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Neuro-Adaptive Traffic Congestion Control for Urban Road Networks

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Abstract— **The rapid increase of private vehicles combined with the limited capabilities of the urban road infrastructure has made congestion one of the main problems of major cities worldwide, having a severe impact on both the economy and the environment. In this work, we shall attempt to solve the traffic management problem by examining in a unified manner the traffic network, the route guidance of the vehicles and the regulation of the traffic lights, as the basic elements of a single controlled system. In particular, we propose a decentralized adaptive control system, comprised of three main modules: i) the network congestion estimator, ii) the reference travel time estimator, and iii) the rate controller, that is capable of efficiently regulating the travel time along the traffic network while avoiding congestion at the junctions. The design of decentralized control algorithms and their implementation as traffic management applications for portable computing devices (e.g., 3$^{\text{rd}}$ and 4$^{\text{th}}$ generation mobile phones, tablets, computers embedded in "smart" vehicles) is expected to improve drastically the traffic condition of urban road networks. Meanwhile, in future traffic networks, where the navigation of the vehicles will be conducted by autopilots in the absence of human-drivers, the use of such a distributed autonomous management system will be essential.**

I. Motivation

During the last decades, the rapid increase of vehicles has created congestion in the traffic network of major cities worldwide. Traffic congestion has led to a lack of capacity utilization and a downgrading of the costly infrastructure of the urban traffic networks, especially during rush-hours, resulting in an increase in average travel time, a reduction in safety and an increase in pollutants emitted. A recent survey conducted in the United States showed that the annual national cost of traffic congestion is around 101 billion dollars for 2010, including 4.8 billion hours of waiting time on the road network and 7.5 billion liters of unnecessary fuel consumption [1]. It has also been shown that the extension of the urban traffic network and the consequent increase in its traffic capacity does not eliminate the problem. Additionally, an improvement of the urban traffic network in many cases becomes impossible due to the dense construction in urban areas. Therefore, in order to achieve optimal traffic conditions, it is necessary to fully utilize the existing infrastructure by developing modern control and management methods; an alternative that may be implemented through telematics science and which has been strongly boosted by the rapid developments in telecommunications technology and computers.

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Traffic congestion in the urban traffic network occurs when a large number of vehicles attempt to use a shared transport infrastructure with limited capacity. Saturated traffic systems have many negative effects, such as increased travel time, psychological pressure on drivers, increased risk of accidents, increased fuel consumption and severe air pollution. Traffic congestion is usually attributed to one or more of the following phenomena: i) high flow of vehicles from the suburban traffic network combined with the usually increased flow of the urban network has the effect of exceeding its capacity as well as the consequent congestion a few meters downstream of its entrances; ii) reduction of traffic lanes (road narrowing) can cause congestion when flow is greater than the bottleneck; iii) congestion may occur when the outflow from the urban network can not be served by the suburban network because the number of vehicles leaving the urban network exceeds the capacity of the host network; and iv) several random events, such as an accident, a damaged or a very slow vehicle, or even bad weather conditions, result in non-recurring bottlenecks.

To address the urban traffic congestion problem, where the increase in capacity with the construction of new roads is not possible due to lack of space, further efforts are needed on the traffic flow control with various automated control methods to ensure maximum use of this infrastructure. It is obvious that the design of efficient traffic management systems, coupled with the proper use of telecommunications and computer technology, can bring about a significant reduction or even elimination of traffic congestion as well as its negative consequences.

II. Related Works

In order to achieve a desired traffic flow response, traffic measuring mechanisms and intervention methods capable of influencing traffic conditions in a systematic and coherent way are required. With the development of broadband telecommunications networks in urban centers and the rapid improvement of mobile computing devices (3$^{\text{rd}}$ and 4$^{\text{th}}$ generation mobile phones, tablets, computers embedded in new "smart" vehicles, etc.), both in computing power, sensing and communication capabilities (e.g., GPS, Wi-Fi, gyroscopes, etc.), the retrieval and transmission of information on the state of the urban road network is feasible nowadays, which significantly facilitates the development and implementation of traffic control systems in real time. On the other side, the main control actions that can be applied to an urban road network are based on the traffic lights at its nodes and the route guidance/navigation of the vehicles.

