Partially Optimal Cubic Subspace Clustering

Research Project Machine Learning

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Problem Statement (1)

Finite sample set S, cost function $\mathrm{c}\colon \binom{S}{3} \to \mathbb{R}$.
Instance of the **Cubic Clique Partition Problem**:

$$\max_{\mathrm{y}\colon \binom{S}{2} \to \{0,1\}} \sum_{\{a,b,c\} \in \binom{S}{3}} \mathrm{c}_{\{a,b,c\}}\, \mathrm{y}_{\{a,b\}}\, \mathrm{y}_{\{b,c\}}\, \mathrm{y}_{\{a,c\}}$$

subject to $\mathrm{y}_{\{a,b\}} + \mathrm{y}_{\{b,c\}} - 1 \leq \mathrm{y}_{\{a,c\}}$ for all distinct $a, b, c \in S$.

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Find a **partially optimal solution**, i.e. fix some labels $\mathrm{y}_{\{a,b\}}$ for distinct $a, b \in S$

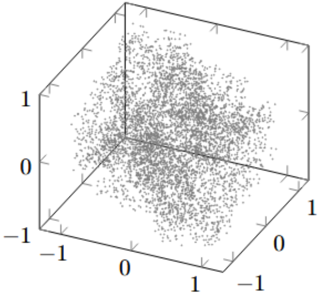
$$\begin{cases} \mathrm{y}_{\{a,b\}} = 1 & \text{join } a, b \\ \mathrm{y}_{\{a,b\}} = 0 & \text{cut } a, b \\ \mathrm{y}_{\{a,b\}} = ? & \text{unknown} \end{cases}$$

in such way that there still exists an optimal solution.

Problem Statement (2)

Subspace Instances of the Cubic Clique Partition Problem

Samples S: points $S \subset \mathbb{R}^3$



(a) Samples S

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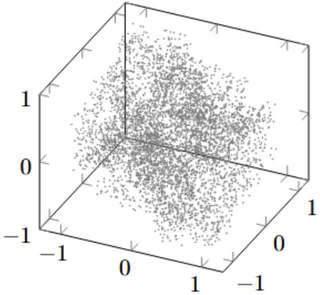
Point generation: 3 distinct planes containing the origin, noise σ



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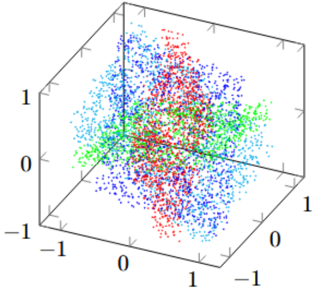

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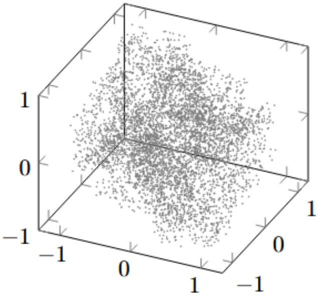

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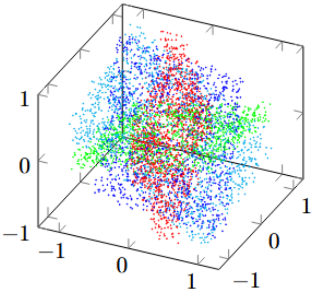

(b) Optimal clustering y*

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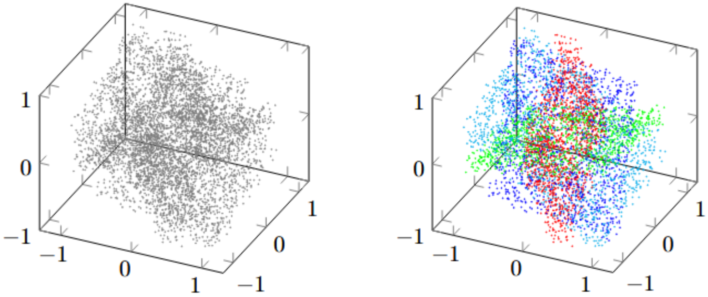
Optimal clustering y^*: original planes
Cost function c? (no concrete plane info given)



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(b) Optimal clustering y^*

Research Goals and Contributions

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read 'Partial Optimality in Cubic Correlation Clustering' [?]; implement the algorithm for establishing partial optimality to the cubic clique partition problem;	

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apply the algorithm to the constructed subspace instances, assess partial optimality, accuracy with respect to truth, and computation time	systematic empirical assessment of the algorithm application the the problem subspace instances with my cost function proving its quality