2018 26th Mediterranean Conference on Control and Automation (MED)
Zadar, Croatia, June 19-22, 2018

Robust H_∞ filter for uncertain continuous-time systems with finite frequency ranges

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***Abstract*— This article examines the H_∞ robust filtering problem for uncertain continuous-time systems with finite frequency (FF) specifications. Our goal is to design a new filter that ensures H_∞ performance in specific finite frequency ranges. By virtue of the generalized Kalman-Yakubovich-Popov lemma (gKYP), the Finsler's lemma, the polynomially parameter-dependent Lyapunov function and a decoupling technique, sufficient conditions to characterize this problem in terms of linear matrix inequalities (LMIs) are established. Finally, we demonstrate by numerical example that our method can achieve a much smaller approximation error than the existing results.**

***Index Terms*— Finite frequency; H_∞ performance; Continuous systems; LMI.**

I. Introduction

The filtering problem has been widely studied and has found many practical applications in signal processing and communication, such as electrical circuit system. Generally speaking, the filtering problem has been investigated for uncertain linear systems [1], [2], [3], [4], [5], stochastic systems [6], switched linear systems [7], time-delay systems [8], [9] and mixed H_2/H_∞ filtering [10]. For the uncertain systems, quadratic approach is considered in [11], [12] to design robust H_2 and H_∞ filters trough LMI formulation [13]. Although the quadratic approach makes the LMI based design problem, it often leads easy to conservative results. In addition, it should be noted that the aforementioned techniques deal with the entire frequency (EF) domain. However, if the frequency ranges of noises are known beforehand. For these cases, designing a filter in the full frequency domain may introduce some undesirable conservatism. In this view, the generalized Kalman-Yakobovich-Popov (gKYP) lemma [14] may be used to reduce the filter design to frequency domain of the noise. In recent years, there has been a growing interest for the FF H_∞ filters, and various results have been proposed in this area (see [15], [16], [17], [18] and the references therein).

The aim of this paper is to design a new filter guaranteeing an H_∞ performance bound over finite frequency ranges. We use the gKYP, Finsler's lemma, and the homogeneous polynomially parameter-dependent matrices of arbitrary degree approach for establish new sufficient characterizations for this problem in terms of LMIs over different frequency ranges. Finally, we demonstrate via numerical example that our method can achieve much smaller approximation error than existing results.

This work was not supported by any organization

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II. Preliminaries

A. Notations and lemmas

We present below some basic notations and lemmas that will be used throughout this document. All matrices are assumed to have compatible dimensions; exponent "T" stands for matrix transposition. In symmetric block matrices or long matrix expressions, we use an asterisk " $*$ " to represent a symmetry-induced term. Notation $\mathcal{P} > 0$ means that the matrix $\mathcal{P}$ is definite semi-positive. The symbol I indicates an identity matrix with the appropriate dimension. Generally, $sym(A)$ denotes $A + A^T$, $diag\{..\}$ represents the diagonal block matrix. $\bar{\sigma}(H)$ denotes the singular maximum value of the H transfer matrix.

Lemma 1 *(Finsler's lemma)[2] Let $\zeta \in \mathbb{R}^n$, $\mathcal{M} \in \mathbb{R}^{n\times n}$ and $\mathcal{X} \in \mathbb{R}^{m\times n}$ with rank $(\mathcal{X}) = r < n$ and $\mathcal{X}^\perp \in \mathbb{R}^{n\times(n-r)}$ be full-column-rank matrix satisfying $\mathcal{X}\mathcal{X}^\perp = 0$. Then, the following conditions are equivalent:*

1) $\zeta^T\mathcal{M}\zeta < 0, \forall \xi \neq 0 : \mathcal{X}\zeta = 0$
2) $\mathcal{X}^{\perp T}\mathcal{M}\mathcal{X}^\perp < 0$
3) $\exists \delta \in \mathbb{R} : \mathcal{M} - \delta\mathcal{X}^T\mathcal{X} < 0$
4) $\exists \mathcal{Y} \in \mathbb{R}^{n\times m} : \mathcal{M} + \mathcal{Y}\mathcal{X} + \mathcal{Y}^T\mathcal{X}^T < 0$

B. Problem formulation

Consider a continuous-time system described by the following model:

$$\begin{array}{rcl} \dot{x}(t) &=& A_\alpha x(t) + B_\alpha w(t) \\ y(t) &=& C_\alpha x(t) + D_\alpha w(t) \\ z(t) &=& L_\alpha x(t) \end{array} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector, $y(t) \in \mathbb{R}^{n_y}$ is the measured output, $z(t) \in \mathbb{R}^{n_z}$ is the signal to be estimated, $w(t) \in \mathbb{R}^{n_w}$ is the noise signal that is assumed to be arbitrary signal in $L_2[0,\infty)$, whose energy is known and located in one of the following frequency sets

$$\Omega = \left\{ \begin{array}{lll} \mu \in \mathbb{R}|\ |\mu| \leq \mu_l, & \mu_l \geq 0, & (LF) \\ \mu \in \mathbb{R}|\ \mu_1 \leq \mu \leq \mu_2, & 0 \leq \mu_1 \leq \mu_2, & (MF) \\ \mu \in \mathbb{R}|\ |\mu| \geq \mu_h, & \mu_h \geq 0, & (HF) \end{array} \right. \tag{2}$$

with LF, MF and HF stand for low-, middle-, and high-frequency ranges, respectively.

