

Partially Optimal Cubic Subspace Clustering

Research Project Machine Learning

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- 1 Introduction
- 2 Partial Optimality for Cubic Clique Partition Problem
- 3 Cubic Subspace Instance Construction
- 4 Experiments and Evaluation
- 5 Conclusion

Problem Statement (1)

Finite sample set S , cost function $c: \binom{S}{3} \rightarrow \mathbb{R}$.

Instance of the **Cubic Clique Partition Problem**:

$$\min_{y: \binom{S}{2} \rightarrow \{0,1\}} \sum_{abc \in \binom{S}{3}} c_{abc} y_{ab} y_{bc} y_{ac}$$

subject to $y_{ab} + y_{bc} - 1 \leq y_{ac}$ for all distinct $a, b, c \in S$.

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Find a **partially optimal solution**, i.e. fix some labels y_{ab} for distinct $a, b \in S$

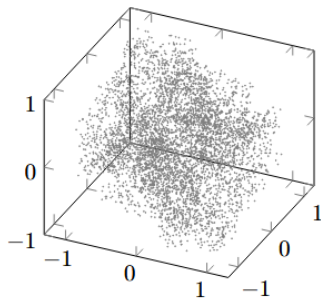
$$\begin{cases} y_{ab} = 1 & \text{join } a, b \\ y_{ab} = 0 & \text{cut } a, b \\ y_{ab} = ? & \text{unknown} \end{cases}$$

in such way that there still exists an optimal solution.

Problem Statement (2)

Subspace Instances of the Cubic Clique Partition Problem

Samples S : points $S \subset \mathbb{R}^3$



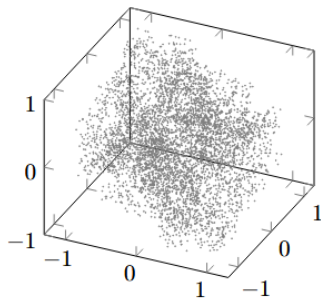
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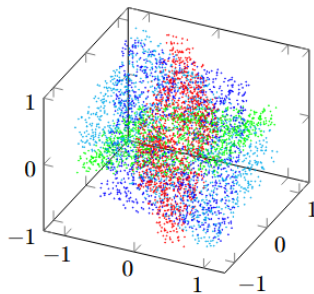
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Point generation: 3 distinct planes containing the origin, noise σ



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(b) Optimal labeling y^*

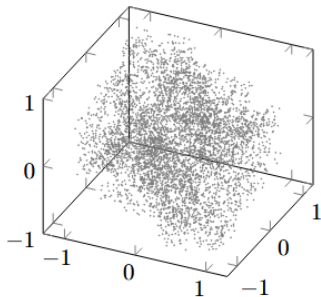
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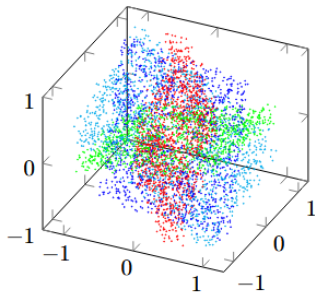
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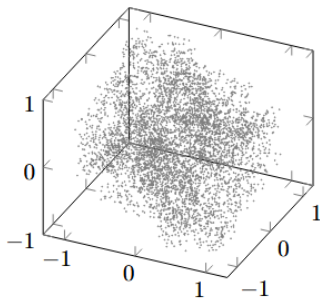
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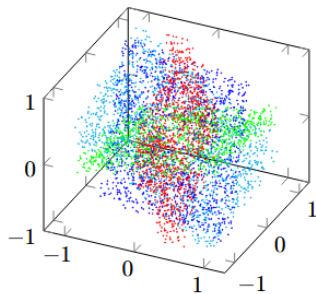
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Optimal labeling y^* : original planes

Cost function c ? (no concrete plane information given)



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Related Work:

TODO: 1 + 2 (citation at the end!)

