A Hybrid Systems Model Predictive Control Framework for AUV Motion Control

Rui Gomes[1] and Fernando Lobo Pereira[1]

Abstract— A computationally efficient architecture to control formations of Autonomous Underwater Vehicles (AUVs) is presented and discussed in this article. The proposed control structure enables the articulation of resources optimization with state feedback control while keeping the onboard computational burden very low. These properties are critical for AUVs systems as they operate in contexts of scarce resources and high uncertainty or variability. The hybrid nature of the controller enables different modes of operation, notably, in dealing with unanticipated obstacles. Optimization and feedback control are brought in by a novel Model Control Predictive (MPC) scheme constructed in such a way that time-invariant information is used as much as possible in a priori off-line computation.

I. INTRODUCTION

The main goal of this article consists in the design of a robust, computationally efficient, optimization-driven state-feedback controller for Autonomous Underwater Vehicles (AUVs). The research reported here constitutes in an improvement and refinement of the work from the authors in [13]. The MPC scheme considered here reduces the on-line computational burden by a priori processing off-line time-invariant data which will be stored onboard, and by using low computational mechanisms to use the stored data according to the on-line circumstances. This is the essential feature of the attainable set based MPC scheme first introduced in [14], [11]. In this way, the following pertinent control features for most of the mission scenarios involving AUVS are guaranteed: (i) optimization driven, (ii) state feedback, and (iii) computationally parsimonious. While the first feature is essential due to the scarcity of onboard resources, the second one is relevant as it allows to steer the AUV in environments variable in time. The last feature is important due to the real-time requirements, and to the relevance of sparing power in the light of the first item.

The motivation to investigate sophisticated control schemes relies on the need to satisfy increasingly complex requirements arising in (i) knowledge of biological, geophysical phenomena, (ii) Ocean and ocean-atmosphere interface monitoring, (iii) surveillance for security and defense, and (iv) territory management [24], [7], are some of the large classes of challenges that are currently being addressed.

Here, we will focus in a very simple context in which an AUV is endowed with motion adaptivity to unforeseen static environment features (i.e., features that were not accounted for in the planning stage), motion robustness to relatively small perturbations, and, simultaneously, a certain level of

sub-optimality can be guaranteed with a low computational budget and in "real-time".

This article is organized as follows: In the next section we provide a succinct overview of the relevant state-of-the-art. Then, we present and discuss the overall MPC AUV motion control problem formulation. Then, in section IV, we present and discuss the computationally efficient MPC scheme for AUV motion control and point out some of its properties. This section includes also a robust version of the basic scheme as well as some results on the required approximation procedures. In section V, we present and discuss the control architecture in which the proposed Attainable Set MPC scheme as well as some simulation results. The last section presents some conclusions and an outline of future work.

II. STATE-OF-THE-ART

Motion control problems have been studied from many angles and perspectives, even those arising from vehicles moving in fluids which are particularly challenging: distributed nature, nonlinearities, significant uncertainties and perturbations. A very good reference is [8]. It is not surprising the huge extent of pertinent literature that, obviously, can not be covered in this section. Moreover, extended versions of these control systems have been considered for multiple vehicles. Non-linear control theory offers important results underlying popular design techniques. As a small sample we consider [19] giving a review of existing methods on tight spacecraft formation flying using state feedback, [27] discusses an approach for formation control using a virtual structure, and [22] presents a back-stepping controller robust to input constraints and parameter uncertainties for spacecraft formations.

MPC is a well-known control strategy that defines a control strategy by solving sequences of finite horizon open-loop optimal control problems in a receding horizon fashion. The optimal control framework endows MPC schemes with a huge flexibility in handling control system with complex dynamics while subject to constraints. As seen above, optimization based control is essential for AUVs control. A selected sample of MPC schemes addressing a wide variety of issues is: [10], [9] address underwater communication constraints for formation control in a context of cooperative control of a team of distributed agents, [16] and [17] discuss a decentralized scheme for the coordinated control of formations, being invariance and stability used to ensure collision avoidance, [6] concerns leader-follower formations in which the control inputs are forced to satisfy suitable constraints that restrict the set of the leader possible paths by

[1]SYSTEC, Faculty of Engineering, Porto University, 4200-465 Porto, Portugal `rgomes@fe.up.pt`, `flp@fe.up.pt`

using fast-marching methods, and [29] path following control uses multi-objective nonlinear MPC for path following, and trajectory tracking.

In spite of the much faster dynamics, MPC schemes have also been used to control Unmanned Air Vehicles (UAVs): [4] designed a distributed nonlinear controller for collision-free formation flight, [26] addressed applications of output-feedback for the problem of two UAVs tracking an evasive moving ground vehicle, [3] - comprehensive framework for the cooperative guidance of fleets of autonomous vehicles combining collision and obstacle avoidance, formation flying and area exploration, and [1] designed a path tracking controller a tilt-rotor UAV carrying a suspended load.

