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Inner-Loop Reference Governor Design with an Application to Human-in-the-Loop Control

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Abstract— **This paper presents a novel reference-governor-based approach to enforcing constraints in open-loop systems, where the constraint enforcement scheme modifies a control signal from a feedback controller. The reference governor is not typically applicable to these systems, since reference governors modify reference signals, which are inputs to a feedback controller.**

The design is based on duplicating the feedback controller of the closed-loop system and implementing a reference governor to modify the reference input to the duplicate controller. Effectively, this bypasses the nominal controller, whose internal dynamics may not be asymptotically stable. We present a method to ensure stability of the closed-loop system by ceding some control authority to the nominal controller.

A numerical example is considered in which the nominal controller is a human operator of a steer-by-wire system. The operator is modeled as a PID controller with unknown parameters. Results show that the inner-loop governor design is able to satisfactorily enforce constraints in such an application.

I. Introduction

The conventional reference governor [1] is an add-on constraint-enforcement scheme, used in enforcing constraints of closed-loop systems. It is designed on top of closed-loop systems, whose controllers have been designed for good performance and other characteristics but without taking constraints into account. The reference governor modifies the reference input to the controller by taking into account the current state of the closed-loop system to form a prediction of future system behavior and to modify the reference in order to enforce constraints.

When first proposed, the reference governor was considered as an inside-the-loop element which would modify the control input to an open-loop system [2], *i.e.*, the output signal of a stabilizing controller, and not the reference input. The vast majority of theoretical work on reference governors (see [1] and references therein) has been on the now-conventional placement of reference governors outside of the control loop. However, it is not always possible to design a system for conventional placement of the reference governor. An obvious example is a human-in-the-loop system, in which it is not possible to modify the input to the human operator. Experimental results, such as in [3], suggest that it is possible to operate a vehicle safely with a reference governor inside the control loop. Theoretical work on inside-the-loop reference governor placement includes [4] and [5]; in both, a passivity approach was exploited to guarantee the stability of the interconnection of the reference governor and the constrained, open-loop system. The applicability of these approaches is limited since passivity is a stringent condition.

In this work, we introduce a constraint-enforcement scheme that is placed inside the loop but has the constraint-enforcement characteristics of the conventional reference governor. Our design consists of a conventional governor, whose input is the desired reference, and a duplicate controller, whose closed-loop behavior is the same in steady-state as the nominal controller. The design effectively bypasses the nominal controller and allows for a conventional reference governor design. The benefit of this approach over other methods that have been developed is that it only requires asymptotic stability of the internal dynamics of the nominal controller. Furthermore, if the internal dynamics are not asymptotically stable, we provide an approach to ensure stability of the entire closed-loop system by ceding some control authority from the duplicate controller to the nominal controller. Although giving control authority to the nominal controller implies that constraints will not always be enforceable, this is necessary since stability must be ensured. Our approach provides a method to ensure system stability, while enforcing constraints to the full extent of the control authority available to the duplicate controller.

The design of the inner-loop design is presented in steps. First we consider an inner-loop reference governor applied to a open-loop controller and extend the design to a feedback system in which the internal dynamics of the feedback controller are asymptotically stable. We then make necessary modifications to the design in the case where the internal dynamics are not asymptotically stable and the case where the controller dynamics are uncertain. In all cases, we show that in the absence of constraints, the modified closed-loop system exhibits the same behavior as the nominal closed-loop system.

To investigate the properties of our inner-loop governor design, we consider a numerical example in which a human, modeled as a PID controller whose parameters are uncertain, operates a steer-by-wire system by tracking a desired angle of the car's wheels. Since the desired angle cannot be modified, we use our approach to introduce another PID controller, whose input can be modified, and a reference governor to modify the reference input to the duplicate controller.

The paper is organized as follows. Section II presents the conventional reference governor. Section III presents the inner-loop governor schemes. Section IV presents an inner-loop governor for uncertain-parameter systems. Section V presents a numerical simulation of a human-in-the-loop steering application. Section VI is the conclusion.

