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# A Group-based Navigation Algorithm for Multiple Anti-Ship Missile Systems

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*Abstract*— **In this paper, feasibility of cooperative navigation for anti-ship missile systems is investigated with the purpose of reducing cost and complexity of each missile system in a cooperative mission. We consider a homing engagement scenario of intercepting a high-value target using multiple missile systems equipped with low cost GPS-based navigation systems without on-board seekers. In order to ensure precise interception performance, the ultimate goal of proposed concept is to improve poor navigation performance of each missile system using the concept of cooperative navigation with additional measurements provided by simple and cheap sensors. In the proposed concept, the local GPS information is shared through communication. We then determine the estimated relative geometry such as relative range and relative angle between the elements in the group based on the shared GPS information. By comparing the estimated values with the additional measurements of relative range and angle, the navigation solution of each missile system is improved. In this study, the proposed concept is realized by means of the multi-rate Extended Kalman Filter (EKF). The performance and the feasibility of proposed method are validated through comparing Fisher Information Matrix (FIM) and numerical interception simulations.**

## I. INTRODUCTION

For the last few years, modern warfare has been evolving towards impersonality by letting precision guided weapons star in the game. Huge development effort has been put into improving the autonomy, accuracy and capabilities of the systems regardless of costs until the start of privatization in the gun industry. Then, optimization, efficiency and cost reduction became a concern as well as keeping the highest war power standards. This is being accomplished by the development of technology facing the implementation of not only new concepts and alternatives to the existing systems but also not-yet-used off-the-shelf models and algorithms.

Nowadays, the guidance system in a missile can escalate up to more than half of the cost of the whole system what makes it the perfect candidate for a deeper development towards the previously commented cost optimization. Another thing to take into account in current most common practice is that all the sensors and kill power of the precision guided weapon is concentrated inside one single missile system which reduces dramatically the chances of better navigation and performance without going the extra mile in sensor technology development as well as in case of being intercepted by the defense systems the mission has no other option than to be a complete failure.

Hence, for cost reduction and system robustness, a cooperative navigation concept is recommended by spreading the sensors throughout different missiles. It means that the multiple missiles would be flying and operating as a group and they are sharing their navigation information through communication channels. This strategy would increase not only the amount of data itself but also the usefulness of this data thanks to the interactions between the multiple missile systems. As a result of this improvement in data amount and significance, the sensors no longer need to be that much accurate nor it is required to have huge amounts of them and redundancies within a single system, what is decreasing each element's cost significantly. At the same time, the probability to kill of the mission is increased too.

Despite this potential benefit, there has not been much extensive research in cooperative navigation. In [1] and [2] the algorithm of the approach relies on the integration of GPS and visual tracking information to enable a master improve its deputies navigation. In this algorithm, there is still the need of one element to have good sensors and capabilities in which the whole system relies. A master would be a critical point in the system and building it would collide with the idea of cost reduction. There is one extra case [3] which the proposed solution adds the first comment about the possibility, but does not dig into it, of all elements to be chiefs and deputies at the same time, provided that each element could track at least another UAV in the system. This way the separation between chief and deputy concepts is blurred and the navigation improvement is expanded throughout the elements in the group.

In this study, we consider a situation of intercepting a high-value target such as a frigate using multiple missile systems. Since previous studies have mainly performed for UAV systems only, these cannot be directly transferred to our problem. Therefore, the aim of this study is to investigate a feasibility of new cooperative navigation concept which is suitable for multiple missile systems. In this study, it is assumed that each anti-ship missile is equipped with a low cost GPS system [4] only (without on-board seekers) to reduce cost and complexity of system so that the navigation performance is relatively poor. Instead, each missile can share its navigation information with other missiles through a communication channel and it utilizes a simple and a cheap sensor that provides relative geometry information such as the relative range and the relative angle between the missiles

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in the group. Then, the fundamental working principle of proposed concept is that each missile computes the relative geometry information using the shared navigation information from other missile systems and corrects its navigation solution by comparing the computed relative information with the additional measurements of relative range and relative angle. This process is realized by using the concepts of multi-rate Extended Kalman Filter (EKF) [5]. In this study, the Fisher Information Matrix (FIM) [6] of proposed concept is determined in order to show that the proposed concept can help improve the navigation performance of each missile system compared with the conventional approach. Finally, the proposed concept is tested with numerical simulations.

