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Modelling and control of a self-balancing electric motorcycle: preliminary results

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***Abstract*—The development of electronic safety systems for two-wheeled vehicles has started with considerable delay with respect to their four-wheeled counterparts because motorcycle dynamics is more complex than four-wheeled vehicles one. In fact, in-plane and out-of-plane dynamics are strongly coupled in bikes. For these reasons the design of such a riderless control system has not been thoroughly investigated at low speed and without the use of steering torque in scientific literature. In this paper a riderless self-balancing two wheel drive electric motorcycle mathematical model - based on Lagrange's equations - with a sliding mode control strategy is put forward to cover this deficiency at low speed. Moreover, the study would find out whether at low speed driving front wheel torque could help vehicle stabilization when steering handlebar can not be actuated. For these reasons, in the proposed model the steering axis is locked over time and both front and rear wheel driving torques could be chosen as control inputs. The paper also presents a model validation with a multibody software.**

I. Introduction

The motorcycle safety system topic has drawn much attention in recent years from motorcycle companies all over the world. In this field the main challenge is vehicle stabilization with electronic control systems - even when vehicle moves slowly or the rider stops, e.g. during a red traffic light. Stabilization of bicycle at zero forward velocity has been studied by some researchers utilizing different external apparatuses to counteract the force of gravity to keep the bicycle balanced at upright position. Yavin [?] simulated the results for an autonomous bicycle by assuming that a rotor was mounted on the crossbar that generated the tilting torque, which was the inverse of the gravitational torque on the bicycle. Lee and Ham [?] proposed a control law to control a load mass balance system mounted on the middle of the bicycle to achieve stabilization. A different, and relatively more applicable method to stabilize the bicycle at its upright position is using a gyroscope. Beznos *et al.* [?] described a bicycle with a gyroscopic stabilization capable of autonomous motion along a straight line as well as along a curve. A sliding mode controller to control the gyroscopic moment and stabilize a bicycle at zero-forward velocity is presented in [?]. Beyond the gyroscope, Niki and Murakami [?] and Tanaka and Murakami [?] proposed a self-balancing bicycle robot by steering control. To improve the stability performance, Keo and Yamakita [?] proposed both balancing and steering control for the bicycle stabilization. In [?], a rear wheel torque controller got a stable speed and a steering angle balance control method kept the system upright.However, in order to analyse the stability and design innovative model-based control strategies for vehicles stabilization, the dynamics of bicycles or motorcycles need to be studied. Limebeer and Sharp [?] proposed the modelling of bicycles and motorcycles and Corno *et al.* [?] a control-oriented motorcycle analytical model which considers both longitudinal and lateral forces exerted by the tires and has as inputs the steering torque and the front and rear wheel ones. Based on the dynamic and steering controller, Yi et al. [?], [?] proposed a trajectory tracking and stability control for autonomous motorcycle for agile manoeuvres using steering angular velocity and rear thrust as control inputs which works fine at low speed too. Recently, electrification of vehicle propulsion is also applied to motorcycles allowing the birth of all-wheel driven motorcycles. The research has investigated whether this feature helps a better management of vehicle stabilization. Yang and Murakami [?] proposed an electric motorcycle model, where there are two steering actuators and two driving ones. When the motorcycle moves with normal or high speed, both front and rear steering motors can be effectively controlled by swaying to keep the balance; when it stops or moves with slow speed, the front and rear steerings are rotated in the same direction and the self-balancing is achieved by driving motors similar to Segway stabilization control. In [?], [?] control strategies that increase the stability of electric motorcycle by acting only on driving and braking torques have been presented. These strategies take into account the rider's intentions and are applied for cornering stability. In mentioned articles the front wheel torque is only braking one, whereas in this paper authors want to take advantage of driving front wheel torque. Notice that all of the aforementioned studies involve in some way either the steering actuation or only the rear driving torque or in other cases a braking not driving front wheel torque.

