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Min-max-min Robust Optimization for Adaptivity-Constrained Economic Dispatch*

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***Abstract*— With large-scale penetration of renewable energy generators, the prediction uncertainty of power demand has highly increased in recent years. In this paper, we propose a new approach for economic dispatch to obtain the tolerance interval of electric facilities' operations, which minimizes the worst-case dispatch cost while guaranteeing the adaptivity of actual dispatch schedule to any realization of net power demand predicted. Representing the dispatch cost as a quadratic function of the power outputs by fuel-based generators and the charging/discharging power of batteries, we formulate this problem as a min-max-min robust optimization problem. To solve it, we first show that the worst-case dispatch schedule consists of either maximum or minimum values in the tolerance intervals at each time slot. Then, we reduce the problem to a minimax problem and solve it using Benders decomposition to deal with the high computational cost. Finally, we verify the effectiveness of our proposed method by numerical simulations.**

NOMENCLATURE

Parameters

ΔT	Length of each time slot
$\overline{D}_t$, $\underline{D}_t$	Upper/lower limit of net power demand at the tth time slot
$\overline{P}$, $\underline{P}$	Upper/lower limit of generator's power output
$\overline{R}$, $\underline{R}$	Upper/lower limit of generator's ramp rate
$\overline{U}$	Upper limit of battery-charging power
$\overline{V}$	Upper limit of battery-discharging power
B	Capacity of battery
B_0	Initial stored energy
C_1^{c}, C_2^{c}	Battery's degradation cost coefficients for charging
C_1^{d}, C_2^{d}	Battery's degradation cost coefficients for discharging
C_1, C_2	Fuel cost coefficients of generator
e^{c}	Charging efficiency of battery
e^{d}	Discharging efficiency of battery
T	Number of time slots

Decision Variables

η	Auxiliary variable for Benders decomposition
$\overline{p}_t$, $\underline{p}_t$	Maximum/minimum tolerable power output by generator at the tth time slot
$\overline{u}_t$, $\underline{u}_t$	Maximum/minimum tolerable charging power of battery at the tth time slot
$\overline{v}_t$, $\underline{v}_t$	Maximum/minimum tolerable discharging power of battery at the tth time slot
d_t	Net power demand at the tth time slot
p_t	Power output by generator at the tth time slot
u_t	Charging power of battery at the tth time slot
v_t	Discharging power of battery at the tth time slot

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I. INTRODUCTION

Economic dispatch (ED), defined as "the operation of generation facilities to produce energy at the lowest cost to reliably serve consumers" [1], composes various scheduling problems in power systems, by which the system can be prepared for any uncertainty assumed. For example, many unit commitment problems include ED to guarantee the reliability of generators' on/off states against the prediction uncertainty of power demand [2], [3], [4], [5]. However, as for ED, most of them focus on the existence of dispatch schedules against all demand scenarios expected, which does not guarantee their adaptivities to the actual demand. This is because they consider each multi-period power demand scenario to be fixed, when each dispatch solution at a certain time slot becomes meaningless unless the corresponding expected demand scenario is realized along the entire planning periods, i.e., after and before the time slot. Consequently, if demand at the next time slot deviates from the prediction, there might not exist any dispatch solution which meets it. While control-based optimization problems can deal with the adaptivity of real-time dispatch solutions, [6], [7] (see also their references), far less extensive works exist which address that of predetermined multi-period dispatch schedules.

In this paper, we propose a new approach for ED to obtain the tightest *tolerance interval*, not a real value as a dispatch solution for a certain demand scenario, for each electrical facility at each time slot, which guarantees that any realization of power demand at the next time slot can be met. Furthermore, among various series of such tolerable intervals along the whole time period, we aim at finding the one which yields the least worst-case dispatch cost.

A research [2] is closely related to our purpose in that an approach considering the feasibility of a redispatch problem is used when solving ED for a given forecast. However, our approach differs from it since we seek to minimize the dispatch cost associated with the worst-case scenario, not the one initially expected. Other relevant methods are devised in [8], [9], [10], where the necessity for the optimal dispatch schedule is obtained as intervals. Nevertheless, they do not take into account dynamic constraints, and thus the adaptivity in a practical sense, of dispatch schedules.

As the setting for this paper, we assume that power systems include fuel-based generators whose upper and lower limits of ramp rate impose dynamical constraints on their power outputs, storage batteries whose capacities impose those on their charging/discharging power, and renewable generation systems whose power outputs are uncertain. Additionally, we suppose that the prediction uncertainty of net power demand at each time slot, i.e., power consumption minus power output of renewable energy systems, which should be met by operations of the generators and batteries, is given as an confidence interval.

The rest of this paper is structured as follows. In Section II, we formulate the problem mathematically to obtain the tolerance intervals for electrical facilities, whose solution method is presented in Section III. Then we show the availability of our proposed method by numerical experiments in Section IV.

