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Hybrid NMPC for Switching Systems Applied to a Supermarket Refrigeration System*

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Abstract— **Refrigeration systems are characterized by switching behavior and discrete control functions as well as fast and slow dynamics, which make their control a challenging task. This paper proposes an efficient hybrid nonlinear model predictive control (HNMPC) scheme, which can be applied to a broad class of switching systems including refrigeration systems. The proposed approach is based on the embedding transformation technique and the sum-up rounding scheme and hence requires no assumptions on the switching sequence. A mixed integer optimal control problem is formulated based on reformulations of the switching dynamics. In order to circumvent the complexity of solving the resulting discontinuous optimization problem, the switching system is embedded into a family of continuous systems. The resulting problem is purely continuous and can be solved using state of the art Nonlinear Programming (NLP) solvers. The solution is either of bang-bang type or a rounding scheme with tight upper bound on the integer approximation is employed to approximate the solution to the closest feasible integer solution. The simulation results show the excellent performance of the proposed scheme in preventing the problem of display valves synchronization and reducing the switching frequency of the compressors to its minimum. In addition, the computational effort is very low compared to other schemes in the literature.**

I. INTRODUCTION

Switching systems, as an important class of hybrid systems have received considerable attention in the recent years because of their significance for numerous applications [1], [2]. In switching systems, the dynamical system evolves according to a set of vector fields that describe the continuous evolution in different sections of the state space which are called modes of operation. Two types of switches can be distinguished in switching systems: explicit and implicit. Explicit switching is also called controlled switching where the switches can be manipulated as degrees of freedom. In contrast, implicit switching arises when the system switches its dynamics depending on a transition condition which is a function of the state vector and the active mode.

Model predictive control (MPC) technique is an efficient way to deal with multi-variable and constrained systems. Several approaches have been proposed in the literature to extend MPC scheme to switching systems. A large portion of the literature focuses on piecewise affine (PWA) systems [3]. PWA models linear switching systems and can also be used to model static nonlinearities as well as to approximate nonlinear systems by successive linearizations. Equivalently, in [4] a framework for Mixed Logical Dynamical (MLD) systems was proposed for modeling and optimal control of linear hybrid systems. Here the optimization problem is recast into a Mixed Integer Quadratic Programming (MIQP) problem or Mixed Integer Linear Programming Problem (MILP) depending on the type of the norm used in the objective function. The sources of computational complexity of the MLD approach was investigated in [5], and it was shown that the number of discretization time intervals is the dominating source of complexity which can be mitigated in the receding horizon framework. The MLD approach works for linear and mildly non-linear systems, but it does not scale well to large problems.

Predictive control algorithms that operate directly on a nonlinear hybrid model have been investigated less and are mostly application specific, where the structure of the system is exploited to overcome the difficulty of the online solution of the underlying Mixed Integer Nonlinear Program (MINLP), e.g. [6], [7], [8].

Many optimization-based methods have been developed for the control of supermarket display cases. A piecewise affine approximation of small scale refrigerator system was presented in [9] within the MLD framework, in which an MIQP has to be solved online. An idea for HNMPC was presented in [7], where the sequence of switchings of the display cases valves were fixed and only the times of occurrence of the switchings were optimized. The underlying hybrid model includes a traditional PI controller for regulating the suction pressure around its reference value. A large scale supermarket refrigeration system was considered in [10] based on a hierarchical structure with two levels in which the upper level optimizes the parameters of the low level simple control schemes.

This paper presents an HNMPC algorithm for switching systems which contain implicit as well as explicit switches, where a Mixed Integer Optimal Control Problem (MIOCP) is solved online in each HNMPC iteration. This is achieved by offline reformulations of the process model in order to set up a well posed MIOCP which is solvable within the time restrictions imposed by the online implementation. The nature of the switches are identified and the state dependent switches are transformed into explicit switches by using principles from propositional logic [11], [12] in order to map the switching conditions into a set of constrained Boolean variables. This treatment of the switching conditions leads to a kind of problems called Mathematical program with Vanishing Constraints (MPVC) [13] which needs a special

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treatment, as will be shown later.

