Benefits of sectoral cryptocurrency portfolio optimization

Maria Čuljak[a], Bojan Tomić[b], Saša Žiković[c,*]

[a] Croatian Financial Services Supervisory Agency, Franje Račkoga 6, 10000 Zagreb
[b] EFFECTUS University college for Law and Finance, John F. Kennedy 2, 10000 Zagreb
[c] Faculty of Economics and Business, University of Rijeka, Ivana Filipovića 4, 51000 Rijeka

Abstract

When creating a portfolio, investor should consider the dynamics of the income ratio of the portfolio asset selected in order to identify and quantify the taken risk of the investment. This research paper will formally identify and describe the benefits of sectoral cryptocurrency classification portfolio optimization and it's performance. Six optimization targets will be formed: MinVar, MinCVaR, MaxSR, MaxSTARR, MaxUT and MaxMean. The formed portfolio is compared with the performance of the CRIX index over the same period. The results suggest that five of the six portfolio strategies performed better if they included cryptocurrencies from financial, exchange and business services sectors.

Keywords: cryptocurrency, portfolio optimization, sectoral classification, investments

JEL classification: E4, E5, L5

1. Introduction

Continuous development of information technology and the Internet, as well as an increase of distrust in the global financial and payment system in response to the recent economic crisis, intensifies work on the existing ideas of

*Corresponding author
Email address: szikovic@efri.hr (Saša Žiković)

digital money. As a result, on October 31, 2008, a document entitled ”Bitcoin: A Peer-to-Peer Electronic Cash System” describes a new decentralized transaction system that does not include an intermediator between entities of interest. The first description of the article’s summary states a person signing up as Satoshi Nakamoto on the Internet at http://www.metzdowd.com along with a link to the entire article of the same author. Already at the outset of the system characteristics described by the author, one can see how revolutionary, far-reaching and promising idea of his project is. The computer program, or the Bitcoin protocol, was released to the public on January 9, 2009, creating the first cryptocurrency - bitcoin. Its technology enabled almost instantaneous transaction execution, with negligible fees without intermediaries or central body, which attracted great attention as well as a large number of users themselves. An important characteristic of the Bitcoin protocol is its open source system, the system whose initial code is open and free to the public. This means that anyone who has an interest can freely study and work on the system - if his suggestions are in the direction of improvement, the community will accept changes and improve the protocol. Again, this also means that anyone who wants can freely take the existing protocol, modify it, in some context change or adapt to their needs, create a new cryptocurrency and release it to the public. The latter allowed the creation of a number of new cryptocurrencies with different properties and spreading their use first in payment transactions and then in the context of their trading on the new secondary market.

2. An overview of current research

At its core, the Bitcoin platform is a decentralized transaction system, that is, a distributed ledger, secured by cryptography and managed by consensus, which records all transactions that occurred between participants in the community. The general ledger consists of transactions grouped into so-called blocks from which the ledger is actually a blockchain. At the end of each block is a summary of that block, that is, a summary of all previous transactions written as a syntax of the hash or digest function (Härdle et al., 2019). This record is distributed throughout the network and represents the record for the next block of transactions. In addition to summarizing all transactions, the network also distributes a new block of transactions. Since the digest function is a one-way mathematical algorithm that has the same input data converts to an output record with a unique structure, if there is a discrepancy in the result of the nodes function, this would mean that there has been a change in data, either in previous transactions, or in a new transaction block created. In other words, one of the participants in the network has changed the balance sheet of one of the accounts in the network, and such a block of transactions is rejected and classified as incorrect. The process just described is a solution to a problem that has long been a stumbling block in the context of the development of digital currencies, which is double expense.

Digital record and blockchain technology allows transparency because every transaction record is visible and publicly available. With the implementation of such technology, the consumption of each unit of money would be public, thus completely eliminating the possibility of malpractice, corruption, etc. Except in the context of the value of units that can be expressed up to 8 decimal places (Symitsi and Chalvatzis, 2018), the application of such technology is very wide. For example, accounting information systems are already being developed based on a distributed ledger because it is actually an electronic record that can serve a variety of purposes, such as a record of balances on accounts of customers and suppliers, proof of ownership of financial instruments, music record, etc. The technological advances and practical applications of blockchain-based cryptocurrencies can also be seen as a new type of digital asset (Glaser et al., 2014). There are different categorizations and definitions of cryptocurrencies, however for the time being none of them is fully accepted or there is a consensus as to what type of existing assets cryptocurrencies represent.

Although the design of the cryptocurrency market in its initial phase was based solely on the parameterization of the existing Bitcoin protocol (Elendner et al., 2016), it is the open source feature and practical implementation of blockchain technology - characteristics that have been recognized by young and innovative companies to raise the capital needed for their development on the one hand, and the positive public reaction to the idea of decentralization, on the other.

In this paper, research is conducted in which cryptocurrency relationships are examined with the aim of constructing and modeling their portfolio. Fundamentally, investors apply different techniques, models and strategies to construct their own portfolio whose performance dynamics should outperform the market, that is, a portfolio that should yield more than market equilibrium returns. Such a definition entails the pursuit of undervalued assets, which would ultimately result in a market that is information-efficient, that is, a market whose aggregate value reflects all relevant and available information related to individual assets. If the standard definitions of investing are placed in the context of the cryptocurrency market, their discrepancy may be noted. The first thing that cannot be put in the context of the cryptocurrency market is the syntagma market in equilibrium. For a market to be in equilibrium, it would mean that there is a market consensus on the expected rate of return on the assets being invested. For there to be market consensus, it implies the existence of its fundamental value, because how to create a consensus on the expected return on an asset if two independent investors value the same assets differently. Secondly, for the market to be information-efficient, as stated above - it is necessary to first define what information that may need, but already affect the price of cryptocurrencies, and only then examine whether there is a positive or negative price response to their publication. (e.g., forking Bitcoin system which means that the number of existing bitcoin units is duplicated and the value of bitcoin remains the same as before the release of that information). Therefore, due to the absence of at least approximately equal valuation, undervalued or overvalued assets do not exist, and therefore there is neither market consensus on expected rates of return nor a market in equilibrium that is information efficient. However, despite all of the above, the cryptocurrency market and its entire infrastructure is continuously growing year on year. Due to its availability, more and more institutional and individual investors of different profiles invest and trade in cryptocurrencies, which makes the need for analysis conducted in this paper even more so.

Among the first studies on this topic is conducted by (Trimborn, 2015), who in his work optimizes the cryptocurrency portfolio of constituents of the CRyptocurrency IndeX - CRIX index with the aim of minimizing variance. The author faces limitations of missing data and insufficiently of long time series. For the first constraint, a bootstrapping parameter method is used to estimate missing values, and for the second, it applies the General Autoregressive Conditional Heteroskedasticity (GARCH) model while estimating the expected volatility of time series. From a volatility point of view, the results of the approximated data portfolio favor an optimized portfolio where volatility is lower than CRIX. On the other hand, excluding the estimated data, the results favor the CRIX index where volatility is lower and cumulative return is higher.

