

Delayless Controllers for Triangular Decoupling with Simultaneous Disturbance Rejection of General Neutral Time Delay Systems*

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Abstract—The problem of Triangular decoupling with simultaneous Disturbance Rejection (TDDR) is studied for the class of general neutral multi delay systems with measurable disturbances via delayless controllers. The delayless controllers are of the measurement output feedback type with compensation of the measurable disturbances. The cases of dynamic and static controllers are studied separately. For both cases, the necessary and sufficient conditions for the problem to have a solution are established and the general form of the delayless controller matrices solving the problem are derived.

I. INTRODUCTION

The problem of triangular decoupling and disturbance rejection (TDDR) has first been introduced in [1] for the case of linear time invariant delayless systems. The TDDR problem for general neutral multi delay systems via realizable proportional measurement output feedback, has been solved in [2]. Using realizable dynamic multi-delay controllers, the pure problem of TD for general neutral multi delay systems has been solved in [3] while the pure problem of DR has been solved in [4]. The combined problem of TDDR has been solved in [5] for the general class of linear neutral multi-delay systems using realizable dynamic multi delay controllers.

The dynamic delayless controllers and the static delayless controllers are quite attractive due to their ease and elegant implementability particularly in low level hardware architectures. Clearly, the solution of design problems via delayless controllers appears to be a quite difficult problem of significant theoretical importance. In [6], the DR problem for left invertible general neutral multi delay systems via delayless dynamic measurement output feedback controllers and delayless dynamic compensation of the measurable disturbances has been solved. In [7], the diagonal decoupling problem for general neutral multi delay systems via delayless dynamic measurement output feedback controllers has been solved. In [11], the problem of exact model matching with simultaneous disturbance rejection for general neutral multi delay systems via delayless dynamic and static measurement output feedback controllers has been solved.

Here, the TDDR problem is studied via delayless controllers with measurable output feedback and compensation of the measurable disturbances. The cases of dynamic and

static controllers are studied separately. The necessary and sufficient conditions for these two problems to have a solution are established. The respective general solutions of the delayless controller matrices are derived in explicit analytic forms. The theoretical results are successfully tested to an idle speed control model for internal combustion engines [8]-[11].

II. SYSTEM DESCRIPTION

The forced response of the general class of linear neutral multi-delay differential systems (see [6] and [8]) can be derived in the frequency domain by solving the following vector equations

$$sX(s) = A(e^{-sT})X(s) + B(e^{-sT})U(s) + D(e^{-sT})\hat{U}_D(s) \quad (1a)$$

$$Y(s) = C(e^{-sT})X(s), \Psi(s) = L(e^{-sT})X(s) \quad (1b)$$

where $X(s) = \mathcal{L}\{x(t)\}$, $U(s) = \mathcal{L}\{u(t)\}$, $Y(s) = \mathcal{L}\{y(t)\}$, $\hat{U}_D(s) = \mathcal{L}\{\hat{u}_D(t)\}$, $\Psi(s) = \mathcal{L}\{\psi(t)\}$, $T = [\tau_1 \ \dots \ \tau_q]$, $e^{-sT} = [\exp(-s\tau_1) \ \dots \ \exp(-s\tau_q)]$ and where $\exp[\cdot]$ is the exponential of the argument quantity and $\mathcal{L}\{\bullet\}$ is the Laplace transform of the argument signal. The vector $x(t) \in \mathbb{R}^n$ denotes the vector of state variables, $u(t) \in \mathbb{R}^m$ denotes the vector of control inputs, $\hat{u}_D(t) \in \mathbb{R}^z$ denotes the vector of measurable disturbances, $y(t) \in \mathbb{R}^p$ is the vector of performance outputs and $\psi(t) \in \mathbb{R}^r$ is the vector of measurement outputs. The quantities τ_i ($i = 1, \dots, q$) are positive reals denoting point delays. Let $\mathbb{R}_e(e^{-sT})$ denote the set of multi-variable rational functions of $\exp(-s\tau_1), \dots, \exp(-s\tau_q)$. In other words, these functions are rational functions of the elements of e^{-sT} . The set $\mathbb{R}_e(e^{-sT})$ is a field. The elements of $A(e^{-sT})$, $B(e^{-sT})$, $D(e^{-sT})$, $C(e^{-sT})$ and $L(e^{-sT})$ are members of $\mathbb{R}_e(e^{-sT})$.

The input and disturbance to measurement output transfer matrices are denoted as follows

$$L_B(s, e^{-sT}) = L(e^{-sT})[sI_n - A(e^{-sT})]^{-1}B(e^{-sT}) \quad (2a)$$

$$L_D(s, e^{-sT}) = L(e^{-sT})[sI_n - A(e^{-sT})]^{-1}D(e^{-sT}) \quad (2b)$$

The input and disturbance to performance output transfer matrices are denoted as follows

$$H_B(s, e^{-sT}) = C(e^{-sT})[sI_n - A(e^{-sT})]^{-1}B(e^{-sT}) \quad (2c)$$

$$H_D(s, e^{-sT}) = C(e^{-sT})[sI_n - A(e^{-sT})]^{-1}D(e^{-sT}) \quad (2d)$$

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III. SOLVABILITY CONDITIONS AND GENERAL SOLUTIONS WITH DELAYLESS DYNAMIC CONTROLLERS

The design goal is that of triangular decoupling with simultaneous disturbance rejection, namely to derive a closed loop system where the transfer matrix relating the external inputs to the performance outputs is triangular while the transfer matrix mapping the measurable disturbances to the performance outputs is equal to zero, i.e. the influence of the disturbances is eliminated. The controller is considered to be

