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Embedded Iterative Learning Contouring Controller Based on Precise Estimation of Contour Error for CNC Machine Tools

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***Abstract*— In machine tool application, a machined shape of workpiece should be considered in machine tool control systems. The contour error relates to it directly. Although most existing contouring controllers are based on feedback control, this paper proposes an embedded iterative learning contouring controller (EILCC) by considering actual contour error (ACE) which is the minimum distance between actual position and desired trajectory. The proposed method modifies original trajectory by ACE compensation with a PID manner, iteratively. The proposed controller can be directly applied to existing commercial machines without any change of their original controllers. The proposed method has been verified by simulation and experiment in commercial CNC machine tool with a right-angled sharp-corner trajectory which normally produces a large contour error around the corner due to unsmoothness. Comparison with a previous EILCC without ACE was done to evaluate its performance. Experimental results have shown that the maximum contour error was reduced by 78.8 % and 9.7 % as compared to the typical feedback controller (FBC) and EILCC without ACE (with estimated contour error), respectively.**

I. INTRODUCTION

Due to advances in mechatronic systems, complex and highly accurate mechanical components with high curvature surface are often required. Machining process is one of the best choices to produce such components because of flexibility of product shape and material, high surface quality and accuracy. Hence, machine tool feed drive systems are required to precisely track desired contours of such complex mechanical components with high speed. The tracking is generally not so perfect that position errors remain [1]. Achieving high precision in machining highly depends on control performance of each individual feed drive axis [2]. Under independent drive axial control, load disturbance or performance variance of each axis causes contour error [3]. Much research is conducted in the past to reduce contour error because it is directly related to a product shape.

Cross-coupled methods for feed drive control with many techniques are proposed to reduce the contour error [4], [5]. In addition, many iterative learning controller (ILC) methods are designed to improve the performance in wide application [6], [7], [8], [9], [10], [11]. A contouring controller reduces the contour error through an iterative estimation of the instantaneous curvature of the reference trajectory and coordinates transformation [12]. Furthermore, an iterative learning controller (ILC) which considers both the tracking and contour error reduces contour error in a few iteration [13]. An embedded iterative learning contouring controller (EILCC) is proposed with contour error approximation by coordinate transformation and it could be applied to commercial machine without any change in an existing controller [14].

All of the above mentioned methods use contour error approximation to design contouring controller or modify reference trajectory [15]. For high curvature surface application, it requires many iterations to reach the required accuracy. Contouring control with the actual contour error is difficult to apply in real-time control system because the actual contour error cannot be obtained immediately for complex contour profiles. Hence, it requires contour error estimation to obtain approximated values. On the other hand, embedded iterative control system is possible to obtain actual contour error by offline calculation.

This research proposes an embedded iterative learning contouring controller (EILCC) by considering the actual contour error (ACE) as shown in Fig. 1. By calculating the minimum distance between an actual position and a reference trajectory, the ACE is used to achieve the desired contour precisely. The EILCC with ACE modifies the original trajectory (NC program) through an actual contour error as a learning controller. The modified trajectory is executed by a CNC machine to reduce the contour error, iteratively. The proposed method is proven to provide better performance than the EILCC without the ACE.

The rest of this paper is organized as follows: section II gives a description of the dynamic model of three axis machine tools, contour error definition, controller design, actual contour error, and convergence analysis. Simulation and experimental results which compare between the proposed and previous methods, are described in section III and IV, respectively, followed by concluding remarks in section V.

