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MPC-based coordinated control design for look-ahead vehicles and traffic flow

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***Abstract*— The paper proposes a control design method for the coordination of a macroscopic traffic system and individual vehicles. Some of the vehicles are equipped with a look-ahead control system, with which energy-efficient traveling can be achieved. In a mixed traffic system vehicles equipped with look-ahead control and conventional uncontrolled vehicles are traveling together. The traffic system is assumed to have ramp metering control, which has a significant effect on the inflow on the highway. The purposes of the coordinated control are to improve the energy efficiency of the entire traffic flow and simultaneously stabilize the controlled highway, reduce the queue length at the controlled gates and avoid congestion. The design is based on the MPC method, with which the prediction of the traffic flow is also taken into consideration. The efficiency of the method is illustrated through VISSIM simulation examples.**

I. Introduction and motivation

Energy-efficient cruise control strategies for road vehicles have been in the focus in research and development centers in the vehicle industry. In the look-ahead control, which is one of these strategies, the forthcoming road and traffic information is exploited to design the speed of the vehicle, reduce energy requirement and fuel consumption while the traveling time is as short as possible [1], [2].

The modeling and analysis of mixed traffic, in which controlled vehicles and conventional uncontrolled vehicles are traveling together, require novel methodologies. The analysis of the traffic-flow, in which semi-automated and automated vehicles were traveling together with conventional vehicles, was proposed by [3], [4]. Since the semi-automated vehicles supported a smooth traffic flow through their filtering effects, a control law was proposed by [5], [6]. A method which is specially elaborated for look-ahead vehicles was presented by [7].

The purpose of the look-ahead cruise control is to find a balance between the minimization of the energy consumption and the traveling time of the vehicle, while the terrain characteristics and the speed limits of a finite forthcoming road horizon is considered. In our approach the impact of parameter R_1 on the speed profile of the individual vehicles is crucial. The mathematical background of the method was proposed in [8]. The parameter R_1 provides a priority between the energy and the time. If $R_1 = 0$ then time is minimized, while at $R_1 = R_{1,max}$ the energy consumption is reduced. Since the maximum value of $R_{1,max} = 1$ can cause the significant variation of the speed, it is recommended to select $R_{1,max} < 1$ in practice. Moreover, in the local traffic around the automated vehicle the speed must be selected to guarantee the safe distance from the surrounding vehicles. Thus, the selection of R_1 leads to an optimization

$$\max_{[0,R_{1,max}]} R_1, \tag{1}$$

where $R_{1,max}$ is a constant parameter, while R_1 is varied dynamically, see [9]. Since the speed profiles of the look-ahead vehicles may differ from those of the conventional vehicles, the characteristics of the traffic flow change. In practice, it is represented by the fundamental diagram, on which the energy-efficient scaling parameter R_1 has a high impact. If this parameter is increased, then the traffic flow decreases due to the decreased vehicle speed. However, the critical density $q_{i,crit}$, which is related to the maximum traffic flow, is higher. The selection of R_1 has an impact on both the outflow and the critical density, thus also on the stability of the traffic flow. Moreover, as another parameter, the rate of the look-ahead vehicles in the entire traffic flow κ has an impact on the fundamental diagram. A detailed analysis on the dependences is found in [10].

In this paper a method is developed in which the control of the macroscopic traffic flow and the cruise control of the local vehicles are coordinated. The contribution of the paper is an optimization strategy, which incorporates the nonlinearities and the parameter-dependency of the traffic system and the multi-optimization of the look-ahead vehicles. Consequently a trade-off between the parameters of the microscopic and the macroscopic models has been created. In the modeling part the Sum-of-Squares (SOS) programming method, while in the control design the Model Predictive Control (MPC) are applied. The role of the MPC control is to consider the prediction of the traffic flow in the design.

