

# PHY 4210-01 Senior Lab

## Lab P2: Electron Spin Resonance

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### **Abstract**

The Lande factor,  $g_s$ , (or the gyromagnetic ratio of spin) for the electron was determined through the use of electron spin resonance and Helmholtz coils. The g-factor of a diphenyl-picryl-hydrazyl (DPPH) sample was obtained following the measurement of the frequency dependence of the resonance field. The line width of the resonance signal was then calculated.

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# 1 Data Analysis

## 1.1 Frequency Dependence of Resonance Field

Voltage was compared to frequency to obtain a graphical relationship for the frequency dependence of the resonance field. The amplitude voltage was obtained by measuring the peak-to-peak voltage from the oscilloscope and dividing it in half. The peak of ?? is the specific resonance frequency for the field. This value is a voltage amplitude of 1.01 V and a frequency of  $4.18 \times 10^7$  Hz. It is important to note that the electron spin resonance device divides the frequency by a factor of a thousand, and thus the Hewlett-Packard frequency counter displayed a corrected frequency. The calculations require an uncorrected frequency, i.e. the counter's frequency multiplied by one thousand.

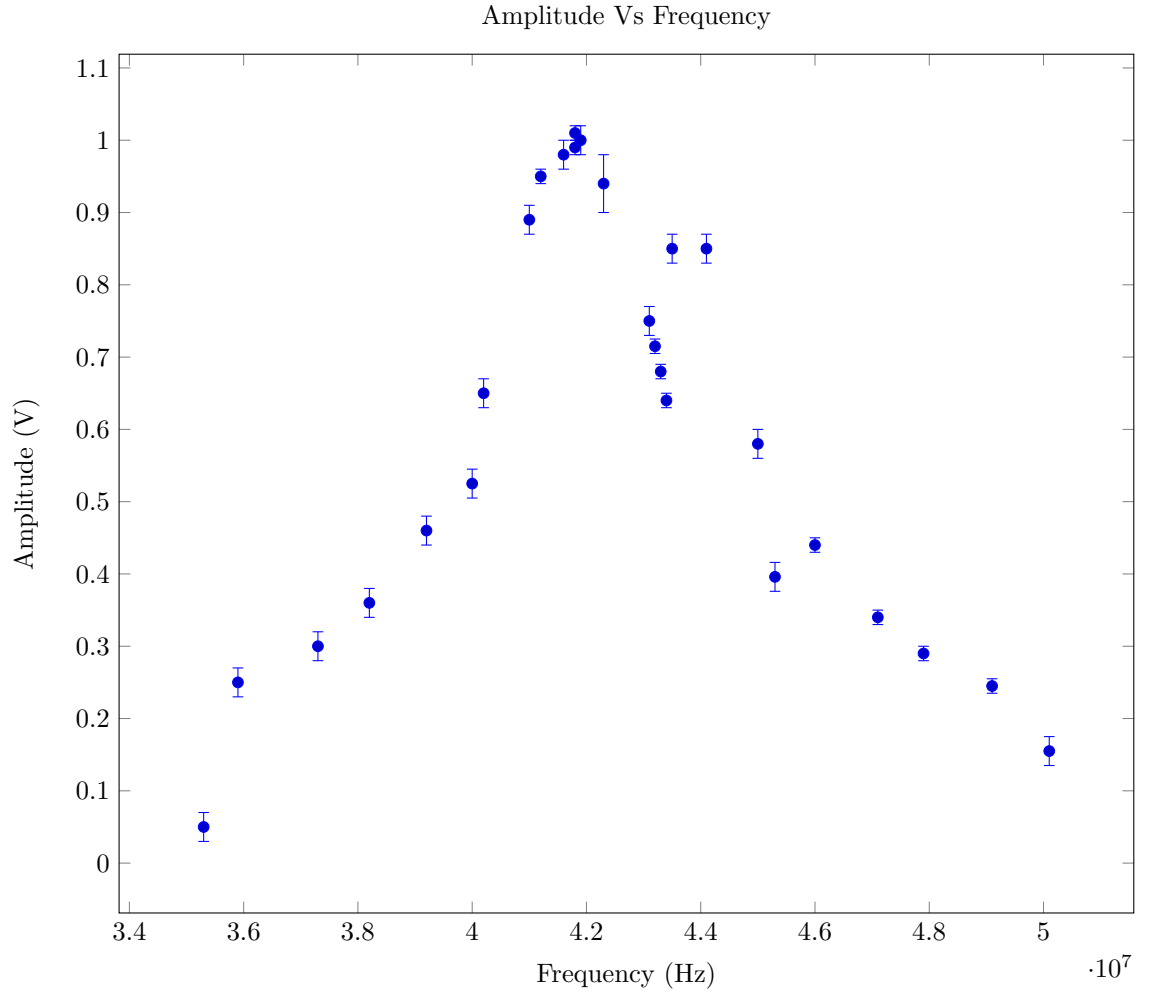


Figure 1: Graphic representation of the frequency dependence of the resonance field.

