

Partially Optimal Cubic Subspace Clustering

Research Project Machine Learning

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- 1 Introduction
- 2 Partial Optimality for Cubic Clique Partition Problem
- 3 Cubic Subspace Instance Construction
- 4 Experiments and Evaluation
- 5 Conclusion

Problem Statement (1)

Finite sample set S , cost function $c: \binom{S}{3} \rightarrow \mathbb{R}$.

Instance of the **Cubic Clique Partition Problem**:

$$\min_{y: \binom{S}{2} \rightarrow \{0,1\}} \sum_{abc \in \binom{S}{3}} c_{abc} y_{ab} y_{bc} y_{ac}$$

subject to $y_{ab} + y_{bc} - 1 \leq y_{ac}$ for all distinct $a, b, c \in S$.

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Find a **partially optimal solution**, i.e. fix some labels y_{ab} for distinct $a, b \in S$

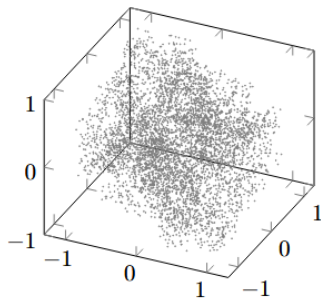
$$\begin{cases} y_{ab} = 1 & \text{join } a, b \\ y_{ab} = 0 & \text{cut } a, b \\ y_{ab} = ? & \text{unknown} \end{cases}$$

in such way that there still exists an optimal solution.

Problem Statement (2)

Subspace Instances of the Cubic Clique Partition Problem

Samples S : points $S \subset \mathbb{R}^3$



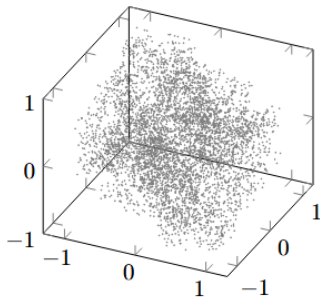
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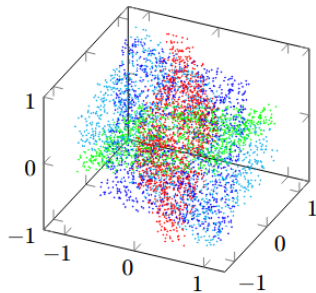
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Point generation: 3 distinct planes containing the origin, noise σ



(a) Samples S



(b) Optimal labeling y^*

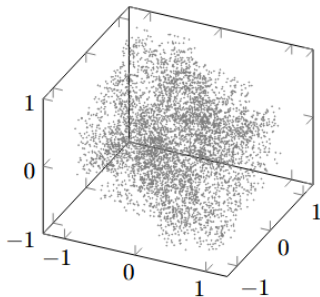
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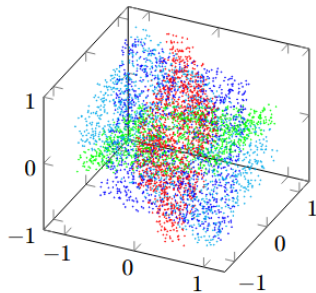
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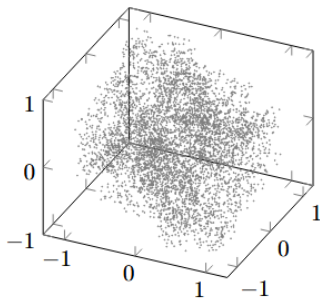
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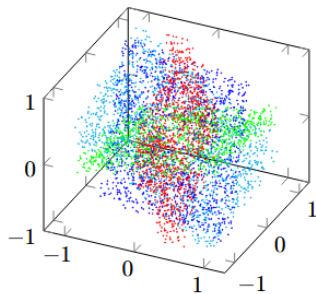
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Optimal labeling y^* : original planes

Cost function c ? (no concrete plane information given)



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Related Work:

TODO: 1 + 2 (citation at the end!)