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A. Traffic control with traffic lights

Traffic regulation with traffic lights is one of the key and most effective control measures that can be applied to urban road networks. Its main principle of operation relies on continuous regulation based on a pre-determined rule, called control strategy. The control strategy determines the operation of the traffic light to regulate vehicle entry at the junction, according to its design criteria and objectives. In addition, control of light signals is applied in combination to more than one node (coordinated control), thereby achieving greater efficiency and fair sharing of resources among competing drivers [4]–[11].

In fixed time strategies, the necessary calculations are based on simple statistical models and do not occur in real time as they employ past data. The fact that these strategies are based on past data and not on current traffic data is a significant disadvantage, as it leads to an oversight of the control problem, since: i) the demand is not constant within the day and may change from day to day; ii) the number of vehicles entering or leaving the urban road network is changing; iii) the past data undergoes the aging phenomenon, which makes optimized strategies outdated; and iv) the adequate reaction of the strategy cannot be determined in advance without the use of real measurements in the event of accidents and other unforeseen incidents that may disrupt traffic flow.

Flow control with traffic lights is an effective method, but if it is not accurate, traffic congestion is difficult to eliminate or the capacity of the urban road network may not be fully exploited. Nevertheless, the aforementioned drawbacks are eliminated when the control strategy is based on real-time measurements taken from sensors located in the area around the controlled node. In this case, the control strategy attempts to add to the flow measured upstream of the node as much flow as is necessary so that the downstream flow reaches the capacity of the receiving path. But if the downstream flow becomes greater than the critical flow then the junction flow becomes equal to the minimum possible to avoid congestion in its downstream portion.

The majority of traffic light control strategies are based on centralized computational systems. Initially, the control of traffic lights on an isolated node was implemented using fuzzy logic [4], [5] and tail theory [6]. Then, in [7], a new control model was developed and tested, combining three basic computational elements: i) genetic algorithms to solve optimization problems, ii) simulators based on cellular automata for viable strategic solutions, and (iii) a Beowulf Cluster computing system with exceptional cost/performance ratio. Moreover, in [8], by modeling the traffic network as a multi-agent system, the authors proposed a reinforcement learning based scheme. In addition, the findings in [9] have led to a distributed control system that optimizes the operation of the traffic light to reduce the overall travel time and the delays in bottlenecks. Finally, in [10] and [11], a hybrid control scheme with neural networks is proposed to regulate the operation of traffic junctions.

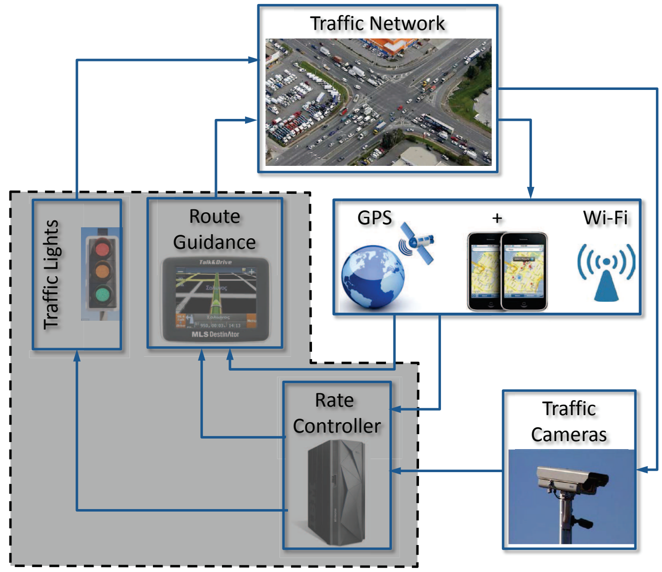

Fig. 1. The architecture of the proposed traffic management scheme.

