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A Game-Theoretic Framework for Distributed Voltage Regulation over HVDC grids

A. R. del Nozal[1], L. Orihuela[1] and P. Millán[1]

***Abstract*— In this paper we propose a non-cooperative game-theoretical framework to design the strategic behaviour of buses involved in a HVDC grid that are connected to renewable resources. This framework takes under consideration voltage stability issues as well as economic factors when the current carried by the power lines is constrained. In order to reach an agreement in the decisions of the buses, an iterative distributed Nash equilibrium seeking algorithm is implemented. Existence and stability conditions for the equilibrium are introduced for the unconstrained case and simulations show the robustness of the algorithm for the constrained scenario.**

***Index Terms*— Game theory. Non-cooperative games. HVDC. Distribution power networks. Distributed optimization.**

I. INTRODUCTION

Nowadays, conventional power system is facing the problems of gradual depletion of fossil fuel resources, poor energy efficiency and environmental pollution. These problems have raised the use of a non-conventional/renewable energy sources like natural gas, biogas, wind power or solar photovoltaic cells. These resources are usually directly connected to distribution power networks. The issues related to the integration of these Distributed Generation Units (DGU) to the distribution power grid has been studied since late 1990s and even these days it is an important field of study and it attracts the interest of many researchers [14].

Optimal Power Flow (OPF) has been widely studied for managing a power system network adjusting the value of free variables (i.e. renewable power injection, energy storage management, etc). OPF was first introduced in 1962 [3] and the terminology has strongly evolved in the last decades. A recent critical review about the last advances in AC OPF can be found in [2]. A related work concretely focused on smartgrids and microgrids was published [1].

Game theory has been studied and applied for DGU management in smartgrids by several researchers. In [16] a game theoretic framework is introduced for modeling the strategic behaviour of the buses that are connected to renewable energy resources. The game evolves relying in recursive linearizations of the power flow equations achieving a Nash equilibrium. Nevertheless, the convergency to this equilibrium is not guaranteed. The work of [4] deals with the same problem using an approximation to treat the AC grid as a DC one, reducing in this way the number of variables. This study is focused on economic factors and stability voltage conditions are not tackled. Another approach is found in [15] where a salck bus is not required because all sources in the system adjust their output to meet energy demands and voltage requirements.

Regarding the intermittent behaviour of renewable resources, energy storage devices and its management play a critical role in smartgrids. In [11] a method to control distributed energy storage units charge/discharge in a DC microgrid is presented. Model predictive control has been widely used to control energy storage units. See for instance [12], [6], [13].

This paper studies a situation in which a set of buses make appropriate decisions in order to minimize local cost functions. This cost function relays in economical and stability factors of the network. Each bus is able to control a DGU directly affecting to the voltage level of the bus and to the loses in the grid. In order to reach a suitable equilibrium, the buses have to negotiate among themselves through playing a non-cooperative game. The final agreement of the negotiation, namely, the Nash equilibrium, is the solution of the distributed OPF. Necessary and sufficient conditions are given for the existence, uniqueness and stability of the Nash equilibria. When the situation with constrained decisions is considered, the paper presents simulation examples that show the robustness of the algorithm. This work extends preliminary theoretical results introduced in [9] and applies these results to HVDC grids management.

In comparison with other approaches found in the literature as [16], [4] or [10], our work takes under consideration physical limitations in lines, prioritizes voltage regulation over economic factors and includes power loses in adjacent lines in the local cost function achieving a more realistic and robust solution. These considerations originate variable bounds for the decision variable during the negotiation impacting on the approach of a more complex problem. In [5] a DC Optimal Power Flow is distributedly solved. However, the decision variable chosen is the power flow without having control of the DC voltage level at each bus.

The paper is organized as follows. Section II states the problem and introduce some assumptions that must be taken under consideration. In Section III, the necessity of a distributed solution of our problem is exposed and an iterative algorithm in order to solve the problem is defined. Section IV deals with stability conditions that the problem must fulfill in order to reach to a Nash equilibrium. Finally, Section V shows simulation examples and Section VI concludes the paper and introduces further work.

