Unified Dark-QCD Model: Variables, Equations, Falsifiability, and Execution Plan

Date: 2025-07-28

Overview

This note specifies a unified Dark-QCD proposal. It contains:

1. a full **variable glossary** (every symbol, why it appears, and what theory motivates it);
2. the **core equations** (mediator mass/couplings, cosmological propagation, collapse, and the horizon-entropy link to Λ);
3. a **systems view** showing how the variables interact;
4. **falsifiable predictions** and the checklist to test them;
5. an **execution plan** for numerical and observational verification.

1) Variable Glossary

Variables are grouped by layer. Where possible it is indicated whether a quantity is *derived* (not a free knob) once the underlying gauge theory is chosen.

A. UV (Walking Dark-QCD) Inputs

- N_D — colors of dark SU(N_D).
- N_f — light dark quark flavors.
- $N_f^\star$ — conformal edge.
- $\Delta \equiv (N_f^\star - N_f)/N_f^\star$ — distance from conformality.
- Λ_D — dark confinement scale.
- f_σ — dilaton decay constant (derived via walking).
- C_f, κ_w — walking coefficients: $f_\sigma/\Lambda_D = C_f e^{\kappa_w (N_f^\star - N_f)}$.

B. IR Composite Spectrum and Couplings

- $m_X = \kappa \Lambda_D$ with $\kappa = \mathcal{O}(3-10)$.
- σ — pseudo-dilaton (mediator).
- m_ϕ — dilaton mass (PCDC): $f_\sigma^2 m_\phi^2 \simeq \kappa_{\rm PCDC}\, \Delta\, \Lambda_D^4$.
- $g_X \simeq m_X/f_\sigma, \alpha_D = m_X^2/(4\pi f_\sigma^2)$.
- $V(r) = -\alpha_D e^{-m_\phi r}/r$ (attractive).
- $\sigma_T(v)$ — momentum-transfer cross section (velocity-dependent; resonances).

C. Cosmology and Propagation (Gravity)

- $M_*^2 = M_P^2 + \xi\sigma^2$ (non-minimal coupling).
- $\beta \simeq 1/f_\sigma$ (universal trace coupling).
- $\mathcal{L}_3 = \frac{c_3}{\Lambda_3^3}(\partial\sigma)^2\Box\sigma$ (screening).
- $\alpha_M = \frac{2\xi\sigma\dot\sigma}{H(M_P^2+\xi\sigma^2)}$.

- $\frac{D_L^{\mathrm{GW}}}{D_L^{\mathrm{EM}}} = \exp[\frac{1}{2}\int_0^z \frac{\alpha_M}{1+z'} dz'] \equiv \Xi(z)$.
- LOS modulation: $\Xi \to \Xi(1+\eta\mathcal{I})$.
- Local tests: $|\alpha_{M,0}| < 10^{-2}$, $r_V \gg$ AU.

D. Collapse and Interior Crystal

- Ordering inside EH: $\mu > \mu_c \sim \xi_c \Lambda_D, T_c(\mu) \gtrsim 10 - 100\,\mathrm{MeV}$.
- EH forms first; crystal (ordered, gapped) later inside.

E. Dark Energy / Horizon Entropy

- Global constraint: $\alpha S_{\mathrm{hor,tot}} = S_\star \Rightarrow \Lambda_{\mathrm{eff}}$ tracks BH entropy; flips negative post-BH.

2) Core Equations

Mediator Mass/Couplings

$f_\sigma^2 m_\phi^2 \simeq \kappa_{\mathrm{PCDC}}\,\Delta\,\Lambda_D^4 \Rightarrow m_\phi \simeq \frac{\Lambda_D^2}{f_\sigma}\sqrt{\kappa_{\mathrm{PCDC}}\Delta}$
$g_X \simeq m_X/f_\sigma, \quad \alpha_D = m_X^2/(4\pi f_\sigma^2)$
$f_\sigma/\Lambda_D = C_f e^{\kappa_w(N_f^* - N_f)}$
$V(r) = -\alpha_D e^{-m_\phi r}/r$, $\sigma_T(v)$ by partial waves (resonances)

Cosmological Propagation

$M_*^2 = M_P^2 + \xi\sigma^2, \alpha_M = \frac{2\xi\sigma\dot\sigma}{H(M_P^2+\xi\sigma^2)}$
$\ddot\sigma + 3H\dot\sigma + V'(\sigma) + \beta T_X = 0, \beta \simeq 1/f_\sigma, T_X \simeq -\rho_X$.
$\frac{D_L^{\mathrm{GW}}}{D_L^{\mathrm{EM}}} = \exp[\frac{1}{2}\int_0^z \frac{\alpha_M}{1+z'} dz']$

Collapse / Crystal

$\mu > \mu_c \sim \xi_c\Lambda_D,\ T_c(\mu) \gtrsim 10 - 100\,\mathrm{MeV}$.

Dark Energy / Entropy

$\alpha S_{\mathrm{hor,tot}} = S_\star \ \Rightarrow\ \Lambda_{\mathrm{eff}} \approx \Lambda_0 \to -|\Lambda_-|$ (post-BH).

3) Systems View

1. Composite DM + attractive Yukawa via σ (all derived).
2. Crystal forms inside EH when $\mu > \mu_c$.
3. Small M_* drift $\Rightarrow$ percent-level GW amplitude change; screening protects local tests; $c_{\mathrm{GW}} = c$.

4) Falsifiable Predictions

- $\langle\sigma_T/m_X\rangle_{1000\,\mathrm{km/s}} < 0.1$, $\langle\sigma_T/m_X\rangle_{30\,\mathrm{km/s}} < 1\ \mathrm{cm^2/g}$.
- Siren split $0.3\% - 1.5\%$ by $z \sim 1$; $|\alpha_{M,0}| < 10^{-2}$; $r_V \gg$ AU.
- No echoes; EH-first collapse; late-time Λ flip only.

5) Execution Plan

1. Pick $(N_D, N_f) \to (\Lambda_D, f_\sigma, \Delta)$.
2. Derive $(m_X, m_\phi, \alpha_D, \beta, \xi, \Lambda_3)$.
3. Compute $\sigma_T(v)$ (resonant, attractive) at dwarf/cluster velocities.
4. Solve $\sigma(a) \to \alpha_M(a) \to \Xi(z)$; apply local/screening cuts.
5. Check BH ordering; adopt asymmetric relic density.