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# An approach to event-triggered control of unstable dissipative systems exemplified via PI controller

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***Abstract*—The recent advent of "smart" technologies in the industry has increased the interest on techniques able to lighten the burden on communication networks. In this framework, the debate on event triggered controllers has been heated by many recent and promising developments from a number of different approaches. In this paper we investigate the potential of an original input-output event triggering strategy via an ideal implementation. Special attention is put on the worst case scenario, where systems in the control loop are not stable in free running configuration. A detailed numerical example featuring a Proportional-Integral controller is then used to test the obtained performance against a common technique.**

***Index Terms*—Event triggered control, asynchronous sampling, industry 4.0, networked systems.**

## I. Introduction

The recent trend in industry towards systems integration, along with the advent of "smart" technologies, has made the performance of communication network critical for correct plant functioning. Indeed, according to this paradigm the amount of data exchanged among devices and information system is crucial to boost efficiency, but in turn it can become a treat for standard networks, unable to bear with such a burden. For instance, Industry 4.0 approach sees in communication technologies the main role towards tight integration of production devices up to enterprise resource planning systems [1].

With such a motivation, researchers have started to develop new and more efficient protocols to transmit data, as well as novel communication devices. However, even if the market constantly provides new products, no standard solution has been established yet, and thus new investigations of the problem are lively carried on. For example, in the last decade the debate on aperiodic (or, more often, asynchronous) sampling has been heated by both academic and industry researchers, because of its promising benefits in terms of communication channel usage. Indeed, asynchronous sampling not only may be result of a more considerate data transmission, that avoids sending redundant information, but also it may turn out necessary to better programming the communication channel allocation. In particular, event triggered controllers aim to restrict data exchange to the only instants when information renovation is strictly necessary to guarantee the proper functioning of the plant [2], [3]. Such situations are usually related to loss of certainty about the plant dynamics, and they are referred to as "events". When an event occurs, the involved players renew their information by sending an update to the others, which in turn modify their behaviour according to this data. There exist many different approaches to event triggered control which differ for the event generation rule that guarantees the correct operation of devices [4]–[6]. However, all of them share a model based approach as common feature, while they may differ for features such as communication protocols or the centralized/distributed triggering mechanism.

In this paper we introduce a novel technique to design an event triggered controller from a stabilizing continuous time controller assumed to be already known, thus fitting this work in the so-called "emulation approach" [2], [4]. The proposed technique will envision a input-output perspective, and it will focus on dissipative systems [7]. In particular, special attention will be put on the case where device and controller quickly diverge from the nominal behaviour, because they are not fed with the actual inputs but with their interpolated samples instead. For instance, this is what happens when a integral controller is driven by a constant input equal to last sample. A numerical example will be detailed to provide a better understanding of the proposed technique, along with a comparison with a standard method and useful remarks about practical implementation.

## II. Problem set up and assumptions

We assume a physical device $\mathcal{P}$ and its controller $\mathcal{Q}$ are connected via a network $\mathcal{N}$. When $\mathcal{N}$ can be freely used by them both without any limitation, the feedback system is asymptotically stable to its scalar output set-point, assumed here with no loss of generality in zero. Delays during the transmissions are supposed negligible for the sake of simplicity. If, instead, $\mathcal{N}$ may experience limited bandwidth, because of many concurrent communications, it turns out important $\mathcal{P}$ and $\mathcal{Q}$ only exchange not redundant data, and possibly with no or few other simultaneous transmissions. Hence, $\mathcal{P}$ and $\mathcal{Q}$ must be able to still work fine with asynchronous communications.

To solve the asymptotic convergence to set-point problem, in this paper we pursue a *event-triggered control approach*. Therefore, a supervisor $\mathcal{S}$ is supposed to be connected to the same network. It reads all the data exchanged between $\mathcal{P}$ and

$\mathcal{Q}$, and it can possibly receive from them other supporting data, if needed. The role of $\mathcal{S}$ is to monitor the system dynamic, and to detect when a new exchange of data is necessary to guarantee asymptotic convergence to the set-point. Triggered by the supervisor, the outputs of $\mathcal{P}$ and $\mathcal{Q}$ are *sampled* and transmitted via $\mathcal{N}$. So, in order for $\mathcal{S}$ to correctly decide the next sampling instant, the supervisor must also know how the new samples will be used to feed device and controller, i.e., their *interpolating techniques* have to be shared.

