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Fuzzy Sliding-Mode Observer for Lateral Dynamics of Vehicles with Consideration of Roll Motion

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Abstract— **This paper focuses on the methodology of estimating dynamic states of the vehicle, by a sliding-mode observer (SMO). The vehicle is represented by the fuzzy model of Takagi Sugeno (TS). The main contribution of this article is the development of a robust observer with the synthesis of sufficient stability conditions of this observer. This representation is used to take into account the non-linearities of the lateral forces. The stability of the fuzzy observer is given by the Lyapunov approach; the stability conditions of this observer are expressed in terms of linear matrix inequalities (LMI). The simulation results show the effectiveness of the proposed observer in estimating the states of the vehicle dynamics.**

I. INTRODUCTION

In the last two decades, the automotive industry and the research laboratories in this area, have sought a lot to make smarter, more reliable and safer vehicles. Since the last quarter of the twentieth century, most vehicles produced are equipped with different elements, which by their presence or their operation, could minimize the severity impact of an accident. They are called palliative or passive safety elements, for instance : airbags, safety belt, side protection bars ... and driver-assisted preventive or active safety elements, such as Adaptive Cruise Control (ACC), Anti-Lock Braking System (ABS), Dynamic Stability Program (DSC) and more recently, the Electronic Stability Program (ESP). The majority of these systems use the wheel speed, yaw rate, lateral acceleration, steering angle and roll angle of the vehicle to determine its dynamic state [1]–[4]. The aforementioned parameters can be directly measured by instrumental sensors, like optical sensors or GPS sensors. However, there are constraints related to the use of these sensors, which are unreliability, their expensive price and degradation and loss of signals under particular weather conditions and in some cases the unavailability of these sensors. Therefore, the vehicle parameters may be estimated using only the accessible and available measurements.

In this paper, our main objective is to accurately measure or estimate the states of vehicle dynamics. Several strategies have been proposed to estimate the dynamics of the vehicle, some methods propose to directly integrate sensors, others use a physical vehicle model to design a model-based observer, a combination of these two approaches is also used to estimate vehicle parameters. Vehicle-model-based methods may be sensitive to changes in vehicle parameters, but this effect can be reduced by using robust observers. So, the challenge here is to improve the observer's performance. The Sliding Mode Observer (SMO) has received a great attention because it offers robustness properties in presence of uncertainties [13]. This observer (SMO) forces the estimation error to be always close to zero [6] [8] [9], and this error is completely insensitive to disturbances. This interesting property has been used for both state estimation and for fault detection and isolation.

In this manuscript, a model of the vehicle dynamics with three degrees of freedom, which is known as "bicycle model" is constituted. This model is presented by a fuzzy model of Takagi-Sugeno (TS) widely used to solve the problem of non-linearity in physical systems [7]. Then, a sliding mode observer (SMO), for estimating the state vector of the vehicle lateral dynamics model with taking into account the roll motion, is designed. This SMO is based on the convex interpolation of classical Luenberger observers with additive terms [8], in order to be robust to parametric uncertainties and to overcome the effect of disturbances. Sufficient asymptotic stability conditions are given using the quadratic Lyapunov function in linear matrix inequality (LMI) [16], which can be solved very efficiently using LMI optimization techniques.

This document is organized as follows: The second section deals with the dynamic model of the vehicle; while the third section presents this model in the form of the fuzzy model of TS. The fourth section presents the design of the sliding mode observer. The fifth section is devoted to simulations and analysis of results. Finally, the conclusion is given in the last section.

Notation: Throughout the paper, the following useful notation is used: M^T denotes the transpose of the matrix M, $M > 0$ means that M is a symmetric positive definite matrix, $||M||$ represents the norm for matrix M and the symbol $*$ denotes the transpose elements in the symmetric positions. Sometimes to simplify the calculation, we will write the variables that depend on the time without ".(t)".

