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# Admissible Control Parametrization of Uncertain Finite-time Processes With Application to Li-ion Battery Management

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***Abstract*— Many control problems, such as charging of batteries or operation of (bio-)chemical processes, are repetitively operated finite-time processes. Such processes are often subject to disturbances and the controllers used depend on parameters for tuning. Guaranteeing safe and reliable operation that achieves the desired outcome in finite time despite uncertainties is a challenging task. We propose a set-based procedure based on a reformulation as a feasibility to identify admissible controller parameters, acceptable uncertainties and disturbances. The approach allows to directly consider bounded uncertainty, nonlinear system dynamics, as well as safety and performance requirements. It provides an outer approximation of the set of admissible controller parameters, for which the desired performance can be guaranteed. We illustrate the approach with a simulation example from battery management considering the problem of safe charging of an off-the shelf Li-ion battery.**

***Index Terms*— controller verification, batch processes, set based, constraint satisfaction, uncertainty, battery management, battery charging**

## I. Introduction

In many control tasks it is desired to achieve a certain performance in finite time while satisfying safety constraints process requirements [1]. Examples are the batch operation of chemical processes subject to temperature constraints, robotic manipulation tasks with a predefined precision, the charging of batteries while avoiding over temperature, or setpoint changes of engines subject to emission constraints.

The question arises how performance and constraint satisfaction can be guaranteed taking constraints and disturbances into account and for which controller parameters, initial conditions and uncertainties this can be guaranteed. This task is often complicated by the fact that the process or the controller is nonlinear.

One way to validate the performance and identify feasible control parameters, disturbances and initial conditions is the use of Monte Carlo methods. Monte Carlo methods are based on performing a large number of simulations for different disturbance scenarios, controller parameters and initial conditions. Various results with respect to Monte Carlo methods for the identification of admissible controller parameters and initial conditions exist, see for example [2]–[8]. The challenge of such approaches is that it is difficult to guarantee the satisfaction of constraints and performance for all relevant uncertainties and initial conditions.

One way to identify admissible sets of parameters, initial conditions and feasible uncertainties such that performance bounds and constraint satisfaction can be guaranteed is the use of set-based approaches. Set-based approaches provide sets of possible solutions, such as parameters, instead of a single value or a probability distribution [9], [10]. Only those realizations that respect all constraints and achieve the desired performance are considered feasible.

Set-based approaches can be based on interval arithmetic [11] or viability theory [12]. We focus on a reformulation of the problem as a feasibility problem, which basically certifies if the regions of the variable space do or do not contain valid solutions [13], [14].

In this work we outline how set-based feasibility methods can be used to identify admissible controller parametrizations for finite-time processes subject to uncertainties. The aim is to identify a sets of controller parameters which can keep the system in a safe region while achieving the desired objectives. We illustrate the approach considering a specific charging strategy of Li-ion batteries, the so called constant current constant voltage charging (CCCV). The charging strategy is the predefined control strategy, for which suitable control parameters, such as the maximum admissible current, need to be identified. Due to manufacturing imperfections and due to the so called battery aging, the physical and thus the electrical parameters vary. Such uncertainty urges the need for methods, which can provide guarantees over the uncertainty range. The particular control objective is to guarantee that a certain minimum charging state is reached in a given finite time, while satisfying constraints on the maximum temperature of the battery and staying below a maximum voltage. The example considers the model of an off-the-shelf battery.

The remainder of the paper is organized as follows. In Section 2 we present the problem formulation of control of uncertain finite-time processes and illustrate it considering two application scenarios. In Section 3 we describe the employed feasibility formulation and the used set-based approximation method. Section 4 illustrates the approach through an example, the charging of a Li-ion battery before we conclude with a brief discussion. Finally, in Section 5 we conclude with a summary and present our current fields of interest.

