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On concept abstraction algorithms

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***Abstract*— We address two fundamental issues in the computational theory of mind (CTM): the extreme accuracy of visual signal perception and the quick recall of relevant information from a vast memory. The key component of our approach is an algorithm that converts the spatial signal into a temporal representation. The model is motivated by human vision, but it is valid for concept abstraction and analogical thinking in general.**

I. Introduction

Human and animal minds are constantly searching for relations among event sequences, observed or internal. In fact all objects (as well as concepts and their interactions) are defined by the relations among their constituents. Relations are much more stable and useful than raw signal pieces. Remembering the repeating relations is effective in the survival game. We ask the following questions: 1) how does the mind generate invariant relations, 2) how should invariant relations be encoded to enable fast retrievable memory with a huge capacity, and 3) what are the enabling algorithms for these processes?

We first tackle the questions about visual perception related relation generation. We see many objects in our visual scene simultaneously, and when focusing we see some very clearly. Our two eyes each receive a different set of light signals, but we don't see doubles. Even in just one eye, we have millions of photo receptors in different places, and they work together to provide a seamless perception of the visual scene. How do they collaborate?

Our hypothesis, which provides a vital cue to many of the puzzles of the mind behavior, is that all these photo receptors are sampling the same set of signal values that characterize the visual scene, thereby helping each other in the spirit of the law of large numbers, where the repeated observations for a fixed statistical parameter help reducing the estimation inaccuracy.

Similarly, in unsupervised object recognition, a fundamental issue is the definition of an object. If a cat is an object, is an ear of a cat also an object? How about the ear tip? This raises an algorithmic question about signal processing in the early stages of the visual pathway. The spatial visual signals need to be converted to time signal representations to travel and communicate. Such time signals, representing a small visual region, should be able to add to those of the neighboring regions in order to represent the larger region. Such additivity would demand the processing algorithms at this level be linear. We develop such processing algorithms in Section II and discuss relation generation with such models in Section III.

Finally, to enable the memory network to have a large capacity, we make the network nodes active. The usual memory networks use only the link weights to code the content and the nodes are assumed to serve as the homogeneous connection hubs only. In contrast, the nodes in our proposed memory network are cascaded resonating circuits with a wide range of resonance frequencies. This will be discussed in Sections IV-VII.

Visual signal processing is not very different from concept abstraction in the mind. This is not strange since Nature repeats successful mechanisms. To this end we use the following outline bullets to provide a background for some statements in the paper.

- Intelligence is based on matching signals with relational experiences, not pixel or sound experiences;
- For this, signals must be converted to a format aimed at relation generation;
- Relational similarity testing must be quick and robust;
- Such testing is ubiquitous at all levels of experience matching;
- Language is the algebra of relation sets (and thus an intelligence amplifier);
- We develop algorithms for signal conversion, relation generation, memory retrieving and concept abstraction.

II. An analysis of visual signal sampling mechanism

Our visual scene is stable and accurate despite the movements of our body and eye balls. We believe this is due to the fact that each of a large number of photoreceptor cells is sampling the same set of signal values. These values are closely related to the phase shift in each Fourier element of the signal marked by the spatial frequency pair (u, v).

To illustrate the process of acquring the phase related signal we look at the following simplified model. Consider

$$\dot{\psi}(x,t) = \int_{D_s} A(x,x')\psi(x',t)dx' \tag{1}$$

with $\psi(x,t)$ the "system state" at location x and time t, D_s a small region covered by the concerned signal sampler, or "neuron". $A(x,x')$ is the symmetric (for ease of discussion)

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spatial signal with the spectral decompostion

$$A(x,x') = \sum_i \alpha_i \phi_i(x)\phi_i(x'). \tag{2}$$

This leads to (assuming an uniform initial condition $\psi(x,0) \equiv 1$)

$$\int_{D_s} \psi(x,t)dx = \sum_i e^{\alpha_i t} \left(\int_{D_s} \phi_i(x)dx \right)^2. \tag{3}$$

As can be seen the signal $\int_{D_s} \phi_i(x)dx$ is being "carried" by the time function $\exp(\alpha_i t)$. Note that the model treats the image as an operator acting on the system state which enables the separation of α_i and $\phi_i(x)\phi_i(x')$. This is a major departure from the conventional approches of spatial signal processing where the singal is viewed as a passive object. This also confirms to the biology reality where the light signals keep acting on the retina.

