2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Robust Fault Detection H_∞ Filter for Markovian Jump Linear Systems

Leonardo de Paula Carvalho[1], André Marcorin de Oliveira[2] and Oswaldo Luiz do Valle Costa[3]

Abstract— **This paper studies the robust fault detection problem associated with Markovian Jump Linear Systems (MJLS) in the discrete-time domain. The approach presented consists in using a MJLS filter as a residual generator designed via Linear Matrix Inequalities (LMI) and formulated as an H_∞ filter problem. We tackle three situations: designing an H_∞ filter that depends on the Markov mode; the so-called mode-independent case; and the design of robust H_∞ filters in the sense that the system matrices are uncertain. A numerical example is presented in order to illustrate the feasibility of the proposed solution.**

I. Introduction

Systems subject to abrupt changes on their dynamics have been receiving a great deal of attention in the literature. Particularly regarding the behavior originated by faults, the necessity of sensing and acting on the system in order to ensure its safety and proper working scheme is of utmost importance. In this sense, the Robust Fault Detection and Isolation (RFDI) algorithms are used with the objective of detecting non-expected behaviours in many distinct fields in engineering such as chemical, nuclear, aerospace, and automotive applications, for instance, see the works [17], [13], [5], [12], respectively. Thus, in the occurence of faults, the main purpose is to increase the safety level, detect the failure, and rearrange the control law in order to minimize the losses, or the chance of a possible accident, [11]. In short, the general working scheme behind the RFDI is to generate a residual signal, predetermine a threshold, and whenever the residual signal is greater than the threshold, assume that a fault occured, [7]. However, to guarantee an FDI algorithm that provides a fast detection and low occurrence of false alarms, the plant identification process must reach a satisfactory level of precision, and also the observer must be robust in order to cope with possible variations on the plant and the presence of disturbances.

All the communication between the components responsible for the RFDF are made through a network, and nowadays the wireless networks are becoming more popular mainly for its lower implementation cost and the higher flexibility whenever compared to the usual wired networks, see [14]. However, using a wireless network increases the chance of network communication loss, since the wireless network is susceptible to problems like, collision [1], channel fading [19], burst failure [2], etc. For that reason, the use of Markovian Jump Linear System (MJLS) [10] is justified, due to the possibility of modelling the dynamical behaviour of systems whose signals are degraded by networks, see for instance, the work [15].

In relation to the MJLS framework applied to FDI theory, we can mention the work [21] that tackled the problem of designing H_∞ residual filters for discrete-time MJLS considering that the Markov chain can be measured. More recently, the work [18] considered also the synthesis of H_∞ residual filters for continuous-time MJLS, but with the assumption that the modes of operation of the filter are unmatched in relation to the system being observed. Regarding the aforementioned works, we consider that this topic is still not completely investigated, since both the works [21] and [18] study a sub-optimized H_∞ residual filters, and for that reason some alternative design conditions to the ones given in [21] for H_∞ residual filters would be desirable in order to cope with potential conservative results.

Bearing in mind the previous discussion, we study in this work the synthesis of residual H_∞ filters for discrete-time MJLS. Our main result consists of LMI design conditions for achieving H_∞ mode-dependent fault detection filters. Moreover, we investigate the mode-independent residual filter formulation, and also the design of H_∞ robust filters with relation to plant uncertainties. A numerical example is presented at the end of this paper in order to illustrate our results.

This paper is organized as follows: Section II presents the notation used in this work, Section III presents the theoretical background necessary in order to understand the following sections, Section IV describes the RFD problem, Section V-A presents the Section V-A presents the mode-dependent case, Section V-B presents the result for the mode-independent case, Section V-C presents the case of designing robust H_∞ filters, Section VI illustrates the obtained results with a numerical example, and Section VII concludes the paper with some final comments. In the Appendix II the proof for the Theorem 1 is presented.

II. Notation

The notation is standard. The operator $(')$ denotes the matrix or vector transpose, $(\bullet)$ indicates each symmetric block of a symmetric matrix. The set of Markov chain states is represented by $\mathbb{K} = \{1, 2, \ldots, N\}$. The convex combinations of the matrix X_j and the weight ρ_{ij} is given by

This work was partly supported by Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - CAPES for the first author, São Paulo Research Foundation - FAPESP, grants FAPESP-2015/09912-8 and FAPESP-2017/06358-5 for the second author, and National Council for Scientific and Technological Development - CNPq, grant CNPq-304091/2014-6, the FAPESP/BG Brasil through the Research Center for Gas Innovation, grant FAPESP/BG-2014/50279-4, and the project INCT, grants FAPESP/INCT-2014/50851-0 and CNPq/INCT-465755/2014-3, for the third author.

