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Sideslip angle estimation of ground vehicles in a finite frequency domain through H_∞ approach

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Abstract— **This paper studies the sideslip angle estimation problem of a ground vehicle in a finite frequency (FF) domain. The main objective is to estimate the sideslip angle using the yaw rate measurements. Filter design conditions are developed based on the uncertain lateral dynamics model, and time domain interpretations of the kalman Yakubovich Popov lemma (GKYP lemma). The designed filter ensures that the estimation error system is stable and has a prescribed H_∞ attenuation level, over a specified FF domain of the vehicle steering angle. Simulation results are presented to demonstrate the effectiveness of the proposed approach.**

I. INTRODUCTION

The vehicle stability control system relies on the knowledge of the sideslip angle and the yaw rate in emergency situations, to handle the vehicle stability and prevent accident, [1]). The yaw rate is easily measured by using low cost sensors. However, the sideslip angle requires an expensive equipment to be directly measured. Recently, a lot of researches has been focused on development of new strategies concerning the vehicle sideslip angle estimation. The standard and the extended Kalman filters have been adopted in [2] and [3]. The authors in [4] have developed a robust energy-to-peak sideslip angle observer, based on the uncertain linear parameter varying (LPV) vehicle model.

On the other hand, it is worth noting that the vehicle wheel steering angle, which is the input of the LPV lateral dynamics model, works in a low frequency domain, and the filter obtained in the entire frequency domain may not be the optimal. The sideslip angle estimation in a low frequency domain has been investigated in [5].

Motivated by the fact that the FF H_∞ filtering approach has not been sufficiently investigated to estimate the vehicle sideslip angle, we aim to design in this paper a new filter that solves the aforesaid topic. Based on the uncertain LPV lateral dynamics model and the measurements of yaw rate, a new filter analysis conditions are developed by means of the time domain interpretations of GKYP lemma. The obtained conditions ensure that the filtering error system is stable with a prescribed H_∞ attenuation level over a specified FF domain of the vehicle steering angle. Moreover, the developed conditions are linearized so that the desired filter can be obtained by solving a set of LMIs.

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The main feature of the present work in comparison with [5], is that several additional slack variables are introduced to our conditions by applying Finsler's lemma twice, which leads to performance improvement and more freedom degrees in the solution space. Moreover, the proposed observer structure in [5] relies on the knowledge of the input signal (the wheel steering angle) and the measurement signal (the yaw rate). However, our filter only uses the measured yaw rate to estimate the sideslip angle, the vehicle steering angle is supposed unknown. We will see in the simulation part that our proposed method can achieve much better H_∞ attenuation level in comparison with [5].

The rest of this paper is organized as follows. Problem formulation and some preliminaries are presented in section II. Section III presents our proposed H_∞ filtering approach in details. In section IV, we provide simulation results. Conclusions are given in section V.

Notations: A^T, A^{-1}, and $A_\perp$ denote the transpose, the inverse and the null space of a matrix A respectively. $R^{n\times m}$ stands for the $n\times m$ dimensional real matrices. R^n denotes the space of real vectors of dimension n. Symbol $(\bullet)$ indicates a symmetric structure LMIs. $He(A)$ denotes $A+A^T$.

NOMENCLATURE

- m The total mass of the vehicle = $880Kg$.
- l_f Distance from centre of gravity to front axle = $0.808m$.
- l_r Distance from centre of gravity to rear axle = $1.082m$.
- I_z Yaw moment of inertia = $728.6Kg.m^2$
- c_f Nominal front-tyre corning stiffness = $15000N/rad$.
- c_r Nominal rear-tyre corning stiffness = $15000N/rad$.
- $\bar{v}_x$ Maximum longitudinal velocity at the vehicle centre of gravity (m/s) .
- $\bar{v}_x$ Minimum longitudinal velocity at the vehicle centre of gravity (m/s) .
- δ The steering angle (rad).
- η Vehicle sideslip angle (rad).
- ε The yaw rate (rad/s).

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Problem Formulation

We consider a simplified two-degree-of-freedom bicycle model to describe the lateral dynamics. Due to the change of road conditions and the vehicle states, the tire cornering stiffness is always varying, so we have:

$$c_f+\Delta c_f N(t)=\bar{c}_f,$$
$$c_r+\Delta c_r N(t)=\bar{c}_r.$$

where $\mid N(t) \mid \leq 1$, the approximated values $\bar{c}_f$ and $\bar{c}_r$ consist of nominal terms (c_f, c_r) and uncertain terms ($\Delta c_f N(t)$, $\Delta c_r N(t)$). Δc_f and Δc_r can be estimated beforehand.

Moreover, the longitudinal velocity is supposed to be non-constant between two given bounds as: $v_x \in [\underline{v}_x, \bar{v}_x]$. Our estimation method is based on the vehicle lateral dynamics which can be described by the following uncertain LPV state-space model, (see [5] for more details):

$$
\begin{aligned}
\dot{x}(t) &= A(\alpha)x(t) + B(\alpha)\delta(t),\\
y(t) &= Cx(t),\\
z(t) &= Hx(t).
\end{aligned}
\tag{1}
$$

where: $x(t) = \begin{bmatrix} \eta^T & \varepsilon^T \end{bmatrix}^T$ is the state vector, $y(t)$ represents the measurement output and $z(t)$ is the signal to be estimated,

$$A(\alpha) = \bar{A}(\alpha) + E_A(\alpha)M(t)F_A(\alpha) = \sum_{i=1}^{3} \alpha_i \left(\bar{A}_i + E_{Ai}M(t)F_A\right),$$

$$B(\alpha) = \bar{B}(\alpha) + E_B(\alpha)M(t)F_B(\alpha) = \sum_{i=1}^{3} \alpha_i \left(\bar{B}_i + E_{Bi}M(t)F_B\right),$$

