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Two-Level Hierarchical Control for Wastewater Treatment Utilizing Neural-Network Predictors and Nonlinear Optimization

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Abstract— **This paper addresses the problem of identifying optimal setpoints for a biological wastewater treatment plant that can react to changing influent flow due to weather conditions. We propose and compare three methods that identify an optimal fixed setpoint for dry weather, and a moving direction for weather changes. The optimal fixed setpoint is the solution of a nonlinear optimization problem. Neural Network Autoregressive eXogenous (NNARX) models predict the correct moving direction in real time. The methods involve choosing the dissolved oxygen setpoint, or the nitrate/nitrite setpoint, or both. Simulations use the Benchmark Simulation Model Number One (BSM1) model of a wastewater treatment plant with the supplied influent data for three weather conditions. The proposed methods can improve effluent quality or energy cost or both.**

I. INTRODUCTION

In wastewater treatment plants the effluent quality should satisfy certain environmental standards while operating in an energy efficient manner. A contemporary challenge for control systems design is to meet pollution constraints even during extreme weather events. Researchers often use the Benchmark Simulation Model Number One (BSM1) to design and compare control strategies; the benchmark contains five tanks in series followed by a settler. The standard BSM1 comes with a dissolved-oxygen (DO) proportional-integral (PI) control loop and a nitrate/nitrite PI control loop. Control researchers typically replace these PI controls with more advanced methods, and evaluate the results by using the standard measures of Effluent Quality Index (EQI) for pollution levels and the Overall Cost Index (OCI) for energy consumption.

Many researcher have looked at designing controls for BSM1 for both designing the setpoint and setpoint tracking i.e. a *two level hierarchical control structure* where the setpoint control defines the upper level. The upper-level strategy often aims to control nitrate/nitrite in the fifth tank, $S_{NH,5}$, by manipulating the setpoint for DO in the fifth tank, $S_{O,5}$. Some have simply applied PI controls at both levels [1]. Model Predictive Control (MPC) in combination with PI has been used at the upper level [2]. MPC has also been tried at both levels [3] .

Rather than use a feedback-control approach for setpoint design, some have utilized optimization techniques. Genetic algorithms (GAs) in the higher level have been tried in [4] and [5]. In [4], a PI controller in the lower level follows an ammonia set-point determined by GA optimization in the higher level. The method in [6] proposes a unique approach that divides the control structure into three layers: the supervisory control layer, the optimizing control layer and, and the low-level control layer. The method utilizes MPC, extended Kalman filters, and grey-box parameter estimation.

Since humans are fairly adept at operating wastewater plants, another strategy is to encode decision-making in fuzzy logic rules. The method in [7] uses a fuzzy controller to control the DO setpoint and the ratio of aerobic and anoxic zones. The method in [8] studies control of the external carbon dosage as well as the DO setpoint control. DO and nitrate were controlled in [9] using a supervisory and fuzzy control. Fuzzy logic in [10] controls both the DO setpoint and the air flow. A similar method has been used in [11] where a fuzzy controller at the higher level regulates setpoints for S_O loops based on $S_{NH,5}$.

We have previously proposed a low-order adaptive control as the lower level and a fuzzy control with optimal membership functions as the higher level [12]. Our method in [13] uses a low-order adaptive control for the lower level and an adaptive method based on (one-step-ahead) predictions of the DO and ammonium for the upper level.

Although the methods mentioned in our literature review have had varying degrees of success in reducing EQI or OCI or both, satisfactory performance during extreme weather events still remains an open problem to our knowledge. Here, we propose adding higher-level controls to find optimal setpoints for the DO loop and the nitrate/nitrite loop, replacing the low-level PI controls with adaptive controls, and adding an extra control loop to remove large violations of $S_{NO,5}$. We then analyze the stability of the closed-loop system. The predicted optimal set points are a compromise between a fixed set point and a moving one by using a weighting gain. The NNARX models predict the moving direction by using the measurable outputs and measurable disturbances. The weighting gains and the fixed optimal set points are the solution of nonlinear optimization functions. Simulation results with BSM1 show that even if reduction of both EQI and OCI simultaneously is not possible, we can at least reduce one without causing a significant increase in the other. Moreover, in certain cases both can be reduced.

