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Regularization approach for an immersion-based observer design

Gildas Besançon[1] and Alexandru Ţiclea[2]

Abstract— **This paper is about immersion-based observer design for nonlinear systems. Starting from the fact that direct immersion may result in stability issues, which may in turn affect the observer performance, a method is proposed to overcome such problems, by somehow 'stabilizing' the transformation (or 'regularizing' it). A couple of examples are presented to illustrate the effectiveness of the method, including the challenging case of state and parameter estimation in a speed sensorless induction machine.**

Keywords: nonlinear observer, immersion, regularization, induction machine.

I. INTRODUCTION

The problem of observer design for nonlinear systems attracts more and more attention. Among the approaches which have been developed, a lot of studies have been devoted to *transformations*, with the purpose of turning the nonlinear representation under consideration into a form more suitable for observer design [1], [2], [3], [4], [5], [6], [7], [8, ...]. In that context, many methods deal with *diffeomorphisms*, ensuring a dynamical behaviour of the transformed system equivalent to the one of the original system. But diffeomorphisms have also been relaxed to *immersions*, taking advantage of state dimension increase to simplify the system structure [9], [10], [11], [12], [13], [14], [15], [16, ...]. Most of those approaches do not consider observability singularities, and some discussions about this can be found in few recent works like [17], [18], showing how to avoid some singularities by immersion. Immersion can also provide a pretty general methodology for observer design, even for *non uniformly observable systems* (in the sense of systems with an observability typically depending on the input [19]), as for instance shown in [20].

However, it turns out that this more general type of transformation may change some of the dynamical properties of the system, and in particular related to stability. In the present paper, we would like to consider this stability issue, more particularly considering the immersion technique of [20], and propose some solution for it , that we call *regularization*: in short indeed, the stability of an equilibrium may be lost when embedded into some manifold of larger dimension, and we propose here a method to ensure that trajectories of the extended system remain attracted to the manifold where the original system lives, before designing an observer. The application of such a regularization is then illustrated with two examples: first, a second order numerical example, and then the problem of state and parameter estimation in an induction motor with no speed measurement (see e.g. [21]). In both cases, the approach is explained and simulation results are provided.

The paper is organized as follows: section II recalls the type of immersion that is considered and the related observer design. Section III then emphasizes the stability issue in this approach, and presents the regularization method which is here proposed. Illustrative examples with corresponding simulation results are given in section IV, and some conclusions finally end the paper in section V.

II. CONSIDERED IMMERSION-BASED OBSERVER DESIGN

A general class of nonlinear systems can be given by a state-space representation of the form:

$$\begin{array}{rcl} \dot{x}(t) & = & f(x(t), u(t)) \\ y(t) & = & h(x(t)) \end{array} \tag{1}$$

where x classically denotes the state vector of $\mathbb{R}^n$, y the vector of measured outputs, and u some vector of known signals (typically control inputs), while f and h are smooth functions of their arguments.

For systems of this kind, it was shown in [20] that in the case of control affine structure of f, the so-called *local weak observability* condition [22] ensures that the system can be transformed by immersion into a form:

$$\begin{array}{rcl} \dot{z}(t) & = & A(u(t))z(t) + B(z(t), u(t)) \\ y(t) & = & Cz(t) \end{array} \tag{2}$$

where A is an upper shift matrix by blocks, and B has a Jacobian matrix of lower triangular form, while $z \in \mathbb{R}^N$ for $N \geq n$.

