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MRAC of a 3-DoF Helicopter with Nonlinear Reference Model

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***Abstract*— Model Reference Adaptive Control (MRAC) technique, which is considered to be an effective tool for the control of unknown dynamical systems behavior, is widely used in practical applications. In principal, a known stable linear model dynamics is taken as a reference model such that its response is tracked by the unknown dynamical system by means of an adaptive control scheme. In this paper, rather than using a linear reference model, we propose a nonlinear reference model to be used in the MRAC of nonlinear plant dynamics. First, a stable nonlinear reference model is formed by using State Dependent Riccati Equation (SDRE) approach. Then an adaptation rule is derived to ensure the convergence of the response of nonlinear plant dynamics to that of the nonlinear reference model. The proposed method is tested experimentally using a 3-DoF helicopter test bed with different parameters and working conditions.**

I. INTRODUCTION

State Dependent Riccati Equation (SDRE) approach has received great attention in recent decades and has been utilized in a wide variety of application areas, including aviation, robotics, biology, etc. [1]–[3]. The main reason of this increasing interest is probably due to the fact that this approach provides a great design flexibility and a simple systematic design methodology, incorporating the inherent nonlinearities of the underlying dynamics, in order to design a suboptimal controller for a nonlinear system [4]. SDRE technique is based on the consideration that the nonlinear dynamic equations are evaluated with the state values at each instant time, thereby converting the nonlinear system into a linear time-invariant (LTI) system, which is valid at that of the instant time. This rigorous transformation enables a linear-quadratic regulator (LQR) controller to be designed and updated online to control the time-varying dynamics of the nonlinear system. However, a suboptimal control law can only be obtained using SDRE approach and this inevitably results in that SDRE approach can only guarantee the local stability. It is also clear that the SDRE method requires the full information about the dynamical system, including the system parameters and possible disturbances.

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Model Reference Adaptive Control (MRAC), on the other hand, has been developed as an effective mean to control uncertain systems by means of a suitable controller designed for a reference model. In the MRAC method, the response of the unknown plant is forced by the adaptive controller to track the output of the reference system. Thus, the controller gains should be updated recursively to handle the circumstances arising from the time-varying parameters and/or the uncertainties. In most of the previous studies (see [5]–[7] and the references therein), an LTI model has been used as the reference model. The MRAC techniques have also been extended to discrete time systems [8], [9]. In addition, the adaptive control architecture has been studied to design MRAC for various classes of nonlinear systems [10], [11].

Recently, the feasibility of establishing a promising cooperation between MRAC and SDRE for controlling nonlinear systems has been explored [12]–[14]. The aim of this endeavor is to prevent possible deterioration in the adaptation of the response of the uncertain plant to the response of the LTI reference model exhibiting slower dynamics than the plant. The drawback may be overcome using a nonlinear reference model rather than an LTI reference model.

The proposed method in this paper provides an adaptive control algorithm to be used for a class of uncertain nonlinear systems, together with a systematic structure including both the MRAC and SDRE control approaches. First, the SDRE method is designed in order to obtain a stabilizing controller for the nonlinear reference model. Then the control input is manipulated using a recursive adaptation procedure to control the nonlinear dynamics of the real plant. To test the proposed SDRE based MRAC control approach experimentally, a validated test platform, known as 3-DoF helicopter [15], [16], is used in this study. The proposed algorithm is implemented in different working frequencies and the parameter uncertainty is introduced by attaching the counterweight of the helicopter to different locations. These different cases are analyzed to investigate the effect of the working frequency and parameter uncertainty on the performance and robustness of the proposed controller. The results reveal that higher working frequency results in lower control efforts and the proposed method manifest high robustness against parameter uncertainty.

Section II introduces the MRAC design methodology proposed in this study. The proposed method is presented with a new adaptation rule where the MRAC is designed for uncertain nonlinear plants with stable nonlinear reference model. In Section III, 3-DoF helicopter model is introduced and the proposed control method is applied to the 3-DoF

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helicopter. Section IV gives the experimental results. Conclusions are given in Section V.

II. MRAC DESIGN WITH SDRE FOR NONLINEAR SYSTEMS

The MRAC methods with stable linear reference dynamics (so-called reference model) are studied thoroughly (see [5]–[11] and the references therein) with theoretical backgrounds. Necessary adaptation rules are derived so that the response of an unknown plant dynamics (linear or nonlinear) is controlled via the MRAC. It is stated that the MRAC with a stable linear reference model could be achieved by means of proper adaptation gain(s). The section extends the MRAC philosophy to unknown nonlinear plant dynamics with nonlinear reference model, and introduces a new adaptation to design the MRAC control for the unknown plant dynamics.

