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Control Synthesis for Multi-UAV Slung-Load Systems with Uncertainties

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Abstract— **This paper develops a new control synthesis for the slung-load transportation system by multiple unmanned aerial vehicles. Effects of the unmatched uncertainties that cannot be directly controlled by the adaptive control are suppressed using the parameter-robust linear quadratic Gaussian method. The stability condition of the tracking and parameter estimation error is proven with Lyapunov analysis. The numerical simulations support the enhancement of robustness and system performance in the presence of both the unmatched and matched uncertainties in the slung-load system.**

I. INTRODUCTION

The large scale of unmanned aerial vehicle (UAV) applications has proliferated vastly within the last few years. The operational experience of UAVs has proven that their technology can bring a dramatic impact to military and civilian areas. This includes, but is not limited to: obtaining real-time, relevant situational awareness; helping human operators; and reducing risk to the mission and operation. Potential applications of UAVs under consideration are quite wide, e.g. border patrol [1], [2], [3], airborne surveillance [4], [5], [6], police law enforcement [7], [8], and forest-fire localisation [9]. One important application that has been attracting an increasing attention is logistics.

This paper addresses a logistics problem using a group of small UAVs. While small UAVs are advantageous in their accessibility and convenience, their payload is limited. This implies that cooperation of multiple UAVs is inevitable to relieve payload constraints. There could be a few ways to enable a group of small UAVs to cooperate for the logistics. One of the most obvious approaches is to utilise a slung-load system in which the vehicles are connected to the payload with suspended cables. Hence, this paper considers the slung-load system as the approach for UAV cooperation in logistics.

Dynamics model of the slung-load system may contain both unmatched and matched uncertainties. Note that the matched and unmatched uncertainties are the uncertainties that lie in the span and the null space of the control input matrix, respectively. UAVs are most likely to deliver various masses of the load. Mass of the load could not be exactly known prior to the operation or could change during the operation. The issue with such uncertainty on the mass is that it generates unmatched uncertainty to the system. The dynamics of the slung-load system is highly coupled and encompasses highly nonlinear terms. This means that a dynamics model of the slung-load system typically contains modelling uncertainty, which also generates matched uncertainties to the system.

Control of a slung-load system is challenging. The motion of each UAV in the slung-load system could be constrained due to the cables connected to the UAV. Oscillation of a vehicle or payload is transmitted through the connected strings to the other vehicles with constrained motion, potentially resulting in instability. The existence of both unmatched and matched uncertainties makes control of a slung-load system even much more challenging.

To this end, this paper aims to develop a control synthesis to tackle the challenge in the slung-load system control under the presence of unmatched and matched uncertainty. The model reference adaptive control (MRAC) method is an option for being a well known parameter-robust controller. However, most MRAC methods are limited to cancel out the effect of the matched uncertainties. The unmatched uncertainties can be neither estimated, nor cancelled out by common MRAC methods.

While the majority of the previous works focused on the matched uncertainties, there have been some studies specifically on the unmatched uncertainties. As controlling the unmatched uncertainties directly in the MRAC formulation is not possible, the bound of the unmatched uncertainties has been investigated by [10]. The detailed effects on the robustness are analysed with linear matrix inequality in [11]. Under the assumption that the unmatched uncertainties are bounded, modifications on the MRAC, such as $\mathcal{L}_1$ adaptive and bi-objective control, have been proposed in [12], [13]. The modifications are focused mainly on preventing the tracking error from drifting. To suppress the specific effect of unmatched uncertainties, adaptive sliding mode control is suggested by [14], and later extended to multiple sliding surfaces in [15] and to cope with time delays in [16]. The adaptive sliding mode control, unlike the MRAC, pursues the states to converge on the sliding surfaces regardless of the reference model. In [17], the concurrent learning MRAC method is used to fully utilise the reference model and to control the unmatched uncertainties by switching the baseline control gain after estimating the unmatched uncertainties. In the actual implementation, however, it is difficult to know the correct values of the unmatched uncertainties and thus the gain switching time remains illusive.

This paper proposes a new control synthesis to handle both matched and unmatched uncertainty. The proposed approach consists of two parts: the baseline control and adaptation. In this study, the baseline control is designed using the parameter-robust linear quadratic Gaussian (PRLQG) method to increase robustness to the unmatched uncertainty and the adaptation using an adaptive control to cancel out the

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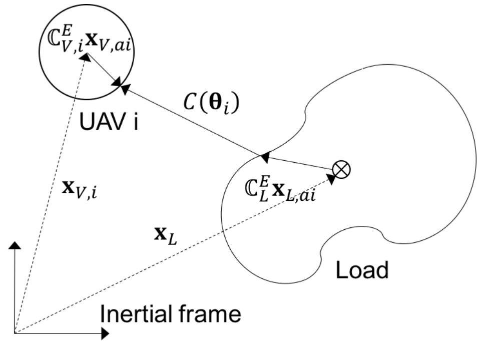


Fig. 1: Transportation system nomenclature

matched uncertainty. The main difference of the our approach from previous approaches is that our approach design a PRLQG controller to generate a reference model for the desired performance and it is combined with adaptive control. The rational behind this proposition is that our previous studies [18], [19] showed that the PRLQG method handles uncertainty in the payload mass. Note that the PRLQG method determines the controller gain to increase the stability margin against the parameter variation in the system state matrix [20], [21].