- a) independent from the point delays
- b) dynamic, i.e. it depends upon s
- c) the measurement outputs and the measurable disturbances are fed back to the input of the system
- d) the external inputs are introduced to the controller through a matrix precompensator

According to the above, the controller is of the form

$$U(s) = K_1(s)\Psi(s) + K_2(s)\hat{U}_D(s) + G(s)\Omega(s) \quad (3)$$

where $\Omega(s)$ is the $m \times 1$ vector of external inputs. The elements of $K_1(s)$, $K_2(s)$ and $G(s)$ belong to $\mathbb{R}(s)$. To preserve the solvability of the closed loop system, namely to preserve the closed loop system matrix regularity, it is required that

$$\det[sI_n - A(e^{-sT}) - B(e^{-sT})K_1(s)L(e^{-sT})] \neq 0 \quad (4)$$

Using the controller (3) and the system (1), the problem of the TDDR problem is formulated as follows

$$\begin{aligned} & C(e^{-sT})[sI_n - A(e^{-sT}) - B(e^{-sT})K_1(s)L(e^{-sT})]^{-1} \\ & \times [B(e^{-sT})G(s) \mid B(e^{-sT})K_2(s) + D(e^{-sT})] = \\ & = [\text{triang}\{t_{i,j}(s, e^{-sT})\} \mid 0_{m \times \zeta}] \end{aligned} \quad (5)$$

where $\text{triang}\{\bullet\}$ denotes an $m \times m$ lower triangular matrix form, where $t_{i,j}(s, e^{-sT})$ ($i, j \in \{1, \dots, m\}$) denote the appropriate rational functions of s with numerator and denominator polynomial coefficients being multivariable rational functions of $e^{-s\tau_1}, \dots, e^{-s\tau_q}$. Note that $t_{i,i}(s, e^{-sT}) \neq 0$ for all $i \in \{1, \dots, m\}$ and $t_{i,j}(s, e^{-sT}) = 0$ for all $j > i$. From equation (5) and the constraint (4) it is observed that $G(s)$ must be invertible, and that the open loop transfer matrix is also invertible i.e.

$$p = m, \det[H_B(s, e^{-sT})] \neq 0 \quad (6)$$

The equation (5) can be broken down as follows

$$\begin{aligned} & [\text{triang}\{t_{i,j}(s, e^{-sT})\}]^{-1} H_B(s, e^{-sT}) = \\ & = [G(s)]^{-1} \{I_m - K_1(s)L_B(s, e^{-sT})\} \end{aligned} \quad (7a)$$

$$\begin{aligned} & [\text{triang}\{t_{i,j}(s, e^{-sT})\}]^{-1} H_D(s, e^{-sT}) = \\ & = [G(s)]^{-1} \{-K_2(s) - K_1(s)L_D(s, e^{-sT})\} \end{aligned} \quad (7b)$$

Let

$$\Gamma(s) = \begin{bmatrix} \gamma_1(s) \\ \vdots \\ \gamma_m(s) \end{bmatrix} = [G(s)]^{-1} \quad (8a)$$

$$\Phi_1(s) = \begin{bmatrix} \phi_{1,1}(s) \\ \vdots \\ \phi_{1,m}(s) \end{bmatrix} = [G(s)]^{-1} K_1(s) \quad (8b)$$

$$\Phi_2(s) = \begin{bmatrix} \phi_{2,1}(s) \\ \vdots \\ \phi_{2,m}(s) \end{bmatrix} = [G(s)]^{-1} K_2(s) \quad (8c)$$

$$\text{triang}\{\eta_{i,j}(s, e^{-sT})\} = [\text{triang}\{t_{i,j}(s, e^{-sT})\}]^{-1} \quad (8d)$$

Using, the definitions (8a)-(8d) and the invertibility condition (6), the equations in (7a)-(7b) can be rewritten as

$$\text{triang}\{\eta_{i,j}(s, e^{-sT})\} = [\Gamma(s) \quad \Phi_1(s) \quad \Phi_2(s)] R_B(s, e^{-sT}) \quad (9a)$$

$$[\Gamma(s) \quad \Phi_1(s) \quad \Phi_2(s)] R_D(s, e^{-sT}) = 0 \quad (9b)$$

where

$$R_B(s, e^{-sT}) = \begin{bmatrix} [H_B(s, e^{-sT})]^{-1} \\ -L_B(s, e^{-sT}) [H_B(s, e^{-sT})]^{-1} \\ 0_{\zeta \times m} \end{bmatrix} \quad (9c)$$

$$R_D(s, e^{-sT}) = \begin{bmatrix} R_{D,1}(s, e^{-sT}) \\ R_{D,3}(s, e^{-sT}) \\ I_\zeta \end{bmatrix} \quad (9d)$$

where $R_{D,1}(s, e^{-sT}) = [H_B(s, e^{-sT})]^{-1} H_D(s, e^{-sT})$ and $R_{D,2}(s, e^{-sT}) = L_D(s, e^{-sT}) - L_B(s, e^{-sT}) [H_B(s, e^{-sT})]^{-1} H_D(s, e^{-sT})$. Let e_i be the i -th column of I_m , where $i = 1, \dots, m$. Let $J_i = [e_{i+1} \dots e_m]$ where $i = 1, \dots, m-1$ and $J_m = 0$. The two equations in (9a) and (9b) can be expressed as follows