For the enhancement of vehicle stability also at low speeds, it is of growing importance to devise control oriented models of bike dynamics. For these purposes, in the paper a validated model of the motorcycle dynamics has been derived with the specific goal of a model simple but able to capture all the dynamics relevant to the capsize motion which are the requirements for designing a model-based self-balancing control method, even when vehicle stops or moves slowly (0-1 m/s). Moreover, the work would also find out whether front wheel torque can help in some way bike stabilization when the steering handlebar can not be actuated. Going in this direction, the paper presents a 4 degrees of freedom (DoF) model dynamically similar to an inverted pendulum that considers both rear and front wheel driving torques instead of rear and steering ones. Steering axis is initially set on a strictly positive steering angle and is kept constant over time. The idea is to reproduce a configuration similar to Segway or wheelchair which are stable at low speed [**?**], [**?**]. In fact, when the steering axis is rotated up to its maximum positive angle and then locked, front wheel driving torque actuation should help motorbike balancing even if steer torque is not available. In the presented model the equations of motion are obtained by the Lagrangian approach: the result is a nonlinear second order ODE system. Based on this model a sliding mode controller has been designed and then tested on a multibody software.

The rest of the paper is organized as follows. Section **??** is devoted to the mathematical model of the two wheel drive electric motorcycle. Section **??** presents the control system design for stability control. Numerical simulation results of the autonomous motorcycle and model validation are presented in Section **??**. Finally, concluding remarks and the future works are described in Section **??**.

II. Vehicle mathematical model

The rider-less motorcycle model has two parts: a rear frame and a front steering assembly. Model assumptions are: 1. the contact of the tread and the ground is point-contact, thus the thickness of tires is supposed to be ignored; 2. the frame of the motorbike is regarded as a point mass; 3. the ground is flat and the vertical motion is neglected (no suspension motion); 4. there is no side sliding when the motorcycle is running; 5. front contact point and instantaneous rotation axis do not change when the lean angle changes; 6. steering angle is positive and constant over time. Notice that these assumptions are not so restricted in the viewpoint of motorcycle low speed.

Figure **??** shows a schematic diagram of the model. Let P and Q denote rear and front contact point, respectively. To identify the motorcycle in a generic configuration and derive the model, two different reference frames have been adopted (see Fig.**??**):

- The inertial reference frame $\Sigma = (Oxyz)$: a right-handed time-invariant reference frame fixed in the space;
- the body reference frame $S = (Px'y'z')$: a reference system fixed in the rear contact point of the main frame of the motorcycle with the z'-axis parallel to the vehicle vertical axis and pointing downwards; the x'-axis indicates the forward direction and the y'-axis completes a right-handed frame. The reference frame origin P has coordinates $(x_0, y_0, 0)^T$ with respect to the inertial one.

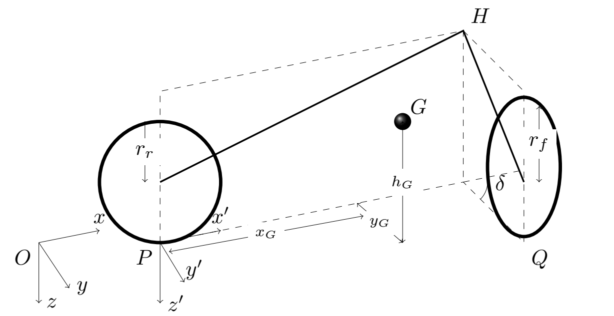

Figure 1. Motorcycle model scheme in trivial configuration with reference frames used to derived the model.

In a generic configuration the rear frame is no more parallel to x, but forms an angle θ, named *yaw angle* and taken about the vertical z-direction. Moreover, the *roll angle* α is the one that motorcycle's rear plane makes with the vertical one. We take α positive when the bike leans to the right according to right hand rule (see Fig. **??**).

Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ and $(\mathbf{i_S}, \mathbf{j_S}, \mathbf{k_S})$ be the unit vector sets for the two coordinate systems respectively and $R(\theta)$ and $R(\alpha)$ the rotation matrices:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} R(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}. \tag{1}$$

It is straightforward to obtain that

$$\begin{bmatrix} \mathbf{i_S} \\ \mathbf{j_S} \\ \mathbf{k_S} \end{bmatrix} = [R(\theta)R(\alpha)]^T \begin{bmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{bmatrix}. \tag{2}$$

The model has 4 degrees of freedom: the x and y position of the contact point between the rear tire and the road expressed in the inertial reference frame; the yaw angle θ and the roll angle α. Rear and front assembly directions differ by the *steering angle* δ. This angle do not change over time, so our model can be considered as a single body one. The input variables of the model are the rear wheel torque T_r and the front one T_f: both of them can be positive or negative. Moreover, lateral tire forces have been also included in the model. Seffen *et al.* [**?**] use similar Lagrangian approach under the same assumptions - except to steering actuation - obtaining a four DoF analytical model. However, model control input is only steering torque. In what follows, the symbols c_θ, s_θ and t_θ stand for $\cos\theta$, $\sin\theta$ and $\tan\theta$, respectively.

A. Mathematical model derivation

The equations of motion are given by Lagrange's equations:

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = Q_q, \tag{3}$$

where $L(q,\dot{q}) = T(q,\dot{q}) - V(q)$ is the Lagrangian function, $T = T(q;\dot{q})$ is the kinetic energy, $V = V(q)$ is the potential

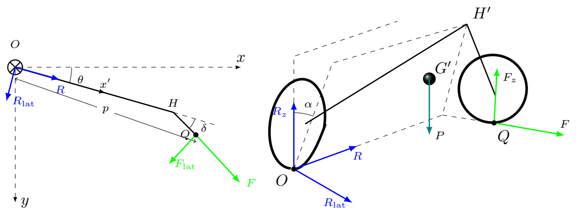

Figure 2. Two Degrees of Freedom of the proposed model: yaw angle θ on the left and roll angle α on the right. Figure also shows forces involved in the model.

energy, $Q_q = [Q_x \;\; Q_y \;\; Q_\alpha \;\; Q_\theta]^T$ is the vector of the generalized external forces and $q = [x \;\; y \;\; \alpha \;\; \theta]^T$ is the generalized coordinates vector.

1) Model Lagrangian function: The kinetic quantities needed to compute kinetic and potential energy are the mass centre velocity and the system angular velocity. Let G be the mass centre of the body (see Fig. **??**). In its local coordinate system we have $G_S = (x_g, 0, h_g)$. Thus, the mass centre velocity with respect to the inertial frame Σ is obtained by differentiating the expression of its inertial position with respect to time:

$$\begin{aligned}\mathbf{v_G} =& (\dot{x} - h_g c_\alpha s_\theta \dot{\alpha} - (h_g s_\alpha c_\theta + x_g s_\theta)\dot{\theta})\mathbf{i} + \\ &+ (\dot{y} + h_g c_\alpha c_\theta \dot{\alpha} + (x_g c_\theta - h_g s_\alpha s_\theta)\dot{\theta})\mathbf{j} + \\ &+ (h_g s_\alpha \dot{\alpha})\mathbf{k}.\end{aligned} \tag{4}$$

On the other hand, the angular velocity of system is

$$\omega_S = \dot{\alpha}\mathbf{i_S} + \dot{\theta}\mathbf{k_S} \tag{5}$$

in the local reference frame.

Now, the kinetic energy of a rigid body is the sum of its kinetic energy associated to the movement of the centre of mass and the kinetic energy associated to the movement of the particles relative to the centre of mass, that is,

$$T = T_{\text{trasl}} + T_{\text{rot}} = \frac{1}{2} m \mathbf{v_G}^2 + \frac{1}{2}\langle \omega_S, I_G \omega_S \rangle \tag{6}$$

where m is the body mass, I_G its inertia tensor in the local reference frame and $\langle , \rangle$ indicates the scalar product. Substituting the kinematics quantities (**??**) and (**??**) in (**??**), the kinetic energy terms become