B. Traffic control with route guidance

The established tactics to guide road traffic on urban road networks, where there are multiple routes leading from one starting point to another destination point, requires the collection of real-time network status information and processing in large computing systems (centralized approach) to calculate the optimal route duration, taking into account the traffic congestion of the network [12], [13]. Moreover, in addition to the cost of installing and maintaining the highly complex computerized network, this strategy becomes inefficient as the number of navigation devices increases (scalability problem). On the other hand, the complexity of the problem is diminished if instead of minimizing the duration of each route, an effort is made to optimize the overall traffic condition of the road network, thus seeking an average optimum. Average optimal status is defined as the minimum total travel time for all network users and differs significantly from the optimum route calculation for each user individually [14], [15]. Therefore, in a problem of calculating average optimal solution, it is expected that some users will be guided through non-optimal paths to achieve the overall optimal operation of the network.

The disadvantages of centralized control strategies are highlighted in detail in [16]–[20] and [22]–[27], where distributed route guidance algorithms are presented. In particular, the decentralized best-of-average [21] calculation algorithm was successfully applied in [16]–[20], where distributed route guidance strategies for users of a traffic network based on the principle of average optimum status were proposed. Finally, decentralized vehicle guidance protocols that develop mechanisms for disseminating traffic information to the users were presented in [22]–[27].

III. Contribution

Contrary to the aforementioned results in the relevant literature, where the traffic control problem is treated distinctly using either traffic lights or route guidance, we shall attempt to solve the problem in a unified manner by studying for the first time: i) the traffic network, (ii) the vehicle guidance and (iii) the regulation of traffic lights as essential elements of a single controlled system (see Fig.1). Specifically, we will study at the same time a fixed interconnected network, including the traffic lights, and a changing topology network, made up of the traffic network users. The traffic network is modeled as a directed graph, the nodes and edges of which correspond to the actual traffic junctions and the roads connecting adjacent nodes respectively. The traffic lights installed at the junctions regulate the traffic flow while the route guidance system decides the nodes that the traffic network user should pass to reach its destination. Therefore, at each node, two interacting control decisions will be implemented, that is a flow-regulating protocol (via the traffic lights) and one that will select the direction (via the route guidance system). These algorithms will be decentralized in order to be efficient in the case of a large number of users of the traffic network and will employ information available in their area of operation to avoid congestion of the urban telecommunications network. However, this requires that each user/vehicle and each traffic light have an appropriate wireless communication system (Wi-Fi) with other users or traffic lights in their range to receive and send the necessary information. In any case, the required telecommunications infrastructure is already available, since in almost every vehicle and traffic light there is a computing device that can either measure the speed of a vehicle or send and receive from neighboring devices status information on the current section of the road network. Therefore, the design of decentralized control algorithms to be implemented as traffic management applications in the aforementioned devices is estimated to improve significantly the traffic condition of urban road networks, without at the same time requiring the replacement of existing infrastructure. Also, in the future traffic networks, where the navigation of the vehicles will be carried out by autopilots in the absence of human drivers, the use of such an autonomous traffic management system will be necessary.

IV. Problem Formulation

The mathematical representation of a natural phenomenon is necessary for its systematic and thorough study. The details that characterize the key components of the phenomenon, their interactions and their dynamics depend on the context of the study in which this model is integrated. Therefore, to meet various needs, different models of the same phenomenon can be developed. In the related literature, two basic models of traffic flow in road networks have been proposed: a) the microscopic modeling [2], which describes the individual movement of each vehicle separately as it travels through the grid, and b) the macroscopic modeling [3], which describes the traffic flow as a fluid characterized by macroscopic variables such as density and average speed of vehicles. Microscopic models are complex and have a high computational cost. Macroscopic models, on the other hand, are simpler, and more suitable for real-time traffic control, as they have less computational cost.

