2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Efficient Optimal Control of Plug-in-Hybrid Electric Vehicles including explicit Engine on/off Decisions

Roland Schmid[1], Johannes Bürger[1] and Naim Bajcinca[2]

Abstract— **This paper presents an optimal control strategy of a parallel Plug-in-Hybrid Electric Vehicle which minimizes the vehicle's fuel consumption including optimal engine on/off decisions. For that purpose a consistent approach based on Pontryagin's Minimum Principle is applied by considering future driving cycle information. The paper discusses an iterative method which involves two steps: First, an analytical co-state for a given engine on/off strategy is calculated. Second, an update of the engine on/off decision points based on the calculated co-state is performed by comparing the associated Hamiltonians for the corresponding engine on/off modes. Through the compact analytical nature, the developed approach is computationally efficient and facilitates the real-time implementation on control units employed in series-production parallel Plug-in-Hybrid Electric Vehicles.**

I. Introduction

The recent high demand of ecofriendly and efficient means of transportation has rapidly increased the popularity of Plug-in-Hybrid Electric Vehicles (PHEV). This can be explained by their capability of both reducing emissions and/or fuel consumption. However, series-production PHEVs are often operated suboptimally because usually no generic approach is applied which exploits the degrees of freedom in a hybrid powertrain optimally with respect to a cost function (engine load point shift or pure electric drive). To address this problem, optimal control strategies can be applied which consult future trip information in order to optimize a specific cost function (fuel consumption) explicitly over the degrees of freedom. Provided that the entire future driving cycle is known, a global, optimal solution can be found by using Dynamic Programming (DP) approaches as presented in [1], [2] and [3]. However, DP is inadequate for series-production automotive applications, because extensive computational demands are required. As an alternative approach, Pontryagin's Minimum Principle (PMP) can be applied to find the optimal control strategy as shown in [3], [4], [5] and [6]. For that purpose, the optimal co-state has to be calculated which depends on the future driving cycle. This can be done by using the shooting method as proposed in [7]. However, this method has the disadvantage that it is based on an iterative calculation method for calculating the optimal co-state and hence is computational expensive.

For further improvement of the vehicle's energy management, an explicit engine on/off control can be applied to avoid inefficient engine operations. However the integration of discrete decision variables results in a mixed integer optimization problem which is computationally very intense and scales exponentially with horizon length. For that reason, in [8] a sequential optimization method is proposed which first calculates the optimal engine on/off strategy based on Pontryagin's Minimum Principle and subsequently optimizes the vehicle's power split by solving a convex optimization problem.

The goal of this paper is the development of an optimal control strategy of a parallel Plug-in-Hybrid Electric Vehicle which minimizes the vehicle's fuel consumption by considering engine on/off decisions explicitly and by considering future driving cycle information. For that purpose an optimal control law is derived by applying Pontryagin's Minimum Principle in two aspects: First an analytical calculation of the optimal co-state is applied, which relies on the knowledge of the future driving cycle (resulting in specified battery energy demands) and satisfies the necessary boundary conditions. Second, an analytical, PMP-based method is integrated into the developed control law, which determines the optimal engine on/off decision.

The developed approach has the advantage relative to [8] that it provides a consistent analysis using PMP. Furthermore the developed approach provides a more efficient method, due to the fact that the update equations are based on analytical PMP. Moreover the presented approach is easier to integrate on series-production vehicles.

This paper is organized as follows. In section II the considered type of parallel Plug-in-Hybrid Electric Vehicles is described and an appropriate modelling approach is presented. Subsequently section III presents the considered optimization method which is applied to minimize the vehicle's fuel consumption. Section IV extends the method of section III by including an explicit engine on/off control strategy. Section V presents possible real time implementations of the developed methods for series-production Plug-in-Hybrid Electric Vehicles and gives an outlook of further research directions. Finally section VI provides a conclusion.