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Tasks and Solutions:

- ① Read [?], implement the partial optimality algorithm
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(high accuracy, significant noise tolerance)

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→ implementation in C++ (with some adjustments)
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→ point generation, appropriate cost function c
(high accuracy, significant noise tolerance)
- 3 Apply algorithm to the subspace instances, assess partial optimality, accuracy and computation time
→ experiments and evaluation (prove the quality of c)

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Apply partial optimality conditions → solve subproblems

Partial Optimality Algorithm

Partial Optimality Algorithm:

Input: labeling y without fixed labels

while condition applied **do**

 apply subproblem-CUT-condition exhaustively

 apply one of JOIN-conditions (in effective order)

end while

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Output: partially optimal labeling y with some fixed labels

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Reduction to subproblems:

- 1 Subproblem-CUT-condition: fix CUT labels for element pairs from different sample subsets; solve each subset as an independent problem and accumulate the results in c_\emptyset ;
- 2 JOIN-Conditions: fix JOIN labels for elements of the sample subset; add the join-cost to c_\emptyset ; solve the problem where the subset is considered as one sample;

Subproblem-CUT and Subset-JOIN

Subproblem-CUT: cut sample subsets R_1, R_2, \dots, R_k that are only connected via non-negative costs (applied if $k > 1$)

Subproblem-CUT and Subset-JOIN

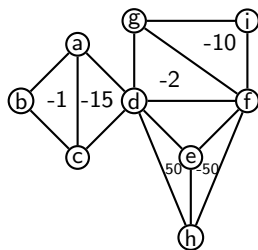
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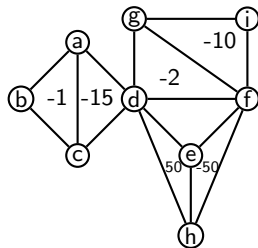


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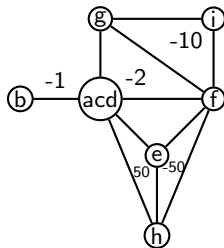
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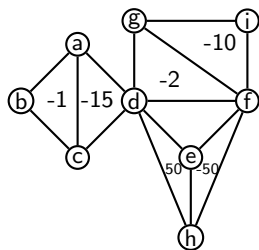


$$c_{\emptyset} = -15$$

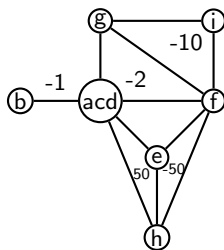
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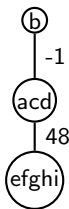
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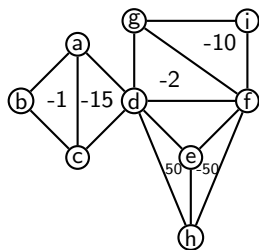


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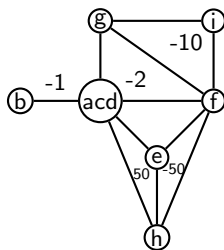
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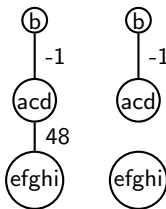
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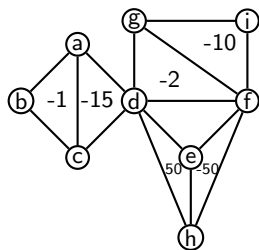


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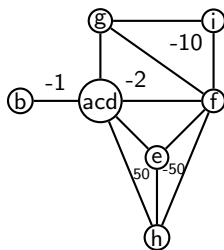
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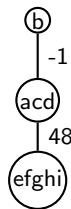
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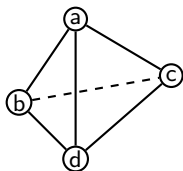
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Pyramid Instance and CUT-conditions

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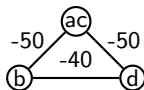
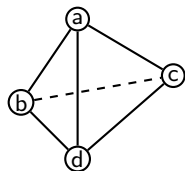
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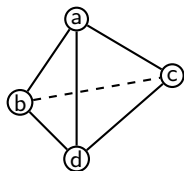


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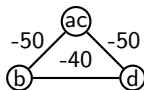
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Subset-JOIN

