Lab 4: Interferometry

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Abstract

Throughout this lab, an interferometer is used to determine the indexes of refraction of several media. This was accomplished by measuring the phase difference between two versions of an incident beam split across two different paths; an uninterrupted path and a path through a subject medium.

In the second experiment, the index of refraction of air at atmospheric pressure was calculated with a very small error to be $n_{\text{atm}} = 1.0391 \pm 49.0 \times 10^{-9}$. In the third experiment, the index of refraction of glass was calculated with a rather high error to be $n_g = 1.50 \pm 10.3$.

1 Experiment 1: Introduction to Interferometry

1.1 Objective

The purpose of the first experiment was to gain familiarity with the interferometry equipment. The simple experiment of determining the wavelength of the laser makes use of the basic setup without introducing too many extraneous components. The relative simplicity of this experiment allows us to focus on calibrating the interferometer and becoming well-versed in its operation.

1.2 Theory

1.2.1 Part I: Wavelength

The first part of this experiment asks us to find the wavelength of the incident laser. The interferometer has the effect of creating two beams incident on the viewing screen, one having traveled a greater distance than the other. Because the two beams have traveled different distances, they are out of phase by a certain amount. The phase difference between the beams can be altered by modifying the length of one of the beam's paths. This is accomplished by rotating the micrometer to change the position of the adjustable mirror. As the adjustable mirror is moved backward, the image of the waves' interference pattern changes. When the phase difference completes half a revolution, the image on the screen appears the same as before the micrometer was rotated. Each time the image on the screen reverts back to its initial state, a fringe is said to have passed by. Since only half a revolution results in a similar image, the distance the mirror is adjusted is equal to half the wavelength of the wave. This relationship is shown by

$$\lambda = 2\frac{d_m}{N} \tag{1}$$

where d_m is the distance traveled by the adjustable mirror and N is the number of fringes that have passed.

1.2.2 Part II: Polarization

The second part of the experiment asks us to observe the effects of polarizing the incident beam. A polarizer accepts incident light and only transmits the components of that light whose orientation matches that of the polarizer. With only one polarizer present, the image is slightly dimmer with any angle. The image dims because a portion of the incident light is absorbed by the polarizer, but because the incident light is not polarized, the same amount of light is absorbed no matter the angle of the polarizer. With two polarizers, however, the light can be completely blocked. Once the light is oriented in a certain direction through the first polarizer, if the angle of the second polarizer is perpendicular to that of the first, all of the light incident on the second polarizer is absorbed, producing no transmission. Additionally, the amount of light transmitted through the second polarizer is inversely proportional to the angle of the second polarizer relative to that of the first.

1.3 Equipment

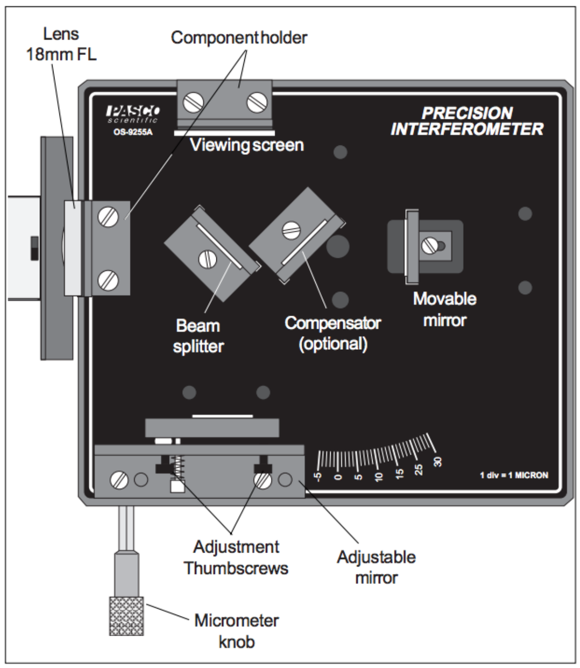

Figure 1: A diagram of the interferometer setup in Michelson mode; the configuration used for Experiment 1.

Figure 2: The list of equipment used in Experiment 1.