In this work, we adopt a macroscopic model characterized by a set of input/output flow nodes $N_B = \{1, 2, \ldots, n\}$ and a set of internal nodes $N_I = \{1, 2, \ldots., m\}$ that model the road junctions, as well as a set of directed links $L = \{1, 2, \ldots, k\}$ modeling the roads that connect the nodes. Each link $k \in L$ has an associated queue with maximum capacity B_k. Within this traffic network, a vehicle has to travel from an initial to a destination node. The flow at each node is controlled by an appropriately designed protocol u, whose rate dictates the number of vehicles that travel via each specific direction of the node. Every pair of nodes is also characterized by a path $P\left(n_i, n_d\right)$, defined as a set of links/roads the vehicle follows from node n_i towards its destination n_d. The amount of time that elapses between this transition is called travel time T. The travel time along a path $P\left(n_i, n_d\right)$ is comprised of the path propagation time as well as the queueing delay formed in every non-empty link/road along $P\left(n_i, n_d\right)$. If the income flow rate to a link/road exceeds its maximum capacity, vehicles are queued in the road, consequently leading to congestion collapse when this incident holds for a long time. The congestion level of each link/road (denoted by $p_j \in [0,1]$) depends on the value of the incoming flow rate and the current queue length. Thus, an aggregate path congestion measure $p \in [0,1]$ for a path $P\left(n_i, n_d\right)$ may be defined by the intermediate congestion levels met in the path. Notice that $p = 0$ represents the almost empty road scenario. However, $p = 1$ does not necessarily mean that the traffic network faces already congestion. In practise, $p = 1$ implies that the corresponding path approaches congestion.

Based on the aforementioned traffic model, our primary goal is to design decentralized rate controllers, implemented on the traffic lights at the junctions, capable of guaranteeing high sensitivity to time quality characteristics (e.g., almost constant per link delay) and preventing congestion. Such design implies the existence of a control algorithm, capable of regulating the travel time T close to a desired level T_d, avoiding congested roads. Towards this direction, it is reasonable to claim that the dynamics of the travel time is modeled by a smooth, bounded but unknown function of the travel time itself T, the flow rate u and the aggregate path congestion p as follows:

$$\dot{T} = f\left(T, u, p\right). \tag{1}$$

Besides the uncertainty in $f\left(T, u, p\right)$, notice that the future path congestion p is also involved, which unfortunately is unknown and thus has to be estimated by an appropriately designed route guidance algorithm. Furthermore, the a priori availability of feasible travel time T_d values is not straightforward, owing to the highly uncertain and dynamically changing character of the traffic network.

V. Main Results

The proposed traffic control management scheme consists of three modules:

Network Congestion Estimator: The purpose of this module is to provide an estimate of the future level of congestion that the user will experience while running the road network.

Reference Travel Time Estimator: Due to the high uncertainty and the dynamic nature of the traffic network, setting a feasible reference time becomes impossible. For this purpose, a neural network based estimator will be designed to receive the status of the network path (congestion level) as input, and calculate an achievable reference time.

Rate Controller: The goal of this module is to calculate the appropriate control signals for the operation of the traffic lights and the route guidance so that the reference time can be reached. For this purpose, a neural network based controller will be designed to accept the status of the traffic network (congestion levels) and the reference signal as inputs and output the flow of vehicles at the nodes to be adjusted by the traffic lights, and the direction of the vehicles at the junctions, to be regulated by the route guidance system.

A. Future Path Congestion Level Estimator

As mentioned earlier, each vehicle when travelling along $P(n_i, n_d)$, experiences an aggregate path congestion $p \in [0,1]$, which is involved in the travel time dynamics (1). Therefore, a module is required to provide the rate controller that is implemented at each traffic light with an estimate of the future path congestion level $\hat{p}$ prior to initiating the corresponding directed flow. In this work, we adopt a stable low pass filter:

$$\dot{\hat{p}} = -a\hat{p} + a\mu,\ \hat{p}(0) = 0 \tag{2}$$

where $a > 0$ is a positive design constant that regulates the convergence rate of $\hat{p}$ and $\mu \in [0,1]$ is a variable depending on the normalized queue length of the road. From the aforementioned recursive formula (2) it becomes apparent that $\hat{p}(t) \in [0,1]$, which complies with the actual values of $p \in [0,1]$. Finally, the route guidance algorithm decides based on the distributed consensus protocol [28] which direction to follow in each junction such that a feasible travel time estimation is extracted at the next module.

