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Intercept Point Prediction for Midcourse Guidance of Anti-ballistic Missile

Sungjun Ann, Seokwon Lee, and Youdan Kim
Department of Mechanical and Aerospace Engineering
Seoul National University, Seoul, Republic of Korea
(sungjun9037, blueswl, and ydkim)@snu.ac.kr

***Abstract*—Interception of the ballistic missile is one of the most challenging issue in the missile defense system. Except the powered phase intercept, the speed of the ballistic missile is usually much faster than that of the intercept missile. Therefore, hit-to-kill and head-on interception is necessary to increase kill probability. These conditions require a precise guidance scheme. This study focuses on the exo-atmospheric midcourse guidance for the solid propellant missile against the ballistic target in the free flight phase. Due to the characteristics of the solid propellant, the acheivable intercept area is limited, and the performance of the guidance is highly affected by the predicted intercept point(PIP). In this study, the range of the achievable PIP is obtained by the trajectory optimization technique, and the proper PIP selection scheme is proposed to intercept the target. Numerical simulations are conducted to demonstrate the pereformance of the PIP selection for the midcourse guidance.**

***Index Terms*—Ballistic Target, Exo-atmospheric Interception, Solid Propellant, Midcourse Guidance, Predicted Intercept Point(PIP).**

I. Introduction

Recently, the threat from a ballistic missile has increased. The trajectory of a ballistic missile, consists of powered phase, free flight phase, and re-entry phase [1]. The missile during the free flight phase is only affected by the gravitational force. In this case, it is relatively easy to predict the trajectory of the missile. However, the speed of the ballistic missile is extremely fast and even exceeds that of the defense missile. Therefore, tail-chase engagement is impossible, instead head-on interception is required. For the extremely fast target, hit-to-kill condition which accomplishes the engagement by directly hitting the target is necessary to increase the probability of kill. In this study, solid propellant missile is considered as a defense missile against a ballistic missile in the free flight phase. The solid propellant cannot control the thrust termination, and therefore, not only the conditions of the intercept but also the characteristics of the solid propellant should be considered to hit the target successfully.

In the previous research, Zarchan summarized the whole process of the midcourse guidance for the exoatmospheric intercept [2]. However, the characteristics of the solid propellant was not considered. The guidance laws considering the solid propellant have been studied for the space related mission such as orbit delivery [3],[4] or orbit transfer [5]. However, the characteristics of the target was not considered. Because the speeds of the target and the missile are different, the missile cannot maintain the orbit of the target.

In this study, a midcourse guidance of solid propellant anti-ballistic missile is considered. The guidance law which can achieve the desired position and velocity at the desired time is modified to deal with the characteristics of the solid propellant. The desired position and velocity for the guidance is obtained by the Kepler's propagator and solution of the Lambert's problem. Moreover, a PIP selection scheme considering the relation between the amount of the propellant and the desired orbit is proposed to handle uncertainty. Using the proposed PIP selection, the guidance law can achieve hit-to-kill under uncertainty on the initial velocity.

This paper is organized as follows. The equations of motion considering exo-atmospheric environment and the characteristics of the missile are expressed in Sec.II-A. In Sec.II-B, the achievable range of the PIP is obtained by trajectory optimization. The midcourse guidance law is introduced in Sec.II-C, and the PIP selection scheme for precise interception is proposed in Sec.II-D. In Sec.III, results of numerical simulation are shown to demonstrate the performance of the proposed PIP selection scheme. Finally, concluding remark is given in Sec.IV.

II. Midcourse Guidance for the Exo-atmospheric Interception

A. Equations of Motion

In this study, the missile and the target are considered as point mass in three dimensional space. Because of the exo-atmospheric environments, the aerodynamic forces such as drag and lift can be neglected. For the target in free flight phase, the gravity is the only external force. As the gravity model, the spherical gravitational acceleration is considered. On the other hand, the missile is affected by both of the thrust and the gravity. Due to the chracteristics of the solid propellant, the direction of the thrust is considered as the input of the system. The equations of the motion can be represented as follows,

$$\dot{\vec{r}}_M = \vec{v}_M \tag{1}$$

$$\dot{\vec{v}}_M = -\frac{\mu_e}{\|\vec{r}_M\|^3}\vec{r}_M + \frac{T_M}{m_M}\vec{u}_M \tag{2}$$

