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Fault-Tolerant Disturbance Observer Based Control for Altitude and Attitude Tracking of a Quadrotor

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Abstract— **This paper presents an attitude and altitude fault-tolerant tracking control for a quadrotor using the Disturbance Observer Based Control (DOBC). A Newton-Euler model of the quadrotor is described and the fault-tolerant control problem is treated for small angles. An appropriate estimation gain is proposed for the considered system, which allows to impose desired closed loop performances. These closed loop performances are analysed, showing the boundedness properties, and highlighting the trade off between fault estimation and control effort. Numerical simulations are provided for testing the effectiveness of the proposed DOBC solution, which is compared with the classical backstepping controller in case of multiple actuation faults.**

I. INTRODUCTION

The quadrotor system is nowadays one of the most employed Unmanned Aerial Vehicle (UAV), due to its usefulness in a wide range of operations; in fact, UAVs are cheap, versatile and light. A growing interest in UAVs has been shown among the research community, including industry, governments and academia [1]. In particular, they are essential in tasks where a pilot could be in danger, or in cases where the physical presence of a human is not possible. UAVs are already used in tasks like inspection of industrial plants or natural territories, traffic monitoring, recognition and surveillance, search and rescue operations in hostile environments [2], [3]. Due to their fast dynamics and underactuated behaviour, UAVs require fast and efficient control algorithms, especially because their stability is often affected by sudden command changes.

The primary problems when treating with the control of quadrotors are the model uncertainties, the external disturbances and the presence of faults, especially acting on actuators. For these reasons, nonlinear robust control techniques are widely investigated for this kind of systems, and the Disturbance Observer Based Control (DOBC) is one of the most promising [4]. The DOBC has been introduced in [5] and [6] for nonlinear affine Single-Input Single-Output (SISO) systems with external constant disturbances. During the years, the DOBC technique has been extended to different classes of systems and different classes of disturbances. In [7], disturbance has been modelled by an exogenous model, while disturbances with bounded variations are taken into account in [8]. The problem of Multi-Input Multi-Output (MIMO) systems has been dealt in [9], while the DOBC for MIMO systems with mismatched disturbances has been analysed in [10]. Due to nonlinearities and disturbances, which are present in different engineering applications, especially in the aerospace sector, the robustness of the DOBC technique is particularly important [11], [12], [13]. DOBC and, in general, active robust techniques are available in the literature on applications involving drones, with the aim of dealing with different problems. In this field, the main issue is the robust control under external disturbances [14] which has been achieved in the literature with the prediction of the uncertainty and the disturbances [15]. Another widely studied important issue is the time delay on drones, and in particular on quadrotors [16]: this is due to fact that the time delays affect the stability of these vehicles, which can significantly worsen their stability properties. Moreover, the actuator saturation is a common and unavoidable issue to take into account in every real system [17]. Since quadrotors are increasingly used in different applications, the path following [18] and obstacle avoidance need to be also considered [19]. Finally, reliability of UAVs against faults and failures is one of the most important objectives, widely studied in literature [1], [20].

This paper proposes a nonlinear fault-tolerant DOBC for a quadrotor, for which the attitude and altitude tracking problem is faced and solved for small angles. The proposed controller and estimator is based on the quadrotor model obtained with Newton-Euler method, as in [21] and [22]. Dealing with the DOBC, an estimation gain has to be found in order to satisfy some required assumption, which is typically the stability of the closed loop system. In this paper, an appropriate estimation gain is designed for the quadrotor vehicle, in order to impose the closed loop performances. In detail, the closed loop performances are proven, and the steady state error bounds are investigated. The provided analysis shows that there is a linear relation between the observer and the controller performances. In particular, it is shown as, for equal benefits, an observer with significant performances allows to design a more relaxed controller.

The paper is organized as follows. The mathematical model of a quadrotor is presented in Section II. Following the same approach presented in [6], the fault estimator is proposed in Section III. Afterwards, in Section IV, the composite controller is formulated. The numerical simulations are presented in Section V, where the proposed fault-tolerant technique is compared with the classical backstepping one. Finally, conclusions and future works are discussed in

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Section VI.

