Partially Optimal Cubic Subspace Clustering

Research Project Machine Learning

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1 Introduction

2 Partial Optimality for Cubic Clique Partition Problem

3 Cubic Subspace Instance Construction

4 Experiments and Evaluation

5 Conclusion

Problem Statement (1)

Finite sample set S, cost function $\mathrm{c}\colon \binom{S}{3} \to \mathbb{R}$.
Instance of the **Cubic Clique Partition Problem**:

$$\min_{\mathrm{y}\colon \binom{S}{2} \to \{0,1\}} \sum_{abc \in \binom{S}{3}} \mathrm{c}_{abc}\, \mathrm{y}_{ab}\, \mathrm{y}_{bc}\, \mathrm{y}_{ac}$$

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Find a **partially optimal solution**, i.e. fix some labels y_{ab} for distinct $a, b \in S$

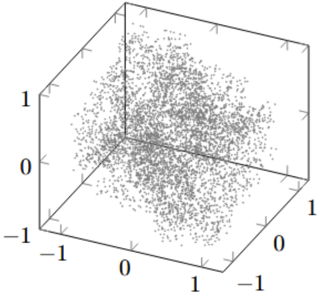
$$\begin{cases} \mathrm{y}_{ab} = 1 & \text{join } a, b \\ \mathrm{y}_{ab} = 0 & \text{cut } a, b \\ \mathrm{y}_{ab} = \,? & \text{unknown} \end{cases}$$

in such way that there still exists an optimal solution.

Problem Statement (2)

Subspace Instances of the Cubic Clique Partition Problem

Samples S: points $S \subset \mathbb{R}^3$



(a) Samples S

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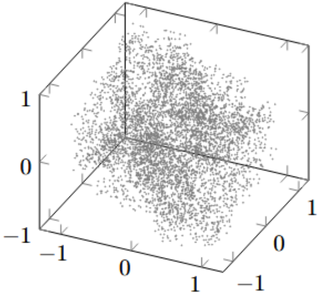

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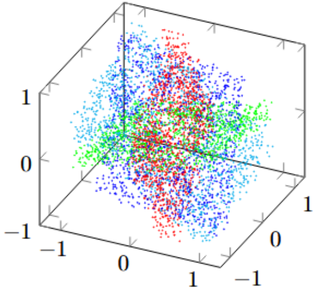

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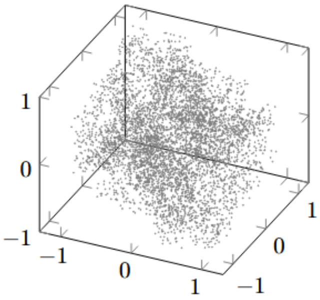

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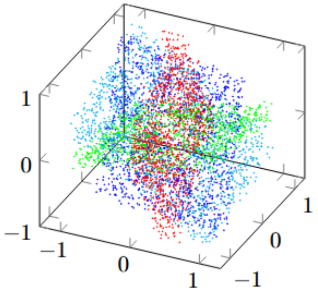

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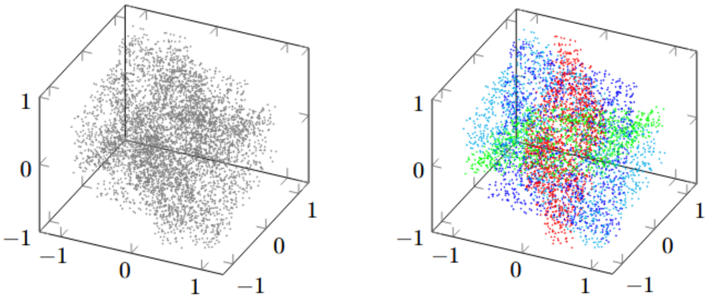
Optimal labeling y*: original planes
Cost function c? (no concrete plane information given)



(a) Samples S (b) Optimal labeling y*

Research Goals and Contributions

Related Work:

- Stein, Di Gregorio, and Andres, "Partial Optimality in Cubic Correlation Clustering"
- Lange, Karrenbauer, and Andres, "Partial Optimality and Fast Lower Bounds for Weighted Correlation Clustering"
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3. Apply algorithm to the subspace instances, assess partial optimality, accuracy and computation time
   $\rightarrow$ experiments and evaluation

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Apply partial optimality connditions $\rightarrow$ solve subproblems

Partial Optimality Algorithm:

Input: labeling y without fixed labels

while condition applied **do**

apply subproblem-CUT-condition exhaustively

apply one of JOIN-conditions (in effective order)

end while

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Output: partially optimal labeling y with some fixed labels