[1],[2],[3]Depto. Eng. Telecomunicações e Controle, Escola Politécnica da Universidade de São Paulo, CEP 05508-010 São Paulo, SP, Brazil [1]carvalho.lp@usp.br, [2]marcorin@usp.br, [3]oswaldo@lac.usp.br

$\varepsilon_i(X) = \sum_{j=1}^{N} \rho_{ij} X_j$ for $i \in \mathbb{K}$. The symbol $\xi()$ represents mathematical expectations. Considering the stochastic signal $z(k)$, its norm is defined by $\|z\|_2^2 = \sum_{k=0}^{\infty} \varepsilon\{z(k)'z(k)\}$. The set of signals $z(k) \in \mathbb{R}^n$ defined for all $k \in \mathbb{N}$, such that $\|z\|_2 < \infty$ is indicated by $\mathcal{L}^2$. We consider the convex set

$$\Upsilon = \left\{ Q; Q = \sum_{l=1}^{V} \mu_l Q^l =, \quad \mu_l \geq 0, \sum_{l=1}^{V} \mu_l = 1 \right\} \tag{1}$$

where V is the number of vertex in the politope. Whenever $Q \in \Upsilon$, we associate the index l with the convex set (1).

III. Mean Square Stability and H_∞ Norm

We define in this section the concept of mean square stability and H_∞ norm. For that we consider the following general discrete-time Markovian Jump Linear System (MJLS) as below

$$\mathcal{G} : \begin{cases} x(k+1) = A_{\theta_k} x(k) + J_{\theta_k} w(k) \\ \quad z(k) = C_{z\theta_k} x(k) + E_{z\theta_k} w(k) \end{cases} \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $y(k) \in \mathbb{R}^q$ is the measured output vector, $z(k) \in \mathbb{R}^p$ is the estimated output, $w(k) \in \mathbb{R}^m$ is the exogenous input. We define a transition probability matrix by $\Omega = [\rho_{ij}]_{i,j \in \psi}$ where ρ_{ij} is defined as follows $\rho_{ij} = Pr[\theta_{k+1} = j | \theta_k = i]$ and $\sum_{j=1}^{N} \rho_{ij} = 1$.

A. Mean Square Stability

The definition of Mean Square Stability presented in [4] is:

Definition: The system (2) is Mean Square Stable (MSS) if for any initial condition $x(0) = x_0 \in \mathbb{R}^n$, initial distribution $\theta(0) = \theta_0 \in \mathbb{K}$ its holds that

$$\lim_{k \to \infty} \varepsilon\{x(k)'x(k) | x_0, \theta_0\} = 0. \tag{3}$$

B. H_∞ Norm

Assuming that $\mathcal{G}$ is MSS, the H_∞ norm of $\mathcal{G}$ is given by (see [6])

$$\|\mathcal{G}\|_\infty^2 = \sup_{0 \neq w \in \mathcal{L}_2, \theta_0 \in \mathbb{K}} \frac{\|z\|_2^2}{\|w\|_2^2} \tag{4}$$

where w represents the inputs and z represents the outputs. Notice that $\mathbb{K} = \{1\}$, that is, there is only one state for the Markov chain, corresponds to the deterministic case.

IV. Problem Formulation

The MJLS we consider in this work is represented by

$$\mathcal{G}_a : \begin{cases} x(k+1) = A_{\theta_k} x(k) + B_{\theta_k} u(k) + B_{d\theta_k} d(k) + B_{f\theta_k} f(k) \\ \quad y(k) = C_{\theta_k} x(k) + D_{d\theta_k} d(k) + D_{f\theta_k} f(k) \\ \quad x(0) = x_0, \end{cases} \tag{5}$$

where $x(k) \in \mathbb{R}^n$ is the state, $y(k) \in \mathbb{R}^q$ is the measured output, $u(k) \in \mathbb{R}^m$ is the known input, $d(k) \in \mathbb{R}^p$ is the exogenous input and $f(k) \in \mathbb{R}^t$ is the fault vector which is considered as an unknown time function. We also consider that $f(k), d(k) \in \mathcal{L}^2$.

Usually the Fault Detection system is divided into two distinct stages, a residual generator and a residual evaluation.

A. Residual Generator

For the purpose of generating the residual signal $r(k)$ a Markovian observer is considered as a fault detection filter with the following definition

$$\mathcal{F} : \begin{cases} \eta(k+1) = A_{\eta\theta_k} \eta(k) + M_{\eta\theta_k} u(k) + B_{\eta\theta_k} y(k) \\ \quad r(k) = C_{\eta\theta_k} \eta(k) + D_{\eta\theta_k} y(k) \\ \quad \eta(0) = \eta_0 \end{cases} \tag{6}$$

where $\eta(k) \in \mathbb{R}^n$ represents the filter states and $r(k) \in \mathbb{R}^l$ is the filter residue. We point out that this filter structure also depends on the Markov mode θ_k.

The main purpose of this paper is to design the matrices $A_{\eta i}$, $B_{\eta i}$, $C_{\eta i}$, $D_{\eta i}$, $M_{\eta i}$ so that the residual generator (6) is mean square stable when $u = 0$, $d = 0$ and $f = 0$ and also minimize the value of γ in

$$\sup_{w \neq 0} \frac{\|r - \hat{f}\|_2}{\|w\|_2} < \gamma \tag{7}$$

where $w(k) = [u'(k) \quad d'(k) \quad f'(k)]$, see [21].