We now argue that the integral of the eigenfunction over a fixed region provides a phase-related signal. For the simplicity of illustration we consider a 1D function $f(x)$ on the real line. The Fourier series coefficients of $f(x)$ form a sequence of complex numbers each with a magnitude and a phase. Consider a signal sampler that sums the Fourier series terms over an interval $[\alpha, \beta]$. We have for the nth Fourier term $a_n \sin(n\omega x) + b_n \cos(n\omega x) = r_n \sin(n\omega x + \phi_n)$, that (ignoring r_n, which becomes a factor in the carrier mode/frequency as mentioned)

$$\begin{aligned} & g^n_{\alpha\beta}(f) \\ = & \int_\alpha^\beta \sin(n\omega x + \phi_n)dx \\ = & \frac{2}{n\omega} \sin\left(\frac{\alpha+\beta}{2} n\omega + \phi_n\right) \sin\left(\frac{\beta-\alpha}{2} n\omega\right) \end{aligned} \tag{4}$$

where all items except ϕ_n are fixed by the sampler position and size in the visual sensory system and are independent of the function $f(x)$. The difference between two functions $f_1(x)$ and $f_2(x)$ will be reflected in their ϕ_n's and eventually in the differences between $g^n_{\alpha\beta}(f_1)$ and $g^n_{\alpha\beta}(f_2)$. Note that all the samplers at different positions are getting the information about the same ϕ_n's. Note also that the phase shifts are much more important than the magnitude as demonstrated in some image processing examples. Furthermore the purpose of such signal processing is not to reconstruct the original image but to be able to tell the similarities and differences of the inputs. The above differences would be accurately estimated and the high resolution perception of the image differences is enabled by "all eyes on the same ball". One may note that the $1/(n\omega)$ factor makes an individual high frequency component small. However the numbers of the high frequency samplers would be larger due to the smaller physical sizes, an observation motivated the method of Mel-frequency cepstrum in audio signal processing.

In the theory about the Fourier transform of causal functions the Kramers-Kronig relations and the Bode gain-phase relation assert that the magnitude and the phase determines each other due to the causality constraint [1]. In the case of 2D images the finiteness of the image provide the constraints [10]. An image is the superposition of 2D Fourier elements and each 2D sampler is getting the phase-like information for all elements with strong enough presence.

To detect the differences we note that each $g^n_{\alpha\beta}(f)$ is carried by a different frequency. The resonators in the Resonator Chain Unit (RCU) networks described later will be able to receive the corresponding quantities. In the brain such quantities are likely to be thresholded into binary ones to enable or disable certain RCUs. Then the large numbers would compensate for such quantization to achieve accuracy.

We note that such "all eyes on the same ball" mechanism explains the visual accuracy, stability and object recognition robustness. For example, the phase shifts ($\phi_n(t)$) for a moving visual scene ($V(x,t)$) are collected from many retina receptors swept by the moving image.

We now develop a basic image processing algorithm unit. The model is

$$\psi_{tt}(x,t) + \gamma\psi_t(x,t) = \alpha\psi_{xx}(x,t) + V(x)\psi(x,t) \tag{5}$$

$$h(t) = \int_{D_s} \psi(x,t)dx \tag{6}$$

with $\psi(x,t)$ a function of the location vector x and the time t, $V(x)$ the image and γ, α real constants. We use this to convert the spatial signal $V(x)$ into a temporal one $h(t)$ with a wavy nature so as to communicate with other parts. We note that $V(x)$ can be time varying $V(x,t)$ to model eye and image movements. In biology reality the whole region is bounded by the eye sight of visual scene and is gradually blurred outside the fovea. Biology study found that the eyes are in constant motion[17], supporting that the receptors in different locations should be sampling the same set of signal values.

To see how the above equations convert signals we use the ansatz

$$\psi(x,t) = X(x)T(t) \tag{7}$$

to substitute in the above equation to see

$$X(T'' + \gamma T') = (\alpha X'' + V(x)X)T. \tag{8}$$

If $T'' + \gamma T' = \lambda T$ and $\alpha X'' + V(x)X = \lambda X$ then the equation is satisfied. Note that

$$(\alpha\Delta + V(x))X = \lambda X \tag{9}$$

is similar to the time independent Schrodinger equation. Under quite reasonable assumptions one can employ spectral decomposition and for the infinite region an eigenfunction $X(x)$ should be alternating similar to a sinusoid. The "phase shifts" of the eigenfunctions reflect the changes of the "potential" or image $V(x)$.

