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A new robust observer approach for unknown input and state estimation

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***Abstract*— This paper proposes a new approach for a robust estimation of unknown inputs as well as state variables in a dynamical system subject to noise or uncertainties. Following a recently proposed *control-based* methodology for observer design, the idea here is to take advantage of robust control methods, and in particular of H_∞ techniques. This provides a new method which is applied for an example of tunneling current estimation in an STM-like device. Simulation results are finally provided together with a comparison to the formerly available H_∞ filtering, but also to the robust sliding-mode technique .**

I. INTRODUCTION

The problem of state estimation is not only well known among the researchers in automatic control systems field, but also really important for engineering applications of all kind. Many solutions have been proposed for both linear and nonlinear systems based on different approaches such as high gain observers [1], or optimal ones, namely Kalman [2] and H_∞ filters [3]. Another particular class of observers, nonlinear and discontinuous, has been developed using the so-called *sliding-mode technique*, and showing good robustness against uncertain parts of the dynamics: for example Utkin's observer [4], Slotine's observer [5], or more advanced as Super-Twisting [6] and Generalized-Super-Twisting [7] observers.

The need of an observer comes naturally when someone wants to extrapolate the information of measurable (external) signals, such as the input (u) and output (y), in order to obtain some non-measurable (internal) characteristics of a system. Having in mind this description, the classical observer problem can be stated as follows: given a system defined by a set of differential equations, infer an estimate $\hat{x}$ for the state x of the system using the information carried by input (u) and output (y). This problem can be extended to that of estimating *unknown inputs* (see for example [8] or [9]).

In the present paper, a new approach for observer design is proposed, allowing to address *robust estimation of both unknown inputs and state variables*. This approach is based on a technique recently introduced in [10], which basically converts the observer problem into a control one: briefly speaking, one can design a controller for a model of the system such that the output of this model follows the output of the real system. As soon as this condition is fulfilled, one gets an estimate for the state of the system. If the driving variable is some unknown input, then the control law gives an estimate for it. This approach has already been proven to be efficient in various examples such as for wind speed estimation in wind turbine [11], surface estimation in a Scanning Tunneling Microsope (STM) device [12] or unknown input disturbance estimation in a magnetic levitation process [13].

By choosing here an H_∞ approach as a control strategy for the observer design, we can take advantage of this framework so as to get a *robust* estimation. Notice that this approach gives unknown input estimation without the need of any state extension of the system, as often done in available results for such a problem. It is here illustrated with an example of tunneling current estimation in an STM.

The paper is organized as follows. In Section II the proposed technique for robust observer design based on H_∞ control is presented. In Section III the proposed example of application to tunneling current estimation is given, followed by the corresponding simulation results in Section IV (including a comparison with a couple of formerly available approaches). Finally, Section V concludes the paper.

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II. ROBUST OBSERVER APPROACH

In this section, the main contribution of the paper is presented. In short, relying on the paradigm of designing observers based on a control strategy as proposed in [10]: for a system, possibly subject to some unknown input, one can consider a model for which a controller is designed such that its output follows the output of the system. This means that the observer problem is converted into a control one, so that instead of designing an observer in a classical way, a tracking problem is solved. In this paper, an H_∞ *controller* is proposed as a control strategy, so as to ensure a *robust solution* for the observer issue. Notice that the observer paradigm applies to nonlinear systems, as recalled below, but owing to the classical H_∞ framework, here the focus is on the linear case, leaving nonlinear extensions to future developments.

A. Principle of observer design based on control strategy

Let us consider a nonlinear system described by equation:

$$\begin{aligned} \dot{x} &= f(x,u,v) \\ y &= h(x) \end{aligned} \tag{1}$$

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where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ classically stand for state, input and output of the system, while v describes some unknown input. Moreover, f and h are assumed to be smooth functions.

