

Partially Optimal Cubic Subspace Clustering

Research Project Machine Learning

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- 1 Introduction
- 2 Partial Optimality for Cubic Clique Partition Problem
- 3 Cubic Subspace Instance Construction
- 4 Experiments and Evaluation
- 5 Conclusion

Problem Statement (1)

Finite sample set S , cost function $c: \binom{S}{3} \rightarrow \mathbb{R}$.

Instance of the **Cubic Clique Partition Problem**:

$$\min_{y: \binom{S}{2} \rightarrow \{0,1\}} \sum_{abc \in \binom{S}{3}} c_{abc} y_{ab} y_{bc} y_{ac}$$

subject to $y_{ab} + y_{bc} - 1 \leq y_{ac}$ for all distinct $a, b, c \in S$.

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Find a **partially optimal solution**, i.e. fix some labels y_{ab} for distinct $a, b \in S$

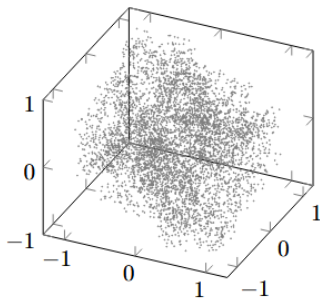
$$\begin{cases} y_{ab} = 1 & \text{join } a, b \\ y_{ab} = 0 & \text{cut } a, b \\ y_{ab} = ? & \text{unknown} \end{cases}$$

in such way that there still exists an optimal solution.

Problem Statement (2)

Subspace Instances of the Cubic Clique Partition Problem

Samples S : points $S \subset \mathbb{R}^3$



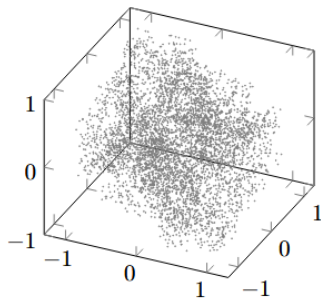
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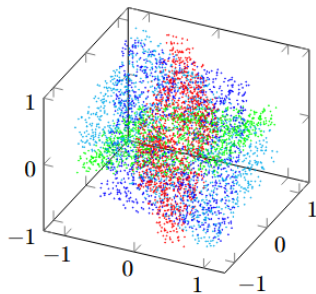
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(b) Optimal labeling y^*

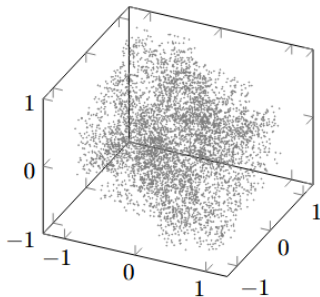
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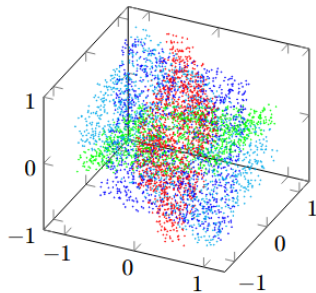
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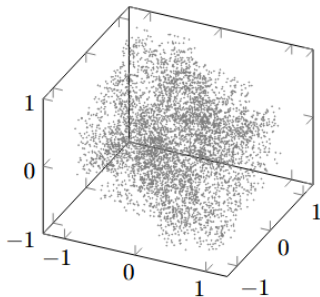
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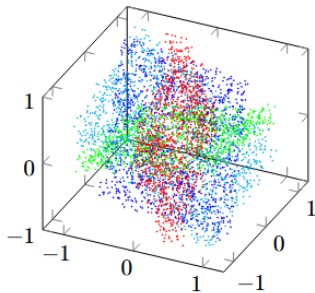
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Cost function c ? (no concrete plane information given)



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Related Work:

TODO: 1 + 2 (citation at the end!)