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Tasks and Solutions:

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→ implementation in C++ (with some adjustments)
- ② Construct subspace instances of increasing difficulty
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(high accuracy, significant noise tolerance)
- ③ Apply algorithm to the subspace instances, assess partial optimality, accuracy and computation time
→ experiments and evaluation (prove the quality of c)

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Apply partial optimality conditions → solve subproblems

Partial Optimality Algorithm

Partial Optimality Algorithm:

Input: labeling y without fixed labels

while condition applied **do**

 apply subproblem-CUT-condition exhaustively

 apply one of JOIN-conditions (in effective order)

end while

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Output: partially optimal labeling y with some fixed labels

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Reduction to subproblems:

- 1 Subproblem-CUT-condition: fix CUT labels for element pairs from different sample subsets; solve each subset as an independent problem and accumulate the results in c_\emptyset ;
- 2 JOIN-Conditions: fix JOIN labels for elements of the sample subset; add the join-cost to c_\emptyset ; solve the problem where the subset is considered as one sample;

Subproblem-CUT and Subset-JOIN

Subproblem-CUT: cut sample subsets R_1, R_2, \dots, R_k that are only connected via non-negative costs (applied if $k > 1$)

Subproblem-CUT and Subset-JOIN

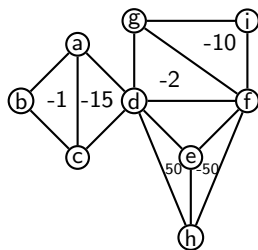
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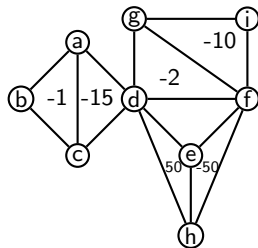


$$c_{\emptyset} = 0$$

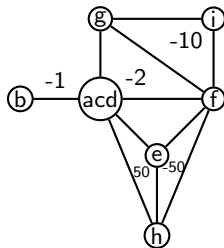
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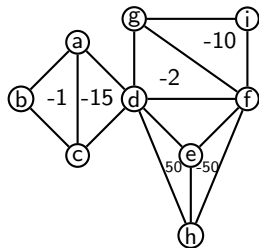


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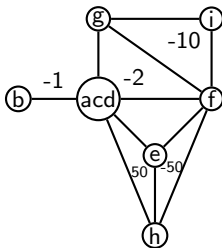
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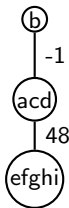
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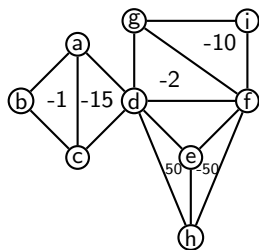


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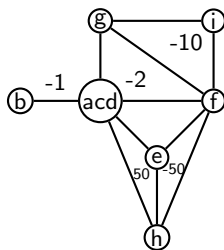
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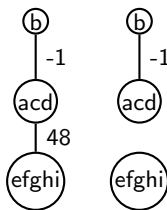
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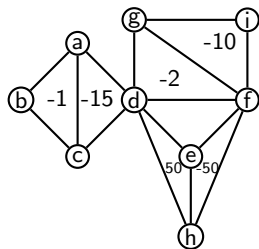


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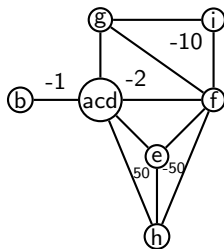
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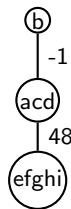
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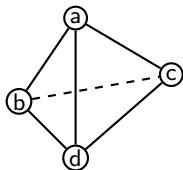
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Pyramid Instance and CUT-conditions

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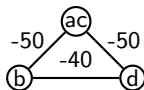
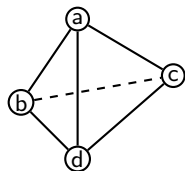
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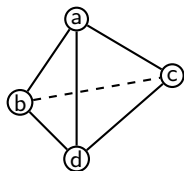