Under the assumption that the system described by equation (1) satisfies some observability (or at least detectability) conditions as emphasized in [14], an observer based on a 'control-strategy' can be synthesized as follows:

$$\begin{array}{rcl} \dot{\hat{x}} & = & \Phi(\hat{x}, u, \hat{v}) \\ \hat{y} & = & h(\hat{x}) \\ \hat{v} & = & \kappa(e_y, t) \end{array} \quad (2)$$

where $\Phi(\hat{x}, u, \hat{v})$ is the model chosen to describe the system, while $\hat{v}$, generated by a controller κ, is the control signal used to drive the model such that its output ($\hat{y}$) tracks the system output (y) (see Figure 1). It will thus in general depend on the tracking error $e_y = y - \hat{y}$ with any (possibly dynamical) law.

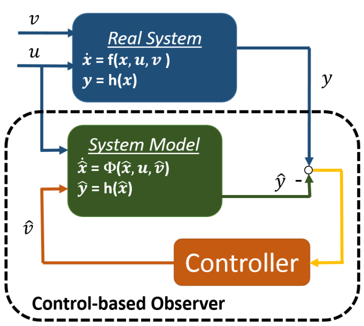


Fig. 1: Control-based Observer principle

B. Robust Observer based on H_∞ control strategy

Thinking of classical H_∞ techniques, let us focus now on linear systems, possibly subject to unknown inputs to be estimated, in addition to other uncertain inputs to be handled in the observer problem:

$$\begin{array}{rcl} \dot{x} & = & Ax + B_0 u + B_2 v \\ y & = & Cx + D_0 u + D_1 w + D_2 v \end{array} \quad (3)$$

where, as reminded before, x, u, and y are the state, the input and the output of the system, v stands for the unknown input to be estimated, and w gathers other disturbances (like noises). Matrices A, B_0, B_2 C, D_0, D_1 and D_2, having appropriate dimensions, completely characterize the linear system.

Notice that, since the system is linear, u can be omitted in equation (3) by considering $\bar{y}$ as the output obtained without the effect of u: $\bar{y} = y - \int_0^t e^{A(t-\tau)} B_0 u(\tau) d\tau$. Also $D_0 u$ being known, it can be dropped in the same equation (up to a change of output), meaning that an observer model to be considered for our design can be of the form:

$$\begin{array}{rcl} \dot{\hat{x}} & = & A\hat{x} + B_2 \hat{v} \\ \hat{y} & = & C\hat{x} + D_2 \hat{v} \end{array} \quad (4)$$

where $\hat{x}$ is the estimated state, $\hat{y}$ is the output of the observer model, and $\hat{v}$ the output of the H_∞ controller driving the observer model, so as to track $\bar{y}$ by attenuating the effect of w.

1) H_∞ controller design: Let us recall the H$_\infty$ closed-loop interconnection framework presented in Figure 2:

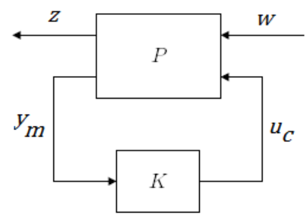


Fig. 2: Closed-loop interconnection

The notations in Figure 2 traditionally have the following meaning:

- w is called generalized disturbance and represents the signal that affects the system, but cannot be influenced by the controller.
- u_c is called control input and is the output of the controller
- z is called controlled variable and is used to describe if the controller should have some specified properties.
- y_m is called measurement outputs and defines the signal that enters the controller.

