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Linear-quadratic Robust Path Tracking for a Dubins Vehicle*

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***Abstract*— Previous theoretical results on a robust linear tracking are applied to a practical problem of trajectory tracking by a Dubins' ground vehicle. The tracking control is constructed as the optimal strategy in an auxiliary linear-quadratic control problem for a linearized vehicle model. Numerical and experimental results are presented and compared. Practical improvements are proposed to circumvent real world problems such as non-ideal dynamics and control saturation. Robustness of tracking method in presence of disturbances is also shown experimentally.**

I. Introduction

Tracking problems for nonlinear systems are widely presented in the literature. General consideration can be found in [1], [2], [3], [4], [5], [6] and references therein. Such problems are formulated for various types of vehicles: ground [7], aerial [8], space [9] and marine - both surface [10] and underwater [11], [12]. Tracking problems arise in guidance [13], motion planning [14] and other applications. There exists a large literature on various aspects of trajectory tracking in robotics [15], [16], [17], [18], [19].

In many publications, the original nonlinear system is linearized either by Taylor series expansion along a nominal trajectory [20], [21], or by a feedback linearization [22], [23], [24]. Once the system is linearized, all the variety of linear control theory results can be exploited subject to additional conditions. This includes the solution of a generalized linear tracking problem which was formulated in [25], [26]. In this formulation, the cost functional is a Lebesgue-Stieltjes integral of a weighted squared discrepancy between an actual and a prescribed system motion. This integral is generated by a measure, consisting of discrete and continuous components. The discrete measure represents a desire of a control designer to guide the system close to prescribed discrete points, while the continuous component corresponds to the problem of tracking a given trajectory at some time intervals in the sense of L_2. A tracking algorithm in the sense of minimization of such a functional, robust with respect to an unknown disturbance, was proposed in [25] and developed in [27] and [28]. The robust tracking strategy is constructed as the optimal strategy in an auxiliary linear-quadratic differential game (LQDG). Penalty coefficients for the control and the disturbance expenditure are small, i.e. a cheap-control approach is utilized. Various specific problems can be obtained from this generalized tracking problem, depending on the controlled system, cost functional structure or prescribed trajectory.

In the present paper, the tracking problem is solved for a ground robot modeled as a Dubins car [29], [30]. By linearization, this problem is reduced to a particular case of the robust linear tracking problem. Some practical adjustments of the tracking feedback strategy are proposed. Numerical and experimental results are compared.

II. Problem Statement

Consider a Dubins vehicle [29] moving in the plane with constant speed v_0 and minimum turn radius r. Let (x, y) be the vehicle's planar coordinates, ψ be its heading angle measured counterclockwise with respect to the x-axis. Assume that the vehicle is controlled by the bounded angular speed mixed with an unknown additive disturbance. Then, its motion is described by the system of differential equations

$$\begin{array}{rcl} \dot{x} & = & v_0 \cos\psi, \\ \dot{y} & = & v_0 \sin\psi, \\ \dot{\psi} & = & U, \end{array} \tag{1}$$

where U is a scalar control.

In order to guarantee that the minimum turn radius is r, the control should satisfy

$$|U| \leq \frac{v_0}{r} \triangleq U_r, \tag{2}$$

Define the state vector

$$X = (X_1, X_2, X_3)^T = (x, y, \psi)^T, \tag{3}$$

where. Then the system (1) can be rewritten as

$$\dot{X} = f(X) + BU, \tag{4}$$

where

$$f(X) = f(\psi) \triangleq (v_0 \cos\psi, v_0 \sin\psi, 0)^T, \tag{5}$$

$$B \triangleq (0, 0, 1)^T. \tag{6}$$

A feasible Dubins path

$$X^*(t) = (x^*(t), y^*(t), \psi^*(t))^T, \tag{7}$$

generated by some admissible control $U = U^*(t)$, is given for $t \in [0, t_f]$. The problem is to form the motion of the vehicle tracking the prescribed path (7). The tracking

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accuracy is understood in the sense of squared L_2-norm of discrepancy:

$$G(X(\cdot)) = \int_0^{t_f} ||X(t) - X^*(t)||^2 \mathrm{d}t, \tag{8}$$

where $||Y||$ is an Euclidian norm of the vector $Y \in R^3$.

