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Optimal Control of Roll-Pitch Seeker with Singularity Avoidance

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Abstract— **In order to provide guidance information, various types of optics seekers are incorporated in most modern missiles. A seeker moves independently of the missile dynamics to track a target image, providing the target information to the missile. Among them, two-axis seekers are widely used because they can rotate freely toward the target. According to the direction of rotating axes, the two-axis seekers are categorized into yaw-pitch and roll-pitch types. The yaw-pitch gimbal structure has been researched widely since it provides an intuitive way to track the target. On the other hand, due to smaller size with a larger gimbal angle, the roll-pitch structure is actively studied and widely applied to missiles recently. Despite the advantages, the inherent structural characteristic of the roll-pitch seeker has made the research relatively late. When the target is aligned with the missile axial direction, a singularity occurs at which the roll angle of the gimbal becomes indefinite. Furthermore, when the target moves around the singular point, the roll gimbal command angle abruptly changes, dissipating control energy while showing an abrupt oscillation in gimbal response. This paper analyzes the singularity problem in the roll-pitch seeker and introduces an optimal control method to properly relieve the problem.**

I. Introduction

A guided missile requires target information to hit a target in its terminal phase. The information is continuously fed into the guidance unit through a seeker. An optics seeker actuates to orient toward the target, maintaining the target image projected at the center of image plane. Through a continuous observation of the target, the line-of-sight (LOS) and the line-of-sight rate (LOS rate) information is obtained. In order to maintain the target image inside the seeker field-of-view throughout the terminal guidance phase, the motion of the seeker has to be completely decoupled form the motion of the missile.

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The simplest structure of the optics seeker is of strap-down seeker. It is stationary, oriented in the forward direction of the missile. Since the strap-down structure does not have actuators to turn the seeker, it is the simplest and the cost of the system is relatively low. However, the center point of the image plane is fixed, and the target image may be projected on the periphery of the image plane. This may cause distortion or loss of target information. Therefore, this type of seeker is used in low performance guidance missiles.

To follow a target, most seekers mount an optics sensor on rotating gimbals. A single-axis seeker, which is also called a linear scanning seeker, has an optics sensor on a gimbal which rotates around an axis perpendicular to the axial axis of the missile. The single-axis seeker only scans the forward plane of the missile, which is inadequate to locate the target image at the center. In order to follow the target precisely a seeker requires at least two axes to rotate. An optic sensor mounted on the gimbal rotating around two axes has enough freedom to orient toward the target position. A three-axis seeker does also exist, but due to large size and complicated structure it is mainly used in the missile with large diameter[1].

There are two types in two-axis seeker category according to the direction of the rotation axis. The yaw-pitch type has gimbal structures rotating in yaw and pitch direction. It has been widely researched since the location of target image on the image plane is intuitively interpreted into command angle of each gimbal rotation. However, the size of the yaw-pitch type is big and it has large blind angle, which is normally 25° on each end, due to the structural limitation[1-2]. On the other hand, the roll-pith type has gimbal structures rotating in roll and pitch direction. The roll-pitch seeker is not only designed in small size but also tracks the target without having blind angle in frontal hemisphere of the missile[1-3]. The roll gimbal can wind up without bounds, and the pitch gimbal rotates up to 90º. Also, the actuating rate of the gimbals is faster in the roll-pitch seeker[1]. By the advantages, the roll-pitch seekers become being widely used in many high performance missiles such as modern air-to-air missiles (AAM).

In spite of the advantages, there is a drawback called the singularity problem[4] or the zenith pass problem[1] in the roll-pitch seeker. When the target image is at the singular point, the command angle for the roll gimbal to track the target goes indefinite. The singular point appears where the axial axis of the missile crosses the image plane with the pitch gimbal rests at 0°. When the target image is located near the singular point, a problem arises. The singularity problem may cause a critical effect on missile guidance. Near the singularity, the sensitivity of roll command angle to the target position increases. This causes an abrupt oscillatory motion of the roll gimbal around the singular point, which affects the guidance performance. In order to implement the roll-pitch seeker, a control scheme is necessary to avoid the singularity problem.

