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# Robust feedback linearization for input-constrained nonlinear systems with matched uncertainties

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Abstract— This paper deals with robust feedback linearization control of nonlinear systems with matched uncertainties and subject to constraints on the control input. The proposed approach not only finds an appropriate pole placement for the linearized system to ensure the satisfaction of input constraints, but also tries to improve the speed of response by effectively exploiting the available control authority. The procedure guarantees the satisfaction of input signal constraints based on the invariant set theorem. Simulation results on an uncertain input-constrained nonlinear spring-mass system demonstrate the effectiveness of the proposed approach in terms of both response speed and constraint satisfaction.

## I. INTRODUCTION

One of the main issues in control design is to take into account model uncertainties which can occur for several reasons, e.g., due to unmodeled dynamics, uncertain parameters and/or disturbances. In these cases, if bounds for the uncertainties are known, a sufficiently large control input can be applied to the system in order to compensate the effects of uncertainties. Sliding mode control [1], [2], [3] and Lyapunov-based control schemes [4], [5], [6] are based on this idea. On the other hand, in almost all industrial plants, the input signal has physical limitations that must be fulfilled. A great deal of research work on constrained systems has been carried out over the last two decades [7], [8], [9], [10], [11], [12], [13], [14]. Most of the proposed approaches to constraint handling can essentially be classified into three main types of control design methods:

- Lyapunov-based control [7], [9], [10]
- Model predictive control [11], [12]
- Optimal control [13], [14]

Lyapunov-based methods rely on invariant sets wherein they can guarantee constraint satisfaction. Model predictive control schemes use their ability to predict the future behavior of the system in order to satisfy the input signal constraints and finally optimal control schemes try to synthesize an appropriate control signal by considering the input signal constraints in their optimization problem. In this paper, we are going to present a novel procedure which can be implemented when fast response is also desired, besides satisfaction of input constraints, for a class of uncertain nonlinear systems. Our approach is based on finding an invariant set containing the feasible region of initial state conditions which also satisfies the input signal constraints. We also proposed an iterative procedure that allows to refine the feasible invariant set so as to improve the speed of response.

The rest of the paper is organized as follows. Section II describes the control problem of interest. Section III presents a novel robust feedback linearization scheme that guarantees asymptotic output tracking of a desired reference in presence of bounded matched uncertainties, first considering the unconstrained case and then showing how to deal with control input constraints while being also concerned with tracking speed. Section IV provides a numerical example to demonstrate the effectiveness of the proposed method. Finally, section V ends the paper with concluding remarks and perspectives for future work.

## II. Problem formulation

Consider an uncertain affine nonlinear system in the form

$$\left\{ \begin{array}{rcl} \dot{\mathbf{x}} & = & f(\mathbf{x}) + \Delta f(\mathbf{x}) + [g(\mathbf{x}) + \Delta g(\mathbf{x})]\, u \\ y & = & h(\mathbf{x}) \end{array} \right. \tag{1}$$

where: $\mathbf{x} \in \mathbb{R}^n$ is the state vector; $u \in \mathbb{R}$ the scalar control input; $y \in \mathbb{R}$ the scalar output that should track a desired reference $y_d$; $f, g, : \mathbb{R}^n \to \mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}$ are known nonlinear functions that are sufficiently smooth on their domains of definition; $\Delta f, \Delta g : \mathbb{R}^n \to \mathbb{R}^n$ are unknown uncertainties. Let us make the following assumptions:

- the origin is an equilibrium point for the system (1);
- the zero dynamics of (1) are stable;
- $\mathrm{sgn}\left\{ L_g L_f^{r-1} h(\mathbf{x}) \right\} = 1$ where $L$ stands for Lie derivative, $r$ is the relative degree of (1) and the function $\mathrm{sgn}(x)$ is defined as

$$\mathrm{sgn}(x) = \left\{ \begin{array}{ccc} -1 & if & x < 0 \\ 0 & if & x = 0 \\ 1 & if & x > 0 \end{array} \right. ;$$

