2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Passivity Based Iterative Learning Control Design in the Discrete Repetitive Process Setting

Pavel Pakshin, Julia Emelianova, Milhail Emelianov, Krzysztof Galkowski and Eric Rogers

Abstract— **Repetitive processes are important class of 2D systems with engineering applications and also the stability theory for them provides a setting for iterative learning control design. The application area for this form of control is systems that execute the same finite duration task over and over again, with resetting to the starting location one each execution is complete. Previous research for linear dynamics has used the stability theory of linear repetitive processes to design control laws that have been experimentally verified. This paper applies the recently developed passivity theory for discrete repetitive process to iterative learning control design. Based on this theory, a parametric description of a class of stabilizing controllers is obtained and a new design is developed that enhances the convergence properties of the implemented control law. An example using the model of a flexible link is given to demonstrate the application of the new design.**

I. INTRODUCTION

Repetitive processes [1] have been the subject of considerable interest both in the design of stabilizing control laws and also their application to physical examples. The unique characteristic of these processes is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. The interaction between the current and previous passes leads to the unique control problem where the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

Background on repetitive processes, including their use in modeling physical examples, can be found in [1] and the relevant cited references. These processes evolve over a subset of the upper-right quadrant of the 2D plane and they are one form of 2D systems. As standard control action cannot prevent the increase in oscillations in the sequence of pass profiles, a stability theory for linear dynamics has been developed [1] using an abstract model of the dynamics in a Banach space setting. This abstract model includes all processes with linear dynamics and a constant pass length as special cases.

The stability theory for linear repetitive processes guarantees that a bounded, in the sense of the norm on the underlying function space, initial pass profile produces a bounded sequence of pass profiles. Two cases exist, either this property is required over the finite and fixed pass length or uniformly, i.e., independent of the pass length. The first of these properties is termed asymptotic stability and the second stability along the pass. Moreover, stability along the pass can be analyzed by considering the case when the pass length tends to infinity. Also the results of applying this theory to many special cases have been reported and extended to control law design.

Some repetitive process cannot be adequately represented by linear models, again see [1] for examples and therefore a stability theory for nonlinear repetitive processes is required. Recent years have seen the emergence of results on such a theory for nonlinear 2D systems. For example, the stability of nonlinear 2D systems written in the Roesser or Fornasini-Marchesini state-space model form have been considered in, e.g., [2]. In [3] a stability theory was developed for discrete nonlinear repetitive processes where vector Lyapunov functions were used to characterize physically motivated stability properties.

In the case of standard, often termed 1D in the multidimensional systems literature, nonlinear systems, dissipativity theory [4] is one of the most powerful methods for control design, where a particular form, known as passivity (and its generalizations) see, e.g., [5], [6], can be used to solve the global feedback stabilization problem for a wide class of systems. In [3] new results on a dissipativity approach to the stabilization of discrete and differential nonlinear repetitive processes were obtained by using a vector storage function approach that is different from that in, e.g., [7] and the resulting design is based on divergence properties of this function.

Mathematical models in the form of repetitive processes arise naturally in iterative learning control (ILC). The general application area for ILC is systems that execute the same finite operation repeatedly, where each execution is termed a pass in this paper and the associate duration is termed the pass length (trial and iteration are also used in some of the literature). This form of control based on the premise

This work is partially supported by Russian Foundation for Basic Research, 16-08-00916a and by National Science Centre in Poland, grant No. 2015/17/B/ST7/03703.

Pavel Pakshin is with the Arzamas Polytechnic Institute of R.E. Alekseev Nizhny Novgorod State Technical University, 19, Kalinina Street, Arzamas, 607227, Russia and Lobachevsky State University of Nizhny Novgorod, Prospekt Gagarina, 23, 603950, Nizhny Novgorod, Russia pakshinpv@gmail.com

Julia Emelianova and Mikhail Emelianov are with Arzamas Polytechnic Institute of R.E. Alekseev Nizhny Novgorod State Technical University, 19, Kalinina Street, Arzamas, 607227, Russia EmelianovaJulia@gmail.com

Krzysztof Galkowski is with the Institute of Control and Computation Engineering, University of Zielona Gora, Podgorna 50, 65-246 Zielona Gora, Poland K.Galkowski@issi.uz.zgora.pl

Eric Rogers is with the Department of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, UK etar@ecs.soton.ac.uk

that the performance of such systems can be improved by using information from the previous pass (or a finite number thereof), i.e., improve performance from pass-to-pass.