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Tasks and Solutions:

- ① Read [?], implement the partial optimality algorithm
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→ implementation in C++ (with some adjustments)
- ② Construct subspace instances of increasing difficulty
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- ③ Apply algorithm to the subspace instances, assess partial optimality, accuracy and computation time
→ experiments and evaluation (prove the quality of c)

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Apply partial optimality conditions → solve subproblems

Partial Optimality Algorithm

Partial Optimality Algorithm:

Input: labeling y without fixed labels

while condition applied **do**

 apply subproblem-CUT-condition exhaustively

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end while

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- 2 JOIN-Conditions: fix JOIN labels for elements of the sample subset; add the join-cost to c_\emptyset ; solve the problem where the subset is considered as one sample;

Subproblem-CUT and Subset-JOIN

Subproblem-CUT: cut sample subsets R_1, R_2, \dots, R_k that are only connected via non-negative costs (applied if $k > 1$)

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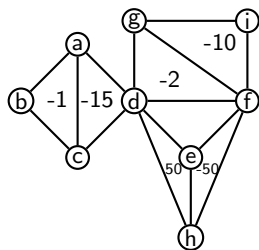
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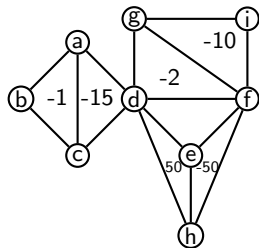


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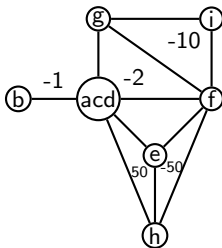
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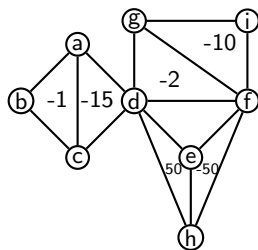


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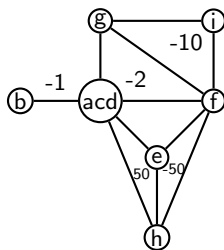
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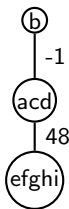
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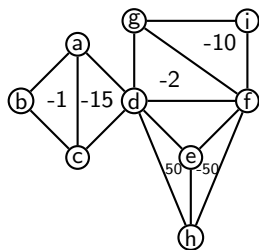


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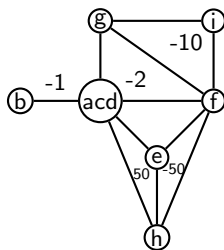
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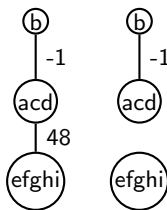
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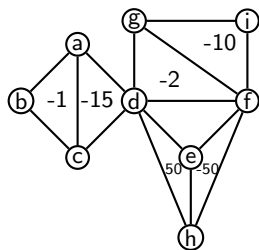


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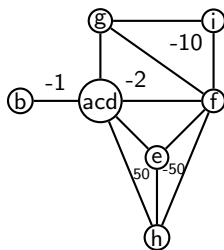
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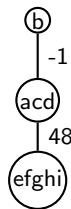
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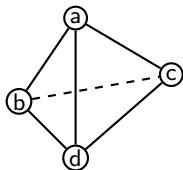
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Pyramid Instance and CUT-conditions

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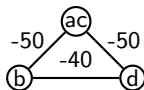
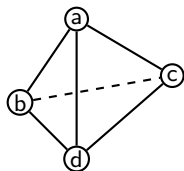
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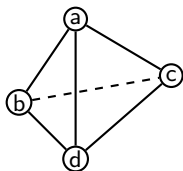


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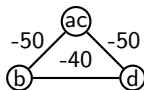
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Subset-JOIN