Tracking Problem. To construct a feedback strategy $U_\zeta(t, X)$ minimizing G for any $\zeta > 0$ such that

$$G(z_\zeta(\cdot)) \leq \zeta, \tag{9}$$

III. Solution

A. Linearization

Define the discrepancy vector

$$z \triangleq X - X^* = (x - x^*, y - y^*, \psi - \psi^*)^T. \tag{10}$$

Along the nominal trajectory (7) it satisfies the linear differential equation

$$\dot{z} = A(t)z + Bu, \tag{11}$$

where

$$A(t) = \frac{\partial f}{\partial X} = \begin{bmatrix} 0 & 0 & -v_0 \sin(\psi^*(t)) \\ 0 & 0 & v_0 \cos(\psi^*(t)) \\ 0 & 0 & 0 \end{bmatrix}, \tag{12}$$

$$u \triangleq U - U^*. \tag{13}$$

Now, by relaxing the constraint on U the original tracking problem is reformulated for the system (11) as a linear tracking problem.

Linear Tracking Problem. For any $\zeta > 0$, to construct a feedback strategy $u_\zeta(t, z)$ such that

$$G(z_\zeta(\cdot)) \leq \zeta, \tag{14}$$

where

$$G(z(\cdot)) = \int_0^{t_f} ||z(t)||^2 \mathrm{d}t, \tag{15}$$

$z_\zeta(t)$ is a solution of (11), generated by $u_\zeta(t, z)$.

B. Solution of linear tracking problem

Note that the linear tracking problem, formulated in the end of the previous section, represents a simplified version of the generalized robust tracking problem considered, e.g., in [28]. It is obtained from the general formulation in the case of no disturbance in the system, no intermediary points to be tracked and a single interval $[0, t_f]$ of trajectory tracking.

Following the lines of [28], the linear tracking problem is solved based on an auxiliary linear-quadratic optimal control problem (LQOCP) for the system (11) with the cost functional

$$J_\alpha = J_\alpha(u(\cdot)) = G(z(\cdot)) + \alpha \int_0^{t_f} u^2(t) \mathrm{d}t, \tag{16}$$

where $\alpha > 0$ is the control penalty coefficient. In the LQOCP formulation the constraint (2) is relaxed.

The solution of LQOCP (11), (16) is well-known (see, e.g., [31]):

$$u_\alpha^0(t, z) = -\frac{1}{\alpha} B^T \Phi^T(t_f, t) R_\alpha(t) \Phi(t_f, t) z, \tag{17}$$

where

$$\Phi(t, t_0) = \begin{bmatrix} 1 & 0 & -v_0 \int_{t_0}^{t} \sin(\psi^*(\xi)) d\xi \\ 0 & 1 & v_0 \int_{t_0}^{t} \cos(\psi^*(\xi)) d\xi \\ 0 & 0 & 1 \end{bmatrix}, \tag{18}$$

is a transition matrix of a homogeneous system (11), $R_\alpha(t)$ satisfies the matrix Riccati equation

$$\dot{R} = RQ_\alpha(t)R - S(t), \quad R(t_f) = 0, \tag{19}$$

$$Q_\alpha(t) = \frac{1}{\alpha} \Phi(t_f, t) B B^T \Phi^T(t_f, t), \tag{20}$$

$$S(t) = \Phi(t, t_f)^T \Phi(t, t_f). \tag{21}$$

Let $z_\alpha(\cdot)$ denote the solution of (11) for $u = u_\alpha^0(t, z)$. Then, applying the results of [28] to the linear tracking problem yields

$$\lim_{\alpha \to 0} G(z_\alpha(\cdot)) = 0. \tag{22}$$

This means that for any $\zeta > 0$ there exists $\alpha = \alpha(\zeta)$ such that for $u_\zeta(t, z) = u^0_{\alpha(\zeta)}(t, z)$, the inequality (14) holds, i.e., the strategy $u^0_{\alpha(\zeta)}(t, z)$ solves the linear tracking problem.

