2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Optimal motion planning for automated vehicles with scheduled arrivals at intersections

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Abstract— **We design and compare three different optimal control strategies for the motion planning of automated vehicles approaching an intersection with scheduled arrivals. The objective is to minimize a combination of energy consumption and deviation from the schedule. The strategies differ in allowed deviations. When taking only vehicles inside the control region into account, the strategy that achieves the lowest energy consumption is the less strict one, albeit at the expense of higher travel times. When traffic conditions beyond the control region are considered, no strategy is able to achieve lower energy consumption or vehicle delay than the strategy that is the most strict in keeping with the schedule. Results suggests that in high traffic situations, from a global energy consumption standpoint, it is best to have vehicles crossing the intersection as soon as possible.**

I. Introduction

Automated and connected vehicles enable new traffic control strategies which can improve traffic safety and efficiency. Several works on automated intersection management use optimal control strategies to assign control inputs (usually accelerations) for vehicles approaching an intersection in order to guarantee a safe crossing [1]–[6]. Typically, vehicles are assumed to be able to keep on their lanes while they travel forward, and maneuvers such as overtaking or lane changing are not allowed. This simplifies the problem as only longitudinal motion has to be considered.

A common feature of most of these strategies is having a term in the cost function that minimizes the summation (or the integral, in the continuous time case) of the squares of the accelerations for each vehicle at each time step, as a proxy for energy (fuel) consumption. Acceleration is usually easier to model and measure than fuel consumption, specially when considering heterogeneous vehicles, and minimizing acceleration tends to produce smooth trajectories. According to [7], under certain modeling assumptions there is a monotonic relation between acceleration and fuel consumption, leading to an almost equivalence in minimizing either of them.

Since minimizing acceleration alone could cause the optimal solution to involve vehicles taking an unreasonable time to cross the intersection, it is usual to either constrain the arrival time or include a term in the cost function that minimizes the deviation from a given desired speed. Problems of this type are termed motion planning problems.

There are various possible design decisions for modeling such motion planning problems. Vehicles may be allowed to cross the intersection in first-in-first-out order (FIFO) [1], follow some pre-established priority relation [3]–[6], [8], or the arrival order may be optimized online [2], [9], [10]. The latter case, although more general, typically leads to non-convex formulations and NP-hard problems [11].

Optimization can be performed in a sequential manner with each vehicle constrained by the decisions of the previous vehicles [1], [4]–[6]. One possibility is to use a simple intersection model so that the entire intersection is considered a conflict zone which can not be occupied simultaneously by vehicles in conflicting movements [1], [3]–[6]. Alternatively, elaborate models (considering conflict points or regions) allow for multiple vehicles to be inside the intersection area [2], [8], [10]. In this case, vehicles with conflicting movements are allowed to be inside the intersection at the same time, as long as they do not occupy the same conflict point simultaneously. Safety is usually guaranteed by enforcing minimum (time) headways or (spatial) gaps.

In [10] the authors proposed a strategy that decomposes the overall problem of coordinating vehicles at an intersection into four subproblems. Solving the first three problems produces an arrival schedule composed by a desired time and speed of arrival at the intersection for each vehicle. Vehicles are able to cross the intersection safely if they follow the arrival schedule. The fourth subproblem is a motion planning problem that consists in finding trajectories for vehicles that enable them to comply with the schedule and also guarantees no collisions while vehicles approach the intersection. The authors employed an heuristic method for solving the fourth subproblem.

In this work we consider the fourth subproblem of [10]. It consists of finding an optimal sequence of control inputs for each vehicle in a given set of vehicles approaching an intersection. Our main contributions are:

- Three different optimal control strategies for the motion planning problem are designed and compared in simulation. They differ mostly on how strict they are about keeping the schedule.
- In particular, one of these strategies is formulated in such a way that allows to model the state of a vehicle at an arbitrary scheduled time (not necessarily a multiple of the usual control interval), even though a discrete time formulation is used.

*The first author was funded by CAPES and CNPq and the third author was funded by CNPq. This work was partially supported by the Swedish Research Council under contract 2016-06079 and the Linnaeus Center ACCESS at KTH.

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- Simulation results show that for high traffic conditions, the strategy that is the most strict in complying with the schedule leads to both lowest delays and energy consumption. This suggests that when demand is high it is beneficial to allow vehicles to cross the intersection as soon as possible, even if this makes a vehicle individual energy expenditure high.

Section II describes the scope of the problem of motion planning of vehicles with scheduled arrivals in more detail. The designed control strategies are presented in Section III. Section IV shows simulation results, and Section V concludes this paper.

