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Optimal Energy Reserve Procurement

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Abstract— **Uncertainties from renewables and demands create power imbalances in real-time electricity markets. This paper studies the problem of procuring reserve services in forward capacity markets from diverse resources to cover imbalance signals e^t. We consider the reserve procurement problem in two scenarios: (a) e^t reveals itself causally, (b) e^t is revealed all at once by an oracle. Each case induces an optimal resource procurement cost. The ratio between the costs in these two cases is defined as the *price of causality*. It captures the additional procurement cost from not knowing the entire imbalance signal in advance. An upper bound on the price of causality is derived, and the exact price of causality is computed in some special cases. The algorithmic basis for these computations is set containment linear programming. A mechanism is proposed to allocate the procurement cost to agents that contribute to the aggregate imbalance signal. This allocation is fair, budget-balanced, and respects the cost causation principle. Our results are validated through simulation studies, where we explore the dependence of the price of causality on unit resource prices.**

I. Introduction

Increasing penetration of renewable resources poses significant challenges in balancing supply and demand in real-time. To address the imbalance, the system operator can procure reserve services from diverse resources in forward capacity markets. These services include frequency regulation, spinning reserves, and non-spinning reserves, all of which require the resource to ramp up and down within a *fixed* capacity range over the contracted delivery window [1]. In purchasing resource capacity, the system operator often ignores dynamic constraints associated with the assets. As a result, resources can be underutilized, and the operator procures more reserves than is necessary.

What is the optimal reserve procurement that covers all imbalance signals? The purchase decision depends on the unit prices of the resources, their dynamic constraints, and critically on the strategy that allocates the imbalance signal to the procured resources. The reserve procurement decision is therefore intimately coupled with the real-time allocation strategy. Since the imbalance signal reveals itself in real-time, the allocation policy needs to be causal. The operator needs to irreversibly commit procured resources to match the imbalance signal at each time step without the luxury of knowing future values of the signal. Clearly, the reserve procurement cost under causal policies will be higher than that under arbitrary (possibly non-causal) policies. This inspires us to quantify the effect of causality on optimal reserve procurement cost.

A second issue is that of paying for the procurement cost. Ideally, the cost allocation should be fair, budget-balanced, and follow the cost causation principle, i.e., those who contribute to imbalance signals are penalized, and those who mitigate imbalance signals are rewarded. We explore cost allocation mechanisms that satisfy these requirements.

A. Related Work

Some works in the literature are closely related to our paper. Subramanian *et al.* studied real-time scheduling for a collection of distributed energy resources in [2]. Madjidian et al. discovered the trade-off between absorbing and releasing energy for collective loads under causal allocation policies in [3]. These works fixed the resource, and considered the optimal causal policy to address the trade-off that arises due to the lack of resources. In contrast, we investigate how to procure enough resource so that their trade-off does not arise in the first place.

Another strand of related work is online optimization [4], [5]. It considers sequential decisions made at each step without secure information about the future. This problem is widely studied in many areas, such as stochastic dynamic programs [6], communication networks [7], renewable energy integration [8], [9], among others. An interesting concept is the competitive ratio [4]. It compares the performance of the optimal online algorithm, where all information is revealed causally, and the performance of the optimal offline algorithm, which is an unrealizable algorithm with full information about the future. This problem is different from our paper in the following manner. Online optimization fixes the resource and chooses the optimal causal policy that optimizes the cost. In contrast, we consider a resource adequacy problem, where enough resources need to be procured to guarantee a feasible allocation exists.

B. Our Contribution

This paper studies the optimal reserve procurement problem. We consider an operator that procures diverse resources to

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cover imbalance signals revealed causally over time. The major contributions of the paper are as follows:

- *Formulate the Price of Causality*: We model the imbalance signals and resources as polytopes. Then, the optimal reserve procurement problem is formulated as optimization under polytope containment constraints. We study the optimal procurement cost under causal policies and arbitrary (possibly non-causal) policies, respectively. The ratio between the two costs is defined as *the price of causality*. It quantifies the additional cost for not knowing the information in the future.
- *Characterize the Price of Casualty*: We show that the optimal reserve procurement cost can be obtained if the allocation policy is arbitrary (possibly non-causal). When the policy is causal, we derive an upper bound on the cost by optimizing over affine causal policies only. In some special cases, the procurement costs for both causal and non-causal cases are available, and the exact price of causality can be derived accordingly. The algorithmic basis for all computations is linear programming.
- *Cost Allocation*: We propose a cost allocation mechanism to allocate the reserve procurement cost. We show that the cost allocation is fair, budget balanced, and respects the cost causation principle, i.e., the imbalance-creating individuals are penalized and imbalance-mitigating individuals are rewarded.