Let $\mathcal{P}$ be described by model

$$\dot{x} = f(x,u) \in \mathbb{R}^n \ , \quad y = p(x) \in \mathbb{R} \tag{1}$$

and let controller $\mathcal{Q}$ have the form

$$\dot{z} = g(z,y) \in \mathbb{R}^m \ , \quad u = q(z) \in \mathbb{R} \ . \tag{2}$$

Assume that, when $\mathcal{P}$ is fed with a constant input $u(t) = u$, the physical device may have a set of possibly unstable fixed points $x_e = x_e(u)$. Similarly, suppose that, when $\mathcal{Q}$ is fed with a constant input $y(t) = y$, the controller may have a set of possibly unstable fixed points $z_e = z_e(y)$. The feedback interconnected system $\mathcal{F}$ reads

$$\dot{x} = f(x, q(z)) \tag{3}$$
$$\dot{z} = g(z, p(x)) \tag{4}$$
$$y = p(x) \tag{5}$$
$$u = q(z) \ , \tag{6}$$

and, for the sake of simplicity, assume its fixed points set $\Omega_e := \{(x_e, z_e)\}$ is such that $p(x_e) = 0 \ \forall x_e \in \Omega_e$.

Let $\mathcal{P}$ and $\mathcal{Q}$ be *dissipative systems* [7], i.e., there exist two *storage functions* $V_p(x) : \mathbb{R}^n \to \mathbb{R}_0^+$, $V_q(z) : \mathbb{R}^m \to \mathbb{R}_0^+$ and two continuous *supply rate functions* $s_p(u,y) : \mathbb{R}^2 \to \mathbb{R}$, $s_q(y,u) : \mathbb{R}^2 \to \mathbb{R}$ such that

$$\dot{V}_p(x(t)) := \frac{dV_p}{dt}(x(t)) \le s_p(u(t), y(t))$$
$$\dot{V}_q(z(t)) := \frac{dV_q}{dt}(z(t)) \le s_q(y(t), u(t))$$
$$\forall t \in \mathbb{R} \ .$$

**Proposition II.1.** *Assume it exists $\alpha \in \mathbb{R} : \ 0 < \alpha < 1$ such that $V_T(x,z) := \alpha V_p(x) + (1-\alpha) V_q(z)$ is radially unbounded. Then, if $s_T(u,y) := \alpha s_p(u,y) + (1-\alpha) s_q(y,u) \le 0$ and $s_T(u,y) = 0 \Rightarrow (u,y) = (p(x_e), q(z_e)) : \ (x_e, z_e) \in \Omega_e$, system $\mathcal{F}$ converges to a subset of $\Omega_e$ and $y \to 0$.*

*Proof.* The result can be seen as a special case of the Lyapunov theorem [7] extended from fixed points to invariant sets. □

*Remark.* Observe that, if system $\mathcal{F}$ satisfies the hypothesis of Proposition II.1, then

$$\lim_{t\to\infty} y(t) = 0 \ , \quad \forall (x(0), z(0)) \in \mathbb{R}^{n\times m}$$

without any possible loss of internal stability.

Supervisor $\mathcal{S}$ must be able to generate a set of sampling instants $t_i$, namely $\mathcal{T} = \{\ldots, t_{i-1}, t_i, t_{i+1}, \ldots\}$, such that the output $y$ of the sampled system

$$\dot{x} = f(x, \tilde{u}) \tag{7}$$
$$\dot{z} = g(z, \tilde{y}) \tag{8}$$
$$y = p(x) \tag{9}$$
$$u = q(z) \tag{10}$$

is still asymptotically converging at set-point zero, under *interpolated inputs* $\tilde{u}$, $\tilde{y}$ generated from actual and previous samples

$$\tilde{u}(t) = \tilde{u}\big(t; u(t_i), u(t_{i-1}), \ldots\big)$$
$$\tilde{y}(t) = \tilde{y}\big(t; u(t_i), u(t_{i-1}), \ldots\big)$$
$$\forall t \in [t_i, t_{i+1}) \ .$$

In the following, for the sake of the simplicity, the interpolated inputs will be generated according to a *Zero-Order-Hold approach* [4] as $\tilde{u}(t) = \tilde{u}_i := \tilde{u}\big(u(t_i), u(t_{i-1}), \ldots\big)$, $\tilde{y}(t) = \tilde{y}_i := \tilde{y}\big(y(t_i), y(t_{i-1}), \ldots\big) \ \forall t \in [t_i, t_{i+1})$.

*Remark.* Under the previous assumptions on $\mathcal{P}$ and $\mathcal{Q}$, when the sampled system (7)-(10) is fed with constant inputs $\tilde{u}(t) = \tilde{u}$, $\tilde{y}(t) = \tilde{y}$, it shares the same fixed points set $\Omega_e$ of $\mathcal{F}$. However, in general, these equilibria are not expected to always show the same stability property.

