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Particle Filter for Underwater Passive Bearings-Only Target Tracking with Random Missing Measurements

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***Abstract*— In underwater passive bearings-only tracking (BOT), it is generally assumed that bearing measurements are valid and available at each time index. However, in practice, some of the measurements could be randomly unavailable or invalid on the account of low signal-to-noise ratio (SNR). This work proposes a particle filter (PF) with modified importance weights to tackle random missing instances in the observed bearings. A measurement model which replaces the missing measurements according to the last received measurement and the knowledge of process dynamics, is formulated. Finally, the proposed PF is implemented on a real-life underwater BOT scenario with random missing measurements. Root mean square error and track-loss count obtained from the simulation show that the proposed PF which accounts for the randomness in missing measurements, performs with improved accuracy compared to the existing PF and other Gaussian filters.**

I. INTRODUCTION

The bearings-only tracking (BOT) problem has wide application in several real-life scenarios like aircraft surveillance [1], underwater target tracking [2] *etc*. For underwater BOT system, passive sonar is generally used to sense the bearings of the target as it has a tactical advantage of being undetected by an external observer [1]. The BOT problem, where the objective is to find the kinematics of a moving target such as range, speed *etc.* using only noise-corrupted bearing measurements, is often referred as target motion analysis (TMA). If a single observer is used to gather the bearing measurements, as is the case in this paper, the problem is known as autonomous TMA [2]. TMA problems have been widely discussed in literature and most of the research in this area are concerned with autonomous TMA for tracking non-manoeuvring targets [3], [4]. In autonomous TMA, if the observer follows a constant velocity model, resulting dynamics becomes unobservable [5]. Tracking a target from only noise-corrupted bearings in TMA requires the observer to manoeuvre [6].

Bayesian framework of filtering has been extensively used to solve the autonomous TMA problems where posterior density of the unobserved target kinematics is determined using the knowledge of transition prior density of target state and the likelihood of received passive bearing measurements. However, often the computation becomes analytically intractable due to the nonlinear measurement model [3], [4]. Hence, the posterior density needs to be approximated and this in turn results into several suboptimal algorithms. The extended Kalman filter (EKF) was the first popular suboptimal algorithm to be used for this kind of problems [7]. Improving upon EKF, several other algorithms appeared in literature like the unscented Kalman filter (UKF) [8], cubature Kalman filter (CKF) [9], particle filter (PF) [2] *etc.* PF is a powerful sequential Monte Carlo method which approximates the posterior density empirically [10]. In PF method, the state space of underlying system is divided into numerous parts which are filled with particles according to a given probability measure. Higher the value of probability, denser the particles are in that part of the state space [11]. The performance of all these filters for solving the BOT problems have been studied in detail [12], [2]. If a processing unit can afford the high computational cost, PF proves to be an otherwise better alternative to other filters for underwater BOT problems [13].

As mentioned, there exist a good number of research which have aptly dealt with the underwater passive BOT problems. These recursive filtering solutions assume that bearings of the target, which are observed through an array of passive sonar, are consistently available at each time index. However, in practice, there could be some external source of noise around the observer, *e.g.* a big ship, which can cause an inconsistent signal received at the input of sensor array. Occurrence of such noise is generally random in nature. These inconsistent signals which have low signal-to-noise ratio (SNR) do not get carried to the input of estimator, and hence considered as random missing measurements. A method to identify the unknown parameter for a linear system in the environment of random missing measurements, has been proposed in [14]. [15] has illustrated a way to identify the system parameter using particle filter under missing measurements. This algorithm doesn't go for the measurement update when observations are not available. To tackle such problem for underwater BOT system, there are very few research available in literature. Recently, a batch filtering solution for underwater BOT problem with missing bearing data has been proposed [16]. This work also ignores the missing measurements and uses whatever bearings are available to track the target path. The underwater BOT is a weakly observable system [17] and ignoring the missed measurements without replacement may lead to higher track-loss for recursive filtering where the predicted estimate is to be corrected with new measurement received at each time index.