The cryptocurrency market can be viewed on its own but also in combination with traditional financial instruments and assets. (Trimborn et al., 2017) conducts research into existing cryptocurrency portfolios with an individual market capitalization of more than 1 million dollars, incorporates traditional instruments - stocks, components of the S&P 100 and DAX30 indexes, shares listed on the Portuguese stock exchange, and runs a portfolio with minimal variance. In order to avoid cryptocurrency liquidity problems the authors approximate the liquidity measure and create a cap on the upper limit on the allocation of each cryptocurrency in the LIquidity Bounded Risk-return Optimization - LIBRO portfolio. Optimization is carried out with and without limitation on units and the performance of the portfolio is compared. In both cases, the results are in favor of cryptocurrency. Including cryptocurrencies in the portfolio improves the reward-risk ratio. Equity-constrained portfolios of equities and cryptocurrencies produce better cumulative returns than non-restricted portfolios.

(Trimborn et al., 2018) extends previous research and introduces Barclays Capital US Aggregate Index and Commodity Market Index (S&P GSCI) into the analysis. In addition to the standard optimization mean-variance model, it introduces Conditional Value at Risk (CVaR) as a measure of risk. All created portfolios that include cryptocurrencies in their composition, with or without LIBRO equity limitation, have performed better than portfolios created only from traditional assets. In addition, certain portfolios that take CVaR as a measure of risk, appear to have a higher cumulative yield than the standard MV model.

One of the most comprehensive studies examining the performance of a portfolio created from cryptocurrency and traditional assets is conducted by (Petukhina et al., 2018). The authors group the existing standard and recent optimization models into four strategies: risk-oriented strategies, return-oriented strategies, risk-return-oriented strategies and combination strategies. The selected models are applied by the authors to portfolios composed of 55 selected cryptocurrencies and 16 variables represented by 5 types of traditional assets. The LIBRO methodology was also included in the research and portfolios were created with and without equity restrictions to control liquidity risk. The performance of all the portfolios created indicates the usefulness of including cryptocurrency in a portfolio made up of traditional assets. The same portfolios achieved a lower cumulative return if the limits on the units controlling the liquidity were raised.

(Lee Kuo Chuen et al., 2018) models market sentiment as the average return of a historical return series and creates a portfolio strategy on a performed sentiment analysis. Also, the research is optimizing the portfolio of ten selected cryptocurrencies along with traditional assets consisting of stock indices, real estate market index and gold. Due to the absence of a normal return distribution, apart from the standard MV model, the authors use CVaR as a measure of risk and compare the performance and allocation of portfolio assets. As in previous research, the inclusion of cryptocurrencies in the portfolio raises the effective limit of possible portfolios, thereby improving the reward-risk ratio. In addition, the strategy created on sentiment analysis has achieved a far higher cumulative return than comparative portfolios, thus confirming the sentiment dynamics in the cryptocurrency market.

In previous recent research, the focus has been on a portfolio that includes multiple cryptocurrencies. Since bitcoin is the first cryptocurrency to have some form of secondary market (Mt. Gox started operating in 2010), and given the rise in its price back in 2013 and 2014, bitcoin has already attracted interest and scientific communities, and is being considered as an individual alternative asset in the area of portfolio modeling. The first papers examining its contribution by including it in a well-diversified portfolio of traditional assets are conducted by (Briere et al., 2015) and (Eisl et al., 2015). In the first paper, the analysis is carried out in the sample, that is, by applying the standard MV model the efficient limit of possible portfolios on the full data sample is derived and its contribution to performance i.e. portfolio diversification is interpreted. Considering the results of the first work - the absence of a normal return distribution, in the second paper an out of sample analysis is performed where CVaR is taken as a measure of risk during optimization. Research results from both papers indicate that bitcoin should be included in the portfolio - the higher risk is compensate by the higher expected return on the portfolio.

Bitcoin's role in the dynamics of portfolios created from traditional assets by continental affiliation (EU, US and China) is examined by (Kajtazi and Moro, 2018). CVaR was used as a measure of risk with the optimization goal of maximizing portfolio return, except for the portfolio with equal allocations. Given the results, it is concluded that the inclusion of bitcoin cryptocurrency in a well-diversified portfolio of traditional assets contributes to improving the risk-reward ratio. Also, the inclusion of BTC as an asset also generates a higher cumulative return.

(Carpenter, 2016) also examines the impact of including BTC in a well-diversified portfolio of traditional assets, however, using the capital asset pricing model (CAPM) to estimate the expected returns of individual assets in the portfolio. Because the implementation of CAPM requires a statistically significant relationship between the additional returns on the index and the total market return, and the relationship fails to prove the relationship between BTC and the observed US capital market index, the author uses the mean of historical BTC returns for expected returns. Also, in the case of abnormally high BTC returns over the period considered, such as in 2014, the return magnitude is adjusted and reduced to a lower value to eliminate extreme values. The analysis is performed out of sample on two data sets, with and without the 2014 BTC return. The results of the survey are divided. The portfolio performance on the overall data set goes in favor of including BTC in the portfolio. However, if the period of extremely high BTC returns is excluded from consideration, the portfolio has a lower risk-reward ratio than the non-bitcoin portfolio as a component. On the other hand, if extremely high returns are adjusted to lower values, optimization includes BTC as a component, thereby contributing to the risk-reward ratio of a well-diversified portfolio of traditional assets.

One of the few studies where the results of previous studies are potentially refuted is conducted by (Klein et al., 2018). To examine the cryptocurrency market's performance as a positive component in portfolio construction, the authors select traditional asset indices, and for each selected index, create and optimize one separate portfolio that includes bitcoin or gold. The goal of optimization is a portfolio with minimal variance. By comparing their performance, the authors conclude that bitcoin has no investment characteristics like gold. A portfolio that includes bitcoin, e.g. The S&P 500-bitcoin, contains a small proportion of bitcoin, unlike the S&P 500-gold portfolio where there is a higher proportion of gold in the portfolio. The risk ratio, measured by standard deviation and CVaR, and the expected portfolio return, also goes more in favor of gold as an alternative asset for portfolio hedging.

All previous papers examine the reactions of incorporating one or more cryptocurrencies into a well-diversified portfolio made up of some form of traditional or alternative assets. However, given their number, the secondary cryptocurrency market can be viewed as a single entity, and it is therefore desirable to examine the possibility of constructing an efficient portfolio made up solely of cryptocurrencies with different allocation goals. One of the first papers examining such a possibility is conducted by (Liu, 2018). The paper creates six portfolios and pursues multiple optimization goals. The observed sample consists of ten cryptocurrencies with a market capitalization of more than 1 billion dollars over a four-year period, August 2015. - April 2018. The results are contrary to what was expected. Other than the portfolio with minimal variance, none of the optimization models met their target. The highest cumulative yield was achieved by the portfolio with the optimization goal of maximizing the utility function, and the highest Sharpe ratio was the portfolio with equally represented units, so the author concludes that in the cryptocurrency market sophisticated models cannot beat the performance of portfolios with equally represented units - looking at them from the point of view of a rational investor.