II. DESCRIPTION OF VEHICLE DYNAMICS MODEL

The complete model of vehicle dynamics is very difficult to use in control and monitoring applications, because of its complexity and high degrees of freedom. For this purpose, a

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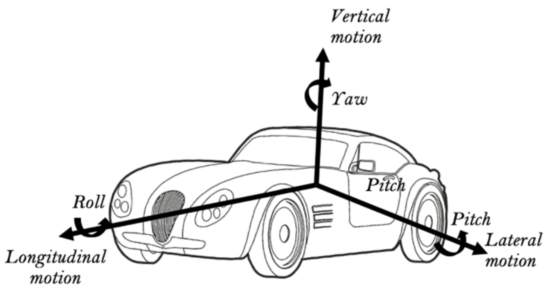

Fig. 1. The vehicle body motions

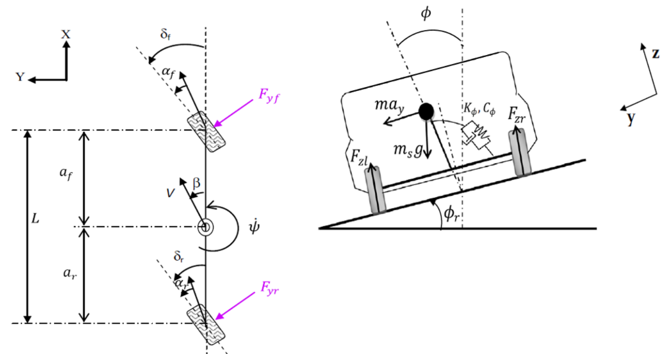

Fig. 2. Bicycle model with Roll rate sensor (3-DOF)

nominal model for the synthesis of observers and controls, is important. The movement of the vehicle is defined by a set of translations and rotational movements (see Fig. 1), usually six main movements [1]. The model used in this document describes the lateral dynamics of the vehicle, with the consideration of roll motion (Fig. 2). This model is obtained by considering the well-known 'bicycle' model with a degree of freedom of roll. With, the lateral velocity v_y, the yaw angle ψ and the roll angle ϕ of the vehicle, are the differential variables. The nonlinear model of vehicle dynamics is described by the following simplified differential equations [10]–[12]:

$$m\dot{v_y} = -m\dot{\psi}v_y + 2(F_{yf} + F_{yr}) \tag{1}$$

$$I_z\ddot{\psi} = +2a_rF_{yr} - 2a_fF_{yf} \tag{2}$$

The suspension is modeled as a torsional spring and damper system acting around the roll axis, illustrated in Fig. 2. The pitch dynamics of the vehicle are ignored or neglected.
If a small angle approximation is used for the roll angle, simplified expression can be obtained:

$$I_x\ddot{\phi} + (K_\phi + C_\phi)\dot{\phi} = mh(v_y + v_x\dot{\psi}) \tag{3}$$

Where m and m_s are the total and spring mass vehicle, respectively, v is the vehicle speed, I_x and I_z are the roll and yaw moments of inertia at center of gravity, a_r and a_f are the distance from center of gravity to rear and front axles, C_ϕ and K_ϕ are the combined roll damping and stiffness coefficients respectively, h is the center of gravity height from roll axis.
The rear and front lateral forces F_{yf} and F_{yr}, can be given according to the magic formula of Pacejka, Depending on the slip angles of the front and the rear tires (α_f and α_r respectively). To simplify the model of the vehicle, we suppose that the forces F_{yf} and F_{yr} are proportional to the front and rear tire slip angles [3] .

$$\begin{cases} F_{yf} = C_f(\sigma)\alpha_f \\ F_{yr} = C_r(\sigma)\alpha_r \end{cases} \tag{4}$$

When the slip angles are very small; in this case, the linear model works very well, but in the case of increasing slip, a nonlinear model must be considered.

