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Approximation of Advance Element in Consecutive Compensator Design for Plant with Control Delay*

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Abstract— **The article describes the algorithm of analytical construction of a consecutive compensator for controlling a plant with delay. The algorithm is based on the approximation of the advance element. The problem of approximation of the advance element appears when the synthesis of the consecutive compensator is performed on the basis of the desired typical polynomial model of the «input-output» system. Ignoring the delay element in a plant while constructing a consecutive compensator results in the appearance of the advance in its composition. An advance element approximation is better than the approximation of a delay element, since the stability problem of the approximant disappears. Approximants in the form of differentiating links of first and second order are used to construct a consecutive compensator with an advanced element in its composition. The article gives the results obtained by the research.**

I. Introduction

The element of delay (ED) in composition of continuous plants has become a challenge in solving problems of system stability and process quality. However, there are different solutions of these problems. The grapho-analytical methods [1, 2] are used for the synthesis of systems that include a plant with delay. These methods are based on a device of logarithmic amplitude frequency and phase frequency characteristics. Methods [3, 4] employ analytical approximations of the transfer function (TF) of the delay element. Smith scheme [5, 6], which was presented in the 1950s, has been widely used recently [7-13]. This method is based on the removal of the delay element beyond the closed loop. The studies [14, 15] are based on the vector-matrix formalism of the state-space method [16] in its integral representation. Each of the methods listed has its own disadvantages.

The authors dealing with the task of a plant with control delay relied on the capabilities of the «input-output» device, as the frequency form of these representations allows to see how the delay element affects the quality of control processes and the stability of the system. The modal control method [17] in this problem is of little use, as both the delay element and the advance element have no modal representation on the complex plane [18]. Therefore, an analytical approach based on the use of typical polynomial models (TPM) is proposed. The quality indicators of TPM (section frequency, phase stability margin, input-output and input-error ratios bandwidth, velocity factor) depend on the coefficients of the denominator of a polynomial (DP) of TPM. Thus, for fixed coefficients of DP of TPM, the problem of forming TPM is the task to determine the value of characteristic frequency. The introduction of the advance element (AE) to the structure of a consecutive compensator (CC) is an important part of the formation of a CC. It is not difficult to understand that the CC with AE in its composition is physically unrealizable. It should be noted that the approximation of the ED does not occur with the stability problem of the approximant. At the same time, the ED approximant in the polynomial representation becomes unstable. We solve the problem that has arisen in the class of approximation representations of AE by differentiating links of first and second orders.

The Section II of the article gives an algorithm of analytical construction of a consecutive compensator with advance element. Section III contains research of quality indicators of the system for particular cases of implementing an approximate advance element. There is an illustrative example of the analytical construction of CC in accordance with the algorithm in Section IV.

II. The Algorithm of Analytical Construction of a Consecutive Compensator with Advance Element

The algorithm of analytical construction of a consecutive compensator with AE is constructed analogically to the algorithm of analytical construction of a consecutive compensator without AE. The algorithm is based on the use of a bank [5, 10-14] dimension, which is determined by the dimension of the plant, and the dimension of the used AE approximant. The TPM is TF "input-output" [5, 13, 15] with the characteristic frequency ω_0

$$\Phi\left(s,\omega_0\right)=\frac{v_n\omega_0^n}{s^n+\sum_{i=1}^{n}v_i\omega_0^i s^{n-i}}=\frac{v_n\omega_0^n}{D\left(s,\omega_0\right)} \qquad (1)$$

In (1), the coefficients v_i $\left(i=\overline{1,n}\right)$ determine the character of the distribution of the roots of DP $D\left(s,\omega_0\right)$ of TF in the complex plane, the characteristic frequency ω_0 determines the size of the localization region for the placement of roots. The

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representation of the transfer function of the direct branch of the system has the form

$$W\left(s,\omega_0\right)=\frac{\Phi\left(s,\omega_0\right)}{1-\Phi\left(s,\omega_0\right)}=\frac{v_n\omega_0^n}{s^n+\sum_{i=1}^{n-1}v_i\omega_0^i s^{n-i}}=\frac{v_n\omega_0^n}{P\left(s,\omega_0\right)} \quad (2)$$

Thus, the analytical construction of a CC with AE can be performed with the following algorithm:

1. Set the transfer function $W_{\mathrm{P}}\left(s,\tau\right)$ of a plant with a delay in the form

$$W_{\mathrm{P}}\left(s,\tau\right)=W_{\mathrm{P}}\left(s\right)e^{-\tau s}=\frac{N_{\mathrm{P}}\left(s\right)}{P_{\mathrm{p}}\left(s\right)}e^{-\tau s} \quad (3)$$

where τ is the delay value.