However, all these MPC approaches suffer from several key drawbacks for AUVs: (i) computationally intensive nature, and (ii) parametrization to ensure convergence and stability that does not take into account onboard sensing capabilities. We address these key issues in this article.

III. AUV TRACKING CONTROL PROBLEM

In this section we consider the problem optimizing the motion of an AUV in the sense that the integral error with respect to a given reference trajectory and the total control effort during a certain period of time should be minimized. The optimal control framework is extremely appropriate to address this problem as well as the various possibilities of extending it to the context of formations of multiple vehicles.

The general optimal control formulation is

$$(P) \text{ Minimize} \quad g(x(t_0 + T)) + \int_{t_0}^{t_0+T} f_0(t, x(t), u(t))dt$$

$$\text{subject to} \quad \dot{x}(t) = f(t, x(t), u(t)) \quad \mathcal{L} - a.e.$$

$$u(t) \in \Omega \quad \mathcal{L} - a.e., \quad x(t_0 + T) \in C_f$$

$$h(t, x(t)) \leq 0, \quad g(t, x(t), u(t)) \leq 0$$

where g is the endpoint cost functional, f_0 is the running cost integrand, f, h, and g represent, respectively, the vehicle dynamics, the state constraints, and the mixed constraints, C is a target that may also be specified in order to ensure stability.

To see how (P) encompasses the AUV formation of N vehicles motion problem of tracking of a given reference trajectory, consider, for the AUV i,

- $x = col(\eta^i, \nu^i)$, $u = \tau^i$, and $g(\cdot) = 0$,
- $f_0(t, x, u) = (\eta^i(t) - \eta_r^i)^T Q(\eta^i - \eta_r^i) + \tau^{iT} R \tau^i)$, where $\eta_r^i(\cdot)$ is the reference trajectory for the ith vehicle.
- By letting for each vehicle (we drop the index i for easier reading) the state be $\eta = [x, y, \psi]^T$ and $\nu = [u, v, r]^T$, and the controls be $\tau = [\tau_u, \tau_r]^T$, the dynamics are, [8] and [25], where $\dot{\eta}$, and $\dot{\nu}$ satisfy:

$$\begin{bmatrix} u\cos(\psi) - v\sin(\psi) \\ u\sin(\psi) + v\cos(\psi) \\ r \end{bmatrix}, \text{ and } \begin{bmatrix} \frac{\tau_u - (m-Y_{\dot{v}})vr - X_{u|u|}u|u|}{m - X_{\dot{u}}} \\ \frac{(m-X_{\dot{u}})ur - Y_{v|v|}v|v|}{m - Y_{\dot{v}}} \\ \frac{\tau_r - (Y_{\dot{v}} - X_{\dot{u}})uv - N_{r|r|}r|r|}{I_{zz} - N_{\dot{r}}} \end{bmatrix},$$

respectively. Here, $X_{\dot{u}}, Y_{\dot{v}}, N_{\dot{r}}$ the hydrodynamic added mass coefficients, $X_{u|u|}, Y_{v|v|}, N_{r|r|}$ the hydrodynamic drag coefficients, and m the vehicle mass.

- This accommodates a wide variety of other constraint types, notably,
  - time endpoints of the state, $\eta^i(t + T) \in C_{t+T}$,
  - control, $\tau^i(s) \in \mathcal{U}^i$,
  - state variable, $(\eta^i(s), \nu^i(s)) \in \mathcal{S}^i$;
  - communication $g_{i,j}^c(\eta^i(s), \eta^j(s)) \in C_{i,j}^c, \forall j \in \mathcal{G}^c(i)$, and
  - formation $g_{i,j}^f(\eta^i(s), \eta^j(s)) \in C_{i,j}^f, \forall j \in \mathcal{G}^f(i)$.

See [12] for additional details on optimal control of AUV formations.

The MPC enables to conciliate, sub-optimization with feedback control. This is relevant due to the need to deal with the combination of the onboard resources scarcity with the significant variability and uncertainty encountered in the underwater milieu. The MPC scheme consists in, at each sampling time, computing the control action for the current control horizon by solving the on-line optimal control problem (P) over the larger prediction horizon with the state variable initialized at the current best estimate updated with the latest sampled value. Then, the optimal control sequence is applied to the plant during the control horizon and, once this interval elapses the new current time is updated and the process is repeated. Details, including implementation, of this standard MPC algorithm can be found in [11].