Now, let us consider the control problem in terms of the variables presented in Figure 2 for which a H$_\infty$ controller has to be designed:

$$\begin{array}{rcl} \dot{\hat{x}} & = & A\hat{x} + B_2 \hat{v} \\ y_m & = & C\hat{x} + D_2 \hat{v} - \bar{y} \\ z & = & \hat{y} - (\bar{y} - D_1 w) \\ u_c & = & \hat{v} \end{array} \quad (5)$$

By considering the vector of external signals, $w_e^T = [\bar{y} \ w]$ we get:

$$\begin{array}{rcl} \dot{\hat{x}} & = & A\hat{x} + B_2 u_c \\ y_m & = & C\hat{x} + [-I \ 0] w_e + D_2 u_c \\ z & = & C\hat{x} + [-I \ D_1] w_e + D_2 u_c \end{array} \quad (6)$$

Thus the problem of designing an observer becomes a standard H$_\infty$ control problem, in other words, given an attenuation γ, we have to find a controller K such that

$$||T_{zw_e}(s)||_\infty < \gamma \quad (7)$$

where $T_{zw_e}(s)$ stands for the closed-loop transfer function between the input w_e and the output z.

This shows how the observer problem is converted into a control one.

2) Performance specification for robust observer design: Performance specification in terms of choosing templates for weighting the sensitivity functions of the closed-loop system is an important step in H_∞ controller design. To that end, one can use this property of H_∞ framework in order to impose some desired performances for the observer such as convergence rate, robustness against noise or parameters uncertainty.

As equation (7) describes it, the aim is to compute a controller which minimizes the H_∞ norm of the transfer between the generalized disturbance (w_e) and controlled variable (z).

In particular, for this linear system, the transfer is described as follows:

$$z = T_{zw_e}(s)w_e \tag{8}$$

At this point, one can design performance specification in terms of weighting functions as follows:

$$w_e = W_w\tilde{w} \text{ and } \tilde{z} = W_z z \tag{9}$$

which leads to the weighted closed-loop interconnection as displayed in Figure 3.

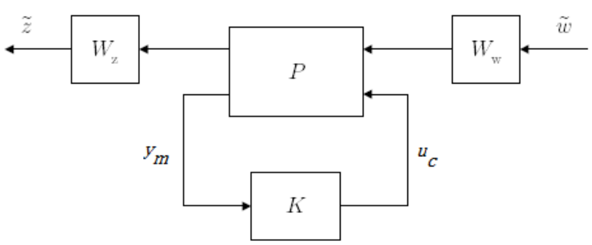


Fig. 3: Weighted closed-loop interconnection

Thus, equation (8) becomes:

$$\tilde{z} = W_z(s)T_{zw}(s)W_w(s)\tilde{w} \tag{10}$$

Keep in mind that, at this point, first the open-loop interconnection is weighted and then a controller is computed such that H_∞-norm of the transfer between $\tilde{z}$ and $\tilde{w}$ is minimized (the so-called Nominal Performance Synthesis Problem).

Finally, the observer is given by the following equations:

$$\begin{array}{rcl} \dot{\hat{x}} & = & A\hat{x} + B_2\hat{v} \\ \hat{v} & = & u_c \end{array} \tag{11}$$

where u_c is the control input given by an H_∞ controller design such that:

$$||T_{\tilde{z}\tilde{w}}(s)||_\infty < \gamma \tag{12}$$

and $\hat{x}$ and $\hat{v}$ are estimates for x and v.

III. APPLICATION TO TUNNELING CURRENT ESTIMATION

In this section the performances of the proposed robust control-based observer are illustrated by applying this technique for a microscopy device called Scanning Tunneling Microscope (STM) [15].

This instrument is based on the motion of a tip driven by piezoelectric actuators in all 3D dimensions (x, y, z axis) over a sample. The tip and the sample are both electrically conductive materials. If the distance between them reaches a subnanometric level and a voltage bias is applied, then a so-called tunneling current appears. By maintaining a certain intensity of tunneling current while the sample is scanned, one can retrieve the information about the shape of the surface having nanometric resolution.