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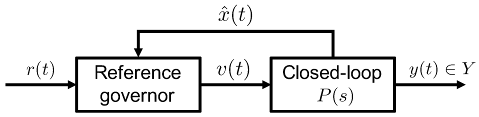

Fig. 1. Schematic of reference governor applied to closed-loop system

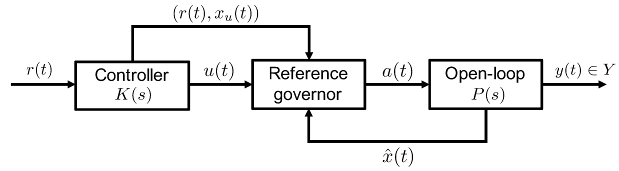

Fig. 2. Schematic of reference governor applied to a dynamic reference

II. REFERENCE GOVERNORS

The reference governor [1] is applied to discrete-time closed-loop systems whose dynamics are given by,

$$x(t+1) = A_d x(t) + B_d v(t), \tag{1a}$$

$$y(t) = E_d x(t) + F_d v(t) \in Y, \tag{1b}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $v(t) \in \mathbb{R}^q$ is a constraint-admissible reference input, $y(t)$ is the constrained output, and $Y \subset \mathbb{R}^p$ is the constraint set.

A schematic of the system set-up is provided in Fig. 1. In the figure, the reference governor receives a desired reference input $r(t) \in \mathbb{R}^q$ and modifies it to the constraint-admissible reference $v(t)$ based on knowledge of a state measurement or estimate $\hat{x}(t)$. The modification is made so that $v(t)$ is as close as possible to $r(t)$, while ensuring constraint enforcement for all present and future time.

Enforcement of the constraint (1b) is achieved by enforcing a set-membership constraint,

$$(\hat{x}(t), v(t)) \in P, \tag{2}$$

where P is a set of initial-state/constant-reference pairs that ensure constraint enforcement for all present and future time. Typically, the set P is an approximation of the maximal output admissible set O_∞, which is the set of all initial-state/constant-reference such that constraints are enforced for all present and future time,

$$O_\infty := \{(x,v) : x(0) = x,\ v(t) = v,\ y(t) \in Y,\ \forall t \in \mathbb{Z}_+\}. \tag{3}$$

To ensure robustness and stability of the reference governor and closed-loop system interconnection, it becomes necessary to slightly modify the set O_∞ to a new set,

$$\tilde{O}_\infty = O_\infty \cap (\mathbb{R}^n \times \tilde{\Omega}), \tag{4}$$

where $\tilde{\Omega}$ is a Minkowski-subset[1] of the interior of the set Ω, which is defined as the set of all constant reference inputs whose corresponding output satisfies constraints in steady state,

$$\Omega = \{v : v(t) \equiv v, \lim_{t\to\infty} y(t) \in Y\}. \tag{5}$$

For more details on the computation of $\tilde{O}_\infty$, refer to [6], [7].

In the formulation of the reference governor, the set P is commonly set to $P = \tilde{O}_\infty$ and the constraint-admissible reference $v(t)$ is computed according to,

$$v(t) = \kappa(t)(r(t) - v(t-1)) + v(t-1), \tag{6}$$

[1]The set is convex, compact, and contains 0 in its interior.

where,

$$\kappa(t) = \max_{\kappa\in[0,1]} \{\kappa : (\hat{x}(t), \kappa(r(t) - v(t-1)) + v(t-1)) \in P\}, \tag{7}$$

so that the reference $v(t)$ is varied along the line segment connecting $v(t-1)$ and $r(t)$.

The reference governor satisfies three properties: (i) pointwise-in-time constraint enforcement, *i.e.*, $y(t) \in Y$ for all $t \in \mathbb{Z}_+$; (ii) recursive feasibility, *i.e.*, for all $t \in \mathbb{Z}_+$, $(x(t), v(t)) \in P$ implies that there exists v^- such that $(x(t+1), v^-) \in P$; and (iii) finite-time convergence to a constant reference, *i.e.*, if $r(t) = r$ for all $t \in \mathbb{Z}_+$, then there exist $v \in \mathbb{R}^q$ and $t_s \in \mathbb{Z}_+$ such that $v(t) = v$ for all $t \geq t_s$; furthermore, if $r \in \tilde{\Omega}$, then $v = r$.