The left side gives us

$$T'' + \gamma T' = \lambda T \tag{10}$$

which is a decayed wave equation. This $T(t)$ is the carrier signal for the spatial phase-like information associated with each eigenvector of the operator $\alpha\Delta + V(x)$. The phase-like information is obtained by integrating $\psi(x,t)$ over a small region D_s.

That the time function $h(t)$ represents the spatial signal $V(x)$ can be seen from the realization theory of linear dynamic systems if we approximate $\alpha\Delta + V(x)$ by a large matrix and double the state space to make the equation first order in time [2]. Since the sampling grid is fixed the differences in the visual scenes can be detected from the time function representations via resonance decompositions. Also the relations among the $h(t)$s will form features such as edges.

III. Magnus expansion exhibits relational information

The time function $h(t)$ contains rich information about the relations among the constituents of the incoming signal $V(x,t)$. This can be seen from the Magnus expansion described below.

Consider a general time varying linear differential equation for the n-dimensional vector function $Y(t)$

$$\frac{d}{dt}Y(t) = A(t)Y(t), Y(0) = Y_0 \tag{11}$$

with an $n \times n$ matrix $A(t)$. This would be the case if in (5) we approximate the operator $\Delta + \alpha V(x,t)$ by a matrix $A(t)$ and double the state space to make the equation first order in time. If $[A(t_1), A(t_2)] = A(t_1)A(t_2) - A(t_2)A(t_1) = 0$ for all t_1, t_2 pairs, for example when $A(t) \equiv A$, we have a matrix exponential solution

$$Y(t) = \exp\left(\int_{t_0}^{t} A(s)\, ds\right) Y_0. \tag{12}$$

In general one could have

$$Y(t) = \exp\left(\Omega(t)\right)\, Y_0, \tag{13}$$

with the series construction

$$\Omega(t) = \sum_{k=1}^{\infty} \Omega_k(t). \tag{14}$$

The Magnus expansion provides a solution to the linear time varying matrix equation above, with the first three terms as

$$\begin{aligned}
\Omega_1(t) &= \int_0^t A(t_1)\, dt_1, \\
\Omega_2(t) &= \frac{1}{2}\int_0^t dt_1 \int_0^{t_1} dt_2\ \left[A(t_1), A(t_2)\right], \\
\Omega_3(t) &= \frac{1}{6}\int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^{t_2} dt_3 \\
&\quad \left([A(t_1), [A(t_2), A(t_3)]] + [A(t_3), [A(t_2), A(t_1)]]\right).
\end{aligned}$$

Intuitively when $A(t)$ at different ts are commutative the solution is provided by $\Omega_1(t)$. In general the solution needs more terms in the exponent and the Magnus series does this in a systematic way. Our usage here is simply to support the idea that the spatial-temporal signal transformation via equations (5) and (6) generates an output time function containing rich relational information among different spatial and temporal parts of the input signal $V(x,t)$. When the output time function $y(t)$ hits the RCU networks described later such relational information forms RCU clusters in memory. These relational memory will be used to form common features via repeated experiences, and will serve as the cues and constraints for retrieval.

In practical algorithms one can use the Fourier phase arrays to represent signals. Properties of Fourier transform related to transformations of the original spatial signals such as translation, scaling, rotation, and differentiation are very useful in obtaining invariant features.

IV. Resonance transients for signal component recognition

Signal transmissions in the brain are five or six orders of magnitude slower than modern CPUs [3]. How could the slow and noisy neurons provide quick and accurate visual perceptions and fast concept retrieval? Here we discuss a plausible mechanism using the resonance transients of the RCU components.