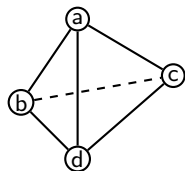


$$c_{\emptyset} = -140$$

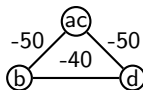
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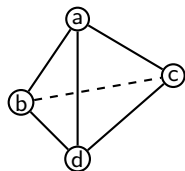
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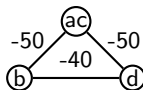
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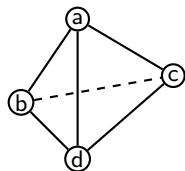
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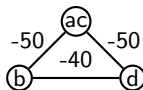
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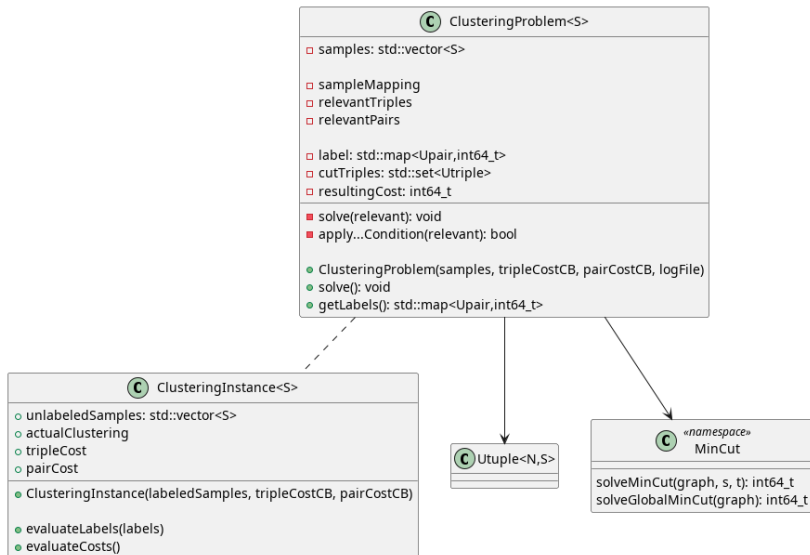
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Samples in the pyramid with $c_{bcd} = 100$ are unjoinable!
Triple-CUT is applied to the triple bcd

Program Structure

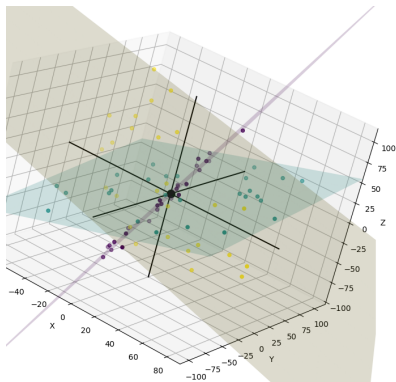


- 1 Introduction
- 2 Partial Optimality for Cubic Clique Partition Problem
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Plane and Point Generation

Plane Generation:

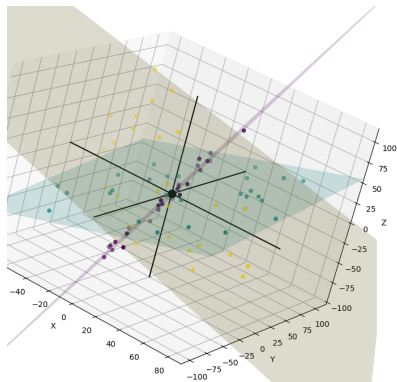
- generate 3 planes
as distinct normal vectors
 $\vec{n}_1, \vec{n}_2, \vec{n}_3$ (normalized)