II. Problem Formulation

In this section, we give our problem in a mathematical form. We first describe a conventional ED problem and show that its solutions are not always adaptive. Then, we model a time series of tolerance intervals of the units and show that any dispatch schedules in it are adaptive. After that, we formulate our problem to find the time series of tolerance intervals with the minimum worst-case dispatch cost. For the sake of simplicity, we describe the problem in the case where one generator and one battery are scheduled. This can be straightforwardly extended to the case when multiple units are in use, which is simulated in Section IV. Hereafter, we denote by $\mathcal{D} := \left[\underline{D}, \overline{D}\right] \subset \mathbb{R}^T$, where $\underline{D} := \left[\underline{D}_1 \ \cdots \ \underline{D}_T\right]^{\mathrm{T}} \in \mathbb{R}^T$ and $\overline{D} := \left[\overline{D}_1 \ \cdots \ \overline{D}_T\right]^{\mathrm{T}} \in \mathbb{R}^T$, a set of predicted demand scenarios for the time slots of interest, which is assumed given.

A. Adaptivity of conventional ED

In this subsection, we explain why conventional ED needs revising in terms of its adaptivity. First of all, let us describe how the conventional ED problem is formulated. One of the essential goals of scheduling the units is to have the power supply-demand balance kept for all time slots, i.e., to make the sum of power generation and battery-discharge power equal to that of battery-charge power and given power demand, which can be written as

$$p - u + v = d \tag{1}$$

where $p := [p_1 \ \cdots \ p_T]^{\mathrm{T}} \in \mathbb{R}^T$, $u := [u_1 \ \cdots \ u_T]^{\mathrm{T}} \in \mathbb{R}^T$, $v := [v_1 \ \cdots \ v_T]^{\mathrm{T}} \in \mathbb{R}^T$, and $d := [d_1 \ \cdots \ d_T]^{\mathrm{T}} \in \mathcal{D}$. At the same time, the following operational conditions for the units must be satisfied:

$$\underline{P} \leq p_t \leq \overline{P}, \quad t = 1, \cdots, T, \tag{2}$$

$$\underline{R} \leq \frac{p_t - p_{t-1}}{\Delta T} \leq \overline{R}, \quad t = 2, \cdots, T, \tag{3}$$

$$0 \leq B_0 + \Delta T \sum_{i=1}^{t} e^{\mathrm{c}} u_i - \frac{1}{e^{\mathrm{d}}} v_i \leq B, \quad t = 1, \cdots, T, \tag{4}$$

$$0 \leq u_t \leq \overline{U}, \quad t = 1, \cdots, T, \tag{5}$$

$$0 \leq v_t \leq \overline{V}, \quad t = 1, \cdots, T \tag{6}$$

where the constraints (2) are for the generator's capacity and (3) for its ramp rate limits, (4) for the battery's capacity, and (5) and (6) for the capacity of the inverter in it. Then, the conventional ED problem to dispatch a given demand scenario d to each unit most economically can be represented as

$$\min_{(p,u,v) \in \mathcal{F}} J(p, u, v) \quad \text{s.t.} \quad (1) \tag{7}$$

where $\mathcal{F} := \{(p, u, v) : \text{(2)-(6) hold}\}$ is a set of all feasible dispatch schedules and

$$J(p, u, v) := \sum_{t=1}^{T} C_2 p_t^2 + C_1 p_t + C_2^{\mathrm{c}} u_t^2 + C_1^{\mathrm{c}} u_t + C_2^{\mathrm{d}} v_t^2 + C_1^{\mathrm{d}} v_t$$

is its cost function representing the sum of the generator's fuel cost and the battery's degradation cost.

In the following, we show that the feasibility of (7) for any $d \in \mathcal{D}$ does not guarantee the adaptivity of its corresponding dispatch solution. Let us suppose that a dispatch solution $(p^{\mathrm{i}}, u^{\mathrm{i}}, v^{\mathrm{i}}) \in \mathcal{F}$ for an initial demand forecast $d^{\mathrm{i}} \in \mathcal{D}$ is obtained from (7) and that the units are operated as scheduled until the t^{c}th time slot, after which actual demand $d^{\mathrm{r}} \in \mathbb{R}^{(T-t^{\mathrm{c}})}$ for the future $T - t^{\mathrm{c}}$ time slots is realized. Then, a new dispatch solution can be obtained from the following redispatch problem:

$$\min_{(p,u,v) \in \mathcal{F}^{\mathrm{r}}(p^{\mathrm{i}}, u^{\mathrm{i}}, v^{\mathrm{i}}, t^{\mathrm{c}})} J(p, u, v) \quad \text{s.t.} \quad p - u + v = d^{\mathrm{f}} \tag{8}$$