III. Vehicle Dynamics Fuzzy Model Representation

The difficulty of obtaining a correct vehicle dynamics model is that the contact forces are difficult to measure and to model, by using the method based on the TS models proposed in [4] [5], which are a very interesting mathematical representation of nonlinear systems, because they allow to represent any nonlinear system, whatever its complexity, by a simple structure based on linear models interpolated by nonlinear positive. They have a simple structure offering interesting properties, that makes them easily exploitable from the mathematical point of view and allows the extension of certain results of the linear domain to the nonlinear systems.

The characteristics of the tires are generally assumed that the front and rear lateral forces (4) are modeled with the magic formula given in [3] by the following rules:

$$if \quad |\alpha_f| \quad is \quad M_1 \quad then \begin{cases} F_{yf} = C_{f1}(\sigma)\alpha_f(t) \\ F_{yr} = C_{r1}(\sigma)\alpha_r(t) \end{cases} \tag{5}$$

$$if \quad |\alpha_f| \quad is \quad M_2 \quad then \begin{cases} F_{yf} = C_{f2}(\sigma)\alpha_f(t) \\ F_{yr} = C_{r2}(\sigma)\alpha_r(t) \end{cases} \tag{6}$$

with σ is the road adhesion coefficient, the slip angles of the front and rear tires are given by [14] :

$$\alpha_f \approx \delta_f - \frac{a_f\dot{\psi}}{v_x} - \beta, \quad \alpha_r \approx \frac{a_r\dot{\psi}}{v_x} - \beta \tag{7}$$

α_f and α_r are considered to be in the same fuzzy set, the proposed rules are only made for α_f, this hypothesis makes it possible to reduce the number of membership functions and considers the rear steering angle as it is neglected. The overall forces are obtained by:

$$\begin{cases} F_{yf} = \sum_{i=1}^{2} \mu_i(|\alpha_f|)C_{fi}\alpha_f(t) \\ F_{yr} = \sum_{i=1}^{2} \mu_i(|\alpha_f|)C_{ri}\alpha_r(t) \end{cases} \tag{8}$$

Where C_{fi} and C_{ri} are the cornering stiffness coefficients of the front and rear wheels, which depend on the road adhesion coefficient σ, and the vehicle parameters. With $\mu_i(i = 1, 2)$

are membership functions, they depend on the front tire slip-angle α_f, which is considered available, they satisfy the following conditions:

$$\begin{cases} \sum_{i=1}^{2} \mu_i(|\alpha_f|) = 1 \\ 0 \le \mu_i(|\alpha_f|) \le 1, \quad i = 1,2 \end{cases} \tag{9}$$

The expressions of the membership functions μ_i used are as follows:

$$\mu_i(|\alpha_f|) = \frac{\omega_i(|\alpha_f|)}{\sum_{i=1}^{2} \omega_i(|\alpha_f|)}; i = 1,2 \tag{10}$$

Where:

$$\omega_i(|\alpha_f|) = \frac{1}{\left(1 + \left|\frac{|\alpha_f| - c_i}{a_i}\right|\right)^{2b_i}} \tag{11}$$

Using an identification method based on the Levenbenrg-Marquadt algorithm combined with the least square method, allows to determine parameters of membership functions and stiffness coefficients parameters, which give us the following values [3] [12]:

$$a_1 = 0.0785, \quad b_1 = 1.7009\ , \quad c_1 = 0.0284$$

$$a_2 = 0.1126, \quad b_2 = 12.0064, \quad c_2 = 0.1647$$

$$C_{f1} = 55234, \quad C_{f2} = 15544$$

$$C_{r1} = 49200, \quad C_{r1} = 13543$$

The non-linear vehicle lateral dynamics described by (1)-(3) becomes:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i\left(|\alpha_f|\right)\left((A_i + \Delta A_i)x(t) + B_i\delta_f(t) + v(t)\right) \\ y(t) = \sum_{i=1}^{2} \mu_i\left(|\alpha_f|\right) C_i x(t) \end{cases} \tag{12}$$

