BE 601 HW #1

Problem #1

A. Find $|A|$

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 \\ 2 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ -1 & -1 & 2 & -1 \end{bmatrix}$$

$$|A| = -20$$

How?: Well let's do it w/ Laplace Expansion

Choose the column or row with the most 0s

In this case row 2, so $i=2$

$$\sum_{i,j=1}^{n} (-1)^{i+j} a_{ij} M_{ij}$$

$$\cancel{|A|} = |A| = \sum_{j=1}^{4} (-1)^{2+j} a_{2j} M_{2j}$$

$$= (-1)^{2+1} a_{21} M_{21} + \cancel{(-1)^{2+2} a_{22} M_{22}}^{\,0} + \cancel{(-1)^{2+3} a_{23} M_{23}}^{\,0} + \cancel{(-1)^{2+4} a_{24} M_{24}}^{\,0}$$

$a_{22} = 0$ $\quad a_{23} = 0$ $\quad a_{24} = 0$

$$= -2 M_{21}$$

$$M_{21} = \begin{vmatrix} -1 & 0 & 1 \\ 2 & 3 & 0 \\ -1 & 2 & -1 \end{vmatrix} = \overset{7}{(-1\cdot 3 \cdot -1 + 0 \cdot 0 \cdot -1 + 1 \cdot 2 \cdot 2)} - (\underset{-3}{-1 \cdot 3 \cdot 1} + 2 \cdot 0 \cdot -1 + -1 \cdot 2 \cdot 0)$$

$$\begin{array}{ccc|cc} -1 & 0 & 1 & -1 & 0 \\ 2 & 3 & 0 & 2 & 3 \\ -1 & 2 & -1 & -1 & 2 \end{array}$$

$$= +10$$

Thus, $|A| = -2(10) = -20$

Find cofactor matrix $\quad C_{ij} = (-1)^{i+j} M_{ij}$

$\cancel{C_{11} =}$

$$C_{11} = +1 \begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 0 \\ -1 & 2 & -1 \end{vmatrix} = 0$$

→ top row is 0

$$C_{12} = -1 \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ -1 & 2 & -1 \end{vmatrix} = -1[[(2\cdot 3 \cdot -1) + (0 \cdot 0 \cdot -1) + (0 \cdot 0 \cdot 2)] - [(-1 \cdot 3 \cdot 0) + (2 \cdot 0 \cdot 2) + (-1 \cdot 0$$

$$= 6$$

cont. →

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Problem 1

Part B(cont.)

$$C_{13} = 1\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ -1 & -1 & -1 \end{vmatrix} = (2\cdot 2\cdot -1 + 0\cdot 0\cdot -1 + 0\cdot 0\cdot -1) - (-1\cdot 2\cdot 0 + -1\cdot 0\cdot 2 + -1\cdot 0\cdot$$
$$= 4$$

$$C_{14} = -1\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 3 \\ -1 & -1 & 2 \end{vmatrix} = -1[(2\cdot 2\cdot 2 + 0\cdot 3\cdot -1 + 0\cdot 0\cdot -1) - (-1\cdot 2\cdot 0 + -1\cdot 3\cdot 2 + 2\cdot 0\cdot 0)]$$
$$= -14$$

$$C_{21} = -1\begin{vmatrix} -1 & 0 & 1 \\ 2 & 3 & 0 \\ -1 & 2 & -1 \end{vmatrix} = -1[(-1\cdot \underset{+3}{3\cdot 1} + \overset{0\cdot 0\cdot -1}{\cancel{2\cdot 0\cdot -1}} + 1\cdot \underset{4}{2\cdot 2}) - (\underset{-3}{-1\cdot 3\cdot} 1 + 2\cdot 0\cdot -1 + 1\cdot 2\cdot 0)]$$
$$= -10$$