The interest for observer design is that under the form (2), an exponential observer can be designed whenever an appropriate excitation condition is satisfied. Let us recall that this condition is given by:

Definition *2.1: Locally Regular Persistent Excitation.* The function $u(.)$ is such that there exist $\alpha > 0$ and $\lambda_0 > 0$ for which for all $\lambda \geq \lambda_0$ and all $t > \frac{1}{\lambda}$:

$$\int_{t-\frac{1}{\lambda}}^{t} \psi_u(\tau,t)^T C^T C \psi_u(\tau,t) d\tau \geq \alpha\lambda\Lambda(\lambda)^{-2} \tag{3}$$

[1]Gildas Besançon is with Univ. Grenoble Alpes, CNRS, Grenoble INP*, GIPSA-lab, 38000 Grenoble, France gildas.besancon@grenoble-inp.fr

[2]Alexandru Ţiclea is with University POLITEHNICA of Bucharest, Romania alexandru.ticlea@upb.ro

*Institute of Engineering Univ. Grenoble Alpes

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where $\Lambda(\lambda)$ is a block-diagonal matrix with diagonal entries of the form $\lambda^i I_{n_i}$ for block index i (and I_{n_i} an $n_i \times n_i$ identity matrix), and ψ_u satisfies $\frac{d\psi_u(\tau,t)}{d\tau} = A(u(\tau))\psi_u(\tau,t)$, $\psi_u(t,t) = I_N$.
The exponential observer for (2) is then given by [20]:

$$\begin{array}{rcl} \dot{\hat{z}}(t) & = & A(u(t))\hat{z}(t) + B(\hat{z}(t), u(t)) \\ & & -\Lambda(\lambda)S^{-1}(t)C^T(C\hat{z}(t) - y(t)) \\ \dot{S}(t) & = & \lambda(-\gamma S(t) - A^T(u(t))S(t) - S(t)A(u(t)) + C^T C) \\ S(0) & > & 0 \end{array} \tag{4}$$

for $\gamma, \lambda > 0$ large enough.

Notice that condition (3) and observer (4) also extend to systems (2) with $A = A(u, y)$.

III. STABILITY ISSUE AND PROPOSED REGULARIZATION APPROACH

Let us point out here a problem related to the immersion-based approach recalled in the previous section, which is the fact that the transformation may affect the *stability* property of the system: this means that a system with a stable operation point may be turned into a system with no longer stability. This in turn may result in observer instability, not due to the observer design, but to the new system instability.

As a simple example, let us consider the following scalar system, with obvious globally asymptotically stable origin:

$$\dot{x}(t) = -x^3(t) \tag{5}$$

For this system, one can consider an immersion defined as follows:

$$\begin{array}{rcl} z_1 & = & x \\ z_2 & = & -x^3 \end{array}$$

With it, the system can easily be re-written as:

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 \\ \dot{z}_2 & = & 3z_1^5 \end{array} \tag{6}$$

Clearly here, strictly negative initial conditions on both z_1 and z_2 result in an infinite decay of both state variables, meaning that the origin $(0,0)^T$ is now unstable.
In fact, its attractivity is only guaranteed on the submanifold $\{z \in I\!R^2 : z_2 = -z_1^3\}$ where the original system lives.

This suggests that instead of transformed system (6), one may consider some system ensuring that $z_2(t) = -z_1^3(t)$ at any time, or at least attractivity of such a manifold, for instance:

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 \\ \dot{z}_2 & = & -3z_1^2 z_2 - (z_2 + z_1^3) \end{array} \tag{7}$$

for which one can check that $lim_{t\to\infty}(z_2(t) + z_1^3(t)) = 0$.

The main point, in the present paper, is to propose a method so as to overcome such a problem in the immersion procedure formerly proposed in [20] for the observer design of previous section.

More precisely, the idea is to enhance the immersion procedure as follows (the reader is referred to [20] for more details on the original version):

Considering a system (1) with a control affine structure $f(x,u) = f_0(x) + \sum_{i=1}^m f_i(x)u_i$,