$$\bar{A}_1 = \begin{bmatrix} \frac{-c_f - c_r}{m\bar{v}_x} & \frac{l_r c_r - l_f c_f}{m\bar{v}_x^2} - 1 \\ \frac{l_r c_r - l_f c_f}{I_z} & \frac{-l_f^2 c_f - l_r^2 c_r}{I_z \bar{v}_x} \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} \frac{-c_f - c_r}{m} \times \frac{\bar{v}_x + \underline{v}_x}{2\bar{v}_x\underline{v}_x} & \frac{l_r c_r - l_f c_f}{m\bar{v}_x\underline{v}_x} - 1 \\ \frac{l_r c_r - l_f c_f}{I_z} & \frac{-l_f^2 c_f - l_r^2 c_r}{I_z} \times \frac{\bar{v}_x + \underline{v}_x}{2\bar{v}_x\underline{v}_x} \end{bmatrix},$$

$$\bar{A}_3 = \begin{bmatrix} \frac{-c_f - c_r}{m\underline{v}_x} & \frac{l_r c_r - l_f c_f}{m\underline{v}_x^2} - 1 \\ \frac{l_r c_r - l_f c_f}{I_z} & \frac{-l_f^2 c_f - l_r^2 c_r}{I_z \underline{v}_x} \end{bmatrix},$$

$$E_{A,1} = \begin{bmatrix} \frac{-\Delta c_f - \Delta c_r}{m\bar{v}_x} & \frac{l_r \Delta c_r - l_f \Delta c_f}{m\bar{v}_x^2} \\ \frac{l_r \Delta c_r - l_f \Delta c_f}{I_z} & \frac{-l_f^2 \Delta c_f - l_r^2 \Delta c_r}{I_z \bar{v}_x} \end{bmatrix},$$

$$E_{A,2} = \begin{bmatrix} \frac{-\Delta c_f - \Delta c_r}{m} \times \frac{\bar{v}_x + \underline{v}_x}{2\bar{v}_x\underline{v}_x} & \frac{l_r \Delta c_r - l_f \Delta c_f}{m\bar{v}_x\underline{v}_x} \\ \frac{l_r \Delta c_r - l_f \Delta c_f}{I_z} & \frac{-l_f^2 \Delta c_f - l_r^2 \Delta c_r}{I_z} \times \frac{\bar{v}_x + \underline{v}_x}{2\bar{v}_x\underline{v}_x} \end{bmatrix},$$

$$E_{A,3} = \begin{bmatrix} \frac{-\Delta c_f - \Delta c_r}{m\underline{v}_x} & \frac{l_r \Delta c_r - l_f \Delta c_f}{m\underline{v}_x^2} \\ \frac{l_r \Delta c_r - l_f \Delta c_f}{I_z} & \frac{-l_f^2 \Delta c_f - l_r^2 \Delta c_r}{I_z \underline{v}_x} \end{bmatrix},$$

$$M(t) = \begin{bmatrix} N(t) & 0 \\ 0 & N(t) \end{bmatrix}, \quad F_A = I, \quad \mid N(t) \mid \leq 1,$$

$$\bar{B}_1 = \begin{bmatrix} \frac{c_f}{m\bar{v}_x} \\ \frac{l_f c_f}{I_z} \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} \frac{c_f}{m} \times \frac{\bar{v}_x + \underline{v}_x}{2\bar{v}_x\underline{v}_x} \\ \frac{l_f c_f}{I_z} \end{bmatrix}, \quad \bar{B}_3 = \begin{bmatrix} \frac{c_f}{m\underline{v}_x} \\ \frac{l_f c_f}{I_z} \end{bmatrix},$$

$$E_{B,1} = \begin{bmatrix} \frac{\Delta c_f}{m\bar{v}_x} & 0 \\ 0 & \frac{l_f \Delta c_f}{I_z} \end{bmatrix}, \quad E_{B,2} = \begin{bmatrix} \frac{\Delta c_f}{m} \times \frac{\bar{v}_x + \underline{v}_x}{2\bar{v}_x\underline{v}_x} & 0 \\ 0 & \frac{l_f \Delta c_f}{I_z} \end{bmatrix},$$

$$E_{B,3} = \begin{bmatrix} \frac{\Delta c_f}{m\underline{v}_x} & 0 \\ 0 & \frac{-l_f \Delta c_f}{I_z} \end{bmatrix}, \text{ and } F_B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

We suppose that the yaw rate is measured, and we aim to estimate the value of the sideslip angle, thus we have:

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \text{ and } H = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

We consider a full-order filter:

$$
\begin{aligned}
\dot{x}_f(t) &= A_f(\alpha)x_f(t) + B_f(\alpha)y(t),\\
z_f(t) &= C_f(\alpha)x_f(t) + D_f(\alpha)y(t),
\end{aligned}
\tag{2}
$$

where x_f and z_f are the state and output of the filter. For less conservative result, we assume that the filter matrices depend on the uncertain parameter α, i.e.:

$$A_f(\alpha) = \sum_{i=1}^{3} \alpha_i A_{fi}, \quad B_f(\alpha) = \sum_{i=1}^{3} \alpha_i B_{fi},$$

$$C_f(\alpha) = \sum_{i=1}^{3} \alpha_i C_{fi}, \quad D_f(\alpha) = \sum_{i=1}^{3} \alpha_i D_{fi}.$$

Matrices $(A_{fi}, B_{fi}, C_{fi}, D_{fi})$ are to be determined.

We combine systems (1) and (2), we obtain the following filtering error system:

$$
\begin{aligned}
\dot{\xi}(t) &= \widetilde{A}(\alpha)\xi(t) + \widetilde{B}(\alpha)\delta(t)\\
e(t) &= \widetilde{C}(\alpha)\xi(t)
\end{aligned}
\tag{3}
$$

where $e(t) = z(t) - z_f(t)$ is the filtering error, and:

$$\xi(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \qquad \widetilde{A}(\alpha) = \begin{bmatrix} A(\alpha) & 0 \\ B_f(\alpha)C & A_f(\alpha) \end{bmatrix},$$

$$\widetilde{B}(\alpha) = \begin{bmatrix} B(\alpha) \\ 0 \end{bmatrix}, \qquad \widetilde{C}(\alpha) = \begin{bmatrix} H - D_f(\alpha)C & -C_f(\alpha) \end{bmatrix}.$$

Consider γ a positive scalar, our main objective is to find a filter of form (2) such that:

- System (3) is asymptotically stable when $\delta(t) = 0$,
- The ℓ_2 gain, from input $\delta(t)$ to the estimation error $e(t)$ in (3), is smaller or equal to γ in the FF domain of $\delta(t)$, under zeros initial conditions ($\xi(0) = 0$).