In this paper, we consider random missing bearing measurements with known latency probability for a typical underwater passive BOT system with non-manoeuvring target. A measurement model which replaces the missing mea-

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surements according to the last received measurement and the knowledge of process dynamics, is proposed. Later, assuming the knowledge of latency probability, a PF for random missing measurements (PF-RM) is designed by developing an importance weight recursion that accounts for the randomness in missing measurements. Finally, the underlying underwater BOT problem is solved using the cubature quadrature Kalman filter (CQKF) [23], conventional PF and the proposed PF-RM. To demonstrate the superiority of the proposed filter, performance of all the filters are compared in terms of the root mean square error (RMSE) and the number of divergent tracks.

II. Problem Formulation

Here, a typical two-dimensional target-observer dynamics is realized in Cartesian coordinates. The motive is to estimate the position vector, $r^t = [x^t \ y^t]^T$ and the velocity vector $v^t = [\dot{x}^t \ \dot{y}^t]^T$ of the target from noise corrupted passive bearing measurements. Hence, the state vector containing target kinematics can be defined as $\mathbf{x}_k^t = [x_k^t \ y_k^t \ \dot{x}_k^t \ \dot{y}_k^t]^T$. Similarly, observer state vector is defined as $\mathbf{x}_k^o = [x_k^o \ y_k^o \ \dot{x}_k^o \ \dot{y}_k^o]^T$. Now, we introduce a relative state vector $\mathbf{x}_k \triangleq \mathbf{x}_k^t - \mathbf{x}_k^o = [x_k \ y_k \ \dot{x}_k \ \dot{y}_k]^T$.

The discrete-time state equation for the target dynamics, where it is assumed to follow a near constant velocity motion model can be expressed as [12]

$$\mathbf{x}_k = F\mathbf{x}_{k-1} + \mathbf{q}_{k-1} - \mathbf{U}_{k-1,k}. \tag{1}$$

F is the state transition matrix defined as

$$F = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $\mathbf{q}_{k-1}$ is an independent and identically distributed (i.i.d.) zero mean Gaussian process noise vector with covariance matrix

$$Q = \begin{bmatrix} \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 \\ 0 & \frac{T^3}{3} & 0 & \frac{T^2}{2} \\ \frac{T^2}{2} & 0 & T & 0 \\ 0 & \frac{T^2}{2} & 0 & T \end{bmatrix} \bar{q},$$

where $\bar{q}$ is the process noise intensity and T, the sampling interval. To incorporate observer accelerations in the target dynamics, a vector of observer deterministic inputs $\mathbf{U}_{k-1,k}$ is considered and given by

$$\mathbf{U}_{k-1,k} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} x_k^o - x_{k-1}^o - T\dot{x}_{k-1}^o \\ y_k^o - y_{k-1}^o - T\dot{y}_{k-1}^o \\ \dot{x}_k^o - \dot{x}_{k-1}^o \\ \dot{y}_k^o - \dot{y}_{k-1}^o \end{bmatrix}.$$

These observer values are often determined by an on-board inertial navigation system powered by a global positioning system (GPS).

The measurement equation can be represented as

$$z_k = h(\mathbf{x}_k) + v_k, \tag{2}$$

where v_k is a sequence of independent zero mean Gaussian random variables with standard deviation σ_θ. From this model, the only available measurement, which is the noise corrupted passive bearings are obtained. Measurements are obtained from the observer's platform to the target with a reference clockwise positive to the y-axis. The true bearing measurements are defined as

$$h(\mathbf{x}_k) = \tan^{-1}\left(\frac{x_k}{y_k}\right). \tag{3}$$

The assumption that a valid bearing measurement is available at each time step does not necessarily hold true always in practice. There could be some external noise source around the observer that would not let the sonar have a positive signal excess (amount of signal in decibel over the noise threshold [18]). Hence the bearing measurements received from passive sonar can have some random missing instances when it has not received the valid signal. Consider β_k be an i.i.d. Bernoulli random number that takes value either 1 or 0 with probability $P(\beta_k = 1) = p = E[\beta_k]$ and $P(\beta_k = 0) = 1 - p$, where p is known latency parameter and $E[\cdot]$ denotes the expectation of a random variable. At any time step k, if z_k^m is the measurement which estimator receives with randomly missed target bearings, then

$$z_k^m = (1 - \beta_k)z_k + \beta_k g(k, z_{k-1}^m, \Delta_k), \tag{4}$$

where $g(k, z_{k-1}^m, \Delta_k)$ determines the replacement for missing measurement when $\beta_k = 1$. For underlying BOT problem, we have proposed $g(\cdot)$ to be defined as

$$g(k, z_{k-1}^m, \Delta_k) = z_{k-1}^m - \Delta_k; \qquad k \geq 2.$$

Here Δ_k represents the change in course of the observer at k^{th} time step and can be defined as

$$\Delta_k = \tan^{-1}\left(\frac{x_k^o - x_{k-1}^o}{y_k^o - y_{k-1}^o}\right) - \tan^{-1}\left(\frac{x_{k-1}^o - x_{k-2}^o}{y_{k-1}^o - y_{k-2}^o}\right); \quad k \geq 2.$$