C. Solution of original tracking problem

Due to (10) and (13), the original problem is solved by using the strategy

$$U_\zeta(t, z) = u^0_{\alpha(\zeta)}(t, z) + U^*(t). \tag{23}$$

IV. Implementation Issues

A. Control saturation

In the auxiliary LQOCP there is no hard control constraint. It is replaced with the soft constraint by introducing the penalty integral term on the control effort. In practical implementation, the control should satisfy (2). Consequently, the tracking strategy (23) should be saturated:

$$\bar{U}_\zeta(t, z) = \mathrm{Sat}_{U_r}(U_\zeta(t, z)) = \begin{cases} U_r, & U_\zeta(t, z) > U_r, \\ U_\zeta(t, z), & |U_\zeta(t, z)| \leq U_r, \\ -U_r, & U_\zeta(t, z) < -U_r. \end{cases} \tag{24}$$

Exploiting the saturated control (24) clearly leads to deteriorating tracking accuracy (see numerical example in Section V). In order to prevent (to decrease) this saturation, it is proposed that a controller with reinforced command should be used. This means that the controller should be designed considering the minimum turn radius of vehicle *smaller* than that on the tracked trajectory:

$$\tilde{r} = \frac{r}{K}, \tag{25}$$

where $K > 0$ is the reinforcement factor. Thus, instead of the control (24), we use

$$\tilde{U}(t, z) = \mathrm{Sat}_{U_{\tilde{r}}}\left(\tilde{u}^0_\alpha(t, z) + U^*(t)\right) \tag{26}$$

where $\tilde{u}^0_\alpha(t, z)$ is calculated as in (17) – (21) by replacing B with

$$\tilde{B} = \left(0, 0, \frac{v_0}{\tilde{r}}\right)^T. \tag{27}$$

B. Angle differentiation

In order to implement the tracking strategy (23), one needs to know the nominal control $U^*(t) = \dot{\psi}^*(t)$. Thus, the desired angle $\psi^*(t)$ should be *differentiated*. A stable differentiation of a noised signal is a well-known ill-posed problem (see, e.g., [32] and references therein). To this end, we propose using a virtual tracking loop for the equation

$$\dot{\psi} = u_\psi. \tag{28}$$

The corresponding LQOCP is formulated with the cost functional

$$J_\psi = \int_0^{t_f} |\psi(t) - \psi^*(t)|^2 dt + \beta \int_0^{t_f} u^2_\psi(t) dt, \tag{29}$$

to be minimized. The penalty coefficient $\beta > 0$ plays the same role as α in (16). Note that the cost functional (29) is the Tikhonov's functional [33] in the problem of a stable differentiation of the function $\psi^*(t)$, and the penalty coefficient β is actually the regularization parameter. Therefore, subject to a proper choice of $\beta > 0$, the time realization of the optimal control in the LQOCP (28) – (29) provides an approximation of the derivative $\dot{\psi}^*(t) = U^*(t)$.

The solution [31] of this optimal control problem is similar to (17):

$$u^0_\psi(t, \psi) = -\frac{1}{2\beta}(2R_\psi(t)\psi + r_\psi(t)), \tag{30}$$

where the functions $R_\psi(t)$ and $r_\psi(t)$ are the solutions of differential equations: the Riccati equation

$$\dot{R} = \frac{1}{\beta}R^2 - 1, \quad R(t_f) = 0, \tag{31}$$

and the linear equation

$$\dot{r} = \frac{1}{\beta}R_\psi(t)r + 2\psi^*(t), \quad r(t_f) = 0, \tag{32}$$

respectively.

Remark 1: If the tracked trajectory (7) is a shortest Dubins' path [29], [34], then the generating control $U^*(t)$ is known, and there is no need in the differentiation of $\psi^*(t)$.