where

$$\begin{aligned} \mathcal{F}^{\mathrm{r}}(p^{\mathrm{i}}, u^{\mathrm{i}}, v^{\mathrm{i}}, t^{\mathrm{c}}) := \big\{&(p, u, v) : \text{(2)-(6) hold}, \\ &p_t = p_t^{\mathrm{i}}, \ u_t = u_t^{\mathrm{i}}, \ v_t = v_t^{\mathrm{i}}, \quad t = 1, \cdots, t^{\mathrm{c}}\big\} \end{aligned}$$

is a set of all feasible redispatch schedules given the past operations up to the t^{c}th time slot, in which p_t^{i}, u_t^{i}, and v_t^{i} are the tth element of p^{i}, u^{i}, and v^{i}, respectively, and

$$d^{\mathrm{f}} := \left[d_1^{\mathrm{i}} \ \cdots \ d_{t^{\mathrm{c}}}^{\mathrm{i}} \ d_1^{\mathrm{r}} \ \cdots \ d_{T-t^{\mathrm{c}}}^{\mathrm{r}}\right]^{\mathrm{T}}$$

is the resulting demand scenario on the time horizon, in which d_t^{i} and d_t^{r} denote the tth elements of d^{i} and d^{r}, respectively. Note that $\mathcal{F}^{\mathrm{r}}(p^{\mathrm{i}}, u^{\mathrm{i}}, v^{\mathrm{i}}, t^{\mathrm{c}}) \subset \mathcal{F}$ varies dependently on d^{i} even with fixed t^{c}, which means there might exist some $d^{\mathrm{i}} \in \mathcal{D}$ and t^{c} which make (8) infeasible for some $d^{\mathrm{f}} \in \mathcal{D}$. This feasibility gap between (7) and (8) comes from the dynamic constraints (3) and (4), where the feasible range of each unit's operation at each time slot is restricted by its operations at the previous time slots. In the next subsection, we solve this adaptivity issue by introducing boxes as an alternative of the original feasible schedule set $\mathcal{F}$.

B. Adaptivity-constrained ED

In this subsection, we model an ED problem that assures the adaptivity of dispatch solutions by introducing a tuple of T-dimensional boxes as a refined set of feasible schedules, i.e., tolerance intervals of the units. Note that this is equivalent to decoupling the dynamic constraints (3) and (4) in each time slot.

First, the tolerance intervals for the generator along the T time slots can be considered as a T-dimensional box $\tilde{\mathcal{P}} := [\underline{p}, \overline{p}] \subset \mathbb{R}^T$ where $\underline{p} := [\underline{p}_1 \cdots \underline{p}_T]^{\mathrm{T}} \in \mathbb{R}^T$ and $\overline{p} := [\overline{p}_1 \cdots \overline{p}_T]^{\mathrm{T}} \in \mathbb{R}^T$ subject to

$$\underline{P} \leq \underline{p}_t \leq \overline{p}_t \leq \overline{P}, \quad t = 1, \cdots, T, \tag{9}$$

$$\frac{\overline{p}_t - \underline{p}_{t-1}}{\Delta T} \leq \overline{R}, \quad t = 2, \cdots, T, \tag{10}$$

$$\frac{\underline{p}_t - \overline{p}_{t-1}}{\Delta T} \geq \underline{R}, \quad t = 2, \cdots, T. \tag{11}$$

The constraints (9) are for the generator's capacity, and (10) and (11) for its ramp rate limits, whose left-hand sides respectively corresponds to the maximum and minimum ramp rate possible at each time slot in $\tilde{\mathcal{P}}$. Note that (9)-(11) guarantee (2) and (3) if $p \in \tilde{\mathcal{P}}$.

Similarly, a pair of series of tolerance intervals along the whole planning period for the battery, one for charging power and the other for discharging power, can be modeled as T-dimensional boxes $\tilde{\mathcal{U}} := [\underline{u}, \overline{u}] \subset \mathbb{R}^T$ where $\underline{u} := [\underline{u}_1 \cdots \underline{u}_T]^{\mathrm{T}} \in \mathbb{R}^T$ and $\overline{u} := [\overline{u}_1 \cdots \overline{u}_T]^{\mathrm{T}} \in \mathbb{R}^T$ and $\tilde{\mathcal{V}} := [\underline{v}, \overline{v}] \subset \mathbb{R}^T$ where $\underline{v} := [\underline{v}_1 \cdots \underline{v}_T]^{\mathrm{T}} \in \mathbb{R}^T$ and $\overline{v} := [\overline{v}_1 \cdots \overline{v}_T]^{\mathrm{T}} \in \mathbb{R}^T$, respectively, subject to

$$B_0 + \Delta T \sum_{i=1}^{t} e^{\mathrm{c}} \overline{u}_i - \frac{1}{e^{\mathrm{d}}} \underline{v}_i \leq B, \quad t = 1, \cdots, T, \tag{12}$$

$$B_0 + \Delta T \sum_{i=1}^{t} e^{\mathrm{c}} \underline{u}_i - \frac{1}{e^{\mathrm{d}}} \overline{v}_i \geq 0, \quad t = 1, \cdots, T, \tag{13}$$

$$0 \leq \underline{u}_t \leq \overline{u}_t \leq \overline{U}, \quad t = 1, \cdots, T, \tag{14}$$

$$0 \leq \underline{v}_t \leq \overline{v}_t \leq \overline{V}, \quad t = 1, \cdots, T. \tag{15}$$