A common form of ILC constructs the current pass input as the sum of the input on the previous pass plus a correction term designed using previous pass information. This design approach was first introduced in [8] and ILC remains a very active area of research. A notable feature is the number of applications where experimental verification is also available and the survey papers [9], [10] are one starting point for the literature.

The novel feature of ILC is that once a pass is complete, all data from this pass is available and hence non-causal temporal information can be used, i.e., in the case of discrete dynamics at sample instant p on pass $k+1$ information at $p+\lambda$, $\lambda > 0$, on pass k can be used. This is known as phase-lead ILC and this form of control law has been extensively researched and implemented in applications.

In cases where the dynamics can be adequately modeled by linear dynamics, the repetitive process stability theory has been used, see, e.g., [11], [12], to design ILC laws with supporting experimental verification. This verification used a gantry robot facility that replicates the pick and place operation commonly encountered in many industrial applications. This previous work also establishes that the repetitive process based design has advantages over alternatives in some cases.

Given the existence of a stability theory for nonlinear repetitive processes and results on passivity based design, this paper considers their application to linear model based ILC design, where the analysis is for the discrete case and the extension to differential dynamics noted as a natural extension. The main result is a new design that has the potential of delivering improved performance over existing alternatives. In particular, based on combination of the vector Lyapunov function approach [13] and 2D systems passivity results [3], a parameterized set of ILC laws is developed, including the option to include nonlinear terms in control law, which provides more options for design to meet performance specifications, such as increasing the rate of the pass-to-pass error convergence. An example, based on a model of flexible link, is given to illustrate the new design. Finally, areas for possible future research are briefly discussed.

II. PROBLEM FORMULATION

Let the nonnegative integer k denote the pass number. Also let $u_k(p) \in \mathbb{R}^l, x_k(p) \in \mathbb{R}^n$ and $y_k(p) \in \mathbb{R}^m$ be the input, state and output vectors, respectively, at instant $0 \le p \le T-1 < \infty$, where T denotes the number of samples along a pass (T times the sampling period gives the pass length). Then in the ILC setting the dynamics of the linear uncontrolled system are described by

$$\begin{aligned} x_k(p+1) &= Ax_k(p) + Bu_k(p), \\ y_k(p) &= Cx_k(p), k = 0, 1, \ldots \end{aligned} \tag{1}$$

with assumed boundary conditions

$$y_0(p) = Cx_0(p),\ 0 \le p \le T-1,\ x_k(0) = x_0,\ k \ge 0 \tag{2}$$

Suppose that $y_{ref}(p)$ denotes the supplied reference vector over $0 \le p \le T-1$. Then $e_k(p) = y_{ref}(p) - y_k(p)$ is the error on pass k and the problem to be considered is the construction of a sequence of pass inputs $\{u_k\}_k$ such that the performance achieved is gradually improving with each successive pass. This can be refined to the following convergence conditions on the input and error, i.e.,

$$\lim_{k\to\infty} ||e_k(p)|| = 0,\ \lim_{k\to\infty} ||u_k(p) - u_\infty(p)|| = 0. \tag{3}$$

where $|| \cdot ||$ denotes the norm on the underlying function space and $u_\infty(p)$ is termed the learned control.

In applications is may be the case that a given ILC law does not produce acceptable performance in terms of, e.g., the pass-to-pass error convergence rate or the transient response along the passes. An alternative to seeking another control law structure is to increase the options for tuning of the control law designed. The new results in this paper result in a parameterized control law, including nonlinear action and hence more possibilities in term of design. These new results make use of the passivity theory for nonlinear repetitive processes, for which the necessary background is given in the next section. One novel feature of the repetitive process setting is that is equally applicable to both discrete and differential dynamics. Discrete dynamics are considered in this paper but with routine modifications also apply to systems described the state-space model

$$\begin{aligned} \dot{x}_{k+1}(t) &= Ax_{k+1}(t) + Bu_{k+1}(t), \\ y_{k+1}(t) &= Cx_{k+1}(t),\ x_k(0) = x_0. \end{aligned} \tag{4}$$

where t denotes the along the pass variable and the remaining notation is the same as for the discrete case.

III. PRELIMINARIES

This section summarizes the required results from the existing passivity based stability theory, which follows in the main [3] for nonlinear dynamics.