II. EILCC WITH ACE FOR MACHINE TOOLS

A. Dynamics Model of Three Axis Machine Tools

The dynamics of a three axis machine tools is represented as follows:

$$
\begin{aligned}
& M\ddot{q}+C\dot{q}=u, \\
& M=\mathrm{diag}\{m_i\},\ C=\mathrm{diag}\{c_i\}, \quad i=x,\ y,\ z, \\
& q=\left[q_x,\ q_y,\ q_z\right]^{\mathrm{T}},\ u=\left[u_x,\ u_y,\ u_z\right]^{\mathrm{T}},
\end{aligned} \tag{1}
$$

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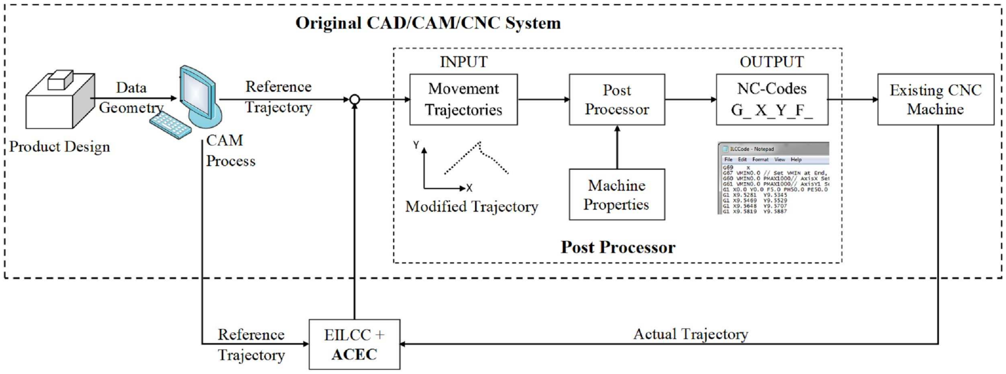


Fig. 1. Proposed EILCC with ACE concept.

where m_i, c_i, q_i, and u_i are the inertia, viscous friction coefficient, each axial position, control voltage for the axis i, respectively.

B. Definition of Contour Error

The contour error is defined as the shortest distance between the actual contour and the desired contour. The difference between tracking error and contour error in machine tools is shown in Fig. 2. The tracking error in each axis is the difference between the reference and actual positions. The reference position from the starting position of the three axis machine tools system at time t in the coordinate frame Σ_w are denoted by r. The closest position of the desired contour to q is denoted by r_c. The tracking error in each axis is defined as

$$e_w = \begin{bmatrix} e_x & e_y & e_z \end{bmatrix}^T = r - q, \tag{2}$$

The coordinate frame Σ_l is attached at r and its axis directional are $\mathfrak{T}$, $\mathfrak{N}$, and $\mathfrak{B}$, respectively. The axis $\mathfrak{T}$ is in the tangential direction of the reference trajectory at r, the direction of $\mathfrak{N}$ is perpendicular to $\mathfrak{T}$, and the $\mathfrak{B}$-axis is the bi-normal component to $\mathfrak{T}$ and $\mathfrak{N}$. The axial directions of frame Σ_l are calculated as follows

$$\mathfrak{T} = \begin{bmatrix} t_x & t_y & t_z \end{bmatrix}^T = \frac{\dot{r}}{\|\dot{r}\|}, \tag{3}$$

$$\mathfrak{N} = \begin{bmatrix} n_x & n_y & n_z \end{bmatrix}^T = \frac{\dot{\mathfrak{T}}}{\left\|\dot{\mathfrak{T}}\right\|}, \tag{4}$$

$$\mathfrak{B} = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^T = \mathfrak{T} \times \mathfrak{N}. \tag{5}$$

The tracking error vector e_l with respect to Σ_l can be expressed as

$$e_l = \begin{bmatrix} e_t & e_n & e_b \end{bmatrix}^T = L^T e_w, \tag{6}$$

$$L = \begin{bmatrix} t_x & n_x & b_x \\ t_y & n_y & b_y \\ t_z & n_z & b_z \end{bmatrix}^T. \tag{7}$$

