2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Cooperative Defense Strategy for Active Aircraft Protection Considering Launch Time of Defense Missile

Huiseong Song, Seokwon Lee, Namhoon Cho, and Youdan Kim

Abstract— **An aircraft can equip itself with a defense missile for active protection against an attack missile. Although a lot of strategies for cooperation between the aircraft and the defense missile have been studied, little attention has been paid to the early phase of the engagement. In this study, a cooperative defense strategy considering both launch and boost phase of the defense missile is proposed based on geometric approach. The strategy is derived to provide kinematic advantages to the defense team before it enters the end-game phase. Launch time of the defense missile is computed from the collision geometry. Numerical simulation demonstrates that the proposed defense strategy is useful for the defense team having no advantages on velocity and maneuverability over the adversary.**

I. INTRODUCTION

Against an air-to-air missile, the aircraft can enhance its survivability both in passive and active ways. Electronic countermeasures (ECM) and various types of decoys such as chaff and flare are widely used for passive protection. The aircraft can also employ evasive maneuvers or launch a counter-missile, which are categorized as active measures in order to cope with an agile attacking missile. Counter-missile, or defense missile, especially has a merit that it can completely remove the potential threat to the aircraft, if successful. For this reason, many recent studies have dealt with the defense missile as an aircraft defense system.

Once the aircraft launches a defense missile, the problem becomes a three-player game between the aircraft, the attacking missile, and the defense missile. Throughout the rest of this paper, each player is named as Target, Attacker, and Defender, respectively. Three player games are substantially different from typical pursuer-evader engagement, because the Target and the Defender can cooperate as a team to protect the Target.

In recent years, various cooperative strategies of the defense team have been proposed. Frameworks to tackle the Target-Attacker-Defender (TAD) problem can be divided into three approaches: optimal control approach, differential game approach, and geometric approach. In the optimal control approach, Shima [1] established a fundamental framework of the TAD problem. Cooperative pursuit-evasion was defined as a problem of minimizing zero-effort miss distance between the Attacker and the Defender based on linearized kinematics. In [2], a trade-off between zero-effort miss distance and control effort of the defense team was considered. Weiss et al. [3] put more emphasis on the guidance effort, which was minimized as long as the defense team achieved two criteria about the interception and evasion. They also derived an optimal countermeasure against a smart Attacker which was assumed to use an optimal maneuver [4]. Most of the research using optimal control theory, however, may degrade the performance especially when the trajectories of each agent are largely deviated from the linearized collision triangle.

Another way to solve the TAD problem is based on differential game theory. This approach basically assumes that not only the defense team but also the adversarial Attacker employs rational strategies during the engagement. Perelman et al. [5] formulated a linear quadratic differential game in the continuous and discrete domains. Garcia et al. [6-7] brought in a nonlinear engagement geometry and formed the miss distance between the Target and the Attacker as a cost/payoff function. As in [6], differential game theoretic framework is often combined with geometric intuition. However, one of the assumptions underlying in both optimal control and differential game approach is that the Attacker's guidance strategy and dynamics are assumed to be known or estimated by the defense team with an online identification scheme. However, in practice, it cannot be guaranteed that the defense team is able to predict the behavior of the offender perfectly.

In that point of view, geometric approach may relieve the assumption because it does not exploit the guidance law of the Attacker. Instead, the defense team attempts to find a geometric configuration taking an advantage over the adversary. Ratnoo and Shima [8] argued that line-of-sight (LOS) guidance was naturally suited for the multi-agent scenario. The Defender was commanded to be on the LOS between the Attacker and the Target. Yamasaki et al. [9] proposed a modified LOS guidance, which commands the Defender to gradually intervene in the LOS. In [10], a restricted turning rate of the Defender was addressed utilizing differential game theory and several geometric analyses.

Although a lot of promising cooperative strategies have been suggested, little attention has been paid to the cooperative strategy in the early phase of the engagement. The scenario usually neglected the launch and boost phase of the Defender in previous research. Therefore, the TAD problem needs to be augmented to incorporate the early phase. It is expected that the cooperation allowing for the early phase can improve the protecting performance of the defense team.

