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PID^m_n Control for IPDT Plants. Part 2: Setpoint Response

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***Abstract*—The paper continues in developing generalized two-degree-of-freedom (2DOF) proportional-integral-derivative (PID) control with the mth order derivative action, $m \in [0, 4]$ and with $n \geq m$th order series binomial filters. By focussing on the setpoint responses, the explicit tuning formulas of a 1DOF PID^m_n control for the integral-plus-dead-time (IPDT) plant models derived by the multiple real dominant pole method are augmented by the optimal prefilter enabling to cancel up to $m+1$ of the $m+2$ tuple dominant closed loop poles and thus to accelerate the setpoint step responses. Since in Matlab/Simulink, the simulative setpoint response evaluation of the PID^m_n with $m > 0$ and IPDT plant is strongly limited by the numerical solver imperfections, the second part of the contribution brings also an experimental evaluation by real time control of a thermal plant. This fully confirms excellent properties of the novel type of control which, due to its high robustness, enables to simplify the plant identification and to work with the simple IPDTs model also in the case of selfregulating processes with significantly more complex dynamics.**

***Index Terms*—Filtration, multiple real dominant pole method, derivative action.**

I. Introduction

Despite the long history of the proportional-integral-derivative (PID) control, it is still possible to meet notes that the derivative part is the most difficult to tune [1], or that the derivative action is not appropriate for noisy, or time delayed processes [2]. Numerous papers deal with the filtration problems in PID control [3]–[8], which represents a central issue especially in controllers with the derivative action. Recently, it has been shown that a new integrated filter+controller tuning [9] allows to deal with filters with an arbitrarily chosen order n and yields much better results than the traditional derivative filter design. It enabled to introduce a PID$^m_n, m = 0, 1, 2, ..., n \geq m$ control including not only the filtered PI and PID controllers [10], but also the less frequent PIDD^2, or PIDD^{2}D^3 controllers as its special cases. Use of higher order derivative actions showed to be interesting both from the decreased excessive control effort, increased transients speed and increased loop robustness points of view [11], [12].

Basic 1DOF PID^m_n derivation for the integral plus time delay (IPDT) plant and its evaluation in [13] extended the tested derivative order up to $m = 4$, whereby it focused on the closed loop simulation of the input disturbance step responses. Simulation of the controller and plant dynamics combined with a dead time showed to be strongly limited by numerical properties of the solvers available in Matlab/Simulink. This 2nd part of the contribution continues with derivation of input prefilters allowing to achieve monotonic setpoint step responses with different degrees of their acceleration and with focusing on evaluation of the developed controllers by real time control. Since in such a task the simulation is limited to emulation of the much simpler controller dynamics, it shows to be much more reliable and (together with the discrete-time controllers derived in [12]) it confirms the assumption that a proper use of appropriately filtered higher order derivatives fulfills both the above formulated expectations.

Thereby, this paper is structured as follows. In Section II, the main results of the first part regarding the 1DOF PID^m_n design are briefly summarized. In Section III they are augmented by appropriate prefilter design enabling to eliminate output overshooting and to speed up the setpoint step responses. Since the loop evaluation by simulation in Section IV is yet more limited by the numerical imperfections, an experimental evaluation by real time control of a thermal process has been included in Section V. The main paper results and future development are then summarized in Conclusions. and it uses the performance measures introduced in [13].

II. PID^m and PID^m_n Controllers for the IPDT Plant Tuned by the MRDP Method

The plant let be approximated by an IPDT model

$$S(s) = \frac{Y(s)}{U(s)} = S_0(s)e^{-T_{dm}s};\ S_0(s) = \frac{K_{sm}}{s} \tag{1}$$

with a gain K_{sm} and a dead time T_{dm}. In the controller derivation, the model index "m" will be firstly omitted, which corresponds to the model parameters equal to the plant parameters K_s and T_d.