The actual contour error is the shortest distance between the actual position q and the reference contour as follows:

$$e_c = \begin{bmatrix} e_{cx} & e_{cy} & e_{cz} \end{bmatrix}^T = r_c - q, \tag{8}$$

$$\grave{e} = \|e_c\|. \tag{9}$$

C. Actual Contour Error

ACE is a direct method to compensate for the actual contour error by finding minimum distance between the actual position and the reference trajectory as shown in Fig. 2. $\grave{q}$ is the candidate closest point which has defined same manner with contour error estimation by rotation matrix in our previous research [14]. $\grave{q}$ is represented as follows:

$$\grave{q} = q + L^{-1} \begin{bmatrix} 0 \\ e_n \\ e_b \end{bmatrix}. \tag{10}$$

Considering that the reference trajectory is represented by a set of discrete points r_l, $l = 1, \ldots, p$, where p is the

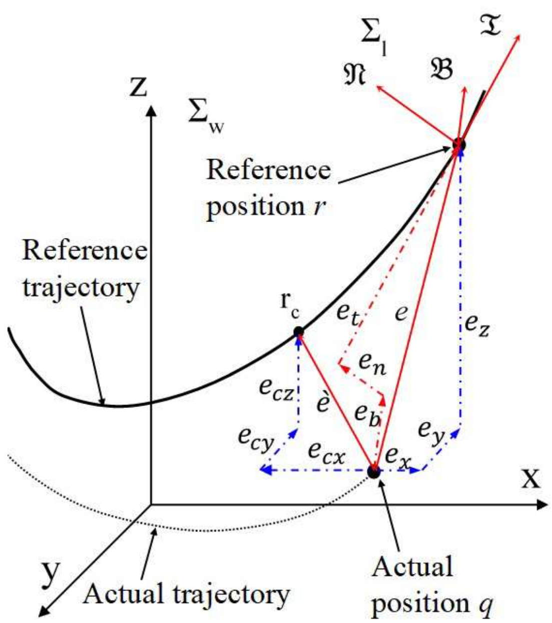


Fig. 2. 3D Tracking and contour errors.

reference point at the current sampling instant, the shortest distance $\grave{d}$ from $\grave{q}$ to the reference trajectory is calculated as follows:

$$\grave{d} = \min_{0<m<p} \left\| r_{p-m} - \grave{q} \right\|, \tag{11}$$

where r_{p-m} is the closest reference position to $\grave{q}$.

The approximated closest position of the desired contour to q is denoted by $\grave{r}_c$ which is obtained by finding minimum distance $\grave{d}_c$ between actual position q and the part of reference trajectory from $(p-m-k)^{\text{th}}$ to $(p-m+k)^{\text{th}}$ discrete positions where k is determined to avoid high computation process. $\grave{r}_c$ is calculated as follows

$$\grave{r}_c = q + \grave{d}_c, \tag{12}$$

$$\grave{d}_c = \min_{-k<h<k} \left\| r_{p-m+h} - q \right\|, \tag{13}$$

The actual closest position r_c is defined by contour error interpolation as shown in Fig. 3 and equations below:

$$r_c = \grave{r}_c + \frac{(\grave{r}_{c+1} - \grave{r}_c) D_1}{D_2}, \tag{14}$$

$$D_1 = \frac{\grave{d}_c^2 - \grave{d}_{c+1}^2 + \|\grave{r}_{c+1} - \grave{r}_c\|^2}{2\|\grave{r}_{c+1} - \grave{r}_c\|}, \tag{15}$$

$$D_2 = \|\grave{r}_{c+1} - \grave{r}_c\| - D_1, \tag{16}$$

where $\grave{r}_{c+1}$, $\grave{d}_{c+1}$, D_1, and D_2 are the discrete position next to the approximated closest discrete position $\grave{r}_c$, the shortest distance from $\grave{r}_{c+1}$ to actual position q, the distance between $\grave{r}_c$ and r_c, and the distance from r_c to $\grave{r}_{c+1}$, respectively.

