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Input shaping for infinite dimensional systems with application on oil well drilling

Dan Pilbauer[1], Delphine Bresch-Pietri[2], Florent Di Meglio[2] Christophe Prieur[1], Tomáš Vyhlídal[3]

***Abstract*— We present an application of the input shaping technique on infinite dimensional systems such as systems described by PDEs. The presented method is a feedforward scheme which allows to target multiple modes of a flexible system and also provide robustness with respect to parameters variations. The method is based on recently developed input shaper with multiple degrees of freedom that are needed to meet all constraints for the given task. We show that, even though the system consists of infinitely many modes, it is required to target only dominant ones. In addition, the method is illustrated on a case example of oil well drilling.**

I. INTRODUCTION

Time delay based input shaping is a well known feedforward technique for pre-compensation of oscillatory modes of flexible mechanical systems. A common application is pre-shaping the trajectory of a crane trolley so that the suspended load does not oscillate when moved from one position to another [1]. Another typical applications are flexible manipulators and industrial robots [2], see also an application in orientation and pointing of solar panels of satellites [3].

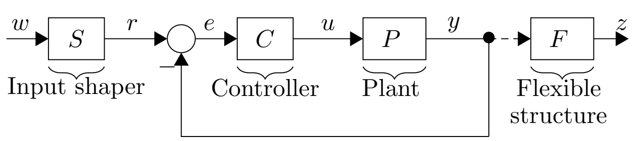

Fig. 1. The classical feedforward application of input shaper

A control scheme with an input shaper is shown in Fig. 1. The original idea of applying an input shapers $S(s)$ in a serial interconnection with a controlled system is to fully or partially compensate its oscillatory modes, determined by one or more couples of complex poles of the flexible structure $F(s)$. From the channel between the shaped system reference w to the output of the flexible structure z, which is not measured as a rule,

$$T_{zw}(s) = \frac{C(s)P(s)S(s)}{1+C(s)P(s)}F(s) \tag{1}$$

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[1]Dan Pilbauer and Christophe Prieur[1], are with Univ. Grenoble Alpes, CNRS, Grenoble INP (Institute of Engineering Univ. Grenoble Alpes), GIPSA-lab, 38000 Grenoble, France, pilbauer@protonmail.com

[2]Florent Di Meglio and Delphine Bresch-Pietri are with MINES ParisTech, PSL Research University, CAS – Centre Automatique et Systemes, 60 bd Saint Michel, 75006 Paris, France

[3]Tomáš Vyhlídal is with CIIRC - Czech Institute of Informatics, Robotics and Cybernetics, and Dept. of Instrumentation and Control Eng., Faculty of Mechanical Engineering, Czech Technical University in Prague, Technická 4, 16607 Praha 6, Czech Republic,

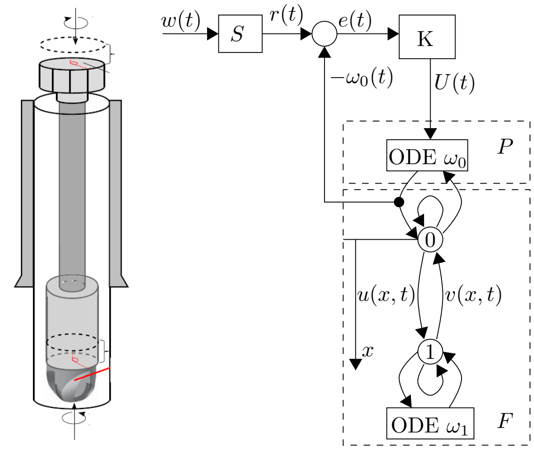

Fig. 2. Left: A scheme of the drill, Right: a block scheme of the drilling system with feedback controller K and feedforward input shaper S

it can clearly be seen that these flexible poles of $F(s)$ can be fully or partially compensated by zeros of the shaper $S(s)$. Thus, the shaper performs the task of a notch filter. The key advantage of the input shapers compared to the classical notch filters is that via involvement of time delays, it is easier to distribute the response in time. This allows to form the responses of monotonously increasing (or decreasing) characters, which are advantageous with respect to the energy demands. This positive aspect of time delay filters has already been recognized by O. Smith in 1950s, who introduced the so-called posi-cast [4], also known as zero vibration (ZV) shaper. Consequently, extensive studies mainly motivated by demands on increasing robustness and decreasing response time have been made proposed in [5], [6]. For an extensive review on input shaping see [7].