II. Problem Delimitation

We define a Control Region (CR) as an area composed by the intersection itself and its approaching lanes up to a given distance upstream of the intersection. We assume vehicles are highly automated [12], connected, and cooperative. While vehicles are inside the CR, they comply with commands received from an Intersection Controller (IC). Vehicles follow fixed paths inside the intersection area. Vehicle behavior outside the CR is beyond the scope of this work.

For each vehicle i in each approaching lane a in the CR, we assume that a feasible scheduled time $t_{a,i}$ and a desired speed $v^{\mathrm{d}}_{a,i}$ are already defined for arrival at the intersection (index a may be omitted in other variables when it is not relevant to distinguish between approaches). Vehicles are expected to comply reasonably well with this schedule (i.e., with deviations not larger than known bounds), which guarantees safety. The problem of obtaining such a schedule is not examined in this paper (see, e.g., [10]).

Vehicle position is measured as the distance d_i from the intersection. When the front bumper of vehicle i is touching the intersection entrance $d_i = 0$. We assume the speed profile of each vehicle inside the intersection is already defined (as such, we are only concerned with vehicle movement at the approaching lanes). Vehicles are assumed to respond instantly to a change in input (acceleration reference). We disregard lateral movement and communication delays, although the schedule may enforce headways large enough to accommodate known bounded uncertainties.

The motion planning problem in this paper consists of obtaining, for each vehicle i that enters the CR, a sequence of control inputs that allows it to comply (reasonably well) with the schedule, while also guaranteeing no collisions with the other vehicles approaching the intersection and minimizing energy consumption.

III. Control Strategies

We propose and investigate three control strategies for the motion planning problem described in Section II. All three formulations consist of discrete optimization problems. In each strategy we optimize a combination of energy consumption and adherence to the schedule.

The main difference between the formulations is how strict they are about keeping the schedule. The motivation for their design is to evaluate how relaxing the adherence to the schedule affects energy consumption. Our reasoning was that a strategy that has more freedom to deviate from the schedule has, as a consequence, more freedom to chose acceleration inputs. Thus, less strict strategies could possibly be able to better optimize fuel consumption, potentially at the cost of increased travel times. We evaluate this trade-off in simulation.

Strategy 1 (S1) employs a single time step with variable duration for each vehicle, which allows the modeling of the vehicle state at the exact scheduled time for arrival. This strategy is able to obtain trajectories that follow the schedule very closely. This formulation, however, does not allow the modeling of a flexible arrival time.

In Strategy 2 (S2) we forgo modeling the vehicle state at the exact scheduled time, instead adopting a fixed time step, and modeling a flexible arrival time. The scheduled arrival time for a vehicle translates into an arrival at a desired time interval. Vehicles are allowed to deviate from the scheduled arrival time by a given number of intervals. By employing a flexible arrival time, this formulation should have more "freedom" to optimize energy consumption. The downside is that the more flexible arrival times can be safe only if the schedule accounts for the possibility of such deviations, which ultimately means using larger headways when computing the schedule.

In Strategy 3 (S3) vehicles are allowed to arrive at any feasible time. Safety is guaranteed by scheduling vehicles to enter the intersection on a pre-established order and obeying safety headways. Vehicle trajectories are optimized sequentially, each vehicle using the trajectories of previous potentially conflicting vehicles as constraints. This formulation allows for much flexibility on vehicle arrival times, but can lead to a globally suboptimal solution and greater travel times.

Following, we present each strategy in detail.

A. Strategy 1 - Arrival at the scheduled time

Consider a set of n_a vehicles arriving at the intersection from approaching lane a. The time horizon of a vehicle i that just entered the CR is its scheduled arrival time at the intersection, $t_{a,i}$, assuming current time instant is $t = 0$. The horizon is divided in a number of time intervals $\tau_{i,k}$, with $k = 1, \ldots, n_i$. Each time interval has a duration given by a fixed time step T_{S}, with the exception of the last time interval, which is shorter in order to satisfy:

$$\sum_{k=1}^{n_i} \tau_{i,k} = t_{a,i} \qquad\qquad i = 1, \ldots, n_a. \tag{1}$$

For example, suppose $T_{\mathrm{S}} = 0.2$ and a given vehicle i has $t_{a,i} = 0.75$, i.e., $n_i = 4$. The time intervals for this vehicle would be $\tau_{i,1} = \tau_{i,2} = \tau_{i,3} = 0.2$ and $\tau_{i,4} = 0.15$. This allows modeling vehicles with different and arbitrary arrival times and also comparing the state of different vehicles inside the approach at any time interval, except the last interval for each vehicle. Safety at the last time interval is equivalent to safety upon arrival at the intersection, which is assumed to be guaranteed by the schedule.

