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# A novel algorithm for the global solution of mixed-integer bi-level multi-follower problems and its application to Planning & Scheduling integration

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*Abstract*— **Optimization problems involving a leader decision maker with multiple follower decision makers are referred to as bi-level multi-follower programming problems (BMF-P). In this work, we present novel algorithms for the exact and global solution of two classes of bi-level programming problems, namely (i) bi-level multi-follower mixed-integer linear programming problems (BMF-MILP) and (ii) bi-level multi-follower mixed-integer convex quadratic programming problems (BMF-MIQP) containing both integer and continuous variables at all optimization levels. Based on multi-parametric programming theory, the main idea is to recast the lower level, follower, problems as multi-parametric programming problems, in which the optimization variables of the upper level, leader, problem are considered as parameters for the lower level problems. The resulting exact multi-parametric mixed-integer linear or quadratic solutions are then substituted into the upper level problem, which can be solved as a set of single-level, independent, deterministic mixed-integer optimization problems. The proposed algorithm is applied for the solution of the challenging problem of planning and scheduling integration.**

## I. INTRODUCTION

Optimization problems involving a leader with multiple followers are referred to as bi-level multi-follower programming problems: the first decision maker (upper level; leader) is solving an optimization problem which includes in its constraint set other optimization problems solved by the second level decision makers (lower level problems; followers). In recent years, leader-follower games have attracted a growing interest not just in game theory, but also across a broad range of research communities. Bi-level multi-follower optimization can be applied to many and diverse problems that require hierarchical decision making such as transportation network planning [4], [23], [44], urban planning and land-use planning [42], economic planning [12], [24], [34], design and control integration [5], [21], [22], [43], [3], and supply chain planning [6], [13], [38], [3].

The problem of Planning & Scheduling integration (Figure 1), can naturally be recast as a bi-level multi-follower problem, where optimal decisions at the upper level (planning) provide constraints for the detailed decision making (scheduling) at each lower level problem that corresponds to each scheduling period [45], [8], [18], [36].

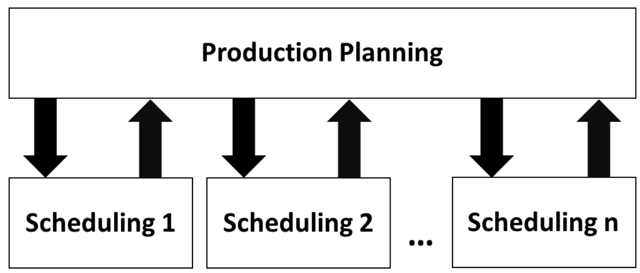


Fig. 1. Representation of the Planning and Scheduling integration problem as a bi-level multi-follower problem

Bi-level multi-follower decision making problems, such as the problem of planning and scheduling integration, can involve decisions in both discrete and continuous variables. Problems belonging in this class are referred to as mixed-integer bi-level multi-follower programing problems (BMF-MIP), and have the general form of (1):

$$
\begin{aligned}
\min_{x_a, y_a} \quad & F_a(x,y) \\
\text{s.t.} \quad & G_a(x,y) \le 0 \\
& \min_{x_{b,1}, y_{b,1}} \quad F_{b,1}(x,y) \\
& \text{s.t.} \quad G_{b,1} \le 0 \\
& \min_{x_{b,2}, y_{b,2}} \quad F_{b,2}(x,y) \\
& \text{s.t.} \quad G_{b,2} \le 0 \\
& \quad \vdots \\
& \min_{x_{b,n}, y_{b,n}} \quad F_{b,n}(x,y) \\
& \text{s.t.} \quad G_{b,n} \le 0 \\
& x = [x_a^T \quad x_{b,1}^T \quad x_{b,2}^T \quad \dots \quad x_{b,n}^T]^T, \quad x \in \mathbb{R} \\
& y = [y_a^T \quad y_{b,1}^T \quad y_{b,2}^T \quad \dots \quad y_{b,n}^T]^T, \quad y \in \mathbb{Z}
\end{aligned}
\tag{1}
$$

where $x_a$ is a vector of the upper level continuous problem variables, $y_a$ is a vector of the upper level integer variables, $x_{b,1}$ to $x_{b,n}$ are vectors of the lower levels continuous problem variables and $y_{b,1}$ to $y_{b,n}$ are vectors of the lower levels integer variables. Note that decision makers $b,1$ to $b,n$ all belong to the same optimization level.

