R = \dot{\psi}_R = \psi_{L_R} = \delta_R = 0$. To remove the constraint on y_L we define the new variable

$$z = \frac{1}{2}\log\frac{y_{L_e} + \sigma}{\sigma - y_{L_e}}.$$

Furthermore, define the variables

$$\begin{aligned} x_1 &= \beta + \frac{l_f}{v_x}\dot{\psi} - a(\beta - \frac{l_r}{v_x}\dot{\psi}), \\ x_2 &= \psi_L, \quad \xi = v_x\beta + T_p v_x\dot{\psi} + v_x\psi_L, \end{aligned} \quad (10)$$

with

$$a = \frac{\dfrac{2C_f l_f^2}{I_z v_x} + \dfrac{2C_f}{m v_x}}{\dfrac{2C_f}{m v_x} - \dfrac{2C_f l_f l_r}{I_z v_x}}. \quad (11)$$

Using the new variables x_1, x_2, ξ and z, the system (1)-(4) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}\xi + a_{14}m_2\epsilon, \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}\xi, \\ \dot{\xi} &= a_{31}(m_2 - m_1) + a_{32}\arctan m_2 + b\tilde{\delta} \\ \dot{z} &= \frac{(e^z + e^{-z})^2}{4\sigma}\xi, \end{aligned} \quad (12)$$

where a_{ij} and b are functions of v_x for all $i \in \{1,2,3\}$ and $j \in \{1, 2, \ldots, 5 - i\}$, and

$$\begin{aligned} \epsilon &= \frac{\arctan(\beta - \frac{l_r}{v_x}\dot{\psi})}{\beta - \frac{l_r}{v_x}\dot{\psi}}, \quad \tilde{\delta} = \delta - \arctan(\beta + \frac{l_f}{v_x}\dot{\psi}), \\ m_1 &= \frac{(l_f - T_p v_x)v_x x_1 - a(l_f + l_r)v_x x_2 + a(l_f + l_r)\xi}{l_f v_x - T_p v_x^2 + a l_r v_x + T_p a v_x^2}, \\ m_2 &= \frac{-(l_r + T_p v_x)v_x x_1 - (l_f + l_r)v_x x_2 + (l_f + l_r)\xi}{l_f v_x - T_p v_x^2 + a l_r v_x + T_p a v_x^2}. \end{aligned} \quad (13)$$

Consider the subsystem

$$\begin{aligned} \dot{x}_1 =& \frac{(a-1)v_x x_1}{l_f - T_p v_x + a l_r + T_p a v_x} - \frac{(a-1)^2 v_x x_2}{l_f - T_p v_x + a l_r + T_p a v_x} \\ &+ (\frac{2C_r l_r l_f}{I_z v_x} - \frac{2C_r}{m v_x} + \frac{2aC_r l_r^2}{I_z v_x} + \frac{2aC_r}{m v_x})m_2\epsilon, \\ \dot{x}_2 =& \frac{v_x x_1}{l_f - T_p v_x + a l_r + T_p a v_x} + \frac{(1-a)v_x x_2}{l_f - T_p v_x + a l_r + T_p a v_x}. \end{aligned} \quad (14)$$

In typical driving scenarios we assume that $|\beta - \frac{l_r}{v_x}\dot{\psi}| \leq 7$, yielding $\epsilon \in [0.2, 1]$. Then the zero equilibrium of the subsystem (14) is globally asymptotically stable for typical vehicles and we assume that the corresponding Lyapunov candidate is L.

The following example can be used as a guidance for proving the stability properties of the system (14).

TABLE I: Typical Vehicle Parameters

η	0.15	B_u	2.5	I_z	1500
C_f	170390	C_r	195940	J_s	0.05
l_f	1.48	l_r	1.12	m	1625
R_s	12				

Example 1: Consider the vehicle parameters given in Table I, together with $T_p = 0.1s$ and $v_x = 10\ m/s$. The subsystem (14) can be rewritten as

$$\begin{aligned} \dot{x}_1 =& (6.16 - 55.36\epsilon)x_1 + (32.3 + 67.91\epsilon)x_2, \\ \dot{x}_2 =& -1.17x_1 - 6.16x_2. \end{aligned}$$

Now consider the Lyapunov function candidate

$$L = 0.5x_1^2 + 5x_2^2,$$

the derivative of which along the trajectories of the system is

$$\dot{L} = (6.16 - 55.36\epsilon)x_1^2 + (20.6 + 67.91\epsilon)x_1x_2 - 61.6x_2^2,$$

which is such that $\dot{L} < 0$ for all $(x_1, x_2) \neq (0, 0)$.

Lemma 1: Consider the system (12) with input

$$\tilde{\delta} = -\frac{\left(\frac{\partial L(x)}{\partial x}\right)^T B + g(x,\xi) + zf(z) + \alpha\xi}{b}, \tag{15}$$

where $L(x)$ is a Lyapunov function for the subsystem (14), $\alpha > 0$, $B = [a_{13}, a_{23}]^T$, $f(z) = (e^z + e^{-z})^2/(4\sigma)$ and $g(x,\xi) = a_{31}(m_2 - m_1) + a_{32}\arctan m_2$ with a given in (11). Suppose σ is a positive constant. Then the zero equilibrium of the closed-loop system (12)-(15) is globally asymptotically stable.