1) Build z_1 from output functions components of h;

k+1) Assume that z_1 up to z_k have been obtained, with possibly enhanced dynamics, and consider a basis of their differentials $\Omega_k = d\phi_1, ...d\phi_{\nu_k}$. If $\nu_k = n$, the construction stops, else build z_{k+1} from all possibly enhanced $L_{f_i} z_{kj}$'s (with z_{kj} denoting component j of z_k, and $i = 0$ to m) whose differentials are independent from ω_k.
If for some l, there exists φ_{k+1l} such that $z_{k+1l} = \varphi_{k+1l}(z_1, ...z_k, z_{k+1\mu})_{\mu\neq l}$, then $\dot{z}_{k+1l}$ is enhanced into:
$$\begin{array}{l} \sum_{i=0}^m L_{f_i}(z_{k+1l})u_i \\ -\delta_{k+1l}(z_{k+1l} - (\varphi_{k+1l}(z_1, ...z_k, z_{k+1\mu})_{\mu\neq l}) \end{array}$$
(with $u_0 := 1$), for some function δ_{k+1l} stabilizing the dynamics: $\dot{\xi}(t) = -\delta_{k+1l}(\xi(t))$,
and z_{k+1l} is replaced by $\varphi_{k+1l}(z_1, ...z_k, z_{k+1\mu})_{\mu\neq l}$ in all subsequent steps. $\triangleleft$

In short, this regularization enforces constraints $z_{k+1l} = \varphi_{k+1l}(z_1, ...z_k, z_{k+1\mu})_{\mu\neq l}$ for the new system representation.
This allows to avoid instabilities that may arise from a possible deviation between z_{k+1l} and φ_{k+1l} without regularization, as this happens in example (6).
In practice, one can select a simple linear correction $\delta_{k+1l}(.) = \delta_{kl} \times (.)$.
If indeed the extended system on the manifold $\{z : z_{kl} = \varphi_{kl}(z_1, ...z_{k-1}, z_{k\mu})_{\mu\neq l}, k \in \kappa\}$ for κ the set of indices corresponding to regularizations, enjoys some stability property, then the regularized system will keep the same stability property. This results from the fact that by construction, the new representation only depends on z_{kl}, $k \in \kappa$, through linear terms, as in:

$$\dot{\bar{z}} = \bar{A}(u)\bar{z} + \tilde{A}(u)\tilde{z} + \bar{B}(\bar{z}, u) \tag{8}$$

where $\tilde{z}$ gathers all components of z subject to regularization, and $\bar{z}$ the remaining ones, while, by construction again:

$$\overbrace{(\tilde{z} - \varphi(\bar{z}))}^{\cdot} = -\Delta(\tilde{z} - \varphi(\bar{z})) \tag{9}$$

where φ is the vector of all φ_{kl}'s, and Δ is a diagonal matrix gathering all regularizing coefficients δ_{kl}.
This yields:

$$\dot{\bar{z}} = \bar{A}(u)\bar{z} + \tilde{A}(u)\varphi(\bar{z}) + \bar{B}(\bar{z}, u) + \tilde{A}(u)(\tilde{z} - \varphi(\bar{z})) \tag{10}$$

where $A(u)(\tilde{z} - \varphi(\bar{z}))$ appears, for bounded $A(u)$, as an exponentially vanishing additive disturbance in the dynamics $\dot{z} = \bar{A}(u)\bar{z} + \tilde{A}(u)\varphi(\bar{z}) + \bar{B}(\bar{z}, u)$, which satisfy the requirements on the state.

IV. SIMULATION EXAMPLES

Let us present here a couple of illustrations of the regularization idea previously introduced, first with some numerical example, and then with some more concrete one.

A. Illustrative numerical example

As a first illustration, let us consider the example of the following second order system:

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 + ux_2^2 \\ \dot{x}_2 & = & -x_1 - ux_1x_2 - x_2^3 \\ y & = & x_1 \end{array} \tag{11}$$

The origin can here be checked to be asymptotically stable, and here the system is not in the form (1), but can be turned into it by the direct immersion:

$$\begin{array}{rcl} z_1 & := & x_1 \\ z_2 : & = & x_2 \\ z_3 & := & x_2^2 \end{array}$$

which typically yields:

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 + uz_3 \\ \dot{z}_2 & = & -z_1 - uz_1z_2 - z_2^3 \\ \dot{z}_3 & = & 2z_2(-z_1 - uz_1z_2) - 2z_3^2 \\ y & = & z_1 \end{array} \tag{12}$$

It can be noticed at this point that this new representation may exhibit instability (e.g. with $z_1 = z_2 = 0$ and $u = 0$), making a related observer design numerically sensitive.
See for instance simulation results of Figure 1, comparing z_1, z_2 to x_1, x_2, with initial discrepancies on state components as:

$$z_1 = x_1 = 1;\ z_2 = 0;\ x_2 = -1;\ z_3 = -0.31. \tag{13}$$

The figure shows an increasing deviation, which means that an observer based on (12) with the above initial errors would diverge.

