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Event-triggered Control for Discrete-time Nonlinear Systems using State-dependent Riccati Equation

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Abstract— **Motivated by the need for the resource-efficient control in networked control systems, this paper considers event-triggered control approaches to discrete-time nonlinear control-affine systems. In particular, we use the discrete-time state-dependent Riccati equation along with event-trigger conditions to obtain control laws that satisfy 1) a stability condition and 2) a performance cost condition, respectively, while reducing the frequency of control input updates. A numerical example of the inverted pendulum is included to illustrate the proposed approach.**

I. Introduction

Thanks to the advancement in computer and communication technology, many of modern control systems exchange information among their elements over networks. The use of network has reduced the deployment cost and has increased the flexibility of the control systems. Thus, it has become easy to collect a lot of data, which enables to perform a wide variety of control tasks which have not been possible before. For this reason, networked control systems are now widespread in our daily lives from robotics [1] to health care systems [2] and power systems [3].

In networked control systems, however, there often exist limits on resources such as energy and communications that can be used to perform control tasks. This is maybe because the sensors or actuators use batteries, or maybe because the communication band is shared with other tasks. These limitations have motivated researchers to seek approaches that replace conventional periodic sampling (e.g., [4]) that may consume energy that can be saved and perform non-essential communications. This has led to alternative control approaches such as the event-triggered control and the self-triggered control, which have been actively studied in the last several years [5]–[8].

Typically, both of the event-triggered control and the self-triggered control consist of two elements; a feedback controller that computes the control input, and a triggering mechanism that determines the next execution time [5]. An event-triggered controller continuously samples the systems' states and determines whether the control input should be updated or not by applying trigger conditions to the current sampled data (e.g., [5], [6]). On the other hand, a self-triggered controller determines online the time instances for the next sampling and control input update so as to guarantee a specific closed-loop behavior using the plant model and previously received data (e.g., [5], [9]).

The purpose of this paper is to study event-triggered control problems for discrete-time nonlinear systems using the discrete-time state-dependent Riccati equation (D-SDRE) technique [10]–[12]. In particular, we present approaches to achieve 1) a stability condition and 2) a performance cost condition, respectively.

The state-dependent Riccati equation (SDRE) technique [13], [14] was first proposed by [15] for continuous-time systems and has become a popular approach to nonlinear systems last few decades. The SDRE technique provides a systematic and effective means of designing nonlinear controllers, observers and filters [14] by yielding a suboptimal solution to the infinite horizon regulation problem and makes the origin a locally asymptotically stable equilibrium point. An advantage of using SDRE includes that it does not require enforcing restrictive assumptions as in the case of many other nonlinear control methods. Additionally, SDRE allows us to tradeoff between the control effort and the state errors by using the quadratic cost function as in the form of linear quadratic regulator. Although SDRE does not guarantee global asymptotic stability in general, its domain of attraction is at least as large as the domain of interest with a certain degree of confidence [16], [17]. Thus, SDRE has been used in many nonlinear control applications such as missile autopilot design [18], pendulum control [19], helicopter control [20], and satellite formation [21].

The discrete-time version of SDRE, D-SDRE has been also actively studied for direct applications to real systems. For example, the optimality of D-SDRE was studied in [10], and the constrained D-SDRE and the constrained D-SDRE subject to uncertainties were studied in [12] and [22], respectively. D-SDRE was also used for developing an observer-based controller for discrete-time nonlinear dynamical systems in [23].

The remainder of the paper is organized as follows: After reviewing D-SDRE technique in Section II, we develop approaches to the event-triggered control for stability and performance in Section III. The approaches are illustrated using numerical examples in Section IV. Finally, Section V concludes the paper.

II. Overview of discrete-time state-dependent Riccati equation technique

Consider the discrete-time nonlinear control-affine system

$$x[t+1] = f(x[t]) + g(x[t])u[t],\ x[t_0] = x_0, \qquad (1)$$

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where

- $x[t] \in \mathbb{R}^n$ is the system state,
- $u[t] \in \mathbb{R}^{n_u}$ is the control input,
- $f(x[t]) \in \mathcal{C}^1$, $f(0) = 0$, and
- $g(x[t]) \in \mathcal{C}^1$, $g(x[t]) \neq 0$ for all $x[t]$.