System matrices

$$\Theta_\alpha = \{A_\alpha,\ B_\alpha,\ C_\alpha,\ D_\alpha,\ L_\alpha\} \tag{3}$$

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belong to a convex bounded polyhedral domain, described by

$$\Gamma = \{\Theta_\alpha | \Theta_\alpha = \sum_{i=1}^{s} \alpha_i \Theta_i;\ \sum_{i=1}^{s} \alpha_i = 1,\ \alpha_i \geq 0\} \quad (4)$$

where

$$\Theta_i := \{A_i,\ B_i,\ C_i,\ D_i,\ L_i\} \quad (5)$$

denotes the ith vertex of the polytope. The purpose of this paper is to design a filter of the form

$$\begin{aligned} \dot{x}_f(t) &= A_f x_f(t) + B_f y(t) \quad x_f(0) = 0 \\ z_f(t) &= C_f x_f(t) + D_f y(t) \end{aligned} \quad (6)$$

where $x_f(t) \in \mathbb{R}^{n_x}$ is the filter state vector, $z_f(t) \in \mathbb{R}^{n_z}$ is the output of the filter. Matrices A_f, B_f, C_f and D_f are the filter matrices to be determined.

Defining $\xi(t) = \begin{bmatrix} x(t)^T & \dot{x}_f^T(t) \end{bmatrix}^T$ and $e(t) = z(t) - \dot{z}_f(t)$, the filtering error system is given by

$$\begin{aligned} \dot{\xi}(t) &= \mathcal{A}_\alpha \xi(t) + \mathcal{B}_\alpha w(t) \\ e(t) &= \mathcal{C}_\alpha \xi(t) + \mathcal{D}_\alpha w(t) \end{aligned} \quad (7)$$

where

$$\begin{aligned} \mathcal{A}_\alpha &= \begin{bmatrix} A_\alpha & 0 \\ B_f C_\alpha & A_f \end{bmatrix},\ \mathcal{B}_\alpha = \begin{bmatrix} B_\alpha \\ B_f D_\alpha \end{bmatrix}, \quad (8) \\ \mathcal{C}_\alpha &= \begin{bmatrix} L_\alpha - D_f C_\alpha & -C_f \end{bmatrix},\ \mathcal{D}_\alpha = -D_f D_\alpha\ . \end{aligned}$$

The transfer function of filtering error system (7) is then

$$H(j\mu) = \mathcal{C}_\alpha [j\mu - \mathcal{A}_\alpha]^{-1} \mathcal{B}_\alpha + \mathcal{D}_\alpha \quad \forall \alpha \in \Gamma,\ \ \forall \mu \in \Omega. \quad (9)$$

Problem description: The robust H_∞ filtering problem for uncertain continuous-time systems with finite frequency specifications is formulated as: find an admissible filter in (6) for the system in (1) such that two conditions are satisfied:

- Filtering error system in (7) is robustly asymptotically stable.
- Under zero-initial conditions, the following finite frequency index holds:

$$\bar{\sigma}(H(j\mu)) < \gamma \qquad \forall \mu \in \Omega,\ \ \forall \alpha \in \Gamma. \quad (10)$$

Before proceeding further, we first give the following Lemma that gives a sufficient condition for error system (7) with a finite frequency (FF) specifications in (10).

Lemma 2 *[14] Consider filtering error system (7), for a given symmetric matrix*

$$\Xi = \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix}$$

the following statements are equivalent

1) *The finite frequency (FF) inequality*

$$\begin{bmatrix} H(j\mu)^T & I \end{bmatrix} \Xi \begin{bmatrix} H(j\mu) \\ I \end{bmatrix} < \gamma \quad (11)$$

$\forall \mu \in (2),\ \ \forall \alpha \in (4).$

2) *There exist Hermitian matrix functions $\mathcal{P}_\alpha$, $\mathcal{Q}_\alpha$ satisfying $\mathcal{Q}_\alpha > 0$ such that*

$$\begin{bmatrix} \mathcal{A}_\alpha & \mathcal{B}_\alpha \\ I & 0 \end{bmatrix}^T \Pi_\alpha \begin{bmatrix} \mathcal{A}_\alpha & \mathcal{B}_\alpha \\ I & 0 \end{bmatrix} + \begin{bmatrix} \mathcal{C}_\alpha & \mathcal{D}_\alpha \\ 0 & I \end{bmatrix}^T \Xi \begin{bmatrix} \mathcal{C}_\alpha & \mathcal{D}_\alpha \\ 0 & I \end{bmatrix} < 0 \quad (12)$$

where

- *For the low frequency (LF) range $|\mu| \leq \mu_l$*

$$\Pi_\alpha = \begin{bmatrix} -\mathcal{Q}_\alpha & \mathcal{P}_\alpha \\ \mathcal{P}_\alpha & \mu_l^2 \mathcal{Q}_\alpha \end{bmatrix} \quad (13)$$

- *For the middle frequency (MF) range $\mu_1 \leq \mu \leq \mu_2$, $\mu_c = \frac{\mu_2 + \mu_1}{2}$*

$$\Pi_\alpha = \begin{bmatrix} -\mathcal{Q}_\alpha & \mathcal{P}_\alpha + j\mu_c \mathcal{Q}_\alpha \\ \mathcal{P}_\alpha - j\mu_c \mathcal{Q}_\alpha & -\mu_1 \mu_2 \mathcal{Q}_\alpha \end{bmatrix} \quad (14)$$

- *For the high frequency (HF) range $|\mu| \geq \mu_h$*

$$\Pi_\alpha = \begin{bmatrix} \mathcal{Q}_\alpha & \mathcal{P}_\alpha \\ \mathcal{P}_\alpha & -\mu_h^2 \mathcal{Q}_\alpha \end{bmatrix} \quad (15)$$

Remark 1 *Condition (12) is the extension of the gKYP lemma for polytopic systems. Note that $\mathcal{P}_\alpha$ and $\mathcal{Q}_\alpha$ are chosen to be parameter-dependent to relax the analysis conditions, decreasing, thus, the conservatism when compared to the case in which $\mathcal{P}$ and $\mathcal{Q}$ are parameter-independent.*

Definition 1 *(***Homogeneous Polynomials***) [17] Matrix $\mathcal{P}_\alpha$ is a homogeneous polynomial matrix of degree g if it depends polynomially on a parameter $\alpha \in \Gamma$ as follows:*

$$\mathcal{P}_\alpha = \sum_{j=1}^{J(g)} \alpha_1^{k_1} \alpha_2^{k_2} ... \alpha_s^{k_N} P_{k_j(g)}, \quad [k_1, k_2, ..., k_N] = K_j(g) \quad (16)$$