According to the regularization approach previously discussed, one may instead consider system (12) with

$$\dot{z}_3 = 2z_2(-z_1 - uz_1z_2) - 2z_3^2 - \delta(z_3 - z_2^2) \quad \text{for some } \delta > 0, \tag{14}$$

for which indeed the problem can be solved.
In fact, following the proposed procedure more closely, one rather ends up with:

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 + uz_3 \\ \dot{z}_2 & = & -z_1 - uz_1z_2 - z_2^3 \\ \dot{z}_3 & = & 2z_2(-z_1 - uz_1z_2) - 2z_2^4 - \delta(z_3 - z_2^2) \\ y & = & z_1 \end{array} \tag{15}$$

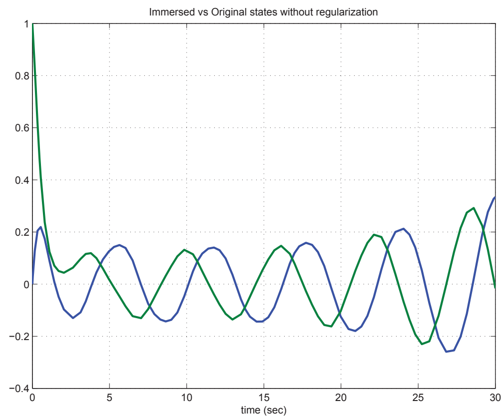

Fig. 1. Comparison of original states of (11) with immersed states of (12) *without regularization* ($z_1 - x_1$ and $z_2 - x_2$ resp.).

for $\delta > 0$.
For this system, like in the case of (14), the discrepancy with original states again vanishes, as it can be seen on the comparison Figure 2 (where $\delta = 2$).

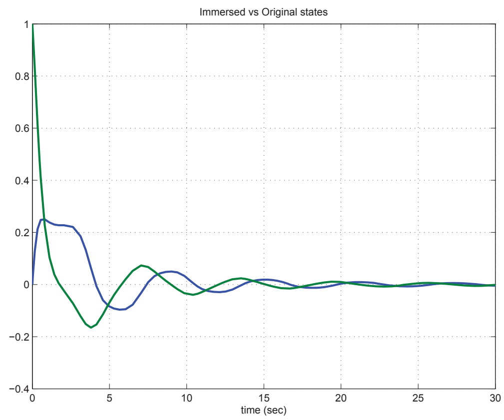

Fig. 2. Comparison of original states of (11) with immersed states of (15) *with regularization* ($z_1 - x_1$ and $z_2 - x_2$ resp.).

In addition, observer (4) still applies (under appropriate exciting input), and related simulation results are shown in Figure 3, with same initial values as in (13), and a simple sine wave input function together with $\gamma = 1, \lambda = 1.1$ as observer parameters.

Remark *4.1:* Notice that this approach also allows to handle algebraic constraints on variables, as in constrained parameter identification: for instance in example (11), one may have $\theta = x_2$ being a constant parameter (that is $\dot{x}_2 = 0$), and our procedure allows to estimate it under the constraint that $\dot{y} = \theta + \theta^2 u$.

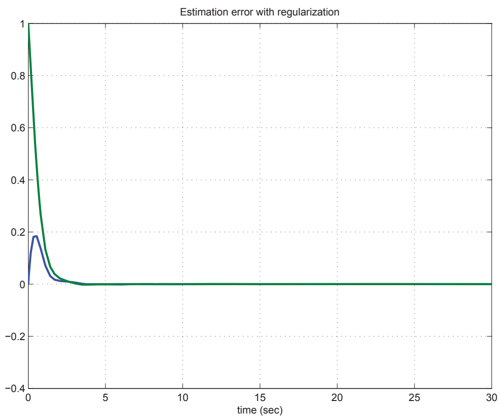

Fig. 3. State estimation errors with regularized observer.