For the validation of the proposed control synthesis, the convergence of the tracking and parameter estimation error are theoretically investigated using Lyapunov analysis. The performance of the control synthesis is also demonstrated through numerical simulations. The simulation results for the slung-load systems with 1, 2, and 4 quadrotors confirm the robustness of the proposed control synthesis against both unmatched and matched uncertainty.

The rest of this paper is composed as follows: in section II, the linear dynamics of the slung-load system and the uncertainties are modelled. In section III, the control synthesismethod is proposed and analysed. In section IV, the settings and results of numerical simulation is given. The paper is finalised with conclusion.

II. SYSTEM DESCRIPTION

The slung-load system is modelled in the spherical coordinate to obtain the minimal controllable linearised dynamics. The type of UAV is chosen as a quadrotor for the modelling. See [18], [19] for the detailed slung-load modelling and [22] for the quadrotor dynamics.

A. Slung-Load Dynamics

The system variables of the slung-load system are shown in Fig. 1. The subscripts V, L, and E stand for vehicle, load, and inertial frame, respectively. The position of each vehicle and the load is denoted by $\mathbf{x}_{V,i} \in \mathbb{R}^3$ and $\mathbf{x}_L \in \mathbb{R}^3$ in the inertial frame, and the vector from the center of mass to the point that the string is attached is $\mathbf{x}_{ai} \in \mathbb{R}^3$ in the body frame. The vector in the body frame is multiplied by a direction cosine matrix $\mathbb{C}^E_{V,i} \in \mathbb{R}^{3\times 3}$ or $\mathbb{C}^E_L \in \mathbb{R}^{3\times 3}$ to present in the inertial frame, which is a function of the attitude of each vehicle or the load, $\boldsymbol{\theta}_{V,i} \in \mathbb{R}^3$ and $\boldsymbol{\theta}_L \in \mathbb{R}^3$ respectively. From the geometric relationship, the string vector is expressed as

$$\begin{aligned} C(\boldsymbol{\theta}_i) &= \mathbf{x}_{V,i} - \mathbf{x}_L - \mathbb{C}^E_L\mathbf{x}_{L,ai} + \mathbb{C}^E_{V,i}\mathbf{x}_{V,ai} \\ &= l_i \begin{bmatrix} \sin\phi_i \\ -\sin\theta_i\cos\phi_i \\ -\cos\theta_i\cos\phi_i \end{bmatrix}, \end{aligned} \tag{1}$$

where $l_i \in \mathbb{R}$ is the length of the i-th string and $\boldsymbol{\theta}_i = [\theta_i, \phi_i]^T \in \mathbb{R}^2$ is the string vector in the spherical coordinate.

The equation of motion of the slung-load system is

$$\begin{aligned} &\begin{bmatrix} \ddot{\boldsymbol{\theta}}_i^T & \ddot{\mathbf{x}}_L^T & \dot{\boldsymbol{\omega}}_{V,i}^T & \dot{\boldsymbol{\omega}}_L^T & T_i/l_i \end{bmatrix}^T = \\ &\begin{bmatrix} \mathbf{M}_{V,i}C'(\boldsymbol{\theta}_i) & \mathbf{M}_{V,i} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & C(\boldsymbol{\theta}_i) \\ \mathbf{0}_{3\times2} & \mathbf{M}_L & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & -C(\boldsymbol{\theta}_i) \\ \mathbf{0}_{3\times2} & \mathbf{0}_{3\times3} & \mathbf{I}_{V,i} & \mathbf{0}_{3\times3} & \mathbf{x}_{V,ai} \times \mathbb{C}^{V,i}_E C(\boldsymbol{\theta}_i) \\ \mathbf{0}_{3\times2} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{I}_L & -\mathbf{x}_{L,ai} \times \mathbb{C}^L_E C(\boldsymbol{\theta}_i) \end{bmatrix}^{-1} \\ &\begin{bmatrix} \mathbf{F}_{V,i} - \sum_i \mathbf{M}_{V,i}(G(\boldsymbol{\theta}_i, \dot{\boldsymbol{\theta}}_i) + \mathbb{G}^E_L\mathbf{x}_{L,ai} - \mathbb{G}^E_{V,i}\mathbf{x}_{V,ai}) \\ \mathbf{F}_L \\ \boldsymbol{\tau}_{V,i} \\ \mathbf{0}_{3\times1} \end{bmatrix}, \end{aligned} \tag{2}$$