$$\begin{aligned}T_{\text{trasl}} = & \frac{1}{2} m \Big[h_g^2 \dot{\alpha}^2 + \left(h_g^2 s_\alpha^2 + x_g^2\right)\dot{\theta}^2 - 2\dot{x}\left(h_g c_\alpha s_\theta \dot{\alpha} + \right. \\ & \left. + (h_g s_\alpha c_\theta + x_g s_\theta)\dot{\theta}\right) + 2 h_g x_g c_\alpha \dot{\alpha}\dot{\theta} + \\ & + 2\dot{y}\left(h_g c_\alpha c_\theta \dot{\alpha} + (x_g c_\theta - h_g s_\alpha s_\theta)\dot{\theta}\right) + \dot{x}^2 + \dot{y}^2 \Big],\end{aligned} \tag{7a}$$

$$T_{\text{rot}} = \frac{1}{2}(I_{\text{xx}}\dot{\alpha}^2 + 2 I_{\text{xz}}\dot{\alpha}\dot{\theta} + I_{\text{zz}}\dot{\theta}^2), \tag{7b}$$

and the potential energy is

$$V = m g G_z = m g h_g \cos\alpha. \tag{8}$$

Using the expressions (**??**) and (**??**), the Lagrangian function L of the motorcycle is

$$L = T_{\text{trasl}} + T_{\text{rot}} - V. \tag{9}$$

2) Contact point modelling and generalized external forces: The potential term $V(q)$ of Lagrangian function is the potential associated to external conservative forces such as the gravity force. On the other hand, the non-conservative external forces (e.g. friction forces) contribute to generalized forces term $Q_q = \sum_h \mathbf{F}_h \frac{\partial P_h}{\partial q}$, where P_h is the application point of the force $\mathbf{F}_h$. The external active forces acting on the body are the conservative gravity force $\mathbf{P}$ applied at the centre of mass G and the rear and front wheel thrusts $\mathbf{R}$ and $\mathbf{F}$ applied at the rear and front contact points P and Q, respectively. In the local frame the last two forces can be written as $\mathbf{R} = R\mathbf{i}$ and $\mathbf{F} = F\cos\delta\mathbf{i} + F\sin\delta\mathbf{j}$ where $R = T_r r_r$ and $F = T_f r_f$ (r_r and r_f are the rear and front wheel radius). The model also includes tyre forces. These forces are generated at the contact patch between tire and road and are the consequence of the sliding of the tread rubber on the asphalt surface. For this reason, forces can be calculated using wheel kinematics and in particular the velocity of the contact point (see Fig. **??**). In this work it has been adopted a linear tire model - the simplest available - where all equations are linearised with respect to a straight running configuration. Let N_i, $i = r, f$ be the tire static load. In this model the lateral force has the roll and slip angles contributions: $F_{\text{lat}} = (k_\alpha \alpha + k_\lambda \lambda)N$ with λ the slip angle, k_α and k_λ the roll and cornering stiffness, respectively. However, the slip angle contribution is smaller than the roll angle one at low speed and for this reason this second term is neglected here [**?**]. Longitudinal slip and longitudinal tire forces are ignored as well. So, tire forces are reduced to

$$F_y = k_\alpha \alpha N_f \tag{10}$$

$$F_z = -N_f. \tag{11}$$

Similar formulas hold for rear tire forces. Notice that lateral forces are friction ones.

If p denotes the length of contact line, which does not change over time because the steering angle is constant, then calculating $Q_q = \sum_h \mathbf{F}_h \frac{\partial P_h}{\partial q}$ the generalized force terms are

$$Q_x = R c_\theta + F\cos(\theta + \delta) - F_y \sin(\theta + \delta) - R_y s_\theta; \tag{12a}$$

$$Q_y = R s_\theta + F\sin(\theta + \delta) + F_y \cos(\theta + \delta) + R_y c_\theta; \tag{12b}$$

$$Q_\alpha = 0; \tag{12c}$$

$$Q_\theta = p(F\sin\delta + F_y\cos\delta). \tag{12d}$$

Finally, applying (**??**) the Lagrange's equations of motion of the model are:

$$\begin{aligned}& m(\ddot{x} - h_g c_\alpha s_\theta \ddot{\alpha} - (h_g s_\alpha c_\theta + x_g s_\theta)\ddot{\theta} + h_g s_\alpha s_\theta \dot{\alpha}^2 - \\ & -2 h_g c_\alpha c_\theta \dot{\alpha}\dot{\theta} + (h_g s_\alpha s_\theta - x_g c_\theta)\dot{\theta}^2) = Q_x;\end{aligned} \tag{13a}$$