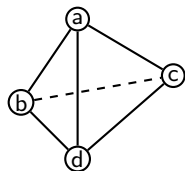


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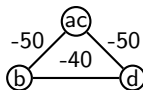
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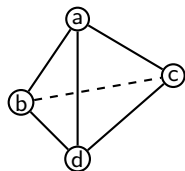
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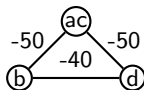
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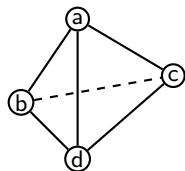
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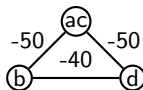
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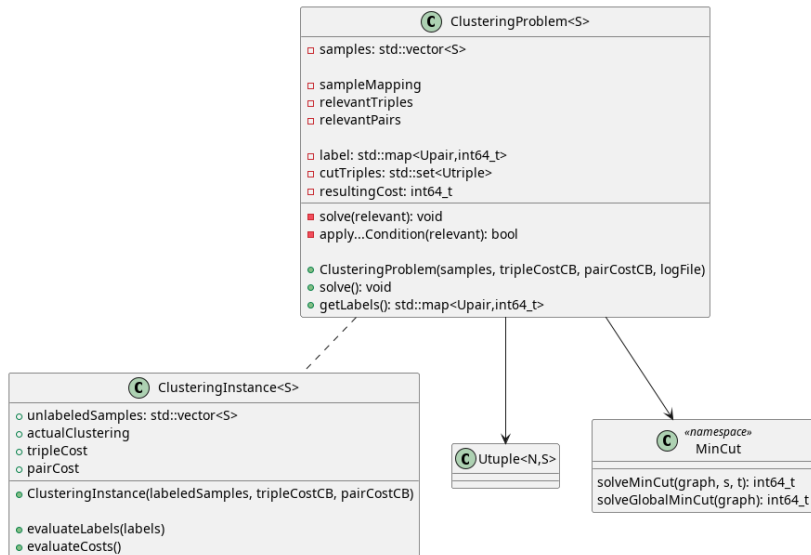
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Samples in the pyramid with $c_{bcd} = 100$ are unjoinable!

Triple-CUT is applied to the triple bcd

Program Structure

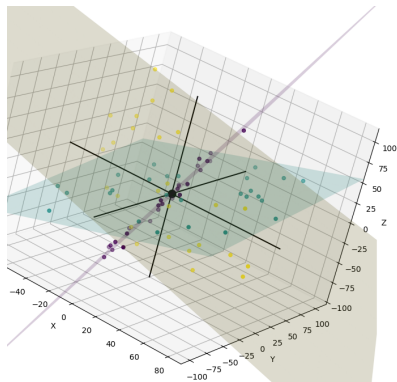


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Plane and Point Generation

Plane Generation:

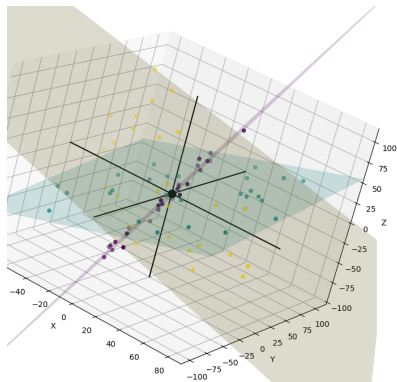
- generate 3 planes
as distinct normal vectors
 $\vec{n}_1, \vec{n}_2, \vec{n}_3$ (normalized)



Plane and Point Generation

Plane Generation:

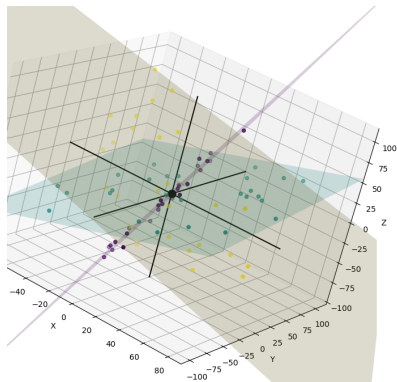
- generate 3 planes as distinct normal vectors $\vec{n}_1, \vec{n}_2, \vec{n}_3$ (normalized)
- compute a $\vec{r}_{i,1}$ (normalized) orthogonal to \vec{n}_i ($i \in \{1, 2, 3\}$)



Plane and Point Generation

Plane Generation:

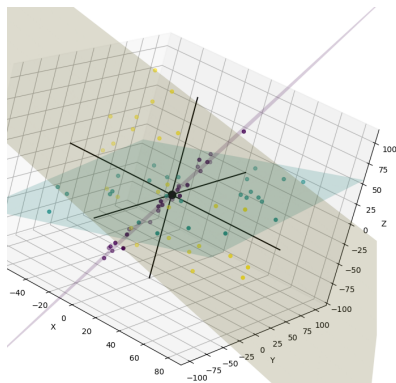
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Plane and Point Generation

Plane Generation:

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Point Generation on the plane $(\vec{n}, \vec{r}_1, \vec{r}_2)$, parameters (D, σ) :

- random variables $k_1, k_2 \in [-D, D]$ (uniform distribution)
- random variable k_n (normal distribution based on σ)
- generate point $p = k_1 \vec{r}_1 + k_2 \vec{r}_2 + k_n \vec{n}$

Cost Function

Triangle $abc \in \binom{S}{3}$

① Smallest side $s < D/2 \rightarrow c_{abc} = 0$

② Largest angle $\alpha > 150^\circ \rightarrow c_{abc} = 0$

③ ha, hb, hc : distances to the best fitting plane

$$ha + hb + hc > 3\sigma + 10^{-6} \\ \rightarrow c_{abc} = \frac{(ha+hb+hc)-(3\sigma+10^{-6})}{3D}$$

④ ho : distance from the origin to the triangle plane

$$ho > \frac{10\sigma}{\#points} + 10^{-6} \rightarrow c_{abc} = 0$$

⑤ for all points p : hp : distance to the best fitting plane

choose p if $hp < \sigma + 10^{-6}$ and $|\vec{p}| > 0.3D$

hp' : distance to the best fitting plane of all chosen points

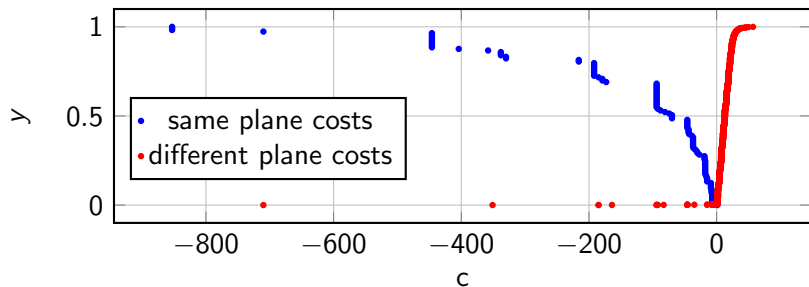
$$\delta_p = \frac{hp' - (\sigma + 10^{-6})}{D}, \text{ SAME} = \{p: \delta_p < 0\}, \text{ rew} = \sum_{p \in \text{SAME}} \delta_p,$$

$$|\text{SAME}| \leq 3 \rightarrow c_{abc} = 0$$

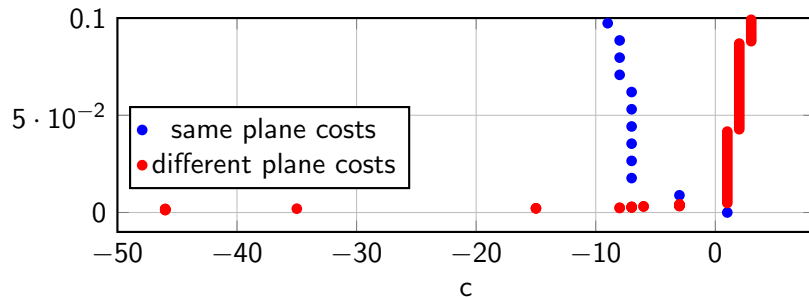
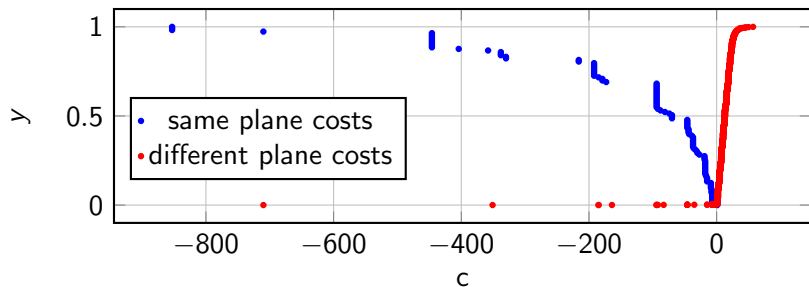
$$\text{else} \rightarrow c_{abc} = 2^{|\text{SAME}|-4} \text{rew}$$

