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Decentralized Control of Uncertain Multi-Agent Systems with Connectivity Maintenance and Collision Avoidance

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Abstract— **This paper addresses the problem of navigation control of a general class of uncertain nonlinear multi-agent systems in a bounded workspace, which is subset of $\mathbb{R}^n$, with static obstacles. In particular, we propose a decentralized control protocol such that each agent reaches a predefined position at the workspace, while using only local information based on a limited sensing radius. The proposed scheme guarantees that the initially connected agents remain always connected. In addition, by introducing certain distance constraints, we guarantee inter-agent collision avoidance, as well as, collision avoidance with the obstacles and the boundary of the workspace. The proposed controllers employ a class of Decentralized Nonlinear Model Predictive Controllers (DNMPC) under the presence of disturbances and uncertainties. Finally, simulation results verify the validity of the proposed framework.**

I. Introduction

During the last decades, *decentralized control of multi-agent systems* has gained a significant amount of attention due to the great variety of its applications, including multi-robot systems, transportation, multi-point surveillance and biological systems. An important topic of research is *multi-agent navigation* in both the robotics and the control communities, due to the need for autonomous control of multiple robotic agents in the same workspace.

The literature on the problem of navigation of multi-agent systems is rich. In [1] and [2], a decentralized control protocol of multiple non-point agents and point masses with collision avoidance guarantees is considered, respectively. The problem is approached by designing navigation functions which have been initially introduced in [3]. However, this method requires preposterously large actuation forces and it may give rise to numerical instability due to computations of exponentials and derivatives. A decentralized potential field approach of navigation of multiple unicycles and aerial vehicles with collision avoidance has been considered in [4] and [5], respectively; Robustness analysis and saturation in control inputs are not addressed. In [6], the collision avoidance problem for multiple agents in intersections has been studied. An optimal control problem is solved, with only time and energy constraints. Authors in [7] proposed decentralized controllers for multi-agent navigation and collision avoidance with arbitrarily shaped obstacles in 2D environments. blue-However, connectivity maintenance properties are not taken into consideration in all the aforementioned works.

In [8], a decentralized receding horizon protocol for formation control of linear multi-agent systems is proposed. The authors in [9] considered the path-following problems for multiple Unmanned Aerial Vehicles (UAVs) in which a decentralized optimization method is proposed through linearization of the dynamics of the UAVs. A DNMPC along with potential functions for collision avoidance has been studied in [10]. A feedback linearization framework along with Model Predictive Controllers (MPC) for multiple unicycles in leader-follower networks for ensuring collision avoidance and formation is introduced in [11]. The authors of [12]–[14] proposed a decentralized receding horizon approach for discrete time multi-agent cooperative control. However, in the aforementioned works, plant-model mismatch or uncertainties and/or connectivity maintenance are not considered. In [15] a centralized and a decentralized linear MPC formulation and integer programming is proposed, respectively, for dealing with collision avoidance of multiple UAVs.

The contribution of this paper is to provide *decentralized* control protocols which guarantee that a team of rigid-bodies modeled by 2nd order *uncertain* Lagrangian dynamics satisfy: collision avoidance between agents; obstacle avoidance; connectivity preservation; singularity avoidance; that agents remain in the workspace; while the control inputs are saturated. This constitutes a general problem that arises in many multi-agent applications where the agents need to perform a collaborative task, stay close and connected to each other and navigate to desired goal points. In order to address the aforementioned problem, we propose a Decentralized Nonlinear Model Predictive Control (DNMPC) framework in which each agent solves its own optimal control problem, having availability of information on the current and estimated actions of all agents within its sensing range. The proposed control scheme, under relatively standard Nonlinear Model Predictive Control (NMPC) assumptions, guarantees that all the aforementioned control specifications are satisfied. Due to space constraints, a more detailed version of this paper that contains: omitted definitions, remarks, extra figures, proofs of lemmas/theorems, and omitted calculations, can be found in [16].

II. Notation and Preliminaries

The set of positive integers is denoted by $\mathbb{N}$. The real n-coordinate space, $n \in \mathbb{N}$, is denoted by $\mathbb{R}^n$; $\mathbb{R}^n_{\geq 0}$ and $\mathbb{R}^n_{>0}$ are the sets of real n-vectors with all elements nonnegative and positive, respectively. The notation $\|x\|$ is used for the Euclidean norm of a vector $x \in \mathbb{R}^n$. Given a real symmetric matrix A, $\lambda_{\max}(A)$ denotes the maximum absolute value of eigenvalues of A. Its maximum singular value is denoted by $\sigma_{\max}(A)$; $I_n \in \mathbb{R}^{n \times n}$ and $0_{m \times n} \in \mathbb{R}^{m \times n}$ are the identity matrix and the $m \times n$ matrix with all entries zeros,

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respectively. The set-valued function $\mathcal{B} : \mathbb{R}^n \times \mathbb{R}_{>0} \rightrightarrows \mathbb{R}^n$, given as $\mathcal{B}(x,r) = \{y \in \mathbb{R}^n : \|y - x\| \leq r\}$, represents the n-th dimensional ball with center $x \in \mathbb{R}^n$ and radius $r \in \mathbb{R}_{>0}$. Given the sets $\mathcal{S}_1$, $\mathcal{S}_2 \subseteq \mathbb{R}^n$, the *Minkowski addition* and the *Pontryagin difference* are defined by: $\mathcal{S}_1 \oplus \mathcal{S}_2 = \{s_1 + s_2 \in \mathbb{R}^n : s_1 \in \mathcal{S}_1, s_2 \in \mathcal{S}_2\}$ and $\mathcal{S}_1 \ominus \mathcal{S}_2 = \{s_1 \in \mathbb{R}^n : s_1 + s_2 \in \mathcal{S}_1, \forall s_2 \in \mathcal{S}_2\}$, respectively. For the definitions of Class $\mathcal{K}$, Class $\mathcal{KL}$ functions, Input-to-State Stability (ISS Stability), ISS Lyapunov Function and positively invariant sets, which will be used thereafter in this manuscript, we refer the reader to [17], [18].