The constraints (12) and (13) are for the battery's capacity, whose left-hand sides are the maximum and minimum stored energy possible at each time slot, respectively, and (14) and (15) for the capacity of its inverter. (12)-(15) guarantee (4)-(6) if $u \in \tilde{\mathcal{U}}$ and $v \in \tilde{\mathcal{V}}$.

Furthermore, we can secure the reliability of three boxes $\tilde{\mathcal{P}}$,$\tilde{\mathcal{U}}$, and $\tilde{\mathcal{V}}$ as a set of possible ED solutions by the constraints

$$\overline{p} - \underline{u} + \overline{v} \geq \overline{D}, \tag{16}$$

$$\underline{p} - \overline{u} + \underline{v} \leq \underline{D}, \tag{17}$$

because actual demand at the tth time slot is within the interval $\left[\underline{D}_t, \overline{D}_t\right]$ for any t.

Consequently, any combination of three boxes $(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})$ satisfying (9)-(17) can be considered as tolerance intervals of the units, and its corresponding adaptivity-constrained ED problem for any given demand scenario $d \in \mathcal{D}$ can be represented as

$$\min_{(p,u,v) \in (\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})} J(p, u, v) \quad \text{s.t.} \quad (1). \tag{18}$$

Note that the adaptivity-constrained problem (18) can be formulated by substituting the feasible region $\mathcal{F}$ in the conventional ED problem (7) with any combination of boxes $(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})$ satisfying (9)-(17) inside $\mathcal{F}$. However, while the adaptivity can be secured by solving (18), the resulting dispatch cost might be too high for some actual demand scenarios depending on the selection of boxes. In the next subsection, we formulate the problem to determine such boxes $(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})$ that minimize the worst-case dispatch cost.

C. Minimization of the worst-case dispatch cost

In this subsection, we model our problem to find the combination $(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})$ which yields the minimum worst-case dispatch cost. Note that we do not consider any longer an initially predicted demand scenario, which is valid because we assume a uniform distribution of realizable demand in the prediction uncertainty $\mathcal{D}$.

The problem to find the dispatch schedule which meets the worst-case realization of $d \in \mathcal{D}$ for a given combination of boxes $(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})$ is written as

$$\max_{d \in \mathcal{D}} \min_{(p,u,v) \in (\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})} J(p, u, v) \quad \text{s.t.} \quad (1). \tag{19}$$

Then, let us denote a set of all the feasible combinations by $\mathcal{X} := \left\{(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}) : (9)\text{-}(17)\right\}$. Our objective is to find the combination $(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}) \in \mathcal{X}$, i.e., that of $\overline{p}$, $\underline{p}$, $\overline{u}$, $\underline{u}$, $\overline{v}$, and $\underline{v}$, which minimizes the worst-case dispatch cost. Consequently, our problem can be formulated in the form of a min-max-min optimization problem as follows:

$$\min_{(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}) \in \mathcal{X}} \max_{d \in \mathcal{D}} \min_{(p,u,v) \in (\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}})} J(p, u, v) \quad \text{s.t.} \quad (1). \tag{20}$$

III. Solution Method

A. Monotonicity of Dispatch Solution

In this subsection, we show that the solutions to the ED problem in (18) are monotonic with respect to demand by parameterizing them with respect to the index set of constraints. Because (18) can be solved independently for each time slot, it is sufficient to show the monotonicity of its solutions when $T = 1$. A generalized analysis method and related proofs can be found in [8].

Let us consider the following parametric quadratic programming

$$\text{QP}(c) : \min_{x \in \mathbb{R}^n} x^{\mathrm{T}} Q x \quad \text{s.t.} \begin{cases} A_{\text{in}} x \leq b_{\text{in}}, \\ A_{\text{eq}} x = c, \end{cases} \tag{21}$$

where $c \in \mathbb{R}$ is its uncertain parameter, $Q \in \mathbb{R}^{n \times n}$ is a positive-definite diagonal matrix,

$$A_{\text{in}} := [I \ -I]^{\mathrm{T}} \in \mathbb{R}^{2n \times n}, \ A_{\text{eq}} := [1 \ \cdots \ 1] \in \mathbb{R}^{1 \times n},$$
$$b_{\text{in}} := [\overline{x}_1 \ \cdots \ \overline{x}_n \ -\underline{x}_1 \ \cdots \ -\underline{x}_n]^{\mathrm{T}} \in \mathbb{R}^{2n}.$$