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## II. Problem setup

We consider discrete-time dynamical systems of the following form[1]:

$$\begin{aligned} x(k_{i+1}) &= f(x(k_i), u(k_i), p), \\ y(k_i) &= g(x(k_i), p), \end{aligned} \tag{1}$$

where $x(k_i) \in \mathcal{X}_i \subset \mathbb{R}^{n_x}$ are the states of the system, $u(k_i) \in \mathcal{U}_i \subset \mathbb{R}^{n_u}$ are the controlled inputs, $p \in \mathcal{P} \subset \mathbb{R}^{n_p}$ are the system parameters and $y(k_i) \in \mathcal{Y}_i \subset \mathbb{R}^{n_y}$ are the system outputs. The variables $n_x$, $n_u$, $n_p$, and $n_y$ correspond to the dimensions of the states, inputs, parameters, and outputs of the system. We assume that the control law $c$ is given by:

$$u(k_{i+1}) = c(u(k_i), y(k_i), q, r(k_i)), \tag{2}$$

where $q \in \mathcal{Q} \subset \mathbb{R}^{n_q}$ are the controller parameters and $r(k_i) \in \mathcal{R}_i \in \mathbb{R}$ correspond to reference values, which need to be achieved within a finite time.

Note that all variables in (1) and 2 can in principle be uncertain at all time instances. This leads to ranges - sets of values. The set of initial conditions $\mathcal{X}_0$ are denoted by

$$x(k_0) \subseteq \mathcal{X}_0 \subset \mathbb{R}^{n_x}. \tag{3}$$

We denote with $k_0$ the time instance the process begins and with $k_n$ the time instance it ends. Between $k_0$ and $k_n$ we define the set $\mathcal{T}$ as the collection of all time instances $k_i$, *i.e.*

$$k_i \in \mathcal{T} = \{k_0, k_1, .., k_n\}, n \in \mathbb{N}.$$

The time instances $k_i \in \mathcal{T}$ are not necessarily equidistant, i.e it is possible that $k_{i-1} - k_i \neq k_i - k_{i+1}$ for $1 \leq i \leq n$. This is the case for processes that acquire measurement data or control the process asynchronously.

The controlled system is required to respect constraints, which we represent in the following generalized form:

$$l_j(x(k_i), u(k_i), y(k_i), b(k_i), k_i) \geq 0. \tag{4}$$

Here $j \in \mathbb{N}$ is the number of constraints and $b(k_i) \in \{0, 1\}$ are binary variables which allow to express qualitative constraints. Note that this formulation allows the consideration of quantitative constraints, i.e. constraints that need to be satisfied at a pre-specified time (interval) or fixed bounds. Moreover, qualitative or semi-quantitative constraints are possible, which for example allow to consider changes in variables at arbitrary, unknown times, such as setpoint changes. For an in depth discussion on quantitative and qualitative constraints see [15], [16].

We consider processes, that need to satisfy production and quality requirements, which can for example be given in the form of a terminal set constraint

$$y(k_n) \in \mathcal{Y}_d \subset \mathbb{R}^{n_y}. \tag{5}$$

Without loss of generality, such constraints can be formulated in the form (4). Finding generally valid controller parametrizations considering the uncertainties is in general challenging. Our motivation is to determine the admissible ranges of controller parameters as well as the admissible set of initial conditions. This leads to the following overall problem.

*Problem 1: (Controller parameter validation)* Given the process (1) with the control law (2) and initial conditions (3) determine the admissible controller parameters $q \in \mathbf{Q}$ such that the constraints (4) and (5) are satisfied.

To illustrate the problem setup consider the following two application scenarios.

*Example 1 - Battery management.* Charging of batteries is an important task, which majorly influences the life time and safe operation of battery powered devices, such as cars. During the charging process, voltage is applied to the battery until a certain charging level is reached in a finite time. As an example consider the charging of a Li-ion battery with the so called CCCV charging strategy, as depicted in Fig. 1. During the first stage of this control strategy, the battery is being charged with constant current. In the second stage, the current is decreased keeping the voltage at a constant level, until the desired charging state is reached. For safe operation it is important to find the maximum current, such that constraints are respected, see Fig. 2. Basically the applied current must be sufficient to reach a desired state of charge while avoiding damage to the battery due to overpotential and overheating.

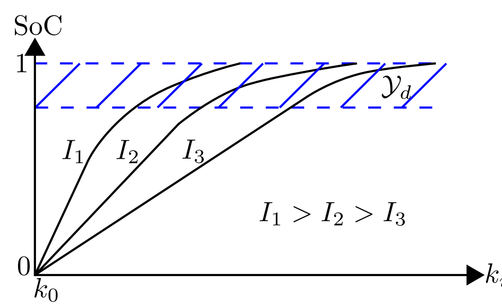


Fig. 1. State of charge (SoC) for three different maximum current values $I_1$, $I_2$, and $I_3$ for a constant current constant voltage charging cycle.