Similarly to the continuous-time case presented in [3] and the discrete-time case in [21], a weighting matrix $W_f(f)$ is used with the intention to increase the system performance, where $\hat{f}(k) = W_f(z) f(z)$. A minimal realization is

$$\mathcal{W}_f : \begin{cases} x_f(k+1) = A_{wf} x_f(k) + B_{wf} f(k) \\ \quad \hat{f}(k) = C_{wf} x_f(k) + D_{wf} f(k) \\ \quad x_f(0) = 0 \end{cases} \tag{8}$$

where $x_f(k) \in \mathbb{R}^t$ is the filter state, and $f(k)$ is the same fault as in (5).

The block diagram presented below represents the equivalent system:

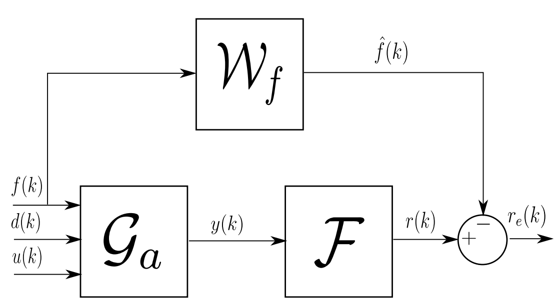

Fig. 1: Block diagram.

Considering $r_e(k) = r(k) - \hat{f}(k)$ the equivalent system can be written in the augmented form as

$$\mathcal{G}_{aug} : \begin{cases} \bar{x}(k+1) = \tilde{A}_{\theta_k} \bar{x}(k) + \tilde{B}_{\theta_k} \bar{w}(k) \\ \quad r_e(k) = \tilde{C}_{\theta_k} \bar{x}(k) + \tilde{D}_{\theta_k} \bar{w}(k) \end{cases} \tag{9}$$

where the augmented state is $\bar{x}(k) = [x'(k)\ \eta'(k)\ x'_f(k)]'$ and $\bar{w} = [u'(k)\ d'(k)\ \hat{f}'(k)]'$ and

$$\tilde{A}_{\theta_k} = \begin{bmatrix} A_{\theta_k} & 0 & 0 \\ B_{\eta\theta_k}C_{\theta_k} & A_{\eta\theta_k} & 0 \\ 0 & 0 & A_{wf} \end{bmatrix}, \tag{10}$$

$$\tilde{B}_{\theta_k} = \begin{bmatrix} B_{\theta_k} & B_{d\theta_k} & B_{f\theta_k} \\ M_{\theta_k} & B_{n\theta_k}D_{d\theta_k} & B_{\eta\theta_k}D_{f\theta_k} \\ 0 & 0 & B_{wf} \end{bmatrix}, \tag{11}$$

$$\tilde{C}_{\theta_k} = \begin{bmatrix} D_{\eta\theta_k}C_{\theta_k} & C_{\eta\theta_k} & C_{wf} \end{bmatrix}, \tag{12}$$

$$\tilde{D}_{\theta_k} = \begin{bmatrix} 0 & D_{\eta\theta_k}D_{d\theta_k} & D_{\eta\theta_k}D_{f\theta_k} - D_{wf} \end{bmatrix}. \tag{13}$$

Summing up, the Robust Fault Detection Filter problem correspond to an optimization problem to obtain the matrices that compose the observer (6) in such a way that the system (9) is MSS and γ is as small as possible in the feasibility of

$$\sup_{\|w\|_2 \neq 0, w \in \mathcal{L}_2} \frac{\|r_e\|_2}{\|w\|_2} < \gamma, \quad \gamma > 0. \tag{14}$$

B. Residual Evaluation

In the evaluation stage it is necessary to set an evaluation function $J(\bar{r}(k))$ and also a threshold $J_{th}(k)$, both as defined in [21]. We consider L as the evaluation time, and with that, we are able to separate the evaluation process into two distinct cases, the first one is defined by $k - L \geq 0$ and the second one, $k - L < 0$. Thus, we define the auxiliary vectors for each case as

$$\begin{cases} \text{for } k - L \geq 0,\ \bar{r}(k) = [r(k)\ r(k-1)\ \dots\ r(k-L)] \\ \text{for } k - L < 0,\ \bar{r}(k) = [r(k)\ r(k-1)\ \dots\ r(0)] \end{cases} \tag{15}$$

and, given the discrepancy between the intervals, the evaluation functions for each case are set as

$$\begin{cases} \text{for } k - L \geq 0,\ J(\bar{r}(k)) = \left\{ \displaystyle\sum_{\sigma=k}^{\sigma=k-L} \bar{r}'(\sigma)\bar{r}(\sigma) \right\}^{\frac{1}{2}}, \\ \text{for } k - L < 0,\ J(\bar{r}(k)) = \left\{ \displaystyle\sum_{\sigma=k}^{\sigma=0} \bar{r}'(\sigma)\bar{r}(\sigma) \right\}^{\frac{1}{2}}. \end{cases} \tag{16}$$