$$C_{22} = -1\begin{vmatrix} 1 & 0 & 1 \\ 0 & 3 & 0 \\ -1 & \overset{}{\underset{2}{\cancel{1}}} & -1 \end{vmatrix} = (1\cdot \overset{3}{\cancel{2}}\cdot -1 + \overset{0}{\cancel{1}}\cdot 0\cdot -1 + 1\cdot 0\cdot -1) - (-1\cdot \overset{3}{\cancel{2}}\cdot 1 + \overset{2}{\cancel{1}}\cdot 0\cdot 1 + -1\cdot 0\cdot -1)]$$
$$= 0$$

$$C_{23} = -1\begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ -1 & -1 & -1 \end{vmatrix} = 0$$ → same as above

$$C_{24} = 1\begin{vmatrix} 1 & -1 & 0 \\ 0 & 2 & 3 \\ -1 & -1 & 2 \end{vmatrix} = (122 + -13-1 + 0) - (0 + -131 + 0)$$
$$= 10$$

$$C_{31} = 1\begin{vmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

$$C_{32} = -1\begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ -1 & 2 & -1 \end{vmatrix} = (0 + 0 + 12-2) - (0 + 0 + -121]$$
$$= -4$$

$$C_{33} = 1\begin{vmatrix} 1 & -1 & 1 \\ 2 & 0 & 0 \\ -1 & -1 & -1 \end{vmatrix} = (0 + 0 + 12-1) - (0 + 0 + -12-1)$$
$$= -4$$

$$C_{34} = -1\begin{vmatrix} 1 & -1 & 0 \\ 2 & 0 & 0 \\ -1 & -1 & 2 \end{vmatrix} = (0 + 0 + 0) - (0 + 0 + 2\cdot 2\cdot -1)$$
$$= \cancel{8} -4$$

$$C_{41} = -1\begin{vmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 3 & 0 \end{vmatrix} = (0 + 0 + 0) - (0 + 0 + 0)$$
$$= 0$$

$$C_{42} = +1\begin{vmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix} = (0 + 0 + 123) + (0 + 0 + 0)$$
$$= 6$$

→

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Problem 1

Part B (cont.)

$$C_{43} = -1 \begin{vmatrix} 1 & -1 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = (0+0+122)-(0+0+0)$$
$$= -4$$

$$C_{44} = 1 \begin{vmatrix} 1 & -1 & 0 \\ 2 & 0 & 0 \\ 0 & 2 & 3 \end{vmatrix} = (0+0+0)-(0+0+32-1)$$
$$= -6$$

So $C = \begin{bmatrix} 0 & 6 & -4 & -14 \\ -10 & 0 & 0 & 10 \\ 0 & -4 & -4 & -4 \\ 0 & 6 & -4 & 6 \end{bmatrix}$ $\quad C^T = \begin{bmatrix} 0 & -10 & 0 & 0 \\ 6 & 0 & -4 & 6 \\ -4 & 0 & -4 & -4 \\ -14 & 10 & -4 & 6 \end{bmatrix}$

Then $A^{-1} = C^T / |A|$

$C^T / -20$

$$A^{-1} = \begin{bmatrix} 0 & 0.5 & 0 & 0 \\ -0.3 & 0 & 0.2 & -0.3 \\ 0.2 & 0 & 0.2 & 0.2 \\ 0.7 & -0.5 & 0.2 & -0.3 \end{bmatrix}$$

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Problem # ~~2~~ 3

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part 1

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 4 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 11 \\ 5 \end{bmatrix}$$

Find P, L, U Essentially I solve for U by eliminating $A_{1,2}$, $A_{1,3}$, $A_{2,3}$

$$\begin{matrix} 1 & -2 & 1 \\ 2 & 4 & 1 \\ 2 & 1 & 1 \end{matrix} \xrightarrow{-(2R_1)+R_2} \begin{matrix} 1 & -2 & 1 \\ 0 & 8 & -1 \\ 2 & 1 & 1 \end{matrix} \xrightarrow{-2R_1+R_3} \begin{matrix} 1 & -2 & 1 \\ 0 & 8 & -1 \\ 0 & 5 & -1 \end{matrix} \xrightarrow{-5/8 R_1 + R_3}$$

From here we can get $L_{2,1} = 2$ thus $L_{3,1} = 2$ $L_{3,2} = 5/8$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 8 & -1 \\ 0 & 0 & -0.375 \end{bmatrix}$$ <- Well this is U

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 0.625 & 1 \end{bmatrix}$$

I think through this P remains unchanged and is still $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

This doesn't match & lu(A)... but if we ~~test~~ consider lu(sparse(A)) it matches...