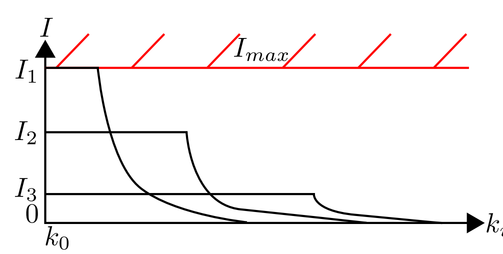


Fig. 2. Current profiles for CCCV charging. The switching from constant current to constant voltage occurs when the battery voltage reaches a predefined maximum.

*Example 2 - Batch reactors* form the production base for the chemical and bio tech industry. Typically, these processes are controlled via simple control scheme which manipulates temperature, pressure or the reactant feed. As

[1]The discrete-time formulation might be given by a discretization of a continuous time system or implicit integration.

desired outcome a certain quality or yield at the end of the batch in a finite time needs to be achieved. Typically batch reactors follow a given recipe, such as a temperate or batch profile, which needs to be tracked. As an example consider the control of oxygen feed for a bacterial culture. The growing bacteria require different amounts of oxygen during the growth at the production phase, in order to ensure favour conditions, cf. Fig. 3. Often for the production stage, the oxygen level is decreased, which leads to a higher production. The controller parameters in this case might be the values for the tracking PID controllers or the maximum oxygen flows such that the constraints on oxygen concentration level are respected during the complete operation of the reactor.

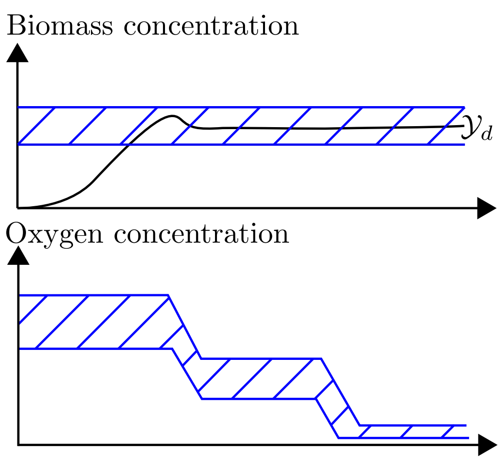


Fig. 3. Bioreactor example: the oxygen is reduced in steps so that the bacterial culture can adapt to the decreasing oxygen levels. The hatched zone at the oxygen diagram is the required profile for the oxygen level and the hatched zone on the biomass diagram is the desired biomass yield.

## III. Set-based Feasibility Approach to Identify Admissible Parameter Sets

To obtain an outer bound of the admissible controller parameters we reformulate the problem as a feasibility problem. As obtaining solutions for nonlinear feasibility problems is non-trivial, we discuss an approach for guaranteed set approximations, which outer bounds the true solutions set.

### A. Feasibility formulation

We combine the system description (1), controller $c$, requirements $\mathcal{Y}_d$, constraints $l_j$ and global bounds on $\mathcal{X}_i$, $\mathcal{Y}_i$, $\mathcal{U}_i$, $\mathcal{P}$, and $\mathcal{Q}$ for the finite time $\mathcal{T}$. Searching for a feasible set of the resulting system of equations is called feasibility problem, cf. also [13], [17]. The resulting feasibility set $\mathcal{F}(\mathcal{Q},\mathcal{X}_0)$ of this problem is the collection of all possible combinations of values that guarantee safe operation, i.e. do not violate the constraints and satisfy the requirements. The feasibility problem becomes

$$\mathcal{F}(\mathcal{Q},\mathcal{X}_0):\begin{cases}\text{find } (x,u,y,p,q,w)\\ \text{s.t. } x(k_{i+1}) = g(x(k_i),u(k_i),w(k_i),p),\\ \quad y(k_i) = h(x(k_i),w(k_i),p),\\ \quad u(k_{i+1}) = c(u(k_i),y(k_i),q,r(k_i)),\\ \quad 0 \le l_j(x(k_i),u(k_i),y(k_i),b(k_i),k_i),\\ \quad x(k_i)\in\mathcal{X}_i, \quad x(k_0)\in\mathcal{X}_0,\\ \quad y(k_i)\in\mathcal{Y}_i, \quad y(k_n)\in\mathcal{Y}_n,\\ \quad u(k_i)\in\mathcal{U}_i, \quad r(k_i)\in\mathcal{R}_i,\\ \quad k_i\in\mathcal{T}, \quad q\in\mathcal{Q}, \quad p\in\mathcal{P}.\end{cases}$$