This paper is structured as follows. First, in Section II the problem formulation is made. Next, the proposed concept is explained in Section III. Then, Section IV presents the simulation results. Finally, we conclude our study in Section V.

## II. Problem Formulation

### A. Target Model

In this study, it is considered a stationary moving target on the sea surface such as a frigate. The planar motion of the target with a constant velocity is described as a point mass model. Actually, velocity vector of the target is not constant neither in direction nor module. However, since missiles' velocity is far larger than the target velocity and its maneuverability is low in that case, we can regard target as a stationary virtual target in the point where the target is expected to be. Therefore, the state variables for the target are set as follows, where velocity components are set to 0 and since it is a surface case, vertical position is constant and taken as a reference. Hence, none are included in the formulation below.

$$\mathbf{x}_T \triangleq \begin{bmatrix} X_T & Y_T \end{bmatrix}^T \tag{1}$$

### B. Missile Model

In this section, a two-dimensional inertia coordinate system is used. Since the target model is defined in Inertial Coordinates, missile's will also be defined in the same reference frame. Therefore, it will have the following state vector. The four states are position with respect to $X$ and $Y$ axis and the associated velocity decoupled into each axis direction.

$$\mathbf{x} \triangleq \begin{bmatrix} X & Y & V_X & V_Y \end{bmatrix}^T \tag{2}$$

It is considered a point mass model of the missile and assumed to be maneuvering at constant speed $V$. This means normal acceleration $a_c$ is only acting on the missile body to change its direction and does not affect its speed modulus. We assume that the missile is operating by using the guidance law developed in [7]. The guidance command is given by

$$a_c = -\frac{V^2}{R}\left(4\lambda + 2\lambda_f\right) + \frac{30V^2\left(4\lambda - \lambda_f\right)}{R\left[\left(4\lambda - \lambda_f\right)^2 + \delta\right]} k\left(\frac{\epsilon_t}{t_{go}}\right) \tag{3}$$

where the parameters $R$, $V$, $\lambda$, $\lambda_f$, $t_{go}$, $\epsilon_t$, and $k$ represent the relative range, the missile speed, the lead angle, the desired lead angle, the time-to-go, the impact time error, and the guidance gain, respectively. Note that the guidance command is given in the line-of-sight reference frame and the state vector in our missiles is defined in inertial coordinates. Therefore, a conversion needs to be applied. The guidance command applied to the state in the system will be defined as

$$\mathbf{u} \triangleq \begin{bmatrix} a_X \\ a_Y \end{bmatrix} = \begin{bmatrix} a_c \sin\gamma \\ a_c \cos\gamma \end{bmatrix} \tag{4}$$

where the parameter $\gamma$ represents the flight path angle. Thus, the system's dynamic model with the process noise can be written as

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{G}\mathbf{w} \tag{5}$$

where

$$\mathbf{F} \triangleq \begin{bmatrix} \mathbf{0}_{2\times2} & \mathbf{I}_{2\times2} \\ \mathbf{0}_{2\times2} & \mathbf{0}_{2\times2} \end{bmatrix}, \ \mathbf{B} \triangleq \begin{bmatrix} \mathbf{0}_{2\times2} \\ \mathbf{I}_{2\times2} \end{bmatrix}, \ \mathbf{G} \triangleq \mathbf{I}_{4\times4} \tag{6}$$

$$\mathbf{w} \triangleq \begin{bmatrix} \omega_X & \omega_Y & \omega_{V_X} & \omega_{V_X} \end{bmatrix}^T \tag{7}$$

where $\mathbf{I}_{2\times2}$ and $\mathbf{0}_{2\times2}$ represent the 2 by 2 identity matrix and null matrix, respectively.

### C. Engagement Scenario

A homing engagement scenario is set as a usual sea-skimming problem. Namely, the coordinates system is bounded in a 2D plane where missiles are set to be flying at a constant altitude over the surface. The target, as defined before, is stationary, at least compared to missiles movement being defined, then, as a true static target or a virtual target depending on movement rate. This case is thought to be for engaging high-value target (HVT) where the cooperative attack of multiple missile systems is necessary for defense systems saturation and therefore, reaching the homing and impact phases of flight. To accomplish this saturation goal, the formation of the swarm needs to be such that attacks the target on most different flanks at the same time. Thus, it is built spreading the elements around the target on the 2D plane dividing the space between them constantly. Missiles fly towards the target placed in a central point.

