Homework 1

Prabhjot Singh Rai

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1 Optimization and probability

Solution 1a: In order to check whether the minima exists for the function, $f(\theta)$, we calculate $f''(\theta)$.

$$\frac{df(\theta)}{d\theta} = \sum_{i=1}^{n} w_i(\theta - x_i)$$
$$g(\theta) = \sum_{i=1}^{n} w_i\theta - \sum_{i=1}^{n} w_i x_i$$

Differentiating $g(\theta)$, we get

$$\frac{dg(\theta)}{d\theta} = \sum_{i=1}^{n} w_i \tag{1}$$

Since it is given that $w_i, ..., w_n$ are positive real numbers, therefore $f''(\theta) > 0$, which shows that the graph of this quadratic function is concave up and minima exists at value of θ when $f'(\theta) = 0$.

$$\frac{df(\theta)}{d\theta} = \sum_{i=1}^{n} w_i(\theta - x_i)$$
$$0 = \sum_{i=1}^{n} w_i\theta - \sum_{i=1}^{n} w_i x_i$$
$$\theta = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Therefore, the value of θ which minimizes $f(\theta)$ is $\frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$. If some of the w_i's are negative, then from equation (1), it's not necessary that $f''(\theta)$ is always positive.If $f''(\theta) < 0$, the function may not have any defined minima, since it will be concave downwards. And if a point of inflection exists ($f''(\theta) = 0$), then again, there would be local minima and maxima but no defined global minima for the function.

Solution 1b. Let n vectors out of $x_i, ...x_d$ be negative where $n <= d$. Let sum of negative vectors be $-\beta$ and sum of positive vectors be α, where α and β are both positive numbers. Evaluating $f(x)$,

$$\begin{aligned}
f(x) &= \sum_{i=1}^{d} max_{s\epsilon\{1,-1\}} s x_i \\
&= max_{s\epsilon\{1,-1\}} s(\alpha) + max_{s\epsilon\{1,-1\}} s(-\beta) \\
&= \alpha + \beta
\end{aligned}$$

Evaluating $g(x)$,