For the nonlinear system (1), the extended linearization is given by:

$$x[t+1] = A(x[t])x[t] + B(x[t])u[t],\ x[t_0] = x_0, \tag{2}$$

where

- $A(x[t])x[t] = f(x[t])$, and
- $B(x[t]) = g(x[t])$.

Remark 1: Such a factorization is possible if and only if $f(x) \in \mathcal{C}^1$ and $f(0) = 0$ [14]. In the multi-variable case, $A(x)$ is not unique [13].

The discrete-time state-dependent Riccati equation (D-SDRE) technique finds a state-feedback control input $u[t] = u(x[t])$ that approximately minimizes the performance index

$$\begin{aligned} J(x_0) &:= \sum_{t=t_0}^{\infty} x[t]^T Q(x[t]) x[t] + u[t]^T R(x[t]) u[t], \\ & Q(x[t]) \geq \varepsilon I > 0,\ R(x[t]) > 0, \end{aligned} \tag{3}$$

by applying the linear quadratic regulator method point-wise.

Thus, we need the following definition and assumption to use D-SDRE technique:

Definition 1 ([13]): $A(x)$ is a controllable parameterization of the nonlinear system if the pair $(A(x), B(x))$ is point-wise controllable in the linear sense for all x.

Assumption 1: A controllable parameterization $(A(x), B(x))$ has been found.

Under Assumption 1, there exists a unique positive definite solution $P(x[t])$ to the discrete-time state-dependent Riccati equation

$$P = A^T PA - A^T PB(B^T PB + R)^{-1} B^T PA + Q, \tag{4}$$

where all matrices are evaluated at $x[t]$. Thus, the control law $u[t] = u(x[t])$ is given by

$$u_{\mathrm{S}}(x[t]) := -K(x[t])x[t], \tag{5}$$

where

$$K(x) := (B^T PB + R)^{-1} B^T PA, \tag{6}$$

and all matrices are evaluated at $x[t]$.

Remark 2: By construction, the closed-loop system

$$x[t+1] = A_{\mathrm{cl}}(x[t])x[t], \tag{7}$$

where $A_{\mathrm{cl}}(x)$ is the closed-loop coefficient matrix

$$A_{\mathrm{cl}}(x[t]) = A(x[t]) - B(x[t])K(x[t]), \tag{8}$$

is point-wise Schur stable for all $x[t]$. Therefore, the closed-loop system is locally asymptotically stable. However, this does not generally guarantee the global asymptotic stability [16].

Since it is usually difficult to achieve or prove global asymptotic stability except for some special cases, we introduce the following definition.

Definition 2 (Region of attraction for asymptotic stability by D-SDRE): If $x = 0$ is an asymptotically stable equilibrium point, then a region of attraction is an open, connected, invariant set defined by

$$\mathcal{D} = \{x_0 : \lim_{t \to \infty} \|x[t]\| = 0 \text{ subject to D-SDRE (2)-(6)}\}. \tag{9}$$

Namely, the region of attraction encloses all initial conditions such that when the system is steered from it, the origin will be reached asymptotically. Approaches to finding the region of attraction are discussed such as in [12], [24], [25]. Although these approaches are not always easy to apply or may result in conservative estimates, it is said that the region of attraction is generally as large as the domain of interest [16], [17].

The rest of the paper assumes the following:

Assumption 2: The performance index (3) is finite subject to D-SDRE (2)-(6), i.e., there exists a finite $\bar{J}$ such that $J^*(x_0) \leq \bar{J}$.

This assumption implies that the initial condition is inside the region of attraction $\mathcal{D}$, i.e., $x_0 \in \mathcal{D}$.