$$\dot{m}_M = -\frac{T_M}{g_0 I_{sp}} \tag{3}$$

$$\dot{\vec{r}}_T = \vec{v}_T \tag{4}$$

$$\dot{\vec{v}}_T = -\frac{\mu_e}{\|\vec{r}_T\|^3}\vec{r}_T \tag{5}$$

$$\vec{u}_M^T \vec{u}_M = 1 \tag{6}$$

where μ_e, T_M, g_0, and I_{sp} represent the standard gravitational parameter of the Earth, the magnitude of the thrust, the gravitational constant at sea level, and specific impulse, respectively. $\vec{r}_M$, $\vec{v}_M$, m_M, and u_M represent position, velocity, mass, and the thrust direction of the missile, respectively. $\vec{r}_T$ and $\vec{v}_M$ represent position and velocity of the target, respectively.

The time line of the missile is shown in Fig. 1. The trajectory of the missile consists of three phases, two powered phases and one coasting phase. In the terminal phase of the missile, control device such as divert and attitude control system(DACS) can be applied. However, the performance of the ideal midcourse guidance is evaluated without additional control surface in the terminal phase. In this study, the terminal phase of the missile is considered as a coasting phase, because midcourse guidance problem is to be solved. Figure 2 shows the time history of T_M and m_M. In the two powered phases, the magnitude of the thrust is considered as constant, and the duration of the thrust is fixed. On the other hand, the duration of the coasting phase is not specified. Namely, t_1 and t_2 are fixed value, but t_f is free variable. For the mass, the seperation of fairing and stage occur at t_1 and t_2, respectively.

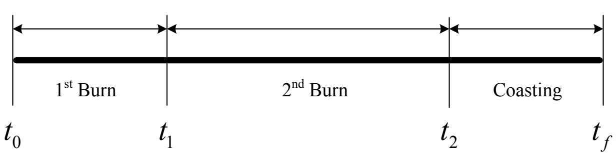

Fig. 1. Time line of the missile

B. Range of the Achievable PIP

As mentioned in Sec. I, the speed of the target exceeds that of the missile, and the direction of the thrust is the only control input. Therefore, the interception can occur only in some specific region.

The trajectory optimization is considered to compute the range of the achievable PIP. In this study, indirect method is adopted to solve the optimization problem. The minimum and maximum PIP can be obtained by considering the following performance indices.

$$J = \int_{t_0}^{t_f} \pm 1 dt \tag{7}$$

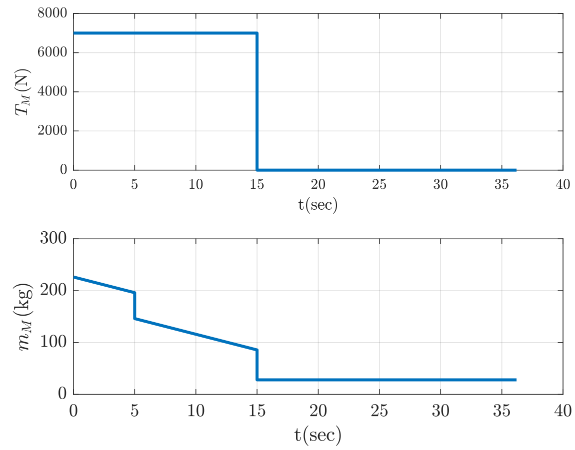

Fig. 2. Time history of the magnitude of the thrust (T_M) and the mass (m_M)

Hamiltonian can be represented as follows,

$$\begin{aligned} H = \; & \pm 1 + \vec{\lambda}_{r_M}^T \vec{v}_M + \vec{\lambda}_{v_M}^T \left(-\tfrac{\mu_e}{r_M^3}\vec{r}_M + \tfrac{T_M}{m_M}\vec{u}_M \right) \\ & + \lambda_{m_M}\left(-\tfrac{T_M}{g_0 I_{sp}} \right) + \vec{\lambda}_{r_T}^T \vec{v}_T + \vec{\lambda}_{v_T}^T \left(-\tfrac{\mu_e}{r_T^3}\vec{r}_T \right) \\ & + \lambda_{u_M}\left(\vec{u}_M^T \vec{u}_M - 1 \right) \end{aligned} \tag{8}$$

where $\vec{\lambda}_{r_M}$, $\vec{\lambda}_{v_M}$, λ_{m_M}, $\vec{\lambda}_{r_T}$, and $\vec{\lambda}_{v_T}$ represent the costates of the corresponding states. λ_{u_M} represents Lagrange multiplier for u_M.