D. Controller Design

The proposed controller for the three axis machine tool is based on the same design manner in our previous study [14]. The proposed controller consists of a feedback controller (FBC) and the EILCC with ACE. The input of the FBC is the modified reference trajectory by considering ACE as shown in Fig. 4, where "Mem", u_c, q, and r are the memories, learning compensation, actual position, and reference for x, y, and z axis. The transfer function of the plant including the FBC is represented as follows:

$$G_i(s) = \frac{P_i(s) K_i(s)}{1 + P_i(s) K_i(s)}, \tag{17}$$

where

$$P_i(s) = \frac{1}{m_i s^2 + c_i s}, \tag{18}$$

where G_i, P_i, K_i, and s are the transfer function from reference command to actual position, the plant, the feedback compensator, and the variable of the Laplace transform for the i^{th} axis, respectively. The FBC $K_i(s)$ is the PID as follows:

$$K_i(s) = K_{P_i}(s) + \frac{1}{s} K_{I_i}(s) + s K_{D_i}(s), \tag{19}$$

where K_{P_i}, K_{I_i}, and K_{D_i} are the proportional, integral, and derivative gains for the i^{th} axis, respectively. Reference signal r_{ij+1} is compensated by iterative learning gain with ACE represented as follows:

$$r_{ij+1}(t) = r_{i1}(t) + \sum_{l=0}^{j} u_{c_{il}}(t), \tag{20}$$

where

$$u_{c_{ij}}(t) = K_{Pl_i} e_{c_{ij}}(t) + K_{Il_i} \int_0^t e_{c_{ij}}(\tau) d\tau + K_{Dl_i} \dot{e}_{c_{ij}}(t), \tag{21}$$

where r_{ij}, $u_{c_{ij}}$, $e_{c_{ij}}$, K_{Pl_i}, K_{Il_i}, and K_{Dl_i} are reference signal, contouring compensated input, contour error for the j^{th} iteration, and the proportional, integral, and derivative learning compensator gain K_l for i^{th} axis, respectively.

E. Convergence Analysis

The convergence property of the EILCC with ACE can be guaranteed to ensure the contour error reduction. The convergence condition of ILC is represented as

$$\varepsilon_i(t) = \left\| \frac{e_{cij+1}(t)}{e_{cij}(t)} \right\| < 1, \tag{22}$$

where $\varepsilon_i(t)$, and e_{cij} and e_{cij+1} are the convergence factor and the contour errors in the j^{th} and $j+1^{\text{th}}$ iteration for i^{th}

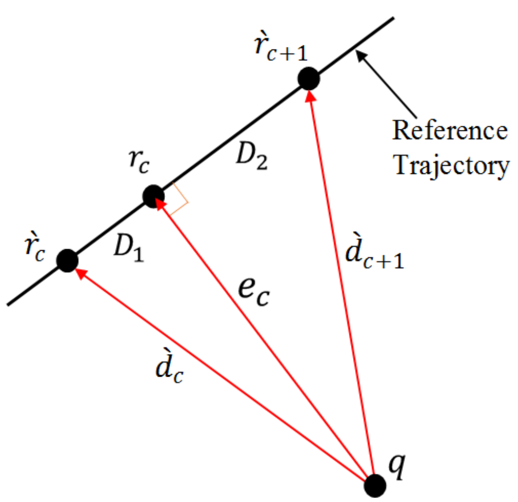


Fig. 3. Actual contour error interpolation.

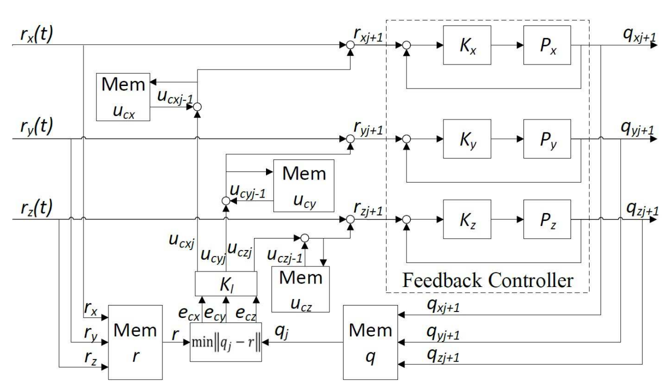


Fig. 4. Proposed EILCC controller.

