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Consensus Based Power Update Algorithm for OFDMA-based Femtocell Networks

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***Abstract*— In this paper, we study the power assignment problem of the downlink of an OFDMA-based two-tier femtocell network. We aim to provide a global fairness among the user equipments in terms of their SINR values. Because of the potential complexity of the femtocell networks, we propose a distributed consensus–based power update algorithm to keep the required information exchange of the base stations minimal. Even though there exist some power control algorithms serving for the same fairness purpose, they are applicable only for the systems where each base station can have at most one user. Since femtocells are designed to serve more than one user equipment at the same time, the proposed power update algorithm can be considered as more real life applicable, and stands out amongst others by its ability to ensure consensus in multi–user/subchannel systems.**

I. INTRODUCTION

In order to cope with the limited availability of the radio spectrum, femtocell technology has been a resorted topic and gaining the attention of researchers in the last decade. Deployment of new femtocells in the transmission area of a macro base station can provide improved service quality for the users who suffer from low signal quality. By reducing the transmission distance and the cell size, femto access points (FAPs) can increase the network capacity and the data rate of the network [1]. However, deployment of femtocells comes with some technical problems to be solved, such as interference management, handoff or security. In a two-tier network, interference may occur in co–tier or cross–tier manner [2]. With the use of orthogonal frequency–division multiple access (OFDMA), intracell interference can be avoided when an OFDMA subchannel is attained at most one user in each cell [3]. Therefore, both co–tier and cross–tier interferences can only occur between the users or base stations which use the same frequency sub-band, which is the case we consider in this paper. Since these base stations have to share the same spectrum, both co-tier and cross-tier interference management stands as a major challenge to be solved.

For the downlink case of an OFDMA-based femtocell network, interference management can be done by adjusting the transmission powers of base stations, changing the subchannel assignment, or performing handovers from one serving station to another. In [4], a decentralized frequency allocation scheme for OFDMA-based two-tier networks is proposed and proven to decrease the spectrum usage for the target QoS. In [5], a game theoretical approach is used for the power control of a two-tier network. In [6] and [7], joint power/subchannel allocation algorithms are proposed by relaxing non-convex problems into convex optimization problems.

Deployment of the femto access points is done by the users without the control of the operators. Therefore, a femtocell network may have various number, maybe hundreds, of base stations with an unknown topology. This uncertainty and the possibility of large numbers make the centralized solutions hard to implement while paving the way for the so called 'self-organizing' decentralized techniques as the best solution [8].

In this paper, we extend the consensus-based power control algorithm (PCA) given in [9], where the base stations and the communication channels between these BSs are respectively considered as nodes and the edges of a graph. The PCA in [9] is analytically and numerically proven to be capable of establishing fairness in terms of the SINR values of the users. This algorithm is applicable for femtocell networks where all base stations have only one user to serve or where all users get service on the same subchannel. However, in the real life scenarios, a base station is more likely to have more than one user equipment. Also by its nature, a joint power/frequency allocation problem should have multi subchannels. In order to make the given power control algorithm more realistic, and more usable in joint resource allocation problems, we adapt the PCA so that it ensures the global consensus of the user SINR values on a single value. The modification is made possible by expanding the corresponding graph of inter–base station communication network in a way that each node split into virtual nodes by the number of available subchannels. According to the proposed algorithm, which will be called as Power Update Algorithm (PUA), each base station updates its transmission power level by using information coming from its neighboring base stations and its own users.

The remaining content of the paper is structured as follows. In Section II-A, the model of system and the required terminology are introduced. The power control problem is explained in Section II-B. The power update algorithm proposed is given in Section III. Convergence analysis and the simulation results of the proposed algorithm are given in Section IV and V, respectively. Finally, some concluding remarks are given in Section VI.