We analyze the transient behavior of a second order dynamic system stimulated by a exponentially decaying sine (EDS) function input. Specifically we check

$$\begin{aligned}
h(t) &= e^{-\lambda t}\sin(nt) * e^{-\mu t}\sin(mt) \\
&= \int_0^t e^{-\lambda t}\sin(n\tau) e^{-\mu t}\sin(m(t-\tau)) d\tau.
\end{aligned} \tag{15}$$

We assume the neural circuits are efficient high Q filters with a very small μ. We also assume the input signal does not decay too fast and thus is with a small λ. The latter can be relaxed. Under these assumptions we have (treating both λ and μ as zero):

$$\begin{aligned}
& h(t) \\
= & \frac{m\sin(nt) - n\sin(mt)}{m^2 - n^2} \\
= & \frac{m}{m^2-n^2}\left[\sin(nt) - \sin(mt)\right] + \frac{m-n}{m^2-n^2}\sin(mt) \\
= & \frac{2m}{m^2-n^2}\left[\sin(\frac{n-m}{2}t)\cos(\frac{n+m}{2}t)\right] \\
+ & \frac{1}{m+n}\sin(mt).
\end{aligned} \tag{16}$$

When m and n are large the impact of the last term is minimal. The first term is a sine wave at frequency $|n+m|/2$ modulated by a low frequency sine wave at frequency $|n-m|/2$. When $n - m \to 0$ it can be shown that this term goes to t, a basic phenomena in resonance. When n and m are close this term results in an envelop starting going down at around $\pi/|n-m|$. When the difference $|n-m|$ increases this point moves closer to the time origin. Note that the amplitude of the components in the incoming signal does not have much effect on the behavior described above.

Check Figure 1 for a small scale experiment where an EDS signal combination is convolved with single EDS signal components to recognize the signals that are in the combination. The responses for those that are in the input combination are marked by red, cyan, magenta and yellow. The responses for the rest are marked by green, blue and

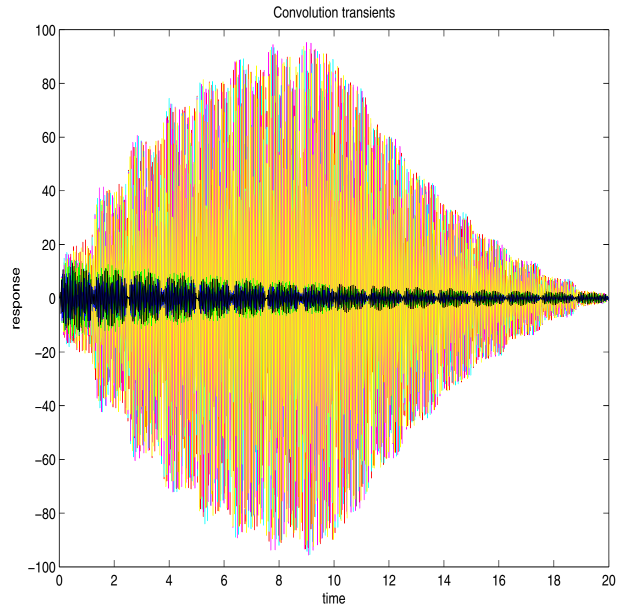

Fig. 1. Using Resonance Transients to Recognize Input Components

black. One can see that the response differences are quite recognizable for the small frequency differences between the two sets.

functions and legends of Figure 1. per MATLAB code

```
t=0.0:0.01:20.0;
a=exp(-0.1*t).*sin(500*t);
b=exp(-0.1*t).*sin(505*t);
c=exp(-0.1*t).*sin(510*t);
d=exp(-0.1*t).*sin(515*t);
e=exp(-0.1*t).*sin(495*t);
f=exp(-0.1*t).*sin(490*t);
g=exp(-0.1*t).*sin(520*t);
x=conv(a+b+c+d,a, red;
y=conv(a+b+c+d,b), cyan;
z=conv(a+b+c+d,c), magenta;
w=conv(a+b+c+d,d), yellow;
u=conv(a+b+c+d,e), green;
v=conv(a+b+c+d,f), blue;
s=conv(a+b+c+d,g), black
```

V. Resonator chain as memory unit

We now focus on how to code the EDS signal combinations in memory. To this end we consider a set of resonators sequentially arranged so that each would get excited in the time order in which the matching frequency occurs in the input signal. We call this arrangement a Resonant Chain Unit, or RCU. RCU uses the resonance mechanism to code information in an unsupervised manner.

