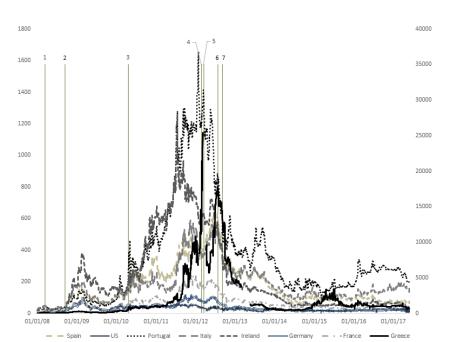
European Sovereign Systemic Risk Zones

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Definition of systemic risk

"A systemic risk is a risk that an event will trigger a loss of confidence in a substantial portion of the financial system that is serious enough to have adverse consequences for the real economy."

G-10 Report on Financial Sector Consolidation (2001)

According to this definition, the current financial crisis is systemic.

Motivation

Detect sovereign systemic risk zones. The early detection and the causal identification of such phenomena may provide valuable early warning signals to countries that move towards dangerous risk paths.

Provide an effective risk mapping in which country-specific fundamentals are mixed together with contagion-based measures, thereby assembling a series of leading indicators that could signal impending sovereign systemic risk abnormalities.

Literature review

- Ang and Longstaff (2013): Systemic sovereign credit risk in the US and Europe, multifactor affine framework. Findings: Heterogeneity among US and European issuers, key role of financial market variables.
- Reboredo and Ugolini (2015): Systemic risk in European sovereign debt markets, conditional value-at-risk measure. Findings: Greek debt crisis was not so severe for non-crisis countries, systemic risk increased for countries in crisis.

Considering changes in regimes in CDS dynamics...

- Caceres, Guzzo and Segoviano (2010): Primary role in sovereign spreads sharp rise, risk aversion (early period of the crisis), countryspecific factors, e.g. public debt and budget deficit (the later stages).
- Arghyrou and Kontonikas (2012): Changes in regime for sovereign debt pricing. Key role of country-specific macro-fundamentals.
- Ait-Sahalia, Laeven and Pelizzon (2014): Eurozone CDS rates exhibited clusters in time and in space.

Data

- Daily 5-yr CDS prices for France, Germany, Greece, Ireland, Italy, Portugal, Spain and US from January 1, 2008 to October 7, 2017 (1505 daily quotes. Only 1414 for the *Greek 5-yr CDS*), splitting the sample in (a) the crisis period from 01/02/2008 to 09/30/2013; (b) the post-Quantitative Easing period from 10/01/2013 to 05/23/2017.
- US and Euro Banks 5-yr CDS indices, US and Euro Other Financials 5-yr CDS indices.
- Macroeconomic factors to investigate their influence to sovereign CDS levels (Augustin (2014)):
 - Debt/GDP,
 - exports/GDP,
 - GDP growth rate,
 - industrial production,
 - inflation, and,
 - unemployment.

Databases: Markit, Thompson Reuters Datastream, Eurostat.

Methodology: Copula-based dependencies

Arakelian and Dellaportas (2012) proposed a flexible threshold model for estimation of bivariate copulas

$$C_t(u, v) = \sum_{j=1}^{J} I_j(t) \sum_{i=1}^{\ell} w_{ij} C_{\theta_j}^i(u_j, v_j)$$
 (1)

where $C_{\theta_j}^i$: the copula function, w_{ij} : the probability of copula i in the interval I_j , $\sum_{i=1}^\ell w_{ij} = 1$ for all j. Thus, our general model (1) allows both the functional form of the copula and the parameters to change within each interval I_j .

A RJMCMC algorithm was proposed which obtained samples from the posterior density of these models, and a Bayesian model-averaging estimation approach constructed a posterior density of Kendall's τ (Kendall (1938)), marginalised over all models and parameters within each model.

MCMC Details

Prior Elicitation

- Non-informative prior model probabilities $f(m) = |M|^{-1}$
- σ^X , $\sigma^Y \sim Gamma(1,1)$
- $\theta \sim N(0, (\gamma_j \gamma_{j-1})|H(\widehat{\theta})|^{-1})$, where $H(\widehat{\theta})$ likelihood's Hessian at $\widehat{\theta}$.