## III. Preliminary results

Assume $\mathcal{F}$ satisfies the hypothesis of Proposition II.1, and consider its sampled version (7)-(10). Notice it always holds true that

$$\frac{dV_T}{dt}(x,z) \le \alpha s_p(\tilde{u}, y) + (1-\alpha) s_q(\tilde{y}, u) =: \tilde{s}_T(\tilde{u}, \tilde{y}, u, y).$$

Moreover, observe that at each possible $t_i$

$$\frac{dV_T}{dt}(x(t_i), z(t_i)) \le \tilde{s}_T(u(t_i), y(t_i), u(t_i), y(t_i)) \le 0 \ ,$$

since $\tilde{s}_T(u,y,u,y) = s_T(u,y)$.

**Proposition III.1.** *Generate the interpolated inputs according to $\tilde{u}(t) = u(t_i) =: u_i$, $\tilde{y}(t) = y(t_i) =: y_i \ \forall t \in [t_i, \infty)$, and assume the solutions of the Ordinary Differential Equations (ODE) problem (7)-(10) are continuous and continuously differential functions. Moreover, assume at $t_i$ it holds*

$$\tilde{s}_T(u_i, y_i, u(t_i), y(t_i)) < 0 \ .$$

*If the only solutions of the sampled version of $\mathcal{F}$ able to make $\tilde{s}_T(\tilde{u}, \tilde{y}, u, y)$ constant are the fixed points $(x_e, z_e) \in \Omega_e$ such that $\tilde{u} = q(z_e)$ and $\tilde{y} = p(x_e)$, then, either $u_i = q(z_e)$, $y_i = p(x_e)$ with $(x_e, z_e) \in \Omega_e$ and the system converges to a fixed point in $\Omega_e$, or it exists $t_{i+1} > t_i$ such that $\tilde{s}_T(u_i, y_i, u(t_{i+1}), y(t_{i+1})) = 0$.*

*Proof.* Only two scenarios are possible. Either $\tilde{s}_T$ remains always negative, or it becomes zero at a certain time, namely $t_{i+1}$. However, if it remains negative, also $\dot{V}_T$ does so, and since $V_T$ is always not negative, necessarily $\dot{V}_T \to 0$. Hence, $(x,z)$ tends to a set $\Omega \subset \mathbb{R}^{n\times m}$ where $\dot{V}_T(x,z) =$

0. This, in turn, implies $\tilde{s}_T(u_i, y_i, q(z), p(x)) \to 0$ and $\dot{\tilde{s}}_T(u_i, y_i, q(z), p(x)) \to 0$, as well. However, by assumption this is possible only if $\tilde{u} = q(z_e)$ and $\tilde{y} = p(x_e)$ with $(x_e, z_e) \in \Omega_e$, which proves the statement. □

**Corollary III.2.** *Let $\mathcal{T}$ and the inputs be designed according to the following rule. For each $t_i \in \mathcal{T}$ let feeds $\tilde{u}$, $\tilde{y}$ be such that $\tilde{u}(t) = q(z(t_i)) =: u_i$, $\tilde{y}(t) = p(x(t_i)) =: y_i$ $\forall t \in [t_i, t_{i+1})$. Then, let $t_{i+1} \in \mathbb{R} \cup \{\infty\} : \; t_{i+1} > t_i$ be such that $\tilde{s}_T(u_i, y_i, u(t), y(t)) < 0$ $\forall t \in [t_i, t_{i+1})$. Under these assumptions sampled system* (7)-(10) *converges to some $(x_e, z_e) \in \Omega_e$ and $y \to 0$.*

*Proof.* Proposition III.1 assures the chosen design is able to make $\tilde{s}_T(u_i, y_i, u(t), y(t)) < 0$ $\forall t \in [t_i, t_{i+1})$ for each $t_i \in \mathcal{T}$. Hence, $\dot{V}_T(x(t), z(t)) < 0$ $\forall t \in \mathbb{R}$, which implies, by reasoning analogous to proof of Proposition III.1, that system (7)-(10) converges to a fixed point in $\Omega_e$ and $y \to 0$. □

## IV. A NOVEL EVENT TRIGGERING STRATEGY

Corollary III.2 defines a triggering rule which makes the output $y$ to asymptotically tend to the set-point in zero. Notice that, as in many other approaches (see, e.g., [4], [5]), at each sampling time $t_i$ the feeds are equal to the loop outputs, i.e. $\tilde{u}(t_i) = q(z(t_i)) = u(t)$, $\tilde{y}(t_i) = p(x(t_i)) = y(t_i)$, as it happens in the original feedback system $\mathcal{F}$. However, it is worth stressing that such values are not the same read at the same instant in $\mathcal{F}$, where the evolution is governed by a "continuous" feedback. This usually is responsible for loss of synchronicity between the responses of $\mathcal{F}$ and its sampled version. In general, if the output $y$ of the sampled system is not asked to resemble that of the original $\mathcal{F}$, the triggering rule in Corollary III.2 can be further developed as follows.

