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Distributed Multi-Equilibria Consensus in the Presence of Byzantine Adversaries and Time Delays

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Abstract— **In this paper, fault tolerant multi-equilibria consensus computation is studied when the thread model is considered as structured Byzantine (StrBYZ). First, a brief review on graph theoretic concepts that we use throughout this paper is provided. Then, the structured Byzantine fault model is presented by considering the general Byzantine misbehaviour. It is shown that non-faulty nodes achieve consensus on K equilibria if there are K number of primary and secondary layers with non-faulty root nodes, each of which contains StrBYZ agents having at least one in-neighbor in the network. The analysis is extended to the case where the network has bounded uniform/non-uniform time delays on its communication links. The results are verified by numerical examples.**

I. INTRODUCTION

The problem of reaching agreement on a common value, i.e. consensus, is an important issue for networked systems [1]–[6]. When the network contains faulty agents, the solution of this problem becomes complicated. For the past few years, fault tolerant consensus has been investigated in detail for synchronous [7] and asynchronous [8] networks. In [9], a fault tolerant algorithm has been proposed to solve the approximate Byzantine consensus problem regardless of the synchronicity of the network. Convergence rate analysis of this algorithm has been discussed in [10] by considering time delays and time varying network topologies. A fault model classification has been presented in [11] where the Byzantine misbehaviour is separated into two different categories: *structured Byzantine (StrBYZ)* and unstructured Byzantine (uStrBYZ). It is shown in [11] that non-faulty nodes achieve consensus in the presence of StrBYZ agents without using a fault tolerant algorithm if the layered structure of the network is resilient against the misbehaviour of these StrBYZ agents. However, non-faulty nodes have to use a fault tolerant algorithm for uStrBYZ faults in order to reach consensus on a common value which must be in the range of initial values of themselves [12]. A parameter independent fault tolerant algorithm has been proposed in [13] that solves the approximate Byzantine consensus problem using a distributed fault detection scheme.

Even though single equilibrium consensus has been extensively studied, the problem of achieving multi-equilibria consensus (or group consensus) has not been sufficiently addressed. A network of non-faulty nodes is said to achieve group consensus if they reach $K \geq 2$ equilibrium points. In [14], continuous time multi-equilibria consensus has been discussed under undirected topologies. Discrete time version of the above study for directed topologies has been presented in [15]. Necessary and sufficient conditions to achieve multi-equilibria consensus in the most general form have been given in [16] and [17]. Nonlinear multi-equilibria consensus has been investigated in [18] under directed topologies.

In this paper, the problem of multi-equilibria consensus in the presence *structured Byzantine (StrBYZ)* faults and time delays is investigated. Note that the Byzantine misbehaviour examined in [7], [8], [19] is dealt with fault tolerant algorithms which require an initial knowledge on the number of faulty nodes (f) in the network. Furthermore, the condition $n \geq 3f + 1$ must be satisfied for the success of these algorithms where n is the number of nodes in the network. These conditions are quite restrictive comparing to the fact that it may be possible for non-faulty nodes to reach consensus without using any fault tolerant algorithm thanks to the resilient topology of the network. In addition, there does not exist a notable study on fault tolerant group consensus computation. In [12], a fault tolerant algorithm, so called L-MSR algorithm, is proposed for non-faulty nodes to achieve approximate Byzantine group consensus with $K \geq 1$ equilibrium points in the presence of *unstructured Byzantine (uStrBYZ)* agents. While [12] addresses the uStrBYZ faulty agents, in this paper, we study the problem of achieving group consensus in the presence of *structured Byzantine (StrBYZ)* agents and time delays without using any fault tolerant algorithm and without making any complex assumptions on the network topology.