B. Concrete application example

As an additional example, let us consider the more concrete case of *full state and parameter estimation in an induction machine without speed measurement* (so-called *sensorless* configuration). This problem of strong pratical interest has motivated a lot of work (see e.g. [21] and references therein).

In [23], an immersion-based possible solution was emphasized by transformation into a *state-affine* form. This was at the price of large dimension increase. In [24], an alternative immersion of much lower dimension has more recently been proposed, giving rise to a possible application of observer (4), but simulation results were in fact limited by numerical sensitivity (only a case of limited unknown parameters was shown).

Let us here revisit this immersion at the light of the proposed regularization approach, and present simulation results for the case of full unknown state and parameters.

First of all, let us recall that a dynamical model for the system under consideration classically reads [25]:

$$\begin{array}{rcl} \frac{di_s}{dt} & = & -(\frac{R_r}{\sigma L_r}+\frac{R_s}{\sigma L_s})i_s + pw_rJi_s \\ & & +\frac{R_r}{\sigma L_s L_r}\phi_s - \frac{1}{\sigma L_s}w_rpJ\phi_s + \frac{1}{\sigma L_s}u \\ \frac{d\phi_s}{dt} & = & -R_s i_s + u \\ \frac{d\omega_r}{dt} & = & -\frac{f_m}{J_m}\omega_r + \frac{p}{J_m}(i_s^T J\phi_s) - \frac{1}{J_m}\tau_l \end{array} \qquad (16)$$

where i_s, ϕ_s are respectively stator current and flux vectors in the so-called $\alpha-\beta$ frame, w_r is the rotor speed, and u is the stator voltage input, while R, L respectively refer to resistance and inductance parameters (with index r for rotor, s for stator), σ to some mutual related inductance, p to the number of pairs of poles, and f_m, J_m to known mechanical parameters. Here J stands for matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

In the speed sensorless case, stator currents i_s are the only available measurements y, meaning that fluxes Φ_s and rotor speed ω_r are to be estimated.

Considering in addition some lack of knowledge (or variations) on electrical parameters, as well as on the load torque, there are also five parameters to be estimated which can be summarized as:

$$\begin{array}{rcl} \rho_1 & := & \frac{R_r}{\sigma L_r}+\frac{R_s}{\sigma L_s}; \\ \rho_2 & := & \frac{1}{\sigma L_s}; \\ \rho_3 & := & \tau_l; \\ \rho_4 & := & \frac{R_s R_r}{\sigma L_s L_r}; \\ \rho_5 & := & \frac{R_r}{\sigma L_s L_r}. \end{array} \qquad (17)$$

By considering them in a vector ρ satisfying $\dot{\rho} = 0$, system (16) admits an extended state $x = \begin{pmatrix} i_s \\ \phi_s \\ \omega_r \\ \rho \end{pmatrix}$, for which, together with output $y = i_s$, it takes general form (1).

Following same arguments as in [24], an immersion can then be defined as follows:

$$z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \qquad (18)$$

with $z_1 := i_s$, $z_2 := \begin{pmatrix} \omega_r \\ \rho_5\phi_s - \rho_2\omega_r pJ\phi_s \\ \rho_1 \\ \rho_2 \end{pmatrix}$ and $z_3 :=$
$$\begin{pmatrix} \phi_s \\ \frac{\rho_2\rho_4}{\rho_5}\omega_r pJi_s - \rho_2\frac{p}{J_m}i_s^T J\phi_s pJ\phi_s + \rho_2(\frac{f_m}{J_m}\omega_r + \frac{\rho_3}{J_m})pJ\phi_s \\ \rho_3 \\ \rho_4 \\ \rho_5 \end{pmatrix}.$$

Here an inverse is easily obtained, and with this, the system becomes (see details in appendix):

$$\begin{array}{rcl} \dot{z} & = & A(u,y)z + B(u,y,z) \\ y & = & Cz \end{array} \qquad (19)$$

for which observer of the form (4) can be used (or the adaptive version proposed in [24]).