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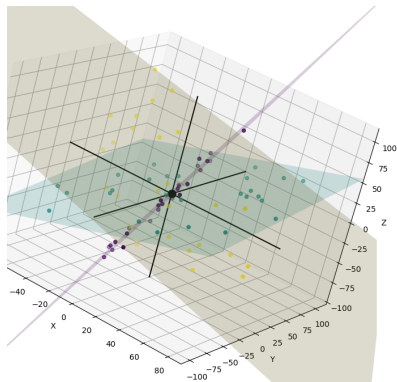
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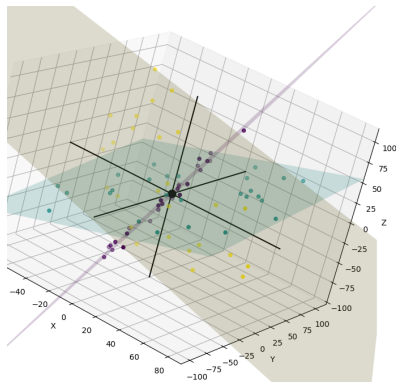
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Point Generation on the plane $(\vec{n}, \vec{r}_1, \vec{r}_2)$, parameters (D, σ) :

- random variables $k_1, k_2 \in [-D, D]$ (uniform distribution)
- random variable k_n (normal distribution based on σ)
- generate point $p = k_1 \vec{r}_1 + k_2 \vec{r}_2 + k_n \vec{n}$

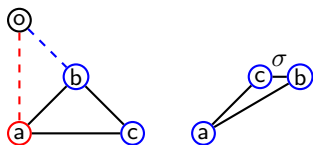
Cost Function (1)

Triangle $abc \in \binom{S}{3}$

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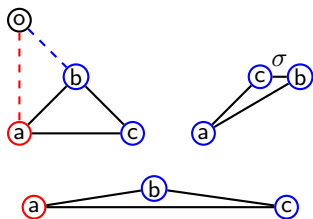
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 $\rightarrow c_{abc} = 0$



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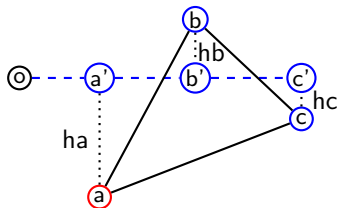
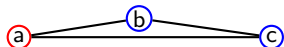
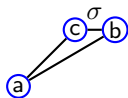
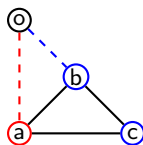
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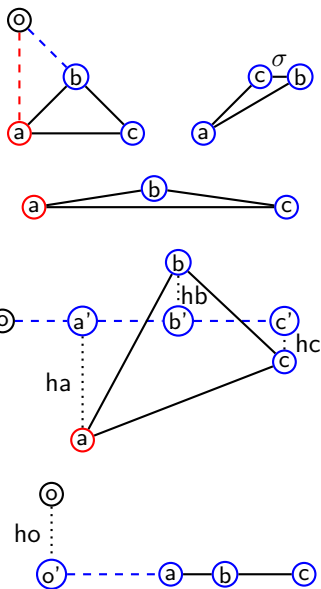
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- ④ ho : distance from the origin to the triangle plane

$$ho > \frac{10\sigma}{\#points} + 10^{-6}$$

$$\rightarrow c_{abc} = 0$$



Cost Function (2)

for all points p : hp : distance to the best fitting plane

choose p if $hp < \sigma + 10^{-6}$ and $|\vec{p}| > 0.3D$

hp' : distance to the best fitting plane of all chosen points

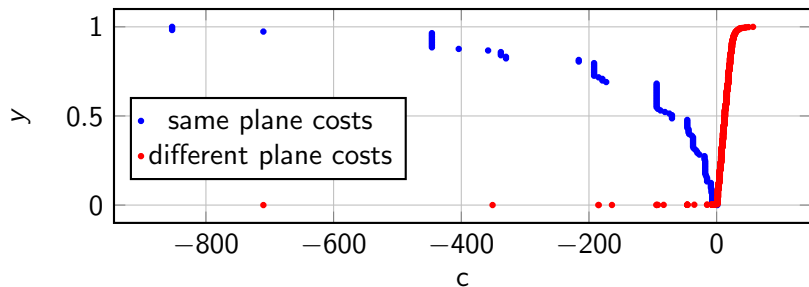
$$\delta_p = \frac{hp' - (\sigma + 10^{-6})}{D}, \text{ SAME} = \{p: \delta_p < 0\}, \text{ rew} = \sum_{p \in \text{SAME}} \delta_p,$$