axis. Based on the cascade ILC [16], the convergence can be guaranteed if the error magnitude at time t in iteration $j+1$ is smaller than the error magnitude in iteration j. From (8), (20) and (13), the convergence factor is derived as follows

$$
\begin{aligned}
e_{cij+1} &= \min\left(r_i - G_i r_{ij+1}\right), \\
&= \min\left\{r_i - G_i\left(r_{ij} + u_{c_{ij}}\right)\right\}, \\
&= \min\left(r_i - G_i r_{ij}\right) - G_i K_{l_i} e_{c_{ij}}, \\
&= e_{c_{ij}} - G_i K_{l_i} e_{c_{ij}}, \\
&= \left(1 - G_i K_{l_i}\right) e_{c_{ij}}, \\
\frac{e_{c_{ij+1}}}{e_{c_{ij}}} &= 1 - \frac{P_i K_i K_{l_i}}{1 + P_i K_i}. \qquad (23)
\end{aligned}
$$

Substituting (23) to (22) leads to

$$
\varepsilon_i = \left\| 1 - \frac{P_i K_i K_{l_i}}{1 + P_i K_i} \right\| < 1. \qquad (24)
$$

Considering the discrete-time form for the implementation, $P_i(s)$ is represented as follows

$$
P_i\left(z^{-1}\right) = \frac{1}{m_i\left(\frac{z^{-1}-1}{t_s z^{-1}}\right)^2 + c_i \frac{z^{-1}-1}{t_s z^{-1}}}, \quad z = e^{j\omega t_s}, \qquad (25)
$$

where z^{-1}, ω, and t_s are a delay operator, the angular frequency, and sampling time, respectively. The feedback and learning compensator in the z-domain are represented as,

$$
K_i\left(z^{-1}\right) = K_{Pi} + K_{Ii}\left(\frac{t_s z^{-1}}{z^{-1}-1}\right) + K_{Di}\left(\frac{z^{-1}-1}{t_s z^{-1}}\right), \qquad (26)
$$

$$
K_{li}\left(z^{-1}\right) = K_{Pli} + K_{Ili}\left(\frac{t_s z^{-1}}{z^{-1}-1}\right) + K_{Dli}\left(\frac{z^{-1}-1}{t_s z^{-1}}\right). \qquad (27)
$$

The convergence speed depends on the parameter ε which should be kept as minimum as possible for fast convergence with a delay factor η due to the system delay. Considering (24) and the delay, the following objective function for minimization is considered:

$$
J_i = \min_{K_{Pli},\, K_{Ili},\, K_{Dli}} \left\| 1 - \frac{z^{\eta} P_i\left(z^{-1}\right) K_i\left(z^{-1}\right) K_{l_i}\left(z^{-1}\right)}{1 + P_i\left(z^{-1}\right) K_i\left(z^{-1}\right)} \right\|_{\infty}, \quad \forall \omega \in \Omega, \qquad (28)
$$

where Ω is the considered domains of the operational frequency. (28) is solved by "fmincon" function in MATLAB® to find learning compensator gains.

III. Simulation

A. Simulation Condition

A three axis machine tools system (Fig. 6) with $m_i = [0.45\ 0.65\ 0.65]$ $\mathrm{Vs^2/mm}$ and $c_i = [0.144\ 0.24\ 0.24]$ Vs/mm was chosen for the simulation. Its table and ball screws in each axis are driven by AC servo motors. The simulation for a biaxial table (X-Y axis) was done by the MATLAB® software. The initial reference trajectory was defined in a G-code form to generate a right-angled sharp-corner trajectory.