The objective of this study is to propose a cooperative defense strategy based on geometric approach. Extending the work of [9], the proposed strategy incorporates a boost phase as well as an end-game phase. Necessary conditions for entering new collision course are established, and corresponding strategy is obtained to determine the launch time of the Defender. The Defender can effectively intercept

H. Song, S. Lee, N. Cho, and Y. Kim are with Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul, 08826, Republic of Korea. (e-mail: altb37@snu.ac.kr, blueswl@snu.ac.kr, nhcho91@snu.ac.kr, ydkim@snu.ac.kr).

the Attacker by forming a favorable collision course before entering the end-game phase.

The rest of the paper is organized as follows. Section II provides the problem statement with assumptions. In Section III, a cooperative defense strategy considering launch time of the Defender is proposed. Section IV demonstrates numerical simulation results. Finally, concluding remarks are presented in Section V.

II. Problem Statement

A. Scenario Description

As an engagement scenario, let us consider a three-player game consisting of three entities: Target, Attacker, and Defender, denoted as T, A, D, respectively. The followings are assumed in this study.

Assumption 1: The kinematic model of each agent is a point-mass moving on an engagement plane.

Assumption 2: Each agent has a constant speed except for the boost phase where the Defender accelerates at a constant rate.

Assumption 3: Scenario begins when the Target detects the Attacker. The Attacker is initially on the collision course toward the Target.

Assumption 4: The defense team can share perfect information with each other.

In the engagement scenario of this study, relatively small defending missile is assumed to take more practical situation into account. The Defender has advantages on neither speed nor maneuverability over the Attacker. The velocity and the acceleration of three entities are denoted as V_i and a_i for $i \in \{T, A, D\}$. At the moment of launch, initial position and velocity of the Defender are identical to those of the Target, which is also a kinematic constraint. It is also assumed that the basic form of the Attacker's guidance law is proportional navigation with a biased term which is unknown to the defense team as

$$a_A = N_A V_A \dot{\lambda}_{AT} + b_A \tag{1}$$

where N_A is a navigation constant of the Attacker, λ_{AT} is the Attacker-Target LOS angle, and b_A is a bias command unknown to the adversary.

A schematic view of the planar engagement is shown in Fig. 1. The origin is defined as the initial point of the Target, x-axis is aligned with the initial LOS of the Target and the Attacker. The flight path angle of agent i is denoted as γ_i, and its rate is given as

$$\dot{\gamma}_i = \frac{a_i}{V_i}, \qquad \text{for } i \in \{T, A, D\} \tag{2}$$

where the acceleration capability of each agent is limited as follows.

$$|a_i| \le a_{i,\max}, \qquad \text{for } i \in \{T, A, D\} \tag{3}$$

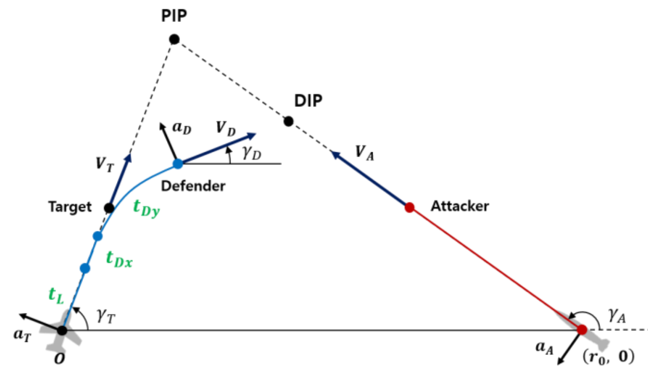


Figure 1. Engagement geometry

Each agent is supposed to have a different level of agility which can be expressed by the difference in speed and maneuverability limit as

$$V_T < V_D < V_A \tag{4}$$

$$a_{T,\max} < a_{D,\max} < a_{A,\max}\ . \tag{5}$$

B. Timeline of cooperative protection

The definition of timeline initiates the formulation of cooperative strategy. Timeline of the defense team consists of following four time intervals as

$$t_{fAD} = t_L + t_{Dx} + t_{Dy} + \Delta t \tag{6}$$

where t_{fAD} is the final time of the Attacker-Defender engagement, t_L is the time when the Target launches the Defender, t_{Dx} and t_{Dy} are the duration time during which geometric advantages are given to the defense team prior to end-game phase, and Δt is the duration of the end-game phase. Each value determines the points at which the Defender switches its maneuver phase.

