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An adaptive approach to zooming-based control for uncertain systems with input quantization

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Abstract— **This paper establishes an adaptive tracking approach for linear systems with parametric uncertainties, when input measurements are quantized due to the presence of a communication network closing the control loop. In order to address the tracking problem, a novel dynamic quantizer with dynamic offset is introduced and embedded into an adaptive hybrid control strategy based on zooming mechanism. A Lyapunov-based approach is used to derive the adaptive adjustments for the control gains and for the dynamic range and dynamic offset of the quantizer: it is proven analytically that the proposed adjustments guarantee asymptotic state tracking. Quantized adaptive control of an electrohydraulic system is given as an example of the effectiveness of the designed control methodology.**

Index Terms- **Hybrid dynamic quantization, input quantization, asymptotic tracking, model reference adaptive control.**

I. INTRODUCTION

With its clearly defined goal of designing control systems that can adapt the control gains to parametric uncertainties and changing conditions, adaptive control constitutes a flourishing research area [1], [2]. A classical control problem is the one of tracking (possibly asymptotically) a desired reference: adaptive control has been shown to tackle this challenge in the presence of unknown or time-varying systems' parameters, providing significant advantages in several application domains [3], [4], [5], [6], [7]. However, despite the emerging research field of networked control systems (NCSs), only limited attention has been devoted to adaptive tracking with networked-induced constraints. In a NCS the control loop is closed through a communication network [8], [9], so that control and feedback signals are exchanged in the form of information packages through the network: because of this, some feedback signals must be quantized [10]. This work will be focusing on how to achieve adaptive (asymptotic) tracking in the presence of networked-induced quantization. In the following we explain how this problem is still an open one.

Much attention has been devoted by the control community to asymptotic stability in non-adaptive (i.e. with fixed-gain control) NCSs in the presence of quantization, with a focus on regulation problems. Established approaches for achieving asymptotic regulation rely on dynamic quantization mechanisms such as logarithmic quantization [11], [12], [13] or zooming-based hybrid control [14], [15]. The latter mechanism takes its name from the analogy with the zooming in digital cameras: since the quantizer has a fixed number of quantization levels (i.e. number of pixels), when the state is outside its range region, the quantizer 'zooms out' so that the state can be captured within the region. This can be achieved by increasing the size of the range. On the other hand, once the state comes close to the origin, we can 'zoom in' by reducing the size of the range so that the quantization resolution becomes finer while the region becomes smaller. Repeating this zooming in, we can obtain asymptotic stabilization.

In the adaptive setting, [16] considered a passification-based adaptive controller with quantized measurements and disturbances, where ultimate boundedness can be obtained. The authors in [17] developed a direct adaptive control framework with a logarithmic quantizer, guaranteeing partial asymptotic stability, i.e. Lyapunov stability of the closed-loop system states and attraction with respect to the plant states. For nonlinear uncertain systems, adaptive quantized approaches based on backstepping [18], [19], neural-networks [20], [21] or passification [22] have been developed with guaranteed global ultimate boundedness. These adaptive approaches to quantization are limited to regulation problems: actually, the fact that the quantizer is anti-symmetric with respect to the origin makes it suitable for regulation, but it prevents from achieving high precision in the tracking case. A solution to this drawback has been recently proposed in [23], [24], where asymptotic tracking has been achieved for the first time via a sliding-mode approach. However, the implementation of sliding-mode control in a NCS is not straightforward: in fact, due to chattering, a sliding-mode approach requires infinite communication bandwidth, because it has to send information infinitely.

From this short overview of the state of the art we see that we need to push further the boundaries of current adaptive control tools, so as to achieve to asymptotic tracking in the presence of quantization and finite communication bandwidth. Solving this open problem is the main motivation to this work. The main contribution of this work is tackling the adaptive tracking problem by introducing a novel dynamic quantizer with dynamic offset. We embed this quantizer in a model reference adaptive control framework with zooming mechanism, and a Lyapunov-based approach is used to derive the adjustments for the control gains, and for the dynamic range and dynamic offset of the quantizer. It is

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proven analytically that the proposed adjustment mechanisms guarantee asymptotic tracking.