Now, the problem of bearings-only tracking boils down to estimating the relative state vector $\mathbf{x}_k$ in the presence of modified measurement model. Thus, objective is to outline a PF algorithm for given system (1) with measurement model (4) which assumes the knowledge of latency probability p. Using Table I and dynamics of the observer state vector $\mathbf{x}_k^o$, tracking scenario is generated and plotted in Fig. 1.

III. Modified Approximate Filtering for Underwater BOT with Random Missing Measurements

A. Particle Filter

As discussed in [19], assuming measurements $z_{1:k}$ are independent and state vector $\mathbf{x}_k$ is given by a Markov process, the marginal posterior density function is empirically

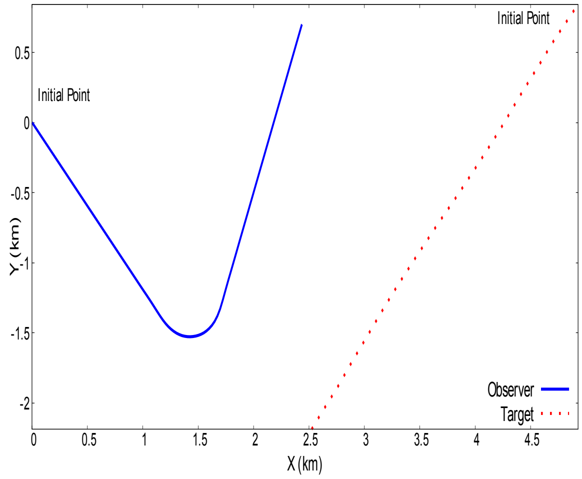

Fig. 1. Target-observer tracking scenario.

approximated as a sum of weighted particles and can be represented as

$$P(\mathbf{x}_k|z_{1:k}) = \sum_{i=1}^{N} w_k^i \delta[\mathbf{x}_k - \mathbf{x}_k^i], \tag{5}$$

where $\delta[\cdot]$ denotes Dirac delta function and particles $\{\mathbf{x}_k^i\}_{i=1}^N$ are i.i.d. random numbers. Particles are drawn from a proposal density $q(\mathbf{x}_{0:k}|z_{1:k})$ which is a known and easy-to-sample density function. The importance weight of particles is chosen using importance principle and can be defined for i^{th} particle as

$$w_k^i = \frac{P(\mathbf{x}_{0:k}^i|z_{1:k})}{q(\mathbf{x}_{0:k}^i|z_{1:k})}. \tag{6}$$

Now, for a sequential case, we need a recursive weight update at each time step which can be formulated with the help of following equations:

$$\begin{aligned} P(\mathbf{x}_{0:k}|z_{1:k}) &= \frac{P(z_k|\mathbf{x}_{0:k})P(\mathbf{x}_{0:k}|z_{1:k-1})}{P(z_k|z_{1:k-1})} \\ &\propto P(z_k|\mathbf{x}_{0:k})P(\mathbf{x}_{0:k}|z_{1:k-1}), \end{aligned} \tag{7}$$

where $P(z_k|z_{1:k-1})$ is a normalizing constant. The proposal density can be assumed to be decomposed as

$$q(\mathbf{x}_{0:k}|z_{1:k}) = q(\mathbf{x}_k|\mathbf{x}_{0:k-1}, z_{1:k})q(\mathbf{x}_{0:k-1}|z_{1:k-1}). \tag{8}$$

