2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Position and speed observer for PMSM with unknown stator resistance*

Dmitry Bazylev, Anton Pyrkin, Alexey Bobtsov

***Abstract*— In this paper a new nonlinear parameterization of permanent magnet synchronous motor (PMSM) model is proposed for the case of an uncertain stator resistance. The assumption of known inductance only is applied. After parameterization the regression model of six parameters is obtained from which it becomes possible to reconstruct the resistance and two necessary parameters involved in the position and speed observers design. The dynamic regressor extension and mixing (DREM) estimator is used to provide good performance and fast estimation of a large regression model which is preferable than the standard gradient approach. Simulation results illustrating proposed approach are given.**

I. INTRODUCTION

An enormous amount of work has been devoted to sensorless (self-sensing) algorithms, where position and speed estimates are used to control the motor. See [1], [14] for a recent review of the literature.

In [15] a very simple, gradient descent–based procedure to carry–out this task was proposed. The analysis, done for the full nonlinear model, proves global (under some conditions, even exponential) convergence of the estimates to a *residual set* and yields very simple robust tuning rules—see also [13] for a simpler stability analysis and the proof of global convergence under (a kind of) *persistent excitation* (PE) condition. In [9] this position observer was combined with an *ad–hoc* linear speed estimator and a standard field–oriented controller, which are often used in applications, yielding very encouraging experimental results. See also [7], [19], [20] where this observer is also used.

Recently, a robust, nonlinear and globally convergent position observer for surface-mount PMSMs was proposed by [4]. In [5] this observer was extended applying DREM procedure of [2]. Only the knowledge of stator currents, voltages, resistance and inductance is required for the rotor position estimation—obviating the need to know other uncertain motor parameters like magnetic flux constant and rotor inertia. The key observation made in [4] is that the position observation problem can be recast in terms of classical *parameter identification*—a result that has been recently generalized in [16], where a class of systems for which this problem reformulation is possible has been identified. This reformulation is achieved for the PMSM in [4] via a new reparameterization of the motor dynamics and some signal manipulation to obtain a classical linear regression form with two unknown *constant parameters*. A fundamental step in all parameter identification problems is, obviously, the selection of a suitable adaptation algorithm. For the sake of simplicity in [4] it is proposed to use a basic gradient observer, which requires the usual, hardly verifiable persistent excitation (PE) condition [10].

The usual assumption in position observer design is that all electrical parameters of the motor are constant, and exactly known. As a rule, precise information about these parameters is also required in controller settings to ensure high performance and accuracy of motor operation. For example, in current controllers [14] stator resistance is required for adequate tuning of controller gains and flux generated by permanent magnets is used for back EMF compensation. In practice, these parameters are uncertain [1]. Moreover, stator resistance can vary significantly due to temperature changes.

In [12] position observer is augmented by stator resistance adaptation, analytical stability conditions useful for tuning procedure are derived. The nonlinear algorithm for PMSM sensorless control with resistance and torque load estimation is proposed in [21]. The presented observer requires the usual PE condition and the knowledge of magnetic flux constant. In [17] an adaptive version of the flux observer of [15] that estimates the *stator resistance* was proposed. Global boundedness of all signal and convergence to a residual set of the flux estimation error was established assuming known the rotor speed and all electrical parameters, including the magnetic flux constant, hence still suffering from the aforementioned lack of robustness problem.

In this paper we propose a new nonlinear parameterization of the PMSM model where the stator resistance is uncertain and the inductance is used as known constant. Such assumption is reasoned by the typical conditions of motor operation in which the temperature-dependent resistance can vary significantly larger than the inductance. After parameterization the regression model of six parameters is obtained from which it is possible to reconstruct the resistance and two necessary parameters involved in the position and speed observers design.

We use the new dynamic regressor extension and mixing (DREM) estimator, which was recently proposed in [2], instead of the standard gradient approach. The new DREM-based observer, as the one with gradient (or least squares) estimators, ensures parameter convergence is *exponentially* fast if its regressor is PE. If the regressor is not PE but the special operator of the regressor [2] is only non-square integrable then the convergence of the DREM-based observer

*This work was supported by the RF Ministry of Education and Science (RF Government resolution 218, April 9, 2010. (R&D project 03.G25.31.0251 dated April 28, 2017 at ITMO University "Creation of high-tech production of configurable frequency converters for new generation of synchronous precision high-speed high-power electromechanical drives").

Dmitry Bazylev, Anton Pyrkin and Alexey Bobtsov are with Department of Control Systems and Informatics, ITMO University, Kronverkskiy av. 49, Saint Petersburg, 197101, Russia bazylevd@mail.ru, a.pyrkin@gmail.com, bobtsov@mail.ru

is still guaranteed, however, convergence is not exponential.

Simulation results presented in the paper illustrate the advantages of the proposed observer. Namely, convergence speed of the new estimator is very fast for a large regression model and its transient behaviour is very well. Due to property fact together with uncertain resistance, the position and speed observers with a DREM estimator are more preferable than the gradient–based estimator used in [4].