A. General stability result

The state-space model of a deterministic discrete nonlinear repetitive process is of the form

$$\begin{aligned} x_{k+1}(p+1) &= f_1(x_{k+1}(p), y_k(p), u_{k+1}(p)), \\ y_{k+1}(p) &= f_2(x_{k+1}(p), y_k(p), u_{k+1}(p)), \end{aligned} \tag{5}$$

where on pass k $x_k(p) \in \mathbb{R}^{n_x}$ is the state vector, $y_k(p) \in \mathbb{R}^{n_y}$ is the pass profile vector, $u_k(p) \in \mathbb{R}^{n_u}$ is the control input vector and f_1, and f_2 are nonlinear functions. Also it is assumed that $f_1(0,0,0) = 0,\ f_2(0,0,0) = 0$ and hence an equilibrium at the origin.

The boundary conditions, i.e., the pass state initial vector sequence and the initial pass profile, are assumed to be known and have the form

$$\begin{aligned} x_{k+1}(0) &= d_{k+1},\ k \ge 0, \\ y_0(p) &= f(p),\ 0 \le p \le T-1, \end{aligned} \tag{6}$$

where $d_{k+1} \in \mathbb{R}^{n_x}$ is a known vector sequence, $f(p) \in \mathbb{R}^{n_y}$ denotes a vector whose entries are known functions of p,

$0 \leq p \leq T-1$. Moreover, if $||q||$ denotes the norm of a vector q, it is assumed that $f(p)$ and d_{k+1} satisfy

$$||f(p)||^2 \leq M_f, \; ||d_{k+1}||^2 \leq \kappa_d \zeta_d^k, \; k = 0, 1, ... \quad (7)$$

where $M_f > 0$ is a finite scalar, $\kappa_d > 0$ is independent of T and $0 < \zeta_d < 1$ determines the rate of convergence of the pass state initial vector sequence. Throughout this paper, it is assumed that the boundary conditions considered satisfy (7).

The unique control problem for repetitive processes is the possible presence of oscillations in the pass profiles that increase in amplitude from pass-to-pass. Hence the stability theory for repetitive processes demands that a bounded initial pass profile produces a bounded sequence of pass profiles. The strongest requirement is for this property to hold for all possible values of the pass length, which can be analyzed mathematically by considering $T \to \infty$.

This paper uses the following definition for stability of discrete nonlinear repetitive processes.

Definition 1: A discrete nonlinear repetitive process described by (5) and (6) is said to be exponentially stable if there exist $\kappa > 0$ and $0 < \lambda < 1$ such that

$$||x_k(p)||^2 + ||y_k(p)||^2 \leq \kappa \lambda^{k+p}, \quad (8)$$

where κ is independent of T, i.e., of the pass length.
In contrast to the 1D case, the full increment of a candidate Lyapunov function for processes described by (5) cannot be used to apply the second Lyapunov method for stability analysis.

This last fact has stimulated research using a vector Lyapunov function and the discrete counterpart of the divergence operator (referred to as the divergence operator in the rest of this paper) instead of the full increment. The analysis below employs a vector Lyapunov function of the following form for discrete nonlinear repetitive processes

$$V(x, y) = \begin{bmatrix} V_1(x_{k+1}(p)) \\ V_2(y_k(p)) \end{bmatrix}, \quad (9)$$

where $V_1(x) > 0, \; x \neq 0, \;\; V_2(y) > 0, \; y \neq 0, \;\; V_1(0) = 0, \; V_2(0) = 0$. The divergence operator of this function along the trajectories of (5) is defined as

$$\begin{aligned} &\mathcal{D}_d V(x_{k+1}(p), y_k(p)) \\ &\qquad = \Delta_p V_1(x_{k+1}(p)) + \Delta_k V_2(y_k(p)), \quad (10) \end{aligned}$$

where

$$\begin{aligned} \Delta_p V_1(x_{k+1}(p)) &= V_1(x_{k+1}(p+1)) - V_1(x_{k+1}(p)), \\ \Delta_k V_2(y_k(p)) &= V_2(y_{k+1}(p)) - V_2(y_k(p)). \end{aligned}$$

The following theorem holds.

Theorem 1: [3] Consider a discrete nonlinear repetitive process described by (5) and (6). Suppose also that there exist positive constants $c_1, \; c_2, \; c_3$ such that the vector Lyapunov function V and its divergence along the trajectories of (5) satisfy the inequalities

$$\begin{aligned} c_1||x_{k+1}(p)||^2 \leq V_1(x_{k+1}(p)) \leq c_2||x_{k+1}(p)||^2, \\ c_1||y_k(p)||^2 \leq V_2(y_k(p)) \leq c_2||y_k(p)||^2, \\ \mathcal{D}_d V(x_{k+1}(p), y_k(p)) \leq -c_3(||x_{k+1}(p)||^2 + ||y_k(p)||^2). \end{aligned}$$

Then this process is exponentially stable.