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### A. Challenges and Previous work

Most research on bi-level optimization, to the best of our knowledge, mainly addressed the case of a single-follower. Bi-level programming problems, with just a single follower, are very challenging to solve, even in the linear case (shown to be NP-hard by [15] and [7]). For classes of problems where the single follower also involves discrete variables, this complexity is further increased, typically requiring global optimization methods for its solution [25], [37], [14], [27].

The problem of a single leader with multiple followers has not received a lot of attention from the research community with some attempts to solve the linear continuous case [39], [40], [19], [20] and very limited heuristic approaches for the solution of non-linear mixed-integer multi-follower problems [41] that do not guarantee global optimality.

In this paper, we focus on the single-leader multi-follower linear or quadratic mixed-integer problem. In the next section, we present a novel algorithm for the exact, global solution of two classes of bi-level multi-follower programming problems, namely (i) bi-level multi-follower mixed-integer linear programming problems (BMF-MILP) and (ii) bi-level multi-follower mixed-integer quadratic programming problems (BMF-MIQP) containing both integer and continuous variables at all optimization levels. In Section 3 we present a case study on the problem of planning and scheduling integration that solved to illustrate the use of the proposed algorithm. Finally, Section 4 concludes and summarizes this paper.

## II. Theory and Algorithm

In a bi-level programming problem it is well known that the feasible set of the follower's problems is parametric in terms of the decision variables of the leader problem. Utilizing this theory, Pistikopoulos and co-workers have presented a series of algorithms based on multi-parametric programming theory, which can address different classes of multi-level single-follower programming problems [9], [1] and have also discussed the case of continuous bi-level problems with multiple followers [10].

The approach presented here is based upon the multi-parametric mixed-integer linear programming (mp-MILP) algorithm of Oberdieck et al. [32], the recently developed multi-parametric mixed-integer quadratic programming (mp-MIQP) algorithm of Oberdieck and Pistikopoulos [31] and new theory on binary parameters in multi-parametric programming problems [28]. The proposed algorithm will be introduced through the general form of the BMF-MILP problem (2), and then illustrated through a case study on a planning and scheduling integration example.

It is assumed that upper level optimization variables that appear in the lower level problems, and lower level integer variables, are bounded.

$$
\begin{aligned}
\min_{x_a, y_a} \quad & Q_a x + H_a y + c_{ca} \\
\text{s.t.} \quad & A_a x + E_a y \le b_a \\
& \min_{x_{b,1}, y_{b,1}} \quad Q_{b,1} x + H_{b,1} y + c_{cb,1} \\
& \text{s.t.} \quad A_{b,1} x + E_{b,1} y \le b_{b,1} \\
& \min_{x_{b,2}, y_{b,2}} \quad Q_{b,2} x + H_{b,2} y + c_{cb,2} \\
& \text{s.t.} \quad A_{b,2} x + E_{b,2} y \le b_{b,2} \\
& \qquad \vdots \\
& \min_{x_{b,n}, y_{b,n}} \quad Q_{b,n} x + H_{b,n} y + c_{cb,n} \\
& \text{s.t.} \quad A_{b,n} x + E_{b,n} y \le b_{b,n} \\
& x = [x_a^T \quad x_{b,1}^T \quad x_{b,2}^T \quad \dots \quad x_{b,n}^T]^T, x \in \mathbb{R} \\
& y = [y_a^T \quad y_{b,1}^T \quad y_{b,2}^T \quad \dots \quad y_{b,n}^T]^T, y \in \{0,1\}
\end{aligned}
\tag{2}
$$

As a first step, the lower level problems are transformed into multi-parametric mixed-integer problems (3). For each transformed lower level problem, the optimization variables of the leader problem along with any decision variables of the rest of the lower level problems that appear in the transformed problem, are considered as parameters.