Consider the following types of nonlinear reference model dynamics

$$\dot{x}_m = A_m(x_m)x_m + B_m(x_m)u_m \tag{1}$$

where the nonlinear system matrices are $A_m(x_m) \in \mathbb{R}^{n\times n}$, $B_m(x_m) \in \mathbb{R}^{n\times m}$ and the controller input, u_m is designed so that the nonlinear reference model is stable. Within this context, the control input may be designed by using the SDRE techniques. For instance, consider the following types of local stabilizing controller,

$$u_m = -K_m(x_m)x_m \tag{2}$$

where the state feedback gain matrix, $K_m(x_m)$ is determined from the following equation,

$$K_m(x_m) = R^{-1}(x_m)B_m^\top(x_m)P(x_m) \tag{3}$$

The state dependent feedback-gain matrix, $K_m(x_m)$: $\mathbb{R}^n \to \mathbb{R}^{m\times n}$ is designed at each instant of time to satisfy local stability while minimizing the following quadratic/non-quadratic cost function;

$$J=\frac{1}{2}\int_0^\infty \left\{x_m^\top(t)Q(x_m)x_m(t)+u_m^\top(t)R(x_m)u_m(t)\right\}dt. \tag{4}$$

$P(x_m)$ is the symmetric, positive-definite solution for the following algebraic SDRE

$$\begin{aligned} &Q(x_m) + P(x_m)A_m(x_m) + A_m^\top(x_m)P(x_m)\\ &- P(x_m)B_m(x_m)R^{-1}(x_m)B_m^\top(x_m)P(x_m) = 0. \end{aligned} \tag{5}$$

The details of the SDRE methodology are well-structured in [4]. As stated in [4], the $\{A_m(x_m), B_m(x_m)\}$ pair is assumed to be point-wise controllable for all x_m. The SDRE control enables the point-wise stability by ensuring that $A_{m_{cl}}(x_m) = A_m(x_m) - B_m(x_m)K_m(x_m)$, is Hurwitz for all x_m.

Consider now the following nonlinear plant dynamics

$$\dot{x}_p(t) = A_p(x_p)x_p + B_p(x_p)u_p,\ x_p(0) = x_{p_0} \tag{6}$$

where $A_p(x_p)$ and $B_p(x_p)$ are assumed to be unknown state dependent matrices. For a known $A_p(x_p)$ and $B_p(x_p)$ matrix pair, the following full-state feedback control may be considered for the perfect model following,

$$u_p = -K_p^*(x_p, x_m)x_p(t). \tag{7}$$

Assumption 1: *There exists an ideal control gain $K_p^*(x_p, x_m) \in \mathbb{R}^{m\times n}$, satisfying perfect model following between the nonlinear reference model (1) and the nonlinear plant dynamics (6) such that $A_p(x_p)-B_p(x_p)K_p^*(x_p, x_m) = A_{m_{cl}}(x_m)$ where $A_{m_{cl}}(x_m) \in \mathbb{R}^{n\times n}$ is Hurwitz matrix for all x_p,x_m and $t \geq 0$.*

The exact value of the gain $K_p^*(x_p, x_m)$ is not necessarily needed, but the existence of the gain is essential.

Condition 2 (see [12]): *$K_p^*(x_p, x_m)$ is continuously differentiable, and its derivative is uniformly bounded, $\|\dot{K}_p^*(x_p, x_m)\| \leq \delta < \infty$ for all $t \geq 0$.*

Assuming that the pair $(A_p(x_p), B_p(x_p))$ is pointwise controllable for all $t \geq 0$, the existence of $K_p^*(x_p, x_m)$ is satisfied. Since $A_p(x_p)$ and $B_p(x_p)$ matrices are not known exactly, an estimate of $K_p^*(x_p, x_m)$ may be used in the adaptive control for the stabilization of the unknown nonlinear plant,

$$u_p = -K_p(t, x_p, x_m)x_p(t) \tag{8}$$

where $K_p(t, x_p, x_m) \in \mathbb{R}^{m\times n}$ is the time varying and state-dependent estimation of the ideal $K_p^*(x_p, x_m)$ in (7), and is adjusted by a newly designed adaptation law.

It is also assumed that there exists a positive definite matrix $G \in \mathbb{R}^{m\times m}$, such that the estimation of $B_p(x_p)$ is known as $\hat{B}_p(x_p) = B_p(x_p)G$.

