Rutherford scattering: measuring the scattering rate as a function of the scattering angle and the atomic number

Objects of the experiment

■ To record the direct counting rate N_d of α particles scattered by a gold foil as function of the angle θ.

■ To determine the corrected counting rates N with respect to the scattering distribution in space.

■ To validate the "Rutherford's scattering formula"

Principles

If α-particles are allowed to strike a thin gold foil, they are deflected from their path ("scattering"), each by an angle θ. The majority of α-particles is scattered by angles less than 1° (Fi.g.1).

A few particles, however, show substantially larger scattering angles θ, in the extreme case up to 180° ("back scattering"). These initially qualitative observations can only be explained by assuming that the gold atoms have a very small nucleus, containing practically the whole atomic mass, and being positively charged.

On the basis of this idea Rutherford calculated the angular distribution of the scattering rate $N(\theta)$. The scattering rate is the number of particles which are scattered during the time unit in a determined interval $d\theta$ around an average angle θ. The result of this calculation is "Rutherford's scattering formula":

$$N(\theta) = N_0 \cdot c_F \cdot d_F \frac{Z^2 \cdot e^4}{(8 \pi \varepsilon_0 E_\alpha)^2 \cdot \sin^4(\frac{\theta}{2})} \tag{I}$$

N_0: particle rate in the foil

c_F: atomic concentration in the foil

d_F: thickness of the foil

Z: nuclear charge number of the scattering material

E_α: energy of the α-particles

e: elementary charge (e = 1.6021×10^{-19} As)

ε_0: dielectric constant   ($\varepsilon_0 = 8.8524 \times 10^{-12}$ As/Vm)

Recording the scattering rate as function of the angle

In this case we need not care about the proportionality factors in equation (I) which are kept constant in our experiment. The relevant shape of this angular distribution curve is described by the function:

$$f(\theta) = \frac{1}{\cdot \sin^4(\frac{\theta}{2})} \tag{II}$$

Fig. 1: Scattering of α-particles on a monolayer of atoms.
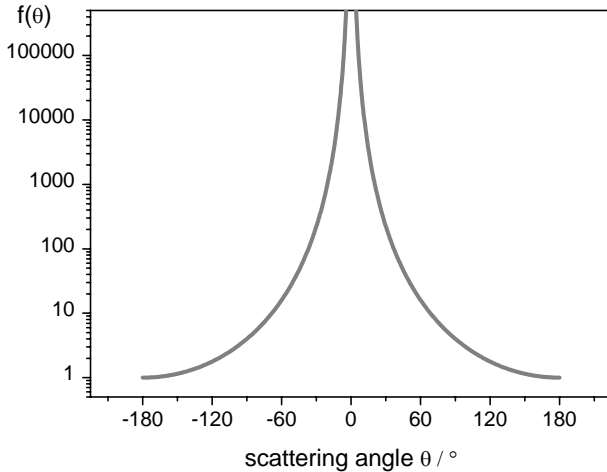

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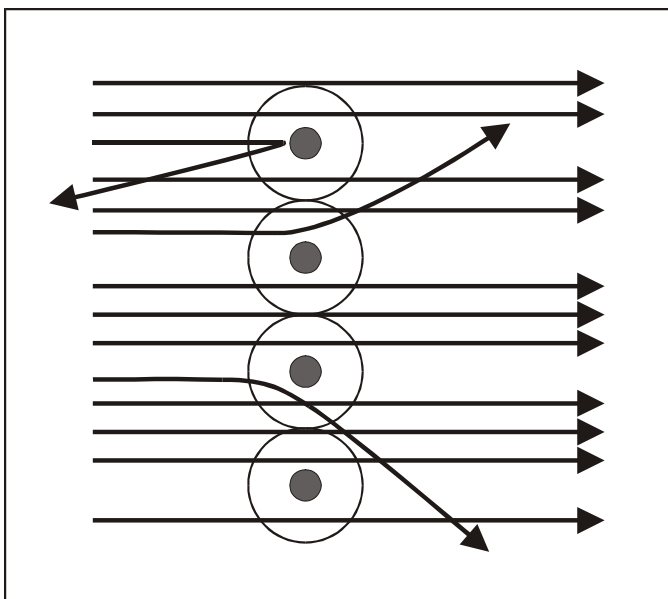
Fig. 2: Theoretical slope of the scattering rate according equation (II).