MCMC Details

Laplace Approximation

Posterior model probability

$$f(m|y) = \frac{f(m)f(y|m)}{\sum_{m \in M} f(m)f(y|m)}, m \in M$$
 (2)

where f(y|m): the marginal probability of model m. Searching in both model and parameter space is possible via RJMCMC algorithm of Green (1995).

$$\widehat{f}(y|m) = (2\pi)^{d/2} |\widehat{\Sigma}_m|^{1/2} f(y|\widehat{\theta}_m, m) f(\widehat{\theta}_m|m)$$
(3)

where $\theta_m=(\theta,\sigma^X,\sigma^Y)$, $dim(\theta_m)=d$, $\widehat{\theta}_m$: MLE, $\Sigma=H^{-1}(\widehat{\theta}_m)$. By performing this approximation for every model m, we are left with the task to sample in the space of (discrete) density function specified by (2) with f(y|m) replaced by (3).

MCMC Details

Simulation Annealing

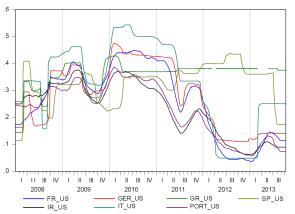
Adopt Jasra et al. (2007) to generate L parallel-sampled auxiliary Markov chains with target densities

$$\pi_I \propto \pi^{\zeta_I}$$
,

where π : posterior density needed to obtain samples from, ζ_I : ordered parameters $0<\zeta_I<\zeta_{I-1}<\ldots<\zeta_1<1$. The densities π_I serve as independent Metropolis-Hasting proposal densities for the main chain with target density π .

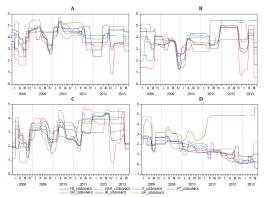
At each iteration, one auxiliary density π_I is chosen at random and together with the current sampled point of π_I is used at the usual acceptance ratio of the main chain. In the terminology of Jasra et al. (2007), this is an exchange move in the population reversible jump algorithm.

Model averaged Kendall's au among sovereign European and US 5-yr CDS spreads



Greece (GR), Italy (IT), Ireland (IR), Portugal (PORT), Spain (SP), France (FR), Germany (GER) relative to the sovereign US 5-yr CDS spread (US). Estimates for Greece have some missing values, as the corresponding sovereign CDS was not traded during those days

Model averaged Kendall's au among sovereign European 5-yr CDS spreads



Greece (GR), Italy (IT), Ireland (IR), Portugal (PT), Spain (SP), France (FR), Germany (GER)) relative to: (A) the Euro Banks 5-yr CDS index (EUBANKS); (B) the Euro Other Financials 5-yr CDS index (EUOTHER); (C) the US Banks 5-yr CDS index (USBANKS); (D) the US Other Financials 5-yr CDS index (USOTHER).

Regression Trees-Random Forests I

To assess the importance of the leading indicators used to explain the dynamics of the sovereign CDS we also adopt the random forest procedure. This ensemble learning inference procedure is essentially a collection of regression trees using different combinations of variables and samples (Breiman (2001); Breiman (2003)). More formally, having a panel data of N observations over a time length T and considering R candidate predictors, the algorithm runs as follows:

- **9** Bootstrap n < N sub-samples with replacement from the original data of N observations.
- ② Run a regression tree on the bootstrap samples by randomly selecting q < R predictors from the total R predictors.

Regression Trees-Random Forests II

- Repeat the steps 1-2 to obtain thousands of regression trees¹. Hence, at each run, the trees are calculated by randomizing over two dimensions, the sub-sample of data and a (small) subset of all the variables.
- Estimate the mean squared error (MSE) from the bootstrap samples, the so-called out-of-bag (OOB):

$$MSE_{OOB} = \frac{\sum_{i=1}^{n} (Y_i - \widehat{Y}_{iOOB})^2}{n},$$
 (4)

where \widehat{Y}_{iOOB} denotes the average prediction for the *i*th observation from all trees for which this observation has been OOB.