It turns out yet that simulations show instabilities, and the point here is to enhance this immersion by the proposed regularization:

Noting that z_{31} is in fact related to z_2, z_{32}, z_{35} as:

$$z_{31} = (z_{35}I - z_{24}z_{21}pJ)^{-1}z_{22} \qquad (20)$$

one can regularize $\dot{z}_{31}$ by adding the following term to its dynamics:

$$-\delta_{31}(z_{31} - (z_{35}I - z_{24}z_{21}pJ)^{-1}z_{22}) \qquad (21)$$

for some $\delta_{31} > 0$, and z_{2j}, z_{3j} denoting subvector j of z_2, z_3 respectively, according to the decomposition of (18).

In a similar way, one can check that z_{32} can also be expressed according to z_1, z_2, z_3. This yields a regularizing term as:

$$\begin{array}{rl} - & \delta_{32}\Big[z_{32} + z_{24}(-\frac{f_m}{J_m}z_{21} + \frac{p}{J_m}z_1^T J\varphi_{31} - \frac{1}{J_m}z_{33})pJ\varphi_{31} \\ & +z_{24}z_{21}pJ\frac{z_{34}}{z_{35}}z_1\Big] \end{array} \qquad (22)$$

where $\varphi_{31} := (z_{35}I - z_{24}z_{21}pJ)^{-1}z_{22}$ and $\delta_{31} > 0$.

For simulation results, let us consider the same numerical values (with appropriate units) as in [24], given by:

$$\begin{aligned}\rho_1 &= 1.76e-2;\\ \rho_2 &= 7.33e-2;\\ \rho_3 &= 1.67e-2;\\ \rho_4 &= 0.62;\\ \rho_5 &= 0\end{aligned}$$

while $f_m = 1e-3$, $J_m = 0.22$, $p = 2$.

Here the input voltage is just chosen as a combination of sine waves, as illustrated in Figure 4, and the observer is implemented under the adaptive form of [24] (allowing to separate tuning coefficient γ between states and parameters, each of them chosen with values between 0.6 and 2.1, while λ is here set to 1.15, and $\delta_{31} = \delta_{32} = 0.1$).

The observer is started at some time during motor operation (set to $t = 0$ in the simulation results which are shown) with initial errors on all unmeasured variables, between 10% and 90% of their actual values.

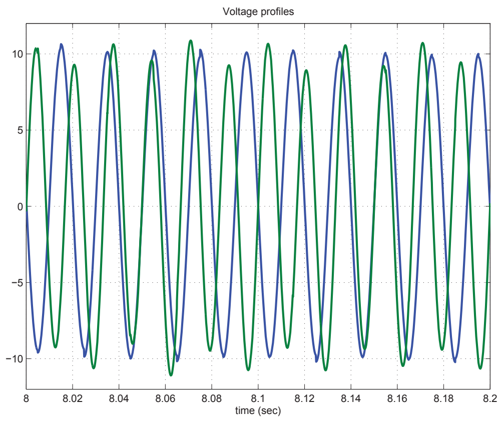

Fig. 4. Excitation input vector u.

Figure 5 then shows the obtained parameter estimation errors, while Figures 6-7 present estimation errors on fluxes and rotor speed respectively. In all cases, the convergence to zero is indeed obtained.

V. CONCLUSIONS

In this paper, the attention has been paid to some possible numerical instability in practical implementation of some formerly proposed immersion-based observer design [24], and a so-called regularization method has been presented. It

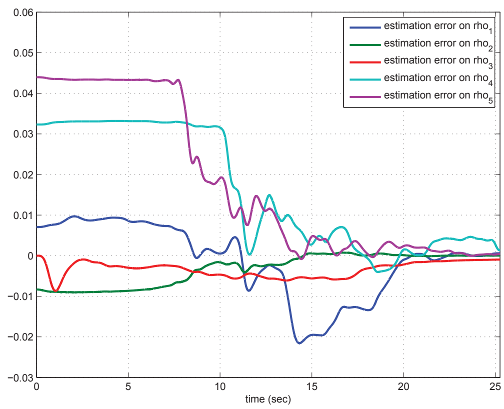

Fig. 5. Parameter estimation errors.