The problem of the position and speed observer is solved in [4], but for reconstruction of the position authors use the trigonometric function $\arctan()$ which is not robust and could not be tuned to attenuate the noise influence. Moreover, the speed observer is obtained using PLL procedure which gives an approximate estimate. In this paper we propose the new hybrid approach which allows to make the dynamic inversion of piece-wise monotonic functions.

The remaining of the paper is organized as follows. In Section II we present the model of the salient PMSM and the problem formulation. Section III contains the new parametrization allowing to reconstruct the resistance estimate and design the flux observer. Section IV contains the main result, namely the new position and speed observers which based on additional parametrization procedure given in Section V. Representative simulation results for typical scenarios of motor operation are given in Section VI. The paper is wrapped-up with concluding remarks in Section VII.

II. PROBLEM FORMULATION

The classical, two–phase $\alpha\beta$ model of the unsaturated, *non–salient* PMSM described by [8], [14], is considered. In the stationary $\alpha\beta$ frame, the following holds

$$\dot{\lambda} = v - Ri, \tag{1}$$

$$J\dot{\omega} = -B\omega + \tau_e - \tau_L, \tag{2}$$

$$\dot{\theta} = \omega, \tag{3}$$

where $\lambda_{\alpha\beta} \in \mathbb{R}^2$ is the total flux, $i_{\alpha\beta} \in \mathbb{R}^2$ the currents, $v_{\alpha\beta} \in \mathbb{R}^2$ are the voltages, $R > 0$ is the stator windings resistance, $J > 0$ is the rotor inertia, $\theta \in \mathbb{S} := [0, 2\pi)$ is the rotor phase, $\omega \in \mathbb{R}$ is the mechanical angular velocity, $B \geq 0$ is the viscous friction coefficient, $\tau_L \in \mathbb{R}$ is the—possibly time–varying—load torque, τ_e is the torque of electrical origin, given by

$$\tau_e = n_p i^\top \mathcal{J}\lambda, \tag{4}$$

with $n_p \in \mathbb{N}$ the number of pole pairs and $J \in \mathbb{R}^{2\times 2}$ is the rotation matrix

$$\mathcal{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

The total flux of surface–mounted PMSM verifies

$$\lambda = Li + \lambda_m \mathcal{C}(\theta), \tag{5}$$

where $L > 0$ is the stator inductance, λ_m is the constant flux generated by permanent magnets and, to simplify the notation, we defined

$$\mathcal{C}(\theta) := \mathrm{col}(\cos(n_p\theta), \sin(n_p\theta)). \tag{6}$$

Hence, a state–space model of the PMSM is given as

$$L\frac{di}{dt} = -Ri - \lambda_m \omega \mathcal{C}'(\theta) + v \tag{7}$$

where $\mathcal{C}'(\theta) := \frac{d\mathcal{C}}{d\theta}$ and we have used (5) and the fact that

$$\mathcal{C}'(\theta) = n_p \mathcal{J}\mathcal{C}(\theta),$$

to obtain the expression for τ_e.

We will use the following assumptions.

- A1 The *measurable* signals are currents $i(t)$ and voltages $v(t)$ of the stator windings.
- A2 The control signal $v(t)$ and the external load torque $\tau_L(t)$ are such that trajectories of the PMSM model (5)-(7) exist for all $t \geq 0$ and are bounded.
- A3 The load torque $\tau_L(t)$ is *unknown* and *constant*.
- A4 The *known* parameters of the PMSM are an inductance L, a rotor inertia J, a viscous friction coefficient B. The constant flux λ_m is *unknown*.
- A5 The measurable signals $i(t)$ and $v(t)$ are *integrable*.

The objective is to reconstruct the stator resistance R, the rotor speed ω and position θ asymptotically.

III. RESISTANCE ESTIMATOR AND FLUX OBSERVER

At the first step we will show the new parameterization, which allows to reconstruct the unknown resistance and design the flux observer using which we establish the adaptive speed and position observers.

Introduce two auxiliary variables [4], [5]

$$\dot{z}_1(t) = v_{\alpha\beta}(t), \quad \dot{z}_2(t) = i_{\alpha\beta}(t).$$

Then, integrating (1) and combining it with (5) yields

$$\begin{cases} \eta_1 - Rz_{21} + z_{11} - Li_1 = \lambda_m \cos(n_p\theta), \\ \eta_2 - Rz_{22} + z_{12} - Li_2 = \lambda_m \sin(n_p\theta), \end{cases} \tag{8}$$

where $\eta = (\eta_1, \eta_2)^\top$ - unknown constant parameters

$$\eta = \lambda_{\alpha\beta}(0) - z_1(0) + Rz_2(0).$$

Let us assume that R is unknown parameter that is time invariant.