Focusing here on tunneling current estimation from a sensor measurement, let us consider the second order transfer function which describes this sensor as follows [16]:

$$\begin{array}{rcl} \dot{x} & = & Ax + B_2v \\ y & = & Cx + D_1w \end{array} \tag{13}$$

where we have $x = [x_1, x_2]^T$ and the following matrices:

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} c_1 & 0 \end{bmatrix}, D_1 = 1. \tag{14}$$

It is worth noting at this point that we will apply the robust control-based observer described in section II-B. To do so, we will consider that v is the unknown input to be estimated, w represents the measurement noise and also that the system has no control input, u, (i.e. $u = 0$).

Next, a model for system (13) is chosen to be:

$$\begin{array}{rcl} \dot{\hat{x}} & = & A\hat{x} + B_2\hat{v} \\ \hat{y} & = & C\hat{x} \end{array} \tag{15}$$

for which a H_∞ controller is designed such that $\hat{y}$ follows system output (in particular, the output of the system which is not affected by noise, (Cx)) as presented in section II-B. Considering the equations of H_∞ framework as described in (6), we get:

$$\begin{array}{rcl} \dot{\hat{x}}_1 & = & \hat{x}_2 \\ \dot{\hat{x}}_2 & = & a_1\hat{x}_1 + a_2\hat{x}_2 + \hat{v} \\ z & = & c_1\hat{x}_1 - y + w \\ y_m & = & c_1\hat{x}_1 - y \end{array} \tag{16}$$

where $\hat{v}$ is computed such that z approaches 0 in spite of the external signals effect.

It can be seen that as soon as z approaches 0 we can conclude that $\hat{v}$ is an estimate for unknown input v and moreover $\hat{x}$ is an estimation of x.

Finally, we design the templates for H_∞ problem which gather the information about observer performances: the main objective here is to assure that the output of the model, $\hat{y}$ follows the output of the real system output not affected by noise, Cx. Moreover the influence of measurement noise for this tracking problem should be as small as possible for both state estimation and unknown input estimation.

To that end, two templates can designed for this particular application. One to assure the tracking performance (a high pass filter for sensitivity function S), and another one to limit the control input of the model, in order to minimize the effect of the noise for the controller output (a low pass filter for sensitivity functions KS).

Those performance specifications for this control problem are illustrated by the control scheme in the Figure 4

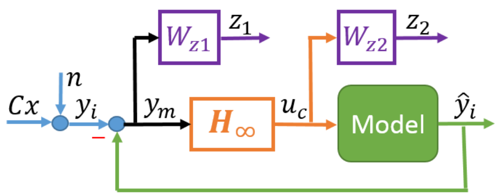


Fig. 4: Performance specifications for observer design

Since the sensitivity function S characterizes the steady state error in tracking and concerns the stability of the system (Maximum peak criterion), the template W_{z1} is chosen such that it has a predefined attenuation level for low frequency (i.e. $S(\omega = 0) = \epsilon_S$). Moreover to satisfy the second specification, in practice, the maximum peak of S is supposed to be less then 2 (i.e. $||S||_\infty < 2$).

Furthermore, sensitivity function KS classically describes the input saturation and the attenuation of measurement noise. Basically, the template W_{z2} is set such that KS has a particular amplification for low frequency (i.e. $||KS||_\infty = u_{max}$ in low frequency) and a certain attenuation for high frequency (i.e. $|KS(j\omega)| \approx \epsilon_{KS}$ when $\omega \to \infty$).

Those choices will obviously, in general, depend on the considered problem. The obtained results with the above approach are illustrated in next section, where a comparison with other approaches is also provided.

IV. ILLUSTRATIVE RESULTS AND COMPARISON WITH TWO OTHER APPROACHES

In this section, some simulation results are presented for the previously discussed example. In addition a comparison with two other possible approaches is proposed, considering the H_∞ filter on the one hand, and a Sliding Mode Observer on the other hand.

A. Simulation results

Here the numerical data for matrix A and C of equation (14), as well as the level of noise, are taken from a real device as in [17], and summarized in Table I.