II. Parallel Plug-in-Hybrid Electric Vehicles

Parallel Plug-in-Hybrid Electric Vehicles possess a parallel hybrid propulsion system which consists of a combination of an electrical engine and a combustion engine as depicted in Fig. 1. The characteristic feature of this system structure is the mechanical coupling of the electrical engine and combustion

[1]Roland Schmid and Johannes Bürger are with BMW AG, Petuelring 130, 80788 München, Germany `roland.rs.schmid@bmw.de`, `johannes.buerger@bmw.de`

[2]Prof. Dr. Naim Bajcinca is head of the Department of Mechatronics in Mechanical and Automotive Engineering, Technical University of Kaiserslautern, Postfach 3049, 67663 Kaiserslatuern, Germany `naim.bajcinca@mv.uni-kl.de`

engine by a clutch. This allows a combined operation of engine and electric motor or pure electric propulsion. Consequently it is possible to operate the vehicle by using a mixture of electrical and chemical energy (obtained by the combustion of fuel) or by electrical energy only (thereby the engine can be decoupled and switched off). Furthermore it is possible to charge the battery during deceleration (recuperation of energy). Furthermore the load point of the combustion engine can be shifted and the electric motor is used for charging or assisting the engine.

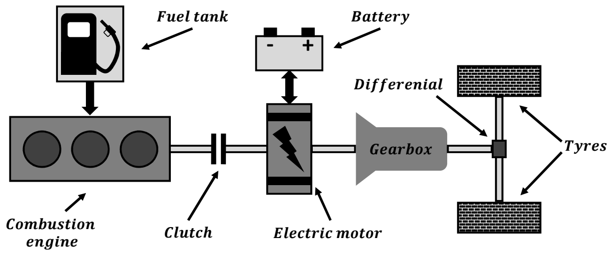

Fig. 1. System structure of parallel Plug-in-Hybrid Electric Vehicles

A. System Modelling

For the utilization of an optimization-based approach of the considered Plug-in-Hybrid Electric Vehicle's energy management, the system's behaviour has to be described by a suitable model. Therefore we utilize a similar model description as presented in [5]. For that purpose the model structure in Fig. 2 is considered, which describes the vehicle's power distribution of the requested power P_{req} into the provided mechanical powers of the internal combustion engine P_c and the electric motor P_e:

$$P_{req} = P_c + P_e. \tag{1}$$

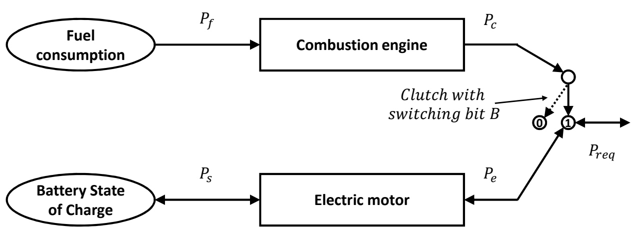

Fig. 2. Modelling of the vehicle's power distribution

It is assumed that the effects of mechanical losses in the gearbox are already considered in P_{req}. Further it is assumed that the gear is selected by separate powertrain functions. Therefore we can consider a specific rotational speed n corresponding to a vehicle velocity. The fuel consumption of the combustion engine can be approximated as a linear function (Willans line approximation - compare [5] page 21) depending on P_c (with coefficients depending the engine speed (n)) and a switching bit $B \in \{0, 1\}$ which represents the current operating state of the engine (clutched and switched on or decluched and switched off) as

$$P_f = B * [a_0(n) + a_1(n)P_c] \tag{2}$$

with

$$B = \begin{cases} 1 & \text{engine is clutched and switched on} \\ 0 & \text{engine is declutched and switched off.} \end{cases} \tag{3}$$

Here P_f represents the required power gained from fuel which is necessary to provide P_c (here (2) represents a linear function with a positive slope and hence $a_1(n) > 0$). Accordingly the vehicle's fuel consumption can be derived by considering the relationship $P_f = \dot{m}h$ ($\dot{m}$ = fuel mass flow, h = heating value of the fuel). Furthermore the mechanical losses of the electric engine are approximated as a quadratic function depending on P_s and P_e (with coefficients depending the engine speed (n))

$$P_e = c_0(n) + c_1(n)P_s + c_2(n)P_s^2 \tag{4}$$

$$P_s = d_0(n) + d_1(n)P_e + d_2(n)P_e^2 \tag{5}$$

where P_s describes the electrical power which has to be provided by the battery in order to provide the demanded mechanical power P_e (Here (4) represents a concave function with $c_2(n) < 0$). The rate of change of electric energy of the battery is given by the required battery power, by definition as:

$$\dot{E} = -P_s(t) \tag{6}$$

$$E(0) = E_{start} \tag{7}$$

$$E(T) = E_{end}. \tag{8}$$

In this paper, the system's current state is represented by the battery's energy $x(t) = E(t)$ and the system's input $u(t) = [P_s(t), B(t)]^T$ is represented by P_s and the switching bit B.

