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Prescribed Interactions among Agents for Swarm Aggregation on a Circle

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***Abstract*— In this work a kinematic model for the evolution of a swarm of agents is proposed. The model benefits of an intuitive structure formed by three main terms, accounting for attraction to a common swarm target, for interactions among all the agents and for peculiar relationship among each agent and a subset of elements. The overall agents evolution is studied and analyzed showing that the swarm tends to a common goal while simultaneously ensuring a set of aggregation properties. Moreover it will be proved that the proposed kinematic model exhibits an equilibrium point on a circular shape. Numerical simulations and experimental tests using a swarm of real mobile robots have been also carried out.**

I. Introduction

Research attention has been attracted in multi-agent systems, like swarms and swarm robotics, in the last decades. Reynolds [1] firstly described swarm behavior studying a flock of birds in flight and associating to this context a behavioral model for each swarm member, based on a set of simple rules that each bird should follow on the basis of the interactions among it and its near agents. Same lines of behavior can be found in fish schools, insect swarms, bacteria swarms, quadruped herds [2], [3]. The common understanding of the above works is that in nature attraction and repulsion forces between individuals lead to swarming behavior.

In a swarm of agents, a set of entities exhibit a collective behavior to aggregate together. In a general point of view, nowadays the term swarm is applied also to inanimate entities when they exhibit parallel behaviors, like robots swarm or stars swarm. The main advantage of swarm based evolutions consists in the simplicity of the rules for each entity, which however provides a complex global behavior with no central coordination.

In this context, probably the most interesting and challenging topic is global shape formation. For instance in a nature scenario, swarm formation could be required for energy saving or path optimization. Considering again the flocks of birds, during their migration, they tend to form a V shape so that to reduce the air resistance and simultaneously decrease each bird fatigue [4]. Following the same lines, among insects, ants tend to optimize their path to the food forming a line formation [5], [6] so that the entire colony can find the shortest path to the food source [7]. Shape formation is also specifically required to accomplish some tasks like sensing grids formation [8]; underwater or hazardous environments autonomous mapping and exploration [9]; areas surveillance or protection [10].

Researchers start their studies about swarm analysis from an abstract point of view, with no context specifications [11], [12], [13], [14]. These works propose a deep analysis on swarm formation properties starting from interactions model assumptions. In particular, steady-state properties and aggregations capabilities for the agents are proved assuming a model where each agent has complete knowledge about all other agents locations. Starting from this abstract point of view, swarm evolution inspires significant interests in various specific research fields, like clustering [15], vehicles or robots moving on the ground [16], [17], [18], [19], [20], [21], [22], underwater or flying in the sky [23], [24]. In particular, in the mobile robotics context, swarm interactions differentiate from other multi-robot systems since in a swarm formed by robots, each agent has to be autonomous and homogeneous with other ones and the robots have to be unconscious about the main task to be solved and they have to collaborate to succeed and to improve the overall swarm performance. Aiming at controlling the swarm formation, [25] proposes a solution based on artificial potential fields and introducing a limiting functions holding the agents on an enclosed curvilinear or ellipse formation. This resulting formation is then moved as a unit, to dynamically change and adapt to non-uniform surfaces. Geometric formations for multivehicle systems have been studied by [26] too. The authors extend the linear cyclic pursuit scenario to the case where each agent is a kinematic unicycle with nonholonomic constraints showing that the system's equilibrium formations are generalized regular polygons. Other applications in this context, including also flight vehicles, are provided in [27], [28].

Usually in a swarm behavioral analysis a microscopic and a macroscopic level can be found. The microscopic level consists in the individual level where, for example, the trajectory of each agent and the distances between agents are managed. On the other side, the macroscopic level is the group level and it accounts only for tasks related to the overall swarm of agents like a mean trajectory of the swarm centroid. This last problem has been faced in [29] where the authors propose to use a weighted sum of the formation and tracking errors as the main signal to be regulated to zero by means of a direct adaptive fuzzy controller augmented with sliding mode and bounding terms.

