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Analytical Tuning of a TSMPC Dedicated to Nonlinear MIMO Systems

Marwa Turki, Khaled Benkhoud, Nicolas Langlois and Adnan Yassine

***Abstract*—In this paper, an analytical tuning method for Takagi-Sugeno-model-based predictive control (TSMPC) is presented. As an advantage, it can be applied to nonlinear multi-input-multi-output (MIMO) controllable processes with constraints and guarantees closed-loop stability while limiting required computational load. Its application to a simulated quad-tilt-wing (QTW) unmanned aerial vehicle (UAV) emphasizes its effectiveness when it is compared to a metaheuristic tuning approach.**

***Keywords*—Nonlinear systems, T-S-model, MPC, quasi-LPV system, analytical tuning, MIMO systems.**

I. Introduction

Constrained MPC is now widely implemented in industry to control complex systems [1]. Since its three parameters (control horizon N_c, prediction horizon N_p and weighting factor λ) influence significantly the closed-loop behavior in a complex manner [2], many tuning approaches have been investigated during the last decades [3]. Most of them do not always permit to identify explicitly their robustness areas face to nonlinear MIMO systems. In this paper, we intend to overcome this limit by investigating an analytical way guaranteeing optimal stability and limiting tuning computational load when systems are described from Takagi-Sugeno (T-S) models.

This paper is outlined as follows: Section II reminds theoretical background on TSMPC formulation using the state-space representation. Section III highlights the analytical tuning approach proposed. In section IV, this latter is applied to a simulated QTW UAV to show its interest.

II. Reminds on TSMPC

The T-S approach is a modeling technique permitting a large class of nonlinear systems to be exactly represented by a set of N_{LTI} linear time-invariant (LTI) local models related to operation areas (OA). Let u, x and y be the manipulated variable, the state variable and output of a given nonlinear system. Let its j^{th} ($j = 1, 2, \cdots, N_{LTI}$) discrete-time LTI augmented local model be defined as follows:

$$\begin{cases} x(k+1) & = A_j x(k) + B_j u(k) \\ y(k) & = C_j x(k) \end{cases} \tag{1}$$

where $A_j \in \mathbb{R}^{n_A \times n_A}$, $B_j \in \mathbb{R}^{n_{in} \times n_A}$ and $C_j \in \mathbb{R}^{n_{out} \times n_A}$ obtained from an initial model represented by $A_{mj} \in \mathbb{R}^{n_1 \times n_1}$, $B_{mj} \in \mathbb{R}^{n_{in} \times n_1}$ and $C_{mj} \in \mathbb{R}^{n_{out} \times n_1}$ [4]. Here n_{in} and n_{out} are the number of system input and output respectively and $n_A = n_1 + n_{out}$. Under the assumption that the dynamic system defined by (A_j, B_j) is controllable, it is possible to design a local predictive controller from the minimization of cost function J :

$$J(k) = \sum_{l=l}^{N_p} \|\hat{y}(k+l) - y_{des}(k+l)\|^2_{R_1} + \sum_{l=1}^{N_c-1} \|\Delta u(k+l|k)\|^2_{R_2}$$

where R_1 and R_2 are weighting matrices, $\hat{y}$ the predicted output, y_{des} the desired output and Δu the increment of control defined by:

$$\Delta u = u(k) - u(k-1)$$

The minimization of J is subject to the constraints:

$$\begin{aligned} & u_{min} \leq u(l) \leq u_{max} \\ & \Delta u_{min} \leq \Delta u(l) \leq \Delta u_{max}, \text{where } k \leq l \leq k + N_c - 1 \\ & x_{min} \leq x(l) \leq x_{max}, \text{ where } k+1 \leq l \leq k + N_p. \end{aligned}$$

The TSMPC concept consists then in designing a controller from the sum of all local weighted predictive ones as follows:

$$u(k) = \sum_{j=1}^{N_{LTI}} \mu_j(\theta(t)) u_j(k) \tag{2}$$

Where $\mu_j(\theta(t))$ is the activation function of the j^{th} local predictive controller and θ the premise variable vector depending on system states and input [5]. The resulting scheme of the proposed TSMPC is shown in Fig 1.

III. Analytical Tuning Approach Proposed

This section details the analytical approach proposed to tune N_c, N_p and λ for each local predictive controller.