III. Problem Formulation

A. System Model

Consider a set $\mathcal{V}$ of N agents, $\mathcal{V} = \{1, 2, \dots, N\}$, $N \geq 2$, operating in a workspace $\mathcal{D} \subseteq \mathbb{R}^n$. The workspace is assumed to be modeled by a bounded ball $\mathcal{B}(x_{\mathcal{D}}, r_{\mathcal{D}})$, where $x_{\mathcal{D}} \in \mathbb{R}^n$ and $r_{\mathcal{D}} \in \mathbb{R}_{>0}$ are its center and radius, respectively. We consider that over time t each agent $i \in \mathcal{V}$ occupies the ball $\mathcal{B}(x_i(t), r_i)$, where $x_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is the position of the agent at time $t \in \mathbb{R}_{\geq 0}$, and $r_i < r_{\mathcal{D}}$ is the radius of the agent's rigid body. The *uncertain nonlinear dynamics* of each agent $i \in \mathcal{V}$ are given by:

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + w_i(x_i(t), t), \tag{1}$$

where $u_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ stands for the control input and $f_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a twice continuously differentiable vector field satisfying $f_i(0_{n\times 1}, 0_{m\times 1}) = 0_{n\times 1}$. The continuous function $w_i : \mathbb{R}^n \times \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ is a term representing *disturbances* and *modeling uncertainties*. We consider bounded inputs and disturbances as $u_i \in \mathcal{U}_i$ and $w_i \in \mathcal{W}_i$, where $\mathcal{U}_i = \{u_i \in \mathbb{R}^m : \|u_i\| \leq \widetilde{u}_i\}$ and $\mathcal{W}_i = \{w_i \in \mathbb{R}^n : \|w_i\| \leq \widetilde{w}_i\}$, for given finite constants $\widetilde{u}_i$, $\widetilde{w}_i \in \mathbb{R}_{>0}$, $i \in \mathcal{V}$.

Assumption 1. The nonlinear functions f_i, $i \in \mathcal{V}$ are *locally Lipschitz continuous* in $\mathcal{D} \times \mathcal{U}_i$ with Lipschitz constants L_{f_i}. Thus, for every $x, x' \in \mathbb{R}^n$ and $u \in \mathcal{U}_i$ it holds that: $\|f_i(x,u) - f_i(x',u)\| \leq L_{f_i}\|x - x'\|$.

We consider that in the given workspace there exist $L \in \mathbb{N}$ *static obstacles*, with $\mathcal{L} = \{1, 2, \dots, L\}$, also modeled by the balls $\mathcal{B}(x_{O_\ell}, r_{O_\ell})$, with centers at positions $x_{O_\ell} \in \mathbb{R}^n$ and radii $r_{O_\ell} \in \mathbb{R}_{>0}$, where $\ell \in \mathcal{L}$. Their positions and sizes are assumed to be known a priori to each agent.

Assumption 2. Agent $i \in \mathcal{V}$ has: 1) access to measurements $x_i(t)$ for every $t \in \mathbb{R}_{\geq 0}$; 2) A limited sensing range $d_i \in \mathbb{R}_{>0}$ such that: $d_i > \max\limits_{i,j\in\mathcal{V}, i\neq j, \ell\in\mathcal{L}} \{r_i + r_j, r_i + r_{O_\ell}\}$.

The latter implies that each agent is capable of perceiving the agent and the obstacle with the largest volume. The consequence of points 1 and 2 of Assumption 2 is that by defining the set of agents j that are within the sensing range of agent i at time t as: $\mathcal{R}_i(t) \triangleq \{j \in \mathcal{V}\backslash\{i\} : \|x_i(t) - x_j(t)\| < d_i\}$, agent i is also able to measure at each time instant t the vectors $x_j(t)$ of all agents $j \in \mathcal{R}_i(t)$.

Definition 1. The multi-agent system is in a *collision/singularity-free configuration* at a time instant $\tau \in \mathbb{R}_{\geq 0}$ if the following hold: for every i, $j \in \mathcal{V}$, $i \neq j$ it holds that: $\|x_i(\tau) - x_j(\tau)\| > r_i + r_j$; for every $i \in \mathcal{V}$ and for every $\ell \in \mathcal{L}$ it holds that: $\|x_i(\tau) - x_{O_\ell}\| > r_i + r_{O_\ell}$ and for every $i \in \mathcal{V}$ it holds that: $\|x_{\mathcal{D}} - x_i(\tau)\| < r_{\mathcal{D}} - r_i$.

Definition 2. The *neighboring set* of agent $i \in \mathcal{V}$ is defined by: $\mathcal{N}_i = \{j \in \mathcal{V}\backslash\{i\} : j \in \mathcal{R}_i(0)\}$. We will refer to agents $j \in \mathcal{N}_i$ as the *neighbors* of agent $i \in \mathcal{V}$.