In order to determine the adaptive control gain $K_p(t, x_p, x_m)$, assuming an initial estimate K_{p_0} of the unknown adaptive control gain $K_p(t, x_p, x_m)$, consider the following time varying and state dependent adaptation law,

$$\begin{aligned} &\dot{K}_p(t, x_p, x_m) = \hat{B}_p^\top(x_p)e(t)x_p^\top(t)P_{ad}(x_p, x_m)\Gamma,\\ &K_p(t_0, x_{p_0}, x_{m_0}) = K_{p_0} \in \mathbb{R}^{m\times n} \end{aligned} \tag{9}$$

where $\Gamma \in \mathbb{R}^{n\times n}$ is the positive definite adaptation rate to be adjusted for the rate of convergence of $x_p(t)$ to $x_m(t)$. The adaptation rate would affect the transient response and stability of the plant.

The matrix-valued function $P_{ad}(x_p, x_m)$ is a symmetric positive definite matrix, i.e. $P_{ad}(x_p, x_m) = P_{ad}^\top(x_p, x_m) > 0$, which is the solution of the following algebraic Lyapunov equation for the stable closed-loop reference model (1) for some arbitrary matrix-valued functions $Q_{ad}(x_p) = Q_{ad}^\top(x_p) > 0$.

$$A_{m_{cl}}^\top(x_m)P_{ad}(x_p, x_m)+P_{ad}(x_p, x_m)A_{m_{cl}}(x_m) = -Q_{ad}(x_p) \tag{10}$$

The following theorem gives the adaptation rule for the SDRE based MRAC design.

Theorem 3: *With the adaptive controller (8) and the adaptation law (9), applied to the nonlinear plant dynamics (6), the closed-loop signals, $x_p(t)$, $e(t)$ and $u_p(t)$ remain bounded and the plant states $x_p(t)$ approach to reference model states $x_m(t)$ such that the tracking error $e(t) \in \mathcal{L}^2$ and $e(t) \to 0$ as $t \to \infty$.*

Proof: *Define first the following state and control parameter errors*

$$\begin{aligned} e(t) &\triangleq x_p(t) - x_m(t), \\ \tilde{K}(t, x_p, x_m) &\triangleq K_p(t, x_p, x_m) - K_p^*(x_p, x_m) \end{aligned} \tag{11}$$

The closed-loop plant dynamics is then represented by

$$\begin{aligned} \dot{x}_p(t) =& A_p(x_p)x_p(t) - B_p(x_p)(K_p(t, x_p, x_m)x_p(t)) \\ \dot{x}_p(t) =& A_p(x_p)x_p(t) - B_p(x_p)(\tilde{K}(t, x_p, x_m) \\ &+ K_p^*(x_p, x_m))x_p(t) \\ \dot{x}_p(t) =& A_{m_{cl}}(x_m)x_p(t) - B_p(x_p)\tilde{K}(t, x_p, x_m)x_p(t) \end{aligned}$$

By adding and subtracting $A_{m_{cl}}(x_m)x_m(t)$ to the right hand side of the equation, we get

$$\dot{x}_p(t) = A_{m_{cl}}(x_m)e(t) + \dot{x}_m(t) - B_p(x_p)\tilde{K}(t, x_p, x_m)x_p(t)$$

then the error dynamic becomes

$$\begin{aligned} &\dot{e}(t) = A_{m_{cl}}(x_m)e(t) - \hat{B}_p(x_p)G^{-1}\tilde{K}(t, x_p, x_m)x_p(t), \\ &e(0) = e_0 = x_{p_0} - x_{m_0}. \end{aligned} \tag{12}$$

Therefore, there exist two error signals in the error dynamics (12) namely tracking error $e(t)$ and feedback gain estimation error $\tilde{K}(t, x_p, x_m)$. Consider now the following positive definite Lyapunov function candidate for the error dynamics (12)

$$\begin{aligned} V(e(t), \tilde{K}(t, x_p, x_m)) =& e^\top(t)P_{ad}(x_p, x_m)e(t) \\ &+ tr(G^{-1}\tilde{K}(t, x_p, x_m)\Gamma^{-1}\tilde{K}^\top(t, x_p, x_m)) \end{aligned} \tag{13}$$

where $\Gamma \in \mathbb{R}^{n\times n}$ is positive definite adaptation gain and $P_{ad}(x_p, x_m) \in \mathbb{R}^{n\times n}$ is symmetric positive definite matrix and solution of (10). The time derivative of $V(e(t), \tilde{K}(t, x_p, x_m))$ along the trajectories is as follows;