We describe the state of vehicle i at time interval k by

- $d_{i,k}$, the distance from the front bumper to the nearest edge of the intersection at the end of the time interval;
- $v_{i,k}$, the instant speed at the end of the time interval; and
- $a_{i,k}$, the acceleration during the time interval.

These state variables should be consistent regarding vehicle dynamics:

$$\begin{aligned} & d_{i,k} = d_{i,k-1} - 0.5 \cdot (v_{i,k-1} + v_{i,k}) \cdot \tau_{i,k} && \text{(2a)} \\ & v_{i,k} - v_{i,k-1} = a_{i,k} \cdot \tau_{i,k} && \text{(2b)} \\ & v_{i,k} \leq v_i^{\max} && \text{(2c)} \\ & a_i^{\min} \leq a_{i,k} \leq a_i^{\max} && \text{(2d)} \\ & k = 2, \ldots, n_i; \ i = 1, \ldots, n_a \end{aligned}$$

with $v_i^{\max}$ an upper bound speed, and $a_i^{\min}$ and $a_i^{\max}$, lower and upper bound accelerations for vehicle i, respectively.

Initial conditions $d_{i,0}$ and $v_{i,0}$ are given by the current (known) state of each vehicle when the algorithm is executed.

Ideally, we would like to have $v_{i,n_i} = v_i^{\mathrm{d}}$ and $d_{i,n_i} = 0$, at time $t_{a,i}$. However, we assume a discretization with constant acceleration during each time interval that may render such solution unfeasible. We relax these requirements by setting ε^{v} and ε^{d} as the maximum allowed deviation for final speed and distance, respectively, obtaining the following bounds for the final conditions:

$$\begin{aligned} & -\varepsilon^{\mathrm{d}} \leq d_{i,n_i} \leq \varepsilon^{\mathrm{d}} && \text{(3a)} \\ & -\varepsilon^{\mathrm{v}} \leq v_{i,n_i} - v_i^{\mathrm{d}} \leq \varepsilon^{\mathrm{v}} && \text{(3b)} \\ & i = 1, \ldots, n_a. \end{aligned}$$

Let $\delta_i^{\min}$ be the minimum distance between vehicle i and vehicle $i+1$ immediately upstream, given by the sum of the vehicle length L_i and a minimum safety gap $g_{\min}$:

$$\delta_i^{\min} = L_i + g_{\min}. \tag{4}$$

We define a family of safety constraints in order to keep vehicles within a safe distance from each other as:

$$\begin{aligned} & d_{i,k} + \delta_i^{\min} \leq d_{i+1,k} && \text{(5)} \\ & k = 1, \ldots, n_i; \ i = 1, \ldots, n_a - 1. \end{aligned}$$

We define also a cost function

$$\begin{aligned} F_1 = \quad & W_{\mathrm{a}} \sum_{i=1}^{n_a} \sum_{k=1}^{n_i} (a_{i,k})^2 \frac{\tau_{i,k}}{t_{a,i}} + W_{\mathrm{d}} \sum_{i=1}^{n_a} (d_{i,n_i})^2 + \\ & W_{\mathrm{v}} \sum_{i=1}^{n_a} (v_{i,n_i} - v_i^{\mathrm{d}})^2 \end{aligned} \tag{6}$$

with W_{a}, W_{d} and W_{v} weights on the acceleration (energy), distance and speed components, respectively. The three terms of F_1 correspond, respectively, to the sums of the squared accelerations for each time interval (which approximates energy expenditure), the distance to the intersection at the last interval, and the deviation from the desired speed at the last interval. The acceleration term is normalized by $\frac{\tau_{i,k}}{t_{a,i}}$, i.e., according to the duration of each interval to obtain a value with a similar order of magnitude as the other terms, which facilitates tunning the weight coefficients.

Formally, S1 consists of minimizing F_1, subject to (2) – (5).

B. Strategy 2 - Arrival in the vicinity of the scheduled time

We develop S2 on top of S1. For S2, however, $\tau_{i,k} = T_{\mathrm{S}}$ for all k, i.e., every time interval has the same duration. We allow vehicles to arrive earlier or later than the scheduled arrival $t_{a,i}$. The deviation must be within α intervals of the time interval corresponding to $t_{a,i}$, which we denote k_i^{t}, the target interval. Because of this, we must model α additional time intervals exceeding k_i^{t}. Therefore, the number of intervals n_i considered for each vehicle i is given by

$$n_i = \left\lceil \frac{t_{a,i}}{T_{\mathrm{S}}} \right\rceil + \alpha. \tag{7}$$

Constraints (2) and (5) remain unchanged, exept for the redefinition of $\tau_{i,k}$ and n_i in this section. Final conditions, however, are reformulated. Instead of (3a) for vehicle position we have:

$$\begin{aligned} & d_{i,k_i^{\mathrm{t}}-\alpha-1} + \varepsilon^{\mathrm{d}} \geq 0 && \text{(8a)} \\ & d_{i,n_i} - \varepsilon^{\mathrm{d}} \leq 0 && \text{(8b)} \end{aligned}$$

which state that a vehicle must not have reached the intersection on the time interval just before the first allowed arrival interval, and must have reached the intersection by the last allowed interval. We say a vehicle has reached the intersection if its position is smaller than ε^{d}.