Conducted research of the options for optimization and diversification of risk in the cryptocurrency market is also conducted by (Brauneis and Mestel, 2018). The authors initially collect data from 500, then from the 20 most liquid cryptocurrencies, and create several different portfolios with associated optimization goals. The study is conducted out of sample with an initial set of observations of 183 historical daily retruns used to estimate the parameters. In addition to the initial settings, the paper also performs an alternative parameterization by changing the sample length of the initial data set used to train the model, using several different time intervals for rebalancing and changing the number of portfolio components. The results of the secondary parameterization portfolios do not deviate from the results of the initial setting portfolios, so they are not considered further. The results obtained confirm the previous research. The highest expected return, as well as the Sharpe ratio, was achieved by the portfolio with equally represented portions of the portfolio, regardless of the frequency of rebalancing. It concludes that an portfolio with equal allocations is the best choice when creating and modeling a portfolio in the cryptocurrency market.

The performance analysis of a sophisticated portfolio optimization model in relation to the passive approach of equal allocations in the cryptocurrency market is also conducted by (Platanakis et al., 2019). Unlike previous works, the analysis was conducted on weekly observations for only four cryptocurrencies: bitcoin, litecoin, ripple and dash for the period from February 2014 until January 2018 The optimization goal used was to maximize the utility function with the short sale limit, and Sharpe and omega ratios were used to evaluate performance. The study was conducted out of sample with two time periods for model training. Given the results of performance measures that do not favor either model, the authors conclude that passive (naive) diversification with equal allocations is a better choice for portfolio construction in the cryptocurrency market.

In all the research papers described above, the cryptocurrency market was viewed as one separate market in which cryptocurrencies have equal characteristics. Each cryptocurrency represented an input variable with equal probability of selection as a component of the portfolio, with its potential allocation in the portfolio defined by the optimization goal. In other words, the initial selection of portfolio components is either conditioned by an existing framework - such as the CRIX cryptocurrency index, or left to the choice resulting from the portfolio optimization of several different cryptocurrencies, most often cryptocurrencies with high market capitalization. Such an approach implies that all cryptocurrencies are equal in all their properties and capabilities, which is not the case. In this paper, the problem will be approached from another aspect. Namely, cryptocurrencies can be defined through six basic categories: payment currencies, blockchain economies, utility tokens, privacy coins, stablecoins and others. Each category is specific and offers certain advantages over the other. Since utilization tokens provide a specific purpose in the practical application of a product or service, it is by far the most created on decentralized computer platforms, i.e. blockchain economies. Accordingly, the cryptocurrency market can also be viewed through sectoral division according to their utilization properties. Comparing portfolio performance with and without sectoral cryptocurrency selection will determine the usefulness of applying such an approach. In addition, cryptocurrencies that have a lower market capitalization and do not represent input variables in previous work will be considered. In other words, cryptocurrencies that are undervalued by their fundamentals will be easier to spot by sectoral observation of the cryptocurrency market. Such an approach is necessary and desirable to eliminate the subordinated position of potential investors, that is, to contribute more to the performance of the investor portfolio in the cryptocurrency market. Also, with the aim of evaluating performance, the performance of the resulting portfolios will be compared with the performance of the CRIX index over the same time period.

3. Data and methodology

For the purpose of this study, publicly available daily price data (in USD) for a total of 65 cryptocurrencies collected from the Coinmarketcap - CMC platform pages, was used. Data were collected for the period of 8/26/2019. to 02/22/2020. making up a sample of a total of 146 daily observations, or 145 daily returns for 65 time series.

To test the utility of cryptocurrency sector observation, an existing portfolio consisting of the top 50 cryptocurrencies by market capitalization includes additional 15 cryptocurrencies, 5 leading cryptocurrencies by each of the three leading utilization sectors by market capitalization: finance, exchanges and business services. Sectoral cryptocurrencies that entered the first 50 by market capitalization were excluded and replaced by the next utilization token by size of market capitalization in the respective sector.

The study forms multiple portfolios with different optimization goals of risk minimization, return maximization and maximization of return and risk ratios. Given the results of previous research by (Briere et al., 2015) and (Lee Kuo Chuen et al., 2018) and the absence of a normal distribution of returns, apart from the standard deviation, will use the conditional Value at Risk - CVaR for the risk measure, i.e. the methodology that follows the work of (Rockafellar and Uryasev, 2000), with a confidence level of 95%. The formed optimization goals are as follows: minimum variance (MinVar), minimum CVaR (MinCVaR), maximize sharpe ratio (MaxSR), maximize stable tail-adjusted return ratio (MaxSTARR), maximize utility function (MaxUT) and maximize mean return (MaxMean). In order to examine the benefits of observing the cryptocurrency market through sector division, the research was conducted in two steps. The first step is to form and test the performance of a portfolio whose components make up the first 50 cryptocurrencies by market capitalization. In the second step, an additional 15 sectoral cryptocurrencies are included in the existing data set. In order to achieve the inclusion of sector cryptocurrencies in the portfolio, in the second step, linear group constraints are created where 20% of the total portfolio allocation must be allocated to sector cryptocurrencies according to the optimization goals. The notation of portfolio optimization goals involving sector cryptocurrencies is as follows: minimum variance-sector (MinVar-S), minimum CVaR-sector (MinCVaR-S), maximize sharpe ratio-sector (MaxSR-S), maximize stable tail-adjusted return ratio- sector (MaxSTARR-S), maximize utility function-sector (MaxUT-S) and maximize mean return-sector (MaxMean-S). Optimization is performed out of sample (backtesting), with the same parameters for each optimization goal. A time period of $k = 10$ days was used to estimate the initial parameters and portfolio allocation. Given the dynamics of the cryptocurrency market, a more frequent monthly rebalance of $K = 30$ days was chosen with the so-called extending window approach $k + K$. For each period $k + 1$, portfolio returns are drawn with respect to the results of the allocation optimization in the previous k, i.e. $k + K$ moment.

3.1. Asset Allocation Models

The basic optimization model used in this paper is based on Modern Portfolio Theory, (Markowitz, 1952). In its original form, the model focuses on minimizing the variance of the asset portfolio for a given level of expected return within certain theoretical assumptions, which is why it is often referred to as the mean-variance (M-V) model. The basic form of the Markowitz formulation (soft return constraints) expressed in the form of linear algebra can be written as follows:

$$\min_{w} \quad \sigma_p^2 \ (w) = w^{\mathrm{T}} \widehat{\Sigma} w \tag{1}$$

$$s.t. \mathbf{1}_N^T w = 1$$

$$x^T w \geq \mu$$

$$w_i \geq 0$$

where σ_p^2 is the variance of the portfolio, $w = (w_1,\ w_2, \ldots,\ w_N)^T$ are the weights of individual assets in the portfolio and $\widehat{\Sigma}$ is the estimated covariance matrix of assets N and their returns T. The above expression involves three additional constraints:$\mathbf{1}_N$ represents a $(N \times 1)$ vector where all elements of the vector represent the portfolio weights and their sum must be one (full investment constraint), x is the $(N \times 1)$ vector of the expected returns of the portfolio assets whose sum, with respect to individual portfolios of the portfolio assets , must be greater than or equal to the desired total portfolio return μ. The last restriction defines a constraint on short selling assets, that is, all portfolio holdings must be positive in size. By further formulation, the model is more adapted to the actual needs where its variants are used in this paper and described.