As the zooming mechanism of state-of-the-art quantizer is typically explained in terms of digital videos, let us give an explanation to the proposed quantizer in similar terms. In fact, dynamic offsets are often used in video encoding: in many H.264-based compression protocols the encoder is regulated on a frame level (offset) to obtain the number of bits that is very close to the allocated one [25]. Several mechanisms have been defined to tune the offset of the quantizer based on the previous frames [26], [27], so as to increase the resolution around similar frames close in time; similarly, our proposed quantizer increases the resolution around a dynamically changing offset in order to achieve asymptotic tracking.

The rest of the paper is organized as follows. Section II formulates the quantized control problem. The adaptive control design is established in Section III, while Section IV presents the main stability and tracking results. In Section V, quantized adaptive control of an electro-hydraulic system serves as an example to illustrate the effectiveness of the proposed ideas.

The notations used in this paper are standard:
$\mathbb{R}$: the set of real numbers;
$\lambda_{\max}(X),(\lambda_{\min}(X))$: the largest (smallest) eigenvalue of matrix X;
$\|X\| = \sqrt{\lambda_{\max}(XX^T)}$: the induced 2-norm of matrix X;
$\|x\| = \sqrt{\sum_{i=1}^{n} |x_i|^2}$: the Euclidean norm of a vector $x \in \mathbb{R}^n$;
$\mathrm{tr}[X]$: the trace of a square matrix X.
$\mathscr{L}_\infty$ class: A vector signal $x(\cdot) \in [0,\infty) \to \mathbb{R}^n$ is said to belong to $\mathscr{L}_\infty$ class ($x \in \mathscr{L}_\infty$), if $\max_{t\geq 0}\|x(t)\| < \infty$, $\forall t \geq 0$;

II. Problem Statement

Let us consider the linear time-invariant system

$$\dot{x}(t) = Ax(t) + Bg_{\eta\mu}(u(t)) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^q$ is the control input, $g_{\eta\mu}(u) : \mathbb{R}^q \to Q$, where $Q \subset \mathbb{R}^q$, is the input quantizer (to be defined later), and the matrices $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times q}$ are *unknown* constant matrices.

A. Linear Reference Model System and Controller Structure

Let us consider the following linear reference model:

$$\dot{x}_\mathrm{m}(t) = A_\mathrm{m}x_\mathrm{m}(t) + B_\mathrm{m}r(t) \tag{2}$$

where $A_\mathrm{m} \in \mathbb{R}^{n\times n}$, $B_\mathrm{m} \in \mathbb{R}^{n\times q}$ are constant *known* matrices with A_m a Hurwitz matrix, $r \in \mathbb{R}^q$ is a bounded continuous reference input signal and $x_\mathrm{m} \in \mathbb{R}^n$ is the desired state to track. The following assumptions are made in order to have a well-posed adaptive problem:

Assumption 1: There exist a constant matrix $K_\mathrm{x}^* \in \mathbb{R}^{n\times q}$ and an invertible constant matrix $K_\mathrm{r}^* \in \mathbb{R}^{q\times q}$ such that

$$A_\mathrm{m} = A + BK_\mathrm{x}^{*T}, \quad B_\mathrm{m} = BK_\mathrm{r}^*. \tag{3}$$

Assumption 2: There exists a known matrix $S \in \mathbb{R}^{q\times q}$ such that

$$\Gamma = K_\mathrm{r}^* S \tag{4}$$

is symmetric and positive definite.

Assumption 3: A and B in (1) belong to a known and bounded uncertainty set Θ.

Remark 1: Assumption 1 is required for the existence of a closed-loop that matches (1) to the reference model (2) (well-posedness). Assumption 2 generalizes the classical condition of knowing the sign of the input vector field in the multivariable case. Both assumptions are, up to now, the most relaxed conditions for ensuring closed-loop signal boundedness in multivariable adaptive control [1], [28], and will be adopted also in our input quantization setting. Assumption 3 is required to obtain a bound to the increasing rate of the tracking error during the zooming in phase, as it will be explained in Section IV.

Being A and B in (1) unknown, the control gains K_x^* and K_r^* in (3) cannot be implemented and must be estimated. Inspired by [1], the following adaptive state-feedback controller is applied:

$$u(t) = K_\mathrm{x}^T(t)x(t) + K_\mathrm{r}(t)r(t). \tag{5}$$

We consider a networked control setup with the controller on the sensor side, so that the control input (5) must be quantized and sent to the actuator via a communication channel. The next section introduces a quantizer appropriate to our control goals.

