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Adaptive Vibration Controller Design for Structural Systems Despite Unknown Seismic Disturbance

Gokhan Kararsiz[1], Hakan Yazici[2], Halil Ibrahim Basturk[3], Rahmi Guclu[4]

***Abstract*— This paper presents active vibration controller design for structures under the effect of unmeasured seismic earthquake disturbances. It is assumed that the disturbance consists of a sum of sinusoidal where amplitude, frequency and phase are unknown. Then, an observer is constructed for unmeasured disturbance with considering the assumptions. Then, an adaptive backstepping controller is designed for the actuator input. The stability of the closed loop system is proven and the effect of the seismic earthquake disturbance is mitigated. To reveal the success of the design, a simulation is prepared where the disturbance excites the natural frequencies of the structure.**

I. INTRODUCTION

It is well known from past experiences that earthquake disturbances have detrimental effect on structures, drastically. In order to isolate this effect, seismic protective systems are offered. In the literature, these systems can be divided into three main parts, namely, passive in other words tuned mass damper [1], semi-active [2], and active control systems [3], respectively. Active vibration control systems have been developing for the isolation of the earthquake induced vibrations by employing an actuator force. The success of the active control system heavily depends on the control strategy that generates the actuator force. Thus, the design of the active vibration controller has received considerable attention by academia and industry.

Many different control approaches have been applied to supress the seismic disturbances such as linear quardatic requlator (LQR) [4], fuzzy-logic [5], sliding mode (SMC) [6], H_∞ controller [7] and adaptive backstepping [8]. In this paper, the adaptive backstepping method is preferred for the controller design because of the ease of the application to subsystems with parameter update laws. This situation enables that the unknown disturbance is handled, practically.

Since, the equipment cost and technical problems such as noisy data or time delay issue for disturbance measurement decrease the applicability of the controller, the effect of the seismic disturbance is assumed to be unmeasured. The disturbance is considered as a finite sum of sinusoidal functions where amplitude, frequency and phase are taken into account as unknown. On the other hand, the number of distinct frequencies of the disturbance sinusoidal are assumed to be known by considering the natural frequencies of the structure. With this information, the disturbance is parametrized by using the method given in [9] and [10]. Then, the problem becomes an adaptive control problem.

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[1,2,4]Gokhan Kararsiz, Hakan Yazici and Rahmi Guclu are with Dept. of Mechanical Eng., Yildiz Technical University, 34349 Besiktas, Istanbul, Turkey {kararsiz, hyazici, guclu}@yildiz.edu.tr

[3]Halil I. Basturk is with the Dept. of Mechanical Eng., Bogazici University, Bebek, 34342, Istanbul, Turkey halil.basturk@boun.edu.tr

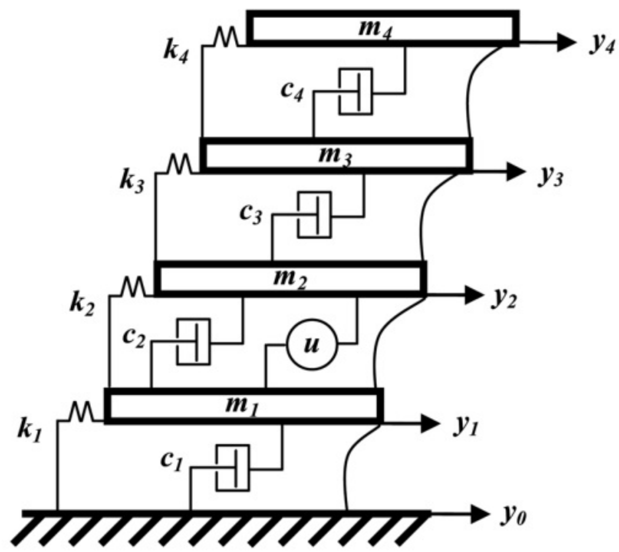

Fig. 1. Schematic sketch of a building model.

The rest of the paper is organized as follows. The mathematical model of the structure and the assumptions are given in Section II. In Section III, the observer for the unmeasured disturbance is designed. Then, an adaptive backstepping controller for the structure is presented in Section IV with the stability theorem. The proof of the theorem is given in Section V. In Section VI, the results of the numerical study is illustrated.