II. BSM1 BENCHMARK

The International Association of Water Quality (IAWA) and the European Cooperation in the field of Scientific and Technical Research (COST) have provided some standard computer models that researchers can use to develop and compare wastewater-treatment controls. The first model is Activated Sludge Model Number 1 (ASM1) which is a 13-state model of a single bioreactor tank. By changing certain parameters, this model can be modified to capture the processes occurring in the three basic kinds of tanks: anoxic, aerobic, and anaerobic. A single aerobic tank can be used to remove organic matter, but processes to remove phosphorus and nitrates require more complex configurations i.e. tanks of different types connected both sequentially and through recycle loops. Not only will each physical plant have different size tanks, there are many possible configurations of tanks. Thus, IAWA and COST have also provided a computer model of a benchmark plant called Benchmark Simulation Model 1 (BSM1) [14],[15]. BSM1 uses ASM1 models of five sequential tanks (two anoxic tanks followed by three aerated tanks) followed by a clarifier, with a recycle loop from the last tank to the first. BSM1 provides three files of input disturbances for a 14-day period that researchers can use to test disturbance rejection: one for dry weather, one for rainy weather, and one for stormy weather. BSM1 comes with two feedback loops implemented with PI controls. The first feedback loop measures the amount of DO in tank 5, $S_{O,5}$, and manipulates amount of air supplied to tank 5, $K_{La,5}$ (controlling the removal of organic matter). The second feedback loop measures the amount of nitrate in tank 2, $S_{NO,2}$, and manipulates the internal recycle flow rate input to tank 1, Q_a (controlling the removal of nitrates and ammonia). The BSM1 with two feedback loops presents a difficult control problem; the dynamics are highly nonlinear, it has 65 states but only 2 measured outputs, the inputs are easily saturated, and the disturbances during storm events are enormous. The second control loop has its control input in one tank but its measured output in a completely different tank.

III. Proposed Methods

A. Lower-level adaptive control

Our proposed controls use the two existing BSM1 control loop input/output pairs, with an additional third control loop (Fig. 1). The additional loop controls the nitrate/nitrite level $S_{NO,5}$ concentration in the last aerated tank to a fixed setpoint of $7\mathrm{mgL}^{-1}$ by manipulating the waste flow rate Q_w, constrained to a maximum of $385\mathrm{m}^3\mathrm{day}^{-1}$. Instead of PI controls (as in the standard BSM1) we propose an adaptive control for each loop

$$u(t) = \theta_1 u_c(t) - \theta_2 y(t) - Ke(t), \tag{1}$$

where u_c denotes the setpoint, e the tracking error, positive constant K the control gain, and θ_1, θ_2 are adaptive parameters with update laws

$$\dot{\theta}_1 = -\gamma(u_c e + \nu|e|\theta_1), \tag{2}$$
$$\dot{\theta}_2 = \gamma(ye - \nu|e|\theta_2). \tag{3}$$

For each control loop, u indicates the manipulated variable and y denotes the controlled variable (see Appendix for stability analysis).

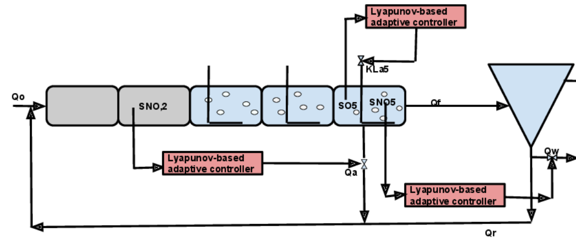

Fig. 1: BSM1 with proposed control method.

B. Nonlinear optimization

The optimal fixed set points and the weighting gains are the solution of a nonlinear optimization problem. We utilize Matlab's Optimization Toolbox for the nonlinear optimization. The nonlinear optimization uses a 65×65 model obtained by Jacobian linearization and the data of inlet flow rate and inlet concentrations for the first two days of dry weather conditions (we rely on the fact that dry weather conditions show a predictable behaviour depending on time of day and the day of the week, making all variables essentially predictable).

The results of nonlinear optimization are then used with neural-network predictors to establish setpoints for dry weather. The resulting solution can then still be used for rain/storm weather events to dynamically adjust the DO setpoint (although the solution is no longer optimal).

