

# Homework 1

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## 1 Optimization and probability

**Solution 1a:** In order to check whether the minima exists for the function,  $f(\theta)$ , we calculate  $f''(\theta)$ .

$$\begin{aligned}\frac{df(\theta)}{d\theta} &= \sum_{i=1}^n w_i(\theta - x_i) \\ g(\theta) &= \sum_{i=1}^n w_i\theta - \sum_{i=1}^n w_ix_i\end{aligned}$$

Differentiating  $g(\theta)$ , we get

$$\frac{dg(\theta)}{d\theta} = \sum_{i=1}^n w_i \tag{1}$$

Since it is given that  $w_1, \dots, w_n$  are positive real numbers, therefore  $f''(\theta) > 0$ , which shows that the graph of this quadratic function is concave up and minima exists at value of  $\theta$  when  $f'(\theta) = 0$ .

$$\begin{aligned}\frac{df(\theta)}{d\theta} &= \sum_{i=1}^n w_i(\theta - x_i) \\ 0 &= \sum_{i=1}^n w_i\theta - \sum_{i=1}^n w_ix_i \\ \theta &= \frac{\sum_{i=1}^n w_ix_i}{\sum_{i=1}^n w_i}\end{aligned}$$

Therefore, the value of  $\theta$  which minimizes  $f(\theta)$  is  $\frac{\sum_{i=1}^n w_ix_i}{\sum_{i=1}^n w_i}$ . If some of the  $w_i$ 's are negative, then from equation (1), it's not necessary that  $f''(\theta)$  is

always positive. If  $f''(\theta) < 0$ , the function may not have any defined minima, since it will be concave downwards. And if a point of inflection exists ( $f''(\theta) = 0$ ), then again, there would be local minima and maxima but no defined global minima for the function.