$$|\text{SAME}| \leq 3 \rightarrow c_{abc} = 0$$

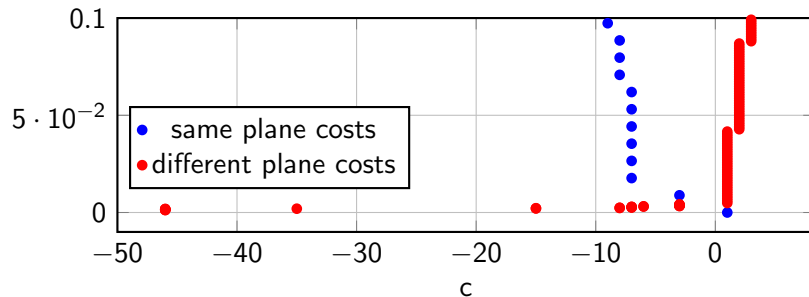
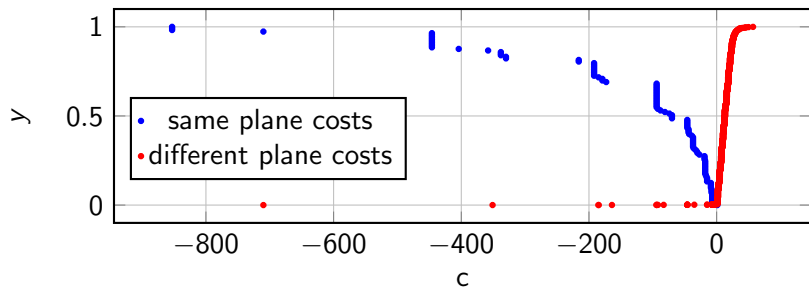
$$\text{else} \rightarrow c_{abc} = 2^{|\text{SAME}| - 4} \text{rew}$$

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Cost Function Evaluation (3x15 points, $\sigma = 1$)



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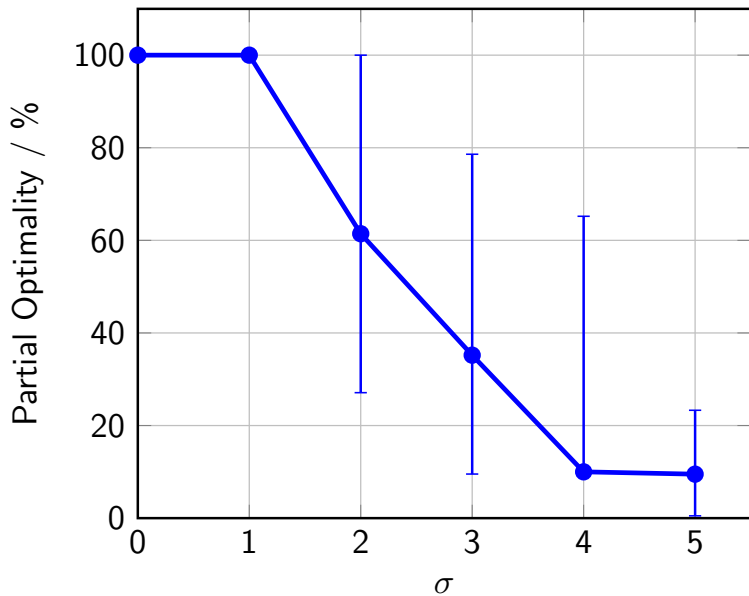
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 - computation time (s)
 - partial optimality (%)
 - accuracy (%) with respect to the truth (correct labeling)