The resulting MIOCP contains only controlled or explicit switches which enables the application of the embedding transformation algorithm [14], [15]. It is shown that within the embedding transformation technique, the forbidden modes can be made infeasible in the reformulation phase, since the resulting Optimal Control Problem (OCP) searches the space of the feasible combinations of the discrete variables rather than their trajectories. In solving the resulting OCP the direct method is employed to obtain a suboptimal solution which is then approximated, if needed, to any desired precision by a switching solution [15], [16]. The applicability of the scheme is shown for the benchmark problem of supermarket display cases, with a realistic nonlinear rigorous switching model.

The paper is organized as follows: section II introduces the problem definition and formulation, it also shows how the implicit switches can be made explicit.Section III illustrates the application of the HNMPC scheme to the supermarket display cases benchmark problem, and finally, part IV concludes the paper and points out ideas for future research.

II. Problem Formulation

We are interested in solving a receding horizon MIOCP within the HNMPC framework. The switching system is described by a set of subsystems and a switching law which determines the active subsystem along the time horizon. In this part it is shown how the implicit switches can be made explicit, and the nature of the arising constraints as well as the reformulation of the resulting MIOCP are discussed. It is assumed that all the subsystems evolve in the same vector space without state jumps, and the variables bounds are enforced globally for all modes. The MIOCP is formulated as follows:

$$\min_{x(t),u(t),v(t)} \quad \varphi(x(t_f)) \tag{1a}$$

$$\textbf{s.t.} \quad \dot{x}(t) = f_{i(t)}(x(t),u(t),v(t)), \quad t \in [t_0, t_f], \tag{1b}$$

$$x(t_0) = x_0, \tag{1c}$$

$$h(x(t),u(t)) \leq 0, \quad t \in [t_0, t_f], \tag{1d}$$

$$g_{i(t)}(x(t),u(t),v(t)) \leq 0, \quad t \in [t_0, t_f], \tag{1e}$$

$$v(t) \in \nu = \{v_1, v_2, \ldots, v_{n_v}\}, t \in [t_0, t_f]. \tag{1f}$$

where the switching dynamics is described by (1b), $i(t) \in \{1, 2, \ldots, \mu\}$ represents the index of the active mode and μ represents the total number of modes. $x \in \mathbb{R}^n$ is the continuous state vector and $u \in \mathbb{R}^m$ denotes the vector of continuous input variables. $v(t)$ is the vector of discrete control variables. The switching signal is represented by a sequence of modes $\sigma = \{i_j\}_{j=0}^{N}$ and the corresponding switching instants $\tau = \{\tau_j\}_{j=0}^{N}$ in an ordered vector of pairs $s = ((\tau_0, i_0), (\tau_1, i_1), \ldots, (\tau_N, i_N))$, such that $0 \leq N \leq \infty$ and $t_0 = \tau_0 \leq \tau_1 \leq \cdots \leq \tau_N = t_f$ with t_0, t_f are the initial and final times, respectively. $\varphi(x(t_f))$ is the objective function represented, without loss of generality, as a Mayer term. Constraints (1d) denote the variables bounds and the mode independent continuous constraints. (1e) are constraints, which are mode dependent or depend explicitly on the discrete variables.

A. Implicit to Explicit Reformulation

We assume that the switches, whether implicit or explicit, are consistent and are not associated with state jumps or resets [17]. In case of implicit switches, the switching sequence depends indirectly on the initial conditions as well as the input variables in a very complex way. Therefore, the optimal mode sequence can be considered as a discrete decision variable which needs to be addressed as a part of the solution of the MIOCP. Exploring the complete discrete mode space or visiting all the combinations is an exhaustive search method and not real time feasible. In order to facilitate the analysis and unify the framework, the implicit dependency of the mode sequence on the control input is made explicit. The transformation employs principles from the propositional logic such that the switching conditions are mapped into a set of binary variables and associated linear constraints. The introduced binary variables modify the underlying dynamics in order to activate only the dynamics corresponding to the active mode and are included as decision variables within the optimization problem. The active mode is characterized by a certain combination of these binary variables, and its dynamic equations are activated only at the respective point in time. Therefore the optimal trajectories of these binary variables reflect directly the optimal mode sequence given that the initial mode is well defined.