TABLE I: Parameters definition

Parameter	Value	Parameter	Value
a_1	$-\omega_i^2$	ω_i	13 kHz
a_2	$-2\zeta_i\omega_i$	ζ_i	0.9
c_1	$Gi\omega_i^2$	G_i	10^9 V/nA

Moreover, the measurement noise is considered to be of 0 mean and standard deviation of $\sigma = 0.05$.

The corresponding output obtained in simulation is displayed in Figure 5.

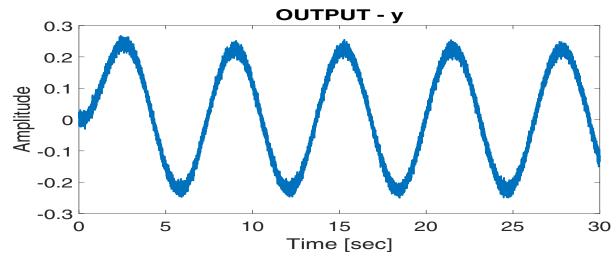


Fig. 5: Noisy output of the system

When applying the observer design as described in the previous section, we obtain the estimation of the unknown input as shown in Figure 6.

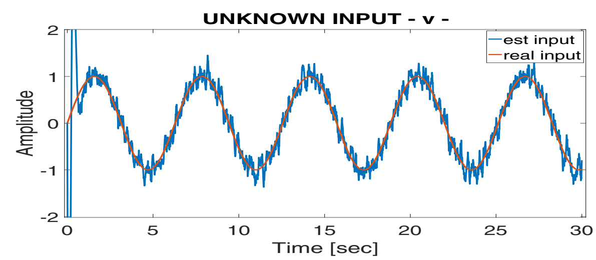


Fig. 6: Estimated unknown input using Control-based Observer

Moreover, we can also get robust estimation for the system states even if the measurement data are affected by noise. Figure 7 shows the estimation of x_1, while Figure 8 displays the estimation of x_2.

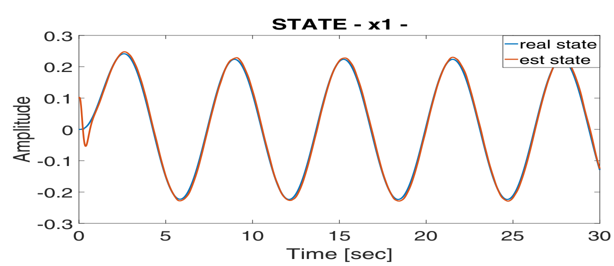


Fig. 7: Estimation of x_{i1} using Control-based Observer

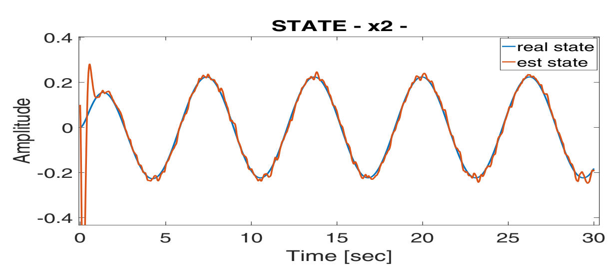


Fig. 8: Estimation of x_{i2} using Control-based Observer

Finally, the magnitudes of the sensitivity functions which describe the observer are given in Figure 9 to illustrate performance specification of the robust observer designed using 'control-based' approach.

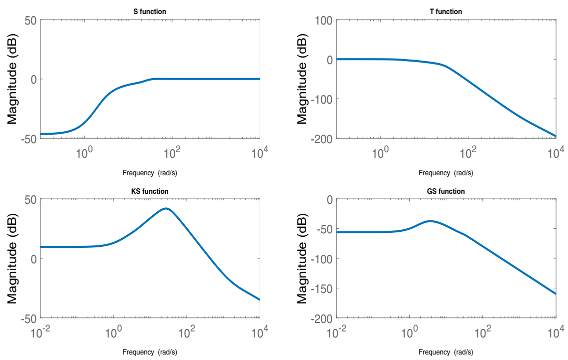


Fig. 9: Sensitivity function for Control-based Observer

B. Comparison with two other approaches

Let us recall here the main features of H_∞ filter and Sliding Mode Observer designs, and compare them with our approach.