Notation. A graph is an ordered pair $\mathscr{G} = (\mathscr{V}, \mathscr{E})$ com-

[1]Authors are with Departamento de Ingeniería, Universidad Loyola Andalucía, Sevilla, Spain {arodriguez,dorihuela,pmillan}@uloyola.es

prising a set $\mathscr{V} = \{0,1,2,\ldots,n\}$ of *vertices* or *buses*, and a set $\mathscr{E} \subset \mathscr{V} \times \mathscr{V}$ of *edges* or *power lines*. An *undirected graph* is a graph in which edges have no orientation, so that if $(i,j) \in \mathscr{E}$ then $(j,i) \in \mathscr{E}$. *The neighborhood of* i, $\mathscr{N}_i \triangleq \{j : (i,j),(j,i) \in \mathscr{E}\}$ is defined as the set of nodes with edges connected to node i.

II. PROBLEM FORMULATION

Consider a direct-current power network comprising a set of buses $\mathscr{V} = \{0,1,2,\ldots,p\}$ connected according to a given undirected graph $\mathscr{G} = (\mathscr{V},\mathscr{E})$ where $\mathscr{E}$ represents the set of power lines between the different buses. The electric lines are characterized by a resistance R_{ij}. Observe that $R_{ij} = R_{ji}$. Let bus 0 represents the slack bus or reference in which voltage is fixed to a reference value. Consider also that each node can exchange information with its neighborhood through the power lines (see for instance [7]). Let us then consider that each bus $i \in \mathscr{V}$ has access to renewable resources[1] generating an amount of power $P_{g,i}$ and tries to feed a certain level of local load $P_{l,i}$ such that

$$P_i \triangleq P_{l,i} - P_{g,i} \tag{1}$$

is the load not covered locally with renewable resources. Note that this term can be negative in the case of $P_{g,i} > P_{l,i}$ and the excess of generation will be used to feed loads in adjacent buses.

Let $I_{j,i}$ denotes the current incoming from bus j to bus i, let $I_{l,i}$ and $I_{g,i}$ denote the current locally demanded by the load and that generated by the renewable resources, respectively, and let V_i the voltage level at bus i. In Figure 1 a current balance at bus i is shown.

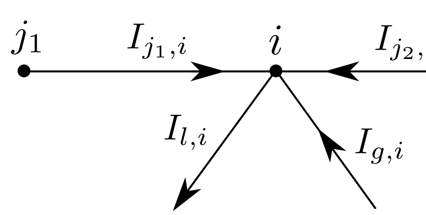

Fig. 1. Current balance at bus i.

The power flow in the grid obeys Kirchhoff's laws:

$$\begin{aligned} V_j - V_i &= R_{ij} I_{ji}, \quad \forall j \in \mathscr{N}_i, & (2)\\ I_{l,i} &= \sum_{j \in \mathscr{N}_i} I_{ji} + I_{g,i}. & (3) \end{aligned}$$

Isolating I_{ji} in (2), substituting into (3) and applying $P_i = V_i\left(I_{l,i} - I_{g,i}\right)$ it yields:

$$V_i^2 \sum_{j \in \mathscr{N}_i} \frac{1}{R_{ij}} - V_i \sum_{j \in \mathscr{N}_i} \frac{V_j}{R_{ij}} + P_i = 0. \tag{4}$$

Please note that if we consider known voltage level in the neighborhood, $V_j \backslash j \in \mathscr{N}_i$, the value of P_i and V_i in (4) are mutually related by a quadratic equation.

[1]No additional constraint would appear if non renewable sources of energy are considered.

The sequel constraints are usually imposed by the typical regulation of the power lines.

Constraint 1. The amount of power generated with renewable resources by each bus $i \in \mathscr{V}$ is upper bounded such that:

$$0 \le P_{g,i} \le \bar{P}_{g,i}, \tag{5}$$

where $\bar{P}_{g,i}$ is the power generation limit for bus i.