In this paper an interaction model for a swarm evolution is described. Each agent is driven by means of three main

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terms: the first one accounts for attraction to a given location representing the swarm centroid target; the second one relate the agents to a subset of other swarm elements to weight the inner-agents distances aiming at yielding to a circular formation; the last term models the interaction with all the swarm agents and its behavior can be modulated by means of an interaction parameter. This parameter can be tuned to opportunely relate the agents, allowing for some overall swarm behaviors.

In particular it will be shown that, based on the interaction parameter, the swarm can exhibit a more aggregated static configuration and aiming at reaching a different circle at the equilibrium. Moreover the swarm centroid will converge to the agents target whatever are the initial conditions and the proposed kinematic model will ensure that a static steady-state configuration will be always reached avoiding collisions among agents during their overall evolution. Further results have been discussed in [30] where a similar model with a reference finite-time tracking for a set of agents has been considered. To the best of authors' knowledge, the most similar model to the proposed one has been presented in [11]. The proposed model mainly differentiates in the agents interaction term which is not formulated as the sum of an attraction effect and of a repulsive effect. Moreover the presented kinematic evolution presents an equilibrium point on a circular agents formation.

The paper is organized as follows: Section II describes the proposed swarm kinematic model and analyzes the problem of the swarm agents clashes avoidance. The main properties of the swarm are discussed in Section III. Section IV shows some numerical and experimental results and the last Section is devoted to conclusions.

II. Swarm model definition

Let $z_i(t),\ i = 1, ..., n$ be a swarm formed by n agents moving on a 2-dimensional space. The swarm interactions are encoded by a time constant fully connected undirected graph which means that each member knows the exact position of all the other ones. Moreover the agents are considered as ideal points moving simultaneously without delay.

The evolution of each agent, $z_i(t),\ i = 1, ..., n$, obeys to the following equation

$$\dot{z}_i(t) = -3z_i(t) + \mathcal{I}_i(t) + \frac{\beta}{n}\sum_{j=1}^{n}\frac{z_i(t) - z_j(t)}{||z_i(t) - z_j(t)||}, \qquad (1)$$

where the term $-3z_i(t)$ accounts for an attraction to a common origin for the agents, $\beta \in \mathbb{R}^+$ models the interactions among the i-th agent and the overall swarm while $\mathcal{I}_i(t)$ rules the interaction of the agent i with two other agents, denoted as its predecessor and successor agents, i.e.

$$\mathcal{I}_i(t) = \begin{cases} z_2(t) + z_n(t), & i = 1, \\ z_{i-1}(t) + z_{i+1}(t), & i = 2, ..., n-1, \\ z_1(t) + z_{n-1}(t), & i = n. \end{cases} \qquad (2)$$

Note that the same model could be easily adapted for modeling the attraction to a target different from the origin by means of a simple reference frame change.

Let $\mathcal{S}_w, w = 1, \ldots, N_c$, be disjoined sets of indexes, at a given time t_0, each one related to agents at collision risk, that is $||z_i - z_j|| \leq \varepsilon, \forall i, j \in \mathcal{S}_w, \forall w = 1, \ldots, N_c$. Let $i, j \in \mathcal{S}_w$ and $\zeta_{i,j}(t) \triangleq z_i(t) - z_j(t)$, then

$$\begin{aligned} \dot{\zeta}_{i,j}(t) &= -3\zeta_{i,j}(t) + \zeta_{i-1,j-1}(t) + \zeta_{i+1,j+1}(t) + \\ &+ \frac{\beta}{n}\left(\sum_{r=1}^{n}\frac{\zeta_{i,r}(t)}{||\zeta_{i,r}(t)||} - \sum_{s=1}^{n}\frac{\zeta_{j,s}(t)}{||\zeta_{j,s}(t)||}\right). \end{aligned}$$

At this point, since the terms $\zeta_{i,q} \approx \zeta_{j,q}, \forall q \notin \mathcal{S}_w$ then the evolution of $\zeta_{i,j}$ depends approximatively by the evolution of the agents which are in $\mathcal{S}_w$. Therefore

$$\dot{\zeta}_{i,j}(t) \approx -3\zeta_{i,j}(t) + \zeta_{i-1,j-1}(t) + \zeta_{i+1,j+1}(t) + \frac{\beta|\mathcal{S}_w|}{n\varepsilon}\zeta_{i,j}(t)$$

where $|\mathcal{S}_w|$ is the cardinality of the set $\mathcal{S}_w$. The term $\zeta_{i-1,j-1}(t) + \zeta_{i+1,j+1}(t)$ can be ignored since as $\varepsilon \to 0$ the last term is dominant, i.e.

