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Aircraft Motion Decoupling of roll and yaw dynamics using Generalized Dynamic Inversion Control

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Abstract— **In this paper, Generalized Dynamic Inversion (GDI) control methodology for linear state equality constraints is proposed for independent motion control of an aircraft along the roll and the yaw axes. In GDI control, dynamic constraints are defined based on the roll and yaw attitude deviation functions and are inverted using Moore Penrose Generalized Inverse (MPGI) to realize the control law. In the particular part of GDI control, constraint differential equation is formulated for the yaw channel, where as the constraint dynamics for the roll axis is constructed by using the involved null control in the auxiliary control part of GDI. The two noninterference control actions is a key advantage of GDI control, and it provides extra degree of design freedom. Numerical simulations are conducted on the lateral dynamics of a transport aircraft to demonstrate the motion decoupled tracking performance of the proposed control strategy.**

I. INTRODUCTION

The performance and strength of the modern aircraft system depends on aircraft maneuverability and accurate tracking. The inherent dynamics of an aircraft is highly coupled which causes the motion of aircraft along or about three axes by providing control input along or about one axis. In the actual flight of aircraft, the lateral and directional modes are closely link to each other and are strongly coupled, which generates a sideslip (yaw rate) due to roll rate and a roll rate due to sideslip. The coupling between lateral and directional modes are inappropriate for an effective maneuvering of the aircraft and must be addressed accordingly.

Several method are proposed in literature for an attempt to produce decoupled flight control system. In [1], [2], [3], the eigen structure assignment method was proposed to achieve the decoupled aircraft motion. The decoupled motion of aircraft through the use of inverse dynamics was proposed in [4], [5]. However control through the dynamic inversion has some limitations such as cancellation of useful nonlinearities, large control efforts, singular configurations of square inversion, large control efforts and simplifying approximations that are required for obtaining the inverse model of the plant [6].

A constrained control design approach based on the non-square inversion principle is Generalized Dynamic Inversion (GDI). Rather to invert the whole system dynamics, it is based on inverting the prescribed set of constraint dynamics that encapsulates the control objectives. Hence this methodology alleviates the complexities required in classical dynamic inversion, which includes inverting the whole system dynamics, singular configurations of square inversion, and avoid cancellation of useful nonlinearities.

In GDI control, the inversion of the dynamical constraints is performed using Moore-Penrose Generalized Inverse (MPGI), see [7], [8] and the associated nullspace parametrization of non square matrices, see [9], in a single control law. The infinite number of solutions of an underdetermined system of equations is parameterized by using the Greville method. The GDI control comprised of two cooperating controllers, i.e., the particular part, that enforces the constraint dynamics and the involved null control vector which provides extra degree of design freedom. The GDI control has been applied to several aerospace engineering and robotic applications, see [10], [11], [12], [13], [14], [15], [16], [17], [18], [19].

In this paper, the two cooperating and non interfering control actions of the GDI control are designed for the roll and yaw axes decoupled motion tracking of an aircraft, in order to avoid the excitation of lateral directional modes. Two linear time varying constraint differential equations are formulated for the yaw and the roll channels respectively, based on the deviation functions of their angular position and body rate. The particular part of GDI is responsible to enforce the asymptotic convergence of the yaw channel constraint dynamics, where as the involved null control vector guarantees the asymptotic stability of the constraint dynamics for the roll channel. Numerical simulations are conducted on the linear model of lateral aircraft dynamics, to demonstrate the effectiveness of the GDI control for achieving the decoupled motion tracking performance.

The remaining parts of the paper are organized as follows. The state space model of the lateral dynamics of an aircraft is presented in section II. The formulation of linear time varying constraint differential equations for the yaw and roll channel dynamics using the particular part and the null control vector respectively is explained in section III. Numerical simulations and the discussions are presented in section IV, where as conclusion is given in section V.