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Cost Function Evaluation (3x15 points, $\sigma = 1$)



Cost Function Evaluation (3x15 points, $\sigma = 1$)



- WSL2 Ubuntu, Intel Core i7-11370H (3.30 GHz), 16 GB RAM

Experiments

- WSL2 Ubuntu, Intel Core i7-11370H (3.30 GHz), 16 GB RAM
- Apply the algorithm to the random cubic subspace instances with $D = 100$ and fixed $\sigma = 0, 1, 2, 3, 4, 5$:
 - 3x7 points (solve 15 instances)
 - 3x10 points (solve 15 instances)
 - 3x15 points (solve 7 instances)
 - 3x20 points (solve 1 instance)

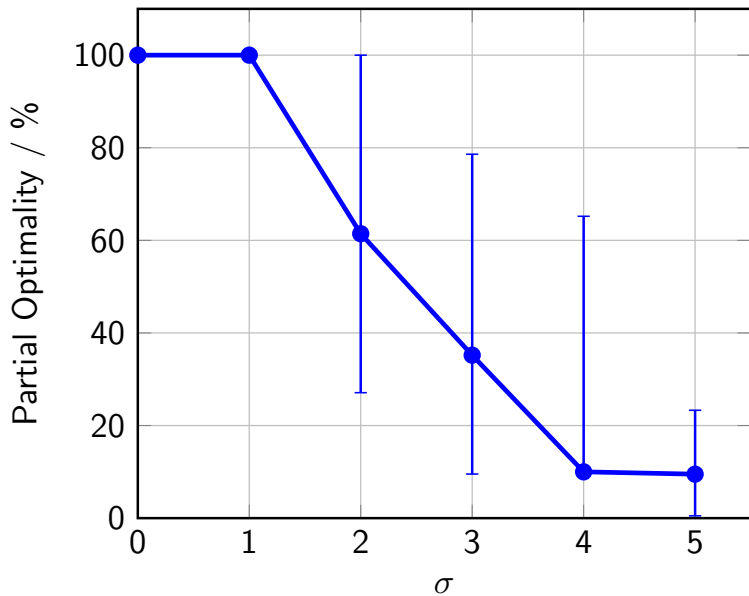
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- Track
 - computation time (s)
 - partial optimality (%)
 - accuracy (%) with respect to the truth (correct labeling)

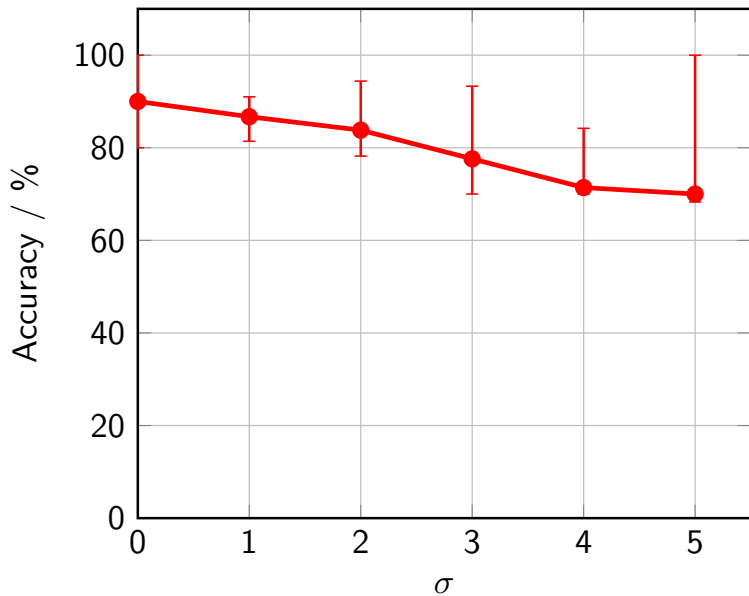
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- Capture
 - 1.quartile (Q1)
 - median (Q2)
 - 3.quartile(Q3)
 - the worst computation time