$$\begin{aligned}& m(\ddot{y} + h_g c_\alpha c_\theta \ddot{\alpha} - (h_g s_\alpha s_\theta - x_g c_\theta)\ddot{\theta} - h_g s_\alpha c_\theta \dot{\alpha}^2 - \\ & -2 h_g c_\alpha s_\theta \dot{\alpha}\dot{\theta} - (h_g s_\alpha c_\theta + x_g s_\theta)\dot{\theta}^2) = Q_y;\end{aligned} \tag{13b}$$

$$\begin{aligned}& - m h_g c_\alpha s_\theta \ddot{x} + m h_g c_\alpha c_\theta \ddot{y} + \left(h_g^2 m + I_{\text{xx}}\right)\ddot{\alpha} + \\ & + (h_g m x_g c_\alpha + I_{\text{xz}})\ddot{\theta} - m h_g^2 s_\alpha c_\alpha \dot{\theta}^2 = m g h_g s_\alpha;\end{aligned} \tag{13c}$$

$$\begin{aligned}& - m(h_g s_\alpha c_\theta + x_g s_\theta)\ddot{x} + m(x_g c_\theta - h_g s_\alpha s_\theta)\ddot{y} + \\ & + (m h_g x_g c_\alpha + I_{\text{xz}})\ddot{\alpha} + (m(h_g^2 s_\alpha^2 + x_g^2) + I_{\text{zz}})\ddot{\theta} - \\ & - m h_g x_g s_\alpha \dot{\alpha}^2 + h_g^2 m s_{2\alpha}\dot{\alpha}\dot{\theta} = Q_\theta,\end{aligned} \tag{13d}$$

that is a nonlinear second order ODE system.

3) State space representation: The equations of motion (**??**) as derived above in Sec. **??** are given in the matrix form

$$\mathbf{M}(\mathbf{q}, \dot{\mathbf{q}})\ddot{\mathbf{q}} = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{B}\mathbf{u} \tag{14}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{q}$ and $\dot{\mathbf{q}}$ denote the generalized coordinate and velocity vectors and $\mathbf{u} = [T_r \; T_f]^T$ the control input vector (obtained by setting $R = T_r/r_r$ and $F = T_f/r_f$). Matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{B}$ are functions of generalized coordinates and velocities. System (**??**) or its equivalent matrix form (**??**) is a second order one, but control techniques work better on first order ODE systems. Defining the state vector $X = [x \; y \; \alpha \; \theta \; \dot{x} \; \dot{y} \; \dot{\alpha} \; \dot{\theta}]^T$, it is possible to recast the proposed model as:

$$\bar{\mathbf{M}}(X)\dot{X} = \bar{\mathbf{A}}(X) + \bar{\mathbf{B}}(X)U \tag{15}$$

where $\bar{\mathbf{M}}(X) \in M^{8\times 8}$ is the new mass matrix, $\bar{\mathbf{A}} \in M^{8\times 1}$ is a column vector obtained from the Christoffel matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ after recasting and $\bar{\mathbf{B}} \in M^{8\times 2}$ is the input matrix of the input vector $U = \mathbf{u}$. Inverting $\bar{\mathbf{M}}(X)$ the system becomes:

$$\dot{X} = \mathbf{A}(X) + \mathbf{B}(X)U. \tag{16}$$

This is the general state-space representation of the proposed model used in control design. Notice that the matrix $\mathbf{A}$ directly depends by X, so the system (**??**) is nonlinear, as stated above.

III. Control design

This Section shows the design of a control system to balance the previous unstable model and avoid motorcycle falling down. The vehicle stabilization is achieved by a sliding mode control strategy. This control method is a robust nonlinear model-based one which is insensitive to unmodelled dynamics and disturbances and for this reason it is particularly suitable for the problem [**?**] - [**?**]. Moreover, this strategy can be implemented for both SISO and MIMO systems, so it could be applied to achieve more than one control target taking into account the system interconnected dynamics. In this paper as first attempt a single input controller has been designed.