### D. Problem Definition

In this study, we consider the anti-ship missile system equipped with a low cost GPS as navigation system without a seeker system for the cost reduction. In this case, obviously, the interception performance highly relies on the accuracy of navigation system. However, since the low cost GPS usually cannot provide accurate navigation solutions, the interception performance is degraded. Thus, the main goal of this paper is to improve the interception performance by enhancing the navigation accuracy of each missile system by using cooperative navigation concept. To this end, it is assumed that each missile system can share its navigation information with other missile systems.

Even though each missile shares navigation information with each other, the navigation errors are not observable if measurements on relative geometry are absence. Thus, in this study, we consider two additional measurements from low cost sensors: the relative range and the relative angle. These measurements will also be noisy and have some errors but by overlapping non-ideal measurements error bounds can be reduced. This is graphically exposed in Fig. 1 which represents the cooperative navigation concept by data sharing of the GPS information and the relative range measurement for three missile systems. The original navigation uncertainty is represented in the figure and it can be seen how the generated additional information in relative range can bound this uncertainty.

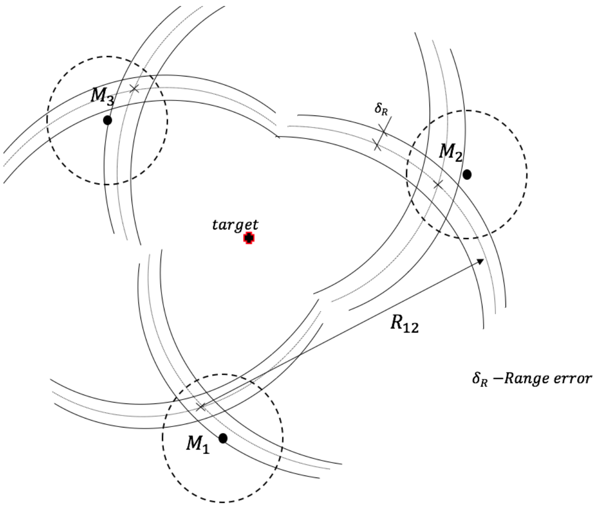


Fig. 1: The graphical interpretation of cooperative navigation using the GPS and the relative range measurement

Similarly, Fig. 2 shows the cooperative navigation with the GPS information and the relative angle measurement.

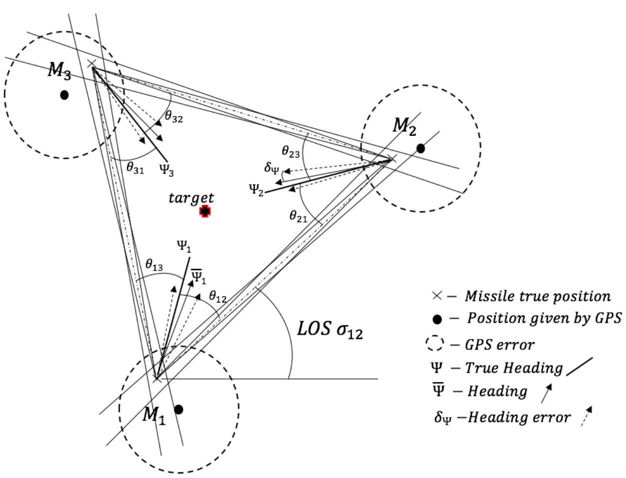


Fig. 2: The graphical interpretation of cooperative navigation using the GPS and the relative angle measurement

Therefore, the main objective of this work is to investigate the feasibility of the new cooperative navigation concept: development of cooperative navigation algorithm by potential utilization of these measurements from several missiles in order to improve their navigation thanks to group cooperation.

## III. The Proposed Algorithm

This section provides the proposed algorithm which is based on the multi-rate Extended Kalman Filter approach.

### A. Derivation of System Equation

The initial and very important step is to modify the matrices in Eq. (5) in order to adapt them from continuous time formulation to be used in discrete time as Kalman filter works. In a similar way in [8], we consider a very small integration step $dt$ and then Eq. (5) can be rewritten as

$$d\mathbf{x} = \mathbf{F}\mathbf{x}dt + \mathbf{B}\mathbf{u}dt + \mathbf{G}d\vec{\beta} \tag{8}$$

where $\vec{\beta}$ is the Brownian motion process. Developing this equation gives