$$
\begin{aligned}
\min_{x_{b,1}, y_{b,1}} \quad & Q_{b,1} x + H_{b,1} y + c_{cb,1} \\
\text{s.t.} \quad & A_{b,1} x + E_{b,1} y \le b_{b,1} \\
& x^L \le x \le x^U \\
\min_{x_{b,2}, y_{b,2}} \quad & Q_{b,2} x + H_{b,2} y + c_{cb,2} \\
\text{s.t.} \quad & A_{b,2} x + E_{b,2} y \le b_{b,2} \\
& x^L \le x \le x^U \\
& \vdots \\
\min_{x_{b,n}, y_{b,n}} \quad & Q_{b,n} x + H_{b,n} y + c_{cb,n} \\
\text{s.t.} \quad & A_{b,n} x + E_{b,n} y \le b_{b,n} \\
& x^L \le x \le x^U
\end{aligned}
\tag{3}
$$

The solution of the multi-paramertric problems (3), using mp-MILP or mp-MIQP algorithms through POP® toolbox [30], provides the complete profile of optimal solutions of the lower level problems as explicit affine functions of the variables of the upper level problem and other lower level problems within corresponding critical regions (4). The problems (3) are interdependent of each other, therefore parallel programming can be used to solve them simultaneously.

$$
[x_{b,n} y_{b,n}] = \begin{cases}
\xi_{b,n}^1 = p_{b,n}^1 + q_{b,n}^1 x, \psi_{b,n}^1 & \text{if } H_{b,n}^1 x \le h_{b,n}^1 \\
\xi_{b,n}^2 = p_{b,n}^2 + q_{b,n}^2 x, \psi_{b,n}^2 & \text{if } H_{b,n}^2 x \le h_{b,n}^2 \\
\qquad \vdots & \qquad \vdots \\
\xi_{b,n}^k = p_{b,n}^k + q_{b,n}^k x, \psi_{b,n}^k & \text{if } H_{b,n}^k x \le h_{b,n}^k
\end{cases}
\tag{4}
$$

where $\xi_{b,n}^i$ is the lower level $b,n$ objective, $H_{b,n}^i x \le h_{b,n}^i$ is referred to as critical region, $CR^i$, and $k$ denotes the number of computed critical regions.

As a next step, the solutions derived in the previous step are used to formulate a set of reformulations of the original bi-level multi-follower problem. Each reformulation is formed by using a different combination of critical regions from the solutions of the follower's problems (4). Considering only the upper level problem, the follower's decision variables are substituted by the affine functions derived. The critical region definitions are added to the upper level problem as a new set of constraints along with constraints for Nash equilibrium, forming single-level deterministic problems for every different combination of critical regions. In the case of quadratic objective functions, the single-level deterministic problems created in this step can contain non-linear constraints as critical region definitions can be non-linear. This makes the single-level problems more challenging to be solved.

Single level problems with the formulation of (5) and all possible combinations of $i, j, \ldots k$ are created.

$$
\begin{array}{ll}
\min\limits_{x_a, y_a} & Q_a x + H_a y + c_{ca} \\
\text{s.t.} & A_a x + E_a y \leq b_a \\
& H^i_{b,1} x \leq h^i_{b,1} \\
& x_{b,1} = p^i_{b,1} + q^i_{b,1} x \\
& y_{b,1} = \psi^i_{b,1} \\
& H^j_{b,2} x \leq h^j_{b,2} \\
& x_{b,2} = p^j_{b,2} + q^j_{b,2} x \\
& y_{b,2} = \psi^j_{b,2} \\
& \vdots \\
& H^k_{b,n} x \leq h^k_{b,n} \\
& x_{b,n} = p^k_{b,n} + q^n_{b,n} x \\
& y_{b,n} = \psi^k_{b,n}
\end{array} \tag{5}
$$

All single level MI(N)LP problems formed are solved using appropriate solvers, such as CPLEX or ANTIGONE solvers. Because those problems are independent of each other, it is possible to use parallel programming to solve them simultaneously.

The solutions of the above single level MI(N)LP problems correspond to different local optimal solutions of the original BMF-MILP. The final step of the algorithm is to compare all the local solutions to obtain the minimum $z$ that would correspond to the exact and global optimum, $z^*$, of the original bi-level multi-follower problem.

The proposed algorithm is also summarized in Table 1.