where $\boldsymbol{\omega} \in \mathbb{R}^3$ is the angular rate in the body frame, $T_i \in \mathbb{R}$ the tensile force, $\mathbf{M} \in \mathbb{R}^{3\times3}$ the mass matrix, and $\mathbf{I} \in \mathbb{R}^{3\times3}$ the inertia. The functions $G(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \mathbb{R}^3$, and $\mathbb{G}^E \in \mathbb{R}^{3\times3}$ are defined as

$$\begin{aligned} G(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) &= \begin{bmatrix} -\dot{\phi}^2\sin\phi \\ (\dot{\theta}^2 + \dot{\phi}^2)\sin\theta\cos\phi + 2\dot{\theta}\dot{\phi}\cos\theta\sin\phi \\ (\dot{\theta}^2 + \dot{\phi}^2)\cos\theta\cos\phi - 2\dot{\theta}\dot{\phi}\sin\theta\sin\phi \end{bmatrix}, \\ \mathbb{G}^E &= (\mathbb{C}^E \times \omega) \times \omega. \end{aligned} \tag{3}$$

The external forces, $\mathbf{F}_{V,i} \in \mathbb{R}^3$ and $\mathbf{F}_L \in \mathbb{R}^3$, include gravitational force as

$$\mathbf{F}_{V,i} = \mathbb{C}^E_{V,i}\mathbf{F}_{M,i} + \mathbf{M}_{V,i}\mathbf{g}, \quad \mathbf{F}_L = \mathbf{M}_L\mathbf{g}, \tag{4}$$

where $\mathbf{g} = [0, 0, g]^T$ is the gravitational acceleration vector. The force and moment generated by the vehicle, $\mathbf{F}_{M,i} \in \mathbb{R}^3$ and $\boldsymbol{\tau}_{V,i} \in \mathbb{R}^3$, are obtained with respect to the quadrotor dynamics as

$$\begin{aligned} \mathbf{F}_{M,i} &= \begin{bmatrix} 0 \\ 0 \\ -4K_{t,0} - K_t(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \end{bmatrix}, \\ \boldsymbol{\tau}_{V,i} &= \begin{bmatrix} K_t(-\Omega_1^2 - \Omega_2^2 + \Omega_3^2 + \Omega_4^2)d \\ K_t(\Omega_1^2 - \Omega_2^2 - \Omega_3^2 + \Omega_4^2)d \\ 4K_{r,0} + K_r(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \end{bmatrix} \\ &\quad + \begin{bmatrix} I_{r_i}q_i(\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4) \\ -I_{r_i}p_i(\Omega_1 - \Omega_2 + \Omega_3 - \Omega_4) \\ 0 \end{bmatrix}, \end{aligned} \tag{5}$$

where K_t is the thrust coefficient, K_r the torque coefficient, d the distance between the rotors, Ω the rotational speed of each rotor, and $\omega_{V,i} = [p_i, q_i, r_i]^T$ the angular rate of the vehicle.

The control allocation of the quadrotor vehicle from the roll, pitch, yaw, and thrust commands to the rotor speeds is

$$\begin{bmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \\ \Omega_4 \end{bmatrix} = K_{c,0} + K_c \begin{bmatrix} 0.5 & 0.5 & -1 & 1 \\ -0.5 & 0.5 & 1 & 1 \\ -0.5 & -0.5 & -1 & 1 \\ 0.5 & -0.5 & 1 & 1 \end{bmatrix} \begin{bmatrix} \delta_{roll} \\ \delta_{pitch} \\ \delta_{yaw} \\ \delta_{thrust} \end{bmatrix}, \tag{6}$$

where K_c is the control allocation coefficient.

B. Linearised Dynamics

The state and control input vector of the slung-load dynamics are defined as

$$\begin{aligned} x(t) = [&\boldsymbol{\theta}_i^T(t), \boldsymbol{\theta}_{V,i}^T(t), \mathbf{x}_L^T(t), \boldsymbol{\theta}_L^T(t), \\ &\dot{\boldsymbol{\theta}}_i^T(t), \boldsymbol{\omega}_{V,i}^T(t), \dot{\mathbf{x}}_L^T(t), \boldsymbol{\omega}_L(t)^T]^T \in \mathbb{R}^n, \\ u(t) = [&\delta_{roll}(t), \delta_{pitch}(t), \delta_{yaw}(t), \delta_{thrust}(t)]^T \in \mathbb{R}^m, \end{aligned} \tag{7}$$

where the number of the states n equals to $10N + 12$ and the number of the control inputs m is $4N$, where N is the number of the UAVs.

The slung-load system is then expressed in the following state-space representation:

$$\dot{x}(t) = Ax(t) + B(u(t) + \Delta(x)) + B_u\Delta_u(x) \tag{8}$$

where the matched uncertainty $\Delta(x) \in \mathbb{R}^m$ includes all the uncertainties that lie in the span of the control input matrix $B \in \mathbb{R}^{n\times m}$, and the unmatched uncertainty $\Delta_u(x) \in \mathbb{R}^{n-m}$ lies in the span of $B_u \in \mathbb{R}^{n\times(n-m)}$, which is the null space of the control input matrix. The state-space representation is minimal and the system matrices (A, B) are controllable.