Manufacturer	Model	Serial Number	Specifications
PASCO Scientific	Precision Interferometer	OS-9255A	n/a
PASCO Scientific	Laser Alignment Bench	OS-9172	n/a
PASCO Scientific	Component Holder	OS-9256A	n/a
PASCO Scientific	Polarizer (2)	OS-9256A	n/a

1.4 Procedures

1.4.1 Part I: Wavelength

The first part of the experiment involves measuring the wavelength of the beam from the laser. Before beginning, the interferometer must be set up in Michelson mode, and an interference pattern must be clearly visible on the viewing screen.

To start off, the micrometer knob ought to be adjusted to a medium setting of approximately 50 µm. To minimize backlash during measuring, turn the micrometer knob one full rotation counterclockwise, and to maximize accuracy of measurement, continue turning counterclockwise until the zero on the knob is aligned with the index mark. Record the micrometer reading, d_i.

Adjust the position of the viewing screen so that one of its marks is aligned with the fringes of the interference pattern. This is the reference mark. Slowly rotate the micrometer knob counterclockwise, and count the number of fringes that pass the reference mark. Continue until at least 20 fringes are counted. One fringe has passed by when the fringes return to the same position before counting began. Record the final reading of the micrometer dial, d_f.

Calculate $d_m = d_f - d_i$, the distance traveled by the adjustable mirror, for the number of counted fringes, N. Repeat these steps several times.

1.4.2 Part II: Polarization

With the same setup as above, place a polarizer between the laser and the beam-splitter. Slowly rotate the polarizer to achieve several different polarization angles, and observe any effects caused by this rotation. Move the polarizer to in front of the movable mirror, and, again, observe the effects of its rotation on the image on the screen.

Now, place one polarizer in front of the fixed mirror and another in front of the movable mirror. Rotate one polarizer, then the other, and observe any effects this rotation has on the image on the screen.

1.5 Data and Analysis

Figure 3: The measurements of the micrometer while counting the fringes during Part I of Experiment 1.

Initial Position (d_i) (µm)	Final Position (d_f) (µm)	Mirror Movement d_m (µm)	Number of Fringes N	Wavelength (λ) (nm)
52.5 ± 0.05	53.2 ± 0.05	$0.7 \pm 70.7 \times 10^{-3}$	20	70 ± 7.07

The distance the mirror traveled is found with $d_m = d_f - d_i$, where d_f is the final position of the micrometer and d_i is its initial position. Both d_f and d_i have an error of $\pm 50\,\mathrm{nm}$. Since d_i is subtracted from d_f, the error of d_m, δd_m, is found with

$$\delta d_m = \sqrt{\delta d_f^2 + \delta d_i^2}$$

$$\delta d_m = \sqrt{(0.05)^2 + (0.05)^2}$$

$$\delta d_m = 70.7\,\mathrm{nm}$$

The wavelength of the incident beam is found with

$$\lambda = \frac{2d_m}{N}$$
$$\lambda = \frac{2(0.7)}{(20)}$$
$$\lambda = 70\,\mathrm{nm}$$

Since N is the constant 20, and d_m is, net, multiplied by $\frac{1}{10}$, its error, δd_m, must be similarly multiplied in order to find the error in the wavelength, $\delta\lambda$

$$\delta\lambda = \frac{2\delta d_m}{N}$$
$$\delta\lambda = \frac{2(70.7 \times 10^{-3})}{N}$$
$$\delta\lambda = 7.07\,\mathrm{nm}$$

1.6 Conclusion

While the error of the wavelength is low compared to the value of the wavelength, it is not accurate. The actual wavelength is 633 nm, but our measurement is for 70 nm. Factors that limited the accuracy of the micrometer measurements include placement of the final reading; the dial can be difficult to read if the marker is in between markings on the Vernier scale.

2 Experiment 2: The Index of Refraction of Air

2.1 Objective

The purpose of this experiment is to use interferometry to determine the index of refraction of air. This is accomplished by measuring the phase difference of a split beam through two different media. In this case, the media are normal air and a semi-evacuated chamber.

2.2 Theory

As discussed earlier, the wavelength of an incident light beam can be determined by measuring the phase difference between two split versions of that beam. The wavelengths of a light beam traveling through two different media are related with Snell's Law,

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1} \tag{2}$$

where λ_1 and λ_2 are the wavelengths of light traveling through media with indexes of refraction n_1 and n_2, respectively. If the first medium is a vacuum, then Equation 2 can be rewritten as

$$\lambda = \frac{\lambda_0}{n} \tag{3}$$

where λ is the wavelength of the incident light beam after traveling through a medium of index of refraction n with respect to its wavelength in a vacuum, $\lambda_0 = 633\,\text{nm}$.