If we are interested only in estimate $P(\mathbf{x}_k|z_{1:k})$ and $q(\mathbf{x}_{0:k}|z_{1:k}) = q(\mathbf{x}_k|z_{1:k})$, (6), with the help of (7), (8) and the Markov first order property of state $\mathbf{x}_k$, can be written as

$$\begin{aligned} w_k^i &\propto \frac{P(z_k|\mathbf{x}_k^i)P(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)P(\mathbf{x}_{0:k-1}^i|z_{1:k-1})}{q(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i, z_{1:k})q(\mathbf{x}_{0:k-1}^i|z_{1:k-1})} \\ &= w_{k-1}^i \frac{P(z_k|\mathbf{x}_k^i)P(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i, z_{1:k})}. \end{aligned} \tag{9}$$

B. Particle Filter for Random Missing Measurements

It is evident from (4) that the measurement z_k^m is a function of previous received measurement z_{k-1}^m and is in turn dependent on z_{k-1}. Hence z_k^m may be correlated with previous state vector $\mathbf{x}_{k-1}$ along with current state vector $\mathbf{x}_k$. A recursive computation of the importance weight for system (1) with measurement model (4) which assumes the knowledge of latency probability p, can be obtained as in the following lemma.

Lemma 1: The importance weight w_k^i at any time step k, can be given as

$$w_k^i = w_{k-1}^i \frac{P(z_k^m|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i)P(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{q(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i, z_{1:k}^m)}, \tag{10}$$

where $\mathbf{x}_k^i$ is drawn from the proposal density $q(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i, z_{1:k}^m)$.

Proof: Consider augmentation of state vector $\mathbf{x}_k$ into $\bar{\mathbf{x}}_k = [\mathbf{x}_k\ \mathbf{x}_{k-1}]^T$ as new state of the underlying system. Using augmented state vector $\bar{\mathbf{x}}_k$ and measurement z_k^m, (9) can be rewritten as

$$\begin{aligned} w_k^i &= w_{k-1}^i \frac{P(z_k^m|\bar{\mathbf{x}}_k^i)P(\bar{\mathbf{x}}_k^i|\bar{\mathbf{x}}_{k-1}^i)}{q(\bar{\mathbf{x}}_k^i|\bar{\mathbf{x}}_{k-1}^i, z_{1:k}^m)} \\ &= w_{k-1}^i \frac{P(z_k^m|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i)P(\mathbf{x}_k^i, \mathbf{x}_{k-1}^i|\mathbf{x}_{k-1}^i, \mathbf{x}_{k-2}^i)}{q(\mathbf{x}_k^i, \mathbf{x}_{k-1}^i|\mathbf{x}_{k-1}^i, \mathbf{x}_{k-2}^i)}. \end{aligned} \tag{11}$$

Using the assumption that state vector $\mathbf{x}_k$ follows the Markov first-order process, *i.e.* $P(\mathbf{x}_k^i|\mathbf{x}_{0:k}^i) = P(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)$ and $q(\mathbf{x}_k^i|\mathbf{x}_{0:k-1}^i, z_{1:k}^m) = q(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i, z_{1:k}^m)$, (11) can be simplified into (10). ■

Note: It is assumed that the received measurement z_k^m depends only on the current state $\mathbf{x}_k$ and previous state $\mathbf{x}_{k-1}$, *i.e.* $P(z_k^m|\mathbf{x}_{0:k}) = P(z_k^m|\mathbf{x}_k, \mathbf{x}_{k-1})$ which represents the real scenario closely if random missing measurements is less likely. If probability of missing measurements is high, z_k^m may be correlated with many past states.

Lemma 2: Likelihood density $P(z_k^m|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i)$ can be recursively computed as

$$\begin{aligned} P(z_k^m|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i) &= (1-p)P_{v_k}(z_k^m - h(\mathbf{x}_k^i)) \\ &\quad + p(P_{v_{k-1}}(z_k^m - h(\mathbf{x}_{k-1}^i) + \Delta_k)), \end{aligned} \tag{12}$$

where $P_{v_k}(\cdot)$ denotes the probability density function for measurement noise v_k.