The case discriminative algorithm has been proposed by Hartmann et al. to relieve the singularity problem[4], and Jiang et al. agreed with the algorithm[1]. Basically, this method is based on the classical control, based on the proportional-derivative (PD) controllers. This algorithm introduces multiple criteria which can describe the singularity, and upon those criteria different control gains are used for the PD controllers. Still it suggests a method to relieve the singularity problem, threshold levels consisting the criteria should be determined in ad-hoc manner. Also, the switching mechanisms make the control command non-differentiable function at the threshold.

In this paper, an optimal control method is introduced to provide mathematical formulation for the singularity problem and proposes a systematic approach to relieve the problem. When the target image is formed far from the singular point, the singularity problem is not an issue. Without singularity consideration, it is desirable for gimbals to follow the target quickly. The maximum rate actuation can be imposed as far as the stability is ensured. This way of control minimizes state errors. However, when the target image is projected near the singular point, it is desirable to save control energy so that the unnecessary roll gimbal actuation is avoided. Proximity of the target image to the singular point recognizes two objectives with different weights. Thus, by formulating a dual objective optimization problem, it is possible to obtain a control scheme which is continuous throughout the whole domain of the image plane.

This paper is composed as follows. Chapter Ⅱ analyzes the singularity problem in roll-pitch seeker. It starts with the frame definitions, upon which the dynamics of missile and seeker is described. Chapter Ⅲ provides the formulation of the optimal control problem. Chapter Ⅳ introduces a method to avoid the singularity problem integrated in the optimal control scheme. Chapter Ⅴ concludes the paper and identifies future works.

II. Singularity in Roll-Pitch Seeker

A. Frame Definitions

In order to describe the dynamics of missile, seeker, and target, it is required to introduce different frames[5-6]. The inertial frame (I-frame) is a fixed frame attached on the earth. The body frame (s-frame) is attached to the missile, with its x-axis (x_s) heads toward the nose direction, y-axis (y_s) orients toward the starboard side, and z-axis (z_s) toward the belly of the missile. The seeker roll gimbal frame (R-frame) is defined by rotating s-frame by amount of roll gimbal rotation. The seeker head frame (h-frame) is obtained after rotating R-frame by the rotation of pitch gimbal. To describe the target position, the reference frame (r-frame) is adopted. The r-frame is defined by rotating I-frame in yaw and pitch direction in sequence to point the target with the x-axis. The relationship and transformation between frames are described in Fig. 1.

In this paper, the behavior of seeker is analyzed regardless of the motion of missile. It is assumed that the missile rests at fixed position with $\psi_b = \theta_b = \phi_b = 0$. The assumption reduces the directional cosine matrix (DCM) between I- and s-frame to a 3-by-3 identity matrix. Thus, I- and s-frame coincides. The DCM from s- to h-frame $\mathbf{T}_{hs}$ and I- to r-frame $\mathbf{T}_{rI}$ are calculated as

$$\mathbf{T}_{hs} = \begin{bmatrix} \cos\lambda_P & \sin\lambda_P \sin\lambda_R & -\sin\lambda_P\cos\lambda_R \\ 0 & \cos\lambda_R & \sin\lambda_R \\ \sin\lambda_P & -\cos\lambda_P\sin\lambda_R & \cos\lambda_P\sin\lambda_R \end{bmatrix}, \quad (1)$$

$$\mathbf{T}_{rI} = \begin{bmatrix} \cos\theta_r\cos\psi_r & \cos\theta_r\sin\psi_r & -\sin\theta_r \\ -\sin\psi_r & \cos\psi_r & 0 \\ \sin\theta_r\cos\psi_r & \sin\theta_r\sin\psi_r & \cos\theta_r \end{bmatrix}. \quad (2)$$

B. Dynamics of Gimbals

The roll gimbal rotates around x_s. The amount and the rate of rotation are expressed in s-frame by the angle λ_R and the angular rate $\dot{\lambda}_R$, respectively. Because s-frame coincides with I-frame, the angular rate of roll gimbal in I-frame is

$$\vec{\boldsymbol{\omega}}_{R_I} = \vec{\boldsymbol{\omega}}_{R_s} = \begin{bmatrix} \dot{\lambda}_R & 0 & 0 \end{bmatrix}^{\mathrm{T}}. \quad (3)$$