Unfortunately, computational complexity of (P) is very high, and, thus, of difficult online implementation. The issue is further aggravated for decentralised implementations required by many practical applications. This hard challenge motivates the novel approach which is an improved and more refined version of the one proposed in [13].

IV. ATTAINABLE SET MPC

In this section we present additional results concerning the attainable set based MPC scheme discussed in [13]. There are two key ideas underlying this novel scheme:

- Replace the infinite dimensional optimal control problem by a finite dimensional one. This requires two steps: (i) time backward propagation of the overall the cost functional, and (ii) time forward propagation the attainable set starting on the current value of the sampled state variable.
- Use time-invariant data to pre-compute off-line all the ingredients - notably a reference short term attainable set, and the value of the value function in an appropriate grid of points - required on-line (feedback synthesis) as a function of a number of parameters that depend on a number of typified circumstances.

  This data is stored onboard in a look-up table, and the operations (rotations, translations, and extrapolations) to adapt on-line either the attainable set or the value function to the current circumstances (t, x) are computationally inexpensive.

A. Short term "equivalent" cost functional and Attainable Sets

Let us consider the optimization of a dynamic control system over a long time horizon $[0, T]$, with, possibly $T =$

∞. In this later case, we consider trajectories converging asymptotically to some equilibrium point.

By defining $V(t,z) := \min\limits_{u \in \mathcal{U}, \xi \in C_f} \{g(\xi) + \int_t^{T_f} l(\tau, x(\tau), u(\tau)) d\tau\}$ with $x(T_f) = \xi$, $x(t) = z$, $\dot{x}(\tau) = f(\tau, x(\tau), u(\tau))$, $\mathcal{L}$-a.e. and by taking into account the Principle of Optimality (i.e., for $T < T_f$, the solution to (P_{T_f}) restricted to the interval $[t, t+T]$ is also a solution to (P_T)), we establish the equivalence on the interval $[t, t+T]$ of the problems:

(P_T) Min $\quad V(t+T, x(t+T)) + \int_t^{t+T} l(\tau, x(\tau), u(\tau)) d\tau$

s. t. $\quad \dot{x}(\tau) = f(\tau, x(\tau), u(\tau)), \quad \mathcal{L} - a.e.$

$\quad u \in \mathcal{U}$, and $x(t)$ is given,

and

(P_{T_f}) Minimize $\quad g(x(T_f)) + \int_t^{T_f} l(t, x(t), u(t)) dt$

subject to $\quad \dot{x}(t) = f(t, x(t), u(t)), \mathcal{L} - a.e.$

$\quad x(T_f) \in C_f, \ x(t)$ is given, $u \in \mathcal{U}$.

where $T < T_f$, and $\mathcal{U} := \{u : [0, T_f] \to \mathbf{R}^m : u(t) \in \Omega\}$, with Ω closed.

In what follows, we need the definition Forward Attainable Set (see [15], [30], [21]). Let $t_0 < t$,

$$\mathcal{A}_f(t; t_0, x_0) := \{x(t) : \dot{x} = f(t, x, u), u \in \mathcal{U}, x(t_0) = x_0\}.$$

Notice further that, with a standard change of variable $\dot{y} = l(t, x, u)$, with $y(0) = 0$, the running cost can be eliminated. Let $\tilde{V}(t, \tilde{x}) = V(t, x) + y$ where $\tilde{x} = (x, y)$. Without relabeling, (P_T) can be formulated only in terms of the Forward Attainable Set and the Value Function.

(P_T^a) Minimize $\quad V(t+T, x(t+T))$

subject to $\quad x(t+T) \in \mathcal{A}_f(t+T; t, x(t))$.

Two important remarks are in order.

- The computation of the Attainable Set is, in general, an extremely costly task, generating large amount of data. This leads to the need of resorting to some kind of approximation. There are three main possibilities: (i) Polyhedral which may be of either inner or outer type, [2], [15]; (ii) Ellipsoidal, [20]; and (iii) Cloud of points as endpoints of trajectory segments generated by constant controls. While the first two seem to be of lower complexity due to their reduced number of parameters, the high on-line computation that they entail, makes the third alternative more attractive given the strict real-time constrains.
- For positional systems, [18], - this is not a very restrictive requirement - the value function may be computed by solving the Hamilton-Jacobi equation (HJE). In general, the value function is, at most, merely continuous, and, thus, the partial derivatives have to be understood in a generalized sense, and the solution concept has to be cast in a nonsmooth context. For details and multiple possibilities, check [5]. Solving HJE numerically is extremely computationally intensive for which here are a number of software packages, e.g., [28], [23].