B. General passivity result

A powerful method in the analysis and control of 1D systems is dissipativity theory [4], especially the particular case of passivity theory [4], [5] where an extension of a Lyapunov function termed a storage function is used.

Introduce, for analysis and control law design purposes only the auxiliary vector for processes described by (5) and (6) $z_k(p) \in \mathbb{R}^{n_u}$ given by

$$z_{k+1}(p) = g(x_{k+1}(p), y_k(p), u_{k+1}(p)), \quad (11)$$

where g is a nonlinear function such that $g(0, 0, 0) = 0$ and define the dissipativity property as follows.

Definition 2: A discrete nonlinear repetitive process described by (5) and (6) is said to be exponentially dissipative if there exists a vector function (9), a scalar function $S(u, z)$ and positive scalars c_1, c_2 and c_3 such that

$$\begin{aligned} c_1||x||^2 &\leq V_1(x) \leq c_2||x||^2, \\ c_1||y||^2 &\leq V_2(y) \leq c_2||y||^2, \\ \mathcal{D}_d V(x_{k+1}(p), y_k(p)) &\leq S(u_{k+1}(p), z_{k+1}(p)) \\ &- c_3(||x_{k+1}(p)||^2 + ||y_k(p)||^2). \end{aligned}$$

In the particular case when $S(u, z) = z^T G u$, where G is a constant square matrix of compatible dimensions, a repetitive process described by (5) and (6) is said to be exponentially G-passive, see [14] for the 1D systems case. Since (5) does not involve full increments, as in the case of 1D discrete systems, it is impossible to use cross terms in the vector storage function (9).

The auxiliary vector z of (11) can be used to achieve certain dissipativity properties and for the case of passivity this is known as passivation (see, e.g., [15] for the 1D systems case). Moreover, the choice of this vector depends on the choice of storage function V and it is a separate complex problem (similar to the choice of a Lyapunov function for 1D nonlinear systems). The problem is to find a pair (z, V) satisfying the definition of passivity and later it will be shown how this pair and corresponding feedback law can be chosen for particular special cases.

The following theorem allows the application of the passivity property to stabilizing control design. (It is an obvious generalization of Theorem 1 from [3].)

Theorem 2: Suppose that a discrete nonlinear repetitive process described by (5), (6) and (11) is exponentially G passive. Suppose also that there exist the vector function $\varphi(z), \; \varphi(0) = 0$ and the matrix function $\Gamma(x, y)$, such that $z^T G \Gamma(x, y) \varphi(z) \geq 0$ if $z \neq 0$. Then the control law

$$u_{k+1}(p) = -\Gamma(x_{k+1}(p), y_k(p)) \varphi(z_{k+1}(p)) \quad (12)$$

results in controlled dynamics that are exponentially stable.

Application of this theory requires the selection of a suitable $V_1(x_{k+1}(p))$ and $V_2(y_k(p))$ and in the nonlinear case there is, as for other nonlinear systems, no general method for selecting these functions. This theory also provides a basis for nonlinear ILC design, but there is also a case for application to linear dynamics where there are still insufficient results on how to tune model based designs to achieve desired performance. The remainder of this paper establishes that applying this stability theory and passivity based control design to linear dynamics produces, where the entries in the vector Lyaunov function can be chosen as quadratic, a parameterized control law can be designed and hence a wider choice in terms of design for performance.

C. Background to passivity based design

It is instructive to consider first 1D discrete linear systems with state equation

$$x(p+1) = Ax(p) + Bu(p),\ p = 0, 1, \ldots, \tag{13}$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $u \in \mathbb{R}^{n_u}$ is the control input vector. Assuming that matrix A is stable, define the matrix $P \succ 0$ (where $\prec 0$ denotes a symmetric negative definite matrix) as a solution of the Lyapunov inequality

$$A^T PA - P + Q \prec 0, \tag{14}$$

where $Q \succ 0$. Consider also a storage function for (13) of the form of $V(x) = x^T Px$. Then the full increment of this function along the trajectories of (13) is

$$\begin{aligned}\Delta V(x(p)) &= x(p)^T (A^T PA - PA)x(p) \\ &\quad + (2x(p)^T A^T PB + u(p)^T B^T PB)u(p).\end{aligned} \tag{15}$$

Using [6], (13) is passive with respect to input $u(p)$ and output $z(p) = 2B^T PAx(p) + B^T PBu(p)$.