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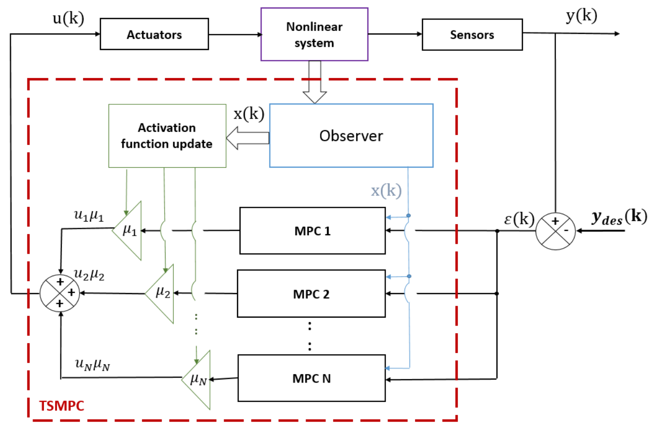

Fig. 1. Structure of the TSMPC using the decoupled multiple model

A. Analytical tuning of N_c

1) The Hessian condition number: In order to compute analytically the optimal value of N_c, the concept of numerical stability is considered. In fact, the numerical stability concerns mainly the condition number of a square nonsingular matrix named the Hessian matrix [6]. This latter is present in the formulation of the MPC sequence [7]. So, with the assumption that $(\Phi^T\Phi+\bar{R})^{-1}$ exists, let consider the Hessian matrix H of size $(n_{in}Nc \times n_{in}Nc)$:

$$H = (\Phi^T\Phi+\bar{R})^{-1} \tag{3}$$

Where $\bar{R}$ is a $(n_{in}N_c \times n_{in}N_c)$ matrix containing on its diagonals the weighting factor λ defined by $\lambda = \begin{bmatrix}\lambda_1 & \lambda_2 & \cdots & \lambda_{n_{in}N_c}\end{bmatrix}$ and Φ a matrix of size $(n_{out}N_p \times n_{in}N_c)$ defined by:

$$\Phi = \begin{bmatrix} C_jB_j & 0 & 0 & \dots & 0 \\ C_jA_jB_j & C_jB_j & 0 & \dots & 0 \\ C_jA_j^2B_j & C_jA_jB_j & C_jB_j & \dots & 0 \\ \vdots & & & & \\ C_jA_j^{N_p-1}B_j & C_jA_j^{N_p-2}B_j & C_jA_j^{N_p-3}B_j & \dots & C_jA_j^{N_p-N_c}B_j \end{bmatrix} \tag{4}$$

To evaluate the conditioning of H, one calculates its condition number defined by:

$$\begin{aligned} cond(H(k)) &= \|H(k)\|_2 . \left\|H(k)^{-1}\right\|_2 \\ &= \frac{\sigma_{max}(k)}{\sigma_{min}(k)}, \end{aligned} \tag{5}$$

where $\sigma_{max}(k)$ and $\sigma_{min}(k)$ are respectively the maximum and the minimum singular values of H. Then, the condition number of a matrix indicates how close it is to be singular [6]. Regarding the literature, different ways exist to improve the condition number of H [7, 4]. Here in this paper, we intend to overcome these limits and enhance the condition number of H by computing analytically N_c.

2) Improving the condition number: a dimension reduction problem : As shown in [8], high MPC horizons guarantee the system closed-loop stability. Ideally, these values tend towards infinity ($N_c \mapsto \infty, N_p \mapsto \infty$). Note then that the issue of improving the condition number of H is related to a matrix dimension reduction one's.
In order to determine an optimal value of N_c, the concept of the effective rank (ER) is considered [9].