Where $K(g)$ is the set of N-tuples obtained as all possible combination of $[k_1, k_2, ..., k_N]$, with k_i being nonnegative integers, such that $k_1 + k_2 + ... + k_N = g$. $K_j(g)$ is the j-th N-tuples of $K(g)$ which is lexically ordered, $j = 1, ..., J(g)$. Since the number of vertices in polytope Γ is equal to s, the number of elements in $K(g)$ as given by $J(g) = \frac{(N+g-1)!}{g!(N-1)!}$. These elements define the subscripts $k_1, k_2, ..., k_N$ of the constant matrices : $\mathcal{P}_{k_1, k_2, ... k_N} \triangleq \mathcal{P}_{k_j(g)}$, which are used to construct the homogeneous polynomial dependent matrices $\mathcal{P}_\alpha$.

For each set $K(g)$, define also set $I(g)$ with elements $I_j(g)$ given by subsets of i, $i \in \{1, 2, ..., N\}$, associated to N-tuples $K_j(g)$ whose k_i's are nonzero. For each i, i=1,...,N, define the N-tuples $K_j^i(g)$ as being equal to $K_j(g)$ but with $k_i > 0$ replaced by $k_i - 1$. Note that the N-tuples $K_j^i(g)$ are defined only in the cases where the corresponding k_i is positive.

Note also that, when applied to the elements of $K(g+1)$, the N-tuples $K_j^i(g+1)$ define subscripts $k_1, k_2, ..., k_N$ of matrices $P_{k_1, k_2, ... k_N}$ associated to homogeneous polynomial parameter dependent matrices of degree g. Finally, define the scalar constant coefficients $\beta_j^i(g+1) = \frac{g!}{(k_1! k_2! ... k_N!)}$, with $[k_1, k_2, ..., k_N] \in K_j^i(g+1)$.

III. Main results

A. Finite Frequency H_∞ Filtering Analysis

We assume that the parameters of the filter's are known, the following theorem presents sufficient conditions to ensure that the estimation error system (7) is stable and has a H_∞ level prescribed in the domain LF $|\mu| \leq \mu_l$ of the measurement noise.

Theorem 1 *: Consider the system in (1). For given $\gamma > 0$, a filter of form (6) exists such that the filtering error system in (7) is asymptotically stable with an H_∞ performance bound γ in the LF domain $|\mu| \leq \mu_l$, if there exist hermitian matrices $\mathcal{P}_\alpha$, $\mathcal{Q}_\alpha > 0 \in \mathbb{C}^{n_x}$ and matrices $M_\alpha, S_\alpha \in \mathbb{R}^{n_x}$, $R_\alpha \in \mathbb{R}^{n_w \times n_x}$ and symmetric matrix $\mathcal{W}_\alpha > 0 \in \mathbb{R}^{n_x}$ for all $\alpha \in \Gamma$ satisfying*

$$\Psi_\alpha = \begin{bmatrix} \Psi_{11\alpha} & \Psi_{12\alpha} & \Psi_{13\alpha} & 0 \\ * & \Psi_{22\alpha} & \Psi_{23\alpha} & \mathcal{C}_\alpha^T \\ * & * & \Psi_{33\alpha} & \mathcal{D}_\alpha^T \\ * & * & * & -I \end{bmatrix} < 0 \tag{17}$$

and

$$\Upsilon_\alpha = \begin{bmatrix} -M_\alpha - M_\alpha^T & \mathcal{W}_\alpha + M_\alpha \mathcal{A}_\alpha - S_\alpha^T \\ * & sym(S_\alpha \mathcal{A}_\alpha) \end{bmatrix} < 0 \tag{18}$$

where

$$\begin{aligned} \bar{\Psi}_{11\alpha} &= -\mathcal{Q}_\alpha - M_\alpha - M_\alpha^T, \\ \bar{\Psi}_{12\alpha} &= \mathcal{P}_\alpha + M_\alpha \mathcal{A}_\alpha - S_\alpha^T, \\ \bar{\Psi}_{13\alpha} &= M_\alpha \mathcal{B}_\alpha - R_\alpha^T, \\ \bar{\Psi}_{22\alpha} &= \mu_l^2 \mathcal{Q}_\alpha + sym(S_\alpha \mathcal{A}_\alpha), \\ \bar{\Psi}_{23\alpha} &= S_\alpha \mathcal{B}_\alpha + \mathcal{A}_\alpha^T R_\alpha^T, \\ \bar{\Psi}_{33\alpha} &= -\gamma^2 I + sym(R_\alpha \mathcal{B}_\alpha). \end{aligned}$$

Proof First, we prove that (12-13) is equivalent to (17). Suppose that (17) holds, let

$$\begin{aligned} \mathcal{M} &= \begin{bmatrix} -\mathcal{Q}_\alpha & \mathcal{P}_\alpha & 0 \\ \mathcal{P}_\alpha & \bar{\mu}_l^2 \mathcal{Q}_\alpha + \mathcal{C}_\alpha^T \mathcal{C}_\alpha & \mathcal{C}_\alpha^T \mathcal{D}_\alpha \\ 0 & \mathcal{D}_\alpha^T \mathcal{C}_\alpha & -\gamma^2 I + \mathcal{D}_\alpha^T \mathcal{D}_\alpha \end{bmatrix}; \quad (19) \\ \zeta &= \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \\ w(t) \end{bmatrix}; \mathcal{Y} = \begin{bmatrix} M_\alpha \\ S_\alpha \\ R_\alpha \end{bmatrix}; \mathcal{X} = \begin{bmatrix} -I & \mathcal{A}_\alpha & \mathcal{B}_\alpha \end{bmatrix}. \end{aligned}$$