Note that Δt has been the only concern in most of earlier studies on the TAD problem. Since the defense team has speed and maneuverability disadvantages, evasive strategy is not of our consideration after the Defender's interception ends in failure. Therefore, the objective of the defense team is solely converted to capture the Attacker with the Defender before the Attacker destroys the Target. Now, cooperative defense strategy is designed with respect to the defined timeline in the following section.

III. Cooperative Strategy

It is clear that the considered situation is not favorable for the protection of the target aircraft because the defense team does not know the exact behavior of the Attacker. Thus, an optimal countermeasure is unlikely to be found. Under the condition, it is natural for the defense team to make all effort on constructing an advantageous geometry. Let us describe the cooperative strategy in the reverse order of time.

A. Strategy in end-game phase for Δt

Let us suppose that the end-game phase begins with a favorable condition formed by previous stage. In the end-game phase, the defense team endeavors to maintain the relative formation.

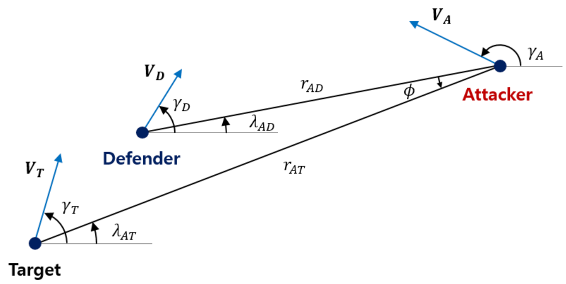


Figure 2. Engagement in end-game phase

In Fig. 2, Target and Defender are flying in formation. Relative LOS angle from the Attacker to the defense team is defined as

$$\phi = \lambda_{AT} - \lambda_{AD} \tag{7}$$

where λ_{ij} is the LOS angle between agent i and j [9]. Now, the angle ϕ determines the relative formation among the agents. Consider guidance commands of the Defender and the Target as

$$a_D = -N_D V_D \dot{\phi} \tag{8}$$

$$a_T = N_T V_T \dot{\phi}\,. \tag{9}$$

The Defender behaves similarly to the conventional proportional navigation guidance (PNG) in the proximity of the Attacker. However, the Defender tries to maintain the initial ϕ and gradually changes its flight path to intercept the Attacker. Note that, although derived from a minimum-energy optimal problem [9], the guidance law of (8) naturally induces the Defender to keep its initial arrangement of the defense team. Therefore, the guidance law (8) is adopted during Δt for the different reason from the original design [9]. At the same time, the Target guided by (9) changes its velocity direction to help the Defender accomplish the mission.

B. *Strategy in boost phase for t_{Dx} and t_{Dy}*

The Target launches the Defender at the beginning of the boost phase, and the position and velocity of the Defender are supposed to be the same as those of the Target at that moment. Before confronting the end-game phase, the Defender should increase its speed up to the maximum and change its heading angle into a desired value.

Let us divide this phase into two parts, t_{Dx} and t_{Dy}, according to the requirements. In order to utilize the Defender's dynamic pressure as much as possible, the Defender starts maneuver after its speed reaches the maximum limit. Therefore, the Defender accelerates along x-direction of its body axis without steering for t_{Dx} as

$$t_{Dx} = \frac{V_D - V_T}{a_{Dx}} \tag{10}$$

where a_{Dx} is the maximum tangential acceleration of the Defender. Note that t_{Dx} is a known constant for given V_T, V_D, and a_{Dx}.