$$\dot{\zeta}_{i,j}(t) \approx -\left(3 - \frac{\beta|\mathcal{S}_w|}{n\varepsilon}\right)\zeta_{i,j}(t).$$

The above equation states that $\zeta_{i,j}(t)$ can be fairly assumed to not converge to zero if $\left(3 - \frac{\beta|\mathcal{S}_w|}{n\varepsilon}\right) < 0$.

A. Model decentralization

Note that the proposed model has been formulated in a centralized fashion, where all the agents have complete knowledge about their own position w.r.t. the same reference frame. In a real context this assumption could be restrictive since each agent needs global information instead of local one. However in many scenarios, for example in a mobile-robot context [31], [32], each robot could benefit of an absolute positioning system (a centralized camera positioning system for indoor applications providing the required information to all agents, or GPS for outdoor environments). Moreover if a decentralized swarm is strongly required by the applications, the proposed model can be easily adapted by using only information about displacements between agents and between each agent and the target location (available by standard sensors).

Considering the general case where the swarm goal $G \in \mathbb{R}^2$ is not the origin the model 1 can be written as

$$\dot{z}_i(t) = -(z_i(t) - G) + \Delta\mathcal{I}_i(t) + \frac{\beta}{n}\sum_{j=1}^{n}\frac{z_i(t) - z_j(t)}{||z_i(t) - z_j(t)||},$$

where $\Delta\mathcal{I}_i(t)$ is the displacement between the i-th agent and its predecessor and successor agents, i.e.

$$\Delta\mathcal{I}_i(t) = \begin{cases} z_1(t) - z_2(t) + z_1(t) - z_n(t), & i = 1, \\ z_i(t) - z_{i+1}(t) + z_i(t) - z_{i-1}(t), & i = 2, ..., n-1, \\ z_n(t) - z_1(t) + z_n(t) - z_{n-1}(t), & i = n. \end{cases}$$

The resulting decentralized model will then be

$$\dot{\Delta}_{i,G}(t) = -\Delta_{i,G}(t) - \Delta\mathcal{I}_i(t) + \frac{\beta}{n}\sum_{j=1}^{n}\frac{\Delta_{i,j}(t)}{||\Delta_{i,j}(t)||}, \qquad (3)$$

by defining

- $\Delta_{i,G}(t) \triangleq z_i(t) - G$ is the displacement between the agent location and the swarm target position G. This term takes into account the environment effects on the agents, i.e. it contains information about the favorable regions to which the agents may want to move and it is typical in social foraging models as discussed in [33];
- $\Delta_{i,j}(t) \triangleq z_i(t) - z_j(t)$ is the displacement between the i-th and the j-th agents.

In a real scenario, each swarm element can use the signal $\Delta_{i,G}(t)$ as a reference trajectory for its displacement from the goal. In other words, as shown in Fig. 1, a control scheme can be designed for each agent where its measured displacement from G, available by standard local sensors placed on the agent, is fed back to a properly chosen control law and forced to follow $\Delta_{i,G}(t)$.

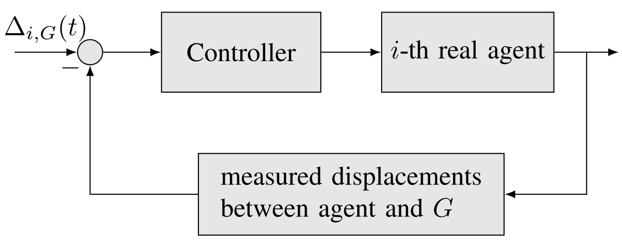

Fig. 1. Displacement control scheme for the i-th real agent.

III. Swarm properties

In this Section, the swarm behavior is analyzed in terms of the centroid evolution and steady-state agents configuration. The explicit dependence of the time t is omitted for the sake of compactness.

Lemma 1: The swarm centroid converges to the origin whatever is the swarm initial condition $\{z_i(t_0)\}, i = 1, \ldots, n$.

