2018 European Control Conference (ECC)
June 12-15, 2018. Limassol, Cyprus

Towards a robust traffic admission control in homogeneous urban vehicular networks under QoS constraints

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***Abstract*— In this paper, we consider the problem of controlling the input flow to a homogeneous urban vehicular network such that certain Quality of Service (QoS) constraints are preserved. In such a network, we model the system with two types of queues: *external* and *internal*. External queues represent vehicles waiting to enter the urban vehicular network under control, and the internal queue is used to describe the network's aggregated behavior based on the Network Fundamental Diagram (NFD). While most of the works assume perfect knowledge of the NFD describing the urban vehicular network, in reality we can only approximate it. On these grounds, by taking a model of the NFD with uncertainties, we propose a robust control design approach in order to gate input flow to a protected urban vehicular network such that travel time Quality of Service (QoS) constraints are preserved within the network. The proposed controller is compared via simulations with controllers assuming perfect knowledge of the NFD and it is shown that it can provide a larger stability region.**

I. INTRODUCTION

Traffic congestion has been a major concern in several urban areas. Such congestions result in, among others, travel time-delays, additional pollutant emissions, energy wasted, and many accidents. Since it is extremely difficult and costly to modify the existing infrastructure, advanced urban traffic control has emerged as one of the most important fields, aiming at developing (optimal) control solutions. These solutions are usually using traffic models that describe urban vehicular networks, which are based on mathematical abstractions. One such an abstraction, widely adopted as a basis for the derivation of traffic control strategies, is the concept of Network Fundamental Diagram, NFD, often called Macroscopic Fundamental Diagram (MFD). In this concept, a homogeneous traffic network is considered as a single-region model whose dynamics are appropriately described by a single NFD. Its application to experimental data is analyzed in [1]–[3]. Furthermore, [4] investigated the properties that a network should satisfy, such that an NFD with low scatter exists, by using data from a field experiment in Yokohama (Japan). It is concluded that if two traffic states from two different time intervals have the same spatial distribution of link density, then the two time intervals have the same average flow. As a result, the assumption that congestion is evenly distributed across the network made by [2] is relaxed.

Direct co-design of travel time estimation/prediction information with urban traffic control solution gives rise to improved transportation service, e.g., [5] proposes link travel time minimization in a predictive way. Ensuring performance (e.g., enforced travel time) metrics via traffic control policies is a relevant, non trivial research path, especially in case of large scale traffic networks. To ensure performance requirements in single-region traffic flow control, [6] and [7] introduced an admission control problem with multiple performance requirements: providing an upper bound on average network travel time and avoiding the blockage of external queues. These performance requirements are jointly considered and used by rolling horizon capacity maximization where performance requirements are relaxed into time-varying upper and lower hard constraints for the internal queue dynamics. The papers show a single-step receding horizon approach for relaxing the general non-convex nonlinear optimization problem to a convex problem. The modeling structure and problem formulation provide means for allocating network loads as well as prioritizing constraints and hence allocating network congestion in real time, even in cases of extreme traffic demands that the performance constraints cannot be jointly satisfied.

The aforementioned works assume that a well-defined NFD is known explicitly, which is not the case in reality. Network heterogeneity increases the scatter in the network production and also, in network outflow [8]. Furthermore, accumulation-based NFD models are inconsistent in terms of information propagation compared to trip-based models, leading to incorrect outflows. The uncertainty is proportional to the average trip length [9]. As a result, the methods developed may reach operating points that are suboptimal and hence has an altered network outflow. A control-oriented approach for handling flow inconsistencies is to formalize the uncertainty in the model and then involve it to the control design leading to a robust solution. A three-dimensional fundamental diagram is suggested for single region networks in [10], distinguishing the NFD of passenger cars and that of buses. The composition of the vehicle classes is considered an uncertain, time-varying parameter, scaling the variation of the NFD. The uncertainty is involved in a linear parameter varying (LPV) framework, for which a constant, robust feedback controller is designed. In [11] an upper-lower structure of NFD uncertainty is suggested. The uncertain dynamics are formalized in an LPV form, and control is realized by an input-constrained proportional-integral con-

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troller, the gains of which are analytically derived. A similar modeling approach is applied for multi-region networks in [12], optimizing outflow with a robust interpolated controller. The above works solve a max-min problem for optimizing network outflow, considering control input constraints, but no further constraints on the performance are considered.

In the current paper, we consider the case for which the NFD model has some uncertainties. Our main contribution is the enhancement of the admission control policy proposed in [6], [7] to consider uncertainties in NFD. More specifically, we propose a robust admission control policy that takes into account the uncertainties in the NFD. Additionally, we propose the use of queueing stability for performance analysis in traffic network systems. We show via simulations that the proposed robust admission control policy outperforms the nominal policy proposed in [6], [7] and in several occasions it can stabilize the traffic network system, while the nominal fails. If both of them fail to stabilize the traffic network system, then the robust admission control policy diverges in a slower rate than that of the nominal one.