The set $\mathcal{N}_i$ is composed of indices of agents $j \in \mathcal{V}$ which are within the sensing range of agent i at time $t = 0$. Agents $j \in \mathcal{N}_i$ are agents which agent i is instructed to keep within its sensing range at all times $t \in \mathbb{R}_{>0}$, and therefore maintain connectivity with them. While the sets $\mathcal{N}_i$ are introduced for connectivity maintenance specifications and they are fixed, the sets $\mathcal{R}_i(t)$ are used to ensure collision avoidance, and, in general, their composition varies through time.

Assumption 3. For sake of cooperation needs, we assume that $\mathcal{N}_i \neq \emptyset$, $\forall i \in \mathcal{V}$, i.e., all agents have at least one neighbor. We also assume that at time $t = 0$ the multi-agent system is in a *collision/singularity-free configuration*, as given in Definition 1.

Given the aforementioned modeling of the system, the objective of this paper is the *stabilization of the agents* $i \in \mathcal{V}$ starting from a collision/singularity-free configuration as given in Definition 1 to a desired configuration $x_{i,\text{des}} \in \mathbb{R}^n$, while maintaining connectivity between neighboring agents, and avoiding collisions between agents, obstacles, and the workspace boundary.

Definition 3. The desired configuration $x_{i,\text{des}} \in \mathcal{D}$ of agent $i \in \mathcal{V}$ is *feasible* if the following hold: 1) It is a collision/singularity-free configuration according to Definition 1; 2) It does not result in a violation of the connectivity maintenance constraint between neighboring agents, i.e., $\|x_{i,\text{des}} - x_{j,\text{des}}\| < d_i$, $\forall i \in \mathcal{V}, j \in \mathcal{N}_i$.

Definition 4. Let $x_{i,\text{des}} \in \mathcal{D}$, $i \in \mathcal{V}$ be a desired feasible configuration as given in Definition 3. Then, the set of all initial conditions $x_i(0)$ according to Assumption 3, for which there exist time constants $\bar{t}_i \in \mathbb{R}_{>0} \cup \{\infty\}$ and control inputs $u_i^\star \in \mathcal{U}_i$, $i \in \mathcal{V}$, which define a solution $x_i^\star(t)$, $t \in [0, \bar{t}_i]$ of the system (1), under the presence of disturbance $w_i \in \mathcal{W}_i$, such that: 1) $x_i^\star(\bar{t}_i) = x_{i,\text{des}}$; 2) $\|x_i^\star(t) - x_j^\star(t)\| > r_i + r_j$, for every $t \in [0, \bar{t}_i]$, $i, j \in \mathcal{V}$, $i \neq j$; 3) $\|x_i^\star(t) - x_{O_\ell}\| > r_i + r_{O_\ell}$, for every $t \in [0, \bar{t}_i]$, $i \in \mathcal{V}$, $\ell \in \mathcal{L}$; 4) $\|x_{\mathcal{D}} - x_i^\star(t)\| < r_{\mathcal{D}} - r_i$, for every $t \in [0, \bar{t}_i]$, $i \in \mathcal{V}$; 5) $\|x_i^\star(t) - x_j^\star(t)\| < d_i$, for every $t \in [0, \bar{t}_i]$, $i \in \mathcal{V}, j \in \mathcal{N}_i$, are called *feasible initial conditions*.

The feasible initial conditions are essentially all the initial conditions $x_i(0) \in \mathcal{D}$, $i \in \mathcal{V}$ from which there exist controllers $u_i \in \mathcal{U}_i$ that can navigate the agents to the given desired states $x_{i,\text{des}}$, under the presence of disturbances $w_i \in \mathcal{W}_i$ while the initial neighbors remain connected, the agents do not collide with each other, they stay in the workspace and they do not collide with the obstacles of the environment. Initial conditions for which one or more agents can not be driven to the desired state $x_{i,\text{des}}$ by a controller $u_i \in \mathcal{U}_i$, i.e., initial conditions that violate one or more of the conditions of Definition 4, are considered as *infeasible* *initial conditions*.

Problem 1. Consider N agents governed by dynamics as in (1), modeled by the balls $\mathcal{B}(x_i, r_i)$, $i \in \mathcal{V}$, operating in a workspace $\mathcal{D}$ which is modeled by the ball $\mathcal{B}(x_{\mathcal{D}}, r_{\mathcal{D}})$. In the workspace there are L obstacles $\mathcal{B}\left(x_{O_\ell}, r_{O_\ell}\right)$, $\ell \in \mathcal{L}$. The agents have communication capabilities according to Assumption 2, under the initial conditions $x_i(0)$, imposed by Assumption 3. Then, given a desired feasible configuration $x_{i,\text{des}}$ according to Definition 3, for all feasible initial conditions, as given in Definition 4, the problem lies in designing *decentralized feedback control* laws $u_i \in \mathcal{U}_i$, such that for every $i \in \mathcal{V}$ and for all times $t \in \mathbb{R}_{\geq 0}$, the following specifications are satisfied: 1) position stabilization is achieved: $\lim\limits_{t\to\infty} \|x_i(t) - x_{i,\text{des}}\| \to 0$; 2) inter-agent collision avoidance: $\|x_i(t) - x_j(t)\| > r_i + r_j, \forall\, j \in \mathcal{V}\backslash\{i\}$; 3) connectivity maintenance between neighboring agents is preserved: $\|x_i(t) - x_j(t)\| < d_i$, $\forall\ j \in \mathcal{N}_i$; 4) agent-with-obstacle collision avoidance: $\|x_i(t) - x_{O_\ell}(t)\| > r_i + r_{O_\ell}$, $\forall\ \ell \in \mathcal{L}$; 5) agent-with-workspace-boundary collision avoidance: $\|x_{\mathcal{D}} - x_i(t)\| < r_{\mathcal{D}} - r_i$.