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Reduction to subproblems:

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2. JOIN-Conditions: fix JOIN labels for elements of the sample subset; add the join-cost to $c_\emptyset$; solve the problem where the subset is considered as one sample;

Subproblem-CUT and Subset-JOIN

Subproblem-CUT: cut sample subsets $R_1, R_2, \ldots, R_k$ that are only connected via non-negative costs (applied if $k > 1$)

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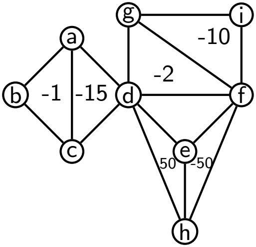

$c_\emptyset = 0$

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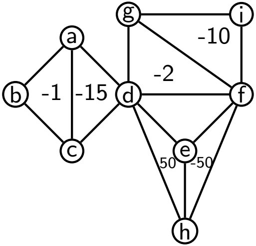

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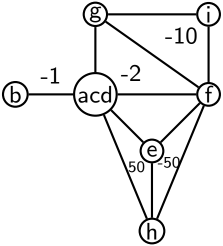

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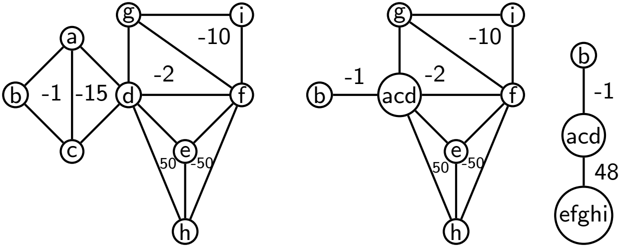

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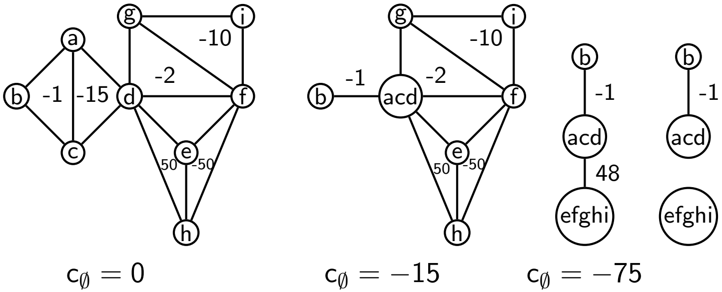

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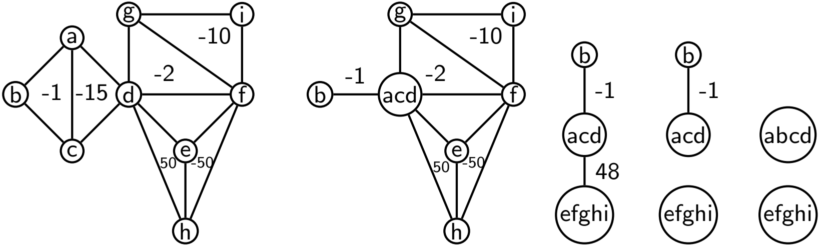

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Other JOIN-conditions

Pair-JOIN-1: join samples i, j if their overall joining reward $\geq$ the sum of rewards and penalties for joining some subset R with $i \in R$ and $\overline{R}$ with $j \in \overline{R}$ ($\approx$ i-j min-cut)

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Pair-JOIN-2: join samples i, k if there exists a sample triple ijk that fulfills 3 conditions ($\approx$ i-jk min-cut, $\approx$ ij-k min-cut, 1 explicit condition)

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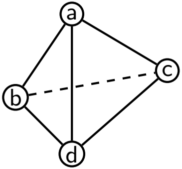
Triple-JOIN: join samples i, j, k if the condition holds (similar to Pair-JOIN-1) ($\approx$ i-jk min-cut)

Pyramid Instance and CUT-conditions

$c_{bcd} = 10$

$c_{abc} = c_{abd} = c_{acd} = -50$



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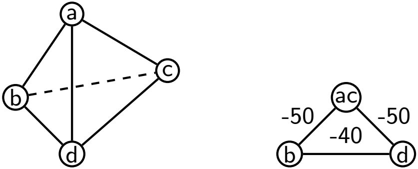

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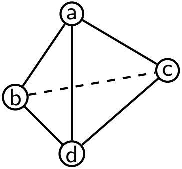

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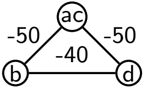
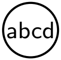