2. Select a roots placement of TPM (with Butterworth's roots placement or Newton binomial) based on the requirement for overshoot, giving it the transfer function of the form (2).

3. Present the desired transfer function of the direct branch of the projected system of the form (2) in multiplicative form with components $W_{\mathrm{CC}}\left(s,\omega_0,\tau\right)$ and $W_{\mathrm{P}}\left(s,\tau\right)$

$$W\left(s,\omega_0,\tau\right)=W_{\mathrm{CC}}\left(s,\omega_0,\tau\right)W_{\mathrm{P}}\left(s,\tau\right) \quad (4)$$

where $W_{\mathrm{CC}}\left(s,\omega_0,\tau\right)$ is the transfer function of the compensator consecutively constructed.

4. Form the analytical representation $W_{\mathrm{CC}}\left(s,\omega_0,\tau\right)$ on the basis of (4), (2), (3)

$$W_{\mathrm{CC}}\left(s,\omega_0,\tau\right)=\frac{W\left(s,\omega_0\right)}{W_{\mathrm{P}}\left(s,\tau\right)}=\frac{v_n\omega_0^n}{P\left(s,\omega_0\right)}\frac{P_{\mathrm{P}}\left(s\right)}{N_{\mathrm{P}}\left(s\right)}e^{\tau s} \quad (5)$$

5. Set the approximate of the AE with TF $e^{\tau s}$ in the form

$$e^{\tau s}\cong 1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i \quad (6)$$

where $(i!)$ is factorial of a positive integer i.

6. Select the degree m of approximant of the AE and present the transfer function (5) of the CC in the form

$$\begin{aligned}W_{\mathrm{CC}}\left(s,\omega_0,\tau\right)&=\frac{v_n\omega_0^n}{P\left(s,\omega_0\right)}\frac{P_{\mathrm{P}}\left(s\right)}{N_{\mathrm{P}}\left(s\right)}e^{\tau s}\cong\\&\cong\frac{v_n\omega_0^n P_{\mathrm{P}}\left(s\right)\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)}{P\left(s,\omega_0\right)N_{\mathrm{P}}\left(s\right)}\end{aligned} \quad (7)$$

7. Check the transfer function (7) for physical realizability comparing the degrees of its numerator polynomials $\deg\left(v_n\omega_0^n P_{\mathrm{P}}\left(s\right)\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)\right)$ and denominator $\deg\left(P\left(s,\omega_0\right)N_{\mathrm{P}}\left(s\right)\right)$ in the form

$$\deg\left(P\left(s,\omega_0\right)N_{\mathrm{P}}\left(s\right)\right)\geq\deg\left(v_n\omega_0^n P_{\mathrm{P}}\left(s\right)\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)\right) \quad (8)$$

8. Select the final version of the TPM based on condition (8).

9. Generate an analytical representation of TF (7) of the CC with a characteristic frequency ω_0

10. Form TF $W\left(s,\omega_0,\tau\right)$ of the direct branch of the system designed, consisting of CC with TF (7) and plant with TF (3)

$$\left.\begin{aligned}W\left(s,\omega_0,\tau\right)&=W_{\mathrm{CC}}\left(s,\omega_0,\tau\right)W_{\mathrm{P}}\left(s,\tau\right)=\\&=\frac{v_n\omega_0^n\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)e^{-\tau s}}{P\left(s,\omega_0\right)}=\\&=W\left(s,\omega_0\right)\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)e^{-\tau s}\end{aligned}\right\} \quad (9)$$