II. AIRCRAFT LATERAL DYNAMICS

The equations of motion of the lateral dynamics of a transport aircraft is approximated by the following Linear Time Invariant (LTI) state space model, see [20]

$$\dot{\mathbf{x}} = \mathcal{A}\mathbf{x} + \mathcal{B}\mathbf{u}, \qquad \mathbf{x}(0) = \mathbf{0}_{5\times 1} \tag{1}$$

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or it is described as

$$\begin{bmatrix}\dot{\beta}\\ \dot{r}\\ \dot{\phi}\\ \dot{p}\\ \dot{\psi}\end{bmatrix} = 10^{-2}\begin{bmatrix}-10.0 & -100.0 & 11.5 & 0 & 0\\ 40.9 & -24.5 & 0 & -4.0 & 0\\ 0 & 0 & 0 & 100.0 & 0\\ -160.4 & 28.5 & 0 & -109.3 & 0\\ 0 & 100 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\beta\\ r\\ \phi\\ p\\ \psi\end{bmatrix} + 10^{-2}\begin{bmatrix}0 & 1.8\\ -0.2 & -24.4\\ 0 & 0\\ 32.2 & 8.7\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix} \quad (2)$$

where β is the sideslip angle, ψ is yaw angle, r is the yaw rate, ϕ is the roll angle, p is the roll rate, δ_a and δ_r are respectively the aileron and the rudder deflections. The motion in yaw and roll axis are always coupled with each other, where the motion is one axis causes a drift in the other axis. There are three types of lateral directional modes, i.e., roll subsidence mode, spiral mode, and Dutch roll mode.

A. Roll subsidence mode

The roll damping mode is inherently a well damped mode. When the aircraft rolls, there is a difference of the lift force, generated by both wings, typically more on the side going down, which creates a moment that tends to restore the equilibrium. After a disturbance, a roll rate grows exponentially until the disturbing moment is balanced by the restoring moment and the steady roll is achieved.

B. Spiral mode

The spiral mode is slow but it creates instability. When a roll angle is generated due to some external disturbance, the tail fin hits on the incoming air and creates the yawing moment. The positive yaw rate generates a positive roll rate, which causes the further rise in the roll angle which produces more yaw rate. If this phenomenon is left uncontrolled, then the aircraft will slowly deviates from its nominal path and tends to spiral into the ground.

C. Dutch roll mode

The Dutch roll mode is the oscillatory combination of the roll and the yaw motion, whose time period varies from few seconds to a minute, depending on the aircraft. This mode is stable is normal aircraft but damped out so slowly that it will create unpleasant effect during the flight.

In the following sections, GDI control is implemented to decoupled the motion of an aircraft along the roll and yaw channels, in order to prevent the excitation of the lateral directional modes, when the aircraft is subject to some external disturbances.

III. GDI CONTROL

To construct the GDI control law, a prescribed set of constraint dynamics are formulated that encapsulates the control objectives, and are inverted using the MPGI based Greville method to realize the control law. In the particular part of GDI control, a linear time varying constraint differential equation is defined for stabilizing the yaw axis dynamics, whose differential order is equal to the relative degree of the state function, which yields

$$\ddot{\psi} + c_1\dot{\psi} + c_2\psi = 0 \quad (3)$$

or (3) is written as

$$\dot{r} + c_1 r + c_2\psi = 0 \quad (4)$$

where c_1 and c_2 are chosen such that it will guarantee the asymptotic stability of the constraint dynamics [19]. By placing the expression of $\dot{r}$ from (2) in (4), its differential form is transformed into an algebraic expression, resulting in

$$\mathcal{A}_1\mathbf{u} = B_1\mathcal{X} \quad (5)$$

where the controls coefficient row vector function $\mathcal{A}_1 : \mathbb{R}^n \times [t_0, \infty) \rightarrow \mathbb{R}^{1\times 2}$ is given as

$$\mathcal{A}_1 = [B_{(2,1)} \;\; B_{(2,2)}]$$

and the controls load scalar function $B_1 \in \mathbb{R}$ is given as

$$B_1 = -A_{(2,1)} - (A_{(2,2)} + c_1) - A_{(2,3)} - A_{(2,4)} - (A_{(2,5)} + c_2)$$