Constraint 2. The amount of current carried by the power lines is upper bounded due to physical limitations, such that:

$$0 \le |I_{ij}| \le \bar{I}_{ij} \quad \forall j \in \mathscr{N}_i, \tag{6}$$

where $\bar{I}_{ij}$ is the current upper-bound.

Constraint 3. In order to avoid stability problems, a normal operation voltage band is fixed, such that:

$$\underline{V}_i \le V_i \le \bar{V}_i, \tag{7}$$

where $\underline{V}_i$ and $\bar{V}_i$ are the voltage operation limits for each of the buses.

According to above constraints, consider the following mild assumptions:

Assumption 1. At least one of the buses is connected to an infinity power DC feeder.

Assumption 2. Each bus $i \in \mathscr{V}$ can select the power generated by a renewable resource within some bounds defined in (5).

Assumption 3. Each bus $i \in \mathscr{V}$ is aware of the resistance R_{ij} and current bounds $\bar{I}_{ij}$ of the adjacent lines $\{(i,j) : j \in \mathscr{N}_i\}$.

Assumption 4. There always exists a set of values of $P_{g,i}$ for all $i \in \mathscr{V}$ fulfilling (5) such that the solution of the set of equations (4) for all $i \in \mathscr{V}$ satisfy constraints (6) and (7).

The goal of the buses will be to optimally choose the power generated ($P_{g,i}$) or, indirectly, the voltage V_i. In order to do so, we define the following local cost function:

$$\begin{aligned} &J_i(V_i, V_j \backslash j \in \mathscr{N}_i) \\ &\quad = \underbrace{\sigma_{1,i} P_i}_{(a)} + \underbrace{\sigma_{2,i}(V_i^{ref} - V_i)^2}_{(b)} + \underbrace{\sigma_{3,i} \sum_{j \in \mathscr{N}_i} \frac{(V_j - V_i)^2}{R_{ij}}}_{(c)}, \end{aligned} \tag{8}$$

where V_i^{ref} is the voltage reference value for each node $i \in \mathscr{V}$ and $\sigma_{1,i}, \sigma_{2,i}$ and $\sigma_{3,i}$ are the weights associated to each of the terms described next:

(a) The amount of power demanded from the grid to feed the power load not covered with the renewable resources is penalized with the cost parameter $\sigma_{1,i}$ (€$/W$). Similarly, when the power generated is higher than the power load, the grid operator will pay for that energy. For our study we consider that the price of sale is equal than the purchase price.

(b) The deviation between the reference value for the voltage level of the node is penalized. Cost parameter $\sigma_{2,i}$ is expressed in (€$/V^2$).

(c) The electrical lost through the adjacent wires is penalized with the cost parameter $\sigma_{3,i}$ (€$/W$).

We are now in a position to state the formulation of the problem. The aim of each bus $i \in \mathscr{V}$ involved in the power network is to solve a local optimization problem by finding a voltage value V_i that minimizes cost function (8) subject to constraints (5)-(7):

$$\begin{aligned}(OP) \quad \min_{V_i} \quad & J_i(V_i, V_j \backslash j \in \mathscr{N}_i) \\ s.t. \quad & 0 \leq P_{g,i} \leq \bar{P}_{g,i}, \\ & 0 \leq |I_{ij}| \leq \bar{I}_{ij} \quad \forall j \in \mathscr{N}_i, \\ & \underline{V}_i \leq V_i \leq \bar{V}_i,\end{aligned}$$

The main goal of this paper is to introduce necessary and sufficient conditions for the solution of the optimization problem stated in (OP). This problem cannot be locally solved since the buses ignore the voltages V_j of their neighborhood. Note that the decision made by each agent will directly affect to the decisions of its neighborhood regarding the coupling terms shown in (4).

In order to distributedly solve this problem, this paper proposes an iterative algorithm based on game theory, which is presented in the next sections.

III. Distributed solution

Consider a game-theoretical scenario defined over the set of buses (players) in which the decisions made by each bus at iteration step k is based on the decisions made by its neighborhood in the previous iteration step $k-1$. It is worth pointing out that at every iteration step, the buses exchange their decisions with the neighborhood. We will define the voltage as the decision variable. As the buses have individual cost functions, a Nash equilibrium is a desirable set of decisions.