The costate equations are obtained as

$$\dot{\vec{\lambda}}_{r_M} = \left[\frac{\mu_e}{\left(\vec{r}_M^T \vec{r}_M\right)^{\frac{3}{2}}} I - \frac{3\mu_e}{\left(\vec{r}_M^T \vec{r}_M\right)^{\frac{5}{2}}} \vec{r}_M \vec{r}_M^T \right] \vec{\lambda}_{v_M} \tag{9}$$

$$\dot{\vec{\lambda}}_{v_M} = -\vec{\lambda}_{r_M} \tag{10}$$

$$\dot{\lambda}_{m_M} = \frac{T}{m_M^2} \vec{\lambda}_{v_M}^T \vec{u}_M \tag{11}$$

$$\dot{\vec{\lambda}}_{r_T} = \left[\frac{\mu_e}{\left(\vec{r}_T^T \vec{r}_T\right)^{\frac{3}{2}}} I - \frac{3\mu_e}{\left(\vec{r}_T^T \vec{r}_T\right)^{\frac{5}{2}}} \vec{r}_T \vec{r}_T^T \right] \vec{\lambda}_{v_T} \tag{12}$$

$$\dot{\vec{\lambda}}_{v_T} = -\vec{\lambda}_{r_T} \tag{13}$$

The optimal control is calculated by $\frac{\partial H}{\partial \vec{u}_M} = 0$ as

$$\vec{u}_M = -\frac{T_M}{2\lambda_{u_M} m_M} \vec{\lambda}_{v_M} \tag{14}$$

To satisfy the input constraint in Eq.(6), λ_{u_M} is obtained as follows,

$$\lambda_{u_M} = \frac{T_M \left\| \vec{\lambda}_{v_M} \right\|}{2 m_M} \tag{15}$$

As boundary conditions, all of the initial state are given, and the constraints of interception are considered as the final conditions. The initial conditions are expressed as

$$\vec{r_M}(t_0), \vec{v}_M(t_0), m_M(t_0), \vec{r}_T(t_0), \vec{v}_T(t_0) \tag{16}$$

The remaining initial costates are unspecified. Mathematical expressions of hit-to-kill and head-on intercept are represented as

$$\vec{r}_M(t_f) = \vec{r}_T(t_f) \tag{17}$$

$$-1 \leq \frac{\vec{v}_M(t_f)\cdot\vec{v}_T(t_f)}{\|\vec{v}_M(t_f)\|\,\|\vec{v}_T(t_f)\|} \leq 0 \tag{18}$$

Then, the augmented performance index can be defined as

$$J_a = \vec{d}^T\vec{\Phi}(\vec{x}(t_f)) + \int_{t_0}^{t_f}(H - \vec{\lambda}^T\dot{\vec{x}})dt \tag{19}$$

where

$$\vec{\Phi}(\vec{x}(t_f)) = \begin{bmatrix} \vec{r}_M(t_f)-\vec{r}_T(t_f) \\ -\frac{\vec{v}_M(t_f)\cdot\vec{v}_T(t_f)}{\|\vec{v}_M(t_f)\|\|\vec{v}_T(t_f)\|} - 1 + s_{v_1}^2 \\ \frac{\vec{v}_M(t_f)\cdot\vec{v}_T(t_f)}{\|\vec{v}_M(t_f)\|\|\vec{v}_T(t_f)\|} + s_{v_2}^2 \end{bmatrix}, \vec{d} = \begin{bmatrix} \vec{d}_r \\ d_{v1} \\ d_{v2} \end{bmatrix}$$

The slack variables, s_{v_1} and s_{v_2}, are utilized to deal with the inequality constraints.