For any given feasible reference trajectory the tracking error $[\beta_e, \dot{\psi}_e, \delta_e, y_{L_e}, \psi_{L_e}]^T$ is defined as

$$\begin{aligned}[\beta_e, \dot{\psi}_e, \delta_e, y_{L_e}, \psi_{L_e}]^T =& [\beta, \dot{\psi}, \delta, y_L, \psi_L]^T \\ &- [\beta_R, \dot{\psi}_R, \delta_R, y_{L_R}, \psi_{L_R}]^T.\end{aligned}$$

Similarly to the definition given in (10), the variables x_{1r}, x_{2r} and ξ_r are defined as

$$\begin{aligned} x_{1R} = \beta_R + \frac{l_f}{v_x}\dot{\psi}_R - a(\beta_R - \frac{l_r}{v_x}\dot{\psi}_R), \\ x_{2R} = \psi_{L_R}, \quad \xi = v_x\beta_R + T_p v_x \dot{\psi}_R + v_x \psi_{L_R}. \end{aligned} \tag{16}$$

Theorem 2: Consider the system (12) with the feedback controller

$$\tau_f = [-\alpha(\dot{\delta} - \dot{\delta}^*) + \delta^* - \delta + \ddot{\delta}^* + B_u D_s \dot{\delta}] J_s D_s + \tau_s, \tag{17}$$

where $\alpha > 0$, τ_s is calculated in (3) and

$$\begin{aligned}\delta^* =& -\frac{\left(\frac{\partial L(x)}{\partial x}\right)^T B + g(x,\xi) + zf(z) + \alpha\xi}{b} \\ &+ \arctan(\beta + \frac{l_f}{v_x}\dot{\psi}),\end{aligned} \tag{18}$$

with $z = (e^z + e^{-z})^2/(4\sigma)$, $B = [a_{13}, a_{23}]^T$, m_{1R} and m_{2R}, the reference value of m_1 and m_2, respectively, and

$$\begin{aligned} g(x, x_R, \xi, \xi_R) =& a_{31}(m_2 - m_{2R} - m_1 + m_{1R}) \\ &+ a_{32}\arctan\frac{m_2 - m_{2R}}{1 + m_2 m_{2R}}. \end{aligned}$$

Suppose σ is a positive constant. Then the tracking error converges to zero.

B. Shared-Control Algorithm

Given the reference trajectory $(\beta_R, \dot{\psi}_R, \delta_R, y_{L_R}, \psi_{L_R})$, on the basis of the value of y_{L_e} the overall state space can be divided into three subsets: the safe subset $\mathcal{S}_s$, the hysteresis subset $\mathcal{S}_h$ and the dangerous subset $\mathcal{S}_d$, the definitions of which are

$$\begin{aligned} \mathcal{S}_s &= \left\{(\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \in \mathbb{R}^6 \mid |y_{L_e}| < \sigma_1\right\}, \\ \mathcal{S}_h &= \left\{(\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \in \mathbb{R}^6 \mid \sigma_1 \le |y_{L_e}| \le \sigma_2\right\}, \\ \mathcal{S}_d &= \left\{(\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \in \mathbb{R}^6 \mid \sigma_2 < |y_{L_e}| < \sigma\right\}, \end{aligned} \tag{19}$$

where σ_1 and σ_2 are user selected parameters and $\sigma > \sigma_2 > \sigma_1 > 0$.

Similarly to [14] and [15], the sharing function k is defined as

$$k(y_{L_e}) = \begin{cases} 1, & (\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \in \mathcal{S}_s, \\ l, & (\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \in \mathcal{S}_h, \\ 0, & (\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \in \mathcal{S}_d, \end{cases} \tag{20}$$

where

$$l = \begin{cases} 1, & \text{if } (\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \text{ enters } \mathcal{S}_h \text{ from } \mathcal{S}_s, \\ 0, & \text{if } (\beta_e, \dot{\psi}_e, \delta_e, \dot{\delta}_e, y_{L_e}, \psi_{L_e}) \text{ enters } \mathcal{S}_h \text{ from } \mathcal{S}_d. \end{cases}$$

Theorem 3: Consider the lateral dynamics of the vehicle (1)-(2)-(4) controlled by the shared-controller (5)-(17)-(20). Let $(\beta_R, \dot{\psi}_R, \delta_R, y_{L_R}, \psi_{L_R})$ be the reference trajectory and τ_h the human input. Assume that the closed-loop system with the human input τ_h is able to track the reference. Then for any $\sigma > 0$ there exist σ_1 and σ_2 such that $0 < \sigma_1 < \sigma_2 < \sigma$ and the closed-loop system has the following properties.

P1) The state constraints are always satisfied, *i.e.* $|y_L(t) - y_{L_R}(t)| < \sigma$ for all $t \ge 0$.