$$\begin{aligned} \dot{V}(e(t), \tilde{K}(.)) =& \dot{e}^\top(t)P_{ad}(x_p, x_m)e(t) \\ &+ e^\top(t)P_{ad}(x_p, x_m)\dot{e}(t) \\ &+ tr(G^{-1}\dot{\tilde{K}}(t, x_p, x_m)\Gamma^{-1}\tilde{K}^\top(t, x_p, x_m) \\ &+ G^{-1}\tilde{K}(t, x_p, x_m)\Gamma^{-1}\dot{\tilde{K}}^\top(t, x_p, x_m)) \end{aligned}$$

$$\begin{aligned} \dot{V}(e(t), \tilde{K}(.)) =& e^\top(t)A_{m_{cl}}^\top(x_m)P_{ad}(x_p, x_m)e(t) \\ &- x_p^\top(t)\tilde{K}^\top(t, x_p, x_m)G^{-1^\top}\hat{B}_p^\top(x_p)P_{ad}(x_p, x_m)e(t) \\ &+ e^\top(t)P_{ad}(x_p, x_m)A_{m_{cl}}(x_m)e(t) \\ &- e^\top(t)P_{ad}(x_p, x_m)\hat{B}_p(x_p)G^{-1}\tilde{K}(t, x_p, x_m)x_p(t) \\ &+ 2tr(G^{-1}\dot{\tilde{K}}(t, x_p, x_m)\Gamma^{-1}\tilde{K}^\top(t, x_p, x_m)) \\ =& e^\top(t)[A_{m_{cl}}^\top(x_m)P_{ad}(x_p, x_m) + P_{ad}(x_p, x_m)A_{m_{cl}}(x_m)]e(t) \\ &- x_p^\top(t)\tilde{K}^\top(t, x_p, x_m)G^{-1^\top}\hat{B}_p^\top(x_p)P_{ad}(x_p, x_m)e(t) \\ &- e^\top(t)P_{ad}(x_p, x_m)\hat{B}_p(x_p)G^{-1}\tilde{K}(t, x_p, x_m)x_p(t) \\ &+ 2tr(G^{-1}\dot{\tilde{K}}(t, x_p, x_m)\Gamma^{-1}\tilde{K}^\top(t, x_p, x_m)) \end{aligned}$$

$$\begin{aligned} \dot{V}(e(t), \tilde{K}(.)) =& - e^\top(t)Q_{ad}(x_p)e(t) \\ &+ 2tr(G^{-1}\dot{\tilde{K}}(t, x_p, x_m)\Gamma^{-1}\tilde{K}^\top(t, x_p, x_m) \\ &- e^\top(t)P_{ad}(x_p, x_m)\hat{B}_p(x_p)G^{-1}\tilde{K}(t, x_p, x_m)x_p(t)) \end{aligned}$$

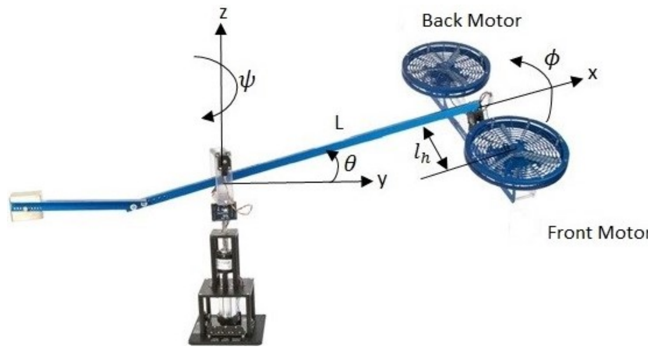

Fig. 1. 3-DoF helicopter test bed [15], [16].

Then by considering the adaptation law (9) and the Lyapunov equation (10), we get

$$\begin{aligned} \dot{V}(e(t), \tilde{K}(.)) =& - e^\top(t)Q_{ad}(x_p)e(t) \\ \leq& -\lambda_{min}(Q_{ad}(x_p))\|e(t)\|_2^2 \leq 0 \end{aligned} \tag{14}$$

where $\lambda_{min}(Q_{ad}(x_p))$ denotes the minimum eigenvalue of the symmetric $Q_{ad}(x_p)$ matrix. Thus, negative semidefinite $\dot{V}(e(t), \tilde{K}(.))$ in (14), yields that the error dynamics (12) is stable and $e(t)$ and $\tilde{K}(t, x_p, x_m)$ are bounded function of time, which conclude the boundedness of $x_p(t)$ and $K_p(t, x_p, x_m)$ that guarantees the boundedness of $\dot{e}(t)$. Furthermore, $\ddot{V}(e(t), \tilde{K}(.)) = -\dot{e}^\top(t)Q_{ad}(x_p)e(t) - e^\top(t)Q_{ad}\dot{e}(t)$ is uniformly bounded function of time, which implies that $\dot{V}(e(t), \tilde{K}(.))$ is uniformly continuous. Then, using Barbalats Lemma one can derive that $\dot{V}(e(t), \tilde{K}(.))$ asymptotically tends to zero as time tends to infinity that implies $\lim_{t\to\infty} e(t) = 0$. ■

The main objective of the proposed method is to adapt the SDRE controller designed for a known nonlinear reference model to an unknown given nonlinear plant dynamics. In the next section, the proposed MRAC is applied to a nonlinear physical system, which is a 3-DoF Helicopter Setup used to test the capabilities of proposed controller.

