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Inhibiting Disturbance Propagation in Large-scale Systems and Its Application to Power System Control

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Abstract— **In this paper, a localized control problem is formulated for large-scale systems: The disturbance propagation into or from a local target part of the system is inhibited by local control. As a preliminary, the parametrization of all stabilizing local controllers is introduced. Then, by specializing the parametrized controller, we give the solution to the localized control problem. Finally, the proposed controller is applied to a power network system. The use and effectiveness are demonstrated in the frequency control of the system.**

I. INTRODUCTION

This paper is devoted to localized control problems for large-scale systems, which are particularly motivated by the application to a power network system. Practical large-scale systems can be structurally varying or expanding, to which additional subsystems are connected one after another [1]. For such systems, there have been proposed solutions to various control problems; expanding construction [2]; plug-and-play control [3], [4]; passivity-based modularized design [5], [6]; localized control [7], [8]; retrofit control [9], [10], [11], and so on. A key observation in them is *localized* design or implementation of controllers: Only the model of a local system is available for the controller design or local action is allowed for measurement and control. Then, stabilization of the system and other control problems are addressed.

In this paper, a local part of the large-scale system is extracted, and a localized control problem is formulated: A local controller is designed and implemented to the target local part such that the disturbance propagation into or from the local part is inhibited. Both of the controller design and implementation are performed in a localized manner. The localized control problem is classified into two detailed problems based on the control objective with disturbance assumptions. 1) One is to inhibit the inward-directed interaction to the target local part, while allowing the outward-directed one. Consider that the disturbance is injected to the environment, which is the remaining global part of the system. Then, the local controller inhibits the disturbance propagation to the target local part as illustrated in Fig. 1. 2) In a similar manner, we formulate a control problem of inhibiting the outward-directed interaction from the target local part, while allowing the inward-directed one. Then, the disturbance injected to the target part is not propagated to the environment as illustrated in Fig. 2. Related control problems to the control problem are addressed and solved in [7], [8], which are called spatial localization.

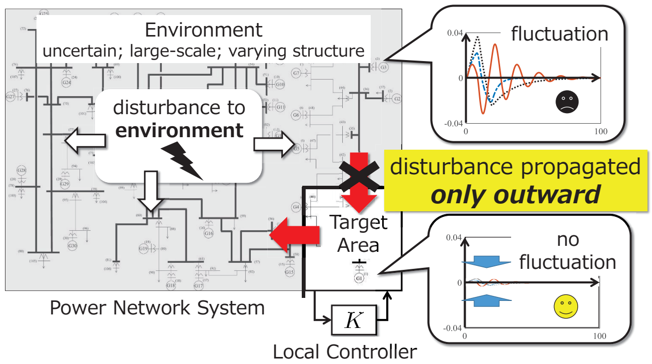

Fig. 1. Inhibiting inward-directed interaction. The objective of the local controller is to inhibit the inward-directed interaction from the environment to the target area, while allowing the other outward-directed one. Then, the disturbance injected to the environment is not propagated to the target area.

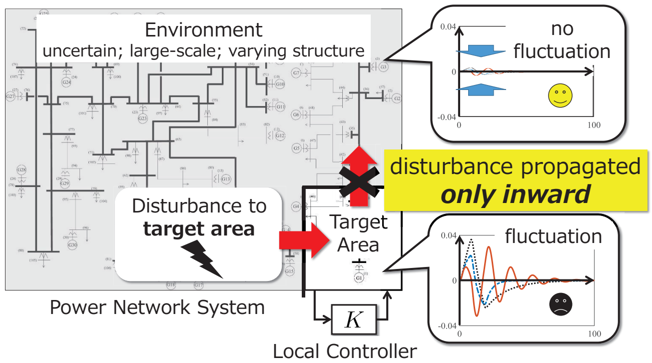

Fig. 2. Inhibiting outward-directed interaction. The objective of the local controller is to inhibit the outward-directed interaction from the target area to the other environmental area, while allowing the other inward-directed one. Then, the disturbance particularly injected to the target area is not propagated to the other environmental area.

In this paper, the propagation inhibition is achieved on the basis of the retrofit control [9], [10], [11], by which the stability of the overall system is guaranteed for any change of the environment. By specializing the parametrized retrofit controller [11], we propose the inhibiting controller for the disturbance propagation.

This paper is organized as follows. In Section II, a general system-description is given, and control objectives are briefly defined. In Section III, the stability of the overall system is analyzed, and the parametrization of all admissible

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controllers is introduced as a summary of the results in [11]. On the basis of the parametrization, the localized control problems of inhibiting the disturbance propagation are formulated. Then, by specializing the controller parameter, we give the solutions to the problems. Finally, in Section IV, the inhibiting control is applied to a power network system [6], and the effectiveness of the control is illustrated.