The rest of the paper is organized as follows. In Section 2, we first present a review on graph terminology. Then, the threat model, so called structured Byzantine, is presented. In Section 3, we investigate the conditions for non-faulty nodes to achieve multi-equilibria consensus in the presence of structured Byzantine agents. An analysis is carried out in Section 4 by considering uniform/non-uniform bounded time delays and some numerical examples are provided to illustrate our results. We conclude with some remarks in Section 5.

II. PRELIMINARIES AND PROBLEM SETUP

In this section, we introduce some graph theoretic concepts that we use throughout the paper. Additionally, we present the linear consensus algorithm and model its dynamics in the presence of StrBYZ agents.

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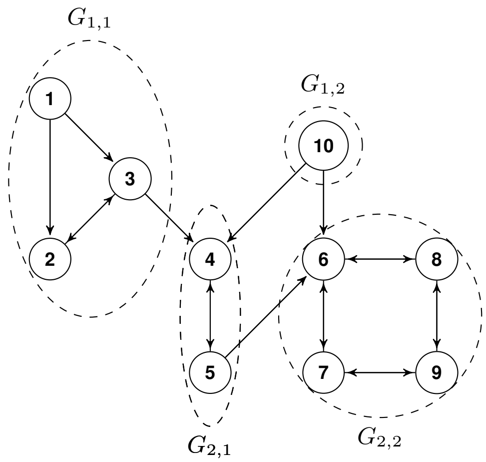

Fig. 1. An illustrative example of primary and secondary layers.

A. Graph Theoretic Concepts

Consider a network with associated digraph denoted by $G = (V, E)$ where V is the node set with the set of indices $I = \{1, 2, \cdots, n\}$. $E \subset V \times V$ represents the edge set in which a directed edge $(i, j) \in E$ implies that node j provides information to node i. For the network G, suppose that there exist l_1 ($l_1 \geq 1$) subsets (so called *primary layers* [16], [17]) in the node set V such that each subset $V_{1,s}$, $s = 1, ..., l_1$, is the largest possible subset that has a spanning tree for its subgraph $G_{1,s}$ and for all $i \in V_{1,s}$ and $j \notin V_{1,s}$, we have $(i, j) \notin E$. In other words, a primary layer in the network G consists of a set of nodes that has a spanning tree for its subgraph, and that do not receive any information from other layers.

If we denote the set of nodes that are not in the primary layers with $\bar{V}$, i.e., $\bar{V} = V \setminus \bigcup_{s=1}^{l_1} V_{1,s}$, then there exist l_2 subsets in $\bar{V}$ (so called *secondary layers* [16], [17]) such that each subset $V_{2,s}$, $s = 1, ..., l_2$, has a spanning tree for its subgraph and there exists exactly one node $i \in V_{2,s}$ which satisfies the following:

- For all $j \in V_{2,s} \setminus i$ and $m \in V \setminus V_{2,s}$, we have $(j, m) \notin E$.
- There exist at least two vertices in two different subgraphs (either primary or secondary layer) p and v such that $(p, i) \in E$ and $(v, i) \in E$.
- i is the root of a spanning tree in $V_{2,s}$.

The notions of primary and secondary layers are illustrated in Fig. 1. There are two primary layers $G_{1,1}$, $G_{1,2}$, where $V_{1,1} = \{1, 2, 3\}$, $V_{1,2} = \{10\}$ respectively denote the sets of nodes of these primary layers, and two secondary layers $G_{2,1}$, $G_{2,2}$, where $V_{2,1} = \{4, 5\}$, $V_{2,2} = \{6, 7, 8, 9\}$ respectively denotes the node sets of these secondary layers, in the network. Note that each primary and secondary layer in the network has a spanning tree. Furthermore, secondary layers $G_{2,1}$ and $G_{2,2}$ have root nodes $i = 4$ and $i = 6$ while primary layers $G_{1,1}$ and $G_{1,2}$ does not receive any information from other layers.