II. Notation and Preliminaries

A. Nominal system model

The system dynamics is modeled through the conservation of vehicles for both the internal and external queues. The first state equation gives the time evolution of the number of vehicles in the protected network (representing the evolution of "internal queues") over a sample step of length T, that is,

$$N_{k+1} = \left[N_k + T \left(\sum_{i \in \mathcal{I}} q_k^{\mathrm{in},i} - \sum_{j \in \mathcal{O}} q_k^{\mathrm{out},j} \right) \right]^+, \quad (1)$$

where $[\cdot]^+$ is the maximum between zero and its argument, N_k denotes the number of vehicles, $q_k^{\mathrm{in},i}$ and $q_k^{\mathrm{out},j}$ denote the inflow at link i and outflow at link j at sample step k in unit [veh/h], respectively. $\mathcal{I}$ denotes the set of entrance queues and $\mathcal{O}$ denotes the set of exit links.

Let $q_k^{\mathrm{in}} \triangleq \sum_{i \in \mathcal{I}} q_k^{\mathrm{in},i}$ and $q_k^{\mathrm{out}} \triangleq \sum_{j \in \mathcal{O}} q_k^{\mathrm{out},j}$, equation (1) can be abstracted to a single internal queue, i.e.,

$$N_{k+1} = \left[N_k + T \left(q_k^{\mathrm{in}} - q_k^{\mathrm{out}} \right) \right]^+. \quad (2)$$

The inner network circulation and also, network outflow is modelled through the NFD concept. The total regional circulating flow $Q(N)$ in unit [veh/h] is approximated by Edie's generalized definition of flow, i.e., the weighted average of link flows multiplied with link lengths. If we assume that the average trip length Υ in the network is constant and the average link length is given by $l = M^{-1} \sum_{i=1}^{M} l_i$, for links $i \in 1, ..., M$, then the output (throughput) of the network can be expressed as [2]:

$$q_k^{\mathrm{out}} = \frac{l}{\Upsilon} Q(N_k). \quad (3)$$

Output flow q_k^{out} is the estimated rate at which vehicles complete trips per unit time either because they finish their trip within the network or because they move outside the network. This function describes steady-state behavior of single-region homogeneous networks if the input to output dynamics are not instantaneous and any delays are comparable with the average travel time across the region [13].

Network inflow q_k^{in} is considered to be the controlled input of the system that follows the admission control policy. The admittance into the network is realized by separate links for which flow characteristics by means of NFD cannot be considered, hence, they are described through a simple queuing model. For entrance gate i:

$$L_{k+1}^i = \left[L_k^i + T \left(\lambda_k^i - q_k^{\mathrm{in},i} \right) \right]^+, \quad (4)$$

where L_k^i is the queue length of the external queue and λ_k^i denotes the uncontrolled arrival rate at time k. We assume the arrival rate is an unknown, deterministic and bounded demand sequence. Summing all external queues $i \in \mathcal{I}$,

$$L_{k+1} = \left[L_k + T \left(\lambda_k - q_k^{\mathrm{in}} \right) \right]^+, \quad (5)$$

where $L_k = \sum_{i \in \mathcal{I}} L_k^i$ and $\lambda_k = \sum_{i \in \mathcal{I}} \lambda_k^i$. Note, that only a controlled number of vehicles enter the network, and no disturbance flows are present, i.e. we assume to gate all external flows entering the network for sake of simplicity. Regarding the overall system, however, λ_k is considered as disturbance.

B. Performance and control input constraints - nominal case

Similar to the classic perimeter control problem, the objective is to optimize network performance through the maximization of network throughput. Moreover, the network performance is characterized by the QoS set [7].

By specifying upper/lower bounds for the indicators, hard constraints can be given for the system. For the traffic networks, two QoS requirements are considered:

a) Average time delay in network: This indicator is modeled by the following formula:

$$\Delta(N_k) = \frac{l}{v(N_k)} - \tau_{nom}, \quad (6)$$

where l denotes the average link length of the network while $v(N_k)$ denotes the actual travel speed of the network. Nominal travel time τ_{nom} is given by $\tau_{nom} = \frac{l}{v_{free}}$, i.e., considering free flow speed v_{free} in the network.

Substituting the generalized definitions of [2], the following formula is obtained for average network speed:

$$v(N_k) = \frac{Q(N_k) l}{N_k}. \quad (7)$$

Note, that $Q(N_k)$ is chosen, such that $v(N_k)$ is an invertible function. In fact, it is intuitive that as the number of vehicles in the network N_k increases, the average speed of the network is expected to decrease. By assuming a continuously differentiable concave NFD over the eventual interval on N and network flow uniformity, invertibility of $v(N_k)$ is therefore a direct consequence of the NFD model.

The free travel speed can be approximated by the following formula:

$$v_{free} = \lim_{N_k \to 0^+} \frac{Q(N_k)}{N_k} \overset{(a)}{=} \lim_{N_k \to 0^+} \frac{\partial Q(N_k)}{\partial N_k}, \tag{8}$$

where (a) is due to L'Hôpital's rule.