The values of $f(\theta)$ decrease rapidly with increasing scattering angle θ. Hence, in Fig. 2 the graphical representation of $f(\theta)$ is plotted in a logarithmic scale over θ. A singularity spot becomes visible at $\theta = 0°$. Therefore we will compare measuring results with the theoretical slope only outside of this region, i.e. for values $|\theta| > 5°$.

As for higher scattering angles θ the counting rates become very small the gate times $t(\theta)$ for determining the counting rate $N(\theta)$ have to be increased with increasing angle θ to obtain an acceptable accuracy. For keeping the total measuring time sustainable the angular range can be restricted to $|\theta| \le 30°$.
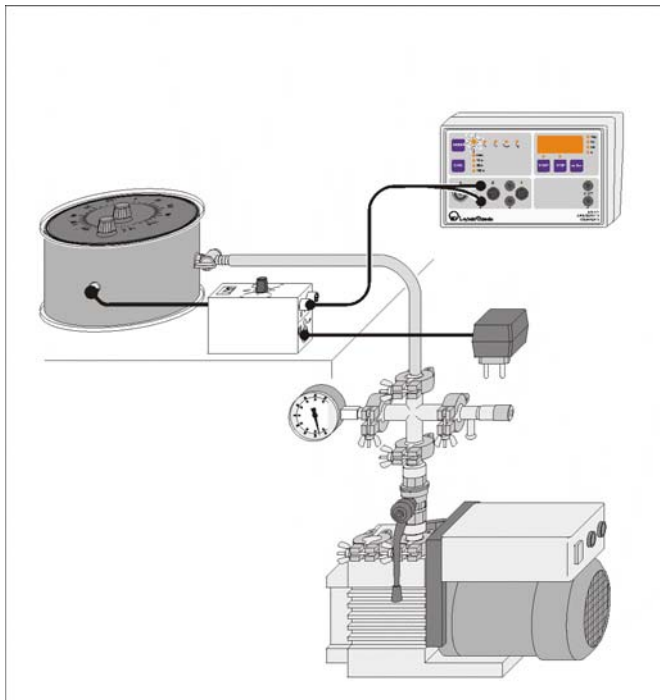

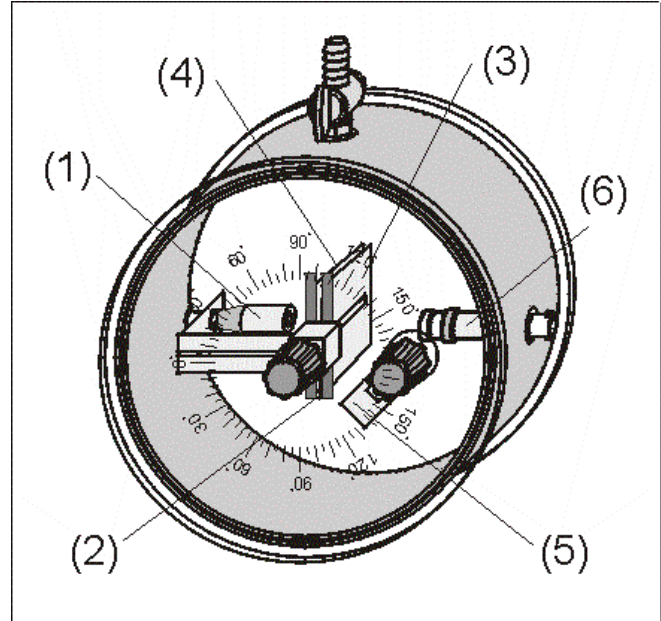Fig. 3: Experimental setup schematically for the Rutherford Scattering Experiment.

Fig. 4: The scattering chamber
(1) Preparation   (4) Gold foil
(2) Holder   (5) swivel arm
(3) Slit   (6) detector

Because of the very low range of α-particles in the air this experiment must be carried out in a closed chamber under vacuum (Fig. 3). Fig. 4 shows the arrangement of the components on the lid of the scattering chamber. Fig. 5 shows the geometry of the experiment.