Denote $\xi_1 = z_{11} - Li_1$ and $\xi_2 = z_{12} - Li_2$ which yields

$$\begin{cases} \eta_1 - Rz_{21} + \xi_1 = \lambda_m \cos(n_p\theta), \\ \eta_2 - Rz_{22} + \xi_2 = \lambda_m \sin(n_p\theta) \end{cases} \tag{9}$$

The sum of squares of two equations in (9) gives

$$\begin{aligned} &\eta_1^2 - 2\eta_1 Rz_{21} + R^2 z_{21}^2 + 2\eta_1\xi_1 - 2Rz_{21}\xi_1 + \\ &\xi_1^2 + \eta_2^2 - 2\eta_2 Rz_{22} + R^2 z_{22}^2 + 2\eta_2\xi_2 - \\ &\quad 2Rz_{22}\xi_2 + \xi_2^2 = \lambda_m^2 \end{aligned} \tag{10}$$

or

$$\begin{aligned} &(\eta_1^2 + \eta_2^2) + R^2(z_{21}^2 + z_{22}^2) - 2R(z_{21}\xi_1 + z_{22}\xi_2) - \\ &2R(\eta_1 z_{21} + \eta_2 z_{22}) + 2\eta_1\xi_1 + 2\eta_2\xi_2 + \\ &\quad (\xi_1^2 + \xi_2^2) = \lambda_m^2. \end{aligned} \tag{11}$$

Applying the filter $\left[\frac{\alpha p}{p+\alpha}\right]$ to (11), where $p=\frac{d}{dt}$, yields

$$\begin{aligned}R\psi_1+\eta_1\psi_2+\eta_2\psi_3+R^2\psi_4+&\\ R(\eta_1\psi_5+\eta_2\psi_6)=y.&\end{aligned} \tag{12}$$

where

$$\begin{aligned}
\psi_1 &= \left[\frac{\alpha p}{p+\alpha}\right](-2z_{21}\xi_1-2z_{22}\xi_2),\ \psi_2=\left[\frac{\alpha p}{p+\alpha}\right](2\xi_1),\\
\psi_3 &= \left[\frac{\alpha p}{p+\alpha}\right](2\xi_2),\ \psi_4=\left[\frac{\alpha p}{p+\alpha}\right](z_{21}^2+z_{22}^2),\\
\psi_5 &= \left[\frac{\alpha p}{p+\alpha}\right](-2z_{21}),\ \psi_6=\left[\frac{\alpha p}{p+\alpha}\right](-2z_{22}),\\
y &= \left[\frac{\alpha p}{p+\alpha}\right](\lambda_m^2-(\eta_1^2+\eta_2^2)-(\xi_1^2+\xi_2^2))\\
&= -\left[\frac{p}{p+1}\right](\xi_1^2+\xi_2^2).
\end{aligned}$$

We have the regression model (12) with 6 unknown constant parameters, where 3 of them are independent.

The next step is to estimate the unknowns. We will use the DREM procedure for 6 parameters [2]. In accordance with DREM we need to form 5 new regression models from the original one (12). One can use:

1) either five LTI filters $[H_k(\cdot)](t):=\dfrac{\varepsilon_k}{p+\varepsilon_k}, k=\overline{1,5}$,

2) or five delay operators $[H_k(\cdot)](t):=(\cdot)(t-d_k)$ with different values of delay $d_k, k=\overline{1,5}$.

3) or their combination.

Applying these dynamic operators to (12) yields

$$\begin{aligned}R\psi_{1k}+\eta_1\psi_{2k}+\eta_2\psi_{3k}+R^2\psi_{4k}&\\ +R(\eta_1\psi_{5k}+\eta_2\psi_{6k})=y_k,&\end{aligned} \tag{13}$$

where $y_k=[H_k](y)$, $\psi_{lk}=[H_k](\psi_l), l=\overline{1,6}, k=\overline{1,5}$.

Combining 5 new regressors (13) and the original regressor (12) we get the extended regressor

$$Y_e=Q_e\mu, \tag{14}$$

where

$$Y_e=\begin{bmatrix}y\\ y_k\end{bmatrix}, \tag{15}$$

$$Q_e=\begin{bmatrix}\psi_1 & \psi_2 & \psi_3 & \psi_4 & \psi_5 & \psi_6\\ \psi_{1k} & \psi_{2k} & \psi_{3k} & \psi_{4k} & \psi_{5k} & \psi_{6k}\end{bmatrix}, \tag{16}$$

$$\mu=\begin{bmatrix}R & \eta_1 & \eta_2 & R^2 & R\eta_1 & R\eta_2\end{bmatrix}^T. \tag{17}$$

Next, multiplying (14) from the left by the adjoint matrix of Q_e results in 6 separate regressor models for each unknown parameter.

$$Y_l=\phi\mu_l, \tag{18}$$

where $\phi=\det\{Q_e\}$ is the determinant of Q_e and $Y_l=\text{adj}\{Q_e\}Y_e$ is computed using adjoint matrix of Q_e.

Since we need to estimate only first 3 parameters μ_1,μ_2 and μ_3, we can use only 3 standard gradient estimators. In the following Proposition the first principal result is established.