Subset-JOIN



$c_{\emptyset} = -140$

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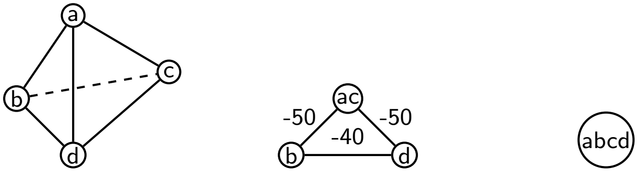

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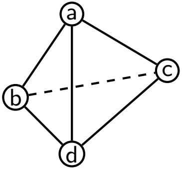

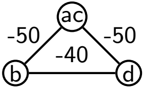

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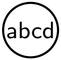

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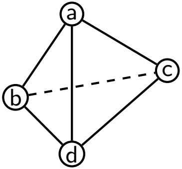
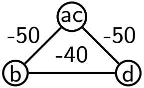
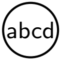






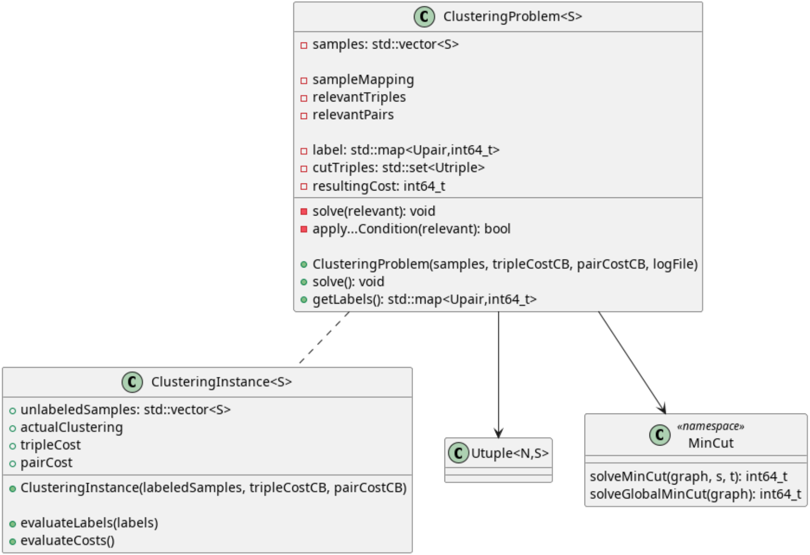
Pair-JOIN-2 Subset-JOIN $c_{\emptyset} = -140$

Pair-CUT: cut samples i, j if the direct joing penalty $\geq$ the sum of rewards for joining some subset R with $i \in R$ and $\overline{R}$ with $j \in \overline{R}$ ($\approx$ i-j min-cut)

Triple-CUT: cut samples i, j, k if the condition holds (similar to Pair-CUT) ($\approx$ i-jk min-cut)

Samples in the pyramid with $c_{bcd} = 100$ are unjoinable!
Triple-CUT is applied to the triple *bcd*

Program Structure



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Plane and Point Generation

Plane Generation:

- generate 3 planes
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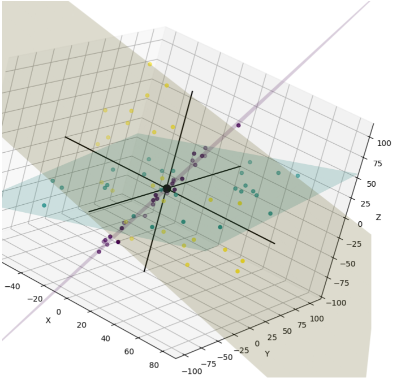
  $\vec{n}_1, \vec{n}_2, \vec{n}_3$ (normalized)



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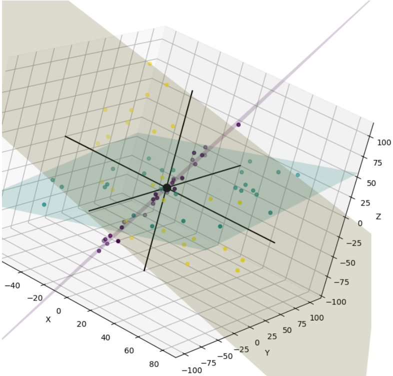

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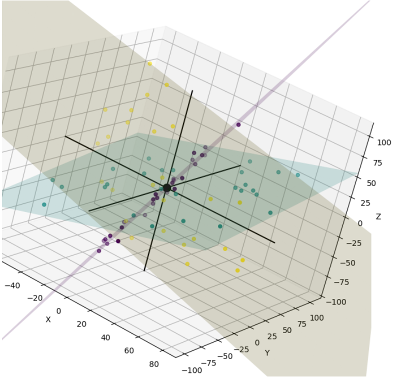