The pitch gimbal rotates around y_R. The amount and the rate of rotation are expressed in h-frame by the angle λ_P and the angular rate $\dot{\lambda}_P$, respectively.

$$\vec{\boldsymbol{\omega}}_{P_h} = \begin{bmatrix} 0 & \dot{\lambda}_P & 0 \end{bmatrix}^{\mathrm{T}}. \quad (4)$$

The angular rate of pitch gimbal in I-frame is

$$\vec{\boldsymbol{\omega}}_{P_I} = \mathbf{T}_{hs}\vec{\boldsymbol{\omega}}_{R_s} + \vec{\boldsymbol{\omega}}_{P_h} = \begin{bmatrix} \dot{\lambda}_R\cos\lambda_P \\ \dot{\lambda}_P \\ \dot{\lambda}_R\sin\lambda_P \end{bmatrix}. \quad (5)$$

Now, the angular momentum of pitch gimbal is

$$\vec{\mathbf{H}}_P = \mathbf{J}_P\vec{\boldsymbol{\omega}}_{P_I} = \begin{bmatrix} J_{Px}\dot{\lambda}_R\cos\lambda_P \\ J_{Py}\dot{\lambda}_P \\ J_{Pz}\dot{\lambda}_R\sin\lambda_P \end{bmatrix}. \quad (6)$$

The motion of pitch gimbal has an effect on the angular momentum of roll gimbal as

$$\begin{aligned} \vec{\mathbf{H}}_R &= \mathbf{J}_R\vec{\boldsymbol{\omega}}_{R_I} + \mathbf{T}_{Rh}\vec{\mathbf{H}}_P \\ &= \begin{bmatrix} J_{Rx}\omega_{Rx} \\ J_{Ry}\omega_{Ry} \\ J_{Rz}\omega_{Rz} \end{bmatrix} + \begin{bmatrix} \cos\lambda_P & 0 & \sin\lambda_P \\ 0 & 1 & 0 \\ -\sin\lambda_P & 0 & \cos\lambda_P \end{bmatrix}\begin{bmatrix} J_{Px}\omega_{Px} \\ J_{Py}\omega_{Py} \\ J_{Pz}\omega_{Pz} \end{bmatrix} \\ &= \begin{bmatrix} J_{Rx}\dot{\lambda}_R + J_{Px}\dot{\lambda}_R\cos^2\lambda_P + J_{Pz}\dot{\lambda}_R\sin^2\lambda_P \\ J_{Py}\dot{\lambda}_P \\ -J_{Px}\dot{\lambda}_R\sin\lambda_P\cos\lambda_P + J_{Pz}\dot{\lambda}_R\sin\lambda_P\cos\lambda_P \end{bmatrix}. \end{aligned} \quad (7)$$

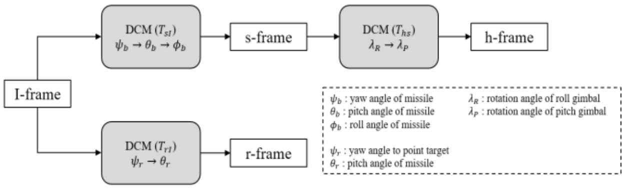

Figure 1. Relationship and transformation between frames

The DCM $\mathbf{T}_{Rh}T_{Rh}$ provides transformation from h- to R-frame. Now, by applying the Euler's equation for the pitch gimbal, the external torque is expressed as

$$\begin{aligned} M_P &= \dot{\vec{\mathbf{H}}}_P + \vec{\boldsymbol{\omega}}_{P_I} \times \vec{\mathbf{H}}_P \\ &= J_{Py}\dot{\omega}_{Py} + (J_{Px} - J_{Pz})\omega_{Px}\omega_{Pz}. \end{aligned} \tag{8}$$