The α-particles emitted from the Am-241 preparation fall through a slit aperture of 5 mm width onto the gold foil and leave this gold foil with various scattering angles. The scattered α-particles are identified with a semiconductor detector. By swinging the detector in steps of 5°, for example, the scattering rate can be determined for all scattering angles from 5° to 60°. With the setup we are going to use, the detector is not swung but rather the preparation, slit and gold foil, which are attached on a common swivel arm. The detector is firmly attached to the side wall of the chamber.

Determining the nuclear charge number of aluminium

If we compare the scattering rates between two different foil materials (e.g. Au and Al) at the same angle θ, we can derive from the scattering formula (I):

$$\frac{N_{Au}}{N_{Al}} = \frac{c_{Au}\, d_{Au}\, Z_{Au}^2}{c_{Al}\, d_{Al}\, Z_{Al}^2} \tag{III}$$

Hence the nuclear charge number of aluminium Z_{Al} can be determined by scattering experiments as following:

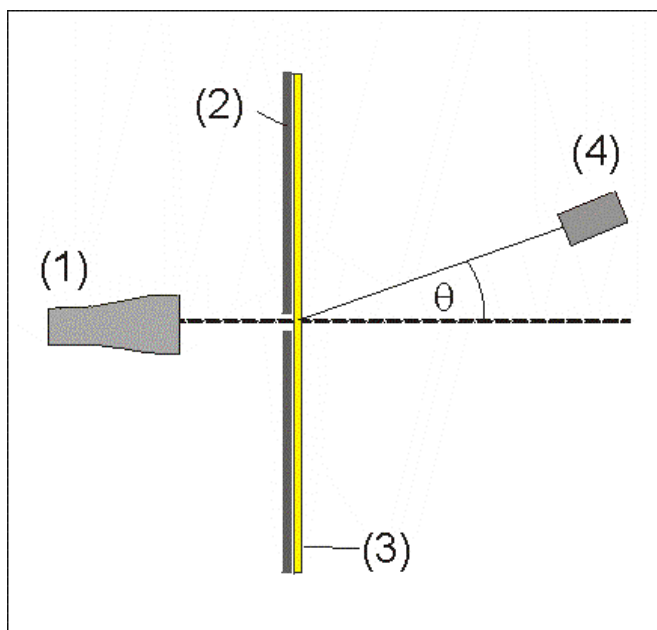$$Z_{Al} = \sqrt{\frac{N_{Al}(\theta)\, c_{Au}\, d_{Au}\, Z_{Au}^2}{N_{Au}(\theta)\, c_{Al}\, d_{Al}}} \tag{IV}$$

Fig. 5: The scattering geometry:
   (1) preparation
   (2) collimator slit
   (3) gold foil
   (4).detector

Apparatus

1 Scattering chamber after Rutherford	559 56
1 Aluminium foil in frame	559 52
1 Vacuum pump S 1.5, 230 V, 50 Hz	378 73
1 T-Piece DN 16 KF	378 005
1 Centering ring (adapter) DN 10/16 KF	378 040
1 Centering ring DN 16 KF	378 045
2 Clamping ring DN 10/16 KF	378 050
1 Air inlet valve with DN 10 KF	378 771
1 Small flange DN 16 with hose nozzle	378 031
1 Vacuum rubber tubing	667 186
1 Discriminator preamplifier	559 931
1 Counter S	575 471
1 Plug-in power supply unit 12 V AC	562 791
1 Am-241 preparation, 333 kBq	559 82
1 HF cable, 25 cm	501 01
1 Measuring cable BNC / 4-mm	575 24

Safety notes

■ Never touch the gold foil!

■ Venting of the chamber after the experiment has to be done very carefully (see operating instructions 559 56, section 3.4), otherwise you may destroy the sensitive gold foil.

Setup

Preparing the scattering chamber:

Vent the scattering chamber (when using at first time) and take off the lid (Fig. 4).