Let τ_{nom} denote the nominal travel time for average link length for a vehicle traveling with v_{free} and it is equal to l/v_{free}. We require that the average time delay in the network is smaller than a threshold value, herein denoted by Δ_{tr}, i.e.,

$$\Delta(N_{k+1}) \le \Delta_{tr} \tag{9}$$

is required as control constraint.

b) Blockage of external queues: Queue blockage can be avoided by satisfying the condition

$$L_k \le L_{cap} \tag{10}$$

for all k, where L_{cap} denotes the capacity of the external queue.

c) Control input constraint: A constraint can be formalized for the admissible flow as follows:

$$0 \le q_k^{in} \le \min(\lambda_k + L_k/T, g_{\max} s) \triangleq q_k^{\text{in,ub}}, \tag{11}$$

where the maximal green time of the entering links is calculated as $g_{\max} = \sum_{i \in \mathcal{I}} g_{\max,i}$, with $g_{\max,i}$ denoting the maximal green time of input link i. The saturation flow of input links is assumed to be constant (for simplicity of exposition), and it is denoted by s.

III. Uncertainty Modeling

Suppose that the nominal NFD function is given by an n^{th}-order polynomial form: $Q(N) = \sum_{i=1}^{n} p_i N^i$. Then, uncertainty of the function can be modeled in a multiplicative manner as follows:

$$\tilde{Q}(N_k, \boldsymbol{\delta}_k) = \sum_{i=1}^{n} p_i (1 + \delta_k^i) N_k^i \ , \tag{12}$$

where $\tilde{Q}(N_k, \boldsymbol{\delta}_k)$ denotes the actual circulation in the network, and vector $\boldsymbol{\delta}_k \triangleq \left[\delta_k^0, \delta_k^1, \ldots, \delta_k^n\right]^T$ gives the uncertainty of NFD function coefficients at discrete step k.

Generally, the multiplicative error δ_k^i is considered to be bounded, i.e. $|\delta_k^i| \le \delta^{i,max}, \delta^{i,max} \in [0,1]$ $\forall i$. However, for a polynomial form, the shape of the uncertain NFD also depends on the correspondence among the δ_k^i's.

The assumptions on the uncertain NFD are as follows:

- $Q(N_{jam}) = \tilde{Q}(N_{jam}, \boldsymbol{\delta}_k) = 0$, i.e., accumulations of traffic jams is equal in the nominal and the uncertain case. Note that at $N_k = 0$, $Q(0) = \tilde{Q}(0, \boldsymbol{\delta}_k) = 0$.
- $\arg\max_N Q(N) = \arg\max \tilde{Q}(N, \boldsymbol{\delta}_k) = N_{cr}$, i.e., the critical accumulation in the nominal and uncertain cases remains equal.

The above considerations imply an upper-lower envelope structure around the nominal NFD; see Fig. 1. This shape is satisfied if $\delta_k^i = \delta_k$, $\forall i$. Hence, in what follows, uncertainty in NFD is described by the scalar parameter δ_k and $\tilde{Q}(N_k, \boldsymbol{\delta}_k) = \tilde{Q}(N_k, \delta_k) \equiv \tilde{Q}(N_k)$.

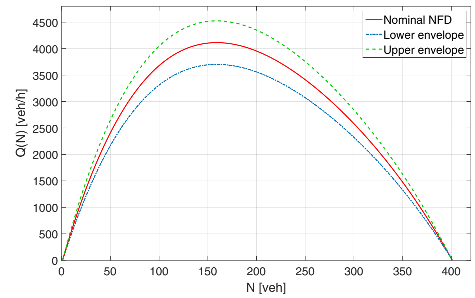


Fig. 1: *Uncertainty in a* 4^{th} *order NFD. Lower and upper envelopes are defined by* $\delta = 0.1$.

Remark 1: The uncertainty structure is similar to that of [11] and [12], but less generic than in [13]. Furthermore, the applied structure is consistent with the concept of [9], i.e., modeling flow uncertainty proportional to the trip length. In case of changing critical accumulation, the problem can still be applicable if some estimate on critical value is available. Around that estimate, we can then define flow uncertainty and perform constrained optimization.

For the robust control design, the outlined uncertainty needs to be interpreted for the state dynamics and the performance constraints. Hence, eqs. (1)-(5) and (19)-(23) are re-specified considering the multiplicative uncertainty δ_k.