$$[\gamma_i(s) \quad \phi_{1,i}(s) \quad \phi_{2,i}(s)] N_i(s, e^{-sT}) = 0 \quad (10a)$$

$$\eta_{i,i}(s, e^{-sT}) = [\gamma_i(s) \quad \phi_{1,i}(s) \quad \phi_{2,i}(s)] R_B(s, e^{-sT}) e_i \quad (10b)$$

where $i = 1, \dots, m$ and where

$$N_i(s, e^{-sT}) = [R_B(s, e^{-sT}) J_i \quad R_D(s, e^{-sT})] \quad (11)$$

If the condition (6) is satisfied, the problem has been reduced to that of solving (10a) under the constraints

$$\det \Gamma(s) \neq 0 \quad (12a)$$

$$[\gamma_i(s) \quad \phi_{1,i}(s) \quad \phi_{2,i}(s)] R_B(s, e^{-sT}) e_i \neq 0 \quad (12b)$$

$$\det\{\Gamma(s) - \Phi_1(s)L_B(s, e^{-sT})\} \neq 0 \quad (12c)$$

where $i = 1, \dots, m$. From (6a) it is observed that if (12a) and (12b) are satisfied then (12c) is also satisfied. To present the solvability conditions, consider the field $\mathbb{R}_0(s, \mathbf{z})$ being the field of rational functions of s with coefficients being multivariable rational functions of the elements of the vector $\mathbf{z} = [z_1 \dots z_q]$ where $z_1, \dots, z_q \in \mathbb{C}$. Let $\text{Rank}_{\mathbb{R}_0}[\cdot]$ denotes the rank of the argument matrix over the field $\mathbb{R}_0(s, \mathbf{z})$ (see [6] and [11]). Using the common properties of \mathbb{R} and $\mathbb{R}(s)$, the definitions of the rank over \mathbb{R} ($\text{rank}_{\mathbb{R}}[\bullet]$) and the orthogonal over \mathbb{R} ($[\bullet]_{\mathbb{R}}^{\perp}$) as well as their extension to rational

functions of s (see [6] and [11]) can be used. Hence, the definition of Rank over $\mathbb{R}(s)$ and the Orthogonal over $\mathbb{R}(s)$, denoted by $\text{Rank}_{\mathbb{R}(s)}[\bullet]$, and $[\bullet]_{\mathbb{R}(s)}^\perp$, can be used.

Using the notation $\mathbf{z} = \mathbf{e}^{-s\mathbf{T}}$, the matrices $H_B(s, \mathbf{z})$, $H_D(s, \mathbf{z})$, $L_B(s, \mathbf{z})$, $L_D(s, \mathbf{z})$ and $N_i(s, \mathbf{z})$ are introduced. Every matrix being function of (s, \mathbf{z}) is a $q+1$ complex variable function matrix. In what follows we may use manipulations in the field $\mathbb{R}_0(s, \mathbf{z})$ instead of $\mathbb{R}_e(s, \mathbf{e}^{-s\mathbf{T}})$.

Let

$$S_i(s) = [N_i(s, \mathbf{z})]_{\mathbb{R}(s)}^\perp \begin{bmatrix} I_m & 0_{m \times r} & 0_{m \times \zeta} \end{bmatrix}^T \quad (13a)$$

$$\Pi_{1,i}(s) = [N_i(s, \mathbf{z})]_{\mathbb{R}(s)}^\perp \begin{bmatrix} 0_{r \times m} & I_r & 0_{r \times \zeta} \end{bmatrix}^T \quad (13b)$$

$$\Pi_{2,i}(s) = [N_i(s, \mathbf{z})]_{\mathbb{R}(s)}^\perp \begin{bmatrix} 0_{\zeta \times m} & 0_{\zeta \times r} & I_\zeta \end{bmatrix}^T \quad (13c)$$

$$\rho_i = m + r + \zeta - \text{Rank}_{\mathbb{R}(s)}[N_i(s, \mathbf{z})] \quad (13d)$$

Clearly if $S_i(s) \neq 0$, then it holds that $\rho_i \geq 1$. Consequently, for $\rho_i \geq 1$ the following matrices are defined

$$S_\sigma^*(s) = \begin{bmatrix} s_{1,\sigma_1}(s) \\ \vdots \\ s_{m,\sigma_m}(s) \end{bmatrix}; \quad \sigma = [\sigma_1 \quad \cdots \quad \sigma_m], \sigma_i \in \{1, \dots, \rho_i\} \quad (13e)$$

where $s_{i,j}(s)$ is the j -th row of $S_i(s)$. Also define

$$\theta_i(s, \mathbf{z}) = [N_i(s, \mathbf{z})]_{\mathbb{R}(s)}^\perp R_B(s, \mathbf{z}) e_i \quad (13f)$$

