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Performance oriented triggering mechanisms with guaranteed traffic characterization for linear discrete-time systems

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Abstract— **This paper is concerned with transmission reduction in linear discrete-time Networked Control Systems with computational capabilities at the sensor only. A triggering mechanism is derived to reduce the number of transmissions while being oriented at the control performance, measured by an infinite horizon linear quadratic cost functional. It is shown how trigger functions can be designed and parametrized such that an a priori demanded performance bound will not be violated. Furthermore the general approach is used to derive a triggering mechanism with guaranteed characterization of the resulting network traffic. Therefore a model for network traffic, known from traffic shaping in communication networks, is presented and a corresponding trigger function is designed. All resulting mechanisms are evaluated in a numerical example.**

I. INTRODUCTION

The motivation for using event-based sampling approaches in Networked Control Systems (NCS), i.e., systems where some information is communicated over a communication medium, is the associated potential to increase the efficiency between transmission effort and resulting performance. An explicit characterization of this gain in efficiency was already shown in [1] for an integrator system with impulsive control.

Recently, there is an increasing interest in investigating this efficiency in more general setups. There are two main approaches to increase the efficiency, leading to different problem setups. The first paradigm is to demand the event-based sampling strategy to have the same average transmission rate as a compared periodic strategy while guaranteeing a better performance. This is guaranteed using a receding horizon approach, employing a baseline periodic optimal policy, in [2]. In [3], the definition of consistent event-triggered policies is introduced to characterize improved efficiency, together with an additional property, and such policies are derived. Another important recent work in this line of research can be found in [4], where it is shown that optimal minimum-variance event-triggered control strategies consist of a controller structure that is independent of the sampling mechanism, which can be seen as a generalization of [5]. The design of the optimal sampling mechanism is analytically solved for the setup in [1] and the work in [6] shows that the design problem can be restated using a partial differential equation.

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A slightly different direction, which is particularly meaningful for discrete-time systems, is to reduce the number of transmissions in such a way that it is guaranteed that transmissions are still initiated if the performance would be sacrificed too strongly. A recent approach can be found in [7]. It considers a discrete-time linear system with output feedback and gives a bound on the resulting closed loop LQG performance in terms of a constant factor of the optimal cost. This direction of increasing the efficiency is followed in the paper at hand as well. A linear discrete-time system is considered and the performance is characterized using an infinite horizon LQR cost. A general performance oriented triggering mechanism will be derived directly from the properties of the optimal controller in the LQR setup. Thereafter, suitable trigger functions are presented for being used in the mechanism to guarantee a desired upper bound on the absolute additional cost due to less transmissions.

Although the increased efficiency in terms of performance per transmissions makes event-based mechanisms interesting for NCS, there is still a drawback which was highlighted in [8]. Therein the usually uncertain network traffic of event-based sampling mechanisms was shown to be a relevant issue for NCS. Thus, another line of research focuses on characterizing the traffic pattern. An analysis technique for stochastic triggering rules and scalar systems is developed in [9] that computes for a given strategy the expected communication rate and control performance. Due to the design procedure in [2], therein it is even possible to give upper bounds on the number of transmissions for the resulting traffic in a window of finite length, which can be very helpful. Another approach is to use formal methods to derive characterizations of the resulting network traffic as in [10].

In the work at hand we will introduce a new approach. This approach is based on a traffic model that is already used for traffic shaping in communication networks, known as the token bucket algorithm, as described in [11]. It will be shown how the model can be used to describe the resulting network traffic of event-based sampling mechanisms. Then, a trigger function for the performance oriented triggering mechanism will be designed, such that it a priori guarantees that the resulting network traffic has a desired specification according to the introduced model.

This traffic specification and corresponding mechanism (Section V), together with the derivation of a performance oriented mechanism from the LQR properties (Section III) and a parametrization for an a priori guaranteed upper bound on the additional cost (Section IV), represent the contributions of the paper at hand.

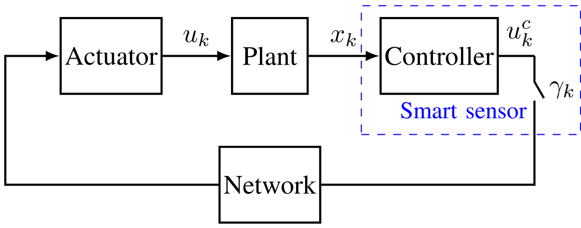

Fig. 1. Setup with controller and transmission decision on sensor side.