By Shur complement, (17) is equivalent to

$$\mathcal{M} + \mathcal{Y}\mathcal{X} + \mathcal{X}^T \mathcal{Y}^T < 0 \tag{20}$$

under condition (4) of Lemma 1, with

$$\mathcal{X}^\perp = \begin{bmatrix} \bar{A}_\alpha & \bar{B}_\alpha \\ I & 0 \\ 0 & I \end{bmatrix}$$

using condition (2) of lemma 1, we obtain (12-13).
On the other hand, let us construct a Lyapunov function inequality, $\mathcal{A}_\alpha$ is stable if and only if there exists $\mathcal{W}_\alpha = \mathcal{W}_\alpha^T > 0$ such that

$$\mathcal{A}_\alpha^T \mathcal{W}_\alpha + \mathcal{W}_\alpha \mathcal{A}_\alpha < 0 \tag{21}$$

which is rewritten in the form

$$\begin{bmatrix} \mathcal{A}_\alpha \\ I \end{bmatrix}^T \begin{bmatrix} 0 & \mathcal{W}_\alpha \\ \mathcal{W}_\alpha & 0 \end{bmatrix} \begin{bmatrix} \mathcal{A}_\alpha \\ I \end{bmatrix} < 0 \tag{22}$$

Define

$$\begin{aligned} \mathcal{M} &= \begin{bmatrix} 0 & \mathcal{W}_\alpha \\ \mathcal{W}_\alpha & 0 \end{bmatrix}; \zeta = \begin{bmatrix} \dot{\xi}(t) \\ \xi(t) \end{bmatrix}; \mathcal{Y} = \begin{bmatrix} M_\alpha \\ S_\alpha \end{bmatrix}; \\ \mathcal{X} &= \begin{bmatrix} -I & \mathcal{A}_\alpha \end{bmatrix}; \mathcal{X}^\perp = \begin{bmatrix} \mathcal{A}_\alpha \\ I \end{bmatrix}. \end{aligned} \tag{23}$$

By lemma 1, (22) and (23) are equivalent to

$$\begin{bmatrix} 0 & \mathcal{W}_\alpha \\ \mathcal{W}_\alpha & 0 \end{bmatrix} + \begin{bmatrix} M_\alpha \\ S_\alpha \end{bmatrix} \begin{bmatrix} -I & \mathcal{A}_\alpha \end{bmatrix} + \begin{bmatrix} -I & \mathcal{A}_\alpha \end{bmatrix}^T \begin{bmatrix} M_\alpha \\ S_\alpha \end{bmatrix}^T < 0 \tag{24}$$

which is nothing but (18). ■

B. Finite Frequency H_∞ Filtering Design

Based on Theorem 1, we select for variables $\mathcal{P}_\alpha$, $\mathcal{Q}_\alpha$ and $\mathcal{W}_\alpha$ the following structures

$$\begin{aligned} \mathcal{P}_\alpha &= \begin{bmatrix} \mathcal{P}_{1\alpha} & \mathcal{P}_{2\alpha} \\ * & \mathcal{P}_{3\alpha} \end{bmatrix}, \mathcal{Q}_\alpha = \begin{bmatrix} \mathcal{Q}_{1\alpha} & \mathcal{Q}_{2\alpha} \\ * & \mathcal{Q}_{3\alpha} \end{bmatrix} > 0, \\ \mathcal{W}_\alpha &= \begin{bmatrix} \mathcal{W}_{1\alpha} & \mathcal{W}_{2\alpha} \\ * & \mathcal{W}_{3\alpha} \end{bmatrix} > 0. \end{aligned} \tag{25}$$

Then, let slack variables M_α, S_α and R_α take the following structures

$$\begin{aligned} M_\alpha &= \begin{bmatrix} M_{1\alpha} & U \\ M_{2\alpha} & U \end{bmatrix}, \quad S_\alpha = \begin{bmatrix} S_{1\alpha} & \eta_1 U \\ S_{2\alpha} & \eta_2 U \end{bmatrix}, \\ R_\alpha &= \begin{bmatrix} R_{1\alpha} & 0 \end{bmatrix}. \end{aligned} \tag{26}$$

where U is invertible matrix, scalar parameters η_1, η_2 will be used as optimization parameters. With a structure of variable matrices, we obtain the following results:

Theorem 2 *Consider the system in (1). For given $\gamma > 0, \eta_1, \eta_2$, a filter of form in (6) exists such that the filtering error system in (7) is asymptotically stable with an H_∞ performance bound γ in the LF domain $|\mu| \leq \mu_l$, if there exist hermitian matrices $\mathcal{P}_\alpha = \begin{bmatrix} \mathcal{P}_{1\alpha} & \mathcal{P}_{2\alpha} \\ * & \mathcal{P}_{3\alpha} \end{bmatrix}$, $\mathcal{Q}_\alpha = \begin{bmatrix} \mathcal{Q}_{1\alpha} & \mathcal{Q}_{2\alpha} \\ * & \mathcal{Q}_{3\alpha} \end{bmatrix} > 0$ and symmetric matrix $\mathcal{W}_\alpha = \begin{bmatrix} \mathcal{W}_{1\alpha} & \mathcal{W}_{2\alpha} \\ * & \mathcal{W}_{3\alpha} \end{bmatrix} > 0$ and matrices $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$, $\hat{D}_f$, $M_{1\alpha}$, $M_{2\alpha}$, $S_{1\alpha}$, $S_{2\alpha}$, $R_{1\alpha}$, and U for all $\alpha \in \Gamma$ such that*