In practical situations, we consider a number of value functions each one associated with a reasonable number of typified situations (which are strongly application dependent). During the real-time execution of the "mission", the relevant value function is identified via sensed data and invoked to determine the next optimal control at any point (t, x). Proper extrapolation techniques can be use to handle situations that do not fully fall in any one of the considered typified situations. "Extraordinary" events, such as the emergence of unmapped obstacles, imposes the need to change the value function. This will be considered in the next section.

Let T be the optimization horizon, Δ the control horizon, and t the current time. Then, the Attainable Set MPC (AS-MPC) scheme is as follows:

1. Initialization: $t = t_0$, $x(t_0)$
2. Solve (P_T^a) over $[t, t+T]$ to obtain u^*
3. Apply u^* during $[t, t+\Delta]$
4. Sample x at $t + \Delta$ to obtain $\bar{x} = x(t+\Delta)$
5. Slide time, i.e., $t = t + \Delta$, update the Attainable Set from the new $x(t)$ by appropriate translation and rotation, update the value function at the new $t + T$ if necessary, and goto 2.

Clearly, the real-time computational burden of this scheme is, in general, very low as it involves only very simple computational operations.

The simplicity of the on-line optimization problem is due to the fact that most of the computational burden is transferred to the off-line stage and this is due to the time invariance of the problem's data. In figure 1, it is shown (i) the forward attainable set for the unicycle, and (ii), the value function in the absence of obstacles. Here, the value function is computed by using its definition, i.e., by solving several optimal control problems with different initial conditions spanning the given state space region of 4 m by 10 m. The value function isolines are shown with the darkest colors closer to the minimum, i.e., the target. Each problem is of the type of (P_T) and took approximately 3 seconds to compute by ACADO solver on a i7-7500CPU @ 2.70GHz computer and gave rise to a set of trajectories starting from the state space partition points and converging to the final target. However, computing the optimal trajectory by solving (P_T^a), requires only 0.05 seconds, i.e., it is 60 times faster. The controls to be applied to the vehicle are found by
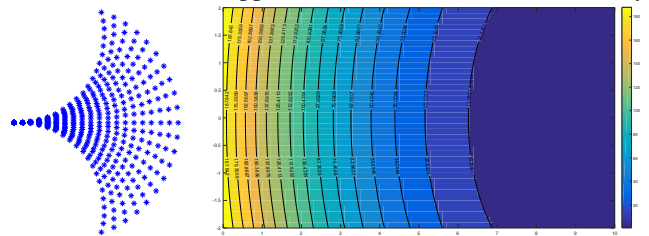

Fig. 1. i) AUV forward attainable set, and ii) Value function

searching for the minimum value within the vehicle's forward attainable set.

Under mild assumptions on the data and, by using the fact that the value function is continuous, it has been shown

that this scheme is robust. Stability is proved by showing that there exists an uniform neighborhood along the reference trajectory for in which the value function satisfies a Lyapunov inequality in a generalized sense.

B. Some Auxiliary results

From the continuity of the value function, we obtain sub-optimality estimates in both global and local senses as it follows from the asymptotic performance convergence result below. Let T, and Δ, be, respectively, the optimization and control horizons, and denote by $(x_{T,\Delta}^*, u_{T,\Delta}^*)$ the associated MPC optimal control process. Let $J(x, u)$ be the value of the cost functional associated with the (x, u) over $[0, \infty)$, by $J(x, u)_{|[\alpha, \beta]}$ be its restriction to the interval $[\alpha, \beta]$, and by $J_k(x, u)$ its restriction to the interval $[k\Delta, (k+1)\Delta]$.

Proposition 1. *Let $T_f = \infty$ and assume that the optimal control horizon has an optimal control process (x^*, u^*) such that $\lim_{t \to \infty} x^*(t) = \xi^*$, being ξ^* an equilibrium point in C_∞. Then,*

(i) $\lim_{\Delta \downarrow 0, T \uparrow \infty} \sum_{k=1}^{\infty} J_k(x_{T,\Delta}^*, u_{T,\Delta}^*) = J(x^*, u^*)$

(ii) $\lim_{k \to \infty} \left| J_k(x_{T,\Delta}^*, u_{T,\Delta}^*) - J(x^*, u^*)_{|[k\Delta, (k+1)\Delta]} \right| = 0$.

The proof is straightforward and an be found in [11].

Since we are using a cloud of points in the state space reached by constant controls as approximation to the Attainable Set, a good estimate of the Hausdorff distance between these sets to determine the nonconservative worst case of the degree of sub-optimality. Denote the Hausdorff distance between A and B by $d_H(A, B)$ which is defined by

$$d_H(A, B) := \max \left\{ \sup_{x \in A} \{d_B(x)\}, \sup_{y \in B} \{d_A(y)\} \right\},$$

being $d_A(a)$ the Euclidian distance between the point a and the set A.