III. PMP-BASED OPTIMIZATION

To find the optimal control law for the energy management of the considered Plug-in-Hybrid Electric Vehicle, which minimizes the vehicle's fuel consumption, the cost function

$$J = \int_0^T L(u(t), t)\, dt = \int_0^T P_f(P_s, B(t), t)\, dt \tag{9}$$

has to be minimized subject to (1) and (6)-(8). For that purpose, Pontryagin's Minimum Principle can be applied.

A. Pontryagin's Minimum Principle

According to Pontryagin's Minimum Principle [9], a candidate for an optimal control input u_{opt} for minimization of (9) is found, if u_{opt} minimizes the Hamiltonian

$$H(u(t), t) = L(u(t), t) + \lambda(t) f(u(t), t) \tag{10}$$

including the co-state λ, while the following conditions are satisfied:

$$\dot{x} = f(u(t), t) = -P_s(t) \tag{11}$$

$$x(0) = x_{start} \tag{12}$$

$$x(T) = x_{end} \tag{13}$$

$$\dot{\lambda} = -\frac{\partial H(u(t), t)}{\partial x}. \tag{14}$$

Given the system modelling equations of subsection II-A we express the Hamiltonian as

$$H(u(t), t) = P_f(t) + s(t)P_s(t) \tag{15}$$

by substituting $-\lambda(t) = s(t)$, to obtain positive co-states and allow the typical interpretation of the co-state as equivalence factor (compare [8]). The Hamiltonian is time-dependent, because at different time steps, different engine speeds are present.

According to Pontryagin's Minimum Principle, an optimal co-state trajectory has to be determined which satisfies the given boundary conditions ((11)-(14)) for the considered driving cycle. We present an analytical method which can be applied in order to calculate the optimal co-state value for a known driving cycle. In this context it is assumed that a forecast of the considered driving cycle is known a priori (for example through data gained from an advanced navigation system). For the analytical derivation of the optimal co-state such that conditions (10)-(14) are satisfied, the optimal co-state s is constant (compare [4]). This is due to the special system modelling approach which results in a Hamiltonian $H(u(t), t)$, which does not rely on the system's state variable $x(t)$. Regarding (14) and (15) this results in a constant value of s:

$$\dot{s} = -\frac{\partial H(u(t), t)}{\partial x} = 0 \;\rightarrow\; s \overset{!}{=} \text{const.} \tag{16}$$

B. Analytical Co-State Calculation

We first consider the case, that the combustion engine is running and clutched during the complete driving cycle ($B(t) = 1 \; \forall t$) and hence the system's corresponding degree of freedom is fixed. The optimal co-state s can then be calculated analytically by considering the known information of the future driving cycle. For that purpose, a discrete description of the system dynamics is considered by approximating

$$\dot{x}(t) \approx \frac{x[k+1] - x[k]}{\Delta T} = -P_s[k] \tag{17}$$

This results in a discrete time representation of (11)-(13) as

$$x[k+1] = x[k] - P_s[k]\Delta T, \text{ for } k = 1, \ldots, N-1 \tag{18}$$

$$x[0] = x_{start} \tag{19}$$

$$x[N] = x_{end} \tag{20}$$

and consequently the system dynamics in each time step k (of length ΔT) can be expressed as

$$x[k+1] = \begin{cases} x[k] + \eta_{recu} P_{req}[k]\Delta T & \text{if } P_{req}[k] \leq 0 \\ x[k] - P_s[k]\Delta T & \text{if } P_{req}[k] > 0 \end{cases} \tag{21}$$