B. Preliminaries

Lemma 1 ([6]): *(Finsler's lemma)* let $E \in R^n$, $\Gamma \in R^{m \times n}$ and $\Sigma = \Sigma^T \in R^{n \times n}$ be given such that $rank(\Gamma) < n$. The following statements are equivalents:

i. $E^T \Psi E < 0$ for all $\Gamma E = 0$, $\Gamma \neq 0$

ii. $\Gamma_\perp^T \Psi \Gamma_\perp < 0$

iii. $\exists M \in R^{n \times m}$ such that $\Psi + M\Gamma + \Gamma^T M^T < 0$

Lemma 2 ([7]): *(The generalized S-procedure)* Let $P \in H^{n \times n}$ and a one vector lossless set $K \subset H^n$ be given such that $P = P^* \geq 0$, then the following two statements are equivalents:

i. $\rho^* P \rho < 0$, $\forall \rho \in \Xi$,

$\Xi = \{\rho \in C^n : \rho \neq 0, \rho^* M \rho \leq 0, \forall M \in K\}$

ii. There exists $M \in K$ such that $P < M$.

Lemma 3 ([8]): Given appropriately dimensioned matrices Υ_1, Υ_2 and Υ_3, with $\Upsilon_1^T = \Upsilon_1$, then

$$\Upsilon_1 + \Upsilon_3 \kappa(t) \Upsilon_2 + \Upsilon_2^T \kappa^T(t) \Upsilon_3^T < 0 \tag{4}$$

hold for all $\kappa^T(t)\kappa(t) \leq I$ if and only if for some $\beta > 0$

$$\Upsilon_1 + \beta \Upsilon_3 \Upsilon_3^T + \beta^{-1} \Upsilon_2^T \Upsilon_2 < 0 \tag{5}$$

Lemma 4 ([9]): The filtering error system in (3) is said to have an ℓ_2 gain γ in a low frequency domain if the inequality:

$$\int_0^{+\infty} e^T(t)e(t)dt \leq \gamma^2 \int_0^{+\infty} \delta^T(t)\delta(t)dt \tag{6}$$

holds for any non-zero $\delta(t) \in \ell_2[0, +\infty)$ and under zero initial conditions, the following holds:

$$\int_0^{+\infty} \dot{\xi}(t)\dot{\xi}^T(t)dt \leq \omega_l^2 \int_0^{+\infty} \xi(t)\xi^T(t)dt \tag{7}$$

for $|\omega| \leq \omega_l$.

III. MAIN RESULT

A. Finite Frequency H_∞ Filtering Analysis

First, we assume that filter's parameters are known, the following theorem provides sufficient conditions under which filtering error system (3) is asymptotically stable with a prescribed H_∞ level γ in the low frequency domain of δ. Based on the new conditions, the filter design method will be proposed later.

Theorem 1: Consider system (1) and filter (2), with input δ which belongs to a known low frequency interval, i.e. $|\ \omega\ | \leq \omega_l$, and let scalars $(\gamma, \mu, \beta_1, \beta_2, \beta_s > 0)$ be given. Then, the estimation error system in (3) is asymptotically stable with H_∞ performance γ if there exist matrices y_1, $y_2(\alpha)$, $y_i(\alpha)$, $(i = 3,4,6,7,9,10)$, y_5, $y_8(\alpha)$, $F(\alpha)$, $G(\alpha)$, $M(\alpha)$, $P(\alpha)$, $Q(\alpha)$, $P_s(\alpha)$, $y_{sj}(\alpha)$, $(j = 1,2)$, and $y_{s3}(\alpha)$, of appropriate dimensions, such that $Q(\alpha) > 0$, $P_s(\alpha) > 0$ and:

$$\hat{\Sigma}_s(\alpha) + He\left\{\hat{\Theta}_s(\alpha)\right\} < 0 \tag{8}$$

$$\hat{\Sigma}(\alpha) + He\left\{\hat{\Theta}(\alpha)\right\} < 0 \tag{9}$$

where:

$$\hat{\Theta}_s(\alpha) = \begin{bmatrix} -y_{s1}(\alpha) & -y_5 & y_{s1}(\alpha)\bar{A}(\alpha) + y_5 B_f(\alpha)C & y_5 A_f(\alpha) & 0 \\ -y_{s2}(\alpha) & -y_5 & y_{s2}(\alpha)\bar{A}(\alpha) + y_5 B_f(\alpha)C & y_5 A_f(\alpha) & 0 \\ -y_{s3}(\alpha) & -y_5 & y_{s3}(\alpha)\bar{A}(\alpha) + y_5 B_f(\alpha)C & y_5 A_f(\alpha) & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\Sigma}_s(\alpha) = \begin{bmatrix} 0 & P_s(\alpha) & \beta_s \begin{bmatrix} y_{s1}(\alpha)E_A(\alpha) \\ y_{s2}(\alpha)E_A(\alpha) \end{bmatrix} & 0 \\ \bullet & 0 & \beta_s y_{s3}(\alpha)E_A(\alpha) & \begin{bmatrix} F_A^T(\alpha) \\ 0 \end{bmatrix} \\ \bullet & \bullet & -\beta_s I & 0 \\ \bullet & \bullet & \bullet & -\beta_s I \end{bmatrix}$$