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II. SYSTEM DESCRIPTION

A. System Model and Notation

We consider the downlink scenario of a two-tier network where the first tier consists of a single macro base station (MBS) and the user equipments (UEs) served by that MBS. Inside the coverage area of the MBS, M_f femto base stations (FBSs) and the UEs served by these FBSs are located. These FBSs and FUEs constitute the second tier of the network. The set of all base stations is denoted as $\mathcal{M} = \{0, 1, \ldots, M_f\}$ where 0 corresponds to the MBS. The set and the number of UEs served by the BS m is respectively denoted as $\mathcal{K}_m$ and K_m, where $m \in \mathcal{M}$. All the BSs, regardless of the tier, have access to the same frequency band which is divided into N OFDMA subchannels. The set of all subchannels is denoted as $\mathcal{N} = \{1, 2, \ldots, N\}$. In any cell, a subchannel can be assigned to at most one UE, which eliminates the intra-cell interference. However, reuse of the same subchannel in other cells creates inter-cell interference which constitutes the reason of the problem.

Transmission power of the BS m on the subchannel n is denoted as p_m^n. The power vector $p_m = [p_m^1, \ldots, p_m^N]^T$ contains the transmission powers of BS m on all the subchannels. Finally, the vector which contains the power assignment information of the entire network is defined as follows:

$$p = [p_0^T, \ldots, p_{M_f}^T]^T \tag{1}$$

$\rho_{m,k}^n$ indicates the connection status between BS m and UE k on subchannel n, where $k \in \mathcal{K}_m$. When the BS m assigns the subchannel n to the UE k, $\rho_{m,k}^n$ becomes 1. Otherwise, it is equal to 0. Then, we denote the vectors $\rho_{m,k} = [\rho_{m,k}^1, \ldots, \rho_{m,k}^N]^T$ and $\rho_m = [\rho_{m,1}^T, \ldots, \rho_{m,K_m}^T]^T$. Here, at most one element of the vector $\rho_{m,k}$ can be 1 and vector ρ_m keeps the complete subchannel assignment information of BS m. Lastly, the following vector contains all the subchannel assignment information of the network:

$$\rho = [\rho_0^T, \ldots, \rho_{M_f}^T]^T \tag{2}$$

The Signal to Interference and Noise Ratio (SINR) of the downlink signal from BS m to UE k on subchannel n, where $k \in \mathcal{K}_m$, can be defined as

$$\Gamma_{m,k}^n(p, \rho) = \rho_{m,k}^n \; \frac{g_{m,k}^n p_m^n}{\sum\limits_{j \in \mathcal{M} \setminus m} g_{j,k}^n p_j^n \; + \; \sigma_k^n} \quad , \quad k \in \mathcal{K}_m \tag{3}$$

where $\rho_{m,k}^n$ term is added as a safety multiplier to ensure that the SINR becomes 0 when the BS m does not serve UE k on the subchannel n. Also, σ_k^n term signifies the thermal noise experienced on subchannel n by UE k. Finally, $g_{m,k}^n$ term defines the channel gain between BS m and UE k on subchannel n. As mentioned above, for given BS m, a subchannel can be assigned at most one UE. Therefore, without loss of generality, we can simply eliminate the k term in $\Gamma_{m,k}^n(p, \rho)$, and let Γ_m^n denote the SINR value on subchannel n in cell m for the particular choice of p and ρ vectors, regardless of the UE using that subchannel.

$\Gamma_{m,ave}(t)$ denotes the average SINR value of the users served by the BS m. It can be calculated as

$$\Gamma_{m,ave}(t) = \frac{\sum\limits_{i \in \mathcal{U}_m} \Gamma_m^i(t)}{U_m}, \tag{4}$$

where $\mathcal{U}_m$ denotes the set of subchannels which are in use in cell m. When we turn the set $\mathcal{U}_m$ into a vector by sorting elements in ascending order, we get u_m vector which has the information of active subchannels in BS m. For example, $\mathcal{U}_2 = \{1, 4, 7\}$ and $u_2 = [1, 4, 7]^T$ respectively denote the set and vector of subchannels that are in use by BS 2. Then, we can define U_m which is the cardinality of the set $\mathcal{U}_m$, and the length of vector u_m as follows:

$$U_m = min(N, K_m). \tag{5}$$

If we patch all the u_m vectors, starting from BS 0 to BS M_f, we get vector u which shows all the active subchannels in the network:

$$u = [u_0^T, \ldots, u_{M_f}^T]^T \tag{6}$$

Then, the length of the vector u, which is equal to number of all active subchannels of the entire network, can be calculated as

$$U = \sum_{i \in \mathcal{M}} min(N, K_m). \tag{7}$$

It is also important to note that the topology of the network is assumed to be fixed during the power update process. In other words, the time needed for the convergence of the power control algorithm is too short compared to the time scale of the UE movements.