II. THE RESERVE PROCUREMENT PROBLEM

Consider a system operator in a power system, who procures resources (generators, batteries, etc.) to balance demand and supply in real-time within a finite horizon T. At the beginning of the horizon, the operator determines the resource procurement so that all possible imbalance signals are covered at the minimum cost. The rest of this section presents the formulation of this problem.

A. Modeling Imbalance Signals and Resources

Consider a sequence of power imbalance signals over time T, i.e., $e = (e^1, \dots e^T)$, where e^t is the imbalance signal at t. Assume that e only takes value in the set E, and E satisfies the following conditions:

Assumption 1: E is a bounded and convex polytope in $\mathbb{R}^T$.

To justify this assumption, E can be either viewed as the support of the distribution of imbalance signals, or the confidence interval such that $e \in E$ with probability $1-\epsilon$.

Diverse resources are available in the forward capacity market. Examples of these resources are batteries, generators, aggregation of thermostatically controlled loads [10], among others. Each resource has some operation constraints, such as maximum charging/discharging rates for batteries and ramping constraints for generators. Therefore, the feasible energy output of each resource can be characterized by a set determined by these constraints. Denote s_i^t as the energy output of resource i at time t, and define the energy output sequence as $s_i = (s_i^1, \dots, s_i^T)$. We assume that s_i takes value in the set S_i, which satisfies the following assumption:

Assumption 2: S_i is a bounded and convex polytope in $\mathbb{R}^T$, and $0 \in S_i$ for all $i = 1, \dots, N$.

The polytope assumption is a consequence of linear constraints of the resource, and $0 \in S_i$ trivially holds if the operator is allowed to disconnect resource i from the grid.

B. Problem Formulation

We refer to S_i as the *unit resource* i, and associate a per unit price p_i to each unit resource i. When α_i units of resource i are purchased, the cost is $\alpha_i p_i$, and the feasible energy output is in the set $\alpha_i S_i$. The resource procurement problem concerns how much units to purchase so that all imbalance signals are covered. We formulate this problem as follows:

$$J^* = \min_{\alpha_1,\dots,\alpha_N} \sum_{i=1}^N \alpha_i p_i \tag{1}$$

$$\begin{cases} E \subseteq \alpha_1 S_1 \oplus \alpha_2 S_2 \oplus \cdots \oplus \alpha_N S_N, & \text{(2a)} \\ \alpha_i \geq 0, \quad \forall i = 1,\dots,N, & \text{(2b)} \end{cases}$$

where J^* is the optimal value of (1), and $\oplus$ denotes the Minkowski sum of sets, i.e., $A \oplus B = \{a+b | a \in A, b \in B\}$. The polytope containment constraint (2a) dictates that any imbalance signal $e \in E$ can be covered by the procured resources collectively.

Constraint (2a) is intimately coupled with an allocation problem: there is a decomposition of e so that $e = e_1 + \cdots + e_N$, and e_i is feasible for all i, i.e., $e_i \in \alpha_i S_i$. However, in real-time market, the imbalance signal reveals itself *causally*. This indicates that the operator has to make an irreversible allocation decision at each time without knowing the imbalance signals in the future. To capture this causal constraint, we define a causal allocation policy as $\gamma_i^t : \mathbb{R}^t \to \mathbb{R}$. At time t, the policy maps $(e^1, \dots, e^t)$ to an energy allocation to resource i at time t. For notation convenience, define $\gamma^t(\cdot) = (\gamma_1^t(\cdot), \dots, \gamma_N^t(\cdot))$ and $\gamma(\cdot) = \big(\gamma^1(\cdot), \dots, \gamma^T(\cdot)\big)$. Under the causal constraint, the reserve procurement problem can be written as follows:

$$J^{**} = \min_{\alpha_1,\dots,\alpha_N,\gamma(\cdot)} \sum_{i=1}^N \alpha_i p_i \tag{3}$$