III. Event-triggered discrete-time state-dependent Riccati equation technique

In the event-triggered control, the state $x[t]$ is continuously monitored. When the state $x[t]$ satisfies a pre-defined event-trigger condition at the time instance t_k, the control input is recomputed and updated according to the control law (5) (Figure 1).

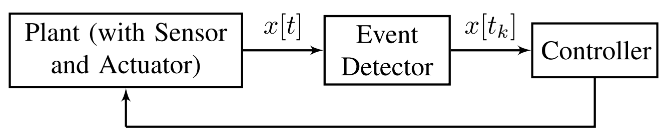

Fig. 1: Schematic of event-triggered control system

Let denote the state at the discrete time instance t_k by

$$x[t_k] = x_k. \tag{10}$$

Assuming that the process of collecting sensor measurements, computing the control input and updating the actuators can be done in zero time [5], the control inputs are held constant between the successive event-triggered time instances:

$$u[t] = u_k = u_{\mathrm{S}}(x_k),\ t \in [t_k, t_{k+1}), \tag{11}$$

where $u_{\mathrm{S}}(x_k)$ is defined in (5).

This paper proposes event-triggered control approaches that use event-trigger conditions defined in terms of the cost-to-go function:

$$J(x[t]) := \sum_{\tau=t}^{\infty} x[\tau]^T Q(x[\tau]) x[\tau] + u[\tau]^T R(x[\tau]) u[\tau], \tag{12}$$

where $u[\tau]$ is a sequence of control inputs and $x[\tau]$ satisfies the extended linearization of the nonlinear system (1) with the control input $u[\tau]$ starting at $t_0 = t$ and $x_0 = x[t]$.

When the sequence of control inputs is given by $u[\tau] = u_S(x[\tau])$ for all $\tau = t, t+1, \cdots$, we denote the cost-to-go function for this input by

$$J^*(x[t]). \tag{13}$$

Namely, $J^*(x[t])$ is the cost-to-go when a sequence of control inputs is obtained by D-SDRE technique (4)-(6) with the initial condition $x[t]$.

Here, note that

$$\min_{u[t],u[t+1],u[t+2],\cdots} J(x[t]) \leq J^*(x[t]) \tag{14}$$

because D-SDRE technique provides only a suboptimal input to minimizing the cost function (3).

Also from the definition of $J^*(x)$ in (13), we have

$$\begin{aligned} &J^*(x_k) \\ &= \sum_{\tau=t_k}^{\infty} x[\tau]^T Q(x[\tau]) x[\tau] + u_S(x[\tau])^T R(x[\tau]) u_S(x[\tau]) \\ &= x_k^T Q(x_k) x_k + u_S(x_k)^T R(x_k) u_S(x_k) \\ &+ \sum_{\tau=t_k+1}^{\infty} x[\tau]^T Q(x[\tau]) x[\tau] + u_S(x[\tau])^T R(x[\tau]) u_S(x[\tau]) \\ &= x_k^T Q(x_k) x_k + u_k^T R(x_k) u_k + J^*(x[t_k+1]). \end{aligned} \tag{15}$$

A. Stability

Let us first consider an event-triggered condition that guarantees the stability of the nonlinear system (1) under Assumptions 1-2.

The following theorem summarizes the sufficient conditions for the event-trigger condition to guarantee stability. In short, the first condition avoids the duration between execution time instances goes to infinity without approaching to zero, the second condition guarantees the states approach to zero at every execution time instances, and the third condition guarantees the states approach to zero between the execution time instances.