Part 2

~~b_{new}~~ Find c b_{new} is just b since Pb is just b

$$Lc = b \rightarrow \quad c_1 = -2$$
$$2c_1 + c_2 = 11 \rightarrow c_2 = 15$$
$$\underbrace{2c_1}_{-4} + \underbrace{0.625c_2}_{9.375} + c_3 = 5 \rightarrow c_3 = -0.375$$

$$c = \begin{bmatrix} -2 \\ 15 \\ -0.375 \end{bmatrix}$$

$$Ux = c \rightarrow \quad x_1 - 2x_2 + x_3 = -2 \quad x_1 = 1$$
$$8x_2 - x_3 = 15 \quad x_2 = 2$$
$$-0.375x_3 = -0.375 \quad x_3 = 1$$

$$x = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

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Problem #3 cont.

✓ linearly dependent

Part 3

ln(A)

$$A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 2 & -2 \\ 0 & 4 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

~~Well Linear independence is easily found~~

~~I think we can determine that by~~

Well we know that L and U are linearly dependent, ~~but P is~~ and P is independent. Since L and U are just decompositions we can determin that ~~A is also li~~ $\underset{\sim}{a}_1$, $\underset{\sim}{a}_2$, $\underset{\sim}{a}_3$ are ~~depen~~ linearly dependent.