B. Dynamic Quantizer Design

A quantizer is a device that converts a real-valued signal into a piecewise constant one taking a finite set of values. A common quantization choice that increases precision without sacrificing the bandwidth is adopted in [15], where a uniform dynamic quantizer is used, whose quantization range M and quantization error Δ can be adjusted by using a hybrid control policy. More precisely, let $z \in \mathbb{R}^q$ be the variable being quantized. The uniform static quantizer is described by a function $g : \mathbb{R}^q \to Q$, where $Q \subset \mathbb{R}^q$. The finite set of values is defined as $\{z \in \mathbb{R}^q : g(z) = i\}$, $i \in Q$.

Remark 2: Commonly adopted quantizers, e.g. in [15], [16], are anti-symmetric with respect to zero: as such, they are appropriate only for regulation problems. If we adopted a standard uniform quantizer for the tracking case we would get

$$g(u) = g\left(K_\mathrm{x}^T x + K_\mathrm{r}r\right) = g\left(K_\mathrm{x}^T(x - x_\mathrm{m}) + K_\mathrm{x}^T x_\mathrm{m} + K_\mathrm{r}r\right). \tag{6}$$

We notice that, if we define the state-tracking error

$$e = x - x_\mathrm{m} \tag{7}$$

then, for $e \to 0$ the quantized input converges to $g(K_\mathrm{x}^T x_\mathrm{m} + K_\mathrm{r}r)$ and asymptotic tracking would be in general impossible due to finite precision of the quantizer around $K_\mathrm{x}^T x_\mathrm{m} + K_\mathrm{r}r$.

With these considerations in mind, we introduce an adjustable offset $\eta(t)$ in the quantizer, so as to achieve quantization anti-symmetry around

$$\eta(t) = K_{\mathrm{x}}^T(t)x_{\mathrm{m}}(t) + K_{\mathrm{r}}(t)r(t) \tag{8}$$

i.e. the offset $\eta(t)$ is adaptive depending on the control gains and the signals from the reference model. We define the following dynamic quantizer:

$$g_{\eta\mu}(u) = \mu g\left(\frac{u-\eta}{\mu}\right) \tag{9}$$

where $\mu > 0$ and the time index t has been (and will be) omitted for compactness. Note that the dynamic quantizer in (9) satisfies the following condition:

$$\mu\left\|g\left(\frac{u-\eta}{\mu}\right)\right\| \le \mu M \tag{10}$$

where μM represents the quantization range and $M > 0$ is the quantization range of the static quantizer $g(u)$. Saturation occurs when the quantized signal is outside the range μM of the quantizer. In case of no saturation, the quantizer must satisfy the additional requirement:

$$\left\|\mu g\left(\frac{u-\eta}{\mu}\right) - u\right\| = \mu\left\|g\left(\frac{u-\eta}{\mu}\right) - \frac{u}{\mu}\right\| \le \mu\Delta \tag{11}$$

where $\mu\Delta$ and $\Delta > 0$ represent the largest quantization error in the dynamic and static uniform quantizers respectively, when no saturation occurs.

III. Adaptive Law Controller Design

Since A_{m} in (2) must be Hurwitz so as to generate a bounded signal state x_{m} from bounded r, there exist positive definite matrices $P \in \mathbb{R}^{n\times n}$, $Q \in \mathbb{R}^{n\times n}$ such that the following inequality holds:

$$A_{\mathrm{m}}^T P + PA_{\mathrm{m}} \le -Q. \tag{12}$$

When the quantized adaptive state-feedback controller given by (5), (9), is applied to (1), the closed-loop system reads as:

$$\begin{aligned}\dot{x} =& Ax + B(K_{\mathrm{x}}^T x + K_{\mathrm{r}} r)\\ &+ \mu B \underbrace{\left[g\left(\frac{K_{\mathrm{x}}^T x + K_{\mathrm{r}} r - \eta}{\mu}\right) - \frac{(K_{\mathrm{x}}^T x + K_{\mathrm{r}} r)}{\mu}\right]}_{\Delta_{\mathrm{u}}}\end{aligned} \tag{13}$$

where Δ_{u} is defined as the quantization error, and in case of no saturation[1] it holds $\|\Delta_{\mathrm{u}}\| \le \Delta$.