The threshold is defined as

$$J_{th}(k) = \sup_{d \in \mathcal{L}^2,\ f=0} \xi(J(\bar{r}(k))). \tag{17}$$

The occurrence of faults can be detected by analyzing the value of $J(\bar{r}(k))$ as follow:

$$\begin{cases} J(\bar{r}(k)) < J_{th}(k), \text{ means that the system is in the nominal mode} \\ J(\bar{r}(k)) \geq J_{th}(k) \text{ means that a fault occurred at the instant } k. \end{cases} \tag{18}$$

V. Main Results

In this section we present the main results of this paper, that consists of a suboptimal LMI constraint to obtain the H_∞ Robust Fault Detection Filter.

A. Mode-Dependent Case

Theorem 1: There exists a mode-dependent Robust Fault Detection Filter in the form of (6) satisfying the constraint (14) for some $\gamma > 0$ if there exist symmetric matrices Z_i, X_i, W_i, and the matrices H_i, Δ_i, O_i, F_i, G_i with compatible dimensions that satisfy the LMI constraint (19) below

$$\begin{bmatrix}
Z_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
Z_i & X_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & W_i & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & \gamma I & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & 0 & \gamma I & \bullet & \bullet & \bullet & \bullet & \bullet \\
0 & 0 & 0 & 0 & 0 & \gamma I & \bullet & \bullet & \bullet & \bullet \\
\varepsilon_i(Z)A_i & \varepsilon_i(Z)A_i & 0 & \varepsilon_i(Z)B_i & \varepsilon_i(Z)B_{di} & \varepsilon_i(Z)B_{fi} & \varepsilon_i(Z) & \bullet & \bullet & \bullet \\
\varepsilon_i(X)A_i + \Delta_i C_i + O_i & \varepsilon_i(X)A_i + \Delta_i C_i & 0 & \varepsilon_i(X)B_i + H_i & \varepsilon_i(X)B_{di} + \Delta_i D_{di} & \varepsilon_i(X)B_{fi} + \Delta_i D_{fi} & \varepsilon_i(Z) & \varepsilon_i(X) & \bullet & \bullet \\
0 & 0 & \varepsilon_i(W)A_{wf} & 0 & 0 & \varepsilon_i(W)B_{wf} & 0 & 0 & \varepsilon_i(W) & \bullet \\
G_i C_i + F_i & G_i C_i & C_{wf} & 0 & G_i D_{di} & G_i D_{fi} - D_{wf} & 0 & 0 & 0 & I
\end{bmatrix} > 0 \tag{19}$$

$i \in \mathbb{K}$. If a feasible solution for (19) is obtained, then a suitable RFD Filter is given by $A_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i$, $M_i = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, for all $i \in \mathbb{K}$.

Proof: See the Appendix.

B. Mode-Independent Case

The mode-independent case is a special situation where the RFD Filter no longer depends on the Markov chain, that is, there is only one filter for the N states. The solution presented in the previous section is already a sub optimized solution for the RFDF problem, however, the mode-independent condition adds more conservadorism to the optimization problem.

In order to design a single RFD Filter for the N modes, it is necessary to fix some of the variables presented in the Theorem 1. The variables $\Delta_i = \Delta$, $O_i = O$, $F_i = F$, $G_i = G$, $H_i = H$, do not vary according to the Markov Chain. However, some matrices for the RFD Filter depend on the term $(\varepsilon_i(Z) - \varepsilon_i(X))$ and, in order to also make this term unique, it is necessary to add the constraint

$$\rho_{ij} = \rho_j, \forall (i, j) \in \mathbb{K} \tag{20}$$

so that this new restriction (20), which corresponds to the so-called Bernoulli case, allows us to rewrite the Theorem 1 in a way that the RFD Filter acquired via the next theorem is mode-independent. The theorem is presented below,

Theorem 2: There exists a mode-independent Robust

Fault Detection Filter in the form of (6) satisfying the constraint (14) for a given $\gamma > 0$ if there exists the symmetric matrices Z_i, X_i, W_i, and matrices H, Δ, O, F, G with compatible dimensions, so that satisfy the same LMI constraint presented in (19), where $i \in [1, 2, \ldots, N]$. If a feasible solution is obtained a suitable RFD Filter is given by $A_\eta = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O$, $B_\eta = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta$, where $\varepsilon(V) = \sum_i \rho_i V_i$, $C_\eta = F$, $D_\eta = G$, $M = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H$.

Proof: The proof is the same as presented in the Appendix for the Theorem 1.

C. Robust Fault Detection Filter with uncertainties

A way to add the system uncertainties in the optimization problem presented in the Theorem (1) is to describe these uncertainties as a politope, [20].