Theorem 1: Consider the system (2) with control inputs (11). Suppose that the first execution is at t_0 of the initial time. Under Assumptions 1-2, if the sequence of trigger time t_k for $k = 1, 2, \cdots$ is determined so as to satisfy

1) bounded interval between two consecutive executions:

$$t_{k+1} - t_k \leq T \tag{16}$$

for a finite fixed $T \geq 1$,

2) strict improvement at each execution:

$$J^*(x_{k+1}) + x_k^T Q(x_k) x_k + u_k^T R(x_k) u_k \leq J^*(x_k), \tag{17}$$

and

3) bounded state error:

$$\|x[t] - x_k\| \leq \sigma \|x_k\|, \ \forall t \in [t_k, t_{k+1}), \tag{18}$$

for a finite fixed $\sigma \geq 0$ for all x_k,

then the state trajectory $x[t]$ that is generated by the event-triggered D-SDRE control satisfies

$$\lim_{t\to\infty} \|x[t]\| = 0. \tag{19}$$

Proof:
(existence of t_{k+1})
First show that for a given t_k, there exists $t_{k+1} \in [t_k+1, t_k+T]$. For this purpose, it suffices to show that $t_{k+1} = t_k + 1$ satisfies the three conditions above.

- The first condition is satisfied with $T = 1$.
- The second condition is also satisfied with equality because $u_S(x_k) = u_k$ and $x_{k+1} = x[t_k + 1]$ (see (11) and (15)).
- The third condition is satisfied because the only t is $t = t_k$, thus

$$0 = \|x[t] - x_k\| \leq \sigma \|x_k\|. \tag{20}$$

(satisfaction of (19))

- The first condition of the bounded interval guarantees that k goes to infinity as t goes to infinity in the following argument.
- The second condition guarantees $J^*(x_{k+1})$ is bounded for all k. Thus, by taking the summation over k for the both sides of (17), we have

$$\begin{aligned} &\sum_{k=0}^{\infty} J^*(x_{k+1}) + \sum_{k=0}^{\infty} x_k^T Q(x_k) x_k + u_k^T R(x_k) u_k \\ &\leq \sum_{k=0}^{\infty} J^*(x_k) \\ \Rightarrow &\lim_{k\to\infty} J^*(x_k) + \sum_{k=0}^{\infty} x_k^T Q(x_k) x_k + u_k^T R(x_k) u_k \\ &\leq J^*(x_0). \end{aligned} \tag{21}$$

Since $\lim_{k\to\infty} J^*(x_k) \geq 0$, this implies

$$\sum_{k=0}^{\infty} x_k^T Q(x_k) x_k + u_k^T R(x_k) u_k \tag{22}$$

is bounded above, and

$$0 = \lim_{k\to\infty} x_k^T Q(x_k) x_k + u_k^T R(x_k) u_k \geq \lim_{k\to\infty} \varepsilon x_k^T x_k. \tag{23}$$

Thus, $\lim_{k\to\infty} \|x_k\| = 0$.

- The third condition implies

$$\begin{aligned} &\lim_{t,k\to\infty} \|x[t] - x_k\| \leq \lim_{k\to\infty} \sigma \|x_k\| = 0 \\ \Rightarrow &\lim_{t\to\infty} x[t] = \lim_{k\to\infty} x_k. \end{aligned} \tag{24}$$

Thus, $\lim_{t\to\infty} \|x[t]\| = 0$.

This completes the proof. ■

Remark 3: The maximum duration between two execution time instances, T, can be chosen arbitrarily large. Without this condition, it may happen $J(x_k) = J(x[t])$ for all $t \geq$

x_k and may not converge. A smaller T guarantees frequent updates and faster convergence. As T becomes smaller, the algorithm approaches to the standard D-SDRE and $T = 1$ coincides with the standard D-SDRE.

Remark 4: The second condition is equivalent to

$$J^*(x_{k+1}) \leq J^*(x[t_k + 1]), \tag{25}$$

i.e., at t_{k+1} the cost-to-go is no more than the cost-to-go at $t_k + 1$ using the suboptimal input obtained by D-SDRE at t_k. In other words, this condition guarantees that holding the input does not act adversely.

Thus, we have the following event-trigger condition for stability.

Event-trigger Condition 1: Suppose that the last trigger time is t_k. Then, $t_{k+1} = t$ that first satisfies one of the following conditions:

- $t = t_k + T$,
- $J^*(Ax[t] + Bu_k) + x_k^T Q(x_k)x_k + u_k^T R(x_k)u_k > J^*(x_k)$, or
- $\|Ax[t] + Bu_k - x_k\| > \sigma\|x_k\|$.