II. PROBLEM SETUP

We consider the linear discrete-time system

$$x_{k+1} = Ax_k + Bu_k,\ k \in \mathbb{N}_0, \tag{1}$$

with initial state $x_0 \in \mathbb{R}^{n_x}$ and input $u_k \in \mathbb{R}^{n_u}$, where $\mathbb{N}_0$ denotes the nonnegative integers and (A, B) is assumed to be stabilizable. We assume (1) to be the plant of a NCS in an architecture with a smart sensor with full state measurement and computational capabilities whereas we assume to have no computational capabilities at the actuation mechanism and try to reduce the transmissions from the sensor to the plant, see Fig.1. At every discrete time instant k, a new control input is computed at the sensor. The computed control input at every time instant k is denoted by u_k^c and given by the static state feedback

$$u_k^c = Kx_k. \tag{2}$$

The computed control input is sent to the actuator if a triggering condition, that will be defined in this paper, is satisfied. To indicate whether a new input vector is sent to the plant, one introduces the binary infinite transmission sequence $\gamma := (\gamma_k)_{k\in\mathbb{N}_0}$, where $\gamma_k = 0$ indicates that no input is sent while $\gamma_k = 1$ represents a transmission. The applied input u_k is given as a hold actuation in the form

$$u_k = \gamma_k u_k^c + (1 - \gamma_k)u_{k-1},\ k \in \mathbb{N}_0 \tag{3}$$

with initial condition $u_{-1} \in \mathbb{R}^{n_u}$.

In this paper we will investigate the standard discrete-time linear quadratic infinite horizon cost functional J of the form

$$J = \sum_{k=0}^{\infty} x_k^\intercal Q x_k + u_k^\intercal R u_k \tag{4}$$

with Q being positive semi-definite, R being positive definite, and $(A, Q^{\frac{1}{2}})$ being detectable. In accordance with this cost functional we choose K to be the optimal controller gain

$$K = -(R + B^\intercal PB)^{-1}B^\intercal PA \tag{5}$$

for the computation of new control inputs according to (2), where P is the unique symmetric positive semi-definite solution of the discrete-time algebraic Riccati equation

$$P = Q + A^\intercal PA - A^\intercal PB(R + B^\intercal PB)^{-1}B^\intercal PA.$$

Thus we know from standard results that the optimal performance, i.e., the minimal cost, is given by $J^* = x_0^\intercal Px_0$ if the input is updated at every time instant.

In the remainder we want to use a property of the linear quadratic regulator to derive a performance oriented triggering mechanism. Afterwards the goal is to develop trigger functions for the mechanism such that a bound on the performance or a traffic specification can be guaranteed.

III. PERFORMANCE ORIENTED TRIGGERING MECHANISM

The goal is to exploit properties of the LQR setup to develop the performance oriented triggering mechanism. For that reason we will start this section with a standard formula from linear quadratic control that presents an expression for the infinite horizon cost (4) in terms of the difference between the applied input u_k and the optimal input u_k^c at all time instants $k \in \mathbb{N}_0$. The expression

$$J = x_0^\intercal Px_0 + \sum_{k=0}^{\infty}(u_k - Kx_k)^\intercal(R + B^\intercal PB)(u_k - Kx_k), \tag{6}$$

sometimes referred to as LQ-formula, finds application for example in constrained predictive control [12] and a derivation (for the continuous-time counterpart with $R = I$) can be found in [13, Lemma 10.11].

With this formula we can now state a general performance oriented triggering mechanism as

$$\gamma_k = \begin{cases} 1 & (u_{k-1} - u_k^c)^\intercal(R + B^\intercal PB)(u_{k-1} - u_k^c) > f_i \\ 0 & \text{otherwise} \end{cases}, \tag{7}$$

where f_i represents a real-valued trigger function. In this paper several trigger functions will be designed that lead to performance oriented triggering mechanisms for desired objectives.

