

ЦПТ (простейшая)

$$\frac{\sum \xi_i - n \cdot \mu_\xi}{\sqrt{n \sigma_\xi}} \sim N(0,1)$$

$$\frac{\bar{x} - \mu_\xi}{\sqrt{\sigma_\xi}} \sqrt{n} \sim N(0,1)$$

$$\begin{aligned} \mu_\xi &= \int_{-\infty}^{\infty} x dF(x) = \int_{-\infty}^{\infty} x p(x) dx = \int_0^{\infty} x e^{-x} dx = \\ &= \int_0^{\infty} x \cdot e^{-x} dx = \int_0^{\infty} -x \cdot d(e^{-x}) = -x \cdot e^{-x} \Big|_0^{\infty} + \int_0^{\infty} e^{-x} dx \\ &= -x \cdot e^{-x} \Big|_0^{\infty} + (-e^{-x}) \Big|_0^{\infty} = 0 - (0 - 1) = 1 \end{aligned}$$

$$\begin{aligned} \mu_\xi^2 &= \int_0^{\infty} x^2 e^{-x} dx = \int_0^{\infty} -x^2 d(e^{-x}) = -x^2 \cdot e^{-x} \Big|_0^{\infty} + \\ &+ 2 \int_0^{\infty} x e^{-x} dx = 0 + 2 = 2 \end{aligned}$$

$$\sigma_\xi^2 = \mu_\xi^2 - (\mu_\xi)^2 = 2 - 1 = 1$$

$$\frac{\bar{x} - 1}{\sqrt{1}} \sqrt{25} \sim N(0,1) \Rightarrow \bar{x} \sim N(1; \frac{1}{25})$$