The coordination of the traffic system and the vehicles is presented on a highway. In the mixed traffic system vehicles equipped with look-ahead control and conventional uncontrolled vehicles are traveling together. In this task there are two intervention possibilities such as the ramp metering control on the highway gates and the parameter setting of the look-ahead cruise control. In an earlier paper a control design method to guarantee stability for only one highway section

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with a fixed energy-efficient parameter was proposed, see [10]. In this paper further performance specifications were also considered such as the total travel distances between the vehicles on the highway and the queue length at the controlled gates The purposes of coordinated control are to improve the energy efficiency of the entire traffic flow and simultaneously stabilize the controlled highway, reduce the queue length at the controlled gates and avoid congestion. The control requires that the energy-efficient parameters of the local vehicles are modified during traveling according to the information of the ramp metering.

The structure of the paper is the following. The parameter-dependent modeling and the stability analysis of the traffic system are presented in Section II. The control oriented-model with the performances and the MPC-based coordinated control strategy are proposed in Section III. The efficiency of the method is demonstrated through simulation examples in Section IV. Finally, Section V summarizes the concluding remarks.

II. Modeling and analysis of the traffic flow with look-ahead vehicles

A. Modeling of the traffic flow

The modeling of traffic dynamics in highway systems is based on the law of conservation. The relationship contains the sum of inflows and the outflows for a given highway segment i. The traffic density ρ_i $[veh/km]$ is expressed in the following way:

$$\rho_i(k+1) = \rho_i(k) + \frac{T}{L_i}\left[q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k)\right], \tag{2}$$

where k denotes the index of the discrete time step, T is the discrete sample time, L is the length of the segment, q_i $[veh/h]$ and q_{i-1} $[veh/h]$ denote the inflow of the traffic in segments i and $i-1$, r_i $[veh/h]$ and s_i $[veh/h]$ are the sum of ramp inflow and outflow values, respectively. In (2) the inflow $q_0(k)$ and the ramp outflow $s_i(k)$ are measured disturbance values, while $r_i(k)$ is a controlled ramp metering inflow, which is also known [11]. The outflow $q_i(k)$ of segment i incorporates the core of the traffic dynamics and depends on several factors, see e.g. [12], [13]. In the following $q_i(k)$ is formulated according to the fundamental relationship [14], which is generally derived through historic measurements [15], such as

$$q_i(k) = \mathcal{F}(\rho_i(k)). \tag{3}$$

Note that $q_i(k)$ is not only the outflow of segment i, but it is also the inflow of segment $i+1$. Although $q_i(k)$ is measured as the inflow of segment $i+1$, in the modeling it must be considered through the formula (3) to receive signals from the dynamics of the traffic flow.

In this paper the function $\mathcal{F}(\rho_i(k))$ is formed in a polynomial form, which is fitted to the historic traffic flow data. Since in the mixed traffic flow both look-ahead controlled vehicles and conventional uncontrolled vehicles are traveling together, the rate of the autonomous vehicles κ and their energy-efficient parameter $R_{1,i}$ must be considered

$$\mathcal{F}(\rho_i(k), R_{1,i}, \kappa) = \sum_{j=1}^{n} c_j(R_{1,i}, \kappa)\rho_i(k)^j \tag{4}$$

where the coefficients in the polynomials are formed as $c_j(R_{1,i}, \kappa) = \sum_{l=1}^{m} \left(d_l R_{1,i}^l \kappa^l\right)$ with constant d_l values. The mixed traffic model using (2) and (4) is as follows:

$$\begin{aligned}\rho_i(k+1) =& \rho_i(k) + \frac{T}{L_i}[-\mathcal{F}(\rho_i(k), R_{1,i}, \kappa) + \\ &+ q_{i-1}(k) + r_i(k) - s_i(k)]\end{aligned} \tag{5}$$

Since highways contain several ramps, it is also required to model the dynamics of the queue on the controlled gates. The length of the queues can be calculated through the following linear relationship, see [16]:

$$l_i(k+1) = l_i(k) + T\left(r_{i,dem}(k) - r_i(k)\right), \tag{6}$$

where l_i in the units of vehicle denotes the queue length, r_i is the control input [veh/h] and the demand is $r_{i,dem}$.