$$\begin{cases} e^t = \displaystyle\sum_{i=1}^N \gamma_i^t(e^1,\dots,e^t), \quad \forall t = 1,\dots,T, & \text{(4a)} \\ \big(\gamma_i^1(\cdot). \dots, \gamma_i^T(\cdot)\big) \in \alpha_i S_i, \quad \forall i = 1,\dots,N, & \text{(4b)} \\ \alpha_i \geq 0, \quad \forall i = 1,\dots,N, & \text{(4c)} \\ \forall e \in E, & \text{(4d)} \end{cases}$$

where J^{**} is the optimal value and $e = (e^1, \dots, e^T)$. This is a joint optimization over the procurement decision and the causal allocation policy, where (4a) ensures the policy is an allocation of e^t, (4b) ensures the allocation to each resource is feasible, and (4d) guarantees all signals in E are covered.

The reserve procurement problem (3) reduces to (1) if $\gamma_i^t(\cdot)$ is permitted to be non-causal. Therefore, the constraint (4) is more restrictive than (2), and $J^{**} \geq J^*$. This means imposing the allocation policy to be causal incurs an increase in the reserve procurement cost from J^* to J^{**}. To capture this phenomena, we define the price of causality as follows:

$$\mathrm{PoC} = \frac{J^{**}}{J^*}. \tag{5}$$

This is the ratio between the optimal procurement cost under causal allocation policies and that under arbitrary policies. Therefore, it captures the additional cost for not knowing the information in the future.

This paper focuses on the optimal reserve procurement problem (1) and (3), and addresses the following questions:
Does causality really have a price?
How to compute the price of causality?
Who pays for the reserve procurement cost?

III. A Motivating Example

Causality *does* have a cost. In this section, we construct an example to show that PoC > 1 is possible.

Consider an example with two batteries to cover an imbalance signal in three periods. The signal is contained in a set E, defined as the convex hull of $(0,0,0)$, $(1,1,-2)$ and $(1,1,4)$. Each unit battery i has a price p_i, where $p_1 = 3$ and $p_2 = 1$. In addition, each battery i has a capacity constraint C_i and a maximum charging/discharging rate r_i. Let $C_1 = C_2 = 3$, $r_1 = 3$ and $r_2 = 1$. Assume all batteries are empty at time 0, then the feasible energy output of the first battery satisfies the following linear constraints:

$$\begin{cases} 0 \leq s_1^1 \leq 3; \\ 0 \leq s_1^1 + s_1^2 \leq 3; \\ 0 \leq s_1^1 + s_1^2 + s_1^3 \leq 3, \\ -3 \leq s_1^t \leq 3, \quad \forall t = 1,2,3. \end{cases} \tag{6}$$

Similarly, the feasible energy output of the second battery is as follows:

$$\begin{cases} 0 \leq s_1^1 \leq 3; \\ 0 \leq s_1^1 + s_1^2 \leq 3; \\ 0 \leq s_1^1 + s_1^2 + s_1^3 \leq 3, \\ -1 \leq s_1^t \leq 1, \quad \forall t = 1,2,3. \end{cases} \tag{7}$$

According to (6) and (7), the set of feasible output S_i is a bounded polytope, and clearly, $0 \in S_i$. In this example, the reserve procurement problem without causal constraint can be formulated as follows:

$$\min_{\alpha_1, \alpha_2} 3\alpha_1 + \alpha_2 \tag{8}$$

$$\begin{cases} E \subseteq \alpha_1 S_1 \oplus \alpha_2 S_2 & \text{(9a)} \\ \alpha_1 > 0, \alpha_2 > 0, & \text{(9b)} \end{cases}$$

where E is the convex hull of $(0,0,0)$, $(1,1,-2)$, and $(1,1,4)$, and S_1 and S_2 are defined by (6) and (7), respectively.

Proposition 1: The polytope containment problem (8) has a unique solution $\alpha_1^* = \alpha_2^* = 1$.

Proof: The idea is as follows. We first show that $J^* \geq 4$, and $\alpha_1 = \alpha_2 = 1$ is the only possible solution that attains $J^* = 4$. Second, we show that $\alpha_1 = \alpha_2 = 1$ satisfies the polytope containment constraint (9a).