A. *PID^m control*

A generalization of the PID control with possible derivative terms up to an integer degree "m" may be proposed

TABLE I
OPTIMAL PID^m PARAMETERS, $m \in [0,4]$

$Parameter$	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
K_o	0.4612	0.78361	1.08268	1.37114	1.65330
τ_{io}	5.8284	3.73205	3.00000	2.61803	2.37980
τ_{1o}	0	0.26289	0.37500	0.43673	0.47525
τ_{2o}	0	0	0.04167	0.07492	0.09972
τ_{3o}	0	0	0	0.00474	0.01020
τ_{4o}	0	0	0	0	0.00042

TABLE II
EQUIVALENT TIME DELAYS RATIOS T_f/T_e, $m \in [0,4], n \in [m,6]$

m	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
0	0.5690	0.3608	0.2647	0.2092	0.1729	0.1474
1	0.7887	0.3943	0.2800	0.2180	0.1787	0.1514
2	-	0.5000	0.3000	0.2279	0.1847	0.1555
3	-	-	0.3618	0.2412	0.1917	0.1599
4	-	-	-	0.2279	0.2012	0.1651

as

$$C_m(s) = K_c\left[\frac{1+T_is}{T_is} + T_{D1}s + ... + T_{Dm}s^m\right] = \\ = K_c + \frac{K_i}{s} + K_{D1}s + ... + K_{Dm}s^m;\ m = 0,1,2,... \tag{2}$$

For $m > 0$ such a controller may not be implemented without a filter, which will be treated later. For the nominal plant parameters, the main closed loop transfer functions are

$$F_{wy}(s) = \frac{Y(s)}{W(s)} = \\ = \frac{K_cK_s(1+T_is+T_iT_{D1}s^2+...+T_iT_{Dm}s^{m+1})}{T_is^2e^{T_ds}+K_cK_s[1+sT_i(1+T_{D1}s+...+T_{Dm}s^m)]} \\ F_{dy}(s) = \frac{Y(s)}{D_i(s)} = \\ = \frac{K_sT_is}{T_is^2e^{T_ds}+K_cK_s[1+sT_i(1+T_{D1}s+...+T_{Dm}s^m)]} \tag{3}$$

The $m+2$ controller parameters are determined to get an $m+2$-tuple real dominant pole s_o of the quasi-polynomial

$$P(s) = T_is^2e^{T_ds}+K_cK_s[1+sT_i(1+T_{D1}s+...+T_{Dm}s^m)] \tag{4}$$

It is, s_o and the parameters have to fulfill the equation system

$$\left[P(s)=0;\ \frac{dP(s)}{ds}=0;\dots;\ \frac{d^{m+2}P(s)}{ds^{m+2}}=0\right]_{s=s_o} \tag{5}$$

Its solution written in an dimensionless form [14] is

$$\begin{array}{l} p_o = s_oT_d = \sqrt{m+2}-(m+2) \\ K_o = K_{co}K_sT_d \\ \tau_{io} = \dfrac{T_{io}}{T_d};\ \tau_{jo} = \dfrac{T_{Dj}}{T_d^j}\ ;\ j=1,2,...,m \end{array} \tag{6}$$

For $m \in [0,4]$ it is summarized in Tab. I.

B. PID$_n^m$ controller and its tuning

For a PID^m controller, nominal plant and an nth order filter

$$Q_n(s) = 1/(T_fs+1)^n;\ n=1,\ 2,\ ..;\ n \geq m. \tag{7}$$

the resulting transfer function $R_n^m(s) = C_m(s)Q_n(s)$ becomes proper. However, the measurement (or quantization) noise impact is significantly lower for strictly proper $R_n^m(s)$.