III. INNER-LOOP GOVERNORS

In this work, we focus our attention to the case where the constrained system $P(s)$, introduced in Fig. 1, is open-loop and not necessarily stable, and the input to $P(s)$ is a control input, not a reference input as in the conventional case. The system is output-constrained, with dynamics given by,

$$\dot{x} = A_{11}x + Ba, \tag{8a}$$

$$y = Ex + Fa \in Y, \tag{8b}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $a(t) \in \mathbb{R}^m$ is the constraint-admissible control input, and $y(t)$ is the output.

As in the case of the closed-loop system formulation, the output of $P(s)$ is subject to constraints and must remain in the constraint set $Y \subset \mathbb{R}^p$. The input $a(t)$ is a modification of the control input $u(t) \in \mathbb{R}^m$, which is generated by the controller $K(s)$. A schematic of such a system structure, in which the controller is open-loop, *i.e.*, not feedback, is presented in Fig. 2. We assume that typically $K(s)$ generates control inputs without regard to potential constraint violation of the output signal and therefore we must design a scheme for constraint enforcement. Due to design considerations or simply because of the structure of the control system, direct modification of the reference input to the controller $K(s)$ is not possible, and hence the controller output is modified instead. The constraint-enforcement scheme we use to achieve this is related to the reference governor and we will discuss different formulations of it in the remainder of this section.

A. Open-loop controller

Consider the controller $K(s)$ with dynamics,

$$\dot{x}_u = A_{22}x_u + B_2 r, \tag{9a}$$

$$u = C_2 x_u + D_2 r, \tag{9b}$$

where $x_u(t) \in \mathbb{R}^{n_u}$ is the internal state of the controller and $r(t) \in \mathbb{R}^q$ is a reference input.

According to the the schematic of Fig. 2, the inner-loop governor is placed so that it modifies the desired control input $u(t)$ to a constraint-admissible control input $a(t)$. When the reference governor is turned off, *i.e.*, $a(t) = u(t)$, the constrained system dynamics become,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} A_{11} & B_1C_2 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x \\ x_u \end{bmatrix} + \begin{bmatrix} B_1D_2 \\ B_2 \end{bmatrix} r, \tag{10a}$$

$$y = \begin{bmatrix} E & FC_2 \end{bmatrix} \begin{bmatrix} x \\ x_u \end{bmatrix} + FD_2 r. \tag{10b}$$

Suppose that A_{22} is asymptotically stable. As we will soon show, the constraint-admissible control input $a(t)$ can be determined as the output of a controller with the same dynamics as $K(s)$, whose input is a modified reference computed according to the ordinary reference governor algorithm. We define the dynamics of this new controller as,

$$\dot{x}_a = A_{22}x_a + B_2 v, \tag{11a}$$

$$a = C_2 x_a + D_2 v. \tag{11b}$$

Coupling this system to (10), the dynamics become,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{22} \end{bmatrix} \begin{bmatrix} x \\ x_a \\ x_u \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ B_2 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix},$$

$$y = \begin{bmatrix} E & E_2 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \\ x_u \end{bmatrix} + \begin{bmatrix} F_1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} \in Y, \tag{12}$$

where $A_{12} = B_1C_2$, $B_1 = BD_2$, $E_1 = FC_2$, and $F_1 = FD_2$. It is clear from (12) that the $x_u(t)$ dynamics are both uncontrollable with respect to the input $v(t)$ and unobservable with respect to the output $y(t)$. With the design of the controller (11), we have effectively bypassed the original controller $K(s)$, whose input is the unmodifiable reference $r(t)$, with an identical controller, whose input is a reference $v(t)$ which can be modified by a reference governor.

The difference in outputs of the two controllers vanishes in the absence of constraint activation, *i.e.*, whenever $v = r$. To see this, define the controller state error as $e(t) := x_u(t) - x_a(t)$ and the output error as $y_e(t) := u(t) - a(t)$. Then the error dynamics are given by,

$$\dot{e} = A_{22}e + B_2(r - v), \tag{13a}$$

$$y_e = C_2 e + D_2(r - v). \tag{13b}$$

It is important that the error dynamics be asymptotically stable so that we avoid integral wind-up and other undesirable behavior during system operation.

