

PHY 4210-01 Senior Lab  
Lab M-1: Magnetic Field Mapping

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**Abstract**

In this experiment the magnetic field inside a Helmholtz coil was measured and compared to theoretical calculations determined from the Smythe derivation of the Biot-Sarvat Law for a plane displaced from the central axis, with coordinates  $z$ ,  $\rho$ , and  $\phi$ . When determining the magnetic field inside a Helmholtz coil, a Hall probe is used to obtain the magnitude of the magnetic field at varying positions inside the coil. Theoretically the axial component of the magnetic field that is produced inside the Helmholtz coil is, to some extent, of uniform magnitude.

## Contents

# 1 Objective of the Experiment

During this lab, the number of turns of wire inside a Helmholtz coil was determined for use in theoretical calculations. Then a 3-dimensional and 2-dimensional mapping of the magnetic field inside the Helmholtz coil was created in order to investigate the presence of a uniform field, running along its axial direction.

# 2 Theory of the Experiment

Recall for a straight current-carrying wire, circular magnetic field lines are generated around the wire in accordance with the curling right-hand rule. The Helmholtz coil contains two regions of circularly wound wires. Due to the the circular symmetry, all components of each infinitesimal segment of the wire will cancel *except* for that in the axial direction. In summary, a circular current produces a linear magnetic field.

The field point of the system has before been typically placed along the axis of the direction of the magnetic field, we will call this the z-direction. This was due to the ease of solving the Biot-Savart Law under these simple conditions, as the direction and strength of the magnetic field will follow along the z-axis of the system, which is where the field point is placed. When this is applied to the co-axial coils of the Helmholtz apparatus the evaluation of the Biot-Savart Law becomes too trivial. One then chooses the field point to be placed off of the z-axis as more information about the magnetic field of the coils can be determined. This is the more general scenario and thus more complex. The off axis form can be used for any point that is off of the z-axis, while the on axis is a specific and simplified form of the general case. The general form is best represented by Smythe's derivation of the Biot-Savart Law.

$$B_z = \frac{\mu_0 IN}{2\pi} \left[ \frac{1}{\sqrt{(a+\rho)^2 + (a-z)^2}} \left[ K_1 + \left( \frac{a^2 - \rho^2 - (a-z)^2}{(a-\rho)^2 + (a-z)^2} \right) E_1 \right] \right. \\ \left. + \frac{1}{\sqrt{(a+\rho)^2 + z^2}} \left[ K_2 + \left( \frac{a^2 - \rho^2 - z^2}{(a-\rho)^2 + z^2} \right) E_2 \right] \right]$$

# 3 Equipment Utilized

## 3.1 The Helmholtz coil

The Helmholtz coil consists of two concentric sets of coils, each with the same radius and separated by a distance equal to their radius. This configuration allows the contribution of each set of coils to produce a uniform field in the center of the coils. The current in each set of coils must be oriented in a particular direction so that their contributions constructively interfere. The circuit is shown in figure ??.

Figure 1: Flow of current through the Helmholtz coil, oriented such that the produced fields are constructive.

### 3.2 The Hall Effect Probe

A DC Gaussmeter (AlphaLab Model GM-1-HS) was connected to a Hall Effect Probe in order to measure the field strength inside the Helmholtz coil. The Hall Effect Probe contains a semiconductor junction that, when exposed to a magnetic field, produces a voltage proportional to the field strength.

### 3.3 Position Controls

The position of the Hall Effect Probe can be modified in the  $\rho$  direction by sliding the ruler bar through the acrylic cube shown in figure ???. The position can be modified in the  $\phi$  direction by rotation the ruler bar about the central pole. However, for the sake of this experiment, this did not have to be modified because measurements were taken in a single  $\rho, z$  plane. The  $z$  coordinate was modified by sliding the acrylic cube and ruler bar up and down the central pole.