The matched uncertainty is parameterized as

$$\Delta(x) = W^{*T}\Phi(x) \tag{9}$$

where $W^* \in \mathbb{R}^{s\times m}$ is a weight matrix, and $\Phi(x) \in \mathbb{R}^s$ is a basis vector.

The unmatched uncertainty is assumed to have a basis linear to $x(t)$ as

$$\Delta_u(x) = N^{*T}x(t) \tag{10}$$

where $N^* \in \mathbb{R}^{n\times(n-m)}$ is a weight matrix.

III. ADAPTIVE PARAMETER-ROBUST LINEAR QUADRATIC GAUSSIAN

The proposed control use the structure of the MRAC method as

$$u(t) = u_{base}(t) + u_{ad}(t) \tag{11}$$

where the baseline control $u_{base}(t)$ determines the reference model with the desired performance, and the adaptive control $u_{ad}(t)$ cancels out the effect of matched uncertainty to track the reference model. The baseline control improves the robustness against the unmatched uncertainty by using the PRLQG method, and the σ-mod adaptive law is used to estimate the matched uncertainty. The stability of the controller is analysed with the Lyapunov analysis.

A. Parameter-Robust Linear Quadratic Gaussian

The unmatched uncertainty $\Delta_u(x)$ is converted to the perturbation of the system matrix as

$$\Delta A = B_u N^{*T} \tag{12}$$

where the perturbation matrix $\Delta A \in \mathbb{R}^{n\times n}$ is implemented as

$$\dot{x}(t) = (A + \Delta A)x(t) + B(u(t) + \Delta(x)) \tag{13}$$

The PRLQG method is applicable for the case that the variation in the system matrix can be decomposed as

$$\Delta A = M_{PR}\epsilon N_{PR}^T \tag{14}$$

where $\epsilon \in \mathbb{R}^{p\times p}$ is the system parameter variation, and $M_{PR} \in \mathbb{R}^{n\times p}$ and $N_{PR} \in \mathbb{R}^{n\times p}$ are constant matrices. Noting that N_{PR} is row-similar to N^*, the PRLQG gain is designed with respect to N_{PR} to enhance the robustness against N^*.

The baseline PRLQG controller is designed as

$$u_{base}(t) = -Kx(t) \tag{15}$$

The gain of the baseline controller $K = R^{-1}B^TP$ is obtained by the following algebraic Riccati equation:

$$PA + A^TP + Q + w_{PR}N_{PR}N_{PR}^T - PBR^{-1}B^TP = 0 \tag{16}$$

where $Q \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ are positive definite weight matrices for states and inputs respectively, and $w_{PR} \in \mathbb{R}$ is a positive weight for the PRLQG method.

The reference model with the desired performance is designed with the baseline PRLQG control as

$$\dot{x}_r(t) = A_r x_r(t) - B_r r(t) \tag{17}$$

where $x_r(t) \in \mathbb{R}^n$ is the reference state, $r(t) \in \mathbb{R}^m$ is the external reference command, and the system matrix $A_r = A - BK$ is Hurwitz stable.

Defining the tracking error between the reference model and the actual system as $e(t) \triangleq x_r(t) - x(t)$, the tracking dynamics is expressed as

$$\dot{e}(t) = A_r e(t) - B(u_{ad}(t) + \Delta(x)) - B_u\Delta_u(x) \tag{18}$$

If $N_{PR}(sI - A)^{-1}B$ is minimum phase, it is proven that as $w_{PR} \to \infty$, the transfer function from $\Delta_u(x)$ to $e(t)$ approaches 0, i.e.

$$\dot{e}(t) \to A_r e(t) - B(u_{ad}(t) + \Delta(x)) \tag{19}$$

The details of the proof are given in [21], and for the bode plot analysis of the slung-load system, refer to our previous work [19].

B. Adaptive Law

The matched uncertainty $\Delta(x)$ is cancelled out by the adaptive control as

$$u_{ad} = -\hat{W}(t)^T\Phi(x) \tag{20}$$

where $\hat{W}(t) \in \mathbb{R}^{s\times m}$ is the estimate of the parameter W^*.

The parameter is estimated with the following σ-mod adaptive law:

$$\dot{\hat{W}}(t) = -\Gamma\left(\Phi(x)e(t)^TP_rB + \sigma(\hat{W}(t) - W_{guess})\right) \tag{21}$$

where $\Gamma \in \mathbb{R}^{s\times s}$ is the adaptation gain, $\sigma \in \mathbb{R}^{s\times s}$ is the σ-mod gain, and $P_r \in \mathbb{R}^{n\times n}$ is a positive definite matrix defined by the following algebraic Riccati equation:

$$P_rA_r + A_r^TP_r + Q_r = 0, \quad Q_r > 0 \tag{22}$$

The σ-mod is applied with respect to $W_{guess} \in \mathbb{R}^{s\times m}$ which is a constant guess of the estimate of the parameter.