IV. Main Results

In this section, a systematic solution to Problem 1 is introduced. Our overall approach builds on designing a decentralized control law $u_i \in \mathcal{U}_i$ for each agent $i \in \mathcal{V}$. In particular, since we aim to minimize the norms $\|x_i(t) - x_{i,\text{des}}\|$, as $t \to \infty$ subject to the state constraints imposed by Problem 1, it is reasonable to seek a solution which is the outcome of an optimization problem.

A. Error Dynamics and Constraints

Define the error vector $e_i : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$ by: $e_i(t) = x_i(t) - x_{i,\text{des}}$. Then, the *error dynamics* are given by:

$$\dot{e}_i(t) = h_i(e_i(t), u_i(t)), \tag{2}$$

where the functions $h_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$, $g_i : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ are defined by: $h_i(e_i(t), u_i(t)) \triangleq g_i(e_i(t), u_i(t)) + w_i(e_i(t) + x_{i,\text{des}}, t)$, $g_i(e_i(t), u_i(t)) \triangleq f_i(e_i(t) + x_{i,\text{des}}, u_i(t))$. Define the set that captures all the *state* constraints on the system (1), posed by Problem 1, by: $\mathcal{Z}_i \triangleq \Big\{x_i \in \mathbb{R}^n : \|x_i(t) - x_j(t)\| \geq r_i + r_j + \varepsilon$, $\forall j \in \mathcal{R}_i(t)$, $\|x_i(t) - x_j(t)\| \leq d_i - \varepsilon$, $\forall j \in \mathcal{N}_i$, $\|x_i(t) - x_{O_\ell}\| \geq r_i + r_{O_\ell} + \varepsilon$, $\forall \ell \in \mathcal{L}$, $\|x_{\mathcal{D}} - x_i(t)\| \leq r_{\mathcal{D}} - r_i - \varepsilon\Big\}$, $i \in \mathcal{V}$, where $\varepsilon \in \mathbb{R}_{>0}$ is an arbitrary small constant. In order to translate the constraints that are dictated for the state z_i into constraints regarding the error state e_i, define the set $\mathcal{E}_i = \left\{e_i \in \mathbb{R}^n : e_i \in \mathcal{Z}_i \oplus (-x_{i,\text{des}})\right\}, i \in \mathcal{V}$. Then, the following equivalence holds: $x_i \in \mathcal{Z}_i \Leftrightarrow e_i \in \mathcal{E}_i$, $\forall i \in \mathcal{V}$.

Property 1. The nonlinear functions g_i, $i \in \mathcal{V}$ are *locally Lipschitz continuous* in $\mathcal{E}_i \times \mathcal{U}_i$ with Lipschitz constants $L_{g_i} = L_{f_i}$. Thus, $\|g_i(e, u) - g_i(e', u)\| \leq L_{g_i}\|e - e'\|$, $\forall e$, $e' \in \mathcal{E}_i$, $u \in \mathcal{U}_i$.

Proof. The proof can be found in [16, App. B, p. 21]. ☐

If the decentralized control laws $u_i \in \mathcal{U}_i$, $i \in \mathcal{V}$, are designed such that the error signal e_i with dynamics given in (2), constrained under $e_i \in \mathcal{E}_i$, satisfies $\lim\limits_{t\to\infty} \|e_i(t)\| \to 0$, then Problem 1 will have been solved.

B. Decentralized Control Design

Due to the fact that we have to deal with the minimization of norms $\|e_i(t)\|$, as $t \to \infty$, subject to constraints $e_i \in \mathcal{E}_i$, we invoke here a class of Nonlinear Model Predictive controllers. NMPC frameworks have been studied in [19]–[25] and they have been proven to be powerful tools for dealing with state and input constraints.

Consider a sequence of sampling times $\{t_k\}$, $k \in \mathbb{N}$, with a constant sampling time h, $0 < h < T_p$, where T_p is the prediction horizon, such that $t_{k+1} = t_k + h$, $k \in \mathbb{N}$. Hereafter we will denote by i the agent and by index k the sampling instant. In sampled data NMPC, a Finite-Horizon Open-loop Optimal Control Problem (FHOCP) is solved at the discrete sampling time instants t_k based on the current state error measurement $e_i(t_k)$. The solution is an optimal control signal $\overline{u}_i^\star(s)$, computed over $s \in [t_k, t_k + T_p]$. The open-loop input signal applied in between the sampling instants is given by the solution of the following FHOCP:

$$\begin{aligned}&\min_{\overline{u}_i(\cdot)} J_i(e_i(t_k), \overline{u}_i(\cdot))\\ &= \min_{\overline{u}_i(\cdot)} \left\{ V_i(\overline{e}_i(t_k + T_p)) + \int_{t_k}^{t_k + T_p} \Big[F_i(\overline{e}_i(s), \overline{u}_i(s))\Big] ds \right\}\end{aligned} \tag{3a}$$

subject to:

$$\dot{\overline{e}}_i(s) = g_i(\overline{e}_i(s), \overline{u}_i(s)), \overline{e}_i(t_k) = e_i(t_k), \tag{3b}$$

$$\overline{e}_i(s) \in \mathcal{E}_{i,s-t_k}, \overline{u}_i(s) \in \mathcal{U}_i, s \in [t_k, t_k + T_p], \tag{3c}$$