During t_{Dy}, the Defender uses the lateral acceleration until a new collision triangle between the Defender and the Attacker is established. One of the necessary conditions of collision can be expressed as

$$V_D \sin(\gamma_D + \phi_0(\gamma_D, t_L)) = V_A \sin(\gamma_A + \phi_0(\gamma_D, t_L)) \tag{11}$$

where ϕ_0 is the initial relative LOS angle from the Attacker to the defense team of the end-game phase as shown in Fig. 3. In order to rearrange the collision triangle as soon as possible, the Defender is commanded to maneuver for a short time by using the maximum y-axis acceleration until the desired course is attained. The duration time can be obtained as

$$t_{Dy} = \frac{V_D(\gamma_T - \gamma_D)}{a_{D,\max}}\,. \tag{12}$$

While the Defender constructs a new collision geometry, the Target, or the aircraft, is commanded to move straight in order to minimize the reaction of the Attacker. The Target and the Attacker maintain the non-rotating collision triangle during the boost phase of the Defender. Note that the LOS angle between the Target and the Attacker is retained to be zero until the end of this phase, that is, $\lambda_{AT} = 0$ and $\phi_0 = -\lambda_{AD}$. On that account, the Defender can estimate the predicted position of the Attacker, and the initial relative LOS angle ϕ_0 can be derived as a function of t_L as described in Appendix A. Since ϕ_0 is a function of γ_D, (11) can be solved for γ_D numerically when t_L is determined.

C. *Strategy in launch phase for t_L*

At the very beginning of the scenario, the Target in a dangerous situation should make a decision about when to launch the Defender. Note that the launch time determines the initial geometry of end-game phase under the given posterior strategies. In particular, an advantageous geometry is required for the Defender which is slower than the Attacker.

It is clear that the Defender cannot pursue the Attacker on an absolute tail-chase geometry. On the other hand, a perfect head-on geometry does not fit the defense team measuring double-LOS (8-9). For this reason, a proper side-on geometry is needed. As illustrated in Fig. 3, the end-game geometry can be expressed by the relative angle between the velocity vectors of the Defender and the Attacker, or the approaching angle ψ, which has the following relationship.

$$\psi = \gamma_A - \gamma_D \tag{13}$$

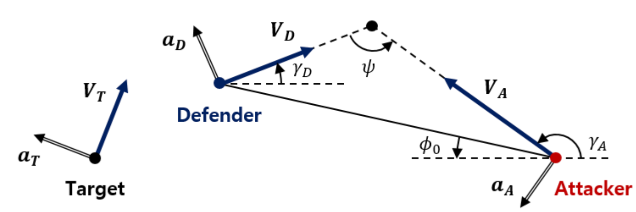


Figure 3. Initial end-game phase geometry

Against the maneuvers of the Attacker, the defense team can take a feasible action by keeping the relative collision triangle. The second derivative of the LOS angle between the Attacker and the Defender can be expressed as

$$\ddot{\lambda}_{AD} = \frac{1}{r_{AD}}(-2\dot{r}_{AD}\dot{\lambda}_{AD} + a_A\cos(\gamma_A+\phi_0) - a_D\cos(\gamma_D+\phi_0)) . \quad (14)$$

Note that if $\dot{\lambda}_{AD}=0$ and $\ddot{\lambda}_{AD}=0$, then the defense team can retain the formation. Equation (11) was already derived from zero LOS rate between the Attacker and the Defender, that is, $\dot{\lambda}_{AD}=0$. To eliminate the remaining terms of (14), the maximum acceleration component of the Defender should be able to compensate that of the Attacker as follows.

$$a_{D,\max}\left|\cos(\gamma_D+\phi_0)\right| \ge a_{A,\max}\left|\cos(\gamma_A+\phi_0)\right| \quad (15)$$

Unfortunately, both (11) and (15) are not achievable at the same time in view of *Remark 1*.

Remark 1: If $V_D < V_A$ and $a_{D,\max} < a_{A,\max}$, then both (11) and (15) cannot be accomplished at the same time.