Theorem 1: Consider the general neutral multi delay system of the form (1a)-(1b). The necessary and sufficient conditions for the solution of the problem of Triangular decoupling with simultaneous disturbance rejection, via a delayless dynamic measurement output controller of the form (3), are the condition (6) and the conditions

$$S_i(s) \neq 0, i = 1, \dots, m \quad (14a)$$

$$\exists \sigma : \det S_\sigma^* \neq 0 \quad (14b)$$

$$\theta_i(s, \mathbf{z}) \neq 0, i = 1, \dots, m \quad (14c)$$

Proof: (Necessity) The condition (6) has already been proven. To prove the necessity of (14a) and (14b), first the general solution of (10a) will be presented. The general solution is

$$\begin{bmatrix} \gamma_i(s) & \phi_{1,i}(s) & \phi_{2,i}(s) \end{bmatrix} = \lambda_i(s) [N_i(s, \mathbf{z})]_{\mathbb{R}(s)}^\perp \quad (15)$$

where $\lambda_i(s)$ is an $1 \times \rho_i$ arbitrary vector with elements in $\mathbb{R}(s)$. Using (13a)-(13f) the following relations are derived

$$\gamma_i(s) = \lambda_i(s) S_i(s) \quad (16a)$$

$$\phi_{1,i}(s) = \lambda_i(s) \Pi_{1,i}(s), \phi_{2,i}(s) = \lambda_i(s) \Pi_{2,i}(s) \quad (16b)$$

$$\eta_{i,i}(s, \mathbf{z}) = \lambda_i(s) \theta_i(s, \mathbf{z}) \quad (16c)$$

Using (16a) and (13e) it is observed that the left hand side of (12a) takes on the form

$$\det \Gamma(s) = \sum_{\sigma_1=1}^{\rho_1} \cdots \sum_{\sigma_m=1}^{\rho_m} \lambda_{1,\sigma_1}(s) \cdots \lambda_{m,\sigma_m}(s) S_\sigma^*(s) \quad (17)$$

The necessity of (14a)-(14b) arrives from (12a). From (16c) and (12b) the necessity of (14c) is derived.

(Sufficiency). To prove sufficiency, assume that (14a)-(14c) hold true. Consider (16a). To satisfy (12a), let $\hat{\sigma} = [\hat{\sigma}_1 \quad \cdots \quad \hat{\sigma}_m]$, $\hat{\sigma}_i \in \{1, \dots, \rho_i\}$ be a choice of σ that satisfies (14b) and let $\tilde{\sigma} = [\tilde{\sigma}_1 \quad \cdots \quad \tilde{\sigma}_m]$, $\tilde{\sigma}_i \in \{1, \dots, \rho_i\}$ be a choice of σ satisfying (14b), in the sense that $\theta_{i,\tilde{\sigma}_i}(s, \mathbf{z}) \neq 0, i = 1, \dots, m$. Consider the following choices

$$\lambda_{i,j}(s) = \begin{cases} 1 & \text{if } j = \hat{\sigma}_i \\ \kappa_i(s) & \text{if } j = \tilde{\sigma}_i \neq \hat{\sigma}_i \\ 0 & \text{if } j \neq \tilde{\sigma}_i, j \neq \hat{\sigma}_i \end{cases} \quad (18)$$

where $\lambda_{i,j}(s)$ are the elements of $\lambda_i(s)$. Let

$$\Gamma(s) = S_\sigma^*(s) + \text{diag} \{ \kappa_i^*(s) \} S_\sigma^*(s) \quad (19a)$$

where

$$\kappa_i^*(s) = \begin{cases} \kappa_i(s), & \text{if } \tilde{\sigma}_i \neq \hat{\sigma}_i \\ 0, & \text{if } \tilde{\sigma}_i = \hat{\sigma}_i \end{cases} \quad (19b)$$

From (19a) and (19b) there always exists an appropriate $\kappa_i(s)$ such that $\det \Gamma(s) \neq 0$ and $\lambda_i(s) \theta_i(s, \mathbf{z}) \neq 0$. ■

Define $\rho = \max\{\rho_i\}$ and

$$M_j(s) = \begin{bmatrix} m_{1,j}(s) \\ \vdots \\ m_{m,j}(s) \end{bmatrix} \quad (20a)$$

$$\Xi_{1,j}(s) = \begin{bmatrix} \xi_{1,1,j}(s) \\ \vdots \\ \xi_{1,m,j}(s) \end{bmatrix} \quad (20b)$$

$$\Xi_{2,j}(s) = \begin{bmatrix} \xi_{2,1,j}(s) \\ \vdots \\ \xi_{2,m,j}(s) \end{bmatrix} \quad (20c)$$

$$\Lambda_j(s) = \text{diag} \{ \lambda_{i,j}(s) \}_{i=1, \dots, m} \quad (20d)$$

where

$$m_{i,j}(s) = \begin{cases} s_{i,j}(s), & \text{if } j \in \{1, \dots, \rho_i\} \\ 0, & \text{if } j \in \{\rho_i + 1, \dots, \rho\} \end{cases}$$

$$\xi_{1,i,j}(s) = \begin{cases} \pi_{1,i,j}(s), & \text{if } j \in \{1, \dots, \rho_i\} \\ 0, & \text{if } j \in \{\rho_i + 1, \dots, \rho\} \end{cases}$$

$$\xi_{2,i,j}(s) = \begin{cases} \pi_{2,i,j}(s), & \text{if } j \in \{1, \dots, \rho_i\} \\ 0, & \text{if } j \in \{\rho_i + 1, \dots, \rho\} \end{cases}$$

and where $\pi_{1,i,j}(s)$ and $\pi_{2,i,j}(s)$ are the j -th rows of the matrices $\Pi_{1,i}(s)$ and $\Pi_{2,i}(s)$, respectively, and the elements $\lambda_{i,j}(s)$ (where $i \in \{1, \dots, m\}$ and $j \in \{1, \dots, \rho\}$) are arbitrary rational functions of s . Based on the proof of Theorem 1 and the above definitions we are now in position to present the following theorem.