One possible solution to get analytical filter tuning is to approximate effect of its n time constants T_f by an equivalent dead time T_e. Its value has been derived [13] by a delay equivalence based on keeping an equal position of the dominant closed loop poles corresponding to PID$_n^m$ control with the delay free integral plant $S_0(s)$ (i.e. $T_{dm}=0$) yielding the characteristic polynomial

$$\begin{array}{l} P(s) = T_is^2(1+T_fs)^n+... \\ +K_cK_s[1+T_is(1+T_{D1}s+...+T_{Dm}s^m)] \end{array} \tag{8}$$

and to PID^m control with an IPDT plant and a dead time $T_d = T_e$. For (8) the MRDP method yields the dominant pole

$$s_n = \frac{\sqrt{(m+2)(n+1)(n-m)}-(m+2)(n+1)}{(n+1)(n+2)T_f} \tag{9}$$

which is required to fulfill equation

$$s_o = s_n \tag{10}$$

formulated for $T_d = T_e$. Solving (10) for T_f yields the delay equivalence

$$T_f = \frac{\sqrt{(m+2)(n+1)(n-m)}-(m+2)(n+1)}{(n+1)(n+2)[\sqrt{m+2}-(m+2)]}T_e \tag{11}$$

Derivation of this equivalence offers following integrated tuning procedure [13]:

1) After identifying the plant model parameters K_{sm} and T_{dm} and taking into account the level of required noise filtration, choose an appropriate value of the tuning parameter $T_e > 0$ corresponding to a required degree of filtration;
2) Specify the derivative degree m and the controller parameters (6) corresponding to the total loop delay
$$T_d = T_{dm} + T_e \tag{12}$$
3) Choose a filter order n and by a delay equivalence $T_f = f(m,n,T_e)$ (11) specify the filter time constants T_f;
4) Check, it the calculated value T_f fulfills the requirement $T_f >> T_s$, with T_s representing the sampling period used for the quasi-continuous control implementation. If not, you should either decrease T_s, or n, which should still fulfill the condition $n \geq m$. In the worst case you may still use the non filtered PI control.
5) By experimentally evaluating the loop properties for different m and n, choose an optimal controller guaranteeing the optimal loop performance.

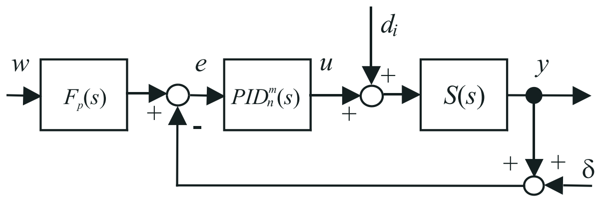

Fig. 1. Considered loop with the PID$_n^m$ controller, a prefilter $F_p(s)$, d_i - input disturbance and δ - measurement noise

TABLE III
OPTIMAL PREFILTER WEIGHTS OF PID^m CONTROL, $m \in [0, 4]$

m	p	b_0	b_1	b_2	b_3	b_4
0	1	0.29289				
1	1	0.21133				
	2	0.42265	0.63398			
2	1	0.16667				
	2	0.33333	0.22222			
	3	0.50000	0.66667	1		
3	1	0.13820				
	2	0.11449	0.27640			
	3	0.41459	0.34346	0.24145		
	4	0.55279	0.68693	0.96580	1.38197	
4	1	0.11835				
	2	0.23691	0.07014			
	3	0.35505	0.21041	0.09415		
	4	0.47340	0.42083	0.37660	0.25935	
	5	0.59175	0.70138	0.94149	1.29676	1.77525

III. OPTIMAL PREFILTER PARAMETERS

The setpoint step response overshooting appearing under 1DOF PID$_n^m$ control may be removed by a prefilter (Fig. 1)

$$F_p(s) = \frac{1 + b_0 T_i s + b_1 T_i T_{D1} s^2 + ... + b_m T_i T_{Dm} s^{m+1}}{1 + T_i s + T_i T_{D1} s^2 + ... + T_i T_{Dm} s^{m+1}} \quad (13)$$

Its optimal numerator parameters may be analytically determined to increase the speed of the setpoint responses by canceling $p \in [1, m+1]$ dominant closed loop poles s_o. They are determined by solving the polynomial equation

$$N_p(s) = \frac{(s - s_o)^p}{(-s_o)^p} = \\ = 1 + b_0 T_i s + b_1 T_i T_{D1} s^2 + ... + b_{m-1} T_i T_{Dm-1} s^p \quad (14)$$

Figures derived for $m \in [0, 4]$ are in Tab.III.