For the next result, the following mild assumptions are required on the dynamics: (i) $u \in L^\infty$ with $u(t) \in \Omega \subset \mathbf{R}^m$, (ii) Ω is compact and convex, (iii) $x \to f(t, x, u)$ is K_f Lipschitz continuous $\forall (t, u) \in \mathbf{R} \times \Omega$, and Lebesgue-Borel measurable in (t, u) for all $x \in \mathbf{R}^n$, (iv) f is continuous in u, and (v) $\forall (t, x) \in \mathbf{R} \times \mathbf{R}^n$, and $\exists K > 0$ such that $\|f(t, x, \Omega)\| \leq K$.

Let Ω_ε denote the set $\{u_i \in \Omega : i = 1, \dots, N_\varepsilon\}$ satisfying the following properties: a) $\Omega \subset \bigcup_{i=1}^{N_\varepsilon} (u_i + \varepsilon B)$, and b) $\forall i \, \exists j$ s.t. $\|f(t, x, u_i) - f(t, x, u_j)\| < \varepsilon$.

Denote by $\mathcal{A}_f(t_1; t_0, x)$ and $\mathcal{A}_f^\varepsilon(t_1; t_0, x)$ the set of points attainable at $t_1 > t_0$ from x at time t_0, by the dynamic system with controls, respectively, in L^∞ with values in Ω, and piecewise constant with values in Ω_ε.

Proposition 2. Let $\Delta > 0$. Then, $\forall (t, x) \in \mathbf{R} \times \mathbf{R}^n$, we have

$$d_H \left(\mathcal{A}_f(t + \Delta; t, x), \mathcal{A}_f^\varepsilon(t + \Delta; t, x) \right) \leq \Delta(\varepsilon + \Delta K_f K).$$

The proof of this result is long and we refer to [11].

Another issue that arises in this approach concerns the fact the point $\bar{x}$ in the state space to which the system is steered at a given time is very likely not to be listed in the look-up table specifying the value function V. Although, typically the value function is not smooth, the fact that the effect of state constraints is incorporated in the form of a penalization, one can assert that it is continuous, and, thus, a generalized gradient, denote below by ∇V, is bounded everywhere.

Proposition 3. Assume that the value of V at $\bar{x}$ is not known, and that there is a grid of points G_δ in $\mathbf{R}^n$ is such that the maximum distance between neighboring points in G_δ is less than $\delta > 0$.

Then, there is a simplex, i.e., a set of $n + 1$ points in independent position, $S_{\bar{x}} = \{x_i : i = 1, \dots n+1\}$ which are the closest to $\bar{x}$ being an estimate $\tilde{V}$ of V at $\bar{x}$ given by

$$\tilde{V}(\bar{x}) = \frac{\sum_{i=1}^{n+1} V_i \|\bar{x} - x_i\|^{-1}}{\sum_{i=1}^{n+1} \|\bar{x} - x_i\|^{-1}}$$

where, for $i = 1, \dots, n+1$, $V_i = V(x_i) + \nabla V(x_i) \cdot \bar{v}_i$, with $\bar{v}_i = \bar{x} - x_i$ and the $n \times (n+1)$ unknowns of the vectors $\nabla V(x_i)$, $i = 1, \dots, n+1$ are given as a solution of the set of $n + 1$ set of equations

$$\nabla V(x_i) \cdot (\bar{v}_i - \bar{v}_k) = \frac{V(x_k) - V(x_i)}{\|x_i - x_k\|}.$$

Moreover, for some $c > 0$,

$$\|V(\bar{x}) - \tilde{V}(\bar{x})\| \leq \max_{x_i, x_j \in S_{\bar{x}}} \{|V(x_i) - V(x_j)|\} + c\delta.$$

The proof of this result is long and we refer to [11].

C. The Robust Attainable Set MPC Scheme

The fact that the vehicle is in open-loop control mode in the interval $[t, t+\Delta]$ makes it vulnerable since, even small but persistent perturbations may prevent the vehicle of reaching the intended point z^* at time $t + \Delta$. This can be addressed by (i) considering an appropriate sub-optimality of z^*, (ii) introducing a number N_T of low complexity intermediate steps of length $\bar{\Delta}$, and (iii) replacing step 3 of the AS-MPC scheme by

3) Compute $z^* = \arg\min\{V(t+\Delta, z) : z \in \mathcal{A}_f(t+\Delta; t, x(t))\}$ For $i = 1$ to N_T, compute

$z_i^* \in \mathcal{A}_f(t+i\bar{\Delta}; t+(i-1)\bar{\Delta}, z_{i-1}^*) \cap \mathcal{A}_b(t+i\bar{\Delta}; t+\Delta, z^*)$
u_i^* driving the state from z_{i-1}^* to z_i^* on $[t+(i-1)\bar{\Delta}, t+\bar{\Delta}]$.