Notation. With $diag\{x_1, \ldots, x_n\}$, we denote the $n \times n$ square matrix where the elements $x_1, \ldots, x_n \in \mathbb{R}$ are placed in diagonal, maintaining the described order. If $v \in \mathbb{R}^n$, with $\|v\|$, we denote the Euclidean norm. Other norms will be correctly described when used. If $A \in \mathbb{R}^{n\times n}$ is a square matrix, with $\|A\|$, we denote the induced 2-norm of A. If $A \in \mathbb{R}^{n\times n}$ is a square matrix of order n with real eigenvalues, with $\lambda_{min}(A)$, we denote the minimum eigenvalue of A.

II. QUADROTOR MATHEMATICAL MODEL

Following [21] and [22], the attitude and altitude dynamics of a quadrotor, neglecting friction and supposing small angles, can be modelled as

$$\begin{aligned} I_x\ddot{\varphi} &= (I_y - I_z)\dot{\theta}\dot{\psi} + l(F_4 - F_2) \\ I_y\ddot{\theta} &= (I_z - I_x)\dot{\varphi}\dot{\psi} + l(F_3 - F_1) \\ I_z\ddot{\psi} &= (I_x - I_y)\dot{\varphi}\dot{\theta} + \frac{d}{c}(-F_1 + F_2 - F_3 + F_4) \\ m\ddot{h} &= -mg + \cos(\varphi)\cos(\theta)(F_1 + F_2 + F_3 + F_4) \end{aligned} \tag{1}$$

where φ, θ and ψ are pitch, roll and yaw angles, h is the z-coordinate of the centre of mass, F_1, F_2, F_3 and F_4 are the upward lift forces generated by the four propellers, I_x, I_y and I_z are the inertia moments along x, y and z body axes respectively, m is the quadrotor mass, g is the gravitational acceleration, l is the distance between the centre of the mass and the propellers, d is the drag coefficient and c is the thrust coefficient.

Defining $\xi_1 = (\varphi\,\theta\,\psi\,h)^T$ and $\xi_2 = (\dot{\varphi}\,\dot{\theta}\,\dot{\psi}\,\dot{h})^T$, model (1) can be rewritten in the following compact form

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= f(\xi_2) + G(\xi_1)BF \end{aligned} \tag{2}$$

with

$$F = (F_1\ \ F_2\ \ F_3\ \ F_4)^T, \tag{3}$$

$$f(\xi_2) = \begin{pmatrix} (I_y - I_z)\dot{\theta}\dot{\psi}/I_x \\ (I_z - I_x)\dot{\varphi}\dot{\psi}/I_y \\ (I_x - I_y)\dot{\varphi}\dot{\theta}/I_z \\ -g \end{pmatrix}, \tag{4}$$

$$G(\xi_1) = \begin{pmatrix} l/I_x & 0 & 0 & 0 \\ 0 & l/I_y & 0 & 0 \\ 0 & 0 & 1/I_z & 0 \\ 0 & 0 & 0 & \cos(\varphi)\cos(\theta)/m \end{pmatrix}, \tag{5}$$

$$B = \begin{pmatrix} 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ -d/c & d/c & -d/c & d/c \\ 1 & 1 & 1 & 1 \end{pmatrix}. \tag{6}$$

Since B is a constant and known matrix, we can define $\rho_B = ||B||$, which is a positive constant. In (1), $\xi_1, \xi_2 \in \mathbb{R}^4$ represent the state variables, while $F \in \mathbb{R}^4$ is the control input. For each motor, we consider a fault which can be expressed as a loss of the generated force. Hence, defining with ΔF_i $(i = 1, \ldots, 4)$ the loss of force for each motor due to a fault, and with $\Delta F = (\Delta F_1\ \Delta F_2\ \Delta F_3\ \Delta F_4)^T$ the fault vector, model (2) can be modified into

$$\begin{aligned} \dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= f(\xi_2) + G(\xi_1)B(F + \Delta F) \end{aligned} \tag{7}$$

Every loss of force ΔF_i $(i = 1, \ldots, 4)$ can also include external disturbances, like additional mass or wind.

III. FAULT OBSERVER

Using the DOBC framework, a fault observer can be implemented for system (7), which is essentially a disturbance observer. With respect to the general theory of MIMO DOBC, an appropriately chosen state transformation is given for the quadrotor. The closed loop performances are proven rigorously, where the boundedness is emphasized.