3) Relation between ER and N_c: In this part, the concept of effective rank is related to MPC in order to calculate the optimum value of N_c. The different steps are:

1) To initialize $N_c \mapsto \infty$ and $N_p \mapsto \infty$ $(Nc < Np)$.
2) To take $A_{ER} = H$ [9].
3) To evaluate Q defined as follows [9]:

$$\begin{aligned} Q &= min\{M_{ER}, N_{ER}\} = min\{n_{in}Nc, n_{in}Nc\} \\ &= n_{in}Nc. \end{aligned}$$

4) To decompose H into singular values and to evaluate $\sigma = \begin{bmatrix}\sigma_1 & \sigma_2 & \cdots & \sigma_\infty\end{bmatrix}^T$:

$$H = U_H D_H V_H \tag{6}$$
$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_\infty \tag{7}$$

5) To evaluate the singular value distribution p_k with $k = [1, \quad 2, \quad \cdots, \quad \infty]$.
6) To calculate the Shannon entropy [9].
7) To evaluate finally the optimal value of N_c by solving the following linear matrix inequality (LMI) problem:

$$\begin{cases} N_c^{opt} = round(\frac{e^{H_{Shannon}(p_1, p_2, \cdots, p_\infty)}}{n_{in}}) \\ \min(N_c^{opt}); \;\; \min(cond(H(k)) - 1) \end{cases} \tag{8}$$

Under the following constraints:

$$\begin{cases} N_c \in \mathbb{N}^*; \;\; 1 \leq N_c^{opt} < N_p \\ \quad U^{min} \leq U \leqslant U^{max} \end{cases}$$

As a conclusion, the smallest control prediction is taken, the better condition number is. Thus, the numerical stability is enhanced.

B. Analytical tuning of N_p

1) Guaranteed closed-loop stability: Here, N_p is computed so that it ensures the optimal closed-loop stability. According to [10], the incremental control can be written as follows:

$$\Delta U(k) = K_y Y_{des} - K_{mpc} x(k) \tag{9}$$

with

$$K_{mpc} = I_{(n_{in} \times N_c)}(\Phi^T\Phi+\bar{R})^{-1}\Phi^T F \tag{10}$$
$$K_y \;\; = I_{(n_{in} \times N_c)}(\Phi^T\Phi+\bar{R})^{-1}\Phi^T I_{(N_p \times 1)} \tag{11}$$

where I is identity matrix and F a matrix of size $(N_p n_{out} \times n_A)$ [4]:

$$F = \begin{bmatrix} C_j A_j & C_j A_j^2 & C_j A_j^3 & \cdots & C_j A_j^{N_p} \end{bmatrix}^T. \quad (12)$$

By substituting this result in (1), we have:

$$x(k+1) = (A_j - B_j K_{mpc})x(k) + B_j K_y Y_{des} \quad (13)$$

So closed loop eigenvalues are obtained from:

$$det[\nu I_{(n_A \times n_A)} - (A_j - B_j K_{mpc})] = 0 \quad (14)$$

The original way to tune analytically N_p consists in computing a feasible gain K_{mpc} assuring the optimal closed-loop stability according to *Schur* stability. This concept has been inspired by the principal of the insensitive region theorem [11]. Given a scalar γ with $0 < \gamma \leq 1$, system (1) is optimally closed-loop stable if:

$$\|A_j - B_j K_{mpc}\|_2 < \gamma \quad (15)$$

where $\|.\|_2$ is the norm 2.
Here let note that the problem of N_p tuning is converted into finding the feasible solution K_{mpc} of the LMI and a minimal γ satisfying (15). This optimization problem with LMI constraints can be solved thanks to Yalmip toolbox [12]:

$$\begin{cases} \min \gamma \\ \begin{bmatrix} -\gamma I_{(n_A \times n_A)} & (A_j - B_j K_{mpc}) \\ (A_j - B_j K_{mpc})^T & -\gamma I_{(n_A \times n_A)} \end{bmatrix} < 0 \end{cases} \quad (16)$$

Let be $(K_{mpc})^{opt}$ then the optimal computed value of K_{mpc}.

2) Relationship between K_{mpc} and $\Phi^T\Phi$: With the assumption that the inverse of the matrix $(\Phi^T\Phi + \bar{R})$ exists, here we admit that the matrix $N = (\Phi^T\Phi)^{-1}\bar{R}$ is a n-order nilpotent matrix ($n \in \mathbb{N}^*$). Then, (3) can be written as follows:

$$\begin{aligned} H = &(\Phi^T\Phi)^{-1} - (\Phi^T\Phi)^{-1}\bar{R}(\Phi^T\Phi)^{-1} \\ &+ [(\Phi^T\Phi)^{-1}\bar{R}]^2(\Phi^T\Phi)^{-1} - \cdots \\ &+ (-1)^{n-1}[(\Phi^T\Phi)^{-1}\bar{R}]^{n-1}(\Phi^T\Phi)^{-1}. \end{aligned} \quad (17)$$