B. Analysis of system stability

Based on the mixed traffic model and considering the effects of κ and $R_{1,i}$ parameters, the stability of the traffic system will be analyzed. The density at the maximum of the fundamental diagram is called critical density $\rho_{crit,i}$. If the density of the traffic is lower than $\rho_{crit,i}$, the system can have an equilibrium point. However, if the density is higher than $\rho_{crit,i}$, the system is unstable.

The examination of the nonlinear system stability is based on the SOS method, in which the polynomial characteristics of the fundamental diagram (4) can be incorporated in [10]. One of the purposes of the analysis it to calculate the maximum inflows r_i and q_{i-1} function of $R_{1,i}$ and κ, with which the stability of the traffic flow can be guaranteed. In the following stability analysis a highway with one segment is examined, and the results are extended to the entire highway. It can guarantee the reduction of the analysis complexity and the computational efforts.

Using (5) the state space representation of the mixed-traffic system is the following form:

$$x(k+1) = f(R_{1,i}(k), x(k)) + g_1 u_{max}(R_{1,i}(k), \kappa(k)) + g_2 d(k) \tag{7}$$

where $x(k) = \rho_i(k)$ is the state of the system, $f(R_{1,i}(k), x(k))$ is a matrix, which incorporates smooth polynomial functions and its initial value is $f(R_{1,i}, 0) = 0$. $u_{max}(R_{1,i}(k), \kappa(k))$ is the function of the maximum controlled inflow $r_i(k)$ and $d_i(k) = q_{i-1}(k) - s_i(k)$ includes the measured disturbances of the system.

The stability analysis is based on the computation of the controlled invariant sets [17]. The parameter-dependent Control Lyapunov Function $\mathrm{V}(R_{1,i}(k), \kappa(k), x(k))$ is chosen in the form $V(x(k)) \cdot b(R_{1,i}(k), \kappa(k))$, where $b(R_{1,i}(k), \kappa(k))$ is an intuitively chosen parameter-dependent basis function. The existence of $\mathrm{V}(R_{1,i}(k), \kappa(k), x(k))$ is transformed into set-emptiness conditions. Moreover, the domains of $R_{1,min} \leq R_{1,i}(k) \leq R_{1,max}$ and $\kappa_{min} \leq \kappa(k) \leq \kappa_{max}$ are also formulated in the set-emptiness conditions. Using the generalized S-Procedure [18] the set-emptiness conditions can be transformed into the SOS existence problem, see [10].

As a result, an optimization problem is derived, in which the SOS conditions must be guaranteed. The optimization problem is to find an $u_{max}(R_{1,i}(k), \kappa(k))$ solution and feasible $\mathrm{V}(R_{1,i}(k), \kappa(k), x(k))$ for the following task:

$$\max\ u_{max}(R_{1,i}(k), \kappa(k)) \tag{8}$$

over $\quad s_1 \dots s_7 \in \Sigma_n;\ V(x(k)), b(R_{1,i}(k), \kappa(k)) \in \mathcal{R}_n$ such that

$$\begin{aligned}
&-\bigg((V(f(R_{1,i}(k), \kappa(k), x(k)) + g u_{max}(R_{1,i}(k), \kappa(k)))- \\
&- V(x(k))) \cdot b(R_{1,i}(k), \kappa(k)) + \nu \cdot V(x(k)) \bigg)- \\
&- s_1 \bigg(V(x(k)) \cdot b(R_{1,i}(k), \kappa(k)) - (1-\varepsilon) \bigg)- \\
&- s_2 \bigg(1 - V(x(k)) \cdot b(R_{1,i}(k), \kappa(k)) \bigg) - s_3 x(k)- \\
&- s_4 \left(R_{1,i}(k) - R_{1,min}\right) - s_5 \left(R_{1,max} - R_{1,i}(k)\right) - \\
&- s_6 \left(\kappa(k) - \kappa_{min}\right) - s_7 \left(\kappa_{max} - \kappa(k)\right) \in \Sigma_n.
\end{aligned} \tag{9}$$

where the set of SOS polynomials in n variables is defined as:
$\Sigma_n := \left\{ p \in \mathcal{R}_n \,\middle|\, p = \sum_{i=1}^{t} f_i^2,\ f_i \in \mathcal{R}_n,\ i = 1, \dots, t \right\}$.