C. Finding the DO setpoint

This section develops a method to predict the direction of movement in a DO setpoint when it is subjected to influent changes. Typically, the goal of DO setpoint control is to improve the nitrification process during a high load of ammonium and reduce the usage of aeration energy (AE) during a low load of ammonium. Our setpoint search algorithm initializes with $y_{\mathrm{ref},1,DO}$ (obtained by solving an optimization problem) and then moves in the direction of ammonium changes in the last tank. The following NNARX model predicts the ammonium concentration in the last tank:

$$\hat{S}_{NH,5,t} = \mathbf{w}_1^T \sigma(\mathbf{H}_1^T \mathbf{q_1}), \tag{4}$$

where the inputs are the delayed ammonium concentration in the last tank, the oxygen transfer coefficient in the last tank, and the inlet ammonium concentration

$$\begin{aligned} \mathbf{q_1}(t) = [&S_{NH,5}(t-1) \quad \dots \quad S_{NH,5}(t-n_a) \\ &K_{La,5}(t-1) \quad \dots \quad K_{La,5}(t-n_b) \\ &S_{NH,o}(t-1) \quad \dots \quad S_{NH,o}(t-n_c)]^T, \end{aligned} \tag{5}$$

$$y_{\mathrm{ref},2,NH} = \hat{S}_{NH,5,t} + \dot{\hat{S}}_{NH,5,t}\Delta t. \tag{6}$$

The predicted optimal setpoint is a compromise between these two desired values

$$y_{\mathrm{ref,DO}} = (y_{\mathrm{ref},1,DO} + G_{NH} y_{\mathrm{ref},2,NH})/(1 + G_{NH}), \tag{7}$$

where G_{NH} is a positive constant control gain (for the "ammonium term") found through nonlinear optimization.

D. Finding the nitrate/nitrite setpoint

This section proposes a nitrate/nitrite setpoint selection process to improve denitrification and reduce the pumping energy (PE) used for returning the sludge. The setpoint search algorithm initializes with $y_{\mathrm{ref},1,NO}$ (obtained by solving an optimization problem) and then moves in the direction which needs less control effort (toward minimum pumping energy). The following NNARX model predicts the nitrate/nitrite concentration in the second tank:

$$\hat{S}_{NO,2,t} = \mathbf{w}_2^T \sigma(\mathbf{H}_2^T \mathbf{q_2}), \tag{8}$$

where the inputs are the delayed nitrate/nitrite concentration in the second tank, the internal recycle flow rate, and the inlet ammonium concentration

$$\begin{aligned}\mathbf{q_2}(t) = [&S_{NO,2}(t-1) \quad \dots \quad S_{NO,2}(t-n_a)\\ &Q_a(t-1) \quad \dots \quad Q_a(t-n_b)\\ &S_{NH,o}(t-1) \quad \dots \quad S_{NH,o}(t-n_c)]^T,\end{aligned} \tag{9}$$

$$y_{\mathrm{ref},2,NO} = \hat{S}_{NO,2,t} + \dot{\hat{S}}_{NO,2,t}\Delta t. \tag{10}$$

The predicted optimal setpoint is a compromise between these two desired values

$$y_{\mathrm{ref,NO}} = (y_{\mathrm{ref},1,NO} + G_{NO} y_{\mathrm{ref},2,NO})/(1+G_{NO}), \tag{11}$$

where G_{NO} comes from solving a nonlinear optimization problem.

IV. Results

A. Pollution and energy measures

The EQI evaluates the discharge of pollutants that have a significant influence on the quality of the receiving water and which are usually included in regional regulations. We evaluate the weighted average of the pollutant compounds over the last seven days of simulation

$$\begin{aligned}\mathrm{EQI} = \frac{1}{1000.T}\int_{t=7days}^{t=14days} &(B_{\mathrm{TSS}}.\mathrm{TSS}(t) + B_{COD}.\mathrm{COD}(t) +\\ &B_{NKj}.S_{NKj}(t) + B_{NO}.S_{NO}(t) + B_{BOD_5}.\mathrm{BOD}_5(t)).Q_e(t)dt,\end{aligned} \tag{12}$$

where Q_e is the outlet flow rate and each B_i is a weighting factor. TSS is the suspended solid concentration, COD_t is total chemical oxygen demand, S_{NKj} is the Kjeldahi nitrogen concentration, and BOD_5 is the biological oxygen demand.

The OCI takes into account total control effort by adding the aeration energy (AE), pumping energy (PE), the sludge production to be disposed (SP), and the mixing energy (ME)

$$\mathrm{OCI} = \mathrm{AE} + \mathrm{PE} + 5.\mathrm{SP} + \mathrm{ME} \tag{13}$$

with

$$\mathrm{AE} = \frac{8}{T.1.8.1000}\int_{t=7\mathrm{days}}^{t=14\mathrm{days}}\sum_{i=1}^{5} V_i.K_{La,i}(t).dt, \tag{14}$$

$$\mathrm{PE} = \frac{1}{T}\int_{t=7\mathrm{days}}^{t=14\mathrm{days}}(0.004.Q_0(t) + 0.008.Q_a(t) + 0.05.Q_w(t)).dt, \tag{15}$$