Let us assume that each loss of force (i.e. fault) has a bounded variation: this implies that no abrupt faults are considered in this paper. Moreover, let us assume that pitch and roll angles are limited within a given domain, which is reasonable in non-acrobatic flight mode. Hence, both these assumptions are physically reasonable, and can be mathematically modelled as follows.

Assumption 1. *$|\dot{\Delta F}_i| \le \rho_{\Delta F_i}$ for $i = 1, \ldots, 4$.*

Assumption 2. *$|\varphi| \le \pi/2$ and $|\theta| \le \pi/2$ for all t.*

From Assumption 1, it follows that we can define $\rho_{\Delta F}$ such that

$$||\dot{\Delta F}||_2 \le ||\dot{\Delta F}||_1 = \sum_{i=1}^{4} |\dot{\Delta F}_i| \le \sum_{i=1}^{4} \rho_{\Delta F_i} = \rho_{\Delta F} \tag{8}$$

From Assumption 2, it follows that $G(\xi_1)$ is invertible and $||G(\xi_1)||$ is upperbounded, namely

$$||G(\xi_1)|| \le max\{l/I_x, l/I_y, 1/I_z, 1/m\} = \rho_G \tag{9}$$

As usual for the DOBC, the estimator is based on a shifted system. Consider a continuously differentiable function $P : \mathbb{R}^4 \times \mathbb{R}^4 \to \mathbb{R}^4$ of the variables ξ_1 and ξ_2. We define the shifted new variable

$$z = \Delta F - P(\xi_1, \xi_2) \tag{10}$$

The design of $P(\xi_1, \xi_2)$ determines the dynamics of the disturbance estimator and hence the estimation error. Amongst the possible choices of $P(\xi_1, \xi_2)$, the following one results in a linear perturbed estimation error dynamics.

Proposition 1. *Consider the quadrotor model* (7) *under Assumptions 1 and 2. Let*

$$P(\xi_1, \xi_2) = AB^{-1} \begin{pmatrix} \frac{\dot{\varphi} I_x}{l} & \frac{\dot{\theta} I_y}{l} & \dot{\psi} I_z & \frac{m\dot{h}}{cos(\phi)cos(\theta)} \end{pmatrix}^T \tag{11}$$

where A is a symmetric positive definite matrix. Then, the fault observer

$$\begin{aligned}\dot{\hat{z}} = &-\frac{\partial P(\xi_1,\xi_2)}{\partial \xi_2}G(\xi_1)B\hat{z} - \frac{\partial P(\xi_1,\xi_2)}{\partial \xi_1}\xi_2 \\ &- \frac{\partial P(\xi_1,\xi_2)}{\partial \xi_2}\left[f(\xi_2) + G(\xi_1)B(F + P(\xi_1,\xi_2))\right]\end{aligned} \tag{12}$$

$$\widehat{\Delta F} = \hat{z} + P(\xi_1,\xi_2) \tag{13}$$

makes the estimation error ultimately bounded.

Proof. Differentiating (10) we obtain

$$\begin{aligned}\dot{z} = \dot{\Delta F} &- \frac{\partial P(\xi_1,\xi_2)}{\partial \xi_2}G(\xi_1)Bz - \frac{\partial P(\xi_1,\xi_2)}{\partial \xi_1}\xi_2 \\ &- \frac{\partial P(\xi_1,\xi_2)}{\partial \xi_2}\left[f(\xi_2) + G(\xi_1)B(F + P(\xi_1,\xi_2))\right]\end{aligned} \tag{14}$$

A state observer can be found in the form

$$\begin{aligned}\dot{\hat{z}} = &-\frac{\partial P(\xi_1,\xi_2)}{\partial \xi_2}G(\xi_1)B\hat{z} - \frac{\partial P(\xi_1,\xi_2)}{\partial \xi_1}\xi_2 \\ &- \frac{\partial P(\xi_1,\xi_2)}{\partial \xi_2}\left[f(\xi_2) + G(\xi_1)B(F + P(\xi_1,\xi_2))\right]\end{aligned} \tag{15}$$

where $\hat{z}$ is the state of the observer. Defining the estimation error $e_z = z - \hat{z}$, the error dynamics is

$$\dot{e}_z = -\frac{\partial P(\xi_1,\xi_2)}{\partial \xi_2}G(\xi_1)Be_z + \dot{\Delta F} \tag{16}$$