To cover $(1,1,4)$, the total maximum rate $\alpha_1 r_1 + \alpha_2 r_2$ is at least 4 (otherwise covering e^3 is not possible). Therefore, a necessary condition is:

$$3\alpha_1 + \alpha_2 \geq 4. \tag{10}$$

In addition, to cover $(1,1,4)$, the total capacity $\alpha_1 C_1 + \alpha_2 C_2$ is at least $1+1+4$. Therefore, another necessary condition is:

$$3\alpha_1 + 3\alpha_2 \geq 6. \tag{11}$$

Combining (10) and (11), the optimal value to the following problem is an upper bound for J^*:

$$\min_{\alpha_1, \alpha_2} 3\alpha_1 + \alpha_2 \tag{12}$$

$$\begin{cases} 3\alpha_1 + \alpha_2 \geq 4, & \text{(13a)} \\ 3\alpha_1 + 3\alpha_2 \geq 6. & \text{(13b)} \end{cases}$$

The optimal value of (12) is 4, and the unique solution is $\alpha_1 = \alpha_2 = 1$. Therefore, $\alpha_1 = \alpha_2 = 1$ is necessary.

Next we show that $\alpha_1 = \alpha_2 = 1$ is sufficient, i.e., the polytope containment constraint (9a) is satisfied. To see this, note that both E and $S_1 \oplus S_2$ are convex. Therefore, (9a) is equivalent to the vertices of E contained in $S_1 \oplus S_2$, which is clearly true. This completes the proof. ■

To show that PoC > 1, it suffices to prove the operator can not find any causal policy to cover all signals in E when $\alpha_1 = \alpha_2 = 1$. Therefore, he has to spend more than J^* to cover the signals causally.

Proposition 2: Let $\alpha_1 = \alpha_2 = 1$, then given any causal allocation policy $\gamma(\cdot)$, at least one of the imbalance signals $(1,1,-2)$, $(1,1,4)$ can not be balanced under $\gamma(\cdot)$.

Proof: The first two elements of $(1,1,-2)$ and $(1,1,4)$ are the same. Since $\gamma(\cdot)$ is causal, the allocation within the first two periods should also be the same for $(1,1,-2)$ and $(1,1,4)$. To cover the signal $(1,1,4)$, the unique allocation is to exclusively use the second battery for the first two periods, and then use both batteries at the third period, i.e., $\gamma_1^1(\cdot) = \gamma_1^2(\cdot) = 0$, $\gamma_1^3(\cdot) = 3$, and $\gamma_2^1(\cdot) = \gamma_2^2(\cdot) = \gamma_2^3(\cdot) = 1$. On the other hand, if we exclusively use the second battery for the first two periods, then $(1, 1, -2)$ cannot be covered, since at time 3, the maximum discharging rate of the second battery is $1 < 2$. This completes the proof. ■

IV. Computing The Price of Causality

This section studies how to compute the price of causality. We show that both J^* and the upper bound of J^{**} can be obtained by linear programming. This provides an upper bound on the price of causality. In addition, some interesting special cases are discussed, where the exact price of causality can be computed.

A. Upper Bound on The Price of Causality

1) Solving J^* by linear programming: Here we show that the polytope containment problem (1) can be transformed to a linear program. To see this, we represent E as the convex hull of a set of vertices, i.e., $E = conv(v_1, \dots, v_m)$, and denote S_i as the intersection of half-spaces $S_i = \{s_i \in \mathbb{R}^T | A_i s_i \le B_i\}$. Since $0 \in S_i$, each element of vector B_i is non-negative, and $\alpha_i S_i = \{s_i \in \mathbb{R}^T | A_i s_i \le \alpha_i B_i\}$

Remark 1: The polytopes E and S_i are represented in different ways: E is a convex full of vertices (vertex representation), and S_i is the intersection of half-spaces (H-representation). These two representations are equivalent, since we can transform from one to another. However, interestingly, the difficulty of solving (1) crucially depends on the choice of polytope representations.

Since E is convex, and the Minkowski sum of $\alpha_i S_i$ is also convex, the set containment constraint (2a) is equivalent to the following conditions:

$$v_j \in \alpha_1 S_1 \oplus \alpha_2 S_2 \oplus \dots \oplus \alpha_N S_N, \quad \forall j = 1, \dots, m. \tag{14}$$

This dictates that each vertex v_j can be feasibly allocated to the resources. Denote $s_{i,j} \in \mathbb{R}^T$ as the allocation of v_j to the ith resource, then the constraint (14) can be written as follows:

$$\begin{cases} v_j = \sum_{i=1}^N s_{i,j}, \quad \forall j = 1, \dots, m, \\ s_{i,j} \in \alpha_i S_i, \quad \forall i = 1, \dots, N, \forall j = 1, \dots, m \end{cases}$$