- the initial state satisfies $\mathbf{x}_0 \in \mathbf{X}_0$ where $\mathbf{X_0}$ is a bounded subset of $\mathbb{R}^n$;
- $\Delta f(\mathbf{x})$ and $\Delta g(\mathbf{x})$ are bounded, i.e. satisfy

$$\begin{array}{rcl} |\Delta f(\mathbf{x})| & \leq & \overline{\Delta f}(\mathbf{x}) \\ |\Delta g(\mathbf{x})| & \leq & \overline{\Delta g}(\mathbf{x}) \end{array} \tag{2}$$

where $\overline{\Delta f}(\mathbf{x})$ and $\overline{\Delta g}(\mathbf{x})$ are known functions that are sufficiently smooth on their domains of definition;

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- $\Delta f(\mathbf{x})$ and $\Delta g(\mathbf{x})$ satisfy the matching condition

$$\Delta f(\mathbf{x}), \Delta g(\mathbf{x}) \in span\{g(\mathbf{x})\}; \tag{3}$$

- the model (1) is sufficiently accurate so that the following inequality holds

$$\left|\left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} L_{\overline{\Delta g}} L_f^{r-1} h(\mathbf{x})\right| < 1, \forall \mathbf{x} \in \mathbb{R}^n; \tag{4}$$

- the input signal is subject to the bound constraint

$$|u| \quad \leq \quad u_{\max}. \tag{5}$$

The system described by (1)-(5) is general enough to cover a wide class of uncertain nonlinear systems and, at the same time, satisfies all the necessary requirements for the application of our proposed method. The goal of this paper is finding a stable robust feedback linearization control scheme for the aforementioned constrained nonlinear system which also provides an acceptable speed of response.

## III. Robust feedback linearization control

In this section, the aim is to find a stable robust control scheme for the system (1) considering (2)-(5). To this end, let us first disregard the input constraints (5) and develop a robust input-output feedback linearization control scheme for the constraint-free case. Robust feedback linearization of nonlinear systems has been widely studied in the literature [15], [16], [17], [18], [19], [20]. Our proposed method is close to [20] with minor changes introduced for compatibility with our subsequent procedure for constrained nonlinear systems.

Theorem 1: Let us consider the control law

$$\begin{aligned}
u &= \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} \left(p - L_f^r h(\mathbf{x})\right) \\
p &= \hat{p} + p_r \\
\hat{p} &= y_d^{(r)} + a_{r-1} e^{(r-1)} + \ldots + a_1 \dot{e} + a_0 e \\
&= y_d^{(r)} + \mathbf{k^T e} \\
\underline{D} &= 1 - \left|\left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} L_{\overline{\Delta g}} L_f^{r-1} h(\mathbf{x})\right| \\
\overline{N} &= \left|\left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} \left(L_{\overline{\Delta g}} L_f^{r-1} h(\mathbf{x})\right) L_f^r h(\mathbf{x})\right| \\
&\quad + \left|L_{\overline{\Delta f}} L_f^{r-1} h(\mathbf{x})\right| \\
&\quad + \left|\left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} \left(L_{\overline{\Delta g}} L_f^{r-1} h(\mathbf{x})\right) \hat{p}\right| \\
p_r &= -\operatorname{sgn}(\mathbf{e^T P w}) \mathbf{\overline{N} \underline{D}^{-1}}
\end{aligned} \tag{6}$$

where: $e \stackrel{\triangle}{=} y_d - y$; $\mathbf{e} \stackrel{\triangle}{=} \left[e, \dot{e}, ..., e^{(r-1)}\right]$; $\mathbf{w} \stackrel{\triangle}{=} [\mathbf{0}, \mathbf{0}, \ldots, \mathbf{1}]^{\mathbf{T}} \in \mathbb{R}^{\mathbf{r}}$; the feasible initial condition of $\mathbf{e}$ is bounded as $\mathbf{e}_0 \in \mathrm{E}_0$ where $\mathrm{E}_0$ is a bounded subset of $\mathbb{R}^r$; the coefficients $a_{r-1}, ..., a_0$ are chosen so that

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{r-1} \end{bmatrix} \tag{7}$$

is a strictly Hurwitz matrix; P is a positive definite matrix which can be obtained by solving the Lyapunov equation

$$\mathrm{A^T P} + \mathrm{PA} = -\mathrm{Q} \tag{8}$$

for an arbitrary fixed positive definite matrix Q. Then, under the assumptions (2)-(4), the controller (6) ensures that the output $y$ asymptotically tracks the desired reference $y_d$.