Definition 2. A Nash equilibrium of the game is a situation in which no bus changes its decisions as long as the rest of the buses keeps the same decisions as well. Mathematically, a Nash equilibrium is defined by a set of decisions $V^* \triangleq [V_1{}^* \, V_2{}^* \, \ldots \, V_n{}^*]^\top$ such that

$$\begin{aligned} V_1{}^* &= \arg\min_{V_1} \quad J_1(V_1^*, V_j^* \backslash j \in \mathscr{N}_1) \\ &\vdots \\ V_p{}^* &= \arg\min_{V_p} \quad J_p(V_p^*, V_j^* \backslash j \in \mathscr{N}_p) \end{aligned}$$

From this definition when the buses find an agreement described by a Nash equilibrium the optimization problem (OP) would be solved by each bus.

A. Iterative algorithm

The iterative algorithm to find the solution of the problem stated in (OP) is introduced here. Let us define $J_i(k) \triangleq J_i(V_i(k), V_j(k-1) \backslash j \in \mathscr{N}_i)$. By substituting (4) into (8) a cost function only dependent on the voltage of the agents is obtained:

$$\begin{aligned} J_i(k) \quad &= \quad a_i V_i(k)^2 - V_i(k) \left(b_i + c_i \sum_{j \in \mathscr{N}_i} \frac{V_j(k-1)}{R_{ij}} \right) \quad (9) \\ &+ \quad d_i + e_i \sum_{j \in \mathscr{N}_i} \frac{V_j(k-1)^2}{R_{ij}}, \end{aligned}$$

where $a_i = \sigma_{2,i} + \sum_{j \in \mathscr{N}_i} (\sigma_{3,i} - \sigma_{1,i})/R_{ij}$, $b_i = 2\sigma_{2,i} V_i^{ref}$, $c_i = 2\sigma_{3,i} - \sigma_{1,i}$, $d_i = \sigma_{2,i} (V_i^{ref})^2$ and $e_i = \sigma_{3,i}$.

In this way, at every iteration step, every agent $i \in \mathscr{V}$ must make a decision:

$$V_i(k) = \arg\min_{V_i(k)} J_i(k),$$

which is based on the decisions made by its neighborhood in the previous step, $V_j(k-1)$. Therefore, this problem can certainly be solved locally. Thus, equation (4) can be rewritten as follows:

$$V_i(k)^2 \sum_{j \in \mathscr{N}_i} \frac{1}{R_{ij}} - V_i(k) \sum_{j \in \mathscr{N}_i} \frac{V_j(k-1)}{R_{ij}} + P_{l,i} - P_{g,i}(k) = 0. \quad (10)$$

In order to state the iterative algorithm a reformulation of Assumption 4 is needed.

Assumption 5. Known $V_j(k-1) \backslash j \in \mathscr{N}_i$ for every bus $i \in \mathscr{V}$, there always exists a pair of values $\{V_i(k), P_{g,i}(k)\}$ solution of (10) fulfilling (5) such that constraints (6) and (7) hold.

Previous Assumption implies that the evolution of the decisions for all the agents involved in the network always fulfill constraints (5)-(7).

Proposition 1. Constraints (5), (6) and (7) can be jointly rewritten as

$$\beta_i(k) \leq V_i(k) \leq \gamma_i(k), \quad (11)$$

where $\beta_i(k)$ and $\gamma_i(k)$ are the most restrictive voltage limits defined by the constraints (see equations (12) and (13)).