$$\begin{aligned}\mathrm{SP} = &\frac{1}{T}.(\mathrm{TSS}_a(14\mathrm{days}) - \mathrm{TSS}_a(7\mathrm{days}) + \mathrm{TSS}_s(14\mathrm{days})\\ &- \mathrm{TSS}_s(7\mathrm{days}) + \int_{t=7\mathrm{days}}^{t=14\mathrm{days}}\mathrm{TSS}_w.Q_w.dt),\end{aligned} \tag{16}$$

$$ME = \frac{24}{T}\int_{t=7\mathrm{days}}^{t=14\mathrm{days}}\sum_{i=1}^{5}\begin{cases}0.005.V_i dt & \text{if } K_{La,i} < 20\mathrm{day}^{-1},\\ 0 & \text{otherwise},\end{cases} \tag{17}$$

where TSS_a is the amount of solids in the reactor, TSS_s is the amount of solids in the settler, TSS_w is the amount of solid in the wastage, and V is the volume of the tank.

B. Control strategies

The simulations test the three proposed methods:

Method 1) A two-level hierarchical control for the DO loop and a fixed setpoint at $1\mathrm{mgL}^{-1}$ for the nitrate/nitrite loop,

Method 2) A two-level hierarchical control for the nitrate/nitrite loop and a fixed setpoint at $2\mathrm{mgL}^{-1}$ for the DO loop,

Method 3) Two-level hierarchical controls for both the DO and the nitrate/nitrite loops.

We compare our proposed methods to both the *standard* method (PI fixed setpoints for both the DO and the nitrate/nitrite loops at $2\mathrm{mgL}^{-1}$ and $1\mathrm{mgL}^{-1}$, respectively) as well as with the third additional loop (Section III-A) with PI control.

C. Method 1

In Method 1 the parameters G_{NH} and $y_{\mathrm{ref},1,DO}$ are the solution of the following Multi-Objective-Optimization problem :

$$\begin{aligned}&\underset{G_{NH},y_{\mathrm{ref},1,DO}}{\text{minimize}} && \mathrm{EQI}(G_{NH}, y_{\mathrm{ref},1,DO}), \mathrm{AE}(G_{NH}, y_{\mathrm{ref},1,DO})\\ &\text{subject to} && N_{tot} \le 18, \mathrm{COD}_t \le 100, S_{NH} \le 4, TSS \le 30,\\ &&& \mathrm{BOD}_5 \le 10, 0 \le G_{NH} \le G_{\max}.\end{aligned}$$

where $G_{\max}$ is an arbitrary limit that prevents excessive optimization time. The resulting DO setpoint changes over time appear in Fig. 2 (the nitrate/nitrite setpoint remains fixed).

D. Method 2

In Method 2 the parameters G_{NO} and $y_{\mathrm{ref},1,NO}$ are the solution of the following Multi-Objective-Optimization problem:

$$\begin{aligned}&\underset{G_{NO},y_{\mathrm{ref},1,NO}}{\text{minimize}} && \mathrm{EQI}(G_{NO}, y_{\mathrm{ref},1,NO}), \mathrm{PE}(G_{NO}, y_{\mathrm{ref},1,NO})\\ &\text{subject to} && N_{tot} \le 18, \mathrm{COD}_t \le 100, S_{NH} \le 4, TSS \le 30,\\ &&& \mathrm{BOD}_5 \le 10, 0 \le G_{NO} \le G_{\max}.\end{aligned}$$

where $G_{\max}$ is an arbitrary limit that prevents excessive optimization time. See Fig. 3 for the resulting time-varying nitrate/nitrite setpoint changes (note the DO setpoint remains fixed).

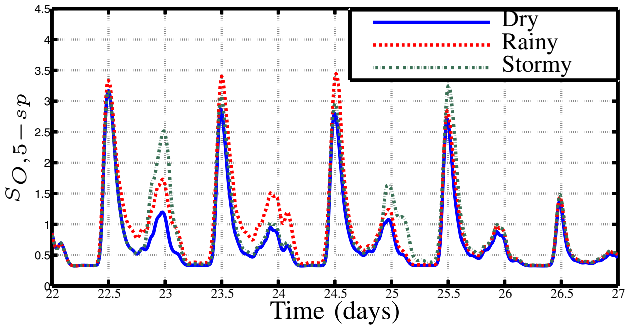

Fig. 2: DO optimal setpoints using Method 1.