Reasons for introducing higher order derivative actions may be demonstrated by the optimal IAE_s values

$$IAE_s = (1 - b_0) T_i \quad (15)$$

corresponding to an unit setpoint step response (which may be derived under assumption of a not changing sign of the control error as $IAE_s = IE_s$). For the maximum number of the cancelled closed loop poles one gets the figures in Tab. IV). Obviously, with increasing parameter m the IAE_s values decrease.

When wishing to examine the most important question, how far such improvements may be achieved in real situations, we have to replace ideal PID^m controllers by the filtered PID$_n^m$, when a part of the total T_d has to be spent on filtration. It is also to note that if the controller tuning guarantees a monotonic error decay, the resulting IAE_s (15) does not change with n, just with T_d (12) determining T_i.

TABLE IV
OPTIMAL IAE_s VALUES CORRESPONDING TO UNIT STEP RESPONSES FOR PID^m PARAMETERS FROM TAB. I AND MAXIMAL PREFILTER DEGREES IN TAB. III

-	$m=0$	$m=1$	$m=2$	$m=3$	$m=4$
IAE_s/T_d	4.12132	2.15470	2.00000	1.17082	0.97155

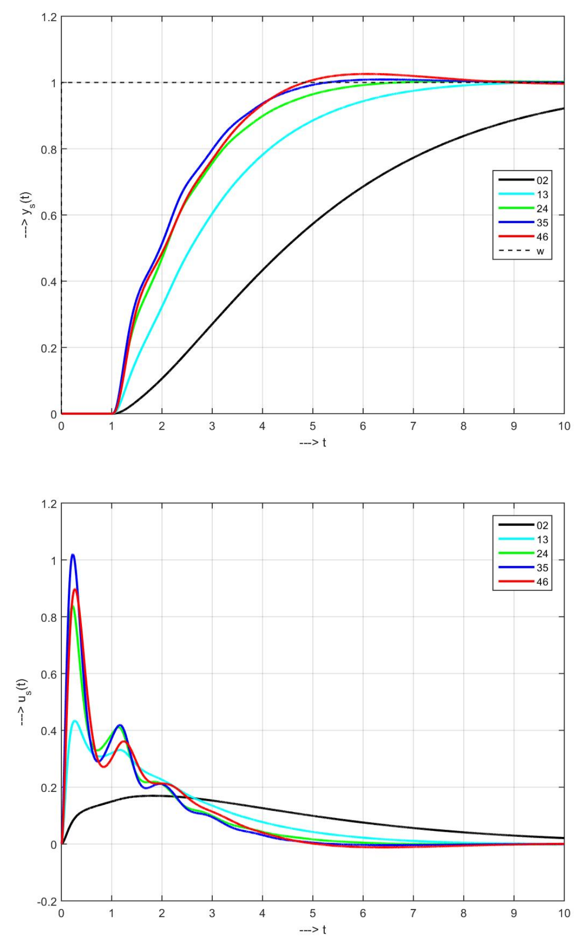

Fig. 2. Setpoint step responses with PID$_n^m$ control tuned by the MRDP method according to (16) for $T_{e0} = 0.25$, $q = 1.5$