Proof: Considering i.i.d. random variable β_k is independent of state vector, the likelihood density $P(z_k^m|\mathbf{x}_k^i)$ can be expressed as marginalization of the joint density function $P(z_k^m, \beta_k|\mathbf{x}_k^i)$ using following steps:

$$\begin{aligned} P(z_k^m|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i) &= \int P(z_k^m, \beta_k|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i)d\beta_k \\ &= \int P(z_k^m|\beta_k, \mathbf{x}_k^i, \mathbf{x}_{k-1}^i)P(\beta_k|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i)d\beta_k \\ &= \int P(z_k^m|\beta_k, \mathbf{x}_k^i, \mathbf{x}_{k-1}^i)P(\beta_k)d\beta_k \\ &= P(z_k^m|\beta_k = 0, \mathbf{x}_k^i, \mathbf{x}_{k-1}^i)P(\beta_k = 0) \\ &\quad + P(z_k^m|\beta_k = 1, \mathbf{x}_k^i, \mathbf{x}_{k-1}^i)P(\beta_k = 1). \end{aligned} \tag{13}$$

Since $P(\beta_k = 1) = p$ and $P(\beta_k = 0) = 1 - p$, (13) can be written as

$$\begin{aligned} P(z_k^m|\mathbf{x}_k^i, \mathbf{x}_{k-1}^i) &= (1-p)P(z_k^m|\beta_k = 0, \mathbf{x}_k^i, \mathbf{x}_{k-1}^i) \\ &\quad + pP(z_k^m|\beta_k = 1, \mathbf{x}_k^i, \mathbf{x}_{k-1}^i). \end{aligned} \tag{14}$$

Using (2), (4) can be expanded as

$$z_k^m = (1-\beta_k)(h(\mathbf{x}_k^i)+v_k)+\beta_k(z_{k-1}^m-\Delta_k). \quad (15)$$

Now, if $\beta_k = 0$, (15) simplifies into $z_k^m = h(\mathbf{x}_k^i)+v_k$ and consequently,

$$P(z_k^m|\beta_k=0,\mathbf{x}_k^i,\mathbf{x}_{k-1}^i) = P_{v_k}(z_k^m - h(\mathbf{x}_k^i)). \quad (16)$$

Similarly, if $\beta_k = 1$, (15) gives $z_k^m = z_{k-1}^m - \Delta_k$ and assuming z_{k-1}^m depends only on z_{k-1}, we have

$$P(z_k^m|\beta_k=1,\mathbf{x}_k^i,\mathbf{x}_{k-1}^i) = P(z_k^m - h(\mathbf{x}_{k-1}^i)+\Delta_k). \quad (17)$$

Finally, we can easily establish (12) by substituting (16) and (17) into (14). ■

Generally, underwater BOT system with non-manouevring target has very small process noise and which demands a different treatment of sampled particles to avoid the state of sample impoverishments [20]. Generally, resampling stage involves sampling of particles from the approximated discrete distribution of the posterior, which results into multiple duplication of some selected particles from previous sampling stage. If a regularization kernel is used to convert the discrete approximation of the posterior into the continuous one, a new particle system with N different locations can be generated in the resampling step [21].

As discussed in [21], we have implemented post-regularized particle filter (post-RPF) method to solve the problem of target tracking with small dynamical noise. In this work, transition prior density $P(\mathbf{x}_k|\mathbf{x}_{k-1}^i)$ is chosen as proposal density $q(\mathbf{x}_k|\mathbf{x}_{k-1},z_k^m)$. However, pdf obtained from a standard non-linear filter can also be used as proposal density function. Further, resampling is not done at each step but only when the effective sample size (N_{eff}) goes below the predefined threshold value (N_T) [22]. Algorithm 1 outlines the PF-RM method of tracking with post-regularized resampling.

Algorithm 1 Particle Filter for Random Missing Measurements

$[\{\mathbf{x}_k^i, w_k^i\}_{i=1}^N] = \texttt{PF-RM}[\{\mathbf{x}_{k-1}^i, \mathbf{x}_k^i, w_{k-1}^i\}_{i=1}^N, p, z_k^m]$

- *for* $i = 1:N$
  - Draw $\mathbf{x}_k^i \sim P(\mathbf{x}_k|\mathbf{x}_{k-1}^i)$ using process dynamics (1)
  - Compute the importance weight w_K^i according to (10) and (12)
- *end for*
- Normalize the computed importance weights: $\{w_k^i\}_{i=1}^N = \{w_k^i\}_{i=1}^N / \sum_{i=1}^N w_k^i$
- Compute the effective sample size as $N_{eff} = 1/\sum_{i=1}^N (w_k^i)^2$
- Set the threshold (N_T) for effective sample size
- *if* $N_{eff} < N_T$
  - Resample as given on page 8 of [21] for post-RPF method:
    - $[\mathbf{x}_k^i, w_k^i\}_{i=1}^N] = \texttt{RESAMPLE}[\mathbf{x}_k^i, w_k^i\}_{i=1}^N]$
- *end if*