I denotes the $n \times n$ identical matrix. Note that QP(c) in (21) models a single-period dispatch problem where c corresponds to demand at each time slot. In other words, solving T sets of the problem (21) is equivalent to solving (18). Now, let $\mathcal{I}$ represent any subset of a set of natural numbers from 1 to $2n$ and e_l be a $2n \times |\mathcal{I}|$ matrix which consists of the columns in the $2n \times 2n$ identical matrix, corresponding to the elements in $\mathcal{I}$, and let

$$x^{\mathrm{c}}(\mathcal{I}, c) := Q^{-1} A_{\mathcal{I}}^{\mathrm{T}} (A_{\mathcal{I}} Q^{-1} A_{\mathcal{I}}^{\mathrm{T}})^{-1} b_{\mathcal{I}}(c) \tag{22}$$

where $A_{\mathcal{I}} := [A_{\mathrm{in}}^{\mathrm{T}} e_{\mathcal{I}} \ A_{\mathrm{eq}}^{\mathrm{T}}]^{\mathrm{T}}$ is of full row-rank, and $b_{\mathcal{I}}(c) := [b_{\mathrm{in}}^{\mathrm{T}} e_{\mathcal{I}} \ c]^{\mathrm{T}}$. Then, using Karush-Kuhn-Tucker condition, we can show that

$$x^*(c) := x^{\mathrm{c}}(\mathcal{I}^*(c), c) \tag{23}$$

is the solution of the QP(c) in (21) where $\mathcal{I}^*(c)$ is the index set corresponding to the active inequality constraints for a fixed c. From (22) and (23), it can be derived that the ith element of $x^*(c)$, denoted by $x_i^*(c), x_i^* : \mathbb{R} \to \mathbb{R}$, is one of $\overline{x}_i$, $\underline{x}_i$, or

$$g_i(c) := \left(\sum_{k \in \hat{\mathcal{I}}^*(c)} q_i q_k^{-1} \right)^{-1} \left(c - \sum_{k \in \overline{\mathcal{I}}^*(c)} \overline{x}_k - \sum_{k \in \underline{\mathcal{I}}^*(c)} \underline{x}_k \right)$$

where q_i denotes the ith diagonal element of Q,

$$\begin{aligned}
\hat{\mathcal{I}}^*(c) &:= \{k : k \notin \overline{\mathcal{I}}^*(c) \cup \underline{\mathcal{I}}^*(c), \ k = 1, \cdots, n\}, \\
\overline{\mathcal{I}}^*(c) &:= \{k : k \in \mathcal{I}^*(c), \ k = 1, \cdots, n\}, \\
\underline{\mathcal{I}}^*(c) &:= \{k : k + n \in \mathcal{I}^*(c), \ k = 1, \cdots, n\}.
\end{aligned}$$

Therefore, for any c_1 and c_2 such that $\underline{x}_i < x_i^*(c_1) < \overline{x}_i$, $\underline{x}_i < x_i^*(c_2) < \overline{x}_i$ and $\mathcal{I}^*(c_1) = \mathcal{I}^*(c_2)$, it holds that $x_i^*(c_1) < x_i^*(c_2)$ if $c_1 < c_2$. This means $x_i^*(c)$ increases with c for the same index set $\mathcal{I}^*(c)$. Furthermore, from the continuity of x^* on $\mathbb{R}$ [11], it follows that $x_i^*(c_1) < x_i^*(c_2)$ if $c_1 < c_2$ for any c_1, c_2 such that $\underline{x}_i < x_i^*(c_1) < \overline{x}_i$, $\underline{x}_i < x_i^*(c_2) < \overline{x}_i$. Note that this holds if any c_3 such that $\underline{x}_i < x_i^*(c_3) < \overline{x}_i$ can be found. In the case when such c_3 does not exist, i.e., $x_i^*(c) = \underline{x}_i$ for any c or $x_i^*(c) = \overline{x}_i$ for any c, x_i^* is still nondecreasing. Consequently, x_i^* is monotonically increasing with respect to c. This proves the monotonicity of solutions in the ED problem (18) with regard to net power demand at each time slot.