In view of (2) and (13), the evolution of the tracking error can be written as:

$$\dot{e} = \dot{x} - \dot{x}_{\mathrm{m}} = A_{\mathrm{m}} e + B\tilde{K}_{\mathrm{x}}^T x + B\tilde{K}_{\mathrm{r}} r + B\mu\Delta_{\mathrm{u}} \tag{14}$$

where $\tilde{K}_{\mathrm{x}} = K_{\mathrm{x}} - K_{\mathrm{x}}^*$ and $\tilde{K}_{\mathrm{r}} = K_{\mathrm{r}} - K_{\mathrm{r}}^*$, correspond to the controller parameter errors. In order to analyze the stability of the closed-loop system (14), the following Lyapunov-like function is considered:

$$V = e^T Pe + \underbrace{\mathrm{tr}\left[\tilde{K}_{\mathrm{x}}\Gamma^{-1}\tilde{K}_{\mathrm{x}}^T\right] + \mathrm{tr}\left[\tilde{K}_{\mathrm{r}}^T\Gamma^{-1}\tilde{K}_{\mathrm{r}}\right]}_{K_{\mathrm{v}}} \tag{15}$$

with $\Gamma \in \mathbb{R}^{q\times q} > 0$ coming from (4).

In view of Assumption 3, lower and upper bounds for the controller parameters $K_{\mathrm{x}}, K_{\mathrm{r}}$ can be found (this can be done by testing the matching conditions (3) over the uncertainty set Θ). As a result, a parameter projection adaptive law can be derived:

$$\begin{aligned}\dot{K}_{\mathrm{x}}^T &= -S^T B_{\mathrm{m}}^T Pex^T + F_{\mathrm{x}}^T\\ \dot{K}_{\mathrm{r}} &= -S^T B_{\mathrm{m}}^T Per^T + F_{\mathrm{r}}\end{aligned} \tag{16}$$

where F_{x} and F_{r} are the projection terms that keep the estimates inside the upper and lower bounds, as defined in [29]. Using (16) and the properties of F_x and F_{r} [29], the time derivative of (15) along (14) is

$$\begin{aligned}\dot{V} =& e^T \underbrace{(A_{\mathrm{m}}^T P + PA_{\mathrm{m}})}_{\le -Q} e + 2\underbrace{\mathrm{tr}[\tilde{K}_{\mathrm{x}}\Gamma^{-1}F_{\mathrm{x}}^T]}_{\le 0}\\ &+ \underbrace{2\mathrm{tr}[\tilde{K}_{\mathrm{r}}^T\Gamma^{-1}F_{\mathrm{r}}]}_{\le 0} + 2e^T PB\mu\Delta_{\mathrm{u}}\end{aligned} \tag{17}$$

which results in

$$\dot{V} \le -e^T Qe + 2e^T PB\mu\Delta_{\mathrm{u}}. \tag{18}$$

Because K_{x}, K_{r} are bounded due to the projection terms in (16), we can define $\rho \in \mathbb{R} \ge 0$ such that:

$$\rho = \max_{t\ge 0}\left\{\mathrm{tr}\left[\tilde{K}_{\mathrm{x}}\Gamma^{-1}\tilde{K}_{\mathrm{x}}^T\right] + \mathrm{tr}\left[\tilde{K}_{\mathrm{r}}^T\Gamma^{-1}\tilde{K}_{\mathrm{r}}\right]\right\} \tag{19}$$

and because of (15) we have

$$e^T Pe \le V \le e^T Pe + \rho. \tag{20}$$

Because P is positive definite, the following inequalities hold

$$\lambda_{min}(P)\|e\|^2 \le e^T Pe \le \lambda_{max}(P)\|e\|^2 \tag{21}$$

with $\lambda_{max}(P) \ge \lambda_{min}(P) > 0$.