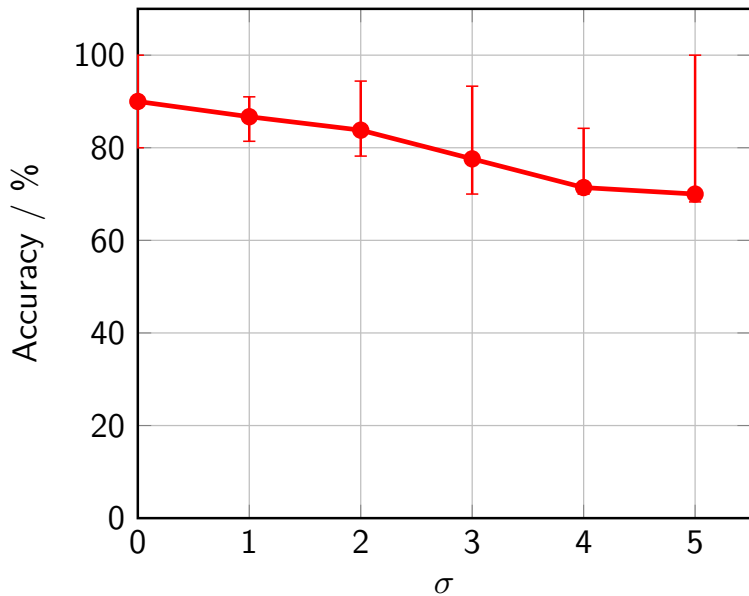
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- Capture
 - 1.quartile (Q1)
 - median (Q2)
 - 3.quartile(Q3)
 - the worst computation time

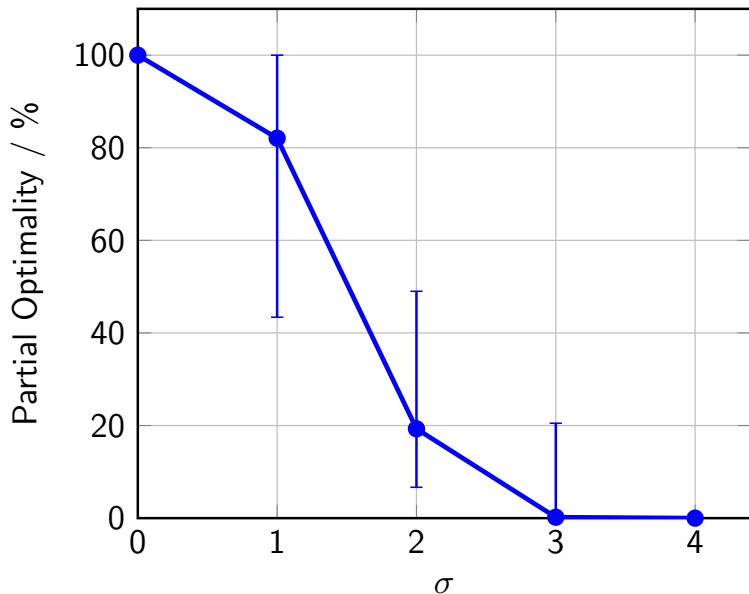
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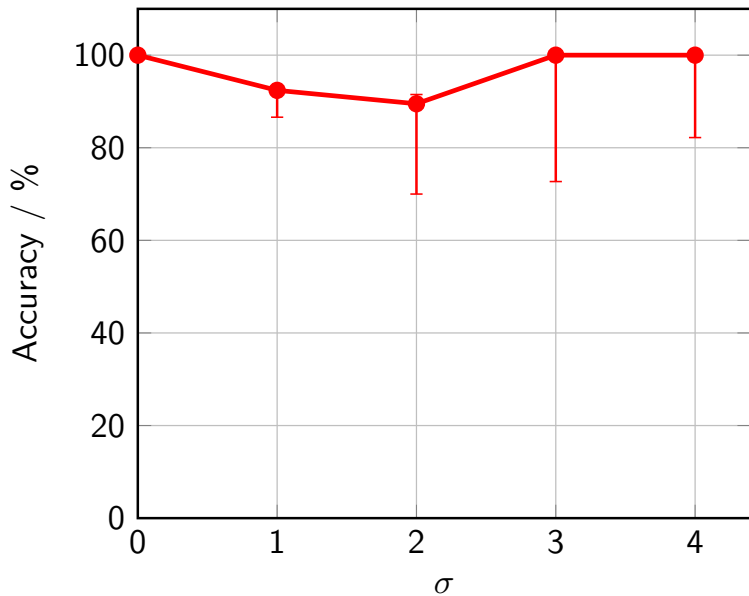
Accuracy (3x7 points)