1) H_∞ filter: In this approach, a second output, z, is added for system (3), which describes the state variables that should be estimated, but no unknown input to be estimated is a priori emphasized, giving rise to a system description as:

$$\begin{aligned} \dot{x} &= Ax + B_0 u + B_1 w \\ y &= Cx + D_0 u + D_1 w \\ z &= C_2 x \end{aligned} \tag{17}$$

The Game Theory, when provides a solution to the estimation, solves the following optimization problem:

$$\underset{z-\hat{z}}{\text{minimize}} \quad J < \frac{1}{\theta} \tag{18}$$

for some cost function, J, given by equation:

$$J = \frac{\int_0^T ||z-\hat{z}||_S^2 dt}{||x(0)-\hat{x}(0)||^2_{P_0^{-1}} + \int_0^T (||w||^2_{Q^{-1}} + ||v||^2_{R^{-1}} dt)} \tag{19}$$

where matrices Q, R, S, P_0 are the parameters which assure the performances specifications of the observer and θ is robustness parameter. They are predefined by the engineer depending on the application.

By solving equation (18), the following representation of the a H$_\infty$ filter is obtained (see for example [18]):

$$\begin{aligned} P(0) &= P_0 \\ \dot{P} &= AP + PA^T + B_1 Q B_1^T - KCP + \theta P C_2^T S C_2 P \\ K &= PC^T R^{-1} \\ \dot{\hat{x}} &= A\hat{x} + B_0 u + K(y - C\hat{x}) \\ \hat{z} &= C_2 \hat{x} \end{aligned} \tag{20}$$

This gives an estimate $\hat{x}$ for x.

As for some unknown input, one way to consider it can be to augment system (17) by adding the equation for unknown input evolution (e.g. $\dot{v} = 0$) and consider the new state for the extended system as $x_E = [x^T v^T]^T$. This leads to a new state representation for the extended system as:

$$\begin{aligned} \dot{x}_E &= A_E x_E + B_{0E} u + B_{1E} w \\ y_E &= C_E x_E + D_{0E} u + D_{1E} w \\ z_E &= C_{2E} x_E \end{aligned} \tag{21}$$

where all the matrices which describe system (21) have been modified according to the state extension. Next, the same procedure is applied as described above in order to obtain the state estimation, now including that of the unknown input.

2) Sliding Mode Observers - Super Twisting Algorithm: This approach is part of the class of so-called second order sliding mode observers [6]. Even if there exist also extensions to Higher Order Sliding Modes [19], it is enough for our second order example to consider the super twisting approach. Here no noise is a priori taken into account, meaning that the model for the system reduces to:

$$\begin{aligned} \dot{x} &= Ax + B_0 u + B_2 v \\ y &= Cx \end{aligned} \tag{22}$$

Then the Super Twisting Algorithm based solution for observer design is given by the equations:

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + B_0 u - L sign(e_y) \begin{bmatrix} |e_y|^{1/2} \\ 1 \end{bmatrix} \\ \hat{y} &= C\hat{x} \end{aligned} \tag{23}$$

where $e_y = y - \hat{y}$ and matrix L is chosen such that the estimated $\hat{x}$ converges to the state of the system.

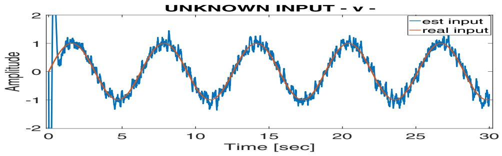
Notice here that the solution is nonlinear and discontinuous.