A. Extension to infinite dimensional systems

Even though a single dominant flexible mode can be identified in most of the cases, there are applications where multiple modes need to be compensated, see e.g., [8], [9]. The intuitive approach to handle this task is to design a single mode shaper for each of the modes and convolve them by a serial interconnection. However, this leads to a long duration of the shaper action time [10]. In order to mitigate this inefficiency, direct methods have been designed to handle this task [11], [12], where the requirement to address multiple modes is performed within a single design procedure. Due to complexity of the design task, where additional constraints on the shaper performance can also be imposed, optimization based approach has been identified as a promising tool to handle it [13], which will also be utilized in this paper. However, compared to [13] where the shaper involves many lumped delays, our objective is to optimize a shape of a single delay of distributed nature. This will be done by adapting the recently proposed technique [14] to the multi-mode problem, see also [15] and [16].

Despite the fact that the state of the art of the input shapers is mainly focused on finite, low order flexible structures, it has been recently shown that it is possible to apply this technique to infinite order systems with a similar performance. Typical feature of his class of systems covering multi-body mechanical systems, flexible structures described with finite-element models [12], spatially-discretized models of long electrical transmission lines [17], or platoon vehicles [18] is a large or even infinite number of oscillatory modes. Thus the potential to apply the multi-mode shaper structure is obvious. Such an attempt has recently been done in [19] in the application to multi-agent dynamical systems.

B. Oil drilling

Motivated by the above outlined potential and preliminary results on compensation of flexible modes of infinite dimensional systems by input shaping, and utilizing the recent results on optimization of distributed delay shapers, the objective of this paper is to design a multi-mode shaper to compensate the dominant oscillatory modes of an oil drilling system. The scheme of a standard device is depicted in Fig. 2. It consists of a rotary table at the top, which is usually attached to a drilling rig located on an onshore or offshore platform or on a drilling ship. Next, it consists of a drill string and a bottom hole assembly. Drill sting vibrations and the so-called stick-slip phenomena bring negative effects on the lifespan of the drill string and equipment. The model's complexity is additionally increased due to its internal coupling and infinite spectrum with neutral distribution. In this paper, we propose to apply input shaping to avoid the oscillations of the drill string by modifying the reference trajectory and providing smooth changes.

The paper is organized as follows. In Section II, background and recent results on input shaping are given. Application of the input shaper together with numerical results and simulation experiments are described in Section III. Section IV concludes the presented results and summarizes the contributions.

II. INPUT SHAPER WITH DISTRIBUTED DELAY

A. Problem statement

The scheme of the proposed control is in Fig. 2 where the main goal is to design a feedforward controller (input shaper) S for an infinite dimensional plant P with low-damped oscillatory modes which does not excite modes of flexible structure F. The plant P is considered to be stabilized by controller C. The input shaper S shapes the reference signal w in such a way it provides non-oscillatory response of the flexible structure F while maintaining its fast response. Usually, the price for the shaped response is response time, which will be always longer. To summarize, the objective in this approach design is the choice of parameters of the input shaper which give minimal response time.

B. Input shapers with distributed delay

An input shaper with distributed delay can be described in general form as

$$r(t) = Aw(t) + (1-A)\int_0^T w(t-\eta)dh(\eta), \tag{2}$$

where the parameter $A \in [0,1]$ and the delay distribution $h(\eta)$ over the finite length segment $\eta \in [0,T]$ satisfies $h(\eta) = 1, \eta \geq T$. We can write the transfer function of the distributed delay (2) as

$$G(s) = A + (1-A)F(s,T), \tag{3}$$

where

$$F(s,T)w(s) = \mathcal{L}\left\{\int_0^T w(t-\eta)dh(\eta)\right\} \tag{4}$$

is the transfer function of the delay. The method of input shaping is based on spectral theory, where the idea is to place a couple of dominant complex zeros of the shaper (2) on a location of poles of the flexible structure. As the input shaper is delay-based, its spectrum consists of infinitely many zeros. However, besides the dominant couple the infinitely many zeros of (3) follows the exponential asymptotic curves departing from the stability boundary with increasing amplitude.