Consider the following control law

$$u(p) = -\Gamma(x(p))z(p), \tag{16}$$

where $\Gamma(x) \succeq 0$ for all $x \in \mathbb{R}^{n_x}$ (where $\succeq 0$ denotes a positive semi-definite matrix). Then the control law of (16) can be uniquely obtained in explicit form as

$$u(p) = [I + \Gamma(x(p))B^T PB]^{-1} 2\Gamma(x(p))B^T PAx(p), \tag{17}$$

and it stabilizes the system (13). Moreover, the matrices Q and $\Gamma(x)$ define a set of stabilizing controllers of the form (17) and by varying these matrices it is possible to alter the transient response properties of the controlled system.

If the matrix A is unstable, the Lyapunov inequality used above has no solution such that $P \succ 0$. In this case, define the pair of matrices $P \succ 0$ and the controller matrix K as a solution to

$$(A + BK)^T P(A + BK) - P + Q \prec 0. \tag{18}$$

Applying the results above to the system

$$x(p+1) = A_c x(p) + Bv(p), \tag{19}$$

where $A_c = A + BK$ is a stable matrix and $v(p) = u(p) - Kx(p)$ gives the set of stabilizing controls as

$$u(p) = [K - [I + \Gamma(x(p))B^T PB]^{-1} 2\Gamma(x(p)B^T PA_c]x(p). \tag{20}$$

Again by varying Q and $\Gamma(x)$ the transient response properties of the controlled system can be modified. In particular, $\Gamma(x)$ can be used to introduce nonlinear terms into the control law and it is this feature that will be used in the analysis to follow in this paper.

IV. Passivity-based ILC design for linear systems

A commonly used ILC law is to select the input on the current pass as that used on the previous pass plus a correction. In this paper the ILC law on pass $k+1$ is of the form

$$u_{k+1}(p) = u_k(p) + \Delta u_{k+1}(p), \tag{21}$$

where $\Delta u_{k+1}(p)$ is the correction term to be designed.

To write the ILC dynamics as a discrete linear repetitive process, introduce, for analysis purposes only, the vector

$$\xi_{k+1}(p+1) = x_{k+1}(p) - x_k(p). \tag{22}$$

Then the controlled dynamics can be written as

$$\begin{aligned}\xi_{k+1}(p+1) &= A\xi_{k+1}(p) + B\Delta u_{k+1}(p-1), \\ e_{k+1}(p) &= -CA\eta_{k+1}(p) + e_k(p) \\ &\quad - CB\Delta u_{k+1}(p-1).\end{aligned} \tag{23}$$

If the control correction term ensures exponential stability of (23), then the ILC law (21) converges in the sense that conditions of (3) hold.

Introduce the notation $A_{11} = A,\ A_{11c} = A + BK_1,\ A_{12} = 0,\ A_{12c} = BK_2,\ A_{21} = -CA,\ A_{21c} = -CA - CBK_1,\ A_{22} = I,\ A_{22c} = I - CBK_2,\ B_1 = B,\ B_2 = -CB,\ v_{k+1}(p) = \Delta u_{k+1}(p-1) - K_1 x_{k+1}(p) - K_2 y_k(p)$ and rewrite (23), for technical purposes only, in the following equivalent form

$$\begin{aligned}\xi_{k+1}(p+1) &= A_{11c}\xi_{k+1}(p) + A_{12c}e_k(p) + B_1 v_{k+1}(p), \\ e_{k+1}(p) &= A_{21c}\xi_{k+1}(p) + A_{22c}e_k(p) + B_2 v_{k+1}(p), \\ &\quad 0 \le p \le T-1;\ k = 0, 1, 2, \ldots.\end{aligned} \tag{24}$$

Also, to simplify the notation introduce the block vectors and matrices $\bar{x}_{k+1}(p) = \begin{bmatrix} \xi_{k+1}(p) \\ e_k(p) \end{bmatrix}$, $\bar{A} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, $K = [K_1\ K_2]$. (The aim of this step is to stabilize $\bar{A}$ (if required) by applying the stabilizing control law defined by K.)