$$\hat{\Theta}(\alpha) = \begin{bmatrix} -y_1 & -y_2(\alpha) & 0 & y_1(H - D_f(\alpha)C) + y_2(\alpha)\bar{A}(\alpha) & -y_1 C_f(\alpha) & y_2\bar{B}(\alpha) & 0 \\ 0 & -y_3(\alpha) & 0 & y_3(\alpha)\bar{A}(\alpha) & 0 & y_3(\alpha)\bar{B}(\alpha) & 0 \\ 0 & -y_4(\alpha) & -y_5 & y_4(\alpha)\bar{A}(\alpha) + y_5 B_f(\alpha)C & y_5 A_f(\alpha) & y_4(\alpha)\bar{B}(\alpha) & 0 \\ 0 & -y_6(\alpha) & -y_5 & y_6(\alpha)\bar{A}(\alpha) + y_5 B_f(\alpha)C & y_5 A_f(\alpha) & y_6(\alpha)\bar{B}(\alpha) & 0 \\ 0 & -y_7(\alpha) & -y_5 & y_7(\alpha)\bar{A}(\alpha) + y_5 B_f(\alpha)C & y_5 A_f(\alpha) & y_7(\alpha)\bar{B}(\alpha) & 0 \\ 0 & -y_8(\alpha) & 0 & y_8(\alpha)\bar{A}(\alpha) & 0 & y_8(\alpha)\bar{B}(\alpha) & 0 \\ 0 & -y_9(\alpha) & -\mu y_5 & y_9(\alpha)\bar{A}(\alpha) + \mu y_5 B_f(\alpha)C & \mu y_5 A_f(\alpha) & y_9(\alpha)\bar{B}(\alpha) & 0 \\ 0 & -y_{10}(\alpha) & 0 & y_{10}(\alpha)\bar{A}(\alpha) & 0 & y_{10}(\alpha)\bar{B}(\alpha) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\hat{\Sigma}(\alpha)$ is given at the top of the next page.

Proof: Define matrices $P(\alpha)$ and $Q(\alpha)$, and $N(\alpha)$ such that $Q(\alpha) > 0$ and:

$$N(\alpha) = \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) \\ I & 0 \end{bmatrix}^T \begin{bmatrix} -Q(\alpha) & P(\alpha) \\ P^T(\alpha) & \omega_l^2 Q(\alpha) \end{bmatrix} \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) \\ I & 0 \end{bmatrix} \geq 0 \tag{10}$$

We pre- and post- multiply (10) by $\begin{bmatrix} \xi^T(t) & \delta^T(t) \end{bmatrix}$ and its transpose, so we have:

$$\begin{bmatrix} \xi(t) \\ \delta(t) \end{bmatrix}^T N(\alpha) \begin{bmatrix} \xi(t) \\ \delta(t) \end{bmatrix} \geq 0, \quad \forall N(\alpha) \in \aleph(\alpha) \tag{11}$$

$$\aleph(\alpha) = \left\{ \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) \\ I & 0 \end{bmatrix}^T \begin{bmatrix} -Q(\alpha) & P(\alpha) \\ P^T(\alpha) & \omega_l^2 Q(\alpha) \end{bmatrix} \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) \\ I & 0 \end{bmatrix}, P(\alpha), Q(\alpha) \in R^{4\times 4}, Q(\alpha) > 0 \right\}$$

Integrating from 0 to $+\infty$ using the stability property

$$trace\left\{ Q(\alpha) \int_0^{+\infty} \left(\dot{\xi}(t)\dot{\xi}^T(t) - \omega_l^2 \xi(t)\xi^T(t) \right) dt \right\} \leq 0 \tag{12}$$

since $Q > 0$, hence we can conclude that inequality (11) guarantees the satisfaction of condition (7) (lemma 4).

Now define the H_∞ performance index as follows:

$$J = \int_0^{+\infty} \left(e^T(t)e(t) - \gamma^2 \delta^T(t)\delta(t) \right) dt \tag{13}$$

Filtering error system (3) has a prescribed H_∞ attenuation level γ if and only if $J < 0$, which can be rewritten as:

$$\begin{bmatrix} \xi(t) \\ \delta(t) \end{bmatrix}^T \begin{bmatrix} \widetilde{C}^T(t)\widetilde{C}(t) & 0 \\ 0 & -\gamma^2 \end{bmatrix} \begin{bmatrix} \xi(t) \\ \delta(t) \end{bmatrix} < 0 \tag{14}$$

From lemma 2, inequalities (11) and (14) are equivalent to:

$$\begin{bmatrix} \widetilde{C}^T(t)\widetilde{C}(t) & 0 \\ 0 & -\gamma^2 \end{bmatrix} + \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) \\ I & 0 \end{bmatrix}^T \begin{bmatrix} -Q(\alpha) & P(\alpha) \\ P^T(\alpha) & \omega_l^2 Q(\alpha) \end{bmatrix} \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) \\ I & 0 \end{bmatrix} < 0 \tag{15}$$

We pre- and post- multiply (15) by $\begin{bmatrix} \xi^T(t) & \delta^T(t) \end{bmatrix}^T$ and its transpose, we get:

$$\begin{bmatrix} \xi(t) \\ \delta(t) \\ \dot{\xi}(t) \end{bmatrix}^T \begin{bmatrix} \varepsilon_{11} & P^T(\alpha)B(\alpha) & 0 \\ \bullet & -\gamma^2 & 0 \\ \bullet & \bullet & -Q(\alpha) \end{bmatrix} \begin{bmatrix} \xi(t) \\ \delta(t) \\ \dot{\xi}(t) \end{bmatrix} < 0 \tag{16}$$

where: $\varepsilon_{11} = \omega_l^2 Q(\alpha) + P^T(\alpha)\widetilde{A}(\alpha) + \widetilde{A}^T(\alpha)P(\alpha) + \widetilde{C}^T(\alpha)\widetilde{C}(\alpha)$

By (3), one has:

$$\begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & -I \end{bmatrix} \begin{bmatrix} \xi(t) \\ \delta(t) \\ \dot{\xi}(t) \end{bmatrix} = 0 \tag{17}$$