We assume that $\mu_i(|\alpha_f|) \ge 0$, and $\sum_{i=1}^{2} \mu_i\left(|\alpha_f|\right) = 1$ for any t, $\forall i = 1,2$. Where $x(t)$ is the state vector, $y(t)$ is the output vector, δ_f is the input vector and $v(t)$ is the disturbance. The matrices ΔA_i are bounded matrices with appropriate dimensions $(||A_i(t)|| \le \rho_i)$, which represent parametric uncertainties in the plant model. A_i, B_i and C_i are the state, input and output matrices which are defined as follows [12] :

$$A_i = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$B_i = \begin{pmatrix} b_{11} \\ b_{21} \\ 0 \\ b_{41} \end{pmatrix}, \quad C_i = \begin{pmatrix} c_{11} & c_{12} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x = \begin{bmatrix} v_y & \dot{\psi} & \phi & \dot{\phi} \end{bmatrix}^T, \quad y = \begin{bmatrix} a_y & \dot{\psi} & \dot{\phi} \end{bmatrix}^T$$

$$a_{11} = -\frac{2(C_{fi} + C_{ri})}{mv_x}, \qquad a_{12} = \frac{2(a_f C_{fi} - a_r C_{ri})}{mv_x} - v_x$$

$$a_{21} = -\frac{2(a_f C_{fi} + a_r C_{ri})}{I_z v_x}, \quad a_{22} = \frac{2(a_f^2 C_{fi} - a_r^2 C_{ri})}{I_z v_x}$$

$$a_{41} = -\frac{2h(C_{fi} + C_{ri})}{mv_x I_x}, \qquad a_{42} = \frac{2m_s h(C_{fi} + C_{ri})}{mv_x I_x}$$

$$a_{43} = \frac{hm_s g - K_\phi}{I_x}, \qquad a_{44} = \frac{C_\phi}{I_x}$$

$$c_{11} = -2\frac{C_{fi} + C_{ri}}{mv_x}, \qquad c_{12} = -2\frac{C_{fi}a_f - C_{ri}a_r}{mv_x}$$

$$b_{11} = \frac{2C_{fi}}{m}, \qquad b_{21} = \frac{2a_f C_{fi}}{I_z}, \qquad b_{41} = \frac{2m_s h C_{fi}}{mI_x}$$

IV. Design of a Sliding Mode Observer

Currently, there are many practical considerations that inhibit production vehicles from using sensors of roll angle and lateral velocity such as high cost, signal degradation, and lost of signal during certain weather conditions. To resolve this problem, we can use the observer theory.

This section is dedicated to the state estimation of the model (1)-(3). The proposed sliding mode fuzzy observer (SMFO) is based on a nonlinear combination of local sliding mode observers involving sliding terms allowing to compensate the model uncertainties. In this study, we consider that the signal input is an assistant steering angle δ_{fa}, which is added to the steering angle given by the driver δ_{fd} $\left(\delta_f = \delta_{fa} + \delta_{fd}\right)$ [11]. The observer mentioned of the Takagi-Sugeno model, with the presence of perturbation has the following form:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^{2} \mu_i\left(|\alpha_f|\right)(A_i\hat{x}(t) + B_i\delta_f(t) + L_i(y(t) - \hat{y}(t) \\ \qquad + \alpha_i(t)) \\ \hat{y}(t) = \sum_{i=1}^{2} \mu_i\left(|\alpha_f|\right) C_i\hat{x}(t) \end{cases} \tag{13}$$

where $\hat{x}(t)$ is the state estimation, $\hat{y}(t)$ is the observer output. The aim is to determine the gain L_i of the local observer i and the term $\alpha_i(t)$ represents a value added to the structure of each local observer, in such a way to force the estimation error converging to zero. In order to establish the conditions for the asymptotic convergence of the observer (14), let us define the state and output estimation errors:

$$e(t) = x(t) - \hat{x}(t) \tag{14}$$

and the output residual as:

$$r(t) = y(t) - \hat{y}(t) = \sum_{i=1}^{2} \mu_i\left(|\alpha_f|\right) C_i e(t) \tag{15}$$

then, the dynamics of the estimation error becomes:

From systems (12)-(14), we have:

$$\dot{e} = \sum_{i=1}^{2}\sum_{j=1}^{2} \mu_i\left(|\alpha_f|\right)\mu_j\left(|\alpha_f|\right)((A_i - L_iC_j)e(t) + \Delta A_i x(t) \\ - \alpha_i) \tag{16}$$

The following lemma is needed to prove theorem 1.

Lemma: We consider two matrices X and Y of appropriate dimensions, then the following property is verified:

$$X^TY+Y^TX \leq \gamma X^TX+\gamma^{-1}Y^TY, \quad \gamma>0 \tag{17}$$

Theorem 1: The state estimation error between the model (12) and the sliding mode observer (13) converges globally asymptotically towards zero, if there exists symmetric positive definite matrices X and Y_{ii}, matrices Y_{ij} and positive scalars λ_1, λ_2 and λ_3 satisfying the following inequalities:

$$\begin{bmatrix} A_i^TX+XA_i-C_i^TZ_i^T-Z_iC_i \\ +Y_{ii}+(\lambda_2\rho_i^2+\lambda_3)I & X \\ * & -\lambda_1 I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \frac{(A_i+A_j)^T}{2}X+X\frac{(A_i+A_j)}{2} \\ -Z_iC_j-C_j^TZ_i^T-Z_jC_i-C_i^TZ_j^T \\ +Y_{ij}+Y_{ij}^T+(\lambda_2\rho_i^2+\lambda_3)I & X \\ * & -\lambda_1 I \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ * & Y_{22} \end{bmatrix} > 0 \tag{20}$$

The terms $\alpha_i(t)$ of the observer (13) are given by the following equations:

- If $|r|<\varepsilon$, then : $\alpha_i(t)=0$
- If $|r|\geq\varepsilon$, then :

$$\alpha_i(t)=\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\rho_i^2\frac{\hat{x}^T\hat{x}}{2r^Tr}X^{-1}\sum_{j=1}^{2}\mu_j\left(|\alpha_f|\right)C_j^Tr \tag{21}$$

and the observer gains are given by:

$$L_i=X^{-1}Z_i \tag{22}$$

Proof: Consider the following Lyapunov function candidate:

$$V=e^TXe \tag{23}$$

Using equations (14) and (16); the time derivative of $V(e(t))$ along the trajectory of (12) is given by:

$$\dot{V}(e)=\dot{e}^TXe+e^TX\dot{e} \tag{24}$$

$$\begin{aligned}\dot{V}(e)=\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(e^T\left(\bar{A}_{ij}^TX+X\bar{A}_{ij}\right)e \\ +x^T\Delta A_i^TXe+e^TX\Delta A_ix-\alpha_i^TXe-e^TX\alpha_i)\end{aligned} \tag{25}$$

with : $\bar{A}_{ij}=A_i-L_iC_j$ Then using the well-known Lemma (17), we obtain:

$$\begin{aligned}\dot{V}(e)\leq\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(e^T\left(\bar{A}_{ij}^TX+X\bar{A}_{ij}\right)e+\lambda_1^{-1}e^TX^2e \\ +\lambda_1x^T\Delta A_i^T\Delta A_ix-\alpha_i^TXe-e^TX\alpha_i)\end{aligned} \tag{26}$$

by the use of the observation error (14), we can find this inequality:

$$\begin{aligned}\dot{V}(e)\leq\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(e^T\left(\bar{A}_{ij}^TX+X\bar{A}_{ij}+\lambda_1^{-1}X^2\right)e \\ +\lambda_1\rho_i^2(\hat{x}+e)^T(\hat{x}+e)-\alpha_i^TXe-e^TX\alpha_i)\end{aligned} \tag{27}$$

$$\begin{aligned}\dot{V}(e)\leq\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(e^T\left(\bar{A}_{ij}^TX+X\bar{A}_{ij}+\lambda_1^{-1}X^2\right)e \\ +\lambda_1\rho_i^2(\hat{x}^T\hat{x}+e^Te)+\lambda_1\rho_i^2(\hat{x}^Te+e^T\hat{x})-\alpha_i^TXe-e^TX\alpha_i)\end{aligned} \tag{28}$$