Propositional logic can be used for the incorporation of logical inference within model predictive control. In which, Big-M formulations are used to transform the logic decisions into inequalities which constrain the evolution of the introduced binary variables in order to represent the switching in the optimization problem [11], [12]. Consider the following transformation:

$$[g(x) \geq 0] \longleftrightarrow [\delta_1 = 1] \textbf{ iff} \begin{cases} g(x) \geq \quad m(1-\delta_1) \\ g(x) \leq -\epsilon + (M+\epsilon)\delta_1 \end{cases} \tag{2}$$

$$M = \max_{x \in X} g(x), \quad \text{and} \quad m = \min_{x \in X} g(x). \tag{3}$$

where ϵ is a small positive number which can be as small as the machine precision. $\delta_1 \in \{0, 1\}$ is the introduced binary variable. Parameters M and m are upper and lower bounds on $g(x)$. Equation (2) presents an example of the mapping of the logical switching conditions into constraints through an equivalence relation to the introduced binary variable.

B. Embedding Transformation

Among several methods to solve MIOCP, the embedding transformation [14], [15] is an effective scheme for optimal control of switching systems especially with forced or explicit switches. In embedding transformation, an optimal control problem is solved for a larger family of systems containing the switching system. A new set of binary multipliers is introduced that represents the different discrete choices that are present in the switching problem. Then the dynamic model equations are evaluated for all the feasible discrete assignments, followed by a multiplication with the respective binary multipliers and then the summation of the products represents the process dynamics. The formulation is called partial outer convexification in which all the binary controls enter the optimization problem linearly, so the convexification is done only with respect to these binary variables. This formulation avoids the evaluation of the dynamics at fractional choices and ensures that all feasible trajectories of the continuous OCP can be approximated to any desired precision by integer solutions [18], [19]. The switching dynamics (1b) is transformed into:

$$\begin{aligned}\dot{x}(t) = &\sum_{j=1}^{n_{v1}} \alpha_j(t) f_1(x(t), u(t), v_1^j) + f_0(x(t), u(t)), \quad (4)\\ &+ \sum_{i=1}^{n_\delta \cdot n_{v2}} \omega_i(t) f^i(x(t), u(t), v_2^i), \qquad x(t_0) = x_0.\end{aligned}$$

where $\alpha_j \in \{0,1\}$, $\sum_{j=1}^{n_{v1}} \alpha_j = 1$ and $\omega_i \in \{0,1\}$, $\sum_{i=1}^{n_\delta . n_{v2}} \omega_i = 1$ are two sets of binary multipliers restricted by the special set order property of type one constraint (SOS-1) to enforce only one choice at any time instant. The system dynamics are separated into three parts, f_0 represents the part of the dynamics which is common in all modes and is purely continuous whereas, f_1 is also independent of the active mode but contains discrete controls $v_1(t) \in \nu_1 = \{v_1^1, v_1^2, \ldots, v_1^{n_{v1}}\}$. The third part f^i is the mode dependent part of the dynamics which might depend on discrete controls $v_2(t) \in \nu_2 = \{v_2^1, v_2^2, \ldots, v_2^{n_{v2}}\}$. The binary multipliers α_j are enumerating all the possible alternatives for the mode independent discrete control variables, whereas ω_i represent all feasible combinations of the modes and the mode dependent discrete control inputs. In case of constraints that depend explicitly on the discrete variables, feasibility is enforced for each possible choice in order to avoid the compensation effects [16]. Thus, the MIOCP is formulated as follows:

$$\min_{x(.),u(.),\omega(.),\alpha(.)} \quad \varphi(x(t_f)) \tag{5a}$$

$$\textbf{s.t.} \quad \text{Equation (4)}, \tag{5b}$$

$$h(x(t), u(t)) \leq 0, \quad t \in [t_0, t_f], \tag{5c}$$

$$\alpha_j(t) d_j(x(t), u(t), v_1^j) \leq 0, \quad t \in [t_0, t_f], \tag{5d}$$

$$\omega_i(t) c_i(x(t), u(t), v_2^i) \leq 0, \quad t \in [t_0, t_f], \tag{5e}$$

$$\alpha_j \in \{0,1\}, \sum_{j=1}^{n_{v1}} \alpha_j = 1, \quad j \in \{1, 2, \ldots, n_{v1}\}, \tag{5f}$$

$$\omega_i \in \{0,1\}, \sum_{i=1}^{n_\delta \cdot n_{v2}} \omega_i = 1, \quad i \in \{1, 2, \ldots, n_\delta \cdot n_{v2}\}. \tag{5g}$$

