

ЛНГ (нормальное)

$$\frac{\sum \xi_i - n \cdot \mu_\xi}{\sqrt{n \delta_\xi}} \sim N(0, 1)$$

$$\frac{\bar{x} - \mu_\xi}{\sqrt{\delta_\xi}} \sqrt{n} \sim N(0, 1)$$

$$\begin{aligned}\mu_\xi &= \int_{-\infty}^{\infty} x dF(x) = \int_{-\infty}^{\infty} x p(x) dx = \int_{-\infty}^{\infty} x e^{-x/\delta} dx = \\ &= \int_0^{\infty} x \cdot e^{-x/\delta} dx = \int_0^{\infty} -x \cdot d(e^{-x/\delta}) = -x \cdot e^{-x/\delta} \Big|_0^{\infty} + \int_0^{\infty} e^{-x/\delta} dx \\ &= -x \cdot e^{-x/\delta} \Big|_0^{\infty} + (-e^{-x/\delta}) \Big|_0^{\infty} = 0 - (0 - 1) = 1\end{aligned}$$

$$\begin{aligned}\mu_\xi^2 &= \int_0^{\infty} x^2 e^{-x/\delta} dx = \int_0^{\infty} -x^2 d(e^{-x/\delta}) = -x^2 \cdot e^{-x/\delta} \Big|_0^{\infty} + \\ &+ \int_0^{\infty} x e^{-x/\delta} dx = 0 + 2 = 2\end{aligned}$$

$$\delta_\xi^2 = \mu_\xi^2 - (\mu_\xi)^2 = 2 - 1 = 1$$

$$\frac{\bar{x} - 1}{\sqrt{1}} \sqrt{25} \sim N(0, 1) \Rightarrow \bar{x} \sim N(1; \frac{1}{25})$$