In addition, let $\beta_{i,k}$ be a binary variable associated with vehicle i and time interval k. If vehicle i has not reached the intersection by the end of interval k, then $\beta_{i,k} = 1$. Otherwise, $\beta_{i,k} = 0$. We model this with constraints

$$\begin{aligned} & d_{i,k} - \varepsilon^{\mathrm{d}} \leq \beta_{i,k} \cdot M_1 && \text{(9a)} \\ & d_{i,k} - \varepsilon^{\mathrm{d}} \geq (\beta_{i,k} - 1) \cdot M_1 && \text{(9b)} \\ & k = 1, \ldots, n_i; \ i = 1, \ldots, n_a. \end{aligned}$$

Then the following constraints for vehicle speed can be employed instead of (3b):

$$\begin{aligned} & v_{i,k} \leq v_i^{\mathrm{d}} + \varepsilon^{v} + (1 - \beta_{i,k-1} + \beta_{i,k}) \cdot M_2 && \text{(10a)} \\ & v_{i,k} \geq v_i^{\mathrm{d}} - \varepsilon^{v} - (1 - \beta_{i,k-1} + \beta_{i,k}) \cdot M_2 && \text{(10b)} \\ & k = 1, \ldots, n_i; \ i = 1, \ldots, n_a \end{aligned}$$

which guarantee vehicle speed is close to v_i^{d} at the end of the interval during which the vehicle enters the intersection. We define $\beta_{i,0} = 1$ and M_1 and M_2 are sufficiently large constants.

Let $t'_{a,i}$ be an approximated time of arrival of vehicle i at the intersection taken as the middle of the arrival time step:

$$t'_{a,i} = \Big[\sum_{k=1}^{n_i} \beta_{i,k} + 0.5 \Big] \cdot T_{\mathrm{S}}. \tag{11}$$

The cost function for S2 is then be defined as

$$F_2 = \quad W_{\mathrm{a}} \sum_{i=1}^{n_a} \sum_{k=1}^{n_i} (a_{i,k})^2 \frac{1}{n_i} + W_{\mathrm{t}} \sum_{i=1}^{n_a} (t'_{a,i} - t_{a,i})^2 +$$

$$W_{\mathrm{v}} \sum_{i=1}^{n_a} \sum_{k=n_i-2\cdot\alpha}^{n_i} (v_{i,k} - v_i^{\mathrm{d}})^2 \frac{1}{2\cdot\alpha+1} \tag{12}$$

with W_{t} the weight for the term related to the arrival time error. Coefficients $\frac{1}{n_i}$ and $\frac{1}{2\cdot\alpha+1}$ on the first and last terms, respectively, normalize the terms to similar orders of magnitude. The first term, similar to the one used in (6), approximates energy expenditure, the second term minimizes the squared deviation from the desired arrival time, and the last term minimizes deviation from v_i^{d} for each time step during which crossing the intersection is feasible.

Formally, S2 consists of minimizing F_2, subject to (2), (4), (5), (8 – 11).

C. Strategy 3 - Optimizing vehicles sequentially

We develop S3 on top of S2. For S3, however, instead of following the schedule, we guarantee safety by keeping vehicle arrival times sufficiently apart according to safety headways to be defined later, then we optimize vehicles sequentially in an arrival order. The states/inputs obtained for each vehicle as a result of the optimization process, as well as the safety headways, are used as constraints for subsequent vehicles. We assume it is not possible for vehicles to arrive safely earlier than the scheduled times $t_{a,i}$. We consider a sufficiently long control horizon so vehicles cam possibly arrive much later than originally scheduled, i.e., $n_i > \frac{t_{a,i}}{T_{\mathrm{S}}}$.

Vehicle position is constrained by (8b), but not (8a), guaranteeing vehicles reach the intersection by the end of the control horizon. Constraints (9) and (10) are used to model vehicle arrival interval and vehicle speed when reaching the intersection.

Consider the set A of lanes approaching the intersection. For each vehicle pair i, j from approaches a and b with potentially conflicting movements at the intersection, we call minimum headway $h_{a,i,b,j}$ the minimum time interval between them that enables entering the intersection safely, assuming they arrive with speed v_i^{d} and follow a known speed profile thereafter. If there is no potential conflict between a vehicle pair, we define $h_{a,i,b,j} = -\infty$.