We have presented an inner-loop governor design for constraint enforcement of systems with open-loop control. Our design transforms the system into a closed-loop form so that we can apply a conventional reference governor for constraint enforcement. Yet the physical design stays the same. That is, unlike in the conventional reference governor, we have not replaced the signal $r(t)$ with $v(t)$; we have instead bypassed the nominal controller $K(s)$ with another controller, whose behavior is the same as the original. We can do this as long as the internal controller dynamics are asymptotically stable, since otherwise we could cause integral wind-up in the internal state of the nominal controller $K(s)$.

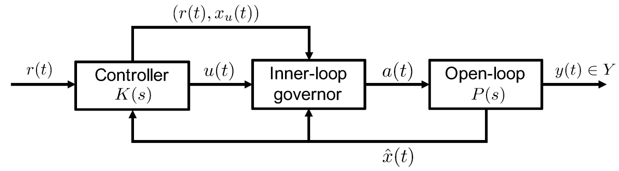

Fig. 3. Schematic of inner-loop governor applied to feedback system

B. Feedback controller with stable internal dynamics

In this subsection, we consider the case where there exists a feedback interconnection between $P(s)$ and $K(s)$ and where the internal controller dynamics are asymptotically stable, *i.e.*, the matrix A_{22} is asymptotically stable, as in the open-loop case, where $K(s)$ receives a feedback signal from $P(s)$. A schematic of the system is provided in Fig. 3. The controller dynamics are given by (9) with an additional state-feedback element. Treating the governor as a pass-through element, *i.e.*, setting $a(t) = u(t)$, the closed-loop dynamics become,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x \\ x_u \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} r, \tag{14}$$

We introduce a replacement controller with the same dynamics as $K(s)$,

$$\dot{x}_a = A_{21}x + A_{22}x_a + B_2 v, \tag{15a}$$

$$a = C_1 x + C_2 x_a + D_2 v, \tag{15b}$$

so that the coupled dynamics become,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{21} & 0 & A_{22} \end{bmatrix} \begin{bmatrix} x \\ x_a \\ x_u \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ B_2 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix},$$

$$y = \begin{bmatrix} E_1 & E_2 & 0 \end{bmatrix} \begin{bmatrix} x \\ x_a \\ x_u \end{bmatrix} + \begin{bmatrix} F_1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} \in Y, \tag{16}$$

where $E_1 = E + FC_1$. As in the case with the open-loop controller, the $x_u(t)$ dynamics are both uncontrollable with respect to the input $v(t)$ and unobservable with respect to the output $y(t)$. The error dynamics are the same as in (13).

C. Feedback controller with unstable internal dynamics

In this subsection, we consider the case where the internal dynamics of $K(s)$ are not asymptotically stable. Here we cannot use the previous approach because the error dynamics (13) are not asymptotically stable.

A guiding principle in the development of the reference governor is the goal of minimal invasiveness. Absent constraints, the reference governor is designed so that the system operates as designed, without any modification from the constraint-enforcement scheme. In practice, this amounts to setting the constraint-admissible reference to the desired value whenever constraint violation is not predicted. In the case of the inner-loop governor, minimal invasiveness corresponds to a desire that the control output of the nominal controller $K(s)$ and the duplicate controller $K'(s)$ become equivalent in the absence of constraints. This can be achieved asymptotically by ensuring that the error dynamics $e(t)$ be asymptotically stable. We begin by modifying the dynamics of the duplicate controller $K'(s)$ to include an input term $u_a(t) = (u_{a,1}(t), u_{a,2}(t))$ where $u_{a,1}(t) \in \mathbb{R}^m$ and $u_{a,2}(t) \in \mathbb{R}^{n_u}$. The internal dynamics become,

$$\dot{x}_a = A_{21}x + A_{22}x_a + u_{a,2} + B_2 v, \tag{17a}$$

$$a = C_1 x + C_2 x_a + u_{a,1} + D_2 v. \tag{17b}$$

With this new controller design, the error dynamics (13) are,

$$\dot{e} = A_{22}e - u_{a,2} + B_2(r - v), \tag{18a}$$

$$y_e = C_2 e - u_{a,1} + D_2(r - v). \tag{18b}$$

Our goal is to ensure that whenever $e(t) \approx 0$, the system dynamics behave almost as if the reference governor were not present. To achieve this, we choose the input $u_a(t)$ so that the error dynamics become asymptotically stable and the output error $y_e(t)$ approaches zero whenever $r(t) = v(t)$ for a prolonged period of time.