Proposition 1: The flux observer is given by

$$\hat\lambda=z_1-\hat R z_2+\hat\eta. \tag{19}$$

with parameter estimators

$$\dot{\hat R}=\gamma_r\phi(Y_1-\phi\hat R), \tag{20}$$

$$\dot{\hat\eta}_1=\gamma_{\eta_1}\phi(Y_2-\phi\hat\eta_1), \tag{21}$$

$$\dot{\hat\eta}_2=\gamma_{\eta_2}\phi(Y_3-\phi\hat\eta_2), \tag{22}$$

where $\gamma_r>0,\ \gamma_{\eta_1}>0, \gamma_{\eta_2}>0$ are the adaptation gains, ensures

$$\begin{aligned}\lim_{t\to\infty}(\lambda(t)-\hat\lambda(t))=0,\ \lim_{t\to\infty}(R-\hat R(t))=0,\\ \lim_{t\to\infty}(\eta_1-\hat\eta_1(t))=0,\ \lim_{t\to\infty}(\eta_2-\hat\eta_2(t))=0.\end{aligned} \tag{23}$$

if $\phi(t)$ is not square integrable. Moreover, if $\phi(t)$ satisfies PE condition the observer (19)-(22) converges exponentially fast.

Proof: The proof of parameter convergence is based on [2] and [4]. It follows from analysis of error models for $\dot{\tilde\eta}_1$, $\dot{\tilde\eta}_2$ and $\dot{\tilde R}$ obtained by replacing (18) in (20)-(22). Solving the resulted differential equations we immediately see that

$$\phi(t)\notin\mathcal{L}_2\ \Leftrightarrow\ \lim_{t\to\infty}\tilde\eta_{1,2}(t)=0,\ \lim_{t\to\infty}\tilde R(t)=0 \tag{24}$$

and, using a well-known property of global exponential stability from [11]

$$\begin{aligned}&\phi(t)\in\text{PE}\ \Leftrightarrow\\ &|\tilde\eta_{1,2}(t)|\le\rho e^{-\delta t}|\tilde\eta_{1,2}(0)|,\ |\tilde R(t)|\le\rho e^{-\delta t}|\tilde R(0)|,\end{aligned} \tag{25}$$

for some $\rho\ge1$ and $\delta>0$.

The claim of the flux observer convergence is established recalling (19), from which it is clear that $\hat\lambda$ inherits the same convergence properties as parameter estimators (20)-(22). ■

IV. Speed and Position Observer

Following [4] we introduce the vector $x=\lambda_m\mathcal{C}(\theta)$ and we can suppose that x is available for measurement by

$$\hat x=\hat\lambda-Li. \tag{26}$$

From (2) we have

$$\omega=\frac{1}{Jp+B}\tau_e-\frac{\tau_L}{B}+\varepsilon(t)=\psi_1(t)+\beta+\varepsilon(t), \tag{27}$$

where

$$\psi_1(t)=\frac{1}{Jp+B}\tau_e \tag{28}$$

is obtained replacing (19) in (4), $\beta=-\tau_L/B$, and the exponentially decaying term $\varepsilon(t)$.

The speed observer may be designed as

$$\hat\omega\ =\ \psi_1(t)+\hat\beta, \tag{29}$$

where $\hat\beta$ is an estimate of β that is to be designed later.

Consequently, the position observer may be chosen as

$$\hat\theta\ =\ \int_0^t\hat\omega(s)ds+\hat\theta_0=\psi_2+\hat\theta_0, \tag{30}$$

where $\psi_2(t) = \int_0^t \hat{\omega}(s)ds$ and $\hat{\theta}_0$ is a difference between real θ and $\int_0^t \hat{\omega}(s)ds$ that is to be designed later. Note that

$$\begin{aligned}\theta_0 :&= \theta(t) - \int_0^t \hat{\omega}(s)ds \\ &= \theta(0) + \int_0^t \omega(s)ds - \int_0^t \hat{\omega}(s)ds \\ &= \theta(0) + \int_0^t (\omega(s) - \hat{\omega}(s))ds. \end{aligned} \tag{31}$$

V. Second Parameterization

Substitution (27) into (7) yields

$$L\frac{di}{dt} = -Ri - (\psi_1(t) + \beta + \varepsilon(t))n_p\mathcal{J}x + v \tag{32}$$

Applying filters $\frac{\varsigma}{p+\varsigma}$ results in

$$y = \beta\psi_3 + \varepsilon(t) \tag{33}$$

where $y = L\frac{\varsigma p}{p+\varsigma}i - \frac{\varsigma}{p+\varsigma}(-Ri + v - \psi_1(t)n_p\mathcal{J}x)$ and $\psi_3 = \frac{\varsigma}{p+\varsigma}(n_p\mathcal{J}x)$.

Proposition 2: The update law

$$\dot{\hat{\beta}} = \gamma_\omega\hat{\psi}_3^\top(\hat{y} - \hat{\beta}\hat{\psi}_3),\ \gamma_\omega > 0, \tag{34}$$

$$\hat{y} = L\frac{\varsigma p}{p+\varsigma}i - \frac{\varsigma}{p+\varsigma}(-\hat{R}i + v - \psi_1 n_p\mathcal{J}\hat{x}), \tag{35}$$

$$\hat{\psi}_3 = \frac{\varsigma}{p+\varsigma}(n_p\mathcal{J}\hat{x}) \tag{36}$$

provides that $\hat{\beta}$ tends to the true value β under given assumptions, which guarantees asymptotic convergence of the position and the speed observers given by (30) and (29), correspondingly, if $\phi(t)$ is PE and $\hat{\theta}_0$ converges asymptotically.