Regression Trees-Random Forests III

To assess the importance attributed at each variable based on the MSE reduction (Breiman (2003)), which is computed for the generic regression tree over all the OOB observations as:

$$MSE_{OOB,tree} = \frac{\sum_{i \in OOB_{tree}}^{n} = 1(Y_i - \widehat{Y}_i^{tree})^2}{n_{OOB}^{tree}},$$

where \widehat{Y}_i^{tree} are the predictions of the regression tree, i are its observations over OOB data only, and n_{OOB}^{tree} is the number of OOB observations in the same regression tree.

• Compute the MSE reduction associated to each X_r covariate in \mathbf{X} by comparing the MSE with and without X_r permuted thereby obtaining the following Variable Importance (VI) measure,

$$VI_r = MSE_{OOB}^{tree} - MSE_{OOB}^{tree}(X_{r,permuted}).$$
 (5)

Regression Trees-Random Forests IV

The idea behind this measure is that, if a regressor X_j does not have a significant contribution in predicting Y, it makes no difference if the values for the predictor are randomly permuted in the OOB data before the predictions are generated.

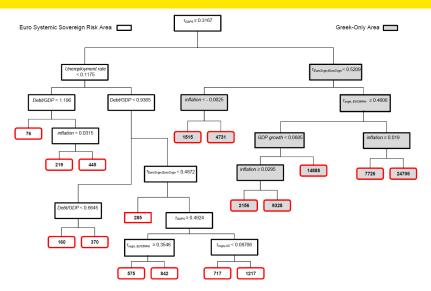
We rank all predictions according to the VI_r measure distinguishing the more influential variable (with the highest MSE reduction value) from the lowest variables (with the lowest MSE reduction values).

• Compute the relative measure, *VIM*, by normalizing the *VI* of each variable over the highest value obtaining the normalized variable importance measure, *VIM*_r which varies from 1 to 100:

$$VIM_r = \frac{VI_r}{VI_r^{highest}} \times 100. \tag{6}$$

 $^{^{1}}$ In our case, it was heuristically shown that the accuracy of random forest converges around at 3,000 trees.

Inside the Risk Zones



The model selected four contagion-based variables and four country-specific fundamentals out of the fourteen candidate covariates. 17/28

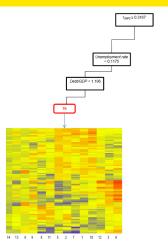
The eight selected variables are the following:

- the Kendall's τ between the single sovereign CDS spread and GIIPS's CDS spreads, $\tau_{svgn,GIIPS}$;
- the Kendall's au between the single sovereign CDS spread and the rest of the Euro sovereign CDS spreads, $au_{svgn,EuroSvgn}$;
- the Kendall's τ between the single sovereign CDS spread and the Euro Other Financials 5-yr CDS index, $\tau_{svgn,EUOthFin}$;
- the Kendall's τ between the single sovereign CDS spread and the sovereign *US 5-yr CDS* spreads, $\tau_{svgn,US}$;
- the Debt/GDP ratio;
- the GDP growth;
- the inflation rate;
- the unemployment rate.

Explaining the heatmaps

The regression tree shows seventeen final nodes. Specifically, to give an overview of the distributions taken on by all variables within each final node, and not only of those selected by the regression tree, we relied on the heatmaps, commonly used to emphasize data above or below specific thresholds as "hot" or "cold" colors, respectively. In our analysis, low values (cold colors) are in blue, high values (hot colors) are in red, while values around the mean (warm colors) are in yellow. By starting from the top node (τ_{GIPS}) and using the corresponding splitting rules (\geq or <), we move along the path traced by the values of the selected variables: if the value in each node agrees with the splitting rule the move is to the left, otherwise, it is to the right. This process leads to the final nodes, where the expected value of the dependent variable is given.