$$\overline{e}(t_k + T_p) \in \Omega_i. \tag{3d}$$

At a generic time t_k then, agent $i \in \mathcal{V}$ solves the aforementioned FHOCP. The notation $\bar{\cdot}$ is used to distinguish predicted states which are internal to the controller, corresponding to the nominal system (3b) (i.e., the system (2) by substituting $w(\cdot) = 0_{n\times 1}$). This means that $\overline{e}_i(\cdot)$ is the solution to (3b) driven by the control input $\overline{u}_i(\cdot) : [t_k, t_k + T_p] \to \mathcal{U}_i$ with initial condition $e_i(t_k)$. Note that the predicted states are not the same with the actual closed-loop values due to the fact that the system is under the presence of disturbances $w_i \in \mathcal{W}_i$. The functions $F_i : \mathcal{E}_i \times \mathcal{U}_i \to \mathbb{R}_{\geq 0}$, $V_i : \mathcal{E}_i \to \mathbb{R}_{\geq 0}$ stand for the *running costs* and the *terminal penalty costs*, respectively, and they are defined by: $F_i\big(e_i, u_i\big) = e_i^\top Q_i e_i + u_i^\top R_i u_i$, $V_i\big(e_i\big) = e_i^\top P_i e_i$; $R_i \in \mathbb{R}^{m\times m}$ and Q_i, $P_i \in \mathbb{R}^{n\times n}$ are symmetric and positive definite controller gain matrices to be appropriately tuned; $Q_i \in \mathbb{R}^{n\times n}$ is a symmetric and positive semi-definite controller gain matrix to be appropriately tuned. The sets $\mathcal{E}_{i,s-t_k}$, Ω_i will be explained later. For the running costs F_i the following hold:

Lemma 1. *There exist functions* α_1, $\alpha_2 \in \mathcal{K}_\infty$ *such that:* $\alpha_1\big(\|\eta_i\|\big) \leq F_i\big(e_i, u_i\big) \leq \alpha_2\big(\|\eta_i\|\big), i \in \mathcal{V}$, *for every* $\eta_i \triangleq \left[e_i^\top, u_i^\top\right]^\top \in \mathcal{E}_i \times \mathcal{U}_i$.

Proof. The proof can be found in [16, App. C, p. 22]. ☐

Lemma 2. *The running costs* F_i, $i \in \mathcal{V}$ *are Lipschitz continuous in* $\mathcal{E}_i \times \mathcal{U}_i$. *Thus, it holds that:* $\big|F_i(e, u) - F_i(e', u)\big| \leq L_{F_i}\|e - e'\|, \forall e, e' \in \mathcal{E}_i, u \in \mathcal{U}_i$, *where* $L_{F_i} \triangleq 2\sigma_{\max}(Q_i) \sup\limits_{e_i \in \mathcal{E}_i} \|e_i\|$.

Proof. The proof can be found in [16, App. D, p. 22]. □

The applied input signal is a portion of the optimal solution to an optimization problem where information on the states of the neighboring agents of agent i is taken into account only in the constraints considered in the optimization problem. These constraints pertain to the set of its neighbors $\mathcal{N}_i$ and, in total, to the set of all agents within its sensing range $\mathcal{R}_i$. Regarding these, we make the following assumption:

Assumption 4. When at time t_k agent i solves a FHOCP, it has access to the following measurements, across the entire horizon $s \in (t_k, t_k + T_p]$: 1) Measurements of the states: $i)$ $x_j(t_k)$ of all agents $j \in \mathcal{R}_i(t_k)$ within its sensing range at time t_k; $ii)$ $x_{j'}(t_k)$ of all of its neighboring agents $j' \in \mathcal{N}_i$ at time t_k; 2) The *predicted states*: $i)$ $\overline{x}_j(s)$ of all agents $j \in \mathcal{R}_i(t_k)$ within its sensing range; $ii)$ $\overline{x}_{j'}(s)$ of all of its neighboring agents $j' \in \mathcal{N}_i$;

In other words, each time an agent solves its own individual optimization problem, it knows the (open-loop) state predictions that have been generated by the solution of the optimization problem of all agents within its range at that time, for the next T_p time units. This assumption is crucial to satisfying the constraints regarding collision avoidance and connectivity maintenance between neighboring agents. We assume that the above pieces of information are *always available*, *accurate* and can be exchanged *without delay*.

Remark 1. The designed procedure flow can be either concurrent or sequential, meaning that agents can solve their individual FHOCPs and apply the control inputs either simultaneously, or one after the other. The conceptual design itself is procedure-flow agnostic, and hence it can incorporate both without loss of feasibility or successful stabilization. The approach that we have adopted here is the sequential one: each agent solves its own FHOCP and applies the corresponding admissible control input in a round robin way, considering the current and planned (open-loop state predictions) configurations of all agents within its sensing range.

The constraint sets $\mathcal{E}_i$, $i \in \mathcal{V}$ involve the sets $\mathcal{R}_i(t)$ which are updated at every sampling time in which agent i solves his own optimization problem. Its predicted configuration at time $s \in [t_k, t_k + T_p]$ is constrained by the predicted configuration of its neighboring and perceivable agents (agents within its sensing range) at the same time instant s.