$$\tilde{\Psi}_\alpha = \begin{bmatrix} \tilde{\Psi}_{11\alpha} & \tilde{\Psi}_{12\alpha} & \tilde{\Psi}_{13\alpha} & \tilde{\Psi}_{14\alpha} & \tilde{\Psi}_{15\alpha} & 0 \\ * & \tilde{\Psi}_{22\alpha} & \tilde{\Psi}_{23\alpha} & \tilde{\Psi}_{24\alpha} & \tilde{\Psi}_{25\alpha} & 0 \\ * & * & \tilde{\Psi}_{33\alpha} & \tilde{\Psi}_{34} & \tilde{\Psi}_{35\alpha} & \tilde{\Psi}_{36\alpha} \\ * & * & * & \tilde{\Psi}_{44\alpha} & \tilde{\Psi}_{45\alpha} & -\hat{C}_f^T \\ * & * & * & * & \tilde{\Psi}_{55\alpha} & \tilde{\Psi}_{56\alpha} \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{27}$$

and

$$\tilde{\Upsilon}_\alpha = \begin{bmatrix} \tilde{\Upsilon}_{11\alpha} & -U - M_{2\alpha}^T & \tilde{\Upsilon}_{13\alpha} & \tilde{\Upsilon}_{14\alpha} \\ * & -U - U^T & \tilde{\Upsilon}_{23\alpha} & \tilde{\Upsilon}_{24\alpha} \\ * & * & \tilde{\Upsilon}_{33\alpha} & \tilde{\Upsilon}_{34\alpha} \\ * & * & * & \tilde{\Upsilon}_{44\alpha} \end{bmatrix} < 0 \tag{28}$$

where

$$\tilde{\Psi}_{11\alpha} = -\mathcal{Q}_{1\alpha} - M_{1\alpha} - M_{1\alpha}^T;$$

$$
\begin{array}{rcl}
\tilde{\Psi}_{12\alpha} &=& -\mathcal{Q}_{2\alpha} - U - M_{2\alpha}^T;\\
\tilde{\Psi}_{13\alpha} &=& \mathcal{P}_{1\alpha} - S_{1\alpha}^T + M_{1\alpha}A_\alpha + \hat{B}_f C_\alpha;\\
\tilde{\Psi}_{14\alpha} &=& \mathcal{P}_{2\alpha} - S_{2\alpha}^T + \hat{A}_f;\\
\tilde{\Psi}_{15\alpha} &=& -R_{1\alpha}^T + M_{1\alpha}B_\alpha + \hat{B}_f D_\alpha;\\
\tilde{\Psi}_{22\alpha} &=& -\mathcal{Q}_{3\alpha} - U - U^T;\\
\tilde{\Psi}_{23\alpha} &=& \mathcal{P}_{2\alpha}^T - \eta_1 U^T + M_{2\alpha}A_\alpha + \hat{B}_f C_\alpha;\\
\tilde{\Psi}_{24\alpha} &=& \mathcal{P}_{3\alpha} - \eta_2 U^T + \hat{A}_f;\\
\tilde{\Psi}_{25\alpha} &=& M_{2\alpha}B_\alpha + \hat{B}_f D_\alpha;\\
\tilde{\Psi}_{33\alpha} &=& \mu_l^2 \mathcal{Q}_{1\alpha} + sym(S_1 A_\alpha + \eta_1 \hat{B}_f C_\alpha);\\
\tilde{\Psi}_{34\alpha} &=& \mu_l^2 \mathcal{Q}_{2\alpha} + \lambda_1 \hat{A}_f + A_\alpha^T S_{2\alpha}^T + \eta_2 C_\alpha^T \hat{B}_f^T;\\
\tilde{\Psi}_{35\alpha} &=& A_\alpha^T R_{1\alpha}^T + S_{1\alpha}B_\alpha + \eta_1 \hat{B}_f D_\alpha;\\
\tilde{\Psi}_{36\alpha} &=& L_\alpha^T - C_\alpha^T \hat{D}_f^T;\\
\tilde{\Psi}_{44\alpha} &=& \mu_l^2 \mathcal{Q}_{3\alpha} + \lambda_2(\hat{A}_F + \hat{A}_f^T);\\
\tilde{\Psi}_{45\alpha} &=& S_{2\alpha}B_\alpha + \lambda_2 \hat{B}_F D_\alpha;\\
\tilde{\Psi}_{55\alpha} &=& -\gamma^2 I + sym(R_{1\alpha}B_\alpha);\\
\tilde{\Psi}_{56\alpha} &=& -D_\alpha^T \hat{D}_f^T;\\
\tilde{\Upsilon}_{11\alpha} &=& -M_{1\alpha} - M_{1\alpha}^T;\\
\tilde{\Upsilon}_{13\alpha} &=& \mathcal{W}_{1\alpha} + M_{1\alpha}A_\alpha + \hat{B}_f C_\alpha - S_{1\alpha}^T;\\
\tilde{\Upsilon}_{14\alpha} &=& \mathcal{W}_{2\alpha} - S_{2\alpha}^T + \hat{A}_f;\\
\tilde{\Upsilon}_{23\alpha} &=& \mathcal{W}_{2\alpha}^T - \eta_1 U^T + M_{2\alpha}A_\alpha + \hat{B}_f C_\alpha;\\
\tilde{\Upsilon}_{24\alpha} &=& \mathcal{W}_{3\alpha} - \eta_2 U^T + \hat{A}_f;\\
\tilde{\Upsilon}_{33\alpha} &=& sym(S_{1\alpha}A_\alpha + \eta_1 \hat{B}_f C_\alpha);\\
\tilde{\Upsilon}_{34\alpha} &=& \eta_1 \hat{A}_f + A_\alpha^T S_{2\alpha}^T + \eta_2 C_\alpha^T \hat{B}_f^T;\\
\tilde{\Upsilon}_{44\alpha} &=& \eta_2(\hat{A}_f + \hat{A}_f^T).
\end{array}
$$

Moreover, if the previous conditions are satisfied, an state-space realization of the H_∞ filter is given by

$$
\begin{array}{rcl}
A_f &=& U^{-1}\hat{A}_f; \quad B_f = U^{-1}\hat{B}_f\\
C_f &=& \hat{C}_f; \quad D_f = \hat{D}_f.
\end{array}
\tag{29}
$$

Define matrices $\mathcal{W}_{s\alpha}$, $\mathcal{P}_{s\alpha}$, $\mathcal{Q}_{s\alpha}$, $s = 1,2,3$, $M_{u\alpha}$, $S_{u\alpha}$, $u = 1,2$, and $R_{1\alpha}$ take homogeneous polynomially parameter-dependent matrix form given in (15), the main result in this section is given in the following Theorem.