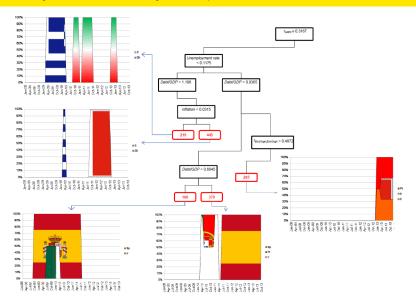
Safe Zone Heatmap



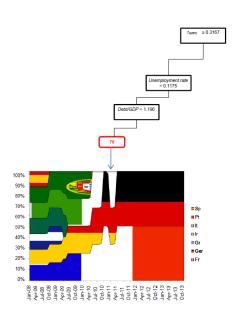
The variables in the heatmap are the following: 1: τ_{GIIPS} , 2: $\tau_{Fr,Ger}$, 3: inflation, 4: industrial production, 5:

 $\tau_{EuroSvgn,EuroSvgn,} \ 6: \ exports/GDP, \ 7: \ \tau_{svgn,EUBanks}, \ 8: \ \tau_{svgn,USBanks}, \ 9: \ GDP \ growth, \ 10: \ \tau_{svgn,US}, \ 11: \ \tau_{svgn,EUOthFin}, \ 12: \ \tau_{EuroSvgn,USOthFin}, \ 13: \ Debt/GDP, \ 14: unemployment \ rate. \ Low \ values (cold \ colors) \ are in \ blue, \ high \ values (hot \ colors)$

Risky Zone Country Composition

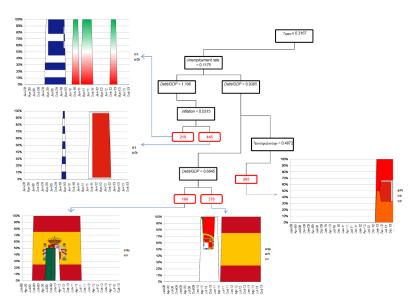


Safe Zone Path



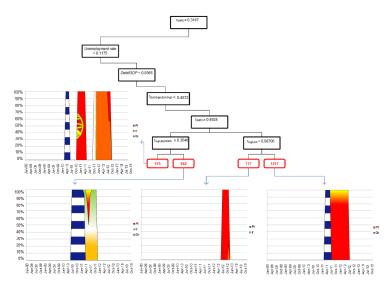
Risky Zone Path

Low unemployment rate with high Debt/GDP ratio or with a high unemployment rate

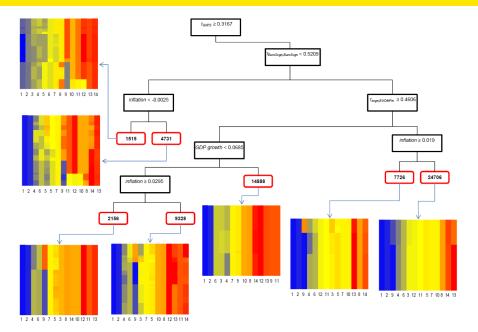


High Risky Zone

 $High\ unemployment\ rate\ together\ with\ high\ Debt/GDP\ ratio\ and\ significant\ sovereign\ contagion$



Greek Only Area Heatmap



Evolution of risk indicators importance



pc-contagion: ↑ importance from Q32008 (LB collapse) ⇒ peak at Q42011 ⇒ \downarrow Q22012 ⇒ high values. Thereafter, both importance metrics showed a downtrend. The time-varying importance assumed by fundamentals is also confirmed becoming relevant with the Greek crisis and contagion-based factors.

Summary - Concluding remarks

- First, Greece is a "world apart" from July 2011 to end of period, i.e. when the country started showing very low dependencies with other peripheral Euro countries with very high levels of CDS quotations mapped onto extremely high values for unemployment rate and Debt/GDP ratio.
- Second, we identified three main systemic risk zones based on contagion and specific country fundamentals, namely:
 - A safe zone (unemployment rate < 11.75% and Debt/GDP ratio < 119.6%);
 - A risky zone with high unemployment rate, or with low unemployment rate coupled with high Debt/GDP ratio;
 - A high risk zone (unemployment rate>11.75%, Debt/GDP ratio > 93.65% and significant sovereign contagion).

Summary - Concluding remarks

 Third, we provided evidence on time-varying importance assumed by fundamentals, which became relevant with the Greek crisis. Instead, contagion-based factors assumed a key importance with the Lehman Brothers collapse, next achieving a new emphasis with the Euro debt crisis erupted in 2010, finally showing the same importance as the fundamental-based variables.

These results have important policy implications for early detection and the causal identification of sovereign systemic risk.

Future work is needed to connect systemic sovereign risk to other systemic risk dimensions, such as banking and other financial intermediaries, and non-financial firms as well.