For the roll gimbal,

$$\begin{aligned} M_R &= \dot{\vec{\mathbf{H}}}_R + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{H}}_R \\ &= J_{Rx}\dot{\omega}_{Rx} + J_{Px}(\dot{\omega}_{Px}\cos\lambda_P - \omega_{Px}\dot{\lambda}_P\sin\lambda_P) \\ &\quad + J_{Pz}(\dot{\omega}_{Pz}\sin\lambda_P + \omega_{Pz}\dot{\lambda}_P\cos\lambda_P) \\ &\quad + (J_{Rz}\omega_{Rz} - J_{Px}\omega_{Px}\sin\lambda_P + J_{Pz}\omega_{Pz}\cos\lambda_P)\omega_{Ry} \\ &\quad - (J_{Ry}\omega_{Ry} + J_{Py}\omega_{Py})\omega_{Rz}. \end{aligned} \tag{9}$$

From (8) and (9), the angular accelerations due to actuating external torques are given as

$$\ddot{\lambda}_P = \frac{M_P}{J_{Py}} - \frac{J_{Px} - J_{Py}}{2J_{Py}}\dot{\lambda}_R^2 \sin 2\lambda_P, \tag{10}$$

$$\ddot{\lambda}_R = \frac{M_R - (J_{Pz} - J_{Px})\dot{\lambda}_P\dot{\lambda}_R \sin 2\lambda_P}{J_{Rx} + J_{Px}\cos^2\lambda_P + J_{Pz}\sin^2\lambda_P}. \tag{11}$$

C. Singularity of Roll-Pitch Seeker

The seeker configuration which points a target is described in Fig. 2. A target image is projected at point T on the image plane. To point the target with x_h axis, the roll gimbal rotates by λ_R with respect to the s-frame and subsequently the pitch gimbal rotates by λ_P with respect to the R-frame. By measuring the position of the image, the rotation angles are determined.

In s-frame, the location of image is measured by Y and Z. From triangle OTT',

$$\mathrm{OT} = \mathrm{OT}'\sin\lambda_P. \tag{12}$$

Here, $\mathrm{OT}' = \mathrm{OP} = l$, which is the radius of image plane dimension. Substituting OT with $\sqrt{\mathrm{OZ}^2 + \mathrm{OY}^2}$ gives

$$\lambda_P = \arcsin\left(\frac{\sqrt{\mathrm{OZ}^2 + \mathrm{OY}^2}}{l}\right). \tag{13}$$

While $0 \le \mathrm{OT} \le 1$ and $0 \le l$ hold, λ_P is always determined definitely in the boundary $0 \le \lambda_P \le \pi/2$. From triangle TOZ,

TABLE I. SEEKER PARAMETERS

Parameter		Value
Notation	*Description*	
J_P	Moment of inertia, pitch gimbal[a]	4.6×10^{-5} kg· m^2
J_R	Moment of inertia, roll gimbal[a]	1.22×10^{-3} kg· m^2
$\lvert\dot{\lambda}_P\rvert_{\max}$	Maximum rate, pitch gimbal	1600° s^{-1}
$\lvert\dot{\lambda}_R\rvert_{\max}$	Maximum rate, roll gimbal	1600° s^{-1}

a. Symmetric, having same value for all axes.

$$-\frac{\mathrm{ZT}}{\mathrm{OZ}} = \tan\angle\mathrm{TOZ} = \tan\lambda_R. \tag{14}$$

The negative sign on the left-hand side comes from the axes direction. Since $\mathrm{ZT} = \mathrm{OY}$, for unbounded roll gimbal angle is obtained as

$$\lambda_R = -\arctan 2(\mathrm{OY}, \mathrm{OZ}) + 2n\pi, \qquad n \in \mathbf{Z}. \tag{15}$$

From the relation above, when the target image is formed at point O, which is the singular point, OY and OZ are zero. In this case, λ_R becomes indefinite. This phenomenon is defined as the singularity problem in a roll-pitch seeker.

III. OPTIMAL CONTROL OF GIMBALS

A. Boresight Error

To obtain target information accurately, it is advantageous for the seeker to have the target image on its central area of the image plane. There is less opportunity to loss the target from field-of-view (FOV) around the central area. However, at the central area the singularity problem becomes an issue, causing undesirable gimbal actuation. In order to maintain the target on the central area with avoiding the singularity problem, it is critical to measure the target position on the image plane. For this reason, the boresight error is measured in real-time.