Proof: The swarm centroid is defined as

$$\overline{z} = \frac{1}{n}\sum_{i=1}^{n} z_i.$$

Therefore, its evolution is governed by

$$\dot{\overline{z}} = -\frac{3}{n}\sum_{i=1}^{n} z_i + \frac{1}{n}\sum_{i=1}^{n}\mathcal{I}_i + \frac{\beta}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{z_i - z_j}{||z_i - z_j||}.$$

Noting that the last term is null, due to the symmetry in the summations, then

$$\dot{\overline{z}} = -3\overline{z} + 2\overline{z} = -\overline{z}$$

which states that the centroid exponentially tends to the origin. ■

The overall swarm aggregation properties will be now studied. In particular it will be shown under which assumptions the agents asymptotically converge to a circular region centered in the origin where they reach a steady-state configuration. Moreover it will be proved that the proposed models presents an equilibrium point on a circular agents formation.

Let us consider the scalar function

$$U = \sum_{k=1}^{n} H_k,$$

where H_k has the following expression

$$H_k = \frac{3}{\beta} z_k^T z_k - \frac{1}{\beta} z_k^T \mathcal{I}_k - \frac{1}{n}\sum_{j=1}^{n} ||z_k - z_j||.$$

Theorem 1: U is a non-increasing function.

Proof: Note that

$$\begin{aligned}\frac{\partial H_k}{\partial z_k} &= \frac{6}{\beta} z_k - \frac{1}{\beta}\mathcal{I}_k - \frac{1}{n}\sum_{j=1}^{n}\frac{z_k - z_j}{||z_k - z_j||}, \\ &= -\frac{1}{\beta}(\dot{z}_k - 3z_k) \qquad (4)\end{aligned}$$

and

$$\frac{\partial H_k}{\partial z_q}_{q\neq k} = -\frac{1}{\beta} z_k\left(\hat{\delta}_{k-1,q} + \hat{\delta}_{k+1,q}\right) - \frac{1}{n}\frac{z_q - z_k}{||z_q - z_k||} \qquad (5)$$

where $\hat{\delta}_{i,q}$ is defined as

$$\hat{\delta}_{i,q} = \begin{cases} \delta(n-q), & i = 0, \\ \delta(i-q), & i = 1, ..., n, \\ \delta(1-q), & i = n+1 \end{cases}$$

and $\delta(\cdot)$ is the classical Kronecker delta function.

It follows that

$$\dot{U} = \sum_{k=1}^{n}\left(\frac{\partial H_k}{\partial z_k}\right)^T \dot{z}_k + \sum_{k=1}^{n}\sum_{q=1, q\neq k}^{n}\left(\frac{\partial H_k}{\partial z_q}\right)^T \dot{z}_q \qquad (6)$$

and then, by substituting (4) and (5) in (6):

$$\begin{aligned}\dot{U} &= \sum_{k=1}^{n}\left(-\frac{1}{\beta}\dot{z}_k + \frac{3}{\beta}z_k\right)^T \dot{z}_k \\ &+ \sum_{q=1}^{n}\sum_{k=1}^{n}\left(-\frac{1}{\beta}z_k(\hat{\delta}_{k-1,q} + \hat{\delta}_{k+1,q}) - \frac{1}{n}\frac{z_q - z_k}{||z_q - z_k||}\right)^T \dot{z}_q\end{aligned}$$

which can be rewritten as

$$\begin{aligned}\dot{U} &= \sum_{k=1}^{n}\left(-\frac{1}{\beta}\dot{z}_k + \frac{3}{\beta}z_k\right)^T \dot{z}_k + \\ &+ \sum_{q=1}^{n}\left(-\frac{1}{\beta}\mathcal{I}_q - \frac{1}{n}\sum_{k=1}^{n}\frac{z_q - z_k}{||z_q - z_k||}\right)^T \dot{z}_q\end{aligned}$$

and finally

$$\dot{U} = -\frac{2}{\beta}\sum_{k=1}^{n}\dot{z}_k^T \dot{z}_k < 0, \quad \forall \dot{z}_i \neq 0_2.$$

■

The above result will be extremely useful for the following analysis where the agents convergence region will be found.