Specifically, we choose a feedback term $u_{a,2}(t) = \Lambda e(t)$, such that $A_{22} - \Lambda$ is asymptotically stable. Since the stabilizing feedback gain Λ can be chosen arbitrarily, in principle it is possible to set the error dynamics as desired. The term $u_{a,1}(t)$ is set to equal $u_{a,1}(t) = \Gamma e(t)$, where there is no restriction on Γ. In summary, the input is given by,

$$u_a(t) = \begin{bmatrix} \Gamma \\ \Lambda \end{bmatrix} e(t). \tag{19}$$

After applying the new controller design (17), (19), the coupled dynamics become,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} A_{11} & \hat{A}_{12} & A_{13} \\ A_{21} & \hat{A}_{22} & A_{23} \\ A_{21} & 0 & A_{22} \end{bmatrix} \begin{bmatrix} x \\ x_a \\ x_u \end{bmatrix} + \begin{bmatrix} B_{12} & 0 \\ B_2 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix},$$

$$y = \begin{bmatrix} E_1 & \hat{E}_2 & E_3 \end{bmatrix} \begin{bmatrix} x \\ x_a \\ x_u \end{bmatrix} + \begin{bmatrix} F_1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} \in Y. \tag{20}$$

where $A_{13} = B_1\Gamma$, $A_{23} = \Lambda$, $\hat{A}_{12} = A_{12} - B_1\Gamma$, $\hat{A}_{22} = A_{22} - \Lambda$, $\hat{E}_2 = E_2 - F\Gamma$, and $E_3 = \Gamma$.

Performing a change of variables, we obtain the dynamics,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ 0 & 0 & \hat{A}_{22} \end{bmatrix} \begin{bmatrix} x \\ x_a \\ e \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ B_2 & 0 \\ -B_2 & B_2 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix},$$

$$y = \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix} \begin{bmatrix} x \\ x_a \\ x_u \end{bmatrix} + \begin{bmatrix} F_1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} \in Y. \tag{21}$$

It is clear from this representation of the dynamics that whenever $e = 0$ and $v = r$, the overall system dynamics are equivalent to the nominal, constraint-free dynamics (14), satisfying our aim.

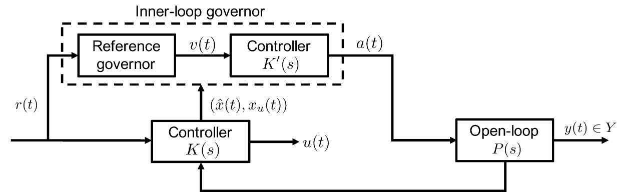

Fig. 4. Schematic showing architecture of inner-loop governor

1) Discussion on the choice of feedback gains Λ and Γ: The feedback gains introduced in (19) can be chosen with very few restrictions. Aggressive choices of the gain Λ leads to giving the nominal controller more authority over the system, which will impact the governor's ability to enforce constraints. Whenever authority is ceded to the nominal controller, it must be accepted that constraint violation may occur since the reference governor will not be able to enforce constraint enforcement for abrupt changes in the reference $r(t)$. For this reason, it appears best to choose a less-aggressive gain Λ.

Similar logic, when applied to choosing Γ, suggests that Γ should be close to nil. Referring to (21), we can see that when $\Gamma = 0$, A_{13} and E_3 are both 0. Diminishing the gain Γ diminishes the impact of the unmodifiable reference $r(t)$ on the constraints.