Proof: See Appendix B.

Allowing the change of the LOS rate, the condition (15) can be relaxed as

$$a_{D,\max}\cos(\gamma_D+\phi_0) - a_{A,\max}\cos(\gamma_A+\phi_0) > -M_a \quad (16)$$

where M_a is a positive constant. The condition (16) means that $\ddot{\lambda}_{AD}$ is bounded so that the rotation of the collision triangle is restrained even in case of maximum maneuver of the Attacker. The following remark provides a way to choose the launch time considering the relaxed condition (16).

Remark 2: ψ is a strictly increasing function of t_L in a given geometry.

Proof: See Appendix C.

According to *Remark 2*, the collision triangle becomes more flattened with large t_L, which causes an undesirable head-on geometry. For the strategy in t_L, therefore, launch time is chosen to be a minimum time satisfying both (11) and (16).

TABLE I. PARAMETER VALUES FOR SIMULATION

Parameter	Symbol	Value	Unit
Speed of Target	V_T	250	m/s
Speed of Attacker	V_A	800	m/s
Speed of Defender	V_D	600	m/s
Lateral acceleration limit of the Target	$a_{T,\max}$	5	g
Lateral acceleration limit of the Attacker	$a_{A,\max}$	25	g
Lateral acceleration limit of the Defender	$a_{D,\max}$	20	g
X-axial acceleration limit of the Defender	a_{Dx}	25	g
Time constant of the Target	τ_T	0.2	s
Time constant of the Attacker	τ_A	0.1	s
Time constant of the Defender	τ_D	0.1	s
Distance between A & D when A starts evasion	M_{AD}	5	km

IV. NUMERICAL SIMULATION

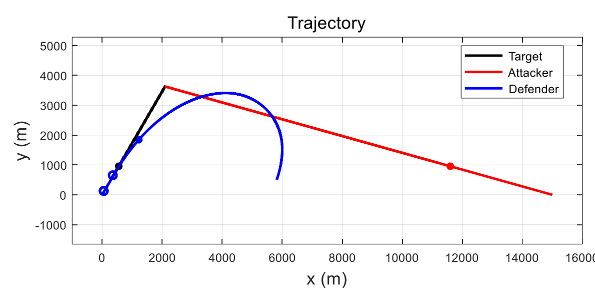
In this section, numerical simulation is carried out to demonstrate the effectiveness of the proposed cooperative strategy. Table 1 summarizes the simulation conditions. Autopilot lag of each agent is assumed to be a first-order linear model with a time constant, τ_i. Two types of Attacker are considered in simulations. Naïve Attacker employing PPN is used in *Simulation 1* and *3*, and evasive Attacker is used in *Simulation 2*. The evasive Attacker is assumed to evade the Defender with an additional acceleration which is perpendicular to the LOS from the Attacker to the Defender [8].

A. Simulation 1

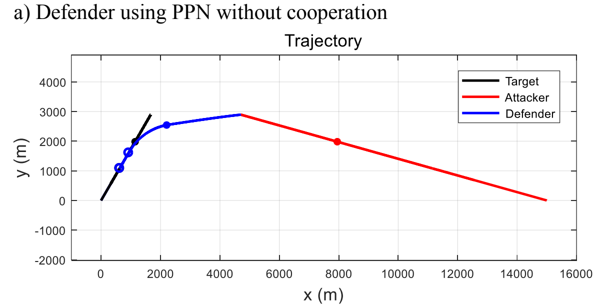
Simulation 1 is performed to clearly show why the cooperation is necessary for active aircraft protection. For the sake of simplicity, a non-maneuvering Target and a naïve Attacker are considered. Initial heading angle of the Target is set as 60 degrees.

In Fig. 4a, the Defender was immediately launched from the Target and was steered by PPN guidance without any cooperation. Although the Attacker was naïve and the Target was non-maneuvering, the Defender failed to intercept the Attacker with large miss distance. It appears that the aircraft protection is not always achievable without cooperation, because the defense team has the velocity and maneuverability disadvantages. Fig. 5a shows the acceleration history. Since the Defender uses little control effort in the early phase of the engagement, it cannot accomplish the mission due to the limited maneuverability in the end.