From the final conditions, the final costates are determined as follows,

$$\vec{\lambda}_{r_M}(t_f) = \vec{d}_r, \vec{\lambda}_{r_T}(t_f) = -\vec{d}_r \tag{20}$$

$$\lambda_{m_M}(t_f) = 0 \tag{21}$$

$$\vec{\lambda}_{v_M}(t_f) = -d_{v_1}\left.\frac{\partial L}{\partial\vec{v}_M}\right|_{t_f} + d_{v_2}\left.\frac{\partial L}{\partial\vec{v}_M}\right|_{t_f} \tag{22}$$

$$\vec{\lambda}_{v_T}(t_f) = -d_{v_1}\left.\frac{\partial L}{\partial\vec{v}_T}\right|_{t_f} + d_{v_2}\left.\frac{\partial L}{\partial\vec{v}_T}\right|_{t_f} \tag{23}$$

where $L = \frac{\vec{v}_M(t_f)\cdot\vec{v}_T(t_f)}{\|\vec{v}_M(t_f)\|\|\vec{v}_T(t_f)\|}$.

The slack variables have to satisfy the following conditions.

$$d_{v_1}s_{v_1} = 0, d_{v_2}s_{v_2} = 0 \tag{24}$$

Moreover, Karush-Kuhn-Tucker(KKT) conditions should be satisfied to guarantee the optimality of solution for the inequality constraints [6].

$$0 \leq d_{v_1}, 0 \leq d_{v_2} \tag{25}$$

Due to the unspecified final time, the value of the hamiltonian at the final time is specified as

$$H(t_f) = 0 \tag{26}$$

Then, the optimal solution is obtained by finding the value of the unspecified initial costates satisfying the boundary conditions. In this study, differential correction method is utilized to find the solution [7].

$$\Delta\vec{p} = -\epsilon\left[\frac{\partial\vec{\Psi}}{\partial\vec{p}}\right]^{-1}\vec{\Psi} \tag{27}$$

$$\vec{p} = \vec{p} + \Delta\vec{p} \tag{28}$$

where

$$\vec{\Psi} = \begin{bmatrix} \vec{r}_M(t_f)-\vec{r}_T(t_f) \\ -\frac{\vec{v}_M(t_f)\cdot\vec{v}_T(t_f)}{\|\vec{v}_M(t_f)\|\|\vec{v}_T(t_f)\|} - 1 + s_{v_1}^2 \\ \frac{\vec{v}_M(t_f)\cdot\vec{v}_T(t_f)}{\|\vec{v}_M(t_f)\|\|\vec{v}_T(t_f)\|} + s_{v_2}^2 \\ \vec{\lambda}_{r_M}(t_f) - \vec{d}_r \\ \vec{\lambda}_{r_T}(t_f) + \vec{d}_r \\ \lambda_{m_M}(t_f) \\ \vec{\lambda}_{v_M}(t_f) + d_{v_1}\left.\frac{\partial L}{\partial\vec{v}_M}\right|_{t_f} - d_{v_2}\left.\frac{\partial L}{\partial\vec{v}_M}\right|_{t_f} \\ \vec{\lambda}_{v_T}(t_f) + d_{v_1}\left.\frac{\partial L}{\partial\vec{v}_T}\right|_{t_f} - d_{v_2}\left.\frac{\partial L}{\partial\vec{v}_T}\right|_{t_f} \\ d_{v_1}s_{v_1} \\ d_{v_2}s_{v_2} \\ H(t_f) \end{bmatrix}, \vec{p} = \begin{bmatrix} \vec{\lambda}r_M(t_0) \\ \vec{\lambda}v_M(t_0) \\ \lambda m_M(t_0) \\ \vec{\lambda}r_T(t_0) \\ \vec{\lambda}v_T(t_0) \\ \vec{d}_r \\ d_{v_1} \\ d_{v_2} \\ s_{v_1} \\ s_{v_2} \\ t_f \end{bmatrix}$$

C. Midcourse Guidance Law

For hit-to-kill and head-on engagement, the midcourse guidance law can control both position and velocity at the interception. Moreover, the missile should satisfy the desired position and velocity during the desired time considering the solid propellant.

The ZEM/ZEV guidance law can generate the command that satisfies the desired position as well as the desired velocity at the final time [8]. In this study, ZEM/ZEV guidance law is modified to consider the characteristics of the solid propellant. To obtain the guidance command, following velocity dynamics is considered instead of Eq.(2).

$$\dot{\vec{v}}_M = -\frac{\mu_e}{\|\vec{r}_M\|^3}\vec{r}_M + \vec{a}_M \tag{29}$$