Proof: From (10) we can obtain an expression of the generated power in terms of voltage:

$$P_{g,i}(k) = V_i(k)^2 \sum_{j \in \mathscr{N}_i} \frac{1}{R_{ij}} - V_i(k) \sum_{j \in \mathscr{N}_i} \frac{V_j(k-1)}{R_{ij}} + P_{l,i}, \quad (14)$$

which is a parabola in $V_i(k)$ with positive leading coefficient. It can be seen that, according to the negative value of the coefficient of the second term, the vertex of the parabola is placed on a positive value of $V_i(k)$. The vertex of this parabola is the minimum generated power that must be positive by equation (5). On the other hand, under Assumption 5, this minimum must be lower than $\bar{P}_{g,i}$. This way, due to the convexity of (14) with respect to $V_i(k)$, it is possible to

$$\beta_i(k) = max\left\{\left(\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)}{R_{ij}} - \sqrt{\left(\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)}{R_{ij}}\right)^2 - \sum_{j\in\mathscr{N}_i}\frac{4(P_{l,i}-\bar{P}_{g,i})}{R_{ij}}}\right) \Big/ \left(\sum_{j\in\mathscr{N}_i}\frac{2}{R_{ij}}\right), V_j(k-1)-R_{ij}\bar{I}_{ij}, \underline{V}_i\right\} \tag{12}$$

$$\gamma_i(k) = min\left\{\left(\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)}{R_{ij}} + \sqrt{\left(\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)}{R_{ij}}\right)^2 - \sum_{j\in\mathscr{N}_i}\frac{4(P_{l,i}-\bar{P}_{g,i})}{R_{ij}}}\right) \Big/ \left(\sum_{j\in\mathscr{N}_i}\frac{2}{R_{ij}}\right), V_j(k-1)+R_{ij}\bar{I}_{ij}, \bar{V}_i\right\} \tag{13}$$

find a feasible region for $V_i(k)$ whose upper and lower limits are given by the roots of the following equation:

$$V_i(k)^2\sum_{j\in\mathscr{N}_i}\frac{1}{R_{ij}} - V_i(k)\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)}{R_{ij}} + P_{l,i} = \bar{P}_{g,i}. \tag{15}$$

Constraint (6) can be easily rewritten as:

$$-\bar{I}_{ij} \le \frac{V_i(k)-V_j(k-1)}{R_{ij}} \le \bar{I}_{ij}, \tag{16}$$

this implying:

$$V_j(k-1) - R_{ij}\bar{I}_{ij} \le V_i(k) \le V_j(k-1) + R_{ij}\bar{I}_{ij}. \tag{17}$$

The feasible set defined by equations (15), (17) and (7) is given by (11) with parameters $\beta_i(k)$ and $\gamma_i(k)$ defined as in (12)-(13). ■

Thus, under Proposition 1, every bus chooses its voltage level by solving the next optimization problem:

$$\begin{aligned}(OP2) \quad \min \quad & a_iV_i(k)^2 - V_i(k)\left(b_i + c_i\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)}{R_{ij}}\right)\\ & + d_i + e_i\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)^2}{R_{ij}},\\ s.t. \quad & \beta_i(k) \le V_i(k) \le \gamma_i(k).\end{aligned}$$

As it will be explained later, parameter a_i is required to be positive. This can be easily achived by suitably choosing the weights $\sigma_{1,i}, \sigma_{2,i}, \sigma_{3,i}$.

The iterative algorithm described in this section is summarized in the Table I. The initial solution for the iterative OPF described can be found assuming that $V_j(-1) = V_i^{ref}\ \forall j \in \mathscr{N}_i$. The tolerance δ affects the speed of convergence.

TABLE I

ITERATIVE ALGORITHM

Algorithm.
Initialization:
Set the weights $\sigma_{1,i}, \sigma_{2,i}, \sigma_{3,i}$ s.t. $a_i > 0$.
Specify power loads $P_{l,i}$.
Set $V_i^{ref}, \underline{V}_i, \bar{V}_i, \bar{P}_{ij}, \bar{P}_i$
Set an initial workable solution for V_i, $\forall\ i \in \mathscr{V}$
Set tolerance δ.
Iterative update:
while $\sum_{i\in\mathscr{V}_i} \lvert\lvert V_i(k+1) - V_i(k)\rvert\rvert > \delta$
solve (OP2) $\forall\ i \in \mathscr{V}$
Exchange $V_i(k)$ with the neighborhood
end while
Find $P_{g,i}^*$ from V_i^*

IV. Stability analysis

This section presents existence and stability of the Nash equilibrium of the game. First of all, let us consider the unconstrained game. Since (9) is a cost function convex in $V_i(k)$, it is possible to find the minimum of the function by determining $V_i(k)$ such that $\partial J_i(k)/\partial V_i(k) = 0$, obtaining

$$V_i(k) = \frac{b_i}{2a_i} + \frac{c_i}{2a_i}\sum_{j\in\mathscr{N}_i}\frac{V_j(k-1)}{R_{ij}},$$

that is the decision that minimizes the cost function for each agent $i \in \mathscr{V}$ in the iteration step k.