In general $\mathcal{F}(\mathcal{Q},\mathcal{X}_0)$ is a non-convex set due to the appearing nonlinearities. To efficiently approximate $\mathcal{F}(\mathcal{Q},\mathcal{X}_0)$ we employ a relaxation procedure. Note, that the consideration of polynomial/rational systems enables the approach to consider other types of nonlinearities by approximating them up to a desired precision, cf. [18], [19].

### B. Set approximation

Various approaches exist to efficiently approximate the set $\mathcal{F}(\mathcal{Q},\mathcal{X}_0)$. For example, its constraints can be approximated as a linear program, which leads to a simpler formulation efficiently solved with tools like [20]. We denote the set of solutions of this relaxed problem as $\mathcal{F}_T(\mathcal{Q},\mathcal{X}_0)$. Note that this solution set is conservative, i. e. $\mathcal{F}(\mathcal{Q},\mathcal{X}_0) \subseteq \mathcal{F}_T(\mathcal{Q},\mathcal{X}_0)$.

Though the relaxed problem is easier to solve, obtaining good bounds of the complete set of feasible points is challenging. Following [21] we employ an approach to derive an outer approximation of the solution set $\mathcal{F}_T(\mathcal{Q},\mathcal{X}_0)$. To this end we define with $\mathbf{Q}$ and $\mathbf{X}_0$ the probing sets for the controller parameters and initial conditions correspondingly. Employing the weak duality principle we notice that if the dual formulation of $\mathcal{F}_T(\mathbf{Q},\mathbf{X}_0)$ is unbounded (i. e. $d\mathcal{F}_T(\mathbf{Q},\mathbf{X}_0)\to\infty$), the set $(\mathbf{Q},\mathbf{X}_0)$ contains no points that correspond to feasible solutions of $\mathcal{F}_T(\mathbf{Q},\mathbf{X}_0)$. This, in turn, implies that there are no solutions of $\mathcal{F}(\mathbf{Q},\mathbf{X}_0)$ because of the conservatism of the relaxation.

The projection of the set $\mathcal{F}_T(\mathcal{Q},\mathcal{X}_0)$ onto the subspace of $q$ and $x_0$ is therefore bounded as

$$\text{proj}_{(q,x_0)}\mathcal{F}_T(\mathcal{Q},\mathcal{X}_0)\subseteq\mathcal{F}_O,$$

where $\mathcal{F}_O$ is defined as

$$\mathcal{F}_O := (\mathcal{Q},\mathcal{X}_0)\setminus\bigcup_{i\in\mathcal{I},d\mathcal{F}_T(\mathbf{Q}_i,\mathbf{X}_{0i})\to\infty}(\mathbf{Q}_i,\mathbf{X}_{0i}).$$

To illustrate the discussed approach, we consider in the following the charging of a rechargeable Li-ion battery. In particular, we examine the influence of the initial temperature on the maximum charging current such that the battery is not overheated during charging and is within the safe voltage region.

## IV. Estimating Feasible Parameter Sets for Safe Charging of Li-ion Batteries

We are interested in finding safe controller parameters for charging Li-ion batteries. We start by presenting the considered equivalent circuit model of an off-the-shelf battery.

Afterwards, we present the employed charging strategy, the core-surface temperature model and the safety constraints considered during operation.

### A. Li-ion battery model

There are many ways to model the dynamics of Li-ion batteries [22]. We consider the so-called equivalent circuit model (ECM), which describes the dynamics of Li-ion batteries using electrical circuit elements such as resistors, capacitors, voltage sources, etc. ECM models are widely used in many applications due to their simplicity [23], [24]. Nevertheless, we would like to point out that the presented framework is capable of dealing not only with ECM models, but with any nonlinear battery model in polynomial form.