TABLE I: Slung-load system specification

		Case 1	Case 2	Case 3
Quadrotor	Number	1	2	4
	Mass (kg)	0.408		
	Size (m)	ϕ 0.356		
	Inertia (kg·m^2)	2.2842·10-3 × 2.4451·10-3 × 4.4562·10-3		
	Thrust Coefficient (N)	$K_t = 8.1763 \cdot 10^{-6}, \quad K_{t,0} = 0.0562$		
	Torque Coefficient (N·m)	$K_r = 2.1703 \cdot 10^{-7}, \quad K_{r,0} = 1.0950 \cdot 10^{-4}$		
	Control Allocation Coefficient (rad/s)	$K_c = 0.7326, \quad K_{c,0} = 1.2694 \cdot 10^{2}$		
Payload	Nominal Mass (kg)	0.1	0.2	0.4
	Size (m)	0.2 × 0.2 × 0.2	0.2 × 0.2 × 1.0	0.2 × 1.0 × 1.0
String	Mass (kg)	None		
	Size (m)	1.4		

C. Lyapunov Analysis

The stability of the tracking and parameter estimation error is analysed by the following continuously differentiable, positive definite Lyapunov function:

$$V(e,\tilde{W}) = \frac{1}{2}e^T(t)P_r e(t) + \frac{1}{2}tr(\tilde{W}^T(t)\Gamma^{-1}\tilde{W}(t)) \quad (23)$$

where $\tilde{W}(t) \triangleq \hat{W}(t) - W^*$ is the parameter estimation error. Defining $\xi(t) \triangleq [e^T(t), \tilde{W}^T(t)]^T$, the lower and upper bounds of the Lyapunov function are given as

$$\begin{aligned} &\frac{1}{2}\min\left(\lambda(P_r), \lambda(\Gamma^{-1})\right)||\xi||^2 \\ \le V(e,\tilde{W}) \le &\frac{1}{2}\max\left(\lambda(P_r), \lambda(\Gamma^{-1})\right)||\xi||^2 \end{aligned} \quad (24)$$

The Lie derivative of the Lyapunov function is computed as

$$\dot{V}(e,\tilde{W}) = e^T(t)P_r\dot{e}(t) + tr(\tilde{W}^T(t)\Gamma^{-1}\dot{\hat{W}}(t)) \quad (25)$$

Substituting the tracking error dynamics in Eqn. (18) and the adaptive law in Eqn. (21) gives

$$\begin{aligned} \dot{V}(e,\tilde{W}) = &-\frac{1}{2}e^T(t)Q_r e(t) + e^T(t)P_r B_u\Delta_u(x) \\ &-tr(\tilde{W}^T(t)\sigma(\hat{W}(t) - W_{guess})) \end{aligned} \quad (26)$$

The Lyapunov is negative definite except $e(t) = 0$ and $\tilde{W}(t) = W_{guess} - W^*$ if

$$\frac{1}{2}\lambda_{min}(Q)||e(t)|| > ||P_r B_u\Delta_u(x)|| \quad (27)$$

If $N_{PR}(sI - A)^{-1}B$ is minimum phase, increasing the weight of PRLQG to infinity leads to $||e(t)|| \gg ||B_u\Delta_u(x)||$, approximating the Lyapunov function as

$$\dot{V}(e,\tilde{W}) \to -\frac{1}{2}e^T(t)Q_r e(t) - tr(\tilde{W}^T(t)\sigma(\hat{W}(t) - W_{guess})) \quad (28)$$

The approximated Lyapunov function is bounded as

$$\dot{V}(e,\tilde{W}) \le -\frac{1}{2}\lambda_{min}(Q_r)||e(t)||^2 - \sigma||\tilde{W} - W_{guess} + W^*||^2 \quad (29)$$

Using the bounds of the Lyapunov function in Eqn. (24):

$$\dot{V}(e,\tilde{W}) \le -\frac{\min(\lambda(Q_r), 2\sigma)}{\max(\lambda(P_r), \lambda(\Gamma^{-1}))}V(e,\tilde{W}) \quad (30)$$

Therefore, the following conditions for stability can be obtained:

1) If $N_{PR}(sI - A)^{-1}B$ and $w_{PR} \to \infty$, the tracking and parameter estimation error is Lyapunov stable with respect to $e(t) = 0$ and $\tilde{W}(t) = W_{guess} - W^*$.
2) If the condition 1) is satisfied and the signal is persistently excited, i.e. $\int e(t)e^T(t)dt > 0$, the tracking and parameter estimation error is asymptotically stable.
3) If the condition 2) is satisfied and $W_{guess} = W^*$, the tracking and parameter estimation error is exponentially stable with respect to $e(t) = 0$ and $\tilde{W}(t) = 0$.

IV. NUMERICAL SIMULATION

The numerical simulation is set with uncertainty in the mass of payload and nonlinear dynamics on the quadrotor dynamics. The tracking performance and parameter estimation of the proposed control synthesis method are evaluated in three slung-load systems: 1-, 2-, and 4-quadorotor systems.