Let $t_i \in \mathcal{T}$ be the actual sampling time, and $u_i := u(t_i)$, $y_i := y(t_i)$. Moreover, denote

$$\partial_{\tilde{u}} \tilde{s}_T(u_i, y_i, u_i, y_i) := \left. \frac{\partial \tilde{s}_T}{\partial \tilde{u}}(\tilde{u}, \tilde{y}, u, y) \right|_{(u_i, y_i, u_i, y_i)}$$
$$\partial_{\tilde{y}} \tilde{s}_T(u_i, y_i, u_i, y_i) := \left. \frac{\partial \tilde{s}_T}{\partial \tilde{y}}(\tilde{u}, \tilde{y}, u, y) \right|_{(u_i, y_i, u_i, y_i)} .$$

**Corollary IV.1.** *Let $\mathcal{T}$ and the inputs be designed according to the following rule. At each sampling time $t_i$, if $\partial_{\tilde{u}} \tilde{s}_T(u_i, y_i, u_i, y_i) = 0$ and $\partial_{\tilde{y}} \tilde{s}_T(u_i, y_i, u_i, y_i) = 0$, then proceed as in Corollary III.2. Otherwise, find $\tilde{u}_i \neq u_i$ and $\tilde{y}_i \neq y_i$ such that*

$$\tilde{s}_T(\tilde{u}_i, \tilde{y}_i, u_i, y_i) < \tilde{s}_T(u_i, y_i, u_i, y_i) \leq 0 ,$$

*and design the feeds as $\tilde{u}(t) = \tilde{u}_i$, $\tilde{y}(t) = \tilde{y}_i$ $\forall t \in [t_i, t_{i+1})$. Under these assumptions sampled system* (7)-(10) *converges to some $(x_e, z_e) \in \Omega_e$ and $y \to 0$.*

*Proof.* The proof follows the same outline and reasoning of that of Corollary III.2, once they have noticed the existence of such $\tilde{u}_i$ and $\tilde{y}_i$ is guaranteed by the assumptions on $\partial_{\tilde{u}} \tilde{s}_T(u_i, y_i, u_i, y_i)$ and $\partial_{\tilde{y}} \tilde{s}_T(u_i, y_i, u_i, y_i)$. □

The triggering rule in Corollary IV.1 differs from that of Corollary III.2 because of the absolute value of $\dot{V}_T$ just right after the new sampling, which implies bigger speed in approaching $\Omega_e$, even though that does not relate to any possible bigger inter-sample time until the next $t_{i+1}$. However, since $y(t)$, $u(t)$ tend to be slower when states $x(t)$, $z(t)$ are closer to $\Omega_e$, the sampled system is expected to reach in a shorter time a condition of long inter-sample times, with benefits in the long run.

The triggering rule in Corollary IV.1 can be summarized as follows.

1) At each new sampling instant $t_i$, find $\tilde{u}_i$ and $\tilde{y}_i$ such that

$$(1 + \rho_1)\tilde{s}_T(u_i, y_i, u(t_i), y(t_i)) \leq \tilde{s}_T(\tilde{u}_i, \tilde{y}_i, u(t_i), y(t_i)) < \tilde{s}_T(u_i, y_i, u(t_i), y(t_i)) \leq 0 \quad (11)$$

for some chosen $\rho_1 > 0$.

2) Feed system (7)-(10) with constant inputs $\tilde{u}(t) = \tilde{u}_i$, $\tilde{y}(t) = \tilde{y}_i$ for $t > t_i$.

3) If at a certain $t^*$

$$\tilde{s}_T(\tilde{u}_i, \tilde{y}_i, u(t^*), y(t^*)) = (1 - \rho_2)\tilde{s}_T(u_i, y_i, u(t^*), y(t^*)) \quad (12)$$

for some chosen $0 < \rho_2 < 1$, take the new sampling time as $t_{i+1} = t^*$, and start back from 1).

Parameters $\rho_1$, $\rho_2$ are used to tune the range of variability of $\tilde{s}_T(\tilde{u}_i, \tilde{y}_i, u(t), y(t))$ during time frame $[t_i, t_{i+1})$. Even though very small values are responsible of small ranges, and so they generate small inter-sample times, in general the relationship between $t_{i+1}$ and the choice of $\rho_1$, $\rho_2$ is case dependent.

*Remark.* The above triggering rule is executed by supervisor $\mathcal{S}$, which, to work properly, must be aware of the actual values of $y(t)$, $u(t)$ at each $t$ in order to detect the right $t^*$. However, this would require a continuous exchange of data between $\mathcal{S}$ and $\mathcal{P}$, $\mathcal{Q}$ in contrast with the sought asynchronous flow of samples. Such a problem, common to many event-triggered controllers [3], [6], [8], can be approached by making $\mathcal{S}$ able to generate sufficiently reliable forecasts of $y(t)$, $u(t)$ based on the past samples and other possible available data. Practical implementations of the proposed triggering rule using forecasts to detect $t^*$ are out of the scope of the present paper, and they will be object of future research.