We define $t_{a,i}^{\mathrm{low}}$ as a lower bound for the arrival time at the intersection

$$t_{a,i}^{\mathrm{low}} = \sum_{k=1}^{n_i} \beta_{i,k} \cdot T_{\mathrm{S}} \tag{13}$$

and $t_{a,i}^{\mathrm{high}}$ as an upper bound:

$$t_{a,i}^{\mathrm{high}} = t_{a,i}^{\mathrm{low}} + T_{\mathrm{S}}. \tag{14}$$

We denote $t_{a,i}^{\mathrm{safe}}$ the earliest possible time for a safe arrival of vehicle i, considering the known upper bounds for the arrival times of the previous vehicles and the minimum headways between them and the current vehicle:

$$\begin{aligned} &t_{a,i}^{\mathrm{safe}} = \max(t_j^{\mathrm{high}} + h_{b,j,a,i}) \\ &a \in A;\ b \in A;\ i = 1, \ldots, n_a;\ j \in P_i. \end{aligned} \tag{15}$$

with P_i the set of vehicles preceding vehicle i in the arrival order. Notice $t_{a,i}^{\mathrm{safe}} \geq t_{a,i}$, as $t_{a,i}^{\mathrm{safe}}$ takes into account the (possibly later than scheduled) arrival times of previous vehicles.

Collision avoidance between vehicles at the intersection is guaranteed by

$$t_{a,i}^{\mathrm{low}} \geq t_{a,i}^{\mathrm{safe}}, \qquad a \in A;\ i = 1, \ldots, n_a \tag{16}$$

Finally, we define the cost function for S3 as

$$\begin{aligned} F_3 = \quad & W_{\mathrm{a}} \sum_{i=1}^{n_a} \sum_{k=1}^{n_i} (a_{i,k})^2 \frac{1}{n_i} + \\ & W_{\mathrm{v}} \sum_{i=1}^{n_a} \sum_{k_i^{\mathrm{t}}}^{n_i} (v_{i,k} - v_i^{\mathrm{d}})^2 \frac{1}{n_i - k_i^{\mathrm{t}}} \end{aligned} \tag{17}$$

The difference between F_2 and F_3 is that in F_3 there is no term for minimizing arrival time deviation, and instead of minimizing the speed deviation on time intervals around interval k_i^{t}, we minimize the speed deviation for any interval $k \geq k_i^{\mathrm{t}}$. Penalizing a speed deviation on any time step after k_i^{t} induces earlier arrivals, since assuming the desired speed reasonably soon lowers F_3.

Formally, S3 consists of minimizing F_3, subject to (2), (4), (5), (8b), (9), (10), (13 – 16).

D. Implementation considerations

The three strategies produce sequences of vehicle states to be used as references over a given time horizon. These sequences are sent to the vehicles, which follow them accordingly until they reach the intersection. Once a vehicle has entered the intersection it ceases following this reference and instead follows a predefined speed profile for crossing the intersection. We assume there is a scheduling algorithm (SA) that produces $t_{a,i}$ and v^{d} for each vehicle, such that if vehicles arrive at the intersection within the defined bounds (constraints) and follow these crossing profiles, safety is guaranteed in the intersection. More specifically, the schedule produced by the SA must be such that any feasible solution for the motion planning problem using that schedule as an input must be absent of collisions. This means that, assuming a proper schedule, any feasible solution for the motion planning problem will be safe. It also follows that a SA must account for possible deviations in the schedule allowed by the motion planning strategy.

Consider an arbitrary SA that takes minimum headways between vehicles into account for defining a schedule. Since the exact instant of vehicle arrival at the intersection for S2 may be significantly different from $t_{a,i}$, the safety headways used by the SA must be large enough to guarantee safety with respect to this uncertainty. A vehicle may arrive at any time $T_{\mathrm{S}} \cdot (\alpha+1)$ before or after the scheduled arrival time. In the case of two consecutive vehicles (not necessarily on the same approach) arriving at the latest and earliest possible arrival times, respectively, this amounts to arrival times $2T_{\mathrm{S}} \cdot (\alpha+1)$ closer than expected. Therefore, headways used by the SA should be increased accordingly to guarantee safety.