Equation (5) is an under-determined system which has infinite number of solutions. This would be parameterized by using the Greville method, which yields

$$\mathbf{u} = \mathcal{A}_1^{+}B_1\mathbf{x} + \mathcal{P}_1\mathbf{u}_a \quad (6)$$

where $\mathbf{u}_a$ is the null control vector, which is designed later to enforce the asymptotic stability of the roll channel constraint dynamics, $\mathcal{A}_1^{+}$ represents the pseudo inverse of $\mathcal{A}_1$, given as

$$\mathcal{A}_1^{+} = \frac{\mathcal{A}_1^T}{\mathcal{A}_1\mathcal{A}_1^T} \quad (7)$$

and $\mathcal{P}_1$ is the null projection vector, defined as

$$\mathcal{P}_1 = \mathbf{I}_{2\times 2} - \mathcal{A}_1^{+}\mathcal{A}_1 \quad (8)$$

By placing the expression of $\mathbf{u}$ given by (6) in (1), the modified closed loop system is given as

$$\dot{\mathbf{x}} = \mathcal{A}\mathbf{x} + \mathcal{B}\big[\mathcal{A}_1^{+}B_1\mathbf{x} + \mathcal{P}_1\mathbf{u}_a\big] \quad (9)$$

or it is also expressed as

$$\dot{\mathbf{x}} = \mathcal{A}_{cl_1}\mathbf{x} + \mathcal{B}_{cl_1}\mathbf{u}_a \quad (10)$$

where the state transition matrix $\mathcal{A}_{cl_1}$ is given by

$$\mathcal{A}_{cl_1} = \mathcal{A} + \mathcal{B}\mathcal{A}_1^{+}B_1$$

and the control input matrix $\mathcal{B}_{cl_1}$ implies,

$$\mathcal{B}_{cl_1} = \mathcal{B}\mathcal{P}_1$$

The null control vector $\mathbf{u}_a$ in the auxiliary part of GDI, acts on the nullspace of the constraint dynamics and provides extra degree of freedom because of the orthogonality. By virtue of this, a second time varying constraint dynamics is formulated for stabilizing the roll dynamics, resulting in

$$\dot{p} + c_3 p + c_4\phi = 0 \quad (11)$$

By placing the expression of $\dot{p}$ from (2) in (11), the differential equation is converted into the algebraic form, given as

$$\mathcal{A}_2 \mathbf{u}_a = B_2 \mathbf{x} \tag{12}$$

where the controls coefficient row vector function $\mathcal{A}_2 : \mathbb{R}^n \times [t_0, \infty) \rightarrow \mathbb{R}^{1\times 2}$ is given as

$$\mathcal{A}_2 = [B_{cl_1(4,1)} \quad B_{cl_1(4,2)}]$$

and the controls load scalar function $B_2 \in \mathbb{R}$ is defined as

$$B_2 = -A_{cl_1(4,1)} - A_{cl_1(4,2)} - (A_{cl_1(4,3)} + c_4) \\ - (A_{cl_1(4,4)} + c_3) - A_{cl_1(4,5)}$$