IV. Inner-loop governors for uncertain-parameter systems

We now consider the case where the controller $K(s)$ is uncertain. This can arise when the controller is only partially known, such as when the controller is a physical actuator or operator that does not communicate with the constraint-enforcement software. We consider the case where the matrices $A_{21} = \bar{A}_{21} + \hat{A}_{21}$ and $A_{22} = \bar{A}_{22} + \hat{A}_{22}$ and,

$$\begin{bmatrix} \hat{A}_{12} & \hat{A}_{22} \end{bmatrix} \in \mathcal{A}, \tag{22}$$

for some polytope $\mathcal{A} \subset \mathbb{R}^{n_u \times (n+n_u)}$. The matrices $\bar{A}_{12}$ and $\bar{A}_{22}$ represent estimated values for A_{12} and A_{22} and are used in the design of the inner-loop governor, *i.e.*, the controller $K'(s)$ is designed under the assumption that $A_{12} = \bar{A}_{12}$ and $\bar{A}_{22} = A_{22}$. Disregarding constraints, the coupled system dynamics, in transformed coordinates, become,

$$\begin{bmatrix} \dot{x} \\ \dot{x}_a \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & B_1\Gamma \\ \bar{A}_{21} & \bar{A}_{22} & \Lambda \\ \hat{A}_{21} & \hat{A}_{22} & A_{22} - \Lambda \end{bmatrix} \begin{bmatrix} x \\ x_a \\ e \end{bmatrix} + \begin{bmatrix} B_1 & 0 \\ B_2 & 0 \\ -B_2 & B_2 \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix}.$$

Conventional robust-control approaches, such as that of [8], can be used to ensure that the system above is asymptotically stable for any $\hat{A}_{21}$ and $\hat{A}_{22}$ satisfying (22). Constraints can be enforced robustly using the parameter governor of [9] or, if the internal state of the controller $K(s)$ is unknown, we may construct the prediction model using the duplicate controller dynamics and use an estimate of the error $e(t)$ for forming a prediction.

As in the results of the previous section, to minimize the impact of the governor on the system dynamics, it is again preferable to choose the gains Λ and Γ so that the impact of the nominal controller on constraint enforcement is minimized. For example, if it is true that the system is asymptotically stable for any possible value of $\hat{A}_{21}$ and $\hat{A}_{22}$, we set $\Lambda = 0$ and $\Gamma = 0$.

V. Case study: Application to a human-in-the-loop steer-by-wire system

We consider the application of the inner-loop governor to a steer-by-wire system, in which the controller is a human operator modeled as an uncertain-parameter PID controller, [10]. The steering system is second-order and given by,

$$\begin{bmatrix} \dot{\theta}_p \\ \ddot{\theta}_p \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\alpha_0 & -\alpha_1 \end{bmatrix} \begin{bmatrix} \theta_p \\ \dot{\theta}_p \end{bmatrix} + \begin{bmatrix} 0 \\ \beta_0 \end{bmatrix} u, \tag{23}$$

where $\theta_p(t)$ and $\dot{\theta}_p(t)$ are the wheel angle and its derivative, respectively, and $u(t)$ is the torque provided by the power-steering motor. The nominal, human controller is PID, *i.e.*,

$$\dot{x}_u = k_i(r - \theta_p), \tag{24a}$$

$$u = x_u + k_p(r - \theta_p) - k_d\dot{\theta}_p, \tag{24b}$$

where $k_p = 5$, $k_i = 130$, $k_d = 0.2$ are the PID gains and the reference $r(t)$ is the desired wheel angle. The gains are fixed but uncertain, and are assumed to range between minimum and maximum values.

Note that the error dynamics are not asymptotically stable and therefore, in our design, we introduce a controller to mimic the behavior of (24) with a stabilizing term for the error dynamics,

$$\dot{x}_a = \bar{k}_i(v - \theta_p) + \lambda e, \tag{25a}$$

$$a = x_a + \bar{k}_p(v - \theta_p) - \bar{k}_d\dot{\theta}_p, \tag{25b}$$

where $\bar{k}_p = 7.3$, $\bar{k}_i = 121$, $\bar{k}_d = 0.11$ are PID gains which satisfy constraints and $\lambda > 0$ is the stabilizing gain.