In this context a fixed rate η_{recu} of recuperated energy is assumed for the cases of negative power requests. Using the expression (21), the system's behaviour over the complete driving cycle can be added up to obtain

$$x[N] - x[0] = -\sum_{i \in \Theta} P_s[i]\Delta T + \sum_{j \in \Phi} \eta_{recu} P_{req}[j]\Delta T \tag{22}$$

where Θ is representing the subset of time steps of the driving cycle, where a positive power is requested and Φ is representing the subset of time steps of the driving cycle, where a negative power is requested. The value of P_s which optimizes (9) explicitly can be obtained in each time step $k \in \Theta$. For that purpose, it is necessary to minimize the Hamiltonian

$$\begin{aligned} &H[k] = P_f[k] + sP_s[k] = \\ &a_0(n) + a_1(n)P_c[k] + sP_s[k] = \\ &a_0(n) + a_1(n)(P_{req}[k] - P_e[k]) + sP_s[k] = \\ &a_0(n) + a_1(n)(P_{req}[k] - c_0(n) - c_1(n)P_s[k] - \\ &c_2(n)P_s[k]^2) + sP_s[k] \end{aligned} \tag{23}$$

for each $k \in \Theta$, which depends on the currently present value of the engine speed n (for simplicity, in the following document, n represents the current engine speed at the considered time step k). Since $a_1(n) > 0$ and $c_2(n) < 0$, $H(k)$ is convex and quadratic. Therefore a unique minimum exists and can be found by setting the gradient with respect to P_s to zero:

$$\frac{\partial}{\partial P_s} H = -a_1(n)c_1(n) - 2a_1(n)c_2(n)P_s[k] + s \overset{!}{=} 0. \tag{24}$$

This results in an optimal value of P_s which can be calculated for each time step as

$$P_{s,opt}[k] = \frac{s - a_1(n)c_1(n)}{2a_1(n)c_2(n)} \tag{25}$$

depending on the optimal co-state and the engine speed present at time step k. Finally, the optimal co-state can be derived by inserting (25) into (22) and solving for s, which results in

$$\begin{aligned} s_{opt} = &\frac{\sum\limits_{i \in \Theta} \left[\frac{a_1[i]c_1[i]}{2a_1[i]c_2[i]}\Delta T\right] - (x[N] - x[0])}{\sum\limits_{i \in \Theta} \left[\frac{1}{2a_1[i]c_2[i]}\right]} \\ &+ \frac{\sum\limits_{j \in \Phi} [\eta_{recu} P_{req}[j]\Delta T]}{\sum\limits_{i \in \Theta} \left[\frac{1}{2a_1[i]c_2[i]}\right]}. \end{aligned} \tag{26}$$

Expression (26) allows a computationally efficient determination of the optimal co-state, since it is a closed-form analytical expression and involves simple arithmetic operations. Therefore it is suitable for real-time applications.

C. Simulation Results

After the analytical derivation of the optimal co-state s_{opt}, an optimization of the energy management of the considered vehicle can be achieved under the assumption that the future driving cycle is known. In that context, for the development process and the evaluation process we utilized simulated data of a standard WLTP (Worldwide Harmonized Light Duty Test Procedure - compare [6]) driving cycle for a typical series-production Plug-in-Hybrid Electric Vehicle which is conducted in a charge sustaining mode. Fig. 3 shows the resulting optimal trajectory of the vehicle's battery state of charge SOC ($SOC(t) = \frac{E(t)}{E_{max}} = \frac{x(t)}{x_{max}}[\%]$) by performing a forward simulation by using the optimal constant co-state using (26). The calculation of (26) is either repeated at each time step for a shrinking horizon or under the assumption that the driving cycle is perfectly known can be determined only once.

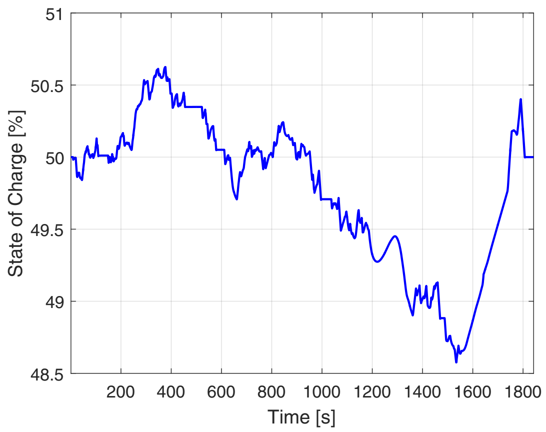

Fig. 3. Optimal state of charge trajectory minimizing the vehicle's fuel consumption