A. Outflow uncertain admission model

The uncertainty influences the internal network dynamics as follows. Substituting (12) into (3), we obtain

$$\tilde{q}^{\text{out}}(N_k) \triangleq \frac{l}{\Upsilon} \tilde{Q}(N_k) = (1 + \delta_k) q^{\text{out}}(N_k). \tag{13}$$

Hence, the internal traffic network queue size is given by

$$N_{k+1} = \left[N_k + T\left(q_k^{\text{in}} - (1 + \delta_k) q^{\text{out}}(N_k)\right)\right]^+ . \tag{14}$$

Furthermore, the speed function of the internal network is modified. Substituting (12) into (7), the uncertain network average speed becomes

$$\tilde{v}(N_k) \triangleq \frac{\tilde{Q}(N_k) l}{N_k} = \frac{(1+\delta_k) Q(N_k) l}{N_k} = (1+\delta_k) v(N_k). \tag{15}$$

B. Robust control problem

The optimization problem is formulated as a one step ahead rolling one as follows:

$$\max_{q_k^{in}} \min_{\delta_k} \quad \tilde{Q}(N_{k+1}) \tag{16a}$$

$$\text{s.t.:} \quad \tilde{\Delta}(N_{k+1}) \le \Delta_{\text{tr}} \tag{16b}$$

$$L_{k+1} \le L_{\text{cap}} \tag{16c}$$

$$0 \le q_k^{\text{in}} \le q_k^{\text{in,ub}} \tag{16d}$$

$$N_{k+1} = \left[N_k + T\left(q_k^{\text{in}} - \tilde{q}^{\text{out}}(N_k)\right)\right]^+ \tag{16e}$$

$$L_{k+1} = \left[L_k + T\left(\lambda_k - q_k^{\text{in}}\right)\right], \tag{16f}$$

where $\tilde{\Delta}(N_{k+1})$ denotes the actual average delay. In the following proposition we cast optimization problem (16) as a relaxed convex optimization one; then, the state-dependent substitution of the worst-case uncertainty for the robust control design is stated.

Proposition 1: Given a single-step control horizon with constraints (16b)-(16f) on variables N_{k+1} and L_{k+1} and q_k^{in}, the QoS constrained network maximization problem can be given as a relaxed convex optimization problem:

$$\max_{N_{k+1}} \min_{\delta_k} \quad \tilde{Q}(N_{k+1}, \delta_k) \tag{17a}$$

$$\text{subject to:} \quad \tilde{N}_{k+1}^{lb}(\delta_k) \leq N_{k+1} \leq \tilde{N}_{k+1}^{ub}(\delta_k) \tag{17b}$$

from which once (17) is solved for N_{k+1}, the optimal control input q_k^{in} can be calculated by (14). •

Optimization problem (17) extends that of the optimization problem in [7] by considering uncertainty i) in the cost function through the uncertain flow function and ii) through the time-varying upper and lower bounds of N_{k+1}, which are used for calculation of optimal inflow $q^{in,opt}$.

Proof 1: The upper and lower bounds are obtained as follows. By substituting the delay function $\Delta(N_k)$ from (6) into (9) for the uncertain speed function, we get

$$\tilde{v}(N_k) \geq \frac{l}{\tau_{nom} + \Delta_{tr}}.$$

Therefore, by substituting in $\tilde{v}(N_k)$ in (15), a constant lower bound can be derived for the nominal speed (for which the function is known and it is monotonic) that includes uncertainty, i.e.,

$$v^{lb,delay} = \frac{l}{(1+\delta_k)(\tau_{nom} + \Delta_{tr})}. \tag{18}$$

Since the speed function is monotonically decreasing, the constant upper bound for the internal queue is obtained by inverting the speed function:

$$\tilde{N}_{k+1}^{ub,delay} = v^{-1}\left(\frac{l}{(1+\delta_k)(\tau_{nom} + \Delta_{tr})}\right). \tag{19}$$

Substituting the upper bound for controlled inflow q_k^{in} from (11) into the equality constraint (14) a non-constant upper bound emerges and it is given by

$$\tilde{N}_{k+1}^{ub,que} = q_k^{in,ub} + N_k - (1+\delta_k) q^{\text{out}}(N_k). \tag{20}$$

As a result, the applied upper bound for the decision variable is given as the minimum of the upper bounds found in (19) and (20), i.e.,

$$\tilde{N}_{k+1}^{ub} = \min(\tilde{N}_k^{ub,que}, \tilde{N}_k^{ub,delay}). \tag{21}$$

The lower bound for N_k aims at avoiding the blockage of the external queues. The bound is obtained by substituting (5) and (14) into (10), whereas the parametric uncertainty is involved through the outflow, i.e,

$$\tilde{N}_{k+1}^{lb,block} = N_k - (1+\delta_k) q^{\text{out}}(N_k) + L_k + \lambda_k - L_{cap}. \tag{22}$$