B. Desired Travel Time Estimation Algorithm

Let us assume that a traffic light is about to initiate a directed vehicle flow at its junction. Let us also denote the average travel time by $\bar{T}$. Assuming a slowly varying $\hat{p}$, we argue that the rate of change of the average travel time $\bar{T}$ depends on a sufficiently smooth and bounded function of $\bar{T}$ and $\hat{p}$ described as follows:

$$\dot{\bar{T}} = \Phi\left(\bar{T}, \hat{p}\right),\ \bar{T}(0) = 0. \tag{3}$$

Therefore, employing the universal approximation capabilities of linear in the weights neural networks [29], we may assume, without any loss of generality, the existence of certain optimal weight values $\theta_t^\star \in \Re^{L_t}$ and an appropriately selected regressor vector $Z_t\left(\bar{T}, \hat{p}\right) \in \Re^{L_t}$, such that (3) is substituted by:

$$\dot{\bar{T}} = \theta_t^T Z_t\left(\bar{T}, \hat{p}\right) + w_t\left(\bar{T}, \hat{p}\right), \tag{4}$$

where $w_t\left(\bar{T}, \hat{p}\right)$ denotes the modeling error that satisfies $\left|w_t\left(\bar{T}, \hat{p}\right)\right| \leq \bar{w}_t,\ \forall\left(\bar{T}, \hat{p}\right) \in \Omega_t$ for any compact set Ω_t. Consequently, employing a gradient based update law, reinforced with an appropriately selected projection law [30] to retain the weight estimates bounded, we design the following estimation algorithm for the desired feasible travel time:

$$T_d = \hat{\bar{T}} \tag{5}$$

where the estimate of the average travel time is obtained by:

$$\dot{\hat{\bar{T}}} = \hat{\theta}_t^T Z_t\left(\bar{T}, \hat{p}\right) \tag{6}$$

and the neural network weights are updated as follows:

$$\dot{\hat{\theta}}_t = \text{Proj}\left\{\gamma_t\left(\bar{T} - \hat{\bar{T}}\right) Z_t\left(\bar{T}, \hat{p}\right), W_t\right\} \tag{7}$$

where $\gamma_t > 0$ is a positive design constant and $W_t \subset \Re^{L_t}$ is a convex and bounded set with respect to which the projection algorithm operates [30].

C. Neuro-Adaptive Rate Controller

In this subsection, we shall present a rate control scheme, based on a control Lyapunov function derivative estimation approach, capable of guaranteeing uniform ultimate boundedness for the tracking error $e = T - T_d$, as well as the uniform boundedness of all other signals in the closed loop system. To proceed, the following assumption on the controllability of the travel time dynamics is required.

Assumption 1: The solution of (1) can be forced to be uniformly ultimately bounded with respect to an arbitrarily small neighborhood of $T = T_d$.

Based on the aforementioned assumption and invoking [31], there exists a sufficiently smooth robust control Lyapunov function $V(e)$ as well as a continuous control signal $u_0(e)$ such that the tracking error e is robustly uniformly stable with respect to a compact set of the origin Ω_e. In this respect, the time derivative of $V(e)$ along the dynamics (1) and (6) is given by:

$$\dot{V} = \frac{dV(e)}{de}\left(f(T,u,p) - \hat{\theta}_t^T Z_t\left(\bar{T}, \hat{p}\right)\right)$$

Adding and subtracting $\frac{dV(e)}{de} f(T, u_0(e), p)$, we get:

$$\dot{V} = A(e,T,u,p) + B\left(e,T,p,\hat{p},\hat{\theta}_t\right)$$

where

$$A(e,T,u,p) = \frac{dV(e)}{de}\left(f(T,u,p) - f(T,u_0(e),p)\right)$$

$$B\left(e,T,p,\hat{p},\hat{\theta}_t\right) = \frac{dV(e)}{de}\left(f(T,u_0(e),p) - \hat{\theta}_t^T Z_t\left(\bar{T}, \hat{p}\right)\right)$$