C. Velocity tracking loop

The assumption of constant vehicle speed $v \equiv v_0$ is not satisfied in real-life applications, which deteriorates a tracking accuracy. This can be circumvented by implementing an additional velocity tracking loop. Let the velocity satisfy the differential equation

$$\dot{v} = u_v, \tag{33}$$

where u_v is an auxiliary control input. Consider a tracking problem for (33) with the cost functional

$$J_v = \int_0^{t_f} |v(t) - v_0|^2 dt + \gamma \int_0^{t_f} u^2_v(t) dt, \tag{34}$$

to be minimized. The penalty coefficient $\gamma > 0$ plays the same role as α in (16) and $\beta > 0$ in (29). The solution is similar to (30) – (32):

$$u^0_v(t, v) = -\frac{1}{2\gamma}(2R_v(t)\psi + r_v(t)), \tag{35}$$

where the functions $R_v(t)$ and $r_v(t)$ are the solutions of differential equations

$$\dot{R}_v = \frac{1}{\gamma}R^2_v - 1, \quad R_v(t_f) = 0, \tag{36}$$

$$\dot{r} = \frac{1}{\beta}R_v(t)r + 2v_0, \quad r(t_f) = 0, \tag{37}$$

respectively.

V. Numerical and Experimental Results

The experiments were conducted in the Cooperative Autonomous SYstem laboratory (CASY) in Technion which serves as a testbed for research in cooperative guidance and control of aerial and ground vehicles. It consists of an overhead motion capture (mocap) system for tracking 6-DOF(degrees of freedom) state of rigid bodies within sub-millimeter accuracy. The speed of the rigid bodies is estimated from the position data obtained at 240Hz by using a SavitzkyGolay digital filter [35]. The robotic platform used for the experiments are Kobuki robots from Yujin robotics (see Fig.1). These robots have a maximum linear speed of 70 cm/s and maximum angular speed of π rad/sec with an inbuilt controller for stabilizing the linear and angular velocities. The software platform used is an experimental framework developed at CASY which integrates motion capture system and other robotic platforms with Simulink and ROS (Robot operating system). The commands from the controller are computed off-board on a host computer and then sent to Kobuki base using 2.4GHz standard WiFi protocol.

A. Comparison of numerical simulation and experimental validation

In this section, a comparison of simulation and experimental results is presented for reinforcement factor $K = 1$, without speed tracking loop and in the absence of external disturbance. The conditions of simulation and experiments are identical and are summarised in table I.

Fig. 1. Kobuki robotic platform

TABLE I
CONDITION FOR EXPERIMENTS

Parameter	Value
speed (v_0)	0.15 m/sec
path radius (r)	0.3 m
vehicle turn radius ($\tilde{r}$)	0.3 m
command saturation (U_r)	0.5 rad/sec
control penalty coefficient (α)	0.3
disturbance penalty coefficient (β)	$>> 1$
total time (t_f)	19.12 sec

For all the experiments a Dubins path from $X_0 = (0.05, -0.13, \pi)^T$ to $X_f(t_f) = (1.1, 0.94, 0.95\pi)^T$ is considered. For the given end points, a RSL type Dubins trajectory is obtained with $U^*(t) \equiv -U_r$ for $t \in [0, 6.615)$, $U^*(t) \equiv 0$ for $t \in [6.615, 12.75)$, $U^*(t) \equiv U_r$ for $t \in [12.75, 19.12)$.

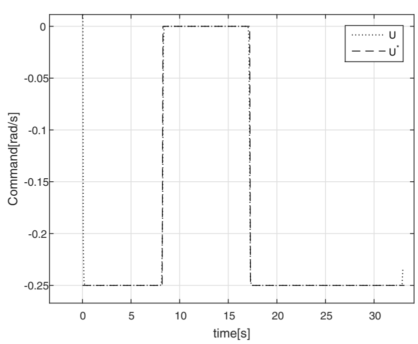

Fig. 2. Stable numerical differentiation of ψ

The reference command(U^*) was obtained using the stable differentiation method as discussed in section IV-B. Fig. 2 shows the concurrency of reference and obtained command.