Notation: (see e.g. [12]) Let $\bar{X}$,

$$\bar{Y} := \begin{bmatrix} \bar{Y}_{11} & \bar{Y}_{12} \\ \bar{Y}_{21} & \bar{Y}_{22} \end{bmatrix},\ \bar{Z} := \begin{bmatrix} \bar{Z}_{11} & \bar{Z}_{12} \\ \bar{Z}_{21} & \bar{Z}_{22} \end{bmatrix},$$

be transfer matrices. Then, the lower linear fractional transformation of $\bar{Y}$ by $\bar{X}$ is defined by

$$\mathcal{F}_{\mathrm{l}}(\bar{Y}, \bar{X}) := \bar{Y}_{11} + \bar{Y}_{12}\bar{X}(I - \bar{Y}_{22}\bar{X})^{-1}\bar{Y}_{21}.$$

In addition, the star product of $\bar{Y}$ and $\bar{Z}$ is defined by

$$\mathcal{P}(\bar{Y}, \bar{Z}) := \begin{bmatrix} \bar{Y}_{11} + \bar{Y}_{12}\bar{Z}_{11}(I - \bar{Y}_{22}\bar{Z}_{11})^{-1}\bar{Y}_{21} & \bar{Y}_{12}(I - \bar{Z}_{11}\bar{Y}_{22})^{-1}\bar{Z}_{12} \\ \bar{Z}_{21}(I - \bar{Y}_{22}\bar{Z}_{11})^{-1}\bar{Y}_{21} & \bar{Z}_{22} + \bar{Z}_{21}\bar{Y}_{22}(I - \bar{Z}_{11}\bar{Y}_{22})^{-1}\bar{Z}_{12} \end{bmatrix}.$$

II. SYSTEM DESCRIPTION AND CONTROL OBJECTIVES

A. General System-Description

We give a description of a plant system which particularly expresses large-scale systems. The plant system $\Sigma(L, G)$ is composed of two dynamical systems L and G. The system L is described by

$$L:\ \begin{bmatrix} \eta \\ \hline z_L \\ y \end{bmatrix} = \bar{L} \begin{bmatrix} \xi \\ \hline w_L \\ u \end{bmatrix},$$

where $\eta \in \mathbb{R}^r$ and $\xi \in \mathbb{R}^s$ are the interaction signals, $z_L \in \mathbb{R}^p$ is the control output, $w_L \in \mathbb{R}^q$ is the disturbance input, $y \in \mathbb{R}^\ell$ is the measured output, and $u \in \mathbb{R}^m$ is the control input. The symbol $\bar{L}$ denotes the transfer matrix:

$$\bar{L} := \left[\begin{array}{c|cc} \bar{L}_{\eta\xi} & \bar{L}_{\eta w} & \bar{L}_{\eta u} \\ \hline \bar{L}_{z\xi} & \bar{L}_{zw} & \bar{L}_{zu} \\ \bar{L}_{y\xi} & \bar{L}_{yw} & \bar{L}_{yu} \end{array}\right].$$

The generalized plant L includes the target *local* system to be controlled, control objective, weighting functions, and so on. In the following discussion, L is simply called the *local* system. In contrast, the system G is called the *global* system implying the rest part of the plant system except L:

$$G:\ \begin{bmatrix} z_G \\ \xi \end{bmatrix} = \bar{G} \begin{bmatrix} w_G \\ \eta \end{bmatrix},$$

where $z_G \in \mathbb{R}^\alpha$ is the control output, $w_G \in \mathbb{R}^\beta$ is the disturbance input, and $\bar{G}$ is the transfer matrix:

$$\bar{G} := \begin{bmatrix} \bar{G}_{zw} & \bar{G}_{z\eta} \\ \bar{G}_{\xi w} & \bar{G}_{\xi\eta} \end{bmatrix}.$$

The global system G can structurally vary and include uncertain parts of the plant system, and therefore the model

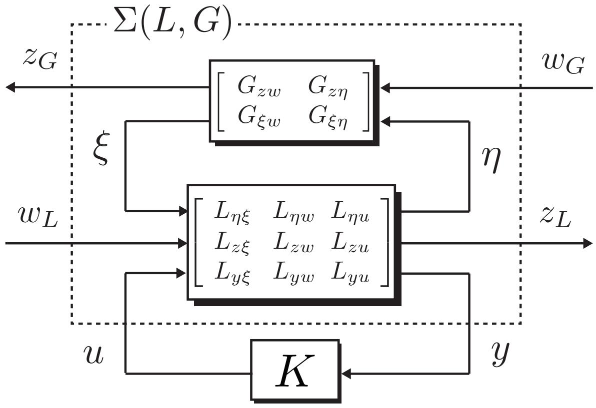

Fig. 3. Overall system Σ_{all}. The plant system $\Sigma(L, G)$ is composed of the local system L and the global system G.