IV. SIMULATION

As already mentioned in [9], [13], for higher $m > 0$ and short T_e the transients simulated in Matlab/Simulink may show numerical imperfections, which are more dominant in the setpoint step responses. One way to avoid such transients is to increase minimal values of T_e with increasing m, for example according to (Fig. 2 and 3)

$$T_e = T_{e0} . q^m \quad (16)$$

However, the main area of interest - the transients with higher order m and relatively short T_e - may not be reliably treated. In this point we are planning to contact Matlab services and

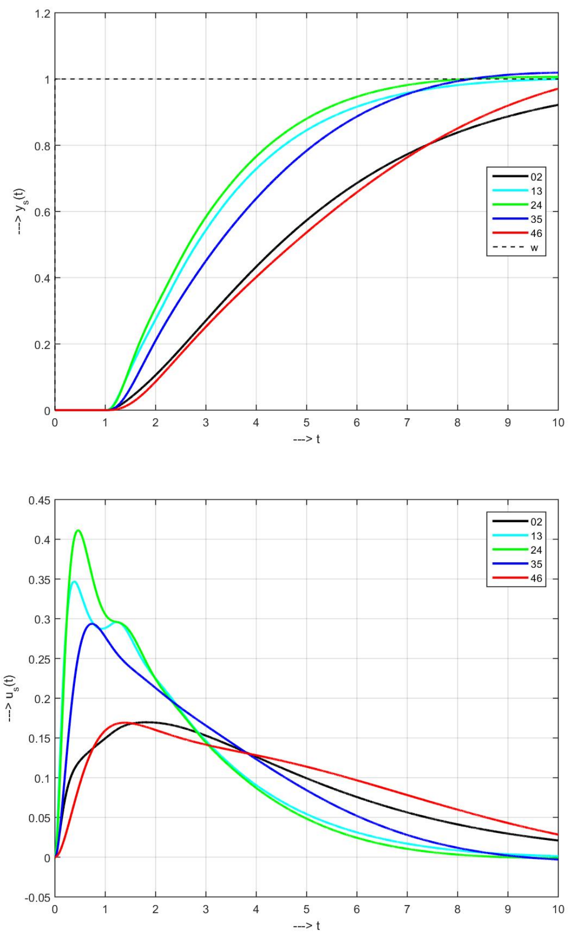

Fig. 3. Setpoint step responses with PID$_n^m$ control tuned by the MRDP method according to (16) for $T_{e0} = 0.25$, $q = 2$

developers, or to check some alternative simulation tools. Another possibility is to evaluate the derived controllers by real time experiments, where the simulation tools are used in a much simpler context - just for simulation of the controller.

V. Experimental Evaluation

The PID$_n^m$ approach will be applied to the thermal channel of the thermo-opto-mechanical TOM1A system described in [15]. Thermal plants are typical with several modes of the heat transfer [16] and monotonic step responses. Despite to their self-regulatory character they may be successfully approximated by the IPDT model. However, the controller design should be sufficiently robust, as, for example, in the model free control [17], or in ADRC [18]. Whereas a temperature increase is supported both by the fast and slow modes (radiation, conduction), in the cooling phase the system dynamics significantly changes. The main question is, if the increased closed loop robustness reported in [11], [12] may also be experimentally confirmed.

A. Plant approximation by the IPDT model

The newest plant model yields different parameters than reported in [15] and requires a new identification. After

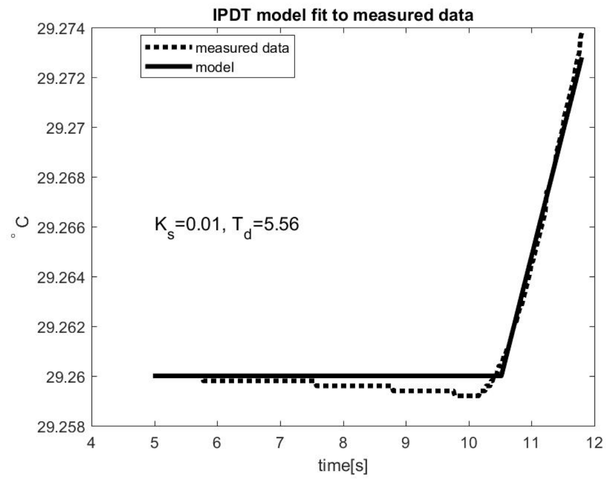

Fig. 4. Approximation of a thermal plant step response by the IPDT model yielding $K_{sm} = 0.01, T_{dm} = 5.56$s