We use again (17), the expression (28) can be rewritten as follows:

$$\begin{aligned}\dot{V}(e)\leq\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(e^T(\bar{A}_{ij}^TX+X\bar{A}_{ij}+\lambda_1^{-1}X^2 \\ +\lambda_2\rho_i^2I)e+\lambda_1\rho_i^2(1+\lambda_4)\hat{x}^T\hat{x}-\alpha_i^TXe-e^TX\alpha_i)\end{aligned} \tag{29}$$

$$\begin{aligned}\dot{V}(e)\leq\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(e^T(\bar{A}_{ij}^TX+X\bar{A}_{ij}+\lambda_1^{-1}X^2 \\ +\lambda_2\rho_i^2I)e+\lambda_1\rho_i^2(1+\lambda_4)\hat{x}^T\hat{x}-2\alpha_i^TXe)\end{aligned} \tag{30}$$

where: $\lambda_2=\lambda_1(1+1/\lambda_4)$

Two situations can therefore be distinguished according to the value of the output residual:

Situation 1: $|r|<0$

In this situation we will write using the relation (21), the following equation:

$$2\alpha_i^TXe=\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\rho_i^2\frac{\hat{x}^T\hat{x}}{r^Tr}r^T\sum_{j=1}^{2}\mu_j\left(|\alpha_f|\right)C_jX^{-1}Xe \tag{31}$$

and since:

$$\begin{aligned}\sum_{j=1}^{2}\mu_j\left(|\alpha_f|\right)C_je&=\sum_{j=1}^{2}\mu_j\left(|\alpha_f|\right)C_j(x-\hat{x}) \\ &=y-\hat{y}=r\end{aligned}$$

then:

$$2\alpha_i^TXe=\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\rho_i^2\frac{\hat{x}^T\hat{x}}{r^Tr}r^Tr \tag{32}$$

$$2\alpha_i^TXe=\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\rho_i^2\hat{x}^T\hat{x} \tag{33}$$

we replace λ_2 by its expression:

$$2\alpha_i^TXe=\lambda_1(\lambda_4+1)\rho_i^2\hat{x}^T\hat{x} \tag{34}$$

Then the expression (30), becomes as follows:

$$\begin{aligned}\dot{V}(e)\leq\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\mu_j(e^T(\bar{A}_{ij}^TX+X\bar{A}_{ij} \\ +\lambda_1^{-1}X^2+\lambda_2\rho_i^2I)e)\end{aligned} \tag{35}$$

TABLE I

SIMULATION VEHICLE PARAMETERS

Constants	Explanation	value	Unit
g	gravity acceleration constant	9.806	m/s^2
m	Vehicle mass	1832	Kg
h	CG height from roll axis	0.90	m
I_z	Yaw moment of inertia at CG	2988	$Kg.m^2$
I_x	Roll moment of inertia at CG	614	$Kg.m^2$
a_f	distance from CG to front axle	1.18	m
a_r	distance from CG to rear axle	1.77	m
C_ϕ	Combined roll damping coefficient	6000	Nms/rad
K_ϕ	Combined roll stiffness coefficient	140000	Nm/rad

Situation 2: $|r| \geq 0$

In this situation, we do not need calculation, because we get directly the expression (35).

Remark

The estimation error cannot converge to zero asymptotically but to a small neighborhood of zero depending on the choice of ε.