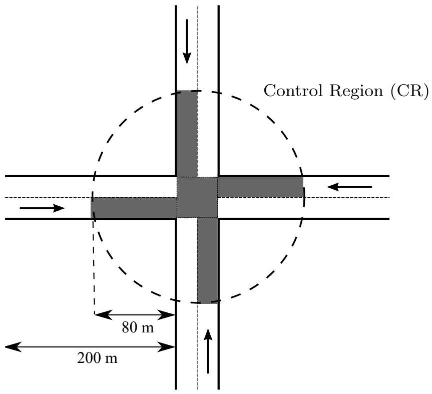

Fig. 1. Intersection layout

TABLE I
PROBLEM INSTANCE

a	i	turn	$a_i^{\min}$	$a_i^{\max}$	$v_{i,0}$	$d_{i,0}$	$t_{a,i}$ $h=$ 0.8	$t_{a,i}$ $h=$ 1.6	v_i^{d}
1	1	0	−4.7	3.0	8.3	3.4	0.4	0.4	8.3
1	2	0	−4.3	2.7	8.3	15.2	1.8	2.1	7.4
1	3	0	−3.9	3.0	7.7	27.4	3.3	4.0	8.2
1	4	0	−3.3	2.7	8.3	39.0	4.7	5.7	7.4
1	5	0	−4.0	3.3	8.3	50.6	6.2	7.6	8.3
1	6	1	−3.8	3.0	8.3	61.3	7.5	14.4	5.5
1	7	0	−4.7	2.5	8.3	78.8	8.4	10.2	7.0
2	1	0	−3.7	3.0	3.5	35.6	12.5	13.2	5.7
2	2	0	−3.2	3.2	6.4	45.3	5.0	5.0	8.3
2	3	−1	−4.1	2.5	8.3	78.1	8.9	9.3	4.2
3	1	0	−3.9	3.2	8.3	41.9	16.0	14.9	8.3
3	2	−1	−4.8	2.6	7.8	71.4	13.9	19.1	4.5

TABLE II
RESULTS OF SOLVING PI1 WITH S1 FOR DIFFERENT WEIGHTS

Scn.	W_{a}	W_{d} W_{v}	total energy ($\mathrm{m^2s^{-4}}$)	total arrival time (s)	avg. speed error (m/s)	avg. position error (m)
1	1	0	31.3	88.7	0.080	0.417
2	0	1	76.2	88.7	0.000	0.001
3	1	1	33.6	88.7	0.059	0.099

IV. SIMULATION

We perform two types of tests for the optimal control strategies: (I) Solving an arbitrary schedule, and (II) solving a series of successive schedules as vehicles enter the CR during traffic simulation. The Gurobi solver [13] is used for solving the optimal control problem in both cases. We use the Aimsun microscopic simulator [14] for the simulation experiments. The arbitrary schedule chosen is actually one problem instance encountered during traffic simulation.

A. Simulation setup

The simulated intersection consists of an intersection with four 200-m long, two-way roads with one lane in each direction, as shown in Fig. 1. The CR is composed by the intersection itself and its approaches up to 80 m from the intersection. The simulation time step is $T_{\mathrm{S}} = 0.2$ s. Demand is 400 veh/h/lane per approach, 60% of which travel in a straight direction through the intersection, and 20% turn to each side. Vehicle entrance times are sampled from an exponential distribution. Speed limit and $v_i^{\max}$ are both 30 km/h, $v_i^{\mathrm{d}} \in [25, 30]$ km/h for vehicles traveling straight, $v_i^{\mathrm{d}} \in [15, 25]$ km/h for turning vehicles, $a_i^{\min} \in [-5, -3]$ m/s^2, $a_i^{\max} \in [2.5, 3.5]$ m/s^2, L_i = 4 m, $g_{\min}$ = 0.5 m, ε^{d} = 0.5 m, ε^{v} = 0.1 m/s. Parameters $a_i^{\min}$, $a_i^{\max}$ and v_i^{d} are sampled from uniform distributions.

Both the scheduling and motion planning problems are solved repeatedly as new vehicles enter the CR. For S1 and S2, vehicles update their plans accordingly, unless they are closer than 40 m to the intersection (these vehicles are still taken into account as constraints for the problems). For S3, once a plan is found it is not changed.

The SA used guarantees that vehicles with potential conflicts are scheduled at least a certain time h apart. For S3, the control horizon is 30 s, and the arrival orders implied by the schedules produced by the the SA are used.

We use several metrics to compare scenarios. By total energy we mean the sum of the square accelerations for each vehicle at each interval, multiplied by the interval length. Total arrival time for experiments in which we only solve one schedule is the sum of the arrival time for each vehicle (assuming, for S2 and S3, arrival at the middle of the corresponding time interval). Arrival time error (only relevant for S2) is the absolute difference between the expected arrival time and $t_{a,i}$. Position error (only relevant for S1) is the absolute position on the last time interval. Speed error is the absolute deviation from v_i^{d} at the interval in which a vehicle reaches the intersection (which, by definition, is the last interval for S1).