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~~Part A~~ Problem #4

Part A

Node 1 | $G_0(V_1-12)+G_0(V_1-0)+G_0(V_1-V_2)+G_0(V_1-V_6)=0$

2 | $G_0(V_2-V_1)\quad G_0(V_2-0)+G_0(V_2-V_3)+G_0(V_2-V_7)=0$

3 | $G_0(V_3-V_2)+G_0(V_3-0)+G_0(V_3-V_4)+G_0(V_3-V_8)=0$

4 | $G_0[(V_4-V_3)+(V_4-0)+(V_4-V_5)+(V_4-V_9)]=0$

5 | $(V_5-V_4)+(V_5-0)+(V_5-0)+(V_5-V_{10})=0$

6 | $(V_6-12)+(V_6-V_1)+(V_6-V_7)+(V_6-V_{11})=0$

7 | $(V_7-V_6)+(V_7-V_2)+(V_7-V_8)+(V_7-V_{12})=0$

8 | $(V_8-V_7)+(V_8-V_3)+(V_8-V_{13})+(V_8-V_9)=0$

9 | $(V_9-V_8)+(V_9-V_4)+(V_9-V_{10})+(V_9-V_{14})=0$

10 | $(V_{10}-V_9)+(V_{10}-V_5)+(V_{10}-0)+(V_{10}-V_{15})=0$

11 | $(V_{11}-12)+(V_{11}-V_6)+(V_{11}-V_{12})+(V_{11}-0)=0$

12 | $(V_{12}-V_{11})+(V_{12}-V_7)+(V_{12}-V_{13})+(V_{12}-0)=0$

13 | $(V_{13}-V_{12})+(V_{13}-V_8)+(V_{13}-V_{14})+(V_{13}-0)=0$

14 | $(V_{14}-V_{13})+(V_{14}-V_9)+(V_{14}-V_{15})+(V_{14}-0)=0$

15 | $(V_{15}-V_{14})+(V_{15}-V_{10})+(V_{15}-\cancel{14}0)+(V_{15}-0)=0$

Node 1: 4 neighbors 12

2

$$4G_0V_1 - G_0V_2 - G_0V_6 = +12G_0$$
$$4G_0V_2 - G_0V_1 - G_0V_3 - G_0V_7 = 0$$
$$4G_0V_3 - G_0V_2 - G_0V_4 - G_0V_8 = 0$$
$$4G_0V_4 - G_0V_3 - G_0V_5 - G_0V_9 = 0$$
$$4G_0V_5 - G_0V_4 \cancel{-G_0G_0V_{10}} = \cancel{+10G_0}\ 0$$
$$4G_0V_6 - G_0V_1 - G_0V_7 - G_0V_{11} = +12G_0$$
$$4G_0V_7 - G_0V_6 - G_0V_2 - G_0V_8 - G_0V_{12} = 0$$
$$4G_0V_8 - G_0V_7 - G_0V_3 - G_0V_{13} - G_0V_9 = 0$$
$$4G_0V_9 - G_0V_8 - G_0V_4 - G_0V_{10} - G_0V_{14} = 0$$
$$4G_0V_{10} - G_0V_9 - G_0V_5 - G_0V_{15} = 0$$
$$4G_0V_{11} - G_0V_6 - G_0V_{12} = +12G_0$$
$$4G_0V_{12} - G_0V_{11} - G_0V_7 - G_0V_{13} = 0$$
$$4G_0V_{13} - G_0V_{12} - G_0V_8 - G_0V_{14} = 0$$
$$4G_0V_{14} - G_0V_{13} - G_0V_9 - G_0V_{15} = 0$$
$$4G_0V_{15} - G_0V_{14} - G_0V_{10} = 0$$

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Problem #4 cont.

Part A cont.

$$G = \begin{bmatrix}
4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 4 & \cancel{0}-1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & \cancel{-4}-1 & 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 4 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 4 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 4 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 4
\end{bmatrix}$$

$$b = \begin{bmatrix} 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 12 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Block Matrix $G = \begin{bmatrix} D & -I & \\ -I & D & -I \\ & -I & D \end{bmatrix}$

$$D = \begin{bmatrix}
\cancel{4} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\
\cancel{0} & \cancel{4} & \cancel{0} & \cancel{0} & \cancel{0} \\
\cancel{0} & \cancel{0} & \cancel{4} & \cancel{0} & \cancel{0} \\
\cancel{0} & \cancel{0} & \cancel{0} & \cancel{4} & \cancel{0} \\
\cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{4}
\end{bmatrix}$$

$$D = \begin{bmatrix}
4 & -1 & 0 & 0 & 0 \\
-1 & 4 & -1 & 0 & 0 \\
0 & -1 & 4 & -1 & 0 \\
0 & 0 & -1 & 4 & -1 \\
0 & 0 & 0 & -1 & 4
\end{bmatrix} \quad I = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

Problem # 4 cont.

Part A cont.

5. Yes, LU factorization will always preserve bandwith.
6. The <u>width</u> of the matrix (~~not including the diagonal~~). /circuit / the nodes between the source and grounds
7. You could theoretically avoid it by not actually using pivots and simply do good old gaussian elimination.

Problem # 4 cont.

Part B Nodes 1-5 are the same, and 6, 8, 10 (in terms of the V connections)
Node 7, 9 and 11-15 change others change as well from 4 → 3 or 2

$$
\begin{array}{c}
1\\2\\3\\4\\5\\6\\7\\8\\9\\10\\11\\12\\13\\14\\15
\end{array}
\quad
G_{new} = \begin{bmatrix}
4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 4 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 3 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 3 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 3
\end{bmatrix}
$$

$$
b = \begin{bmatrix} 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 12 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

b has not changed as the only node to affect that would be 11, however not having a ground should not change it.

$$
\cancel{D_1 = \begin{bmatrix} 4 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & \cdots & & \\ 0 & 0 & 0 & 0 & \end{bmatrix}}
\quad
D_1 = \begin{bmatrix} 4 & -1 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & 4 \end{bmatrix}
\quad
D_2 = \begin{bmatrix} 4 & -1 & 0 & 0 & 0 \\ -1 & 3 & -1 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 4 \end{bmatrix}
$$

$$
D_3 = \begin{bmatrix} 3 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}
\quad
E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & +1 \end{bmatrix}
$$

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Problem #4 cont.

Part B cont.

5. No, I believe I ~~slightly~~ made an error somewhere: certain nodes and there surroundings were unchanged by the removal of resistors, so their voltage should not have changed.