Pair-JOIN-2

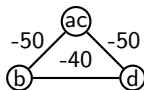
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Subset-JOIN

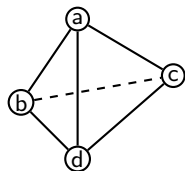


$$c_{\emptyset} = -140$$

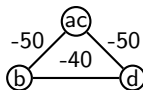
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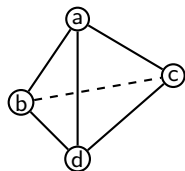
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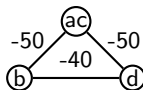
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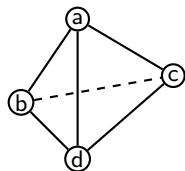
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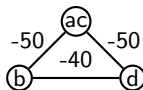
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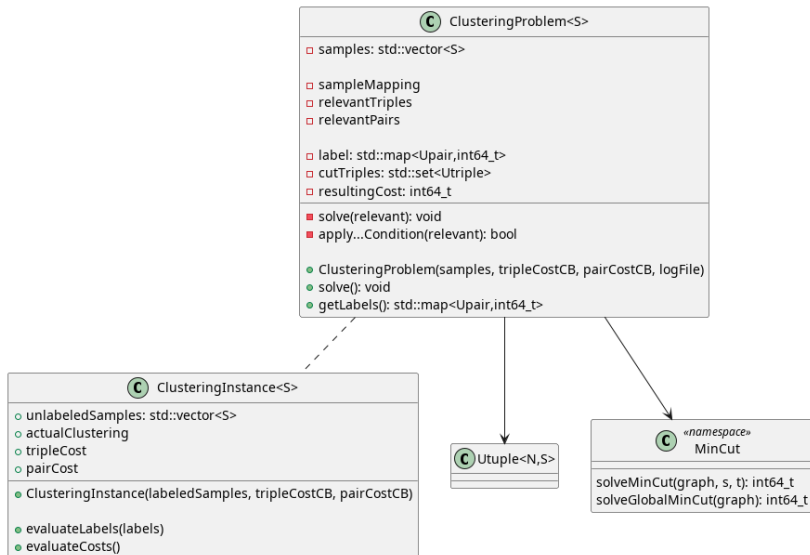
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Triple-CUT: cut samples i, j, k if the condition holds (similar to Pair-CUT) (\approx i-jk min-cut)

Samples in the pyramid with $c_{bcd} = 100$ are unjoinable!
Triple-CUT is applied to the triple bcd

Program Structure

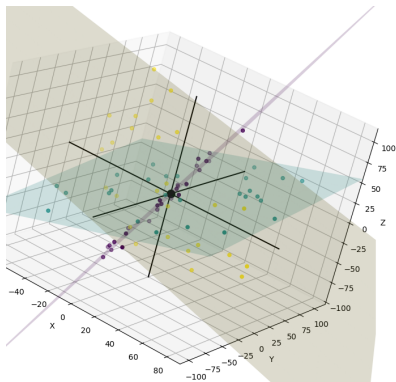


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Plane and Point Generation

Plane Generation:

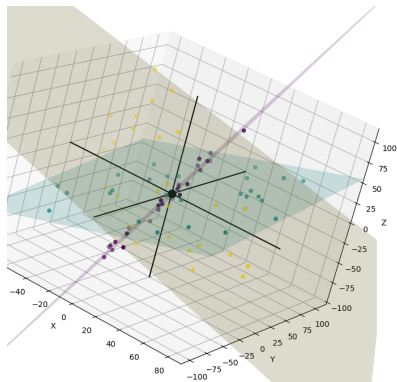
- generate 3 planes
as distinct normal vectors
 $\vec{n}_1, \vec{n}_2, \vec{n}_3$ (normalized)