Notice that the control system wants to test whether vehicle can be self-balanced when its initial velocity is zero and the steering axis is locked at a positive angle. Remember that the steering handlebar can not be actuated. This describes the motorcycle initial configuration after the rider leaves it.

As in the references reported in this Section where sliding mode control systems for autonomous bike at low speed range are presented as in most of models and stability controllers presented in literature, one of control inputs is always steering torque which is not available in this project. Thus, in the paper only wheel torques can be used as control inputs to achieve stabilization that is possible due to the rotated steering axis: this model feature can not be removed. Specifically, front wheel thrust has been chosen as system input and the rear one has been set identically zero during simulations. Notice that motorcycle is balanced when it has a null roll angle. Thus, roll angle α is the controlled variable with setpoint $\alpha_s = 0$.

The sliding mode control scheme involves two steps. The first one is the selection of a hypersurface or a manifold (i.e., the *sliding surface*) such that the system trajectory exhibits desirable behaviour when it is confined to this manifold. In the second step feedback control laws are designed such that the system trajectory intersects and stays on the manifold. More precisely, a (discontinuous) control law should force the system trajectory to intersect the surface in a finite time and a continuous control guarantees that system trajectories stay on it. The final control system is a combination of these two control laws. Further details in [**?**].

Control priority is the roll angle $\alpha (= X(3) = X_3)$ stabilisation, so the sliding surface is

$$s(X) = \dot{X}_3 + \lambda X_3 = X_7 + \lambda X_3, \quad \lambda > 0 \tag{17}$$

where λ is a design parameter that determines the velocity in which system reaches the surface. On the sliding surface, control objective is achieved.

To guarantee that system trajectories stay on the surface $s(X)$, $\dot{s}(X) = \dot{X}_7 + \lambda \dot{X}_3 = 0$ must hold. Then, the designed equivalent control is

$$u_{\text{eq}} = -\mathbf{B}_7^{-1}(\mathbf{A_7} + \lambda X_3). \tag{18}$$

In general the discontinuous component of controller is $u_n = -\mathbf{B}_7^{-1}\eta \text{sgn}(s)$ with η a design parameter and $\text{sgn}(x)$ the signum function. It can be obtained by the η-reachability condition: $s\dot{s} < -\eta|s|$. To avoid implementing problems and reduce control signal chattering a boundary layer for the nonlinear control law u_n is designed. The boundary layer width is denoted by ε.

In conclusion, after setting the design parameters λ, η and ε the control law applied to dynamic system is:

$$u = u_{\text{eq}} + u_{\text{n}}. \tag{19}$$

IV. Simulation results and model validation

This Section shows some simulation results of the control strategy presented above and its validation. To validate analytical model and analise whether it captures roll vechicle dynamics, a equal set of parameters of a real motorcycle - manufactured by Visionar srl - has been used in both models: these parameters are listed in Table **??**. Moreover, roll angle is set equal to $\alpha = 4°$ at the beginnig of simulation and the sliding mode controller of front wheel torque - designed by analytical model - has been tested in the multibody software applying same control design parameters: $\lambda = 5$, $\eta = -5$ and boundary layer width $\varepsilon = 10^{-3}$. The model and controller are implemented in Matlab/Simulink.

The single control input sliding mode controller had to test whether motorcycle can be self-balanced with null initial velocity, no handlebar actuation and locked steering axis. In Figure **??** the dashed red line gives a positive preliminary result to this question. In fact, after less than 2 seconds the controller stabilizes vehicle reaching a null roll angle. In addition, as shown in Figure **??**, the system begins its evolution

Table I
NUMERICAL VALUES OF THE MOTORCYCLE MODEL PARAMETERS

Symbol [SI]	Definition	Value
p [m]	wheelbase	1.416
r_f [m]	front wheel radius	0.347
r_r [m]	rear wheel radius	0.318
x_G [m]	CoM G x-axis local coordinate	0.745
h_G [m]	CoM G height	0.601
δ [deg]	steering angle	40
m [kg]	motorcycle mass	130.5
g [m/s^2]	gravity acceleration	9.806
I_{xx} [kgm^2]	inertia tensor term	8.268
I_{xz} [kgm^2]	inertia tensor term	0.19
I_{zz} [kgm^2]	inertia tensor term	21.025
N_f [N]	front tire load	678.69
N_r [N]	rear tire load	600.69
k_α [1/rad]	rolling stiffness	0.8
α_0 [deg]	initial roll angle	4