3.2. Global Minimum Variance Portfolio Objective

If the limit of the required rate of return is omitted from relation (1), portfolio optimization with the aim of minimizing risk results in a global minimum variance of portolio - GMV. Such a strategy calculates only the return covariance matrix and is based on finding the proportion of individual assets that minimizes the total variance of the portfolio. The GMV formulation used in this paper is given by expression (2), which includes a linear constraint for a sectoral cryptocurrency group. For portfolios that do not include sectors, the linear restriction is omitted.

$$\min_{w} \quad \sigma_p^2\ (w) = w^{\mathrm{T}}\widehat{\Sigma}w \tag{2}$$

$$s.t.\mathbf{1}_N^{\mathrm{T}} w = 1$$

$$w_i \geq 0$$

$$L \leq \mathrm{A}w \leq U$$

where L and U are the lower and upper bounds for the sector cryptocurrency group. A is the constraint matrix for the sector cryptocurrency group.

3.3. Global Minimum CVaR Portfolio Objective

The disadvantage of the expression (2), is the assumption of a normal distribution of the portfolio's asset return for which the parameters are estimated. Considering the results of the study by (Briere et al., 2015) and (Lee Kuo Chuen et al., 2018), where evidence for the presence of a heavy-tailed cryptocurrency return distribution is shown, in this study expression (3) is used (Petukhina et al., 2018) and (Eisl, 2015), which is based on the methodology by (Rockafellar and Uryasev, 2000). In this case, a more reliable measure of CVaR is taken as a measure of risk so that the Mean-Variance model goes into Mean-Conditional Value at Risk (M-CVaR).

We define the cumulative distribution function of a loss function $z = f(w, y)$ as

$$\Psi(w, \zeta) = P\left\{y \,\middle|\, f(w, y) \le \zeta\right\} \tag{3}$$

where w is fixed decision vector (i.e. portfolio weights), ζ loss associated with that vector and y uncertainties (e.g. market variables) that impact the loss. Then, for a given confidence level α, the Value at Risk (VaR_α) associated with portfolio is given as

$$VaR_\alpha(w) = min\left\{y \,\middle|\, \Psi(w, \zeta) \ge \alpha\right\} \tag{4}$$

If $f(w, y)$ exceeds the VaR, then the expected value of the loss is defined as

$$\mathrm{CVaR}_\alpha(w) = \frac{1}{1-\alpha} \int_{y(w) \le VaR_\alpha(w)} y f\left(y \,\middle|\, w\right) dy \tag{5}$$

Expression (5) is adapted to the optimization goal of risk minimization, with a confidence level of 95%. For portfolios that do not include sector cryptocurrencies, their linear restriction is also omitted.

$$\min_w \mathrm{CVaR}_\alpha(w) \tag{6}$$

$$s.t. \mathbf{1}_N^{\mathrm{T}} w = 1$$

$$w_i \geq 0$$

$$L \leq \mathrm{A} w \leq U$$

3.4. Maximize Sharpe and STARR Ratio Portfolio Objective

The basic Markowitz relation (1) minimizes the variance of the portfolio return given the default expected return. On the other hand, by putting the expected return on the portfolio (adjusted for the risk-free interest rate over the same observation period) and the standard deviation of the portfolio, a Sharpe ratio will be obtained, i.e. a ratio indicating how much additional return is received per unit of risk. With the rational investment condition, by changing the investor's tolerance for risk, one will expect a higher expected return for the additional risk unit. In this case, the optimization goal of maximizing the return for a given level of risk is implemented. Portfolios that have the highest expected return for a given level of risk create an efficient frontier of feasible portfolios, hence the portfolio that has the highest Sharpe ratio represents the optimal portfolio, i.e. the tangent portfolio used in this paper (7). For portfolios without sector cryptocurrency, the linear restriction is omitted.

$$\max_{w} \quad \left\{ \frac{w^{\mathrm{T}} \mu - \bar{r}_f}{\sqrt{w^{\mathrm{T}} \widehat{\Sigma} w}} \right\} = \left\{ \frac{w^{\mathrm{T}} \mu}{\sqrt{w^{\mathrm{T}} \widehat{\Sigma} w}} \right\} \tag{7}$$

$$s.t. \mathbf{1}_N^{\mathrm{T}} w = 1$$

$$w_i \geq 0$$

$$L \leq \mathrm{A} w \leq U$$

where $\bar{r}_f$ represents the risk-free interest rate adjusted for the observation period. For the purposes of the research, the risk-free interest rate is omitted, as can be seen from (7) and (8).

If CVaR is used in the denominator of expression (7) instead of standard deviation, the Sharpe ratio goes to Stable Tail-Adjusted Return Ratio (STARR) and is given by (8). The optimization goal is to maximize the STARR ratio with a 95% confidence level and a linear limit for sector cryptocurrencies.

$$\max_{w} \quad \left\{ \frac{w^{\mathrm{T}}\mu - \overline{r}_f}{\mathrm{CVaR}_{\alpha}(w)} \right\} = \left\{ \frac{w^{\mathrm{T}}\mu}{\mathrm{CVaR}_{\alpha}(w)} \right\} \tag{8}$$

$$s.t. \mathbf{1}_N^{\mathrm{T}} w = 1$$

$$w_i \geq 0$$

$$L \leq \mathrm{A}w \leq U$$

3.5. Maximize Quadratic Utility Function Portfolio Objective

The Markowitz model is also based on the assumption of a utility function, that is, the degree of satisfaction an investor achieves by investing in some form of asset. Specifically, the disadvantage of a Sharpe ratio is the assumption that all investors are equally risk-averse, resulting in only one optimal portfolio that delivers the best reward-risk ratio. On the other hand, by changing the ratio of expected return and risk, it is possible to create an indifference curve, i.e. an utility as a function of investor risk preference for lower but safer versus higher but riskier expected returns, whereas different combinations of risks and returns found on the indifference curve equal investor satisfaction. In order to derive the utility function curve, it is necessary to introduce an investor aversion parameter to risk γ. According to (9), a lower parameter value (lower risk aversion) also means a lower penalization of the portfolio risk contribution, which leads to a higher risk portfolio, that is, a higher expected return. Conversely, in the case of more risk aversion, higher risk portfolios will be more penalized, leading to lower risk portfolios and lower expected returns. By gradually increasing the degree of risk aversion and optimization, a portfolio efficient frontier is derived in order to find the desired risk-return profile. The value of the parameter used in the paper is one.

$$\max_{w} \quad \mu \ (w) - \frac{\gamma}{2} w \widehat{\Sigma} w \tag{9}$$

$$s.t. \mathbf{1}_N^{\mathrm{T}} w = 1$$

$$w_i \geq 0$$

$$L \leq \mathrm{A} w \leq U$$

where *gamma* represents the degree of investor aversion to risk.