Plane and Point Generation

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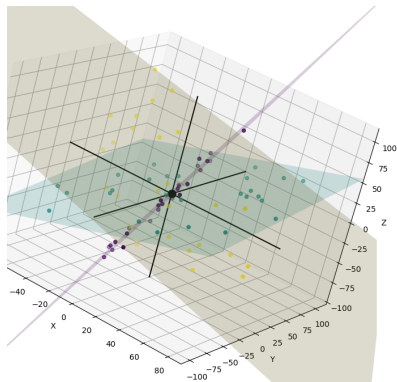
- generate 3 planes as distinct normal vectors $\vec{n}_1, \vec{n}_2, \vec{n}_3$ (normalized)
- compute a $\vec{r}_{i,1}$ (normalized) orthogonal to \vec{n}_i ($i \in \{1, 2, 3\}$)



Plane and Point Generation

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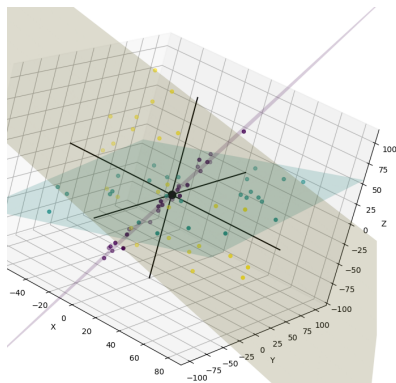
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Point Generation on the plane $(\vec{n}, \vec{r}_1, \vec{r}_2)$, parameters (D, σ) :

- random variables $k_1, k_2 \in [-D, D]$ (uniform distribution)
- random variable k_n (normal distribution based on σ)
- generate point $p = k_1 \vec{r}_1 + k_2 \vec{r}_2 + k_n \vec{n}$

Cost Function

Triangle $abc \in \binom{S}{3}$

① Smallest side $s < D/2 \rightarrow c_{abc} = 0$

② Largest angle $\alpha > 150^\circ \rightarrow c_{abc} = 0$

③ ha, hb, hc : distances to the best fitting plane

$$ha + hb + hc > 3\sigma + 10^{-6} \\ \rightarrow c_{abc} = \frac{(ha+hb+hc)-(3\sigma+10^{-6})}{3D}$$

④ ho : distance from the origin to the triangle plane

$$ho > \frac{10\sigma}{\#points} + 10^{-6} \rightarrow c_{abc} = 0$$

⑤ for all points p : hp : distance to the best fitting plane

choose p if $hp < \sigma + 10^{-6}$ and $|\vec{p}| > 0.3D$

hp' : distance to the best fitting plane of all chosen points

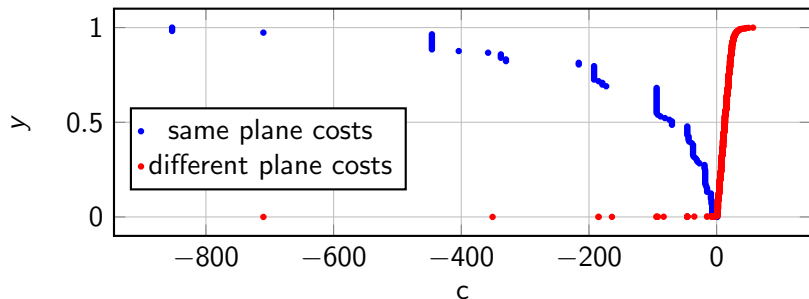
$$\delta_p = \frac{hp' - (\sigma + 10^{-6})}{D}, \text{ SAME} = \{p: \delta_p < 0\}, \text{ rew} = \sum_{p \in \text{SAME}} \delta_p,$$