Theorem 2: The agents enter in a circle of radius equal to β.

Proof: Let

$$h_1 = -\frac{1}{\beta}\sum_{i=1}^{n} z_i^T\left(-3z_i + \mathcal{I}_i\right),$$

$$h_2 = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} ||z_i - z_j||.$$

Using Theorem 1 it follows that $\dot{h}_1 < \dot{h}_2,\ \forall \dot{z}_i \neq 0_2$.

Consider the Lyapunov function

$$V = \frac{1}{2}(h_1 - h_2)^2$$

from which

$$\dot{V} = (h_1 - h_2)(\dot{h}_1 - \dot{h}_2).$$

Note that a sufficient condition for $\dot{V}$ to be definite negative is that $h_1 > h_2$.

By considering

$$-\frac{1}{\beta} z_i^T \mathcal{I}_i \geq -\frac{1}{\beta} z_i^T z_i - \frac{1}{\beta}\mathcal{I}_i^T \mathcal{I}_i,$$

it follows that

$$h_1 \quad \geq \quad \frac{2}{\beta}\sum_{i=1}^{n} z_i^T z_i - \frac{1}{\beta}\sum_{i=1}^{n}\mathcal{I}_i^T\mathcal{I}_i \geq \frac{2}{\beta}\sum_{i=1}^{n} z_i^T z_i.$$

Moreover

$$h_2 \leq 2\sum_{i=1}^{n} ||z_i||.$$

Therefore $\dot{V}$ is definite negative if

$$\frac{1}{\beta}\sum_{i=1}^{n} ||z_i||^2 \geq \sum_{i=1}^{n} ||z_i||.$$

Then it is sufficient that $||z_i|| \geq \beta$. ■

Remark 1: Results stated in Theorem 2 guarantee that agents enter and remain in the compact set $||z_i|| < \beta,\ \forall i = 1, ..., n$. For this reason, using Theorem 1 and the LaSalle criterion, the swarm is ensured to converge to the invariant set defined as

$$\Omega = \left\{ z_i \mid \dot{U} = 0 \right\} = \{ z_i \mid \dot{z}_i = 0 \}$$

which means that the swarm agents converge to their equilibrium points (i.e. the swarm reaches a constant configuration).

Theorem 3: An equilibrium configuration for the swarm (1) is

$$z_i = \rho \left[\begin{array}{cc} \cos\left(\frac{2\pi i}{n}\right), & \sin\left(\frac{2\pi i}{n}\right) \end{array} \right]^T \tag{7}$$

with

$$\rho = \frac{\frac{\beta}{n}\cot\left(\frac{\pi}{2n}\right)}{3 - 2\cos\left(\frac{2\pi}{n}\right)}. \tag{8}$$

Proof: Let $f_{i,j} = \dfrac{z_i - z_j}{||z_i - z_j||}$, then by substituting z_i with (7), it follows that

$$f_{i,j} = \mathrm{sgn}\left(\sin\left(\frac{i-j}{n}\pi\right)\right)\left[-\sin\left(\frac{i+j}{n}\pi\right), \cos\left(\frac{i+j}{n}\pi\right)\right]^T$$

where

$$\mathrm{sgn}(x) = \begin{cases} 1, & x \geq 0, \\ -1, & x < 0. \end{cases}$$

Moreover [34]

$$\sum_{j=1}^{n} f_{i,j} = \cot\left(\frac{\pi}{2n}\right)\left[\begin{array}{cc} \cos\left(\frac{2\pi i}{n}\right), & \sin\left(\frac{2\pi i}{n}\right) \end{array} \right]^T.$$

Therefore, the r.h.s of Eq. (1) can be rewritten as

$$\gamma \left[\begin{array}{cc} \cos\left(\frac{2\pi i}{n}\right), & \sin\left(\frac{2\pi i}{n}\right) \end{array} \right]^T$$

with

$$\gamma = \left(2\cos\left(\frac{2\pi}{n}\right) - 3\right)\rho + \frac{\beta}{n}\cot\left(\frac{\pi}{2n}\right).$$

This solution is then an equilibrium configuration if $\gamma = 0$ from which the proof follows. ■

Remark 2: Note that the equilibrium configuration of Theorem 3 is not assured to be the unique one. For this reason, the agents could reach a different constant steady-state configuration inside the circle of radius β.