Here, $\Delta = N_T \bar{\Delta}$, $z_{N_T}^* = z^*$, and $z_0^* = x(t)$, and $\mathcal{A}_b(\tau; t, z_f) := \{z \in \mathbf{R}^n : z_f \in \mathcal{A}_f(t; \tau, z)\}$, with $\tau < t$, denotes the Backward Attainable Set (see [30], [21]).

All the previous considerations on the AS-MPC scheme migrate to this Robust AS-MPC (RAS-MPC) scheme.

V. THE CONTROL ARCHITECTURE

Given the high variability of the environment due to unexpected events, notably the emergence of unmapped obstacles, requires the embedding of the RAS-MPC controller described in the previous section in a control architecture. This provides the logic presiding the articulation of the different modes of operation. Figure 2 shows the system automaton representing the highest layer of the control architecture. In order to facilitate the explanation of the main idea, we focus solely in the motion control for a single AUV in the plane endowed with the capability of locally optimizing trajectories while avoiding collision with unexpected obstacles with data from a rotating pointed range finder. Moreover, we impose
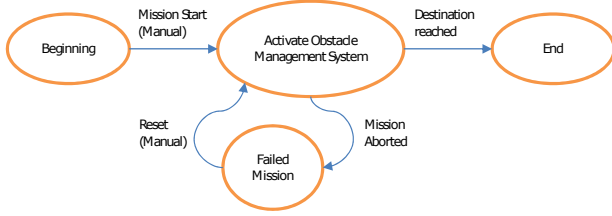
Fig. 2.   Main System Automaton

the general assumption that the set of unmapped obstacles are relatively sparse (albeit some might be close to one another) and each obstacle is locally a circle. The latter can be justified by the fact that this circle is estimated as the one with the smallest radius that includes all detected points.

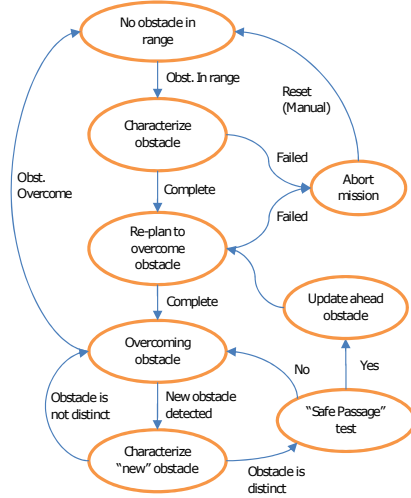In figure 3, we depict the events, and associated transitions that might arise in a still simple conceptual automaton.

Fig. 3.   Obstacle Collision Avoidance Automaton

In order to explain the Obstacle Collision Avoidance Automaton, we assume that range finder sensor detects obstacles from a distance significantly larger than the one transversed by the vehicle in the time interval of length Δ and its direction can vary between the limiting angles δ_{max} and $-\delta_{max}$.

Once an obstacle is detected, an exploratory search of the range finder enables to determine the location of both outer limits of the obstacle required to define the best way to overcome it for the current mission target or waypoint. For this purpose, the Value Function of the RAS-MPC is modified by adding an appropriate positive valued function penalizing the approximation to the obstacle which has a minimum at a safety distance d_s to the obstacle.

It may well happen that while circumventing an obstacle, a possibly new obstacle is detected. In this case, the range finder proceeds with the characterization of the eventual "new" obstacle. If it is concluded that it is really a new obstacle and that the path between both obstacles is the optimal, then it is necessary to determine wether the passage is safe or not.

The overall control system requires the characterize the degree and nature of the AUV situational awareness, being
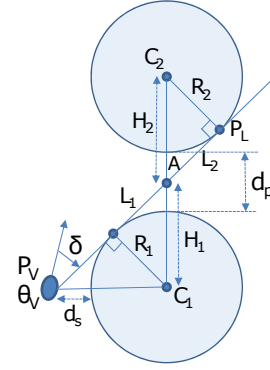

Fig. 4.   Safe passage detection

the articulation provided by the Control Architecture (CA). The perception issues fall outside the scope of this article, and, thus, we just illustrate the way the proposed MPC scheme, accommodates unexpected significant events in the context of the CA by taking into account onboard sensing devices. Figure 4 helps to understand how the criterion for this decision is defined. Observe that the passage is safe if $H_1+H_2-R_1-R_2-2d_s > 0$ where d_s is a design parameter, R_1, R_2, C_1, C_2, and P_L are estimated with the range finder, $H_1=\sqrt{R_1^2+L_1^2}$, $H_2=\sqrt{R_2^2+L_2^2}$, $L_2=|P_L-A|$, $L_1=|A-P_V|+\sqrt{(R_1+d_s)^2-R_1^2}$, P_V is the AUV position, and A is the intersection point of segments $\overline{C_1,C_2}$ and $\overline{P_V,P_L}$.