By integrating equation (17) in (10) and using $(K_{mpc})^{opt}$, the square matrix $\Phi^T\Phi$ becomes:

$$\begin{aligned} (\Phi^T\Phi)^{opt} = &[-\bar{R} + \bar{R}(\Phi^T\Phi)^{-1}\bar{R} - \cdots \\ &+ (-1)^{n-1}(\Phi^T\Phi)^{-1}[(\Phi^T\Phi)^{-1}\bar{R}]^{n-1}] \\ &\times [(K_{mpc})^{opt}F^T(FF^T)^{-1}\Phi - I_{(n_{in}N_c \times n_{in}Nc)}]^{-1} \end{aligned}$$

3) Relationship between $\Phi^T\Phi$ and N_p: From (4), $\Phi^T\Phi$ can be written as:

$$\Phi^T\Phi = \begin{bmatrix} \sum_{i=0}^{N_p-1}(C_jA_j^iB_j)^2 & . & \cdots & . \\ . & \sum_{i=0}^{N_p-2}(C_jA_j^iB_j)^2 & \cdots & . \\ . & . & \cdots & . \\ \vdots & & & \\ . & . & \cdots & \sum_{i=0}^{N_p-N_c}(C_jA_j^iB_j)^2 \end{bmatrix}$$

Let define the first element of $\Phi^T\Phi$ as $X_{N_p} = \sum_{i=0}^{N_p-1}(C_jA_j^iB_j)^2$. Note that the number of elements of X_{N_p} is the optimal value of the prediction horizon N_p^{opt}. This latter is finally obtained from an element-based identification as follows:

$$X_{N_p} = \sum_{i=0}^{N_p^{opt}-1}(CA^iB)^2 = \sum_{i=0}^{N_p^{opt}-1}(\sum_{k=0}^{i} C_{mj}A_{mj}^kB_{mj})^2$$

C. Analytical tuning of λ

In this paper, an original approach for computing analytically λ is presented. The key point here is to find a weighting matrix that minimizes J. Because $\bar{R}$ is a diagonal matrix containing the weighting factors λ_i where $i = \{1, \cdots, n_{in}N_c\}$, minimizing J with respect to λ comes to minimize J with respect to $\bar{R}$. The necessary condition to obtain the minimum value of J with respect of both $\bar{R}$ and ΔU is reached as:

$$\begin{cases} \frac{\partial J}{\partial \Delta U} = 0 \\ \frac{\partial J}{\partial \bar{R}} = 0 \end{cases} \quad (18)$$

The optimal control is then given by:

$$\Delta U = (\Phi^T\Phi + \bar{R})^{-1}\Phi^T(Y_{des} - Fx(k)) \quad (19)$$

Let define a new matrix Ψ as $\Psi = Y_{des} - Fx(k)$. Due to computation complexity, let consider here that N is a three-order nilpotent matrix.
From (19), the derivative of J by $\bar{R}$ is:

$$\begin{aligned} \frac{\partial J}{\partial \bar{R}} = &(\Phi^T\Phi)^{-1}G^TG(\Phi^T\Phi)^{-1} \\ &- 2(\Phi^T\Phi)^{-1}\bar{R}(\Phi^T\Phi)^{-1}G^TG(\Phi^T\Phi)^{-1} \end{aligned} \quad (20)$$

where $G = \Psi^T\Phi$. From (20), the optimal value of $\bar{R}$ is reached when the derivative of J with respect to $\bar{R}$ is equal to zero. Then:

$$\bar{R}_{opt} = \frac{1}{2}(\Phi^T\Phi). \quad (21)$$

Finally, when (21) is solved, the optimal weighting factor vector is:

$$\bar{R}_{opt} = \begin{bmatrix} \lambda_1^{opt} & 0 & \cdots & 0 \\ 0 & \lambda_2^{opt} & \cdots & \vdots \\ \vdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \lambda_{n_{in}N_c}^{opt} \end{bmatrix} \quad (22)$$

As an advantage, this novel approach computes a matrix of weighting factors in such a way they minimize the energy required to reach control objective.

IV. Application to a simulated QTW UAV

In this paper, the considered nonlinear system is a simulated QTW UAV with three inputs and six outputs. All simulations are derived via Matlab Simulink with a sampling time equal $T_s = 0.1\ s$.