Note, that $N^{lb,block}$ may take negative values. Hence, the applied lower bound is given as:

$$\tilde{N}_{k+1}^{lb} = \max(0, \tilde{N}_k^{lb,block}). \tag{23}$$

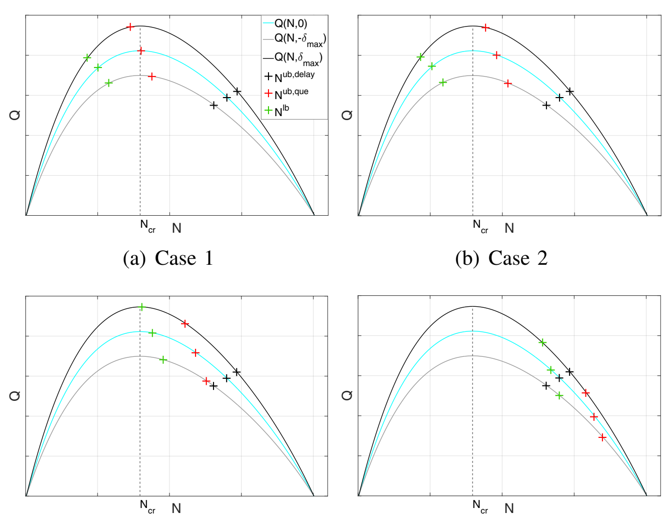
Remark 2: Note that for an extreme arrival rate λ_k it is not possible to guarantee that both performance requirements are fulfilled. Further specification of the constraints consider the flow characteristics of the inner and external dynamics.[1] In our scheme, similar to [7], priority is given to the protected network, i.e., violation of $\tilde{N}_{k+1}^{ub}$ is not permitted. Hence, when $\tilde{N}_{k+1}^{lb} \geq \tilde{N}_{k+1}^{ub}$ by choosing $\tilde{N}_{k+1}^{ub}$ to be the solution of the optimization, we relax the constraint of having $L_{k+1} \leq L_{cap}$ for the external queues. •

Remark 3: The relaxed optimization problem (17) is a max-min problem, in which δ_k is chosen such that the utility function (here $\tilde{Q}(N_{k+1}, \delta_k)$) in minimized. Given the assumptions of the actual NFD with respect to the nominal (N_{cr} remains unchanged) and the effect of the uncertainty δ_k, monotonicity in the same regions as the nominal is preserved and, hence, the uncertainty influences optimization (17) only through the bounds $\tilde{N}_{k+1}^{lb}(\delta_k)$ and $\tilde{N}_{k+1}^{ub}(\delta_k)$. Therefore, the worst-case uncertainty is obtained by restricting the domain of N_{k+1} in optimization (17).

For the analysis of the robust optimization interval, it is helpful to examine the dependence of the robust bounds on $\delta_k \in [-\delta_{max}, \delta_{max}]$ (see also any case in Fig. 2).

1. The lower bound is in indirect proportion with δ_k due to eq. (22), i.e., $\tilde{N}^{lb}(\delta_{max}) \leq \tilde{N}^{lb}(\delta_k) \leq \tilde{N}^{lb}(-\delta_{max})$.
2. The same applies to the queue-related upper bound: $\tilde{N}^{ub,que}(\delta_{max}) \leq \tilde{N}^{ub,que}(\delta_k) \leq \tilde{N}^{ub,que}(-\delta_{max})$, (20).
3. The delay related upper bound behaves the opposite way, i.e. $\tilde{N}^{ub,delay}(-\delta_{max}) \leq \tilde{N}^{ub,delay}(\delta_k) \leq \tilde{N}^{ub,delay}(\delta_{max})$, Hence, the bound is in direct proportion with δ_k which is a result of eq. (19).

Note, that the first two bounds depend on the triplet (N_k, L_k, λ_k), but the third one does not depend on the state of the external queue, L_k, or the arrival , λ_k.



(a) Case 1 (b) Case 2

(c) Case 3 (d) Case 4

Fig. 2: *Robust bounds $\tilde{N}^{lb}(\delta_k)$, $\tilde{N}^{ub,delay}(\delta_k)$ and $\tilde{N}^{ub,que}(\delta_k)$ and for three cases: the nominal NFD and its upper and lower envelopes*

To give an intuition on the effect of the bounds,

[1] Note, that the internal network flow depends on the accumulation, whereas the queuing system of the external network considers a constant flow capacity. Hence, when prioritizing networks, the former is preferred since it requires a specific state region to maintain its performance.

we provide and discuss some examples. Assuming that $\tilde{N}^{ub,delay}(-\delta_{max}) > N_{cr}$, i.e., for any uncertainty, the bounds on network delay remain larger than the critical accumulation, we discuss the following cases:

1. $\tilde{N}^{ub,que}(\delta_{max}) \leq N_{cr}$; see Fig. 2a. By choosing the upper bound on the uncertainty, the optimization interval is restricted the most, even below N_{cr}, and the robust optimum is $N_{k+1}{=}\tilde{N}^{ub,que}(\delta_{max})$.
2. $\tilde{N}^{lb}(-\delta_{max}) \leq N_{cr} \leq \tilde{N}^{ub,que}(\delta_{max})$; see Fig. 2b. In this case, the choice of δ does not influence the robust optimization interval.
3. $N_{cr} < \tilde{N}^{lb}(-\delta_{max}) < \tilde{N}^{ub,delay}(-\delta_{max})$, as in Fig. 2c, i.e., the lower bound can get above the critical density, but still does not reach the upper bound for network delays, and hence no conflict is present in QoS constraints. Then, the optimization interval gets furthest from the desired N_{cr} by choosing $\delta_k = -\delta_{max}$.
4. For $\tilde{N}^{ub,delay}(-\delta_{max}) \leq \tilde{N}^{lb}(-\delta_{max})$, the conflict in bounds is handled by relaxing the lower bound, as discussed in Remark 2. In this way, the upper bound is obtained as the solution to the optimization. However, to obtain the worst-case scenario, δ is chosen such that $\tilde{N}^{lb}(\delta) = \tilde{N}^{ub,delay}(\delta)$ and only one value of N is in the feasible set.