B. Problem Definition

Starting with arbitrary p and ρ vectors, we aim to provide fairness among the users of the network by adjusting the power vector p. This fairness will be provided by making all the users in the network have the same SINR value which means equal data rates for all users. Recall that Γ_m^n denotes the SINR value on subchannel n in cell m, regardless of the UE using that subchannel. Then, the vector which contains SINR values of all the UEs that receive service can be defined as

$$\widetilde{\Gamma} = [\widetilde{\Gamma}_0^T, \ldots, \widetilde{\Gamma}_{M_f}^T]^T, \tag{8}$$

where $\widetilde{\Gamma}_m$ is the vector that contains SINR values on the subchannels which are assigned to a UE by BS m. The length of vector $\widetilde{\Gamma}_m$ is equal to U_m, which can be calculated by (5). With these definitions, the problem is to find the proper power update algorithm that satisfies $\widetilde{\Gamma} \to \mathbf{1}\tilde{x}$, where $\tilde{x}$ is a scalar constant and $\mathbf{1}$ is a column vector of ones with length of U.

III. POWER UPDATE ALGORITHM

The proposed Power Update Algorithm is described as follows:

$$\dot{p}_m^n(t) = -\beta_m^n \frac{\Gamma_m^n(t)}{p_m^n(t)} \left[f_m^n(t)\Gamma_m^n(t) - \sum_{j \in \mathcal{N}_m} f_{m,j}(t)\Gamma_{j,ave}(t) - \sum_{i \in \mathcal{N} \setminus n} \tilde{f}_m^i(t)\Gamma_m^i(t) \right], n \in \mathcal{U}_m. \tag{9}$$

Here, the term β_m^n is a positive value and it determines the update speed of the transmission power of BS m, on subchannel n. Please notice that a high update constant β_m^n may help us to reach consensus more quickly, but it may also cause overshoots in transmission powers. $\mathcal{N}_m$ is the set of neighboring cells of BS m. $\Gamma_{j,ave}(t)$ term, where $j \in \mathcal{N}_m$, denotes the average SINR value of the UEs of BS j. The $f_{m,j}(t)$ terms stand for the inter–cell connection weights between the base stations. $\tilde{f}_m^i(t)$ terms are the intra–cell connection weights between the same cell's subchannels which are considered as imaginary nodes of the underlying graph. $f_m^n(t)$ is the weight that determines the importance of $\Gamma_m^n(t)$ while updating the transmission power of BS m on subchannel n. Some possible choices will be shown in Section III-A.

By using (9), BS m updates its transmission power on subchannel n in the following way: BS m collects information of the average SINR values, which is denoted as $\Gamma_{j,ave}(t)$ where $j \in \mathcal{N}_m$, coming from its neighboring BSs. By the first summation sign inside the bracket, BS m adds up these average SINR values after multiplying them by inter–cell connection weights $f_{m,j}$. Then, by the second summation sign, it adds up the SINR values of its own subchannels other than n, after multiplying them by intra–cell connection weights $\tilde{f}_m^i(t)$. These SINR values are represented by $\Gamma_m^i(t)$, where $i \in \mathcal{N} \setminus n$. Finally, these two summation results are compared with $\Gamma_m^n(t)$ to decide the change in the transmission power $p_m^n(t)$.

For better understanding of the algorithm and the theoretical analysis in Section IV, it should be stressed that BS m does not update its transmission power on n if there is no user assigned to that subchannel.

A. An Example Choice for Weights

The intra–cell connection weights $\tilde{f}_m^n(t)$ are predefined by the corresponding BS in cell m. They represent the status whether the subchannel n is in use or not. An example choice, where BS m gives equal priority to all of its UEs, can be as follows:

$$\tilde{f}_m^n(t) = \begin{cases} 1, & n \in \mathcal{U}_m, \\ 0, & \text{otherwise.} \end{cases}, \quad n \in \mathcal{N} \tag{10}$$

Here, if the subchannel n is assigned to any UE by BS m, that subchannel's intra–cell weight is taken as 1, otherwise it is 0. It should be noted that instead of 1, we could also pick any other positive number, which would change the weighted priority of the subchannel n among the other subchannels, in the cell m.