Then, guidance command minimizing the performance index Eq.(30) is determined as follows,

$$J = \int_{t_0}^{t_f}\frac{1}{2}\vec{a}_M^T\vec{a}_M dt \tag{30}$$

$$\vec{a}_M = \frac{6}{t_{go}^2}Z\vec{E}M - \frac{2}{t_{go}}Z\vec{E}V \tag{31}$$

where $Z\vec{E}M$, $Z\vec{E}V$, and t_{go} are zero-effort-miss, zero-effort-velocity, and time-to-go, respectively. When the gravitational acceleration is considered as constant, these values are calculated using the current position($\vec{r}_M(t)$), velocity($\vec{v}_M(t)$), and time(t) as

$$Z\vec{E}M = \vec{r}_d - \left(\vec{r}_M + t_{go}\vec{v}_M + \frac{1}{2}t_{go}^2\vec{g}\right) \tag{32}$$

$$Z\vec{E}V = \vec{v}_d - (\vec{v}_M + t_{go}\vec{g}) \tag{33}$$

$$t_{go} = t_d - t \tag{34}$$

where $\vec{r}_d$, $\vec{v}_d$, and t_d are the desired position, velocity, and time, respectively. The average value of gravitational acceleration at the current and desired position is chosen as the value of $\vec{g}$.

$$\vec{g} = \frac{1}{2}\left(-\frac{\mu_e}{\|\vec{r}_M\|^3}\vec{r}_M - \frac{\mu_e}{\|\vec{r}_d\|^3}\vec{r}_d\right) \tag{35}$$

In this study, the magnitude of the acceleration from the thrust cannot be adjusted. Therefore, the direction of the

ZEM/ZEV guidance command is utilized as the control input as

$$\vec{u}_M = \frac{\vec{a}_M}{\|\vec{a}_M\|} \tag{36}$$

D. PIP Selection

If there exists uncertainty such as change in the initial states, then the missile could not reach the calculated PIP at the desired time. Because the magnitude and duration of the thrust cannot be changed, the PIP should be changed to deal with the uncertainty. In this section, PIP selection scheme is proposed. The proposed PIP selection scheme consists of three steps.

Kepler's propagator: The missile during the coasting phase and the target are only affected by the gravity. When the current values of $\vec{r}$ and $\vec{v}$ are given, the future trajectory of an object affected by spherical gravitational acceleration during the free flight can be obtained by the Kepler's propagator [9].

$$\begin{bmatrix} \vec{r}(t+\Delta t) \\ \vec{v}(t+\Delta t) \end{bmatrix} = \begin{bmatrix} f & g \\ \dot{f} & \dot{g} \end{bmatrix} \begin{bmatrix} \vec{r}(t) \\ \vec{v}(t) \end{bmatrix} \tag{37}$$

1) Step 1. Determine an initial PIP $(t_{PIP}, \vec{r}_T(t_{PIP}))$: Using the results of the optimization in Sec.II-B, the time at PIP, t_{PIP}, is selected within the range of the achievable PIP.

$$t_{f,\min} \le t_{PIP} \le t_{f,\max} \tag{38}$$

Then, the position of the PIP, $\vec{r}_T(t_{PIP})$, is calculated by the Kepler's propagator using current position and velocity of the target.

2) Step 2. Find the desired position and velocity for the guidance $(\vec{r}_d(t_2), \vec{v}_d(t_2))$: To intercept the target precisely, the position of the missile and the target should be equal at the same time. The time and position of the interception are determined in Step 1, and the current position can be measured. This situation is similar to the Lambert's problem. The Lambert's problem is to find the orbit passing the given two position with the given time of flight [9]. The desired orbit should be reached exactly at the end of the burn arc, t_2, because the final phase of the missile is the coasting arc. Therefore, the desired position and velocity for the guidance command should be determined as the value of the orbit at t_2.

To solve the Lambert's problem, Nelson and Zarchan's method is utilzed [10]. With the given two positions ($\vec{r}_1$, $\vec{r}_2$) and the time of the flight(Δt), the initial velocity($\vec{V}$) can be calculated by updating flight path angle(γ).