TABLE I

MULTI-PARAMETRIC BASED ALGORITHM FOR THE SOLUTION OF BMF-MILP PROBLEMS

| | |
|---|---|
| **Step 1** | Establish integer and continuous variable bounds. |
| **Step 2** | Recast the lower level problems as mp-MILPs, in which the optimization variables of the upper level problem along with the optimization variables of other lower level problems are considered as parameters. |
| **Step 3** | Solve the resulting mp-MILP problems to obtain the optimal solution of the lower level problems as explicit functions of the upper level variables and other lower level variables. |
| **Step 4** | Substitute each multi-parametric solution into the upper level problem and add Nash equilibrium constraints to formulate $k$ single level MILP problems. |
| **Step 5** | Solve all $k$ single level problems and compare their solutions to select the exact and global optimum. |

### A. *Pessimistic and Optimistic Solutions:*

The choice of a pessimistic versus an optimistic solution emerges when the optimal solution of the lower level problems is not unique for the set of optimal upper level variables. This degeneracy can result either from the lower level discrete variables or from the lower level continuous variables.

*Lower level binary variables:* The solution method described above is able to capture all degenerate solutions and therefore supply the decision maker with both the pessimistic and optimistic solutions.

*Lower level continuous variables:* The multi-parametric solution via POP® toolbox is not able to supply the decision maker with the full range of degenerate solutions. Even though there are techniques to handle degeneracy in multi-parametric problems [11], [16], [29], those are not yet implemented in the approach described above.

Therefore, it is assumed that there is a unique optimal solution for the continuous lower level variables corresponding to the upper level optimal solution.

## III. APPLICATION: PLANNING AND SCHEDULING INTEGRATION

Traditionally, process planning and scheduling strategies are derived sequentially and separately. Scheduling decisions are derived after process planning decisions are already taken. This can lead to sub-optimal strategies, therefore researchers have tried to solve this problem holistically [17], [33], [26].

Planning and scheduling optimization problems with seasonal demand variability can be often expressed holistically within a hierarchical structure, where optimal decisions at an upper level (planning) provide constraints for the detailed decision making (scheduling) at a lower level, typically posed as bi-level multi-follower optimization problems [45], [8], [18], [36] (Figure 1). Since discrete decisions are involved most likely at both levels, the resulting formulations typically correspond to BMF-MILP problems.

In this section we are considering the simple case study of the planning and scheduling integration of a production processing plant.

### A. *Problem description*

The problem under investigation considers a production processing plant that produces three different products, A, B and C, through two processing stages. The processing time of product B, for both of the production stages, is proportional to the production target of B, $P_B$. The processing times of products A and C are constant and are presented in Table 2.

We are considering that one planning period consists of two scheduling periods (Figure 2), and at the end of each scheduling period the production facility must supply its customers the demand asked (Table 3).

TABLE II

PROCESSING TIME, $T_{jk}$

| Product | Stage 1 (hr) | Stage 2 (hr) |
|---|---|---|
| A | 3 | 5 |
| B | $kP_B$ | $2k(P_B+0.5)$ |
| C | 6 | 3 |

TABLE III

COSTUMER DEMAND FOR EACH PRODUCT

| Product | Period 1 (kg) | Period 2 (kg) |
|---|---|---|
| A | 4 | 8 |
| B | 5 | 7 |
| C | 6 | 5 |

*1) Production Planning problem:* The aim of the production planning decision level is to determine the production target of each product for every scheduling period, so as to minimize the inventory level after each scheduling period comes to an end and make sure that the demand of each product is fulfilled. The formulation of the production problem is presented below (6). The notation used is defined in Table 4.

$$
\begin{array}{ll}
\min\limits_{P,I} & k_A(I_A^1+I_A^2)+k_B(I_B^1+I_B^2)+k_C(I_C^1+I_C^2) \\
& \qquad +k_4^1c_{2C}^1+k_4^2c_{2C}^2 \\
\text{s.t.} & P_A^1=D_A^1-I_A^0+I_A^1 \\
& P_B^1=D_B^1-I_B^0+I_B^1 \\
& P_C^1=D_C^1-I_C^0+I_C^1 \\
& P_A^2=D_A^2-I_A^1+I_A^2 \\
& P_B^2=D_B^2-I_B^1+I_B^2 \\
& P_C^2=D_C^2-I_C^1+I_C^2
\end{array} \tag{6}
$$