V. SIMULATION RESULTS

To demonstrate the efficiency of the proposed observer to estimate the states of the vehicle dynamics, we have carried out some simulations using the vehicle model (1)-(3) and MATLAB software. In the design, the vehicle parameters considered are given in Tab. I [3] . To take in consideration the stiffness coefficients C_{fi} and C_{ri}, which are supposed to be varying depending on road adhesion σ_i.

The simultaneous resolution of the inequalities (18)-(20) using an LMI resolution tools, leads to the following matrices Z_i, X and the gains L_i:

$$Z_1 = \begin{bmatrix} -8.0336 & -3.9890 \\ 1.6773 & 2.4189 \\ 1.4128 & -37.0150 \\ -0.6010 & 16.0643 \end{bmatrix}, \quad Z_2 = \begin{bmatrix} -19.6696 & 0.0326 \\ 20.049 & 1.9234 \\ -0.0131 & -36.9854 \\ -1.2991 & 16.0702 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} -8.6599 & -4.1459 \\ 0.8287 & 1.8216 \\ 1.3106 & -32.6477 \\ -1.4656 & 42.8363 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -20.9405 & -1.7475 \\ 11.2130 & 1.5949 \\ 0.5331 & -32.6663 \\ 0.1913 & 42.7660 \end{bmatrix}$$

$$X = \begin{bmatrix} 0.9263 & -0.0232 & 0.0205 & 0.0131 \\ -0.0232 & 1.7429 & 0.0172 & -0.0068 \\ 0.0205 & 0.0172 & 1.6083 & 0.3629 \\ 0.0131 & -0.0068 & 0.3629 & 0.6532 \end{bmatrix}$$

All the simulations are realized on the nonlinear model given in (1)-(3) with the front steering angle profile given in Fig. 3 and the following consideration:

- Vehicle speed : $20ms^{-1} \leq v \leq 25ms^{-1}$
- Adhesion coefficient : $0.4 \leq \sigma \leq 1$
- No rear wheel steering angle.
- Front wheel steer angle $[rad]$: $-1 \leq \delta_{fd} \leq 1$
- Vehicle longitudinal acceleration is low or equal to zero.

The model uncertainties are such that [8] [9] [15]:

$$\Delta A_{i,(l,c)} = \xi A_{i,(l,c)} \Omega(t) \tag{36}$$

where $A_{i,(l,c)}$ denotes the $(l,c)^{th}$ element of A_i and $\xi = 0.2$. The function $\Omega(t)$ is a piece-wise constant function which magnitude is uniformly distributed on the interval [0 1]. The simulation results are given in figures 4, 5 and 6 with ε iqual to 0.002.