IV. Simulations

For solution of the underwater passive BOT problem with dynamics and measurement model given by (1) and (4), three filters: the CQKF, conventional PF and the PF-RM are applied. All three filters use the modified measurements $z_{1:k}^m$ which are generated using (3) and (4) for a given value of latency probability p. Resampling stage of the both particle filters follows the steps of post-RPF method [21]. Number of particles used for two filters is, $N = 10000$. Threshold for the effective sample size of particles is set as $N_T = N/4$ for the both particle filters. The process noise intensity $\bar{q} = 1.944 \times 10^{-6} km^2/min^3$ is used for every run of tracking process. Sampling time used in this simulation work is $T = 1$ *min*, and observation period lasts for 30 *min*.

A. Filter Initialization

All the filters are initialized with same set of estimates using the assumptions and methods given in [2]. Initial estimates for the position coordinates of relative state vector are determined using first bearing measurement and initial target range. If $\bar{r}$ is initial range estimate of the target, $\bar{r} \sim \mathbf{N}(r, \sigma_r^2)$, where r is the true initial range and $\mathbf{N}$ denotes normal distribution of given random variable. Initial bearing estimate is defined as $\theta_0 \sim \mathbf{N}(\theta, \sigma_\theta^2)$, where θ is true initial bearing. With prior target speed s, initial estimate for the target speed is used as $\bar{s} \sim \mathbf{N}(s, \sigma_s^2)$. Assuming the target is moving towards the observer, initial course estimate can be calculated as $\bar{c} = \theta_0 + \pi$. Table I lists the value of parameters used in this simulation. Now, the mean estimate of relative state vector $\hat{\mathbf{x}}$ can be initialized as

$$\hat{\mathbf{x}}_0 = \begin{bmatrix} \bar{r}\sin(\theta_0) \\ \bar{r}\cos(\theta_0) \\ \bar{s}\sin(\bar{c}) - \dot{x}_0^o \\ \bar{s}\cos(\bar{c}) - \dot{y}_0^o \end{bmatrix}, \quad (18)$$

where $(\dot{x}_0^o, \dot{y}_0^o)$ is the initial velocity for observer. The initial covariance matrix can be given as

$$\mathbf{P}_0 = \begin{bmatrix} \mathbf{P}_{xx} & \mathbf{P}_{xy} & 0 & 0 \\ \mathbf{P}_{yx} & \mathbf{P}_{yy} & 0 & 0 \\ 0 & 0 & \mathbf{P}_{\dot{x}\dot{x}} & \mathbf{P}_{\dot{x}\dot{y}} \\ 0 & 0 & \mathbf{P}_{\dot{y}\dot{x}} & \mathbf{P}_{\dot{y}\dot{y}} \end{bmatrix}, \quad (19)$$

where

$$\begin{aligned}
\mathbf{P}_{xx} &= \bar{r}^2\sigma_\theta^2\cos^2(\theta_0) + \sigma_r^2\sin^2(\theta_0) \\
\mathbf{P}_{yy} &= \bar{r}^2\sigma_\theta^2\sin^2(\theta_0) + \sigma_r^2\cos^2(\theta_0) \\
\mathbf{P}_{xy} &= \mathbf{P}_{yx} = (\sigma_r^2 - \bar{r}^2\sigma_\theta^2)\sin\theta_0\cos\theta_0 \\
\mathbf{P}_{\dot{x}\dot{x}} &= \bar{s}^2\sigma_c^2\cos^2\bar{c} + \sigma_s^2\sin^2\bar{c} \\
\mathbf{P}_{\dot{y}\dot{y}} &= \bar{s}^2\sigma_c^2\sin^2\bar{c} + \sigma_s^2\cos^2\bar{c} \\
\mathbf{P}_{\dot{x}\dot{y}} &= \mathbf{P}_{\dot{y}\dot{x}} = (\sigma_s^2 - \bar{s}^2\sigma_c^2)\sin\bar{c}\cos\bar{c} \ .
\end{aligned}$$

In order to start the tracking process for the particle filters, initial set of particles are drawn as $\{\mathbf{x}_0^i\}_{i=1}^N \sim \mathbf{N}(\hat{\mathbf{x}}_0, \mathbf{P}_0)$, where $\hat{\mathbf{x}}_0$ and $\mathbf{P}_0$ are defined by (18) and (19) respectively.