The term $f_{m,j}(t)$ denotes the weight of the inter–cell information link from BS j to BS m. An example selection of these parameters can be done by cell j according to the number subchannels that are in use. This choice is mathematically described as

$$f_{m,j}(t) = U_j, \quad j \in \mathcal{N}_m, \tag{11}$$

where U_j is defined in (5). According to this choice of parameters, each BS sends the total number of its active subchannel to the neighboring BSs. It is important to see that all the UEs in the network have the same priority when the intra–cell and inter–cell weights are chosen as (10) and (11). The relative priorities between the UEs could be changed by another choice of weight parameters.

For connection matrix $\mathbf{L}$, which is defined later in Section IV, to be a Laplacian matrix, it should have zero row sums [10]. This can be satisfied by choosing the $f_m^n(t)$ weights as follows:

$$f_m^n(t) = \sum_{i \in \mathcal{N} \setminus n} \tilde{f}_m^i(t) + \sum_{j \in \mathcal{N}_m} f_{m,j}(t) \tag{12}$$

B. Distributed Nature of the Algorithm

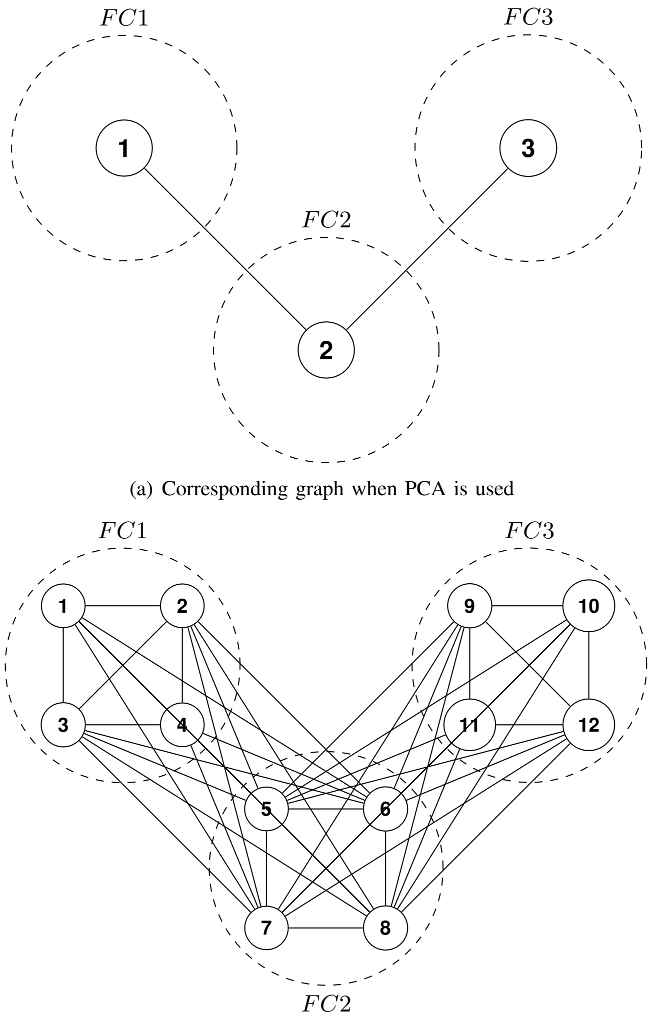
Thanks to its distributed nature, the proposed PUA can be used in complex networks where centralized solutions are not possible. Besides, required information exchange between the BSs is kept in minimum levels. In order to execute the PUA properly, BS m needs to receive the following information from its neighboring BS i, $\forall i \in \mathcal{N}_m$:

i) The number of subchannels that are in use for the given time, U_i, which is defined in (5).
ii) The average of the SINR value of the active subchannels, $\Gamma_{i,ave}(t)$, which is given in (4).

Rest of the information that BS m requires is the SINR values on all the subchannels, which can easily come from its UEs.

