7.6

Donnerstag, 5. Mai 2022 13:18

$$\int_0^1 \int_{x^2}^{x} \int_{x-y}^{x+y} 21(x+y+z)\, dz\, dy\, dx$$

$$= \int_0^1 \int_{x^2}^{x} \left[21xz + 21yz + 21\frac{z^2}{2} \right]_{x-y}^{x+y} dy\, dx$$

$$= \int_0^1 \int_{x^2}^{x} \cancel{21x^2} + 21xy + 21xy + 21y^2 + 21\frac{\cancel{x^2} + 2xy + \cancel{y^2}}{2} - \cancel{21x^2} + \cancel{21xy} - \cancel{21xy} + 21y^2 - 21 \cdot \frac{\cancel{x^2} - 2xy + \cancel{y^2}}{2}\, dy\, dz$$

$$= \int_0^1 \int_{x^2}^{x} 84xy + 42y^2\, dy\, dx$$

$$= \int_0^1 \left[42xy^2 + 14y^3 \right]_{x^2}^{x} dx$$

$$= \int_0^1 42x^3 + 14x^3 - 42x^5 - 14x^6\, dx = \int_0^1 56x^3 - 42x^5 - 14x^6\, dx$$

$$= \left[14x^4 - 7x^6 - 2x^7 \right]_0^1 = 14 - 7 - 2 = 5$$