Theorem 2: The general solutions for the delayless dynamic feedback matrices of the controller (3) solving the TDDR problem are

$$G(s) = \left[\sum_{j=1}^{\rho} \Lambda_j(s) M_j(s) \right]^{-1} \quad (21a)$$

$$K_1(s) = \left[\sum_{j=1}^{\rho} \Lambda_j(s) M_j(s) \right]^{-1} \left[\sum_{j=1}^{\rho} \Lambda_j(s) \Xi_{1,j}(s) \right] \quad (21b)$$

$$K_2(s) = \left[\sum_{j=1}^{\rho} \Lambda_j(s) M_j(s) \right]^{-1} \left[\sum_{j=1}^{\rho} \Lambda_j(s) \Xi_{2,j}(s) \right] \quad (21c)$$

where the diagonal matrices $\Lambda_j(s)$ are arbitrary but they are constrained to satisfy the inequalities

$$\det \left[\sum_{j=1}^{\rho} \Lambda_j(s) M_j(s) \right] \neq 0 \quad (21d)$$

$$\lambda_i(s) \Theta_i(s, z) \neq 0, i = 1, \dots, m \quad (21e)$$

$$\lambda_i(s) = \begin{bmatrix} \lambda_{i,1}(s) & \dots & \lambda_{i,\bar{\rho}_i}(s) \end{bmatrix} \quad (21f)$$

■

IV. SOLVABILITY CONDITIONS AND GENERAL SOLUTIONS WITH DELAYLESS STATIC CONTROLLERS

In this section the TDDR problem with delayless static measurement output feedback controllers is investigated. The delayless static controller is of the form

$$U(s) = K_1 \Psi(s) + K_2 \hat{U}_D(s) + G \Omega(s) \quad (22)$$

where the elements of K_1 , K_2 and G are real numbers being independent from s . Let

$$\Gamma = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_m \end{bmatrix} = G^{-1} \quad (23a)$$

$$\Phi_1 = \begin{bmatrix} \phi_{1,1} \\ \vdots \\ \phi_{1,m} \end{bmatrix} = G^{-1} K_1, \Phi_2 = \begin{bmatrix} \phi_{2,1} \\ \vdots \\ \phi_{2,m} \end{bmatrix} = G^{-1} K_2 \quad (23b)$$

Following the analytic steps of Section III, the TDDR problem via delayless static controllers is reduced to the solution of the following set of equations for $i = 1, \dots, m$

$$\begin{bmatrix} \gamma_i & \phi_{1,i} & \phi_{2,i} \end{bmatrix} N_i(s, \mathbf{e}^{-s\mathbf{T}}) = 0 \quad (24)$$

under the constraints

$$\det \Gamma \neq 0 \quad (25a)$$

$$\begin{bmatrix} \gamma_i & \phi_{1,i} & \phi_{2,i} \end{bmatrix} R_B(s, \mathbf{e}^{-s\mathbf{T}}) e_i \neq 0 (i = 1, \dots, m) \quad (25b)$$

Using the definitions of the rank over \mathbb{R} ($\text{rank}_{\mathbb{R}}[\bullet]$) and the orthogonal over \mathbb{R} ($[\bullet]_{\mathbb{R}}^{\perp}$) the quantities $\text{Rank}_{\mathbb{R}}[N_i(s, \mathbf{z})]$ and $[N_i(s, \mathbf{z})]_{\mathbb{R}}^{\perp}$ are introduced and the following are defined

$$\bar{S}_i = [N_i(s, \mathbf{z})]_{\mathbb{R}}^{\perp} \begin{bmatrix} I_m \\ 0_{r \times m} \\ 0_{\zeta \times m} \end{bmatrix} \quad (26a)$$

$$\bar{\Pi}_{1,i} = [N_i(s, \mathbf{z})]_{\mathbb{R}}^{\perp} \begin{bmatrix} 0_{m \times r} \\ I_r \\ 0_{\zeta \times r} \end{bmatrix} \quad (26b)$$

$$\bar{\Pi}_{2,i} = [N_i(s, \mathbf{z})]_{\mathbb{R}}^{\perp} \begin{bmatrix} 0_{m \times \zeta} \\ 0_{r \times \zeta} \\ I_{\zeta} \end{bmatrix} \quad (26c)$$

$$\bar{\rho}_i = m + r + \zeta - \text{Rank}_{\mathbb{R}}[N_i(s, \mathbf{z})] \quad (26d)$$

Clearly if $\bar{S}_i \neq 0$ then it holds that $\bar{\rho}_i \geq 1$. Consequently, for $\bar{\rho}_i \geq 1$, the following matrices may be defined

$$\bar{S}_{\sigma}^* = \begin{bmatrix} \bar{s}_{1,\sigma_1} \\ \vdots \\ \bar{s}_{m,\sigma_m} \end{bmatrix}; \sigma = \begin{bmatrix} \sigma_1 & \dots & \sigma_m \end{bmatrix}, \sigma_i \in \{1, \dots, \bar{\rho}_i\} \quad (27)$$

where $\bar{s}_{i,j}$ is the j -th row of the matrix \bar{S}_i . Finally define

$$\bar{\theta}_i(s, z) = [N_i(s, \mathbf{z})]_{\mathbb{R}}^{\perp} R_B(s, \mathbf{z}) e_i \quad (28)$$

Based on all above definitions and following the steps of the proof of Theorem 1, the following theorem can be proven.