$\bar{G}$ is unavailable for local controller design. The plant system $\Sigma(L, G)$ is defined by the feedback connection of L and G:

$$\Sigma(L, G):\ \begin{bmatrix} z_G \\ \hline z_L \\ y \end{bmatrix} = \mathcal{P}(\bar{G}, \bar{L}) \begin{bmatrix} w_G \\ \hline w_L \\ u \end{bmatrix}.$$

The local controller K is described by

$$K:\ u = \bar{K}y,$$

where $\bar{K}$ is the transfer matrix of K. Then, the overall system Σ_{all}, which is illustrated in Fig. 3, is described by

$$\Sigma_{\mathrm{all}}:\ \begin{bmatrix} z_G \\ z_L \end{bmatrix} = \bar{\Sigma}_{\mathrm{all}} \begin{bmatrix} w_G \\ w_L \end{bmatrix},$$

where $\bar{\Sigma}_{\mathrm{all}}$ is the transfer matrix is given by

$$\bar{\Sigma}_{\mathrm{all}} := \mathcal{F}_{\mathrm{l}}(\mathcal{P}(\bar{G}, \bar{L}), \bar{K}). \tag{1}$$

B. Control Objectives

In this paper, we aim to inhibit *spatial distribution* of the disturbance effects from or into the local system L. Then, two types of control objectives are considered: 1) One is to inhibit the propagation of the global disturbance w_G to the local control output z_L, and 2) the other is to inhibit that of the local disturbance w_L to the global z_G.

1) Inhibiting Inward Disturbance Propagation: To state the control objective, we focus on the disturbance propagation only from w_G to z_L. To this end, supposing that $w_L = 0$, we have

$$\Sigma_{\mathrm{all,in}}:\ z_L = \bar{\Sigma}_{\mathrm{all,in}} w_G,$$

where $\bar{\Sigma}_{\mathrm{all,in}}$ is the transfer matrix:

$$\bar{\Sigma}_{\mathrm{all,in}} := \begin{bmatrix} 0 & I \end{bmatrix} \bar{\Sigma}_{\mathrm{all}} \begin{bmatrix} I \\ 0 \end{bmatrix}.$$

The input-output system $\Sigma_{\mathrm{all,in}}$ is illustrated in Fig. 4. The control objective is to reduce $\|\bar{\Sigma}_{\mathrm{all,in}}\|$ in some norm.

Note that w_G is propagated to L through ξ, which is the input signal to L. Then, the control objective means the inhibition of the inward signal-propagation to L. In contrast, η, which is the output signal from L, is not inhibited in

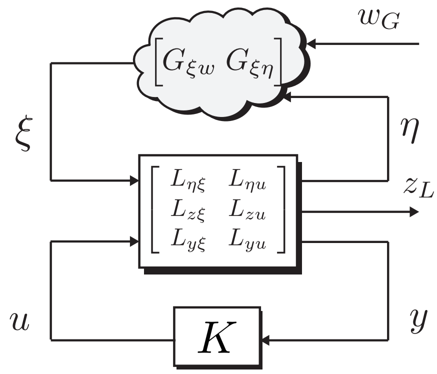

Fig. 4. Overall system $\Sigma_{\mathrm{all,in}}$ in the problem of inhibiting inward disturbance propagation. The disturbance w_G injected to the global part affects the local system L through the interaction signal ξ. Then, the inward disturbance propagation to z_L is inhibited by the local controller K.

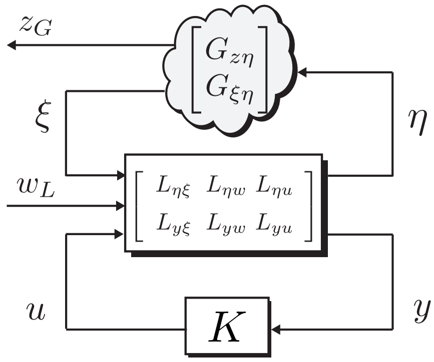

Fig. 5. Overall system $\Sigma_{\mathrm{all,out}}$ in the problem of inhibiting outward disturbance propagation. The control output z_G is defined for the global system G and is proportional to the interaction signal η. The disturbance propagation of w_L into z_G is inhibited by the local controller K.

general, and the outward propagation of the internal signals of L is allowed.

The detailed problem of the propagation inhibition is formulated in Section III.

2) Inhibiting Outward Disturbance Propagation: Consider the disturbance propagation only from w_L to z_G. Then, supposing that $w_G = 0$, we have

$$\Sigma_{\mathrm{all,out}}:\ z_G = \bar{\Sigma}_{\mathrm{all,out}} w_L,$$

where $\bar{\Sigma}_{\mathrm{all,out}}$ is the transfer matrix:

$$\bar{\Sigma}_{\mathrm{all,out}} := \begin{bmatrix} I & 0 \end{bmatrix} \bar{\Sigma}_{\mathrm{all}} \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

The input-output system $\Sigma_{\mathrm{all,out}}$ is illustrated in Fig. 5. The control objective is to reduce $\|\bar{\Sigma}_{\mathrm{all,out}}\|$ in some norm.