The GDI control law that enforces the asymptotic convergence of the error dynamics for the roll channel is written as

$$\mathbf{u}_a = \mathcal{A}_2^+ B_2 \mathbf{x} + \mathcal{P}_2 \mathbf{u}_b \tag{13}$$

where $\mathbf{u}_b$ denotes the null control vector, $\mathcal{A}_2^+$ is the pseudo inverse of $\mathcal{A}_2$ and $\mathcal{P}_2$ is the null projection vector. By placing the value of null control $\mathbf{u}_a$ given by (13) in (10), it implies

$$\dot{\mathbf{x}} = A_{cl_1}\mathbf{x} + B_{cl_1}\left[\mathcal{A}_2^+ B_2 \mathbf{x} + \mathcal{P}_2 \mathbf{u}_b\right] \tag{14}$$

or it can be written as

$$\dot{\mathbf{x}} = \mathcal{A}_{cl_2}\mathbf{x} + \mathcal{B}_{cl_2}\mathbf{u}_b \tag{15}$$

where the state transition matrix $\mathcal{A}_{cl_2}$ is given as

$$\mathcal{A}_{cl_2} = A_{cl_1} + \mathcal{B}_{cl_1}\mathcal{A}_2^+ B_2$$

and the control input matrix $\mathcal{B}_{cl_1}$ is defined as

$$\mathcal{B}_{cl_2} = \mathcal{B}_{cl_1}\mathcal{P}_2 = 0$$

It is inferred from (15), that the null control vector $\mathbf{u}_b$ is not capable to further force the states to the desired positions because the control load matrix $\mathcal{B}_{cl_2}$ becomes zero, which will be validated numerically later in the simulations section.

Remark 1: To decoupled the motion of roll and yaw channels, there is a possibilities to define the first constraint equation for the roll angle ϕ, in the particular part of GDI, followed by the construction of second constraint differential equation for the sideslip angle β, employing the null control vector in the auxiliary part. However in this case, the roll angle ϕ, roll rate p, sideslip angle β and yaw rate r are all stabilized, however their is a steady state error left is yaw angle ψ, which does not converge to zero. Hence the proposed case, in which the constraint differential equations are formulated for the roll and the yaw channels, is providing an optimal way for the decoupled motion tracking and prevents the excitation of lateral directional modes. It is noteworthy to mentioned here that, formulating the two constraint differential equations for the yaw and the sideslip angles, by utilizing the particular and auxiliary part of GDI is not a feasible solution, as it will destabilized the closed loop system by placing one of the eigen values on the right half plane.

IV. Simulation Results

For numerical simulations, the initial value of the state vector $[\beta \;\; r \;\; \phi \;\; p \;\; \psi]^T$ is taken to be $[1 \;\; 1 \;\; 1 \;\; 1 \;\; 1]^T$. The open loop response of the linear lateral dynamics of the aircraft given by (1) is shown in Fig. 1. The simulation results illustrate that the system's response is stable.