A RCU is composed of several resonators with different resonating frequencies. There are fixed time delays between the successive resonators. For an illustrative example consider the case of a RCU consisted of 4 resonators $R1 \sim R4$ with frequencies $f_1 \sim f_4$, respectively. The RCU is build as

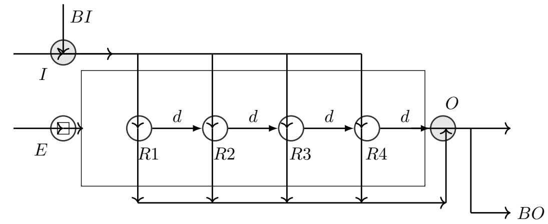

Fig. 2. Basic structure of a Resonator Chain Unit (RCU)

$R_1 \rightarrow d \rightarrow R_2 \rightarrow d \rightarrow R_3 \rightarrow d \rightarrow R_4$. When R_1 is excited by a matching frequency close to f_1 in the input its output goes through the delay d and gets the R_2 resonator ready. But R_2 has to also receive a signal with a matching frequency close to f_2 to get excited. This process goes on until either all resonators are excited, or the unit excitation is aborted. Figure 2 is a schematic structure of a RCU. RCU could be forced to excite by a signal at the circled box E. Such signals could be due to group connections via simultaneity, attention, and other learning-based connections. RCU output can also branch out (BO in Figure 2) to get other RCU input port ready to respond to an input. RCU input port has a control signal BI which receives signals from other RCU's BO port. The control signals have a binary nature namely they serve as on-off switches.

When a RCU is excited it generates output as the sum of all its resonators with the delays between them. Note that in Figure 2 the sum signal can go out only when R4 is excited. The sum signal goes on to other parts looking for similar RCUs to excite. In the areas immediately behind the signal receptors the RCUs reset to rest condition shortly after the excitation input signal vanishes. However the RCUs in the memory region would sustain the excitation states longer in order to form connections with other excited RCUs to form a RCU cluster.

A RCU could also serve as a node in a super RCU to code more complicated sequences. RCUs may also be used to code spatial-temporal information and to construct hierarchies of memory trees to facilitate fast retrieval of information coded by long RCU sequences. The spatial-temporal signal conversation algorithms discussed before is generally applicable to convert the signal of a cluster of spatially connected RCUs to a time function.

RCU interactions are quick since the resonance effect accumulates from the moment when the external signal arrives. In other words, when a sinusoid function hits a group of resonators the resonating response curve deviates from others immediately. While the initial differences are small, they are often enough to inhibit other resonators. This is similar to the selection rule in the ordinal optimization method [11][4] referred as horse race rule. As the name implies, often times the transient behavior of a system reveals its potential when ordinal comparison is the decision base. In our current situation it is known in neuroscience that most neurons when excited tend to inhibit other neighboring neurons and render them silent. If we restrict to a mathematically simplified scenario where a causal sinusoid function convolve with the impulse response function of a second order linear system, it can be seen that with the above convolutional horse race selection the input sinusoid function would very quickly select a receiver that resonances with it. This could be the basis for the quickness of image recognition exhibited in human and many animals. In fact this could also be the basis for the quickness of abstract thoughts where relevant concepts in the memory are recalled very quickly.

Resonance transients based quick recall assumes linear signal processing and enables signal aggregation and decomposition in large scale, which are needed to address various binding problems. In previous works [5][6][7][8][9] we connect the spatial-temporal conversion in linear control theory to the researches relating the geometric shapes to their Laplacian spectrums [13], [16]. Although nonlinear mechanisms are ubiquitous in biology systems including the brain, linear signal processing in certain levels could nevertheless be essential.

VI. RCU NETWORKS

We discuss some uses of the RCU networks. Figure 3 shows a small sector of a RCU network to illustrate some connection possibilities.