## V. AN ILLUSTRATIVE NUMERICAL EXAMPLE

In this section we illustrate how to derive the triggering rule of Corollary IV.1 in practice. Moreover, we will test its performance against traditional approaches in an ideal implementation. Indeed, we stress that supervisor $\mathcal{S}$ would need complete knowledge of the actual outputs of $\mathcal{P}$ and $\mathcal{Q}$, which, in real applications, can only be forecast. Therefore, only remarkable improvements will encourage further development of the proposed technique.

Let $\mathcal{P}$ be

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= ax_2 - bx_1^3 - u\\ y &= x_2\end{aligned}$$

with $a, b > 0$. Notice that for each possible constant input $u$, all the fixed point of $\mathcal{P}$ are unstable, and output $y$ at these equilibria is always zero. Let $\mathcal{Q}$ be a Proportional-Integral (PI) controller

$$\begin{aligned}\dot{z} &= y\\ u &= cz + dy\ ,\end{aligned}$$

with $c, d > 0$ and observe that for any not zero constant input $y$ it has no fixed points. Closed loop system $\mathcal{F}$ reads

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= (a-d)x_2 - bx_1^3 - cz\\ \dot{z} &= x_2\ ,\end{aligned}$$

and it has an entire set of fixed points

$$\Omega_e = \left\{(x_1, x_2, z) \in \mathbb{R}^3 :\ x_2 = 0,\ z = -\frac{b}{c}x_1^3\right\}\ ,$$

where $y = 0$. Let us consider the following candidate storage functions

$$\begin{aligned}V_p(x_1, x_2) &= \frac{1}{4}bx_1^4 + \frac{1}{2}x_2^2\\ V_q(z) &= \frac{1}{2}cz^2\ ,\end{aligned}$$

and notice they are dissipative by choosing as related supply rates the following ones

$$\begin{aligned}\dot{V}_p(x_1, x_2) &= bx_1^3x_2 + ax_2^2 - bx_1^3x_2 - x_2u\\ &= ay^2 - yu =: s_p(u, y)\\ \dot{V}_q(z) &= cz\dot{z} = (u - dy)\,y = uy - dy^2 =: s_q(y, u)\ .\end{aligned}$$

Moreover, observe that

$$\begin{aligned}V_T(x_1, x_2, z) &= \frac{1}{2}V_p(x_1, x_2) + \frac{1}{2}V_q(z)\\ &= \frac{1}{8}bx_1^4 + \frac{1}{4}x_2^2 + \frac{1}{4}cz^2\\ \dot{V}_T(x_1, x_2, z) &= \frac{1}{2}(a-d)y^2\end{aligned}$$

is a suitable Lyapunov function for system $\mathcal{F}$ provided $d > a$. Under such a condition $\mathcal{F}$ converges to one of its fixed points in $\Omega_e$ and $y$ tends to set-point zero. Then, choice

$$s_T(u, y) := \frac{1}{2}(a-d)y^2\ .$$

Consider now the sampled system

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= ax_2 - bx_1^3 - \tilde{u}\\ \dot{z} &= \tilde{y}\\ y &= x_2\\ u &= cz + d\tilde{y}\ .\end{aligned}$$

and observe that

$$\begin{aligned}\tilde{s}_T(\tilde{u}, \tilde{y}, u, y) &:= \frac{1}{2}\left(ay^2 - y\tilde{u}\right) + \frac{1}{2}\left(u\tilde{y} - d\tilde{y}^2\right)\\ &= \frac{1}{2}\left(ay^2 - y\tilde{u} + u\tilde{y} - d\tilde{y}^2\right)\\ &= \frac{1}{2}(a-d)y^2 + \frac{1}{2}\left(dy^2 - y\tilde{u} + u\tilde{y} - d\tilde{y}^2\right)\ .\end{aligned}$$

As already noticed, when feed $\tilde{u}$ on $\mathcal{P}$ is very close to the actual output $u$ of the PI controller $\mathcal{Q}$, and $\tilde{y}$ is very close to $\mathcal{P}$ output $u$, $\tilde{s}_T(\tilde{u}, \tilde{y}, u, y)$ boils down to $s_T(u, y)$ and the system converges to $\Omega_e$. For the sake of notation, let us introduce

$$\begin{aligned}\Psi(u, y, \tilde{u}, \tilde{y}) &:= \frac{1}{2}\left(dy^2 - y\tilde{u} + u\tilde{y} - d\tilde{y}^2\right)\\ &= \frac{1}{2}\left(ax_2^2 - x_2\tilde{u} + cz\tilde{y}\right)\end{aligned}$$