The solution to FHOCP (3a) - (3d) at time t_k provides an optimal control input, denoted by $\overline{u}_i^\star(s;\ e_i(t_k))$, $s \in [t_k, t_k + T_p]$. This control input is then applied to the system until the next sampling instant t_{k+1}:

$$u_i(s; e_i(t_k)) = \overline{u}_i^\star\big(s;\ e_i(t_k)\big),\ s \in [t_k, t_{k+1}). \tag{4}$$

At time t_{k+1} a new finite horizon optimal control problem is solved in the same manner, leading to a receding horizon approach. The control input $u_i(\cdot)$ is of feedback form, since it is recalculated at each sampling instant based on the then-current state. The solution of (2) at time s, $s \in [t_k, t_k + T_p]$, starting at time t_k, from an initial condition $e_i(t_k) = \overline{e}_i(t_k)$, by application of the control input $u_i : [t_k, s] \to \mathcal{U}_i$ is denoted by $e_i\big(s;\ u_i(\cdot), e_i(t_k)\big)$, $s \in [t_k, t_k + T_p]$. The *predicted* state of the system (3b) at time s, $s \in [t_k, t_k + T_p]$ based on the measurement of the state at time t_k, $e_i(t_k)$, by application of the control input $u_i\big(t;\ e_i(t_k)\big)$ as in 4, is denoted by $\overline{e}_i\big(s;\ u_i(\cdot), e_i(t_k)\big)$, $s \in [t_k, t_k + T_p]$.

The satisfaction of the constraints $\mathcal{E}_i$ on the state along the prediction horizon depends on the future realization of the uncertainties. On the assumption of additive uncertainty and Lipschitz continuity of the nominal model, it is possible to compute a bound on the future effect of the uncertainty on the system. Then, by considering this effect on the state constraint on the nominal prediction, it is possible to guarantee that the evolution of the real state of the system will be admissible all the time. In view of latter, the state constraint set $\mathcal{E}_i$ of the standard NMPC formulation, is being replaced by a restricted constrained set $\mathcal{E}_{s-t_k} \subseteq \mathcal{E}_i$ in (3c). This state constraint's tightening for the nominal system (3b) with additive disturbance $w_i \in \mathcal{W}_i$, is a key ingredient of the proposed controller and guarantees that the evolution of the evolution of the real system will be admissible for all times. If the state constraint set was left unchanged during the solution of the optimization problem, the applied input to the plant, coupled with the uncertainty affecting the states of the plant could force the states of the plant to escape their intended bounds. The aforementioned tightening set strategy is inspired by the works [26]–[28], which have considered such a robust NMPC formulation.

Lemma 3. *The difference between the actual measurement $e_i\big(t_k + s;\ u_i(\cdot), e_i(t_k)\big)$ at time $t_k + s$, $s \in (0, T_p]$, and the predicted state $\overline{e}_i\big(t_k + s;\ u_i(\cdot), e_i(t_k)\big)$ at the same time, under a control input $u_i(\cdot) \in \mathcal{U}_i$, starting at the same initial state $e_i(t_k)$ is upper bounded by: $\big\|e_i\big(t_k + s;\ u_i(\cdot), e_i(t_k)\big) - \overline{e}_i\big(t_k + s;\ u_i(\cdot), e_i(t_k)\big)\big\| \leq \dfrac{\widetilde{w}_i}{L_{g_i}}(e^{L_{g_i} s} - 1)$, $s \in (0, T_p]$, where $e^{\cdot}$ denotes the exponential function.*

Proof. The proof can be found in [16, App. E, p. 23]. □

By taking into consideration the aforementioned Lemma, the restricted constraints set are then defined by: $\mathcal{E}_{i,s-t_k} = \mathcal{E}_i \ominus \mathcal{X}_{i,s-t_k}$, with $\mathcal{X}_{i,s-t_k} = \Big\{e_i \in \mathbb{R}^n : \|e_i(s)\| \leq \dfrac{\widetilde{w}_i}{L_{g_i}}\big(e^{L_{g_i}(s-t_k)} - 1\big), \forall s \in [t_k, t_k + T_p]\Big\}$. This modification guarantees that the state of the real system e_i is always satisfying the corresponding constraints $\mathcal{E}_i$.

Assumption 5. The terminal set $\Omega_i \subseteq \Psi_i$ is a subset of an admissible and positively invariant set Ψ_i, where Ψ_i is defined as $\Psi_i \triangleq \big\{e_i \in \mathcal{E}_i : V_i(e_i) \leq \varepsilon_{\Psi_i}\big\}$, $\varepsilon_{\Psi_i} > 0$.

Assumption 6. The set Ψ_i is interior to the set Φ_i, $\Psi_i \subseteq \Phi_i$, which is the set of states within $\mathcal{E}_{i,T_p-h}$ for which there exists an admissible control input which is of linear feedback form with respect to the state $\kappa_i : [0, h] \to \mathcal{U}_i$: $\Phi_i \triangleq \big\{e_i \in \mathcal{E}_{i,T_p-h} : \kappa_i(e_i) \in \mathcal{U}_i\big\}$, such that for all $e_i \in \Psi_i$ and for all $s \in [0, h]$ it holds that: $\dfrac{\partial V_i}{\partial e_i} g_i(e_i(s), \kappa_i(s)) + F_i(e_i(s), \kappa_i(s)) \leq 0$.

Remark 2. According to [29], [30], the existence of the linear state-feedback control law κ_i is ensured if for every $i \in \mathcal{V}$ the following conditions hold: 1) f_i is twice continuously differentiable with $f_i(0_{n\times 1}, 0_{m\times 1}) = 0_{n\times 1}$; Assumption 1 holds; 3) the sets $\mathcal{U}_i$ are compact with $0_{m\times 1} \in \mathcal{U}_i$; and 4) the linearization of system (2) is stabilizable.

Assumption 7. The admissible and positively invariant set Ψ_i is such that $\forall e_i(t) \in \Psi_i \Rightarrow e_i\big(t+s;\ \kappa_i(e_i(t)), e_i(t)\big) \in \Omega_i \subseteq \Psi_i$, for some $s \in [0, h]$.