The result of the optimization (8) defines the maximum Controlled Invariant Set, in which the system is stable with the function $u_{max}(R_{1,i}(k), \kappa(k))$. Thus, for the stability of the system the following inequality must be guaranteed:

$$q_{i-1}(k) + r_i(k) - s_i(k) \leq u_{max}(R_{1,i}(k), \kappa(k)) \tag{10}$$

The analysis of the results were previously proposed in [19]. It was stated that increases in $R_{1,i}$ and κ reduce the maximum number of vehicles of the traffic flow u_{max}. Consequently, in the control design it is necessary to find an appropriate coordination between the stability margin of the traffic system and the energy-optimal cruising of the individual vehicles.

III. MPC-BASED COORDINATED CONTROL STRATEGY

A. Control-oriented modeling and performances

The control design of the highway is based on the the model of the traffic flow (5) and the queue dynamics (6). The fundamental diagram of the traffic dynamics in (5) can be linearized at reduced density values. Thus, the system (5) is linearized at $\rho_i = \rho_{max}/2$, where the linear model is valid between $\rho = 0 \dots \rho_{max}$. In practice, ρ_{max} can be selected for the density at the maximum of the fundamental diagram. The mixed traffic model is as follows:

$$\begin{aligned}
\rho_i(k+1) = \rho_i(k) + \\
+ \frac{T}{L} \left(-\alpha(R_{1,i}, \kappa)\rho(k) + q(k) + r(k) - s(k)\right)
\end{aligned} \tag{11}$$

where $\alpha(R_{1,i}, \kappa)$ is the slope of the fundamental diagram in $\rho_i = 0$. The equation of one highway section is formed as

$$\begin{aligned}
\begin{bmatrix} \rho_i(k+1) \\ l_i(k+1) \end{bmatrix} = \begin{bmatrix} 1-\alpha(R_{1,i}, \kappa) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \rho_i(k) \\ l_i(k) \end{bmatrix} + \\
+ \begin{bmatrix} \frac{T}{L} & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} q(k) - s(k) \\ r_d \end{bmatrix} + \begin{bmatrix} \frac{T}{L} \\ -T \end{bmatrix} r_i(k),
\end{aligned} \tag{12}$$

By defining the state vector of one highway section $x_i(k) = \begin{bmatrix} \rho_i(k) & l_i(k) \end{bmatrix}^T$ the state-space representation can be formed.

Then all the highway sections f are taken into consideration in order to compress the result in the following matrix form:

$$x(k+1) = A(R_{1,i}(k), \kappa(k))x(k) + B_1 w(k) + B_2 u(k), \tag{13}$$

where $x(k) = \begin{bmatrix} x_1(k) & x_2(k) & \dots & x_f(k) \end{bmatrix}^T$ is the state vector, $w(k)$ is the disturbance and $u(k)$ is the control. In the following, the $A(R_{1,i}, \kappa)$ simplified denotion will be used instead of $A(R_{1,i}(k), \kappa(k))$.

In the traffic control problem two main performances are defined.

1./ The total travel distance must be maximized in order to guarantee the maximum outflow of the highway section. The outflow can be improved by increasing $\rho_i(k)$ until reaching the critical density. Although at a low number of inflow vehicles the $\rho_{crit,i}$ cannot be achieved, the total travel distance can be maximized with the following tracking criterion:

$$z_{1,i}(k) = \rho_i(k) - \rho_{crit,i}(k), \qquad |z_1| \to min. \tag{14}$$

In this performance specification $\rho_{crit,i}$ is the reference value. Note that $z_{1,i} > 0$ if $\rho_i(k) \ll \rho_{crit,i}$, but the total travel distance is maximized. $\rho_{crit,i}$ is selected through the previous analysis of the highway.