Note that if the event-trigger condition is met and we update the control input at $t = t_{k+1}$, then

$$J^*(x_{k+1}) = J^*(x[t]) = J^*(Ax[t-1] + Bu_k) \tag{26}$$

satisfies 2) of Theorem 1. Also, observe that this second condition implicitly keeps $x[t+1] \in \mathcal{D}$ for all t, which is a necessary condition for the stability. The satisfaction of 1) and 3) of Theorem 1 is clear.

There are other ways of choosing t_{k+1} for the execution time to guarantee the stability of the closed-loop. In fact, another event-triggered approach is presented in the next subsection along with the additional requirement of satisfying a performance cost condition.

B. Guaranteed Cost Performance

Now, under Assumptions 1-2, we develop an event-triggered condition for the nonlinear system (1) that guarantees the satisfaction of a performance cost condition

$$\begin{aligned} J(x_0) = \sum_{\tau=t_0}^{\infty} x[\tau]^T Q(x[\tau])x[\tau] + u[\tau]^T R(x[\tau])u[\tau] \\ \leq J_{\text{perf}}(x_0). \end{aligned} \tag{27}$$

where

$$J_{\text{perf}}(x_0) := \gamma J^*(x_0),\ \gamma \geq 1. \tag{28}$$

γ is used to balance the performance compared with "without event-trigger" and the amount of resource-savings. The use of $\gamma = 1$ results in a standard D-SDRE without event-trigger. A large γ will result in less executions but leads to inferior performance, and a small γ will lead to a good performance but require more executions.

The following theorem gives sufficient conditions for the event-triggered control to satisfy the performance condition (27). As before, the first condition provides a condition to avoid the duration between execution time instances goes to infinity.

Theorem 2: Consider the system (2) with control inputs (11). Suppose that the first execution is at t_0 of the initial time. Under Assumptions 1-2, if the sequence of trigger time t_k for $k = 1, 2, \cdots$ is determined so as to satisfy

1) bounded interval between two consecutive executions:

$$t_{k+1} - t_k \leq T \tag{29}$$

for a fixed $T \geq 1$, and

2) bounded cost increase beyond the suboptimal cost by D-SDRE:

$$\begin{aligned} \sum_{\tau=t_k}^{t_{k+1}-1} x[\tau]^T Q(x[\tau])x[\tau] + u_k^T R(x[\tau])u_k \\ + \gamma J^*(x_{k+1}) \leq \gamma J^*(x_k),\ \gamma \geq 1, \end{aligned} \tag{30}$$

then the performance condition (27) is satisfied.

Proof:

(existence of t_{k+1})

As before, we can easily show that $t_{k+1} = t_k + 1$ satisfies the first condition with $T = 1$ and the second condition using $x_{k+1} = x[t_k + 1]$.

(satisfaction of (27))

The first condition guarantees that k goes to infinity as t goes to infinity in the following. Taking summation over k for both sides, we have

$$\begin{aligned} &\sum_{k=0}^{\infty} \sum_{\tau=t_k}^{t_{k+1}-1} \left(x[\tau]^T Q(x[\tau])x[\tau] + u_k^T R(x[\tau])u_k \right) \\ &\qquad + \gamma \sum_{k=0}^{\infty} J^*(x_{k+1}) \leq \gamma \sum_{k=0}^{\infty} J^*(x_k), \\ &\Rightarrow \sum_{\tau=t_0}^{\infty} \left(x[\tau]^T Q(x[\tau])x[\tau] + u^T R(x[\tau])u \right) \\ &\qquad \leq \gamma J^*(x_0) - \lim_{k\to\infty} J^*(x_k) \leq \gamma J^*(x_0), \end{aligned} \tag{31}$$

where u is defined in (11).