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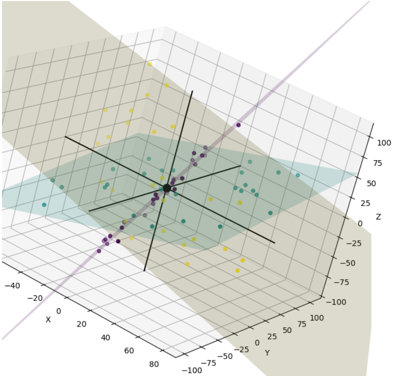

Point Generation on the plane $(\vec{n}, \vec{r}_1, \vec{r}_2)$, parameters (D, σ):

- random variables $k_1, k_2 \in [-D, D]$ (uniform distribution)
- random variable k_n (normal distribution based on σ)
- generate point $p = k_1\vec{r}_1 + k_2\vec{r}_2 + k_n\vec{n}$

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Triangle $abc \in \binom{S}{3}$

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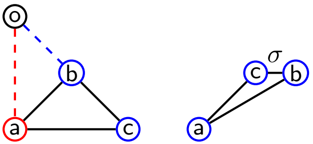
$\rightarrow \mathrm{c}_{abc} = 0$



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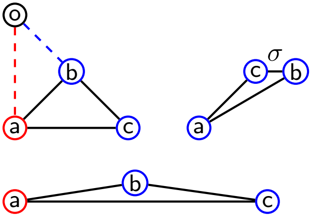

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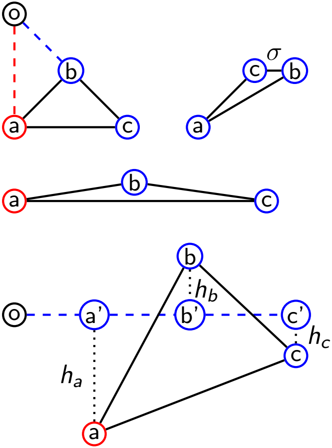
(3) h_a, h_b, h_c: distances
to the best fitted plane
containing the origin;
$h_a + h_b + h_c > 3\sigma + 10^{-6}$
$\rightarrow c_{abc} = \frac{(h_a+h_b+h_c)-(3\sigma+10^{-6})}{3D}$



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(4) h_o: distance from the origin to the triangle plane;
$h_o > \frac{10}{\#points}\sigma + 10^{-6}$
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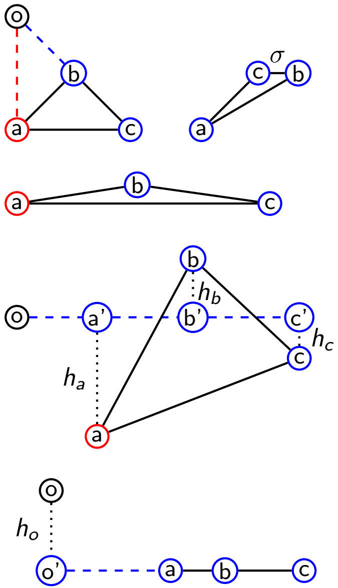

Cost Function (2)

(5) for all points p:
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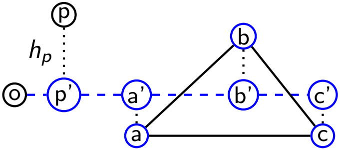

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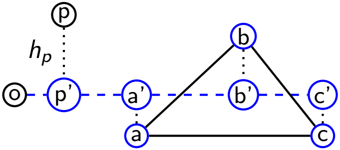

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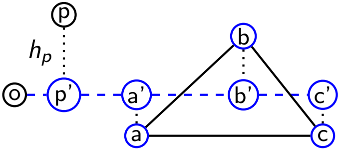

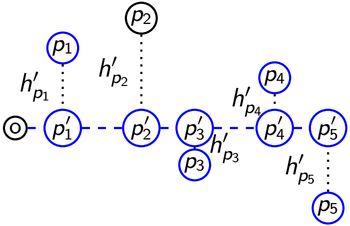

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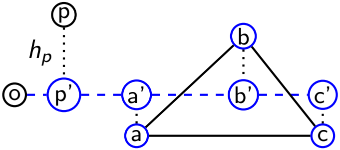
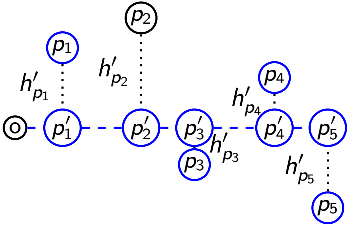
$M := \{p \mid p \text{ chosen } \wedge \delta_p < 0\}$





