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Fast Adaptive Compensation of Multi-Sinusoidal Disturbance in Linear MIMO Systems with Multiple Input Delays*

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***Abstract* — The paper addresses the problem of adaptive compensation of multi-sinusoidal disturbances in linear time-invariant multi-input multi-output (MIMO) plants with multiple input delays. The disturbance is considered as the vector output of a dynamical autonomous model with known order, but unmeasurable state and unknown parameters. The model parameters are related to a priori unknown frequencies, phases and amplitudes of the disturbance harmonics. Two solutions to the problem with different adaptation algorithms are derived. The first solution uses standard gradient adaptation algorithm with relatively slow convergence dependent on disturbance persistent excitation (PE). At the same time, the algorithm is simple and demonstrates the asymptotical stability property despite the inputs delays. The second solution is based on adaptation algorithm driven by disturbance dependent regressors (measurable functions) filtered by a linear operator with memory. Despite the inputs delays the rate of parametric convergence of the algorithm can be arbitrary increased by simultaneous use of linear operator with memory and increase of adaptation gain. Proposed solutions are verified via Matlab/Simulink environment.**

I. INTRODUCTION

The paper deals with the problem of adaptive compensation of multi-sinusoidal disturbances in MIMO linear time-invariant systems with multiple input delays.

The problem of disturbance compensation has been intensively researched during last decades and successfully resolved for different classes of linear [1-4] and nonlinear [5-8] systems. Most of the solutions have been obtained by means of internal model principle firstly proposed in [9, 10]. According to this principle the disturbance is represented as the output of autonomous linear model (exosystem) whose states are related to disturbance derivatives and whose parameters are dependent on amplitudes, frequencies and phase shifts of disturbance. Incorporation of the model states and parameters permits to completely compensate the disturbance and reach zero steady state error.

In classical control theory internal model principle was successfully applied for MIMO adaptive control of linear systems [11-14]. There is the number of papers, wherein the problem of adaptive MIMO system control was extended for nonlinear systems [15], systems with state delay [16], and uncertain systems with input delays [17]. The key and general idea of these solutions is in design of a standalone block of adaptive identification of the disturbance parameters – frequencies, amplitudes, and phases of harmonics. On one hand, the identifiers generate the estimates of these parameters for compensators, and the compensation performance mainly depends on identification process. On the other hand, the identifiability property of the majority of existing algorithms depends on PE condition what affects the final result of compensation. To avoid this dependence the technique of direct adaptation was applied (see works [18,19]). However, MRAC solutions proposed in [18,19] ensure systems stability, if for adaptation gains and/or delays do not exceed upper values called gain margins and delay margin respectively. In [20-22] authors proposed modified augmented error algorithm for SISO adaptive disturbance compensation and tracking to completely overcome the influence of delay.

The *contribution of this paper* is to:

1. Extend the results proposed in [20, 21] for MIMO systems and avoid the gain and time margins restrictions.

2. Extend the results proposed in [23] for MIMO systems and design and apply to the control problem the direct adaptation algorithm with dramatically improved parametric convergence. The algorithm represents a modification of adaptation algorithm initially proposed in [24] for identification problem.

The remaining of the paper is organized as follows. In section II the problem of disturbance compensation is formulated. In section III the delayed disturbance is parameterized for the control design purposes. In section IV the error model modification and corresponding adaptation algorithm based on this error model are presented. Simulation results are demonstrated in section V.

II. PROBLEM STATEMENT

Consider the linear time-invariant plant with multiple input delays[1]:

$$\begin{aligned} \dot{x} &= Ax + B\left(u_d + \delta\right), \\ y &= Cx, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $y \in \mathbb{R}^q$ is the output, $u_d = col\left(u_1(t-\tau_1),...,u_q(t-\tau_q)\right) \in \mathbb{R}^q$ is the delayed

[1] Here and hereafter the argument t is omitted excepting delayed arguments and the case when it is needed to emphasize the dependence on time.

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control vector, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times q}, C \in \mathbb{R}^{q\times n}$ are the known matrices, $\tau_i, i=\overline{1,q}$ are the known constant delays, $\delta \in \mathbb{R}^q$ is the vector of unmeasurable and bounded external disturbances.

The following assumptions are supposed to be satisfied.

Assumption 1. The known matrix A is Hurwitz.