Note that η is the output signal from L and that z_G is proportional to η as $z_G = \bar{G}_{z\eta}\eta$. The control objective means the inhibition of the outward signal-propagation from L, while allowing the inward one through ξ or w_L.

In the propagation inhibition, we aim to *localize* the disturbance effects in a limited target area modeled by $\bar{L}$. The control objective is also called *localization* of the disturbance effects [7], [8] and is illustrated in Fig. 2 of Section I.

III. MAIN RESULT

The control problems of the propagation inhibition are formulated and solved in this section. As a preliminary, we find the set of the local controllers K such that the overall system Σ_{all} is stable for any dynamical system G. Then, by specializing the class, we give the solutions to the problems of the propagation inhibition.

A. Stability Analysis in General Control Problem

We particularly consider that the global system G is an uncertain, large-scale, and varying-structured dynamical system. Only a qualitative assumption is imposed on G rather than quantitative one such as norm-bounded uncertainties. This is the main difference from standard robust control problems (see e.g. [12]). We assume that

$$\bar{G} \in \mathcal{S}_{\mathrm{g}} := \{\bar{G} \,|\, \mathcal{P}(\bar{G}, \bar{L}) \in \mathcal{RH}_\infty\} \tag{2}$$

holds[1]. There is no limitation on e.g., the norm and phase of G as long as $\Sigma(L, G)$ is stable. Then, we formulate a general stabilization problem by the local controller K, which is called the *retrofit control problem* [9], [11]:

Problem 1: Retrofit control problem: Find K such that $\bar{\Sigma}_{\mathrm{all}} \in \mathcal{RH}_\infty$ holds for all $\bar{G} \in \mathcal{S}_{\mathrm{g}}$.

The solution to the retrofit control problem is given in the following theorem [11]. For simplicity, it is assumed that the local system L is stable in the theorem.

Theorem 1: [11] Suppose that $\bar{L} \in \mathcal{RH}_\infty$ holds. Then, all the solutions to Problem 1 are expressed by

$$\bar{K} = \bar{Q}_{\mathrm{k}}(I + \bar{L}_{yu}\bar{Q}_{\mathrm{k}})^{-1}, \tag{3}$$

where $\bar{Q}_{\mathrm{k}} \in \mathcal{RH}_\infty$ satisfies

$$\bar{L}_{\eta u}\bar{Q}_{\mathrm{k}}\bar{L}_{y\xi} = 0. \tag{4}$$

The proof is omitted in this paper. We emphasize that the theorem provides the *parametrization* of all stabilizing local controllers or more precisely all stability-inheriting local controllers. The parametrization is utilized for designing the controllers that inhibit the disturbance propagation in the following two subsections.

In addition to the stability of Σ_{all}, we aim to achieve

$$\bar{\Sigma}_{\mathrm{all,in}} = 0 \tag{5}$$

or alternatively

$$\bar{\Sigma}_{\mathrm{all,out}} = 0 \tag{6}$$

for all $\bar{G} \in \mathcal{S}_{\mathrm{g}}$. Then, $z_L(t) \equiv 0$ or $z_G(t) \equiv 0$ holds for any disturbance w_G or w_L. We give the solutions to the control problems of the propagation inhibition.

B. Control for Inhibiting Inward Disturbance Propagation

We formulate and address the control problem of inhibiting the disturbance propagation to the local system L. The problem is formulated as follows:

Problem 2: Suppose that $\bar{L} \in \mathcal{RH}_\infty$ holds. Then, find $\bar{Q}_{\mathrm{k}}$ such that (4) and (5) hold for all $\bar{G} \in \mathcal{S}_{\mathrm{g}}$.

From Theorem 1, the parametrized controller (3) under (4) achieves $\bar{\Sigma}_{\mathrm{all}} \in \mathcal{RH}_\infty$ for all $\bar{G} \in \mathcal{S}_{\mathrm{g}}$. In addition, we give

[1] It is implicitly assumed that the feedback connection is well-posed.

another expression of the overall system $\Sigma_{\rm all}$. Without loss of generality, the Youla parametrization of a part of $\bar{G} \in \mathcal{S}_{\rm g}$ gives the expression

$$\bar{G}_{\xi\eta} = \bar{Q}_{\rm g}(I + \bar{L}_{\eta\xi}\bar{Q}_{\rm g})^{-1}, \tag{7}$$

where $\bar{Q}_{\rm g} \in \mathcal{RH}_\infty$. Then, the transfer matrix of $\Sigma_{\rm all}$ is equivalently reduced from (1) to

$$\bar{\Sigma}_{\rm all} = \begin{bmatrix} * & \bar{G}_{z\eta}(I + \bar{L}_{\eta\xi}\bar{Q}_{\rm g})(\bar{L}_{\eta w} + \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{yw}) \\ (\bar{L}_{z\xi} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{y\xi})(I + \bar{Q}_{\rm g}\bar{L}_{\eta\xi})\bar{G}_{\xi w} & * \end{bmatrix}. \tag{8}$$