The angular error from the aim point and the target image is called the boresight error. The boresight error is measured in h-fame ($\boldsymbol{\varepsilon}_h$), being decomposed into ε_{yh} and ε_{zh} in direction of y_h and z_h axis respectively. The target position is indicated

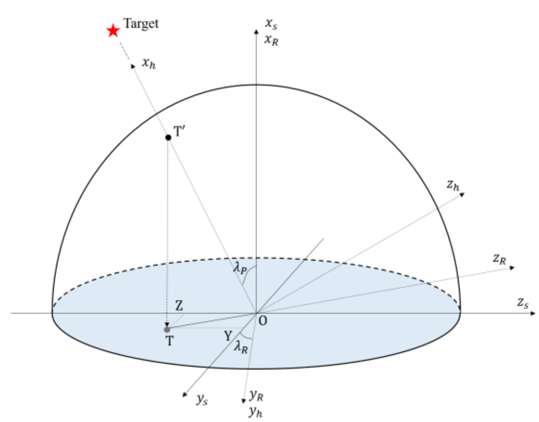

Figure 2. Seeker configuration and target image projection

by the pointing vector $\vec{\mathbf{x}}_{P_r} = \begin{bmatrix}1 & 0 & 0\end{bmatrix}^{\mathrm{T}}$ in r-frame. Since r-frame is rotated by ψ_r and θ_r from I-frame, the pointing vector can be transformed into I-frame as

$$\vec{\mathbf{x}}_{P_I} = \mathbf{T}_{Ir}\vec{\mathbf{x}}_{P_r} = \mathbf{T}_{rI}{}^{\mathrm{T}}\vec{\mathbf{x}}_{P_r} = \begin{bmatrix}\cos\theta_r\cos\psi_r \\ \cos\theta_r\sin\psi_r \\ -\sin\theta_r\end{bmatrix}. \tag{16}$$

Since a stationary missile is assumed, I-frame is identical to s-frame. Therefore, $\vec{\mathbf{x}}_{P_I} = \vec{\mathbf{x}}_{P_s}$. On the other hand, when the target image is formed at the point (OY, OZ), the pointing vector is expressed in h-frame as

$$\begin{aligned}\vec{\mathbf{x}}_{P_h} &= \begin{bmatrix}\cos\varepsilon_{y_h}\cos\varepsilon_{z_h} & \cos\varepsilon_{y_h}\sin\varepsilon_{z_h} & -\sin\varepsilon_{y_h} \\ -\sin\varepsilon_{z_h} & \cos\varepsilon_{z_h} & 0 \\ \sin\varepsilon_{y_h}\cos\varepsilon_{z_h} & \sin\grave{o}_{y_h}\sin\varepsilon_{z_h} & \cos\varepsilon_{y_h}\end{bmatrix}^{\mathrm{T}} \begin{bmatrix}1 \\ 0 \\ 0\end{bmatrix} \\ &= \begin{bmatrix}\cos\varepsilon_{y_h}\cos\varepsilon_{z_h} \\ \cos\varepsilon_{y_h}\sin\varepsilon_{z_h} \\ -\sin\varepsilon_{y_h}\end{bmatrix}.\end{aligned} \tag{17}$$

However,

$$\vec{\mathbf{x}}_{P_h} = \mathbf{T}_{hs}\vec{\mathbf{x}}_{P_s} = \begin{bmatrix}x_{p1} & x_{p2} & x_{p3}\end{bmatrix}^{\mathrm{T}}. \tag{18}$$

From the equality between (17) and (18),

$$\varepsilon_{y_h} = \arcsin\left(-x_{p3}\right), \tag{19}$$

$$\varepsilon_{z_h} = \arctan\left(x_{p2}/x_{p1}\right). \tag{20}$$

B. Optimal Control Problem Formulation

Since the seeker is required to rotate fast toward the target, it is desirable to control the seeker in a way of minimizing the boresight error integration. However, since the seeker control system draws energy from the missile, it is not realistic that the system imposes infinite control energy to actuate the seeker. In order to prevent energy depletion, it is beneficial to minimize the control effort integration as well. Therefore, the objective of the optimal control problem is formulated by augmenting the error and the control effort term for integrand.