11. Conduct a complex computer simulation of the system projected, the direct branch of which is described by TF (9), in order to estimate the maximum permissible value $\omega_0\left(\tau\right)=\arg\left(\sigma\left(\omega_0,\tau\right)\leq\sigma_R\right)$, where σ_R is requirement value for overshoot $\sigma\left(\omega_0,\tau\right)$. Estimate the transient time $t_{\mathrm{t}}\left(\omega_0,\tau\right)$ and the quality factor for velocity $D_1\left(\omega_0\right)$. In addition, estimate the limiting values of the characteristic frequency $\omega_0\left(\sigma,\tau\right)=\arg\left(\sigma\left(\omega_0,\tau\right)=0\right)$ and $\omega_0\left(\Delta\varphi,\tau\right)=\arg\left(\Delta\varphi\left(\omega_0,\tau\right)=0\right)$, at which the system is at the stability boundary.

III. RESEARCH OF QUALITY INDICATORS OF THE SYSTEM FOR PARTICULAR CASES OF IMPLEMENTING AN APPROXIMATE ADVANCE ELEMENT

Let the AE be a physically realizable element, then (9) can be written in the form

$$W\left(s,\omega_0,\tau\right)=W\left(s,\omega_0\right)e^{\tau s}e^{-\tau s}=W\left(s,\omega_0\right) \quad (10)$$

Quality indicators set for a system with a forward branch TF (10) is presented in Table 1. In column 2 of Table 1, the overshoot value is indicated, column 3 contains the cutoff frequency, column 4 gives the phase stability margin, in column 5 the system bandwidth $\Delta\omega$ is calculated by frequency response $M\left(\omega\right)=\left|\Phi\left(j\omega,\omega_0\right)\right|\geq\delta_y$, where δ_y is given number (for example $\delta_y=0.05$ or $\delta_y=0.707$), in column 6 the system bandwidth $\Delta\omega$ is calculated by error frequency response $\delta(\omega)=\left|\Phi_\varepsilon\left(j\omega,\omega_0\right)\right|\leq\delta_\varepsilon$, where δ_ε is given number; in column 7 has the quality factor in velocity, in column 8 contains the transient time. The indicators with the sign (*) are indicators of the TPM with TF of the form (1) at $\omega_0 = 1$, which are determined by modeling. Note that $\Phi_{\varepsilon}(s, \omega_0, \tau)$ is TF from reference to error.

TABLE I. SYSTEM-WIDE INDICATORS OF THE TPM WITH TF (1)

Analytic representation of the denominator polynomial $D(s,\omega_0)$	σ	ω_c	$\Delta\varphi$	**Bandwidth** $\Delta\omega/\omega_0$		$D_1(\omega_0)$	$t_t(\omega_0)$
				$M(\omega) \geq \delta_y$	$\delta(\omega) \leq \delta_\varepsilon$		
1	***2***	***3***	***4***	***5***	***6***	***7***	***8***
$s^n + \sum_{i=1}^{n} v_i \omega_0^i s^{n-i}$	σ^*	$\frac{v_n \omega_0}{v_{n-1}}$	$\pi + \arg\{W(j\omega_c, \omega_0)\}$	$\left(\frac{v_n}{\delta_y}\right)^{1/n}$	$\frac{\delta_\varepsilon v_n}{v_{n-1}}$	$\frac{v_n \omega_0}{v_{n-1}}$	$\frac{t_t^*}{\omega_0}$

For the general representation of approximant (6), the transfer function $W(s, \omega_0, \tau)$ has the form (9). We estimate the influence of the factor $\left(1 + \sum_{i=1}^{m} (i!)^{-1} (\tau s)^i\right) e^{-\tau s}$ on the stability of the system. If the stability of the system is estimated [1] from the stability margin $\Delta\varphi$ in phase, then for a system of the form (9) we can write

$$\Delta\varphi = \pi + \arg\{W(j\omega_c, \omega_0)\} + \arg\left(1 + \sum_{i=1}^{m} (i!)^{-1} (j\tau\omega_c)^i\right) - \tau\omega_c \quad (11)$$

Moreover, expression (11) allows to estimate the maximum value of the characteristic frequency at which the system with the transfer function of straight branch (9) remains stable, with the aid of relation

$$\omega_0 = \arg\left\{\begin{array}{l} \pi + \arg\{W(j\omega_c, \omega_0)\} + \\ + \arg\left(1 + \sum_{i=1}^{m} (i!)^{-1} (j\tau\omega_c)^i\right) - \tau\omega_c = 0 \end{array}\right\} \quad (12)$$