Note that cases 1. and 2. have to do with the performance of the traffic network when the network is under-utilized. Cases 3. and 4. have to do with the stability of the system. When the network has more vehicles that its critical number, N_{cr}, $\delta = -\delta_{\max}$ is usually the solution, apart from case 4. discussed above. The fact that the robust controller provides a larger lower bound is equivalent to having a longer external queue in the nominal case. When $\tilde{N}^{lb}$ becomes larger than $\tilde{N}^{ub}$, then the network operates at $N = \tilde{N}^{ub} < N^{lb}$. This results in a more "aggressive" control for reducing the effect of the external queue and a larger outflow.

C. Discussion on queue stability

Since priority is given to the protected network (see Remark 2), for high arrival rates, the external queue might exceed the desired upper level and keep growing if the outflow of the network is not big enough. As a result, the traffic network as a system might become unstable.

Consider the outlined network as a queuing system, the queue of which is given by $N_k{+}L_k$. The arrival rate of the overall system is λ_k and its service rate is $q^{\text{out}}(N_k)$. The queue of the network is considered rate stable [14, Definition 2.2] if

$$\lim_{k\to\infty} \frac{(N_k + L_k)}{k} = 0, \tag{24}$$

with probability 1. According to the rate stability theorem, (24) is satisfied if and only if

$$\lim_{M\to\infty} \frac{1}{M} \sum_{k=1}^{M-1} \left[\lambda_k - q_k^{\text{out}}\right] \leq 0, \tag{25}$$

where $q_k^{\text{out}} \equiv q^{\text{out}}(N_k)$. Inequality (25) states that the mean arrival rate $\bar{\lambda}$ does not exceed the mean service rate $\bar{q}^{\text{out}}$. The mean arrival rate of the system is given by

$$\bar{\lambda} = \frac{1}{M} \sum_{k=1}^{M-1} \lambda_k, \tag{26}$$

and the mean service rate of the system is given by

$$\bar{q}^{\text{out}} = \frac{1}{M} \sum_{k=1}^{M-1} q_k^{\text{out}}. \tag{27}$$

In our case, $\bar{q}^{\text{out}}$ cannot be defined precisely. The reason for this is twofold: first, the network is operating based on an optimization with time-varying bounds. Therefore, at certain cases, network is not operating at its capacity, rather, effective outflow is determined by the time-varying constraints. Second, the system is compromised by parametric uncertainty.

Compared to the nominal controller, the robust controller is expected to provide larger average service rate in case of overloading of the network and hence, extending the queue stability region of the non-robust approach. This hypothesis is investigated in the numerical simulations in Section IV.

IV. Simulation analysis

The proposed robust controller is applied for a test system, simulated in Simulink assuming a quadratic nominal NFD of the form $Q(N_k) = aN_k^2 + bN_k$. For the 4 h long simulations, the time function of NFD uncertainty is given by Fig. 3.

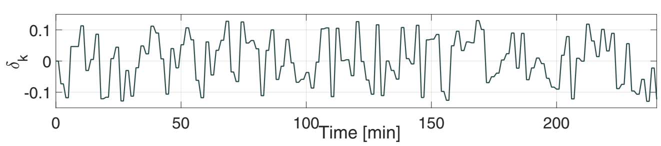


Fig. 3: *NFD uncertainty;* $\delta_{\max} = 0.125$.

The model parameters of the system are given in Table I.

Parameter	Value	Parameter	Value
p_2	-0.1	l	0.15 km
p_1	40	Υ	6 km
p_0	0	s	0.5 veh/s
N_{cr}	200	g_{max}	240s
T	60s	v_{nom}	40 km/h
L_{cap}	200 veh	τ_{nom}	13.5s
		Δ_{tr}	$1.5\tau_{nom}$

TABLE I: Model parameters

To focus on the important properties of the robust formulation, we investigate the performance of two controllers:

1. *Nominal controller.* The optimization problem (17) is solved considering no uncertainty in the NFD.
2. *Robust controller.* For the same NFD, uncertainty is considered with $\delta_{max} = 0.125$.

A comparative analysis of the two controllers is carried out through three different scenarios. For the test cases, sinusoid arrival rate is considered with the same amplitude, but with increasing mean arrival rates (see Table II). This choice aims at highlighting the improved queue stability properties of the robust approach.