Using Finsler's lemma (statements i, and iii) with:

$$E = \begin{bmatrix} \xi(t) \\ \delta(t) \\ \dot{\xi}(t) \end{bmatrix}, \quad \Psi = \begin{bmatrix} \varepsilon_{11} & P^T(\alpha)B(\alpha) & 0 \\ \bullet & -\gamma^2 & 0 \\ \bullet & \bullet & -Q(\alpha) \end{bmatrix},$$

$$\text{and } \Gamma = \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & -I \end{bmatrix}$$

(16) is feasible if and only if there exist matrices $F(\alpha)$, $G(\alpha)$, and $M(\alpha)$ such that:

$$\Psi + He\left\{ \begin{bmatrix} F(\alpha) \\ M(\alpha) \\ G(\alpha) \end{bmatrix} \begin{bmatrix} \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & -I \end{bmatrix} \right\} < 0 \tag{18}$$

We rewrite (18) in the following form:

$$\begin{bmatrix} \widetilde{C}(\alpha) & 0 & 0 \\ \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & 0 \\ I & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix}^T \Sigma(\alpha) \begin{bmatrix} \widetilde{C}(\alpha) & 0 & 0 \\ \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & 0 \\ I & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix} < 0 \tag{19}$$

$$\hat{\Sigma}(\alpha)=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & \beta_1 y_2 E_A(\alpha) & \beta_2 y_2 E_B(\alpha) & 0 & 0\\ \bullet & 0 & P(\alpha)+F^T(\alpha) & M^T(\alpha) & G^T(\alpha) & \beta_1\begin{bmatrix}y_3(\alpha)E_A(\alpha)\\ y_4(\alpha)E_A(\alpha)\end{bmatrix} & \beta_2\begin{bmatrix}y_3(\alpha)E_B(\alpha)\\ y_4(\alpha)E_B(\alpha)\end{bmatrix} & 0 & 0\\ \bullet & \bullet & \omega_l^2 Q(\alpha) & 0 & -F(\alpha) & \beta_1\begin{bmatrix}y_6(\alpha)E_A(\alpha)\\ y_7(\alpha)E_A(\alpha)\end{bmatrix} & \beta_2\begin{bmatrix}y_6(\alpha)E_B(\alpha)\\ y_7(\alpha)E_B(\alpha)\end{bmatrix} & 0 & \begin{bmatrix}F_A^T(\alpha)\\ 0\end{bmatrix}\\ \bullet & \bullet & \bullet & -\gamma^2 & -M(\alpha) & \beta_1 y_8(\alpha)E_A(\alpha) & \beta_2 y_8(\alpha)E_B(\alpha) & F_B^T(\alpha) & 0\\ \bullet & \bullet & \bullet & \bullet & -Q(\alpha)-G(\alpha)-G^T(\alpha) & \beta_1\begin{bmatrix}y_9(\alpha)E_A(\alpha)\\ y_{10}(\alpha)E_A(\alpha)\end{bmatrix} & \beta_2\begin{bmatrix}y_9(\alpha)E_B(\alpha)\\ y_{10}(\alpha)E_B(\alpha)\end{bmatrix} & 0 & 0\\ \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_1 I & 0 & 0 & 0\\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_2 I & 0 & 0\\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_2 I & 0\\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_1 I\end{bmatrix}$$

where:

$$\Sigma(\alpha)=\begin{bmatrix}1 & 0 & 0 & 0 & 0\\ \bullet & 0 & P(\alpha)+F^T(\alpha) & M^T(\alpha) & G^T(\alpha)\\ \bullet & \bullet & \omega_l^2 Q(\alpha) & 0 & -F(\alpha)\\ \bullet & \bullet & \bullet & -\gamma^2 & -M(\alpha)\\ \bullet & \bullet & \bullet & \bullet & -Q(\alpha)-G(\alpha)-G^T(\alpha)\end{bmatrix}$$

Noticing:

$$\begin{bmatrix}-1 & 0 & \widetilde{C}(\alpha) & 0 & 0\\ 0 & -I & \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & 0\end{bmatrix}_{\perp}=\begin{bmatrix}\widetilde{C}(\alpha) & 0 & 0\\ \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & 0\\ I & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & I\end{bmatrix} \quad (20)$$

Using Finsler's lemma again (ii and iii), (19) is feasible if and only if there exists a matrix $y(\alpha)$ such that:

$$\Sigma(\alpha)+He\left\{y(\alpha)\begin{bmatrix}-1 & 0 & \widetilde{C}(\alpha) & 0 & 0\\ 0 & -I & \widetilde{A}(\alpha) & \widetilde{B}(\alpha) & 0\end{bmatrix}\right\}<0 \quad (21)$$

Consider the following structure of matrix $y(\alpha)$:

$$y(\alpha)=\begin{bmatrix}y_1 & y_2(\alpha) & 0\\ 0 & y_3(\alpha) & 0\\ 0 & y_4(\alpha) & y_5\\ 0 & y_6(\alpha) & y_5\\ 0 & y_7(\alpha) & y_5\\ 0 & y_8(\alpha) & 0\\ 0 & y_9(\alpha) & \mu y_5\\ 0 & y_{10}(\alpha) & 0\end{bmatrix} \quad (22)$$

So we can rewrite (21) as follows:

$$\Sigma(\alpha)+He\{\Theta(\alpha)\}<0 \quad (23)$$

$$\Theta(\alpha)=\begin{bmatrix}-y_1 & -y_2(\alpha) & 0 & y_1(H-D_f(\alpha)C)+y_2(\alpha)A(\alpha) & -y_1C_f(\alpha) & y_2B(\alpha) & 0\\ 0 & -y_3(\alpha) & 0 & y_3(\alpha)A(\alpha) & 0 & y_3(\alpha)B(\alpha) & 0\\ 0 & -y_4(\alpha) & -y_5 & y_4(\alpha)A(\alpha)+y_5B_f(\alpha)C & y_5A_f(\alpha) & y_4(\alpha)B(\alpha) & 0\\ 0 & -y_6(\alpha) & -y_5 & y_6(\alpha)A(\alpha)+y_5B_f(\alpha)C & y_5A_f(\alpha) & y_6(\alpha)B(\alpha) & 0\\ 0 & -y_7(\alpha) & -y_5 & y_7(\alpha)A(\alpha)+y_5B_f(\alpha)C & y_5A_f(\alpha) & y_7(\alpha)B(\alpha) & 0\\ 0 & -y_8(\alpha) & 0 & y_8(\alpha)A(\alpha) & 0 & y_8(\alpha)B(\alpha) & 0\\ 0 & -y_9(\alpha) & -\mu y_5 & y_9(\alpha)A(\alpha)+\mu y_5B_f(\alpha)C & \mu y_5A_f(\alpha) & y_9(\alpha)B(\alpha) & 0\\ 0 & -y_{10}(\alpha) & 0 & y_{10}(\alpha)A(\alpha) & 0 & y_{10}(\alpha)B(\alpha) & 0\end{bmatrix}$$