A. Preliminaries in Hybrid Control Policy

The time derivative of V in (18) in case of no saturation can be equivalently expressed as

$$\begin{aligned}\dot{V} \le& -e^T Qe + 2e^T PB\mu\Delta_{\mathrm{u}}\\ \le& -\lambda_{\min}(Q)\|e\|^2 + 2e^T PB\mu\Delta\\ \le& -\lambda_{\min}(Q)\|e\|\left(\|e\| - \underbrace{\frac{2\max_{B\in\Theta}\|PB\|}{\lambda_{\min}(Q)}}_{R}\mu\Delta\right) \implies\end{aligned}$$

$$\dot{V} \le -\|e\|\,\lambda_{\min}(Q)(\|e\| - \mu R\Delta) \tag{22}$$

where R is bounded, in view of Assumption 3. According to (10), the requirement of no saturation is represented by the following condition:

$$\|u-\eta\| \le \mu M.$$

[1] In case of no saturation ($\|u-\eta\| \le \mu M$), it holds $\|g_{\eta\mu}(u) - u\| = \mu\|\Delta_{\mathrm{u}}\| \le \mu\Delta$.

We define

$$\overline{K}_{\mathrm{x}} = \max_{t\geq 0}\|K_{\mathrm{x}}\| \tag{23}$$

which is well defined in view of the projection terms in (16). Considering $\|u-\eta\| = \left\|K_{\mathrm{x}}^T x + K_{\mathrm{r}} r - K_{\mathrm{x}}^T x_{\mathrm{m}} - K_{\mathrm{r}} r\right\| = \left\|K_{\mathrm{x}}^T (x - x_{\mathrm{m}})\right\|$, the condition for no saturation is satisfied if the following condition holds:

$$\|e\| \leq \frac{\mu M}{\overline{K}_{\mathrm{x}}}. \tag{24}$$

We define the following regions:

$$\begin{aligned}
\mathscr{B}_1(\mu) &:= \left\{ e : \|e\| \leq \frac{\mu M}{\overline{K}_{\mathrm{x}}} \right\} \\
\mathscr{I}_1(\mu) &:= \left\{ e : e^T P e \leq \lambda_{\min}(P) \frac{\mu^2 M^2}{\overline{K}_{\mathrm{x}}{}^2} \right\} \\
\mathscr{B}_2(\mu) &:= \left\{ e : \|e\| \leq \mu R \Delta \right\} \\
\mathscr{I}_2(\mu) &= \left\{ e : e^T P e \leq \lambda_{\max}(P) \mu^2 R^2 \Delta^2 \right\}.
\end{aligned} \tag{25}$$

Note that, when

$$\frac{\sqrt{\lambda_{\min}(P)} M}{\overline{K}_{\mathrm{x}}} > \sqrt{\lambda_{\max}(P)} R \Delta$$

then $\mathscr{B}_2(\mu) \subset \mathscr{I}_2(\mu) \subset \mathscr{I}_1(\mu) \subset \mathscr{B}_1(\mu)$.

IV. Main Result

Using the previously explained design, the following stability result can be derived:

Theorem 4.1: Consider the input-quantized model reference adaptive control given by system (1), reference model (2), quantizer (9), adaptive laws (16). If the following holds

$$\frac{\sqrt{\lambda_{\min}(P)} M}{\overline{K}_{\mathrm{x}}} > \sqrt{\lambda_{\max}(P)} R \Delta \tag{26}$$

then there exists an error-based hybrid quantized feedback control policy that makes the closed-loop system (14) globally asymptotically stable with $\lim_{t\to\infty} e(t) = 0$.

Proof: The hybrid quantized feedback control is designed in a constructive way along the proof. Let us distinguish two phases, namely the zooming-out and zooming-in phases [15]. In the zooming-out phase μ is chosen so that $e \in \mathscr{B}_1(\mu)$ and thus boundedness can be guaranteed. During the zooming-in phase the objective is to shrink the region $\mathscr{I}_2(\mu)$ by reducing the dynamic quantizer parameter μ so that state-tracking properties can be concluded. The two phases are analyzed as follows:

Zooming-out phase: Let $\mu(0) = 1$. If $\left\|e(0)\right\| > \frac{M}{\overline{K}_{\mathrm{x}}}$ we have saturation. In this case we increase $\mu(t)$ fast enough to dominate the growth of e, which can be seen from (14) to be equal to $\left|e^{\max_{A,B\in\Theta}\left\|A+BK_{\mathrm{x}}^T\right\|}\right|$, where $\max_{A,B\in\Theta}\left\|A+BK_{\mathrm{x}}^T\right\|$ is bounded in view of Assumption 3. There will be a time instant, call it $t_0 > 0$, at which the following relation is true:

$$\left\|e(t_0)\right\| \leq \sqrt{\frac{\lambda_{\min}(P)}{\lambda_{\max}(P)}} \frac{\mu(t_0) M}{\overline{K}_{\mathrm{x}}} \tag{27}$$

and as a consequence of (21), (25), $e(t_0) \in \mathscr{I}_1(\mu(t_0)) \cap \mathscr{B}_1(\mu(t_0))$. Because $e(t_0) \in \mathscr{B}_1(\mu(t_0))$ and $\mathscr{B}_2(\mu) \subset \mathscr{B}_1(\mu)$, we have $\dot{V} \leq 0$ from (22). Thus, for $t > t_0$ when $e(t) \notin \mathscr{B}_2(\mu(t))$ we have

$$\dot{V} \leq 0 \implies V(t) \leq V(t_0) \implies$$

$$\left\|e(t)\right\| \leq \sqrt{\frac{\mu^2(t_0) M^2}{\overline{K}_{\mathrm{x}}{}^2} + \frac{\rho}{\lambda_{min}(P)}} \tag{28}$$

which implies that $e(t)$ does not necessarily decrease monotonically. Then, for $t > t_0$ we might have two cases: either $e^T P e$ is decreasing, in which case there is no saturation and we go to the zooming-in phase; or $e^T P e$ is increasing. For this second case, because $\mu(t)$ is increased at higher rate than the growth of $e(t)$ to avoid saturation, we can assume that $\forall t \geq t_0$, $e(t) \in \mathscr{B}_1(\mu(t))$. In addition, the following inequality holds from $V(t) \leq V(t_0)$:

$$\left\|e(t)\right\| \leq \sqrt{\frac{V(t_0)}{\lambda_{\min}(P)}} \implies e(t) \in \mathscr{L}_\infty. \tag{29}$$

Zooming-in phase: Let t' be a time instant such that $t \geq t' \geq t_0$, and no saturation occurs, i.e. $e(t) \in \mathscr{B}_1(\mu(t'))$. Then it is true that $\dot{V} \leq 0$ as long as $e \notin \mathscr{B}_2(\mu(t'))$. One can see from (25) that $\mathscr{B}_2(\mu) \subset \mathscr{I}_2(\mu)$. Thus, at time $\tilde{t}$ with $\tilde{t} \geq t'$, when $e(t) \in \mathscr{I}_2(\mu(t'))$, $\mu(\tilde{t})$ is updated

$$\mu(\tilde{t}) = \underbrace{\frac{\overline{K}_{\mathrm{x}} \sqrt{\lambda_{\max}(P)} R \Delta}{\sqrt{\lambda_{\min}(P)} M}}_{\Omega} \mu(t'). \tag{30}$$

Obviously $\Omega < 1$ due to (26). Thus, a zooming-in event occurs, and (30) implies that $\mathscr{I}_1(\mu(\tilde{t})) = \mathscr{I}_2(\mu(t'))$. After the zooming-in event one might have two cases: either $e^T P e$ increases tending to violate $e \in \mathscr{B}_1(\mu(\tilde{t}))$, in which case a new zooming-out phase is activated; or $e^T P e$ keeps decreasing in which case a new zooming-in will eventually be triggered. In the second case, since μ is updated when $e \in \mathscr{I}_2(\mu)$ and because $\mathscr{B}_2(\mu) \subset \mathscr{I}_2(\mu)$, it is true that $\dot{V} \leq 0$ and, as a consequence, (28) holds implying $e(t) \in \mathscr{L}_\infty$. Additionally, because of (7) and because x_{m} is bounded, it is true that $x \in \mathscr{L}_\infty$. By looking at (14) and (17) we can see by using similar argumentation that $\dot{e}$ and $\ddot{V}$ consist of bounded terms, and thus they are bounded.