The feedback controller gains were chosen from default gains of the existing machine as $[K_P,\ K_I,\ K_D] = [0.96\ \mathrm{V/mm},\ 0.01\ \mathrm{V/smm},\ 8\ \mathrm{Vs/mm}]$. The optimal learning compensator gains were calculated from (28) as $[K_{P_l},\ K_{I_l},\ K_{D_l}] = [0.468,\ 0.044,\ 0.443]$. Based on the defined feedback and learning controller gains, the convergence factor $\varepsilon_i = 0.6025$ for $0 < \Omega < 100$ Hz is obtained. The actual contour error (8) is calculated by (10) - (16). According to the optimized learning gain and the ACE, contour error compensation is conducted by (21) to modify the reference trajectory as in (20). The modified reference trajectory is applied to the original feedback controller. The above process is done iteratively until no significant contour error reduction is observed.

B. Simulation Results

The simulation was conducted in 4 times iterations for EILCC to compare the performance with/without ACE (with only estimated contour error in (6)). Simulation results are shown in Fig. 5 and Table I where in Figs. 5 (a) and (b) show the real trajectory for the EILCC without/with ACE, respectively. Figs. 5 (c) and (d) show the contour error for the EILCC without/with ACE, respectively. In all figures, the first iteration refers to the only feedback controller result. The feedback controller without any embedded system produces the largest contour error of 0.1082 mm. By applying the EILCC, contour error was reduced to 0.0214 mm which is equivalent to the error reduction of 80.22 %. The proposed controller exhibits the best performance as it has reduced the contour error to 0.0198 mm which is equivalent to an error reduction of 81.7 % and 7.5 % as compared to the feedback controller and the EILCC, respectively.