Theorem 3: Consider the general neutral multi delay system of the form (1a)-(1b). The necessary and sufficient conditions for the solution of the TDDR problem via a delayless static measurement output controller of the form (22) are the condition (6) and the conditions

$$\bar{S}_i \neq 0, i = 1, \dots, m \quad (29a)$$

$$\exists \sigma : \det \bar{S}_{\sigma}^* \neq 0 \quad (29b)$$

$$\bar{\theta}_i(s, z) \neq 0, i = 1, \dots, m \quad (29c)$$

The general solutions for the delayless static feedback matrices of the controller (22) are

$$G = \left[\sum_{j=1}^{\bar{\rho}} \bar{\Lambda}_j \bar{M}_j \right]^{-1} \quad (30a)$$

$$K_1 = \left[\sum_{j=1}^{\bar{\rho}} \bar{\Lambda}_j \bar{M}_j \right]^{-1} \left[\sum_{j=1}^{\bar{\rho}} \bar{\Lambda}_j \bar{\Xi}_{1,j} \right] \quad (30b)$$

$$K_2 = \left[\sum_{j=1}^{\bar{\rho}} \bar{\Lambda}_j \bar{M}_j \right]^{-1} \left[\sum_{j=1}^{\bar{\rho}} \bar{\Lambda}_j \bar{\Xi}_{2,j} \right] \quad (30c)$$

where the elements of the arbitrary diagonal matrices $\bar{\Lambda}_j$ are constraint by the inequalities

$$\det \left[\sum_{j=1}^{\bar{\rho}} \bar{\Lambda}_j \bar{M}_j \right] \neq 0, \bar{\lambda}_i \bar{\theta}_i(s, \mathbf{z}) \neq 0 (i = 1, \dots, m) \quad (31)$$

where $\bar{\rho} = \max\{\bar{\rho}_i\}$, where

$$\bar{M}_j = \begin{bmatrix} \bar{m}_{1,j} \\ \vdots \\ \bar{m}_{m,j} \end{bmatrix} \quad (32a)$$

$$\bar{\Xi}_{1,j} = \begin{bmatrix} \bar{\xi}_{1,1,j} \\ \vdots \\ \bar{\xi}_{1,m,j} \end{bmatrix} \quad (32b)$$

$$\bar{\Xi}_{2,j} = \begin{bmatrix} \bar{\xi}_{2,1,j} \\ \vdots \\ \bar{\xi}_{2,m,j} \end{bmatrix} \quad (32c)$$

$$\bar{\Lambda}_j = \text{diag} \{ \bar{\lambda}_{i,j} \}_{i=1,\dots,m} \quad (32d)$$

where

$$\bar{m}_{i,j} = \begin{cases} \bar{s}_{i,j}, & \text{if } j \in \{1, \dots, \bar{\rho}_i\} \\ 0, & \text{if } j \in \{\bar{\rho}_i + 1, \dots, \bar{\rho}\} \end{cases}$$

$$\bar{\xi}_{1,i,j} = \begin{cases} \bar{\pi}_{1,i,j}, & \text{if } j \in \{1, \dots, \bar{\rho}_i\} \\ 0, & \text{if } j \in \{\bar{\rho}_i + 1, \dots, \bar{\rho}\} \end{cases}$$

$$\bar{\xi}_{2,i,j} = \begin{cases} \bar{\pi}_{2,i,j}, & \text{if } j \in \{1, \dots, \bar{\rho}_i\} \\ 0, & \text{if } j \in \{\bar{\rho}_i + 1, \dots, \bar{\rho}\} \end{cases}$$

and where $\bar{\pi}_{1,i,j}$ and $\bar{\pi}_{2,i,j}$ are the j -th rows of the matrices $\bar{\Pi}_{1,i}$ and $\bar{\Pi}_{2,i}$, respectively, and the elements $\bar{\lambda}_{i,j}$ of $\bar{\Lambda}_i = [\bar{\lambda}_{i,1} \dots \bar{\lambda}_{i,\rho_i}]$ are arbitrary reals. ■

V. TDDR FOR IDLE SPEED CONTROL OF IC ENGINES VIA DELAYLESS CONTROLLERS

A two-input (idle by-pass valve opening and spark advance) two-output (engine speed and intake manifold pressure) idle speed control model for Internal Combustion (IC) engines is studied. The model of the process belongs to the class described by the equation (1a)-(1b) (see [8]-[11]) with

$$x(\theta) = \begin{bmatrix} \Delta\omega(\theta) \\ \Delta p_m(\theta) \end{bmatrix}, u(\theta) = \begin{bmatrix} \Delta a(\theta) \\ \Delta \delta(\theta) \end{bmatrix}$$

$$A(e^{-sT}) = \begin{bmatrix} -\tau_{e,0}/J\omega_0^2 & 0 \\ -K_\varepsilon a_0/\omega_0^2 & -\eta_v V_d/4\pi V_m \end{bmatrix} + e^{-sT} \begin{bmatrix} 0 & K_\tau/J\omega_0 \\ 0 & 0 \end{bmatrix}$$

$$B(e^{-sT}) = \begin{bmatrix} 0 & K_\delta/J\omega_0 \\ K_\varepsilon/\omega_0 & 0 \end{bmatrix}, D(e^{-sT}) = \begin{bmatrix} 1/J\omega_0 \\ 0 \end{bmatrix}$$

and where ω is the engine speed (rad/s), p_m is the intake manifold pressure (kPa), δ is the spark advance (deg), a is the throttle opening (deg), θ is the crank angle (rad), θ_d is the induction to power delay (rad), $\hat{u}_D(\theta) = \tau_f$ is the disturbance torque (Nm), τ_e is the net engine torque (Nm), K_ε is the coefficient relating the throttle opening to delayed manifold pressure (kPa/s/deg), K_τ is the coefficient relating engine torque to manifold pressure (kPa/s/deg), K_δ is the coefficient relating engine torque to spark timing (Nm/deg), J is the engine inertia (Nm-s²/rad), η_v is the volumetric efficiency, V_d is the engine displacement (m³) and V_m is the manifold volume (m³). The subscript 0 denotes nominal operating point and Δ is the prefix of the increments of the system variables with respect to their nominal operating point. The performance outputs are the state variables (after appropriate rearrangement), while the measurement output is $\Delta\omega(\theta)$. Thus, it holds that

$$C(e^{-sT}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, L(e^{-sT}) = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