For the choice in (11), it follows

$$\partial P(\xi_1,\xi_2)/\partial \xi_2 = AB^{-1}G^{-1}(\xi_1) \tag{17}$$

hence the observation error can be rewritten as

$$\dot{e}_z = -Ae_z + \dot{\Delta F} \tag{18}$$

Choosing the Lyapunov function $V_z = (1/2)e_z^T e_z$, it follows that

$$\begin{aligned}\dot{V}_z &= e_z^T \dot{e}_z = -e_z^T A e_z + e_z^T \dot{\Delta F} \\ &\le -e_z^T A e_z + ||e_z||||\dot{\Delta F}|| \\ &\le -\rho_A e_z^T e_z + \frac{||e_z||^2 \rho_{\Delta F}^2}{4\epsilon} + \epsilon \\ &\le -\left(\rho_A - \frac{\rho_{\Delta F}^2}{4\epsilon}\right) e_z^T e_z + \epsilon\end{aligned} \tag{19}$$

where $\rho_A = \lambda_{min}(A)$ and $\epsilon > 0$ is an arbitrary constant given by the Young's inequality. If $\rho_A > \rho_{\Delta F}^2/(4\epsilon)$, the observation error is ultimately bounded. Defining the estimation error $e_\Delta = \Delta F - \widehat{\Delta F}$, it follows

$$\begin{aligned}e_\Delta = \Delta F - \widehat{\Delta F} &= (z + P(\xi_1,\xi_2)) - (\hat{z} + P(\xi_1,\xi_2)) \\ &= z - \hat{z} = e_z\end{aligned} \tag{20}$$

hence $\dot{e}_\Delta = \dot{e}_z$. Trivially, defining the Lyapunov function $V_\Delta = (1/2)e_\Delta^T e_\Delta$, it follows

$$\begin{aligned}\dot{V}_\Delta &= e_\Delta^T \dot{e}_\Delta = e_z^T \dot{e}_z \le -\left(\rho_A - \frac{\rho_{\Delta F}^2}{4\epsilon}\right) e_z^T e_z + \epsilon \\ &= -\left(\rho_A - \frac{\rho_{\Delta F}^2}{4\epsilon}\right) e_\Delta^T e_\Delta + \epsilon\end{aligned} \tag{21}$$

If $\rho_A > \rho_{\Delta F}^2/(4\epsilon)$, the estimation error is ultimately bounded and the convergence is at least exponential. Since ϵ is a free positive constant, for every $\rho_A > 0$ there exists an $\bar{\epsilon} > 0$ such that $\rho_A > \rho_{\Delta F}^2/(4\bar{\epsilon})$. ■

Remark 1. *The matrix A governs the closed loop estimation error, as in* (18)*. Thus, with a proper choice of A (and hence of $P(\xi_1,\xi_2)$), it is possible to impose proper closed loop estimation performances. If $\dot{\Delta F} = 0$, which means a constant loss of thrust, the error closed loop dynamics is totally governed by the matrix A.*

Remark 2. *Given a positive definite matrix A, which defines $\rho_A > 0$, and given $\epsilon > 0$ such that $\rho_A > \rho_{\Delta F}^2/(4\epsilon)$, then the estimation error e_Δ is bounded in 2-norm by*

$$B_\epsilon = \sqrt{\frac{\epsilon}{\rho_A - \rho_{\Delta F}^2/(4\epsilon)}} = \frac{2\epsilon}{\sqrt{4\epsilon\rho_A - \rho_{\Delta F}^2}} \tag{22}$$

Since the bound is valid for each $\epsilon \in D_\epsilon = (\rho_{\Delta F}^2/4\rho_A, +\infty)$, it is valid for

$$\epsilon_{min} = \underset{\epsilon \in D_\epsilon}{argmin}\, B_\epsilon = \frac{\rho_{\Delta F}^2}{2\rho_A} \tag{23}$$

Thus, the minimum granted bound is $B_{min} = \rho_{\Delta F}/\rho_A$. Therefore, the Proposition 1 establishes a sufficient condition for the boundedness of the estimation error, and B_{min} is an upperbound for the estimation error.