2./ The length of the queue on the controlled ramp metering must be reduced to minimize the waiting time of the vehicles:

$$z_{2,i}(k) = l_i(k), \qquad |z_2| \to min. \tag{15}$$

Since in the control design all of the highway sections are handled together, the performances of the sections are compressed to a vector

$$z(k) = \begin{bmatrix} z_{1,1} & z_{2,1} & z_{1,2} & z_{2,2} & \dots & z_{1,f} & z_{2,f} \end{bmatrix}^T. \tag{16}$$

Since the traffic has relatively slow dynamics, the optimal selection of the current control input has high importance. The expected traffic dynamics or the expected changes in the traffic flow must be incorporated into the control design. Moreover, in the control design constraints must be taken into consideration, e.g. the control inputs $r_i \geq 0$. Consequently, the Model Predictive Control is applied to design the appropriate control interventions.

B. The MPC control design method for the traffic system

The MPC problem is described on a finite time horizon $n \cdot T$ ahead [20], [21]. The performance on the given horizon is calculated in the following way

$$Z = \begin{bmatrix} C \\ CA(R_{1,i},\kappa) \\ CA(R_{1,i},\kappa)^2 \\ \vdots \\ CA(R_{1,i},\kappa)^n \end{bmatrix} x(k) - \begin{bmatrix} z_{ref}(k) \\ z_{ref}(k+1) \\ \vdots \\ z_{ref}(k+n) \end{bmatrix} +$$
$$+ \begin{bmatrix} 0 & \cdots & 0 & \\ CB_1 & \cdots & 0 & \\ CA(R_{1,i},\kappa)B_1 & \cdots & 0 & \\ \vdots & \vdots & \ddots & \vdots \\ CA(R_{1,i},\kappa)^{n-1}B_1 & \cdots & CB_1 & \end{bmatrix} \begin{bmatrix} w(k) \\ \vdots \\ w(k+n) \end{bmatrix} +$$
$$+ \begin{bmatrix} 0 & \cdots & 0 & \\ CB_2 & \cdots & 0 & \\ CA(R_{1,i},\kappa)B_2 & \cdots & 0 & \\ \vdots & \vdots & \ddots & \vdots \\ CA(R_{1,i},\kappa)^{n-1}B_2 & \cdots & CB_2 & \end{bmatrix} \begin{bmatrix} u(k) \\ \vdots \\ u(k+n) \end{bmatrix} \quad (17)$$

Based on the reference values $\rho_{crit,i}(j)$ in (14) the reference signals $z_{ref}(j), j \in \{k, k+n\}$ are defined. The performance on the given horizon in a compact form is as follows:

$$Z = \mathcal{C}(R_{1,i},\kappa) - \mathcal{R} + \mathcal{B}_1(R_{1,i},\kappa)W + \mathcal{B}_2(R_{1,i},\kappa)U \quad (18)$$

where $\mathcal{C}$ contains the current states of the system, $\mathcal{R}$ contains the reference values, W contains the disturbance values, U contains the control input values. Note that both W and U also contain the forthcoming disturbances and control inputs, respectively.

In the general MPC design the following cost function is minimized:

$$J(U) = \frac{1}{2}\sum_{i=1}^{n} Z^T(U)QZ(U) + U^T RU, \quad (19)$$

where Q and R are weighting matrices. Substituting (18) into the function (19), the cost function is transformed as

$$\begin{aligned} J(U) = & \frac{1}{2}U^T(\mathcal{B}_2^T Q\mathcal{B}_2 + R)U + \\ & + (\mathcal{C}^T Q\mathcal{B}_2 + \mathcal{R}^T Q\mathcal{B}_2 + W^T\mathcal{B}_1 Q\mathcal{B}_2)U + \varepsilon = \\ & = \frac{1}{2}U^T\phi U + \beta^T U + \varepsilon, \end{aligned} \quad (20)$$

where ε consists of all the constant components. Since ε is independent of the effect of U on $J(U)$, it can be omitted from the optimization problem. Moreover, the forthcoming disturbances in $J(U)$ must be estimated, see e.g. [22], [23].