Insert the Am-241 preparation into the 4-mm socket of the main swivel arm down to the stop. Place the 5-mm slit aper-

ture and the plastic sheet containing the gold foil on top of one another (with the gold foil between them) and insert them both into the holder so that the slit points towards the preparation. Move the small swivel arm (not used here) close to the chamber's side wall so that the measurements are not disturbed.

Make sure that the detector is fixed on the BNC socket at the inner wall of the chamber, and that the detector's aperture slit is perpendicular to the lid (with the mark at the top). Fit vacuum tubing to the hose nozzle. Close the chamber by placing its lid onto the housing. Ensure the correct positioning by positioning pin in borehole. Then evacuate the chamber. Firmly press lid on, if necessary.

Preparing the electrical counting components:

Connect the scattering chamber, discriminator preamplifier and counter S as shown in Fig. 3. Set discriminator to zero (turn potentiometer fully anticlockwise).

Carrying out the experiment

a) Recording scattering rate as function of the angle

- Prepare the counter S for pulse counting by pressing the push button MODE to activate $N_{A,E}$.

- Select gate time $t(\theta) = 100$ s by pressing the toggle button GATE three times
  Note: $t(\theta) = 100$ s is useful for small angles, i.e. angle up to +/ 15°. By pressing GATE + MODE, longer gate times are can be adjusted, i.e. up to 9999 s (MODE upwards, GATE downwards).

Important: *During measurements protect the sensitive detector from light (e.g. especially from fluorescent light of ceiling lamps)! If necessary, cover the scattering chamber during measurements with a black cloth.*

- Count at least 50 particles (n(θ) >50) at the angles:

  θ = +/-5°, +/-10°, +/-15°, +/-20°, +/-25° and +/-30°

Note: It is recommended to measure symmetrically to $\theta = 0$ to detect unwanted misalignments in the setup.

- Use gate times t(θ) as given in the Table 1. Repeat counting several times for all angles (with exception of +/-5°).

Measuring example

Table. 1: Measured values (with gold foil and slit d = 5 mm):

Angle	Gate time	Pulse counts (single values)	Pulse counts (mean value)	Counting rate (directly)	Counting rate (space corrected)
θ in °	$t(\theta)$ in s	$n(\theta)$	$n_m(\theta)$	$N_d(\theta)$ in 1/s	$N(\theta)$ in 1/s
-30	900	77 99 76 69 81	80,4	0,0893	0,281
-25	600	119 118 150 107 153	129,4	0,216	0,573
-20	200	131 114 118 123 131	123,4	0,617	1,33
-15	100	255 256 255 298 263	265,4	2,65	4,32
-10	100	1772 1731 1790 1825 1730	1769,6	17,7	19,3
-5	100	4641	4641	(46,4)	
+5	100	5608	5608	(56,1)	
+10	100	4047 3983 4146 4123 4107	4081,2	40,8	44,5
+15	100	1000 1036 968 997 1009	1002,0	10,0	16,3
+20	200	278 273 276 284 289	280,0	1,40	3,01
+25	600	263 252 274 250 269	261,6	0,436	1,16
+30	900	147 161 148 158 155	153,8	0,171	0,537

Evaluation and results

a) Recording the scattering rate as function of the angle

After recording the pulse counts $n(\theta)$ the mean values $n_m(\theta)$ can be determined. Using the mean values $n_m(\theta)$ the scattering rates $N_d(\theta)$ are calculated by:

$$N_d(\theta) = \frac{n_m(\theta)}{t(\theta)} \tag{V}$$

These measuring results $N_d(\theta)$ are typical for a plane scattering geometry which is given by the transparent construction of the chamber used in this experiment.

The theoretical function (according to Rutherford's formula), however, is related to a three-dimensional geometry. The relation between these different aspects can considered by the following concept (Fig. 6):

Each plane angle θ corresponds in space to a cone with an aperture of $2 \cdot \theta$ (produced by rotation of the plane structure around the incident beam axis). In the same way the plane angular differential $d\theta$ corresponds in three dimensions to a spatial angular differential $d\Omega$ given by:

$$d\Omega = 2 \cdot \pi \cdot \sin(\theta)\, d\theta \tag{VI}$$

This geometrical corrections allows to derive a relation between the plane scattering rate $N \cdot d(\theta)$ and the spatial scattering rate $N(\theta)$:

$$N(\theta) = 2 \cdot \pi \cdot \sin(\theta) \cdot N\, d(\theta) \tag{VII}$$

Finally, the corresponding spatial values $N(\theta)$ are calculated (Table. 1) and the space corrected values plotted in a diagram (Fig. 7).