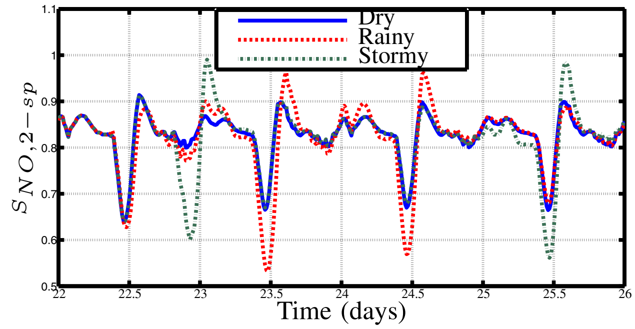

Fig. 3: Nitrate/Nitrite optimal setpoints using Method 2.

E. Method 3

In Method 3 the parameters G_{NH}, G_{NO}, $y_{\text{ref},1,DO}$ and $y_{\text{ref},1,NO}$ are the solution of the following Multi-Objective-Optimization problem:

$$\begin{array}{ll}\underset{G_{NH},G_{NO},y_{\text{ref},1,DO},y_{\text{ref},1,NO}}{\text{mininmize}} & \text{EQI, OCI,}\\ \text{subject to} & N_{tot} \le 18, \text{COD}_t \le 100, S_{NH} \le 4\\ & TSS \le 30, \text{BOD}_5 \le 10,\\ & 0 \le G_{NH}, G_{NO} \le G_{\max}.\end{array}$$

where $G_{\max}$ is an arbitrary limit that prevents excessive optimization time. The DO and nitrate/nitrite setpoints change simultaneously in Fig. 4.

F. Simulation tests

The simulation starts with a closed-loop startup period in dry weather (with constant disturbance inputs) for two weeks. During dry weather conditions, we would like to reduce both control effort (OCI) and pollution (EQI) compared to PI control with constant setpoints. The PI with extra loop strategy also uses the fixed setpoints but adds the extra control loop. Comparing the results show that, this additional control loop reduces both EQI and OCI. Adapt with fixed setpoints in Table. I uses the same fixed set points but replaces PI controls with adaptive ones. Adapt with extra loop applies fixed set points as well as the extra loop to reduce $S_{NO,5}$ violations. Note that method1,2,3 all use this extra loop.

Our methods result in a reduction of EQI by -15% for the Method 1, by -4.466% for Method 2, by -6.13% for Method 3 (Table. I). Note that Method 1 does especially well, reducing both $N_{tot,ave}$ and EQI. This method consumes the lowest aeration energy but costs more pumping energy and sludge production. Method 2 has the lowest operational cost, but has unsatisfactory removal of pollutants. Method 3 achieves satisfactory pollutant-removal at a reasonable cost.

After two weeks of dry-weather simulation, the rain or storm disturbances are introduced. During the rainy weather conditions, all methods result in a simultaneous reduction of EQI and OCI. Method 1 has the lowest value of EQI. Methods 1 and 3 result in less consumed aeration energy compared to the PI control with constant setpoints. However, Method 1 uses a large amount of pumping energy which results in a larger OCI compared to other methods (Table. IV). Method 3 simultaneously improves AE, PE, and SP compared to the default PI controller with constant setpoints.

During the stormy weather condition, Methods 1 and 3 are able to reduce both EQI and OCI. Although Method 1 results in a large amount of pumping energy, a larger reduction of AE resulted in an improvement in OCI by -9%. Methods 2 and 3 consume much less energy compared to the first one. However, Method 2 is not successful in reducing EQI, which is of special importance during the storm event.

V. Conclusions

In this paper, we propose three different methods for choosing setpoints in the BSM1 model of a biological wastewater treatment plant, using adaptive controls to track the setpoints. The main idea is that optimal setpoints can be produced during the (predictable) dry weather conditions using nonlinear optimization, and the setpoint can be changed appropriately during extreme weather by using neural-network predictors. The first method aims to facilitate the nitrification process by identifying an appropriate setpoint for dissolved oxygen that results in desired ammonium changes. The second method aims to minimize pumping energy in the denitrification process, by controlling the nitrate/nitrite setpoint. The third method controls both setpoints simultaneously. Simulations show improved effluent quality and/or reduced energy cost compared to using standard PI controls regulating fixed setpoints. Results of indicate that the third method would be preferred in dry weather (when reducing the energy usage is most important) while the first method would be preferred during rain/storm events (when the effluent quality limit should be met even at the cost of using more energy).

APPENDIX

This stability analysis for each SISO control loop uses the same assumptions required for an analysis of a PI control: namely that nonlinearities, higher-order dynamics, cross-coupled dynamic terms, and disturbances are all bounded.

