Problem #3: Eigenvalues / eigenvectors / linear transformations

Pertinent reading for Problem #3:

Strang (Introduction to linear algebra, 5th ed):

(Download from Blackboard, under /Resources / Linear algebra texts)

      Ch. 6:   pages 288 - 291   (brief intro to eigenvalues, eigenvectors, and transformations)

                pages   292 - 296   (properties of eigenvalues + eigenvectors)

3Blue-1Brown's video on eigenvalues + eigenvectors

               https://www.youtube.com/watch?v=PFDu9oVAE-g

1. Using matlab's *eig* command, find the eigenvalues and eigenvectors of matrix A.

$$A = \frac{1}{3}\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Part B: Interpreting the eigenvalue equation as "Transformations"

$$C \ (object \ matrix) \quad \xrightarrow{\quad A \quad} \quad AC$$

1) If we apply transformation A to a 3-dimensional cube "C", where vertices of the cube are defined as:

$$C = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

This cube will have an offset with respect the origin !

What will this shape look like after 1 transformation by A ? Using matlab, plot:

- The vertices and borders of the pre-image cube "C" with *plot3* and *line* commands. Draw each corner of the cube as small black dots (using the plot3 command), all borders as thin black lines, and also, color the closest to the origin RED.

- Choose 3 faces of that cube and shade them with different colors using matlab's *patch* command. Make them semi-transparent by changing their 'FaceAlpha' values.

- On the same graph, plot the resulting image "AC" after one transformation by A. Draw each corner of the image object as small black dots (using the plot3 command), all borders as thin black lines, and also, color the corresponding post-transformed RED image node.

- Then, for each of the 3 faces you chose in the pre-image cube, shade in the corresponding 3 faces on the post-transformed image object.

- Then, plot a line connecting the points: (0,0,0) to (1, 1, 1). Make this a black dashed line..

Comment on the growth/decay, reflections, rotations, or translations of the object in the direction of the 3 eigenvectors. How do each of the eigenvalues affect the transformation process ?