which satisfies $\Psi(u, y, u, y) = 0$. By using $\Psi$ conditions (11)-(12) boils down to

$$-\frac{1}{2}\rho_1(d-a)y^2 \leq\ \Psi(u, y, \tilde{u}, \tilde{y}) \leq\ \frac{1}{2}\rho_2(d-a)y^2 \tag{13}$$

for a chosen pair $0 < \rho_1$, $0 < \rho_2 < 1$. In order to design the constant values $\tilde{u}_i$, $\tilde{y}_i$ of the feeds during two consecutive sampling times $t_i$, $t_{i+1}$, let us compute

$$\begin{aligned}\frac{\partial\Psi}{\partial\tilde{u}}(u, y, \tilde{u}, \tilde{y}) &= -\frac{1}{2}y = -\frac{1}{2}x_2\\ \frac{\partial\Psi}{\partial\tilde{y}}(u, y, \tilde{u}, \tilde{y}) &= \frac{1}{2}cz = \frac{1}{2}(u - d\tilde{y})\end{aligned}$$

and design at each sampling time $t_i$ the feeds according to

$$\tilde{u}_i = u(t_i) - \frac{1}{2}\varepsilon y(t_i) \tag{14}$$

$$\tilde{y}_i = y(t_i) + \frac{1}{2}\varepsilon(u(t_i) - dy(t_i)) \tag{15}$$

where

$$\varepsilon = \arg\min_{\varepsilon} \Psi(\tilde{u}_i, \tilde{y}_i, u(t_i), y(t_i)) \tag{16}$$

$$\text{subject to } -\frac{1}{2}\rho_1(d-a)y^2(t_i) \leq \Psi(\tilde{u}_i, \tilde{y}_i, u(t_i), y(t_i)) \tag{17}$$

The next sampling time $t_{i+1}$, instead, is designed so that

$$\Psi(\tilde{u}_i, \tilde{y}_i, u(t_{i+1}), y(t_{i+1})) = \frac{1}{2}\rho_2(d-a)y^2(t_{i+1})\ . \tag{18}$$

To illustrate the results of event triggered control (14)-(18) we have chosen $a = 1.0$, $b = 3.0$, $c = 0.5$, $d = 1.5$. Figure 1 shows outputs $y$ and $u$ of $\mathcal{F}$ in the continuous data transmission case.

Figure 2, instead, reports the same outputs when data between $\mathcal{P}$ and $\mathcal{Q}$ are exchanged with a periodic sampling with period $\Delta = 0.10$. It is worth stressing that for $\Delta$ slightly bigger than this value, the sampled system is not able to converge to $y = 0$ any more.

In Figure 3 the system dynamics under a very common and effective triggering rule based on percentage thresholds is reported [9]. The triggering condition at $t_{i+1}$ is obtained

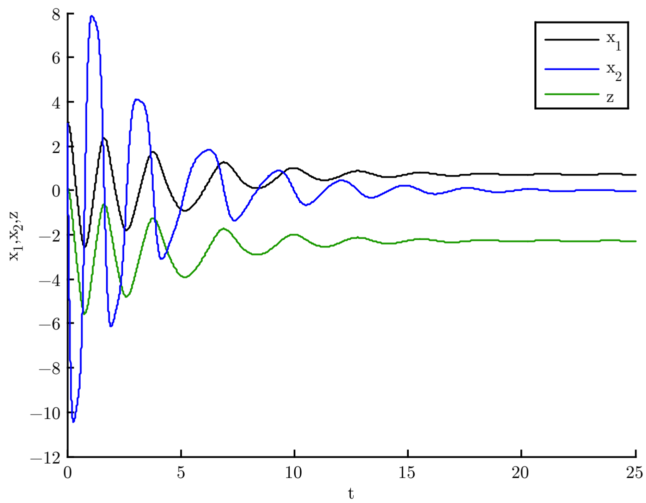


Figure 1. Time evolution of $\mathcal{F}$ state variables in the continuous scenario. Observe $y = x_2$ converges to zero, while the other state varibles tends to values compatible with the desired output zero set-point.

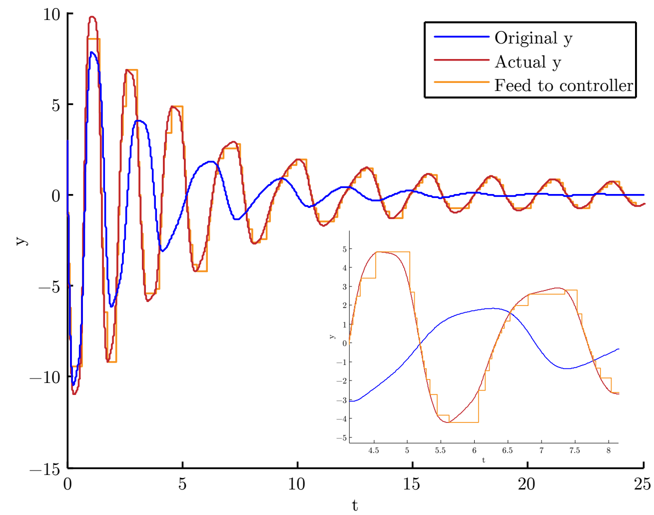


Figure 3. Blue line: Output $y$ of time continuous system. Red line: Actual output $y$ of sampled $\mathcal{F}$ under threshold triggering generation rule. Orange line: Feed provided to controller $\mathcal{Q}$.