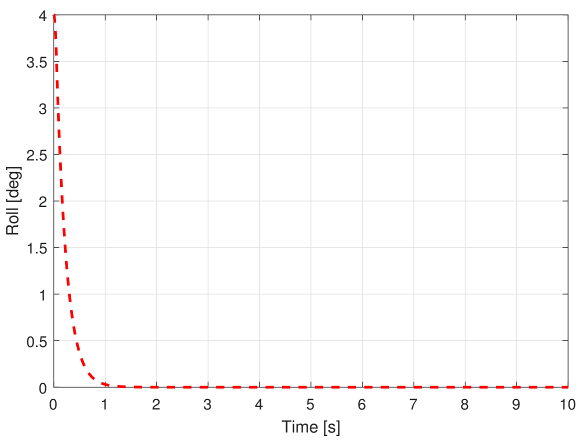

Figure 3. Simulation results of roll angle α for sliding mode control designed by analytical model.

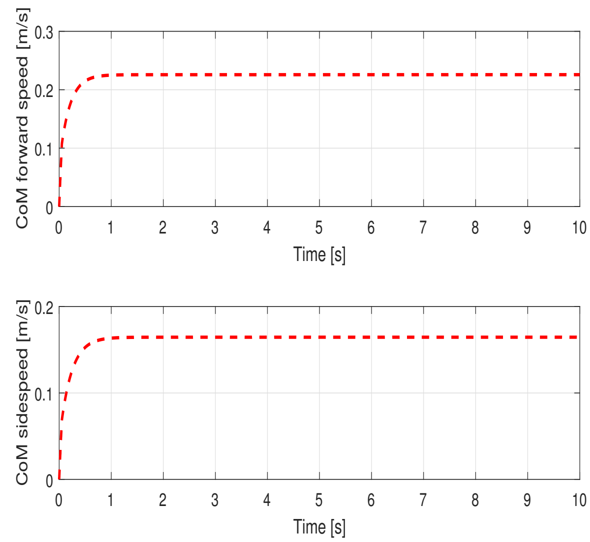

Figure 4. Simulation results of forward (top) and side (bottom) speed of centre of mass in the analytical model.

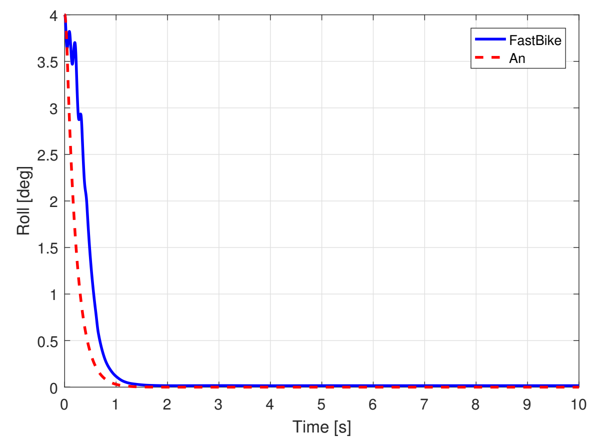

Figure 5. Roll angle α simulation results of sliding mode control: multibody software (blue solid line) and analytical model (red dashed line).

with null velocity and then evolves keeping a low speed - about 0.2 m/s - and so it remains in the model framework.

The presented model and designed sliding mode controller have been validated by FastBike^{RT}, a computer simulation software for real-time dynamic analysis of two wheel vehicles distributed by Dynamotion [**?**]. The software has been suitably modified for low speed range. Its multibody model includes five bodies - the rear assembly, the rider, which is rigidly attached to the rear assembly, the front steering assembly, the rear and the front wheels - and has 9 degrees of freedom - longitudinal, lateral and vertical motion, roll, yaw and pitch angle, steering rotation and rear and front wheel spin. Moreover, the software accounts the deformability of tyres using a nonlinear tyre model.