Figures 3 $-$ 5 compares the tracking for Dubins' path in simulation and experiments. It is shown that numerical simulation results in perfect tracking whereas the results are not so positive for real world experiments. This can be attributed to several facts: the dynamics of the robot is not ideal; the onboard inner loop controller for velocity is prone to errors and has a slower response (see Fig. 6); the control input is saturated almost on the whole control time interval

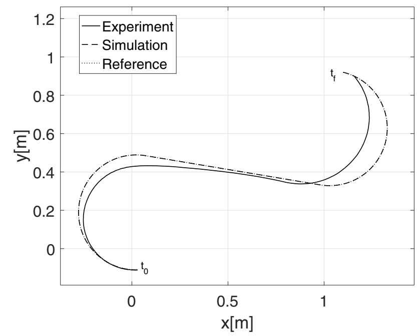

Fig. 3. Trajectory tracking comparison in experiment and simulation

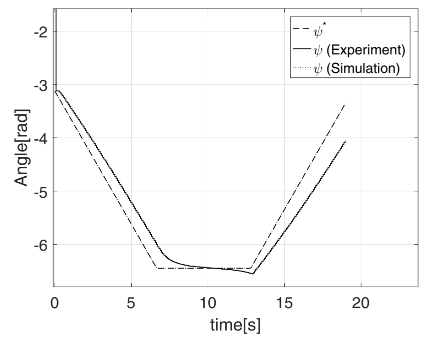

Fig. 4. Angle Tracking in experiment and simulation

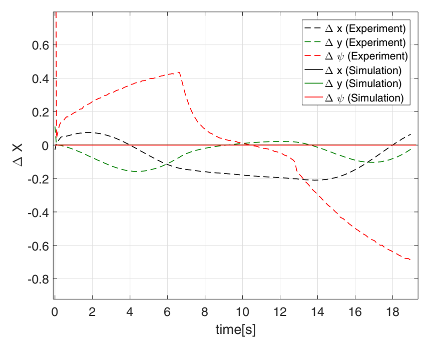

Fig. 5. Tracking errors in experiment and simulation

B. Practical improvements

1) LQ Velocity Controller: In order to improve the response time and accuracy of velocity controller, an additional LQ feedback controller, described in Section IV-C, is used to track a constant velocity with feedback from the camera. Fig. 6 compares the step response of inbuilt controller and the LQ controller for velocity. Note that the settling time improves from 17.44 seconds to 2.56 seconds with an error band of 2%.

2) Command Reinforcement: One of the reasons resulting in the deterioration of tracking is that the controller is saturated for most of the time. Hence there is less room to compensate for deviation from the desired trajectory. As a practical modification to prevent this saturation, it is

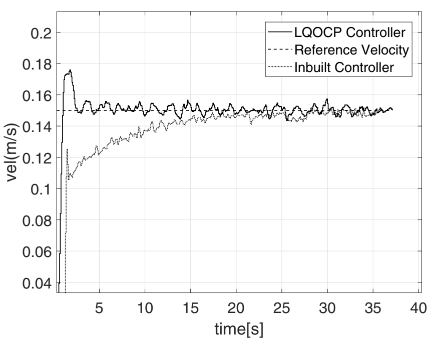

Fig. 6. Velocity Controller Comparison

proposed that a controller with reinforced command should be used as discussed in Section IV-A. A set of experiments were performed to determine the variation of percentage of command saturation with K. It is shown in Fig. 7 that the percentage of time when the command is saturated decreases monotonically with K.

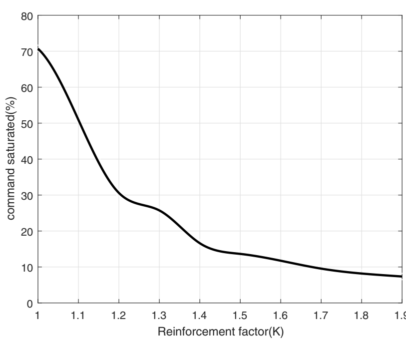

Fig. 7. Percent of saturated command varying with K

For experiments $K = 1.8$ was chosen as for this value, less that 10% of the command is saturated. Fig. 8-10 shows the tracking of Dubins path with above practical improvements implemented. It can be seen that tracking is very accurate and errors is x, y and ψ converge to zero.