Finally, exploiting [31], we conclude that $B\left(e,T,p,\hat{p},\hat{\theta}_t\right) \leq 0,\ \forall e \in \Re - \Omega_e$ and $t \geq 0$. Therefore, we arrive at:

$$\dot{V} \leq A(e,T,u,p),\ \forall e \in \Re - \Omega_e$$

Apparently, the term $A(e,T,u,p)$ cannot be employed in the control design since it involves the unknown dynamics

$f(T,u,p)$ as well as the unknown functions $V(e)$ and $u_0(e)$ the existence of which is secured by Assumption 1. However, based on the universal approximation capabilities of linear in the parameters neural networks, we may write:

$$A(e,T,u,p) = \theta_A^T Z_A(e,T,u,p) + w_A(e,T,u,p)$$

where $\theta_A \in \Re^{L_A}$ denote the optimal weights that minimize the modeling error $w_A(e,T,u,p)$ within a compact set Ω_A (i.e., $|w_A(e,T,u,p)| \leq W_A$, $\forall (e,T,u,p) \in \Omega_A$) for an appropriately selected regressor vector $Z_A(e,T,u,p)$. In this respect, we design the following rate controller:

$$\dot{u} = \frac{1}{u}\left(-\hat{\theta}_A^T Z_A(e,T,u,p) - k_e e^2 - k_u u^2\right),\; k_e, k_u > 0 \tag{8}$$

where the estimate $\hat{\theta}_A$ of the unknown parameters θ_A is provided by:

$$\dot{\hat{\theta}}_A = -\sigma_A \hat{\theta}_A + Z_A(e,T,u,p),\; \sigma_A > 0. \tag{9}$$

Stability Analysis

Let us consider the candidate Lyapunov function:

$$L = V(e) + \frac{1}{2}\left\|\tilde{\theta}_A\right\|^2 + \frac{1}{2}u^2$$

where $V(e)$ is the aforementioned robust control Lyapunov function and $\tilde{\theta}_A = \theta_A - \hat{\theta}_A$ denotes the parametric error. Differentiating with respect to time along the solutions of (1) and (6), substituting the aforementioned adaptive control scheme (8), (9) and invoking Young's inequality, we obtain:

$$\dot{L} \leq -\frac{k_e}{2}e^2 - k_u u^2 - \frac{\sigma_A}{2}\left\|\tilde{\theta}_A\right\|^2 + \bar{d}$$

where $\bar{d} = \frac{\sigma_A}{2}\|\theta_A\|^2 + \frac{W_A^2}{4k_e}$. Therefore, all closed loop signals remain uniformly ultimate bounded. Moreover, the size of the ultimate bounds can be regulated by selecting appropriately the control gains k_e, k_u, σ_A.

However, all aforementioned arguments hold as long as the state e, T, u, p remains within the compact set Ω_A, where the approximation capabilities of neural networks hold (i.e., $|w_A(e,T,u,p)| \leq W_A$). Hence, we have to prove that the proposed control scheme does not force the state to escape Ω_A at any point in time. Towards this direction and following similar technical arguments with [32], we may easily prove the existence of certain control gains values and initial weight estimates that guarantee $(e,T,u,p) \in \Omega_A$, $\forall t \geq 0$, which completes the proof.

Remark 1: Notice that the proposed rate controller (8) involves the control signal in the denominator, raising thus implementation issues as u tends to zero. However, such case represents an almost no vehicle flow for the specific direction at the junction and should be avoided. In this respect, we propose to adopt a projection algorithm when the rate u approaches a predefined small rate $\underline{u}$ to keep $u \geq \underline{u}$ and thus resolve both the implementation and the fairness issues discussed above.

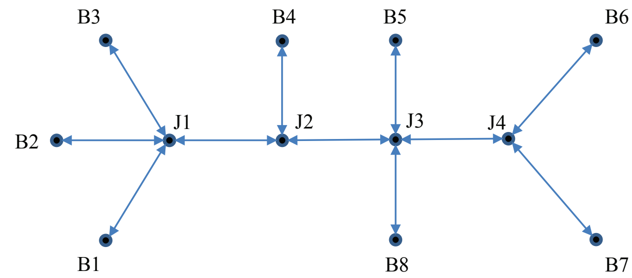

Fig. 2. The underlying graph of the traffic network.