Let us now look at the combined behavior of V for zooming-in and zooming-out phases. For $t \geq t_0$, at both zooming-in and zooming-out phases it holds $\dot{V} \leq 0 \implies V(t) \leq V(t_0)$, thus V is upper-bounded by $V(t_0)$. Because $\ddot{V}$ is bounded, V is upper-bounded by $V(t_0)$ and lower-bounded by 0, and because it holds $\dot{V} \leq 0$ $\forall t \geq t_0$, we can conclude using Barbalat's lemma $\lim_{t\to\infty} \dot{V}(t) = 0$. The following relation holds from (22):

$$\lim_{t\to\infty} \dot{V}(t) \leq -\lim_{t\to\infty} \|e(t)\| \lambda_{\min}(Q)\Big(\|e(t)\| - \mu(t)R\Delta\Big) \implies$$
$$0 \leq -\lim_{t\to\infty} \|e(t)\| \lambda_{\min}(Q)\Big(\|e(t)\| - \mu(t)R\Delta\Big). \tag{31}$$

The above relation is true when

$$\lim_{t\to\infty} \|e(t)\| = 0 \quad \text{or} \quad \lim_{t\to\infty} \|e(t)\| - \mu(t)R\Delta \leq 0. \tag{32}$$

The second relation in (32) implies that $e \in \mathscr{B}_2(\mu)$. However when $e \in \mathscr{I}_2(\mu)$, and because $\mathscr{I}_2(\mu) \supset \mathscr{B}_2(\mu)$, μ is decreasing as in (30) because zooming-in occurs, and consequently $e \notin \mathscr{B}_2(\mu)$. As a consequence $\lim_{t\to\infty} \mu(t) = 0$ and from (32) we conclude $\lim_{t\to\infty} \|e(t)\| = 0$, i.e. asymptotic stability of (14). ■

V. Simulation Results

In this section we study the effectiveness of the proposed adaptive hybrid control policy using the electro-hydraulic system of [29]. The transfer function of the system operating at supply pressure 11.0 MPa is:

$$G(s) = \frac{62.4}{s(s+4.58)}$$

which can be equivalently written in controllable canonical form:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -4.58 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 62.4 \end{bmatrix} u(t) \tag{33}$$

where $x = [x_1, x_2]^T$, with x_1, x_2 representing the displacement and the velocity of the arm respectively, $u(t)$ is the control voltage and $y(t)$ is the measurement of the actuator arm displacement. The desired dynamics are given as follows:

$$\dot{x}_{\mathrm{m}}(t) = \begin{bmatrix} 0 & 1 \\ -15 & -8 \end{bmatrix} x_{\mathrm{m}}(t) + \begin{bmatrix} 0 \\ 31.2 \end{bmatrix} r(t)$$

where the reference input signal $r(t)$ is specified as $r(t) = \sin(0.8\pi t) + \sin(\pi t)$. Moreover, the matrices P, Q in (12) are defined as

$$P = \begin{bmatrix} 1.2830 & 0.1030 \\ 0.1030 & 0.0578 \end{bmatrix}, \quad Q = \begin{bmatrix} 2.1811 & 0.1751 \\ 0.1751 & 0.0983 \end{bmatrix}$$

and S in (4) is chosen equal to 2. The quantizer is chosen so that $M = 10$, $\Delta = 0.01$ and μ initially is 1. Also, Ω in (30) is computed $\Omega = 0.78$. The initial error in the simulations is chosen as $e(0) = [0.1, -0.2]^T$. The controller parameters are assumed to reside between lower and upper bounds as follows: $K_{\mathrm{r}} \in [0.01, 0.8]$, $K_{\mathrm{x}}^{(i,j)} \in [-0.5, 0.5]$, $i = \{1,2\}, j = 1$ (the notation $K^{(i,j)}$ represents the (i,j)-th entry of matrix K). The initial parameter estimates are chosen $K_{\mathrm{x}}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $K_{\mathrm{r}}(0) = 0.6$. At first, we present the simulation results in Matlab-Simulink® for the case of no input quantization: tracking performance is shown in Fig. 1.

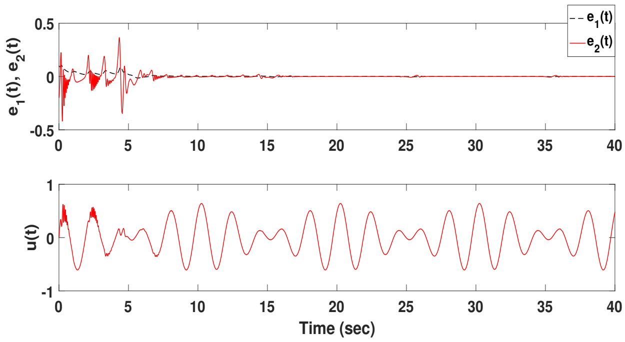

Fig. 1. State tracking error and control input without input quantization

Next, we perform similar simulations for the case of dynamic input quantizer with adjustable offset. Fig. 2 shows that the tracking performance of the dynamic quantizer with adjustable offset is clearly satisfactory: it is hard to notice any difference between Fig. 1 (no quantization) and Fig. 2 (dynamic quantization). This reveals that the same tracking performance can be attained without the need for infinite bandwidth.