III. 3-DOF HELICOPTER DYNAMICS AND SDRE BASED MRAC CONTROLLER DESIGN

The 3-DoF helicopter is a commercial setup of Quanser Inc. and has three degrees of the freedom: the elevation angle θ measured around y-axis, the pitch angle ϕ measured around x-axis and the travel angle ψ measured around z-axis, as shown in Fig. 1. Two distinct DC motors are employed to generate the sufficient thrust forces, named as cyclic thrust force τ_{cyc} and collective thrust force τ_{coll}, to rotate the helicopter around the three axes. A simplified nonlinear model of the 3-DoF helicopter is given in [15] by

$$\begin{aligned} \ddot{\theta} &= -d_1\dot{\theta} - d_2\sin(\theta) + d_3\tau_{coll}\cos(\phi) \\ \ddot{\phi} &= -b_1(\dot{\phi}) - b_2\sin(\phi) - b_3\tau_{cyc} \\ \ddot{\psi} &= -a_1\dot{\psi} - a_2(\alpha\tau_{coll} + 1)\sin(\phi) \\ \dot{\tau}_{cyc} &= -c_1\tau_{cyc} + 0.5c_2(V_b - V_f) \\ \dot{\tau}_{coll} &= -e_1\tau_{coll} + 0.5e_2(V_b + V_f) \end{aligned} \tag{15}$$

TABLE I
PARAMETER VALUES OF THE HELICOPTER MODEL

Parameter	a_1	a_2	b_1	b_2	b_3	c_1	c_2
Value	0.2517	0.2105	0.3290	1.5664	16.200	7.3200	1.000
Parameter	d_1	d_2	d_3	e_1	e_2	α	
Value	0.1011	0.5040	1.3400	6.1600	1.000	4.000	

in which the second order differential equations represent the Euler dynamics of the rotations. Since the dynamics of the travel and elevation angles are coupled with the dynamics of the pitch angle, only two command signals are required. Thus the remaining equations of the simplified model (15) are derived to evaluate the thrust forces generated by the front and back motor voltages, denoted as V_f and V_b. The unknown parameters of the nonlinear model (15) may be identified using a well-known parameter identification software, such as CIFER [15]. The parameters of the nonlinear model (15) previously identified by [15] are given in Table I. The equations of motion for the 3-DoF helicopter test bed can be transformed into the following state-space form to create a nonlinear reference model

$$\begin{aligned}
\dot{x}_{m_1} &= x_{m_4}\\
\dot{x}_{m_2} &= x_{m_5}\\
\dot{x}_{m_3} &= x_{m_6}\\
\dot{x}_{m_4} &= -d_1 x_{m_4} - d_2 \sin\left(x_{m_1}\right) + d_3 x_{m_8} \cos\left(x_{m_2}\right)\\
\dot{x}_{m_5} &= -b_1 x_{m_5} - b_2 \sin\left(x_{m_2}\right) - b_3 x_{m_7}\\
\dot{x}_{m_6} &= -a_1 x_{m_6} - a_2\left(\alpha x_{m_8}+1\right)\sin\left(x_{m_2}\right)\\
\dot{x}_{m_7} &= -c_1 x_{m_7} + 0.5 c_2 u_{m_2} - 0.5 c_2 u_{m_1}\\
\dot{x}_{m_8} &= -e_1 x_{m_8} + 0.5 e_2 u_{m_1} + 0.5 e_2 u_{m_2}
\end{aligned} \tag{16}$$

by defining, respectively, the state vector and the control inputs as

$$\begin{bmatrix} x_{m_1} & x_{m_2} & \cdots & x_{m_8}\end{bmatrix}^\top = \begin{bmatrix}\theta & \phi & \psi & \dot\theta & \dot\phi & \dot\psi & \tau_{cyc} & \tau_{coll}\end{bmatrix}^\top$$

and

$$u_m = \begin{bmatrix} u_{m_1} \, u_{m_2}\end{bmatrix}^\top = \begin{bmatrix} V_f & V_b\end{bmatrix}^\top .$$