Assume that matrix K satisfies the following matrix inequality

$$(\bar{A} + \bar{B}K)^T P(\bar{A} + \bar{B}K) - P + Q + K^T RK \preceq 0 \tag{25}$$

where $P \succ 0$, $Q \succ 0$ and $R \succ 0$ are weighting matrices. This inequality is easily reduced to a Linear Matrix Inequality (LMI) with respect to $X = \mathrm{diag}[X_1\ X_2]$, where $X_1 = P_1^{-1}$ and $X_2 = P_2^{-1}$ and $Y = KX$:

$$\begin{bmatrix} X & (\bar{A}X + \bar{B}Y)^T & X & Y^T \\ \bar{A}X + \bar{B}Y & X & 0 & 0 \\ X & 0 & Q^{-1} & 0 \\ Y & 0 & 0 & R^{-1} \end{bmatrix} \succeq 0, \quad X \succ 0. \tag{26}$$

If these LMis are feasible then $K = YX^{-1}$.

The next theorem describes a set of stabilizing control laws for (23) and hence a parameterized set of ILC laws.

Theorem 3: The dynamics of (23) are exponentially passive with respect to the output

$$z_{k+1}(p) = 2\bar{B}^T P\bar{A}_c\bar{x}_{k+1}(p) + \bar{B}^T P\bar{B}v_{k+1}(p), \quad (27)$$

where

$$\bar{A}_c = \bar{A} + \bar{B}K$$

and a set of feedback stabilizing control laws for the system with this output is given by

$$\Delta u_{k+1}(p-1) = F(\bar{x}_{k+1}(p))\bar{x}_{k+1}(p), \quad (28)$$

where

$$F(\bar{x}_{k+1}(p)) = [I + \Gamma(\bar{x}_{k+1}(p))\bar{B}^T P\bar{B}]^{-1}[(I - \Gamma(\bar{x}_{k+1}(p))\bar{B}^T P\bar{B})K - 2\Gamma(\bar{x}_{k+1}(p))\bar{B}^T P\bar{A}], \quad (29)$$

where $\Gamma(\bar{x}) \succeq 0$ for all $\bar{x} \in \mathbb{R}^{n_x+n_y}$ is an arbitrary matrix of compatible dimensions and the pair of matrices $P = X^{-1}$ and K solve (26).

Proof: Consider the candidate vector storage function as (9) with $V_1(\xi_{k+1}(p)) = \xi_{k+1}^T(p)P_1\xi_{k+1}(p), V_2(e_k) = e_k^T(t)P_2e_k(t)$, where $P_1 \succ 0$ and $P_2 \succ 0$. Calculating the divergence of (9) along the trajectories of (24) gives

$$\mathcal{D}_d V = \bar{x}^T(\bar{A}_c^T P\bar{A}_c - P)\bar{x} + (2\bar{x}^T\bar{A}_c^T P\bar{B} + v^T\bar{B}^T P\bar{B})v, \quad (30)$$

where $P = \mathrm{diag}[P_1\ P_2]$. Choose the output z as (27), then

$$\mathcal{D}_d V(x,y) \leq -\bar{x}^T Q\bar{x} + z^T v \quad (31)$$

and it follows from (31) that (24) is exponentially passive with respect to input v and output (27). By Theorem 2, the control law

$$v_{k+1}(p) = -\Gamma(\bar{x}_{k+1}(p))z_{k+1}(p), \quad (32)$$

results in exponential stability of (24). Then by routine calculations it follows from (32), (27) and the definition of v that each control law of from (28) results in exponential stability of (23). ■

V. EXAMPLE

As an example consider a flexible link with state-space model

$$\dot{x} = A_o x + B_o u, \quad (33)$$

where $x = [\theta\ \alpha\ \dot{\theta}\ \dot{\alpha}]^T$, θ is the servo angle, α is the flexible link angle (Fig.1), $B_o = [0\ 0\ -\frac{K_s(J_1+J_{eq})}{J_1 J_{eq}}\ \frac{1}{J_{eq}}\ -\frac{1}{J_{eq}}]^T$,

$$A_o = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_s}{J_{eq}} & -\frac{B_{eq}}{J_{eq}} & 0 \\ 0 & -\frac{K_s(J_l+J_{eq})}{J_l J_{eq}} & \frac{B_{eq}}{J_{eq}} & 0 \end{bmatrix},$$

K_s is the stiffness of the flexible link, J_{eq} is the moment of inertia of the servo, B_{eq} is the viscous friction coefficient of the servo and J_l is the moment of inertia of the flexible link. The results shown in the remainder of this paper are for the case when $K_s = 1.3\ N\ m/rad$, $J_{eq} = 2.08 \times 10^{-3}\ kg\ m^2$, $B_{eq} = 0.004\ N\ m/(rad/s), J_l = 0.0038\ kg\ m^2$.