$$|\text{SAME}| \leq 3 \rightarrow c_{abc} = 0$$

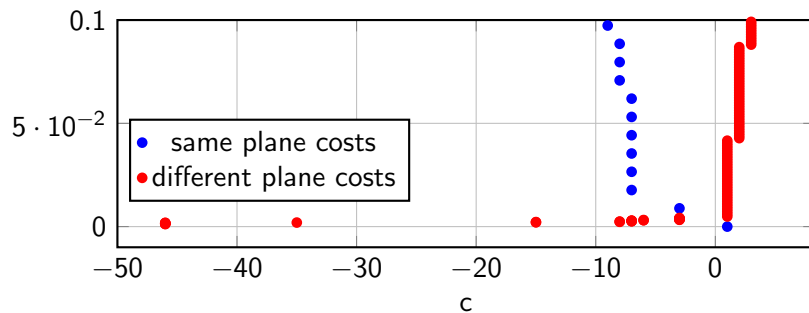
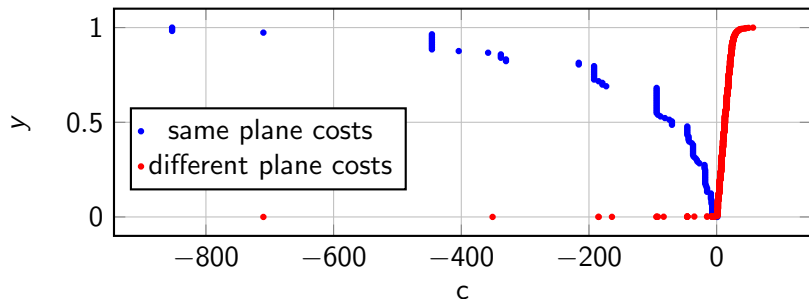
$$\text{else} \rightarrow c_{abc} = 2^{|\text{SAME}|-4} \text{rew}$$

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Cost Function Evaluation (3x15 points, $\sigma = 1$)



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Experiments

- WSL2 Ubuntu, Intel Core i7-11370H (3.30 GHz), 16 GB RAM

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 - 3x7 points (solve 15 instances)
 - 3x10 points (solve 15 instances)
 - 3x15 points (solve 7 instances)
 - 3x20 points (solve 1 instance)

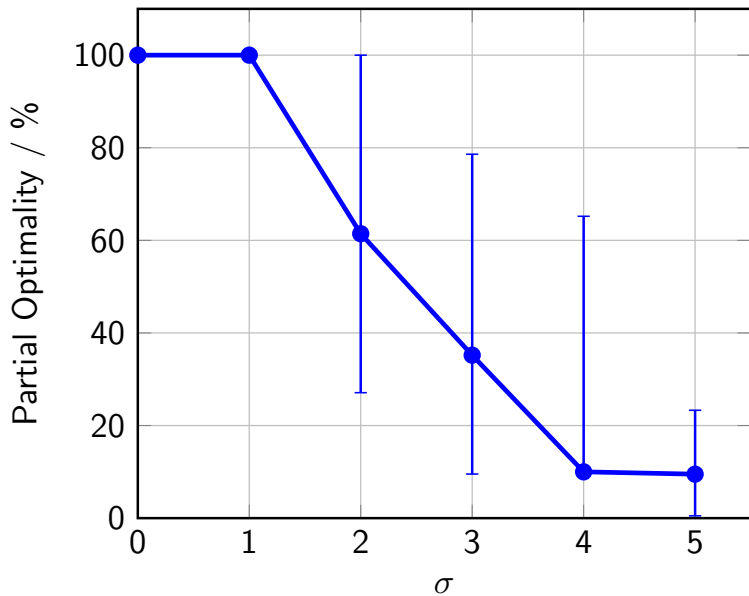
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- Track
 - computation time (s)
 - partial optimality (%)
 - accuracy (%) with respect to the truth (correct labeling)