The transfer matrices from the inputs and the disturbances to the performance outputs are computed to be [11]

$$H_B(s, z) = \begin{bmatrix} (h_B)_{1,1}(s, z) & (h_B)_{1,2}(s, z) \\ (h_B)_{2,1}(s, z) & (h_B)_{2,2}(s, z) \end{bmatrix} \quad (33a)$$

$$H_D(s, z) = \begin{bmatrix} (h_D)_{1,1}(s, z) \\ (h_D)_{2,1}(s, z) \end{bmatrix} \quad (33b)$$

where

$$(h_B)_{1,1}(s, z) = [(K_\varepsilon/J\omega_0)s + (K_\varepsilon\tau_{e,0}/J^2\omega_0^3)]/d(s, z)$$

$$(h_B)_{1,2}(s, z) = -(a_0K_\delta K_\varepsilon/J\omega_0^3)/d(s, z)$$

$$(h_B)_{2,1}(s, z) = (K_\varepsilon K_\tau/J^2\omega_0^2)z/d(s, z)$$

$$(h_B)_{2,2}(s, z) = [(K_\delta/J\omega_0)s + (K_\delta n_v V_d/4J\pi V_m \omega_0)]/d(s, z)$$

$$(h_D)_{1,1}(s, z) = -(a_0K_\varepsilon/J\omega_0^3)/d(s, z)$$

$$(h_D)_{2,1}(s, z) = [(1/J\omega_0)s + (n_v V_d/4J\pi V_m \omega_0)]/d(s, z)$$

$$d(s, z) = s^2 + \left(\frac{n_v V_d}{4\pi V_m} + \frac{\tau_{e,0}}{J\omega_0^2} \right) s + \frac{a_0 K_\varepsilon K_\tau}{J\omega_0^3} z + \frac{n_v V_d \tau_{e,0}}{4J\pi V_m \omega_0^2}$$

The transfer matrix from the inputs to the measurement outputs and the transfer matrix from the measurable disturbances to the measurement outputs are computed to be (see also [11]) as follows

$$L_B(s, z) = \begin{bmatrix} (h_B)_{1,2}(s, z) \\ (h_B)_{2,2}(s, z) \end{bmatrix}, L_D(s, z) = (h_D)_{2,1}(s, z) \quad (34)$$

It holds that $n = m = 2$, $r = 1$ and

$$\det[H_B(s, z)] = \frac{4K_\delta K_\varepsilon \pi V_m \omega_0}{J[4a_0 K_\varepsilon K_\tau \pi V_m z + \omega_0(n_v V_d + 4\pi V_m s)(\tau_{e,0} + J\omega_0^2 s)]} \neq 0$$

From (9c)-(9d) and (11) we get $R_D(s, z) = \begin{bmatrix} 0 & 1/K_\delta & 0 & 1 \end{bmatrix}^T$

$$R_B(s, z) = \begin{bmatrix} \frac{J(n_v V_d + 4\pi V_m s)\omega_0}{4K_\varepsilon \pi V_m} & \frac{a_0 J}{\omega_0} \\ -\frac{K_\tau z}{K_\delta} & \frac{\tau_{e,0} + J\omega_0^2 s}{K_\delta \omega} \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$$

$$N_1(s, z) = \begin{bmatrix} \frac{a_0 J}{\omega_0} & 0 \\ \frac{\tau_{e,0} + J\omega_0^2 s}{K_\delta \omega} & \frac{1}{K_\delta} \\ -1 & 0 \\ 0 & 1 \end{bmatrix}, N_2(s, z) = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{K_\delta} \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[N_1(s, z)]_{\mathbb{R}(s)}^\perp = \begin{bmatrix} 1 & 0 & \frac{a_0 J}{\omega_0} & 0 \\ 0 & 1 & \frac{\tau_{e,0} + J\omega_0^2 s}{K_\delta \omega} & -\frac{1}{K_\delta} \end{bmatrix}$$

$$[N_2(s, z)]_{\mathbb{R}(s)}^\perp = \begin{bmatrix} 1 & -K_\delta & 0 & 1 \\ 0 & -K_\delta & 1 & 1 \\ 1 & -K_\delta & 1 & 1 \end{bmatrix}$$