The dynamics of the system implementing the duplicate controller are given by,

$$\begin{bmatrix} \dot{\theta}_p \\ \ddot{\theta}_p \\ \dot{x}_a \\ \dot{e} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\alpha_0 - \bar{k}_p\beta_0 & -\alpha_1 - \bar{k}_d\beta_0 & \beta_0 & 0 \\ -\bar{k}_i & 0 & 0 & \lambda \\ -k_i + \bar{k}_i & 0 & 0 & -\lambda \end{bmatrix} \begin{bmatrix} \theta_p \\ \dot{\theta}_p \\ x_a \\ e \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \bar{k}_i & 0 \\ -\bar{k}_i & k_i \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix}, \tag{26}$$

where the gain λ is chosen to ensure asymptotic stability of the above dynamics for any $k_i \in [k_i^-, k_i^+] = [100, 200]$.

Since the internal driver state $x_u(t)$ is not known, we cannot assume full knowledge of the error variable $e(t)$. For this reason, we design an estimator for the error, with the dynamics,

$$\dot{\hat{e}} = -k_o(\hat{e} - e) - k_o(k_p - \bar{k}_p)\theta_p - k_o(k_d - \bar{k}_d)\dot{\theta}_p - \bar{k}_i v + (\bar{k}_i + k_o(k_p - \bar{k}_p))r, \tag{27}$$

where $k_o > 0$ is the observer gain. Note that, with the introduction of an estimate of the operator's internal state, the dynamic equation (25a) for $x_a(t)$ must be modified to use $\hat{e}(t)$ for feedback instead of $e(t)$.

The system is state-constrained according to,

$$|\theta_p(t)| \leq \frac{1}{2},\ |\dot{\theta}_p(t)| \leq \frac{\pi}{4}\text{rad/s},\ |\ddot{\theta}_p| \leq 6\text{rad/s}^2. \tag{28}$$

To enforce constraints, a conventional reference governor is applied to modify the desired reference angle $r(t)$ to a constraint-admissible reference angle $v(t)$. To design the constraint-admissible set, the dynamics (26) are discretized using a sampling time of 1ms. Note that the input to the reference governor are the measured states $\theta_p(t)$, $\dot{\theta}_p(t)$, $x_a(t)$, the esimated state $\hat{e}(t)$, the desired reference $r(t)$, and the previously constraint-admissible reference $v(t-1)$.

In order to fully explore the characteristics of the inner-loop governor, we begin by considering the case where the operator's internal integral state is fully known.

We begin by considering different choices of $\lambda > 0$ and their effect on system response. All choices of λ are stable for any value of $\bar{k}_i \in [100, 200]$. We first note that, in steady state, the wheel angle $\theta_p(t)$ approaches the desired angle $r(t)$ regardless of $v(t)$, *i.e.*,

$$\lim_{t\to\infty} \theta_p(t) = r. \tag{29}$$

whenever $r(t) \equiv r$ is held constant. It becomes clear then that the operator ultimately has full authority over steering. In fact, we can vary the amount of authority ceded to the operator by varying λ, since λ affects the speed at which our controller internal state $x_a(t)$ approaches the operator's internal state $x_u(t)$. To show this, we perform numerical simulations for different values of λ, in which the operator provides a step input in $r(t)$ from 0 to 0.45 and $v(t)$ is held constant at 0.

The results of the simulations are presented in Fig. 5. It is shown that the response of $\theta_p(t)$ becomes quicker as λ increases, which implies that overly increasing λ results in the overall system effectively ignoring the output of the reference governor. It is also shown that, as expected, the effort expended by the driver decreases as λ increases. This suggests that, in the system design, the choice of λ may be informed by the maximum effort that the operator may be expected to exert.

Furthermore, we perform a simulation corresponding to a step-change in the reference $r(t)$ from 0 to 0.45 with a choice $\lambda = 1$. In the simulation, the driver's internal state $x_u(t)$ is estimated according to (27) with observer gain $k_o = 10$. The reference governor determines $v(t)$ according to the algorithm (6)-(7).