The matrices that represent the system dynamic in this case have another index l representing the politope vertex, where $i = [1, 2, \ldots, N]$ and $l = [1, 2, \ldots, V]$, see Section II. Replacing the matrices from the system (5), like $A_i = A_i^l$, $B_i = B_i^l$, $C_i = C_i^l$, $D_i = D_i^l$, in the LMI constraint (19), the theorem below is obtained.

Theorem 3: There exist a mode-dependent Robust Fault Detection Filter in the form of (6) satisfying the constraint (14) for a given $\gamma > 0$ if there exist symmetric matrices Z_i,X_i,W_i, and matrices H_i, Δ_i, O_i, F_i, $D_{\eta i}$ with compatible dimensions that satisfy the LMI constraint (19). If a feasible solution is obtained a suitable RFD Filter is given by $A_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}O_i$, $B_{\eta i} = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}\Delta_i$, $C_{\eta i} = F_i$, $D_{\eta i} = G_i =$, $M_i = (\varepsilon_i(Z) - \varepsilon_i(X))^{-1}H_i$, for all $i \in \mathbb{K}$.

Proof: The proof straightforwardly derives from the proof presented in the Appendix for the Theorem 1.

VI. Numerical Example

This numerical example was extracted from [21], considering a two-mode Markovian Jump Linear System in the discrete-time domain. The matrices that compose this system are

$$A_1 = \begin{bmatrix} 0.1 & 0 & 1 & 0 \\ 0 & 0.1 & 0 & 0.5 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.3 & 0 & -1 & 0 \\ -0.1 & 0.2 & 0 & -0.5 \\ 0 & 0 & -0.2 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 0.8 \\ -2.4 \\ 1.6 \\ 0.8 \end{bmatrix}, \quad B_f = \begin{bmatrix} 1 \\ 1 \\ 2 \\ -2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix},$$

$$D_d = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} \quad D_f = \begin{bmatrix} 2 \\ -1 \end{bmatrix},$$

$$\Omega = \begin{bmatrix} 0.3 & 0.7 \\ 0.6 & 0.4 \end{bmatrix},$$

$$A_{wf} = 0.5, \quad B_{wf} = 0.25, \; C_{wf} = 1, \; D_{wf} = 0.5.$$

The RFDF obtained using the Theorem 1 is

$$A_{\eta 1} = \begin{bmatrix} -0.7545 & -0.6279 & 0.1455 & -0.6279 \\ 0.2357 & -0.0906 & 0.2357 & 0.3094 \\ -0.4086 & -0.8038 & -0.2085 & -0.8038 \\ 0.3352 & 0.7392 & 0.3352 & 0.8392 \end{bmatrix},$$

$$A_{\eta 2} = \begin{bmatrix} 0.7212 & -0.0470 & -0.5788 & -0.0470 \\ -0.5803 & -0.3480 & -0.4803 & -1.0480 \\ -0.0859 & -0.6524 & -0.2859 & -0.6524 \\ -0.1360 & 0.5317 & -0.1360 & 0.0317 \end{bmatrix},$$

$$B_{\eta 1} = \begin{bmatrix} 0.6279 & 0.8545 \\ 0.1906 & -0.2357 \\ 0.8038 & 0.4086 \\ -0.7392 & -0.3352 \end{bmatrix},$$

$$B_{\eta 2} = \begin{bmatrix} 0.0470 & -0.4212 \\ 0.5480 & 0.4803 \\ 0.6524 & 0.0859 \\ -0.5317 & 0.1360 \end{bmatrix},$$

$$C_{\eta 1} = \begin{bmatrix} -0.0431 & -0.1188 & -0.0431 & -0.1188 \end{bmatrix},$$

$$C_{\eta 2} = \begin{bmatrix} -0.0353 & -0.1157 & -0.0353 & -0.1157 \end{bmatrix},$$

$$D_{\eta 1} = \begin{bmatrix} 0.1188 & 0.0431 \end{bmatrix}, \quad D_{\eta 2} = \begin{bmatrix} 0.1157 & 0.0353 \end{bmatrix},$$

$$M_1 = 0, \; M_2 = 0.$$

and the H_∞ norm obtained is 0.5612.

In order to show that the theoretical results presented in the paper are a valid approach to the RFDF problem, a simulation was performed with a single realization, using the previous results and the following addition criteria: the unknown signal $d_k(k)$ is a white noise with mean equal to 0 and standard deviation equal to 0.7071. The weighted fault $\hat{f}(k)$ used is as in Fig. 2

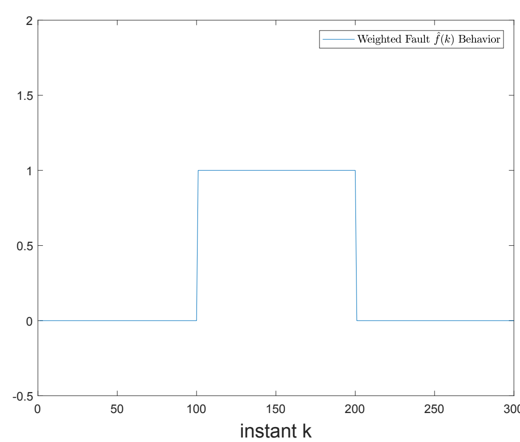

Fig. 2: Weighted Fault behavior.