A form of non-linear optimization problem in continuous time is defined as

$$J = \int_{t_0}^{t_f}\left[\boldsymbol{\varepsilon}^{\mathrm{T}}(t)\mathbf{Q}(t)\boldsymbol{\varepsilon}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}(t)\mathbf{u}(t)\right]dt, \tag{21}$$

$$\begin{aligned}\text{s.t.}\quad & \dot{\mathbf{x}} = f(t,\mathbf{x},\mathbf{u}),\ \mathbf{x}(t_0) = \mathbf{x}_0,\ \mathbf{x}(t_f) = \mathbf{x}_f,\ \boldsymbol{\varepsilon}(t_f) = \boldsymbol{\varepsilon}_f \\ & \mathbf{L}_{\mathbf{x}} \le \mathbf{x} \le \mathbf{U}_{\mathbf{x}},\quad \mathbf{L}_{\mathbf{u}} \le \mathbf{u} \le \mathbf{U}_{\mathbf{u}}.\end{aligned} \tag{22}$$

Here, J is an objective function, $\mathbf{x}$ is the state vector, $\boldsymbol{\varepsilon}$ is the error vector, $\mathbf{u}$ is the control vector, t_0 and t_f are initial and final time respectively, and $\mathbf{Q}$, $\mathbf{R}$ are weight matrices. In order to describe the dynamics of the seeker, the state vector is

$$\mathbf{x} = \begin{bmatrix}\lambda_P & \lambda_R & \dot{\lambda}_P & \dot{\lambda}_R\end{bmatrix}^{\mathrm{T}}. \tag{23}$$

The $\mathbf{x}_0$ describes the initial seeker configuration, which can be arbitrary according to the initial seeker states. At the final time, the seeker is stationary at the angle λ_{Pf} and λ_{Rf}, which are (13) and (15). Accordingly, the final state is

$$\mathbf{x}_f = \begin{bmatrix}\lambda_{Pf} & \lambda_{Rf} & 0 & 0\end{bmatrix}^{\mathrm{T}}. \tag{24}$$

From the states, the boresight error is measured,

$$\boldsymbol{\varepsilon} = \begin{bmatrix}\varepsilon_{y_h} & \varepsilon_{z_h}\end{bmatrix}^{\mathrm{T}}. \tag{25}$$

Also, for the control vector,

$$\mathbf{u} = \begin{bmatrix}M_P & M_R\end{bmatrix}^{\mathrm{T}}. \tag{26}$$

The function f is obtained by (10) and (11). The weight matrices determine the weight on the error and control effort term. Furthermore, those matrices enable varying weights on each element of the error and control. If **Q**>>**R**, the optimal control let the seeker gimbal to actuate as fast as possible to reduce the accumulated error. However, if **Q**<<**R**, the seeker moves smoothly to minimize the control energy, permitting the boresight error to be reduced slowly.

The seeker parameters are summarized in Table 1. Bounds for the control input is not assumed. The case with $\mathbf{Q}=\mathbf{I}_2$ and $\mathbf{R}=\mathbf{0}$ results in Fig. 3, and with $\mathbf{Q}=\mathbf{0}$ and $\mathbf{R}=\mathbf{I}_2$ in Fig. 4.

IV. Singularity Avoidance Algorithm

To avoid the singularity problem, it is required for the roll gimbal to be insensitive to the target position near the singular point. In the case discriminative method proposed earlier[3], the rate of roll gimbal actuation is suppressed if the target image is formed in a defined area with pre-determined size. However, this method incorporates non-differentiable control scheme at the boundary of different areas. Also, gains in PD controller and rate estimators should be tuned finely to achieve acceptable performance, although which is hardly an optimal one. In this chapter, the optimal control scheme is applied to avoid the singularity problem.