A. Simulation Settings

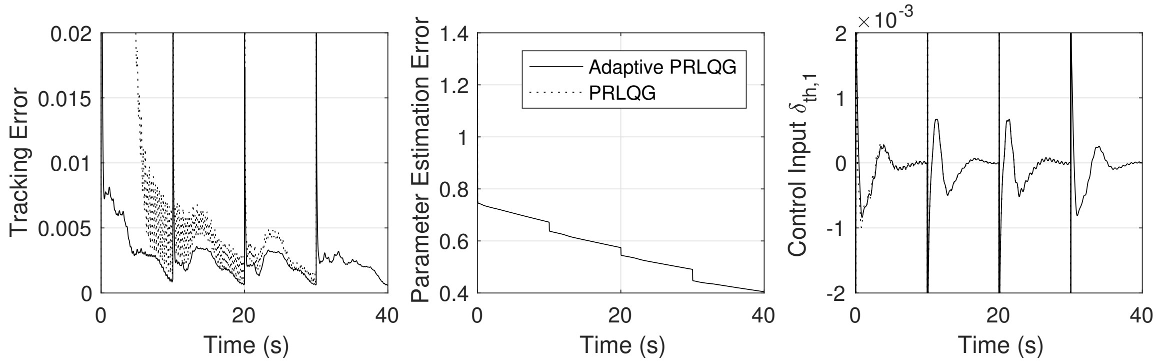
The dynamic coefficients of the quadrotor, and the dimension of the payload and string are specified in the Table I. The strings are attached on the top surface of the payload, and below each quadrotor with a distance of $||\mathbf{x}_{V,ai}|| = 0.1$ m from the center of mass.

The actual mass of payload is considered as 120% of the nominal mass, i.e. $\Delta m_L = 0.2m_{L,nominal}$, creating the variation on the system matrix A as

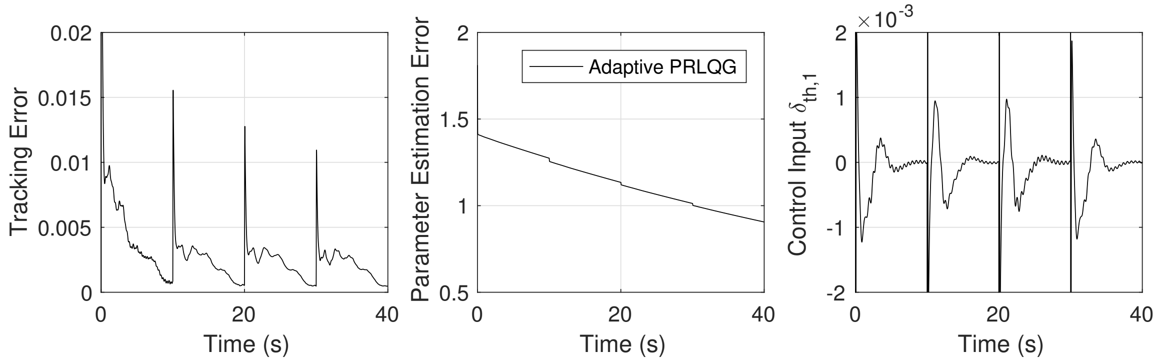
$$\begin{aligned} \Delta A &= \mathbf{U}_1\left(\mathbf{I}_{N\times N} \otimes \Delta A_i\right)\mathbf{U}_2 \\ \Delta A_i &= -\frac{\Delta m_L g}{N}\begin{bmatrix} \mathbf{U}_3\mathbf{M}_{V,i}^{-1}\mathbf{U}_4 \\ ||\mathbf{x}_{V,ai}||\mathbf{I}_{V,i}^{-1}\mathbf{U}_4 \end{bmatrix} \end{aligned} \quad (31)$$