In [20], a zero vibration shaper with equally distributed delay (DZV) was proposed, considering the transfer function of the delay $F(s) = \frac{1-e^{-sT}}{sT}$. Next to the retarded spectrum, a smoother transition and filtering effect are the main benefits in comparison with classical ZV shaper. On the other hand, the price to pay is the increased response time of the input shaper.

In order to increase robustness, i.e. provide distributed delay alternatives to the classical extra-insensitive (EI), zero-derivative-vibration (DZV) shapers, and additionally a least squares approach for distributed delay shapers have been proposed in [21].

A technique using smooth kernel function was proposed by [14] where transfer function of the shaper is given by

$$G(s) = A + \int_0^T g(\theta)e^{-s\theta}d\theta. \tag{5}$$

The kernel function $g(\theta)$ is chosen as the polynomial $g(\theta) = \sum_{i=0}^{N_p} a_i\theta^i$, where $N_p \in \mathbb{N}$ is the degree of the polynomial and A, a_i are gains to be assigned. We can rewrite (5) as

$$G(s) = A + \sum_{i=0}^{N_p} a_i g_i(s), \tag{6}$$

where the functions $g_i(s)$ ("moments" of $e^{-s\theta}$) are given by

$$g_i(s) = \int_0^T \theta^i e^{-s\theta} d\theta. \tag{7}$$

Note, that the shaper is still linear in parameters A, a_i, which significantly simplifies the design procedure. This approach has more degrees of freedom in the design. This flexibility allows the designer to select more constraint in view of satisfying the requirements of the application under consideration.

In order to remove undesirable oscillations, e.g., achieve zero-pole cancellation, we firstly define constraints on placing zeros of the shaper at the expected position of the oscillatory mode to be compensated denoted as

$$\hat{s}_n = -\alpha_n \pm j\beta_n,$$

where $\alpha_n,\ \beta_n \geq 0$, $\hat{s}_n \in \mathbb{C}$ and $n \in [1, N]$ for a given number $N \in \mathbb{N}$ of oscillatory modes to be compensated. By placing zeros of the shaper, we achieve zero-pole cancellation, which removes entirely the undesired oscillatory modes. These requirements corresponds to

$$G(\hat{s}_n) = 0 \Rightarrow A + \sum_{i=0}^{N_p} a_i g_i(\hat{s}_n) = 0, \tag{8}$$

which can be turned into two real equations for the case of placing a complex zero,

$$\Re\left\{G(\hat{s}_n)\right\} = 0, \tag{9}$$

$$\Im\left\{G(\hat{s}_n)\right\} = 0. \tag{10}$$

As will be shown for the case of infinite dimensional systems we need to place multiple zeros. However, each additional zero decreases the number of degrees of freedom. In order to arrive at a feasible solution, this may lead to an increase of the time delay of the shaper.

Additionally, input shaper (5) requires more constraints that are common for every shaper [15]. First requirement is that the static gain equals one, which can be written in from

$$G(0) = 1 \Rightarrow A + \sum_{i=0}^{N_p} a_i g_i(0) = 1. \tag{11}$$

The additional linear equality constraint that might be necessary corresponds to the requirement of continuity of both the step response and its derivatives at times $t = 0$ or (and) $t = T$. At $t = 0$ it is expressed by

$$A = 0, \quad g(0) = a_0 = 0, \tag{12}$$

and for $t = T$ by

$$g(T) = \sum_{i=0}^{N_p} a_i T^i = 0. \tag{13}$$

The next fundamental requirement that needs to be considered is the *non-decreasing step response* [22], [23], or equivalently, the *non-negative impulse response*, which can be formulated as

$$g(\alpha) \geq 0,\ \forall \alpha \in [0,\ T]. \tag{14}$$

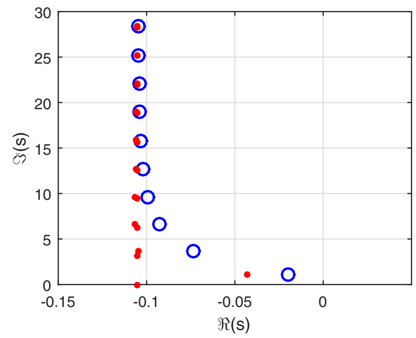

Fig. 3. Red dots: Spectrum of the closed loop system (26)-(27). Blue circles: spectrum of flexible part of the system F, where the dynamics of ODE ω_0 at the top is not considered