As seen the important point in expressions (11), (12) is awareness of the cutoff frequency ω_c. In the general case, this problem for the representation of the approximant in the form (6) is difficult to solve, so we are limited to particular cases of realization of the approximant. In the first case, approximate (6) is given in the form

$$1 + \sum_{i=1}^{m} (i!)^{-1} (\tau s)^i = 1 + \tau s \quad (13)$$

which allows us to write (9) in the form

$$W(s, \omega_0, \tau) = W(s, \omega_0)(1 + \tau s) e^{-\tau s} \quad (14)$$

In the second case, we define the approximant (6) in the form

$$1 + \sum_{i=1}^{m} (i!)^{-1} (\tau s)^i = 1 + \tau s + 0.5(\tau s)^2 \quad (15)$$

which allows us to write (9) as follows

$$W(s, \omega_0, \tau) = W(s, \omega_0)\left(1 + \tau s + 0.5(\tau s)^2\right) e^{-\tau s} \quad (16)$$

In the first case, the approximant is an ideal differentiating link of the first order, and in the second - the ideal differentiating link of the second order. Hence, for the first case the assertion holds true.

Proposition 1. An estimate $\hat{\omega}_c$ of the cutoff frequency ω_c of a system with the transfer function of a direct branch of the form (14) is given by

$$\hat{\omega}_c = \omega_0 \Big/ \left((v_{n-1}/v_n)^2 - (\omega_0\tau)^2\right)^{1/2} \quad (17)$$

Proof. The cutoff frequency ω_c satisfies the relation

$$\omega_c = \arg\left(\left|W(s, \omega_0, \tau)\right|\Big|_{s=j\omega=j\omega_c} = 1\right) \quad (18)$$

where TF $W(s, \omega_0, \tau)$ for the first realization of the AE approximant is written in the form

$$W(s, \omega_0, \tau) = \frac{v_n \omega_0^n (1 + \tau s) e^{-\tau s}}{s^n + v_1 \omega_0 s^{n-1} + \cdots + v_{n-1} \omega_0^{n-1} s} \quad (19)$$

Knowing the definition of the cutoff frequency (18) and equality $\left|e^{-j\tau\omega}\right| = 1$, we can write an equality that proves the correctness of the relation (17)

$$\left|W(j\omega, \omega_0, \tau)\right| = \left.\left|\frac{v_n \omega_0^n (1 + \tau s)}{\left(s^{n-1} + v_1 \omega_0 s^{n-2} + \cdots + v_{n-1} \omega_0^{n-1}\right) s}\right|_{s=j\omega}\right|_{\omega=\omega_c} \cong$$

$$\cong \frac{v_n \omega_0 \left(1 + (\tau\hat{\omega}_c)^2\right)^{1/2}}{v_{n-1} \hat{\omega}_c} = 1$$

For the second case, an assertion similar to Proposition 1 is valid.

Proposition 2. An estimate $\hat{\omega}_c$ of the cutoff frequency ω_c of a system with TF of a direct branch of the form (16) is given by

$$\hat{\omega}_c = \arg\max\left(v_n \omega_0 \left(1 + 0.25(\tau\omega_c)^4\right)^{1/2} = v_{n-1}\omega_c\right) \quad (20)$$

Proof. The cutoff frequency ω_c satisfies the relation (18), where TF $W(s, \omega_0, \tau)$ for the second realization of the AE approximant is written in the form

$$W(s, \omega_0, \tau) = \frac{v_n \omega_0^n \left(1 + \tau s + 0.5(\tau s)^2\right) e^{-\tau s}}{s^n + v_1 \omega_0 s^{n-1} + \cdots + v_{n-1} \omega_0^{n-1} s} \quad (21)$$

Knowing the definition of the cutoff frequency (18) and equality $\left|e^{-j\tau\omega}\right| = 1$, we can write an equality that proves the correctness of the relation (20)

$$\left|W\left(j\omega,\omega_0,\tau\right)\right| = \left.\left.\left|\frac{v_n\omega_0^n\left(1+\tau s+0.5\left(\tau s\right)^2\right)}{\left(s^{n-1}+v_1\omega_0 s^{n-2}+\cdots+v_{n-1}\omega_0^{n-1}\right)s}\right|_{s=j\omega}\right|_{\omega=\omega_c} \cong$$

$$\cong \frac{v_n\omega_0\left(1+0.25\left(\tau\omega_c\right)^4\right)^{1/2}}{v_{n-1}\omega_c} = 1$$