It should be noticed that the binary multipliers now are included as new decision variables. The bijection mapping between the binary multipliers and the original discrete controls is given in the following relations:

$$v_1(t) = \sum_{j=1}^{n_{v1}} \alpha_j(t) v_1^j, \qquad v_2(t) = \sum_{i=1}^{n_{v2} \cdot n_\delta} \omega_i(t) v_2^i. \tag{6}$$

The constraints (1e) are separated into two parts, mode independent constraints (5d), and mode dependent constraints (5e). Constraints $d_j(.)$ arise from evaluating the mode independent part of the constraints at every possible choice of the $v_1(t)$ variables. These kind of constraints represent restrictions that are imposed directly on the discrete controls, e.g. a maximum number of switches. In contrast, (5e) are the mode dependent constraints which arise from the transformation of the implicit switches or from the mode dependent discrete controls. These constraints are evaluated at every choice of the binary variables $v_2(t)$. (5d, 5e) are only enforced when the corresponding binary multiplier assumes a value of one.

By relaxation of the binary multipliers $\alpha_j \in [0,1]$, $\omega_i \in [0,1]$ such that they attain values from a convex compact set, the system (4) is embedded into a family of continuous systems. Therefore, the set of trajectories of the switching system is dense in the trajectories of the relaxed continuous system [14]. It is computationally more efficient to solve the relaxed problem, as the solution of the relaxed system is either a valid solution of the switching system in case of a bang-bang solution, or can be approximated up to the desired precision by an integer solution. An efficient rounding scheme with tight upper bound on the integer approximation error, namely sum-up rounding scheme, was presented by [18], [16]. The sum-up rounding scheme rounds the relaxed control trajectory to an integer feasible trajectory by sequentially rounding-up the control variable that shows the highest integrated difference between the relaxed counterpart and all the previously rounded intervals. For the case within the HNMPC formulation, the control variable which assumes the biggest relaxed value within the current interval will be rounded up and then passed to the process. Therefore, the switching times are approximated within the accuracy of one sampling interval of the HNMPC.

C. Vanishing Constraints

The constraints (5d, 5e) include a direct multiplication of the binary variables and the residual functions evaluated at corresponding discrete assignments. The presented complementarity formulation leads to vanishing of inactive constraints i.e., in case the binary control assumes a value of exactly zero. Thus a relatively difficult and highly nonconvex problem arises, a mathematical program with vanishing constraints (MPVC) [13]. MPVCs are characterized by critical points at which the gradient based solvers tend to perform very badly due to ill-conditioning and numerical problems related to the machine precision [17], [19]. Two main reformulations are reported in [19], which are either based on regularization or smoothing by a nonlinear complementarity functions (NCP-functions), e.g. the Fischer-Burmeister function. The main idea is to recover the constraint qualification at the critical points by an enlargement of the feasible region.

$$\alpha_j(t) d_j(x(t), u(t), v_1^j) \leq \tau, \quad t \in [t_0, t_f] \tag{7}$$

$$\omega_i(t) c_i(x(t), u(t), v_2^i) \leq \tau, \quad t \in [t_0, t_f] \tag{8}$$

Constraints (7, 8) are the regularized version of the original constraints (5d, 5e), Where $\tau > 0,\ \tau \to 0$ is the regularization parameter.

III. Supermarket Refrigeration System

Here, the proposed HNMPC scheme is applied to a process with switching dynamics and discrete inputs as well as operational constraints. A version with 3 display cases and 3 compressors is considered under parametric uncertainty. Firstly, a brief description of the process is presented, then HNMPC problem is set up and finally the simulation results are shown.

A. Process Description

The main component of the supermarket refrigeration system is the central rack of compressors which maintains the flow of the liquid refrigerant to the display cases. Each display case has an associated inlet on/off valve for the refrigerant which keeps the air temperature within the required range in order to achieve good preservation of the edible goods. The overall model of the supermarket refrigeration system is constructed by connecting the sub-models of its three main components, namely the display cases, the suction manifold and the compressors. Each display case is described by three temperature states, namely the temperature of the goods T_{goods}, of the evaporator wall T_{wall}, and of the air curtain T_{air}, as well as the mass of the refrigerant inside the evaporator m_{ref}. The suction manifold is modelled by the suction pressure dynamic equation P_{suc} and finally the total refrigerant volume drawn by the compressor rack V_{comp}. Please refer to Larsen et al. [20] for a complete description of the process including a rigorous model as well as simulation results using the traditional control scheme.