P2) The tracking error converges to zero, *i.e.* $\lim_{t\to\infty} \beta_e = \lim_{t\to\infty} \dot{\psi}_e = \lim_{t\to\infty} \delta_e = \lim_{t\to\infty} y_{L_e} = \lim_{t\to\infty} \psi_{L_e} = 0$.

P3) If the human input is bounded, then the shared-control input is bounded.

P4) $\tau(t) = \tau_h(t)$ if $(\beta(t), \dot{\psi}(t), \delta(t), \dot{\delta}(t), y_L(t), \psi_L(t)) \in \mathcal{S}_s$ at time t.

Remark 1: Even though the paper studies the lateral shared-control problem for the lane keeping case, a similar approach, together with path planning methods, could be applied to more general areas, such as lane changing scenarios.

V. SIMULATION RESULTS AND ANALYSIS

This section introduces the driver model used in the simulations and provides simulation results for the system (1)-(2)-(4) with and without the shared-controller.

A. Driver Model

We use the so-called two-level driver model to model the performance of typical human drivers [16], the dynamics of which is described by the equations

$$\begin{aligned} \dot{d} =& \begin{bmatrix} -\frac{1}{T_i} & 0 \\ \frac{K_c(T_l - T_i)}{T_i^2} & -\frac{1}{T_n} \end{bmatrix} d \\ &+ \begin{bmatrix} 0 & 1 \\ K_a & -\frac{K_c T_l}{T_i} \end{bmatrix} \begin{bmatrix} D\rho \\ \frac{y_L}{v_x T_p} \end{bmatrix}, \\ T_h =& \frac{d_2}{T_n}, \end{aligned} \tag{21}$$

where $d = [d_1, d_2]^T \in \mathbb{R}^2$ are the states of the driver model and D is a constant between 10 m to 20 m according to

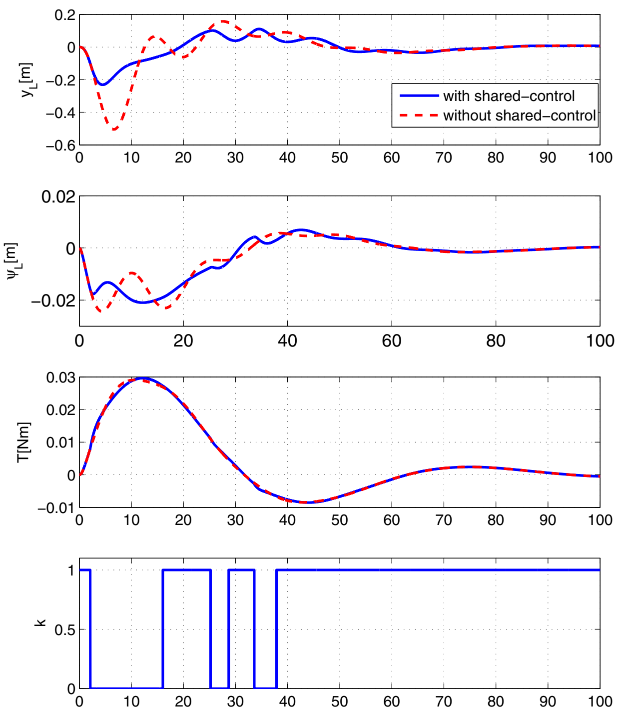

Fig. 2: Time histories of the lateral deviation y_L, heading error ψ_L, steering torque T and the value of the sharing function k for the system (1)-(2)-(4) with the shared-controller (5)-(17)-(20) (blue, solid) and with the human driver only (red, dashed).

the radius of curvature. Note that the definitions of all the parameters are given at the beginning of the paper. Table II shows a set of typical parameter values.

TABLE II: Typical Parameter Values for the Driver Model

T_l	T_i	T_n	D	T_p	K_a	K_c
1.16	0.14	0.11	15	2	56.97	36.13

B. Trajectory Tracking

In this case study we assume that the reference trajectory is winding and tortuous at the beginning and becomes a straight line gradually, with the curvature of the reference path defined as

$$\rho(t) = 0.02e^{-0.04t}\sin(0.1t). \tag{22}$$

In addition, we assume that $\sigma = 0.3\ m$, while σ_1 and σ_2 have been selected as $0.08\ m$ and $0.15\ m$, respectively.

Simulation results are displayed in Fig. 2, from which we see clearly that with the shared-control the lateral deviation is significantly reduced, demonstrating the effectiveness of the shared-controller. In addition, Fig. 2 shows that both the lateral deviation and the yaw error ψ_L of the vehicle converge to zero for the closed-loop system with and without the shared-controller.

VI. CONCLUSIONS

We have studied the globally asymptotic stabilization problem for a class of nonlinear systems. The solution has been used to develop a lateral assistant control for the nonlinear model of a vehicle. With the developed shared-controller the vehicle is able to track any given feasible references while guaranteeing that the lateral deviation is smaller than a preselected bound. Simulation results for the closed-loop system with the shared-controller show the effectiveness of the proposed shared-control law. Future research will focus on shared-control design for the lateral dynamics of vehicles with trailers.

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