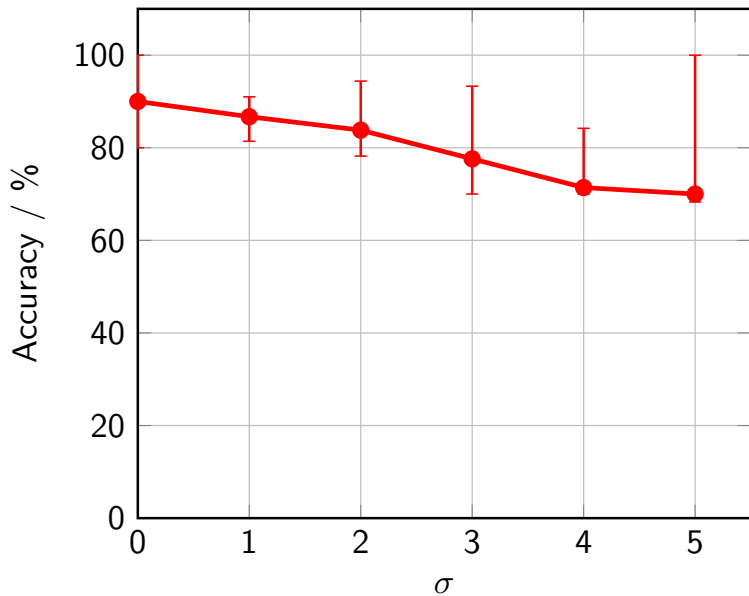
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- Capture
 - 1.quartile (Q1)
 - median (Q2)
 - 3.quartile(Q3)
 - the worst computation time

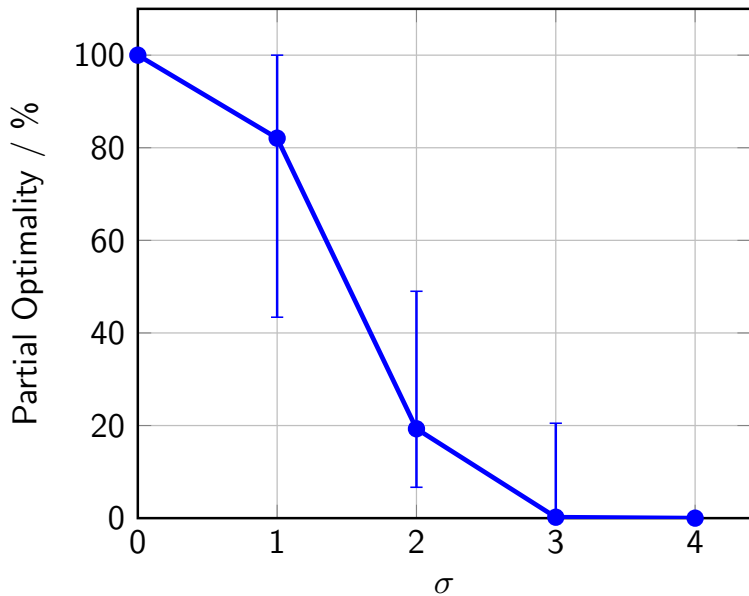
Partial Optimality (3x7 points)



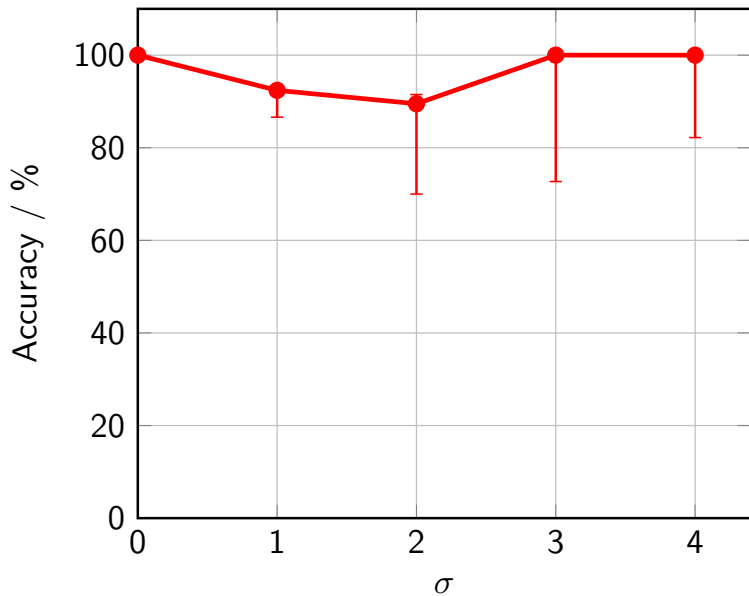
Accuracy (3x7 points)