Then by lemma 3, for $\kappa(t)=M(t)$, (23) is feasible if and only if there exist $\beta_1,\beta_2>0$ such that (9) holds. This completes the proof for the low frequency H_∞ norm. Following the same lines, we can easily prove inequality (8) for the asymptotic stability of system (3).

B. Finite Frequency H_∞ Filter Design

In this subsection, we will present a low frequency H_∞ filter design approach in order to estimate the sideslip angle.

Theorem 2: Consider system (1), with input $\delta(t)$ which belongs to a known low frequency interval ($|\ \omega\ |\leq\omega_l$), and let scalar ($\gamma,\mu,\beta_1,\beta_2,\beta_s>0$) be given. Then, there exists a filter of form (2) that satisfies the asymptotic stability of estimation error system (3) with H_∞ performance γ if there exist matrices y_1, y_{2i}, y_{ki}, $(k=3,4,6,7,9,10)$, y_5, y_{8i}, F_i, G_i, M_i, P_i, Q_i, P_{si}, y_{sji}, $(j=1,2)$, and y_{s3i}, such that $Q_i>0$, $P_{si}>0$ and:

$$\hat{\Sigma}_{sii}+He\left\{\hat{\Theta}_{sii}\right\}-X_{sii}<0 \quad (24)$$

$$\hat{\Sigma}_{sij}+\hat{\Sigma}_{sji}+He\left\{\hat{\Theta}_{sij}\right\}+He\left\{\hat{\Theta}_{sji}\right\}-X_{sij}-X_{sij}^T<0,,(i<j) \quad (25)$$

$$\hat{\Sigma}_{ii}+He\left\{\hat{\Theta}_{ii}\right\}-X_{ii}<0 \quad (26)$$

$$\hat{\Sigma}_{ij}+\hat{\Sigma}_{ji}+He\left\{\hat{\Theta}_{ij}\right\}+He\left\{\hat{\Theta}_{ji}\right\}-X_{ij}-X_{ij}^T<0,,(i<j) \quad (27)$$

$$X=\begin{bmatrix}X_{11} & X_{12} & X_{13}\\ X_{21} & X_{22} & X_{23}\\ X_{31} & X_{32} & X_{33}\end{bmatrix}<0 \quad (28)$$

$$X_s=\begin{bmatrix}X_{s11} & X_{s12} & X_{s13}\\ X_{s21} & X_{s22} & X_{s23}\\ X_{s31} & X_{s32} & X_{s33}\end{bmatrix}<0 \quad (29)$$

where: $i,j\in(1,2,3)$,

$$\hat{\Sigma}_{sij}=\begin{bmatrix}0 & P_{si} & \beta_s\begin{bmatrix}y_{s1i}E_{Aj}\\ y_{s2i}E_{Aj}\end{bmatrix} & 0\\ \bullet & 0 & \beta_s y_{s3i}E_{Aj} & \begin{bmatrix}F_{Aj}^T\\ 0\end{bmatrix}\\ \bullet & \bullet & -\beta_s I_2 & 0\\ \bullet & \bullet & \bullet & -\beta_s I_2\end{bmatrix}$$

$$\hat{\Sigma}_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \beta_1 y_{2i} E_{Aj} & \beta_2 y_{2i} E_{Bj} & 0 & 0 \\ \bullet & 0 & P_i + F_i^T & M_i^T & G_i^T & \beta_1 \begin{bmatrix} y_{3i} E_{Aj} \\ y_{4i} E_{Aj} \end{bmatrix} & \beta_2 \begin{bmatrix} y_{3i} E_{Bj} \\ y_{4i} E_{Bj} \end{bmatrix} & 0 & 0 \\ \bullet & \bullet & \omega_l^2 Q_i & 0 & -F_i & \beta_1 \begin{bmatrix} y_{6i} E_{Aj} \\ y_{7i} E_{Aj} \end{bmatrix} & \beta_2 \begin{bmatrix} y_{6i} E_{Bj} \\ y_{7i} E_{Bj} \end{bmatrix} & 0 & \begin{bmatrix} F_{Ai}^T \\ 0 \end{bmatrix} \\ \bullet & \bullet & \bullet & -\gamma^2 & -M_i & \beta_1 y_{8i} E_{Aj} & \beta_2 y_{8i} E_{Bj} & F_{Bi}^T & 0 \\ \bullet & \bullet & \bullet & \bullet & -Q_i - G_i - G_i^T & \beta_1 \begin{bmatrix} y_{9i} E_{Aj} \\ y_{10i} E_{Aj} \end{bmatrix} & \beta_2 \begin{bmatrix} y_{9i} E_{Bj} \\ y_{10i} E_{Bj} \end{bmatrix} & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_1 I_2 & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_2 I_2 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_2 I_2 & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\beta_1 I_2 \end{bmatrix}$$