Vehicle delay is the difference between the time a vehicle takes to cross the network and the time it would take to do so with free flow speed. Solution time is the time to solve one problem instance. Experiments were performed on a machine with a 2.4 GHz Intel i7-4700MQ CPU and 16 GB of RAM.

B. Solving one schedule

We define the turning direction of a vehicle as -1 if it turns left, 0 if it travels straight, and 1 if it turns right. Consider the two schedules represented in Table I by the the combination of v_i^{d} and either one of the two sets of $t_{a,i}$, one calculated by a SA enforcing at least $h = 0.8$ s between vehicles, and the other with $h = 1.6$ s. We call the problem of finding a motion plan for the first schedule as Problem Instance 1 (PI1), and for the second schedule as PI2.

Table II shows the results of solving PI1 with S1 for different weight combinations on F_1. As could be expected, Scenarios 1 and 2 show the best performance in the criteria being optimized, energy and deviation of the schedule, respectively. Scenario 3 consists of a compromise, with results fairly close to Scenario 1 in terms of energy.

Table III shows the results of solving PI1 with S2 with different values of α, and $W_{\mathrm{a}} = 1$ and $W_{\mathrm{t}} = W_{\mathrm{v}} = 0$ for all scenarios, (i.e, only energy is optimized). As α increases, energy goes down, and even the worse case has less

TABLE III

RESULTS FOR SOLVING PI1 WITH S2, VARYING α, AND $W_{\mathrm{a}} = 1$, $W_{\mathrm{t}} = W_{\mathrm{v}} = 0$

Scn.	α	total energy ($\mathrm{m}^2\mathrm{s}^{-4}$)	total arrival time (s)	avg. arrival time error (s)	avg. speed error (m/s)
4	0	26.3	88.8	0.225	0.008
5	1	24.2	88.4	0.187	0.041
6	2	22.4	88.0	0.270	0.069
7	3	21.0	87.8	0.370	0.090

TABLE IV

RESULTS OF SOLVING PI2 WITH S2 AND $W_{\mathrm{a}} = 1$, $W_{\mathrm{t}} = W_{\mathrm{v}} = 0$

Scn.	α	total energy ($\mathrm{m}^2\mathrm{s}^{-4}$)	total arrival time (s)	avg. arrival time error (s)	avg. speed error (m/s)
8	1	59.5	121.8	0.12	0.010

TABLE V

RESULTS FOR SOLVING PI1 WITH S3 AND $W_a = 1$, $W_v = 0$

Scn.	total energy ($\mathrm{m}^2\mathrm{s}^{-4}$)	total arrival time (s)	avg. speed error (m/s)
9	27.182	103.2	0.008

TABLE VI

COMPARISON OF S1, S2 AND S3 IN TRAFFIC SIMULATION.

Scn.	Strat.	α	total energy ($\mathrm{m}^2\mathrm{s}^{-4}$)	avg delay (s)	avg. speed (m/s)	avg. solution time (s)
1	S1	-	5030	1.2	8.1	< 0.01
2	S2	0	5184	2.2	7.9	< 0.01
3	S2	1	5730	3.6	7.7	0.01
4	S2	2	7424	6.3	7.4	0.02
5	S3	-	5504	3.4	7.8	0.18

energy expenditure than Scenario 1. This happens because the increase of the allowed margin for vehicle arrival time provides more freedom for the controller to minimize energy. Notice that even though vehicle arrival times go down as α increases these scenarios use the same schedule (and h), meaning safety is being compromised for larger values of α.

Consider that if α = 1, a vehicle may arrive up to 0.4 s earlier or later than scheduled. In the worst case of a late vehicle followed by an early one, they can arrive at the intersection up to 0.8 s closer than expected. As such, to meet the same safety criteria, minimum headways should be increased by at least 0.8 s for S2 in this case. Table IV shows the results for solving PI2 with S2 and $\alpha = 1$ and $W_{\mathrm{a}} = 1$ and $W_{\mathrm{t}} = W_{\mathrm{v}} = 0$, which is a fairer comparison to Scenario 1. Total arrival time increases significantly, since headways are higher. Energy also increases, mostly due to the fact that with higher headways most vehicles are scheduled to arrive later and need to decelerate/accelerate more.

Table V shows the results for solving SP1 with S3, with $W_{\mathrm{a}} = 1$ and $W_{\mathrm{v}} = 0$, and a control horizon of 30 seconds. Results are very similar when $W_{\mathrm{v}} = 1$ (omitted). Energy is lower compared to scenarios with S1, at the expense of higher total arrival time. Notice that both energy and arrival times are lower than when we solve PI2 with S2, indicating that S3 may be a better energy saving strategy than S2.