The diagonal parts of this $\bar{\Sigma}_{\rm all}$ are not considered in the following discussion and therefore omitted. It follows that $\bar{\Sigma}_{\rm all,in}$ is expressed by

$$\bar{\Sigma}_{\rm all,in} = (\bar{L}_{z\xi} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{y\xi})(I + \bar{Q}_{\rm g}\bar{L}_{\eta\xi})\bar{G}_{\xi w}. \tag{9}$$

Now, letting

$$\begin{aligned} \bar{\Sigma}_{\rm in} &:= \bar{L}_{z\xi} + \bar{L}_{zu}\bar{Q}_{\rm k}\bar{L}_{y\xi}, \\ \bar{\Delta}_{\rm in} &:= (I + \bar{Q}_{\rm g}\bar{L}_{\eta\xi})\bar{G}_{\xi w}, \end{aligned} \tag{10}$$

we reduce the expression (9) to

$$\bar{\Sigma}_{\rm all,in} = \bar{\Sigma}_{\rm in}\bar{\Delta}_{\rm in}. \tag{11}$$

We see that $\bar{\Sigma}_{\rm in}$ contains only the local model $\bar{L}_{**}$ and controller parameter $\bar{Q}_{\rm k}$, but is independent of $\bar{Q}_{\rm g}$. Then, $\bar{\Sigma}_{\rm in}$ is to be designed and shaped, while $\bar{\Delta}_{\rm in}$ is not to be. If it is achieved that

$$\bar{\Sigma}_{\rm in} = 0, \tag{12}$$

we have (5) for all $\bar{G} \in \mathcal{S}_{\rm g}$, and vice versa. The solution to Problem 2 is given in the proposition.

Proposition 1: The solution to Problem 2 is given by $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$ such that (4) and (12) hold.

In the proposition, a condition to achieve (5) is stated. Under the condition, the resulting controller (3) achieves $z_L(t) \equiv 0$ independently of $\bar{G} \in \mathcal{S}_{\rm g}$.

Remark 1: Note that Problem 2 has the solution only if

$$\bar{L}_{\eta u}\bar{Q}_{\rm k} = 0 \tag{13}$$

holds. We can show this fact as follows. In place of (13), we consider the constraint

$$\bar{Q}_{\rm k}\bar{L}_{y\xi} = 0, \tag{14}$$

which is also sufficient for (4). This condition (14) seems to provide another solution to Problem 2. However, $\bar{Q}_{\rm k}$ satisfying (14) cannot be the solution. If (14) holds in place of (13), the transfer matrix of $\Sigma_{\rm all,in}$ given in (9) is reduced to

$$\bar{\Sigma}_{\rm all,in} = (\bar{L}_{z\xi} + \bar{L}_{z\xi}\bar{Q}_{\rm g}\bar{L}_{\eta\xi})\bar{G}_{\xi w},$$

which is independent of $\bar{Q}_{\rm k}$. Therefore, (5) is not achieved in general, and (13) must hold rather than (14) for the propagation inhibition.

Remark 2: It is implicitly necessary for the constraint (13) that $m > r$ holds, i.e., the dimension of u must be strictly greater than that of η. This implies that flexible actuation in L is required for the propagation inhibition.

Remark 3: In practical problems, (12) is so tight that it cannot be satisfied by any $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$. However, even if (12) does not completely hold, the stability of $\Sigma_{\rm all}$ is guaranteed as long as (13) holds. In addition, the minimization of $\|\bar{\Sigma}_{\rm in}\|$ tends to improve the performance of $\Sigma_{\rm all,in}$, which can be called *weak* inhibition. Note that $\bar{\Sigma}_{\rm all,in}$ of (11) has the cascaded structure of $\bar{\Sigma}_{\rm in}$ and $\bar{\Delta}_{\rm in}$. Then, the decrease of $\|\bar{\Sigma}_{\rm in}\|$ can lead that of $\|\bar{\Sigma}_{\rm all}\| = \|\bar{\Sigma}_{\rm in}\bar{\Delta}_{\rm in}\|$. Practical design of the *weakly* inhibiting controller is to minimize $\|\bar{\Sigma}_{\rm in}\|$ by $\bar{Q}_{\rm k}$ satisfying (13).

C. Control for Inhibiting Outward Disturbance Propagation

We formulate and solve the problem of inhibiting the disturbance propagation from the local system L. In a similar manner to Problem 2, the problem achieving (6) is formulated as follows:

Problem 3: Suppose that $\bar{L} \in \mathcal{RH}_\infty$ holds. Then, find $\bar{Q}_{\rm k}$ such that (4) and (6) hold for all $\bar{G} \in \mathcal{S}_{\rm g}$.