$$\hat{\Theta}_{sij} = \begin{bmatrix} -y_{s1i} & -y_5 & y_{s1i}\bar{A}_j + b_{fi} C & a_{fi} & 0 \\ -y_{s2i} & -y_5 & y_{s2i}\bar{A}_j + b_{fi} C & a_{fi} & 0 \\ -y_{s3i} & -y_5 & y_{s3i}\bar{A}_j + b_{fi} C & a_{fi} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\Theta}_{ij} = \begin{bmatrix} -y_1 & -y_{2i} & 0 & y_1 H - d_{fi} C + y_{2i}\bar{A}_j & -y_1 c_{fi} & y_{2i}\bar{B}_j & 0 \\ 0 & -y_{3i} & 0 & y_{3i}\bar{A}_j & 0 & y_{3i}\bar{B}_j & 0 \\ 0 & -y_{4i} & -y_5 & y_{4i}\bar{A}_j + b_{fi} C & a_{fi} & y_{4i}\bar{B}_j & 0 \\ 0 & -y_{6i} & -y_5 & y_{6i}\bar{A}_j + b_{fi} C & a_{fi} & y_{6i}\bar{B}_j & 0 \\ 0 & -y_{7i} & -y_5 & y_{7i}\bar{A}_j + b_{fi} C & a_{fi} & y_{7i}\bar{B}_j & 0 \\ 0 & -y_{8i} & 0 & y_{8i}\bar{A}_j & 0 & y_{8i}\bar{B}_j & 0 \\ 0 & -y_{9i} & -\mu y_5 & y_{9i}\bar{A}_j + \mu b_{fi} C & \mu a_{fi} & y_{9i}\bar{B}_j & 0 \\ 0 & -y_{10i} & 0 & y_{10i}\bar{A}_j & 0 & y_{10i}\bar{B}_j & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In this case, a suitable H_∞ filter realization is given by:

$$A_f = y_5^{-1} \sum_{i=1}^{3} \alpha_i a_{fi}, \qquad B_f = y_5^{-1} \sum_{i=1}^{3} \alpha_i b_{fi},$$
$$C_f = y_1^{-1} \sum_{i=1}^{3} \alpha_i c_{fi}, \qquad D_f = y_1^{-1} \sum_{i=1}^{3} \alpha_i d_{fi}.$$

Proof: Suppose that (24-29) are satisfied and define :

$$y_k(\alpha) = \sum_{i=1}^{3} \alpha_i y_{ki}, \ (k = 2,3,5,6,7,8),$$

$$y_{sk}(\alpha) = \sum_{i=1}^{3} \alpha_i y_{ski}, \ (k = 2,3,5,6,7,8),$$

$$F(\alpha) = \sum_{i=1}^{3} \alpha_i F_i, \qquad G(\alpha) = \sum_{i=1}^{3} \alpha_i G_i, \qquad M(\alpha) = \sum_{i=1}^{3} \alpha_i M_i,$$

$$P(\alpha) = \sum_{i=1}^{3} \alpha_i P_i, \qquad Q(\alpha) = \sum_{i=1}^{3} \alpha_i Q_i, \qquad P_s(\alpha) = \sum_{i=1}^{3} \alpha_i P_{si},$$

Then, we have:

$$\begin{aligned} \hat{\Sigma}(\alpha) + He\left\{\hat{\Theta}(\alpha)\right\} &= \sum_{i=1}^{3}\sum_{j=1}^{3} \alpha_i \alpha_j \left(\hat{\Sigma}_{ij} + He\left\{\hat{\Theta}_{ij}\right\}\right) \\ &= \sum_{i=1}^{3} \alpha_i^2 \left(\hat{\Sigma}_{ii} + He\left\{\hat{\Theta}_{ii}\right\}\right) \\ &\quad + 2 * \sum_{i=1}^{3}\sum_{i<j}^{3} \alpha_i \alpha_j \left(\frac{\hat{\Sigma}_{ij} + He\left\{\hat{\Theta}_{ij}\right\} + \hat{\Sigma}_{ji} + He\left\{\hat{\Theta}_{ji}\right\}}{2}\right) \\ &\leqslant \sum_{i=1}^{3} \alpha_i^2 X_{ii} + \sum_{i=1}^{3}\sum_{i<j}^{3} \alpha_i \alpha_j \left(X_{ij} + X_{ij}^T\right) \\ &= \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \alpha_3 I \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \alpha_3 I \end{bmatrix} < 0 \end{aligned}$$

$$\begin{aligned} \hat{\Sigma}_s(\alpha) + He\left\{\hat{\Theta}_s(\alpha)\right\} &= \sum_{i=1}^{3}\sum_{j=1}^{3} \alpha_i \alpha_j \left(\hat{\Sigma}_{sij} + He\left\{\hat{\Theta}_{sij}\right\}\right) \\ &= \sum_{i=1}^{3} \alpha_i^2 \left(\hat{\Sigma}_{sii} + He\left\{\hat{\Theta}_{sii}\right\}\right) \\ &\quad + 2 * \sum_{i=1}^{3}\sum_{i<j}^{3} \alpha_i \alpha_j \left(\frac{\hat{\Sigma}_{sij} + He\left\{\hat{\Theta}_{sij}\right\} + \hat{\Sigma}_{sji} + He\left\{\hat{\Theta}_{sji}\right\}}{2}\right) \\ &\leqslant \sum_{i=1}^{3} \alpha_i^2 X_{sii} + \sum_{i=1}^{3}\sum_{i<j}^{3} \alpha_i \alpha_j \left(X_{sij} + X_{sij}^T\right) \\ &= \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \alpha_3 I \end{bmatrix}^T \begin{bmatrix} X_{s11} & X_{s12} & X_{s13} \\ X_{s21} & X_{s22} & X_{s23} \\ X_{s31} & X_{s32} & X_{s33} \end{bmatrix} \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \alpha_3 I \end{bmatrix} < 0 \end{aligned}$$

This implies that conditions of theorem 1 hold, and completes the proof.

IV. SIMULATION RESULTS

We suppose that the wheel stiffness coefficients vary as follows:

$$-0.4c_f \leq \Delta c_f \leq 0.4c_f, \quad \text{and} \quad -0.4c_r \leq \Delta c_r \leq 0.4c_r.$$

The longitudinal velocity v_x is assumed to be within the range $[5m/s; 30m/s]$.