The results are presented in Figs. 6-7. In Fig. 6, we see that the settling time is increased to about 4s. As was shown previously, a higher value of λ would result in a smaller settling time. We also see that the operator torque achieves a maximum value about double that of the nominal. The reason for the higher torque is the integral wind-up that occurs when the operator does not quickly achieve the desired value

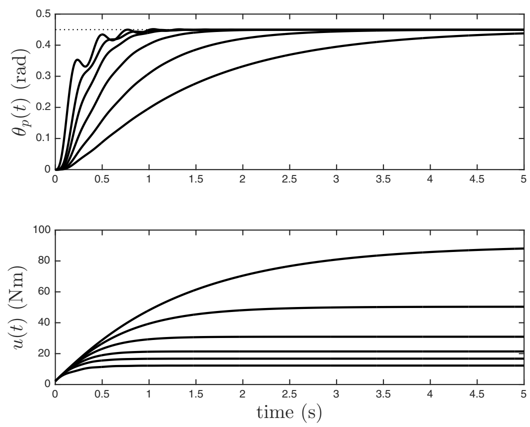

Fig. 5. Responses of $\theta_p(t)$ (top) and $u(t))$ (bottom) to a step input $r(t) = 0.45$ and $v(t) = 0$ held constant, corresponding to $\lambda = 0.70, 1.43, 2.92, 5.95, 12.14, 10^5$; the dashed arrow corresponds to direc-

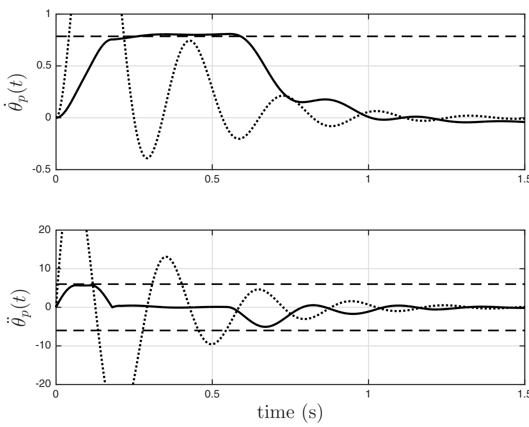

Fig. 7. Nominal (dotted) and constraint-enforced (solid) responses of $\dot{\theta}_p(t)$ (top) and $\ddot{\theta}$ (bottom) with constraints (dashed)

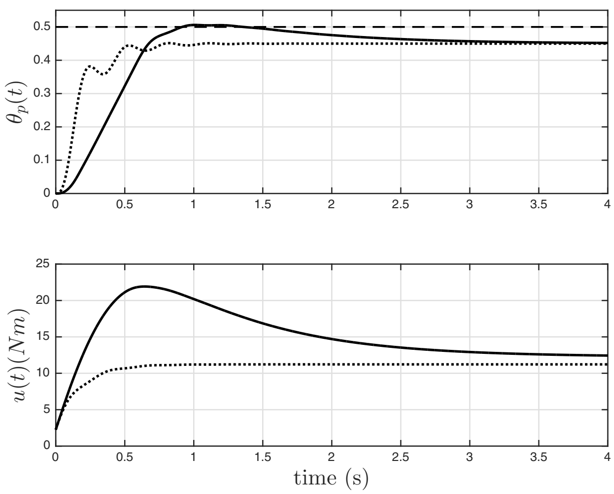

Fig. 6. Nominal (dotted) and constraint-enforced (solid) responses of $\theta_p(t)$ (top) and $u(t)$ (bottom) with constraint (dashed)

of $\theta_p(t)$. In Figs. 6-7, we see small constraint violations which are the result of the use of an estimator for the error $e(t)$ with an inexact model of the nominal controller. This is typical of constrained systems which use an estimated variable in constraint enforcement; addressing this can be done by improving the observer or tightening constraints, which has been done in [3].

VI. Conclusion

In this paper, we have considered the design of an inner-loop constraint enforcement scheme based on the reference governor. Previous theoretical treatments of this problem have focused on exploiting stringent passivity properties to ensure stability of the interconnection between the reference governor and the constrained system.

Our scheme consists of duplicating the feedback controller of the closed-loop system with a controller whose input is a reference that is modified by a reference governor. We have designed our scheme to ensure closed-loop stability of the entire closed-loop system, consisting of both nominal and duplicate feedback controllers. The main result is that passivity is not required and that stability can be ensured by ceding some control authority to the nominal controller.

Numerical simulations were performed for a case study of a human-in-the-loop steer-by-wire system, where the human operator was modeled as a PID controller with uncertain parameters. The first set of simulations showed the trade-off between enforcing system stability and achieving constraint satisfaction. The second set of simulations showed that the inner-loop governor is able to effectively enforce system constraints.

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