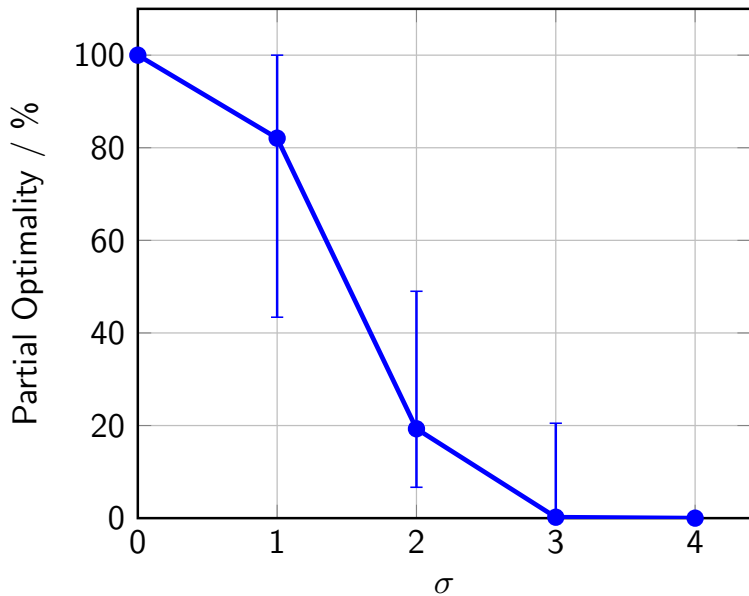
Partial Optimality (3x7 points)



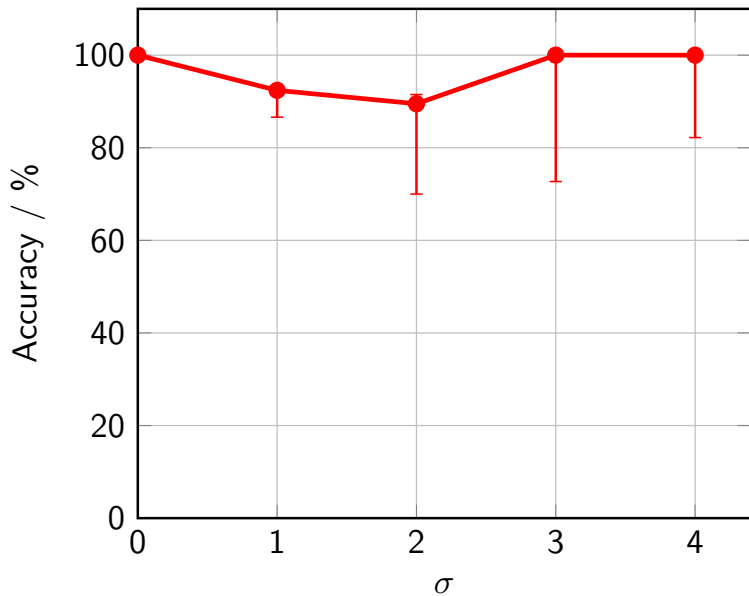
Accuracy (3x7 points)