Assumption 2. The vector of external disturbances δ can be presented as the output of linear exosystem

$$\begin{cases} \dot{z} = \Upsilon z,\ z(0), \\ \delta = Hz \end{cases} \tag{2}$$

where $z \in \mathbb{R}^m$ is the unmeasurable state vector, the constant matrix $\Upsilon \in \mathbb{R}^{m\times m}$ has its all eigenvalues on the imaginary axis, $H \in \mathbb{R}^{q\times m}$ is a constant matrix. Without loss of generality the pair (H,Υ) is assumed to be observable.

According to assumption 2 the elements of disturbance vector δ belong to the class of biased multi-sinusoidal signals.

Assumption 3. The dimension m corresponding to the number of harmonics of the exosystem (2) is known, parameters of matrices Υ and H are unknown.

The objective is to design a state-feedback control providing the boundedness of all closed-loop signals and the convergence of the state x to zero[2]:

$$\lim_{t\to\infty} \|x(t)\| = 0\,. \tag{3}$$

III. Disturbance Parameterization

For controller design purposes we parameterize the disturbance vector as [6, 25]

$$\delta = \Theta\xi + \varsigma, \tag{4}$$

where $\Theta = col(\Theta_1^T,...,\Theta_q^T) \in \mathbb{R}^{q\times m}$ is the matrix of unknown parameters, $\Theta_i^T \in \mathbb{R}^m$, $i=\overline{1,q}$ are the rows of Θ, $\xi \in \mathbb{R}^m$ is the regressor generated by filter (*virtual disturbance observer*)

$$\dot{\xi} = G\xi + L\delta \tag{5}$$

with arbitrary selected Hurwitz matrix $G \in \mathbb{R}^{m\times m}$, matrix $L \in \mathbb{R}^{m\times q}$ selected such that the pair (G,L) is controllable, exponentially decaying term ς due nonzero initial conditions. Since the stability of the closed-loop system does not depend on the term ς, here after all exponentially decaying terms will be omitted.

Substitution of (4) into (5) gives the disturbance model presented in alternative basis (canonical form):

$$\dot{\xi} = \left(G + L\Theta\right)\xi\,. \tag{6}$$

In contrast to (2) this form contains known matrices G and L. However, the state ξ is not measurable due to unknown matrix Θ what motivates the use of the following *disturbance observer* [8, 26]:

$$\begin{cases} \hat{\xi} = \eta + Nx, \\ \dot{\eta} = G\eta + (GN - NA)x - NBu_d, \end{cases} \tag{7}$$

where $\hat{\xi}$ is the estimate of ξ, $\eta \in \mathbb{R}^m$ is the auxiliary signal, $N \in \mathbb{R}^{m\times n}$ is an arbitrary matrix satisfying condition $NB = L$.

Remark 1. Existence of matrix N, especially for the case when $q \ge n$, is related to required controllability of observer (7). In this regard, matrix N can be chosen together with Hurwitz matrix G from the following Kalman criteria:

$$rank\left(col(NB,\ GNB, G^2NB,...,G^{m-1}NB)\right) = m.$$

By differentiating the observation error $e_\xi = \xi - \hat{\xi}$ in view of (1), (5) and (7) we prove the exponential vanishing of e_ξ. Finally, parameterized disturbance can be presented in the form of linear regression with measurable regressor $\hat{\xi}$:

$$\hat{\delta} = \Theta\hat{\xi}\,.$$

To synchronize all the delayed inputs with corresponding elements of the disturbance vector δ we design the disturbance predictor by applying the fundamental solution of (6) [9, 27]:

$$\xi(t+\tau_i) = R_i\xi.$$

By taking into account exponential convergence of $\hat{\xi}$ to ξ we rewrite the last equality as

$$\hat{\xi}(t+\tau_i) = R_i\hat{\xi}$$

with matrix exponent $R_i = \exp((G + L\Theta)\tau_i)$, $i=\overline{1,q}$, and parameterize predicted elements of vector δ:

$$\hat{\delta}_i(t+\tau_i) = \Theta_i^T\hat{\xi}(t+\tau_i) = \Psi_i^T\hat{\xi},\ \ i=\overline{1,q}\,, \tag{8}$$

where $\hat{R}_i = \exp((G + L\hat{\Theta})\tau_i)$ are the estimates of R_i, Ψ_i^T are the rows with new unknown parameters.