$$\begin{aligned}\mathbf{x}(t_2) = \mathbf{\Phi}(t_2,t_1)\,\mathbf{x}(t_1) + \textstyle\int_{t_1}^{t_2} \mathbf{\Phi}(t_2,\tau)\,\mathbf{B}(\tau)\,\mathbf{u}(\tau)\,d\tau \\ + \textstyle\int_{t_1}^{t_2} \mathbf{\Phi}(t_2,\tau)\,\mathbf{G}(\tau)\,d\vec{\beta}(\tau)\end{aligned} \tag{9}$$

where $\mathbf{\Phi}(t_2,t_1)$ represent the state transition matrix of Eq. (5). Next, let, $t_2 = t_{k+1}$ and $t_1 = t_k$ then, after developing the equation, we can get the discrete process model as

$$\mathbf{x}(t_{k+1}) = \mathbf{\Phi}_k \mathbf{x}(t_k) + \mathbf{\Gamma}_k \mathbf{u}(t_k) + \omega_k \tag{10}$$

with

$$\mathbf{\Phi}_k = \begin{bmatrix} \mathbf{I}_{2\times 2} & \Delta t \mathbf{I}_{2\times 2} \\ \mathbf{0}_{2\times 2} & \mathbf{I}_{2\times 2} \end{bmatrix}, \quad \mathbf{\Gamma}_k = \begin{bmatrix} \frac{1}{2}\Delta t^2 \mathbf{I}_{2\times 2} \\ \Delta t \mathbf{I}_{2\times 2} \end{bmatrix} \tag{11}$$

$$\mathbf{Q}_k = \sigma_p^2 \begin{bmatrix} \frac{1}{4}\Delta t^3 \mathbf{I}_{2\times 2} & \frac{1}{2}\Delta t^2 \mathbf{I}_{2\times 2} \\ \frac{1}{2}\Delta t^2 \mathbf{I}_{2\times 2} & \Delta t \mathbf{I}_{2\times 2} \end{bmatrix}, \quad \omega_k \sim N(0, \mathbf{Q}_k) \tag{12}$$

where $\Delta t = t_{k+1} - t_k$. The notations $\mathbf{\Phi}_k$, $\mathbf{\Gamma}_k$, $\mathbf{Q}_k$, $\omega_k$, and $\sigma_p$ are the discrete system matrix, the discrete control matrix, the discrete covariance matrix, the discrete process noise, and the standard deviation of continuous process noise, respectively.

### B. Measurement Equations

Basically, each missile can achieve its navigation information. Single navigation information of the $i$-th missile includes the position and velocity of missile as

$$\mathbf{z}_G^i = \mathbf{x}^i + \mathbf{v}_G^i \tag{13}$$

where $\mathbf{v}_i$ denotes the measurement noise vector of GPS as

$$\mathbf{v}_i = \begin{bmatrix} \nu_X^i & \nu_Y^i & \nu_{V_X}^i & \nu_{V_Y}^i \end{bmatrix}^T \tag{14}$$

For the relative range measurement, the nonlinear measurement equation is the usual distance measurement equation in 2D by triangulation. Thus, the measurement vector for $i$-th missile is written as

$$\mathbf{z}_R^i = \begin{bmatrix} \sqrt{(X_1 - X_i)^2 + (Y_1 - Y_i)^2} \\ \vdots \\ \sqrt{(X_j - X_i)^2 + (Y_j - Y_i)^2} \end{bmatrix} + \begin{bmatrix} \nu_R^{i1} \\ \vdots \\ \nu_R^{ij} \end{bmatrix}, \quad \forall j \neq i \tag{15}$$

where subindex $i$ indicates the missile being updated and subindex $j$ is the one identifying the partner whose information is being used. The parameter $\nu_R^{ij}$ represents the measurement noise of relative range. For notation convenience, it can also be rewritten in shorthand form as

$$\mathbf{z}_R^i = \mathbf{h}_R^i\left(\mathbf{x}^i, \mathbf{x}^j\right) + \nu_R, \quad \forall j \neq i \tag{16}$$

In a similar way, the relative angle measurement for the $i$-th missile can be formulated as

$$\mathbf{z}_\theta^i = \begin{bmatrix} \tan^{-1}\left(\frac{Y_1 - Y_i}{X_1 - X_i}\right) \\ \vdots \\ \tan^{-1}\left(\frac{Y_j - Y_i}{X_j - X_i}\right) \end{bmatrix} + \begin{bmatrix} \nu_\theta^{i1} \\ \vdots \\ \nu_\theta^{ij} \end{bmatrix}, \quad \forall j \neq i \tag{17}$$

$$\mathbf{z}_\theta^i = \mathbf{h}_\theta^i\left(\mathbf{x}^i, \mathbf{x}^j\right) + \nu_\theta, \quad \forall j \neq i \tag{18}$$

Here, note that the GPS measurements $\mathbf{z}_G$ is given by linear form and the relative measurements $\mathbf{z}_R$ and $\mathbf{z}_\theta$ are given by nonlinear form. In the proposed algorithm, those measurements are combined in a single form by using Extended Kalman Filter structure.