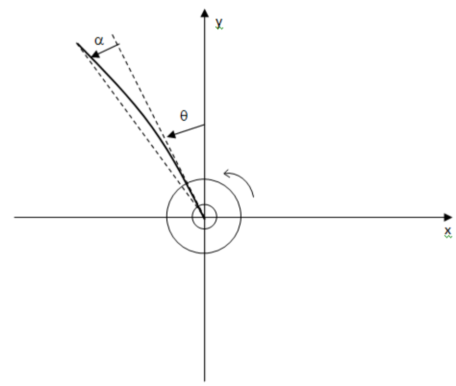

Fig. 1. Rotary flexible link angles.

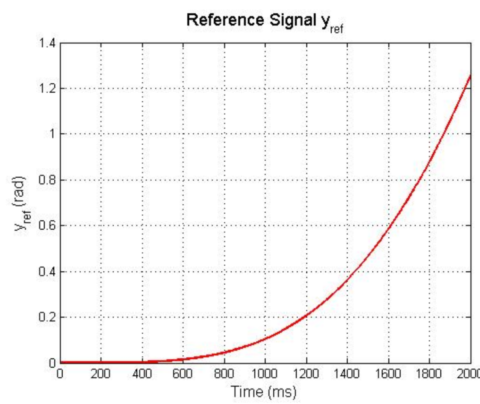

Fig. 2. The reference trajectory

The required reference trajectory is the servo angle $\theta(t)$, chosen to simulate a "pick and place" process of duration 2 sec and is shown in Fig. 2.

For the flexible link model (33) $A = \exp(A_o T_s)$, $B = \int_0^{T_s} \exp(A_o\tau)B_o d\tau$, where T_s is sampling time, and $C = [1\ 0\ 0\ 0]$. Since $CB \neq 0$ it is possible to use the model (23) for ILC design and on applying Theorem 3 the set of stabilizing control laws is given by

$$\Delta u_{k+1}(p-1) = [I + \Gamma(\bar{x}_{k+1}(p))\bar{B}^T P\bar{B}]^{-1}[(I - \Gamma(\bar{x}_{k+1}(p))\bar{B}^T P\bar{B})K - 2\Gamma(\bar{x}_{k+1}(p))\bar{B}^T P\bar{A}]\bar{x}_{k+1}(p), \quad (34)$$

where $\bar{A} = \begin{bmatrix} A & 0 \\ -CA & I \end{bmatrix}$, $\bar{B} = \begin{bmatrix} B \\ -CB \end{bmatrix}$ and matrices K and P are given by the solution of the LMI (26).

Choosing $T_s = 0.01$ sec and applying Theorem 3 for the case with $\Gamma = 0$ and

$$Q_1 = \mathrm{diag}[10^{-5}\ 10^{-5}\ 20\ 0.5],\ \ Q_2 = 10^4,\ \ R = 10^{-9}$$

gives

$$F = \begin{bmatrix} -25.565 & -1.475 & -0.338 & -0.0093 & 2.126 \end{bmatrix}.$$

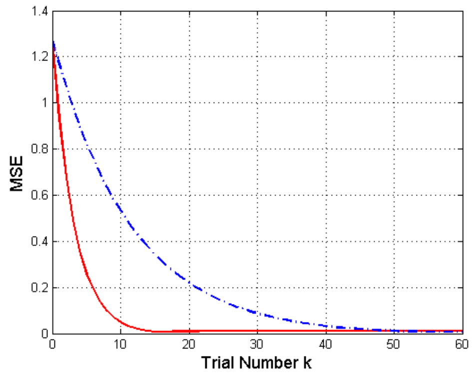

Fig. 3. Comparing the performance of the controllers based on using of discrete model: $\Gamma = 0$ (dashed line), $\Gamma = 1 + ||\bar{x}||^2$ (solid line).

To measure the performance of this ILC law and the one designed below, introduce

$$E(k) = \sqrt{\frac{1}{T}\sum_{p=0}^{T}||e_k(p)||^2}$$

and Fig. 3 plots this function for the above design (dashed plot), which shows monotonic pass-to-pass error convergence, where after 40 passes the error is zero.