II. PROBLEM STATEMENT

A four-storey structure is considered to illustrate the effect of the earthquake excitation. The model of the system is depicted in Figure 1. It is well known that the first storey is exposed to the maximum inner storey shear force during an earthquake. Therefore, the actuator (U) is located in the first storey. In this model, the horizontal displacements are given by y_1, y_2, y_3 and y_4, respectively. The symbols m_1, m_2, m_3 and m_4 represent the mass of the storeys. The stiffness and damping coefficients of the storeys are denoted with k_1, k_2, k_3, k_4 and c_1, c_2, c_3 c_4 for each floor, respectively. The seismic disturbance is represented with x_0 and $\dot{x}_0$.

The equations of motion is given by

$$\begin{aligned}
m_1\ddot{y}_1(t) =& -(k_1+k_2)y_1(t) - (c_1+c_2)\dot{y}_1 \\
&+ k_2y_2(t) + c_2\dot{y}_2 + k_1y_0 + c_1\dot{y}_0 - U(t) \quad (1)\\
m_2\ddot{y}_2(t) =& -(k_2+k_3)y_2(t) - (c_2+c_3)\dot{y}_2 + k_2y_1(t) \\
&+ c_2\dot{y}_1 + k_3y_3(t) + c_3\dot{y}_3 + U(t) \quad (2)\\
m_3\ddot{y}_3(t) =& -(k_3+k_4)y_3(t) - (c_3+c_4)\dot{y}_3 + k_3y_2(t) \\
&+ c_3\dot{y}_2 + k_4y_4(t) + c_4\dot{y}_4 \quad (3)\\
m_4\ddot{y}_4(t) =& -k_4y_4(t) - c_4\dot{y}_4(t) + k_4y_3(t) + c_4\dot{y}_3(t) \quad (4)
\end{aligned}$$

To ease the mathematical burden in the controller design, the plant is represented in a more compact form as follows,

$$\begin{aligned}
\dot{z}_1(t) =& A_1z_1(t) + B(\alpha z_2(t) + \nu(t) - \frac{U(t)}{m_1}) \quad (5)\\
\dot{z}_2(t) =& A_2z_2(t) + B(\mu z_1(t) + \zeta z_3(t) + \frac{U(t)}{m_2}) \quad (6)\\
\dot{z}_3(t) =& A_3z_3(t) + B(\gamma z_2(t) + \psi z_4(t)) \quad (7)\\
\dot{z}_4(t) =& A_4z_4(t) + B(\chi z_3(t)) \quad (8)
\end{aligned}$$

where the corresponding matrices and vectors are given in the appendix.

A. Plant Assumptions

Assumption 1: All states are available for measurement except the seismic disturbance.

Assumption 2: It is assumed that the profile of the seismic disturbance ($y_0(t)$) consists of a final sum of sinusoidal where amplitude, phase and frequency is unknown, but, the number of distinct frequency is known.

$$y_o(t) = \sum_{i=1}^{n} g_i \sin(\omega_i t + \phi_i). \quad (9)$$

Substituting (9) into (53) and after trigonometric calculations to (9), we obtain,

$$\nu(t) = \sum_{i=1}^{n} \bar{g}_i \sin(\omega_i t + \bar{\phi}_i), \quad (10)$$

where

$$\bar{g}_i = \frac{\sqrt{(k_tg_i)^2 + (c_tw_ig_i)^2}}{m_u^2}, \quad (11)$$

$$\bar{\phi}_i = tan^{-1}\{\frac{\frac{k_tg_i}{m_u}\sin(w_it+\phi_i) - \frac{c_tw_ig_i}{m_u}\cos(w_it+\phi_i)}{\frac{k_tg_i}{m_u}\cos(w_it+\phi_i) + \frac{c_tw_ig_i}{m_u}\sin(w_it+\phi_i)}\}. \quad (12)$$

III. Seismic Disturbance Observer

The seismic disturbance can be represented as an output of a linear exosystem

$$\dot{W}(t) = SW(t) \quad (13)$$
$$\nu(t) = h^TW(t) \quad (14)$$

where $W(t) \in \mathbb{R}^{2q}$. The eigenvalues of S are designed with considering the unknown frequencies of the road disturbance $\nu(t)$. Moreover, the unknown amplitude and phase depend on the unknown initial condition of (13), which leads that the output vector h^T can be considered as unknown. Then, we can obtain an observable pair (h^T, S).