In our model-reference adaptive control approach the reference model is

$$\dot{y}_m(t) = -a_m y_m(t) + b_m u_c(t) \tag{A.1}$$

where u_c is the desired value of the manipulated variable.

Consider a first-order approximation of the system

$$\dot{y}(t) = -ay(t) + bu(t) + d(t), \tag{A.2}$$

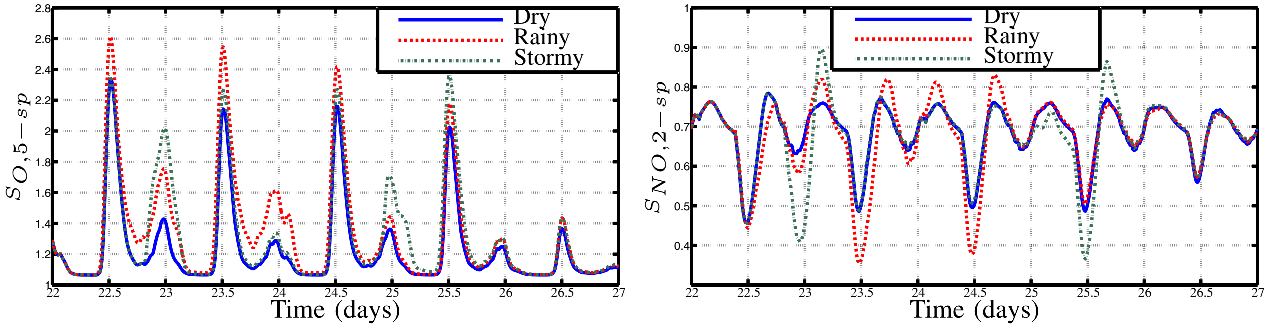

Fig. 4: DO and Nitrate/Nitrite optimal setpoints using Method 3.

TABLE I: Comparison of effluent quality in dry weather: an optimal setpoint decision can reduce EQI

Controller type	$N_{tot,ave}$(mg Nl^{-1})	COD_{ave}(mg CODl^{-1})	$S_{NH,ave}$(mg Nl^{-1})	TSS_{ave}(mg SSl^{-1})	$BOD_{5,ave}$(mg l^{-1})	EQI (kg poll.unitsd^{-1})
PI with fixed setpoints	16.9245	48.2201	2.5392	13.0038	2.7568	7552.3603
PI with extra loop	15.1860	51.1459	0.9946	15.2453	2.9275	7.1876e+03(**-4.83%**)
adapt with fixed setpoints	16.7659	47.6044	2.3942	12.5437	2.6710	7.4848e+03(**-0.89%**)
adapt with extra loop	15.0492	51.1665	0.8989	15.2649	2.9252	7.1377e+03(**-5.5%**)
method1	13.2260	50.6955	1.3747	14.8679	2.8889	6.4122e+03(**-15.1%**)
method2	15.2464	51.2396	0.9095	15.3176	2.9329	7.2156e+03(**-4.46%**)
method3	14.9125	51.2558	0.99	15.3159	2.9359	7.0896e+03(**-6.13%**)

TABLE II: Comparison of operational cost in dry weather: an optimal setpoint decision can reduce OCI

Controller type	AE (kWhd^{-1})	PE (kWhd^{-1})	SP ($kgSS$d^{-1})	OCI
PI with fixed setpoints	7.2414e+03	1.4954e+03	2.441e+03	2.11797e+04
PI with extra loop	7.3714e+03	1.6878e+03	1.4135e+03	1.6127e+04(**-23.86%**)
adapt with fixed setpoints	7.2266e+03	1.5259e+03	2.4418e+03	2.2135e+04(**+4%**)
adapt with extra loop	7.3542e+03	1.7153e+03	1.4070e+03	1.6105e+04(**-24%**)
method1	6.8432e+03	2.1722e+03	1.9474e+03	1.8753e+04(**-11.46%**)
method2	7.3649e+03	1.6288e+03	1.3387e+03	1.5687e+04(**-26%**)
method3	7.0830e+03	1.6501e+03	1.4103e+03	1.5784e+04(**-25.48%**)

TABLE III: Comparison of effluent quality during rain: an optimal setpoint decision can reduce EQI