As mentioned above, the PUA is inspired from [9] where each cell is taken as a node in the inter–cell communication graph. The novelty of the PUA is that it expands the underlying graph by U_m times for each cell m. Difference between the two algorithms is shown by an illustrative example in Fig. 1. When the network consists of 3 femto BSs and $U_m = 4$ served UEs for each cell, Fig. 1(a) shows the underlaying communication graph of the network when the PCA of [9] is used. When we use the proposed PUA given in (9), each node turns into $N = 4$ nodes and the corresponding graph expands as shown in Fig. 1(b).

While changing the transmission power on subchannel n, BS m takes the SINR values on subchannel $i \in \mathcal{N} \setminus n$ into account. This is the major difference between our proposed algorithm (9) and the one given in [9] which only considers the information coming from the neighboring base stations.



(a) Corresponding graph when PCA is used

(b) The expanded graph when PUA is used

Fig. 1. Comparison of the corresponding graphs of PCA and PUA

In order to update the transmission power on one subchannel, BS m needs to use the PUA (9) exclusively for that subchannel. Since there are U_m UEs that receives service from BS m, it should execute of the PUA U_m times in parallel.

IV. THEORETICAL ANALYSIS

The convergence analysis of the proposed algorithm is carried out by using the weights given in (10), (11), and (12). By substituting the equations (4)-(5)-(12) into (9), we can rewrite the PUA as follows:

$$\dot{p}_m^n(t) = -\beta_m^n \frac{\Gamma_m^n(t)}{p_m^n(t)} \left[\left(U_m - 1 + \sum_{j \in \mathcal{N}_m} U_j \right) \Gamma_m^n(t) - \sum_{j \in \mathcal{N}_m} \sum_{i \in \mathcal{U}_j} \Gamma_j^i(t) - \sum_{i \in \mathcal{U}_m \setminus n} \Gamma_m^i(t) \right], n \in \mathcal{U}_m. \tag{13}$$

Notice that we consider only the subchannels that are assigned to a UE, $n \in \mathcal{U}_m$, $\forall m \in \mathcal{M}$. Then, (13) can be written in vector notation as

$$\dot{\widetilde{p}}(t) = -\mathbf{B}\mathbf{I_d}^{-1}\mathbf{L}\widetilde{\Gamma}(t), \tag{14}$$

where $\widetilde{p}(t)$ and $\widetilde{\Gamma}(t)$ are the reduced version of the vectors $p(t)$ and $\Gamma(t)$, respectively. While the $p(t)$ vector contains N power values for each cell, $\widetilde{p}(t)$ vector contains only the power values of the subchannels that are in use. So, the lengths of $p(t)$ and $\widetilde{p}(t)$ vectors are $N(M_f + 1)$ and U, respectively. In the case of $U_i = N \quad \forall i \in \mathcal{M}$, they would have the same length. By the same logic, $\widetilde{\Gamma}(t)$ vector contains only the power values of subchannels which are in use, and its length is equal to U, as well. $\mathbf{I}_d = \text{diag}(I_1, \ldots, I_U)$ is the diagonal matrix with normalized interference values of the active subchannels, where $I_i = p_i/\gamma_i$. $\mathbf{B} = \text{diag}(\beta_1, \ldots, \beta_U)$ is the diagonal matrix created with update constants. For given $\mathcal{N}$, $\mathcal{M}$, and u_m, where $m \in \mathcal{M}$, the index i of the I_i and β_i corresponds a unique BS–subchannel pair. So, we can define a mapping rule as $i \rightarrow (m, n)$. Finally, $\mathbf{L} = [l_{i,j}]$ is the Laplacian matrix which contains the connection information of the underlying graph. Also, the same $i \rightarrow (m, n)$ mapping rule is valid for the i and j indices of $l_{i,j}$ terms. By the weight choices (10), (11), and (12), the connection matrix $\mathbf{L} = [l_{i,j}]$ satisfies the following conditions to be the graph Laplacian of the network:

i) All row-sums of $\mathbf{L}$ are zero: $\sum_{j=1}^{U} l_{i,j} = 0, \forall i \in \{1, 2, \ldots, U\}$.
ii) $\mathbf{L}$ is a symmetric matrix: $l_{i,j} = l_{j,i}, \forall (i, j) \in \{1, 2, \ldots, U\}$.