A. Optimal Control for Singularity Avoidance

When the target image is far from the singular point, the seeker has to rotate rapidly to follow the target. Therefore, it is reasonable to have more weight on the error integrand in the objective function. Since the magnitude of boresight error is $\boldsymbol{\varepsilon}^{\mathrm{T}}\boldsymbol{\varepsilon}$, the weight matrix $\mathbf{Q}$ is in a form of

$$\mathbf{Q}(t) = q\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}, \qquad q:\ \text{constant}. \tag{27}$$

On the other hand, when the target image is near the singular point, the roll gimbal should become insensitive to the target position. It is toward minimizing control energy term. Particularly, it is only required to suppress the actuation of roll gimbal, not pitch gimbal. This is because the pitch gimbal angle provides cue to determine the proximity to the singular point. In fact, the singular point is reached when the pitch gimbal angle λ_P goes to zero. If the pitch gimbal actuation were suppressed as for the roll gimbal, from the initial state λ_P=0 the seeker cannot escape from the singularity.

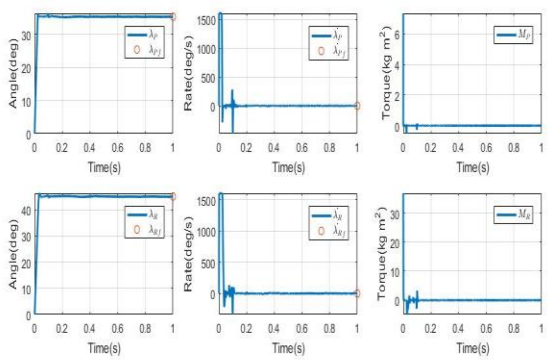

Figure 3. Optimal control of the seeker with **Q**=**I**$_2$ and **R**=**0**.

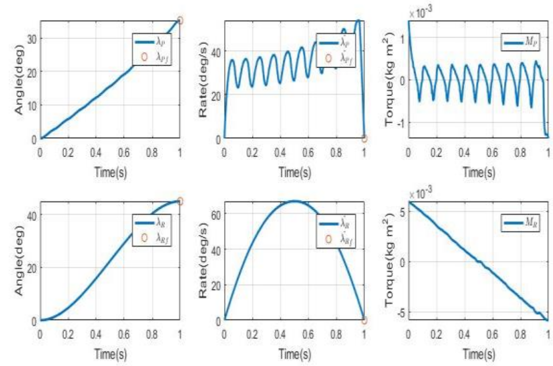

Figure 4. Scheduling r with regard to pitch gimbal angle.

If the control energy for the pitch gimbal were considered, it is beneficial not to actuate the pitch gimbal to minimize the control energy.

In order to only consider the control energy imposed on the roll gimbal, the weight matrix **R** is defined as

$$\mathbf{R}(t) = r(t)\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{28}$$

Now, to suppress the roll gimbal actuation near the singular point, the weight r is designed to have a larger value when the pitch gimbal angle goes down to zero. A decaying exponential function is adopted for the singularity avoidance algorithm, as

$$r(t) = \exp\left(-\lambda_P(t)\right). \tag{29}$$

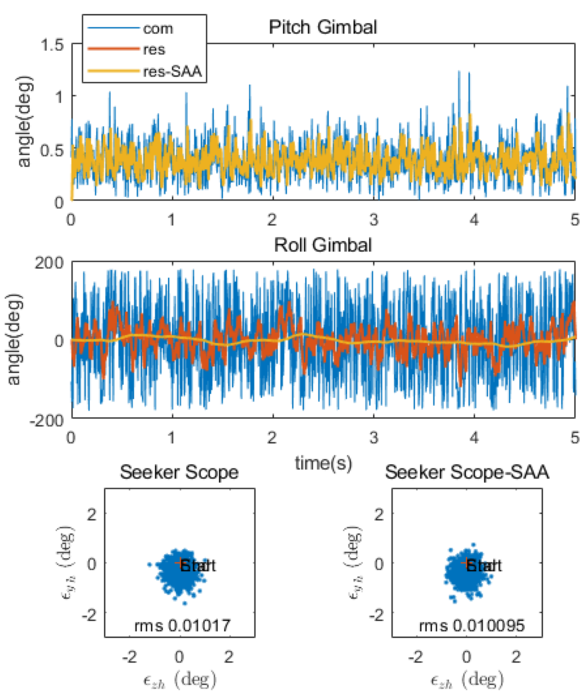

Figure 5. Optimal control of the seeker with **Q**=**0** and **R**= **I**$_2$.