IV. Simulations and Laboratory Experiments

The proposed kinematic model has been tested in both numerical and real contexts to further validate its main properties.

A. Numerical simulations

A swarm of $n = 20$ agents has been tested setting the interaction parameter as $\beta = 5$. Figs. 2 and 3 show the overall results in terms of the agents evolution highlighting that the agents reach a constant steady-state configuration on the circle of equilibrium with radius defined by Eq. (8).

B. Real experiment

In the experimental setup a swarm of 6 agents has been used tuning the interaction parameter as $\beta = 0.6$. Real mobile robots have been considered in this case. In particular the *Elisa*-3 robots by *GCtronic* [35] have been used in an environment of $0.8m \times 0.6m$. Each Elisa-3 is a differential drive mobile robot the model of which may be assumed as

$$\begin{cases} \dot{x}_r(t) = v_r(t)\cos(\theta_r(t)) \\ \dot{y}_r(t) = v_r(t)\sin(\theta_r(t)) \\ \dot{\theta}_r(t) = \omega_r(t) \end{cases}$$

where x_r, y_r, θ_r represent the robot pose in terms of position and orientation while v_r and ω_r are the model inputs, i.e. the imposed linear speed and rotation speed respectively.

In this case, a control scheme has been designed for each robot where its pose is fed back to a properly chosen control

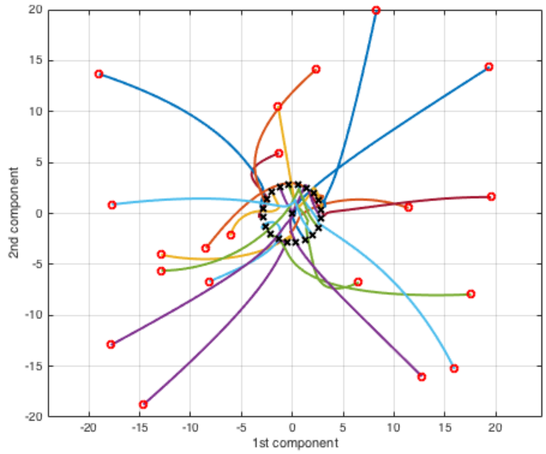

Fig. 2. Agents trajectories. Red circles are agents initial conditions, black crosses are agents final locations.

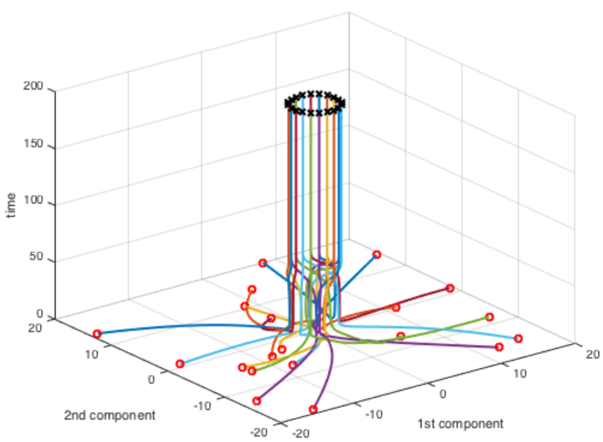

Fig. 3. Agents evolution w.r.t. time. Red circles are agents initial conditions, black crosses are agents final locations.

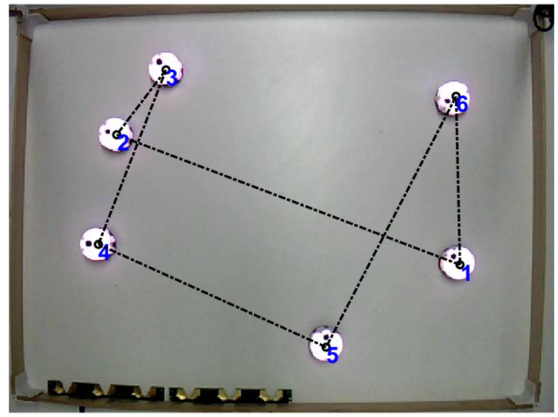

Fig. 4. Starting agents configuration.