Case no.	Mean arrival rate [veh/min]
1	88
2	97
3	105

TABLE II: Case study excitations

Case study no. 1. A low mean arrival rate is chosen. Although controlled inflow is different between the nominal and the robust controllers, a more oscillative behaviour (Fig. 4) is observed for the robust controller.

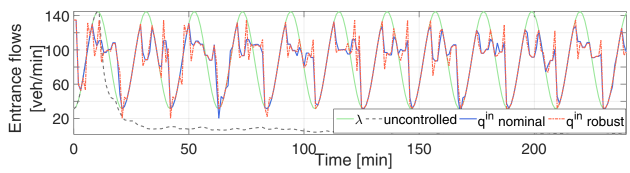


Fig. 4: *Arrival rate and inflow values for case no. 1.*

It is clear that the two controllers lead to very similar values both in internal accumulations (Fig. 5) and external queues (Fig. 6).

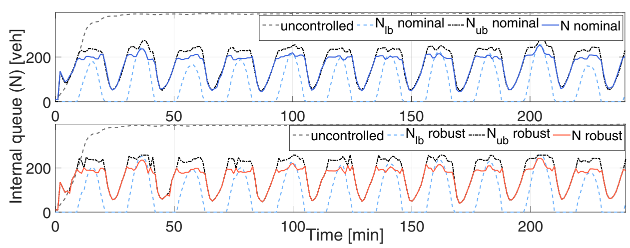


Fig. 5: *Accumulation in network for case no. 1.*

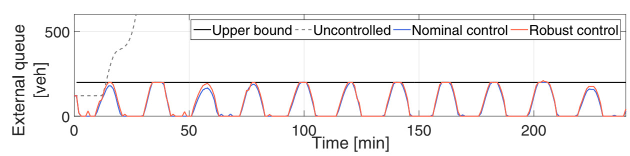


Fig. 6: *External queues for case no. 1.*

Furthermore, the service rates of the system are very similar too, as it is demonstrated in Fig. 7.

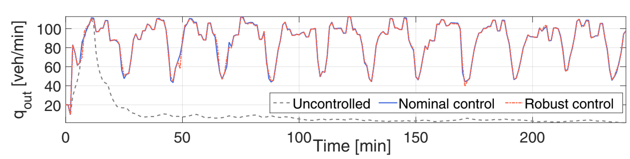


Fig. 7: *Network outflow values, case no. 1.*

The similar behavior of the two controllers can be also observed on the aggregated values, tabulated in Table III. It can be observed that the nominal control results in slightly better performance.

	Uncontrolled	Nominal control	Robust control
TTD [veh km]	$5.69{\cdot}10^5$	$5.02{\cdot}10^6$	$5.02{\cdot}10^6$
$\mathrm{TTS_{int}}$ [veh h]	1520.19	634.93	619.53
$\mathrm{TTS_{ext}}$ [veh h]	33972.17	281.48	292.89

TABLE III: Aggregated results, mean arrival rate: 88 veh/min

Case study no. 2. This study emulates a situation with an increased arrival rate, exceeding the mean service rate of the nominal case, but staying below that of the robust case.

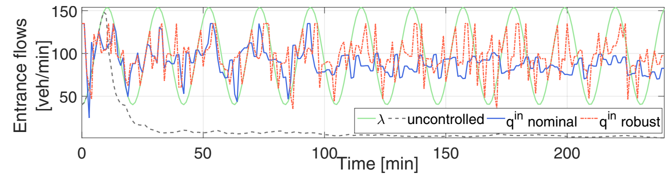


Fig. 8: *Arrival rate and inflow values for case no. 2.*

Fig. 9 highlights that after 100 mins, the internal accumulation for the nominal controller stays at the upper bound, but cannot maintain a satisfactory outflow level for the network, therefore, the external network gets blocked and the overall queue becomes unstable; see Fig. 10. The robust controller, however, even though violates the bound for queue blockage in case of extreme arrivals, according to Remark 2, it maintains a stable queue for the overall system.