The residual obtained in a single iteration is presented in the fig. 3

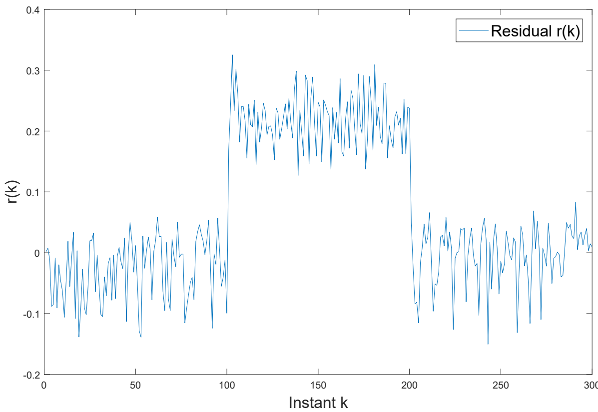

Fig. 3: Residual $r(k)$ behavior.

The next graphic is the evaluation of $J(\bar{r}(k))$ for a situation where the fault presented in the fig. 2 occurs (red curve) and the other one where there is no fault (blue curve). The graphic obtained is presented in the fig. 4

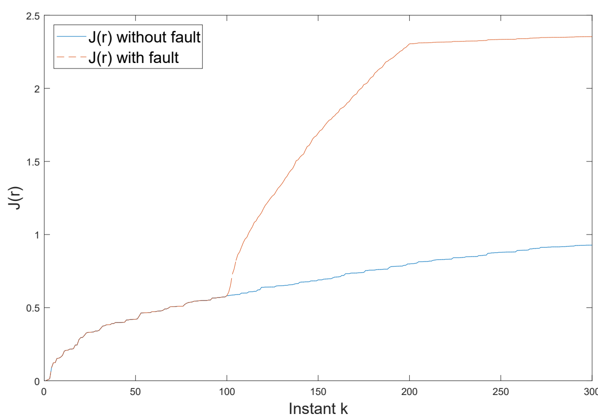

Fig. 4: $J(\bar{r}(k))$ behavior.

Using the evaluation criterion presented in the Section IV-B, the threshold J_{th} value was obtained from the situation where there is no fault (blue curve in the Fig.3) and also considering $L = 300$. The value obtained was $J_{th} = \{\xi(\sum_{300}^{0} \bar{r}(\sigma)\bar{r}(\sigma)')\}^{1/2} = 0.6106$. The evaluation function was applied in the cases where the fault occurred (red curve in the Fig.3). The fault started at $k = 100$, as presented in the Fig. 2 and in the instant $k = 102$ the evaluation function value was $J(\bar{r}(k)) = \{\sum_{\sigma=102}^{0} \bar{r}(\sigma)\bar{r}(\sigma)'\}^{1/2} = 0.6542$, which exceeds the threshold, meaning that the fault was detected after two instants of time.

From 5000 Monte Carlo simulations we obtained that the average other results obtained in the simulation is that the average amount of steps necessary to the fault to be detected is 2.1890 steps after the fault starts.

VII. Conclusion

In this paper, the Robust Fault Detection Problem associated with Markovian Jump Linear System in the discrete-time domain is studied. The proposed solution consists in designing a residual generator via LMI and formulated as an H_∞ MJLS filter. The mode-independent case and the uncertain case are encompassed under our formulation. The numerical example illustrates that our approach can provide a viable solution to the RFDF problem. The next step along this line of research is to formulate the H_2 RFDF in order to evaluate its efficiency when compared to the H_∞ approach. Another possible work would be the formulation of a Hybrid H_∞ and H_2 RFDF.

Appendix I
Bounded Real Lemma

The lemma known as Bounded Real Lemma for Markovian Jump Linear Systems was first presented in [16], and is stated below

Lemma: The system (2) is MSS and satisfies the norm constraint $\|G\|_\infty^2 < \gamma$ if and only if there exist matrices $P_i = P_i' > 0$ such that

$$\begin{bmatrix} A_i & J_i \\ C_{zi} & E_{zi} \end{bmatrix}' \begin{bmatrix} \varepsilon_i(P) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_i & J_i \\ C_{zi} & E_{zi} \end{bmatrix} - \begin{bmatrix} P_i & 0 \\ 0 & \gamma I \end{bmatrix} < 0. \quad (21)$$