bringing the system to a steady state, an input step has been applied. Parameters of the model (1)

$$K_{sm} = 0.01;\ T_{dm} = 5.56\text{s} \tag{17}$$

have been determined to approximate the steepest segment of the step response curve. It is to note that the initial part of the step response in Fig. 4 shows a slight undershooting, which corresponds to a non-minimum phase behavior reported already in [19]. As it will be shown by experiments, this represents one of the limit factors in accelerating the transients.

B. Dominant issues in a real time control - windup, bumpless transfer and non-minimal phase behavior

As reported in the Simulation section, the numerical imperfections of the Matlab/Simulink solvers dominate for short T_e. Therefore, it may be helpful to find firstly the application limits due to the special loop properties also in the real time control. As, for example, apparent from the setpoint steps for chosen controllers tuned with a relatively short $T_e = 0.2$s (Fig. 5), due to the non-minimal phase behavior system the transients may be accelerated by increased m just to some degree. For PID$_4^2$ the non-minimal properties start to dominate during an upwards temperature step. But, they do not occur for slower downward temperature step. In the initial phase of transients starting with zero initial conditions (up to $t = 200$), all transients are influence by the not yet solved bump-less-transfer problem. Transients in Fig. 6 achieved for $T_{dm} \in [0.5, 8]$s document that for the relatively short value of $T_e = 0.2$s for the upward steps the PID$_4^2$ controller magnifies the non-minimum phase behavior up to $T_{dm} = 4$s and it disappears just for transients slowed down by choosing significantly longer $T_{dm} = 8$s than the identified value (17). However, with exception of $T_{dm} = 0.5$s, all the downward steps show nearly monotonic transients and confirm a relatively high closed loop robustness regarding the dead time estimation. Fig. 7 may finally document that even for the relatively short value $T_e = 0.8$s in PID$_6^4$ control no

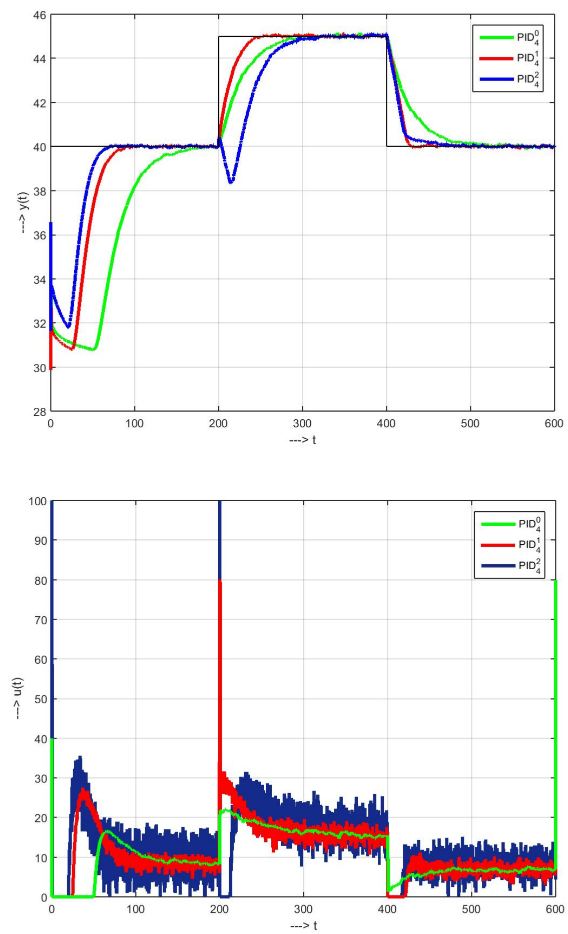

Fig. 5. Setpoint step responses of the PID$_n^m$ controllers for $T_e = 0.2$s, $T_s = 0.02$s,$K_{sm} = 0.01, T_{dm} = 5$s