B. Fault Model

Suppose that the network $G = (V, E)$ contains n nodes where f of them are StrBYZ faulty agents. Let F denote the set of faulty agents in the network and the cardinality of the set F is denoted by $|F| = f$. Note that the sets of in-neighbors, out-neighbors and inclusive neighbors of node $i \in V$ are defined as $N_i^+ = \{j \in V : (i, j) \in E\}$, $N_i^- = \{j \in V : (j, i) \in E\}$ and $N_i = N_i^+ \cup \{i\}$, respectively.

Before introducing the fault model studied in this paper, the notion of Byzantine node should be carefully examined. In [7], a Byzantine node is defined as the one that it does not send the same value to all of its neighbors at some time-step, or if it applies some other function $f_i'(\cdot)$ at some time-step. It is obvious from this definition that the misbehaviour of a Byzantine node can be seperated into two categories:

(i) It sends different information to each of its out-neighbors.

(ii) It may send an arbitrary incorrect information to all of its out-neighbors.

With this in mind, we present the fault model that we use throughout the paper as follows:

Definition 1: (Structured Byzantine) A node $i \in F$ is said to be StrBYZ if it sends different information to its out-neighbors while employing $|N_i^-|$ different update rules starting with $|N_i^-|$ different initial conditions.

Remark 1: Note that an StrBYZ agent uses the information that it receives from its in-neighbors to generate different state values for each of its out-neighbors. *Structured Byzantine (StrBYZ)* fault model can be encountered in the applications of wireless networks [20]. Similar misbehaviour called *node replication attack* (also called clone attack) considered in [21] where faulty agent creates some copies having the same ID as some other nodes in the network.

There is a great deal of studies in the literature [7], [8], [9], [10] most of which assume the Byzantine misbehaviour in the sense of category (i). Therefore, it is required to use fault tolerant consensus algorithms to deal with the faulty behaviour of Byzantine agents and these fault tolerant algorithms require strict conditions on the network topology. Furthermore, none of these algorithms can solve the multi-equilibria consensus problem in the presence of Byzantine agents. By considering the Byzantine behaviour in the sense of (i), we will show that the use of fault tolerant consensus algorithms is not necessary and we will derive the conditions on the network which are required so that multi-equilibria consensus is achieved using a traditional consensus algorithm.

III. MULTI-EQUILIBRIA CONSENSUS IN THE PRESENCE OF STRBYZ ADVERSARIES

Consider a network $G = (V, E)$ of n agents where f of the agents are StrBYZ. Let $G_{nf} = (V_{nf}, E_{nf})$ and F respectively denote the network of the non-faulty agents and the set of StrBYZ agents in the network G. Then, update rule for a non-faulty agent i can be written as:

$$x_i[k+1] = \sum_{j\in N_i\cap V_{nf}} w_{ij}x_j[k] + \sum_{j\in N_i\cap F} w_{ij}x_{ij}[k],\ i \in V_{nf}, \tag{1}$$

where $x_i[k]$ denotes the state value of node i and $x_{ij}[k]$ denotes the state value of node j that is sent to node i at time step k.

For an StrBYZ agent j, there are $|N_j^-|$ different copies that use $|N_j^-|$ different updates of (1) by choosing different weights and starting with different initial conditions. Therefore, a faulty agent j uses the following update rule:

$$x_{ij}[k+1] = \sum_{l\in N_j\cap V_{nf}} \tilde{w}_{ij,l}x_l[k] + \sum_{l\in N_j\cap F} \tilde{w}_{ij,l}x_{jl}[k],\ j\in F,\ i\in N_j^-. \tag{2}$$

For the coefficients in the update rules (1) and (2), the following is assumed to be satisfied:

Assumption 1: The averaging coefficients w_{ij} in the update rule (1) are assumed to satisfy the following conditions:

- $w_{ij} = 0$ whenever $j \notin N_i$, $i \in V$, $k \in \mathbb{Z}_{\geq 0}$;
- $w_{ij} > 0$, $\forall j \in N_i, i \in V$, $k \in \mathbb{Z}_{\geq 0}$;
- $\sum_{j=1}^n w_{ij} = 1$, $\forall i \in V$, $k \in \mathbb{Z}_{\geq 0}$;

and for the averaging coefficients $\tilde{w}_{ij,l}$ in the update rule (2), the following conditions hold:

- $\tilde{w}_{ij,l} = 0$ whenever $l \notin N_j$, $i \notin N_j^-$, $j \in F$;
- $\tilde{w}_{ij,l} > 0$, $\forall l \in N_j$, $j \in F$, $i \in N_j^-$;
- $\sum_{j=1}^n \tilde{w}_{ij,l} = 1$, $\forall i \in V$.

In multi-equilibria consensus, the network reaches $K \geq 2$ equilibrium points depending on the network structure. For a directed network of n non-faulty nodes having l_1 primary layers and l_2 secondary layers, it is stated in [16], [17] that the number of equilibrium states when the agents of the network use the update rule (1) is computed as $K = l_1 + l_2$. In other words, agents in the same primary or secondary layer reach on the same consensus equilibrium. Therefore, the number of primary and secondary layers in the network determines the number of equilibrium states. For instance, consider the network given in Fig. 1. Since there are two primary ($l_1 = 2$) and two secondary layers ($l_2 = 2$) and $F = \emptyset$ holds, the number of equilibria is computed as $K = 4$ when the agents of the network use the update rule (1).

In Definition 1, an StrBYZ agent is defined as one that sends different state values to each of its out-neighbors. Since each StrBYZ agent generates different values by using (2), it is assumed throughout this paper that $|N_j^-| \geq 2$ holds for all $j \in F$. Otherwise, StrBYZ node j produces just one state value and does not cause any misbehaviour in the network.

When there exist StrBYZ agents in the network, the topological structure of the network might change and non-faulty nodes cannot reach consensus on $K = l_1 + l_2$ equilibrium points. Suppose that $|N_j^-| \geq 2$ holds for all StrBYZ agents $j \in F$ in the network G that has l_1 primary and l_2 secondary layers. An StrBYZ agent $j \in G_{1,s} \cap F$, where $G_{1,s}$ is the s-th primary layer, provides information to at least two nodes in the same primary layer using the update rule (2). Since it uses the information of its in-neighbors to update the state values of itself and the primary layer $G_{1,s}$ has a spanning tree, it is convenient for non-faulty nodes of $G_{1,s}$ to reach consensus on the same equilibrium. Furthermore, by definition, secondary layers in the network G receive information from at least two other primary or secondary layers through the root nodes of themselves. If the root node of the secondary layer $i \in G_{2,s}$ is an StrBYZ agent that sends different information to each of its neighbors, non-faulty nodes in the secondary layer $G_{2,s}$ will be led as if there are two root nodes in $G_{2,s}$. With these in mind, we present the conditions on the network G for non-faulty nodes to reach consensus on $K = l_1 + l_2$ equilibria in the presence of StrBYZ agents.

Proposition 1: Consider a network $G = (V, E)$ having l_1 primary layers and l_2 secondary layers. Non-faulty agents of the network G reach consensus on $K = l_1 + l_2$ equilibria in the presence of StrBYZ agents if the following conditions hold:

(i) Each StrBYZ agent has at least one in-neighbor from the primary or the secondary layer that it belongs to.

(ii) Root nodes of all secondary layers in the network G are non-faulty.

Note that Proposition 1 states the conditions for non-faulty nodes to achieve multi-equilibria consensus when the network does not have any time delays on its communication links. However, it is a well-known fact that communication delays negatively effect the performance of the system. Furthermore, they may cause lack of stability in the overall system dynamics. Therefore, one should carefully examine the system dynamics under time delays.

