Problem 4: Finding the sine and cosine Fourier coefficients

Suppose we have a periodic function $f(x)$ that's slightly different than our overview PDF file:

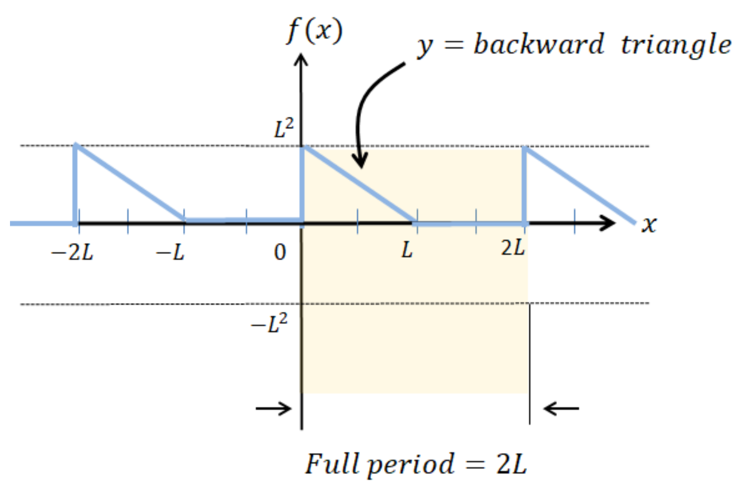

Figure 1: A sawtooth-shaped $f(x)$ that is neither odd nor even

We would like to express $f(x)$ as both a linear combination of the orthogonal basis functions sines or cosines.

$$\psi_m(x) = \left\{ \cos\left(\frac{m\pi}{L}x\right) \right\} \qquad and\ /\ or \qquad \phi_n(x) = \left\{ \sin\left(\frac{n\pi}{L}x\right) \right\}$$

Where: $m = 0,\ 1,\ 2\ \cdots,\qquad and \quad n\ =\ 1,\ 2, \cdots\ positive\ integers$

Let's write out the reconstruction of our target function (x) :

DC term !!

$$f(x) = a_0\ \psi_0(x)\ +\ a_1\,\psi_1(x)\ +\ a_2\,\psi_2(x)\ +\ \cdots +\ a_m\,\psi_m(x)$$
$$+\ b_1\,\phi_1(x)\ +\ b_2\,\phi_2(x)\ +\ \cdots +\ b_n\,\phi_n(x)$$

We can easily rewrite this in a compact form…. of which you might have seen before in undergrad !

$$\boxed{f(x) = \sum_{m=0}^{\infty} a_m \cos\left(\frac{m\pi}{L}x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L}x\right)} = \underbrace{\sum_{m=0}^{\infty} a_m\,\psi_m(x)}_{\substack{even\\ basis}} + \underbrace{\sum_{n=1}^{\infty} b_n\,\phi_n(x)}_{\substack{odd\\ basis}}$$

<u>Your tasks:</u> *(Turn in all blue-highlighted paper-written work, yellow-highlighted echoed parts, matlab codes, and your plots in your matlab publish PDF files)*

1\. . Using inner products definition over the:

<u>full interval</u> $x = [0,\ 2L]$, where $L = 3.$

and taking advantage of the orthogonality / $(\text{norm})^2$ relationships among cosines, sines, write down the general expression for the sine and cosine Fourier coefficients (*do the integral on paper…. while leaving <u>the constant "L" as an arbitrary one</u> !!!) , where:*

- $a_m \ = \ The \ m^{th}$- cosine Fourier coefficient
- $b_n \ = \ The \ n^{th}$- sine Fourier coefficient

2\. Numerically evaluate and echo the following in your diary:

- The first 6 cosine coefficients $a_0, a_1 \cdots, a_5$.
- The first 5 sine coefficients $b_1 \cdots, b_5$.

3\. Using the first 6 cosine coefficients (this includes the DC term) and the first 5 sine coefficients , we can reconstruct $f(x)$ with the following expression:

$$f(x) \ \approx \ \sum_{m=0}^{5} a_m \ \ cos\left(\frac{m\pi}{L}x\right) \ + \ \sum_{n=1}^{5} b_n \ sin\left(\frac{n\pi}{L}x\right)$$

- Using either the *line* or the *plot* function, plot the original $f(x)$in the interval $x = \ [0,\ 2L]$
- Hold the figure
- Overlay the Fourier series reconstruction of $f(x)$ onto the plot as a different color.
- Add legend to differentiate between the 2 traces !