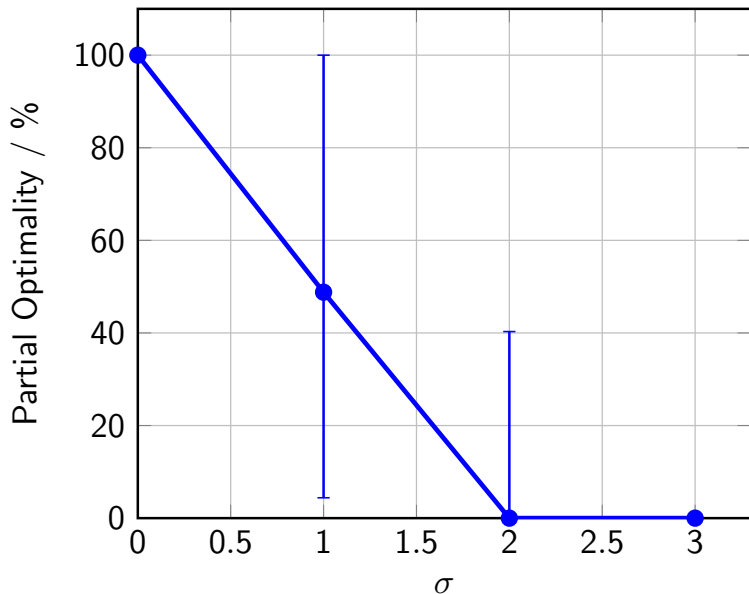
Partial Optimality (3x10 points)



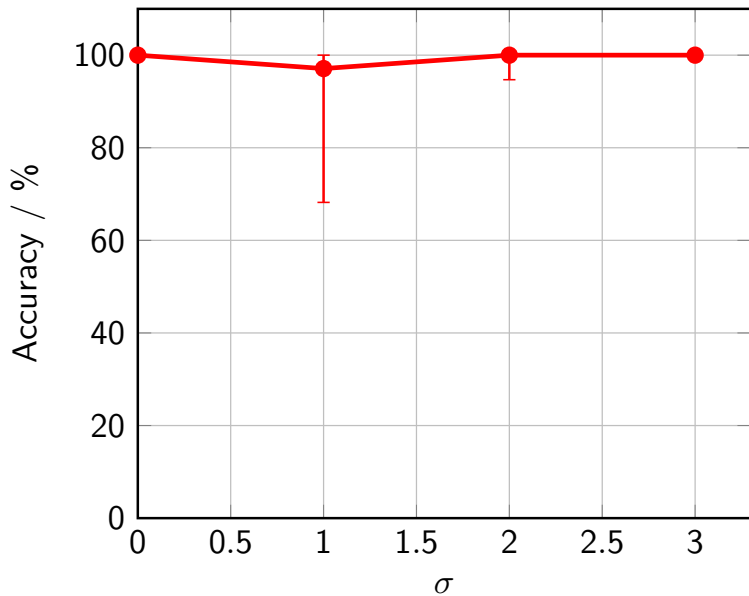
Accuracy (3x10 points)



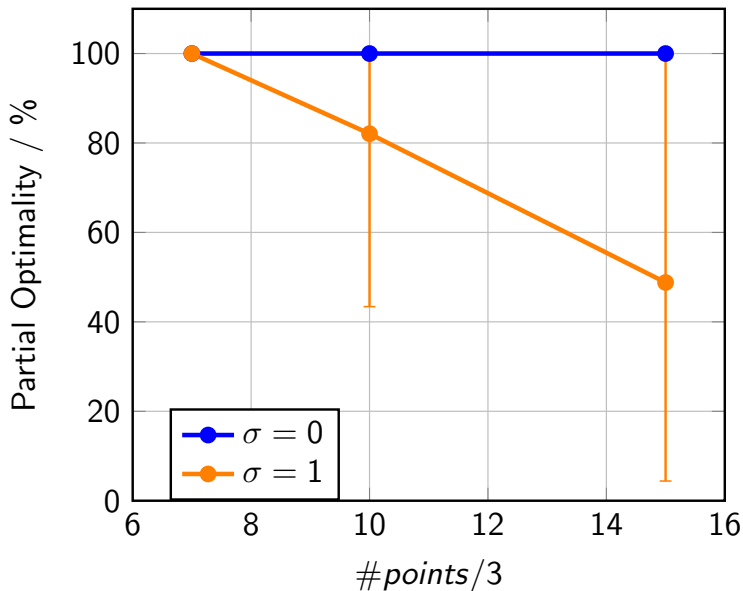
Partial Optimality (3x15 points)