Proof: After $r$-order differentiation of $y$ with respect to time and considering the matching condition (3), one gets

$$\begin{aligned} y^{(r)} &= L_f^r h(\mathbf{x}) + L_{\Delta f} L_f^{r-1} h(\mathbf{x}) \\ &+ \left(L_g L_f^{r-1} h(\mathbf{x})\right) u + \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) u \end{aligned}$$

which, by substituting $u$ from (6), yields

$$\begin{aligned} y^{(r)} &= p + L_{\Delta f} L_f^{r-1} h(\mathbf{x}) \\ &+ \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} \left(p - L_f^r h(\mathbf{x})\right) \end{aligned}$$

which, exploiting $p = \hat{p} + p_r$ and after some manipulations, provides

$$\begin{aligned} y^{(r)} &= \hat{p} + \left(1 + \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1}\right) p_r \\ &+ L_{\Delta f} L_f^{r-1} h(\mathbf{x}) \\ &+ \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} \left(\hat{p} - L_f^r h(\mathbf{x})\right) \end{aligned} \tag{9}$$

Next, using the definitions of $\hat{p}$ and $A$, (9) can be written in state space form as

$$\begin{aligned} \dot{\mathbf{e}} &= \mathrm{A}\mathbf{e} \\ &+ \mathbf{w}\left[\left(1 + \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1}\right) p_r \right. \\ &+ L_{\Delta f} L_f^{r-1} h(\mathbf{x}) \\ &\left. + \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} \left(\hat{p} - L_f^r h(\mathbf{x})\right)\right] \end{aligned} \tag{10}$$

Let us now consider the candidate Lyapunov function

$$V(\mathbf{e}) \quad = \quad \mathbf{e}^T \mathrm{P} \mathbf{e} \tag{11}$$

Since, by construction, A is strictly Hurwitz, it can be concluded that P is a positive definite matrix. Hence, (11) turns out to be a well defined candidate Lyapunov function. By calculating the time derivative of (11) and considering (10), we obtain

$$\begin{aligned} \dot{V} &= -\mathbf{e}^T \mathrm{Q} \mathbf{e} + 2\mathbf{e}^T \mathrm{P} \mathbf{w}\left[L_{\Delta f} L_f^{r-1} h(\mathbf{x}) \right. \\ &+ \left(1 + \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1}\right) p_r \\ &+ \left(L_{\Delta g} L_f^{r-1} h(\mathbf{x})\right) \left(L_g L_f^{r-1} h(\mathbf{x})\right)^{-1} \left(\hat{p} - L_f^r h(\mathbf{x})\right) \end{aligned} \tag{12}$$

Then, substituting $p_r$ from (6) into (12) ensures that $\dot{V}$ is negative definite so that (6) guarantees asymptotic tracking in that $\lim_{t\to\infty}\mathbf{e}(t)=0$. ■

Remark 1: To prevent chattering [21], [22] $\mathrm{sgn}(\cdot)$ in the bottom equation of (6) (calculation of $p_r$) can be replaced by $\mathrm{sat}(\cdot)$ where

$$\mathrm{sat}(x)=\begin{cases} -1 & if \quad x<-1 \\ x & if \quad -1<x<1 \\ 1 & if \quad x>1 \end{cases}$$

This replacement makes the control law softer and more compatible with industrial plants. It must be noticed that this replacement leads to steady state error [23].

Let us now also consider the input-constrained case (5). To this end, let us first introduce the following concept of Lyapunov region.