*2) Scheduling problems:* The objective of each scheduling problem is to minimize the makespan by assigning time slots to each product. The formulation of the scheduling problems is a modification of the formulation developed by [35] and presented below. The notation used is defined in Table 4.

$$
\begin{array}{lll}
\min\limits_{y,c} & c_{2C} & \\
\text{s.t.} & \sum_{i=1}^3 y_{i,k}=1 & \forall k \\
& \sum_{k=1}^3 y_{i,k}=1 & \forall i \\
& c_{i,j}\geq c_{i,j-1}+\sum_{k=1}^3 w_{i,k,j} & j>1,\forall i \\
& c_{i,j}\geq c_{i-1,j}+\sum_{k=1}^3 w_{i,k,j} & i>1,\forall j \\
& w_{i,k,j}\geq \theta_{k,j}-\theta_{k,j}^U(1-y_{i,k}) & \forall i,j,k \\
& w_{i,k,j}\leq \theta_{k,j}-\theta_{k,j}^L(1-y_{i,k}) & \forall i,j,k \\
& y_{i,k}\theta_{k,j}^L\leq w_{i,k,j}\leq y_{i,k}\theta_{k,j}^U & \forall i,j,k \\
& \theta_{k,j}^L\leq \theta_{k,j}\leq \theta_{k,j}^U & \forall i,j,k
\end{array} \tag{7}
$$

where $i$ is the time slot (1,2,3), $j$ is the production stage (1,2), and $k$ is the product (A,B,C).

The first constraint ensures that each product is assigned one time slot, wheres the second constrained ensures that just one product is assigned in each time slot. The third and fourth constraint define the completion times of products at different stages.

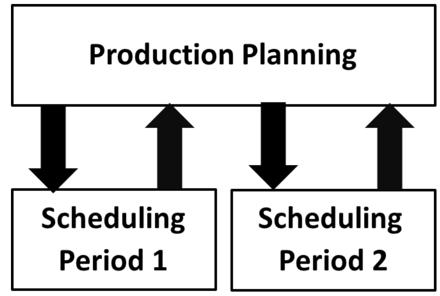


Fig. 2. Representation of the Production Planning and Scheduling integration case study problem

TABLE IV

NOTATION

| | |
|---|---|
| $k$ | Products (A, B, C) |
| $i$ | Time slot (1, 2 ,3) |
| $j$ | Production stage (1, 2) |
| $n$ | Scheduling period (1,2) |
| $P_k^n$ | Production target for product $k$ in scheduling period $n$ |
| $T_{j,k}$ | Processing time of product $k$ in stage $j$ |
| $D_k^n$ | Demand of product $k$ in scheduling period $n$ |
| $I_k^n$ | Inventory level of product $k$ after scheduling period $n$ |
| $c_{i,j}^n$ | Completion time of product $k$ in stage $j$ and period $n$ |
| $y_{i,k}$ | Binary 0-1 variable for product $k$ being assigned in time slot $i$ |
| $CR$ | Critical region |

Nonlinearities arise from the term $w_{i,k,j}=y_{i,k}T_{j,k}$, as $T_{j,B}$ is a function of $P_B$. The last four constraints are used to eliminate this nonlinearities resulting into a mixed-integer linear programming problem (more information on the reformulation can be found in [35]).

### *B. Solution method*

To solve the problem defined above the method presented in Section II is followed.

**Step 1:** The two follower problems are reformulated as mp-MILP problems, in which the optimization variables of the upper level problem $P^1,P^2$ and $I^1,I^2$ are considered as parameters. Note that the decision variables of each of the follower problems does not appear in the other follower problem therefore are not considered as parameters.

**Step 2:** The mp-LP problems are then solved using a mp-LP algorithm through POP toolbox [30] and yield the optimal parametric solutions given in Table 5 and 6 and presented in Figure 3. In this example each parametric solution consists of only two critical regions. Since the scheduling problems are structurally identical one could just solve one of the problems and then derive the solution for the rest.