A. T-S modelling of QTW UAV

In this section, a nonlinear model of a QTW UAV, which incorporates the dynamics of attitude vertical flight, is briefly reviewed, and its T-S model is established. As shown in Fig 2, it has two fundamental motion modes: the vertical and the horizontal flights. A transition operation mode, that interposes these two flights, is also noted. The switching from one mode to another is affected using the well-known tilting mechanism. During the vertical takeoff and landing (VTOL) modes, the QTW drone depends only on its rotors and behaves like a Quadrotor with H-type structure [13]. The tilt angles of the wings are nearly equal to 90° with the horizontal plane. At the horizontal flight mode, it behaves like a conventional plane with a tilt angle of wings almost equal to zero degree. The dynamic model behavior of the

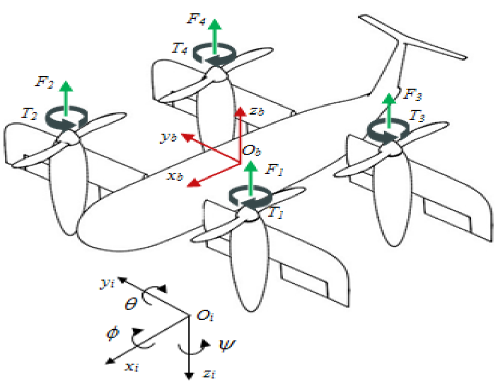

Fig. 2. Configuration of the convertible QTW UAV.

QTW UAV attitude movement during the VTOL mode has been intensively studied in [14, 15], and can be expressed as:

$$\begin{cases} \ddot{\phi} = \dfrac{\tau_\phi}{I_{xx}} + \dfrac{I_{yy} - I_{zz}}{I_{xx}} \dot{\Theta}\dot{\psi} \\ \ddot{\Theta} = \dfrac{\tau_\Theta}{I_{yy}} + \dfrac{I_{zz} - I_{xx}}{I_{yy}} \dot{\phi}\dot{\psi} \\ \ddot{\psi} = \dfrac{\tau_\psi}{I_{zz}} + \dfrac{I_{xx} - I_{yy}}{I_{zz}} \dot{\phi}\dot{\Theta} \end{cases} \tag{23}$$

Let be $\phi \in [\![-\pi/2, +\pi/2]\!]$, $\Theta \in [\![-\pi/2, +\pi/2]\!]$ and $\psi \in [\![-\pi, +\pi]\!]$ the roll, pitch and yaw Euler angles, respectively. These angles represent the rotation motions of the QTW body around the axis x, y and z, respectively. l is the distance between the rotor and the QTW's center of gravity, I_{xx}, I_{yy} and I_{zz} are the inertias in the body reference frame along x-axis, y-axis and z-axis, respectively. The QTW UAV is controlled by independently varying the speed of these four rotors. Hence, the relationship between system inputs and speed rotors can be defined as follows:

$$\begin{bmatrix} \tau_\phi \\ \tau_\Theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} kl & -kl & kl & -kl \\ kl & kl & -kl & -kl \\ k\lambda_{tr} & -k\lambda_t r & -k\lambda_{tr} & k\lambda_{tr} \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{24}$$

where $k > 0$ is the lift coefficient, ω_i denotes the angular speed of the i^{th} motor, and λ_{tr} is the torque/force ratio which depends on the propellers geometry. The dynamical model (23) can be written as the following state-space representation:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1\dot{\Theta} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2\dot{\phi} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_3\dot{\theta} & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 & 0 \\ b_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3 \end{bmatrix} u(t)$$

where $x(t) = \begin{bmatrix} \phi, \dot{\phi}, \Theta, \dot{\Theta}, \psi, \dot{\psi} \end{bmatrix}^T$; $u(t) = \begin{bmatrix} \tau_\phi, \tau_\Theta, \tau_\psi \end{bmatrix}^T$; $a_1 = \frac{I_{yy} - I_{zz}}{I_{xx}}$; $a_2 = \frac{I_{zz} - I_{xx}}{I_{yy}}$ and $a_3 = \frac{I_{xx} - I_{yy}}{I_{zz}}$. In this paper we intend to control essentially $y_1 = \phi$, $y_2 = \Theta$ and $y_3 = \psi$. Based on the sectors non linearity approach [16], two nonlinear continuous terms $\left(\dot{\phi}, \dot{\psi}\right)$ can be observed. Therefore, we set the premise variables as $\theta_1(x(t)) = \dot{\phi}$ and $\theta_2(x(t)) = \dot{\psi}$. Therefore, the nonlinear model (23) is written as four local linear models $\left(r = 2^p = 2^2\right)$ as:

$$A_{m1} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1\underline{\theta}_2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2\underline{\theta}_1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_3\underline{\theta}_2 & 0 & 0 \end{bmatrix}; \quad A_{m2} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1\bar{\theta}_2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2\underline{\theta}_1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_3\bar{\theta}_2 & 0 & 0 \end{bmatrix};$$

$$A_{m3} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1\underline{\theta}_2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2\bar{\theta}_1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_3\underline{\theta}_2 & 0 & 0 \end{bmatrix}; \quad A_{m4} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_1\bar{\theta}_2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & a_2\bar{\theta}_1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a_3\bar{\theta}_2 & 0 & 0 \end{bmatrix};$$

$$B_{mi} = \begin{bmatrix} 0 & 0 & 0 \\ b_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_3 \end{bmatrix}; \quad C_m = \boldsymbol{I}_6 \quad .$$

B. TSMPC synthesis

Let remind that the control law is:

$$u(k) = \sum_{j=1}^{4} \mu_j(\theta(t)) u_j(k)$$

where

$$\mu_1 = F_0^1 F_0^2, \ \mu_2 = F_0^1 F_1^2$$

$$\mu_3 = F_1^1 F_0^2, \ \mu_4 = F_1^1 F_1^2$$

And each premise can be described by two partition functions:

$$\theta_1\left(x\left(t\right)\right) = F_0^1\left(\theta_1\left(x\right)\right)\underline{\theta}_1 + F_1^1\left(\theta_1\left(x\right)\right)\bar{\theta}_1 \tag{25}$$

$$\theta_2\left(x\left(t\right)\right) = F_0^2\left(\theta_2\left(x\right)\right)\underline{\theta}_2 + F_1^2\left(\theta_2\left(x\right)\right)\bar{\theta}_2 \tag{26}$$

where

$$\begin{cases} F_0^1 & = \frac{\bar{\theta}_1 - \theta_1(x(t))}{\bar{\theta}_1 - \underline{\theta}_1}; \quad F_1^1 \quad = \frac{\theta_1(x(t)) - \underline{z}_1}{\bar{z}_1 - \underline{\theta}_1}; \\ F_0^2 & = \frac{\bar{\theta}_2 - z_2(x(t))}{\bar{\theta}_2 - \underline{\theta}_2}; \quad F_1^2 \quad = \frac{z_2(x(t)) - \underline{\theta}_2}{\bar{\theta}_2 - \underline{\theta}_2}. \end{cases} \tag{27}$$

$$\bar{\theta}_p = \max\left\{\theta_p\left(x\left(t\right)\right)\right\} \quad \text{and} \quad \underline{\theta}_p = \min\left\{\theta_p\left(x\left(t\right)\right)\right\}, \ \forall p = 1, 2$$

C. Simulation results and discussion

In this section some simulation results are shown and discussed. For comparison purpose, λ is computed from a metaheuristic approach [15] as well and the following performance indexes are considered:

- **The stability degree index (SDI)** is used to evaluate the system closed-loop stability [17].
- **The variance of control signal (VARU)** makes it possible to observe the mean value of the square deviations of u(k) from its average.
- **The rise time (RT)** is from 10% to 90%.
- **The settling time (ST)** is within 2%.
- **The overshoot (OV)** is the overshoot in the output signal.
- **The static error (SE)**.
- **The control signal energy (CSE) and the control effort energy (CEE)** [18].

The parameters used in simulations for both approaches are given in Table I. The output and control signals are shown in Fig 3 and 4 respectively. From Fig 3 and Tables II, III and VI, better results are obtained in terms of rise time, settling time and overshoot with the analytical approach. Moreover this latter one permits the TSMPC to reduce energy consumed to reach control objective. As consequence, it leads to the best trade-off between rapidity and energy consumption.