3.6. Maximize Return Portfolio Objective

In contrast to the strategy that minimizes variance, the study also implements an optimization strategy that maximizes expected portfolio returns, i.e. does not include a predefined level of risk. In this case, the optimization algorithm does not take into account the variance and covariance matrix, but uses the average returns of the previous period to estimate the highest expected portfolio return in the next period. The assets of the portfolio with the highest average return in the previous period will have the highest allocation in the portfolio. Because the strategy does not consider risk as an input in optimization, it is considered high-risk. The formulation used in this paper to maximize the expected return is given by (10). It includes a linear constraint for a group of sector cryptocurrencies. For portfolios without cryptocurrencies by sector, the linear restriction is not included.

$$\max_{w} \quad \mu_p \ (w) = w^{\mathrm{T}} \tag{10}$$

$$s.t. \mathbf{1}_N^{\mathrm{T}} w = 1$$

$$w_i \geq 0$$

$$L \leq \mathrm{A}w \leq U$$

where μ_p is the expected portfolio return.

3.7. Performance Metrics

The results of several different absolute and relative measures of success are presented in order to evaluate the success of each optimization strategy: Sharpe ratio, MSquared, Regression alpha, Jensen's alpha, Treynor ratio and Information ratio, where the values are calculated annually and relate to total time series of portfolio returns. The annual average geometric return was used to calculate the realized portfolio return: $R_{Gi} = prod(1 + R_{di})^{\frac{scale}{n}} - 1$, where R_{di} is the daily realized return of the observed portfolio i at time t, n is the total number of existing observations and $scale$ number of observations in a year 252. The annual standard deviation is given by the $\sigma_{ai} = \sigma_{di} \times \sqrt{252}$, where σ_{di} is the standard deviation of daily portfolio returns. If the risk-free interest rate is omitted from the calculation as an indicator of opportunity profitability, the Sharpe ratio SR used in this paper is given by (11).

$$SR = \frac{R_{Gi}}{\sigma_{ai}} \tag{11}$$

Apart from ranking investment opportunities, the Sharpe ratio does not provide additional information. Only for investments with the same level of risk, the Sharpe ratio indicates exactly how much one investment is better than the other. The measure that corrects this shortcoming is MSquared M^2 (12). It indicates the difference in the ratio of return to risk between investments, regardless of the level of risk. Risk-free interest rate is omitted.

$$M^2 = R_{Gi} \times \frac{\sigma_{aM}}{\sigma_{ai}} \tag{12}$$

where σ_{aM} is the annual standard deviation of the market (in this case of the CRIX index). By comparing the values of the above measures, information can be obtained about the superiority of one strategy over another in the cryptocurrency market. However, the above measures were calculated in the sample, i.e. on the total sample of data. The results indicate the average values of the measures calculated on the portfolio returns of each optimization strategy. In order to consider investment opportunities, it is also advisable to consider the results of the linear regression model (13) between the time series of portfolio returns as dependent variables and the CRIX index return as an independent variable. Where CRIX index return represents the benchmark of the market, since the estimated slope value is an input parameter for the Treynor ratio (14) and Jensen's alpha (15).

$$R_{di} = \alpha_i + \beta_i \times R_{dM} + \epsilon_i \tag{13}$$

where α_i is the regression intercept, β_i is the slope of the regression line, R_{dM} is the daily realized market return (CRIX index) and ϵ_i is the residual deviation from the regression line. Substituting the standard deviation in expression (10) for the estimated slope value of the regression equation $\overline{\beta}_i$ as a measure of volatility risk, we obtain the Treynor ratio TR. The measure used in this paper is given by (14).

$$TR = \frac{R_{Gi}}{\overline{\beta}_i} \tag{14}$$

As part of the Capital Asset Pricing Model CAPM (Sharpe, 1963), the slope value of (13) is the estimated magnitude of the regression equation that relates the risk of a potential investment to the equilibrium expected return on a risky investment. If the potential investment is more variable relative to the market, rational investors will expect a premium on their investment to compensate for the higher risk assumed and the estimated slope of the direction $\overline{\beta}_i$ will be above one. The basic relation of the CAPM model indicates that the expected return rate on investment equals the risk-free interest rate increased by the market risk premium (benchmark standard), which is corrected by its systematic risk, i.e. $\overline{\beta}_i$. In addition, the CAPM model neglects the existence of a specific investment risk ϵ_i, that is, it only takes into account its systematic risk arising from the movement of the entire market. By rearranging the basic equation of the CAPM model, one can derive expression (15) representing Jensen's alpha, the measure used in this study. Jensen's alpha is a measure whose value indicates whether an investment has achieved a higher or lower return than the expected or required return per CAPM model. Assuming that CRIX index is an adequate approximation of the cryptocurrency market movement, if Jensen's alpha portfolios are positive in size, the optimization strategy can be said to have outperform the market by yielding higher returns than required by the CAPM model. In (15), the risk-free interest rate is omitted.

$$\alpha_i = R_{Gi} - \beta_i \times R_{GM} \tag{15}$$

where R_{GM} is the annual average geometric return of the CRIX index. The last measure of performance used in this study is the Information ratio IR (16). Information ratio is a relative measure that compares the difference between the annual average geometric portfolio return and the CRIX index (active premium) and the annual standard deviation of the active premium between the portfolio yield and the CRIX index (tracking error). If IR is a positive value, it would mean that the optimization strategy performed better than the CRIX index, which approximates the movement of the cryptocurrency market.

$$IR = \frac{R_{Gi} - R_{GM}}{\sigma_{iM}} \tag{16}$$

where σ_{iM} is the standard deviation of the active premium between the portfolio return and the CRIX index. In addition, the study presents the cumulative portfolio return of individual optimization strategies and CRIX index assuming an initial investment of one dollar, as well as their worst drawdown, which indicates the highest loss relative to the highest value of cumulative return in the observed period.

4. Results

Here the empirical results of out-of-sample backtesting for each of the optimization models implemented are presented and discussed. The interpretation of the results is done in two phases. At the first phase, the results are reviewed and interpreted by a comparative method between asset allocation models according to the initial selection of the portfolio components. In addition, the success of a particular strategy is determined by the implementation of performance measures that include the CRIX index as a benchmark of the market over the observation period. In second phase, the results of allocation models are compared and interpreted between portfolios to determine the benefits of sectoral cryptocurrency market observation.

Table 1 shows the results of the previously described performance measures for six asset allocation models for 50 cryptocurrencies selected by CMC market size. Table 2 shows the results of portfolio performance measures that, in addition to the 50 cryptocurrencies per CMC, include an additional 15 cryptocurrencies per related financial, exchange and business services sector. The last column in the tables shows the results of the CRIX index over the same period as the benchmark of the cryptocurrency market. All values except the regression beta and worst drawdown of each optimization strategy are reported annually. In case of negative values the Traynor ratio is omitted.