At reasonably low pressures, the pressures at which this experiment takes place, the index of refraction of a gas is linearly proportional to its pressure. By slightly altering the pressure of a gaseous medium by a known, proportional amount, the wavelength of a light beam traveling through the altered medium will be affected by a similar relative amount.

If the pressure of the medium is decreased, the wavelength of light passing through it will be decreased by a proportional amount. This decrease in wavelength manifests itself as a decrease in the number of fringes counted while changing the pressure of the medium. Rewriting Equation 1 for the number of fringes, the following two equations can be obtained:

$$N_i = \frac{2d}{\lambda_i} \tag{4}$$

$$N_f = \frac{2d}{\lambda_f} \tag{5}$$

where d is now the length of the cell in which the pressure of the medium is altered, N_i is the number of fringes counted when the light traveled through the initial medium, and N_f is the number of fringes counted when the light traveled through the modified, final medium. The difference between the number of fringes, N, can then be written as

$$N = \frac{2d}{\lambda_i} - \frac{2d}{\lambda_f} \tag{6}$$

By applying Equation 3 to the initial and final wavelengths in Equation 6, it can be rewritten as

$$N = \frac{2d(n_i - n_f)}{\lambda_0} \tag{7}$$

Since the relationship between a medium's pressure and its index of refraction is linear, the index of refraction of a medium is equal to the slope of its n vs. P graph, where P is the pressure of the medium. The equation for this graph can be found by rewriting Equation 7 as $n_i - n_f = \frac{N\lambda_0}{2d}$ and multiplying both sides by the inverse of the pressure difference between the two media, $\frac{1}{P} = \frac{1}{P_i - P_f}$,

$$\frac{n_i - n_f}{P_i - P_f} = \frac{N\lambda_0}{2d}\frac{1}{P_i - P_f} \tag{8}$$

2.3 Equipment

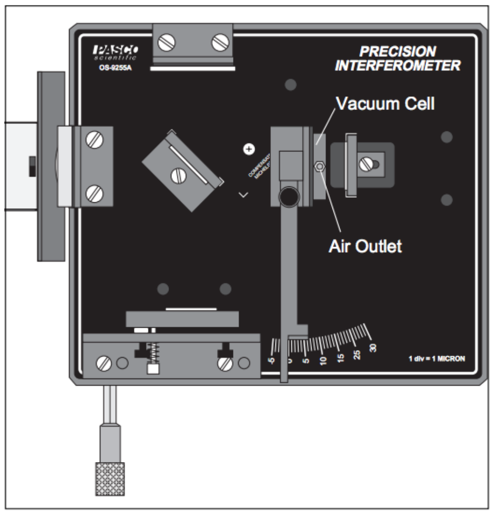

Figure 4: A diagram of the setup for Experiment 2. Since it is very similar to the setup of Experiment 1, only the components added to the interferometer for Experiment 2 have been labeled.

Figure 5: A list of the equipment used in Experiment 2.

Manufacturer	Model	Serial Number	Specifications
PASCO Scientific	Precision Interferometer	OS-9255A	n/a
PASCO Scientific	Laser Alignment Bench	OS-9172	n/a
PASCO Scientific	Rotational Pointer	OS-9256A	n/a
PASCO Scientific	Vacuum Cell	OS-9256A	thickness: 8 mm
PASCO Scientific	Vacuum Pump	OS-9256A	n/a

2.4 Procedures

Before starting, ensure that the interferometer is in Michelson mode, properly aligned, and in the same configuration as Experiment 1. To set up the interferometer for Experiment 2, place the rotational pointer between the movable mirror and the beam-splitter, then attach the vacuum cell to the magnetic backing of the rotational pointer. Attach the air hose to the vacuum cell, then adjust the alignment of the fixed mirror so that the center of the interference pattern is clearly visible on the viewing screen.

Ensure that the vacuum cell is at atmospheric pressure by flipping the vacuum release toggle switch, and record this initial value of the vacuum cell's pressure, P_i. Slowly pump air out of the cell, and record the number of fringes that pass the reference point on the viewing screen, N. Continue pumping until the vacuum pump's limit is reached, and record this final pressure, P_f. Repeat these steps to collect several trials' worth of data.