V. Conclusion

In this paper, an analytical approach has been proposed to tune constrained MPC. Its main advantages are: 1) Applicability to MIMO nonlinear controllable systems described by T-S models, 2) Guaranteed closed-loop stability, 3) Minimized cost function and 4) Limited tuning computational load. An application to the control of a simulated QTW UAV has emphasized its interest. Future work will consist in studying disturbance rejection and then in testing this approach on a real QTW UAV.

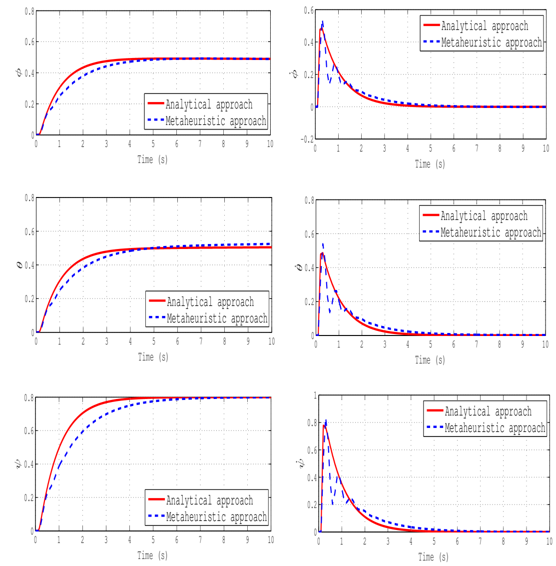

Fig. 3. Output signals vs. time

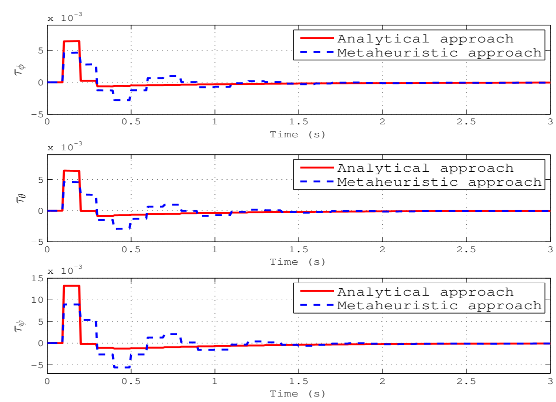

Fig. 4. Control signals vs. time

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TABLE I. TSMPC PARAMETERS

	Metaheuristic approach				Analytical approach			
	OA1	OA2	OA3	OA4	OA1	OA2	OA3	OA4
N_c	5	5	5	5	5	5	5	5
N_p	29	29	29	29	29	29	29	29
	0.9450	0.0495	0.1655	3	0.0095	0.0033	0.0133	0.051
λ	1	1.6982	2.2678	2.9962	0.008	0.0024	0.065	1.022
	0.0297	3	0.0018	2.9807	0.0067	0.005	0.0065	0.0065

TABLE II. y_1: PERFORMANCE COMPARISON

	Metaheuristic approach	**Analytical approach**
RT (s)	2.1895	1.9747
ST (s)	3.8713	2.9816
OV (%)	1.2457	0
SE	0.0285	0.0091
SDI (%)	1.7764	1.1102
VARU ($e^{-}7$)	5.4067	5.1076
CSE ($e^{-}4$)	1.6959	1.3961
CEE ($e^{-}4$)	2.9198	2.7208

TABLE III. y_2: PERFORMANCE COMPARISONS

	Metaheuristic approach	**Analytical approach**
RT (s)	2.0893	1.3022
ST (s)	3.8225	2.6328
OV (%)	2.1384	1.4898
SE	0.0149	0.0214
SDI (%)	3.1086	3.1086
VARU ($e^{-}7$)	5.3987	5.1076
CSE ($e^{-}4$)	1.6935	1.3961
CEE ($e^{-}4$)	2.9121	2.7208

TABLE IV. y_3: PERFORMANCE COMPARISONS

	Metaheuristic approach	**Analytical approach**
RT (s)	2.0358	1.3415
ST (s)	3.7998	2.6161
OV (%)	2.0401	0
SE	0.0245	0.0002
SDI (%)	1.9984	1.2273
VARU ($e^{-}7$)	5.4068	5.4026
CSE ($e^{-}4$)	1.6959	1.5961
CEE ($e^{-}4$)	2.9200	2.9107

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