Remark 3. *From* (21) *and* (23)*, it follows that the transient performance can be upperbounded by*

$$||e_\Delta||^2 \le \left(||e(t_0)||^2 - \frac{\rho_{\Delta F}^2}{\rho_A^2}\right) e^{-\rho_A(t-t_0)} + \frac{\rho_{\Delta F}^2}{\rho_A^2} \tag{24}$$

IV. COMPOSITE CONTROLLER

With the fault observer (12), (13), a composite control law can be designed with a backstepping approach, in order to obtain bounded tracking errors. Since the observer dynamics can be consistently faster than the control law dynamics, the analysis will be made for a steady state observer, i.e., bounded estimation error. This condition permits to highlight an important trade off between observer and controller performances.

Proposition 2. *Consider the quadrotor model* (7) *under Assumptions 1, 2, and the fault observer* (12), (13)*. If $||e_\Delta|| \le \rho_{\Delta F}/\rho_A$, then, the backstepping control law*

$$\begin{aligned}F = -\widehat{\Delta F} + B^{-1}G^{-1}(\xi_1)\Big[&-f(\xi_2) + \ddot{\xi}_{1d} \\ &-K_1(S_2 - K_1S_1) - S_1 - K_2S_2\Big]\end{aligned} \tag{25}$$

where $K_1, K_2 \in \mathbb{R}^{4\times4}$ are symmetric positive definite matrices, $\ddot{\xi}_{1d}$ is a C^2 reference for ξ_1, $S_1 = \xi_1 - \xi_{1d}$ and

$$S_2 = \xi_2 - \dot{\xi}_{1d} + K_1S_1 \tag{26}$$

makes the tracking error ultimately bounded.

Proof. Differentiating S_1 we have

$$\dot{S}_1 = \dot{\xi}_1 - \dot{\xi}_{1d} = \xi_2 - \dot{\xi}_{1d} = S_2 - K_1S_1 \tag{27}$$

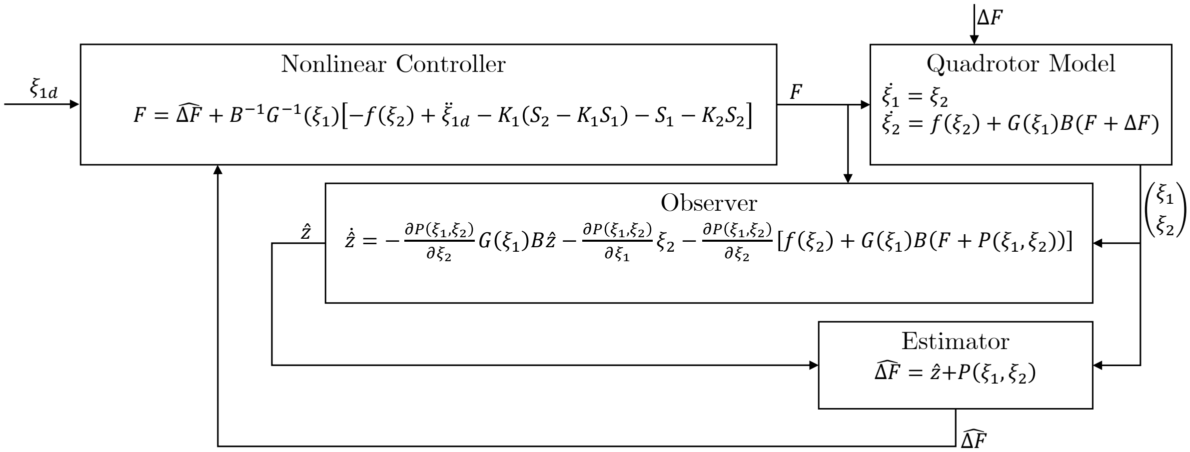

Fig. 1. The proposed fault-tolerant control scheme.

By choosing the Lyapunov function

$$V = (1/2)S_1^T S_1 + (1/2)S_2^T S_2 \tag{28}$$

we have

$$\begin{aligned}\dot V &= S_1^T \dot S_1 + S_2^T \dot S_2\\ &= S_1^T(S_2 - K_1S_1) + S_2^T\left\{f(\xi_2) + G(\xi_1)B\left[F + \widehat{\Delta F} + e_\Delta\right]\right.\\ &\quad \left. -\ddot\xi_{1d} + K_1(S_2 - K_1S_1)\right\}\end{aligned} \tag{29}$$