Substitution of (8) into (1) motivates the application of certainty equivalence principle and design of the following adjustable control with involved predictive properties:

$$u = col(u_1,u_2,...,u_q) = -\hat{\Psi}\hat{\xi},, \tag{9}$$

where $\hat{\Psi} = col(\hat{\Psi}_1^T,...,\hat{\Psi}_q^T) \in \mathbb{R}^{q\times m}$ is the matrix estimate of $\Psi = col(\Psi_1^T,...,\Psi_q^T)$.

IV. Error Model and Adaptation Algorithms

After substitution of (9) and (8) into (1) we get the dynamical error model [13]:

$$\dot{x} = Ax + B\Xi_d\tilde{\Psi}_d^*, \tag{10}$$

[2] Here and hereafter $\|\cdot\|$ denotes Euclidean norm.

where

$$\tilde{\Psi}_d^* = col\left(\tilde{\Psi}_1(t-\tau_1), \tilde{\Psi}_2(t-\tau_2), ..., \tilde{\Psi}_q(t-\tau_q)\right)$$

is the $m \cdot q$ dimensional vector of parametrical errors with $\tilde{\Psi}_i = \Psi_i - \hat{\Psi}_i$,

$$\Xi_d = \begin{bmatrix} \hat{\xi}^T(t-\tau_1) & O_{1\times m} & \cdots & O_{1\times m} \\ O_{1\times m} & \hat{\xi}^T(t-\tau_2) & \cdots & O_{1\times m} \\ \vdots & \vdots & \ddots & \vdots \\ O_{1\times m} & O_{1\times m} & \cdots & \hat{\xi}^T(t-\tau_q) \end{bmatrix}$$

is the extended $q \times m \cdot q$ matrix regressor. Subscript d in matrices $\tilde{\Psi}_d^*$ and Ξ_d denotes and emphasizes the delayed value.

Model (10) and Lyapunov function

$$V = 0.5x^T Px + 0.5\tilde{\Psi}_d^{*T}\Gamma^{-1}\tilde{\Psi}_d^*$$

with positively defined matrix $P = P^T$ being the solution of Lyapunov equation $A^T P + PA = -Q$ ($Q = Q^T \succ 0$) and positively defined $m \cdot q \times m \cdot q$ matrix $\Gamma = diag\left\{\Gamma_i,\, i = \overline{1,q}\right\}$ containing $m \times m$ matrices $\Gamma_i = \Gamma_i^T \succ 0$ give the following matrix adaptation algorithm [13, 18]:

$$\dot{\hat{\Psi}}_d^* = \Gamma \Xi_d^T B^T Px\,.$$

It worth noting that the algorithm generates the vector of estimates

$$\hat{\Psi}_d^* = col\left(\hat{\Psi}_1(t-\tau_1), \hat{\Psi}_2(t-\tau_2), ..., \hat{\Psi}_q(t-\tau_q)\right)$$

with delays. At the same time adjustable control (9) is fed with nondelayed estimates $\hat{\Psi}_i^T$, and direct substitution of delayed estimates into (9) can cause instability, if the maximum delay $\max_i\{\tau_i\},\, i = \overline{1,q}$ or the maximum eigenvalue of Γ $\max_i\{\lambda_{\gamma i}\},\, i = \overline{1,q}$ exceeds some threshold. Results presented in [18] demonstrate this limitation in the framework of SISO adaptive compensation.

To design an adaptation algorithm generating $\hat{\Psi}$ without delay we introduce augmented vector

$$\hat{x} = \varphi + x\,, \tag{11}$$

where φ is the vector generated by filter

$$\dot{\varphi} = A\varphi + \bar{L}\hat{x} + B\Xi_d\left(\hat{\Psi}_d^* - \hat{\Psi}^*\right), \tag{12}$$

$\hat{\Psi}^* = col\left(\hat{\Psi}_1, \hat{\Psi}_2, ..., \hat{\Psi}_q\right)$ is the nondelayed vector of estimates, $\bar{L} \in \mathbb{R}^{n\times n}$ is the design matrix such that matrix $\bar{A} = A + \bar{L}$ is Hurwitz and possesses desired algebraic spectrum.

By taking derivative of (11) in view of (10) and (12) we obtain augmented error model

$$\dot{\hat{x}} = \bar{A}\hat{x} + B\Xi_d\tilde{\Psi}^*, \tag{13}$$

where $\tilde{\Psi}^* = \Psi^* - \hat{\Psi}^*$. The model can directly be used for design of an adaptation algorithm

$$\dot{\hat{\Psi}}^* = F(t)\,, \tag{14}$$

where F is a measurable nonlinear function responsible for performance of estimation process.