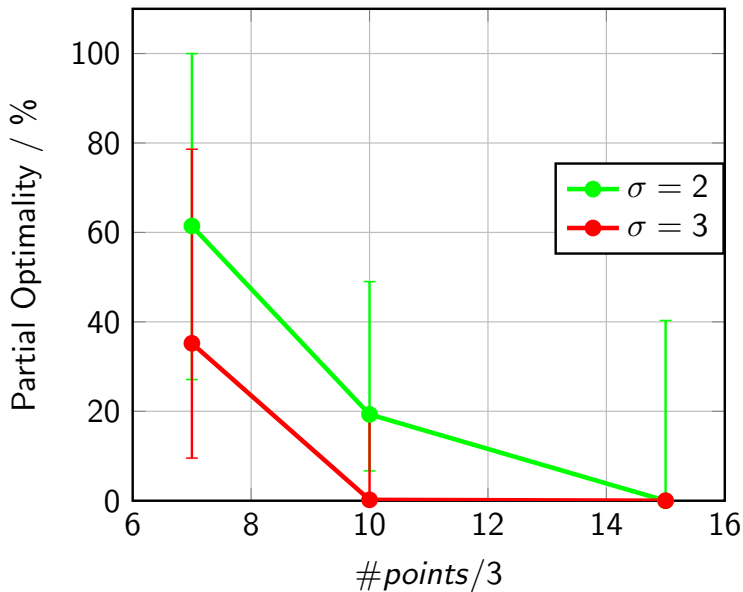
Accuracy (3x15 points)



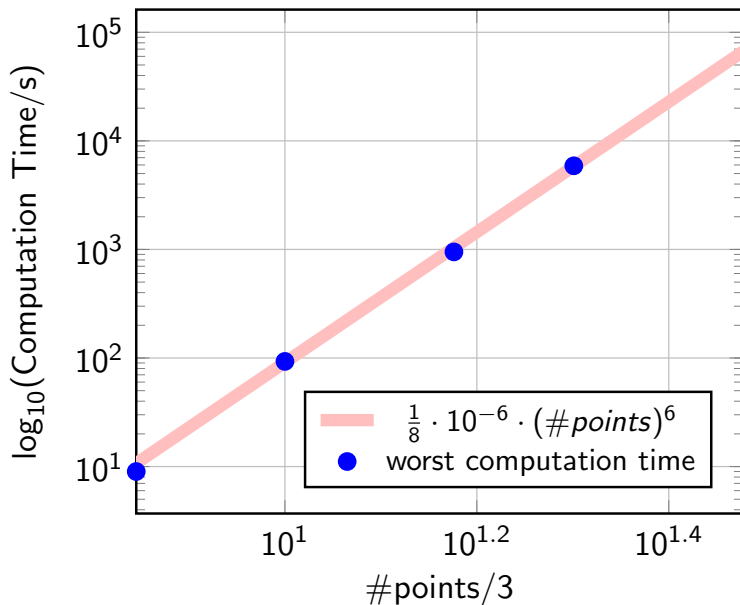
Partial Optimality



Partial Optimality



Computation Time (worst case)



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- ❶ Implementation of the partial optimality algorithm:
 - arbitrary sample type
 - sparse cost representation
 - pair labeling and triple cuts
 - reasonable adjustments of the partial optimality conditions
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Conclusion

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 - high accuracy (over 75%)
 - significant noise tolerance ($\sigma \geq 1$)
 - $O(k \cdot n^6)$ for $n = \# \text{points}$ and a small k

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Future Work:

- optimize and parallelize the algorithm
- overcome the partial optimality loss for c
- determine better parameters for c
- update c with advanced criteria

My program, scripts and presentation:

<https://github.com/Vovsanka/ResearchProjectML>

TODO: citation