By choosing F as in (25), it follows that

$$\begin{aligned}\dot V &= -S_1^T K_1 S_1 - S_2^T K_2 S_2 - S_2^T G(\xi_1) B e_\Delta\\ &\le -S_1^T K_1 S_1 - S_2^T K_2 S_2 + \|S_2\| \, \|G(\xi_1)\| \, \|B\| \, \|e_\Delta\|\\ &\le -S_1^T K_1 S_1 - S_2^T K_2 S_2 + \|S_2\| \, \rho_G \rho_B \frac{\rho_{\Delta F}}{\rho_A}\\ &\le -\rho_{K_1} S_1^T S_1 - \rho_{K_2} S_2^T S_2 + \frac{\|S_2\|^2 \rho_G^2 \rho_B^2 \rho_{\Delta F}^2}{4\rho_A^2 \epsilon_c} + \epsilon_c\\ &= -\rho_{K_1} S_1^T S_1 - \left(\rho_{K_2} - \frac{\rho_G^2 \rho_B^2 \rho_{\Delta F}^2}{4\rho_A^2 \epsilon_c}\right) S_2^T S_2 + \epsilon_c\end{aligned} \tag{30}$$

where $\epsilon_c > 0$ is a free constant given by the Young's inequality, $\rho_{K_1} = \lambda_{min}(K_1)$ and $\rho_{K_2} = \lambda_{min}(K_2)$. For every symmetric positive definite K_1, there is $\rho_{K_1} > 0$. If

$$\rho_{K_2} - \rho_G^2\rho_B^2\rho_{\Delta F}^2/(4\rho_A^2\epsilon_c) > 0 \tag{31}$$

we have

$$\dot V \le -\alpha_c\left(S_1^T S_1 + S_2^T S_2\right) + \epsilon_c \tag{32}$$

with $\alpha_c = min\{\rho_{K_1}, \rho_{K_2} - \rho_G^2\rho_B^2\rho_{\Delta F}^2/(4\rho_A^2\epsilon_c)\} > 0$, hence the tracking error is ultimately bounded. Finally, since ϵ_c is a free parameter, for every $\rho_{K_2} > 0$ there exist an $\bar\epsilon_c > 0$ such that $\rho_{K_2} - \rho_G^2\rho_B^2\rho_{\Delta F}^2/(4\rho_A^2\bar\epsilon_c) > 0$. ∎

Remark 4. *Let us consider the condition $\rho_{K_2} - \rho_G^2\rho_B^2\rho_{\Delta F}^2/(4\rho_A^2\epsilon_c) > 0$. It is clearly visible that there is a trade off between the estimator and controller performances. Indeed, a well designed estimator (i.e. $\rho_A >> 1$), imposes a relaxed constraint on ρ_{K_2}, and it can improve the overall closed loop performances. In the limit case of $\rho_A \to \infty$, it is sufficient that $\rho_{K_2} > 0$. Finally, a reduced control effort can avoid the actuator saturation problem.*

Remark 5. *In Proposition 2 we analysed the overall closed loop performances with a steady state estimator, since the estimator dynamics can be designed to be significantly faster than the controller dynamics, which is tied to the system dynamics. Moreover, it is plausible to consider the system to initially operate in a fault-free condition, hence the estimation error starts inside the bound.*

The complete control scheme is reported in Fig. 1.

TABLE I
QUADROTOR PARAMETERS ACCORDING WITH MODEL (1)

Parameter	Value	Measurement Unit
m	2	kg
l	0.2	m
I_x	1.2416	$kg \cdot m^2$
I_y	1.2416	$kg \cdot m^2$
I_z	1.2416	$kg \cdot m^2$
d	$7.5 \cdot 10^{-7}$	$N \cdot m \cdot s^2$
c	$3.13 \cdot 10^{-5}$	$N \cdot s^2$
g	9.81	m/s^2

V. NUMERICAL SIMULATIONS

In order to check its performances, the proposed control method has been tested in different faulty simulation scenarios. The quadrotor parameters used for the simulation are reported in Table I. The fault-tolerant DOBC controller is compared with the well known integrator backstepping controller, using the same K_1 and K_2. In particular, for both controllers $K_1 = diag\{2, 2, 0.1, 2\}$ and $K_2 = diag\{2, 2, 0.1, 2\}$ are used. In the DOBC case, a matrix $A = diag\{10, 10, 10, 10\}$ is chosen. The lumped disturbances and their estimations for the fault-tolerant DOBC technique are reported in Fig. 2. The lumped disturbance $\widehat{\Delta F}_1$ is a fluctuating disturbance starting after $8s$, while the faults $\widehat{\Delta F}_2$ and $\widehat{\Delta F}_3$ occur at