The reserve procurement problem (1) can be then transformed to the following linear program:

$$\min \sum_{i=1}^N \alpha_i p_i \tag{15}$$

$$\begin{cases} v_j = \sum_{i=1}^N s_{i,j}, \quad \forall j = 1, \dots, m, & (16a) \\ A_i s_{i,j} \le \alpha_i B_i, \quad \forall i, j, & (16b) \\ \alpha_i \ge 0, \quad \forall i = 1, \dots, N. & (16c) \end{cases}$$

This is a linear program with decision variable $\alpha_1, \dots, \alpha_N$ and $s_{i,j}$, $\forall i, j$.

2) Upper bound of J^{**} by linear programming: Under causal constraint, the reserve procurement problem (3) is essentially a robust optimization over functions [11]. This is a rather challenging problem in general. Here we restrict the causal allocation policy to be affine, and derive an upper bound on J^{**} by solving a linear program.

Definition 1: The causal allocation policy $\gamma(\cdot)$ is affine if for any i and t, there exist $C_i^t \in \mathbb{R}^t$ and $D_i^t \in \mathbb{R}$ such that $\gamma_i^t(e_1, \dots, e_t) = [e_1, \dots, e_t] C_i^t + D_i^t$.

Since $\gamma_i^t(\cdot)$ takes signals up to t as the input, the size of C_i^t increases with respect to t. As the causal policy is affine, we show that the reserve procurement problem (3) can be transformed to a linear program:

Theorem 1: Assume that $\gamma(\cdot)$ is affine, then the reserve procurement problem (3) can be transformed to the following linear program:

$$\min \sum_{i=1}^N \alpha_i p_i \tag{17}$$

$$\begin{cases} v_j^t = \sum_{i=1}^N [v_j^1, \dots, v_j^t] C_i^t + D_i^t, \quad \forall j, t, & (18a) \\ A_i [\gamma_i^1(v_j^1), \dots, \gamma_i^T(v_j^1, \dots, v_j^T)]^T \le \alpha_i B_i, & (18b) \\ \alpha_i \ge 0, \quad \forall i = 1, \dots, N, & (18c) \end{cases}$$

where the decision variable is $\alpha_1, \dots, \alpha_N$, C_i^t and D_i^t for $\forall i, t$.

Proof: Since the allocation policy is linear, the reserve procurement problem can be written as follows:

$$\min \sum_{i=1}^N \alpha_i p_i \tag{19}$$

$$\begin{cases} e^t = \sum_{i=1}^N [e^1, \dots, e^t] C_i^t + D_i^t, \quad \forall t, & (20a) \\ A_i [\gamma_i^1(e^1), \dots, \gamma_i^T(e^1, \dots, e^T)]^T \le \alpha_i B_i, & (20b) \\ \alpha_i \ge 0, \quad \forall i = 1, \dots, N, & (20c) \\ \forall e \in E. & (20d) \end{cases}$$

Note that constraints (20a)-(20c) is linear with respect to e. Therefore, (20a)-(20c) holds for all $e \in E$ if and only if it holds for all vertices of E. This indicates that (20) is equivalent to (18), which completes the proof. ■

Based on Theorem 1, the solution to (17) provides a causal allocation policy that is optimal over all affine causal policies. The optimal value of (17) provides an upper bound on J^{**}.

B. Special Cases with Exact Price of Causality

In some interesting special cases, the exact price of causality can be obtained.

1) Resources in a Hyper-Rectangle: First, assume that S_i is a hyper-rectangle: $S_i = \{s_i \in \mathbb{R}^T | \underline{s}_i^t \le s_i^t \le \bar{s}_i^t, \forall t\}$. Under Assumption 1 and Assumption 2, the price of causality is 1:

Proposition 3: For $\forall i = 1, \dots, N$, let S_i be a hyper-rectangle: $S_i = \{s_i \in \mathbb{R}^T | \underline{s}_i^t \le s_i^t \le \bar{s}_i^t, \forall t\}$. Under Assumption 1 and Assumption 2, we have PoC $= 1$.