Let o be the index of the element n in the vector of u_m. Then, for given $\mathcal{N}$, $\mathcal{M}$, and u_m, where $m \in \mathcal{M}$, there exists a unique (m, n) pair for each i satisfying the following equation:

$$i = \sum_{l=0}^{m-1} U_l \ + \ o \tag{15}$$

In order to clarify the mapping rule $i \rightarrow (m, n)$, let's have a look at the example system defined with $N = 3$, $\mathcal{M} = \{0, 1, 2\}$, $u_0 = [1, 3]$, $u_1 = [2]$, $u_2 = [2, 3]$. Then, the normalized interference matrix $\mathbf{I}_d$ would be written as

$$\mathbf{I}_d = \begin{bmatrix} I_1 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 \\ 0 & 0 & I_3 & 0 & 0 \\ 0 & 0 & 0 & I_4 & 0 \\ 0 & 0 & 0 & 0 & I_5 \end{bmatrix} = \begin{bmatrix} \frac{p_0^1}{\gamma_0^1} & 0 & 0 & 0 & 0 \\ 0 & \frac{p_0^3}{\gamma_0^3} & 0 & 0 & 0 \\ 0 & 0 & \frac{p_1^2}{\gamma_1^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{p_2^2}{\gamma_2^2} & 0 \\ 0 & 0 & 0 & 0 & \frac{p_2^3}{\gamma_2^3} \end{bmatrix}. \tag{16}$$

Let the (m_i, n_i) and (m_j, n_j) be the corresponding pairs of i and j indices in the connection matrix $\mathbf{L} = [l_{i,j}]$. Then, $l_{i,j}$ can be defined as

$$l_{i,j} = \begin{cases} -1, & i \neq j, m_i = m_j, \\ -1, & i \neq j, m_i \neq m_j, m_j \in \mathcal{N}_{m_i}, \\ 0, & i \neq j, m_i \neq m_j, m_j \notin \mathcal{N}_{m_i}, \\ (U_{m_i} - 1) + \sum_{c \in \mathcal{N}_{m_i}} U_c, & i = j. \end{cases} \tag{17}$$

A. Stability Analysis

In (13), the diagonal entries of $\mathbf{B}$ and ${\mathbf{I_d}}^{-1}$ matrices are positive. Therefore, the only way to have $\dot{\widetilde{p}}(t) = 0$ is having $\mathbf{L}\widetilde{\Gamma}(t) = 0$. Since we have chosen the $f_m^n(t)$ weights by (12), sum of each row in connection matrix $\mathbf{L}$ is 0. That makes 0 a simple eigenvalue of $\mathbf{L}$. Thus, the $\widetilde{\Gamma}^*(t) = \mathbf{1}\tilde{x}$ is the solution of $\mathbf{L}\widetilde{\Gamma}(t) = 0$, where $\tilde{x}$ is a scalar constant and $\mathbf{1}$ is a column vector of ones with length of U. In other words, the change in power vector $\widetilde{p}(t)$ becomes zero when all the UEs have the same SINR value. However, the existence of this equilibrium state does not ensure its stability. For the stability of $\widetilde{\Gamma}^*(t)$, we get the following result:

Theorem 4.1: Suppose that the network is associated with a connected graph at any given time. Then, the proposed algorithm that uses the weights (10), (11), and (12) converges to a fair solution $\widetilde{\Gamma}^*(t)$ where $\widetilde{\Gamma}_i = \widetilde{\Gamma}_j \ \ \forall i, j \in \{1, 2, \ldots, U\}$.