■

Remark 5: Clearly, the satisfaction of (27) implies the boundedness of $J(x_0)$, thus the stability of the system. This can be seen from the inequality (31), which implies the boundedness of

$$\sum_{\tau=t_0}^{\infty} x[\tau]^T Q(x[\tau])x[\tau] + u_k^T R(x[\tau])u_k \tag{32}$$

thus,

$$\begin{aligned} 0 &\leq \lim_{\tau\to\infty} \varepsilon x[\tau]^T x[\tau] \\ &\leq \lim_{\tau\to\infty} x[\tau]^T Q(x[\tau])x[\tau] + u_k^T R(x[\tau])u_k = 0, \end{aligned} \tag{33}$$

which implies $\lim_{t\to\infty} \|x[t]\| = 0$, showing the stability of the closed-loop. Therefore, Theorem 2 can also be used to guarantee the stability of the closed-loop for an arbitrary but fixed $\gamma \geq 1$.

Thus, we have the following event-trigger condition for performance.

Event-trigger Condition 2: Suppose that the last trigger time is t_k. Then, $t_{k+1} = t$, where $t > t_k$ is the time instance that first satisfies one of the following conditions:

- $t = t_k + T$, or
- $$\sum_{\tau=t_k}^{t} x[\tau]^T Q(x[\tau])x[\tau] + u_k^T R(x[\tau])u_k + \gamma J^*(Ax[t] + Bu_k) > \gamma J^*(x_k). \quad (34)$$

IV. Example

This section illustrates the proposed approach using an example of the inverted pendulum described in [10] (Figure 2).

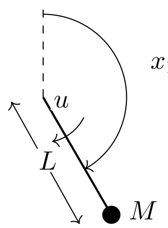

Fig. 2: Inverted pendulum

In this example, we aim at finding an event-triggered control sequence using D-SDRE for the pendulum from a certain initial level to the unstable equilibrium point. Assuming that the origin corresponds to the unstable equilibrium, the model is given by [10]

$$x_1[t+1] = x_1[t] + T_s x_2[t],$$
$$x_2[t+1] = \frac{T_s g}{L}\sin(x_1[t]) + \left(1 - \frac{T_s p}{ML}\right) x_2[t] + u[t]. \quad (35)$$

A state-dependent parameterization of the system (35) is

$$\begin{bmatrix} x_1[t+1] \\ x_2[t+1] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & T_s \\ \frac{T_s g \sin(x_1[t])}{L x_1[t]} & \left(1 - \frac{T_s p}{ML}\right) \end{bmatrix}}_{:=A(x)} \begin{bmatrix} x_1[t] \\ x_2[t] \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{:=B(x)} u[t], \quad (36)$$

where we define

$$\frac{\sin(x_1[t])}{x_1[t]} := 1, \text{ when } x_1[t] = 0. \quad (37)$$

Suppose that the following parameter values and the initial condition are given:

$$T_s = 0.05,\ M = 0.1,\ L = 0.1,\ g = 10,\ p = 0.05, \quad x_0 = [2.75,\ 0]^T. \quad (38)$$

This initial condition means that the pendulum starts near the stable equilibrium position as indicated in Figure 2.

To apply D-SDRE, we choose weights $Q = I$ and $R = 1$.

Clearly, $(A(x), B(x))$ is point-wise controllable everywhere. $x_0 \in \mathcal{D}$ can be easily checked by applying D-SDRE control sequence. This is confirmed from Figure 3, where we observe that the trajectory of D-SDRE approaches to the origin as time goes to infinity.

The proposed approaches for the stability with $\sigma = 0.5$ and $\sigma = 1$, and for the performance with $\gamma = 1.2$ and 2, are compared with the standard D-SDRE by simulations as shown in Figures 3-5 and Table I. For the bounded interval, $T = 60$ of the simulation length is used.

From Figures 3-4, we observe that the states approach to zero as desired in all cases. Figure 5 shows the control inputs illustrating how much control updates were removed by the event-trigger condition.