For $\mu = 0.6$, we apply th. 2 in different frequency ranges, the obtained values of γ are grouped in table I. It can be easily seen that our approach can achieve better H_∞ performance indexes in comparison with [5].

Now, we consider the FF range $|\, \omega \,| \leq 0.8\pi$, and the entire frequency range. By applying theorem 2, we obtain the following filter parameters:

$$A_{f1} = \begin{bmatrix} -0.7020 & 0.0733 \\ 0.0733 & -0.1847 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} -1.4855 & 0.1689 \\ 0.1689 & -0.2000 \end{bmatrix},$$

$$A_{f3} = \begin{bmatrix} -1.9672 & 0.2020 \\ 0.2020 & -0.1864 \end{bmatrix}, \quad y_5 = \begin{bmatrix} 0.5564 & -0.0686 \\ -0.0985 & 0.0201 \end{bmatrix},$$

$$B_{f1} = \begin{bmatrix} 0.4436 \\ -0.1951 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.2836 \\ -0.1625 \end{bmatrix}, \quad B_{f3} = \begin{bmatrix} 0.0153 \\ -0.1134 \end{bmatrix},$$

$$C_{f1} = \begin{bmatrix} -0.9407 & 0.3221 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} -0.4700 & 0.1402 \end{bmatrix},$$

$$C_{f3} = \begin{bmatrix} -0.0229 & 0.0824 \end{bmatrix}, \quad y_1 = 0.9971,$$

$D_{f1} = 0.1290$, $\quad D_{f2} = 0.0557$, and $D_{f3} = 0.1161$.

for the frequency domain $|\, \omega \,| \leq 0.8\pi$.

$$A_{f1} = \begin{bmatrix} -0.5429 & 0.0528 \\ 0.0528 & -0.0736 \end{bmatrix}, \quad A_{f2} = \begin{bmatrix} -0.6700 & 0.0723 \\ 0.0723 & -0.1123 \end{bmatrix},$$

$$A_{f3} = \begin{bmatrix} -0.8241 & 0.1181 \\ 0.1181 & -0.1266 \end{bmatrix}, \quad y_5 = \begin{bmatrix} 0.3353 & -0.0391 \\ -0.0206 & 0.0112 \end{bmatrix},$$

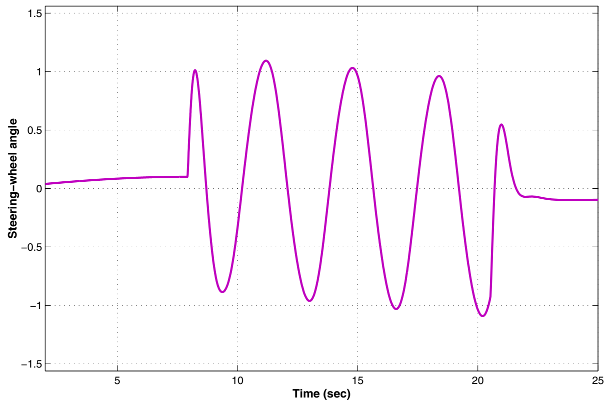

Fig. 1.
Steering wheel angle

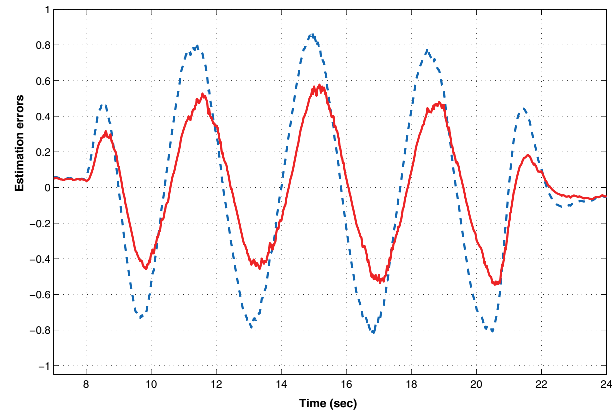

Fig. 2.
Estimation errors: full frequency domain (dashed line) and $\theta \leq 0.8\pi$ (continuous line)

$B_{f1} = \begin{bmatrix} 0.3208 \\ -0.0736 \end{bmatrix}$, $B_{f2} = \begin{bmatrix} 0.1701 \\ -0.0767 \end{bmatrix}$, $B_{f3} = \begin{bmatrix} 0.1609 \\ -0.0787 \end{bmatrix}$,
$C_{f1} = \begin{bmatrix} -0.9659 & 0.1355 \end{bmatrix}$, $C_{f2} = \begin{bmatrix} -0.4397 & 0.1078 \end{bmatrix}$,
$C_{f3} = \begin{bmatrix} -0.0909 & 0.0094 \end{bmatrix}$, $y_1 = 0.9910$
$D_{f1} = -0.0136$, $D_{f2} = 0.0453$, and $D_{f3} = 0.1322$.
for the full frequency domain.

Connecting these filters to the original system in (1), we depict in figure (2) the corresponding estimation errors subject to $\delta(t)$ given in figure (1).

ω	$\mid \omega \mid \leq 0.1$	$\mid \omega \mid \leq 1$	$\mid \omega \mid \leq 0.8\pi$	$\mid \omega \mid \leq 10$	*Entire*
th. 2	0.6325	0.6325	0.7746	0.9487	1.4832
[5]	1.6056	1.6974	1.7261	2.1532	2.9719

TABLE I: Achieved H_∞ attenuation level γ for different frequency ranges in comparison with [5]

It is clear from table I and figure (2) that the H_∞ attenuation level in the FF ranges is much better than in the entire frequency domain, which demonstrates the effectiveness of considering the wheel steering angle frequency during filter design procedure.

V. CONCLUSION

In this work, we have investigated the sideslip angle estimation problem, via FF H_∞ approach, based on lateral dynamics model. H_∞ filter design conditions have been developed in terms of LMIs in the FF domain of the vehicle wheel steering angle, using time domain interpretations of GKYP lemma,generalized S-procedure and Finsler's lemmas. Finally, simulation results have been presented to show the advantages of the proposed approach in comparison with another existing result.

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