The cutoff frequency obtained is estimated in the form of (17) and (20). This allows us to estimate the permissible maximum value of the characteristic frequency ω_0 for a fixed value τ of the delay from the condition that the zero stability margin along the phase of the considered systems with TF of the straight branch (9) in the forms (14) and (16). Thus, for the case (9) in the form (14) for the target value ω_0, we have

$$\omega_0 = \arg\left.\left\{\begin{aligned}&\pi+\arg\left\{W\left(j\omega_c,\omega_0\right)\right\}+\\&+arctg\left(\tau\omega_c\right)-\tau\omega_c=0\end{aligned}\right\}\right|_{\omega_c=\omega_0/\left(\left(v_{n-1}/v_n\right)^2-\left(\omega_0\tau\right)^2\right)^{1/2}} \quad (22)$$

For the case (9) in the form (16) for the target value ω_0, we obtain

$$\omega_0 = \arg\left\{\begin{aligned}&\pi+\arg\left\{W\left(j\omega_c,\omega_0\right)\right\}+\\&+arctg\left(\tau\omega_c/1-0.5\left(\tau\omega_c\right)^2\right)-\tau\omega_c=0\end{aligned}\right\} \quad (23)$$

where $\omega_c = \arg\max\left(v_n\omega_0\left(1+0.25\left(\tau\omega_c\right)^4\right)^{1/2} = v_{n-1}\omega_c\right)$

Consider the case of a static reference signal $g(t)=g_0=const$ and a ramp signal $g(t)=\dot{g}_0 t$ for estimating the influence of the factor on the quality indicators of the system with TF $W(s,\omega_0)$ direct branch (Table 1).

We solve the problem using the analysis of the error of the system with TF of the direct branch of the general form (9), which is based on TF $\Phi_\varepsilon(s,\omega_0,\tau)$

$$\left.\begin{aligned}\Phi_\varepsilon\left(s,\omega_0,\tau\right) &= \frac{\varepsilon(s)}{g(s)} = \frac{\varepsilon(s)}{\varepsilon(s)+y(s)} = \\&= \frac{1}{1+W\left(s,\omega_0,\tau\right)} = \\&= \frac{1}{1+W\left(s,\omega_0\right)\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)e^{-\tau s}} = \\&= \frac{s^n+\sum_{i=1}^{n-1}v_i\omega_0^i s^{n-i}}{s^n+\sum_{i=1}^{n-1}v_i\omega_0^i s^{n-i}+v_n\omega_0^n\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)e^{-\tau s}}\end{aligned}\right\} \quad (24)$$

where $g(s)$, $\varepsilon(s)$, $y(s)$ are Laplace images representing external influences, errors in its reproduction and output respectively. The Laplace error image can be written as

$$\varepsilon(s) = \Phi_\varepsilon\left(s,\omega_0,\tau\right)g(s) \quad (25)$$

For the steady-state value of the error $\varepsilon_{ss}(t)$, by virtue of the theorem on the finite value of the original [1], the representation is

$$\varepsilon_{ss}(t) = \lim_{t\to\infty}\varepsilon(t) = \lim_{s\to 0}s\varepsilon(s) = \lim_{s\to 0}s\Phi_\varepsilon\left(s,\omega_0,\tau\right)g(s) \quad (26)$$