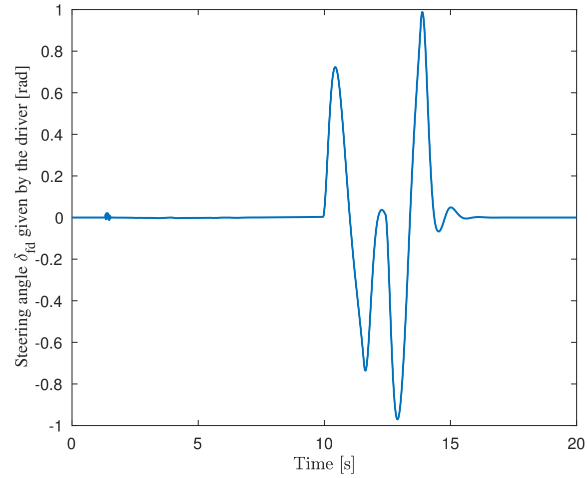

Fig. 3. Steering angle given by driver $\delta_{fd}[rad]$

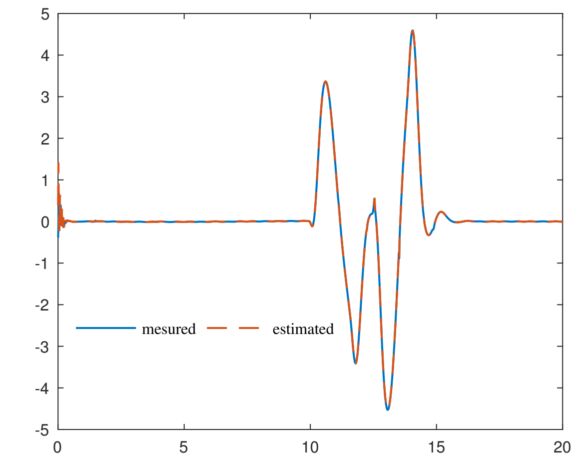

Fig. 4. The output of the lateral acceleration a_y and its estimated $[m/s^2]$

We can clearly see that the two lines (the measured and the estimated) are superimposed, except in the near of the origin; this is because of the choice of the initial values of the observer which are: $\hat{x}^T(0) = [0 \ \ 0 \ \ 0 \ \ 0]$; however, the initial values of the system are: $x^T(0) = [0.2 \ \ 0.2 \ \ 0.2 \ \ 0.2]$, which shows the effectiveness of the proposed observer to estimate the vehicle parameters. In these simulations, we simply choose $v(t) = 0.2sin(4\pi)$.

VI. CONCLUSION

In this paper, we have proposed a robust sliding mode observer design to estimate the states of a car-type vehicle system, which is represented by Takagi-Sugeno uncertain model. This observer can estimate the states of the lateral dynamics of the vehicle with taking into account the roll movement even in the presence of uncertainties and disturbances. The coincidence of the estimation error to zero is

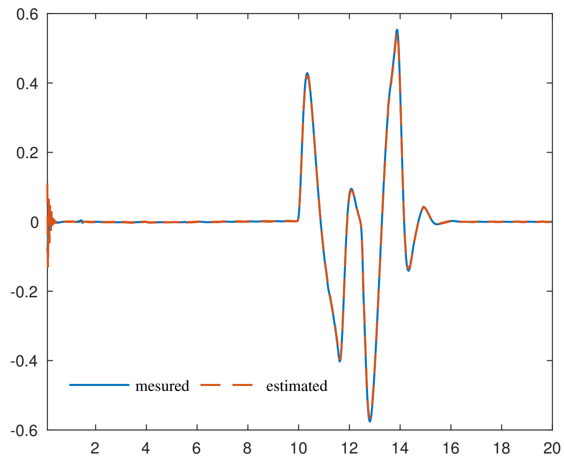

Fig. 5. The output of the yaw rate $\dot{\psi}$ and its estimated $[rad/s]$

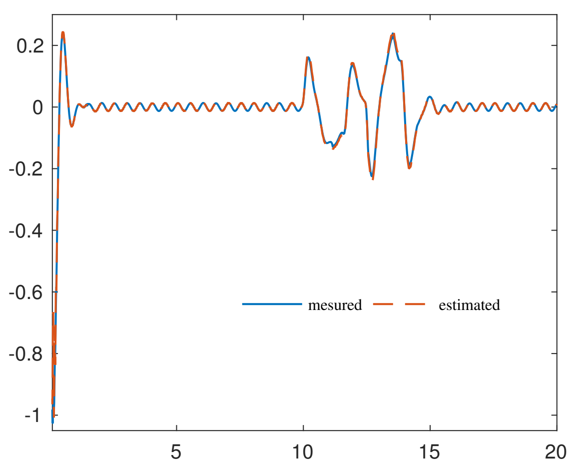

Fig. 6. The output of the roll rate $\dot{\phi}$ and its estimated $[rad/s]$

studied with the Lyapunov approach and LMI constraints, which are provided to calculate the gains of the mentioned observer. The vehicle simulations show clearly the quality of states estimation of the vehicle dynamics and the proposed approach can be adapted to driving conditions.

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