Let $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$ satisfy (14). From Theorem 1, we show the stability of $\Sigma_{\rm all}$ for all $\bar{Q}_{\rm g} \in \mathcal{RH}_\infty$. In addition, recalling the expression in (8), we have the expression of $\bar{\Sigma}_{\rm all,out}$ by

$$\bar{\Sigma}_{\rm all,out} = \bar{G}_{z\eta}(I + \bar{L}_{\eta\xi}\bar{Q}_{\rm g})(\bar{L}_{\eta w} + \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{yw}). \tag{15}$$

The expression of $\bar{\Sigma}_{\rm all,out}$ is further reduced to

$$\bar{\Sigma}_{\rm all,out} = \bar{\Delta}_{\rm out}\bar{\Sigma}_{\rm out}, \tag{16}$$

where $\bar{\Delta}_{\rm out}$ and $\bar{\Sigma}_{\rm out}$ are defined by

$$\begin{aligned} \bar{\Delta}_{\rm out} &:= \bar{G}_{z\eta}(I + \bar{L}_{\eta\xi}\bar{Q}_{\rm g}), \\ \bar{\Sigma}_{\rm out} &:= \bar{L}_{\eta w} + \bar{L}_{\eta u}\bar{Q}_{\rm k}\bar{L}_{yw}, \end{aligned}$$

respectively. We note that $\bar{\Sigma}_{\rm out}$ is to be designed and shaped by $\bar{Q}_{\rm k}$, while $\bar{\Delta}_{\rm out}$ is not to be.

We conclude that (6) holds for all $\bar{G} \in \mathcal{S}_{\rm g}$ if $\bar{Q}_{\rm k}$ is designed such that

$$\bar{\Sigma}_{\rm out} = 0 \tag{17}$$

holds. Then, the following proposition is given.

Proposition 2: The solution to Problem 3 is given by $\bar{Q}_{\rm k} \in \mathcal{RH}_\infty$ such that (4) and (17) hold.

Under the condition of Proposition 2, $z_G(t) \equiv 0$ holds independently of $\bar{G} \in \mathcal{S}_{\rm g}$. The disturbance propagation from L to G is inhibited by the resulting controller.

The outward propagation inhibition is further studied in the following three remarks, which are the counterparts of Remarks 1–3 for the inward propagation inhibition.

Remark 4: The constraint (14) must hold rather than (13) for the solvability of Problem 3.

Remark 5: The constraint (14) implicitly requires that $\ell > s$ holds, i.e., the dimension of y must be strictly greater than that of ξ. Furthermore, the measured output y must include rich information on ξ by many sensing modules. In [9], the local control problem of inhibiting the outward disturbance propagation is addressed and solved. A realization of the solution is given by using the direct measurement of ξ. This is an example of using rich information.

Remark 6: Even if (17) is not satisfied by any $\bar{Q}_{\mathrm{k}} \in \mathcal{RH}_\infty$, it is worth minimizing $\|\bar{\Sigma}_{\mathrm{out}}\|$. The minimization tends to decrease $\|\bar{\Sigma}_{\mathrm{all,out}}\| = \|\bar{\Delta}_{\mathrm{out}}\bar{\Sigma}_{\mathrm{out}}\|$.

IV. APPLICATION TO POWER SYSTEM CONTROL

A. Linearized Power System Model

A power network model is briefly introduced in this subsection. The derivation process is omitted, see e.g. [6] for details. We consider the IEEJ EAST 30-machine model [13]: The baseline power system consists of 30 generators and 107 buses with some loads. In addition, two battery storages and one renewable energy (RE) farm are installed into some of the buses, which behave as the control and disturbance inputs, respectively. The symbols $\mathcal{V} := \{1, 2, \ldots, 107\}$, $\mathcal{V}_{\mathrm{gen}} := \{1, 2, \ldots, 30\}$, $\mathcal{V}_{\mathrm{B}} := \{1, 31\}$, and $\mathcal{V}_{\mathrm{R}} = \{34\}$ are the label sets of the buses, generators, battery storages, and RE farm, respectively.

The power system model is described by the dynamics for the generator node in $\mathcal{V}_{\mathrm{gen}}$ and the power flow equations between nodes, see e.g. [14] for details. Let $(\delta_i, \omega_i, P_{\mathrm{m}i}, \theta_j)$ be the internal variables of the power system: the rotor angle, the frequency deviation of the rotor, the mechanical power input given by a first-order turbine-governor dynamics [15], and the phase angle of the terminal bus, respectively. They are defined for all $i \in \mathcal{V}_{\mathrm{gen}}$ and all $j \in \mathcal{V}$. Further let $P_{\mathrm{B}k}$ and P_{R} be the active power injected from the storages $k \in \mathcal{V}_{\mathrm{B}}$ and the RE farm, respectively. Then, we obtain the linearized model near the steady-state. Denoting the error variables of $(\delta_i, P_{\mathrm{m}i}, \theta_j, P_{\mathrm{B}k}, P_{\mathrm{R}})$ by $\tilde{\cdot}$ and letting