An intuitive approach to design a first trigger function is to simply use the constant function f_0, i.e.,

$$f_0(k) = c_0, k \in \mathbb{N}_0 \tag{8}$$

with a nonnegative real value c_0. The interpretation of the triggering mechanism (7) with trigger function (8) is that a new input is sent over the network only if the performance decrease using the currently applied input was greater than c_0. While this obviously presents a performance oriented triggering mechanism we are interested in designing other ones with additional desired properties.

This will be the goal of the next two sections. The next section focuses on a priori guarantees for performance bounds whereas the subsequent section comes up with a trigger function that allows to make a priori guarantees on the resulting network traffic, which is a very interesting problem in the context of event-triggered control since the resulting traffic is usually unknown due to its explicit state dependency.

IV. TRIGGERING WITH BOUNDED PERFORMANCE LOSS

As just stated the goal of this section is to develop a trigger function for the general triggering mechanism (7) and to parametrize it such that one can a priori guarantee a bound on the cost increase due to reducing the number of transmissions. More precisely, we seek for a trigger function that guarantees that the infinite horizon cost (4) satisfies the bound

$$J \leq J^\star + \eta \tag{9}$$

for a given $\eta \geq 0$, referred to as the performance loss.

Remark 1: The bound in (9) aims for bounding performance loss as an absolute quantity and not as a constant factor of the optimal cost and thus of the initial condition as for example in [7], [14]. This can be beneficial for the efficient utilization of resources in the case of several plants sharing a communication network.

The trigger function that we propose to be used is

$$f_1(k) = c_1 r_1^k,\ k \in \mathbb{N}_0 \tag{10}$$

with scalar design parameters c_1 and r_1. Note the explicit dependency of time in (10). This is a close similarity to the time-dependent triggering approach in [15]. The following theorem gives a parametrization for (10) such that (9) holds.

Theorem 2: Assume system (1) is controlled with (2), (3) where K is the LQR controller (5). If the transmission sequence γ is defined by triggering mechanism (7) with trigger function (10), where $r_1 \in [0, 1)$ and $c_1 \leq \eta(1 - r_1)$, the closed-loop infinite horizon cost J is bounded according to (9).

Proof: From triggering mechanism (7) and trigger function (10) we know that

$$(u_k - Kx_k)^\intercal (R + B^\intercal PB)(u_k - Kx_k) \leq c_1 r_1^k$$

for all $k \in \mathbb{N}_0$. Therefore we can use the LQ formula (6) and the parameter bounds to derive

$$\begin{aligned} J &= J^\star + \sum_{k=0}^{\infty} (u_k - Kx_k)^\intercal (R + B^\intercal PB)(u_k - Kx_k) \\ &\leq J^\star + \sum_{k=0}^{\infty} c_1 r_1^k = J^\star + c_1 \frac{1}{1 - r_1} \leq J^\star + \eta \end{aligned}$$

using the properties of the sum of a geometric series. ■

By looking at the proof one can notice that the guarantee due to the trigger function is used elementwise in the LQ formula. In other words, the infinite sum that appears in the LQ formula is bounded by bounding every element of the sum with (10). Thus already from the fact, that for all time instants $k \in \mathbb{N}_0$ with $\gamma_k = 1$ it is known that $u_k - Kx_k = 0$, it is clear that the bound (9) is not tight with trigger function (10).

Therefore, a second triggering function is derived that follows similar ideas but being less conservative in the sense that it will make the gap between the guaranteed and the actual performance smaller. For that reason we introduce another sequence $\tau := (\tau_{k_\tau})_{k_\tau \in \mathbb{N}_0}$, that consists of $\tau_0 := 0$ and $(\tau_{k_\tau})_{k_\tau \in \mathbb{N}}$ being the time instants when new information is sent over the network.

Based on this sequence we define the modified trigger function as

$$\begin{aligned} &f_2(k, k_\tau, (u_j)_{j \in [\tau_{k_\tau}, k-1]}, (u_j^c)_{j \in [\tau_{k_\tau}, k-1]}) \\ &= c_2 r_2^{k_\tau} - \sum_{j=\tau_{k_\tau}}^{k-1} (u_j - Kx_j)^\intercal (R + B^\intercal PB)(u_j - Kx_j) \end{aligned} \tag{11}$$

for $k \in 0 \cup [\tau_{k_\tau} + 1, \tau_{k_\tau+1}], k_\tau \in \mathbb{N}_0$ with scalar design parameters c_2 and r_2. One sees that this trigger function is again dependent on time, directly as well as implicitly by the time instants of successful transmissions. Furthermore it can be seen that the sampling mechanism needs to store applied and computed input values until a new input value is sent. This increases the memory that is employed with the goal to reduce the number of transmissions and shows a relationship to dynamic event-triggered control, see for example [16]. The next theorem will provide a parametrization for trigger function (12) such that (9) still holds.