The state model (16) can be now rewritten in the form of the nonlinear reference model dynamics (1) with the proper SDC matrices given as

$$A_m(x_m)=\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
-d_2 L(x_m) & 0 & 0 & -d_1 & 0 & 0 & 0 & N(x_m)\\
0 & -b_2 M(x_m) & 0 & 0 & -b_1 & 0 & -b_3 & 0\\
0 & -H(x_m)M(x_m) & 0 & 0 & 0 & -a_1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & -c_1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -e_1
\end{bmatrix}$$

and

$$B_m\left(x_m\right) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -0.5c_2 & 0.5c_2\\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5e_2 & 0.5e_2\end{bmatrix}^\top$$

where $M\left(x_m\right) = \sin\left(x_{m_2}\right) x_{m_2}^{-1}$, $L\left(x_m\right) = \sin\left(x_{m_1}\right) x_{m_1}^{-1}$, $N\left(x_m\right) = d_3 \sin\left(x_{m_2}\right)$ and $H\left(x_m\right) = a_2\left(\alpha x_{m_8}+1\right)$.

In order to achieve a tracking objective, the SDC parameterized system is augmented as follows:

$$\hat{A}_m\left(\hat{x}_m\right) = \begin{bmatrix} A_m\left(x_m\right) & 0\\ -C & 0\end{bmatrix} \text{ and } \hat{B}_m\left(\hat{x}_m\right) = \begin{bmatrix} B_m\left(x_m\right)\\ 0\end{bmatrix}$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

which yields the following augmented nonlinear reference model dynamics

$$\dot{\hat{x}}_m = \hat{A}_m(\hat{x}_m)\hat{x}_m + \hat{B}_m(\hat{x}_m)u_m \tag{17}$$

The nonlinear reference model is stabilized through the SDRE control law (2). The unknown 3-DoF helicopter dynamics is controlled by the proposed SDRE based MRAC control algorithm defined by (8) and the control gains are adapted by using the adaptation rule given by (9).

The next section examines the robustness of the proposed the SDRE based MRAC control approach and presents the results of its tracking performance.

IV. EXPERIMENTAL RESULTS

The proposed SDRE based MRAC approach is implemented to the 3-DoF helicopter test bed. The weighting matrices (as defined in the cost function (4)) designed for SDRE and MRAC controllers are selected for the experimental study respectively as follows,

$$\begin{aligned} Q(x_m) = Q &= \text{diag}(40, 5, 5, 1, 10, 15, 1, 1, 4, 2)\\ R(x_m) = R &= \text{diag}(1, 1). \end{aligned}$$

The positive semi-definite matrix in the Lyapunov equation (10) is selected as follows:

$$Q_{ad}(x_p) = Q_{ad} = \text{diag}(100, 0.1, 100, 1, 1, 1, 1, 1, 1, 1).$$

The adaptation rate for the controller design, defined in adaptation law (9), is selected to be $\Gamma = 10^{-6}$.

For tracking task, a reference trajectory comprised of a set of consecutive step inputs is generated. Since only travel and elevation motions of the 3-DoF helicopter are controlled in experiments, separate reference inputs are designed for the rotational motions around both the travel and elevation axes and are applied to the closed-loop system simultaneously. These reference trajectories are indicated by the dashed line shown in Fig. 2 and Fig. 3 which show the elevation and travel responses of 3-DoF helicopter to these desired inputs for a working frequency of $f_w = 50$ Hz.

In addition, to produce different plant dynamics, the counterweight of 3-DoF helicopter can be positioned in different locations at the side of the arm opposite to that where the motors are mounted, as shown in Fig. 1. In this study, this attribute of the laboratory helicopter enables the robustness of the proposed control method to be tested. To perform a simple and reasonable robustness analysis, the nonlinear reference model is used to control the 3 DoF Helicopter dynamics in two different parameter set. The first set is formed

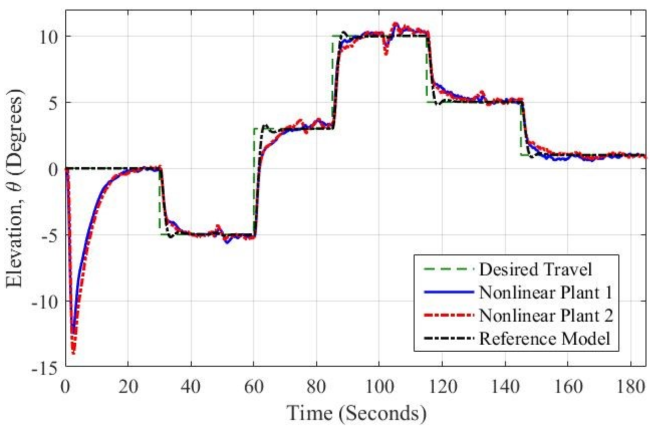

Fig. 2. Elevation responses of NP1 and NP2 for $f_w = 50$ Hz.