## 1.2 Propagation of Uncertainty in the Frequency Dependence of the Resonance Field

### 1.3 Experimental Value of Gyromagnetic Ratio

The gyromagnetic ratio is calculated using the following equation, where  $\nu$  is the frequency,  $h$  is Planck's constant,  $\mu_B$  is the Bohr magneton, and  $B_0$  is the magnetic field strength.

$$g_s = \frac{h \times \nu}{\mu_B \times B_0} \quad (1)$$

The magnetic field used in calculating equation 1 must be calculated as well. It is determined from the measured current using equation 2, where  $\mu_0 = 4\pi \times 10^{-7} \frac{Vs}{Am}$ , the number of turns is  $n = 320$ , and the radius of the coils is  $r = 6.8cm$ .

$$B_0 = \mu_0 \left(\frac{4}{5}\right)^{3/2} \times \frac{n}{r} \times I \quad (2)$$

Rather than measuring the current directly, the current is calculated by measuring the voltage drop across a resistor, of which the resistance is also measured. This calculation is shown below in equation.

$$I = \frac{V}{R} \quad (3)$$

By substituting equation 3 into 2, and then substituting equation 2 into equation 1, we arrive at an expression for the gyromagnetic ratio in terms of known constants and measured quantities. This final expression is shown in equation 4.

$$g_s = \frac{h \times \nu}{\mu_B \times \left(\mu_0 \left(\frac{4}{5}\right)^{3/2} \times \frac{n}{r} \times \frac{V}{R}\right)} \quad (4)$$

### 1.4 Propagating Uncertainty in Gyromagnetic Ratio

The error in the experimental value of the gyromagnetic ratio is determined by propagating uncertainty in equation 4. There are no uncertainties associated with fundamental constants such as  $h$ ,  $\mu_B$ , and  $\mu_0$ . It is assumed that the number of coil turns,  $n$ , also has no associated uncertainty because it was reported in the manual as such. The uncertainty in the radius is constant for all measurements, but the frequency, voltage, and resistance will differ for each measurement. Equation 5 shows this error propagation.

$$\delta g_s = g_s \times \sqrt{\left(\frac{\delta \nu}{\nu}\right)^2 + \left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta R}{R}\right)^2} \quad (5)$$

An example calculation for the value of  $g_s$  and its propagated uncertainty is shown below for a measurement taken with the large coil:

$$\begin{aligned} g_s &= \frac{6.626 \times 10^{-34} \times (3 \times 10^7)}{\mu_B \times \left(4\pi \times 10^{-7} \left(\frac{4}{5}\right)^{3/2} \times \frac{320}{0.068} \times \frac{0.44}{1.7}\right)} \\ &= 1.93 \end{aligned}$$

$$\begin{aligned}
\delta g_s &= g_s \times \sqrt{\left(\frac{1.00 \times 10^4 \text{Hz}}{3.00 \times 10^7 \text{Hz}}\right)^2 + \left(\frac{0.5 \text{cm}}{6.7 \text{cm}}\right)^2 + \left(\frac{0.1 \text{V}}{2 \text{V}}\right)^2 + \left(\frac{0.1 \Omega}{1.7 \Omega}\right)^2} \\
&= 1.93 \times \sqrt{(3.3 \times 10^{-4})^2 + (0.006)^2 + (0.05)^2 + (0.06)^2} \\
&= .15
\end{aligned}$$

We can calculate the discrepancy between the experimental and theoretical values as follows. Recall the theoretical value of  $g_s$  for DPPH is 2.0036.

$$\begin{aligned}
\Delta g_s &= |g_{s_{exp}} - g_{s_{theo}}| \\
&= |1.93 - 2.0036| \\
&= 0.074
\end{aligned}$$

Evidently, this difference  $\Delta g_s$  is less than  $1\sigma = .15$ .

## 1.5 Rejection of Data

## 1.6 Determining Line Width of Resonance Signal

$\delta B_0$  is representative of an absorption line, and is obtained when the energy is measured at a fixed frequency as function of the magnetic field. The line width  $\delta B_0$  is used as an expression of the uncertainty in the energy of the transition. This is best represented by the equation  $\delta E = g \times \mu_0 \times \delta B_0$ . Using the uncertainty principle a relation is then found for  $\delta B_0$ .

$$\delta B_0 = \frac{\hbar}{2 \times g_J \times \mu_B \times T}$$

where T is the lifetime of the level and  $g_J$  is the Land  $g$  factor. Experimentally  $\delta B_0$  can be determined by the following equation:

$$\delta B_0 = B \times \left(\frac{\delta I}{I_{mod}}\right)$$

where  $\delta I$  is represented as  $\frac{\delta U}{U_{mod}} \times I_{mod} \times 2\sqrt{2}$ .

$$\begin{aligned}
\delta I &= \frac{\delta U}{U_{mod}} \times I_{mod} \times 2\sqrt{2} \\
&= \frac{0.55}{2} \times 0.156 \times 2\sqrt{2} \\
&= 0.121
\end{aligned}$$

$$\begin{aligned}
\delta B_0 &= B \times \left( \frac{\delta I}{I_{mod}} \right) \\
&= 6.23 \times 10^{-4} \times \left( \frac{0.121}{0.156} \right) \\
&= 4.85 \times 10^{-4} T \\
&= 4.85 \times 10^{-1} mT
\end{aligned}$$

## 2 Results

### 2.1 Discrepancy in the Resonance Field

### 2.2 Discrepancy in Gyromagnetic Ratio

### 2.3 Discrepancy in Line Width

## 3 Sources of Error

## 4 Conclusion

## 5 Appendices

### 5.1 Appendix A: Data

### 5.2 Appendix B: Source Code