Theorem 3: Assume system (1) is controlled with (2), (3) where K is the LQR controller (5). If the transmission sequence γ is defined by triggering mechanism (7) with trigger function (12), where $r_2 \in [0, 1)$ and $c_2 \leq \eta(1 - r_2)$, the closed-loop infinite horizon cost J is bounded according to (9).

Proof: From triggering mechanism (7) and trigger function (12) one can now conclude that

$$\sum_{k=\tau_{k_\tau}}^{\tau_{k_\tau+1}-1} (u_k - Kx_k)^\intercal (R + B^\intercal PB)(u_k - Kx_k) \leq c_2 r_2^{k_\tau} \tag{12}$$

for all $k_\tau \in \mathbb{N}_0$. We can use this again together with the LQ formula (6), the parameter bounds and the properties of the sum of a geometric series to show

$$\begin{aligned} &J^\star + \sum_{k=0}^{\infty} (u_k - Kx_k)^\intercal (R + B^\intercal PB)(u_k - Kx_k) \\ &= J^\star + \sum_{k_\tau=0}^{\infty} \sum_{k=\tau_{k_\tau}}^{\tau_{k_\tau+1}-1} (u_k - Kx_k)^\intercal (R + B^\intercal PB)(u_k - Kx_k) \\ &\leq J^\star + \sum_{k_\tau=0}^{\infty} c_2 r_2^{k_\tau} = J^\star + c_2 \frac{1}{1 - r_2} \leq J^\star + \eta. \end{aligned}$$

■

In contrast to the proof of Theorem 2 the only remaining gap to tightness of the bound is that due to the triggering mechanism one can not guarantee equality in (12) since this is not possible in a discrete-time setup.

Remark 4: Both results give a guarantee on the boundedness of the closed-loop infinite horizon cost. By standard arguments, employing the assumption that $(A, Q^{\frac{1}{2}})$ is detectable, this can be used to conclude that $\lim_{k \to \infty} x_k = 0$.

V. TRIGGERING WITH GUARANTEED TRAFFIC CHARACTERIZATION

In the previous section we focused on trigger functions that a priori guarantee a desired level of performance, differing in the amount of conservatism. We will not demand such a bound in this section. Instead, the goal is to demand another a priori guarantee that is relevant for event-triggered control, which is an a priori traffic characterization. This is a property that is in particular important for NCS where the design of suitable communication networks can be performed more efficiently with an increasingly precise traffic characterization. Nevertheless, we are still aiming for a performance-oriented mechanism, which excludes explicitly mechanisms that simply satisfy the specification by not sending at all.

While currently the traffic characterization is often limited to statements about the minimal inter-event time or bounds on average sampling rates this work focuses on guaranteeing a traffic behavior that is specified with a model that is actually used for traffic shaping in communication networks. This model will be introduced in the first subsection while the second subsection will make use of (7) to design a performance-oriented triggering mechanism with guaranteed traffic characterization.

A. Token bucket traffic characterization

The token bucket model or token bucket algorithm is usually applied to permit a specified burstiness of a flow with varying bit rates, see e.g. [11]. In this subsection we will show how the idea of the token bucket model can be used to describe the network traffic resulting from event-based sampling mechanisms.

The token bucket model can be illustrated by a virtual bucket with limited capacity. At each discrete time instant $k \in \mathbb{N}_0$ a constant amount of tokens is added to the bucket. This amount is called the token generation rate and specified by the positive integer g^t. On the other hand if a message is transmitted over the network the number of tokens in the bucket is reduced by the cost per transmission, being specified by the positive integer c^t. As for a real bucket it holds that the capacity of the virtual bucket is bounded. The lower bound is given by 0, so it is never possible to have a negative amount of tokens. The upper bound of the capacity is called the bucket size and the corresponding parameter is the positive integer b^t. Thus, no new tokens can be added if the number of tokens would exceed the capacity of the bucket. With this description we can now give the definition of a sequence that satisfies a token bucket characterization.