The considered model is of the so called "R-RC" type, as depicted in Fig. 4.

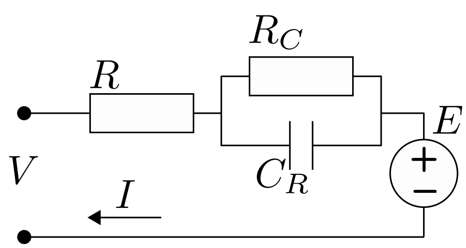


Fig. 4. Considered equivalent circuit model, representing the electrical behavior of the Li-ion battery.

The output voltage of the battery $V$ is given by:

$$V(k_i) = E(k_i) - V_C(k_i) - I(k_i)R. \quad (6)$$

Note, that the voltage source $E$ is in general nonlinear, as it represents the so-called open circuit voltage (OCV) of the battery. The OCV is a function of the state of charge (SoC) $s$ of the battery:

$$E(k_i) = \eta(s(k_i)). \quad (7)$$

The relation between $s$ and $E$ depends on many factors, such as the chemistry of the battery [25]. For an overview of OCV-SoC models we refer to [26]. We employ the following SoC model:

$$s(k_{i+1}) = s(k_i) + \delta(k_i)I(k_i). \quad (8)$$

To illustrate the framework, we consider a Nokia BP-4L battery for which the OCV ca be approximated by a polynomial/rational function $\eta$, which we adopted from [27]:

$$\begin{aligned} z(s(k_i)) &= \xi + s(k_i)(1-2\xi), \\ \eta(z(k_i)) &= \kappa_1 + \kappa_2 z(k_i) + \kappa_3 z(k_i)^3 + \\ &\qquad \kappa_4 z(k_i)^4 + \kappa_5 z(k_i)^{-1} + \kappa_6 z(k_i)^{-2}. \end{aligned}$$

Here $\xi$ is a scaling coefficient. Fig. 5 depicts the resulting relation between the OCV and SoC. We performed a least square fit to the model of the $\eta$ from [27] and the resulting coefficients are presented in Appendix I.

The voltage $V_C$ over the capacitor $C_R$ is modelled by:

$$V_C(k_{i+1}) = V_C(k_i) + \delta(k_i)\left(\frac{-V_C(k_i)}{C_R R_C} - \frac{I(k_i)}{C_R}\right) \quad (9)$$

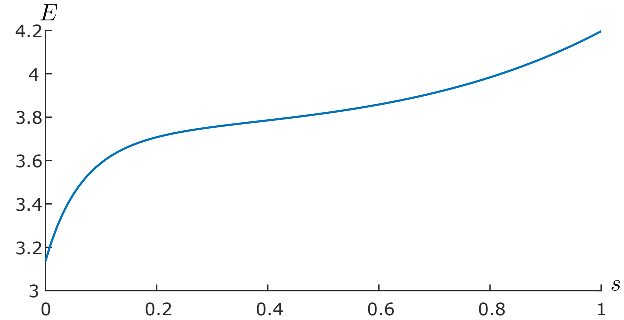


Fig. 5. Relationship between the open circuit voltage $E$ and the state of charge $s$ of the considered battery.

where $\delta$ is the discretization time, i. e. $\delta(k_i) = k_{i+1} - k_i$. Taken together (6), (7), (8) and (9) model the electrical behavior of a rechargeable Li-ion battery.

### B. Charging and safety requirements

The charging is performed by the so-called constant current constant voltage (CCCV) charging protocol. CCCV charging is characterized by two stages, as presented in Fig. 1 and Fig. 2. During the first stage a constant current (CC) is applied to the battery until a maximum voltage $V_{\max}$ of the battery is reached. In the second stage (constant voltage (CV) stage) the applied voltage is kept constant through decreasing the applied current. The charging is completed once a predetermined threshold for minimum current is reached. At this point the battery is considered to be sufficiently charged.

For safety reasons several additional aspects need to be taken into account. Firstly, high charging current can damage the battery. Secondly, high voltage jumps should be avoided. Thirdly, the temperature of the battery should not leave the safe region of operation [23].