2.5 Data and Analysis

The slope of the n vs. P graph, the $m = \frac{N\lambda_0}{2d}$ term from Equation 8, where $N = 13$, $\lambda_0 = 633\,\text{nm}$, and $d = 8\,\text{mm} \pm 500\,\mu\text{m}$ is

$$m = \frac{N\lambda_0}{2d}$$
$$m = \frac{(13)(633 \times 10^{-9})}{2(8 \times 10^{-3})}$$
$$m = 514 \times 10^{-6}$$

The general equation for determining the error in a function, y, of multiple variables, $x_1, \ldots, x_n$, is

$$\delta y = y\sqrt{\left(\frac{\partial y}{\partial x_1}\delta x_1\right)^2 + \ldots + \left(\frac{\partial y}{\partial x_n}\delta x_n\right)^2} \tag{9}$$

Applying Equation 9 to $m(d) = \frac{N\lambda_0}{2d}$, where $d = 8\,\text{mm} \pm 500\,\mu\text{m}$, the error in the slope, δm, can be found:

$$\delta m = m\sqrt{\left(\frac{\partial m}{\partial d}\delta d\right)^2}$$

$$\delta m = m\sqrt{\left(\left(\frac{N\lambda_0}{2d^2}\right)\delta d\right)^2}$$

$$\delta m = (514 \times 10^{-6})\sqrt{\left(\frac{(13)(633 \times 10^{-9})}{2(8 \times 10^{-3})^2}\delta d\right)^2}$$

$$\delta m = 16.5 \times 10^{-9}$$

The slope of the n vs. P graph is $m = 514 \times 10^{-6} \pm 16.5 \times 10^{-9}$.

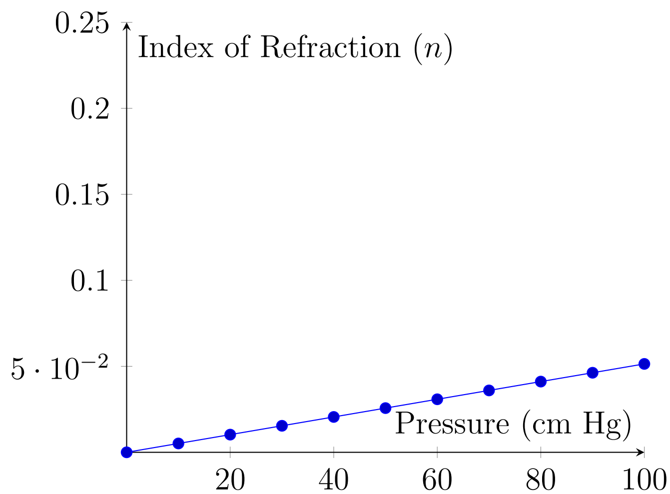

Figure 6: The plot of n vs. P.

The index of refraction of air at a given pressure is found with

$$n = 1 + n(P) \tag{10}$$

where $n(P) = mP = 5.14 \times 10^{-3}P$. Therefore, the index of refraction of air at atmospheric pressure ($P = 76cmHHg$) is

$$\begin{aligned} n_{\text{atm}} &= 1 + n(76) \\ n_{\text{atm}} &= 1 + ((5.14 \times 10^{-3})(76)) \\ n_{\text{atm}} &= 1.0391 \end{aligned}$$

The error of the index of refraction of air is found by applying Equation 9 to Equation 10:

$$\begin{aligned} \delta n &= n\sqrt{\left(\frac{\partial n}{\partial m}\delta m\right)^2} \\ \delta n_{\text{atm}} &= (39.1 \times 10^{-3})\sqrt{\left((P)(16.5 \times 10^{-9})\right)^2} \\ \delta n_{\text{atm}} &= (39.1 \times 10^{-3})\sqrt{\left((76)(16.5 \times 10^{-9})\right)^2} \\ \delta n_{\text{atm}} &= 49.0 \times 10^{-9} \end{aligned}$$

2.6 Conclusion

The property that the pressure of a gaseous medium is linearly related to the medium's index of refraction was used to find the index of refraction of air at atmospheric pressure. This was accomplished by plotting the index of refraction of air as a function of the pressure of air. The slope of this linear relationship was found to be $m = 514 \times 10^{-6} \pm 16.5 \times 10^{-9}$. Therefore, the index of refraction of air at atmospheric pressure, when $P = 76$cm Hg, is $n_{\text{atm}} = 1.04$, a value very near 1.