Let $G \in \mathbb{R}^{2qX2q}$ be a Hurwitz matrix and (G, l) be a controllable pair. Since (h^T, S) is observable and the spectra of S and G are disjoint the unique solution $M \in \mathbb{R}^{2qX2q}$ of Sylvester equation

$$MS - GM = lh^T, \quad (15)$$

is invertible [12]. Employing the state transformation, $p(t) = MW(t)$ to (13), we get;

$$\dot{p}(t) = Gp(t) + l\nu(t), \quad (16)$$
$$\nu(t) = \theta^T p(t), \quad (17)$$

where $\theta^T = h^TM^{-1}$. Up to now, we represent the unknown disturbance (10) as an product of the unknown constant vector θ^T and the unknown state $p(t)$ in (17). In order to estimate $p(t)$, an observer is designed in the following lemma.

Lemma: The seismic disturbance can be represented in the form

$$\nu(t) = \theta^T\xi(t) + \theta^T\delta(t), \quad (18)$$

where

$$\begin{aligned}
\xi(t) =& \eta(t) + Nz_1(t), \quad (19)\\
\dot{\eta}(t) =& G\xi(t) - N(A_1z_1(t) + B(\alpha z_2(t) - \frac{1}{m_1}U(t))), \quad (20)
\end{aligned}$$

with

$$N = \frac{1}{B^TB}lB^T, \quad (21)$$

where $l = NB$.

Moreover, the estimation error $\delta(t)$ is defined as

$$\delta(t) = p(t) - \xi(t), \quad (22)$$

and satisfies the equation

$$\dot{\delta}(t) = G\delta(t). \quad (23)$$

Proof: The equation (23) is obtained by taking the time derivative of (22) and using (5), (16) and (19)-(20). Plugging (22) into (17) yields (18).

With this lemma, the seismic disturbance is constructed by the unknown vector θ, the observer filter ξ and the exponentially decaying term δ. Now, we can apply the adaptive control method with considering this representation.

IV. CONTROLLER DESIGN

The aim of the controller is to cancel the effect of the seismic disturbance. Here, we consider the plant with the disturbance observer. Then, adaptive controller is designed with using backstepping method.

A. Backstepping Design

It is reasonable to cancel the effect of disturbance at the first floor, because the earthquake motion is transmitted to the structure from the ground. Moreover, it is observed that maximum share force is occurred at the first floor, during an earthquake. Therefore, the effect of the seismic disturbance is expected the strongest on the first floor. To stabilize the plant, the first floor dynamics (5) can be forced to have a form $\dot{z}_1(t) = (A_1 - BK)z_1(t)$ by selecting the desired input as

$$U_d(t) = m_1\Big(Kz_1(t) + \alpha z_2(t) + \hat{\theta}^T\xi(t)\Big), \qquad (24)$$

where $K \in \mathbb{R}^{2q}$ is a controller gain which is chosen such that $(A_1 - BK)$ is Hurwitz.

Throughout the paper, the symbols '$\hat{}$' and '$\tilde{}$', used in (24), represent the estimation and the estimation error which is given by

$$\tilde{\theta} = \theta - \hat{\theta}. \qquad (25)$$

If $U(t)$ is solely applied to the system, the input signal would be unbounded. The main reason of this opposite effect of $U(t)$ to $\dot{z}_2(t)$ as it seen in (6) and coupled dynamics of ($z_1(t)$, $z_2(t)$) systems. Therefore, we define an error function between $U(t)$ and U_d,

$$e(t) = U(t) - U_d(t), \qquad (26)$$

and deriving error function with respect to time with considering (5), (18) and (24), yields,