For the CCCV charging protocol the overheating is only possible during the first stage of the charging protocol, i. e. at the CC stage, and therefore we only focus on this stage. To guarantee the temperature safety requirement we consider the following dynamical model for the surface and the core temperature of the battery [28]:

$$\begin{aligned} T_C(k_{i+1}) &= T_C(k_i) + \delta(k_i)(\gamma(T_S(k_i) - \\ &\qquad T_C(k_i)) + \beta I(k_i)^2), \\ T_S(k_{i+1}) &= T_S(k_i) + \delta(k_i)(-\gamma(T_S(k_i) - \\ &\qquad T_C(k_i)) + \alpha(T_{\text{amb}} - T_S(k_I)). \end{aligned}$$

Here $T_C$ is the core temperature, $T_S$ is the surface temperature, $T_{\text{amb}}$ is the temperature of the ambient environment. The coefficients $\alpha$, $\beta$, $\gamma$ represent the heat transfer between the surface and the environment, the heat generated by the current and the heat transfer between the core and the surface of the battery. The values for the thermal coefficients are based on the scaled version in [27] and [28], and presented in Table I along with the uncertain-but-bounded parameter values of the ECM model.

Due to aging of the battery, its internal resistance changes, which in turn, leads to changes in the dynamics and the model. First, the magnitude of the voltage jump when the

TABLE I

PARAMETER VALUES OF THE ECM AND THE THERMAL MODEL.

| Variable | Value | Measurement unit |
|---|---|---|
| $R$ | $[0.25, 0.28]$ | $\Omega$ |
| $C_R$ | $[5031, 5365]$ | $F$ |
| $R_C$ | $[0.093, 0.108]$ | $\Omega$ |
| $\alpha$ | 0.046 | $W/J$ |
| $\beta$ | 0.0196 | $(\Omega K)/J$ |
| $\gamma$ | 0.098 | $W/J$ |

current is applied can change, and second, the amount of heat that is generated by the current can change as well. This leads to variations of parameters, which can lead to constraint violation, see Fig. 6.

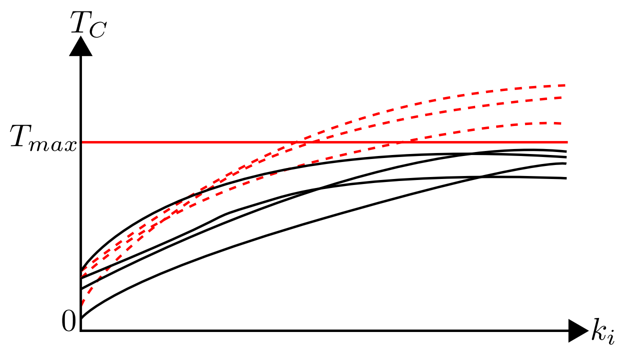


Fig. 6. Influence of aging on the temperature during charging. Black lines depict the temperature changes of healthy batteries, while red dashed lines represent possible profile for aged batteries, that could violate the maximal temperature constraints.

### C. Identifying feasible CCCV controller parameters

Our goal is to identify safe regions of maximal charging current to the of initial conditions (core temperature) and admissible battery parameters. As a performance requirement, we want the battery to achieve a minimal charge level at the end of the charging, i. e. $s(k_n) \geq 0.55$. Additionally, we pose a safety requirement on the maximum output voltage and on the maximum core temperature, i. e. $V(k_i) < 4.2$ and $T_C(k_i) < 20.4$.

For the estimation of the admissible sets we used the MATLAB toolbox ADMIT [29], which is capable of executing the approach outlined in Section III. Fig. 9 shows the result of an outer-approximation for the initial conditions presented in Table II.

TABLE II

INITIAL CONDITIONS OF THE VERIFICATION EXAMPLE.

| Variable | Value | Measurement unit |
|---|---|---|
| $s(k_0)$ | $[0, 0.01]$ | $-$ |
| $T_{\text{amb}}$ | 20 | $^oC$ |
| $T_S(k_0)$ | 20 | $^oC$ |
| $V_C(k_0)$ | $[-0.01, 0]$ | $V$ |
| $I$ | $[0.5, 2.0]$ | $A$ |
| $T_C(k_0)$ | $[20, 20.4]$ | $^oC$ |

We are especially interested in maximum charging currents such that the temperatures are constrained. The outer-bounding algorithm leads to the admissible region for the current of $I \in [0.692, 0.884]$. Fig. 7 and Fig. 8 show the estiamted SoC and output voltage values.