In order to test our assumption that the relationship between pressure and index of refraction is linear, several trials would be conducted to verify our results. The index of refraction of a gas also depends on temperature as it depends on pressure. If temperature rather than pressure were used to find the index of refraction of air, instead of evacuating an isolated chamber of air, the temperature of the air in the chamber would be altered.

3 Experiment 3: The Index of Refraction of Glass

3.1 Objective

While the purpose of Experiment 2 is to measure the index of refraction of air, a gas, the purpose of Experiment 3 is to measure the index of refraction of glass, a solid. In order to achieve this, the wavelength of the light through the material is, again, altered so that Snell's Law may be used to determine the medium's index of refraction. In this case, the distance traveled by the light through the medium in question is adjusted.

3.2 Theory

Similar to Experiment 2, Snell's Law, Equation 2, is used to determine the index of refraction of the medium through which the light passes. When light passes through a different medium, its wavelength is altered. The longer the light has this altered wavelength, as the distance traveled through the different medium increases, the greater its phase shift from the unaltered beam. This change in phase shift is quantified by counting the number of fringes passing the reference point, N, as the length of the medium is increased. The length of the medium is increased by increasing the angle of the medium's plane with respect to the plane perpendicular to the normal of the incident beam.

The total distance traveled by the light beam is represented as $d = d_a(\theta) + d_g(\theta)$, where $d_a(\theta)$ and $d_g(\theta)$ are the distances traveled through air and glass, respectively, as functions of θ, the angle between the plane of the glass and the plane perpendicular to the normal of the beam. Keeping in mind that the index of refraction is also different for each of the two media, the number of fringes that have passed the reference point can be found by expanding Equation 4 to account for the change in medium:

$$N = \frac{2n_a d_a(\theta) + 2n_g d_g(\theta)}{\lambda_0} \tag{11}$$

where n_a and n_g are the indexes of refraction of air and glass respectively and λ_0 is the wavelength of the incident beam in a vacuum. Equation 11 can then be solved for the index of refraction of the glass, n_g:

$$n_g = \frac{(2t - N\lambda_0)(1 - \cos(\theta))}{2t(1 - \cos(\theta)) - N\lambda_0} \tag{12}$$

where t is the thickness of the glass through which the light travels.

3.3 Equipment

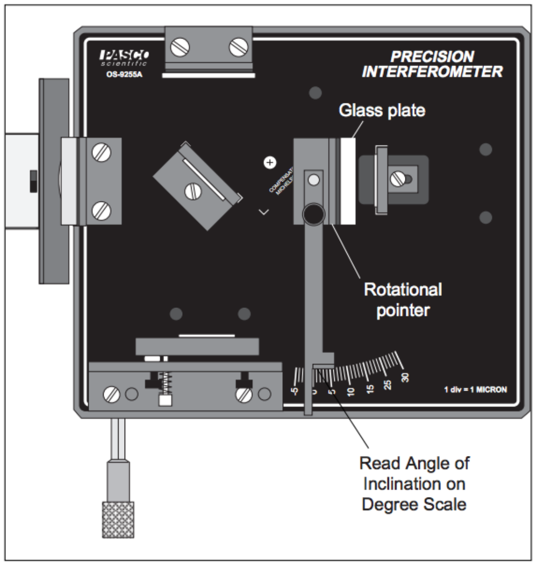

Figure 7: A diagram for the setup of Experiment 3. Since this setup is similar to the one from Experiment 1, only the components unique to Experiment 3 have been labeled.

Figure 8: A list of the equipment used in Experiment 3.

Manufacturer	Model	Serial Number	Specifications
PASCO Scientific	Precision Interferometer	OS-9255A	n/a
PASCO Scientific	Laser Alignment Bench	OS-9172	n/a
PASCO Scientific	Rotational Pointer	OS-9256A	n/a
PASCO Scientific	Rotating Table	OS-9256A	n/a
PASCO Scientific	Glass Plate	OS-9256A	thickness: 5 mm

3.4 Procedures

Before beginning, ensure the interferometer is in Michelson mode and that it is properly calibrated. Place the rotating table between the beam-splitter and movable mirror, then attach the glass plate to the magnetic backing of the rotational pointer. Position the rotational pointer so that it is pointing at the zero degree marking on the base of the interferometer.