IV. Engine On/Off Decision

The results of section III result in an optimal control for the power split ratio of the hybrid powertrain. However, an explicit engine on/off decision can be included for further reduction of the vehicle's fuel consumption. To this end, an analytical method based on Pontryagin's Minimum Principle is presented in this section, which is inspired by the approach presented in [8]. In this context we utilize the previously presented Hamiltonian shown in (15) and apply the function to both cases of a running engine or a engine which is switched off. Consequently the Hamiltonian can be represented as

$$\begin{aligned} H_{on/off}[k] &= P_f[k] + sP_s[k] = \\ & B[k](a_0(n) + a_1(n)P_c[k]) + s_{opt}P_{s,opt}[k] \end{aligned} \tag{27}$$

with

$$B[k] = \begin{cases} 1 & \text{if engine is on} \\ 0 & \text{if engine is off} \end{cases} \tag{28}$$

Since the Hamiltonian $H_{on/off}$ is independent of the system state, this implies that the optimal co-state is constant similar to (16), but different to (26), in general. The engines state (engine on/ engine off) which results in the smallest Hamiltonian (for an optimized power split ratio when the engine is on) is selected. Therefore, the Hamiltonians for the cases if the engine is switched on (H_{on}, $B[k] = 1$) or off ($H_{off}[k]$, $B[k] = 0$) are compared. Consequently we apply the following control law:

$$\text{Engine State}[k] = \begin{cases} \text{ON} & \text{if } H_{on}[k] < H_{off}[k] \\ \text{OFF} & \text{if } H_{on}[k] \geq H_{off}[k] \end{cases} \tag{29}$$

with the Hamiltonians represented by

$$\begin{aligned} & H_{on}[k] = P_f[k] + s_{opt}P_{s,opt}[k] = \\ & a_0(n) + a_1(n)P_c[k] + s_{opt}P_{s,opt}[k] = \\ & a_0(n) + a_1(n)(P_{req}[k] - P_e[k]) + s_{opt}P_{s,opt}[k] = \\ & a_0(n) + a_1(n)(P_{req}[k] - c_0(n) - c_1(n)P_{s,opt}[k] \\ & - c_2(n)P_{s,opt}[k]^2) + s_{opt}P_{s,opt}[k] \end{aligned} \tag{30}$$

and

$$\begin{aligned} & H_{off}[k] = s_{opt}P_{s,off}[k] = \\ & s_{opt}(d_0(n) + d_1(n)P_{req}[k] + d_2(n)P_{req}[k]^2). \end{aligned} \tag{31}$$

Here, (30) shows the Hamiltonian for the case of a running combustion engine what can be represented by the Hamiltonian in (23) in combination with the optimal values of s and P_s which can be calculated by applying (26) and (25). Furthermore (31) represents the Hamiltonian for the case of a switched off combustion engine, where P_f can be dropped out and the total amount of requested power has to be provided by the electrical motor what necessitates the calculation of the corresponding power $P_{s,off}$ through (5) due to losses in the electrical system. Regarding these relations, the optimal engine on/off decision can be achieved by an iterative procedure which is depicted in Fig. 4:

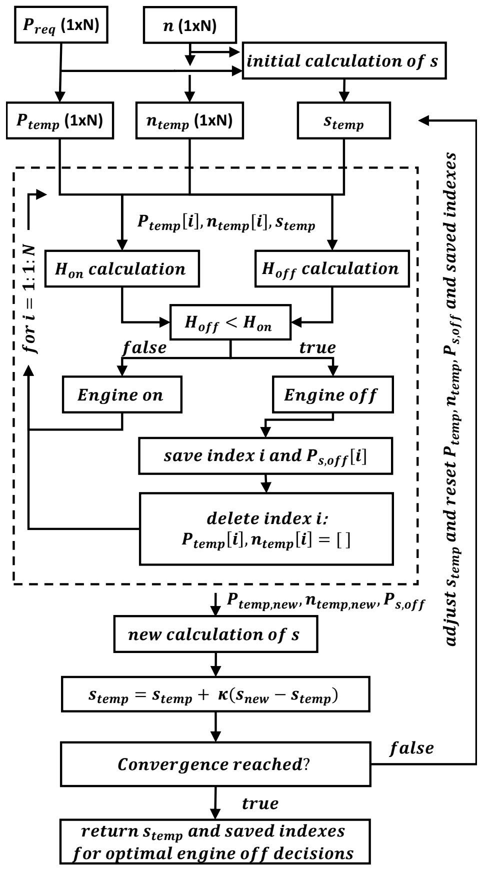

Fig. 4. Engine on/off decision - Algorithm

The main idea of the presented algorithm is to derive an explicit engine on/off decision for each time step of the considered future driving cycle. For this purpose, an initial co-state s_{temp} must be calculated for the considered driving cycle in the case of $B(t) = 1 \ \forall t$ according to the analytical method in section III. By applying this optimal co-state and the corresponding optimal value of $P_{s,opt}$ in each time step, the corresponding Hamiltonian functions H_{on} and H_{off} can be compared in each time step, in order to derive an engine on/off decision. If an engine off decision is made, the actual time step is saved as an indicator for the engine off decision in the control during the driving cycle. Furthermore the amount of electrical power which would be necessary to facilitate pure electric driving has to be determined as

$$P_{s,off}[k] = d_0(n) + d_1(n)P_{req}[k] + d_2(n)P_{req}[k]^2 \quad (32)$$

and saved for later calculations. After the execution of an engine on/off decision in every time step of the driving cycle, a re-optimization of the power split ratio of the remaining time steps is necessary, in which the engine remains switched on. For this purpose, the optimal co-state s_{new} is recalculated for the remaining time steps using the method of section III under the consideration of the additional energy which is consumed during pure electrical driving. For this purpose, the additionally required energy has to be incorporated into the previously presented method by adapting (26) with an extra term as:

$$\begin{aligned} s_{opt} = & \frac{\sum\limits_{i\in\Theta}\left[\frac{a_1[i]c_1[i]}{2a_1[i]c_2[i]}\Delta T\right] - (x[N]-x[0])}{\sum\limits_{i\in\Theta}\left[\frac{1}{2a_1[i]c_2[i]}\right]} \\ & + \frac{\sum\limits_{j\in\Phi}[\eta_{recu}P_{req}[j]\Delta T] - \sum\limits_{l\in\Psi}[P_{s,off}[l]\Delta T]}{\sum\limits_{l\in\Theta}\left[\frac{1}{2a_1[i]c_2[i]}\right]} \end{aligned} \quad (33)$$

where Ψ represents the set of time steps where pure electric driving is applied. Subsequently the engine on/off decision is executed once again for the complete driving cycle (including the already found time steps where the engine shall be switched off). For this purpose, similar to [8], the co-state is updated according to

$$s_{temp} = s_{temp} + \kappa(s_{new} - s_{temp}) \quad (34)$$

and the whole procedure is repeated until the co-state converges. If the optimal constant co-state is found, it necessarily represents a solution to the considered PMP problem including engine on/off decisions.

A. Simulation Results

We can now apply the algorithm including engine on/off decisions in a simulation for a WLTP driving cycle. In Fig. 5 the optimal trajectories of the vehicle's battery SOC are depicted for the two cases of a PMP-based optimization of only the power split (as presented in section III) and a PMP-based optimization including an explicit engine on/off decision. In this context, the time steps where an explicit engine off decision is made are especially highlighted.

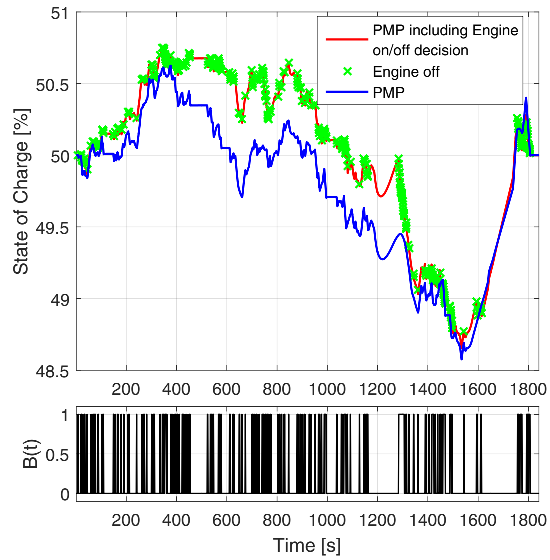

Fig. 5. Comparison of the optimal state of charge trajectories minimizing the vehicle's fuel consumption