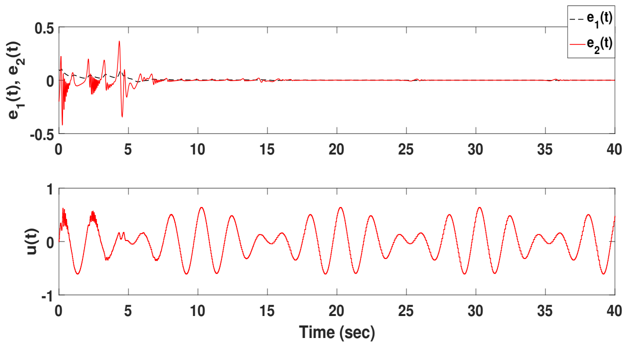

Fig. 2. State tracking error and control input with input quantization

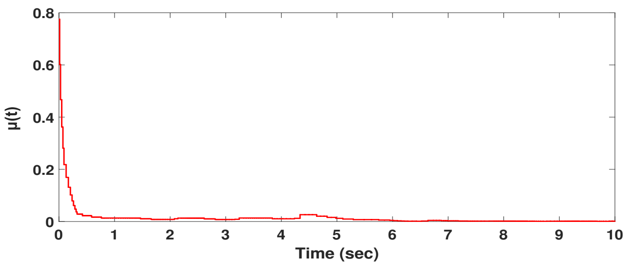

Fig. 3. Hybrid control parameter $\mu(t)$ versus time (for the first 10 seconds of the simulation)

The parameter μ in Fig. 3 is decreasing abruptly since the first seconds of the experiment, indicating that the condition $e \in \mathscr{I}_2(\mu)$ triggers (30) very often and state-tracking is achieved quite fast. One can see in Fig. 3 that μ is not strictly decreasing, indicating intermediate zooming-out phases in between zooming-in time intervals, which complies with our theoretical result in (28).

To illustrate the effectiveness of the dynamic quantizer with adaptive offset, we repeat the simulations for the case of the static quantizer, antisymmetric with respect to zero. Fig. 4 reveals that asymptotic tracking is not achieved because of finite precision of the static quantizer.

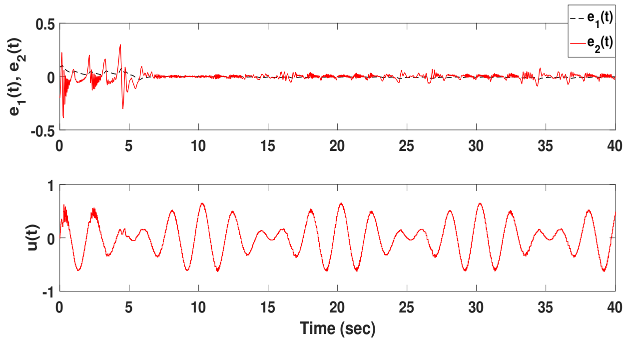

Fig. 4. State tracking error and quantized control input without dynamic offset

VI. Conclusion

The adaptive asymptotic state-tracking problem in the model reference adaptive control framework has been investigated in the quantized input measurements. In view of the uncertainty in the system, an adaptive hybrid control policy has been derived for a novel dynamic quantizer with adjustable offset. With respect to state-of-the-art policies, the proposed hybrid control policy comprises adjustment laws for the control gains and for the dynamic offset and dynamic range of the quantizer. Asymptotic state-tracking was proven via Lyapunov analysis. A practical example of an electro-hydraulic system has been used to demonstrate the effectiveness of the proposed hybrid control scheme.

Future work will include the extension of the proposed approach to switched systems, via dwell-time techniques as in [30], [31]. In addition, the study of quantization in networked environments with asynchronous switching [32], [33] will also be investigated.

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