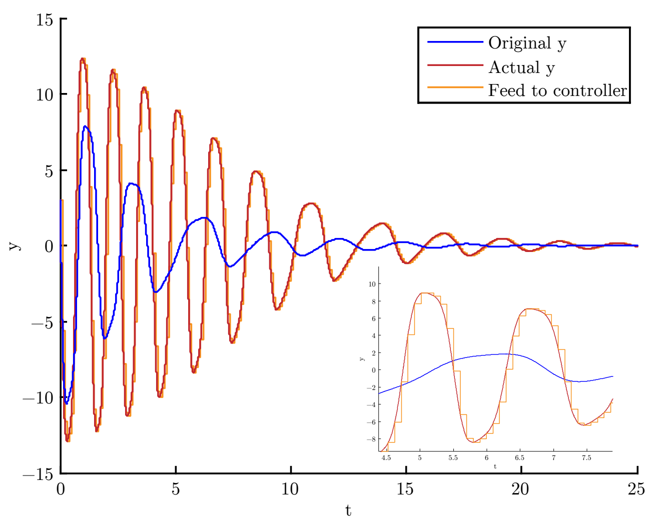


Figure 2. Blue line: Output $y$ of time continuous system. Red line: Actual output $y$ of periodically sampled $\mathcal{F}$. Orange line: Feed provided to controller $\mathcal{Q}$.

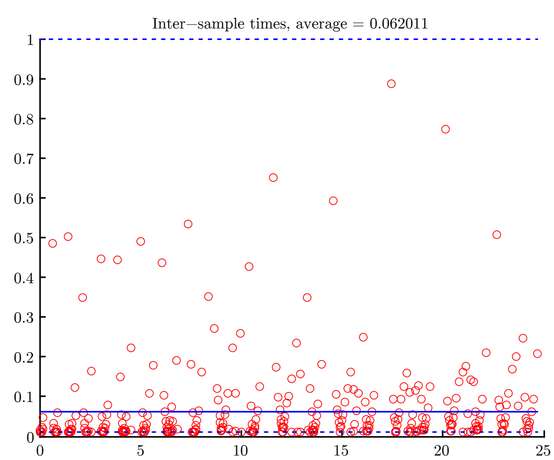


Figure 4. Time distribution of inter-sample times in the case of threshold triggering rule (19). Blue solid line: Average value. Blue dotted lines: Upper and lower bounds on the inter-sample time.

by the fulfillment of condition

$$|u(t_{i+1}) - u(t_i)| = \theta_u |u(t_{i+1})| \vee |y(t_{i+1}) - y(t_i)| = \theta_y |y(t_{i+1})| \ . \tag{19}$$

Here, the thresholds have been set to $\theta_u = 0.40$, $\theta_y = 0.40$, that are close to the limit values after which the system does not converge to the set-point in zero any more. Moreover, as in practice, lower and upper bounds have been enforced on the inter-sample time to avoid Zeno effects [2] or loss of active communications between the devices. However, the chosen bounds cast all the inter-sample time in $[0.01, 1.00]$, and so they only slightly affect the triggering process, as shown in Figure 4. Notice the average inter-sample time over a simulation time span of 25 is 0.06 time units, which is even worse than in the case of periodic sampling. Here, the problem is caused by a common drawback of this kind of triggering rules, which are very sensitive to the stability of the involved systems, i.e. of $\mathcal{P}$ and $\mathcal{Q}$, when they run "free" after the new sample. Since in this example both physical device and PI controller under constant feed tend to rapidly diverge from the conditions granting $y = 0$, threshold condition (19) is quickly met with detrimental effect on the inter-sample time.