### C. Multi-rate Kalman Filtering

Since the sampling rates of above measurements are different each other, we adopt the multi-rate Extended Kalman Filtering approach in order to fuse all information together. The proposed algorithm is a five-step process that consists of three filtering stages where the different parts of the information are used and works as follows.

Step 1 is the state vector propagation through the dynamic model described in previous section.

$$\begin{aligned} \hat{\mathbf{x}}_k &= \mathbf{\Phi}_{k-1}\hat{\mathbf{x}}_{k-1} + \mathbf{\Gamma}_{k-1}\mathbf{u}_{k-1} \\ \hat{\mathbf{P}}_k &= \mathbf{\Phi}_{k-1}\hat{\mathbf{P}}_{k-1}\mathbf{\Phi}_{k-1}^T + \mathbf{Q}_{k-1} \end{aligned} \tag{19}$$

Once propagated, each missile uses its own GPS measurement information to go through the first filtering stage in the algorithm. In Step 2, GPS linear Kalman Filter where the first update is performed as

$$\begin{aligned} \mathbf{K}_{k,G} &= \hat{\mathbf{P}}_k\mathbf{H}_{k,G}^T\left(\mathbf{H}_{k,G}\hat{\mathbf{P}}_k\mathbf{H}_{k,G}^T + \mathbf{R}_{k,G}\right)^{-1} \\ \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k + \mathbf{K}_{k,G}\left(\mathbf{z}_{k,G} - \hat{\mathbf{x}}_k\right) \\ \hat{\mathbf{P}}_k &= \left(\mathbf{I} - \mathbf{K}_{k,G}\mathbf{H}_{k,G}\right)\hat{\mathbf{P}}_k\left(\mathbf{I} - \mathbf{K}_{k,G}\mathbf{H}_{k,G}\right)^T \\ &\quad + \mathbf{K}_{k,G}\mathbf{R}_{k,G}\mathbf{K}_{k,G}^T \end{aligned} \tag{20}$$

Up to this stage the filtering is just about individual performance.

In Step 3, now group-based phase starts. If the communication channels are connected, then $i$-th missile can achieve other missile's navigation information as

$$\mathbf{z}_O^i = \begin{bmatrix} \hat{\mathbf{x}}_k^j \\ \vdots \\ \hat{\mathbf{x}}_k^N \end{bmatrix}, \quad \forall j \neq i \tag{21}$$

Note that this is the information to be shared for the cooperative performance. It is relevant to mention that the information used after each missile system has been self-updated with its own GPS is the one shared among the system so all missiles in the group use the exact same information in their own 2nd and 3rd update stages to ensure good time synchronicity and missile element independence and helps to robustness. Based on the information shared, each missile determines the estimated relative range and the relative angle by using Eqs. (15) and (17). These estimated parameters are utilized for the next update stages.

TABLE I: Simulation basic parameters

| Definition | Symbol | Value |
|---|---|---|
| Missile Velocity | V | 1,000 km/h |
| Range | R | 10,000 m |
| Altitude | h | 20 m ASL |
| Filter frequency | f | 50 Hz |
| Timestep length | $\Delta$t = 1/f | 0.02 s |
| Initial heading | $\Psi_0$ | Pointing towards target. |

In Step 4, after taking all the measurements (relative range and relative angle) the state vector goes first into the range EKF where it is updated for the second time as

$$\begin{aligned} \mathbf{K}_{k,R} &= \hat{\mathbf{P}}_k\mathbf{H}_{k,R}^T\left(\mathbf{H}_{k,R}\hat{\mathbf{P}}_k\mathbf{H}_{k,R}^T + \mathbf{R}_{k,R}\right)^{-1} \\ \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k + \mathbf{K}_{k,R}\left(\mathbf{z}_{k,R} - \mathbf{h}_R\left(\hat{\mathbf{x}}_k\right)\right) \\ \hat{\mathbf{P}}_k &= \left(\mathbf{I} - \mathbf{K}_{k,R}\mathbf{H}_{k,R}\right)\hat{\mathbf{P}}_k\left(\mathbf{I} - \mathbf{K}_{k,R}\mathbf{H}_{k,R}\right)^T \\ &\quad + \mathbf{K}_{k,R}\mathbf{R}_{k,R}\mathbf{K}_{k,R}^T \end{aligned} \tag{22}$$