$$\begin{aligned}\dot{e}(t) =& \dot{U}(t) - \frac{m_1\alpha B}{m_2}U(t) - m_1KB(\theta^T\delta(t) + \tilde{\theta}^T\xi(t)) \\ &+ H(t), \end{aligned} \qquad (27)$$

where

$$\begin{aligned}H(t) = &-m_1(K(A_1 - BK)z_1(t) + \alpha(A_2z_2(t) \\ &+ B(\mu z_1(t) + \zeta z_3(t))) + \dot{\hat{\theta}}^T\xi + \hat{\theta}^T\dot{\xi}) - \frac{z_1^T(t)}{m_1}P. \end{aligned} \qquad (28)$$

B. Adaptive Controller Design and Stability Theorem

Considering $\dot{U}(t) - \frac{m_1\alpha B}{m_2}U(t)$ as a control input in (27), the adaptive controller is obtained by

$$\dot{U}(t) = \frac{m_1\alpha B}{m_2}U(t) - ce(t) - H(t), \qquad (29)$$

$$\dot{\hat{\theta}}(t) = \rho(z_1^T(t)P - e(t)m_1K)B\xi. \qquad (30)$$

where $\rho > 0$ is a update gain and P is a positive definite matrix which satisfies the solution of the matrix equation

$$\Big(A_1 - BK\Big)^T P + P\Big(A_1 - BK\Big) = -2I. \qquad (31)$$

The stability statement is given in the following.

Theorem: Consider the closed loop system consisting of the plant (5)-(8), forced by the unknown disturbance (9), the disturbance observer (18)-(20) and the adaptive controller (29) with the update law (30). Under Assumptions 1-2, the following holds;

(a) The control input $U(t)$ is bounded for all initial conditions.

(b) The equilibrium $z_1(t) = z_2(t) = z_3(t) = z_4(t) = \delta(t) = \tilde{\theta} = 0$ is stable and the signals z_1, $e(t)$ and $\delta(t)$ converge to zero as $t \to \infty$.

The proof of the theorem is given in the next section.

V. STABILITY PROOF

In this section, the proof of the theorem is stated.

Proof: Substituting (18) and (24) into (5) and (29) into (27), the following system is obtained by

$$\dot{z}_1(t) = (A_1 - BK)z_1(t) + B(\tilde{\theta}^T\xi + \theta^T\delta - \frac{e(t)}{m_1}), \qquad (32)$$

$$\dot{e}(t) = -ce(t) - m_1KB(\tilde{\theta}^T\xi + \theta^T\delta). \qquad (33)$$

To show the stability of the closed loop system, the following Lyapunov function is established by

$$V(t) = \frac{1}{2}(z_1^T(t)Pz_1(t) + e^2 + \frac{1}{\rho_\theta}\tilde{\theta}^T\tilde{\theta} + \frac{1}{\rho_\delta}\delta^T(t)P_G\delta(t)), \qquad (34)$$

where

$$G^TP_G + P_GG = -2\epsilon, \qquad (35)$$

with

$$\epsilon = 1 + \frac{\lambda_{max}(PB\theta^T\theta B^TP^T)}{2} - \frac{\lambda_{max}(m_1^2B\theta^T\theta B)}{2}. \qquad (36)$$

Deriving $V(t)$ with respect to time with considering (23), (30), (31), (32), (33), (35), yields

$$\begin{aligned}\dot{V}(t) = &- z_1^T(t)z_1(t) + z_1^T(t)PB\theta\delta(t) - ce^2(t)\dots \\ &- e(t)m_1KB\theta^T\delta(t) - \delta^T(t)\delta(t). \end{aligned} \qquad (37)$$

Using Young's inequality for the cross terms, we get

$$\dot{V}(t) \leq -z_1(t)^Tz_1(t) - ce^2(t) - \delta^T\delta. \qquad (38)$$

From (38), we conclude,

$$V(t) \leq V(0). \qquad (39)$$

Defining

$$\vartheta(t) = \begin{bmatrix} z_1(t) & e(t) & \tilde{\theta} & \delta(t) \end{bmatrix}. \qquad (40)$$

Using (34) and (39), we obtain,

$$|\vartheta|^2 \leq J_1|\vartheta(0)|^2 \qquad (41)$$

for some $J_1 > 0$. For all ϑ, the right hand side of (23), (25), (30), (32) and (33) are continuous in time and ϑ which leads that the right hand side of (38) is continuous in time and ϑ. Moreover, the right hand side of (38) is zero when $\vartheta = 0$. By LaSalle-Yoshisawa theorem, (38) guarantees that $z_1(t)$, $e(t)$ and $\delta(t)$ converge to zero while time goes to ∞.