B. Problem Reformulation

In this subsection, we reformulate the min-max-min problem (20) into a minimax problem. From the monotonicity of the dispatch solutions shown in the previous subsection, we can replace the security constraints (16) and (17) with the following equalities, respectively:

$$\overline{p}_t - \underline{u}_t + \overline{v}_t = \overline{D}_t, \quad t = 1, \cdots, T, \tag{24}$$

$$\underline{p}_t - \overline{u}_t + \underline{v}_t = \underline{D}_t, \quad t = 1, \cdots, T. \tag{25}$$

This is because we do not have to consider any generator output p_t greater (less) than the most economical dispatch solutions associated with $\overline{D}_t$ ($\underline{D}_t$), which corresponds to $\overline{p}_t$ in (24) ((25)), not in (16) ((17)). The same holds for u_t and v_t. Then, it readily follows that the worst-case power demand at tth time slot is either $\overline{D}_t$ or $\underline{D}_t$. Therefore, (20) can be reduced to the following minimax problem with a binary variable s_t representing $\overline{D}_t$ when 1 or $\underline{D}_t$ when 0 for the tth time slot:

$$\min_{(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}) \in \mathcal{Z}} \ \max_{s \in \{0,1\}^T} J'(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}, s) \tag{26}$$

where

$$\begin{aligned}
\mathcal{Z} &:= \big\{(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}) : (9) - (15), (24), (25)\big\}, \\
s &:= [s_1 \ \cdots \ s_T]^{\mathrm{T}}, \\
J'(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}, s) &:= \sum_{t=1}^{T} C_2 (s_t \overline{p}_t + (1 - s_t) \underline{p}_t)^2 \\
&\quad + C_1 (s_t \overline{p}_t + (1 - s_t) \underline{p}_t) \\
&\quad + C_2^{\mathrm{c}} (s_t \underline{u}_t + (1 - s_t) \overline{u}_t)^2 \\
&\quad + C_1^{\mathrm{c}} (s_t \underline{u}_t + (1 - s_t) \overline{u}_t) \\
&\quad + C_2^{\mathrm{d}} (s_t \overline{v}_t + (1 - s_t) \underline{v}_t)^2 \\
&\quad + C_1^{\mathrm{d}} (s_t \overline{v}_t + (1 - s_t) \underline{v}_t).
\end{aligned}$$

C. Benders Decomposition

In this subsection, we solve the minimax problem (26) and obtain the tolerance intervals of dispatch schedules.

To deal with the high computational cost caused by a number of binary variables in the problem, we adopt Benders decomposition [12]. In the method, the original large-scale problem is partitioned into two subproblems, one of which gives an assumed solution and the other generates the most violated constraint for the solution, if any, which is added to the former. We repeat this until no constraint is generated. Benders decomposition can be applied to the minimax problem (26) as follows.

First, let us consider the following quadratically constrained linear programming as the first subproblem of (26) for any fixed s, which we denote by s_0:

$$\min_{(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}) \in \mathcal{Z}, \eta \in \mathbb{R}} \eta \quad \text{s.t.} \quad \eta \geq J'(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}, s_0) \tag{27}$$

which can be solve by an arbitrary algorithm. Note that the infeasibility of the subproblem (27) indicates that there does not exist a single dispatch solution which can be adjusted to meet all the demand scenarios predicted. In a practical sense, this can be overcome by adding power sources to the grids, e.g., non-spinning reserves of another generator. In this paper, we assume that the problem (27) is always feasible and denote each solution by $\tilde{\mathcal{P}}^*, \tilde{\mathcal{U}}^*, \tilde{\mathcal{V}}^*$, and η^*. Next, let us consider the following quadratic integer programming as the second subproblem of (26) to detect one of the worst-case demand scenarios given the assumed solution of (27):

$$\max_{s \in \{0,1\}^T} J'(\tilde{\mathcal{P}}^*, \tilde{\mathcal{U}}^*, \tilde{\mathcal{V}}^*, s) \tag{28}$$

whose solutions and the maximum objective value are denoted by s^{w} and η^{w}, respectively. Note that the subproblem (28) can easily be solved because no dynamic constraint is imposed and thus we can simply compare the objective values when $s_t = 0$ and $s_t = 1$ for each t.

If $\eta^{\mathrm{w}} = \eta^*$, it can be said that $(\tilde{\mathcal{P}}^*, \tilde{\mathcal{U}}^*, \tilde{\mathcal{V}}^*)$ minimizes the worst-case dispatch cost, which the demand scenario corresponding to s_0 can cause. If $\eta^{\mathrm{w}} > \eta^*$, on the other hand, it follows that they are not the solutions in the original problem (26) because the larger worst-case dispatch cost η^{w} does exist. In this case, the violated constraint should be identified and added to the first subproblem (27) which is then solved again. The constraint to be generated can be written as

$$\eta \geq J'(\tilde{\mathcal{P}}, \tilde{\mathcal{U}}, \tilde{\mathcal{V}}, s^{\mathrm{w}}). \tag{29}$$

By solving iteratively (27) and (28) until no violated constraint is found, we can obtain the solutions in the minimax problem (26). The entire algorithm to solve (26) can be summarized as the flow chart in Fig. 1. Note that this framework always gives the solution of (26), ultimately that of (20), if (27) is feasible, as we assumed.