After a detailed analysis of the resulting energy management strategies it can be noticed that the resulting SOC-trajectories vary from each other. This results from the fact, that generally points below a certain power threshold are used for pure electrical driving and this threshold is dependent on the engine speed and co-state (compare [8]). Consequently the algorithms in Table I result in different fuel consumptions of the vehicle and it can be shown that the application of a PMP-based optimization for energy management including an explicit engine on/off decision achieves a further reduction of 3.63% of the vehicle's fuel consumption as expected. However, even if the vehicle's fuel consumption can be reduced by an explicit engine on/off decision, this results in increased computation time. However the computational demand is low, given that the entire driving cycle is considered(see Table I). The calculations were executed in Matlab R2015b on a standard office laptop including a Intel(R) Core (TM) i7-6600U CPU.

TABLE I
COMPUTATION TIME

Algorithm	Computation time [s]
PMP without Engine on/off decision	0.2730
PMP incl. Engine on/off decision	1.7260

V. IMPLEMENTABILITY AND FURTHER EXTENSIONS

This section addresses the implementability of the considered algorithms on real control units employed in series-production parallel Plug-in-Hybrid Electric Vehicles. Therefore a forecasting algorithm estimates the vehicle's future driving information such as the vehicle's requested power and engine speed based on information gained from an advanced state-of-the-art navigation system. Furthermore, as in [10], we propose to compute an outer loop solution first, which is repeated at certain fixed time spans. This overcomes erroneous forecast data (e.g. resulting from traffic jams or changing routes) or initially unpredictable disturbances caused by the vehicle's driver (e.g. unexpected driving styles). This outer loop generates a state of charge trajectory ($E_{opt,outer}$) which is tracked by an inner loop controller using the same algorithm as shown in Fig. 6 which will be repeatedly executed for a reduced optimization horizon. Thereby the computational demands can be reduced for the iterative optimization routine and a real-time-implementability of the presented algorithm can be achieved.

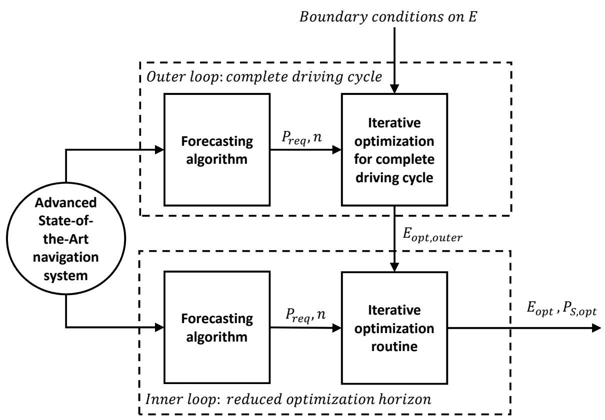

Fig. 6. Proposed real-time optimization strategy

Furthermore, the method of section IV can be extended to include fixed given electrical driving strategies resulting from a separate electrical driving decision function. Therefore priorly known fixed engine off time steps are explicitly considered by the algorithm by deciding to switch the engine off and incorporating the required electric energy (similar to (32) and (33)). In addition, the cost of engine starts and a limitation of the switching behaviour of the combustion engine will be included in future work, in order to avoid suboptimalities due to frequent engine on/off switches. Moreover, future work will evaluate the impact of uncertainties of the predicted driving cycle data on the optimality of the solution.

VI. Conclusion

This paper presents efficient optimization methods for minimizing the fuel consumption of parallel Plug-in-Hybrid Electric Vehicles. The developed algorithm is based on Pontryagin's Minimum Principle and considers an explicit engine on/off decision to achieve a further reduction of the vehicle's fuel consumption. In this context the presented algorithm is evaluated on a simulative basis and it is shown that a further reduction of the vehicle's fuel consumption can be achieved. In addition, an outlook is presented which proposes additional extensions in order to achieve a highly flexible real-time implementability on real vehicles.
Future work will focus on the implementation and evaluation of the presented methods on a real vehicle.

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