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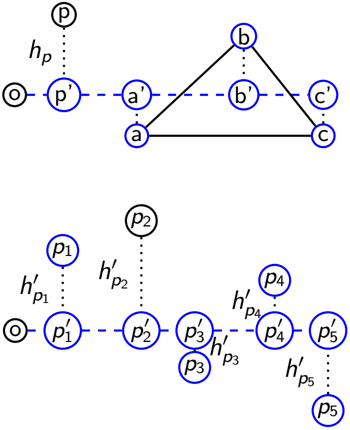

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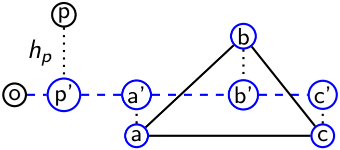

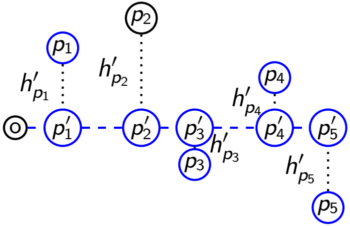

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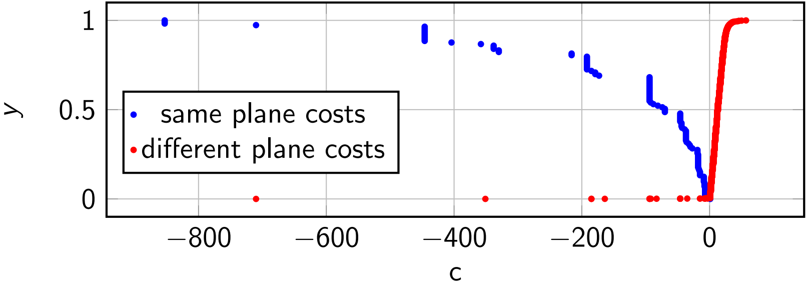
Cost Function Evaluation (3x15 points, $\sigma = 1$)



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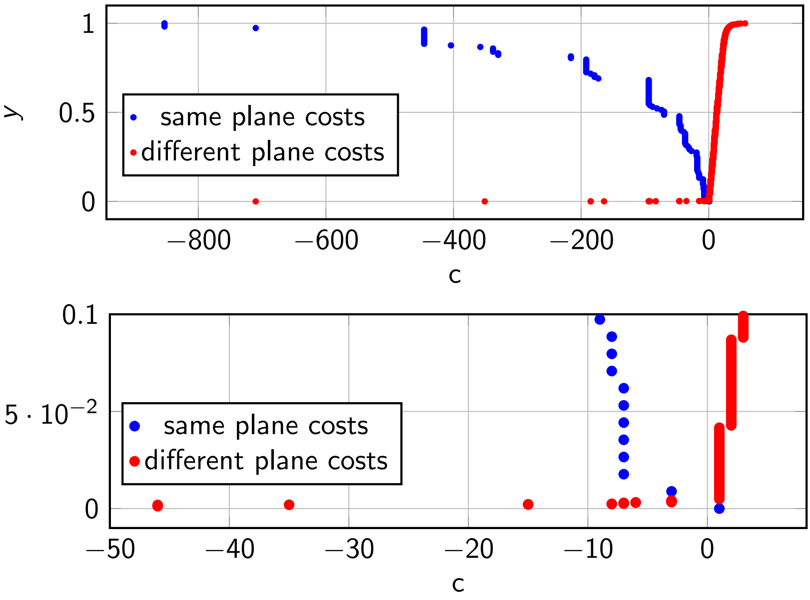