If the regression beta between portfolio returns and the CRIX index is considered in the context of the CAPM model, most portfolios have a very low systematic risk. Moreover, four strategies have achieved a negative beta, which means that they are moving in the opposite direction of the CRIX index. Positive regression alphas indicate that, in the case of a cryptocurrency market stagnation, each of the observed portfolios on average achieves more returns than the market returns. As expected, the highest average alpha was achieved by the MaxMean portfolio. On the other hand, it is worth pointing out that the highest annual geometric return as well as the cumulative return in the total period, achieved the portfolio with the aim of minimizing CVaR. Therefore the best values of the performance measures are expected to be related to the MinCVaR portfolio. So SR stands at a high of 2,68 which is by far the best of all other optimization strategies. The difference between the MSquared return and the risk ratio relative to the CRIX index also benefits the portfolio, which minimizes the conditional value at risk. Jensen's alpha which suggests if the strategy has outperformed the market for all optimization solutions is positive. In other words, all strategies yielded returns higher than required by the CAPM model. The highest ratio of active premium IR and standard deviation was also achieved by the MinCVaR portfolio.

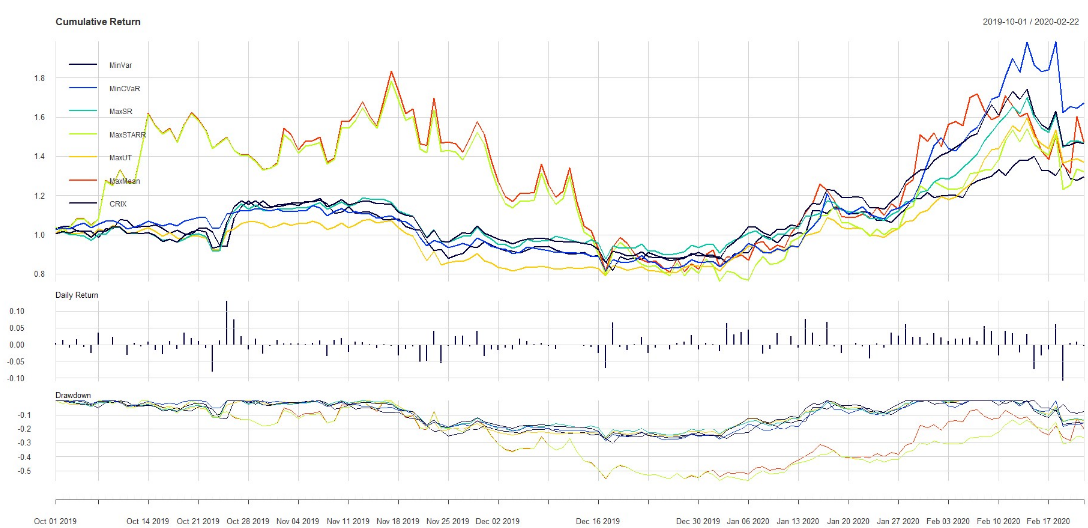
Table 1: Asset allocation models without sectoral cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar	MinCVaR	MaxSR	MaxSTARR	MaxUT	MaxMean	CRIX
Beta	β_i	0,05	-0,05	0,02	-0,014	-0.05	-0,01	1
Annualized Alpha	a_{ai}	1,12	1,91	1,16	1,53	0,97	2,29	/
Annualized Return	R_{Gi}	0,94	1,44	0,95	0,62	0,72	0,95	0,57
Annualized Std Dev	σ_{ai}	0,49	0,54	0,48	0,94	0,46	1,04	0,47
Worst Drawdown	WD	0,27	0,29	0,26	0,57	0,27	0,56	0,31
Cumulative Return	CY	1,46	1,67	1,47	1,32	1,37	1,47	1,29
Sharpe Ratio	SR	1,92	2,68	1,95	0,66	1,58	0,91	1,20
MSquared	M^2	0,91	1,27	0,92	0,31	0,75	0,43	0,57
Treynor Ratio	TR	18,69	/	59,03	/	/	/	0,57
Jensen's Alpha	α_i	0,91	1,47	0,94	0,63	0,75	0,95	/
Information Ratio	IR	0,56	1,20	0,56	0,05	0,23	0,34	/

On the other hand, the lowest geometric and cumulative returns were achieved by an optimization strategy for maximizing the ratio of returns and CVaR, so the values of other performance measures are consistent with these values. The highest annual standard deviation was achieved by the portfolio with the optimization goal of maximizing expected returns, and the lowest portfolio maximizing the utility function. In comparison with the CRIX index, all implemented optimization goals achieved a higher cumulative return in the same observation period. However, the CRIX index achieved a lower standard deviation level in five out of six cases. Four portfolios have smaller worst drawdowns than the index as well as a higher SR.

Figure 1 shows the dynamics of the daily cumulative returns of individual strategies, the total daily returns of all strategies, and an underwater chart for drawdown to further illustrate the performance of portfolio optimization goals.

Figure 1: Performance summary various strategies without sectoral cryptocurrencies



By including an additional 15 sectoral cryptocurrencies that would not initially be selected as a component of the portfolio by market capitalization, the results differ by all measures shown in Table 2. Similar to the above, the regression beta values suggest low systematic risk of all optimization strategies. The highest average annual alpha, geometrical as well as the total cumulative return, was achieved by the portfolio with the aim of maximizing it. Which is in line with expectations due to the higher risk assumed as standard deviation, i.e. worst drawdown. However, the return of the MaxMean-S portfolio adequately compensated for the higher risk assumed, which ultimately resulted in a high SR of 4,66. MSquared also points out the difference between the MaxMean-S portfolio and the CRIX index. Jensen's alpha suggests that all observed portfolios have yielded higher than expected returns per CAPM ratio. The significant difference between the realized geometric return of the MexMean-S portfolio and the CRIX index, comparing to the standard deviation of the active premium which is extremely low due to the equal volatility between the observed investments, also influenced the highly positive Information ratio. In the second order of the best size of all performances, except for the risk measures, it was achieved by a portfolio that maximizes the ratio of return and risk expressed as CVaR.