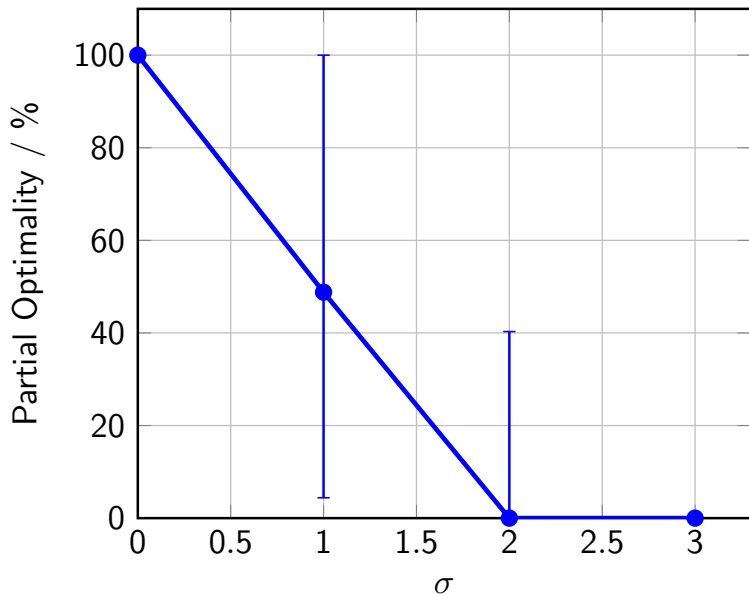
Partial Optimality (3x10 points)



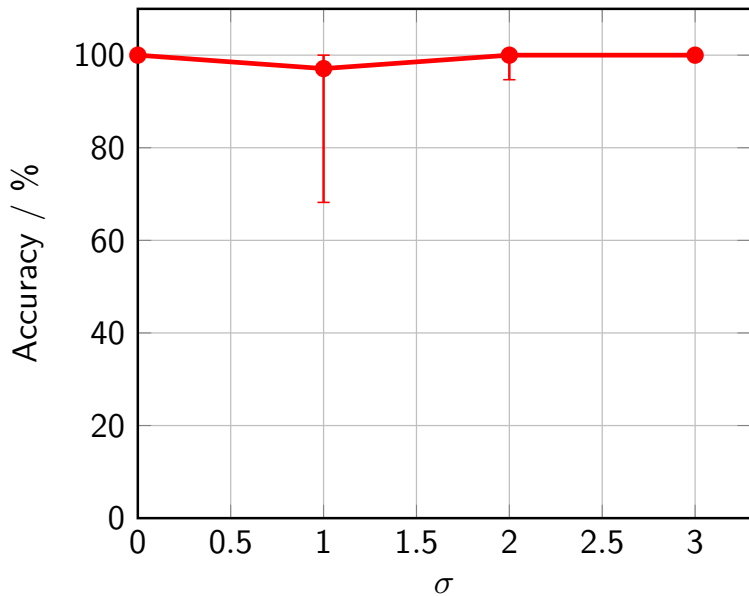
Accuracy (3x10 points)



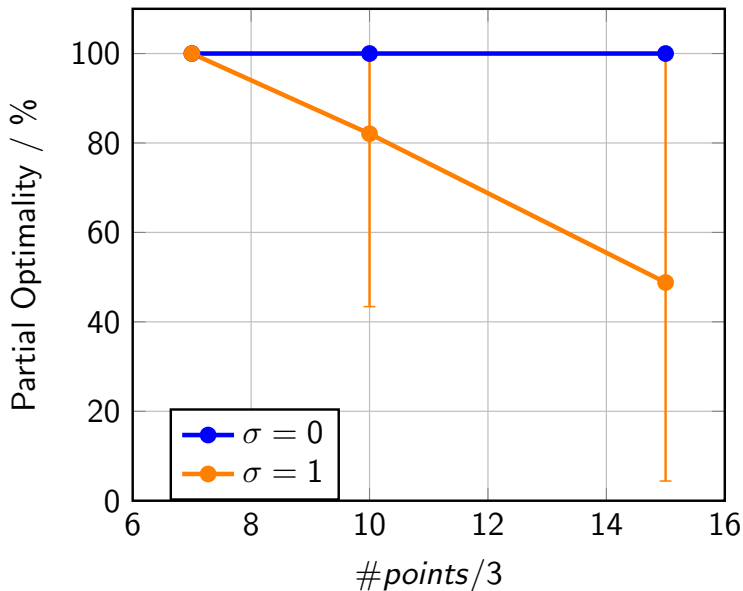
Partial Optimality (3x15 points)