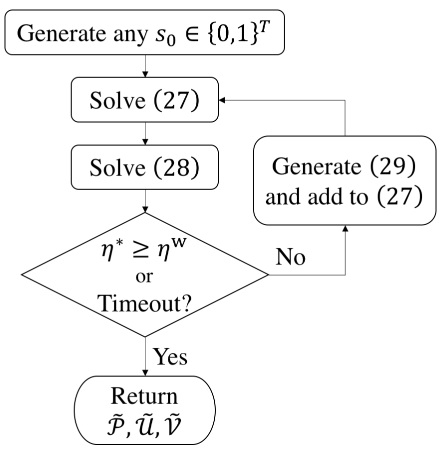


Fig. 1. Decomposition framework for solving (26)

IV. Numerical Simulations

A. Tolerance intervals for economic dispatch

In this subsection, we verify the effectiveness of our proposed method by numerical experiments for a single-bus power system with three generating units and a unit of storage batteries, whose specifications are shown respectively in the TABLE I and TABLE II. Let us suppose that they are scheduled on an hourly basis for the next twenty-four hours. The confidence interval of net power demand at each time slot is as shown in Fig. 2a. Using our proposed method, we obtained a series of tolerance intervals along the twenty-four time slots for each unit as in Fig. 2, whose worst-case dispatch cost is 2.6079×10^5. We used the all-maximum demand scenario as the initial guess, i.e., a vector of ones as s_0 in (27), and the entire calculation took six iterations.
In the following, we demonstrate the adaptivity of dispatch solutions in the tolerance intervals. Let us suppose that the all-maximum demand scenario is forecast for the whole time period but realized only up to the twelfth time slot, i.e., 11:00 a.m., until when the units are operated as scheduled, after which the all-minimum demand scenario is realized as shown in Fig. 2a. We can see that the dispatch solutions still exist after the time slot, which are plotted in wider grey lines in Fig. 2. For comparison, let us solve another ED and re-dispatch problems under the same condition but without considering the adaptivity of dispatch solutions, i.e., without using the tolerance intervals. The initial day-ahead dispatch schedule for each unit is illustrated in Fig. 3, where we tagged with circles the minimum power outputs and discharging power, the maximum charging power feasible at 12 noon, and the corresponding battery-stored energy at 1:00 p.m. From the indicated values, it can be observed that the actual demand at 12 noon, labeled in Fig. 2a, cannot be met.

TABLE I
Specification of generation units

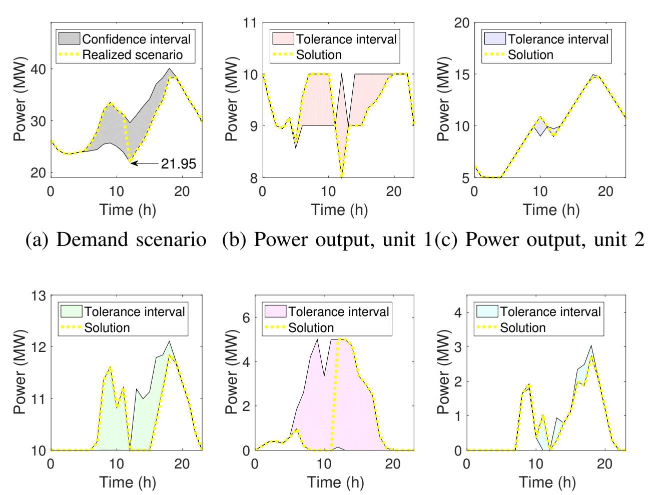
Unit	$\overline{P}$ (MW)	$\underline{P}$ (MW)	$\overline{R}$ (MW/h)	$\underline{R}$ (MW/h)	C_1 (/MWh)	C_2 (/MW^{2}h)
1	10	0	1	-1	150	5
2	20	5	1	-1	150	20
3	30	10	2	-2	160	30

TABLE II
Specification of a storage battery unit

B_0 (MWh)	B (MWh)	$\overline{U}$ (MW)	$\overline{V}$ (MW)	e^{c} (%)	e^{d} (%)	C_1^{c} (/MWh)	C_2^{c} (/MW^{2}h)	C_1^{d} (/MWh)	C_2^{d} (/MW^{2}h)
25	50	5	5	0.8	0.7	120	12	130	13

B. Application to unit commitment

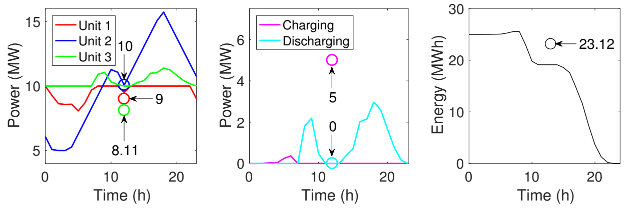
The proposed concept of tolerance intervals can be straightforwardly applied to unit commitment (UC) problems. In this subsection, we solve a security-constrained UC problem adopting the proposed solution method for a single-bus power system with ten units of generators and a unit of batteries, whose specifications are shown in TABLE III and TABLE II, respectively. C^{SU}, C^{SD}, R^{SU}, and R^{SD} denotes a generator's startup cost, shutdown cost, startup ramp limit, and shutdown ramp limit, respectively. The confidence interval of power demand is as shown in Fig. 2a and each generation unit is assumed to be initially in its



