CS221 Fall 2018 Homework [logic]

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Problem 4

(a) In order to convert the knowledge base into conjunctive normal form (CNF), we can follow the following steps:

$$
\begin{aligned}
KB &= \{(A \vee B) \to C, A\} \\
&= \{\neg(A \vee B) \vee C, A\ \} && \text{Eliminating Implications} \\
&= \{(\neg A \wedge \neg B) \vee C, A\} && \text{Push } \neg \text{ inwards} \\
&= \{(\neg A \vee C) \wedge (\neg B \vee C), A\} && \text{Distribute } \vee \text{ C} \\
&= \{(\neg A \vee C) \wedge (\neg B \vee C), A, \neg A \vee C, \neg B \vee C\} && \text{If A } \wedge \text{ B is in the knowledge base,} \\
& && \text{then we can derive both A and B} \\
&= \{(\neg A \vee C) \wedge (\neg B \vee C), A, A \to C, \neg B \vee C\} && \text{If A } \wedge \text{ B is in the knowledge base,}
\end{aligned}
$$

By Modus ponens inference rule, when we have A and $A \to C$ in our knowledge base, we can derive the following:

$$\frac{A, A \to C}{C}$$

Hence, C is derived:

$$= \{(\neg A \vee C) \wedge (\neg B \vee C), A, A \to C, \neg B \vee C, C\}$$

(b) The steps would be the following:

$$
\begin{aligned}
KB &= \{A \vee B, B \to C, (A \vee C) \to D\} \\
&= \{A \vee B, \neg B \vee C, \neg(A \vee C) \vee D\} && \text{Eliminating Implications} \\
&= \{A \vee B, \neg B \vee C, (\neg A \wedge \neg C) \vee D\} && \text{Push } \neg \text{ inwards} \\
&= \{A \vee B, \neg B \vee C, (\neg A \vee D) \wedge (\neg C \vee D)\} && \text{Distribute } \vee \text{ D} \\
&= \{A \vee B, \neg B \vee C, (\neg A \vee D) \wedge (\neg C \vee D), (\neg A \vee D), (\neg C \vee D)\} && \text{If A } \wedge \text{ B is in the knowledge} \\
& && \text{then we can derive both A an}
\end{aligned}
$$

By resolution, we can use $A \vee B, \neg A \vee D$, we can derive $B \vee D$

$$\frac{A \vee B, \neg A \vee D}{B \vee D}$$

Similarly, by resolution, we can use $\neg B \vee C, B \vee D$ to derive $C \vee D$

$$\frac{\neg B \vee C, B \vee D}{C \vee D}$$

Hence, new knowledge base would be:

$$= \{A \vee B, \neg B \vee C, (\neg A \vee D) \wedge (\neg C \vee D), (\neg A \vee D), (\neg C \vee D), B \vee D, C \vee D\}$$

We can use resolution again on $\neg C \vee D$ and $C \vee D$ to derive D:

$$\frac{\neg C \vee D, C \vee D}{D}$$

Final knowledge base:

$$= \{A \vee B, \neg B \vee C, (\neg A \vee D) \wedge (\neg C \vee D), (\neg A \vee D), (\neg C \vee D), B \vee D, C \vee D, D\}$$

Hence, D is derived.

Problem 5

(b) According to constraint 1, constraint 3 and constraint 4, every number has one and only one successor, which is odd if number is even and is even if the number is odd. Therefore, the number of odd and even numbers should be equal in our model to satisfy both the constraints, hence for models having odd number of finite numbers the set of 7 constraints is not consistent.

For 2n integers, where n even integers and n odd integers:

$$n_{e1}, n_{e2}, n_{e3}...n_{en}, n_{o1}, n_{o2}, n_{o3}...n_{on}$$

Starting with finding successor of n_{e1}, let the successor be n_{ok}, where k is an integer from $1, 2, 3...n$. By constraint 5, n_{ok} is larger than n_{e1}.

Finding successor of n_{ok}, let the successor be n_{el}, where l is an integer from $1, 2, 3, ...n$. By constraint 5, n_{el} is larger than n_{ok}.

By constraint 6, n_{el} is larger than n_{e1}. This means there exists an even number(n_{el}) larger than n_{e1}. Further, there exists an odd number larger than n_{el}, let's assume n_{op}, and again by constraint 6, n_{op} is larger than n_{e1}. (.... 1)

Since the number of odd and even numbers are same, and every number has to have a successor, every number will also be a successor of some number in the finite model. Therefore, there exists a number for which n_{e1} is a successor. Let the number be n_{ot}, where t is some number from $1, 2, 3, ...n$ (t is one of the odd numbers out of the odd numbers in the finite model). (.... 2)

Since it is a finite model, there would exist an instance when n_{op} is equal to n_{ot} (p is equal to t). Therefore, from 1, n_{ot} is larger than n_{e1}. Also, from 2, successor of n_{ot} is n_{e1}. Therefore, n_{e1} is larger than n_{ot}. From constraint 6, n_{e1} is larger than n_{e1}. But this contradicts our 7th constraint that a number is not larger than itself. Hence there exists no finite model for which the resulting set of constraints is consistent.