TABLE I

THE PARAMETERS USED IN THE TRACKING

Initial range (r)	5 km
Target speed (s)	4 knots
Target course	-140^o
Observer speed	5 knots
Observer initial course	140^o
Observer final course	20^o
Observer manoeuvre	From 13^{th} to 17^{th} min
σ_r	2 km
σ_s	2 knots
σ_θ	1.5^o
σ_c	$\pi/\sqrt{12}$

B. Performance Analysis

Once filters are initialized, Algorithm 1 is used to carry out the PF-RM method whereas, the conventional PF is implemented as the standard post-RPF discussed in [21]. CQKF with fourth-order quadrature $(n' = 4)$ points is implemented as given in [23]. As one of the performance metrics, the position error for a filter at k^{th} time step and during m^{th} Monte Carlo (MC) run, is defined as

$$pos\_err_{k,m} = \sqrt{(x^t_{k,m} - \hat{x}^t_{k,m})^2 + (y^t_{k,m} - \hat{y}^t_{k,m})^2},$$

where $(\hat{x}^t_{k,m}, \hat{y}^t_{k,m})$ is the estimated target position during m^{th} MC run. Similarly, RMSE of position at time step k is defined as $rmse_k = \sqrt{\frac{1}{M}\sum_{m=1}^{M}(pos\_err_{k,m})^2}$, where M is the total number of MC runs. A track is said to be divergent when estimates do not converge towards the true path. In this work, if position error at final step is more than 1 km (*i.e.* $pos\_err_{kmax} > 1$ km, where $kmax$ is the final time step), that track is considered to be divergent. Further, diverged tracks and corresponding MC runs are not considered while calculating the RMSE of position for a given total number of MC runs.

Since both particle filters are basically post-RPF, computational time for two filters is almost same. However, importance weights are computed differently and that results into distinct performance in terms of RMSE of position and consistency in tracking the true path. Target path estimated by PF-RM algorithm, considering latency probability to be $p = 0.30$, is plotted with true path in Fig. 2. It can be seen that after observer manoeuvre, estimated target path rapidly follows the true path and gradually coincides with it. For the same value of latency probability $(p = 0.30)$, RMSE of position and velocity for three filters are calculated over 500 MC runs and plotted in Fig. 3 and Fig. 4 respectively. From the plots, it is clear that RMSE curves for the PF-RM converge faster and are significantly lower than other filters. Table II analyses the performance of filters in terms of divergent tracks. It displays the track-loss counted over 500 MC runs with different values of latency probability.

From the observations in Table II, it is quite evident that the PF-RM performs with significantly more accuracy than the other two filters under the presence of random missing measurements. It should also be noted that in the absence

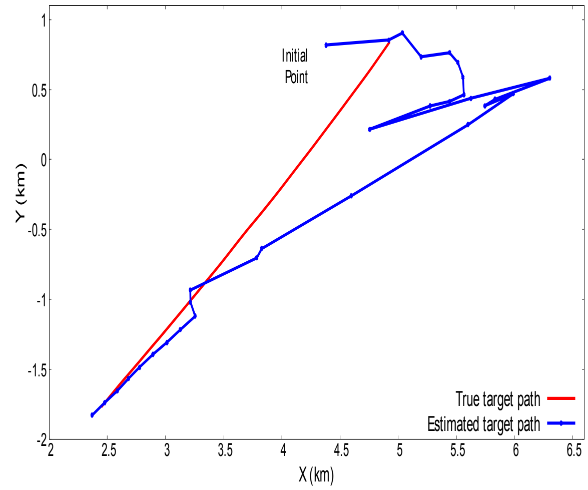

Fig. 2. Target path estimated by PF-RM algorithm considering $p = 0.30$.