Condition (14) is a semi-infinite polynomial inequality, i.e., a requirement to be satisfied for a continuum of α values. The requirement can be solved via Pólya's relaxation (for more details see [14]).

Optionally, next constraints can be added, such as continuity of both the step response and its derivatives or limiting the jerk and jounce, which would be important in case where additional requirements on the system are needed, e.g., less wear on mechanical parts or smoother run of motors. Since this is not the main aim of this paper we omit details, and refer the interested readers to [14].

We may now define an optimization problem as

$$\begin{array}{l} \min_{\mathbf{x}} T, \\ \text{subject to} \\ \left\{ \begin{array}{l} A_1(T)\mathbf{x} = b_1(T), \\ A_2(T)\mathbf{x} \geq b_2(T). \end{array} \right. \end{array} \tag{15}$$

where gain vector $\mathbf{x} = [A\ a_0\ \dots\ a_{N_p}]$ and equality constraints $A_1(T)\mathbf{x} = b_1(T)$ come from (9)–(12) and inequality constraints are given by (14). The dependance of the matrices $A_1,\ A_2$ on the parameter T is here solved by fixing the total delay length and looking for a feasible solution. If the solution is not found, the total time or the number of parameters N_p is increased.

III. APPLICATION TO DRILLING

In order to show applicability of the proposed method, a case example is provided in the following. We apply the method on an oil drilling model. Avoiding oscillations of such systems is crucial because they can cause damage to the equipment and increase the non-productive time. A degenerate version of this behavior is often called stick-slip, an highly undesirable oscillatory limit cycle.

A. System description

We use a model similar to [24] where we consider dynamics of the drive at the top of the rig [25], [26], corresponding to the ODE governing ω_0. The model also encompasses the linearized dynamics of the interaction of the bit with the surface at the bottom of the well, corresponding to the ODE

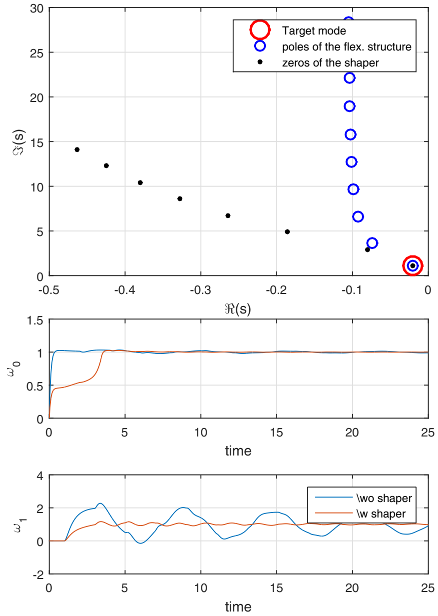

Fig. 4. Top: Spectrum of the shaper with one exact zero placed, Middle: Step change in reference velocity ω_0 without input shaper (blue) and with input shaper (red), Bottom: Response of the velocity at the bottom ω_1 to the step response of the reference to the top velocity ω_0

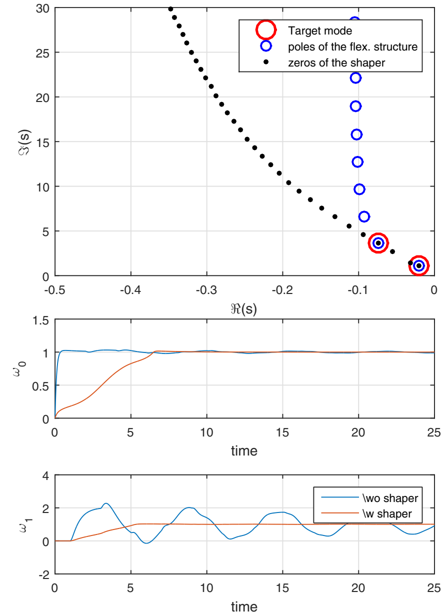

Fig. 5. Top: Spectrum of the shaper with two exact zeros placed, Middle: Step change in reference velocity ω_0 without input shaper (blue) and with input shaper (red), Bottom: Response of the velocity at the bottom ω_1 to the step response of the reference to the top velocity ω_0