The terminal sets Ω_i are chosen as: $\Omega_i \triangleq \big\{e_i \in \mathcal{E}_i : V_i(e_i) \leq \varepsilon_{\Omega_i}\big\}$, where $\varepsilon_{\Omega_i} \in (0, \varepsilon_{\Psi_i})$.

Lemma 4. *For every $e_i \in \Psi_i$ there exist functions $\alpha_1, \alpha_2 \in \mathcal{K}_\infty$ such that: $\alpha_1\big(\|e_i\|\big) \leq V_i(e_i) \leq \alpha_2\big(\|e_i\|\big)$, $\forall i \in \mathcal{V}$.*

Proof. The proof can be found in [16, App. G, p. 24]. □

Lemma 5. *The terminal penalty functions V_i are Lipschitz continuous in Ψ_i, thus it holds that: $\big|V_i(e) - V_i(e')\big| \leq L_{V_i}\|e-e'\|, \forall e, e' \in \Psi_i$, where $L_{V_i} = 2\sigma_{\max}(P_i) \sup\limits_{e_i \in \Psi_i} \|e_i\|$.*

Proof. The proof is similar to the proof of Lemma 2. □

We can now give the definition of an *admissible input* for the FHOCP (3a)-(3d).

Definition 5. A control input $u_i : [t_k, t_k + T_p] \to \mathbb{R}^m$ for a state $e_i(t_k)$ is called *admissible* for the FHOCP (3a)-(3d) if the following hold: 1) $u_i(\cdot)$ is piecewise continuous; 2) $u_i(s) \in \mathcal{U}_i,\ \forall s \in [t_k, t_k + T_p]$; 3) $e_i\big(t_k + s;\ u_i(\cdot), e_i(t_k)\big) \in \mathcal{E}_i \ominus \mathcal{X}_{i,s},\ \forall s \in [0, T_p]$; 4) $e_i\big(t_k + T_p;\ u_i(\cdot), e_i(t_k)\big) \in \Omega_i$;

Under these considerations, we can now state the theorem that relates to the guaranteeing of the stability of the compound system of agents $i \in \mathcal{V}$, when each of them is assigned a desired position.

Theorem 1. *Suppose that for every $i \in \mathcal{V}$: 1) assumptions 1-7 hold; 2) a solution to FHOCP (3a)-(3d) is feasible at time $t = 0$ with feasible initial conditions, as defined in Definition 4; 3) the upper bound of the disturbance w_i satisfies the following: $\widetilde{w}_i \leq \dfrac{\varepsilon_{\Psi_i} - \varepsilon_{\Omega_i}}{\frac{L_{V_i}}{L_{g_i}}(e^{L_{g_i}h} - 1)e^{L_{g_i}(T_p - h)}}$. Then, the closed loop trajectories of the system (2), under the control input (4) which is the outcome of the FHOCP (3a)-(3d), converge to the set Ω_i, as $t \to \infty$ and are ultimately bounded there, for every $i \in \mathcal{V}$.*

Proof. The proof of the theorem consists of two parts: firstly, recursive feasibility is established, that is, initial feasibility is shown to imply subsequent feasibility; secondly, and based on the first part, it is shown that the error state $e_i(t)$ reaches the terminal set Ω_i and it remains there for all times. The feasibility analysis and the convergence analysis can be found in [16, App. H, p. 25], [16, App. I, p. 30], respectively. □

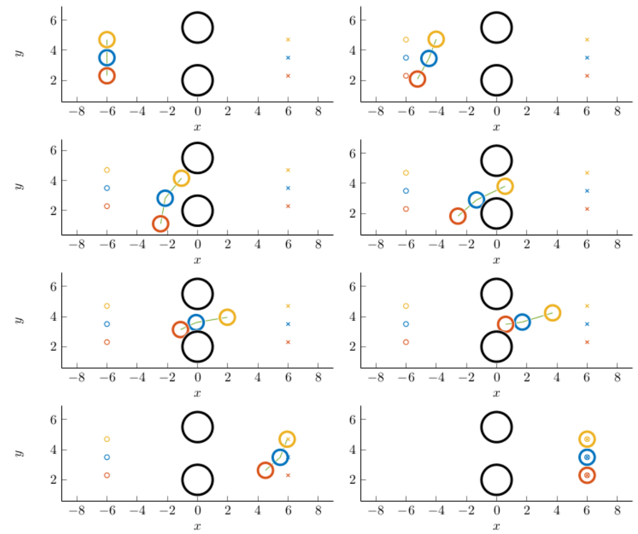

Fig. 1: The trajectories of the three agents in the $x-y$ plane. Agent 1 is with blue, agent 2 with red and agent 3 with yellow. A faint green line connects agents deemed neighbors. The obstacles are depicted with black circles. The indicator "O" denotes the initial configurations. The indicator "X" marks the desired configurations.