Proof: First, we apply Proposition 1 that yields the exponential convergence of $\hat{\lambda}$, $\hat{R}$ and $\hat{\eta}$. Then, step by step, we conclude that the following functions are reconstructed asymptotically:

1) ψ_1 from (28) and (4) using assumption A2;
2) $\hat{y}$ using assumption A2 and the fact that multiplication of asymptotically convergent functions results in the same stability properties;
3) $\hat{\psi}_3$ recalling (26).

Next, supposing that ψ_3 preserves PE condition due to given $\phi(t)$ we establish asymptotic properties for $\hat{\beta}$ and, consequently, for the speed observer (29). Therefore, function ψ_2 in (30) is bounded. The proof is completed by definition of θ_0 presented in (31). ■

Thus, the speed observer is (29) and (34).

To use the position observer (30) we will need to find the estimate $\hat{\theta}_0$.

Let us remind that $x(t) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda_m \begin{pmatrix} \cos(n_p\theta) \\ \sin(n_p\theta) \end{pmatrix}$ is supposed to be known (already reconstructed using (26)) whence we are going to get θ without inverse trigonometric functions.

Immediately we have the estimate of the uncertain constant λ_m as

$$\hat{\lambda}_m = \sqrt{x_1^2 + x_2^2}. \tag{37}$$

Using $\theta = \theta_0 + \psi_2$ for the vector $x(t)$ we have

$$\begin{aligned} x(t) &= \lambda_m \begin{pmatrix} \cos(n_p(\theta_0 + \psi_2)) \\ \sin(n_p(\theta_0 + \psi_2)) \end{pmatrix} \\ &= \lambda_m \begin{pmatrix} \cos(n_p\psi_2) & -\sin(n_p\psi_2) \\ \sin(n_p\psi_2) & \cos(n_p\psi_2) \end{pmatrix} \begin{pmatrix} \cos(n_p\theta_0) \\ \sin(n_p\theta_0) \end{pmatrix}. \end{aligned} \tag{38}$$

Introduce the new vector

$$\bar{x} = \frac{1}{\hat{\lambda}_m} \begin{pmatrix} \cos(n_p\psi_2) & \sin(n_p\psi_2) \\ -\sin(n_p\psi_2) & \cos(n_p\psi_2) \end{pmatrix} x(t) \tag{39}$$

which is constant by definition

$$\bar{x} = \begin{pmatrix} \cos(n_p\theta_0) \\ \sin(n_p\theta_0) \end{pmatrix} \tag{40}$$

Consider two observers $\hat{x}_1 = \cos(n_p\hat{\theta}_0)$, $\hat{x}_2 = \sin(n_p\hat{\theta}_0)$ and two update laws

$$\dot{\hat{k}}_1(t) = \gamma_1(-\bar{x}_1 + \cos(n_p\hat{k}_1(t))), \tag{41}$$

$$\dot{\hat{k}}_2(t) = \gamma_2(\bar{x}_2 - \sin(n_p\hat{k}_2(t))), \tag{42}$$

where $\hat{k}_1$ and $\hat{k}_2$ are estimates of θ_0

For errors $\tilde{x}_1 = \bar{x}_1 - \hat{x}_1$ and $\tilde{x}_2 = \bar{x}_2 - \hat{x}_2$ one can compute

$$\dot{\tilde{x}}_1 = -\gamma_1\left[\sin(n_p\hat{k}_1(t))\right]\tilde{x}_1, \tag{43}$$

$$\dot{\tilde{x}}_2 = -\gamma_2\left[\cos(n_p\hat{k}_2(t))\right]\tilde{x}_2. \tag{44}$$

If the angle θ_0 is in the interval $(0; \frac{\pi}{n_p})$ and the estimate $\hat{k}_1$ is projected to this interval then $\sin(n_p\hat{k}_1(t)) > 0$ and $\tilde{x}_1$ asymptotically converges to zero which means that $\hat{k}_1$ converges to k_1. If the angle θ_0 is in the interval $(\frac{\pi}{n_p}; \frac{2\pi}{n_p})$ and the estimate $\hat{k}_1$ is projected to this interval then $\sin(n_p\hat{k}_1(t)) < 0$. Let us make a minor modification of the update law (41)

$$\dot{\hat{k}}_1(t) = \gamma_1(-\bar{x}_1 + \cos(n_p\hat{k}_1(t)))\,\textit{sign}\bar{x}_2, \tag{45}$$

which yields

$$\dot{\tilde{x}}_1 = -\gamma_1\left[\sin(n_p\hat{k}_1(t))\,\textit{sign}\bar{x}_2\right]\tilde{x}_1. \tag{46}$$

If the estimate $\hat{k}_1$ is projected to a necessary interval either $(0; \frac{\pi}{n_p})$ or $(\frac{\pi}{n_p}; \frac{2\pi}{n_p})$ which is defined by sign of $\bar{x}_2$ then $\left[\sin(n_p\hat{k}_1(t))\,\textit{sign}\bar{x}_1\right]$ is always positive definite and the model (46) is asymptotically stable. Note that at the points $\theta_0 = 0$ and $\theta_0 = \frac{\pi}{n_p}$ the system (46) is neutrally stable.