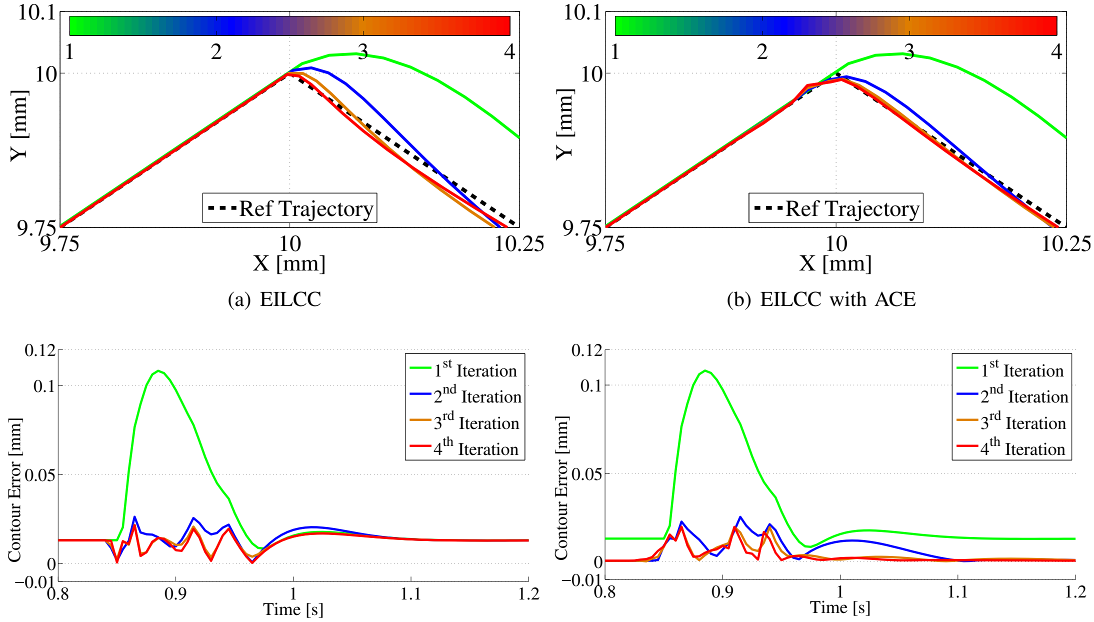
IV. Experiment

A. Experimental Condition

A three axis commercial CNC machine (Fig. 6.(a)) with ballscrew mechanisms attached to a table and three servo motors, which are controlled by Mitsubishi M70 controller, was used for experiment to verify the effectiveness of the proposed controller. The actual position of X-Y-Z table was measured by linear encoder with resolution 5 μm and microcontroller ATMega16 as interface attached to each axis. The EILCC was programmed in a separate personal computer (Windows OS) and embedded to the machine by the DNC system with RJ45 connection. The experimental interface design is shown in Fig. 7. The experimental scenario uses two types of controllers, the EILCC without/with ACE. Each scenario tracks a 3D sharp corner trajectory which is implemented in G-code form on the EILCC system.

B. Experiment Results

Experiments with 5 times iterations for the EILCC and 4 times iterations for the EILCC with ACE were conducted, and the minimum error is reached in the last iteration for both cases as shown in Fig. 8. Figs. 8 (a) and (b) show



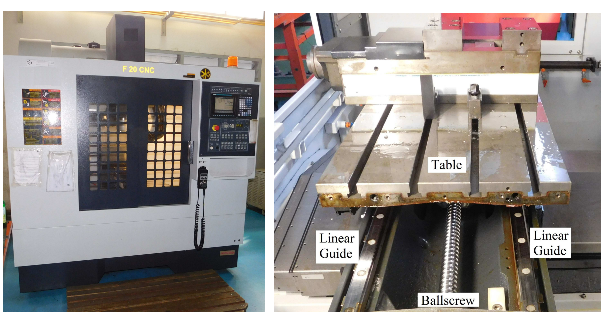
(a) EILCC (b) EILCC with ACE

(c) EILCC (d) EILCC with ACE

Fig. 5. Simulation results: (a) and (b) are real trajectory profiles; (c) and (d) are contour errors.

the real trajectories based on the EILCC without/with ACE, respectively. Figs. 8 (c) and (d) show the contour error based on the EILCC without/with ACE, respectively, around the sharp corner area (4 - 4.6 second). In both controllers, the first iteration is controlled only by the feedback controller. Under the EILCC without ACE, the initial maximum contour error was 0.132 mm which was reduced to 0.031 mm in the 5^{th} iteration. On the other hand, by the EILCC with ACE, the final maximum contour error was 0.028 mm in the 4^{th} iteration which is equivalent to the error reduction 78.8 % from the initial largest contour error.

All the simulation and experimental results are summarized in Table I, which shows the similar reduction tendency between simulation and experiment. The proposed method verified that a linear process assumption can be considered to converge the contour error, although actual process has nonlinear properties.



(a) CNC Machine (b) Table mechanism

Fig. 6. Three axis CNC machine.

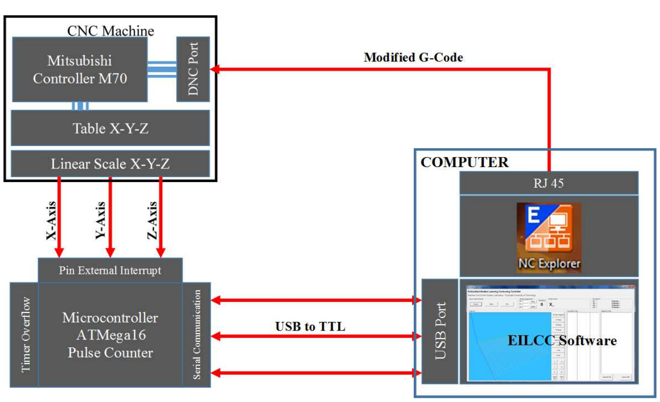


Fig. 7. Experimental interface design.