governing ω_1, and the drill itself as a distributed system described by two PDEs. It writes

$$\dot{\omega}_0(t) = -a_0\omega_0(t) + b_0 v(0,t) + U(t), \tag{16}$$
$$u(0,t) = q_0 v(0,t) + c_0\omega_0(t), \tag{17}$$
$$u_t(x,t) = -\lambda u_x(x,t), \tag{18}$$
$$v_t(x,t) = \mu v_x(x,t), \tag{19}$$
$$v(1,t) = q_1 u(1,t) + c_1\omega_1(t), \tag{20}$$
$$\dot{\omega}_1(t) = -a_1\omega_1(t) + b_1 u(1,t), \tag{21}$$

where ω_0, ω_1 are velocities of the drill at the top and at the bottom, respectively. $u(x,t)$, $v(x,t) \in [0,\infty) \times [0,1]$ are describing propagation of the velocity through the drill with parameters λ and μ denoting speed of propagation through the drill. Parameters a_0, a_1, b_0, b_1, c_0, c_1, q_0, q_1 are constants of the system.

The ODE-PDE-ODE system (16)–(21) can be transformed into delay differential equations (DDEs) as follows. We first notice that from (17)–(19)

$$\begin{aligned} u(1,t) =& q_0 v\left(0, t-\tau_1\right) + c_0\omega_0\left(t-\tau_1\right) = \\ & q_0 v\left(1, t-\tau_3\right) + c_0\omega_0\left(t-\tau_1\right), \end{aligned} \tag{22}$$

where $\tau_1 = 1/\lambda, \tau_2 = 1/\mu$ and $\tau_3 = 1/\lambda + 1/\mu$. Then, by substituting $v\left(1, t-\tau_3\right)$ by (20), shifted by time $t-\tau_3$, we can write

$$\begin{aligned} u(1,t) = & q_0\left(q_1 u\left(1, t-\tau_3\right) + c_1\omega_1\left(t-\tau_3\right)\right) \\ & + c_0\omega_0\left(t-\tau_1\right). \end{aligned} \tag{23}$$

Again, we can substitute $u\left(1, t-\tau_3\right)$ by (21), shifted by time $t-\tau_3$. Following a few algebraic operations and plugging the result into (21), the final DDE is

$$\begin{aligned} &\dot{\omega}_1(t) - q_0 q_1\dot{\omega}_1\left(t-\tau_3\right) = -a_1\omega_1(t) + \\ &\left(q_0 q_1 a_1 + b_1 c_1 q_0\right)\omega_1\left(t-\tau_3\right) + b_1 c_0\omega_0\left(t-\tau_1\right) \end{aligned} \tag{24}$$

Following the same procedure for the second ODE of the variable ω_0, we obtain

$$\begin{aligned} &\dot{\omega}_0(t) - q_0 q_1\dot{\omega}_0\left(t-\tau_3\right) = \\ &- a_0\omega_0(t) + \left(q_0 q_1 a_0 + b_0 c_0 q_1\right)\omega_0(t-\tau_3) \\ &+ b_0 c_1\omega_1(t-\tau_2) + U(t) - q_0 q_1 U(t-\tau_3) \end{aligned} \tag{25}$$