Controller type	$N_{tot,ave}$(mg Nl^{-1})	COD_{ave}(mg CODl^{-1})	$S_{NH,ave}$(mg Nl^{-1})	TSS_{ave}(mg SSl^{-1})	$BOD_{5,ave}$(mg l^{-1})	EQI (kg poll.unitsd^{-1})
PI with fixed setpoints	14.7465	45.4337	3.226	16.1768	3.4557	9037.7895
PI with Extra loop	13.9836	48.2548	1.3040	16.8209	3.2946	8.6875e+03(**-3.8758%**)
adapt with fixed setpoints	15.2699	45.1015	2.8797	14.3997	3.0918	8.9434e+03(**-1%**)
adapt with extra loop	13.8607	48.2758	1.2108	16.8425	3.2918	8.6292e+03(**-4.5209%**)
method1	12.5006	47.8471	1.6552	16.4819	3.2574	8.0154e+03(**-11.3124%**)
method2	14.0479	48.4600	1.2221	16.9768	3.3155	8.7381e+03(**-3.3%**)
method3	13.8153	48.5147	1.3026	17.0031	3.3236	8.6527e+03(**-4.26%**)

TABLE IV: Comparison of operational cost during rain: an optimal setpoint decision can reduce OCI

Controller type	AE (kWhd^{-1})	PE (kWhd^{-1})	SP ($kgSS$d^{-1})	OCI
PI with fixed setpoints	7.1707e+03	1.9377e+03	2.3576e+03	21136.3723
PI with Extra loop	7.3319e+03	1.9888e+03	1.9132e+03	1.8887e+04(**-10.64%**)
adapt with fixed setpoints	7.1578e+03	2.0149e+03	2.5327e+03	2.3760e+04(**+12%**)
adapt with extra loop	7.3166e+03	2.0304e+03	1.9067e+03	1.8881e+04(**-10.67%**)
method1	6.8722e+03	2.4965e+03	2.3161e+03	2.0949e+04(**-0.9%**)
method2	7.3279e+03	1.8964e+03	1.8530e+03	1.8489e+04(**-12.5%**)
method3	7.0767e+03	1.8846e+03	1.9031e+03	1.8477e+04(**-12.6%**)

TABLE V: Comparison of effluent quality for storms: an optimal setpoint decision can reduce EQI

Controller type	$N_{tot,ave}$(mg Nl^{-1})	COD_{ave}(mg CODl^{-1})	$S_{NH,ave}$(mg Nl^{-1})	TSS_{ave}(mg SSl^{-1})	$BOD_{5,ave}$(mg l^{-1})	EQI (kg poll.unitsd^{-1})
PI with fixed setpoints	15.8676	47.6626	3.0622	15.2737	3.205	8.3027e+03
PI with Extra loop	14.6900	50.7200	1.3240	17.1056	3.2812	8.2338e+03(**-0.83%**)
adapt with fixed setpoints	15.9604	46.1077	2.9067	13.5869	2.8750	8.2151e+03(**-1%**)
adapt with extra loop	14.5501	50.8755	1.2180	17.2255	3.2963	8.2043e+03(**-1.185%**)
method1	12.9653	49.2927	1.6959	15.9927	3.1081	7.2991e+03(**-12.1%**)
method2	14.7875	51.4349	1.2311	17.6439	3.3700	8.4018e+03(**+1.2%**)
method3	13.3584	49.5593	1.7148	16.1992	3.1386	7.4952e+03(**-9.73%**)

TABLE VI: Comparison of operational cost for storms: an optimal setpoint decision can reduce OCI

Controller type	AE (kWhd^{-1})	PE (kWhd^{-1})	SP ($kgSS$d^{-1})	OCI
PI with fixed setpoints	7.2892e+03	1.7371e+03	2.6055e+03	22293.6884
PI with extra loop	7.4344e+03	1.8534e+03	1.7393e+03	1.7984e+04(**-19.33%**)
adapt with fixed setpoints	7.2727e+03	1.8066e+03	2.5600e+03	2.3454e+04(**+5.2%**)
adapt with extra loop	7.4150e+03	1.9040e+03	1.7377e+03	1.8007e+04(**-19.23%**)
method 1	6.9520e+03	2.3539e+03	2.1688e+03	2.0150e+04(**-9.2%**)
method 2	7.4272e+03	1.7803e+03	1.6924e+03	1.7669e+04(**-20.744%**)
method 3	6.8596e+03	2.2566e+03	2.0818e+03	1.9525e+04(**-12.42%**)

where we assume terms in $d(t)$ are bounded i.e. $|d(t)| \leq d_{\max}$ where $d_{\max}$ is a finite positive constant.