Next, a sufficient and necessary condition for the existence of a unique Nash equilibrium is given. Let us introduce the following compact notation $V \triangleq [V_1, V_2, \dots, V_p]^\top$, $A \triangleq [b_1/2a_1, b_2/2a_2, \dots, b_p/2a_p]^\top$, $B \triangleq diag\{c_1/2a_1, c_2/2a_2, \dots, c_p/2a_p\}$ and

$$C \triangleq \begin{bmatrix} 0 & C_{12} & \dots & C_{1p}\\ C_{21} & 0 & \dots & C_{2p}\\ \vdots & \vdots & \ddots & \vdots\\ C_{p1} & C_{p2} & \dots & 0\end{bmatrix},$$

where $C(i,j) = 1/R_{ij}$ if edge $(i,j) \in \mathscr{E}$ and $C(i,j) = 0$ otherwise.

The sequel lemma is based on Lemma 1 in [9]:

Lemma 1. The game under no constraints admits a unique Nash equilibrium V^*, given by $V^* = (I-CB)^{-1}A$, if and only if condition

$$\det(I-CB) \ne 0 \tag{18}$$

holds.

Proof: From Definition 1, a Nash equilibrium of the game satisfies

$$V_1^* = \frac{b_1}{2a_1} + \frac{c_1}{2a_1}\sum_{j\in\mathscr{N}_1}\frac{V_j^*}{R_{ij}}, \dots, V_p^*\frac{b_p}{2a_p} + \frac{c_p}{2a_p}\sum_{j\in\mathscr{N}_p}\frac{V_j^*}{R_{ij}}.$$

Using the notation introduced above, we can stack all the equations and rewrite the expression as $V^* = A + CBV^*$. By isolating the decision variables it holds $V^* = (I-CB)^{-1}A$. This system of equations has a unique solution if and only if matrix $(I-CB)$ is a full rank matrix fulfilling (18). ■

Let's $\Omega(k) \in \mathbb{R}^p$ be the feasible set for any possible decision vector $V(k)$ following Proposition 1. Then we can state the next Theorem that is based on Theorem 2 in [4]:

Theorem 1. Assume that $V^* \in \Omega(k)\ \forall k$. Then, the iterative algorithm introduced in Table I is stable and converges to the unique Nash equilibrium V^* if the following condition is satisfied

$$\max_{i,j \neq i \in \mathcal{N}_i} \frac{c_i p}{2 a_i R_{ij}} < 1.$$

The proof of this Theorem is omitted due to space issues.

V. Simulation examples

In order to demonstrate the performance of the proposed algorithm, a five terminal HVDC grid shown in Figure 2 was selected. This grid is introduced in [8]. Consider that buses 1 and 2 are offshore generators and buses 3 and 4 are a pool of load that must be fed. For our study, bus 0 is the slack bus and can provide power from an infinity power network to this grid. The reference voltage is $V_i^{ref} = 250\,kV$ for all $i \in \mathscr{V}$ and the line parameters are $R_{12} = 0.5\,\Omega$, $R_{13} = 0.8\,\Omega$, $R_{14} = 0.8\,\Omega$, $R_{23} = 1\,\Omega$, $R_{34} = 1.6\,\Omega$ and $R_{04} = 0.64\,\Omega$. Finally the voltage band defined in (7) is given by $\bar{V}_i = 275\,kV$ and $\underline{V}_i = 225\,kV$ for all $i \in \mathscr{V}$.