The minimization of the cost function $J(U)$ also guarantees the performances (16). However, the cost function itself does not guarantee the stability of the system. Thus, constrains are built in the MPC optimization problem by using the results of the SOS-based stability analysis, see Section II. The SOS analysis results in an inequality (10), which must be guaranteed to provide stability.

Moreover, the states of the system $\rho_i(k)$ and $l_i(k)$ must be positive, which is a further constraint on the MPC problem:

$$x(k) \geq 0 \quad (21)$$

for all $1 \leq k \leq n$ time steps.

Finally, from (10), (20), and (21) the MPC control design problem is formed in the following way:

$$\min_{u(k)\dots u(k+n)} \frac{1}{2}U^T\phi U + \beta^T U \quad (22)$$

such that

$$\begin{aligned} u_{max}(R_{1,i}(k),\kappa) \geq q_{i-1}(k) + r_i(k) - s_i(k), \quad & \forall i,k \\ x(k) \geq 0, \quad & \forall k \\ U \in \mathbf{U} & \end{aligned} \quad (23)$$

where $\mathbf{U}$ contains the achievable control inputs. Although the matrices ϕ, β depend on $R_{1,i}, \kappa$ due to $A(R_{1,i},\kappa)$, these parameters are fixed in all computation steps. Thus, it leads to a linear MPC problem, which can be solved using standard quadratic programming methods, e.g. [24], [25]. The result of the computation (22) is a series of control inputs on the horizon $T \cdot n$. The control inputs are computed online during the cruising of the vehicle.

C. Intervention of the energy-efficient parameter into the control design

Since in the entire traffic system the energy-efficient parameter R_1 has an important role, its intervention possibility is also built into the control design. The selection of $R_{1,i}$ in each section is important not only in the force/energy requirement of the vehicles, but also in the dynamics and stability of the traffic system, see (5) and (10). In the followings the selection of $R_{1,i}$ in the coordinated control strategy with the MPC-based control design is proposed. Considering the inflows $q_{i-1}(k)$, $r_i(k)$, $s_i(k)$ and the length of the queue of the controlled ramp metering $l_i(k)$, the energy-efficient parameter $R_{1,i}$ is computed for each section. Depending on the ramp metering two different scenarios are distinguished:

1./ If the highway section i is controlled by the ramp metering r_i, then $R_{1,i}$ must depend on l_i. If l_i increases significantly then $R_{1,i}$ must be reduced to guarantee larger traffic flow on this section. However, the increased flow results in increased r_i, with which the length of queue is reduced, see (6). Consequently, the parameter $R_{1,i}$ must be selected as the function of l_i according to Figure 1. Here $l_{i,min}$ and $l_{i,max}$ are design parameters.

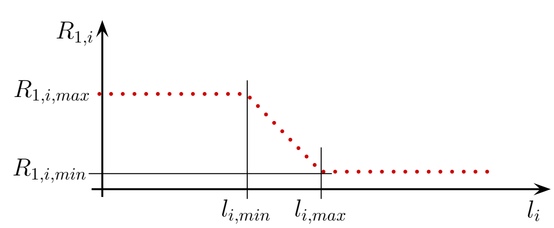


Fig. 1. Effect of $l_i(k)$ on $R_{1,i}(k)$

2./ If the highway section i does not include any controlled ramp metering $r_i(k)$, then $R_{1,i}$ must depend on the inflows and outflows. In this case it is necessary to avoid the instability of the system while the maximum $R_{1,i}$ is selected. From the SOS programing method the maximum traffic flow $u_{max}(R_{1,i}(k), \kappa(k))$ is calculated using (8). Exploiting the experiences, the maximum traffic flow can be expressed in the following form:

$$u_{max}(R_{1,i}(k), \kappa(k)) = u^0_{max} - u^1_{max} R_{1,i}(k)\kappa(k), \tag{24}$$

where u^0_{max} and u^1_{max} are selected constants, while $R_{1,i}(k)$ and $\kappa(k)$ are functions of k. Since $r_i(k) = 0$, the upper limit of (10) can be transformed into the following form:

$$u^0_{max} - u^1_{max} R_{1,i}(k)\kappa(k) = q_{i-1}(k) - s_i(k), \tag{25}$$

where $R_{1,i}(k)$ is selected as follows:

$$R_{1,i}(k) = \min\left(1, \max\left(\frac{u^0_{max} - q_{i-1}(k) + s_i(k)}{u^1_{max}\kappa(k)}, 0\right)\right), \tag{26}$$