Applying the Schur complement to (21), we obtain that

$$\begin{bmatrix} P_i & \bullet & \bullet & \bullet \\ 0 & \gamma I & \bullet & \bullet \\ \varepsilon_i(P)A_i & \varepsilon_i(P)J_i & \varepsilon_i(P) & \bullet \\ C_{zi} & E_{zi} & 0 & I \end{bmatrix} > 0 \quad (22)$$

and the LMI constraint (22) can also be described as the inequality below (see, for instance,[9])

$$\begin{bmatrix} \tilde{P}_{\theta_k} & \bullet & \bullet & \bullet \\ 0 & \gamma I & \bullet & \bullet \\ \tilde{A}_{\theta_k} & \tilde{B}_{\theta_k} & \varepsilon_{\theta_k}(\tilde{P})^{-1} & \bullet \\ \tilde{C}_{\theta_k} & \tilde{D}_{\theta_k} & 0 & I \end{bmatrix} > 0. \quad (23)$$

Appendix II
Proof Theorem 1

The first step to derive the subotimal condition is to impose the following structure, similar to the structure in [8], for the matrices P and P^{-1}

$$P_i = \begin{bmatrix} X_i & U_i & 0 \\ U_i' & \hat{X}_i & 0 \\ 0 & 0 & P_i^{33} \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} Y_i & V_i & 0 \\ V_i' & \hat{Y}_i & 0 \\ 0 & 0 & Q_i^{33} \end{bmatrix}, \quad (24)$$

and also0 consider the following structure for the matrices $\varepsilon_i(P)$ and $\varepsilon_i(P)^{-1}$

$$\varepsilon_i(P) = \begin{bmatrix} \varepsilon_i(X) & \varepsilon_i(U) & 0 \\ \varepsilon_i(U)' & \varepsilon_i(X) & 0 \\ 0 & 0 & \varepsilon_i(P^{33}) \end{bmatrix}, \quad \varepsilon_i(P)^{-1} = \begin{bmatrix} R_{1i} & R_{2i} & 0 \\ R_{2i}' & R_{3i} & 0 \\ 0 & 0 & R_{6i} \end{bmatrix}. \quad (25)$$

We define the following matrices α and δ as

$$\alpha = \begin{bmatrix} I & I & 0 \\ V_i'Y_i^{-1} & 0 & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \delta = \begin{bmatrix} R_{1i} & X_{pi} & 0 \\ 0 & U_{pi}' & 0 \\ 0 & 0 & \varepsilon_i(P^{33}) \end{bmatrix}. \quad (26)$$

Considering $U_i = Z_i - X_i$ in (24), we get from (24), (26) that $V_i = V_i'$ and $V_i = Z_i^{-1}$. Moreover, we have that $R_{1l}^{-1} = \varepsilon_i(Z)$, and so we have that

$$\alpha' P_i \alpha = \begin{bmatrix} Y_i^{-1} & Y_i^{-1} & 0 \\ Y_i^{-1} & X_i & 0 \\ 0 & 0 & P_i^{33} \end{bmatrix},$$

$$\delta' \tilde{A}_i \alpha =$$

$$\begin{bmatrix} \varepsilon_i(Z)A_i & \varepsilon_i(Z)A_i & 0 \\ \varepsilon_i(X)A_i + \varepsilon_i(U)'B_{ni}C_i + \varepsilon_i(U)'A_{ni}V_i'Z_i & \varepsilon_i(X)A_i + \varepsilon_i(U)'B_{ni}C_i & 0 \\ 0 & 0 & \varepsilon_i(P^{33})A_{wf} \end{bmatrix},$$

$$\delta' \tilde{B}_i =$$

$$\begin{bmatrix} \varepsilon_i(Z)A_i & \varepsilon_i(Z)A_i & 0 \\ \varepsilon_i(X)A_i + \varepsilon_i(U)'B_{ni}C_i + \varepsilon_i(U)'A_{ni}V_i'Z_i & \varepsilon_i(X)A_i + \varepsilon_i(U)'B_{ni}C_i & 0 \\ 0 & 0 & \varepsilon_i(P^{33})A_{wf} \end{bmatrix},$$

$$\delta' \varepsilon_i(P)^{-1} \delta = \begin{bmatrix} \varepsilon_i(Z) & \varepsilon_i(Z) & 0 \\ \varepsilon_i(Z) & \varepsilon_i(X) & 0 \\ 0 & 0 & \varepsilon_i(P^{33}) \end{bmatrix},$$

$$\tilde{C}_i \alpha = \begin{bmatrix} D_{ni}C_i + C_{ni}V_i'Z_i & D_{ni}C_i & C_{wf} \end{bmatrix}.$$

Applying the change of variables $P_i^{33} = W_i$, $\varepsilon_i(P^{33}) = \varepsilon_i(W)$, $\varepsilon_i(U)'B_{ni} = \Delta_i$, $\varepsilon_i(U)'A_{ni}V_i'Z_i = O_i$, $\varepsilon_i(U)'M_i = H_i$, $C_{ni}V_i'Z_i = F_i$ and also substituting $\varepsilon_i(Z) = R_{1l}^{-1}$ in (19), allow us to get the following inequality

$$\begin{bmatrix} \alpha' \tilde{P}_i \alpha & \bullet & \bullet & \bullet \\ 0 & \gamma I & \bullet & \bullet \\ \delta' \tilde{A}_i \alpha & \delta' \tilde{B}_i & \delta' \varepsilon_i(P)^{-1} \delta & \bullet \\ \tilde{C}_i \alpha & \tilde{D}_i & 0 & I \end{bmatrix} > 0, \qquad (27)$$

and the inequality (27) is equivalent to the inequality (19). Multiplying to the right by $diag[\alpha^{-1}, I, \delta^{-1}, I]$ and to the left by its transpose, we get the inequality (23) and with that we can guarantee that $\|\mathcal{G}\|_\infty < \gamma$.