Remark 2. It can be shown that the augmentation scheme can be successfully applied for unstable plants or plants with small gain/phase margins and slow transients. Indeed, the correct choice of matrix $\bar{L}$ defining the algebraic spectrum of $\bar{A}$ ensures the stability of error model (13) and can accelerate model transients. Therefore, the estimation process becomes independent from plant stability and potentially can be accelerated independently from original plant dynamics.

Now we are in position to propose two adaptation algorithms based on the model (14) and the following Lyapunov function:

$$V = 0.5\hat{x}^T\bar{P}\hat{x} + 0.5\tilde{\Psi}^{*T}\Gamma^{-1}\tilde{\Psi}^*, \tag{15}$$

where $\bar{P} = \bar{P}^T \succ 0$ is the solution of Lyapunov equation $\bar{A}^T\bar{P} + \bar{P}\bar{A} = -\bar{Q}$ with chosen matrix $\bar{Q} = \bar{Q}^T \succ 0$. By evaluating the time derivative of (15) in view of (13) and (14) we get:

$$\dot{V} = -0.5\hat{x}^T\bar{Q}\hat{x} + \tilde{\Psi}^{*T}\Xi_d^T B^T\bar{P}\hat{x} - \tilde{\Psi}^{*T}\Gamma^{-1}F. \tag{16}$$

The last equality gives a room for choice of function F and matrix $\bar{Q}$ to improve the convergence to zero of both $\hat{x}$ and $\tilde{\Psi}^*$.

1. ***Gradient algorithm***

If $F = \Gamma\Xi_d^T B^T\bar{P}\hat{x}$, the adaptation algorithm can be presented in the form

$$\dot{\hat{\Psi}}^* = \Gamma\Xi_d^T B^T\bar{P}\hat{x} \tag{17a}$$

or in simpler row wise form

$$\dot{\hat{\Psi}}_i^T = B_i^T\bar{P}\hat{x}\hat{\xi}^T(t-\tau_i)\Gamma_i\,,\; i = \overline{1,q}, \tag{17b}$$

where B_i^T are the rows of matrix B^T, i.e. $B^T = col(B_1^T, B_2^T, ..., B_q^T)$. Combining all the rows together we design adaptation algorithm generating $\hat{\Psi} = col(\hat{\Psi}_1^T, ..., \hat{\Psi}_q^T)$.

Substitution of (17a) back into (16) in view of (13) gives

$$\dot{V} = -0.5\hat{x}^T\bar{Q}\hat{x} < 0\,.$$

The Lyapunov function analysis reveals the following properties of algorithm (17) and the closed-loops system [13]:

a) $\|\hat{x}\| \in L_\infty, \|x\| \in L_\infty, \left\|\tilde{\Psi}^*\right\| \in L_\infty$;

b) $\|\hat{x}\| \in L_2$, i.e. $\|\hat{x}\|$ tends to zero asymptotically fast;

c) $\left\|\tilde{\Psi}^*\right\|$ is a nonincreasing function;

d) If Ξ_d is PE, $\left\|\tilde{\Psi}^*\right\|$ tends to zero exponentially fast. If Ξ_d is PE, there is an optimal maximum eigenvalue of Γ $\max\limits_i\left\{\lambda_{\gamma i}\right\}, i=\overline{1,q}$, for which the rate of parametric convergence is maximum;

e) $\|x\|$ tends to zero asymptotically fast.

The last property is a serious drawback of the algorithm. Indeed, it is practically difficult to guarantee PE condition of disturbance in MIMO systems especially of a high order, and even if this condition is met, there is no way to improve transients performance.

To relax these restrictions and increase the rate of convergence we propose the scheme based on the previous works [24, 28] devoted to identification problem and a linear operator with memory.