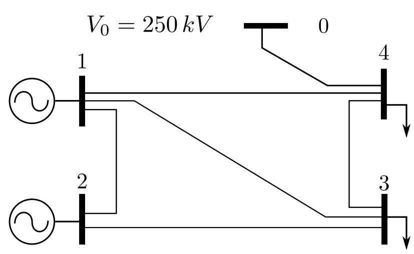

Fig. 2. A five terminal HVDC grid example.

Example 1. The parameters for this example are given in Table II.

TABLE II

Parameters considered in Example 1.

Bus	1	2	3	4
$\bar{P}_{g,i}$	1200 MW	900 MW	0	0
$\bar{P}_{l,i}$	0	0	800 MW	1000 MW
$\sigma_{1,i}$	1€/W	1€/W	1€/W	1€/W
$\sigma_{2,i}$	1000€/V^2	1000€/V^2	1000€/V^2	1000€/V^2
$\sigma_{3,i}$	10€/W	10€/W	10€/W	10€/W

Note that the power available at buses 1 and 2 is enough to feed the loads of the right side of the grid. For this case no limitation in the power lines is considered. By running the proposed algorithm, results in Table III are obtained.

TABLE III

Results for Example 1. Unconstrained case.

Bus	$V_i(kV)$	$P_{g,i}(MW)$	Line	$I_{ij}(A)$
0	250.000	454.65	$0-4$	1818.62
1	250.522	1088.6	$1-2$	313.98
2	250.365	267.34	$1-3$	1923.48
3	248.984	0	$1-4$	2107.89
4	248.836	0	$2-3$	1381.80
$\sum J_i$	2995.6M€		$3-4$	92.21

Let us introduce for the next simulation the sequel constraints: $\bar{I}_{13} = 1600A$ and $\bar{I}_{14} = 1996A$. The results obtained are detailed in Table IV.

TABLE IV

Results for Example 1. Constrained case.

Bus	$V_i(kV)$	$P_{g,i}(MW)$	Line	$I_{ij}(A)$
0	250.000	558.26	$0-4$	2233.06
1	249.952	631.76	$1-2$	799.50
2	250.352	620.75	$1-3$	1600
3	248.672	0	$1-4$	1726.73
4	248.571	0	$2-3$	1680
$\sum J_i$	4137M€		$3-4$	63.21

Note that in this case, the demand of power from the grid arises, being more profitable for the agents to obtain the power from the infinity DC power source instead of from renewable resources, directly affecting to the loses at power lines. It is worth pointing out that the constrained game presents a slower convergency rate to the equilibrium than the unconstrained game. Figure 3 plots a measure of the convergence time through a parameter α defined as:

$$\alpha(k) = \sum_{i=1}^{4} \frac{|V_i(k) - V_i^*|}{V_i^*},$$

where V_i^* is the voltage value of node i once reached the Nash equilibrium. However, the equilibrium is met in less than 20 iterations.

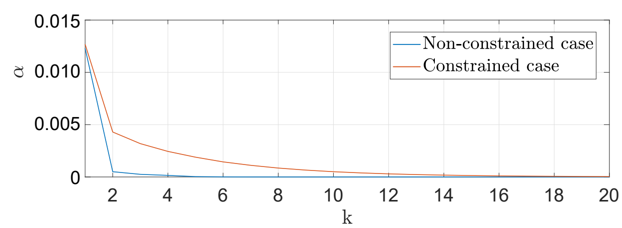

Fig. 3. Comparative between the convergence time for the constrained and non-constrained case.

Example 2. This example explores the effect that the weights in the cost function (9) has in the Nash equilibrium of the game. Consider the next limitation in the lines: $\bar{I}_{13} = 1600A$, $\bar{I}_{14} = 1996A$ and $\bar{I}_{23} = 1996A$. The parameters for the two cases are listed in Table V.

TABLE V

Parameters considered in Example 2.