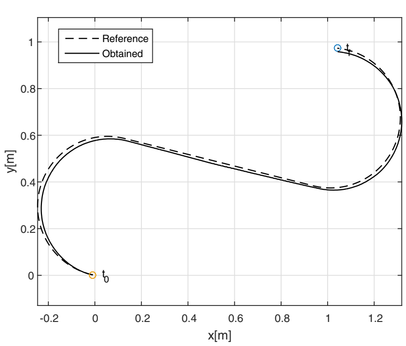

Fig. 8. Trajectory

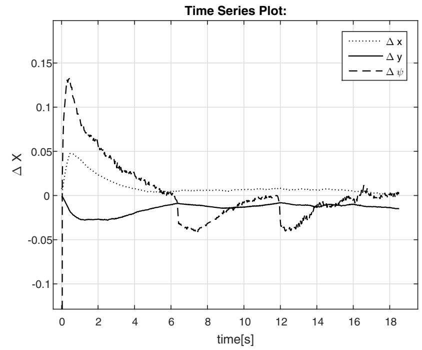

Fig. 9. Tracking errors

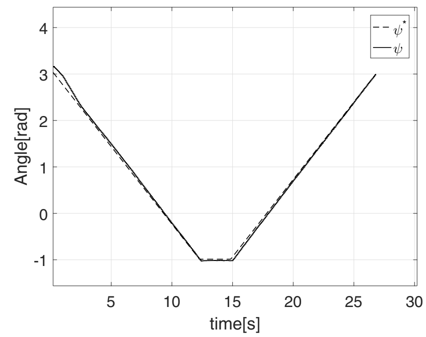

Fig. 10. Angle Tracking

C. Robust response in presence of external disturbance

The controller is expected to perform in presence of uncertainty and external disturbances, hence a test was performed to test its robustness. Fig. 11 shows the tracking of desired trajectory when an external disturbance of $-0.4\ rad/sec$ was constantly applied for the interval $(\frac{t_f}{2} - 3)$ to $(\frac{t_f}{2} + 3)$ seconds. It can be seen that for higher value of α the applied disturbance detours the robots while for $\alpha = 0.01$ the trajectory tracking is almost immune to the same disturbance. Therefore the controller is robust to external disturbances for lower values of α.

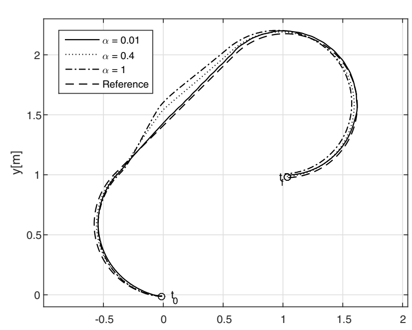

Fig. 11. Robust response in presence of external disturbance

VI. Conclusions

Trajectory tracking algorithm, based on an auxiliary linear-quadratic optimal control problem, was applied to a Dubins' ground vehicle. The implementation of the algorithm was three-fold. First, it was employed in the main tracking loop where the zero functions were tracked for the linearization errors of two coordinates and the angle. Second, in the velocity stabilization loop where a constant velocity was tracked. Third, in the angle differentiation block where a given trajectory angle was tracked in order to restore an input control (the angle derivative).

The paper is focused on the challenges of a real-life implementation of this linearized tracking strategy. The tracking accuracy in the experiments was lower than in the numerical simulation. This can be explained by several reasons, among which are a non-ideal robot dynamics, control saturation, a slow response of an onboard velocity controller, etc. In order to reduce the saturation region, a minimal radius constraint was reinforced in the control design. The modified tracking strategy showed a sufficient accuracy in experiment with results comparable to the one of numerical simulations.

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