Figure 5 shows simulation results obtained with the proposed CA. The AUV departs from A to get to B. At time t_1, the obstacle O_1 is detected. The Value Function is changed in a neighborhood of O_1 by adding a penalization function to prevent collision. This forces the vehicle to overcome the obstacle by the right. While circumventing O_1, at time t_2, the object O_2 is detected. Its characterization reveals that a path between O_1 and O_2 is optimal. Since O_1 is still the closest obstacle, the current perturbed Value Function is kept while the system decides whether there is a safe passage. At time t_3, there is enough information to conclude that the passage between O_1 and O_2 is safe and the Value Function is now locally changed to prevent collisions with either obstacle. The path is now chosen by the left of O_2 as it minimizes the Value Function. The same rationale is applied at time t_4 when O_3 is detected. A safe passage between O_2 and O_3 is detected and the path to B is straightforward. Had the distance between O_2 and O_3 been such that the passage was unsafe, a not-so-optimal solution would have been obtained as the traveled distance by the left of O_3 would be longer than the one by the right of O_2.

Obviously, this scheme with locally based decisions does not guarantee the overall optimality. However, the sparser the unmapped obstacles are, the better approximation to optimality is achievable.

Figure 6 presents simulation results on a triangle formation of AUVs with one leader and two followers tracking a sinusoidal trajectory. The leader can communicate with each follower but the followers can not communicate between them. At some point in time, the leader's range finder detects an obstacle and changes its path to avoid the collision and in such a way that each one of the followers also can do it
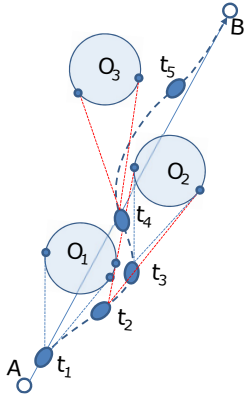
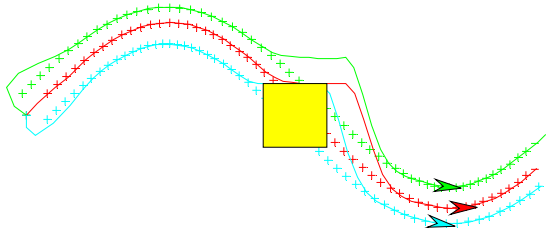Fig. 5.   Obstacle avoidance simulations results

Fig. 6.   Tracking with RAS MPC formation control with obstacle avoidance

without distorting the formation.

VI. CONCLUSIONS

In this article, a novel RAS-MPC scheme that enters into account the specific requirements arising in AUV motion control is introduced. The key drivers of the approach concerns the mitigation of the real-time computational burden and the ability of adapting to unmapped obstacle avoidance. While the former is motivated by limited onboard energy and computational power, in a context of strict real-time constraints, the presented CA shows the flexibility of the RAS-MPC scheme to handle unmapped obstacle. The mathematical details had to be omitted due to the lack of space. However, the obtained simulation results are a good illustration. The next step concerns the control implementation in the AUV onboard control software for field testing.

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REFERENCES

[1] R. Andrade, G. Raffo, and J. Rico. Model predictive control of a tilt-rotor uav for load transportation. In *European Control Conf.*, pages 2165–2170, June 2016.

[2] V. Baturin, E. Goncharova, F. Pereira, and J. Sousa. Polyhedral approximations to the reachable set of impulsive dynamic control systems. *Autom. & Remote Control*, 69(3), 2006.

[3] S. Bertrand, J. Marzat, H. Piet-Lahanier, A. Kahn, and Y. Rochefort. MPC strategies for cooperative guidance of autonomous vehicles. In *AerospaceLab Journal, Issue 8*, December 2014.

[4] Z. Chao, L. Ming, Z. Shaolei, and Z. Wenguang. Collision-free UAV formation flight control based on nonlinear MPC. In *Conf. on Electronics, Communications and Control*, pages 1951–1956, Sept 2011.

[5] F. Clarke, Yu. Ledyaev, R. Stern, and P. Wolenski. *Control Theory and Nonsmooth Analysis*. Springer Verlag, New York, 1998.