Remark 1. For the steady-state error value $\varepsilon_{ss}(t)$ for the case of a static reference signal $g(t)=g_0=const$ in the system with TF $\Phi_\varepsilon(s,\omega_0,\tau)$, the error of the form (24) can be written as equation

$$\varepsilon_{ss}(t) = 0,\ for\ \forall g_0 \quad (27)$$

Proposition 3. For the steady-state error value $\varepsilon_{ss}(t)$ for the case of a ramp signal $g(t)=\dot{g}_0 t$ in the system with TF $\Phi_\varepsilon(s,\omega_0,\tau)$, the error of the form (24) can be written as equation

$$\varepsilon_{SS}(t) = \frac{v_{n-1}}{v_n\omega_0}\dot{g}_0 \quad (28)$$

Proof. Substituting (24) into (26), taking into account the fact that $g(s)=L\left(g(t)=g_0\right)=g_0/s$, gives

$$\left.\begin{aligned}\varepsilon_{SS}(t) &= \dot{g}_0\lim_{s\to 0}\frac{1}{s}\Phi_\varepsilon\left(s,\omega_0,\tau\right) = \\&= \dot{g}_0\lim_{s\to 0}\frac{s^{n-1}+\sum_{i=1}^{n-1}v_i\omega_0^i s^{n-1-i}}{s^n+\sum_{i=1}^{n-1}v_i\omega_0^i s^{n-i}+v_n\omega_0^n\left(1+\sum_{i=1}^{m}\left(i!\right)^{-1}\left(\tau s\right)^i\right)e^{-\tau s}} = \\&= \frac{v_{n-1}\omega_0^{n-1}}{v_n\omega_0^n}\dot{g}_0 = \frac{v_{n-1}}{v_n\omega_0}\dot{g}_0\end{aligned}\right\} \quad (29)$$

Remark 2. The relation (29) allows for the control system of a plant with a delay by means of a CC having in its composition an approximant of the AE to give an analytical representation of the quality factor D_1 of the velocity of the form

$$D_1 = \frac{\dot{g}_0}{\varepsilon_{SS}} = \frac{v_n\omega_0}{v_{n-1}} \quad (30)$$

As seen the quality factor of the velocity (30) coincides with the quality factor in the velocity of the system with the input-output transfer function (1) given in Table 1 (column 7).

IV. Illustrative Example

Consider the proposed algorithm for analytical construction of a CC with a link of AE by example.

1. We define TF of a plant $W_P(s,\tau)$ with a delay in the form (3)

$$W_P(s,\tau)=W_P(s)e^{-\tau s}=\frac{N_P(s)}{P_P(s)}e^{-\tau s}=\frac{62.8}{(0.04s+1)(0.3s+1)s}e^{-0.1s}.$$

2. We select Newton binominal like roots placement of TPM for which $v_i=C_n^i$ setting it in the form (2)

$$W(s,\omega_0)=\frac{\Phi(s,\omega_0)}{1-\Phi(s,\omega_0)}=\frac{C_n^n\omega_0^n}{s^n+\sum_{i=1}^{n-1}C_n^i\omega_0^i s^{n-i}}=\frac{\omega_0^n}{P(s,\omega_0)}.$$

At this step, the dimension n is not fixed.

3. We make the desired TF of the direct branch of the projected system of the form (2) in the multiplicative form (4) with the components $W_{CC}(s,\omega_0,\tau)$ and $W_P(s,\tau)$.

4. We form the analytical representation $W_{CC}(s,\omega_0,\tau)$ in the form (5)

$$W_{CC}(s,\omega_0,\tau)=\frac{W(s,\omega_0)}{W_P(s,\tau)}=\frac{\omega_0^n}{P(s,\omega_0)}\frac{P_P(s)}{N_P(s)}e^{\tau s}=$$
$$=\frac{\omega_0^n}{P(s,\omega_0)}\frac{(0.04s+1)(0.3s+1)s}{62.8}e^{0.1s}.$$

5. We set the approximate of the AE with TF $e^{\tau s}$ in the form (6) in the two investigated forms

5.1 $e^{0.1s}\cong 1+\sum_{i=1}^{m}(i!)^{-1}(0.1s)^i\Big|_{m=1}=1+0.1s$;

5.2 $e^{0.1s}\cong 1+\sum_{i=1}^{m}(i!)^{-1}(0.1s)^i\Big|_{m=2}=1+0.1s+0.005s^2$.

6. For the selected approximants of the AE We write TF (5) of the CC in forms

6.1 $W_{CC}(s,\omega_0,\tau)=\dfrac{\omega_0^n}{P(s,\omega_0)}\dfrac{(1+0.1s)(0.04s+1)(0.3s+1)s}{62.8}$;

6.2 $W_{CC}(s,\omega_0,\tau)=\dfrac{\omega_0^n}{P(s,\omega_0)}\dfrac{(1+0.1s+0.005s^2)(0.04s+1)(0.3s+1)s}{62.8}$.