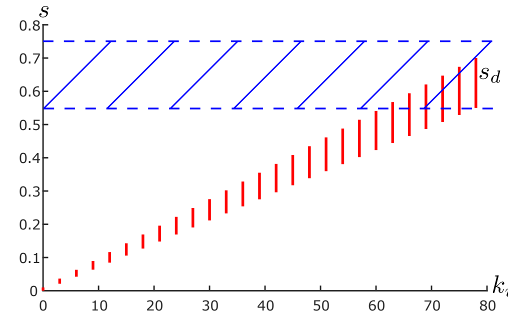


Fig. 7. Estimated tube of admissible SoC values for the bounds on the maximum current considering the uncertainties.

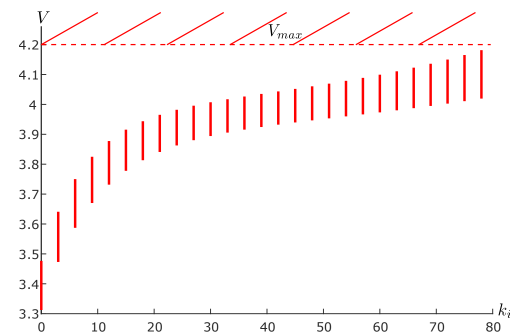


Fig. 8. Outer-approximated tube of possible voltage values.

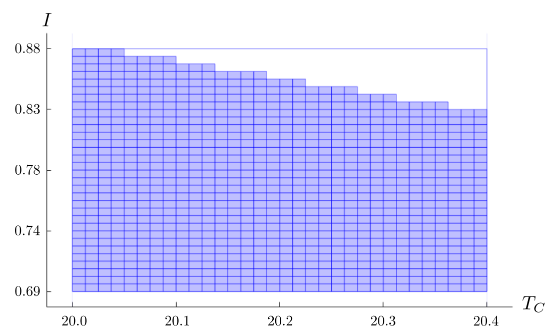


Fig. 9. Outer approximation of the feasible combinations of initial temperature $T_C(0)$ vs. applied charging current $I$, that satisfies all constraints.

As can be seen from Fig. 9, initial temperatures above 20.4 are not admissible, as they violate the constraint for maximal temperature. Fig. 10 depicts the trajectories for several admissible values of the initial temperature.

## V. CONCLUSIONS

In this contribution, we presented an approach to identify regions of admissible controller parameters for controlled uncertain finite-time/batch processes. The approach allows to estimate admissible regions for the initial and controller

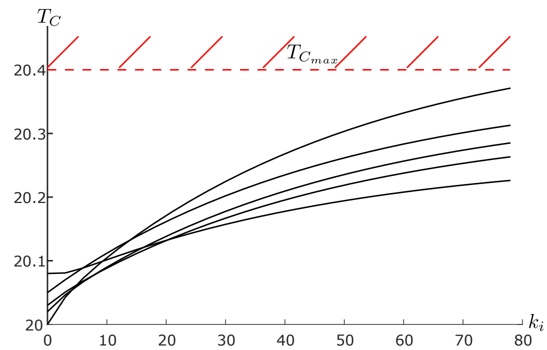


Fig. 10. Monte Carlo runs for different values of the initial core temperature and the charging current for admissible initial conditions.

parameter values, which guarantee satisfaction of the safety constraints. The employed set-based formulation allows direct inclusion of uncertainties.

The results have been illustrated considering charging of an off-the-shelf Li-ion battery. The considered approach is capable of handling also other types of models and charging strategies. While a higher polynomial resolution of the approximation will lead to an increase in the computational effort, this can be overcome by parallelization. Furthermore, complexity reduction techniques can be implemented, cf. [14]. The authors consider comparing the results with models of different complexity. Furthermore, more complex charging strategies are also of interest.

## APPENDIX I

The fitted coefficient values for $\eta(s)$ for the considered battery, Nokia BP-4L are: $\xi = 0.15$, $\kappa_1 = 3.0545$, $\kappa_2 = 0.8153$, $\kappa_3 = -0.5668$, $\kappa_4 = 1.0457$, $\kappa_5 = 0.2601$, $\kappa_6 = -0.0399$.

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