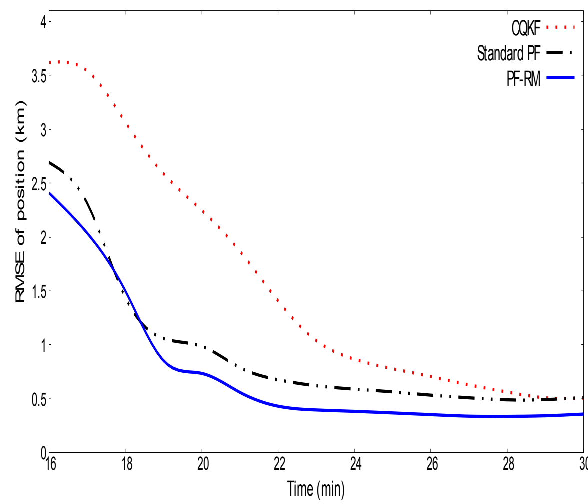

Fig. 3. RMSE of position for different filters after observer manouevre $(p = 0.30)$.

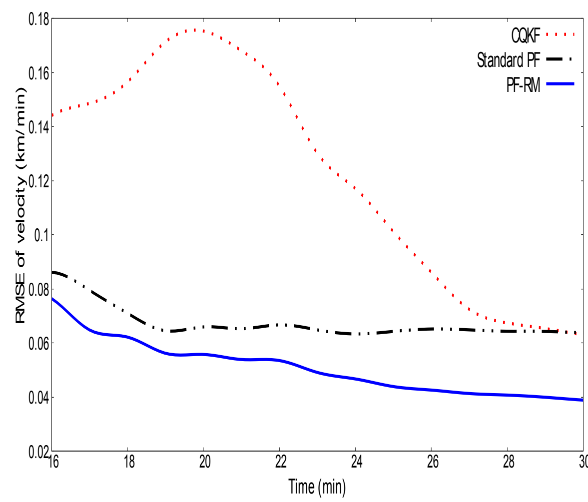

Fig. 4. RMSE of velocity for different filters after observer manouevre $(p = 0.30)$.

TABLE II
TRACK-LOSS VERSUS LATENCY PROBABILITY FOR DIFFERENT FILTERS

Latency probability (p)	Track-loss (%)		
	PF-RM	PF	CQKF
0.30	10	63	56
0.20	8	48	45
0.10	3	26	28
0.00	1	1	5

of missing measurements ($p = 0$), both the particle filters converge to the same count of track-loss. Fig. 5 shows the RMSE performance for all the filters with no missing measurements. It can be observed that RMSE of the PF-RM coincides with that of the standard PF as expected.

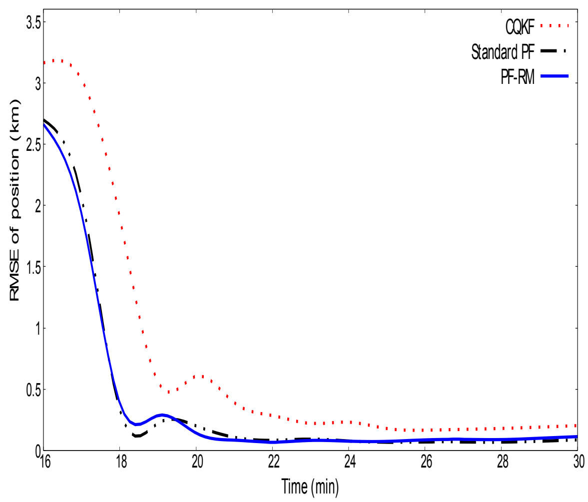

Fig. 5. RMSE of position for different filters after observer manoeuvre ($p = 0.00$)

V. CONCLUSIONS

The conventional filters lose its applicability in underwater passive BOT system if measurements are randomly missed. In this technical work, the particle filter is modified to deal with such scenarios. The paper formulates a modified measurement model which provides observations for the missing instances, and proposes a recursion equation of importance weight developed in accordance with the latency probability and replaced values of random missing measurements. Though the proposed PF algorithm is derived for a specific BOT problem, it can be used for a general system with random missing measurements assuming the knowledge of modified measurement model and latency probability.

To validate the performance of the proposed filter and showcase its superiority, one real-life underwater BOT problem is solved using the CQKF, conventional PF and the proposed PF-RM. Simulation results suggest that it is better to use the PF-RM if random missing in bearing measurements is likely.

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