Definition 1: If the function $V(\mathbf{x})$ satisfies the conditions of the direct stability Lyapunov theorem, and $G=\{\mathbf{x}\,|V(\mathbf{x})<c\}$ then due to negativity of $\dot{V}$, $G$ is an invariant set and is called Lyapunov region.

The following control design algorithm provides an appropriate selection of the coefficients $a_{r-1},\ldots,a_0$ in (6) so as to guarantee the satisfaction of (5).

Procedure 1: Initialize two flags as $f_1=0$ and $f_2=0$.
Step 1: Select an appropriate (i.e. symmetric with respect to the real axis) set of $r$ pole rays as depicted in Fig. 1, for the specific case $r=3$.
Step 2: Select a feasible set of $r$ poles each lying on a selected pole ray. Set $v=1$ for the chosen pole pattern and find the feedback gain $\mathbf{k}(v=1)=[\hat{a}_0,\hat{a}_1,\ldots,\hat{a}_{r-1}]^T$ corresponding to this pole placement. The matrix $\mathrm{A}(v)$ corresponding to $v$ is defined as

$$\mathrm{A}(v)=\begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\hat{a}_0 v^{-r} & -\hat{a}_1 v^{-(r-1)} & \cdots & -\hat{a}_{r-1}v^{-1} \end{bmatrix}$$

Step 3: Select suitable positive variation steps, $s_1$ and $s_2$ with $s_2 \ll s_1 < 1$. For example, $s_1=0.2$ and $s_2=0.02$ could be appropriate choices.
Step 4: Solve the Lyapunov equation (8) with $\mathrm{A}=\mathrm{A}(v)$ for the positive definite matrix P to get the Lyapunov function $V(\mathbf{e},v)=\mathbf{e}^T\mathbf{P}\mathbf{e}$.
Step 5: Initialize an arbitrary value $c>0$.
Step 6: Using the Lyapunov function, find a Lyapunov region $G=\{\mathbf{e}\,|V(\mathbf{e},v)<c\}$.
Step 7: Find the maximum and the minimum amount of $y_d^{(r)}$, $\left(y_d^{(r)}\right)_{max}$ and $\left(y_d^{(r)}\right)_{\min}$, the maximum and the minimum amount of $p_r$, $(p_r)_{\max}$ and $(p_r)_{\min}$, the maximum and the minimum amount of $L_f^r h(\mathbf{x})$, $\left(L_f^r h(\mathbf{x})\right)_{\max}$ and $\left(L_f^r h(\mathbf{x})\right)_{\min}$, the minimum amount of $L_g L_f^{r-1}h(\mathbf{x})$, $\left(L_g L_f^{r-1}h(\mathbf{x})\right)_{\min}$, over the Lyapunov region $G$.

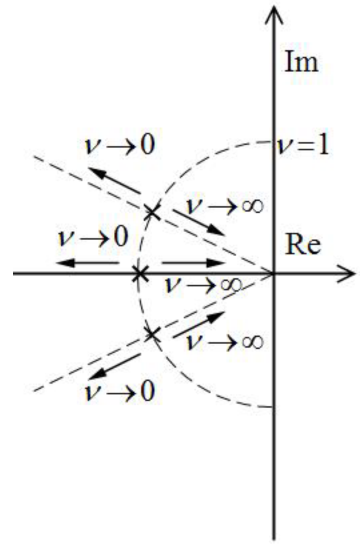


Fig. 1. Pole rays in the complex plane

Step 8: If the hyperplanes

$$\mathbf{k}^T\mathbf{e}=\left(L_g L_f^{r-1}h(\mathbf{x})\right)_{\min}u_{\max}-(p_r)_{\max} -\left(y_d^{(r)}\right)_{\max}+\left(L_f^r h(\mathbf{x})\right)_{\min}$$

and

$$\mathbf{k}^T\mathbf{e}=-\left(L_g L_f^{r-1}h(\mathbf{x})\right)_{\min}u_{\max}-(p_r)_{\min} -\left(y_d^{(r)}\right)_{\min}+\left(L_f^r h(\mathbf{x})\right)_{\max}$$