Fig. 4b and Fig. 5b show the performance of the proposed guidance strategy. The Defender which was assumed to be launched at a fixed time, 5 seconds, utilized its maximum ability in incipient stage. The Defender could get ready to enter the end-game phase in case the Attacker could maneuver rapidly. Finally, the Defender cooperating with the Target completed the mission with small miss distance.

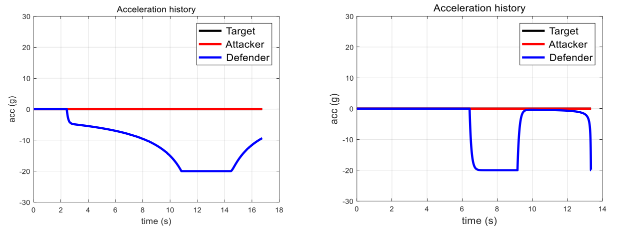


a) Defender using PPN without cooperation



b) Defender using the proposed strategy with $t_L = 5s$

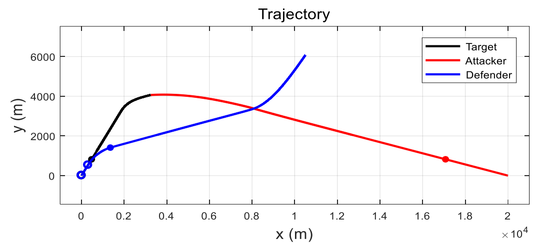
Figure 4. Trajectories of each agent (*Simulation 1*)



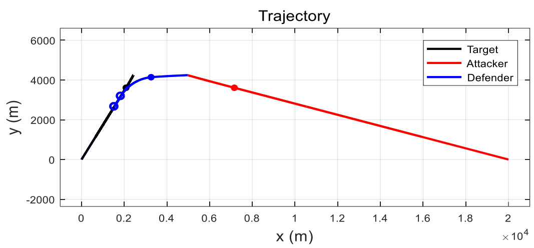
a) Defender using PPN b) Defender using proposed strategy

Figure 5. Time histories of accelerations of each agent (*Simulation 1*)

B. Simulation 2



a) Trajectories with $\psi_c = 148°$ and $t_L = 0s$



b) Trajectories with $\psi_c = 161°$ and $t_L = 12.52s$

Figure 6. Trajectories of each agent (*Simulation 2*)

Simulation 2 demonstrates that the launch time of the Defender can be significant to capture a maneuvering Attacker. In simulation setting, the Target-Attacker initial distance was set to $r_0 = 20$ km, and the other parameters were set to be the same as those of *Simulation 1*.

From (11) with $t_L = 0s$, minimum approaching angle was computed as 148 degrees. Fig. 6a shows the trajectories when the approaching angle command ψ_c was 148 degrees without considering condition (16). Fig. 6b shows the trajectories when (16) was taken in determining t_L with $M_a = 50$. For the latter case, ψ_c and t_L were numerically computed as 161 degrees and 12.52 seconds, respectively. The Defender succeeded in capturing the Attacker in the latter case, whereas it failed in the former case. As a result of *Simulation 2*, it was verified that the defense team using the proposed strategy can cope with the worst case in which the Attacker evades rapidly.

C. Simulation 3

In *Simulation 3*, the performance of the proposed strategy is evaluated by investigating the miss distance between a naïve Attacker and the Defender. Simulations were conducted with varieties of initial heading angles of the Target and launch time: heading angles from 20 to 160 degrees with the interval of 20 degrees, and launch time from 0 seconds to the maximum of each case at the interval of 1 second.