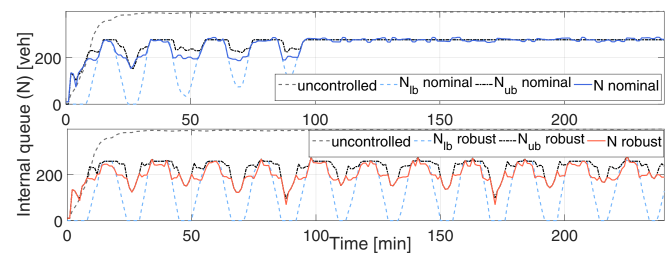


Fig. 9: *Accumulation in network for case no. 2.*

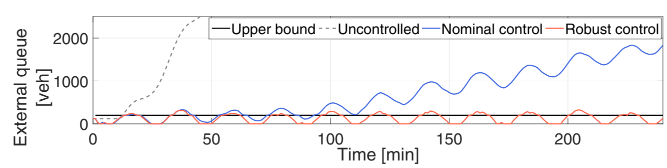


Fig. 10: *External queues for case no. 2.*

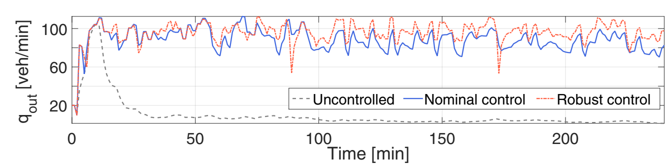


Fig. 11: *Network outflow values, case no. 2.*

The aggregated performance also highlight the queue stability differences between the two controllers. For the nominal case, external TTS gets extremely high, whereas the robust controller keeps it bounded; see Table IV.

	Uncontrolled	Nominal control	Robust control
TTD [veh km]	$5.64{\cdot}10^5$	$5.10{\cdot}10^6$	$5.51{\cdot}10^6$
$\mathrm{TTS_{int}}$ [veh h]	1524.91	1014.70	810.34
$\mathrm{TTS_{ext}}$ [veh h]	38382.62	2621.12	521.79

TABLE IV: Aggregated results, mean arrival rate: 97 veh/min

Case study no. 3. This study emulates a situation with an increased arrival rate, exceeding the mean service rate of both the nominal and robust cases. Both controllers fail to keep rate stability due to the definite overloading of the network. The two controllers show very similar behavior both in terms of the internal states and the destabilization of the external queue. When the external queue is blocked, and the internal network is overloaded as well, the approach outlined in Remark 2 takes effect. However, as opposed to the second case study, even the robust approach is not capable of stabilizing the overall queue. This leads to an oscillation in the considered worst case δ_k of case 4 ,which results is an oscillation in the input as well. Note, however, that the robust controller still leads to shorter external queues.

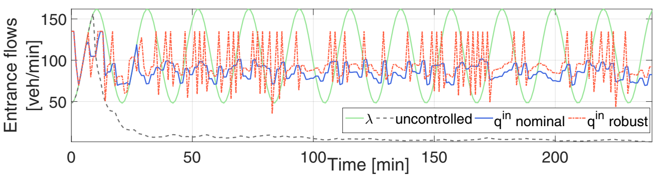


Fig. 12: *Arrival rate and inflow values for case no. 3.*

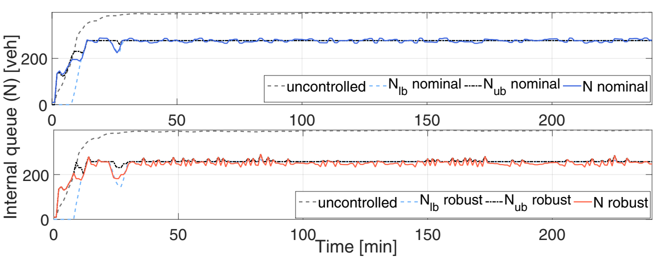


Fig. 13: *Accumulation in network for case no. 3.*

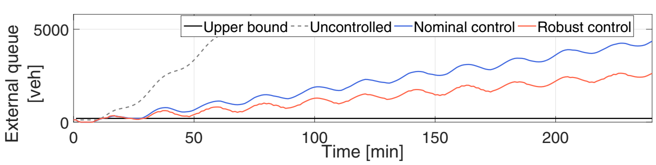


Fig. 14: *External queues for case no. 3.*

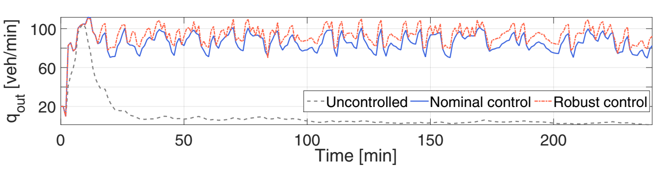


Fig. 15: *Network outflow values, case no. 3.*

The aggregated performance also highlight the performance differences between the two controllers.

	Uncontrolled	Nominal control	Robust control
TTD [veh km]	$5.58{\cdot}10^5$	$4.94{\cdot}10^6$	$5.37 \cdot 10^6$
TTS_{int} [veh h]	1528.6.78	1079.22	979.53
TTS_{ext} [veh h]	$4.23{\cdot}10^4$	8410.22	5231.61

TABLE V: Aggregated results, mean arrival rate: 105 veh/min

V. Conclusions and Future Directions

In this paper, we proposed a robust control design approach for controlling input flow to a protected urban vehicular network, such that certain Quality of Service (QoS) constraints are preserved. The NFD describing the urban vehicular network is assumed to be uncertain. The proposed controller was compared via simulations with a nominal controller assuming perfect knowledge of the NFD. It was shown numerical that under QoS constraints, the robust controller provides a larger rate stability region than the nominal approach. Furthermore, the case studies illustrate that the proposed approach has improved performance in terms of network throughput, average time delay and external queue length.

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