B. Model Adaptation

The considered system is hybrid in nature and contains switches of both types, implicit and explicit. The switches are identified to be consistent and are not associated with state jumps or resets. On one hand, explicit or controlled switches are introduced by the display valves signals which can assume only one of two positions, either fully open or closed $v_i \in \{0, 1\}$, and directly influence the dynamics of m_{ref} inside the evaporator. On the other hand, the process dynamics are forced to switch based on P_{suc} in order to activate the required number of compressors to keep P_{suc} within the operational bounds. Given the assumption that all compressors are of equal size and have the same compression capacity (C_{comp}), the total number of the involved compressors at any time instant is given by $U_{comp} = \sum_{j=1}^{nc} U_{comp,j}$, where nc is the total number of compressors. This implicit dependency is illustrated by a simple hybrid automaton (1), which has $nc + 1$ locations due to the available levels of compression plus the zero level. The process dynamics in mode i is given by $f^i(x, v)$ which represents the process model with fixed compression capacity corresponding to the level indicated by $U_{comp} \in \{0, 1, \ldots, nc\}$.

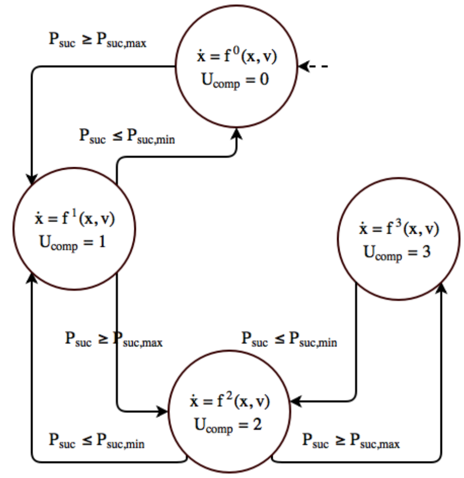

Fig. 1. Hybrid automaton of the supermarket refrigerator

In order to reduce the compressor switches and to prolong their lifetime as well as reduce the power consumption, the compression capacity is allowed to increment or decrement in a step of only one compressor as shown in (1). This exploits the fact that the pressure reacts very fast compared to the temperature dynamics and leads to simplified switching behaviour which is beneficial for the solution of the optimization problem.

The implicit switches in (1) are transformed into explicit as explained in section (II-A). Two binary variables δ_1 and δ_2 are introduced to represent the switching logic as follows:

$$[P_{suc} - P_{suc,max} \geq 0] \longleftrightarrow [\delta_1 = 1]$$
$$\textbf{iff} \quad \begin{cases} P_{suc} - P_{suc,max} \geq m_1(1 - \delta_1) \\ P_{suc} - P_{suc,max} \leq -\epsilon + (M_1 + \epsilon)\delta_1 \end{cases} \tag{9}$$
$$[P_{suc} - P_{suc,min} \leq 0] \longleftrightarrow [\delta_2 = 1]$$
$$\textbf{iff} \quad \begin{cases} P_{suc} - P_{suc,min} \leq M_2(1 - \delta_2) \\ P_{suc} - P_{suc,min} \geq \epsilon + (m_2 - \epsilon)\delta_2 \end{cases} \tag{10}$$

where M_1, M_2 and m_1, m_2 are the upper and lower bounds of the respective switching functions. The combinations of the variables δ_1 and δ_2 together with the current mode determine the next active mode. Two binary variables can assume four combinations out of which three are feasible and the last one which represents the case when P_{suc} hits the upper and the lower bound together is rejected. Accordingly, (11) is added to the process model, which approximates the change of the compression capacity due to the number of active compressors. The total number of active compressors is relaxed and added as an augmented state which increments or decrements to the next level very fast within one HNMPC sampling interval Δt.

$$\dot{U}_{comp} = 1/\Delta t(\omega_1 - \omega_2), \quad 0 \leq U_{comp} \leq nc. \tag{11}$$

The binary multipliers $\omega_0, \omega_1, \omega_2 \in \{0, 1\}$ represent the three feasible combinations of the binary variables δ_1, δ_2. Notice that: $\omega_0 = 1$ implies that there is no change in the level of compression. This formulation is advantageous for cases when a larger number of compressors is present, as the number of the introduced binary multipliers is independent of the compression levels, provided that the compressors are all similar.