7. We verify TF of 6.1 and 6.2 on physical realizability comparing the degrees of its polynomials of the numerator and denominator in the forms

7.1 $W_{CC}(s,\omega_0,\tau)=\dfrac{\omega_0^n}{P(s,\omega_0)}\dfrac{(1+0.1s)(0.04s+1)(0.3s+1)s}{62.8}$
$\Rightarrow \deg(P(s,\omega_0))=n=4$

7.2 $W_{CC}(s,\omega_0,\tau)=\dfrac{\omega_0^n}{P(s,\omega_0)}\dfrac{(1+0.1s+0.005s^2)(0.04s+1)(0.3s+1)s}{62.8}$.
$\Rightarrow \deg(P(s,\omega_0))=n=5$

8. We select the final version of the TPM based on the fulfillment of item 7 conditions

8.1 $W(s,\omega_0)=\dfrac{\omega_0^4}{s^4+4\omega_0 s^3+6\omega_0^2 s^2+4\omega_0^3 s}$;

8.2 $W(s,\omega_0)=\dfrac{\omega_0^5}{s^5+5\omega_0 s^4+10\omega_0^2 s^3+10\omega_0^3 s^2+5\omega_0^4 s}$.

9. On the basis of items 7 and 8, we formulate an analytical representation of TF of a CC for two variants of the AE approximation for the purpose of its technical implementation

9.1 $W_{CC}(s,\omega_0,\tau)=\dfrac{\omega_0^4}{62.8}\dfrac{(1+0.1s)(0.04s+1)(0.3s+1)s}{s^4+4\omega_0 s^3+6\omega_0^2 s^2+4\omega_0^3 s}$;

9.2

$$W_{CC}(s,\omega_0,\tau)=\frac{\omega_0^5}{62.8}\frac{(1+0.1s+0.005s^2)(0.04s+1)(0.3s+1)s}{s^5+5\omega_0 s^4+10\omega_0^2 s^3+10\omega_0^3 s^2+5\omega_0^4 s}.$$

We verify TF of 9.1 and 9.2 on physical realizability comparing the degrees of its polynomials of the numerator and denominator and we see that the consecutive compensator is physical realizability.

10. We form TF of the direct branch of the system designed, composed of a CC and a plant for two implementations of the approximate AE

10.1 $W(s,\omega_0,\tau)=\dfrac{\omega_0^4(1+0.1s)e^{-0.1s}}{s^4+4\omega_0 s^3+6\omega_0^2 s^2+4\omega_0^3 s}$;

10.2 $W(s,\omega_0,\tau)=\dfrac{\omega_0^5(1+0.1s+0.005s^2)e^{-0.1s}}{s^5+5\omega_0 s^4+10\omega_0^2 s^3+10\omega_0^3 s^2+5\omega_0^4 s}$.

11. We conduct a complex computer simulation of the system projected, the direct branch of which is presented in item 10, for the investigation its quality indicators.

11.1. We determine experimentally the value $\omega_0=21.3\ \mathrm{s}^{-1}$ at which the requirement $\sigma(\omega_0,\tau)\le\sigma_R=20\%$ is satisfied and the transient time $t_t(\omega_0,\tau)=0.8\,\mathrm{s}$ is provided, the velocity quality factor $D_1(\omega_0)=5.25\ \mathrm{s}^{-1}$. We estimate the limiting values of $\omega_0(\sigma,\mathrm{\tau})=\arg(\sigma(\omega_0,\mathrm{\tau})=0)$ and $\omega_0(\Delta\varphi,\mathrm{\tau})=\arg(\Delta\varphi(\omega_0,\mathrm{\tau})=0)$ for which we obtain $\omega_0(\sigma,\mathrm{\tau})=14\ \mathrm{s}^{-1}$ and $\omega_0(\Delta\varphi,\mathrm{\tau})=36.93\ \mathrm{s}^{-1}$ respectively. Thus, the range of variation of the characteristic frequency is formed, with the system being stable.