do not cross the Lyapunov region $G$ and $f_1=1$, go to Step 9. If the aforementioned two hyperplanes do not cross the Lyapunov region $G$ and $f_1=0$, set $c=c+s_1$ and go to Step 6. Otherwise, set $f_1=1$, $c=c-s_2$ and go to Step 6.
Step 9: If $E_0$ is not a subset of $G$, set $v=v+s_2$, $f_2=1$, and go to Step 4.
Step 10: If $E_0\subset G$ and $f_2=0$, set $v=v-s_1$ and go to Step 4. If $E_0\subset G$ and $f_1=1$, go to Step 11.
Step 11: The control law (6) with the coefficients $\mathbf{k}^T=\left[\hat{a}_0 v^{-r},\hat{a}_1 v^{-(r-1)},\ldots,\hat{a}_{r-1}v^{-1}\right]$ will guarantee the satisfaction of the input constraints (5). ■

Remark 2: The inner loop from Step 6 through Step 9 tries to find the maximum value of $c$ which can guarantee that the hyperplanes in Step 8 do not cross the Lyapunov region. The trial-and-error method adopted in this procedure is based on starting with an unfeasible value of $c$. Hence, at the beginning of the procedure it is assumed that $f_1=0$ and if the hyperplanes do not cross the Lyapunov region $G$, $c$ will be increased by $c=c+s_1$ until the hyperplanes of Step 8 cross $G$. Then, $c$ is slowly reduced by $c=c-s_2$ in order to find the largest value of $c$ which also