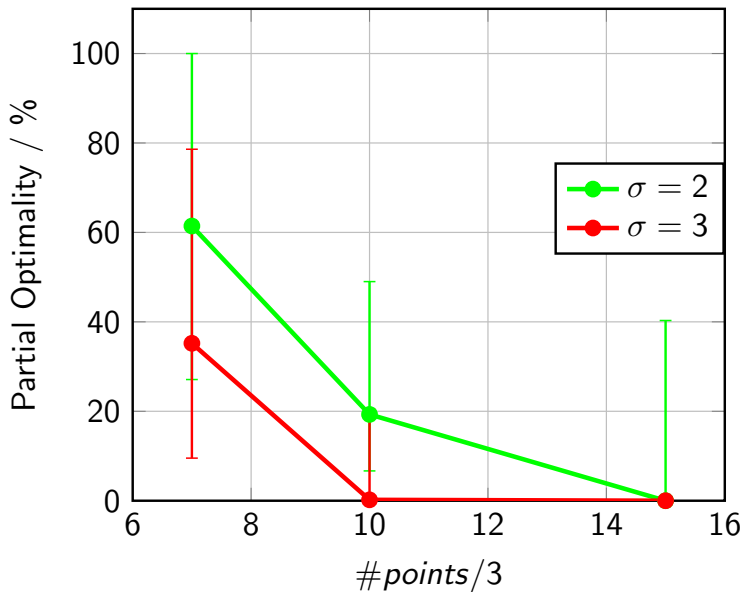
Accuracy (3x15 points)



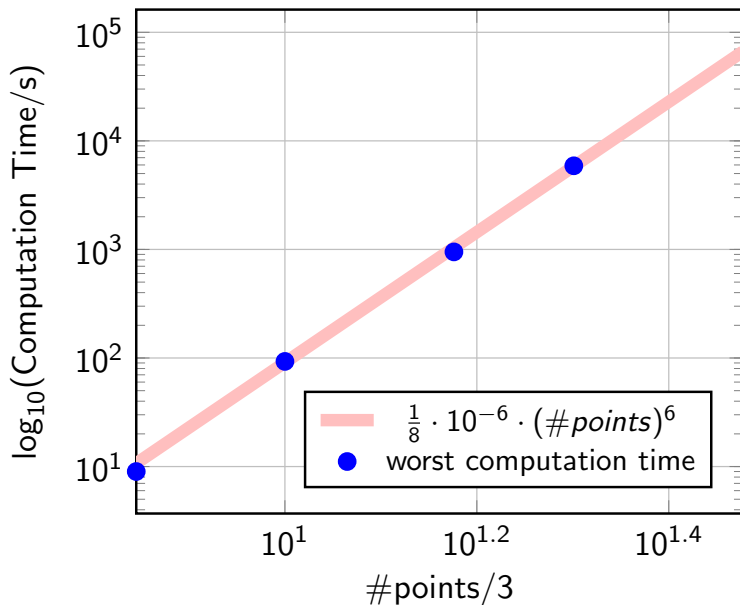
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Partial Optimality



Computation Time (worst case)



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Future Work:

- optimize and parallelize the algorithm
- overcome the partial optimality loss for c
- determine better parameters for c
- update c with advanced criteria

My program, scripts and presentation:

<https://github.com/Vovsanka/ResearchProjectML>

TODO: citation