And then, finally, for the third when it is filtered with the relative angle EKF in Step 5.

$$\begin{aligned} \mathbf{K}_{k,\theta} &= \hat{\mathbf{P}}_k\mathbf{H}_{k,\theta}^T\left(\mathbf{H}_{k,\theta}\hat{\mathbf{P}}_k\mathbf{H}_{k,\theta}^T + \mathbf{R}_{k,\theta}\right)^{-1} \\ \hat{\mathbf{x}}_k &= \hat{\mathbf{x}}_k + \mathbf{K}_{k,\theta}\left(\mathbf{z}_{k,\theta} - \mathbf{h}_\theta\left(\hat{\mathbf{x}}_k\right)\right) \\ \hat{\mathbf{P}}_k &= \left(\mathbf{I} - \mathbf{K}_{k,\theta}\mathbf{H}_{k,\theta}\right)\hat{\mathbf{P}}_k\left(\mathbf{I} - \mathbf{K}_{k,\theta}\mathbf{H}_{k,\theta}\right)^T \\ &\quad + \mathbf{K}_{k,\theta}\mathbf{R}_{k,\theta}\mathbf{K}_{k,\theta}^T \end{aligned} \tag{23}$$

That is the definite state and next time step begins.

### D. Fisher Information Matrix

The Fisher Information Matrix for the proposed algorithm is derived to determine the effectiveness of proposed method in terms of informativeness. More information implies that the error covariance becomes smaller as explained by the Cramer-Rao lower bound (CRLB). Thus, the inverse of the FIM is essentially equivalent to the lower bound of the estimation error covariance. Since the purpose of the FIM is to measure the amount of information it needs to be modelled by using this information sources. In this case, the GPS, the relative range and the relative angle measurement. In this study, the FIM of proposed concept is developed in a similar way in [9] where it is built based on the derivatives of the measurement equations.

## IV. Simulation Results

In this section, the proposed concept is tested with several numerical simulations in order to show the feasibility of proposed method. For these simulations, the parameters shown in Table 1 are selected. Also, the filter parameters are tuned as given in Table 2.

First, FIMs are determined under the proposed method and GPS-only case. About the information in the system,

TABLE II: Filter parameters for performance test

| Parameter | Value |
|---|---|
| Initial State Estimation | Initialised as a first GPS measurement |
| Initial Error Covariance Estimation | $\begin{bmatrix} 500^2 & 0 & 0 & 0 \\ 0 & 500^2 & 0 & 0 \\ 0 & 0 & 100^2 & 0 \\ 0 & 0 & 0 & 100^2 \end{bmatrix}$ |
| Process noise | 0.1 |
| GPS frequency | 1 Hz |
| GPS position measurement error | 10 m |
| GPS velocity measurement error | 0.5 m/s |
| Range measurement error | 5 m |
| Angular measurement error | 1 degree |
| Measurement Covariance | Diagonal Matrix with $\sigma^2$ in each element according to filter and measurement length |

shown in Fig. 4 there is proof that the amount of useful information the system keeps gathering increases with time. This is extracted from the constant positive gradient of the FIM determinant over time. Comparing the computation for the GPS-only case shown in Fig. 3 and the case with the cooperative phases of the algorithm it can be noticed how the determinant value is bigger. Therefore, there is no unproductive information input on one side, and the results and conclusions extracted from them can be directly related to the performance of the filter on the other. So, the suggested information sharing is not redundant and contribute positively to the system performance.

Next, we determine the intercept performance of proposed method compared with the GPS-only case. In this case, the filtering algorithm provides the estimated missile velocity and missile position. At each single missile system, the guidance command is then computed using the estimated missile states. In order to test that, 500-cases Monte Carlo simulations have been run for two cases. With these results in figures 5 and 6 which show the scattered error of each missile in each simulation, it can be proved from their trend, that the proposed concept actually helps improve navigation and intercept performance compared to the GPS-only case.