Definition 5: A binary transmission sequence γ *satisfies a token bucket characterization with bucket size b^t, token generation rate g^t, and transmission cost c^t* if a transmission over the network is only performed at time instant k if the number of tokens at k is larger or equal than the cost per transmission, where the number of tokens increases or decreases according to g^t and c^t and the bucket is upper bounded by b^t.

This token bucket characterization gives the possibility to compute important quantities for the design of suitable communication networks like a peak and a sustainable transmission rate directly from the given parameters of the token bucket model. Thus, it is a nice interface between control and communication if it is possible to design, based on the parameters, a sampling mechanism that generates network traffic satisfying the characteristics and employing the assigned resources as good as possible.

B. Design of a corresponding triggering mechanism

The general idea is to use the performance oriented triggering mechanism and adapt the threshold, as a difference to the baseline mechanism with trigger function f_0 in (8), such that the resulting transmissions satisfy a desired token bucket traffic specification. The key idea of the adaptation is to simulate the current number of tokens in a virtual bucket based on the previous transmissions and use this information to adjust the threshold. The resulting trigger function is given as

$$f_3(\beta_{k-1}, \gamma_{k-1}) = c_3 \frac{c_4 - \beta_k}{\max\{\beta_k - c_5, 0\}},\ k \in \mathbb{N}_0 \qquad (13)$$

with $\beta := (\beta_k)_{k \in \mathbb{N}_0}$ being a sequence with $\beta_0 = c_6$ and

$$\beta_k = \min\left\{c_4, \max\left\{0, \beta_{k-1} + c_7 - \gamma_{k-1} c_5\right\}\right\},\ k \in \mathbb{N}. \qquad (14)$$

Remark 6: Since the term $\max\{\beta_k - c_5, 0\}$ can be zero, the triggering mechanism (7) needs to be multiplied with this term on both sides to avoid division by zero.

The following theorem gives a parametrization for the five parameters of trigger function (13), (14) such that the resulting traffic can be a priori guaranteed to satisfy a token bucket characterization with desired bucket size b^t, generation rate g^t, and transmission cost c^t.

Theorem 7: Assume system (1) is controlled with (2), (3) where K is the LQR controller (5). If the transmissions are initiated by triggering mechanism (7) with trigger function (13), (14), where $c_3 > 0, c_4 = b^t, c_5 = c^t, c_6 \in [0, b^t]$, and $c_7 = g^t$, the resulting transmission sequence γ satisfies a token bucket characterization with bucket size b^t, token generation rate g^t, and transmission cost c^t according to Definition 5.

Proof: With the given parametrization, the auxiliary state variable β_k in the trigger function evolves according to (14) as

$$\beta_{k+1} = \begin{cases} b^t & \beta_k + g^t - \gamma_k c^t \geq b^t \\ 0 & \beta_k + g^t - \gamma_k c^t \leq 0 \\ \beta_k + g^t - \gamma_k c^t & \text{otherwise} \end{cases}$$

for $k \in \mathbb{N}_0$ with $\beta_0 \in [0, b_t]$. Thus, β_k models the number of tokens in a bucket of size b^t according to the transmission sequence γ.

On the other hand the trigger function (13) is designed such that according to triggering mechanism (7) (and Remark 6 for division by zero issues) it is guaranteed that no transmission is initiated at time instant k when $\beta_k - c^t \leq 0$, i.e., when the cost for a transmission is higher than the number of tokens in the bucket at time instant k. This guarantees that the resulting transmission sequence satisfies the token bucket characterization with bucket size b^t, token generation rate g^t, and transmission cost c^t according to Definition 5. ■

The proof already highlighted the key property of the triggering mechanism. It does not allow a transmission if the bucket does not contain enough tokens. As soon as there are enough tokens the transmission decision is based on the possible performance loss when no new input value is transmitted. This general sensitivity can be adjusted by the design parameter c_3. Another nice feature of the mechanism is the fact that as soon as the number of tokens in the bucket has reached the bucket size it is guaranteed that a transmission is demanded by the numerator in (13). This means that if it is not worth to save communications for a later point in time due to the traffic characterization, a message should be transmitted and in such a case the optimal control input is applied. Thus the mechanism presents by design a trade-off between saving communication resources according to the traffic specification and increasing control performance.