1) Partial Outer Convexification: As shown in section (II-B), the underlying dynamic equations are reformulated such that the resulting MIOCP is searching the space of the binary multipliers representing different assignments of the discrete controls. Therefore not only non-physical combinations of the discrete inputs are rejected but also unwanted assignments can be made infeasible and discarded from the optimization problem. For instance, synchronized switching of the display valves can be rejected.

$$\dot{x}(t) = \sum_{j=1}^{n_\alpha} \alpha_j(t) f_1(x(t), v^j) + \sum_{i=1}^{n_\omega} \omega_i(t) f^i(x(t)), \quad (12)$$

$$x(t_0) = x_0,$$

$$\alpha_j \in \{0,1\}, \ \sum_{j=1}^{n_\alpha} \alpha_j = 1, \ \ j \in \{1,2,\dots,n_\alpha\}, \quad (13)$$

$$\omega_i \in \{0,1\}, \ \sum_{i=1}^{n_\omega} \omega_i = 1, \ \ i \in \{1,2,\dots,n_\omega\}. \quad (14)$$

The dynamic equations are separated into two parts, the mode independent part $f_1(x(t), v^j)$ represents the display cases, and the mode dependent part $f^i(x(t))$ which is comprised of two ODE equations for P_{suc} and the augmented state U_{comp}. $f_1(x(t), v^j)$ is evaluated at 4 feasible assignments of display valve signals v^j, namely, all the valves are closed or only one of the three valves is switched. Thus, the total number of the binary multipliers is limited to $n_\alpha = 4$. The function $f^i(x(t))$ is evaluated at the 3 feasible choices of the binary variables δ_1, δ_2 and accordingly $n_\omega = 3$.

C. HNMPC Formulation

In this section, the ingredients of the HNMPC are presented and the underlying optimization problem is formulated.

1) The process constraints: In addition to the dynamic equations (12), operational bounds are imposed on the state variables. Moreover, regulated vanishing constraints arise as a result of the convexification of (9, 10).

$$\underline{x}(t) \le x(t) \le \overline{x}(t), \ \ t \in [t_0, t_f], \quad (15)$$

$$\omega_i(t) c_{k,i}(x(t), \delta^i) \le \tau_{vc}, \quad t \in [t_0, t_f], \quad (16)$$

$$\alpha_j \in [0,1], \ \sum_{j=1}^{n_\alpha} \alpha_j = 1, \ \ j \in \{1,2,\dots,n_\alpha\}, \quad (17)$$

$$\omega_i \in [0,1], \ \sum_{i=1}^{n_\omega} \omega_i = 1, \ \ i \in \{1,2,\dots,n_\omega\}. \quad (18)$$

where, $\underline{x}(t)$ and $\overline{x}(t)$ are the lower and the upper bounds of the state vector. The functions $c_{k,i}(x(t), \delta^i), i \in \{0,1,2\}, k \in \{0,1,2,3\}$ contain the vanishing constraints (9, 10) evaluated at the 3 feasible assignments of the binary variables $\{\delta_1, \delta_2\}$, which results in 12 constraints. The vanishing constraints are relaxed by $\tau_{vc} = 0.03$. Constraints (17, 18) are the relaxed counterparts of (13, 14).

2) Objective Function: The purpose of the supermarket refrigeration system is to keep the goods temperature within certain bounds in order to achieve high quality preservation. This is translated into a tracking term in the objective function for $T_{goods,i}$ for all 3 display cases as follows:

$$\varphi(x(t)) = \frac{1}{\tau_p} \int_0^{\tau_p} \sum_{i=1}^{nd} P_i (T_{goods,i} - T_{goods,ref})^2 dt \quad (19)$$

where τ_p is the prediction horizon, and P_i is a penalization term which assumes a value of one for all display cases.

D. Simulation Results

It is assumed that full state information is available and the environmental heat load $\dot{Q}_{air,load}$ is considered to be uncertain within $\pm 20\%$ of its nominal modelled value. The sampling period was chosen to be $\Delta t = 30s$, and the prediction horizon was set to $\tau_p = 20\Delta t$. All the operating bounds were relaxed using slack variables which are heavily penalized within the objective function. In addition, a back-off from T_{air} operating bounds with value of ± 0.5 degree was introduced to avoid infeasibilities due to the rounding errors. Direct collocation was the method of choice, due to its numerical efficiency, in which the states and the control inputs are parametrized leading to a large scale Nonlinear Program (NLP). The resulting NLP was solved using the IPOPT [21] interior-point solver under CasADi [22]. All the simulations were performed under Windows 7 hosted on $32GB$ RAM, $i7 - 6700 \ \ 3,4GHz$ CPU machine.