If we define a new state vector as $\Omega = [\omega_1\ \omega_0]^T$, the system in the matrix form is

$$\begin{aligned} \dot{\Omega}(t) + &\begin{bmatrix} -q_0 q_1 & 0 \\ 0 & -q_0 q_1 \end{bmatrix}\dot{\Omega}(t-\tau_3) = \begin{bmatrix} -a_1 & 0 \\ 0 & -a_0 \end{bmatrix}\Omega(t) \\ &+ \begin{bmatrix} q_0 q_1 a_1 + b_1 c_1 q_0 & 0 \\ 0 & q_0 q_1 a_0 + b_0 c_0 q_1 \end{bmatrix}\Omega\left(t-\tau_3\right) \\ &+ \begin{bmatrix} 0 & b_1 c_0 \\ 0 & 0 \end{bmatrix}\Omega\left(t-\tau_1\right) + \begin{bmatrix} 0 & 0 \\ b_0 c_1 & 0 \end{bmatrix}\Omega\left(t-\tau_2\right) \\ &+ \begin{bmatrix} 0 \\ 1 \end{bmatrix}U(t) + \begin{bmatrix} 0 \\ -q_0 q_1 \end{bmatrix}U\left(t-\tau_3\right) \end{aligned} \tag{26}$$

The system is commonly controlled via PI or PID controller. Here, we compare the results with a PI controller tuned to achieve fast tracking at the top velocity, which is the case in the field. The controller is in the form

$$K:\ U(t) = k_P\left(r(t) - \omega_0(t)\right) + k_I\int_0^t\left(r(\xi) - \omega_0(\xi)\right)d\xi \tag{27}$$

We can check the spectrum of the closed loop (26)–(27) which is given in Fig. 3 (red dots). However, for application of input shaping we must mention that the targeted system (flexible structure) corresponds only to the part F shown in Fig. 2 bounded by dashed box. The zeros of the shaper

TABLE I

RIGHTMOST POLES OF THE FLEXIBLE STRUCTURE

n	$\Re(s_n)$	$\Im(s_n)$
1	-0.0199	± 1.074
2	-0.0737	± 3.642
3	-0.0931	± 6.578
4	-0.0992	± 9.6294

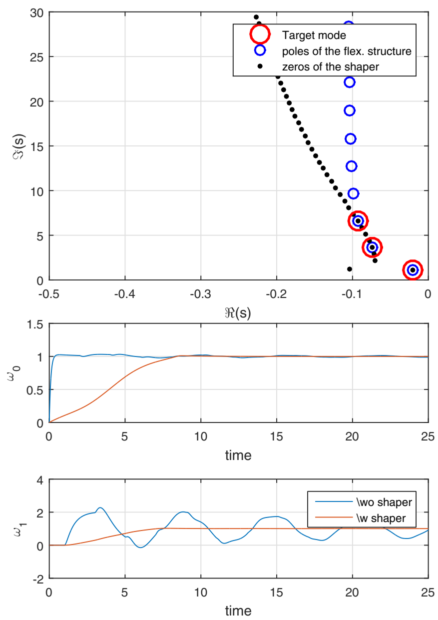

Fig. 6. Top: Spectrum of the shaper with three exact zeros placed, Middle: Step change in reference velocity ω_0 without input shaper (blue) and with input shaper (red), Bottom: Response of the velocity at the bottom ω_1 to the step response of the reference to the top velocity ω_0

should target only the poles of the flexible structure and not the overall system. The poles of the flexible part of the system is shown in Fig. 3 (blue circles). Spectra of the systems have been computed by QPmR algorithm [27].

B. Simulation results

This section presents simulation results for the following set of parameters: $c_0 = c_1 = 2$, $q_0 = q_1 = -0.9$, $a_0 = 0.17$, $a_1 = 1.8, \lambda = \mu = 1$, $b_0 = a_0$, $b_1 = a_1$. Four rightmost poles of the flexible structure are shown in Tab. I. Firstly, we design the input shaper with one zero compensating the rightmost pole of the flexible system. The result is shown in Fig. 4. The response is significantly slower than without the input shaper but the residual vibrations after the response time are much smaller. However, there are still significant residual oscillations after the response time which is due to the fact that the other rightmost poles of the flexible may play a role. If we now place two poles, as shown in Fig. 5, it is easy to see that the response is much smoother and with almost no visible residual vibrations. If we go further, we can place three or four zeros of the shaper, see Figs. 6 and 7. If we place three the response is totally smooth without any oscillation after the response time. If we place four zeros, we can observe that the response is very similar.