IV. RESILIENT MULTI-EQUILIBRIA CONSENSUS UNDER BOUNDED TIME DELAYS

In this section, we study the problem of multi-equilibria consensus when there exist uniform/non-uniform bounded time delays and StrBYZ agents in the network.

A. Multi-Equilibria Consensus in the Presence of StrBYZ agents and Uniform Bounded Time Delays

When a directed network $G = (V, E)$ of n agents, which contains f number of StrBYZ agents, has uniform bounded time delays on its communication links, the update rule (1) can be rewritten as:

$$x_i[k+1] = w_{ii}x_i[k] + \sum_{j\in N_i^+\cap V_{nf}} w_{ij}x_j[k-\tau] + \sum_{j\in N_i^+\cap F} w_{ij}x_{ij}[k-\tau],\quad i \in V_{nf}, \tag{3}$$

where $x_i[k]$ denotes the state value of node i, $x_{ij}[k]$ denotes the state value of node j that is sent to node i at time step k and $\tau \leq \tau_{max}$ denotes the uniform bounded time delay between nodes i and j.

In a similar way, the update rule (2) can be rewritten as:

$$x_{ij}[k+1] = \tilde{w}_{ij,l}x_l[k] + \sum_{l \in N_j \cap V_{nf}} \tilde{w}_{ij,l}x_l[k-\tau] + \sum_{l \in N_j \cap F} \tilde{w}_{ij,l}x_{jl}[k-\tau], \quad j \in F,\ i \in N_j^-, \tag{4}$$

where Assumption 1 holds for the coefficients in the update rules (3) and (4).

Suppose that the network G has l_1 primary and l_2 secondary layers. It is shown in [17] that the number of equilibrium points is found as $K = l_1 + l_2$ when agents of the network G, which are assumed to be non-faulty, use the update rule (3) with uniform bounded time delays. Combining this with Proposition 1, we present the following result:

Proposition 2: Consider a network $G = (V, E)$ having l_1 primary layers and l_2 secondary layers. Non-faulty agents of the network G reach consensus on $K = l_1 + l_2$ equilibria in the presence of StrBYZ agents if the following conditions hold:

(i) Each StrBYZ agent has at least one in-neighbor from the primary or the secondary layer that it belongs to.
(ii) Root nodes of all secondary layers in the network G are non-faulty.
(iii) $\tau \leq \tau_{max}$ holds for some $\tau_{max} < \infty$.

Proposition 2 states that if each StrBYZ agent in G has at least one in-neighbor from the layer that they belong and root nodes of all secondary layers in the network G are non-faulty, then the number of equilibrium states is computed as $K = l_1 + l_2$ when non-faulty agents of the network G use the update rule (3) with uniform tim delays in the presence of StrBYZ agents. In the following, an illustrative example is presented to show the effect of uniform bounded time delays.

Example 1: Consider the network with 9 nodes given in Fig. 2 in which nodes 1, 9 are StrBYZ and there is no communication delay among its agents, i.e., $\tau = 0$. There are $l_1 = 2$ primary layers ($G_{1,1}$, $G_{1,2}$) and $l_2 = 1$ secondary layer ($G_{2,1}$) in the network. Since each StrBYZ agent has at least one in-neighbor from the layer that they belong and the root node of the secondary layer $G_{2,1}$ is non-faulty, it can be concluded from Proposition 1 that non-faulty nodes reach consensus on $K = l_1 + l_2 = 3$ different equilibrium points as it can be seen from Fig. 3.

Reconsider the network given in Fig. 2. This time, suppose that there are uniform bounded time delays in the communication links of the network G for which $\tau = 3$ holds. As it can be seen from Fig. 4, non-faulty nodes reach consensus on $K = l_1 + l_2 = 3$ equilibria, which agrees with Proposition 2.