Based on the above scenarios, in the coordinated control strategy first the current $R_{1,i}$ values are computed for each section and second the MPC problem (22) is solved to achieve the optimal control inputs $r_i(k)$.

IV. Simulation Examples

Finally, the efficiency of the traffic control strategy is illustrated through a simulation example, which is performed in the high fidelity microscopic traffic software VISSIM. The purpose of the example is to show that the MPC-based coordinated strategy is able to control the highway ramps and the energy-efficient parameter of the vehicles guarantees the performances of the traffic system. The results are compared to an uncoordinated scenario, in which $R_{1,i}$ values are fixed and the freeway ramps are uncontrolled.

In the simulation a $20km$ long section of the highway $M1$ between Budapest and Vienna is demonstrated. The highway section is divided into 5 segments, and it contains two controlled on-ramps and one off-ramp, see Figure 2. During the simulation it is necessary to minimize the lengths of the queues on the on-ramps, while the traffic flow and the energy-saving of the vehicles are maximized. The simulation

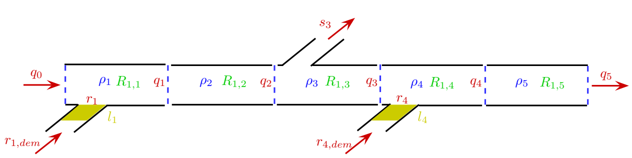
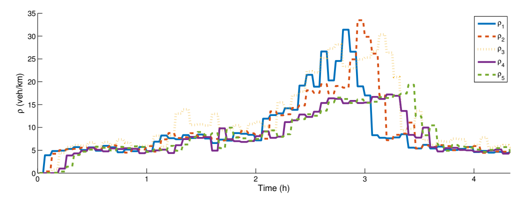


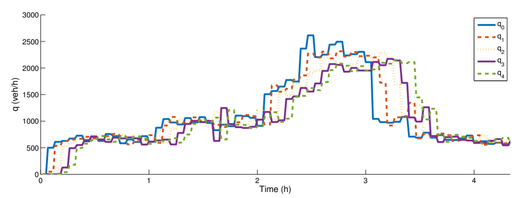
Fig. 2. Simulation scenario

parameter is $T = 30$ sec sampling time in the prediction with $n = 12$ points, which leads to a total of 6 min prediction horizon. However, $T = 30$ sec is too small sampling time for the intervention in the traffic control system, thus $T = 120$ sec is selected with $n = 3$ points for the control horizon [20]. Since the control input is computed as a flow value, it is transformed into green time with a 120 sec cycle. Moreover, in the simulation $\kappa = 20\%$ value along the highway is considered. During the simulation the signals in $w(k)$ are considered.

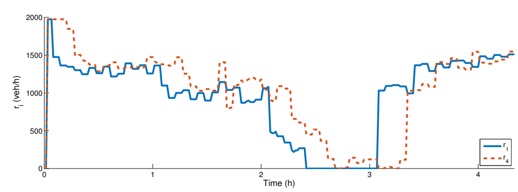
The results of the simulation with the coordinated control are shown in Figure 3. The simulation shows increasing traffic, whose maximum is approximately at 3 $hours$, see the density and the flow values at Figures 3(a)-(b). The critical density of the traffic is $\rho_{crit} = 25$ veh/km, whose tracking influences the ramps, see Figure 3(c). The efficiency of the prediction can be seen in the dynamics of the r_i intervention. For example at 2.4 h the value of r_1 significantly decreases, because q_0 increases in the future rapidly. However, ρ_1 is approximately 15 veh/km at $t = 2.4$ h. The variation