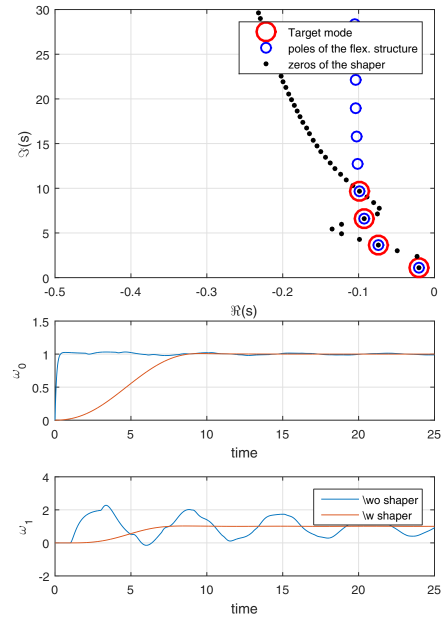

Fig. 7. Top: Spectrum of the shaper with four exact zeros placed, Middle: Step change in reference velocity ω_0 without input shaper (blue) and with input shaper (red), Bottom: Response of the velocity at the bottom ω_1 to the step response of the reference to the top velocity ω_0

The results can be also confirmed by looking at the so called residual vibration function, i.e., amplitude of the oscillation after the response time T. However, this function has been always evaluated for cases where we target only one pole of the flexible structure [14], [21], [28]. Therefore, we need to modify it for multimode input shaper. We can express this function by substituting complex variable $s = \hat{\omega}$ and using (5)-(7). Then we can write

$$V(\hat{\omega}) = |G(\hat{\omega})|e^{\hat{\omega}T} \tag{28}$$

where $\hat{\omega}$ is defined as continuous function crossing each zero of the input shaper that has been placed. This is visually shown in Fig. 8, where points are connected with lines. The starting point is at the origin of the complex plane $\hat{\omega}(0) = 0 + j0$ and the last point goes to infinity with constant real part given by the last placed zero. Fig. 9 shows how the residual vibration function changes for shapers with one, two, three and four zeros placed exactly on the desired locations. As can be seen, when only one zero is placed, the amplitude for the second, third and fourth poles of the flexible structure is still significant. When two zeros are placed, the residual vibrations decreases much faster with increasing $\hat{\omega}$. Placing more zeros improves performance even more.

IV. CONCLUSIONS

As the main contribution, the input shaper with distributed delay is applied to an infinite dimensional system. The input shaper is designed in such a way that it compensates multiple dominant poles of the flexible structure while maintaining fast response. The proposed design of the parameters of the shaper is based on constrained optimization and solved

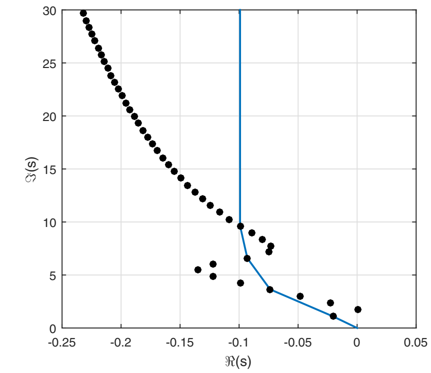

Fig. 8. Spectrum of an input shaper with four placed zeros (black dots) and a line representing values of $\hat{\omega}$ where the residual function is evaluated

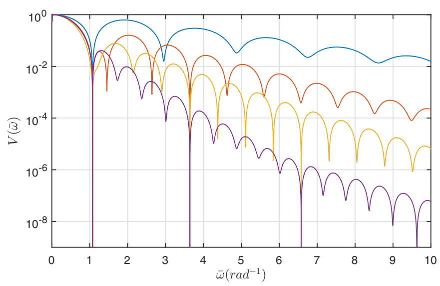

Fig. 9. Residual vibration function (28), for each case, blue line: one zero placed, red line: two zeros placed, yellow line: three zeros placed, purple line: four zeros placed

as linear programing problem. The application of the input shaper is demonstrated on a simplified model of oil drilling. We illustrated that it is not necessary to target all of the modes of the flexible structure to obtain a non-oscillatory response but it is necessary to target more poles than the dominant ones. Moreover, as the real part of shaper's zeros tends to go to minus infinity with increasing moduli, it appears it will not excite any high frequency modes, even though they are not directly targeted.

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