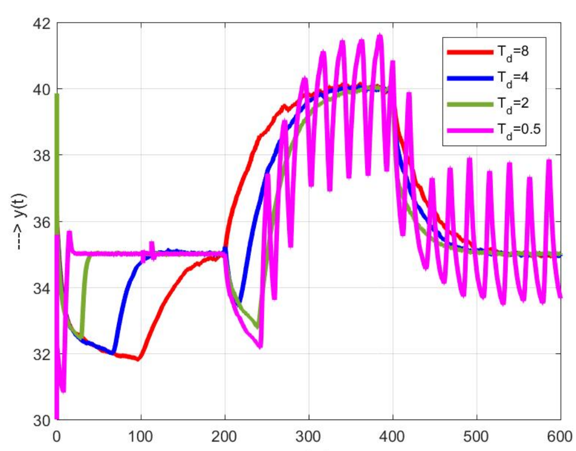

Fig. 6. Setpoint step responses of the PID$_4^2$ controllers for $T_e = 0.2$s, $T_s = 0.02$s, $K_{sm} = 0.01, T_{dm} \in [0.5, 8]$s

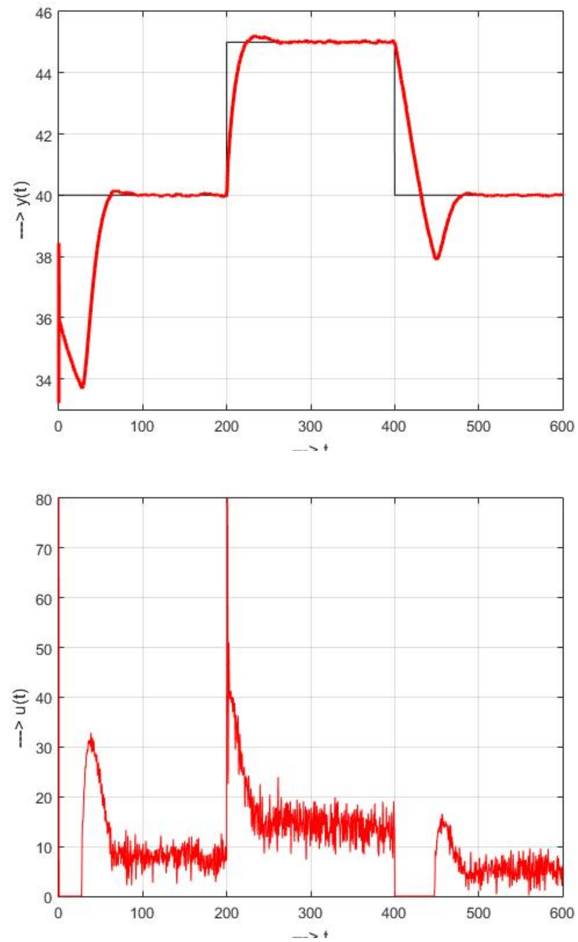

Fig. 7. Setpoint step responses of the PID$_6^4$ control for $T_e = 0.8$s, $T_s = 0.02$s, $K_{sm} = 0.01, T_{dm} = 5$s

numerical imperfections occur, just the windup effect caused by the control constrained to $u \in [0, 80]$. The asymmetry of dynamical properties is reflected by much higher windup appearing for the downwards steps.