Since the disturbance is bounded from Assumption 2, (18) is bounded. In addition, considering the boundedness of $\delta(t)$ and $\tilde{\theta}$, it leads from (18) that ξ is bounded. Then, we conclude from (24) that $U(t)$ is bounded. The proof of the part (a) of the theorem is completed.

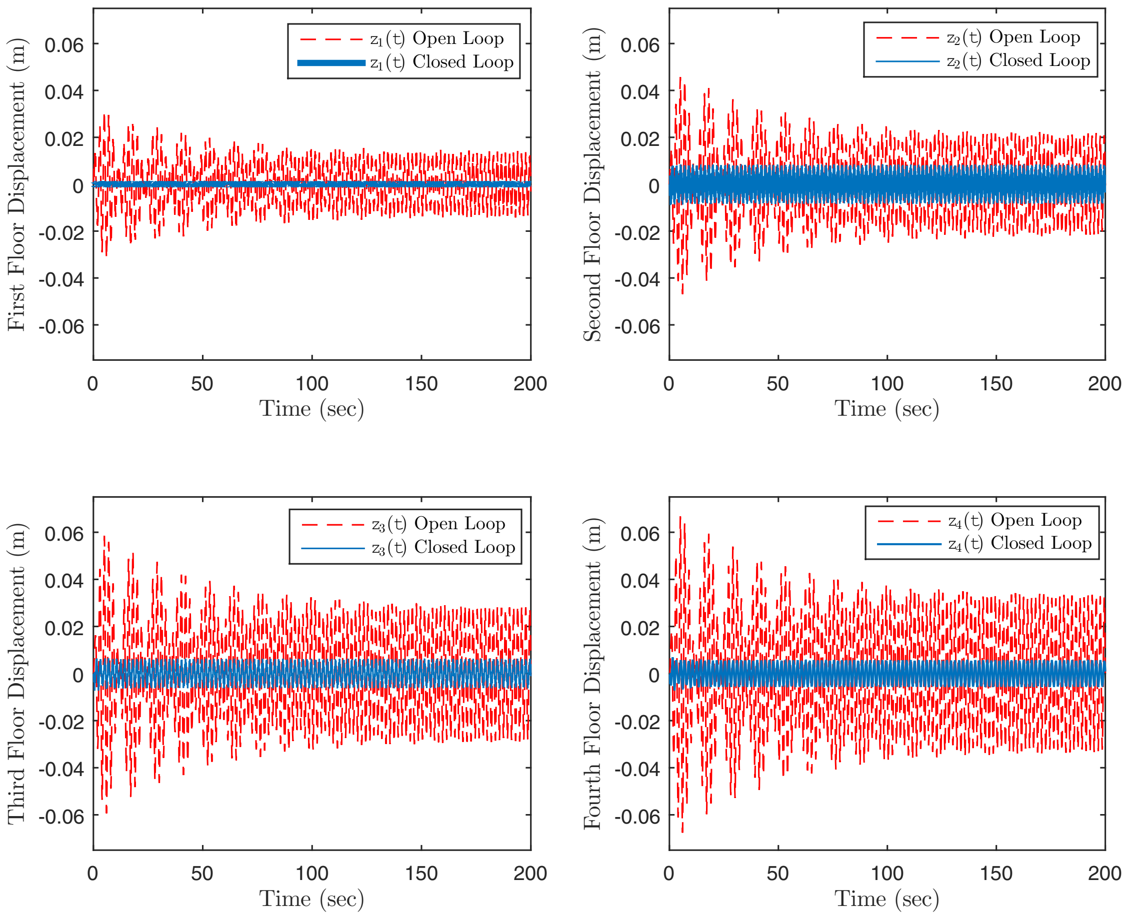

Fig. 2. Displacement responses for each floor.