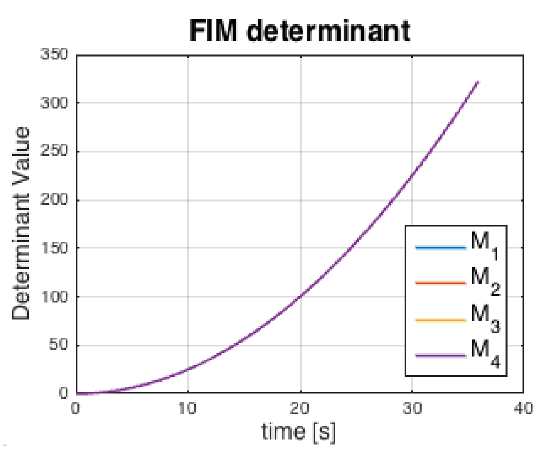


Fig. 3: The pattern of FIM over time under GPS-only.

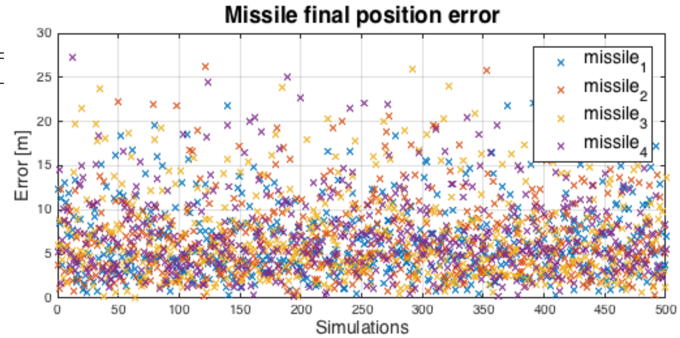


Fig. 5: Closed loop simulation results for only GPS filtering. Target miss distance.

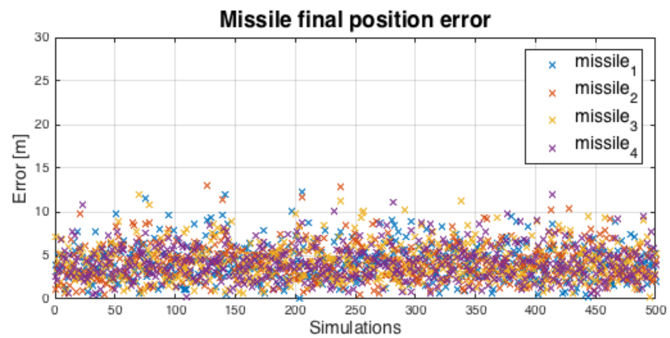


Fig. 6: Closed loop simulation results for the whole algorithm enabled. Target miss distance.

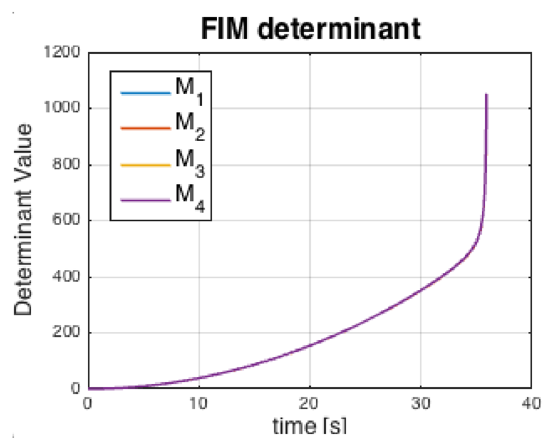


Fig. 4: The pattern of FIM over time under proposed concept.

## V. Conclusion

In this paper, the feasibility of new cooperative navigation for multiple missile systems which is equipped with a low cost GPS sensor is investigated in order to reduce development cost and system complexity. In the proposed concept, each missile system shares its navigation information to other missile systems through the communication channel. And then, by using the estimated relative geometry information which can be determined using other missile's navigation information, each missile system additional improve its navigation information. In order to ensure the observability, two additional measurements (i.e., relative range and relative angle) are considered. Based on the concept of multi-rate Extended Kalman Filter, the proposed cooperative algorithm is realized. The feasibility of proposed concept is first determined by computing the Fisher Information Matrix (FIM). Additionally, the performance of proposed concept is tested with the numerical simulations. The results obtained indicate that the proposed concept can improve the navigation performance as well as the intercept performance compared with previous approach.

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