Figures 5 and 6 report the actual outputs $y$, $u$ under the proposed triggering rule, when $\rho_1 = \frac{1}{\rho_2} = 5$. Notice that, differently from common approaches, feeds provided to $\mathcal{P}$ and $\mathcal{Q}$ are not interpolated by $u$ and $y$, respectively, as a consequence of choice (14) and (15). Figure 7 shows the time distribution of the inter-sample times. Observe the average sampling time over the same simulation time is 0.35 time units, which is remarkably better than both periodic

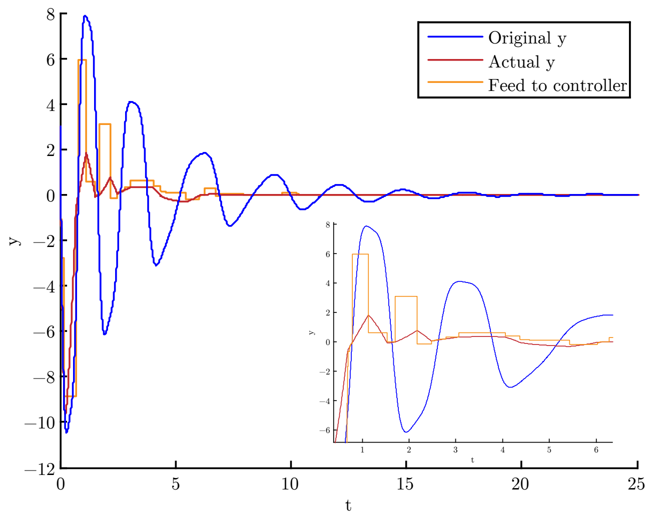


Figure 5. Blue line: Output $y$ of time continuous system. Red line: Actual output $y$ of $\mathcal{P}$ under the proposed triggering generation rule. Orange line: Feed provided to controller $\mathcal{Q}$.

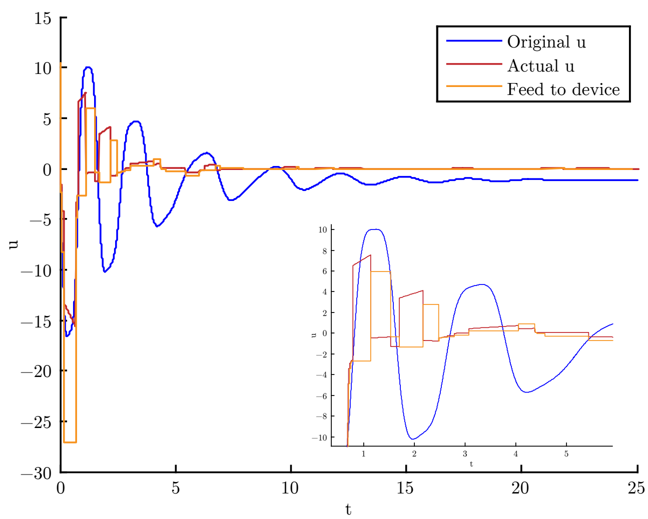


Figure 6. Blue line: Output $u$ of time continuous system. Red line: Actual output $u$ of $\mathcal{Q}$ under the proposed triggering generation rule. Orange line: Feed provided to device $\mathcal{P}$.

sampling and the threshold technique. Even if this result is obtained with an ideal implementation (i.e., with not use of forecasts to have $\mathcal{S}$ aware of the actual values of $y$, $u$), it is still a very encouraging performance. Hence, a centralized implementation of this technique based on reliable outputs forecasting by the supervisor has very likely good margin to perform better than standard techniques.

## VI. Conclusion and Future Research

In this paper a new technique to design a event triggered controller from the knowledge of a time continuous one has been presented. A input-output approach focused on dissipative systems has been pursed. Moreover, special attention has been put on devices and controllers which quickly diverge from the desired behaviour when driven by constant feeds,

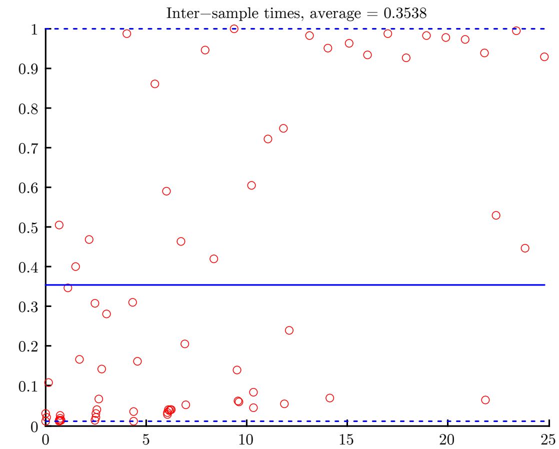


Figure 7. Time distribution of inter-sample times in the case of proposed triggering rule. Blue solid line: Average value. Blue dotted lines: Upper and lower bounds on the inter-sample time.

instead of the actual time continuous inputs, even if obtained from asynchronous samples. A detailed illustrative example featuring a PI controller and a device unstable under constant inputs has been used to better explain the technique. A comparison with another very common approach has been provided, as well. Remarks on the practical implementation issues via centralized supervisor close the paper. In particular, the good results obtained from the ideal realization of our technique encourage the development of real use cases, which will be object of future research.

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