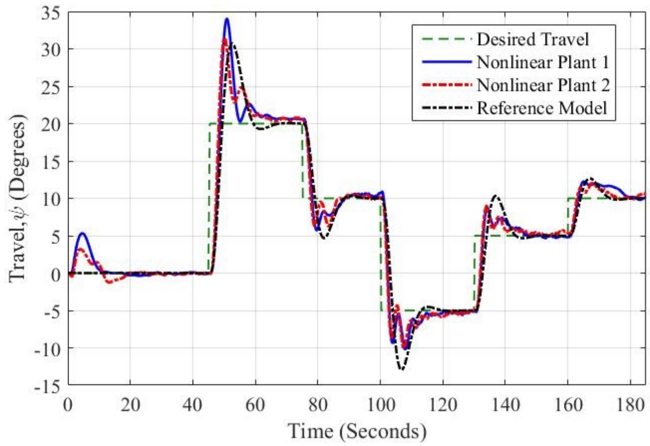

Fig. 3. Travel responses of NP1 and NP2 for $f_w = 50$ Hz.

by using 35 g counterweight position and it is indicated by Nonlinear Plant 1 (NP1) in the experimental results. The second set is formed by changing the counterweight position to 56 g closer to the travel axis, which yields a new nonlinear plant dynamics as indicated by Nonlinear Plant 2 (NP2).

Fig. 2 and Fig. 3 shows clearly that the SDRE based MRAC technique enables the closed-loop systems for both NP1 and NP2 to follow the ideal response of the reference model. However, the comparison between the responses of the reference and plant models shown in Fig. 3 also reveals that the tracking performance of the proposed control technique is slightly degraded in transient region if the maximum overshoot of the reference response becomes quite high, as is the case for the travel response between 40 s and 60 s. Based on these results, it is obvious that the proposed control technique is robust to the uncertainties of the plant dynamics. On the other hand, closer counterweight position to the travel axis results in heavier weight of the helicopter. This increment requires higher control efforts to enable the helicopter to follow the desired trajectories in travel and elevation axes. This can be confirmed by the comparison of the control inputs for NP1 and NP2, as shown in Fig. 4 and Fig. 5, respectively.

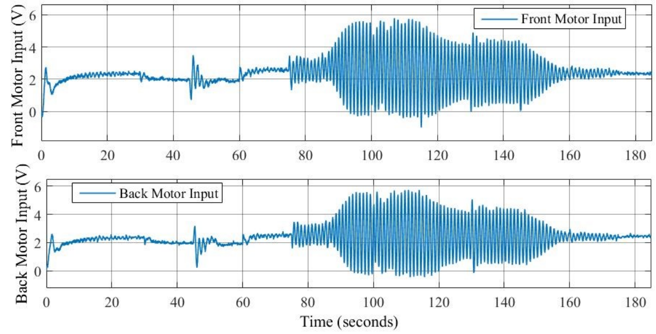

Fig. 4. Controller inputs of NP1 for $f_w = 50$ Hz.

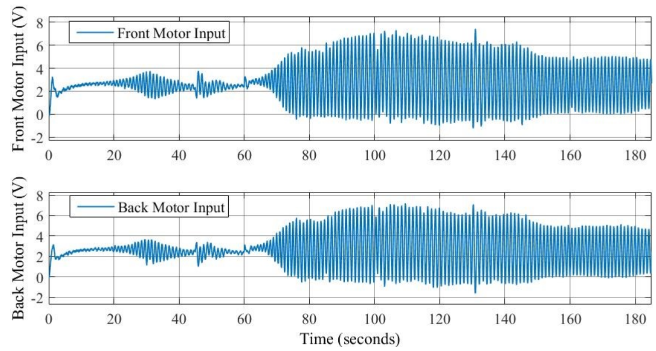

Fig. 5. Controller inputs of NP2 for $f_w = 50$ Hz.

The effect of working frequencies on the efficacy of the proposed controller is also investigated. Thus the proposed controller is implemented to the test bed for the different working frequencies of 50 Hz and 150 Hz. By comparing the control inputs of NP1 for both working frequencies shown in Fig. 4 and Fig. 6, it is found that lower working frequencies lead to higher computational efforts. Fig. 5 and Fig. 7 show the control inputs to NP2 for $f_w = 50$ Hz and $f_w = 150$ Hz, respectively and a similar result can be also found from the comparison of these results. However, higher working frequencies may result in undesirable system response, such as higher maximum overshoot in transient region as shown in Fig. 8 and Fig. 9.