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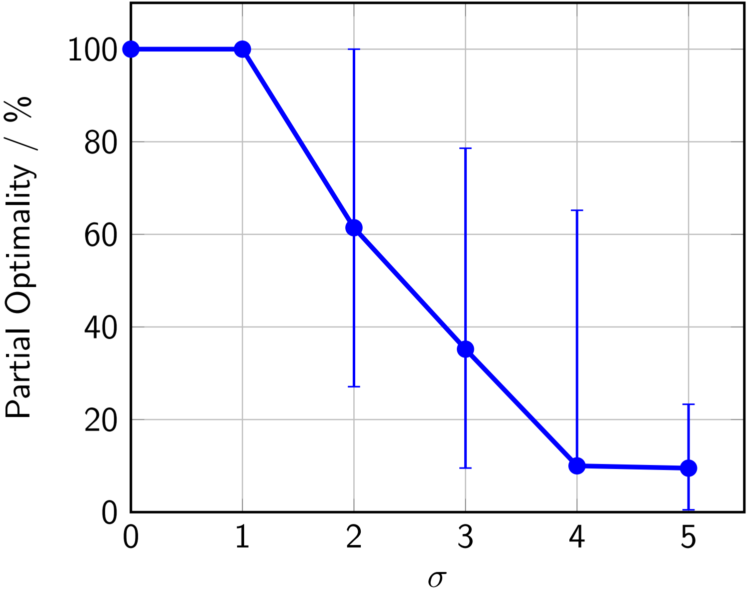
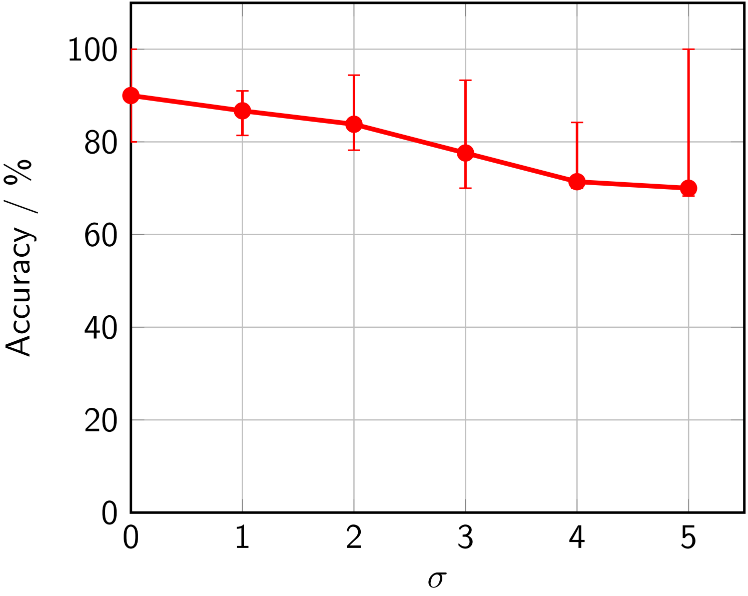
- Capture
  - 1.quartile (Q1)
  - median (Q2)
  - 3.quartile(Q3)
  - the worst computation time

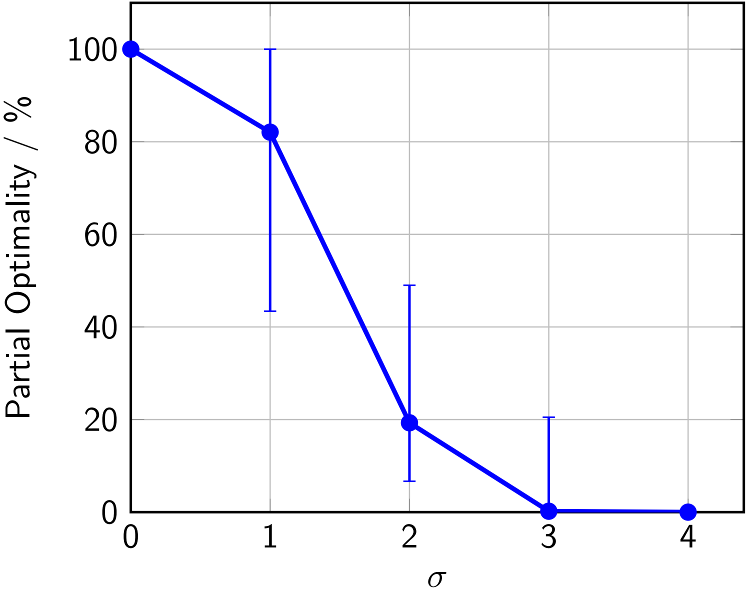
Partial Optimality (3x7 points)



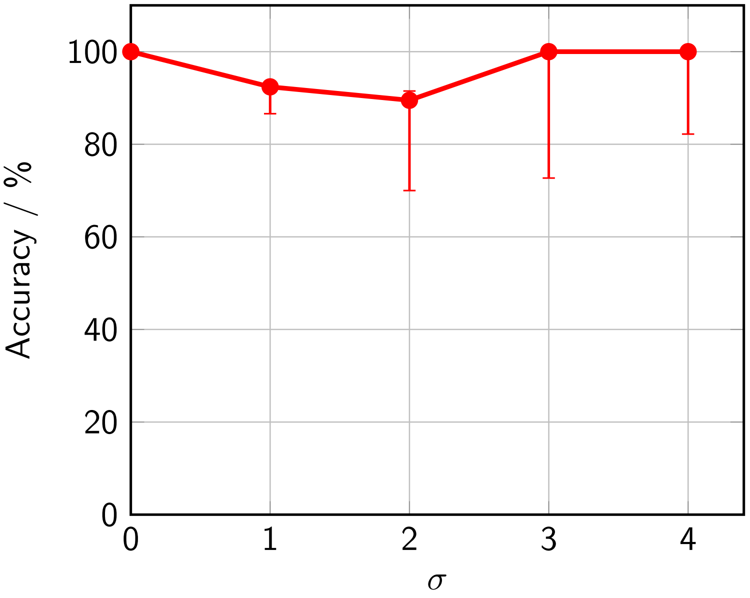
Accuracy (3x7 points)