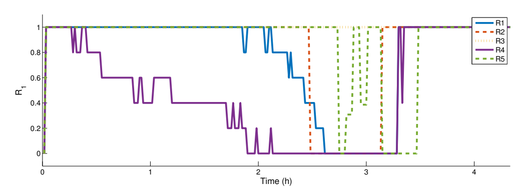
(a) Traffic density



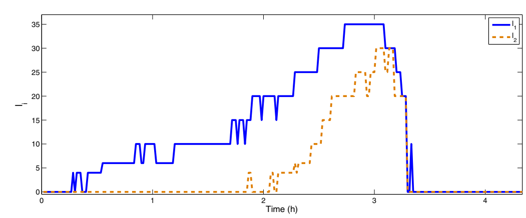
(b) q_{i-1} inflow of the sections



(c) Ramp control inflow



(d) Look-ahead parameter $R_{1,i}$

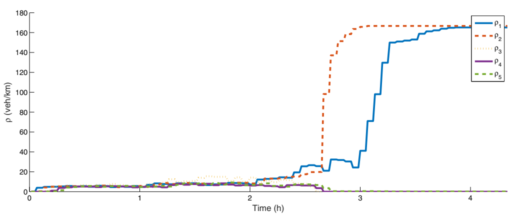


(e) Queue length on the controlled gates

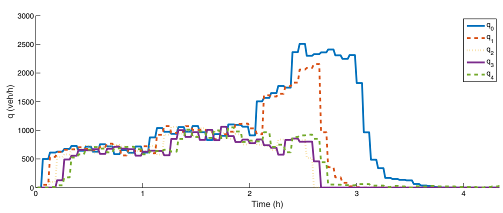
Fig. 3. Results of the VISSIM simulation

in economy parameters and the lengths of the queues are illustrated in Figure 3(d) and Figure 3(e), respectively. The effect of the queue length on $R_{1,i}$ values is shown in the figures. When the queue length increases, the parameters $R_{i,1}$ decrease simultaneously, see the signals of l_1, l_4 and $R_{1,1}, R_{1,4}$. Moreover, $R_{1,2}, R_{1,3}, R_{1,5}$ are selected to avoid the instability of the traffic and improve the flow capacity, e.g. at rush hour traffic their values decrease, see between $2.6\ h$ and $3.5\ h$.

Comparing the efficiency of the coordinated MPC control, Figure 4 illustrates selected results of the uncoordinated scenario. In this case the on-ramps are uncontrolled and $R_1 = 1$ for all controlled vehicles. Figure 4(a) shows the traffic density, which significantly increases at rush hour traffic until the saturation of the freeway, see between $2.6 \dots 3.5\ h$. Thus, the flow values of the sections are decreased, which results in a congestion, see Figure 4(b). The illustrations demonstrate that the coordination of the individual automated vehicles and the freeway control can provide an efficient operation for the entire traffic-vehicle system, while the lack of the coordinated control can result in traffic jams.



(a) Traffic density



(b) q_{i-1} inflow of the sections

Fig. 4. Results of the VISSIM simulation without coordination

V. Conclusions

In the paper a control design method for the coordination of the macroscopic traffic system and the individual vehicles has been proposed. The purposes of the coordinated control are to improve the energy efficiency of the entire traffic flow and simultaneously stabilize the controlled highway, reduce the queue length at the controlled gates and avoid congestion. The modeling and the stability analysis are based on the SOS programing method. In the mixed traffic system vehicles equipped with look-ahead control and conventional uncontrolled vehicles are traveling together. The result of the analysis is the maximum Controlled Invariant Set of the traffic flow and an inequality with the inflow and outflow of the system. In the control design the MPC method is applied, with which the prediction of the traffic flow and that of the traveling of the vehicles are taken into consideration. The traffic system is assumed to have ramp metering control, which has a significant effect on the inflow on the highway. In the coordinated control the energy-efficient parameter is built into the control design in order to guarantee the dynamics and stability of the traffic system besides reducing the force/energy of the individual vehicles.

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