(a) Demand scenario (b) Power output, unit 1 (c) Power output, unit 2

(d) Power output, unit 3 (e) Charging power (f) Discharging power

Fig. 2. Tolerance intervals with minimum worst-case dispatch cost



(a) Power outputs (b) Charging and discharging powers (c) Stored energy

Fig. 3. Economic dispatch without considering tolerance intervals

off state. The resultant interval tolerances for the units are illustrated in Fig. 4.

TABLE III
SPECIFICATION OF GENERATION UNITS

Unit	C^{SU} $(\times 10^4)$	C^{SD} $(\times 10^4)$	C_1 (/MWh)	C_2 (/MW^{2}h)	R^{SU} (MW/h)
1	1	0.8	150	5	0.9
2	1.2	1	150	6	1.8
3	1.5	1.2	150	10	2.7
4	1.6	1.4	150	12	3.6
5	2	1.8	160	15	4.5
6	2.2	2	160	18	5.4
7	2.5	2.3	160	20	6.3
8	2.7	2.5	170	25	7.2
9	3	2.8	170	30	8.1
10	3.3	3	170	35	9
Unit	R^{SD} (MW/h)	$\overline{P}$ (MW)	$\underline{P}$ (MW)	$\overline{R}$ (MW/h)	$\underline{R}$ (MW/h)
1	0.6	1	0.5	0.25	-0.3
2	1.2	2	1	0.5	-0.6
3	1.8	3	1.5	0.75	-0.9
4	2.4	4	2	1	-1.2
5	3	5	2.5	1.25	-1.5
6	3.6	6	3	1.5	-1.8
7	4.2	7	3.5	1.75	-2.1
8	4.8	8	4	2	-2.4
9	5.4	9	4.5	2.25	-2.7
10	6	10	5	2.5	-3

V. CONCLUSION

In this paper, we have proposed a new approach for economic dispatch to find the tolerance interval for each electrical facility. The solution guarantees that any dispatch schedules in it can be adjusted so any future demand can be met, while minimizing the worst-case dispatch cost. As economic dispatch is essential for power systems, the proposed method is expected to have a wide range of applications. Future work will include developing solution methods for adaptivity-constrained economic dispatch under dynamic uncertainty sets of demand scenarios.

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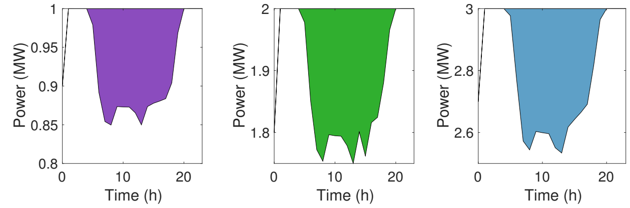
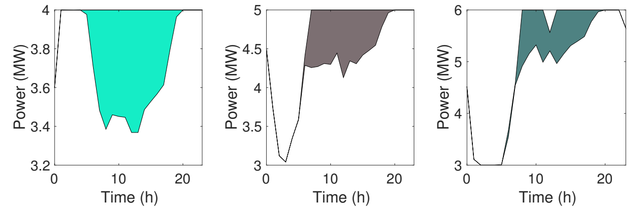
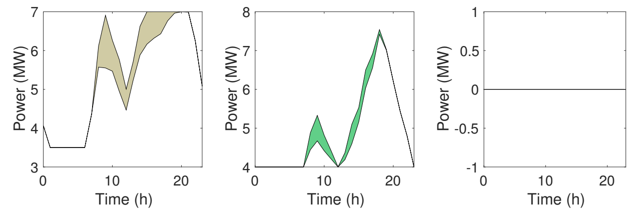


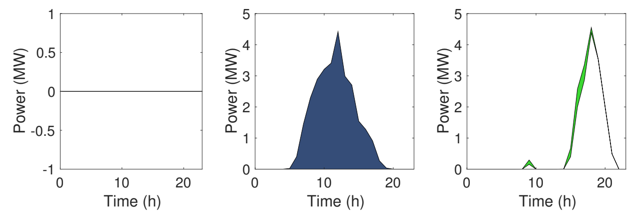
(a) Power output, unit 1 (b) Power output, unit 2 (c) Power output, unit 3



(d) Power output, unit 4 (e) Power output, unit 5 (f) Power output, unit 6



(g) Power output, unit 7 (h) Power output, unit 8 (i) Power output, unit 9



(j) Power output, unit 10 (k) Charging power (l) Discharging power

Fig. 4. Security-constrained unit commitment using tolerance intervals

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