C. Robustness tests with increased dead time values

In order to work with $T_e << T_d$ without the non-minimum phase behavior, or attacking the control constraints and facing the windup problem, the inherent plant dead time has been increased by an additional Simulink delay $T_a = 7$s added also to (12). Step responses corresponding to three different controllers are in Fig. 8. Since each of this controllers depends on T_e in a different way, the question arises, how to organize the comparative framework in order to guarantee a maximal correctness. Although it would be possible to discuss whether the solution applied complied fully with this requirement, from the obtained results it is clear that (with the exception of the initial phase of the particular step responses) by increasing the derivative action degree m, the oscillations of the transient responses (indicating a lower loop robustnes) decrease. The numerical problems preventing the loop analysis by simulation disappeared and there seems to

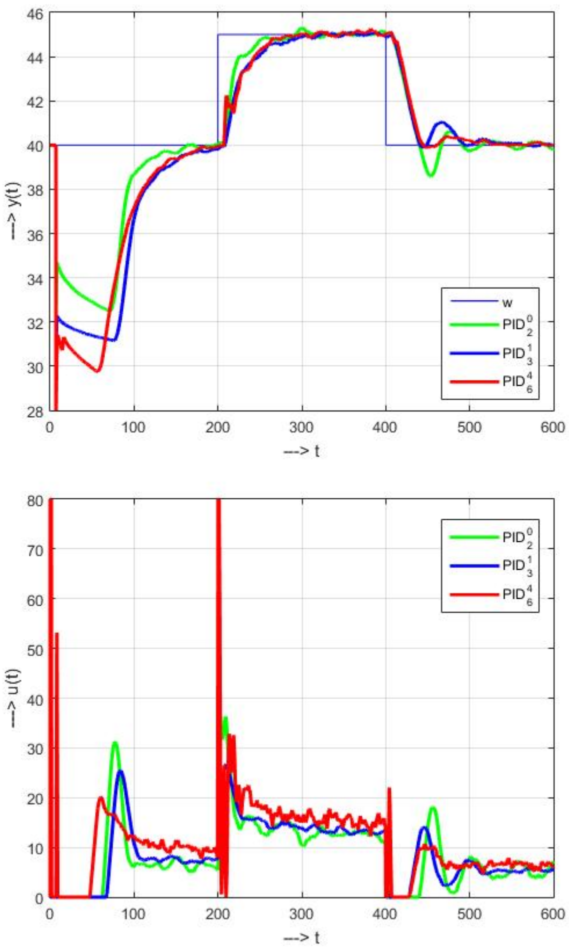

Fig. 8. Setpoint step responses of different controllers with a loop dead-time increased significantly by a Simulink delay block $T_a = 7$s, $T_s = 0.02$s,$K_{sm} = 0.01, T_{dm} = 5$s

be no difference between simple PI (PID$^0_0$) control and the highest tested PID$^4_6$ controllers.

VI. Conclusions

The article showed that, despite the problems with the simulation of PID^m_n control, its use increases loop robustness and improves the transient shapes of systems with long delays and uncertainties. The numerical problems dominating in simulation of such loops do not appear in real time control, where the solvers restrict to the (much simpler) controller simulation. Thus, up to the solution of the simulation problems, the research of the PID^m_n controllers will preferably be based on real time experiments, or use alternative simulation tools.

In control of the thermal channel of the TOM1A system the performance limits are represented by the non-minimum phase system behavior and windup which dominate for relatively short T_e. They show also on the necessity to get a bumpless transfer from an initial state to the automatic regime.

Such tests of the newly proposed controllers in the real time situations may not only be interesting from the control point of view, but also for understanding the issues of numerical simulation. As shown, for example, in [20], the numerical imperfections of the continuous-time solvers applied to dead-time compensators may be avoided by developing the discrete-time controllers. Of course, such controllers are necessary also with respect to their implementation by different types of (embedded) computer control. However, as shown in [12], due to the discrete-time controller complexity, which is rapidly increasing with increasing m, also the quasi-continuous-time controllers deserves the attention of researchers.

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