As for the unknown input estimation, it can be handled. But if this input acts on the second state (i.e. the relative degree is equal to the dimension of the state), as in our example, then the correction term that is supposed to give its estimate has to be filtered using a low pass filter (due to the chattering effect). Finally, the gain design for such an observer remains, in general, an issue.

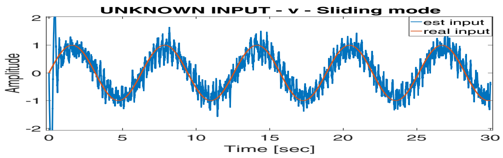
To complete this comparison discussion, some simulation results are finally given: in Figure 10 the estimation of the unknown input in our example is given for the three methods, while state estimates can be seen on Figure 11 for x_1, and Figure 12 for x_2 respectively. It can be noticed that, with the chosen tuning, a better reconstruction is achieved with our control-based observer for unknown input v, but also for x_2.

V. Conclusion

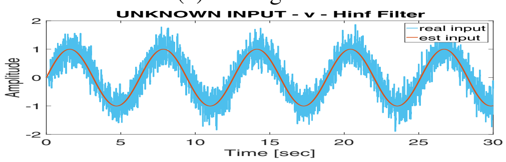
This paper has proposed a new approach for robust observer design for uncertain dynamical systems with possible unknown input estimation. It has in particular emphasized the possibility to take advantage of H_∞ control techniques by relying on a control-based approach. It has been compared



(a) Control-based Observer

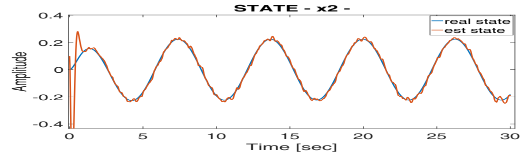


(b) Sliding mode Observer

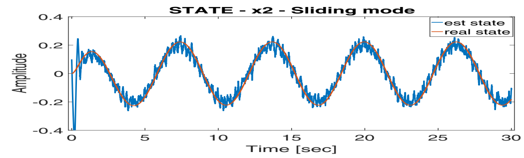


(c) H_∞ Filter

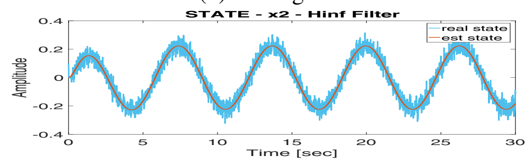
Fig. 10: Unknown inputs estimation



(a) Control-based Observer

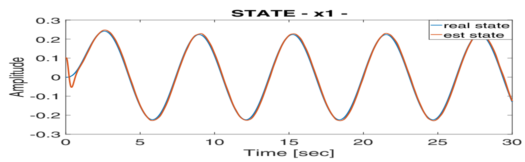


(b) Sliding mode Observer

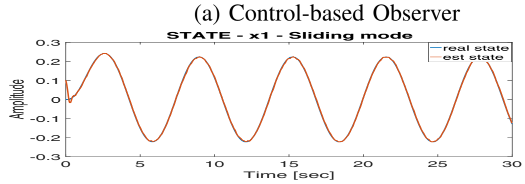


(c) H_∞ Filter

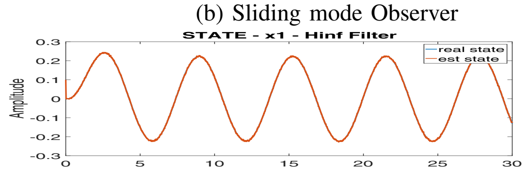
Fig. 12: Estimation of state x_2



(a) Control-based Observer



(b) Sliding mode Observer



(c) H_∞ Filter

Fig. 11: Estimation of state x_1

with a couple of other robust approaches, and illustrated with simulation results. It is worth noting that the proposed observer is linear and continuous, and can directly estimate unknown inputs in addition to the state, with no need of model dimension increase. Its extension to more general (nonlinear) classes of systems is part of future developments.

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