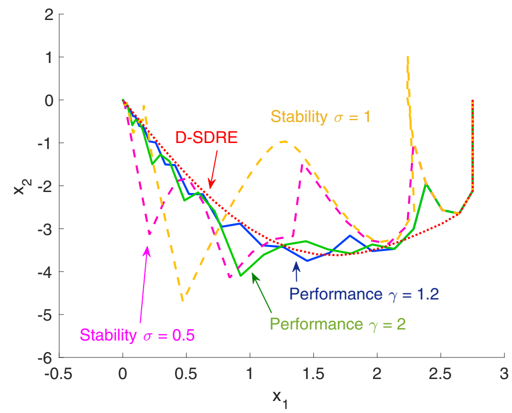

Fig. 3: Phase portrait

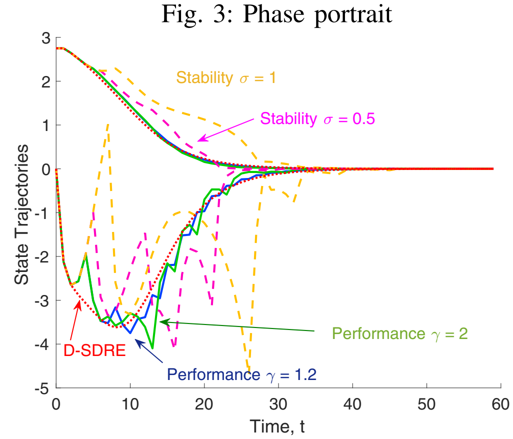

Fig. 4: State trajectories

Table I summarizes the simulation results in 60 time steps. Here, $J(x_0)$ is approximated by truncating at 60th step and the number of updates excludes at the initial execution at time t_0. From this table, the proposed event-trigger condition seems rather conservative yielding the actual performance cost much smaller than the specified cost. Also, notice that the case "Performance $\gamma = 1.2$" provides smaller cost with less number of control updates compares with "Stability $\sigma = 0.5$". This suggests that using the performance event-trigger condition with a right value of γ may be preferred to using the stability event-trigger condition. When comparing "Performance $\gamma = 2$" with "Performance $\gamma = 1.2$", we see that $\gamma = 2$ requires much less updates while achieving nearly the same cost of $J(x_0)$.

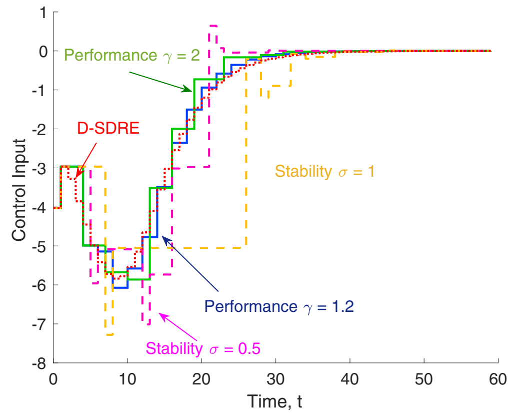

Fig. 5: Control trajectories

TABLE I: Summary of Performance

	$J(x_0)$	$\gamma J^*(x_0)$ [Specified upper bound on $J(x_0)$]	Number of updates
D-SDRE	570.03	N/A	60
Stability $\sigma = 0.5$	654.34	N/A	31
Stability $\sigma = 1$	809.67	N/A	21
Performance $\gamma = 1.2$	580.57	684.04	29
Performance $\gamma = 2$	584.65	1140.1	17

V. Conclusions

The state-dependent Riccati equation technique is one of the simplest and popular approaches in nonlinear controls. Motivated by the need for the resource-efficient control approaches in networked control systems, this paper proposed to use discrete-time state-dependent Riccati equation technique along with event-triggered algorithm for discrete-time nonlinear systems. The presented approach successfully reduced the number of executions while achieving the goals of stabilization and performance.

Future work will investigate the event-triggered control for the discrete-time nonlinear system with uncertainties and possibly extend the proposed approach to the self-triggered control with event-triggered sampling as in [26].

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