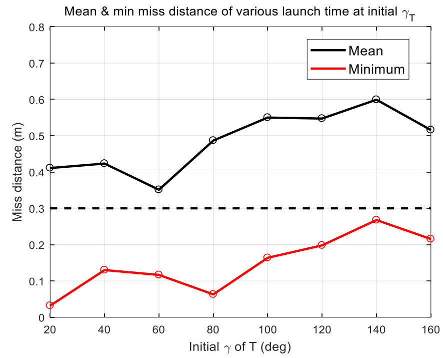


Figure 7. Miss distance for various t_L at initial γ_T (*Simulation 3*)

In Fig. 7, a black point indicates that the average of the miss distance with respect to the launch time at the initial heading angle, whereas a red point is its minimum. This result apparently shows that the performance of interception can be improved by adjusting the launch time of the Defender in every initial geometry. In addition, it is observed that the miss distance mildly increases with the initial heading angle. It can be inferred that an initial tail-chase geometry between the Target and the Attacker even degrades the interception performance of the Defender.

V. CONCLUSION

In this study, a cooperative strategy of the defense team considering launch time of the defense missile was proposed based on geometric approach. The engagement scenario was augmented to include the early phase of the defense missile and formulated in practical manner. The strategies prior to the end-game phase were mainly addressed, which has little been considered in previous Target-Attacker-Defender problem. Especially launch time of the defense missile was numerically determined by kinematic relations among the agents. In the boost phase, the defense missile is commanded to provide the defense team with geometric advantages in minimum time before it enters the end-game.

Numerical simulations were conducted to verify the performance of the proposed strategy. Simulations explained why the launch time is important to form an advantageous geometry, and indicated that the timely launch of the defense missile could improve the efficiency of the aircraft protection. Future work will address an optimal strategy of the aircraft prior to the launch of the defense missile.

APPENDIX

A. Description of ϕ_0

The non-rotating collision triangle between the Target and the Attacker is retained until the moment of $t = t_L + t_{Dx} + t_{Dy}$. Note that, although the actual positions are different from the expected positions, the end-game phase can compensate the error. Therefore, the initial relative angle ϕ_0 is obtained as follows.

$$\tan\phi_0 = \frac{-V_A \sin\gamma_A (t_{Dx} + t_{Dy}) + \{\frac{V_D^2 - V_T^2}{2a_{Dx}}\sin\gamma_T - \frac{V_D^2}{a_{D,\max}}(\cos\gamma_T - \cos\gamma_D)\}}{r_0 + V_A\cos\gamma_A(t_L + t_{Dx} + t_{Dy}) - \{(\frac{V_D^2 - V_T^2}{2a_{Dx}} + V_T t_L)\cos\gamma_T - \frac{V_D^2}{a_{D,\max}}(\sin\gamma_D - \sin\gamma_T)\}} \tag{A.1}$$

B. Proof of Remark 1

Without the loss of generality, let us consider only equality condition of (15). By taking squares on both sides of (11) and (15), we have

$$\cos^2(\gamma_A+\phi_0)=\frac{a_{D,\max}^2(V_A^2-V_D^2)}{a_{D,\max}^2V_A^2-a_{A,\max}^2V_D^2} \quad \text{(B.1)}$$

$$\sin^2(\gamma_A+\phi_0)=\frac{V_D^2(a_{D,\max}^2-a_{A,\max}^2)}{a_{D,\max}^2V_A^2-a_{A,\max}^2V_D^2}\ . \quad \text{(B.2)}$$

If $V_D<V_A$ and $a_{D,\max}<a_{A,\max}$, then the right side of either (B.1) or (B.2) is not a positive, that is, the solution of ϕ_0 does not exist. ■

C. Proof of Remark 2

From (13), showing *Remark 2* is equivalent to prove that γ_D is a strictly decreasing function of t_L. By taking the derivative of (A.1), we have $\frac{\partial\phi_0}{\partial t_L}>0$. By taking the derivative of (11), the following equation is obtained.

$$V_D\cos(\gamma_D+\phi_0)\frac{\partial\phi_0}{\partial t_L}=\{V_A\cos(\gamma_A+\phi_0)-V_D\cos(\gamma_D+\phi_0)\}\frac{\partial\gamma_D}{\partial t_L} \quad \text{(C.1)}$$

Since the value in the bracket of (C.1) is the Defender-Attacker closing velocity in the initial end-game phase, according to the defined geometry, $\frac{\partial\gamma_D}{\partial t_L}<0$. ■

REFERENCES

[1] T. Shima, "Optimal cooperative pursuit and evasion strategies against a homing missile," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 2, pp. 414-425, 2011.