Theorem 3 *Consider system (1). given a scalars $\gamma > 0$, η_1, η_2, a filter of form (6) exists such that the filtering error system in (7) is asymptotically stable with an H_∞ performance bound γ in the LF domain $|\mu| \le \mu_l$. If there exist Hermitian matrices $\mathcal{P}(k_j(g)) = \begin{bmatrix} \mathcal{P}_{1k_j(g)} & \mathcal{P}_{2k_j(g)} \\ * & \mathcal{P}_{3k_j(g)} \end{bmatrix}$, $\mathcal{Q}(k_j(g)) = \begin{bmatrix} \mathcal{Q}_{1k_j(g)} & \mathcal{Q}2k_j(g) \\ * & \mathcal{Q}_{3k_j(g)} \end{bmatrix} > 0$ and symmetric matrix $\mathcal{W}(k_j(g)) = \begin{bmatrix} \mathcal{W}_{1k_j(g)} & \mathcal{W}_{2k_j(g)} \\ * & \mathcal{W}_{3k_j(g)} \end{bmatrix} > 0$, matrices $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$, $\hat{D}_f$, $M_{tk_j(g)}$, $S_{tk_j(g)}$ (where $t = 1,2$), $R_{1k_j(g)}$, U and the real scalars η_1 and η_2 such that the following LMIs hold for all $K_l(g+1) \in K(g+1)$, $l = 1, ..., J(g+1)$:*

$$
\begin{bmatrix} \mathcal{Q}_{1k_j(g)} & \mathcal{Q}_{2k_j(g)} \\ * & \mathcal{Q}_{3k_j(g)} \end{bmatrix} > 0; \begin{bmatrix} \mathcal{W}_{1k_j(g)} & \mathcal{W}_{2k_j(g)} \\ * & \mathcal{W}_{3k_j(g)} \end{bmatrix} > 0 \tag{30}
$$

$$
\sum_{i \in I_l(g+1)} \left[\check{\Psi}_k + \check{\Phi}_k\right] < 0 \tag{31}
$$

$$
\sum_{i \in I_l(g+1)} \check{\Upsilon}_k < 0 \tag{32}
$$

where

$$
\check{\Psi}_k = \begin{bmatrix}
\check{\Psi}_{11k} & \check{\Psi}_{12k} & \check{\Psi}_{13k} & \check{\Psi}_{14k} & \check{\Psi}_{15k} & 0\\
* & \check{\Psi}_{22k} & \check{\Psi}_{23k} & \check{\Psi}_{24k} & \check{\Psi}_{25k} & 0\\
* & * & \check{\Psi}_{33k} & \check{\Psi}_{34k} & \check{\Psi}_{35k} & \check{\Psi}_{36k}\\
* & * & * & \check{\Psi}_{44k} & \check{\Psi}_{45k} & -\beta_j^i(g+1)\hat{C}_f^T\\
* & * & * & * & \check{\Psi}_{55k} & \check{\Psi}_{56k}\\
* & * & * & * & * & -\beta_j^i(g+1)I
\end{bmatrix}
$$

$$
\check{\Phi}_k = \begin{bmatrix}
\check{\Phi}_{11k} & -\mathcal{Q}_{2k_j(g)} & \mathcal{P}_{1k_j(g)} & \mathcal{P}_{2k_j(g)} & 0 & 0\\
* & -\mathcal{Q}_{3k_j(g)} & \mathcal{P}_{2k_j(g)}^T & \mathcal{P}_{3k_j(g)} & 0 & 0\\
* & * & \mu_l^2 \mathcal{Q}_{1k_j(g)} & \mu_l^2 \mathcal{Q}_{2k_j(g)} & 0 & 0\\
* & * & * & \mu_l^2 \mathcal{Q}_{3k_j(g)} & 0 & 0\\
* & * & * & * & 0 & 0\\
* & * & * & * & * & 0
\end{bmatrix}
$$

$$
\check{\Upsilon}_k = \begin{bmatrix}
-M_{1k_j(g)} - M_{1k_j(g)}^T & \check{\Upsilon}_{12k} & \check{\Upsilon}_{13k} & \check{\Upsilon}_{14k}\\
* & \check{\Upsilon}_{22k} & \check{\Upsilon}_{23k} & \check{\Upsilon}_{24k}\\
* & * & \check{\Upsilon}_{33k} & \check{\Upsilon}_{34k}\\
* & * & * & \check{\Upsilon}_{44k}
\end{bmatrix}
$$