Bus	1	2	3	4
$\bar{P}_{g,i}$	1200 MW	900 MW	0	0
$\bar{P}_{l,i}$	0	0	800 MW	1000 MW
		Case 1		
$\sigma_{1,i}$	1€/W	1€/W	1€/W	1€/W
$\sigma_{2,i}$	1€/V^2	1€/V^2	1€/V^2	1€/V^2
$\sigma_{3,i}$	1€/W	1€/W	1€/W	1€/W
		Case 2		
$\sigma_{1,i}$	1€/W	1€/W	1€/W	1€/W
$\sigma_{2,i}$	10000€/V^2	1€/V^2	1€/V^2	1€/V^2
$\sigma_{3,i}$	1€/W	1€/W	1€/W	1€/W

Note that in the second case, according to $\sigma_{2,2}$, agent 2 requires a voltage really close to the reference, V_i^{ref}, in order to minimize its local cost function.

TABLE VI

RESULTS FOR EXAMPLE 2.

	Case 1		Case 2	
Bus	$V_i(kV)$	$P_{g,i}(MW)$	$V_i(kV)$	$P_{g,i}(MW)$
0	250.000	441.19	250.000	525.92
1	250.468	471.07	250.056	386.19
2	251.266	900	250.872	900
3	249.282	0	248.917	0
4	248.871	0	248.653	0
$\sum J_i$	15.87M€		47.43M€	

Observe that for the minimization of J_2 the voltage level at bus 2 is closer to the reference in case 2. As a counterpart, the other agents obtain a value of the cost function greater than in the first case. This can be checked in the next table that shows the difference between the cost function in cases 1 and 2.

TABLE VII

COMPARATIVE BETWEEN THE TWO CASES IN EXAMPLE 2.

bus	1	2	3	4
$J_i(\text{Case 1}) - J_i(\text{Case 2})\,(M€)$	115.422	-0.901	0.343	0.585

Example 3. This example shows the robustness of the method for a 24 hours case study. Let us consider the values for $\sigma_{1,i}, \sigma_{2,i}, \sigma_{3,i}$, proposed in Table II. We will consider a PV power plant connected to bus 1 and a Wind power plant connected to bus 2 with a maximum generation power of $1200\,MW$ and $900\,MW$, respectively. A typical residential load profile will be used with a maximum power demand of $800\,MW$ and $1000\,MW$ at buses 3 and 4 respectively.

In Figure 4 the generation and load profiles considered are exposed as well as the result of the voltage values of the buses once reached the Nash equilibrium of the problem.

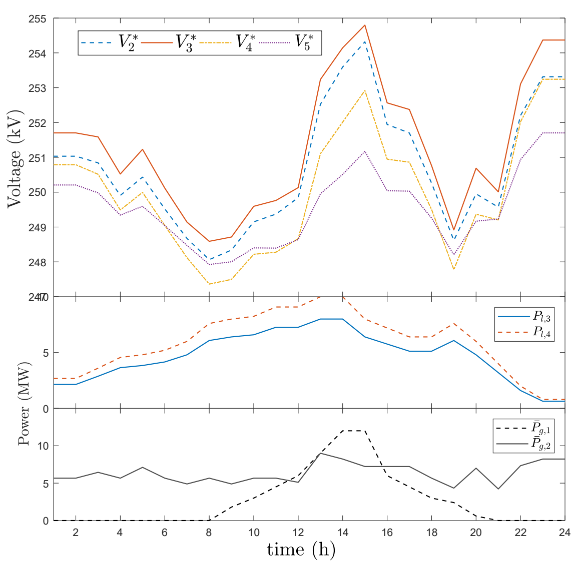

Fig. 4. Generation and load profiles.

VI. CONCLUSIONS AND FUTURE WORK

A game-theoretical framework for the voltage regulation problem in HVDC grids has been introduced. Every bus of the network is able to minimize a cost function that penalizes the power required from the grid, the voltage deviations and the loses through the lines. Conditions for the convergence of the game to a Nash equilibrium has been introduced. Simulation results have reflected the robustness of the method for the constrained scenario.

This work establishes a preliminary result and the mathematical basis to more complex problems considering AC networks. Providing necessary and sufficient conditions under which the decision variables always belong to the feasible region during the negotiation will be tackled as a future work.

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