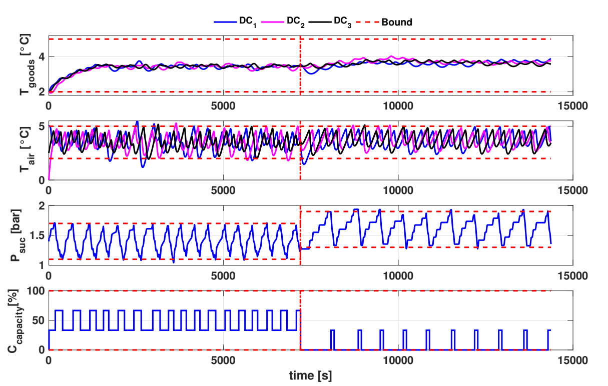

Fig. 2. Process variables and compression capacity (at nominal conditions)

Figures (2, 3, 4) show the simulated trajectories of the main controlled variables $T_{goods}, T_{air}, P_{suc}$ as well as the number of the active compressors in three different cases according to the parametric uncertainty in $\dot{Q}_{air,load}$ namely, at same level in the HNMPC prediction model and $\pm 20\%$ levels compared to the value used for the predictions. The figures show simulations for $2h$ in the day conditions and then switch to another $2h$ of simulation for the night mode. The goods temperature T_{goods} was specified to track $3.5°C$ during the day time and $4.0°C$ during the night.

The simulation results show that the process needs only two compressors, with one operating almost continuously and the second being used only during the day time when

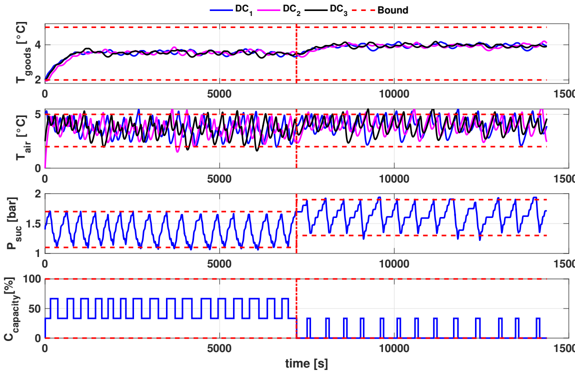

Fig. 3. Process variables and compression capacity ($\dot{Q}_{air,load}$ 20% higher than nominal value)

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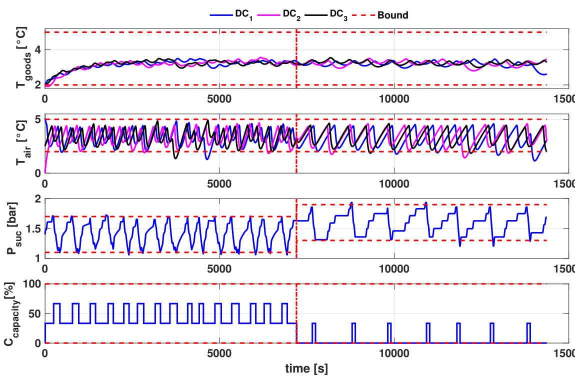

Fig. 4. Process variables and compression capacity ($\dot{Q}_{air,load}$ 20% lower than nominal value)

the load is relatively high compared to the night time. As can be noticed in the three simulations, the compressors switches are minimized and the synchronization of the display valves does not happen. The average CPU-time for one HNMPC iteration was $1.19\ s$, and in the worst case it was $4.32\ s$ when the $\dot{Q}_{airload}$ assumed a 20% lower than the value used in prediction.

IV. CONCLUSIONS

A scheme for HNMPC was proposed and applied to a supermarket refrigeration system with 3 display cases and 3 compressors. The scheme avoids the drawbacks of the traditional decentralized controller and fulfils the requirements of high quality control. Furthermore it can be easily extended to control large scale refrigeration systems without an exponential increase in the problem size by omitting the synchronization assignments for the display valves and considering incrementing and decrementing the current compression level instead of choosing the suitable level directly. The simulation results show a computational performance which is orders of magnitude better than for methods which rely on solving the underlying MINLP directly. Future work will consider economic HNMPC as well as further applications to the ramp-up and shut down of processing units.

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