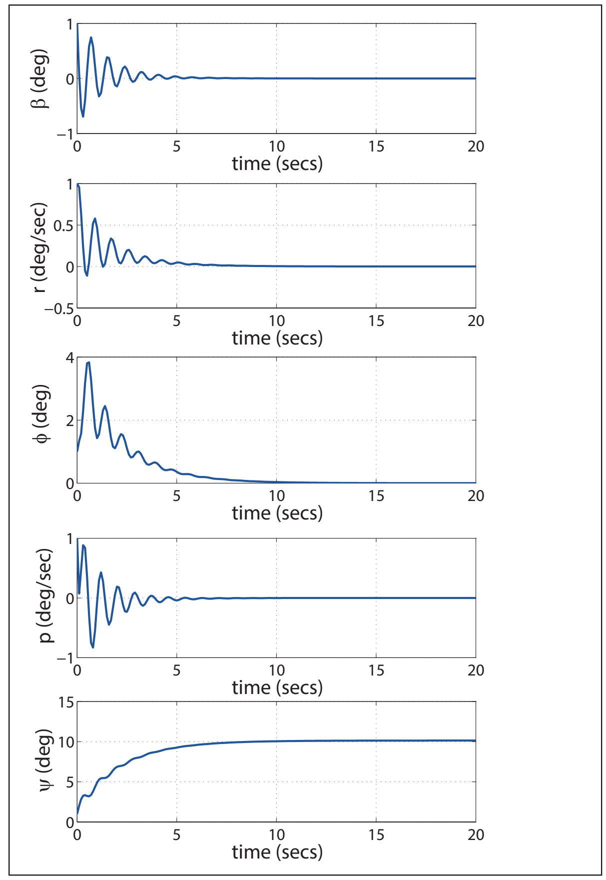

Fig. 1. Open loop response

A. Response of yaw axis constraint dynamics

In this plot, the asymptotic convergence of the constraint dynamics based on the deviation function of yaw angle and its rate, is guaranteed by employing the particular part of GDI control. The initial value of the state vector $[\beta \;\; r \;\; \phi \;\; p \;\; \psi]^T$ is taken to be $[1 \;\; 1 \;\; 1 \;\; 1 \;\; 1]^T$. The closed loop response after implementing the particular given by (6) is shown in Fig. 2. The proposed GDI controller enforces the states ψ and r towards zero, whose convergence trend is shown in Fig. 2, where as the unstable behaviour of the roll angle and its rate is quiet visible. The corresponding control deflections are shown in Fig. 3.

B. Response of roll axis constraint dynamics

In order to stabilize the roll dynamics, a second constraint based on the error dynamics of the roll axis is integrated by using the concept of null control vector. The closed loop response together with the inclusion of null control is shown in Fig. 4. It can be seen that by using the concept