Proof: Let's define a candidate Lyapunov function as

$$V(\widetilde{p}(t)) = \widetilde{p}(t)^T \mathbf{B}^{-1} \widetilde{p}(t) = \sum_{i=1}^{U} \frac{1}{\beta_i} \widetilde{p}_i(t)^2. \qquad (18)$$

Since $V(\widetilde{p}(t))$ is sum of squares, it is globally positive definite. For the isolated equilibrium points $\widetilde{\Gamma}^*(t)$ to be stable, we need to show that $\dot{V}(\widetilde{p}(t))$ is negative semi-definite. The derivative of $V(\widetilde{p}(t))$ can be calculated as follows:

$$\begin{aligned} \dot{V}(\widetilde{p}(t)) &= \widetilde{p}(t)^T \mathbf{B}^{-1} \dot{\widetilde{p}}(t) + \dot{\widetilde{p}}(t)^T \mathbf{B}^{-1} \widetilde{p}(t) \\ &= -2\widetilde{p}(t)^T {\mathbf{I_d}}^{-1} \mathbf{L}\widetilde{\Gamma}(t) \\ &= -2\widetilde{\Gamma}(t)^T \mathbf{L}\widetilde{\Gamma}(t) \end{aligned}$$

Since weight choice is made by (12), and the network is associated with a connected graph with undirected edges, $\mathbf{L}$ has the simple eigenvalue of 0 and its other eigenvalues are positive. Therefore, $\dot{V}(\widetilde{p}(t))$ takes negative values for all $\widetilde{\Gamma}(t)$ except $\widetilde{\Gamma}^*(t)$. It becomes 0 only when $\widetilde{\Gamma}^*(t)$. In other words, the algorithm converges to a point where all the UEs have the same SINR value. ■

Remark 1: Theorem 4.1 requires that the associated network graph is connected. Research is currently underway to relax this condition to partially connected networks.

V. SIMULATIONS

Performance of the proposed algorithm (9) is tested by simulations. The topology used for simulations is shown in Fig. 2 which consists of a single MBS and 4 FBSs. Each femtocell has 6 UEs while the macrocell has 9 UEs. The relative positions of UEs and BSs are also shown in Fig. 2. There are $N = 8$ OFDMA subchannels available for each cell. Each element of the power vector $\mathbf{P}$, given in (1), is initially chosen randomly from the intervals of $[\frac{p_{f,mask}}{2}, p_{f,mask}]$ and $[\frac{p_{m,mask}}{2}, p_{m,mask}]$ for FUEs and MUEs, respectively. Here, $p_{f,mask}$ and $p_{m,mask}$ are the spectral mask values which define the maximum value of signal power on a single subchannel of FBSs and MBS, respectively. The received noise power is the same for all UEs and on each subchannel and it is taken as $\sigma = 1,8 \cdot 10^{-12,4} mW$. The channel gains are calculated by using $g_{m,k}^n = \chi^{(n)} d_{m,k}^{-\eta}$ equation, where $d_{m,k}$ is the physical distance between BS m and UE k. η term signifies the pathloss constant, and it is taken as $\eta = 3$. The $\chi^{(n)}$ term is chosen randomly, by using Rayleigh distribution, for each subchannel as [1.098 1.201 1.129 1.122 1.199 1.161 1.174 1.123]. Finally, in the simulations, the update constant β_m^n in (9) is taken as $\beta_m^n = 0.03, \ \forall m, n$.

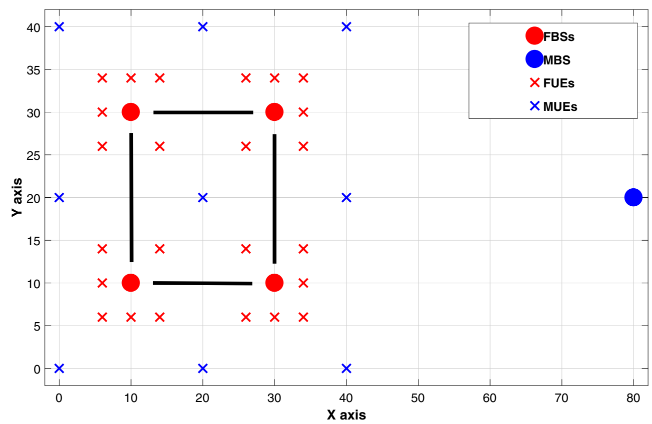


Fig. 2. Simulation topology with edges.