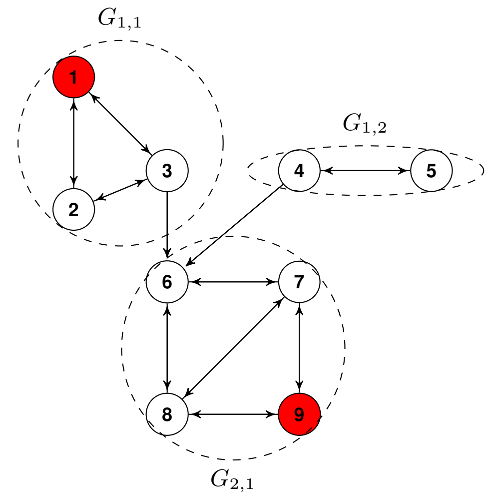

Fig. 2. A network of 9 nodes where node 1 and node 9 are StrBYZ.

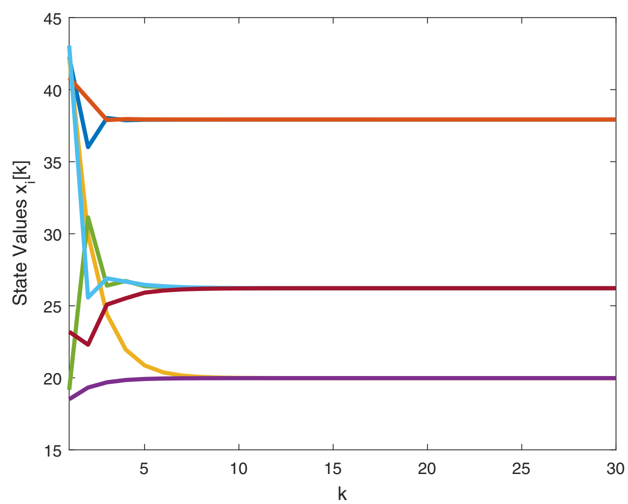

Fig. 3. The simulation result for he network given in Fig. 2 when $\tau = 0$.

B. Multi-Equilibria Consensus in the Presence of StrBYZ agents and Non-uniform Bounded Time Delays

Non-uniform bounded time delays can also be encountered in a networked system. When a directed network $G = (V, E)$ of n agents, which contains f number of StrBYZ agents, has non-uniform bounded time delays on its communication links, the update rule (1) can be rewritten as:

$$x_i[k+1] = w_{ii}x_i[k] + \sum_{j \in N_i^+ \cap V_{nf}} w_{ij}x_j[k-\tau_{ij}] + \sum_{j \in N_i^+ \cap F} w_{ij}x_{ij}[k-\tau_{ij}], \quad i \in V_{nf}, \tag{5}$$

where $x_i[k]$ denotes the state value of node i;, $x_{ij}[k]$ denotes the state value of node j that is sent to node i at time step k and $\tau_{ij} \leq \tau_{max}$ denotes the non-uniform bounded time

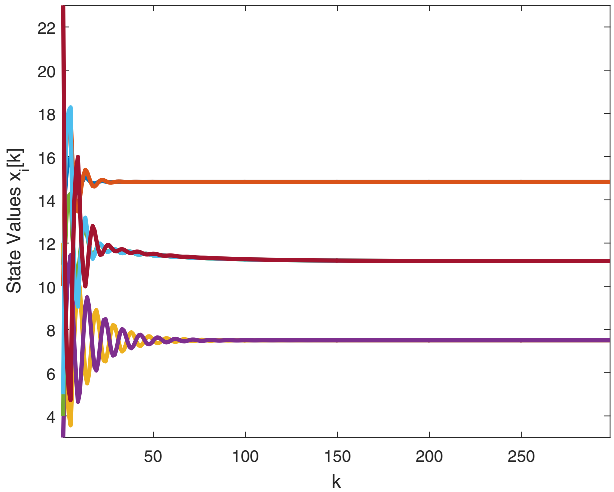

Fig. 4. The simulation result for he network given in Fig. 2 when $\tau \leq \tau_{max} = 3$.