The $\vec{V}$ is defined by $\vec{r}_1$, $\vec{r}_2$, and γ as follows,

$$\vec{V} = V\sin\gamma \cdot i + V\cos\gamma \cdot j \tag{39}$$

where $i = \frac{\vec{r}_1}{|\vec{r}_1|}$, and $j = \frac{(\vec{r}_1 \times \vec{r}_2)\times\vec{r}_1}{|(\vec{r}_1 \times \vec{r}_2)\times\vec{r}_1|}$. The magnitude of the velocity(V) is calculated as follows,

$$V = \sqrt{\frac{\mu_e}{|\vec{r}_1|}\frac{|\vec{r}_2|\,(1-\cos\theta_f)}{|\vec{r}_1|\cos^2\gamma - |\vec{r}_2|\cos(\theta_f+\gamma)\cos\gamma}} \tag{40}$$

where $\theta_f = \cos^{-1}\left(\frac{\vec{r}_1\cdot\vec{r}_2}{|\vec{r}_1||\vec{r}_2|}\right)$. If the desired orbit is an elliptical trajectory, then the time of flight(Δt) can be calculated using Eq.(43). Using Eq.(41), γ is updated until the calculated Δt is equal to the desired time of flight(Δt_d).

$$\gamma_{n+1} = \gamma_n + \frac{(\gamma_n - \gamma_{n-1})(\Delta t_d - \Delta t_n)}{(\Delta t_n - \Delta t_{n-1})} \tag{41}$$

The parameters of the method corresponding to the values of this study are defined as follows,

$$\vec{r}_1 = \vec{r}_M(t), \vec{r}_2 = \vec{r}_T(t_{PIP}), \Delta t_d = t_{PIP} - t, \vec{V} = \vec{v}_d(t) \tag{42}$$

Finally, the desired position and velocity at t_2 ($\vec{r}_d(t_2)$,$\vec{v}_d(t_2)$) is calculated using Kepler's propagator with the desired position and velocity at the current time ($\vec{r}_d(t)$,$\vec{v}_d(t)$). The process of this section is summarized as follows,

1) Lambert's problem : $r_M(t)$, $r_T(t_{PIP})$, $\Delta t = t_{PIP} - t$ $\to v_d(t)$
2) Kepler's propagator : $r_d(t)$, $v_d(t) \to r_d(t_2)$, $v_d(t_2)$

$$\Delta t = \frac{|\vec{r}_1|}{V\cos\gamma}\left\{\begin{array}{l} \frac{\tan\gamma(1-\cos\theta_f)+(1-\lambda)\sin\theta_f}{(2-\lambda)\frac{|\vec{r}_1|}{|\vec{r}_2|}} \\ -\frac{\cos\gamma}{\lambda\left(1-\frac{2}{\lambda}\right)^{\frac{3}{2}}}\ln\frac{\sin\gamma-\cos\gamma\cot\left(\frac{\theta_f}{2}\right)-\left(1-\frac{2}{\lambda}\right)^{\frac{1}{2}}}{\sin\gamma-\cos\gamma\cot\left(\frac{\theta_f}{2}\right)+\left(1-\frac{2}{\lambda}\right)^{\frac{1}{2}}} \end{array}\right\} \tag{43}$$

where $\lambda = \frac{|\vec{r}_1|V^2}{\mu_e}$.

3) Step 3. Modify the PIP (t_{PIP}): The relation between the remaining amount of the propellant and the desired orbit determines the performance of the interception. If the remaining amount of the propellant is not enough to reach the desired orbit at the desired time, then the missile cannot intercpet the target. On the other hand, the remaining amount of the propellant is enough to reach the desired orbit at the desired time, the missile needs to decelerate to discard the surplus propellant.

When uncertainty such as variation of the current state is considered, t_{PIP} should be properly changed to deal with the uncertainty. If the propellant is insufficient, then t_{PIP} has to be longer, and the interception occurs at the lower altitude. On the other hand, if the propellant is excessive, then t_{PIP} has to be shorter, and the interception occurs at the higher altitude.

III. Numerical Simulations

In this study, inertial coordinate system is considered to express the motion of the missile and the target. The origin of the coordinate system is located at the surface of the Earth, which is equal to the initial position of the missile except for the altitude,. x-axis is defined as the direction toward target, and z-axis is equal to the upward direction from the center of the Earth. The angles of the thrust direction, i.e., pitch θ and yaw ψ, are defined as follows.

$$\theta = \tan^{-1}\left(\frac{u_{M,z}}{\sqrt{u_{M,x}^2 + u_{M,y}^2}}\right), \psi = \tan^{-1}\left(\frac{u_{M,y}}{u_{M,x}}\right) \tag{44}$$

where $u_M^T = \begin{bmatrix} u_{M,x} & u_{M,y} & u_{M,z} \end{bmatrix}^T$.