2. ***Algorithm with improved parametric convergence***

By selecting

$$F=\Gamma\Xi_d^T B^T\bar{P}\hat{x}+\bar{\Gamma}\mathcal{L}\left\{\Xi_d^T\Xi_d\right\}\tilde{\Psi}^*, \tag{18}$$

where $\mathcal{L}\{\cdot\}$ is a scalar linear operator with memory: $L_\infty \to L_\infty$[3], $\bar{\Gamma}=diag\left\{\bar{\Gamma}_i,\ i=\overline{1,q}\right\}$ is the $m\cdot q\times m\cdot q$ matrix containing $m\times m$ matrices $\bar{\Gamma}_i=\bar{\Gamma}_i^T\succ 0$,

$$\Xi_d=\begin{bmatrix} W_{11}(s)\left[\hat{\xi}^T(t-\tau_1)\right] & W_{21}(s)\left[\hat{\xi}^T(t-\tau_1)\right] & \cdots & W_{q1}(s)\left[\hat{\xi}^T(t-\tau_1)\right] \\ W_{12}(s)\left[\hat{\xi}^T(t-\tau_2)\right] & W_{22}(s)\left[\hat{\xi}^T(t-\tau_2)\right] & \cdots & W_{22}(s)\left[\hat{\xi}^T(t-\tau_2)\right] \\ \vdots & \vdots & \ddots & \vdots \\ W_{1q}(s)\left[\hat{\xi}^T(t-\tau_q)\right] & W_{2q}(s)\left[\hat{\xi}^T(t-\tau_q)\right] & \cdots & W_{qq}(s)\left[\hat{\xi}^T(t-\tau_q)\right] \end{bmatrix}$$

is the filtered regressor matrix, $W_{ij}(s), i=\overline{1,q}, j=\overline{1,q}$ are the elements of strictly positive real (SPR) transfer matrix $W(s)=B^T\bar{P}\left(sI-\bar{A}\right)^{-1}B,\quad s=d/dt$ is the differential operator, we streamline the presentation toward to implementable form of adaptation algorithm:

$$\begin{aligned}\dot{\hat{\Psi}}^* &= \Gamma\Xi_d^T B^T\bar{P}\hat{x}+\bar{\Gamma}\mathcal{L}\left\{\Xi_d^T\Xi_d\right\}\tilde{\Psi}^* = \\ &\Gamma\Xi_d^T B^T\bar{P}\hat{x}+\bar{\Gamma}\mathcal{L}\left\{\Xi_d^T\Xi_d\Psi^*\right\}-\bar{\Gamma}\mathcal{L}\left\{\Xi_d^T\Xi_d\right\}\hat{\Psi}^*.\end{aligned}$$

[3] Linear operators are assumed to be scalar for the sake of simplicity. The presented algorithm can be extended for matrix linear operators and called multiple model algorithm proposed.

By taking into account that

$$\Xi_d\Psi^*=B^T\bar{P}\hat{x}+\Phi,$$

where

$$\Phi=\sum_{j=1}^{q}\begin{bmatrix} W_{1j}(s)\left[\hat{\Psi}_j^T\xi(t-\tau_j)\right] \\ W_{2j}(s)\left[\hat{\Psi}_j^T\xi(t-\tau_j)\right] \\ \vdots \\ W_{qj}(s)\left[\hat{\Psi}_j^T\xi(t-\tau_j)\right]\end{bmatrix},$$

we finally get:

$$\begin{aligned}\dot{\hat{\Psi}}^* &= \Gamma\Xi_d^T B^T\bar{P}\hat{x}+ \\ &\bar{\Gamma}\left(\mathcal{L}\left\{\Xi_d^T B^T\bar{P}\hat{x}\right\}+\mathcal{L}\left\{\Xi_d^T\Phi\right\}-\hat{\Psi}^*\mathcal{L}\left\{\Xi_d^T\Xi_d\right\}\right).\end{aligned} \tag{19}$$

Substitution of (18) into (16) gives the Lyapunov function derivative

$$\dot{V}=-0.5\hat{x}^T\bar{Q}\hat{x}-\tilde{\Psi}^{*T}\Gamma^*\mathcal{L}\left\{\Xi_d^T\Xi_d\right\}\tilde{\Psi}^*,$$

with $\Gamma^*=\Gamma^{-1}\bar{\Gamma}$. Temporary assuming that

$$\mathcal{L}\left\{\Xi_d^T\Xi_d\right\}\geq\beta I_{q\cdot m\times q\cdot m}>0, \tag{20}$$

where β is a positive constant, $I_{q\cdot m\times q\cdot m}$ is the $q\cdot m\times q\cdot m$ identity matrix, and we derive the condition for exponential decaying of both $\|\hat{x}\|$ and $\left\|\tilde{\Psi}^*\right\|$:

$$\dot{V}\leq-0.5\min_i\left\{\lambda_{\bar{q}}\right\}\|\hat{x}\|^2-\min_j\left\{\lambda_{\gamma*}\right\}\beta\left\|\tilde{\Psi}\right\|^2,$$

$$V\leq 0.5\max_i\left\{\lambda_{\bar{p}}\right\}\|\hat{x}\|^2+\frac{0.5}{\min\limits_j\left\{\lambda_\gamma\right\}}\left\|\tilde{\Psi}^*\right\|^2,$$

$$\dot{V}\leq-\kappa V \tag{21}$$

where $\min\limits_i\left\{\lambda_{\bar{q}}\right\}$, $i=\overline{1,n}$, $\min\limits_j\left\{\lambda_{\gamma*}\right\}$, $\min\limits_j\left\{\lambda_\gamma\right\}$ ($i=\overline{1,q}$, $j=\overline{1,q\cdot m}$) are the minimum eigenvalues of $\bar{Q}$, Γ^* and Γ respectively, $\max\limits_i\left\{\lambda_{\bar{p}}\right\}$ is the maximum eigenvalue of $\bar{P}$,

$$\kappa=\frac{\min\left\{0.5\min\limits_i\left\{\lambda_{\bar{q}}\right\},\ \min\limits_j\left\{\lambda_{\gamma*}\right\}\beta\right\}}{\max\left\{0.5\max\limits_i\left\{\lambda_{\bar{p}}\right\},\ \dfrac{0.5}{\min\limits_j\left\{\lambda_\gamma\right\}}\right\}}. \tag{22}$$

Inequality (21) immediately proves the following properties of algorithm (19) and the closed-loop system:

a) $\|\hat{x}\| \in L_\infty, \|x\| \in L_\infty, \left\|\tilde{\Psi}^*\right\| \in L_\infty$;

b) $\|\hat{x}\| \in L_2$;

c) $\left\|\tilde{\Psi}^*\right\|$ is a nonincreasing function;

d) If $\beta > 0$, $\left\|\tilde{\Psi}^*\right\|$ and $\|\hat{x}\|$ tend to zero exponentially fast. The rate of parametric convergence can be increased arbitrary by simultaneous increase of $\min_j \{\lambda_{\gamma *}\}$ and minimum eigenvalue of matrix denoted as $\min_j \{\lambda_{\bar{A}}\}$. Indeed, as it can be seen from (22), coefficient κ responsible for the convergence rate can be increased by increase of nominator with tuning parameters $\min_i \{\lambda_{\bar{q}}\}$, $\min_j \{\lambda_{\gamma *}\}$ and freezing of denominator with eigenvalues of $\bar{P}$ and Γ. Since matrices $\bar{Q}$ and $\bar{P}$ are related via Lyapunov equation, these manipulations can be possible by simultanious increase of $\min_j \{\lambda_{\bar{A}}\}$ and $\min_j \{\lambda_{\gamma *}\}$;

e) $\|x\|$ tends to zero asymptotically fast.

Remark 3. The property d of the algorithm (19) is rather stronger than the property d of (17) and permits to improve the rate of disturbance compensation despite the affection of input delay.

Remark 4. If the matrix Ξ_d is not singular, condition (20) can be ensured by appropriate choice of linear operator with memory. The operator can be selected, for example, as the delay operator or the first order transfer function

$$\mathcal{L}\{\cdot\} = \frac{1}{s+\mu}\{\cdot\}, \quad \mu = const > 0 .$$

If the matrix Ξ_d is singular, i.e. condition (20) does not hold, and vector ξ is not PE, parametric convergence cannot be improved arbitrary. However, it does not affect the final result of disturbance compensation.

V. Simulation

Consider state equality of the plant

$$\dot{x} = Ax + B(u_d + \delta), \quad x(0) = col(1,0)$$

with control $u_d = col(u_1(t-\tau_1), u_2(t-\tau_2), u_3(t-\tau_3))$, matrices

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 3 & 5 \end{bmatrix},$$

disturbance vector $\delta = col(3\cos(2t), 4\cos(3t), 2\sin(2t))$ (amplitudes, phases and frequencies are unknown), and input delays $\tau_1 = 10$ sec, $\tau_2 = 3$ sec, $\tau_3 = 20$ sec.