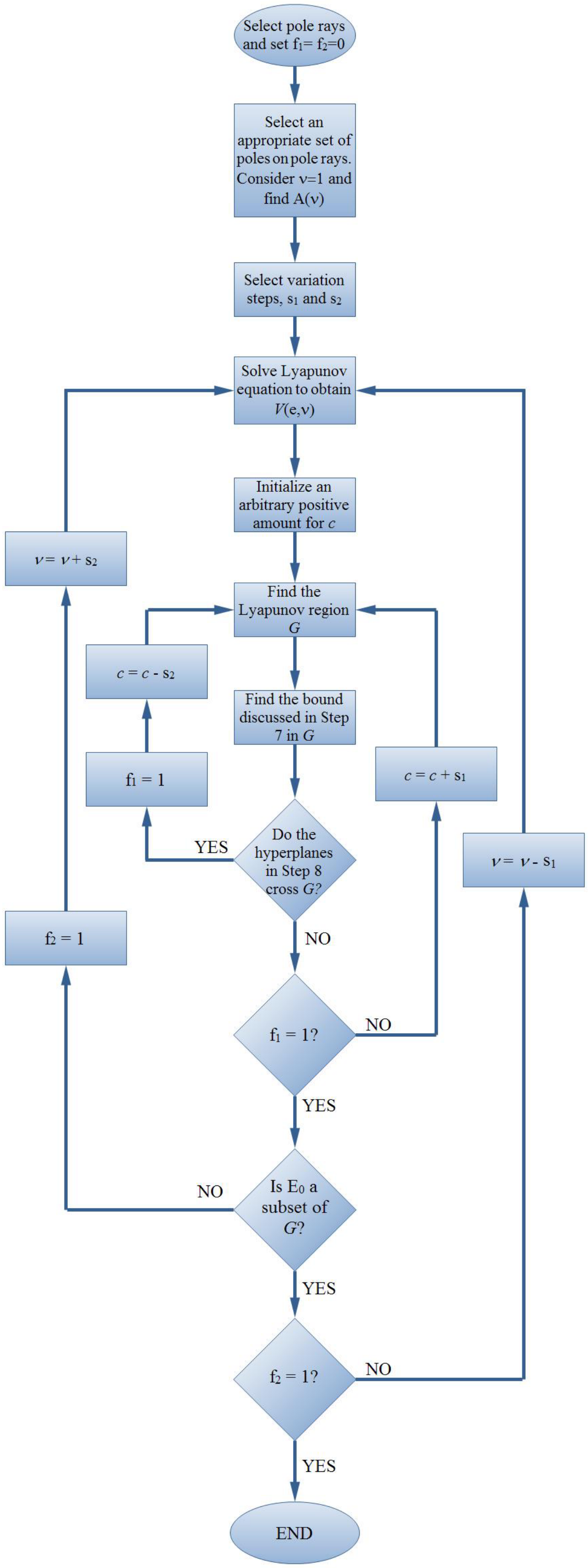


Fig. 2. Flowchart of Procedure 1

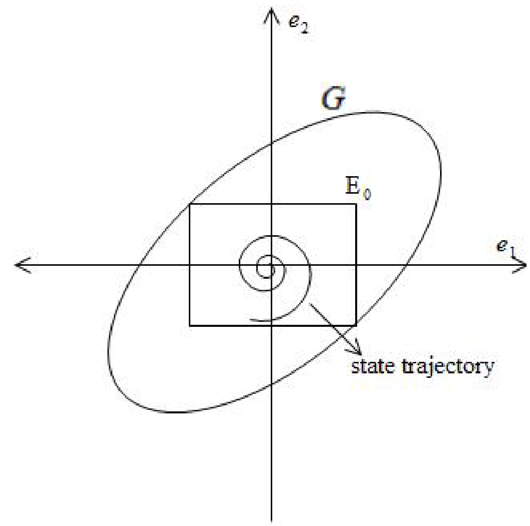


Fig. 3. Phase plane diagram of a second order system when $E_0 \subset G$

leads to a Lyapunov region $G$ that does not cross the hyperplanes. We need to find this optimal value of $c$ to reach the largest $G$ that can satisfy the input constraints. If the feasible initial condition set $E_0$ is a subset of $G$, then this robust feedback linearization control will satisfy the input constraints. Since $G$ is an invariant set and $E_0 \subset G$, the state trajectory cannot leave the Lyapunov region of Step 6 and the input signal always satisfies the constraints (5). Fig. 3 illustrates this situation. The outer loop from Step 4 through Step 11 checks if $E_0 \subset G$. Like the previous loop, this procedure tries to find the smallest $v$ that leads to a Lyapunov region $G$ such that $E_0 \subset G$. In this way, the outcome of Procedure 1 leads to a fast response (smallest $v$) while respecting the input signal constraints.

Remark 3: Step 7 requires obtaining the minimum and/or maximum of several nonlinear functions over region $G$. Though the analytical solution of these optimization problems might not be easy in general, Procedure 1 is actually performed offline so that heuristic and iterative methods such as genetic algorithm (GA), particle swarm optimization (PSO), tabu search, etc. can be used to accomplish Step 7.

## IV. Simulation results

Consider a nonlinear spring-mass system modeled by

$$m\ddot{y} + k_1 y + k_3 y^3 = u \tag{13}$$

Assume that the coefficients in (13) are not exactly known, but (13) can be rewritten as [24]

$$\ddot{y} = -c_1 y - c_2 y^3 + bu \tag{14}$$

where

$$\begin{array}{ccccc} 0.9 & \leq & c_1 & \leq & 1.1 \\ 0.09 & \leq & c_2 & \leq & 0.11 \\ 0.9 & \leq & b & \leq & 1.1 \end{array} \tag{15}$$