Using the relations (13a) to (13d) we get $\rho_1 = 2$, $\rho_2 = 3$,

$$S_1(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, S_2(s) = \begin{bmatrix} 1 & -K_\delta \\ 0 & -K_\delta \\ 1 & -K_\delta \end{bmatrix}$$

$$\Pi_{1,1}(s) = \begin{bmatrix} \frac{a_0 J}{\omega_0} \\ \frac{\tau_{e,0} + J\omega_0^2 s}{K_\delta \omega} \end{bmatrix}, \Pi_{1,2}(s) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\Pi_{2,1}(s) = \begin{bmatrix} 0 \\ -\frac{1}{K_\delta} \end{bmatrix}$$

$$\Pi_{2,2}(s) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \theta_1(s, z) = \begin{bmatrix} \frac{J(n_v V_d + 4\pi V_m s)\omega_0}{4K_\epsilon \pi V_m} \\ -\frac{K_\tau z}{K_\delta} \end{bmatrix}$$

$$\theta_2(s) = \frac{1}{\omega_0} \begin{bmatrix} a_0 J - \tau_{e,0} - J\omega_0^2 s \\ -\tau_{e,0} - \omega_0 - J\omega_0^2 s \\ a_0 J - \tau_{e,0} - \omega_0 - J\omega_0^2 s \end{bmatrix}$$

Following the steps of Section 3, the conditions of Theorem 1 are satisfied and according to Theorem 2 the controller matrices are derived to be

$$G(s) = \frac{1}{p(s)} \begin{bmatrix} K_\delta(\lambda_{2,1}(s) + \lambda_{2,2}(s) + \lambda_{2,3}(s)) & \lambda_{1,2}(s) \\ \lambda_{2,1}(s) + \lambda_{2,3}(s) & -\lambda_{1,1}(s) \end{bmatrix}$$

$$K_1(s) = \frac{1}{K_\delta \omega_0 p(s)} \begin{bmatrix} \kappa_{1,1}(s) & \kappa_{2,1}(s) \end{bmatrix}^T$$

$$K_2(s) = - \begin{bmatrix} 1 & K_\delta^{-1} \end{bmatrix}^T$$

where

$$p(s) = \lambda_{1,2}(s)(\lambda_{2,1}(s) + \lambda_{2,3}(s)) +$$

$$K_\delta \lambda_{1,1}(s)(\lambda_{2,1}(s) + \lambda_{2,2}(s) + \lambda_{2,3}(s))$$

$$\kappa_{1,1}(s) = a_0 J K_\delta \lambda_{1,1}(s)(\lambda_{2,1}(s) + \lambda_{2,2}(s) + \lambda_{2,3}(s)) +$$

$$\lambda_{1,2}(s)[(\tau_{e,0} + J\omega_0^2 s)\lambda_{2,1}(s) +$$

$$+ (\tau_{e,0} + \omega_0 + J\omega_0^2 s)(\lambda_{2,2}(s) + \lambda_{2,3}(s))]$$

$$\kappa_{2,1}(s) = (\tau_{e,0} + J\omega_0^2 s)\lambda_{1,2}(s)(\lambda_{2,1}(s) + \lambda_{2,3}(s))/K_\delta$$

$$+ \lambda_{1,1}(s)[a_0 J \lambda_{2,1}(s) + a_0 J \lambda_{2,3}(s) - \omega_0(\lambda_{2,2}(s) + \lambda_{2,3}(s))]$$

The elements of the resulting triangular closed loop system matrix are

$$t_{1,1}(s, z) = 4K_\delta K_\epsilon \pi V_m / [JK_\delta(n_v V_d + 4\pi s V_m)\omega_0 \lambda_{1,1}(s) -$$

$$- 4K_\epsilon K_\tau \pi V_m z \lambda_{1,2}(s)]$$

$$\tilde{t}_{2,1}(s, z) = (K_\delta \omega_0 \{4K_\epsilon K_\tau \pi V_m z \lambda_{2,2}(s) +$$

$$[Jn_v V_d \omega_0 + 4\pi V_m (K_\epsilon K_\tau z + J\omega_0 s)](\lambda_{2,1}(s) + \lambda_{2,3}(s))\}) /$$

$$\{[JK_\delta(n_v V_d + 4\pi V_m s)\omega_0 \lambda_{1,1}(s) - 4K_\epsilon K_\tau \pi V_m z \lambda_{1,2}(s)]$$

$$+ (-a_0 J + \tau_{e,0} + J\omega_0^2 s)\lambda_{2,1}(s) + (\tau_{e,0} + \omega_0 + J\omega_0^2 s)\lambda_{2,2}(s) +$$

$$(-a_0 J + \tau_{e,0} + \omega_0 + J\omega_0^2 s)\lambda_{2,3}(s)\}$$

$$t_{2,2}(s, z) = -\omega_0 / [(-a_0 J + \tau_{e,0} + J\omega_0^2 s)\lambda_{2,1}(s)$$

$$+ (\tau_{e,0} + \omega_0 + J\omega_0^2 s)\lambda_{2,2}(s) +$$

$$(-a_0 J + \tau_{e,0} + \omega_0 + J\omega_0^2 s)\lambda_{2,3}(s)]$$

The closed loop transfer matrix provides independent control for the intake manifold pressure and satisfactory performance of the overall process.

The results of the present section cover the respective results in [11] as a special case. It is noted that in [11] the requirement of exact model matching with a specific triangular model and disturbance rejection have been fulfilled for the present idle machine model.

VI. CONCLUSION

In this paper, for the class of general neutral multi delay systems with measurable disturbances, the TDDR problem via delayless dynamic controllers with output measurement feedback and compensation of the measurable disturbances, has been solved. Using a pure algebraic approach, the necessary and sufficient conditions for the solvability of the problem have been established and the general forms of the delayless dynamic controller matrices solving the problem have been derived in explicit analytic forms. For the same system class the TDDR problem has been solved via delayless controllers with static measurement output feedback and static compensation of the measurable disturbances.

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