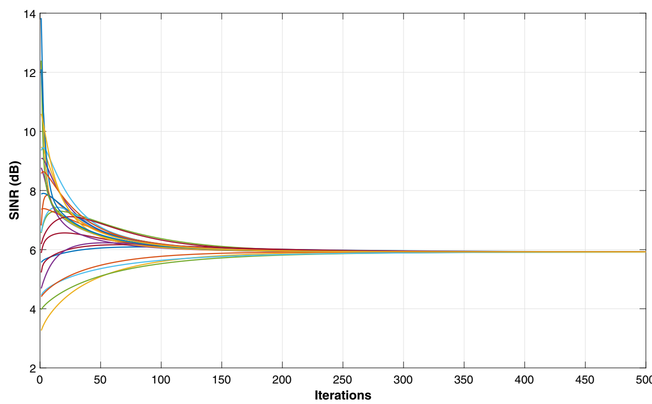


Fig. 3. Convergence of the SINR values of the FEUs.

The black lines shown in Fig. 2 represent the communication links between the BSs. It is assumed that all these links can provide two–way connections continuously. In other words, if BS i receives information from BS j, then BS j should be receiving information from BS i. Therefore, the corresponding graph is undirected and edges have no arrows. In the simulation topology, the MBS is located away from the nearest FBS's signal reach. Thus, it is assumed that the MBS is isolated from the FBSs in terms of sharing information.

Since $N = 8$ and all femtocells have 6 UEs, there are 2 unused subchannels in each femtocell. Similarly, one of the MUEs is left unserved in this scenario. Also, the initial choice for the subchannel assignment vector ρ, given in (2), is made randomly. Then, throughout the power update iterations, ρ is always kept unchanged.

Simulation results are given in Fig. 3. At the end of

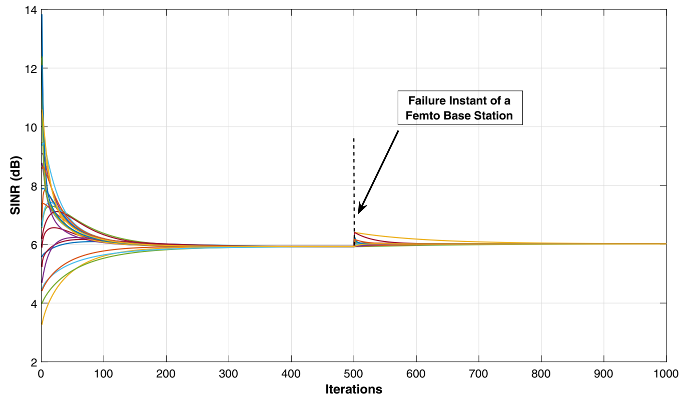


Fig. 4. Behavior of the network when a FBS fails to work.

power update process, it is shown that all 24 FUEs reach the consensus on a common SINR value. Since the MBS is not a part of the underlaying communication graph of the topology given in Fig. 2, SINR values of the MUEs are not expected to reach the consensus with the SINR values of FUEs. Therefore, they are not shown in the figure.

In real life, FBSs are quite likely to be turned off since they are deployed by the will of end users. Also, they can easily fail to work for many reasons. We expect our power update algorithm to restore the SINR fairness in such scenarios. The simulation for such situations is carried out by removing the FBS on the lower left–hand side of Fig. 2, instantly. Results are shown in Fig. 4. After the failure, the remaining FUEs reach consensus on their SINR values.

VI. CONCLUSIONS

This paper proposes a consensus based power update algorithm for the OFDMA–based femtocell networks. After explaining the proper usage of the algorithm, in detail, we mathematically show that the only equilibrium points of the system are located where all the users have the same SINR value. Furthermore, this results are supported by the simulations where the network has achieved restoring the fairness even after the failure of a base station.

The main contribution of this paper is the global consensus ability of the proposed algorithm. All UEs in the network where BSs can have more than one user, whether they use the same OFDMA subchannel or not, can reach consensus on the same SINR value. There are some algorithms that ensure the consensus in systems where each BS have just one UE. However, as far as we know, no other algorithm in the literature can achieve the same.

Because of the complicated nature of the problem, theoretical analysis of the PUA is carried out by using the example weights given in Section III-A. For the future work, more general implementations of the algorithm will be investigated. Also, we plan to integrate the PUA into a joint frequency/power update algorithm.

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