C. *Solving successive schedules*

The experiments in Section IV-B show that for a specific schedule, S3 achieves the best results with respect to energy consumption, but it does not achieve a total arrival time as low as S1. However, all three optimization strategies only consider vehicles inside the CR, which is a region fairly close to the intersection, and the state of vehicles beyond that is not taken into account. In this section, we perform traffic simulations and evaluate the state of vehicles in the entire network.

Table VI shows simulation results with all three strategies for the scenario described in Section IV-A. Energy expenditure takes into account the entire time a vehicle is in the network (not just the CR). The SA uses $h = 0.8$ s for Strategies 1 and 3, and $h = 0.8 + 2 \cdot (1 + \alpha)$ for Strategy 2. The SA minimizes vehicle arrival times, producing schedules that are very tight for a large portion of vehicles (several vehicles must arrive as soon as possible). In all scenarios $W_{\mathrm{a}} = W_{\mathrm{t}} = W_{\mathrm{d}} = W_{\mathrm{v}} = 1$. Queues did not exceed the simulated area. Simulation time was 30 minutes for each scenario, during which 785 motion planning problems were solved.

Scenario 1 has a much lower delay than any other. This was already expected, since S1 allows no flexibility on the arrival times. In scenarios with S2, the increase in α and, hence, in h is followed by an increase in vehicle delay.

Scenario 1 is also the best in terms of energy expenditure, even though scenarios 2–5 should have more freedom to optimize energy. The reason for this is that, by being very strict about complying with the schedule, S1 makes vehicles leave the CR as fast as possible, which decreases the possibility that a vehicle outside the CR (which is not taken into account by any strategy) has to slow down because of a vehicle in front of it. Since traffic demand is relatively high, an strategy which allows vehicles to exit the CR more rapidly is also the one that has lower global energy expenditure.

Table VII shows energy and delay results for S1 and S3 for different traffic demands. For lower demands (below 320 veh/h/lane) S3 fares better with respect to energy consumption, although S1 is always the best strategy for minimizing delay. This happens because as demand decreases, so does the likelihood of a vehicle outside the CR being delayed by the presence of a vehicle in front of it.

It is noteworthy that solving S3 takes significantly more time than the other strategies. Both S1 and S2 consist of one separate motion planning problem for each lane approaching the intersection, while S3 models all lanes in one problem instance. In S1 all constraints are linear, which makes it relatively easy (computationally) to solve. In S2 and S3, binary variables are used to model the time interval in which a vehicle arrives. In S2 the number of binary variables associated to each vehicle depends on the allowed deviation

TABLE VII

COMPARISON OF S1 AND S3 WITH VARYING DEMAND

	S1		S3	
Demand (veh/h/lane)	total energy (m^2s^{-4})	avg delay (s)	total energy (m^2s^{-4})	avg delay (s)
400	5174	1.2	5452	3.1
360	4471	0.9	4392	2.5
320	3765	0.9	3722	2.3
280	3204	0.6	2933	1.8
240	2937	0.6	2622	1.8
200	2542	0.6	2276	1.7

from the scheduled arrival time. When this increases, the problem becomes harder to solve. For small deviations, such as the ones used in this work, this effect is small, and S2 is not much harder than S1. In S3, vehicles may arrive much later than originally scheduled, and the number of binary variables associated to a vehicle is (mostly) limited by the control horizon. This larger number of binary variables is responsible for making the problem harder to solve than S1 and S2. The number of variables (both continuous and binary) is proportional to the number of vehicles for all three strategies. As such, solution time is expected to increase linearly to the number of vehicles.

V. CONCLUSION

Three different optimal control strategies were evaluated for solving a motion planning problem to guarantee the safe scheduled arrival of vehicles at an intersection.

The most flexible strategy (S3) with respect to arrival times provided the lowest energy consumption when solving a specific instance of the problem. However, when successive problems were solved as new vehicles approached the intersection, no strategy outperformed strategy S1 in a high traffic situation. This happens because clearing the control region as fast as possible ensures traffic conditions are better at the arriving lanes. This is beneficial for vehicles that have not yet entered the control region as it is less likely they have to decelerate when approaching, particularly if traffic demand is high.

These results suggest that, at least for high traffic demands and free flow downstream, S1 is the most suitable of the three for obtaining speed profiles for vehicles to approach the intersection, as it leads to both lowest arrival time and lowest energy expenditure. Taking into account traffic conditions outside the CR is important when evaluating these types of strategies, since vehicles upstream the CR may be impacted by the control strategy.

The results of this work will be used on ongoing research on intersection management alongside an algorithm for the problem of obtaining an optimal schedule.

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