$$x = \left[\, \tilde{\delta}_1 \ \cdots \ \tilde{\delta}_{30} \ \omega_1 \ \cdots \ \omega_{30} \ \tilde{P}_{\mathrm{m}1} \ \cdots \ \tilde{P}_{\mathrm{m}30} \,\right]^\top,$$
$$\theta = \left[\, \tilde{\theta}_1 \ \cdots \ \tilde{\theta}_{107} \,\right]^\top, u = \left[\, \tilde{P}_{\mathrm{B}1} \ \tilde{P}_{\mathrm{B}31} \,\right]^\top, w_G = \tilde{P}_{\mathrm{R}},$$

we describe the linearized model by the descriptor equation:

$$\begin{bmatrix} \dot{x} \\ 0 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \\ B_u \end{bmatrix} u + \begin{bmatrix} 0 \\ B_w \end{bmatrix} w, \qquad (18)$$

where the coefficient matrices are given by

$$A_{11} := \begin{bmatrix} 0 & I & 0 \\ -M^{-1}X & -M^{-1}D & M^{-1} \\ 0 & -T^{-1}K & -T^{-1} \end{bmatrix} \in \mathbb{R}^{90\times 90},$$
$$A_{12} := \begin{bmatrix} 0 & 0 \\ M^{-1}X & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{90\times 107},$$
$$A_{21} := \begin{bmatrix} X & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^{107\times 90},$$
$$A_{22} := -F - \mathrm{diag}(X, 0) \in \mathbb{R}^{107\times 107}.$$

In the coefficient matrices, M, D, T, and K, and X represent diagonal matrices determined by the inertia and damping of the generators, time constant and gain in the first-order turbine-governor dynamics, and reactance, respectively. The matrices B_u and B_w depend on the set $\mathcal{V}_{\mathrm{B}}$ and $\mathcal{V}_R$, respectively. The $(1, 1)$ and $(31, 2)$-elements of B_u and the 34th element of B_w are one, while the others are zero. In addition, F is a Laplacian matrix that represents the interconnection between the buses.

B. Application of Inward Propagation Inhibition

The objective of the numerical experiment is to design a local controller that suppresses the frequency deviation ω_1 caused by the disturbance w_G. The measurable output y, control input u, and available model are limited on those of a spatially local area. The disturbance suppression must be achieved *without impairing the stability* of the overall power system. To this end, the local model L is extracted from (18), and the proposed inhibiting controller is applied to L.

We extract the local variables from x and θ as $x_L := [\, \tilde{\delta}_1 \, \omega_1 \, \tilde{P}_{\mathrm{m}1} \,]^\top$ and $\theta_L := [\, \theta_1 \, \theta_{31} \,]^\top$. The measurable and control outputs are defined by

$$y = \begin{bmatrix} x_L \\ \theta_L \end{bmatrix}, \quad z_L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_L \\ \theta_L \end{bmatrix}. \qquad (19)$$

Letting x_G and θ_G be the other rest parts of x and θ except x_L and θ_L, respectively, we decompose the overall system (18) into

$$\begin{bmatrix} \dot{x}_L \\ 0 \end{bmatrix} = A_L \begin{bmatrix} x_L \\ \theta_L \end{bmatrix} + A_{LG} \begin{bmatrix} x_G \\ \theta_G \end{bmatrix} + \begin{bmatrix} 0 \\ B_{Lu} \end{bmatrix} u, \qquad (20)$$
$$\begin{bmatrix} \dot{x}_G \\ 0 \end{bmatrix} = A_{GL} \begin{bmatrix} x_L \\ \theta_L \end{bmatrix} + A_G \begin{bmatrix} x_G \\ \theta_G \end{bmatrix} + \begin{bmatrix} 0 \\ B_{Gw} \end{bmatrix} w_G, \qquad (21)$$

where the coefficient matrix $[\, A_L, A_{LG}; A_{GL}, A_G \,]$ is constructed from $[\, A_{11}, A_{12}; A_{21}, A_{22} \,]$ by exchanging the columns and rows, and B_{Lu} and B_{Gw} are constructed from B_u and B_w, respectively.