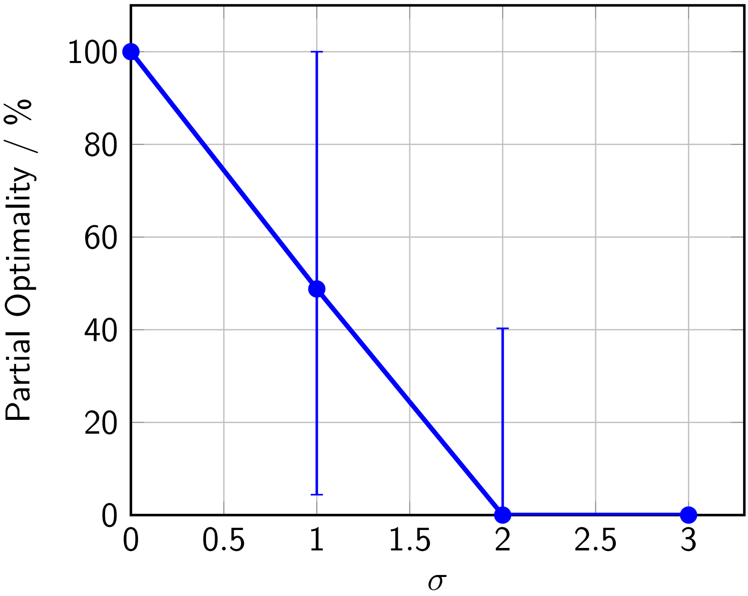
Partial Optimality (3x10 points)



Accuracy (3x10 points)



Partial Optimality (3x15 points)



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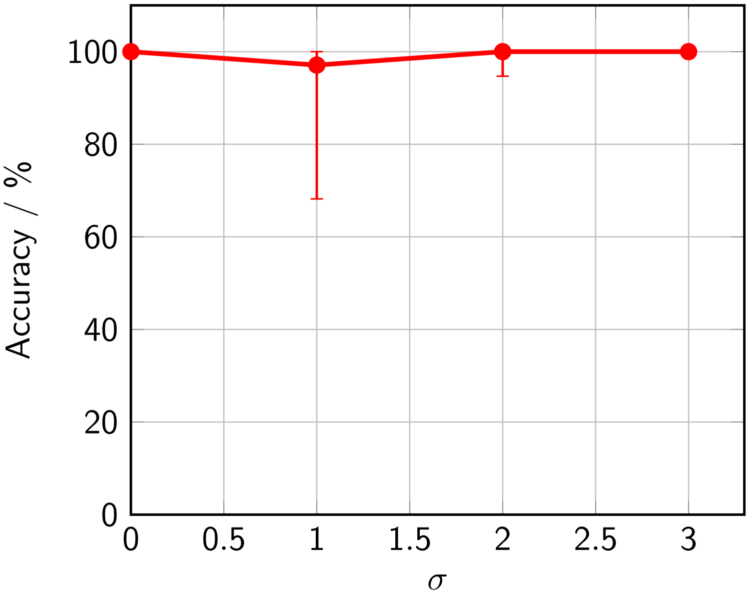