To ensure the plate is perpendicular to the optical path, remove the lens from in front of the laser and hold the viewing screen in between the glass plate and the movable mirror. Adjust the rotating table, while keeping the rotational pointer in place, until there is one bright dot on the viewing screen. Replace the lens once the plate in calibrated.

Slowly rotate the table by moving the rotational pointer and count the number of fringes passing the reference point as the table is rotated from 0 degrees to at least 10 degrees. In our experiment, sufficiently many fringes were counted up to an angle of 5 degrees.

3.5 Data and Analysis

Figure 9: The total number of fringes counted, N, after the rotational pointer had been rotated θ degrees.

Angle Displacement (θ) (degrees)	Fringes Passed (N)
5 ± 0.5	100

The index of refraction of the glass can be found with Equation 12 where $t = 5\text{mm} \pm 500\,\mu\text{m}$ and $\lambda_0 = 633\,\text{nm}$,

$$n_g = \frac{(2t - N\lambda_0)(1 - \cos(\theta))}{2t(1 - \cos(\theta)) - N\lambda_0}$$
$$n_g = \frac{(2(5 \times 10^{-3}) - (100)(633 \times 10^{-9}))\,(1 - \cos(5^\circ))}{2(5 \times 10^{-3})\,(1 - \cos(5^\circ)) - (100)(633 \times 10^{-9})}$$
$$n_g = 1.50$$

In order to find the error in the index of refraction, δn, Equation 9 can be applied to Equation 12. First, Equation 12 can be rewritten as a more easily differentiable function,

$$n_g(t,\theta) = 3 - 2cos(\theta) - N\lambda_0\frac{1}{t} + N\lambda_0\frac{1}{t}\cos(\theta) - \frac{1}{\cos(\theta)} - \frac{2}{N\lambda_0}t + \frac{2}{N\lambda_0}t\cos(\theta) \tag{13}$$

Equation 13 can then be derived with respect to both t and θ to be later used in Equation 9,

$$\frac{\partial n_g}{\partial t} = N\lambda_0\frac{1}{t^2} - N\lambda_0\frac{1}{t^2}\cos(\theta) - \frac{2}{N\lambda_0} + \frac{2}{N\lambda_0}\cos(\theta)$$

$$\frac{\partial n_g}{\partial t} = 2\sin(5^\circ) - (100)(633\times10^{-9})\frac{1}{(5\times10^{-3})}\sin(5^\circ) - \tan(5^\circ)\sec(5^\circ) - \frac{2}{(100)(633\times10^{-9})}(5\times10^{-3})\sin(5^\circ)$$

$$\frac{\partial n_g}{\partial t} = -123$$

$$\frac{\partial n_g}{\partial \theta} = 2\sin(\theta) - N\lambda_0\frac{1}{t}\sin(\theta) - \tan(\theta)\sec(\theta) - \frac{2}{N\lambda_0}t\sin(\theta)$$

$$\frac{\partial n_g}{\partial \theta} = 2\sin(5^\circ) - (100)(633\times10^{-9})\frac{1}{(5\times10^{-3})}\sin(5^\circ) - \tan(5^\circ)\sec(5^\circ) - \frac{2}{(100)(633\times10^{-9})}(5\times10^{-3})\sin(5^\circ)$$

$$\frac{\partial n_g}{\partial \theta} = -13.7$$

Now, the error of the index of refraction can be found with

$$\delta n_g = n_g \sqrt{\left(\frac{\partial n}{\partial t}\delta t\right)^2 + \left(\frac{\partial n}{\partial \theta}\delta\theta\right)^2}$$

$$\delta n_g = (1.5)\sqrt{\left((-123)(500 \times 10^{-6})\right)^2 + ((-13.7)(0.5))^2}$$

$$\delta n_g = 10.3$$

3.6 Conclusion

The index of refraction of the glass was found to be $n_g = 1.50 \pm 10.3$. Although the value of the index of refraction of the glass is a reasonable value of 1.5, the error is quite unreasonable. This error most likely arose from miscalibration of the interferometer. Since more than sufficiently many fringes were counted long before the recommended minimum angle of 10°, it is probable that something was significantly misaligned.