Obviously that signs of $\bar{x}_1$ and $\bar{x}_2$ define in which quadrant the angle $n_p\theta_0$ is situated. Denote 4 sets

$$\begin{aligned} \mathcal{A}_1 &= \left(0; \frac{\pi}{n_p}\right),\ \mathcal{A}_2 = \left(\frac{\pi}{n_p}; \frac{2\pi}{n_p}\right), \\ \mathcal{A}_3 &= \left[0; \frac{\pi}{2n_p}\right) \cup \left(\frac{3\pi}{2n_p}; \frac{2\pi}{n_p}\right),\ \mathcal{A}_4 = \left(\frac{\pi}{2n_p}; \frac{3\pi}{2n_p}\right). \end{aligned}$$

and establish two update laws

$$\dot{\hat{k}}_1(t) = \gamma_1(-\bar{x}_1 + \cos(n_p\hat{k}_1(t)))\, sign\bar{x}_2,$$
$$\begin{cases} \hat{k}_1 \in \mathcal{A}_1 \text{ if } \bar{x}_2 \geq 0 \\ \hat{k}_1 \in \mathcal{A}_2 \text{ if } \bar{x}_2 < 0 \end{cases} \tag{47}$$
$$\dot{\hat{k}}_2(t) = \gamma_2(\bar{x}_2 - \sin(n_p\hat{k}_2(t)))\, sign\bar{x}_1,$$
$$\begin{cases} \hat{k}_2 \in \mathcal{A}_3 \text{ if } \bar{x}_1 \geq 0 \\ \hat{k}_2 \in \mathcal{A}_4 \text{ if } \bar{x}_1 < 0 \end{cases} \tag{48}$$

which give two estimates of θ_0. Note that both observers are not converging on whole interval $\left[0; \frac{2\pi}{n_p}\right)$. To cover full set of definition for θ_0 the following algorithm is proposed

$$\hat{\theta}_0(t) = \begin{cases} \hat{k}_1(t) \text{ if } |\bar{x}_1| \leq |\bar{x}_2| \\ \hat{k}_2(t) \text{ if } |\bar{x}_1| > |\bar{x}_2| \end{cases} \tag{49}$$

The most important feature of such approach is the possibility to tune the accuracy by γ_1 and γ_2 if the measured vector $\bar{x}$ is corrupted by noise and other additional terms. The direct inverse trigonometric functions are very sensitive to the latter and can not give a relevant estimates.

VI. SIMULATION RESULTS

The objective of simulations is to verify the performance and robustness of the new observer with R estimator for different gain settings. Here we use the motor BMP0701F taken from [6] with parameters listed in Table I. The motor is driven in speed control mode and the examination of the observer is carried out in open-loop design.

Parameter (units)	BMP0701F
Inductance L (mH)	40.03
Resistance R (Ω)	8.875
Drive inertia J (kgm^2)	60×10^{-6}
Pairs of poles n_p (–)	5
Magnetic flux λ_m (Wb)	0.2086
Maximum current I (A)	2.3

TABLE I: Parameters of the motor BMP0701F

Quantities displayed in simulations are the following: parameter estimates $\hat{R}$, $\hat{\eta}_1$ and $\hat{\eta}_2$, generated by DREM estimator, estimate $\hat{\beta}$ given by (34) and position and speed errors $\tilde{\theta}$ and $\tilde{\omega}$.

The reference speed for both tests is shown in Fig. 1. The motor operates under a constant load torque $\tau_L = 1$ Nm and the viscous friction coefficient $B = 0.01$ Nms. Initial states of the motor and observer are set to zero. Parameters of LTI filters in DREM estimator are $\varepsilon_1 = 10$, $\varepsilon_2 = 70$, $\varepsilon_3 = 130$, $\varepsilon_4 = 200$, $\varepsilon_5 = 260$.

Fig. 2 demonstrates the observer behaviour for the first set of design parameters. As seen from the figure, the estimates of stator resistance $\hat{R}$ and parameters $\hat{\eta}_1$ and $\hat{\eta}_2$ converge fast to their true values introducing no oscillations. The observer shows good performance with convergence of position and speed errors to zero. In Fig. 3 the adaptation gain γ_ω is reduced while the rest design parameters take increased values. One can see the transient behaviour of the proposed observer is similar to the previous test.

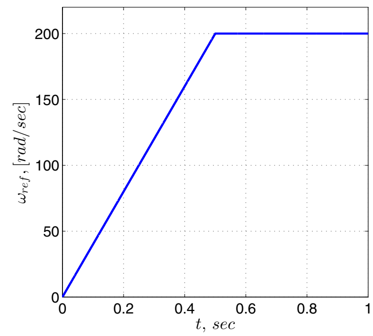


Fig. 1: Speed reference

VII. CONCLUSIONS

In this paper a new nonlinear parameterization of the PMSM model is proposed for the case of an uncertain stator resistance. The assumption of known inductance only is applied. After parameterization the regression model of six parameters is obtained from which it becomes possible to reconstruct the resistance and two necessary parameters involved in the position and speed observers design. The dynamic regressor extension and mixing (DREM) estimator is used to provide good performance and fast estimation of a large regression model which is preferable than the standard gradient approach. Simulation results illustrating proposed approach are given.