Proof: Let $\alpha^* = (\alpha_1^*, \dots, \alpha_N^*)$ be the optimal solution to the resource procurement problem (1). It suffices to construct a causal policy that covers all $e \in E$ under α^*. To this end, define $\big(\gamma_1^t(e^{1:t}), \dots, \gamma_N^t(e^{1:t})\big)$ as any vector that:

$$\begin{cases} e^t = \sum_{i=1}^N \gamma_i^t(e^{1:t}), \quad \forall t, \\ \alpha_i^* \underline{s}_i^t \le \gamma_i^t(e^{1:t}) \le \alpha_i^* \bar{s}_i^t, \quad \forall i, t. \end{cases}$$

The above allocation policy is causal, and it exists. If it does not exist, then it indicates that there is some t such that $e^t < \sum_{i=1}^N \alpha_i^* \underline{s}_i^t$ or $e^t > \sum_{i=1}^N \alpha_i^* \bar{s}_i^t$. This contradicts (2a). For the same reason, it covers all $e \in E$. This completes the proof. ■

Proposition 3 indicates that the dynamics of resources cause the price of causality to be greater than 1.

2) Signals in the Minkowski Sum of S_i: Consider the resource to be a group of batteries. Each battery i has a capacity constraint C_i and a maximum charging/discharging rate r_i. Denote the initial state of charge for the ith battery (in percentage value) as θ_i, then S_i contains all $s_i \in \mathbb{R}^T$ that satisfy the following constraints:

$$\begin{cases} -r_i \le s_i^t \le r_i, \quad \forall t, & (21) \\ 0 \le \theta_i C_i + \sum_{k=1}^{t} s_i^k \le C_i, \quad \forall i, t, & (22) \end{cases}$$

where (21) is the charging/discharging rate constraint, and (22) is the capacity constraint.

Assume E is the Minkowski sum of S_i, i.e., $E = S_1 \oplus S_2 \oplus \cdots \oplus S_N$. We show that J^{**} can be obtained by solving a linear program.

Theorem 2: Assume S_i is determined by (21) and (22), and $\sum_{i=1}^N C_i \le 2\sum_{i=1}^N r_i$. Let $E = S_1 \oplus S_2 \oplus \cdots \oplus S_N$, then there exists $T_0 > 0$, so that for $\forall T \ge T_0$, J^{**} is the optimal value for the following linear program:

$$\min_{\alpha_1, \dots, \alpha_N} \sum_{i=1}^N \alpha_i p_i \tag{23}$$

$$\begin{cases} \sum_{i=1}^N \alpha_i r_i \ge \sum_{i=1}^N r_i, & \text{(24a)} \\ \sum_{i=1}^N \alpha_i \min(C_i, 2r_i) \ge \sum_{i=1}^N C_i, & \text{(24b)} \end{cases}$$

where $\min(a, b)$ is the lesser of a and b.

The proof of Theorem 2 is long, so we defer it to the journal version of this draft. Under the assumption of Theorem 2, we can solve (15) and (23) to derive J^* and J^{**}, respectively. This gives the exact price of causality.

V. A Cost Allocation Mechanism

The reserve procurement cost J^{**} is caused by balancing the real-time supply and demand. Therefore, it should be allocated to those who contribute to imbalance signals. This section considers the cost allocation problem.

Consider an electricity market with I players (consumers, generators, etc). Each player commits a day-ahead energy and has a real-time deviation from the committed energy. Denote the deviation as a vector $d_i \in \mathbb{R}^T$, then the imbalance signals are the sum of deviations, i.e., $e = \sum_{i=1}^N d_i$.

Let $\pi_i : \mathbb{R}^T \times \mathbb{R}^T \to \mathbb{R}$ be the cost allocation mechanism, and write it as $\pi_i(d_i, e)$. This mechanism maps the deviation of i and the imbalance signal to the cost allocation to i. Ideally, a just and reasonable cost allocation should satisfy equity, budget balance and cost causation principles. Each of these axioms are formally defined as follows:

Axiom 1 (Equity): The cost allocation $\pi_i(d_i, e)$ is equal if players with the same deviation have the same cost allocation, i.e., if $d_i = d_j$, then $\pi_i(d_i, e) = \pi_j(d_j, e)$.

Axiom 2 (Budget Balance): The cost allocation $\pi_i(d_i, e)$ is budget balanced, i.e., $\sum_{i=1}^I \pi_i(d_i, e) = J^{**}$.

Axiom 3 (Penalty for Cost Causation): Those who cause imbalances should pay for it, i.e., if $d_i \cdot e > 0$, $\pi_i(d_i, e) > 0$.