The vector $\hat{\xi}$ is calculated by observer (7) with matrices

$$G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -4 & -6 & -4 \end{bmatrix}, \; L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 3 & 5 \end{bmatrix}, \; N = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

The control is presented by (9), where estimates are generated by adaptation algorithm (17), (11), (12) or (19), (11), (12) with the matrices

$$\bar{A} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}, \; \bar{P} = \begin{bmatrix} 1.75 & 1 \\ 1 & 0.75 \end{bmatrix}. \quad .$$

Simulation results for gradient adaptation algorithm (17), (11), (12) with gain matrix $\Gamma = 5I_{12\times 12}$ are shown in Fig. 1 and demonstrate the boundness of all the signals in the closed-loop system and asymptotical decaying of $\|x\|$.

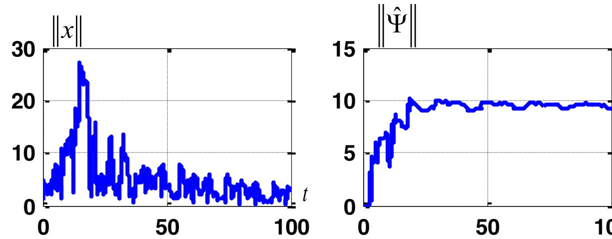

Figure 1. Transients in the system closed by gradient adaptation algorithm (17), (11), (12)

Simulation results for adaptation algorithm with improved parametric convergence (19), (11), (12) with matrices

$$\Gamma = 5I_{12\times 12}, \; \bar{\Gamma} = 500 I_{12\times 12},$$

and linear operator

$$\mathcal{L}\{\cdot\} = \frac{1}{s+0.1}\{\cdot\}$$

are shown in Fig. 2. Results demonstrate faster parametric convergence and relatively fast compensation despite the influence of input delays.

It worth noting that norms $\left\|\hat{\Psi}\right\|$ in the first and second experiments approach different values, what illustrates key property of direct adaptive control: the estimates need not be converged to nominal values.

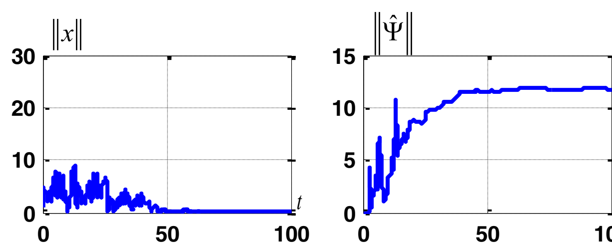

Figure 2. Transients in the system closed by adaptation algorithm with improved parametric convergence (19), (11), (12) for input delays are $\tau_1 = 10$ sec, $\tau_2 = 3$ sec, $\tau_3 = 20$ sec

Fig. 3 illustrates simulation results repeated for adaptation algorithm (19), (11), (12) and plant with doubled delays ($\tau_1 = 20$ sec, $\tau_2 = 6$ sec, $\tau_1 = 40$ sec). As it can be seen, despite the increased delays, the system does not lose the stability properties as well as rate of parameters transients.

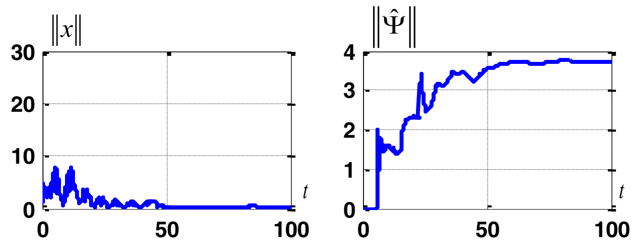

Figure 3. Transients in the system closed by adaptation algorithm with improved parametric convergence (19), (11), (12) for input delays are $\tau_1 = 20$ sec, $\tau_2 = 6$ sec, $\tau_3 = 40$ sec

VI. CONCLUSION

Thus, two solutions to the problem of adaptive multi-sinusoidal disturbance compensation for the class of linear MIMO systems with multiple delays are proposed. The first solution uses gradient-based adaptation algorithm and provides complete compensation with rate that can be arbitrary small. The second solution is obtained by extension of the first scheme by a linear operator with memory applied to the matrix $\Xi_d^T \Xi_d$ and capturing past information about disturbance. The extension permits to increase the rate of parametric convergence and to force the transients in adaptive compensator.

The further steps of our research are related to the logical extensions of this problem for unstable plants and the plants with an unmeasurable states and unknown parameters.

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