$$
\begin{array}{rcl}
\check{\Psi}_{11k} &=& -M_{1k_j(g)} - M_{1k_j(g)}^T;\\
\check{\Psi}_{12k} &=& -\beta_j^i(g+1)U - M_{2k_j(g)}^T;\\
\check{\Psi}_{13k} &=& -S_{1k_j(g)}^T + M_{1k_j(g)}A_i + \beta_j^i(g+1)\hat{B}_f C_i;\\
\check{\Psi}_{14k} &=& -\beta_j^i(g+1)\eta_2 U - S_{2k_j(g)}^T + \beta_j^i(g+1)\hat{A}_f;\\
\check{\Psi}_{15k} &=& -R_{1k_j(g)}^T + M_{1k_j(g)}B_i + \beta_j^i(g+1)\hat{B}_f D_i;\\
\check{\Psi}_{22k} &=& -\beta_j^i(g+1)\eta_1 U - \beta_j^i(g+1)\eta_1 U^T;\\
\check{\Psi}_{23k} &=& -\beta_j^i(g+1)\eta_2 U^T + M_{2k_j(g)}A_i + \beta_j^i(g+1)\eta_1 \hat{B}_f C_i;\\
\check{\Psi}_{24k} &=& -\beta_j^i(g+1)U + \beta_j^i(g+1)\eta_1 \hat{A}_f;\\
\check{\Psi}_{25k} &=& M_{2k_j(g)}B_i + \beta_j^i(g+1)\eta_1 \hat{B}_f D_i;\\
\check{\Psi}_{33k} &=& sym[S_1 A_i + \beta_j^i(g+1)\eta_2 \hat{B}_f C_i];\\
\check{\Psi}_{34k} &=& \beta_j^i(g+1)\eta_1 \hat{A}_f + A_i^T S_{2k_j(g)}^T + \beta_j^i(g+1)C_i^T \hat{B}_f^T;\\
\check{\Psi}_{35k} &=& A_i^T R_{1k_j(g)}^T + S_{1k_j(g)}B_i + \beta_j^i(g+1)\eta_2 \hat{B}_f D_i;\\
\check{\Psi}_{36k} &=& \beta_j^i(g+1)L_i^T - \beta_j^i(g+1)C_i^T \hat{D}_f^T;\\
\check{\Psi}_{44k} &=& \beta_j^i(g+1)[\hat{A}_f + \hat{A}_f^T];\\
\check{\Psi}_{45k} &=& S_{2k_j(g)}B_i + \beta_j^i(g+1)\hat{B}_f D_i;\\
\check{\Psi}_{55k} &=& -\beta_j^i(g+1)\gamma^2 I + sym[R_{1k_j(g)}B_i];\\
\check{\Psi}_{56k} &=& -\beta_j^i(g+1)D_i^T \hat{D}_f^T;\\
\check{\Phi}_{11k} &=& -\mathcal{Q}_{1k_j(g)};\\
\check{\Upsilon}_{12k} &=& -\beta_j^i(g+1)U - M_{2k_j(g)}^T;\\
\check{\Upsilon}_{13k} &=& \mathcal{W}_{1k_j(g)} + M_{1k_j(g)}A_i + \beta_j^i(g+1)\hat{B}_f C_i - S_{1k_j(g)}^T;\\
\check{\Upsilon}_{14k} &=& \mathcal{W}_{2k_j(g)} - S_{2k_j(g)}^T + \beta_j^i(g+1)\hat{A}_f;\\
\check{\Upsilon}_{22k} &=& -\beta_j^i(g+1)\eta_1 U - \beta_j^i(g+1)\eta_1 U^T;
\end{array}
$$

$$
\begin{aligned}
\check{\Upsilon}_{23k} &= \mathcal{W}^T_{2k_j(g)} - \beta^i_j(g+1)\eta_2 U^T + M_{2k_j(g)} A_i \\
&+ \beta^i_j(g+1)\eta_1 \hat{B}_f C_i; \\
\check{\Upsilon}_{24k} &= \mathcal{W}_{3k_j(g)} - \beta^i_j(g+1)U^T + \beta^i_j(g+1)\eta_1 \hat{A}_f; \\
\check{\Upsilon}_{33k} &= sym[S_{1k_j(g)} A_i + \beta^i_j(g+1)\eta_2 \hat{B}_f C_i]; \\
\check{\Upsilon}_{34k} &= \beta^i_j(g+1)\eta_2 \hat{A}_f + A_i^T S^T_{2k_j(g)} + \beta^i_j(g+1)\eta_1 C_i^T \hat{B}_f^T; \\
\check{\Upsilon}_{44k} &= \beta^i_j(g+1)\eta_2\{\hat{A}_f + \hat{A}_f^T\}.
\end{aligned}
$$

Then the homogeneous polynomially parameter-dependent matrices given by (16) ensure Theorem 2 for all $\alpha \in \Gamma$. Moreover, if the LMIs of Theorem 3 are fulfilled for a given degree $\bar{g}$, then the LMIs corresponding to any degree $g > \bar{g}$ are also satisfied.

Proof The proof is similar to that of Theorem 3 in [17] and is thus omitted. ■

Remark 2 *When scalars η_1 and η_2 of Theorem 3 are fixed to be constant, then (30), (31) and (32) are LMIs which are effectively linear in the variables. To select values for these scalars, optimization can be used to optimize some performance measure (for example γ , the disturbance attenuation level).*

Remark 3 *As the degree g of the polynomial increases, conditions become less conservative as new free variables are added to LMIs. Although the number of LMIs is also increasing, each LMIs becomes easier to fill because of the additional degrees of freedom provided by the new free variables; as a result, better H_∞ guaranteed costs can be obtained.*

Remark 4 *If we take $\mathcal{Q}_\alpha = 0$, we can use Theorem 3 to solve the robust H_∞ filtering problem in the entire frequency domain for uncertain continuous-time linear systems.*

Remark 5 *In Theorem 3, $\bar{\mu}_l$ is a constant related to the frequency, which is given beforehand. Conditions (30), (31) and (32) are LMIs, and thus the above optimization problem is convex and can be easily solved by Yalmip [19] and SeDuMi [20] in MATLAB 7.6.*

IV. Simulation Study

Based on the example presented in [5][15], consider a continuous-time system of form (1) described by

$$
\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} 0 & -1+0.3\beta \\ 0.01+0.05\beta & 0.9 \end{bmatrix} x(t) \\
&+ \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} w(t) \qquad (33) \\
y(t) &= \begin{bmatrix} -100+10\beta & 100 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \end{bmatrix} w(t) \\
z(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)
\end{aligned}
$$

where β is the uncertain parameter satisfying $|\beta| \leq 1$. By solving Theorem 3 , the minimum H_∞ attenuation level is listed in Tables 1 and 2.

Table 1: Comparison of filtering performance obtained in different methods for LF range ($|\mu| \leq 20$).