Axiom 4 (Reward for Cost Mitigation): Those who mitigate imbalances should be rewarded, i.e., if $d_i \cdot e < 0$, $\pi_i(d_i, e) < 0$.

Axiom 3 and Axiom 4 dictates that those who contribute to the imbalance should be penalized, and those who mitigate the imbalance signal should be rewarded. This is referred to as the cost causation principle [12].

We show the following allocation satisfies all the axioms:

Proposition 4: The cost allocation mechanism $\pi_i(d_i, e) = (d_i \cdot e)\dfrac{J^{**}}{||e||^2}$ satisfies Axiom 1-4.

The proof of Proposition 4 easily follows from the definition

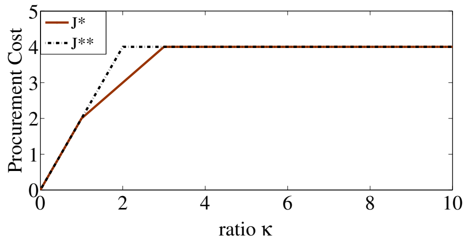

Fig. 1. The procurement costs under different price ratio κ.

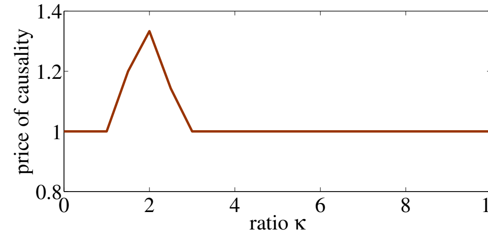

Fig. 2. The price of causality under different price ratio κ.

of the axioms, and is thus omitted.

VI. Case Studies

In the case study, we consider a reserve procurement problem with two batteries in three time periods. The capacity of the batteries are $C_1 = 1$ and $C_2 = 3$, and the maximum charging/discharging rates are $r_1 = r_2 = 1$. Assume that the initial state of charge of each battery is 50%, then S_1 is the set of $s_1 \in \mathbb{R}^3$ such that:

$$\begin{cases} -1 \le s_1^t \le 1, \quad \forall t = 1, 2, 3. \\ 0 \le 0.5 + s_1^1 + s_1^2 + s_1^3 \le 1. \end{cases}$$

S_2 is the set of $s_2 \in \mathbb{R}^3$ such that:

$$\begin{cases} -1 \le s_2^t \le 1, \quad \forall t = 1, 2, 3. \\ 0 \le 1.5 + s_2^1 + s_2^2 + s_2^3 \le 3. \end{cases}$$

Assume the signals are contained in the Minkowski sum of S_1 and S_2, i.e., $E = S_1 \oplus S_2$.

In the sequel, we study how the price vector (p_1, p_2) affects the price of causality. To this end, define $\kappa = p_2/p_1$ as the ratio between the two prices, we will calculate the price of causality under different κ.

Since all assumptions in Theorem 2 are satisfied (with $T_0 = 2$) for the case studied in this section, for any fixed κ, we can compute J^* and J^{**} by solving (15) and (23), respectively. In the simulation, we fix $p_1 = 1$, change κ from 0 to 10 at a step of 0.5, then compute the price of causality for the reserve procurement problem under each κ. The procurement cost J^* and J^{**} are shown in Figure 1, and the price of causality under different κ is shown in Figure 2. It is interesting to note that when κ is too small or too big ($\kappa < 1$ or $\kappa > 3$), one of the two batteries is getting too expensive, thus only the other battery is procured, i.e., either $\alpha_1 = 0$ or $\alpha_2 = 0$. In this case, the allocation problem is trivial, and the price of causality is 1. On the other hand, when κ takes some intermediate value ($1 \le \kappa \le 3$), both batteries are procured, which gives rise to the price of causality. An interesting future research direction would be to study under what price, causality affects the reserve procurement cost, i.e., the prices for which PoC > 1.

VII. Conclusion

This paper studies the optimal reserve procurement problem. We consider an operator that purchases diverse resources from a forward capacity market to match an imbalance signal revealed causally over time. We show that causality incurs an additional resource procurement cost, and the resulting price of causality is upper bounded. The upper bound can be derived by computing a linear program, and the exact price of causality is available in some special cases with batteries. A cost-allocation mechanism is proposed, which satisfies the equity, budget balance, and cost causation axioms. Simulation results show interesting dependence of the price of causality on the resource prices, and future research includes sensitivity analysis on other resource parameters, computing the exact price of causality, and applying the concept to other research areas.

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