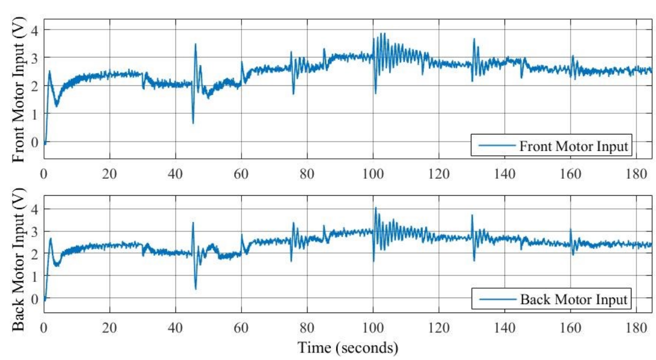

Fig. 6. Controller inputs of NP1 for $f_w = 150$ Hz.

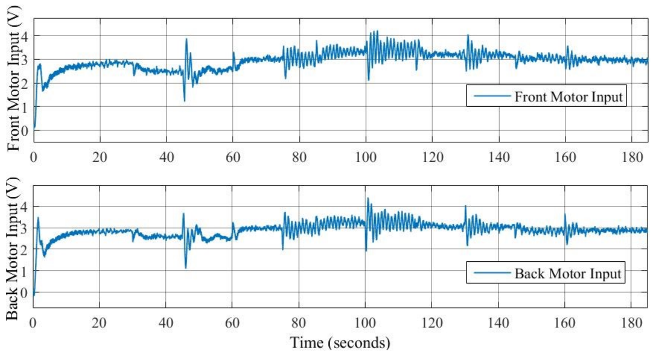

Fig. 7. Controller inputs of NP2 for $f_w = 150$ Hz.

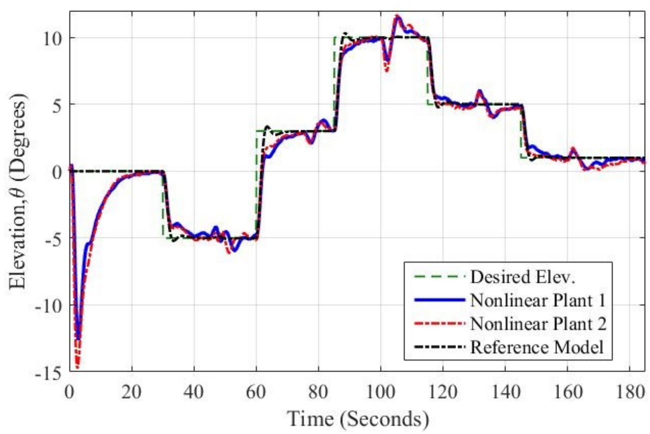

Fig. 8. Elevation responses of NP1 and NP2 for $f_w = 150$ Hz.

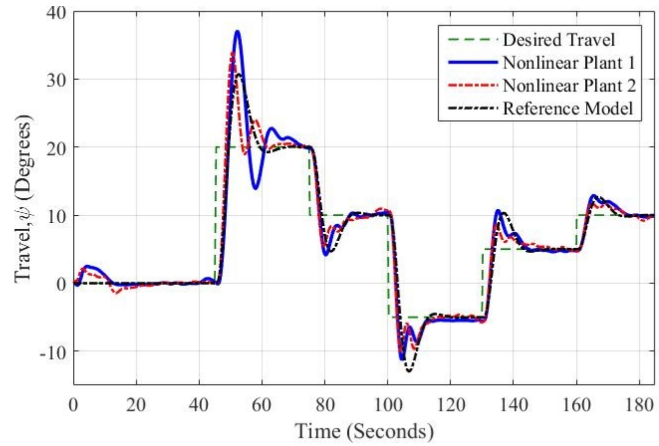

Fig. 9. Travel responses of NP1 and NP2 for $f_w = 150$ Hz.

V. CONCLUSIONS

In this paper, the SDRE based MRAC for nonlinear systems proposed in [12] has been implemented in real time using a 3-DoF nonlinear helicopter test bed with different sampling periods. The key advantage of synthesizing MRAC with SDRE is that it can be designed with a systematic procedure and easily applied to real-time nonlinear systems. The results validate the effectiveness and robustness of the proposed algorithm for the real time adaptive control of nonlinear systems with different working conditions.

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