The following representation is defined to show the boundedness of the states $z_2(t)$, $z_3(t)$ and $z_4(t)$.

$$\dot{X} = AX + B_U U(t) + B_W \nu(t) \tag{42}$$

where reminding (51) and (52) the matrices A, B_U and B_W are given by

$$A = \begin{bmatrix} A_1 & \alpha & 0 & 0 \\ \mu & A_2 & \zeta & 0 \\ 0 & \gamma & A_3 & \psi \\ 0 & 0 & \chi & A_4 \end{bmatrix} \tag{43}$$

$$B_U = \begin{bmatrix} 0 & \frac{1}{m_1} & 0 & \frac{1}{m_2} & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{44}$$

$$B_W = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{45}$$

$$X = \begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & z_4(t) \end{bmatrix}^T \tag{46}$$

Noting that A is Hurwitz and recalling that $U(t)$ and $\nu(t)$ is bounded it follows from (42) that X in other words $z_1(t)$, $z_2(t)$ $z_3(t)$ and $z_4(t)$ are bounded.

VI. Simulations

A simulation is prepared to reveal the effectiveness of the designed controller. The system parameters are given in the Table I.

TABLE I
PARAMETER VALUES FOR SIMULATION

$m_1 = 450$ ton	$k_1 = 18\ MN/m$
$m_2 = 345$ ton	$k_2 = 340\ MN/m$
$m_3 = 345$ ton	$k_3 = 326\ MN/m$
$m_4 = 345$ ton	$k_4 = 280\ MN/m$
$c_1 = 26\ kNs/m$	$c_3 = 467\ kNs/m$
$c_2 = 490\ kNs/m$	$c_4 = 410\ kNs/m$

It is well-known that the natural frequency responses can be vastly destructive for structures. Under the effect of earthquake, the structure can be excited by these natural frequencies. For this problem, we select 4-storey-structure. Therefore, the plant has 4 degrees of freedom which are calculated from the parameters provided in Table I as 0.54, 3.67, 6.87 and 9.16 Hz. In order to simulate the effect

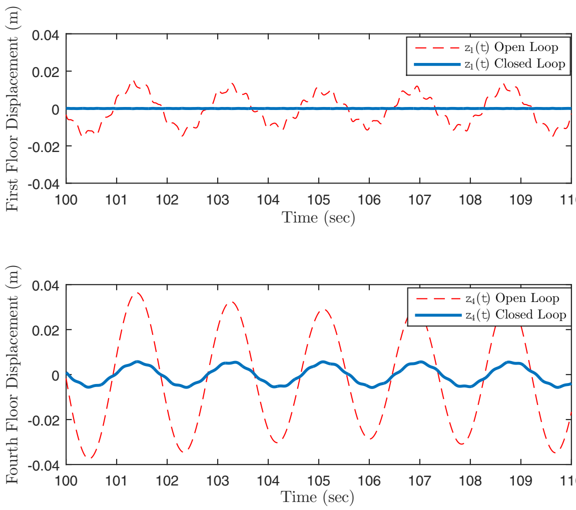

Fig. 3. Zoomed section of the displacement response.

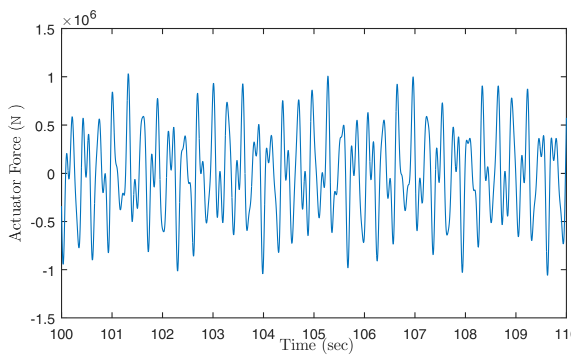

Fig. 4. Actuator force.

