BE601 HW2 Problem 4

$$f(x) = \begin{cases} L-x & 0 < x < L \\ 0 & L < x < 2L \end{cases}$$

$$a_0 = \frac{1}{2L}\int_0^{2L} f(x)\,dx = \frac{1}{2L}\int_0^{L}(L-x)\,dx + \frac{1}{L}\int_L^{2L} 0\,dx$$

$$= \frac{1}{2L}\left[Lx - \tfrac{1}{2}x^2\right]_0^L$$

$$\frac{1}{2L}\left[L^2 - L^2/2\right]$$

$$a_0 = \frac{L}{2} - L/4 = L/4$$

$$a_n = \frac{1}{L}\int_0^{2L} f(x)\cos nx\,dx$$

$$= \frac{1}{L}\int_0^{L}(L-x)\cos nx\,dx + \cancel{\frac{1}{L}\int_L^{2L} 0\,dx} \to 0$$

integ $\left[\int_a^b u\frac{dv}{dx}dx = [uv]_a^b - \int_a^b \frac{du}{dx}v\right.$

$$\frac{1}{L}\left[(L-x)\frac{\sin nx}{n}\Big|_0^L - (-1)\int_0^L \frac{\sin nx}{n}dx\right]$$

$$0 + 0 \qquad -\frac{\cos nx}{n}\Big|_0^L$$

$$1/Ln \cdot \frac{-\cos nx}{n}\Big|_0^L \longrightarrow \frac{1}{n^2 L}(\cos Ln - 1)$$

$$a_m = \frac{1}{m^2 L}(\cos Lm - 1)$$

$$b_n = \frac{1}{L}\int_0^L (L-x)\sin nx\,dx + \cancel{\int_L^{2L} 0\,dx} \to 0$$

$$(L-x)\frac{-\cos nx}{n}\Big|_0^L + \int_0^L -\frac{\cos nx}{n}dx$$

$$-L\frac{\cos 0}{n} \qquad \frac{\sin Ln}{n}$$

$$+L/n$$

$$1/L\left[L/n + \frac{\sin Ln}{n}\right]$$

$$b_n = \frac{1}{n} + \frac{\sin Ln}{n}$$