**Step 3:** The solutions obtained are then substituted into the upper level problem to formulate four new single-

TABLE V

MULTI-PARAMETRIC SOLUTION OF THE FIRST FOLLOWER - SCHEDULING PERIOD 1

| CR | Definition | Objective-Makespan |
|---|---|---|
| 1.1 | $4\leq P_B^1\leq 5.5$ | $P_B^1+12$ |
| 1.2 | $5.5\leq P_B^1\leq 7$ | $2P_B^1+6.5$ |

TABLE VI

MULTI-PARAMETRIC SOLUTION OF THE SECOND FOLLOWER - SCHEDULING PERIOD 2

| CR | Definition | Objective-Makespan |
|---|---|---|
| 2.1 | $4 \le P_B^2 \le 5.5$ | $P_B^2 + 12$ |
| 2.2 | $5.5 \le P_B^2 \le 7$ | $2P_B^2 + 6.5$ |

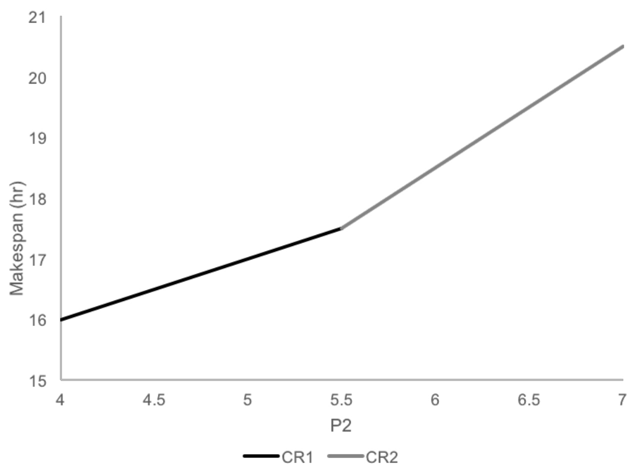


Fig. 3. Multi-parametric solution of the followers problem

level deterministic linear programming (LP) problems that correspond to reformulations of the original bi-level multi-follower problem. The four problems are created by taking different combinations of the followers critical regions (Table 7).

**Step 4:** The single level problems created in **Step 3** are then solved using CPLEX linear programming solver, and result to the solutions presented in Table 8.

After the comparison procedure the global optimum is found to be 377 and lies at the point were critical region 1.1 meets critical region 1.2, resulting to both combinations 2 and 4 giving the same solution. The global optimal solution is also presented in Figure 4.

TABLE VII

COMBINATIONS OF CRS USED FOR THE SINGLE LEVEL REFORMULATIONS

| Combination | Critical regions |
|---|---|
| 1 | 1.1 & 2.1 |
| 2 | 1.1 & 2.2 |
| 3 | 1.2 & 2.1 |
| 4 | 1.2 & 2.2 |

TABLE VIII

SINGLE LEVEL SOLUTIONS

| Combination | Makespan 1 | Makespan 2 | Planning Obj. |
|---|---|---|---|
| 1 | | infeasible | |
| 2 | 17.5 | 19.5 | 377 |
| 3 | 19.5 | 17.5 | 381 |
| 4 | 117.5 | 19.5 | 377 |

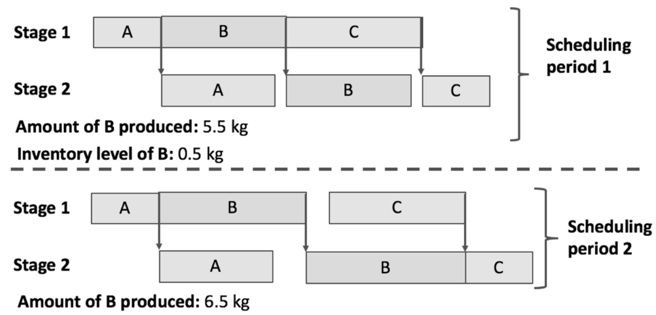


Fig. 4. Global optimal solution of the original bi-level multi-follower planning and scheduling integration problem

## IV. CONCLUSIONS, APPLICATIONS AND FUTURE WORK

We have presented the only, to our knowledge, exact and global optimization algorithm for the solution of BMF-MILP or BMF-MIQP problems.

This algorithm could be applied to many and diverse decision problems that concern the control community such as hierarchical model predictive control [2], scheduling and control integration or simultaneous design and control.

Future work would include the computational implementation of the algorithm into a toolbox, that would allow computational studies and the wide use of the algorithm by the research community.

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