Degree	*FF* ($|\mu| \leq 20$) *Theorem 2* [15]	*Theorem 3*
$g=1$	**1.1402**	**1.0088** $\eta_1 =$ 10.8554 $\eta_2 =$ 1.3558
$g=2$	–	**0.9523** $\eta_1 =$ 12.1585 $\eta_2 =$ 0.9513

Values of $\mathbf{H_\infty}$ performance are highlighted in bold.

Table 2: Comparison of filtering performance obtained in different methods for EF range.

Degree	*Th* 2 [5]	*Th* 4 [15]	**Th 3 (Q = 0)**
$g=1$	**2.1700**	**1.1511**	**1.1185** $\eta_1 =$ 0.8945 $\eta_2 =$ 0.3723
$g=2$	–	–	**1.0913** $\eta_1 =$ 0.8050 $\eta_2 =$ 0.4732

Values of $\mathbf{H_\infty}$ performance are highlighted in bold.

Table 1 gives the robust H_∞ filtering performance levels obtained by the finite frequency (FF) approaches proposed in this work and in [15]. In addition, Table 2 shows that even in the entire frequency (EF) range, our proposed approach outperforms some recent results in the literature, which study the entire frequency (EF) robust H_∞ filtering problem.
It is clearly shown that Theorem 3 yields less conservative results than the finite frequency method proposed in [15], as well as the entire frequency methods in Theorem 3 (with $\mathcal{Q}_\alpha = 0$) and [15][5].
The filter parameters in different frequency ranges are:

1) Low Frequency (LF) Case: For degree $\mathbf{g=1}$, the obtained disturbance attenuation level is $\gamma = \mathbf{1.0088}$ and the corresponding filter matrices are:

$$
\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{cc|c} -2.9842 & -1.0490 & 0.2342 \\ -1.8046 & -3.4523 & 0.1418 \\ \hline 0.0085 & -0.0731 & -0.0102 \end{array}\right] \quad (34)
$$

When degree $\mathbf{g=2}$, $\gamma = \mathbf{0.9523}$ and the corresponding filter matrices are:

$$
\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{cc|c} -1.2394 & -1.4694 & 0.1553 \\ -0.2101 & -2.3163 & 0.0264 \\ \hline 0.0031 - 0.0520 & & -0.0097 \end{array}\right] \quad (35)
$$

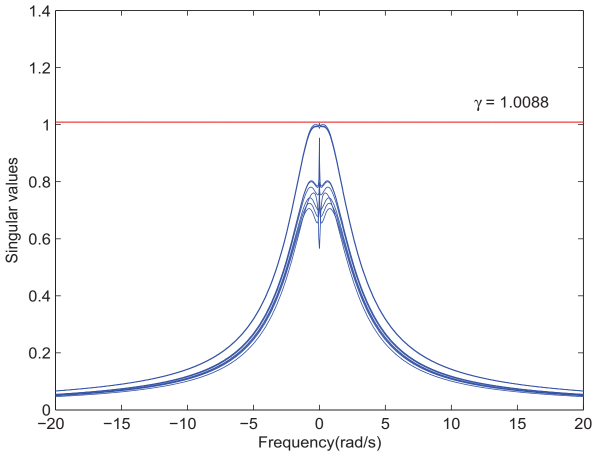
2) Entire Frequency (EF) case: For degree $\mathbf{g=1}$, $\gamma = \mathbf{1.1185}$ and the corresponding filter matrices are:

$$
\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{cc|c} -0.4698 & -3.5690 & 0.0111 \\ 0.1278 & -3.0759 & -0.0030 \\ \hline 0.0441 & -0.2786 & -0.0108 \end{array}\right] \quad (36)
$$

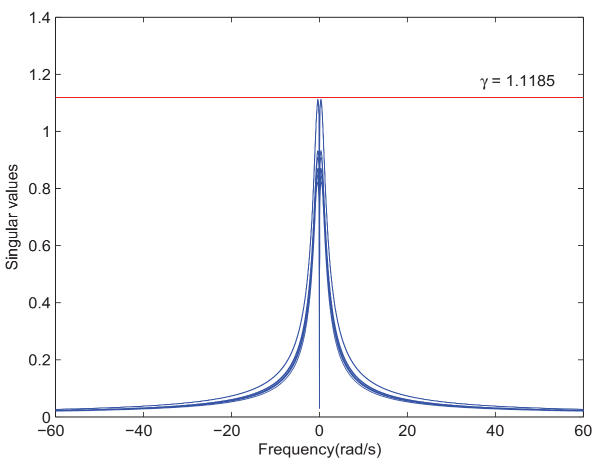
Finally, when $\mathbf{g=2}$, $\gamma = \mathbf{1.0913}$ and the corresponding filter matrices are:

$$
\left[\begin{array}{c|c} A_f & B_f \\ \hline C_f & D_f \end{array}\right] = \left[\begin{array}{cc|c} -1.5402 & 0.5849 & 0.0257 \\ -0.0813 & -1.4089 & 0.0014 \\ \hline 0.1439 & -0.4503 & -0.0122 \end{array}\right] \quad (37)
$$

Illustrate the effectiveness of these designed filters. We consider the polytopic case (degree $g = 1$), by respectively connecting (34) and (36) to the systems of (33), the frequency responses of the filtering error systems are represented in the figures 1 and 2, respectively. All the singular values in these figures are below the filtering performance perturbation attenuation level H_∞ reaches γ, which demonstrates the effectiveness of the proposed method.



Singular value curves for LF range ($|\mu| \leq 20$) with filter (34).



Singular value curves for EF range with filter (36).

V. Conclusion

In this paper, we have discussed the H_∞ filtering problem for linear invariant systems in continuous time with polytopic uncertainties in the low frequency ranges. Based on the gKYP lemma, the Finsler's lemma, the polynomially-dependent Lyapunov function of parameters and some sufficient conditions are given to characterize this problem in terms of linear matrix inequalities (LMI). Finally, through a numerical example, we have shown that the proposed method has advantages over some existing works.

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