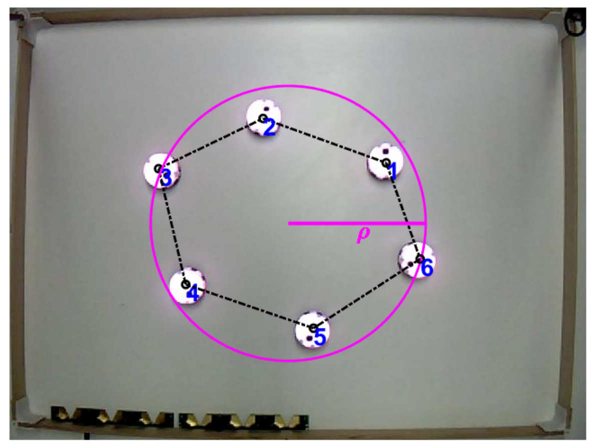

Fig. 5. Steady-state agents configuration.

law and forced to follow the kinematic trajectory given by the proposed model. In particular the input/output linearization control law, described in [36], has been used. The agents initial conditions are shown in Fig. 4 where the edges connect each agent with its predecessor and successor. Fig. 5 depicts the swarm final configuration highlighting that the steady-state formation is approximatively on the equilibrium circle with radius ρ. Fig. 6 shows the real agents evolution and the reference trajectories as imposed by the kinematic model. The agents trajectories w.r.t. time are shown in Fig. 7 where it can be noted that no collisions among agents exist. Note that, in a real scenario, the clashes avoidance can be ensured only if the kinematic model is modified, as shown in [33], to accomplish with a constraint on the minimum allowed distance between agents related to each real agent size.

V. Conclusions

In this work a kinematic model for the evolution of a swarm of agents has been discussed. The overall agents evolution has been studied and it has been proved that the swarm tends to a static steady-state configuration in a circle centered on a given target. It has been shown that the proposed kinematic model exhibits an equilibrium point on a circular shape too. Numerical simulations and experimental tests using a swarm of real mobile robots have been presented.

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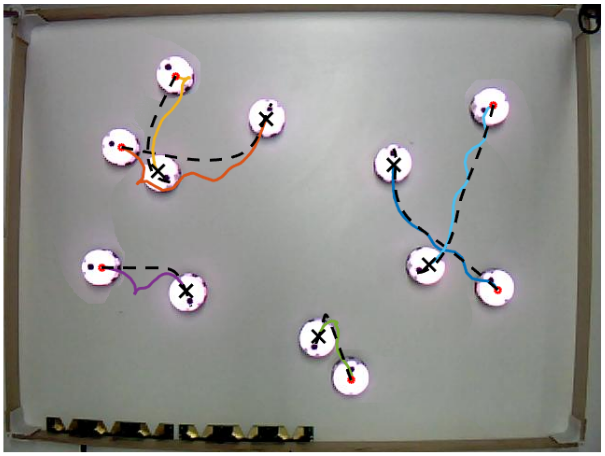

Fig. 6. Agents evolution. Red circles are agents initial conditions, black crosses are agents final locations. Dotted lines represent the reference trajectories imposed by the kinematic model.

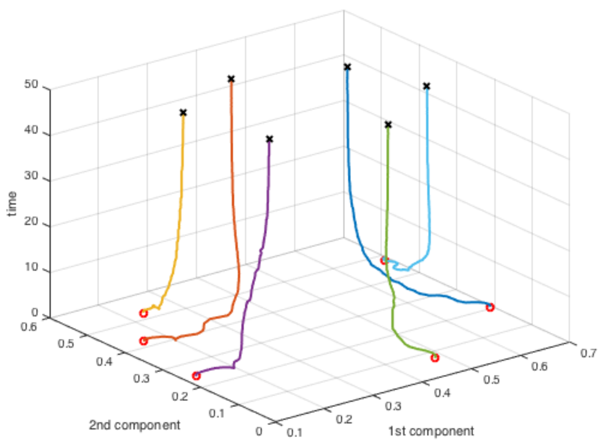

Fig. 7. Agents evolution w.r.t. time. Red circles are agents initial conditions, black crosses are agents final locations.

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