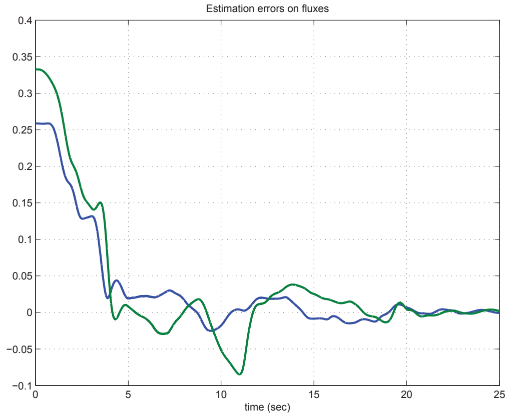

Fig. 6. Flux estimation errors.

has been illustrated with a couple of simulation examples, including the challenging problem of full state and parameter estimation in an asynchronous machine, with only stator currents measurements and no speed sensor. This could also apply to *synchronous* machines, as an extension of [26] for instance.

APPENDIX

Matrix A and function B of equation (19) can be recovered from [24] as follows:

$$A(u,y) = \begin{pmatrix} 0 & A_{12}(u,y) & 0 \\ 0 & 0 & A_{23}(u,y) \\ 0 & 0 & 0 \end{pmatrix} \tag{23}$$

where:

$$\begin{aligned} A_{12}(u,y) &= \begin{pmatrix} pJy & I_2 & -y & u \end{pmatrix} \\ A_{23}(u,y) &= \begin{pmatrix} \frac{p}{J_m}y^TJ & 0 & -\frac{1}{J_m} & 0 & 0 \\ 0 & I_2 & 0 & -y & u \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

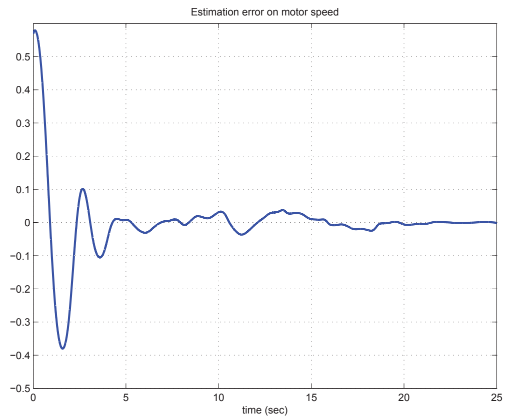

Fig. 7. Speed estimation error.

and 0 stands for zero matrices of appropriate dimensions;

On the other hand:

$$B(u,y,z)=\begin{pmatrix}0\\ B_2(u,y,z)\\ B_3(u,y,z)\end{pmatrix} \tag{24}$$

with $B_2(u,y,z)=\begin{pmatrix}-\frac{f_m}{J_m}z_{21}\\ -z_{24}z_{21}pJu\\ 0\end{pmatrix}$,

and $B_3(u,y,z)=\begin{pmatrix}-\frac{z_{34}}{z_{35}}y+u-\lambda_{31}(z_{31}-\varphi_{31})\\ \Phi\\ 0\end{pmatrix}$,

where here 0 stands for zero vectors of appropriate dimension, and Φ is the time derivative of z_{32} corrected by an additive term of the form $-\lambda_{32}(z_{32}-\varphi_{32})$ as in equation (22), which clearly depends on y, u, and $\omega_r=z_{21}$, $\phi_s=z_{31}$, as well as $\rho=\begin{pmatrix}z_{23} & z_{24} & z_{33} & z_{34} & z_{35}\end{pmatrix}^T$.

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