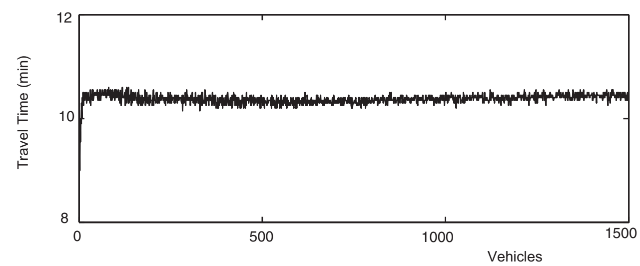

Fig. 3. The travel time from B1 to B6 for 1500 vehicle.

VI. Simulation Results

To demonstrate the performance of the proposed traffic congestion control scheme, we performed MATLAB simulations on a multi-node road network whose graph is given in Fig.2. We considered 8 input/output nodes marked as B1-B8 as well as 4 internal nodes at the junctions J1-J4 respectively. The average traveling velocity was set at 40 km/h, the average input rate at each node B1-B8 was considered 4 vehicles per minute and the average queue length at each road/link was assumed 50 vehicles. Moreover, it should be noticed that since the network graph does not involve any cycles then the routing algorithm was implemented by simply detecting the line subgraph that connects the desired and the goal nodes.

The travel time between B1 and B6 achieved by the proposed algorithm for 1500 vehicles is shown in Fig.3. Notice that the travel time admitted an almost constant value of 10.5 minutes for all vehicles. In the same vein, the travel time error was kept relatively small as depicted in Fig.4. On the other side, although the utilization in the bottleneck of the traffic network (i.e., the roads between the internal nodes J1-J4) was constantly 100%, the path congestion level experienced during the aforementioned vehicle transitions was less than one as illustrated in Fig.5, which verifies the robustness of the proposed scheme against model uncertainties as well as that no congestion collapse arose within the traffic network. Finally, similar behavior was also achieved for the rest of the paths within the considered traffic network.

VII. Conclusions

In this paper, a neuro-adaptive traffic congestion control scheme was presented, capable of regulating the travel time

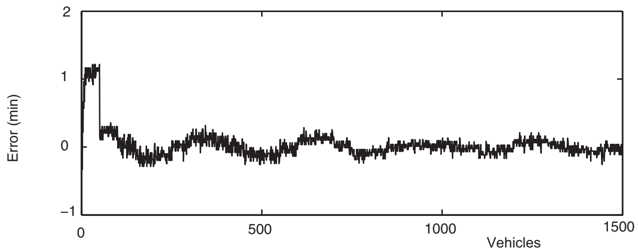

Fig. 4. The travel time error from B1 to B6 for 1500 vehicle.

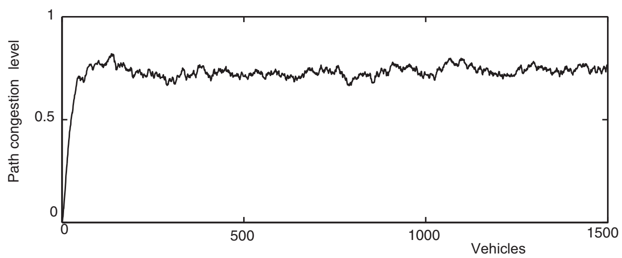

Fig. 5. The path congestion level experienced by 1500 vehicle travelling from B1 to B6.

in urban road networks. The controller is implemented in a decentralized manner at the traffic lights and the route guidance systems and is comprised of three main modules: i) the network congestion estimator, ii) the reference travel time estimator, and iii) the rate controller. Robustness against model uncertainties has also been proven. Future research efforts will be devoted towards studying the effect of delays and exogenous disturbances (in the form of stochastic traffic events) in the closed loop system as well as achieving a fair sharing of resources among competing users.

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