Consider a control design

$$u(t) = \theta_1(t)u_c(t) - \theta_2(t)y(t) - Ke(t), \tag{A.3}$$

where positive constant K is the control gain and θ_1, θ_2 are adaptive parameters. For simplicity, we drop the argument (t) in further equations. The parameter errors are

$$z_1 = b\theta_1 - b_m, \quad z_2 = b\theta_2 + a - a_m,$$

and their derivatives are

$$\dot{z}_1 = b\dot{\theta}_1, \qquad \dot{z}_2 = b\dot{\theta}_2.$$

The error dynamics become

$$\begin{aligned} \dot{e} =& \dot{y} - \dot{y}_m = -ay + bu + d + a_m y_m - b_m u_c, && \text{(A.4)} \\ =& -ay + b\theta_1 u_c - b\theta_2 y - bKe - a_m e \\ & + a_m y - b_m u_c + d, && \text{(A.5)} \\ =& -a_m e - bKe + y(a_m - b\theta_2 - a) \\ & + u_c(b\theta_1 - b_m) + d, \\ =& -a_m e - bKe - yz_2 + u_c z_1 + d. && \text{(A.6)} \end{aligned}$$

Consider the adaptive control Lyapunov function

$$V(e, \theta_1, \theta_2) = \frac{1}{2}\left(e^2 + +\frac{1}{b\gamma}z_1^2 + \frac{1}{b\gamma}z_2^2\right), \tag{A.7}$$

where γ is a positive constant determining the rate of adaptation. The time derivative of V is

$$\begin{aligned} \dot{V} &= e\dot{e} + \frac{1}{\gamma}z_1\dot{\theta}_1 + \frac{1}{\gamma}z_2\dot{\theta}_2, && \text{(A.8)} \\ &= -a_m e^2 - bKe^2 + de - eyz_2 + eu_c z_1 + \frac{1}{\gamma}z_1\dot{\theta}_1 + \frac{1}{\gamma}z_2\dot{\theta}_2, \\ &= -a_m e^2 - bKe^2 + de + \frac{z_1}{\gamma}(\gamma u_c e + \dot{\theta}_1) + \frac{z_2}{\gamma}(-\gamma ye + \dot{\theta}_2). \end{aligned}$$

Applying the adaptive-parameter update laws with e-modification:

$$\dot{\theta}_1 = -\gamma(u_c e + \nu|e|\theta_1), \tag{A.9}$$

$$\dot{\theta}_2 = \gamma(ye - \nu|e|\theta_2), \tag{A.10}$$

results in

$$\dot{V} = -a_m e^2 - bKe^2 + de + \frac{z_2}{\gamma}(-\gamma\nu|e|\theta_2) + \frac{z_1}{\gamma}(-\gamma\nu|e|\theta_1),$$

which can be bounded

$$\begin{aligned} \dot{V} \leq& |e|\Bigg(\begin{bmatrix} |z_1| \\ |z_2| \end{bmatrix}^T \begin{bmatrix} -\nu/b & 0 \\ 0 & -\nu/b \end{bmatrix} \begin{bmatrix} |z_1| \\ |z_2| \end{bmatrix} \\ & - |e|(a_m + bK) + \begin{bmatrix} -\nu b_m/b \\ -\nu(a_m - a)/b \end{bmatrix}^T \begin{bmatrix} |z_1| \\ |z_2| \end{bmatrix} + d_{\max}\Bigg), \\ \leq& |e|(-\mathbf{z}^T\mathbf{K_1}\mathbf{z} - (\mathbf{a_m} + \mathbf{bK})|\mathbf{e}| - \mathbf{K_2}\mathbf{z} + \mathbf{d}_{\max}), && \text{(A.11)} \\ \leq& |e|(-K_1\|z\|^2 - (a_m + bK)|e| + k_{2,\max}\|\mathbf{z}\| + d_{\max}), && \text{(A.12)} \end{aligned}$$

where we have positive constants $K_1 = \nu/b$ and $k_{2,\max} > \|\mathbf{K_2}\|$. Assuming we have chosen K such that $a_m + bK > 0$, then $\dot{V} < 0$ when $|e| > \delta_e$ or $\|\mathbf{z}\| > \delta_z$ where

$$\delta_e = \frac{1}{a_m + bK}\left(\frac{k_{2,\max}^2}{4K_1} + d_{\max}\right), \tag{A.13}$$

$$\delta_z = \frac{k_{2,\max}}{2K_1} + \sqrt{\frac{k_{2,\max}^2}{4K_1^2} + \frac{d_{\max}}{K_1}}, \tag{A.14}$$

and thus all signals are uniformly ultimately bounded on the $(|e|, \|\mathbf{z}\|)$ plane with an ultimate bound given by Lyapunov surface $V(|e|, \|\mathbf{z}\|) = V(\delta_e, \delta_z)$.

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