V. Simulation Results

For a simulation scenario, consider $N = 3$ unicycle agents with dynamics: $\dot{x}_i(t) = \begin{bmatrix} \dot{x}_i(t) \\ \dot{y}_i(t) \\ \dot{\theta}_i(t) \end{bmatrix} = \begin{bmatrix} v_i(t)\cos\theta_i(t) \\ v_i(t)\sin\theta_i(t) \\ \omega_i(t) \end{bmatrix} + w_i(x_i, t)I_{3\times 1}$, where: $i \in \mathcal{V} = \{1, 2, 3\}$, $x_i = [x_i, y_i, \theta_i]^\top$, $f_i(z_i, u_i) = [v_i \cos\theta_i, v_i \sin\theta_i, \omega_i]^\top$, $u_i = [v_i, \omega_i]^\top$, $w_i = 0.1\sin(2t)$. We set $\widetilde{u}_i = 15$, $r_i = 0.5$, $d_i = 4r_i = 2.0$ and $\varepsilon = 0.01$. The neighboring sets are set to $\mathcal{N}_1 = \{2, 3\}$, $\mathcal{N}_2 = \mathcal{N}_3 = \{1\}$. The agents' initial positions are $x_1 = [-6, 3.5, 0]^\top$, $x_2 = [-6, 2.3, 0]^\top$ and $x_3 = [-6, 4.7, 0]^\top$. Their desired configurations in steady-state are $x_{1,\text{des}} = [6, 3.5, 0]^\top$, $x_{2,\text{des}} = [6, 2.3, 0]^\top$ and $x_{3,\text{des}} = [6, 4.7, 0]^\top$. In the workspace, we place 2 obstacles with centers at points $[0, 2.0]^\top$ and $[0, 5.5]^\top$, respectively. The obstacles' radii are $r_{\mathcal{O}_\ell} = 1.0$, $\ell \in \mathcal{L} = \{1, 2\}$. The matrices Q_i, R_i, P_i are set to $Q_i = 0.7(I_3 + 0.5\dagger_3)$, $R_i = 0.005 I_2$ and $P_i = 0.5(I_3 + 0.5\dagger_3)$, where $\dagger_N$ is a $N \times N$ matrix whose elements are uniformly randomly chosen between the values 0.0 and 1.0. The sampling time is $h = 0.1$ sec, the time-horizon is $T_p = 0.6$ sec, and the total execution time given is 10 sec. Furthermore, we set: $L_{f_i} = 10.7354$, $L_{V_i} = 0.0471$, $\varepsilon_{\Psi_i} = 0.0654$ and $\varepsilon_{\Omega_i} = 0.0035$ for all $i \in \mathcal{V}$.

The frames of the evolution of the trajectories of the three agents in the $x-y$ plane are depicted in Fig. 1; Fig. 2 depicts the evolution of the error states of agents; Fig. 3 shows the evolution of the distances between the neighboring agents; Fig. 4 and Fig. 5 depict the distance between the agents and the obstacle 1 and 2, respectively. Finally, Fig. 6 shows the input signals directing the agents through time. It can be observed that all agents reach their desired goal by satisfying all the constraints imposed by Problem 1.

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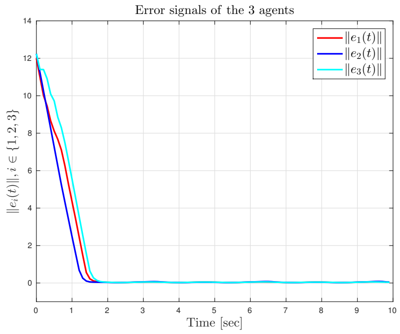

Fig. 2: The evolution of the error signals of the three agents.

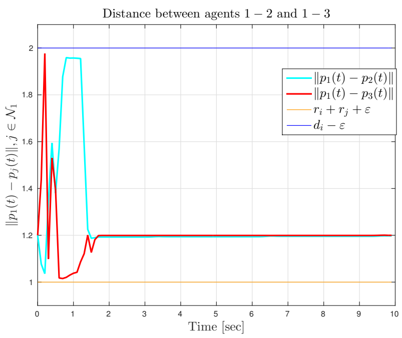

Fig. 3: The distance between the agents $1-2$ and $1-3$ over time. The maximum and the minimum allowed distances are $d_i - \varepsilon = 1.99$ and $r_i + r_j + \varepsilon = 1.01$, respectively for every $i \in \mathcal{V}$, $j \in \mathcal{N}_i$.

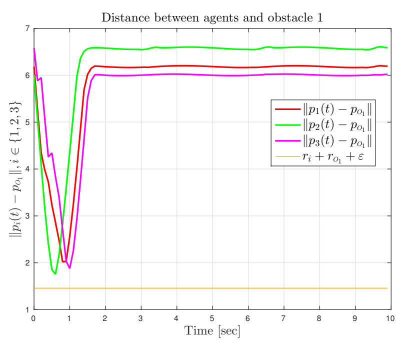

Fig. 4: The distance between the agents and obstacle 1 over time. The minimum allowed distance is $r_i + r_{O_1} + \varepsilon = 1.51$.

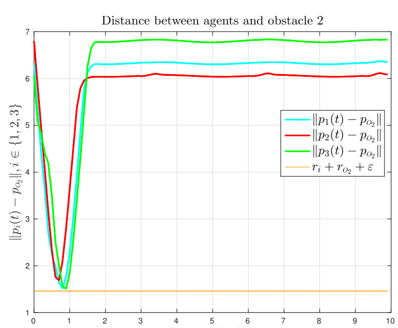

Fig. 5: The distance between the agents and obstacle 2 over time. The minimum allowed distance is $r_i + r_{O_2} + \varepsilon = 1.51$.

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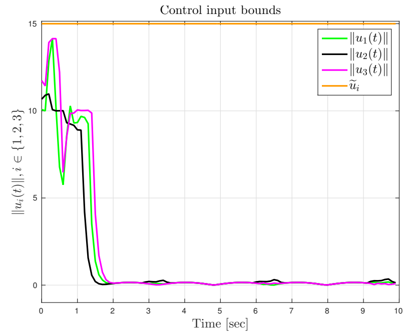

Fig. 6: The norms of control inputs signals with $\widetilde{u}_i = 15$

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