An option to increase the speed of pass-to-pass convergence is through the choice of $\Gamma(\bar{x})$. As one case set $\Gamma = 1 + ||\bar{x}||^2$, where by exponential stability $\bar{x} \to 0$ as $k \to \infty$ and as $k \to \infty$ $F(\bar{x})$ tends to

$$F = \begin{bmatrix} -13.288 & -1.135 & -0.278 & -0.0085 & 3.58 \end{bmatrix}.$$

For the same Q and R significantly faster pass-to-pass error convergence occurs as also shown in Fig. 3 (solid line). These confirm that monotonic pass-to-pass error convergence occurs for both designs but the choice $\Gamma = 1 + ||\bar{x}||^2$ has better performance in terms of the convergence rate. In particular, the initial error is reduced by a factor of 10 in 5 passes for $\Gamma = 1 + ||\bar{x}||^2$ but 10 passes are required under $\Gamma = 0$. Moreover, convergence occurs after 10 passes for $\Gamma = 1 + ||\bar{x}||^2$ but the stability along the pass with $\Gamma = 0$ requires 40 passes. Further tuning of this ILC is possible by varying the matrices Q and $\Gamma(\bar{x})$ and hence the ILC design performance can be tailored to the requirements of the example under consideration.

VI. CONCLUSIONS

This paper has developed a new ILC design for linear dynamics using the stability and passivity theory for discrete nonlinear repetitive processes. The result is a parameterized control law and hence a large range options in terms of tuning the design for a given example. Further research should aim to develop mechanisms to select the tuning parameter(s) to best effect and if successful proceed experimental validation. Moreover, the approach should extend naturally to various forms of nonlinear dynamics, e.g., in the sensors and/or actuators during implementation or nonlinear processes.

REFERENCES

[1] E. Rogers, K. Gałkowski, and D. Owens, *Control Systems Theory and Applications for Linear Repetitive Processes. Lecture Notes in Control and Information Sciences, Vol. 349*. Berlin: Springer-Verlag, 2007.

[2] N. Yeganefar, N. Yeganefar, M. Ghamgui, and E. Moulay, "Lyapunov theory for 2-D nonlinear Roesser models: Application to asymptotic and exponential stability," *IEEE Transactions on Automatic Control*, vol. 58, pp. 1299–1304, 2013.

[3] P. V. Pakshin, J. Emelianova, K. Gałkowski, and E. Rogers, "Dissipivity and stabilization of nonlinear repetitive processes," *Systems & Control Letters*, vol. 91, pp. 14–20, 2016.

[4] J.C.Willems, "Dissipative dynamical systems part I: General theory," *Arch. Rational Mech. Analysis*, vol. 45, pp. 325 – 351, 1972.

[5] C. Byrnes, A. Isidori, and J. Willems, "Passivity, feedback equivalence and the global stabilization of minimun phase nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 1228 – 1240, 1991.

[6] W. Lin and C. Byrnes, "Passivity and absolute stabilization of a class of discrete-time nonlinear systems," *Automatica*, vol. 31, pp. 263 – 267, 1995.

[7] W. Haddad, V. Chellabona, and S. Nersesov, "Vector dissipativity theory and stability of feedback interconnections for large-scale nonlinear dynamical systems," *Int.J. Control*, vol. 77, pp. 907–919, 2004.

[8] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning," *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123–140, 1984.

[9] H.-S. Ahn, Y.-Q. Chen, and K. L. Moore, "Iterative learning control: Brief survey and categorization," *IEEE Transactions on Systems, Man and Cybernetics, Part C: Applications and Reviews*, vol. 37, no. 6, pp. 1099–1121, 2007.

[10] D. Bristow, M. Tharayil, and A. Alleyne, "A survey of iterative learning control," *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96–114, 2006.

[11] L. Hladowski, K. Galkowski, Z. Cai, E. Rogers, C. Freeman, and P. Lewin, "Experimentally supported 2D systems based iterative learning control law design for error convergence and performance," *Control Engineering Practice*, vol. 18, pp. 339– 348, 2010.

[12] W. Paszke, E. Rogers, K. Galkowski, and Z. Cai, "Robust finite frequency range iterative learning control design with experimental verification," *Control Engineering Practice*, vol. 23, pp. 1310–1320, 2013.

[13] J. Emelianova, P. Pakshin, K. Gałkowski, and E. Rogers, "Vector Lyapunov function based stability of a class of applications relevant 2D nonlinear systems," *IFAC Proceedings Volumes (IFAC-PapersOnline)*, vol. 47, no. 3, pp. 8247–8252, 2014.

[14] A. Fradkov and D. Hill, "Exponential feedback passivity and stabilizability of nonlinear systems," *Automatica*, vol. 34, pp. 697 – 703, 1998.

[15] H. Khalil, *Nonlinear Systems. Third Edition*. New Jersey: Prentice Hall, 2002.