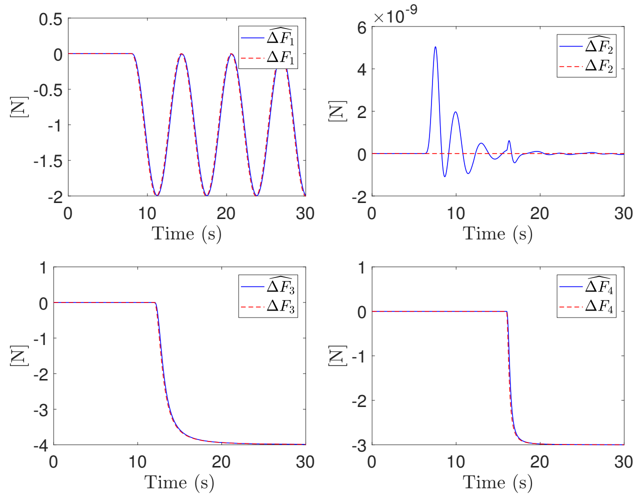

Fig. 2. Estimator performances.

$12s$ and $16s$, respectively. The amplitude of the lumped disturbances is approximately up to 40% of the actual control efforts. Note that, since ΔF mainly represents a loss of force caused by actuators faults, each component assumes negative sign. The tracking variables' references and the closed loop performances are reported for both techniques in Fig. 3, where it is shown that there are no differences between the backstepping and the DOBC techniques until at least one ΔF_i $(i = 1, \ldots, 4)$ acts. With the passing of time and with the increasing of the fault effects, there is a significant difference between the considered controllers. In particular, while the DOBC technique is able to maintain the tracking references, the backstepping one shows increasing errors, especially on the altitude component. The control efforts for both techniques are reported in Fig. 4, where it is possible to see that, although there are no straightforward differences, there is an increasing discrepancy between the imposed control force in each component.

VI. CONCLUSIONS

In this work, we present a fault-tolerant DOBC for a quadrotor vehicle, considering the attitude and altitude tracking problem for small angles. A suitable design of the estimation gain $P(\xi_1, \xi_2)$ is developed, and the closed loop performances are analysed and compared with the well known backstepping controller. Some future works are possible, starting with extending the control model by removing the small angle approximation and adding non-matching lumped disturbances. Moreover, the composite controller can be expanded in order to take into account the transient dynamics of the observer. Finally, the controller could be tested on a real quadrotor.

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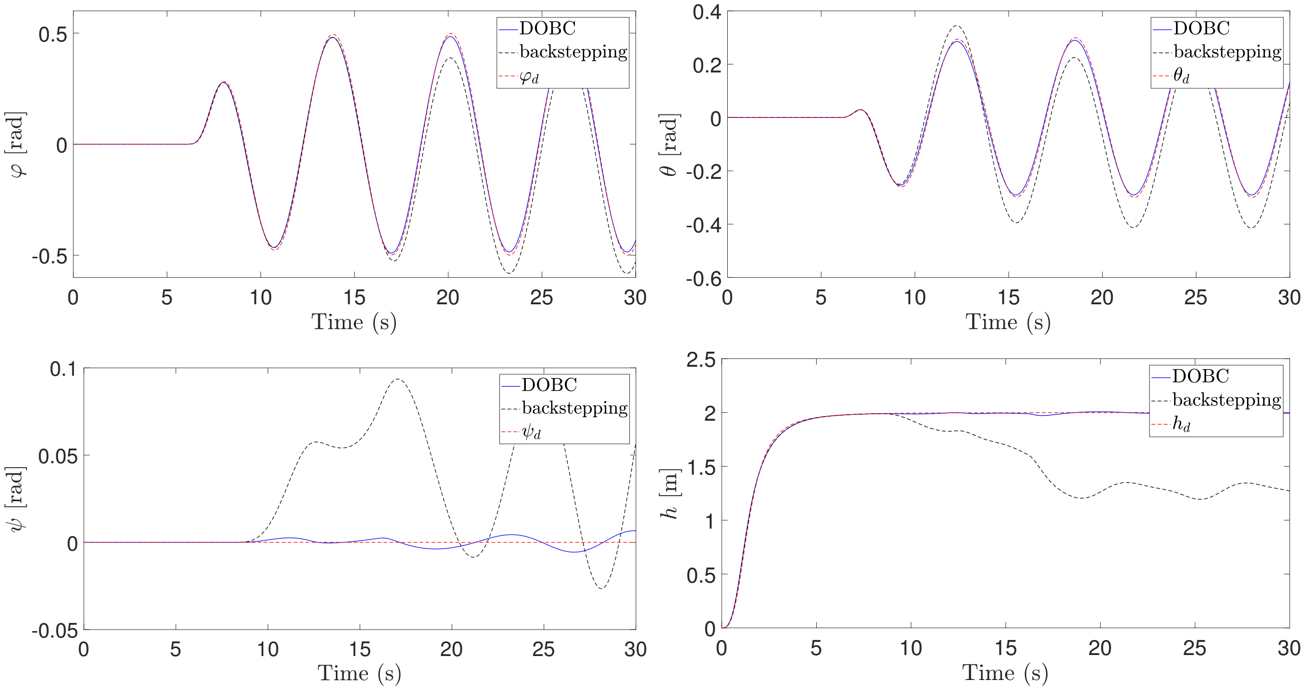