This section consists of two parts. The results of the optimization are given to show the achievable range of the

PIP. Considering the uncertainty on the initial velocity, numerical simulations for the midcourse guidance is performed to investigate the performance of the PIP selection.

The initial conditions are set as follows,

$$r_M(0) = \begin{bmatrix} 0 \\ 0 \\ 56 \end{bmatrix} \text{km},\ v_M(0) = \begin{bmatrix} 1.8569 \\ -0.0717 \\ 1.1045 \end{bmatrix} \text{km/s},$$

$$r_T(0) = \begin{bmatrix} 192.9094 \\ 0 \\ 140.5462 \end{bmatrix} \text{km},\ v_T(0) = \begin{bmatrix} -2.9593 \\ -0.0083 \\ -0.9686 \end{bmatrix} \text{km/s},$$

$$m_M(0) = 226.4\text{kg}$$

A. Achievable Range of the PIP

The trajectory optimization is performed to find the achievable range of the PIP. The achievable minimum and maximum PIP are obtained as follow,

$$t_{f,min} = 36.1028, t_{f,max} = 45.6212 \tag{45}$$

The results of 3D trajectories, time histories of the input and hamiltonian are shown in Figs.3-5. In case of $t_{f,min}$, the trajectory of the misisle is similar to that of target. On the other hand, the missile takes a long way to hit the target in case of $t_{f,max}$. Moreover, ψ of $t_{f,max}$ case is nearly $180°$, which means that the missile tries to decelerate. For the $t_{f,max}$ case, it is expected to hit the target perpendicularly. But the angle of the interception is nearly $180°$ because of the fixed duration of the thrust. The optimality of both case is verified by the hamiltonian, which is constant over each phase of the missile.

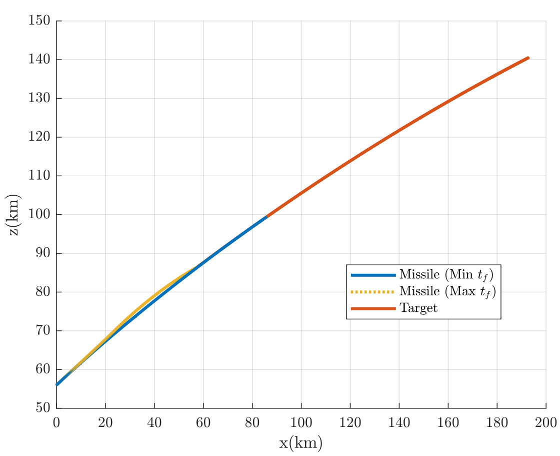

Fig. 3. 3D Trajectories (optimization results of $t_{f,min}$ and $t_{f,max}$)

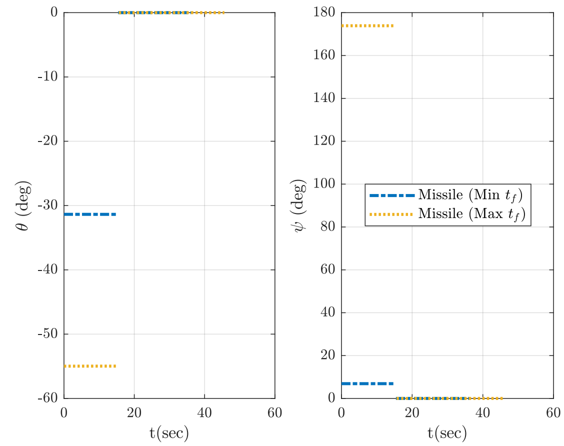

Fig. 4. Time histories of the input (optimization results of $t_{f,min}$ and $t_{f,max}$)

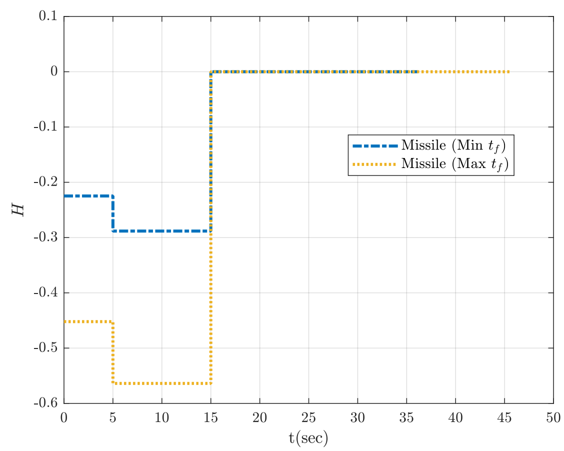

Fig. 5. Time histories of the hamiltonian (optimization results of $t_{f,min}$ and $t_{f,max}$)