Remark 8: Since we do not make any assumptions on the parameters of the desired token bucket characterization apart from being positive integers it is clear that one can not give an a priori guarantee about the performance or the evolution of the state, since it could be possible that the characterization does allow transmissions only very rarely. There are two directions how one can tackle this issue in the future. On the one hand one could calculate bounds for token bucket parameters that are dependent of the system dynamics to give guarantees. On the other hand one can add a different type of communication that is for example scheduled periodically with a higher priority to give performance or stability guarantees, i.e., a similar approach as it has been used for CAN in [17].

VI. NUMERICAL EXAMPLE

In this section a numerical example of system (1) with

$$A = \begin{bmatrix} 1.0142 & -0.0018 & 0.0651 & -0.0546 \\ -0.0057 & 0.9582 & -0.0001 & 0.0067 \\ 0.0103 & 0.0417 & 0.9363 & 0.0563 \\ 0.0004 & 0.0417 & 0.0129 & 0.9797 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0000 & 0.0556 & 0.0125 & 0.0125 \\ -0.0010 & 0.0000 & -0.0304 & -0.0002 \end{bmatrix}^{\mathsf{T}} \tag{15}$$

and $x_0 = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^{\mathsf{T}}$ is investigated. The matrices A and B correspond to the linearized batch example used in [18] after discretization with sampling time 0.01s. As weighting matrices for the cost functional we choose Q and R to be identity matrices of suitable dimension. The optimal controller gain for this setup is computed and the resulting optimal infinite horizon cost with communication at every time instant is given by $J^* = 354.35$.

In the following the triggering mechanisms derived in the paper will be analyzed. At first, the mechanisms with guaranteed performance are investigated. Therefore we demand a performance bound in the sense of (9), i.e., we request that the performance is not worse than $J^* + \eta$ with the choice $\eta = 100$. For the parametrization of trigger functions f_1 and f_2 we choose $r_1 = r_2 = 0.8$. Thus, using Theorem 2, respectively Theorem 3, one knows that the desired performance is guaranteed with the choice $c_1 = c_2 = 20$. In Fig. 2 one can see the cost until time instant k, i.e., $\sum_{j=0}^{k} x_j^{\mathsf{T}} Q x_j + u_j^{\mathsf{T}} R u_j$, depicted over k with blue plus signs for function f_1 and with red circles for function f_2. From Theorem 2 and 3 one knows that it is guaranteed that for k going to infinity the two lines will not cross the black dotted line which depicts the desired performance bound, while the dashed black line represents a lower bound for k going to infinity since it is the optimal cost with transmission at every time instant. As already mentioned in Section IV, trigger function f_1 introduces quite much conservatism in the sense that it performs actually better than demanded which is highly correlated with consuming more communication resources. Therefore we developed f_2 for a less conservative mechanism and Fig. 2 confirms this idea, since with this mechanism the infinite horizon cost matches the desired performance bound much better.

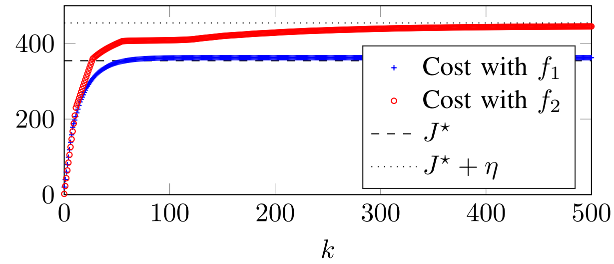

Fig. 2. Cost until time instant k using trigger functions f_1, f_2.

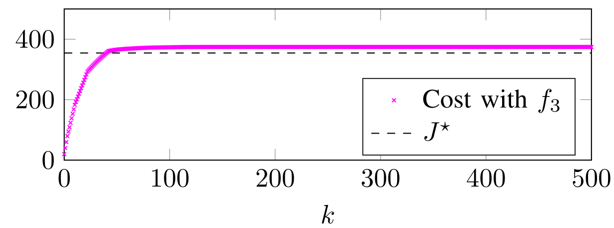

Fig. 3. Cost until time instant k using trigger function f_3.