11.2. We determine experimentally the value $\omega_0=32.3\ \mathrm{s}^{-1}$ at which the requirement $\sigma(\omega_0,\tau)\le\sigma_R=20\%$ is satisfied with the transient time $t_t(\omega_0,\tau)=1.06\,\mathrm{s}$ provided, the velocity quality factor is $D_1(\omega_0)=6.46\,\mathrm{s}^{-1}$. We estimate the limiting values of $\omega_0(\sigma,\mathrm{\tau})=\arg(\sigma(\omega_0,\mathrm{\tau})=0)$ and $\omega_0(\Delta\varphi,\mathrm{\tau})=\arg(\Delta\varphi(\omega_0,\mathrm{\tau})=0)$ for which we obtain $\omega_0=17\ \mathrm{s}^{-1}$ and $\omega_0(\Delta\varphi,\mathrm{\tau})=39.66\ \mathrm{s}^{-1}$ respectively. Thus, the range of variation of the characteristic frequency is formed, with the system being stable.

The results of item 11.1 are illustrated by the curves of the processes shown in Fig.1.

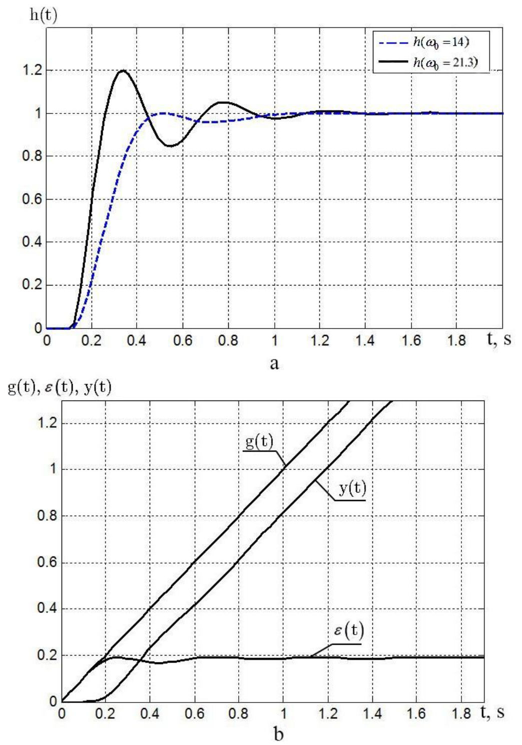

Figure 1. Curves of transient processes (1.a) and curves of ramp signal, error, output signal (1.b)

The results of item 11.2 are illustrated by the curves of the processes shown in Fig.1.

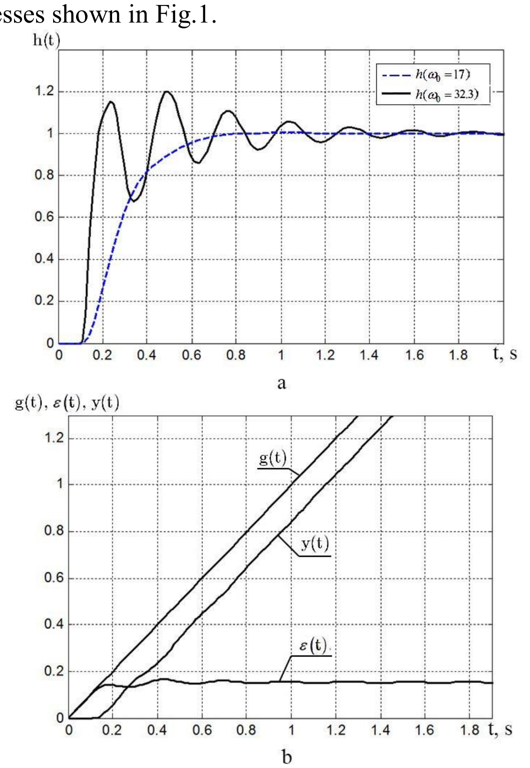

Figure 2. Curves of transient processes (2.a) and curves of ramp signal, error, output signal (2.b)

V. Conclusion

The algorithm we have developed allows the use of approximants in the form of differentiating links of first and second order to construct a consecutive compensator with an advance element in its composition. The approximation of the advance element has advantages over the approximation of the delay element, since the stability problem of the approximant used is removed.

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