REFERENCES

[1] P.P. Acarnley and J.F. Watson, Review of position-sensorless operation of brushless permanent-magnet machines, *IEEE Trans Industrial Electronics,* Vol. 53, No. 2, pp. 352–362, April, 2006.

[2] S. Aranovskiy, A. Bobtsov, R. Ortega, A. Pyrkin, Performance enhancement of parameter estimators via dynamic regressor extension and mixing, *IEEE Transactions on Automatic Control* 2016. DOI: 10.1109/TAC.2016.2614889

[3] S. Aranovskiy, A. Bobtsov, R. Ortega, A. Pyrkin, A. Chaillet, Flux and position observer of permanent magnet synchronous motors with relaxed persistency of excitation conditions, *1st IFAC Conf. on Modelling, Identification and Control of Nonlinear Systems*, St. Petersburg, Russia, June 24–26, pp. 311-316, 2015.

[4] A.A. Bobtsov, A.A. Pyrkin, R. Ortega, S.N. Vukosavic, A.M. Stankovic, E.V. Panteley, A robust globally convergent position observer for the permanent magnet synchronous motor. *Automatica*, vol. 61, pp. 47-54, 2015.

[5] A. Bobtsov, D. Bazylev, A. Pyrkin, S. Aranovskiy, R. Ortega, A robust nonlinear position observer for synchronous motors with relaxed excitation conditions *Int. Journal of Control*, vol. 90, pp. 813-824, 2017.

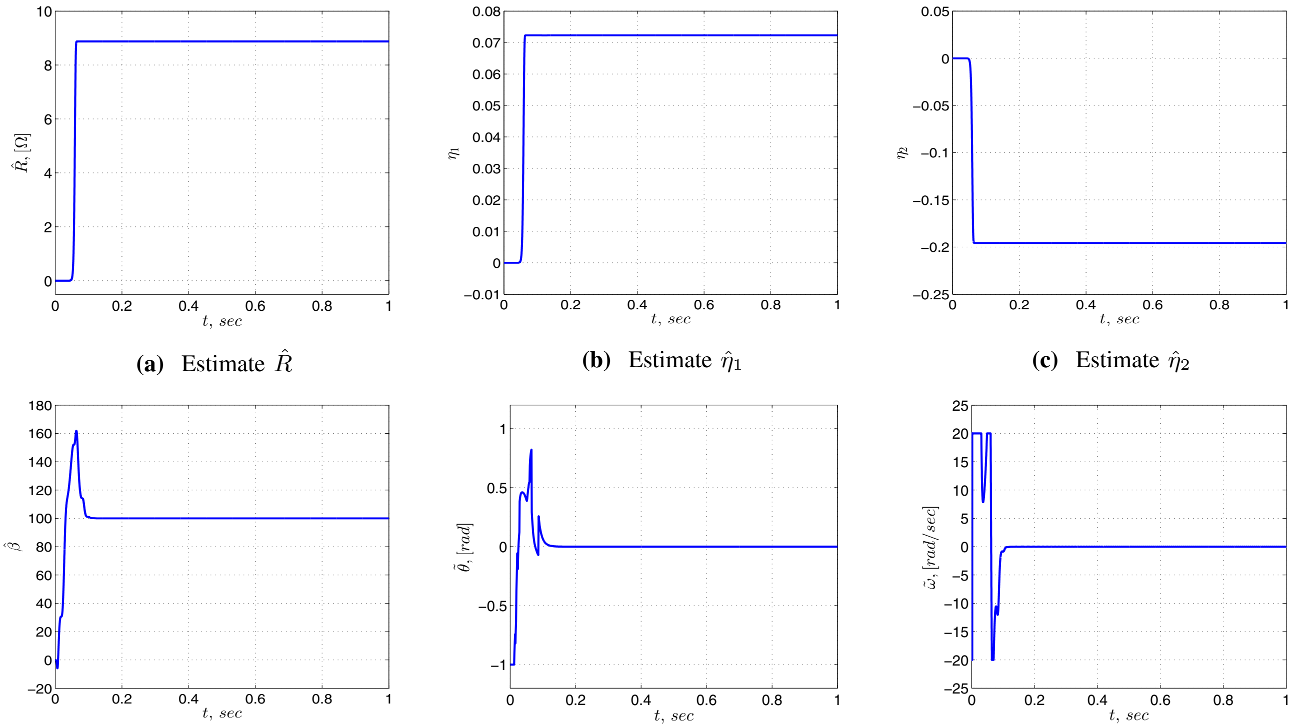
[6] BMP Synchronous motor, Motor manual, V1.00. (2012), *Schneider Electric*, 12.2012.

[7] W. Dib, R. Ortega and J. Malaize, Sensorless control of permanent-magnet synchronous motor in automotive applications: Estimation of the angular position, *IECON 2011–37th Annual Conference on IEEE Industrial Electronics Society*, Melbourne, Australia, 2011, pp. 728–733.

[8] P.C. Krause, *Analysis of electric machinery*, McGraw Hill, New York, 1986.