of these natural frequencies, the earthquake disturbance is chosen as

$$x_0(t) = 0.1\sin(2\pi 0.54t) + 0.2\sin(2\pi 3.55t + \frac{\pi}{6}) \dots$$
$$\dots + 0.15\sin(2\pi 6.56t + \frac{\pi}{3}) + 0.12\sin(2\pi 9.16t + \frac{\pi}{12}). \tag{47}$$

For the disturbance observer, we assume that the earthquake disturbance has 4 distinct frequencies (n) which correspond to 4 natural frequencies. Thus, we select the controllable pair (G, l) as $G, l \in \mathbb{R}^8$.

The displacement response against the disturbance (47) for each floor is given in Figure 2. Red dashed and solid blue line represent open loop and closed loop responses, respectively. From this time response, it is clearly seen that the effects of the 4 natural frequencies are mitigated for each floor despite the unknown disturbance. Since it is not easy to see the details of displacement responses in Figure 2, the zoomed results for first and fourth floors are given by Figure 3 for 10 seconds. The actuator response for the zoomed section is shown in Figure 4.

VII. CONCLUSIONS

In this paper, a four-storey-structure, which is subjected to unknown seismic disturbance, is concerned with adaptive backstepping method. The unknown disturbance is represented with a sum of sinusoidal where amplitude, frequency and phase are unknown. Then, an observer and an adaptive controller are designed to compensate the effect of the unknown seismic disturbance. It is proven that the response of the structure is stable. Finally, the performance of the controller is demonstrated with a simulation. For future work, the parametric uncertainty of structures and the dynamics of actuators can be considered.

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APPENDIX

Corresponding matrices and vectors for (5)-(8) are given by

$$z_1(t) = \begin{bmatrix} y_1(t) & \dot{y}_1(t) \end{bmatrix}^T, \quad z_2(t) = \begin{bmatrix} y_2(t) & \dot{y}_2(t) \end{bmatrix}^T \tag{48}$$

$$z_3(t) = \begin{bmatrix} y_3(t) & \dot{y}_3(t) \end{bmatrix}^T, \quad z_4(t) = \begin{bmatrix} y_4(t) & \dot{y}_4(t) \end{bmatrix}^T \tag{49}$$

$$A_1 = \begin{bmatrix} 0 & 1 \\ -\frac{k_1+k_2}{m_1} & -\frac{c_1+c_2}{m_1} \end{bmatrix}, \tag{50}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -\frac{k_2+k_3}{m_2} & -\frac{c_2+c_3}{m_2} \end{bmatrix}, \tag{51}$$

$$A_3 = \begin{bmatrix} 0 & 1 \\ -\frac{k_3+k_4}{m_3} & -\frac{c_3+c_4}{m_4} \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 1 \\ -\frac{k_4}{m_4} & -\frac{c_4}{m_4} \end{bmatrix}, \quad (52)$$

$$B = \begin{bmatrix} 0 & 1 \end{bmatrix}^T, \quad \nu(t) = \begin{bmatrix} \frac{k_1}{m_1} & \frac{c_1}{m_1} \end{bmatrix} \begin{bmatrix} y_0 \\ \dot{y}_0 \end{bmatrix}, \qquad (53)$$

$$\alpha = \begin{bmatrix} \frac{k_2}{m_1} & \frac{c_2}{m_1} \end{bmatrix}, \quad \mu = \begin{bmatrix} \frac{k_2}{m_2} & \frac{c_2}{m_2} \end{bmatrix}, \qquad (54)$$

$$\zeta = \begin{bmatrix} \frac{k_3}{m_2} & \frac{c_3}{m_2} \end{bmatrix}, \quad \gamma = \begin{bmatrix} \frac{k_3}{m_3} & \frac{c_3}{m_3} \end{bmatrix}, \qquad (55)$$

$$\psi = \begin{bmatrix} \frac{k_4}{m_3} & \frac{c_4}{m_3} \end{bmatrix}, \quad \chi = \begin{bmatrix} \frac{k_4}{m_4} & \frac{c_4}{m_4} \end{bmatrix}. \qquad (56)$$