- **Dictionary style retrieval.** A RCU with BO port can be used to control the access of other RCUs. An important usage is to carry a dictionary style retrieval. A query signal is a string of EDSs. The first portion of such a query signal could excite some RCUs to let their BO port sending an "on" signal to open the entrance of some other RCUs. Now the query string drops the first portion and moves to look for opened RCUs that match the second portion, and so on.
- **Learning with RCU networks.** RCUs could form a cluster and excitation of enough number of the member RCUs of a cluster would excite the entire cluster. The E port connects the RCU members in a cluster. Such connections would gradually decay and enable learning new connections.
- **Relation recording.** We consider the playground of mind as a huge fabric in the 3 dimensional space with randomly and densely distributed appositions between the axons and dendrites ready to be connected. The relational signals are EDS strings and naturally form RCU clusters for recording and for link control. The links that have been used more often would be easier to access, since such links would have more widely spread access branches than the less used links.
- **Simultaneity recording.** Node $E1, E2, E3$ in Figure 3 denote the output branches that could be sent to other RCU's external triggering point E which, upon receiving enough inputs would excite the RCU. This enables a RCU memory group to be excited together when needed. Different groups may overlap to cover the entire set of simultaneity. Overlapping is used to hook up small groups to form a large group. In visual image coding such overlapping coding is crucial to keep the neighboring information of the image patches. But the mechanism works generally for clusters of RCUs for keeping their connectivity information when multiple concepts interact.

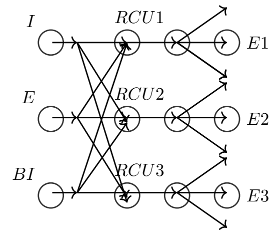

Fig. 3. A small sector of a RCU network

Although RCU networks are feasible and serve as a plausible computation structure for biological intelligence, one can bypass them in computer algorithms for general artificial intelligence. For example the similarity testing of the Fourier phase arrays could be used to replace the Laplacian-Integral-RCU operations.

VII. ABSTRACTION AND ANALOGICAL THINKING

Abstraction is the process of sifting the similarities from the instances or the differences among the instances. One of the impressive examples of abstract similarity testing is the psychology phenomena referred as analogical reminding, where a sequence of current events reminds a possibly remote experience that is only similar in an abstract manner [12][14][15]. Analogical reminding is featured by the quickness, the abstractness, and the involuntariness of a memory recall carried out by the relatively slow neurons. The quickness calls for large scale content addressable memory. The abstractness needs the ability to sift commons from instances. And the involuntariness demands automatical recall. The spatial/temporal signal conversion described above could help achieving these.

To explain the idea of abstraction we use an example of shape relation similarity testing. The shapes in the visual field are represented as clusters of connected RCUs. Using the above conversion scheme one has the temporal representation of a RCU cluster, which codes the spectral information of the shape images in the operator/matrix $\alpha\Delta + V(x)$. The relation between two such matrices can be coded in another matrix/network/RCU cluster. Now we consider simple analogies in which one tests the similarity between the relations that relate more concrete concepts. Consider two pairs of geometrical shapes (A, B) and (X, Y). Suppose it makes sense to say that "A is to B as X is to Y", for example "a square A is to a rectangle B as a circle X is to an ellipse Y". The following diagram shows the relational matrices R_{AB} and R_{XY} (which would be coded into RCU clusters) between the shape pairs.

Now the relational similarity statement that "A is to B as X is to Y" can be understood as the similarity of matrices R_{AB} and R_{XY}, which is algorithmically the same as the similarity testing of the two concrete shapes such as two squares.

$$\begin{array}{ccc} A & \overset{R_{AX}}{\dashrightarrow} & X \\ {\scriptstyle R_{AB}}\downarrow & & \downarrow{\scriptstyle R_{XY}} \\ B & \overset{R_{BY}}{\dashrightarrow} & Y \end{array}$$

More concretely let A be a square and X be a circle, and B and Y are their stretched versions namely a horizontal rectangle and a horizontal ellipse, respectively. In terms of the RCU cluster representations one checks the similarity of the similar components between A and B and those between X and Y, as well as the similarities of the differences between A and B and those between X and Y. The concept of "stretching a shape" would be represented by the commonalities emerged from such comparisons, which are the Fourier phase array changes from one to the other, forming the RCU cluster representation of the abstract concept of "stretching a shape". Such a process could iterate and generate many "abstract" relations for later use. Note that the relations represented by the dashed lines could generate the representation for the concept of "rounded smoothing".

The relational similarity testing does not need conscious instructional effort and leads to an involuntary experience, not unlike recognizing facial expressions. Recognizing facial expressions such as a smile is crucial for human interactions. Consider two faces A and X, and their smile versions B and Y. The relational matrices R_{AB} and R_{XY} are similar and the similarity defines the concept of "smile". Human babies perhaps sift this out from smile faces early on. The relational network of smile could make connections to relevant concepts and it is possible that the silly smiles and sounds made by adults when holding a baby form the neurological base for the original sense of humor, which would be triggered by concrete or abstract silliness depends on the storage of abstractions in the mind.