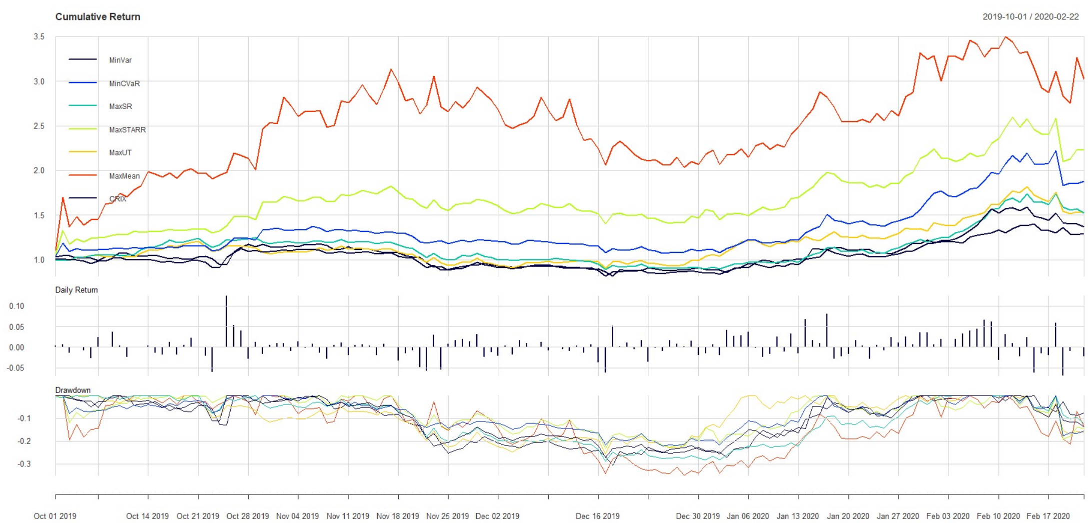
Table 2: Asset allocation models with sectoral cryptocurrencies

		Asset Allocation Models						
Performance Metrics		MinVar-S	MinCVaR-S	MaxSR-S	MaxSTARR-S	MaxUT-S	MaxMean-S	CRIX
Beta	β_i	0,05	0,03	-0,08	0,10	-0,12	0,22	1
Annualized Alpha	a_{ai}	0,86	2,39	1,37	3,74	1,51	10,10	/
Annualized Return	R_{Gi}	0,73	1,99	1,09	3,02	1,09	5,84	0,57
Annualized Std Dev	σ_{ai}	0,45	0,53	0,43	0,67	0,48	1,17	0,47
Worst Drawdown	WD	0,27	0,22	0,29	0,23	0,25	0,35	0,31
Cumulative Return	CY	1,37	1,88	1,52	2,23	1,53	3,02	1,29
Sharpe Ratio	SR	1,62	3,79	2,53	4,50	2,27	4,99	1,20
MSquared	M^2	0,77	1,79	1,20	2,13	1,07	2,36	0,57
Treynor Ratio*	TR	14,91	76,54	/	31,22	/	26,88	0,57
Jensen's Alpha	α_i	0,70	1,98	1,12	2,97	1,16	5,72	/
Information Ratio	IR	0,26	2,04	0,77	3,09	0,74	4,31	/

In terms of performance measures, the strategy to minimize standard deviation of the portfolio has performed the worst. On the other hand, the lowest standard deviation was achieved by the MaxSR-S optimization strategy, where the standard deviation of the MinVar-S portfolio is slightly higher. The lowest worst drawdown belongs to the MinCVaR-S portfolio, which is in line with the optimization goal. In comparison with the CRIX index, all of the optimization goals achieved a higher cumulative return in the same observation period. It is also worth pointing out that only two MaxUT-S and MaxMean-S strategies achieved a higher standard deviation than the CRIX index. In addition, all portfolios achieved a higher Sharpe ratio than the CRIX index during the same period.

Figure 2 shows the dynamics of the daily cumulative returns of individual strategies, the total daily returns of all strategies, and the underwater chart for drawdown, further illustrating the performance of portfolio optimization goals.

Figure 2: Performance summary various strategies with sectoral cryptocurrencies



5. Interpretation of the results and discussion

Interpreting the results involves considering the performance of strategies between portfolios that differ in composition. The first thing to notice is the height of the regression alpha, which for all portfolios except the MinVar-S, achieved a better result if sectoral cryptocurrencies were included in the portfolio. Portfolios with additional cryptocurrencies earn on average more returns than portfolios without sectoral components. Geometric return has the same relationships. Only the MinVar-S portfolio has a lower return than the portfolio return without sectoral cryptocurrency, thus confirming the utility of observing the cryptocurrency market through sectoral affiliation.

In terms of risk, four strategies involving sectoral cryptocurrencies have achieved a lower standard deviation than portfolios without them. However, the higher risk was offset by the higher return achieved, implying a higher Sharpe ratio.

Worst drawdown also points to the benefits of including additional cryptocurrencies where only the MaxSR-S portfolio has a higher worst drawdown than the MaxSR portfolio.

The inclusion of sector cryptocurrencies has also led to an increase in cumulative return for all strategies except the MinVar-S portfolio. The biggest difference was made by the MaxMean-S portfolio, where its cumulative return increased by 1,55. By applying a return maximization strategy and considering sectoral cryptocurrencies as a components of the portfolio, it is possible to achieve a cumulative return higher by 105% over a 146-day period than the same strategy that does not consider sectoral cryptocurrencies. A significant increase in cumulative return was also achieved by the MaxSTARR-S portfolio of 0,91, or 69%, compared to MaxSTARR. The results of the Sharpe ratio and MSquared measures are consistent with the above. The largest increase in SR and M^2 refers to the MaxMean-S portfolio. Only the MinVar-S portfolio has a lower SR and M^2 relative to an equivalent strategy with no additional cryptocurrencies. Given that five of the six portfolios that include additional cryptocurrencies had a higher geometric return and the regression beta did not increase significantly, so Jensen's alpha performed better for all portfolios except MinVar-S. The inclusion of additional sector cryptocurrencies in existing portfolios contributes to the improvement of portfolio performance compared to the market represented by the CRIX index. Treynor ratio and Information ratio also performed significantly better for all sector portfolios except MinVar-S portfolios. Considering all the above, it can be determined that five of the six portfolios created according to different optimization goals achieved better results if they included the cryptocurrencies of the financial, exchange and business services sectors. Such results contribute significantly to the research of investment opportunities in the cryptocurrency market to date. In addition, the positive results suggest more observations from which certain conclusions need to be drawn.

The first observation is certainly the existence of distinction within the category of cryptocurrencies and also the category of utilization tokens. If cryptocurrency litecoin is used solely as a means of payment, comparing litecoin with a decentralized computer platform like ethereum and treating the two assets equally in the context of investment opportunities, is simply not practical or even impossible. On blockchain economies like ethereum, among other things, tokens as a type of cryptocurrency, can be created to provide a different purpose and utility in the practical application of the product and service. This also means that products and services are different, i.e. utilization tokens differ in their fundamentals. For an example, a utilization token with a strictly defined purpose in a specific online game should not be equated with a utilization token that has a wider purpose, such as tokens created for decentralized finances. If viewed together, the market recognizes such anomalies and "properly" values the assets observed. This situation has led to significantly better portfolio results that include sector cryptocurrencies, i.e. other observations that need to be made from the results of this paper.