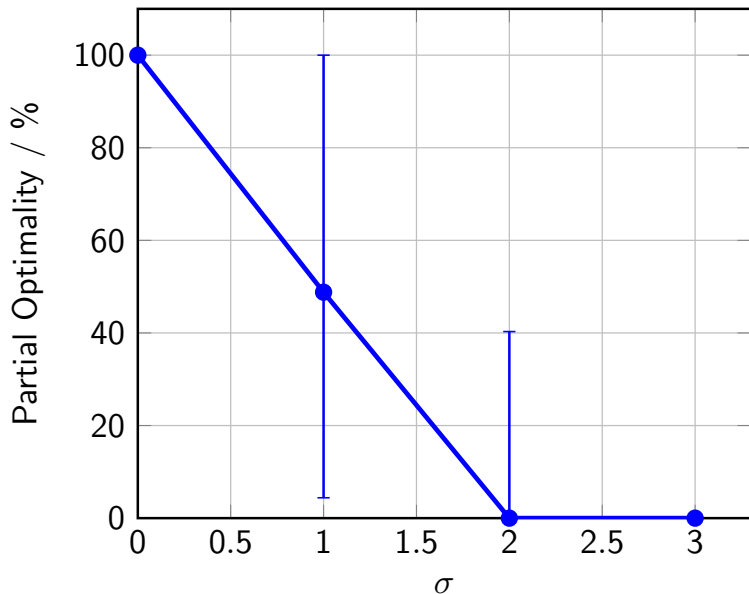
Partial Optimality (3x10 points)



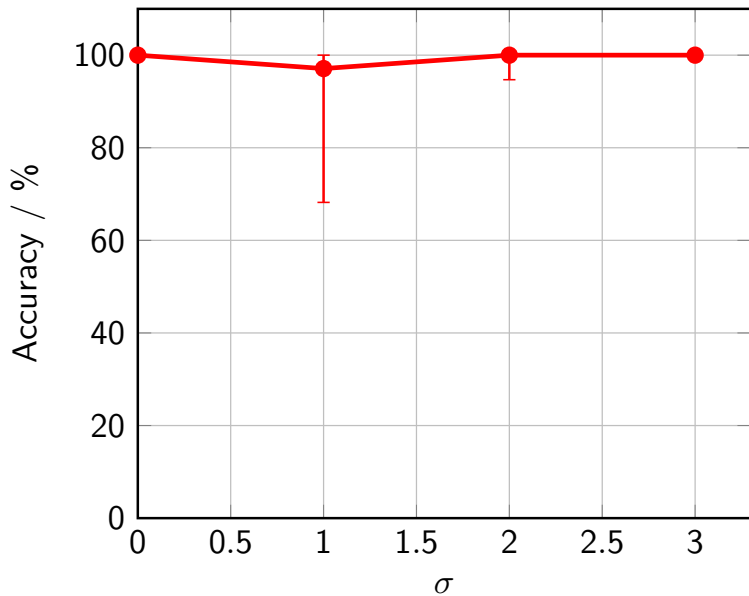
Accuracy (3x10 points)



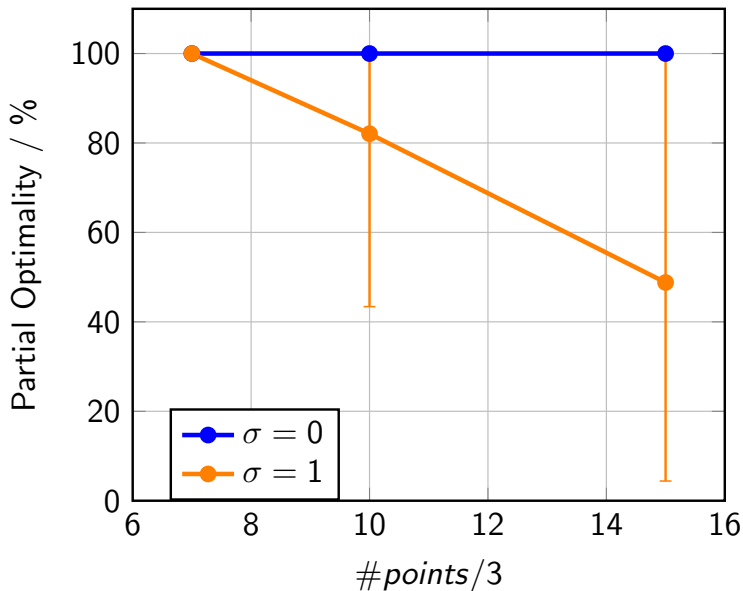
Partial Optimality (3x15 points)