Fig. 3. Attitude and altitude variables.

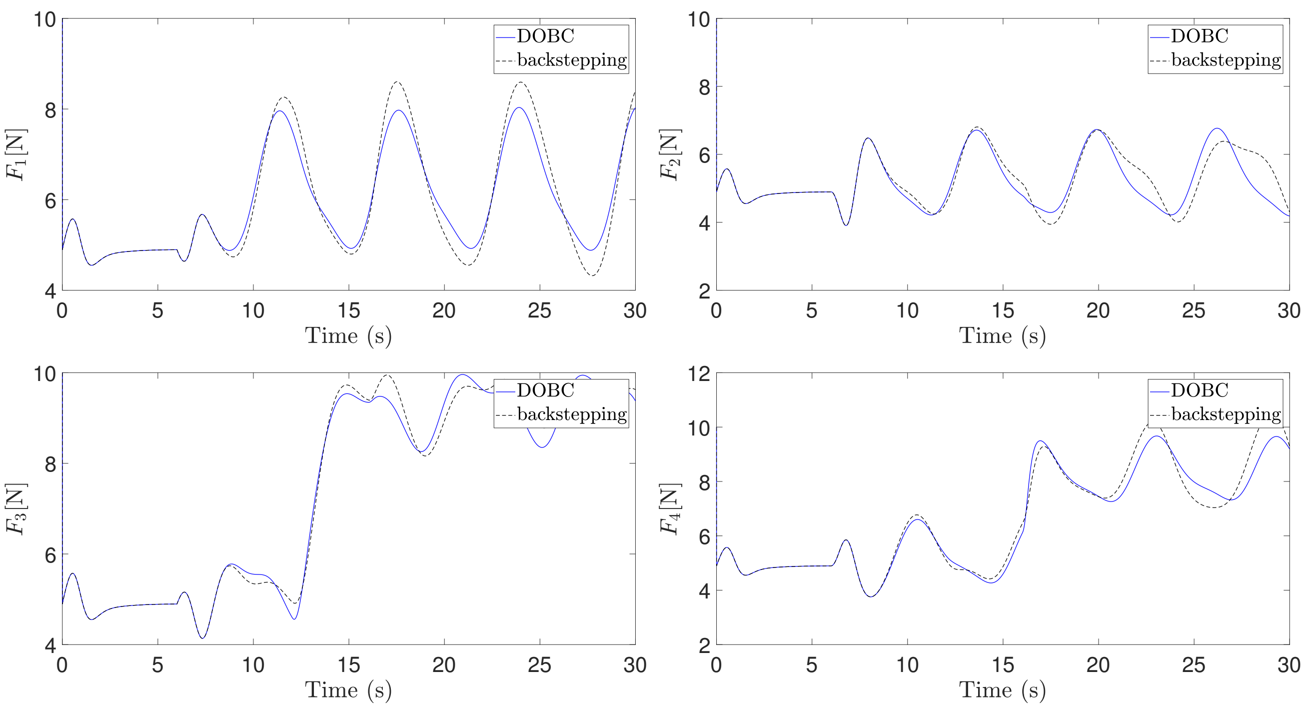

Fig. 4. Control variables.