[6] L. Consolini, F. Morbidi, D. Prattichizzo, and M. Tosques. Leader-follower formation control of nonholonomic mobile robots with input constraints. *Automatica*, 44(5):1343 – 1349, 2008.

[7] E. Fiorelli, N. Leonard, P. Bhatta, D. Paley, R. Bachmayer, and D. Fratantoni. Multi-AUV control and adaptive sampling in monterey bay. In *IEEE J. of Oceanic Eng.*, pages 935–948, 2004.

[8] T. Fossen. *Guidance and Control of Ocean Vehicles*. Wiley, 1994.

[9] E. Franco, L. Magni, T. Parisini, M. Polycarpou, and D. Raimondo. Cooperative constrained control of distributed agents with nonlinear dynamics and delayed information exchange: A stabilizing receding-horizon approach. *IEEE Trans. Automatic Control*, 53:324–338, 2008.

[10] E. Franco, T. Parisini, and M. Polycarpou. Cooperative control of discrete-time agents with delayed information exchange: A receding-horizon approach. In *Decision and Control Conf.*, volume 4, pages 4274–4279, 2004.

[11] R. Gomes. *AUV Formation Control: A Model Predictive Control Approach*. PhD thesis, Faculty of Engineering, Porto University, 2017.

[12] R. Gomes, P. Calado, F. Pereira, and J. Sousa. Motion Coordination of Autonomous Underwater Vehicles Under Acoustic Communications. In *IFAC, Milan, Italy*, August 2011.

[13] R. Gomes and F. Pereira. A Reach Set MPC Scheme For The Cooperative Control Of Autonomous Underwater Vehicles. In *Procs PhysCon 2017, Florence, Italy*, July 2017.

[14] R. Gomes and F. Pereira. A Robust Reach Set MPC Scheme For Control of AUVs. In *Procs ROBOT 2017, Seville, Spain*, November 2017.

[15] T. Graettinger and B. Krogh. Hyperplane method for reachable state estimation for linear time-invariant systems. *J. of Optim. Theory and Appl.*, 69:555–588, 1991.

[16] T. Keviczky, F. Borrelli, and G. Balas. Decentralized receding horizon control for large scale dynamically decoupled systems. *Automatica*, 42:2105–2115, 2006.

[17] T. Keviczky, F. Borrelli, K. Fregene, D. Godbole, and G. Balas. Decentralized receding horizon control and coordination of autonomous vehicle formations. *IEEE Trans. Control Systems Tech.*, 16:19–33, 2008.

[18] N. Krasovskii and A. Subbotin. *Game-Theoretical Control Problems*. Springer Series in Soviet Mathematics. Springer-Verlag New York, 1988.

[19] R. Kristiansen and P. Nicklasson. Spacecraft formation flying: A review and new results on state feedback control. *Acta Astronautica*, 65(11-12):1537 – 1552, 2009.

[20] A. Kurzhanski and I. Vályi. *Ellipsoidal Calculus for Estimation and Control*. Birkhuser, 1997.

[21] A. Kurzhanski and P. Varaiya. Dynamic optimization for reachability problems. *J. Optim. Theory & Appl.*, 108(2):227–251, 2001.

[22] Y. Lv, Q. Hu, G. Ma, and J. Zhou. 6 dof synchronized control for spacecraft formation flying with input constraint and parameter uncertainties. *ISA Trans.*, 2011.

[23] I. Michel, A. Bayen, and C. Tomlin. Computing reachable sets for continuous dynamics games using level sets methods. *IEEE Trans. on Automatic Control*, 50:980–1001, 2005.

[24] D. Paley, F. Zhang, D. Fratantoni, and N. Leonard. Glider control for ocean sampling: The glider coordinated control system. *Trans. on Control System Technology*, 12(4):735–744, 2008.

[25] T. Prestero. Verification of a six-degree of freedom simulation model for the REMUS AUV. Master's thesis, MIT / WHOI, 2001.

[26] S. Quintero, D. Copp, and J. Hespanha. Robust UAV coordination for target tracking using output-feedback model predictive control with moving horizon estimation. In *American Control Conf.*, pages 3758–3764, July 2015.

[27] W. Ren and R. Beard. Virtual structure based spacecraft formation control with formation feedback. In *AIAA Guidance, Navigation and Control Conf., Monterey CA*, pages 2002–4963, 2002.

[28] J. Sethian. *Level Set Methods and Fast Marching Methods*. Cambridge University Press, 2 edition, 1999.

[29] Chao Shen, Y. Shi, and B. Buckham. Path-following control of an AUV using multi-objective model predictive control. In *2016 American Control Conf. (ACC)*, pages 4507–4512, July 2016.

[30] P. Varaiya. Reach set computation using optimal control. In *Proc. KIT Workshop*, 1998.