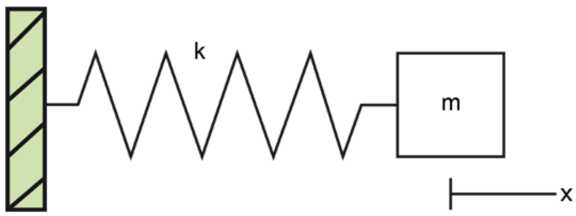


Fig. 4. Nonlinear spring-mass system

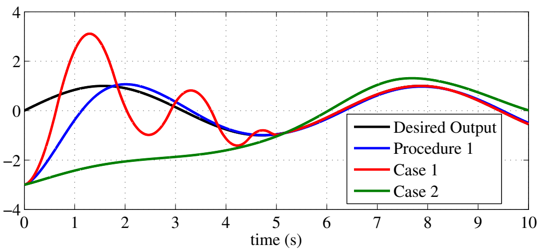


Fig. 5. System output

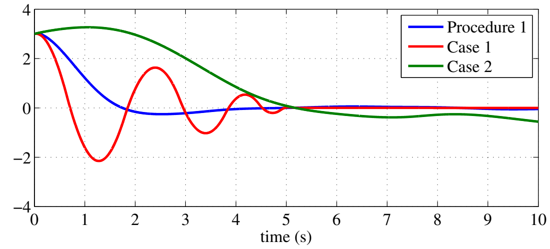


Fig. 6. Tracking error

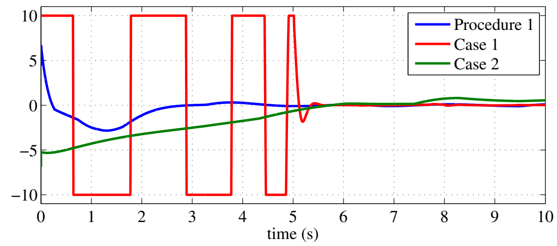


Fig. 7. Input signal

Let $x_1 = y$ and $x_2 = \dot{y}$. Consider that $y_d = \sin(t)$ and the feasible initial state region is limited by $|x_1(0)| \leq 3$ and $|x_2(0)| \leq 0.5$. The relative degree of this system is $r = 2$, there is no zero dynamics and the feasible initial condition for $e$ is $|e_1(0)| \leq 3$, $0.5 \leq |e_2(0)| \leq 1.5$. Assume that the control input is bounded as $|u(t)| \leq 10$. Regarding (14), (15) can be expressed as

$$\begin{aligned}\dot{\mathbf{x}} = &\left( \begin{bmatrix} x_2 \\ -x_1 - 0.1x_1^3 \end{bmatrix} \right. \\ &+ \left. \begin{bmatrix} 0 \\ -(c_1-1)x_1 - (c_2-0.1)x_1^3 \end{bmatrix} \right) \\ &+ \left( \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ b-1 \end{bmatrix} \right) u \end{aligned} \tag{16}$$

In our simulations, we have considered that these coefficients are varying randomly in their admissible ranges. Applying Procedure 1 to the system (16) with two pole rays on the real axis and $s_1 = 0.2, s_2 = 0.02$, we obtained $[a_0, a_1] = [3, 2]$ which, replaced in the robust feedback linearization law (6), allowed to guarantee asymptotic tracking while satisfying the input signal constraints. To show the advantages of designing the control law parameters in (6) by means of Procedure 1, we have also considered the following two cases where the same parameters have been heuristically chosen without Procedure 1:

- Case 1: High-gain control law, obtained by setting $[a_0, a_1] = [300, 20]$ in (6);
- Case 2: Small-gain control law, obtained by setting $[a_0, a_1] = [0.03, 0.2]$ in (6).

Fig. 5 compares the output response with the desired output, while Fig. 6 displays the tracking error and Fig. 7 the input signal. Figs. 5 and 6 show that by adopting Procedure 1 for offline control design, the robust control law (6) exhibits a satisfactory performance while satisfying the input constraints (see Fig. 7). Compared to Case 1, the proposed method avoids input signal saturation and reduces the control effort while providing better performance. Moreover, asymptotic tracking is no longer guaranteed by (6) whenever the control input saturates. On the other hand, the performance of the proposed method is clearly much better than the design of Case 2.

## V. CONCLUSIONS

This paper has proposed a novel robust feedback linearization control scheme that is able to cope with control input limitations by means of a suitable procedure. The main feature of the proposed algorithm is its iterative adjustment of certain parameters in the control law that allows not only to satisfy the input signal constraints, but also to efficiently exploit the available control authority in order to improve the speed of response. Future work will possibly extend this control scheme to MIMO nonlinear systems and/or other types of uncertainties.

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