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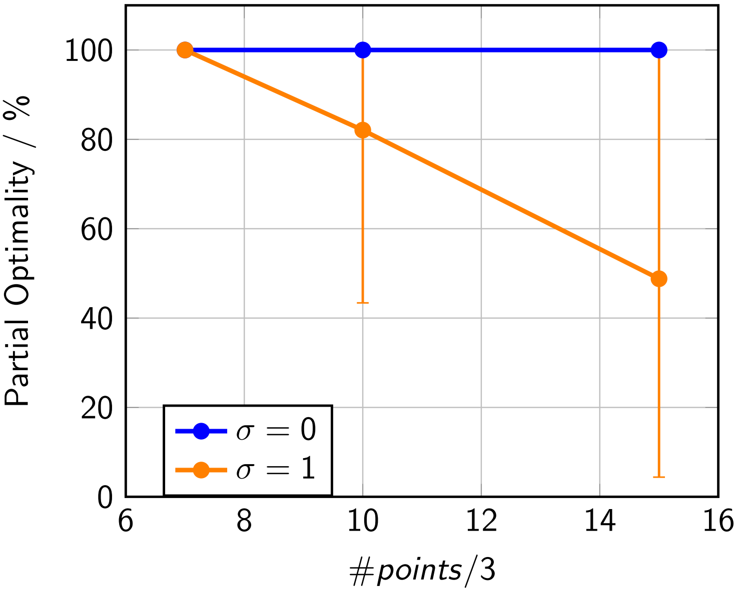

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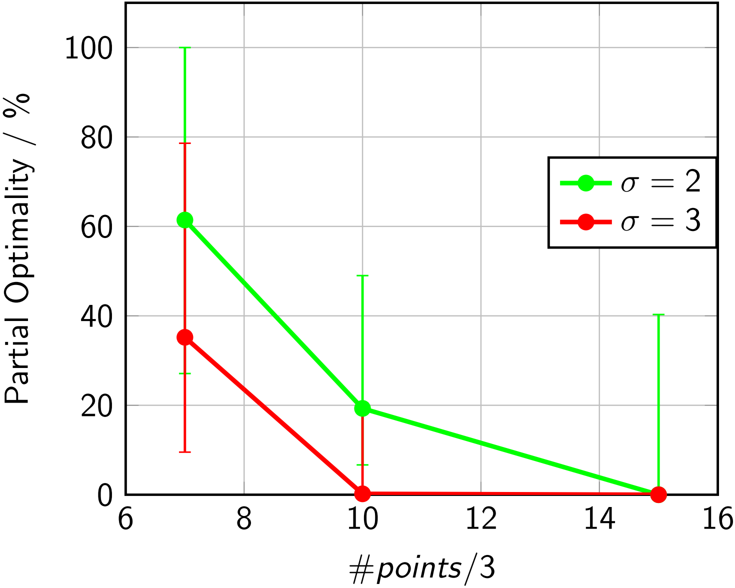
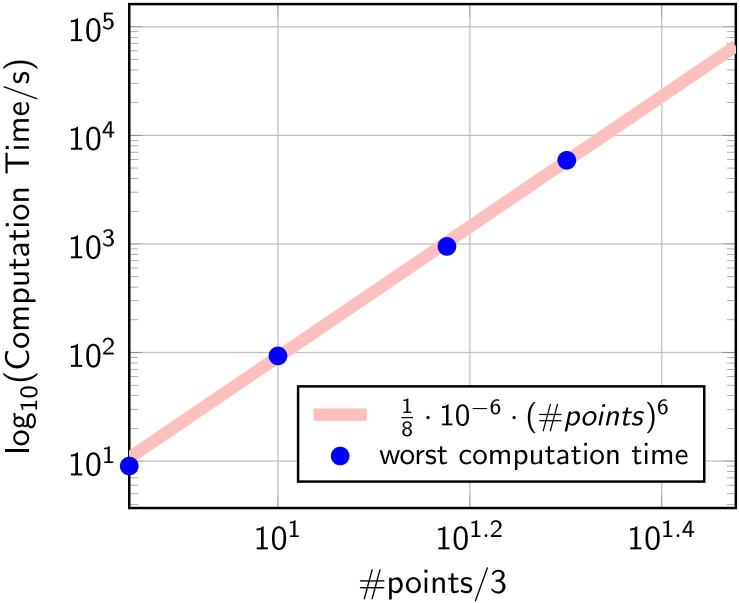


Computation Time (worst case)



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Future Work:

- optimize and parallelize the algorithm
- overcome the partial optimality loss for c
- determine better parameters for c
- update c with advanced criteria

References

My program, scripts and presentation:

`https://github.com/Vovsanka/ResearchProjectML`

Bibliography:

- Lange, Jan-Hendrik, Bjoern Andres, and Paul Swoboda. "Combinatorial persistency criteria for multicut and max-cut". In: *CVPR* (2019).
- Lange, Jan-Hendrik, Andreas Karrenbauer, and Bjoern Andres. "Partial Optimality and Fast Lower Bounds for Weighted Correlation Clustering". In: *ICML* (2018).
- Stein, David, Silvia Di Gregorio, and Bjoern Andres. "Partial Optimality in Cubic Correlation Clustering". In: *ICML* (2023).