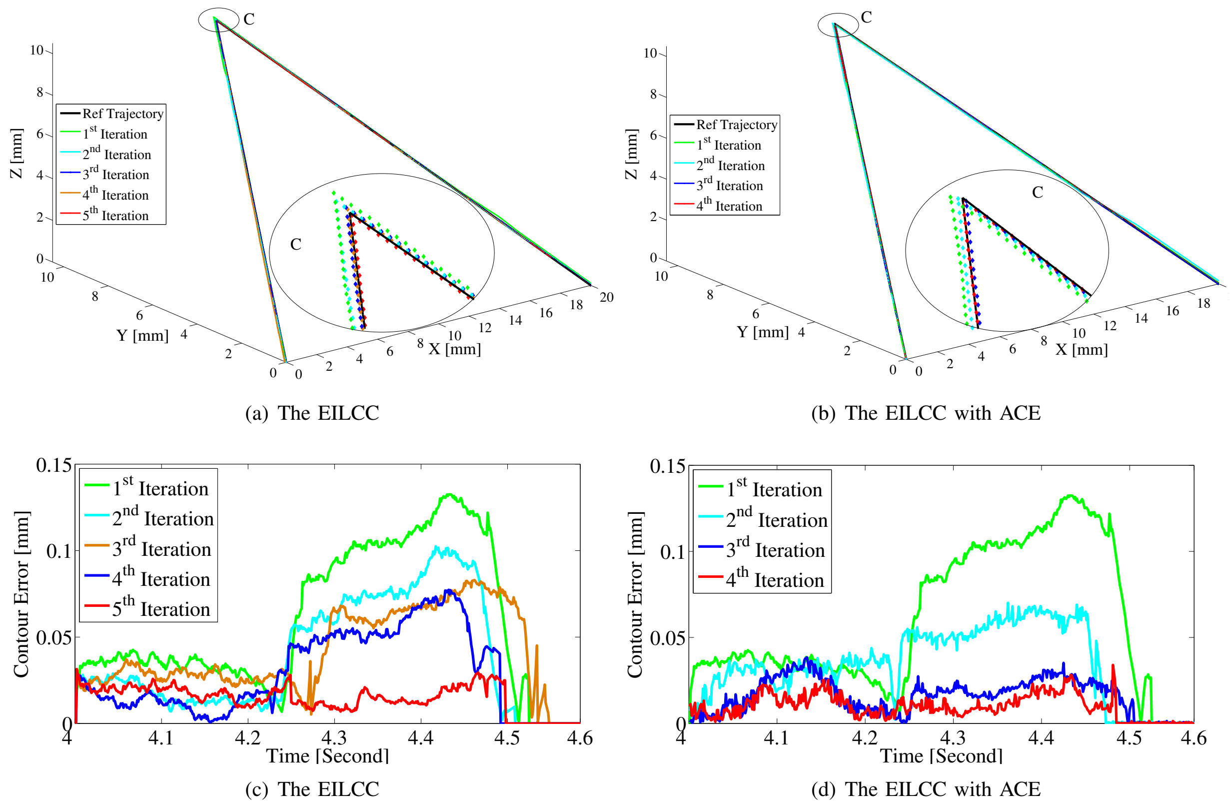
TABLE I

SIMULATION AND EXPERIMENT RESULTS

Controller	Contour Error [mm]			
	Simulation		Experiment	
	Max	Mean	Max	Mean
Feedback controller	0.1082	0.0149	0.132	0.037
EILCC without ACE	0.0214	0.013	0.031	0.009
EILCC with ACE	0.0198	0.012	0.028	0.006

V. CONCLUSION

An EILCC with ACE has been verified by simulation and experiment on a commercial CNC machine tool. The proposed method uses the actual contour error to enhance previous the EILCC performance. Simulation and experimental results show that the proposed method reduces the maximum contour error more effective than FBC and the



(a) The EILCC (b) The EILCC with ACE

(c) The EILCC (d) The EILCC with ACE

Fig. 8. Experiment results: (a) and (b) are real trajectory profiles; (c) and (d) are contour errors.

EILCC without ACE. In the future work, the EILCC with ACE will be developed to construct an ILC considering nonlinear process to enhance the contouring performance.

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