Fig. 5 shows the seeker response when the target position measurement is corrupted by noise near the singularity. With proposed method, abrupt response of roll gimbal is effectively suppressed. Furthermore, with the algorithm implemented the seeker prevents target image to be dispersed by abrupt roll gimbal motion, yielding an improved root-mean-squared (rms) boresight error.

B. *Optimization Result*

Without loss of generality, the weight q in (30) is set to 100. The optimization is performed from t_0 =0 to t_f =0.1, by using GPOPS-II Version 1.0[7].

An engagement scenario is established as

- Seeker: stationary at the origin of I-frame,
- Target: stationary at (6000m, 3000m, -3000m),
- No measurement noise is assumed.

Fig. 6 shows the optimization results. It is shown that the roll gimbal actuates slowly during the initial phase. The pitch gimbal angle provide a cue that the seeker is in proximity with the singular point. As the pitch gimbal increases to follow the target, the control scheme transits dramatically to actuate the roll gimbal fast. This implies that the singularity avoidance algorithm works well. Furthermore, the transition of control is differentiable throughout the process.

As a result of the optimal seeker control, the target image is captured within 10° range in 0.025 seconds. Fig. 7 shows the history of target image plotted in 1,000 Hz frequency. It shows the proposed algorithm effectively controls the seeker to rapidly track the target with avoiding the singularity problem.

V. Conclusion

This paper proposed an optimal control of roll-pitch seeker with avoiding the singularity problem. An optimal solution is obtained under an engagement scenario. However, in order to apply the method to real system, optimal solutions should be obtained in real-time. In order to achieve the calculation speed, it would be useful to construct a response surface by using a neural network. Furthermore, measurement noise

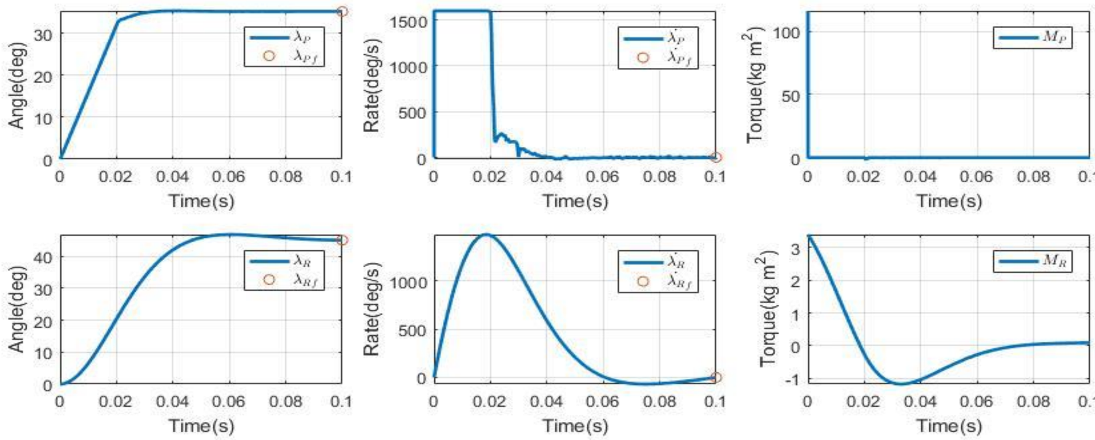

Figure 6. Optimal control with the singularity avoidance algorithm .

might be included to analyze the performance in stochastic system.

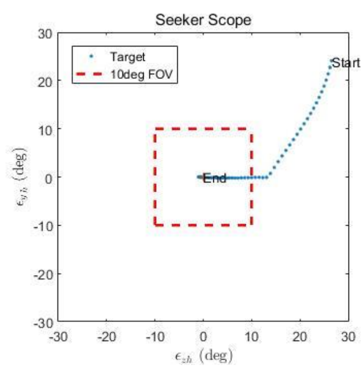

Figure 7. Target image history projected on seeker image panel.

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