[9] J. Lee, K. Nam, R. Ortega, L. Praly and A. Astolfi, Sensorless control incorporating a nonlinear observer for surface–mount permanent magnet synchronous motors, *IEEE Transactions on Power Electronics*, Vol. 25, No. 2, pp. 290–297, 2010.

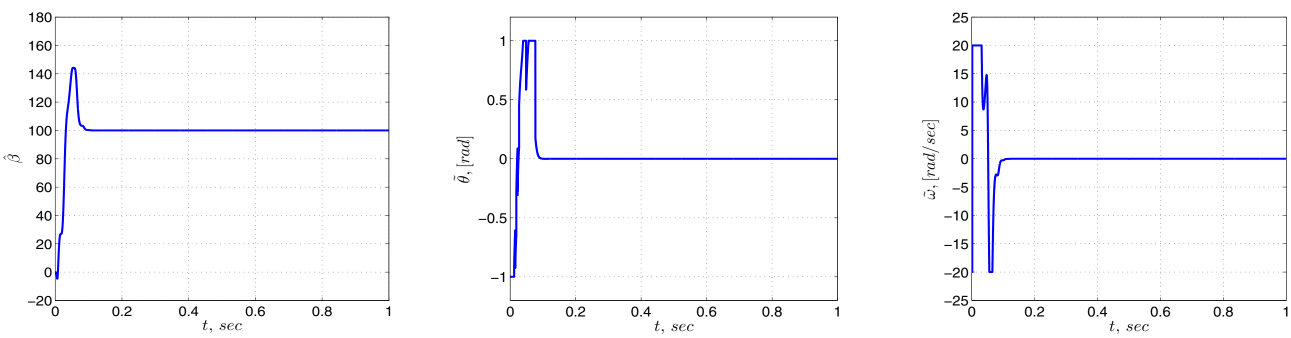
[10] L. Ljung, *Systems Identification: Theory for the User*, Prentice Hall, 1987.



(a) Estimate $\hat{R}$ **(b)** Estimate $\hat{\eta}_1$ **(c)** Estimate $\hat{\eta}_2$

(d) Estimate $\hat{\beta}$ **(e)** Error $\tilde{\theta}$ **(f)** Error $\tilde{\omega}$

Fig. 2: Transients with $\varsigma = 500$, $\gamma_{1,2} = \gamma_r = \gamma_{\eta_1,\eta_2} = 100$, $\gamma_\omega = 400$, $\alpha = 100$.



(a) Estimate $\hat{\beta}$ **(b)** Error $\tilde{\theta}$ **(c)** Error $\tilde{\omega}$

Fig. 3: Transients with $\varsigma = 700$, $\gamma_{1,2} = \gamma_r = \gamma_{\eta_1,\eta_2} = 200$, $\gamma_\omega = 300$, $\alpha = 200$.

[11] S. Sastry and M. Bodson, Adaptive Control: Stability, Convergence and Robustness. Prentice Hall, Englewood Cliffs, NJ, 1989

[12] M. Hinkkanen, T. Tuovinen, L. Harnefors and J. Luomi, A combined position and stator-resistance observer for salient pmsm drives: design and stability analysis. IEEE Trans. on Power Electronics, 27(2), pp. 601-609, 2012

[13] J. Malaize, L. Praly and N. Henwood, Globally convergent nonlinear observer for the sensorless control of surface-mount permanent magnet synchronous machines, *51st IEEE Conference on Decision and Control (CDC'12)*, Maui, Hawaii, pp. 5900–5905, 2012.

[14] K. Nam, *AC motor control and electric vehicle applications*, CRC Press, 2010.

[15] R. Ortega, L. Praly, A. Astolfi, J. Lee and K. Nam, Estimation of rotor position and speed of permanent magnet synchronous motors with guaranteed stability, *IEEE Transaction on Control Systems Technology*, Vol. 19, No. 3, pp. 601–614, 2011.

[16] Ortega, R., Bobtsov, A., Pyrkin, A., and Aranovskiy, A. (2015) A parameter estimation approach to state observation of nonlinear systems, *Systems and Control Letters*, Vol. 85, pp 84–94.

[17] J. Romero, R. Ortega, M. Hernandez, T. Devos and F. Malrait, Adaptive Flux Observer for the Permanent Magnet Synchronous Motor, *Schneider electric Int. Report*, July 2014.

[18] S. Sastry and M. Bodson. *Adaptive Control: Stability, Convergence and Robustness*, Prentice Hall, Englewood Cliffs, NJ, 1989.

[19] D. Shah, G. Espinosa, R. Ortega and M. Hilairet, Sensorless speed control of non-salient permanent magnet synchronous motors, *Int. J. on Robust and Nonlinear Control*, Vol. 24, pp. 644–668, 2014.

[20] P. Tomei and C. Verrelli, Observer–based speed tracking control for sensorless PMSM with unknown torque, *IEEE Trans. Automat. Contr.*, Vol. 56, No. 6, pp. 1484–1488, 2011.

[21] C.M. Verrelli, P. Tomei, E. Lorenzani, G. Migliazza, and F. Immovilli, Nonlinear tracking control for sensorless permanent magnet synchronous motors with uncertainties, *Control Engineering Practice*, Vol. 60, No. 1, pp. 157–170, 2017.