Next, we will investigate the numerical example with trigger function f_3, that is able to guarantee a desired network traffic according to a token bucket specification. The desired network traffic is specified with the bucket size $b^t = 80$, the token generation rate $g^t = 1$, and the cost per transmission $c^t = 20$. Based on this demand the parameters c_4, c_5, and c_7 are already fixed for trigger function f_3. The remaining parameters are chosen in accordance with Theorem 7 as $c_3 = 0.8$ and $c_6 = 80$. In Fig. 3 one can see the cost until time instant k depicted over k with magenta crosses and the optimal performance $J^\star$ as the black dashed line. Although we did not demand any guarantees on the performance it is still interesting to see that the given traffic characterization results in a infinite horizon closed-loop performance that is in between the performance with f_1 and f_2 in Fig. 2. In addition, Fig. 4 confirms that the resulting network traffic meets the demanded specification. It shows the evolution of the number of tokens in a bucket that is initialized with 80 tokens. At every time instant one token is added if there is capacity in the token and 20 tokens are removed if there is a transmission at that time. Since the number of tokens is always greater than zero one can see that the specification is met. One can also see that if the bucket is full, a new

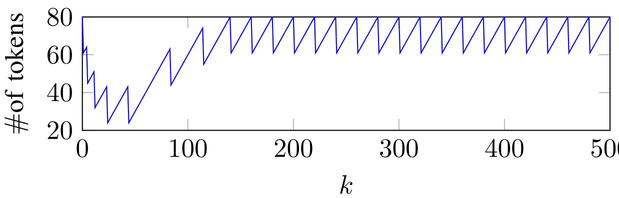

Fig. 4. Evolution of tokens for transmission sequence γ using f_3.

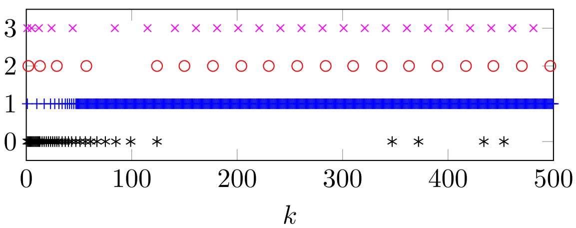

Fig. 5. Resulting triggering times with trigger functions f_0, f_1, f_2, f_3.

message is sent as designed in the mechanism since it is not worth to save transmission resources. On the other hand directly after initialization the whole number of tokens being in the bucket is consumed very fast, since the performance improvement is immense as long as the effect of the initial condition is present.

At last a comparison of the resulting triggering times is given for all trigger functions, including the pure performance oriented approach f_0 in Fig. 5. One can see that this pure performance-oriented approach also triggers very frequently in the beginning which is again due to the effect of the initial condition. The opposite behavior can be seen by f_1. Here the triggering is very frequently for large times although the system seems to be already converged which is probably not the desired behavior for such a triggering mechanism. Thus, again function f_2 presents the more interesting approach that still gives a performance guarantee but rather does this by triggering enough often in the beginning and triggering rather infrequently when it is converged. The triggering pattern of f_4 directly shows as stated before that in the beginning all available tokens are used quite fast, followed by a phase where triggering needs to be reduced due to the traffic characterization and a phase where triggering is only performed when the bucket is full which leads to a periodic behavior.

VII. CONCLUSIONS

In this paper we developed a performance oriented triggering mechanism for discrete-time systems where the computation of a new control input is performed at the sensor side. The key idea behind this mechanism is given by the LQ formula, that allows to compute the cost increase due to reusing the previous input only based on the new optimal and the old input. Building up on this general mechanism two trigger functions were derived that guarantee an a priori desired infinite horizon closed-loop performance, where the difference between the mechanisms is on the one hand the conservativity and on the other hand the amount of information that needs to be stored and processed for the transmission decision. In addition, another trigger function was derived that can be parametrized such that the resulting network traffic satisfies a token bucket characterization that is known from standard traffic shaping mechanisms.

As already indicated in Remark 8, future work will focus on the combination of such a mechanism with additional periodic transmissions. Furthermore the mechanisms are currently designed for impulsive disturbances. While simulation results indicate a nice behavior for persistent disturbances, a theoretical study of such a setup is also an open point.

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