In the cryptocurrency market beginning of 2020 and in 2019, was marked by a rise in the value of utilization tokens, which to some extent represented Decentralized Finance - DeFi. None of the 15 additional cryptocurrencies (tokens) selected by sector were in the top 50 by CMC, so they have not been considered in previous studies. All that previous research has considered was cryptocurrencies as one unit and relied solely on the optimization algorithm when selecting portfolio components. Such an approach implies a consensus on the magnitude of the equilibrium expected return of the selected cryptocurrencies. However, previously it has been stated that such a return does not even exist due to the absence of adequate valuation, that is, the intrinsic value of cryptocurrencies. Taking into account previous research, if one draws a parallel with thinking of the traditional capital market and the CAPM model, it can be concluded that all cryptocurrencies are properly valued, i.e. all cryptocurrencies are on the Security Market Line-SML. The results presented in this paper suggest the opposite. The conclusion of this study is supported by positive average regression and realized alpha portfolios, as well as portfolios with additional sectoral cryptocurrencies.

In line with the results obtained, the last observation emphasizes the utility and necessity of observing the cryptocurrency market by sectoral affiliation with the aim of finding potentially "undervalued" cryptocurrencies. If portfolio components are selected solely by market capitalization, it would mean that these cryptocurrencies have already achieved the value that makes them a potential portfolio component. The possibility of price growth of such cryptocurrency is certainly much lower than the possibility of cryptocurrency growth which ranks 150th in terms of market capitalization. Sectorally, cryptocurrencies with much lower market capitalization are emerging and investors can more easily spot them. Looking at the overall capitalization of the sector, it is easier to spot and identify current trends in the cryptocurrency market, as was the growth trend in 2019 of DeFi cryptocurrencies.

6. Conclusion

Examining the utility of observing cryptocurrencies through their sectoral affiliation when constructing a portfolio is the primary theme of this paper. The results of the methodological approach are contributing to considering investment opportunities in the cryptocurrency market. The methodology for exploring the benefits of sectoral allocation and portfolio construction has been implemented in two phases. In the first phase, the performance of the portfolio limited in composition to market capitalization is created and interpreted. In the second phase the cryptocurrencies of the three leading sectors by market capitalization are included: finance, exchanges and business services. Consideration of the cryptocurrency market by sectoral affiliation is justified by the theoretical assumption that there are significant price trends of certain sectors in the cryptocurrency market. Such an approach easiliy recognizes cryptocurrencies that belong to the same sector and have lower market capitalization (higher price growth potential).

The results suggest that portfolios in which 20% of the share is allocated to cryptocurrencies of lower market capitalization are achieving higher values across all implemented performance measures in five of the six optimization strategies created. It is concluded that it is desirable and necessary to observe the cryptocurrency market through their type or their utility, and such an approach can be achieved by categorizing cryptocurrencies into their sectors. Potential investors, and portfolio managers in particular, should not consider cryptocurrencies by market capitalization. Cryptocurrencies have characteristics and capabilities that define them according to their nominal purpose. Accordingly, portfolio managers are encouraged to consider cryptocurrencies by their characteristics (the type and purpose they provide) when constructing a portfolio, in order to eliminate their subordinate position and to contribute to portfolio performance in the cryptocurrency market.

References

[1] Brauneis, A., & Mestelb, R. 2018. Cryptocurrency-portfolios in a mean-variance framework. Finance Research Letters, Vol. 28, 259-264.

[2] Briere, M., Oosterlinck, K. & Szafarz, A. 2013. Virtual currency, tangible return: Portfolio diversication with bitcoins. Journal of Asset Management, Vol. 16, no. 6, 365-373.

[3] Carpenter, A. 2016. Portfolio diversification with Bitcoin. Journal of Undergraduate Research in France, Vol. 6, no. 1, 1-27.

[4] Chuen, D. L. K., Guo, L. & Wang, Y. 2017. Cryptocurrency: A New Invest-

ment Opportunity? The Journal of Alternative Investments, Vol. 20, no. 3, 16-40.

[5] Eisl, A., Gasser, S. & Weinmayer, K. 2015. Caveat emptor: Does bitcoin improve portfolio diversication? Working Paper, WU. Available at SSRN: https://ssrn.com/abstract=2408997 or http://dx.doi.org/10.2139/ssrn.2408997.

[6] Elendner, H., Trimborn, S., Ong, B. & Lee, T. M. 2016. The Cross-Section of Crypto-Currencies as Financial Assets: An Overview. SFB 649 Discussion paper 2016-038. Berlin: Humboldt-Universität zu Berlin.

[7] Glaser, F., Zimmermann, K., Haferkorn, M., Weber, M. & Siering, M. 2014. Bitcoin - Asset or currency? Revealing users' hidden intentions. ECIS 2014 Proceedings - 22nd European Conference on Information Systems. Vol. 3, 1-14.

[8] Kajtazi, A., & Moro, A. 2019. The role of bitcoin in well diversified portfolios: A comparative global study. International Review of Financial Analysis, Vol. 61, 143-157.

[9] Klein, T., Hien, P. & Walther, T. 2018. Bitcoin Is Not the New Gold: A Comparison of Volatility, Correlation, and Portfolio Performance. International Review of Financial Analysis, Vol. 59, 105-116; University of St.Gallen, School of Finance Research Paper No. 2018/14; QMS Research Paper 2018/01. Available at SSRN: https://ssrn.com/abstract=3146845 or http://dx.doi.org/10.2139/ssrn.3146845.

[10] Liu, W. 2018. Portfolio Diversification across Cryptocurrencies. Finance Research Letters, Vol. 29, 200-205.

[11] Markowitz, H. 1952. Portfolio selection. The journal of finance, Vol. 7, no. 1, 77-91.

[12] Petukhina, A., Trimborn, S., Härdle, Wolfgang, K. & Elendner, H. 2018. Investing with Cryptocurrencies - evaluating the potential of portfo-

lio allocation strategies. IRTG 1792 Discussion Paper 2018-058. Berlin: Humboldt-Universität zu Berlin.

[13] Platanakis, E., Sutcliffeb, C. & Urquhartc, A. 2018. Optimal vs naïve diversification in cryptocurrencies. Economics Letters, Vol. 171, pp. 93-96.

[14] Rockafellar, R. T. and S. Uryasev 2000. Optimization of conditional value-at-risk. Journal of risk 2, 21-42.

[15] Symitsi, E. & Chalvatzis, K.J. 2018. The Economic Value of Bitcoin: A Portfolio Analysis of Currencies, Gold, Oil and Stocks. Research in International Business and Finance, Vol. 48. 97-110.

[16] Trimborn, S. 2015. Towards Cryptocurrency Index - Analysis of the market. Master's Thesis, Humboldt-Universität zu Berlin.

[17] Trimborn, S., Li, M. & Härdle, Wolfgang, K. 2017. Investing with Cryptocurrencies - A Liquidity Constrained Investment Approach. SFB 649 Discussion paper 2017-014. Berlin: Humboldt-Universität zu Berlin.

[18] Trimborn, S., Li, M. & Härdle, Wolfgang, K. 2018. Investing with Cryptocurrencies - A Liquidity Constrained Investment Approach. SFB 649 Discussion paper 2017-014. Berlin: Humboldt-Universität zu Berlin.