References

[1] Jamal N Al-Karaki and Ahmed E Kamal. Routing techniques in wireless sensor networks: a survey. *IEEE wireless communications*, 11(6):6–28, 2004.

[2] Leopoldo Angrisani, Matteo Bertocco, Daniele Fortin, and Alessandro Sona. Experimental study of coexistence issues between ieee 802.11 b and ieee 802.15. 4 wireless networks. *IEEE Transactions on Instrumentation and Measurement*, 57(8):1514–1523, 2008.

[3] Jie Chen and J Ron Patton. Standard h filtering formulation of robust fault detection. *IFAC Proceedings Volumes*, 33(11):261–266, 2000.

[4] Oswaldo LV Costa and Marcelo D Fragoso. Stability results for discrete-time linear systems with markovian jumping parameters. *Journal of mathematical analysis and applications*, 179(1):154–178, 1993.

[5] Christian Favre. Fly-by-wire for commercial aircraft: the airbus experience. *International Journal of Control*, 59(1):139–157, 1994.

[6] Marcelo D Fragoso and Oswaldo LV Costa. A unified approach for stochastic and mean square stability of continuous-time linear systems with markovian jumping parameters and additive disturbances. *SIAM Journal on Control and Optimization*, 44(4):1165–1191, 2005.

[7] Janos Gertler. *Fault Detection and Diagnosis*. Springer, 2015.

[8] Alim PC Gonçalves, André R Fioravanti, and José C Geromel. Filtering of discrete-time markov jump linear systems with uncertain transition probabilities. *International Journal of Robust and Nonlinear Control*, 21(6):613–624, 2011.

[9] Alim Pedro de Castro Gonçalves. *Controle dinâmico de saída para sistemas discretos com saltos Markovianos (in portuguese)*. PhD dissertation, Universidade de Campinas-UNICAMP, 2009.

[10] João P Hespanha, Payam Naghshtabrizi, and Yonggang Xu. A survey of recent results in networked control systems. *Proceedings of the IEEE*, 95(1):138–162, 2007.

[11] Inseok Hwang, Sungwan Kim, Youdan Kim, and Chze Eng Seah. A survey of fault detection, isolation, and reconfiguration methods. *IEEE Transactions on Control Systems Technology*, 18(3):636–653, 2010.

[12] Rolf Isermann, Ralf Schwarz, and Stefan Stolzl. Fault-tolerant drive-by-wire systems. *IEEE Control Systems*, 22(5):64–81, 2002.

[13] Keehoon Kim and Eric B Bartlett. Nuclear power plant fault diagnosis using neural networks with error estimation by series association. *IEEE Transactions on Nuclear Science*, 43(4):2373–2388, 1996.

[14] Knut Ovsthus, Lars M Kristensen, et al. An industrial perspective on wireless sensor networks—a survey of requirements, protocols, and challenges. *IEEE Communications Surveys & Tutorials*, 16(3):1391–1412, 2014.

[15] Carvalho L. d. P. Gonçalves C Palma, J. M. and Cristian Duran-Faundez. Minimizing the number of transmissions in a multi-hop network for the dynamical system filtering problem and the impact on the mean square error.

[16] Pete Seiler and Raja Sengupta. A bounded real lemma for jump systems. *IEEE Transactions on Automatic Control*, 48(9):1651–1654, 2003.

[17] Venkat Venkatasubramanian, Raghunathan Rengaswamy, Kewen Yin, and Surya N Kavuri. A review of process fault detection and diagnosis: Part i: Quantitative model-based methods. *Computers & chemical engineering*, 27(3):293–311, 2003.

[18] G. Wang and C. Yin. Fault detection for Markovian jump systems with partially available and unmatched modes. *IEEE Access*, 5:14943–14951, 2017.

[19] Hong Shen Wang and Pao-Chi Chang. On verifying the first-order markovian assumption for a rayleigh fading channel model. *IEEE Transactions on Vehicular Technology*, 45(2):353–357, 1996.

[20] Maiying Zhong, Steven X Ding, James Lam, and Haibo Wang. An lmi approach to design robust fault detection filter for uncertain lti systems. *Automatica*, 39(3):543–550, 2003.

[21] Maiying Zhong, H Ye, Peng Shi, and G Wang. Fault detection for markovian jump systems. *IEE Proceedings-Control Theory and Applications*, 152(4):397–402, 2005.