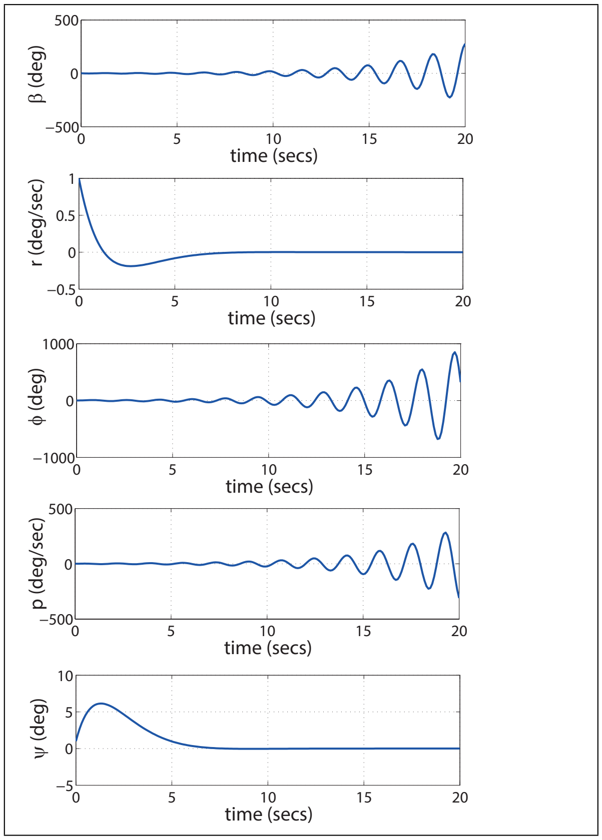

Fig. 2. Yaw constraint dynamics

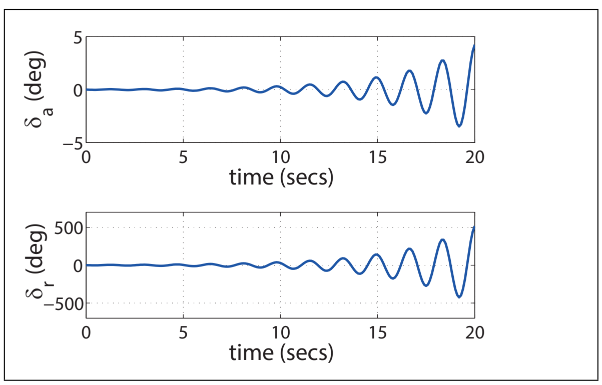

Fig. 3. Control commands

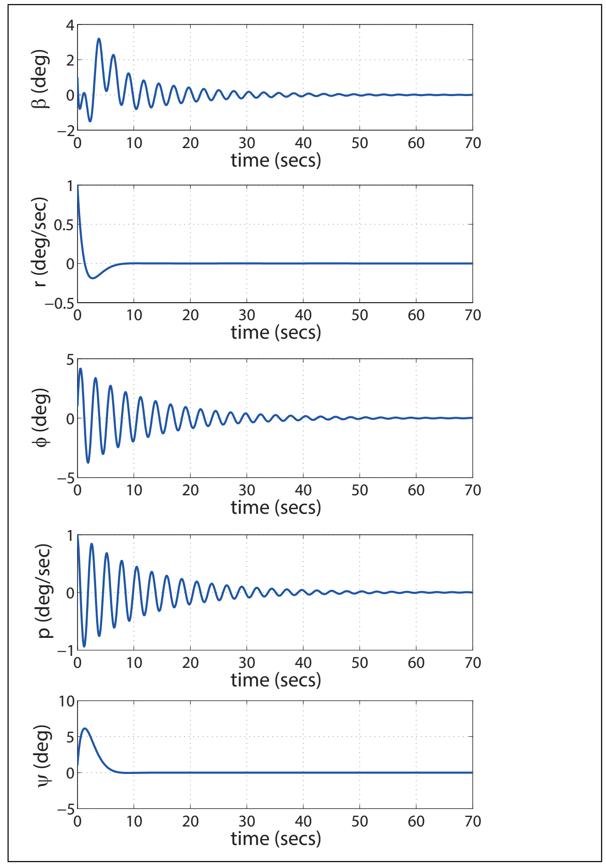

Fig. 4. Yaw and roll constraint dynamics

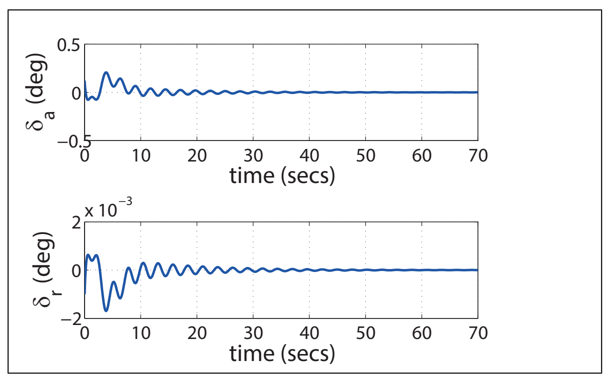

Fig. 5. Final control deflections

of null control vector, all the system state trajectories are asymptotically converging towards zero. The corresponding control deflections are shown in Fig. 5. It is noteworthy to mention here that after employing the null control $\mathbf{u}_a$ for roll dynamics, the input matrix $\mathcal{B}_{cl_2}$ of the closed loop system given by (15) in ineffective, whose magnitude is given as

$$\mathcal{B}_{cl_2} = 1^{-15} \begin{bmatrix} 0.0002 & 0.0025 \\ -0.0003 & -0.0336 \\ 0 & 0 \\ -0.3123 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence, it is inappropriate to design the control input $\mathbf{u}_b$ due to ineffectiveness of the input matrix $\mathcal{B}_{cl_2}$.

V. CONCLUSION

In this paper, GDI control system is designed successfully for the decoupled motion tracking control of the roll and the yaw channel state trajectories. The GDI control is applied to the lateral linear dynamics of an aircraft. The particular part of GDI control is utilized for assuring the asymptotic stability of the yaw axis dynamics, where as the null control vector is employed to guarantee asymptotic converges of the roll dynamics. The effectiveness of proposed control algorithm is verified through numerical simulations. The closed loop results validate the satisfactory performance of the GDI control law, by demonstrating the motion decoupled tracking response between the roll and the yaw channel dynamics.

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