where the unit permutation matrices $\mathbf{U}$'s are

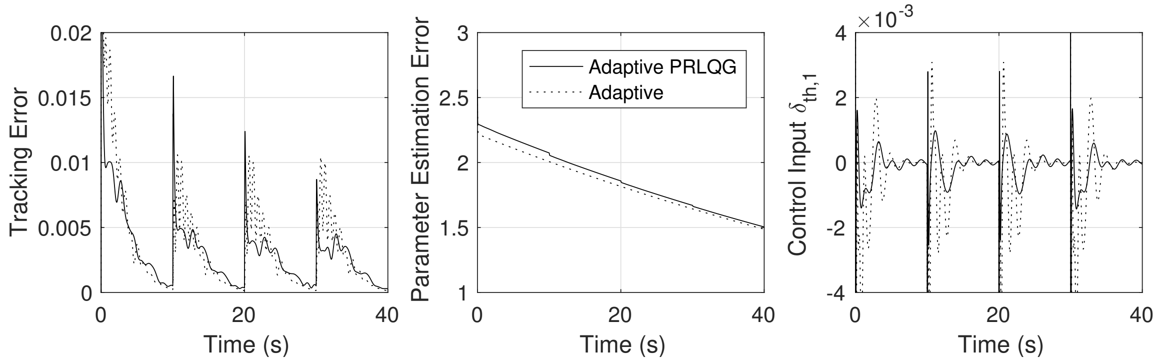
$$\begin{aligned} \mathbf{U}_1 &= \begin{bmatrix}\mathbf{I}_{5N\times 5N} & \mathbf{0}_{5N\times(5N+12)}\end{bmatrix}^T \\ \mathbf{U}_2 &= \begin{bmatrix}\mathbf{0}_{5N\times(5N+6)} & \mathbf{I}_{5N\times 5N} & \mathbf{0}_{5N\times 6}\end{bmatrix} \\ \mathbf{U}_3 &= \begin{bmatrix}\mathbf{I}_{2\times 2} & \mathbf{0}_{2\times 1}\end{bmatrix} \\ \mathbf{U}_4 &= \begin{bmatrix}\mathbf{I}_{2\times 2} & \mathbf{I}_{2\times 2} & \mathbf{0}_{2\times 1} \\ \mathbf{0}_{1\times 2} & \mathbf{0}_{1\times 2} & 0\end{bmatrix} \end{aligned} \quad (32)$$



(a) Case 1: Single quadrotor transportation system



(b) Case 2: 2-quadrotor transportation system



(c) Case 3: 4-quadrotor transportation system

Fig. 2: Tracking and parameter estimation error

The variation ΔA leads to the unmatched uncertainty as

$$N^* = -\mathbf{U}_2^T \left(\mathbf{I}_{N\times N} \otimes \frac{\Delta m_L g}{N} \mathbf{U}_3 \mathbf{M}_{V,i}^{-1} \mathbf{U}_4 \right)^T \tag{33}$$

The nonlinear uncertainty is considered on the dynamics of a quadrotor as

$$\begin{aligned} W^* &= \mathbf{I}_{N\times N} \otimes [1 \quad 0 \quad 0 \quad -0.8] \\ \Phi(x) &= [x_3^2 \quad x_8^2 \quad \cdots \quad x_{5(N-1)+3}^2]^T \end{aligned} \tag{34}$$

where $x_{5(i-1)+3}$ is the roll rate of each quadrotor as defined in Eqn. (7).

The external reference command $r(t)$ is applied on the position of each quadrotor with the change in every 10s as

$$\begin{aligned} r(t) &= [x_{cmd}, y_{cmd}, z_{cmd}]^T \\ &= \begin{bmatrix} 1 \\ 1 - h(t-10) - h(t-20) + h(t-30) \\ -h(t-10) \end{bmatrix} \end{aligned} \tag{35}$$

where $h(t)$ is the unit step function.

The gains of PRLQG are designed as $Q = 1$ for the position and attitude states, $Q = 0.5$ for their derivatives, and $R = 10$ for the control inputs. The weight of PRLQG is chosen as $w_{PR} = 10$. The adaptation gains are set as $\Gamma = 10^3$ and $\sigma = 10^{-5}$.

B. Simulation Results

The tracking error $||e(t)||_2$, parameter estimation error $||\tilde{W}^T(t)\tilde{W}(t)||_2$, and thrust control input of a quadrotor are shown in Fig. 2. The figures (a), (b), and (c) show the response of the slung-load system with 1, 2, and 4 quadrotors, respectively.

The simulation is conducted with three different controls for the reference: the proposed control synthesis method, the PRLQG method without adaptation, and the adaptive control on the baseline controller with LQG gain. The three control schemes are applied in all the scenario cases, but the diverging responses are not shown in the figure. Note that only the responses of the proposed control remain stable and are able to be plotted on all the cases. The PRLQG method diverges on the point $t = 30$ s in Case 1, and has rapid oscillations in the quadrotor attitude with the strictly large tracking errors compared with the control synthesis method. The adaptive control is stable when the number of quadrotor is large and the effect of the variation of the payload's mass is smaller than the other cases. Whereas the matched parameter is estimated similarly with the adaptive PRLQG, the tracking error shows undesirable response for the presence of unmatched uncertainties. The matched and unmatched uncertainties are successfully suppressed only by the control synthesis among the three control schemes.

The tracking error of the control synthesis is not only stable, but also shows similarities with respect to different slung-load systems: the sudden increase of tracking error at the signal excitation gradually diminishes as the parameter estimation converges, and the scale of the peak error is bounded. This implies that the proposed control method can be applied for the slung-load system with general number of UAVs.

V. CONCLUSIONS

In this paper, we proposed an control synthesis to effectively suppress the effect of uncertainty of the multi-UAV slung-load system. The rationale behind the idea was that although the slung-load system has considerable unmatched uncertainties which are neither estimated or controlled with common adaptive controls, the baseline controller can be designed to be robust to the unmatched uncertainties through the PRLQG method. The stability conditions of the proposed control method were obtained through Lyapunov analysis. The numerical simulations demonstrate that the proposed adaptive PRLQG approach effectively cancels out the effect of both unmatched and matched uncertainties in three different slung-load systems with a potential to be extended to incorporate more UAVs.