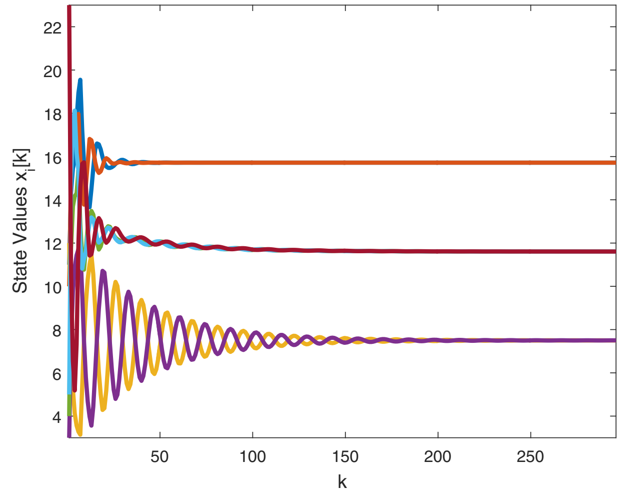

Fig. 5. The simulation result for he network given in Fig. 2 when $\tau_{ij} \leq \tau_{max} = 5$.

delay between nodes i and j.

Similarly, the update rule (2) can be rewritten as:

$$x_{ij}[k+1] = \tilde{w}_{ij,l}x_l[k] + \sum_{l \in N_j \cap V_{nf}} \tilde{w}_{ij,l}x_l[k-\tau_{ij}] + \sum_{l \in N_j \cap F} \tilde{w}_{ij,l}x_{jl}[k-\tau_{ij}], \quad j \in F,\ i \in N_j^-, \tag{6}$$

where Assumption 1 is assumed to be satisfied for the coefficients in the update rules (5) and (6). Following the discussion given in Section IV-B, similar results can be obtained for the case where the network has non-uniform bounded communication delays.

Proposition 3: Consider a network $G = (V, E)$ having l_1 primary layers and l_2 secondary layers. Non-faulty agents of the network G reach consensus on $K = l_1 + l_2$ equilibria in the presence of StrBYZ agents if the following conditions hold:

(i) Each StrBYZ agent has at least one in-neighbor from the primary or the secondary layer that it belongs to.
(ii) Root nodes of all secondary layers in the network G are non-faulty.
(iii) $\tau_{ij} \leq \tau_{max}$ holds for some $\tau_{max} < \infty$ and all $i, j \in V$.

Next, we will present another illustrative example for non-uniform bounded time delays.

Example 2: Reconsider the network G with 9 nodes given in Fig. 2. Suppose that non-faulty agents use the update rule (6) in the presence of non-uniform bounded time delays for which $\tau_{ij} \leq \tau_{max} = 5$ holds for all $i, j \in V$. As it can be seen from Fig. 5, non-faulty nodes reach consensus on $K = l_1 + l_2 = 3$ equilibria, which agrees with Proposition 3.

V. CONCLUSION

In this paper, we have studied the problem of resilient multi-equilibria consensus in the presence of StrBYZ faulty agents and uniform/non-uniform bounded time delays. It is shown that non-faulty nodes can achieve multi-equilibria consensus on K equilibria without using a fault tolerant algorithm if the network topology is resilient against the misbehaviour of StrBYZ faults. To this end, we have presented the conditions on the network G so that the non-faulty agents of G achieve multi-equilibria consensus where the number of equilibrium points is given by $K = l_1 + l_2$. We have also studied the case where the network has uniform/non-uniform bounded time delays on its communication links.

In the future, we plan to extend these results to approximate Byzantine group consensus for which non-faulty nodes reach K equilibria that are in the range of the initial values of non-faulty nodes. Furthermore, the multi-equilibria consensus problem can be extended to the case where non-faulty nodes use a nonlinear consensus protocol. The problem of achieving approximate Byzantine consensus is also to be investigated for higher order systems.

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