Validation tests are carried out comparing analytical model response to the multibody software one when same sliding mode controller is applied. Figures **??**, **??** and **??** show simulation results of this comparison. Specifically, in Figure **??** the simulations show a good match in roll angles: the same controller can stabilize the motorcycle multibody model of the software - which is a more complex model - behaving in a similar way with respect to the response of the analytical one. This means that the mathematical model captures the main roll dynamics. As reported in Figure **??**, the yaw angle has a linear growth in both models, but in FastBike software it is slightly bigger. Moreover, control inputs have the same magnitude and both of them are under the physical limit imposed by the real electric motor ($T_f = 120$ Nm), as shown in Figure **??**.

V. CONCLUSION AND FUTURE WORKS

In this paper, a 4 DoF mathematical model for a two wheel drive electric motorcycle has been presented. The study has been carried out for low speed range of vehicle (0.1-1 m/s) when rider stops and leaves the bike. The mathematical model has been derived in the perspective of the stability control system design. Consequently, the analytical model

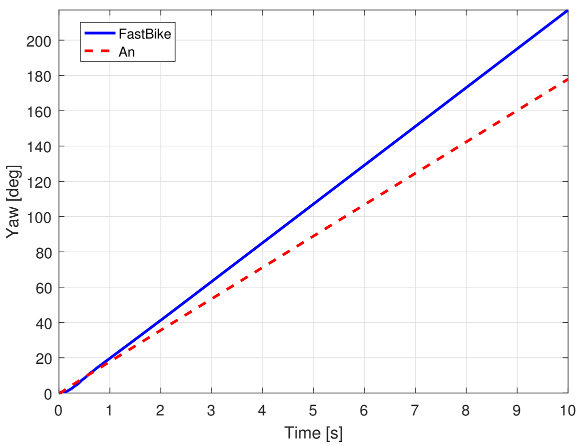

Figure 6. Yaw angle θ simulation results of sliding mode control: multibody software (blue solid line) and analytical model (red dashed line).

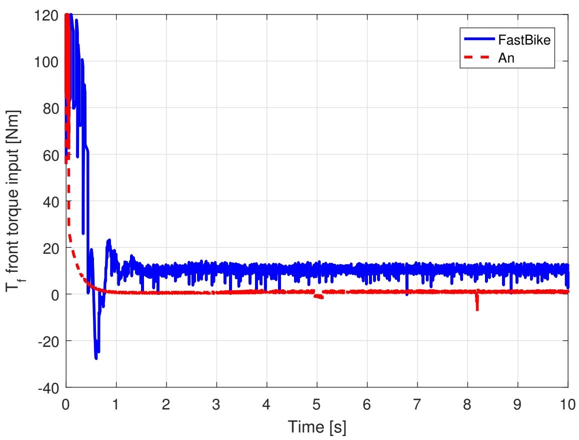

Figure 7. Front torque control input T_f simulation results of sliding mode control: multibody software (blue solid line) and analytical model (red dashed line).

has been developed with the specific target of a model as simple as possible, but able to capture the main motorcycle roll dynamics. Based on the analytical model, a sliding mode controller has been designed with positive simulation results with respect to motorcycle balancing. The work has been developed under the hypothesis that the handlebar can not be actuated, but both rear and front wheel torque are available. Specifically, the front wheel torque has been chosen as single control input of the control strategy as preliminary study.

The model validation with a multibody software has highlighted a good match of the balancing variable (the roll angle) which means the presented analytical model captures the main capsize motion dynamics and can be used for model-based control systems. In addition, the model has been derived considering both rear and front wheel torques which results in the possibility of design MIMO or more advanced control strategies, eventually satisfying more than one control aim.

The presented work is a positive preliminary study on the topic. Confining motorcycle path in a small area could be a further control requirement needed to system application in real situations. For this purpose, it is under study the design of control strategy using more advanced control techniques which can take into account the presence of two control inputs - both front and rear wheel torque - instead of a single one to achieve both motorcycle balancing and its track confinement. The results will be reported once they are available.

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