REFERENCES

[1] D. Kingston, R. Beard, and R. Holt, "Decentralized perimeter surveillance using a team of UAVs," *IEEE Transactions on Robotics*, vol. 24, no. 6, pp. 1394–1404, 2008.

[2] D. Bein, W. Bein, A. Karki, and B. Madan, "Optimizing border patrol operations using unmanned aerial vehicles," 2015, pp. 479–484.

[3] S. Minaeian, J. Liu, and Y.-J. Son, "Vision-based target detection and localization via a team of cooperative uav and ugvs," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 7, pp. 1005–1016, 2016.

[4] R. Pitre, X. Li, and R. Delbalzo, "UAV route planning for joint search and track mission - an informative-value approach," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 3, pp. 2551–2565, 2012.

[5] B. Lim, J. Kim, S. Ha, and Y. Moon, "Development of software platform for monitoring of multiple small uavs," vol. 2016-December, 2016.

[6] P. Li and H. Duan, "A potential game approach to multiple uav cooperative search and surveillance," *Aerospace Science and Technology*, vol. 68, pp. 403–415, 2017.

[7] A. Puri, "A survey of unmanned aerial vehicles (UAV) for traffic surveillance," Department of Computer Science and Engineering, University of South Florida, Tech. Rep., 2004.

[8] E. Carapezza and D. Law, "Sensors, c3i, information, and training technologies for law enforcement," in *Proc. SPIE*, January 1999.

[9] A. Belbachir and J.-A. Escareno, "Autonomous decisional high-level planning for uavs-based forest-fire localization," vol. 1, 2016, pp. 153–159.

[10] B. R. Barmish and G. Leitmann, "On Ultimate Boundedness Control of Uncertain Systems in the Absence of Matching Assumptions," *IEEE Transactions on Automatic Control*, vol. 116, no. 1, pp. 153–155, 1982.

[11] B.-j. Yang, T. Yucelen, and J.-y. Shin, "An LMI-based Analysis of an Adaptive Flight Control," *AIAA Infotech@Aerospace 2010*, 2010.

[12] C. Cao and N. Hovakimyan, "Design and Analysis of a Novel L1 Adaptive Control," *IEEE Transactions on Automatic Control*, vol. 53, no. 2, pp. 586–591, 2008.

[13] N. T. Nguyen and S. N. Balakrishnan, "Bi-objective optimal control modification adaptive control for systems with input uncertainty," *IEEE/CAA Journal of Automatica Sinica*, vol. 1, no. 4, pp. 423–434, 2014.

[14] T. I. Fossen and S. I. Sagatun, "Adaptive control of nonlinear systems: A case study of underwater robotic systems," *Journal of Robotic Systems*, vol. 8, no. 3, pp. 393–412, 1991.

[15] A. C. Huang and Y. C. Chen, "Adaptive sliding control for single-link flexible-joint robot with mismatched uncertainties," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 5, pp. 770–775, 2004.

[16] B. Meng, C. Gao, S. Tang, and Y. Liu, "Adaptive Variable Structure Control for Linear Systems with Time-varying Multi-delays and Mismatching Uncertainties," *Physics Procedia*, vol. 33, no. 60974025, pp. 1753–1761, 2012.

[17] J. F. Quindlen, G. Chowdhary, and J. P. How, "Hybrid model reference adaptive control for unmatched uncertainties," *2015 American Control Conference (ACC)*, pp. 1125–1130, 2015.

[18] H.-I. Lee, B.-Y. Lee, D.-W. Yoo, G.-H. Moon, and M.-J. Tahk, "Dynamics Modeling and Robust Controller Design of the Multi-Uav Transportation System," *29th Congress of the International Council of the Aeronautical Sciences*, 2014.

[19] H.-I. Lee, D.-W. Yoo, B.-Y. Lee, G.-H. Moon, and D.-Y. Lee, "Parameter-Robust Linear Quadratic Gaussian Technique for Multi-Agent Slung Load Transportation (Under Revision)," *Aerospace Science and Technology*, 2017.

[20] M. Tahk and J. L. Speyer, "Modeling of Parameter Variations and Asymptotic LQG Synthesis," *IEEE Transactions on Automatic Control*, vol. 32, no. 9, pp. 793–801, 1987.

[21] ——, "Parameter Robust Linear- Quadratic-Gaussian Design Synthesis with Flexible Structure Control Applications," *Journal of Guidance, Control, and Dynamics,*, vol. 12, no. 4, pp. 460–468, 1988.

[22] G. Hoffmann, H. Huang, and S. Waslander, "Quadrotor helicopter flight dynamics and control: Theory and experiment," *American Institute of Aeronautics and Astronautics*, 2007.

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