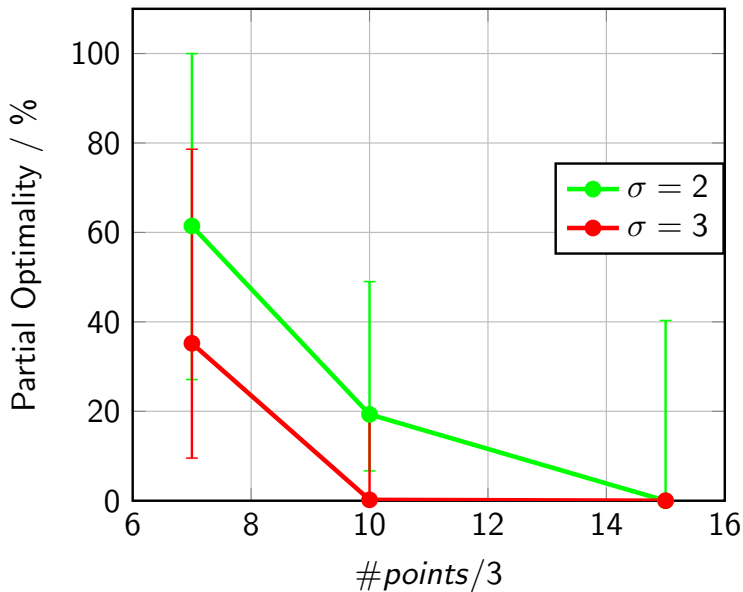
Accuracy (3x15 points)



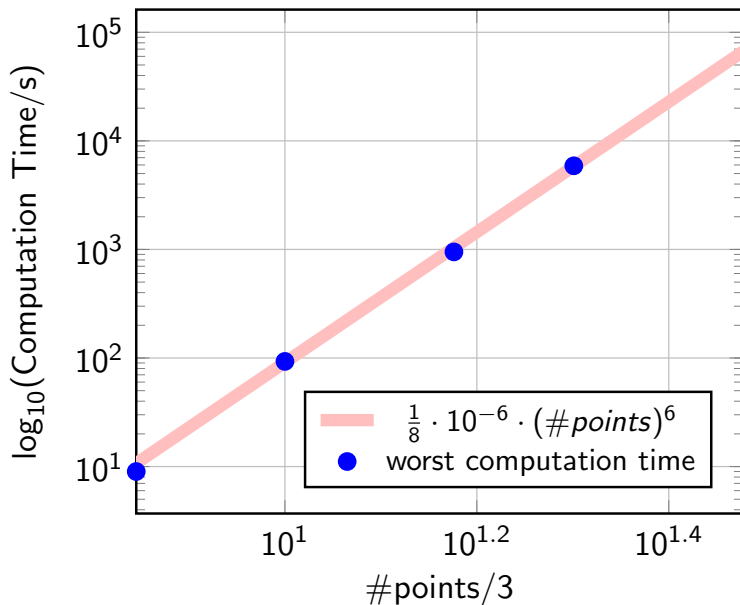
Partial Optimality



Partial Optimality



Computation Time (worst case)



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Future Work:

- optimize and parallelize the algorithm
- overcome the partial optimality loss for c
- determine better parameters for c
- update c with advanced criteria

My program, scripts and presentation:

<https://github.com/Vovsanka/ResearchProjectML>

TODO: citation