[2] O. Prokopov and T. Shima, "Linear quadratic optimal cooperative strategies for active aircraft protection," *Journal of Guidance, Control, and Dynamics*, vol. 36, no. 3, pp. 753-764, 2013.

[3] M. Weiss, T. Shima, D. Castaneda, and I. Rusnak, "Minimum effort intercept and evasion guidance algorithms for active aircraft defense," *Journal of Guidance, Control, and Dynamics*, vol. 39, no. 10, pp. 2297-2311, 2016.

[4] M. Weiss, T. Shima, D. Castaneda, and I. Rusnak, "Combined and cooperative minimum-effort guidance algorithms in an active aircraft defense scenario," *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 5, pp. 1241-1254, 2017.

[5] A. Perelman, T. Shima, and I. Rusnak, "Cooperative differential games strategies for active aircraft protection from a homing missile," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 3, pp. 761-773, 2011.

[6] E. Garcia, D. W. Casbeer, and M. Pachter, "Active target defense differential game with a fast defender," *American Control Conference*, Chicago, IL, USA, July, 2015.

[7] E. Garcia, D. W. Casbeer, and M. Pachter, "Cooperative strategies for optimal aircraft defense from an attacking missile," *Journal of Guidance, Control, and Dynamics*, vol. 38, no. 8, pp. 1510-1520, 2015.

[8] A. Ratnoo and T. Shima, "Line-of-sight interceptor guidance for defending an aircraft," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 2, pp. 522-532, 2011.

[9] T. Yamasaki, S. N. Balakrishnan, and H. Takano, "Modified command to line-of-sight intercept guidance for aircraft defense," *Journal of Guidance, Control, and Dynamics*, vol. 36, no. 3, pp. 901-905, 2013.

[10] D. W. Casbeer, E. Garcia, Z. E. Fuchs, and M. Pachter, "Cooperative target defense differential game with a constrained-maneuverable defender," *IEEE 54th Annual Conf. Decision and Control (CDC)*, Osaka, Japan, Dec. 2015.

[11] V. Turetsky and T. Shima, "Target evasion from a missile performing multiple switches in guidance law," *Journal of Guidance, Control, and Dynamics*, vol. 39, no. 10, pp. 2364-2373, 2016.

[12] O. Ariff, R. Zbikowski, A. Tsourdos, and B. A. White, "Differential geometric guidance based on the involute of the target's trajectory," *Journal of Guidance, Control, and Dynamics*, vol. 28, no. 5, pp. 990-996, 2005.

[13] S. Bezick, I. Rusnak, and W. S. Gray, "Guidance of a homing missile via nonlinear geometric control methods," *Journal of Guidance, Control, and Dynamics*, vol. 18, no. 3, pp. 441-448, 1995.

[14] T. Yamasaki and S. N. Balakrishnan, "Triangle intercept guidance for aerial defense," *AIAA Guidance, Navigation, and Control Conference*, Toronto, Ontario, Canada, Aug, 2010.

[15] V. Shalumov and T. Shima, "Weapon-target-allocation strategies in multiagent target-missile-defender engagement," *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 10, pp. 2452-2464, 2017.

[16] S. R. Kumar and T. Shima, "Cooperative nonlinear guidance strategies for aircraft defense," *Journal of Guidance, Control, and Dynamics*, vol. 40, no. 1, pp. 124-138, 2017.

[17] R. Fonod and T. Shima, "Target evasion strategy against a finite set of missile guidance laws," *European Control Conference (ECC)*, Aalborg, Denmark, June, 2016.