The relational networks R_{AB} and R_{XY} are generated automatically (due to resonance among components of the temporal representations A, B, X, Y) and stored in the memory as part of the experiences. Such relational networks, and the relational networks for these relational networks, are all generated in subconscious and sitting there ready to be excited. This may help explaining the analogical reminding phenomena mentioned before.

Furthermore, since the above scheme in testing the relational similarity is the same as the "concrete" shape similarity, the process can be repeated to extract multiple levels of relations that exists between different lower level relations. The spatial-temporal signal conversion plays a central role in this recursion. Since a RCU cluster can be represented by a matrix, the algorithm capable of converting the information carried in a matrix or by a time function back and forth enables such recursion. This process generates many RCU clusters representing all sorts of relations which serve as cues and constraints for memory recall.

Finally we quote the renowned scientist Stanislaw Ulam from [18]. "There must be a trick to the train of thought, a recursive formula. A group of neurons starts working automatically, sometimes without external impulse. It is a kind of iterative process with a growing pattern. It wanders about in the brain, and the way it happens must depend on the memory of similar patterns. Very little is known about this. Perhaps before a hundred years have passed this will all be part of a fascinating new science."

VIII. Concluding Remarks

Unsupervised learning of concepts is essential for analogical thinking, which is considered a hallmark of human intelligence. We argue that signals in the brain constantly generate relations among their constituents. Unsupervised learning is then accomplished by fast similarity testing of the stored relation sets using resonance transients and time sequenced memory units. We discuss specific algorithms to achieve these functions.

References

[1] J. Bechhoefera. Kramers-Kronig, Bode, and the meaning of zero. *American Journal of Physics*, 79(1053), 2011.

[2] R.W. Brockett. *Finite Dimensional Linear Systems*. Classics in Applied Mathematics. SIAM, 2015.

[3] A. Shashua et al. Is the brain a good model for machine intelligence? *Nature, February 2012*, February 2012.

[4] P. Glynn and S. Juneja. Ordinal optimization: A nonparametric framework. *Proceedings of the 2011 Winter Simulation Conference*, 2011.

[5] W. Gong. Can one hear the shape of a concept? In *Proceedings of 31st Chinese Control Conference*, pages 22–26, 2012.

[6] W. Gong. Transient response functions for graph structure addressable memory. In *Proceedings of 52nd IEEE Conference on Decision and Control*, pages 6978–6985, 2013.

[7] W. Gong. Spatial/temporal coding for involuntary concept abstraction using linear dynamic systems. In *Proceedings of 2015 European Control Conference*, 2015.

[8] W. Gong and Y.-C. Ho. On fast retrieval of relational experiences. In *Proceedings of the 2017 American Control Conference*, 2017.

[9] W. Gong and B. Jiang. Relation generation and fast similarity testing for unsupervised learning. In *56th IEEE Annual Conference on Decision and Control, Melbourne, Australia*, pages 4231–4236, 2017.

[10] M. Hayes. IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP '81, Atlanta, Georgia. IEEE, 1981.

[11] Y.-C. Ho, Q. Zhao, and Q. Jia. *Ordinal Optimization: Soft Optimization for Hard Problems*. Springer, 2007.

[12] D. Hofstadter and E. Sander. *Surfaces and Essences: Analogy as the Fuel and Fire of Thinking*. Basic Books, 2013.

[13] M. Kac. Can one hear the shape of a drum? *American Mathematical Monthly*, 73(4, part2):1–23, 1966.

[14] S. Pinker. *The Stuff of Thought*. Viking Penguin, 2007.

[15] S. Pinker. The cognitive niche: Coevolution of intelligence,sociality, and language. *PNAS*, 107(May):8993–8999, 2010.

[16] M. Reuter, F. Wolter, and N. Peinecke. Laplace-beltrami spectra as "shape-dna" of surfaces and solids. *Computer-Aided Design*, 38(4):342–366, 2006.

[17] M. Rucci and J.D. Victor. The unsteady eye: an information-processing stage, not a bug. *Trends in neurosciences*, 38 4:195–206, 2015.

[18] S.M. Ulam. *Adventures of a Mathematician*. University of California Press, 1991.