Note here that the dynamical systems (20) and (21) are connected each other with *low-rank interactions*: A_{LG} and A_{GL} are dramatically sparse. Actually, in the IEEJ model [13], the interaction between (20) and (21) is expressed only by the *scalar*-valued signals θ_{31} and θ_{32} rather than all of $[\, x_L^\top \ \theta_L^\top \,]^\top$ and $[\, x_G^\top \ \theta_G^\top \,]^\top$. Then, letting $\eta = \theta_{31}$ and $\xi = \theta_{32}$, we have $B_L\xi = A_{LG}[\, x_G^\top \ \theta_G^\top \,]^\top$ for some vector B_L. The expression (20) with the control objective is reduced to the local model L described by

$$L: \begin{cases} \begin{bmatrix} \dot{x}_L \\ 0 \end{bmatrix} = A_L \begin{bmatrix} x_L \\ \theta_L \end{bmatrix} + B_L\xi + \begin{bmatrix} 0 \\ B_{Lu} \end{bmatrix} u, \\ \eta = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_L \\ \theta_L \end{bmatrix}, \\ z_L = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_L \\ \theta_L \end{bmatrix}, \\ y = \begin{bmatrix} x_L \\ \theta_L \end{bmatrix}. \end{cases}$$

The global model G is internally connected to L through signals ξ and η. We note that the disturbance w_G is propagated through ξ from G to this L.

The weak inhibition of the inward disturbance propagation, which is formulated by the disturbance suppression in this L, is achieved by finding the minimizer $\bar{Q}_{\mathrm{k}}$ of $\bar{\Sigma}_{\mathrm{in}}$, which

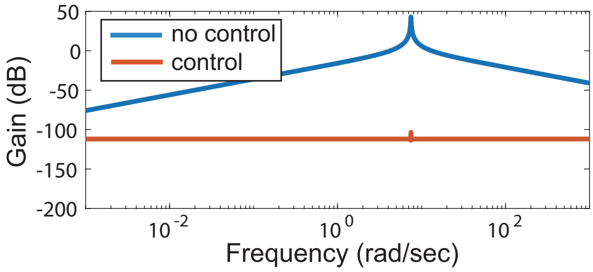

Fig. 6. The gain plot of $\bar{\Sigma}_{\rm in}$.

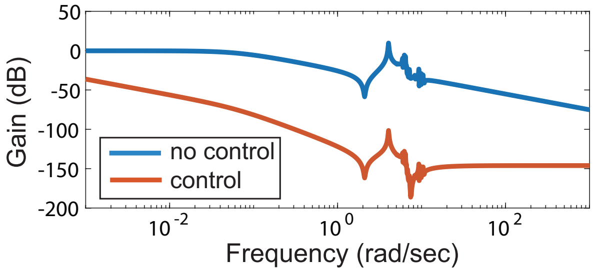

Fig. 7. The gain plot of $\bar{\Sigma}_{\rm all,in}$.

is defined in (10). In addition, a constraint (13) is imposed on $\bar{Q}_{\rm k}$ to guarantee the stability of the overall system. To this end, letting $\bar{L}_{\eta\underline{u}} =: [\,\bar{L}_{\eta u1}\,\bar{L}_{\eta u2}\,] \in \mathbb{C}^{1\times 2}$, we show that for any parameter $\bar{Q}_{\rm pow} \in \mathcal{RH}_\infty$ satisfying

$$\bar{Q}_{\rm k} = \begin{bmatrix} \bar{L}_{\eta u2} \\ -\bar{L}_{\eta u1} \end{bmatrix} \bar{Q}_{\rm pow},$$

the constraint (13) holds. The minimization of $\bar{\Sigma}_{\rm in}$ by $\bar{Q}_{\rm k}$ under the constraint (13) is reduced to that by $\bar{Q}_{\rm pow}$. The reduced problem is solvable in a numerically efficient way by applying the H_∞ control method for descriptor systems, see e.g. [16], [17], [18].

C. Result of Inward Propagation Inhibition

The result of the controller design is compared with that of the no-controller case. In Fig. 6, the gain plot of $\bar{\Sigma}_{\rm in}$ is drawn for the two cases: 1) proposed controller: $\bar{Q}_{\rm pow} \neq 0$ and 2) no-controller: $\bar{Q}_{\rm pow} = 0$. We see that $\bar{\Sigma}_{\rm in}$ cannot be completely canceled out, and the propagation inhibition is not perfectly achieved. However, the gain of $\bar{\Sigma}_{\rm in}$ decreases for all frequency range. Then, it is expected that the gain of the overall system $\bar{\Sigma}_{\rm all,in}$ decreases as well, and it is actually shown in Fig. 7. By the local controller, the performance of the disturbance suppression is improved for all frequency range. In addition to the performance improvement, it should be emphasized again that the stability of the overall system is guaranteed independently of any change or uncertainty in the global system G. This plays an essential role for design problems for practical large-scale systems.

V. CONCLUSION

The problems of inhibiting the disturbance propagation were formulated and solved on the basis of the retrofit control [9], [11], which was the localized design of stabilizing local controllers. In the numerical experiment, the proposed inhibiting controller was applied to a power system. Then, the effectiveness was demonstrated in a practical power system model developed in [13].

In the numerical experiment, only one local controller was designed and implemented to a local part of the large-scale system. However, the proposed design methods are simultaneously applicable to other parts of the system *without impairing the stability* of the overall system. In future work, multiple controllers are designed such that a common *global* objective as well as individual *local* objectives are simultaneously achieved. The controllers design can be done in a distributed manner [19].

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