B. PIP Selection

To investigate the performance of proper PIP selection, variations of the initial velocity of the missile are considered as the uncertainty. The uncertainties considered in this simulation are defined as follows,

1) Nominal : $v_M(0)$
2) Velocity (-) case : $0.95v_M(0)$
3) Velocity (+) case : $1.05v_M(0)$

The results of the midcourse guidance without PIP correction are summarized in Table. I. Any PIP can be utilzed within the range obtained in the previous section. In this simulation, the PIP of $t_{f,min}$ case is selected. The miss distance and time at that moment are represented as r_{miss} and t_{miss}, respectively.

The missile successfully intercepts the target in the nominal case. If the initial speed of the missile decreases, the missile fails to intercept the target, because the amount of the propellant of the missile is insufficient to reach the given PIP. On the other hand, if the initial speed of the missile increases, the miss distance is relatively small. However, the direction of the thrust is suddenly changed as shown in Fig. 6. The change of ψ to nearly $180°$ means that the missile decelerates. In other words, the amount of the propellant of the missile is excessive to reach the given PIP. Both of the cases with the variation of initial speed are not physically desirable for the interception. The results of the midcourse guidance with

TABLE I
SIMULATION RESULTS WITHOUT PIP CORRECTION

	Nominal	Velocity (-) case	Velocity (+) case
r_{miss} (m)	$6.9327 \cdot 10^{-4}$	$3.6520 \cdot 10^{3}$	0.3238
t_{miss} (sec)	36.1028	36.2028	36.1028
t_{PIP} (sec)	36.1028	36.1028	36.1028

TABLE II
SIMULATION RESULTS WITH PIP CORRECTION

	Nominal	Velocity (-) case	Velocity (+) case
r_{miss} (m)	$6.9327 \cdot 10^{-4}$	0.2456	0.0182
t_{miss}	36.1028	36.8686	35.6622
t_{PIP} (sec)	36.1028	36.8686	35.6622

PIP correction are summarized in Table. II.

Both cases satisfy the hit-to-kill condition, and the thrust directions are reasonable. For the velocity (-) case, t_{PIP} increases instead of increasing the amount of the propellant. The interception occurs at the lower altitude. On the other hand, t_{PIP} decreases to consume the remaining propellant for the velocity (+) case.

IV. CONCLUSION

Predicted Intercept Point (PIP) selection scheme was proposed to intercept the ballistic missile precisely in the free flight phase. The characteristics of the solid propellant anti-ballistic missile resulted in failure of the interception if the PIP was not properly modified. The midcourse guidance using the proposed PIP selection scheme achieved the hit-to-kill misison although uncertainty on the initial velocity was considered.

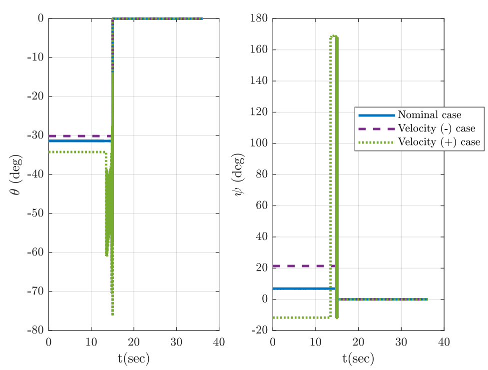

Fig. 6. Time histories of the input (without PIP correction)

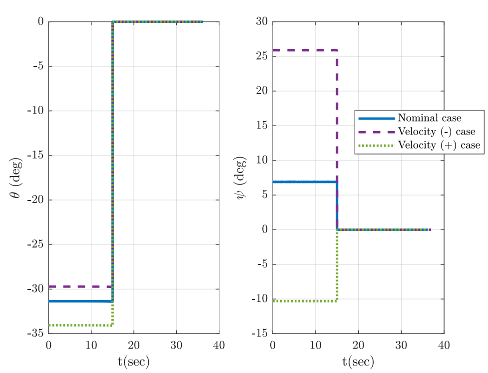

Fig. 7. Time histories of the input (with PIP correction)

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