The measuring value pairs $\{\theta\, /\, N(\theta)\}$ can be compared with the shape of the theoretical curve of equation (II):

$$f(\theta) = \frac{A}{\cdot \sin^4(\frac{\theta - B}{2})} \tag{VIII}$$

The proportionality factor A represents a vertical shift (at logarithmic scale). The coefficient B is representing a small displacement along the horizontal angular scale.

Note: A small inaccuracy of the collimator-slit adjustment or non centric distribution of the radiation, coming from the preparation in the holder, may cause a shift of the curve along the horizontal axis (angle shift < 3°).

Due to such effects it is useful to record scattering rates as well in the positive as in the negative angular range, to get information of both branches with respect to an accurate determination of the symmetry-axis displacement.
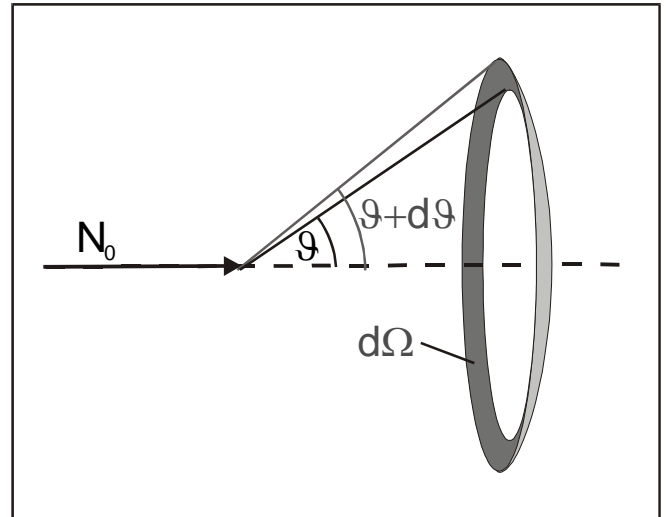

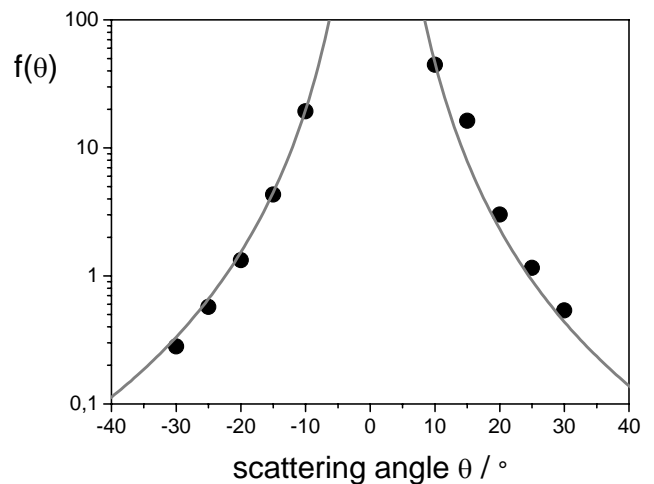Fig. 6: The α-particles are scattered into the angular region $\vartheta + d\vartheta$.

Fig. 7: Results (circles) with a shifted fitting curve (grey) according to equation (VIII) for A = 0.00171 and B = 1.04.

b) Determining the nuclear charge number of aluminium

Typical results for the counting rates at gold and silver are:

$N_{Au}(10°) = 0{,}301$ 1/s

$N_{Al}(10°) = 0{,}027$ 1/s

With $d_{Au} = 2\ \mu m$, $d_{Al} = 7\ \mu m$, $c_{Au} \approx c_{Al}$ and $Z_{Au} = 79$ we obtain with equation (IV):

$Z_{Al} = 12{,}7$ (exact value: $Z_{Al} = 13$).