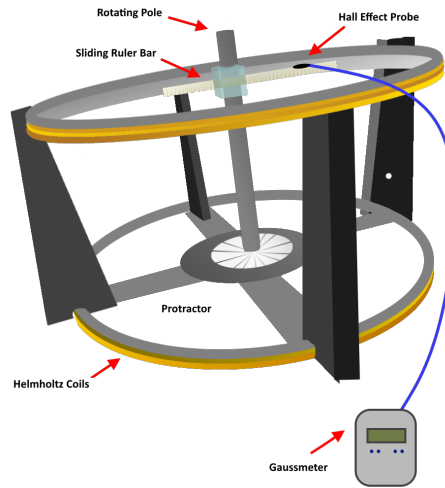


Figure 2: Two concentric Helmholtz coils separated by a distance equal to their radius. Rotating pole and sliding ruler allow for modification of the probe's position.

## 4 Procedure

Note that, per suggestion of the laboratory manual, the procedural steps of this experiment have been omitted. The discussion section provides sufficient detail on what actions were taken.

### 4.1 Measuring the External Field

The Helmholtz coil is oriented such that the Earth's magnetic field is parallel to the  $z$ -axis of the coils. This allows us to produce an applied magnetic field that is exactly anti-parallel to the Earth's field. From there, we can compute the applied field by subtracting the Earth's field from the total resultant field.

Note that there was an apparent offset in the Gaussmeter reading, as the 0.36G measurement for Earth's field was consistently higher than the expected value for Earth's field of 0.24G. However, if there truly exists such an offset in the measurement device, it would appear in both the measurement of Earth's field and in the measurement of the total field inside the Helmholtz coil. Subtracting these two to obtain the strength of the applied field would cancel any contribution from such an offset.

## 4.2 Procedural Modifications

Upon initial inspection of the equipment, it appeared the center pole running along the z axis of the Helmholtz coil was misaligned. In order to mitigate this error and ensure that coordinates were modified independently, a chord was used to realign the pole as closely as possible to the true z axis. However, since this alignment was not quantified, it is possible that there the pole is misaligned to some degree. This would result in a systematic error intrinsic to the experimental set-up. If the pole deviates from the z-axis, the experimentally recorded z-values are underestimated, causing the experimental field strengths to trend lower than the theoretical field strengths.

The majority of field strength measurements collected for the 3-dimensional mapping were taken on the same day of experimentation. After resuming this data collection on the next day, the values appeared to be systematically higher. Possible causes of this offset were investigated. Before taking measurements and intermittently during the data collection, the hall effect probe was zeroed and observed with the power supply off in order to ensure a consistent reading of the Earth's magnetic field. The reference measurement taken at the start of this lab session was similar to those taken during the previous session (zeroed field measurements were between 0.36G and 0.4G on both days), so a discrepancy in the Earth's field strength measurement was eliminated as the source of this error. Note that any small variation in the Earth's field measurement could be due to misalignment of the probe (a systematic error in measurement that would under-report the field strength) or simply a random error in measurement due to the limited performance of the probe.

An ammeter was also used to ensure a 2A current was consistently applied on both days of data collection, thus a change in the applied current was eliminated as a source of error. Because the source of this error was ultimately not determined and eliminated, the effect had to be compensated for with a procedural modification. In order to recreate the data points from the previous lab session, the current from the power supply was modified until the field strength matched previous measurements in several locations. This ultimately required lowering the applied current from 2000mA to 1790mA.

Upon further investigation, it appeared the current from the power supply was unstable, as it would decrease and increase every few minutes. This produced a source of random intrinsic error, which was mitigated by fine tuning the current value before each measurement after the issue was discovered.

## 4.3 Additional Sources of Error

Because the experimental set-up was restricted to a small area, the contribution from the field produced by the power supply may be non-negligible. From the

perspective of the experimenter, the power supply sits behind and to the right of the Helmholtz coil. Therefore, by the curling right hand rule, this would produce an upward magnetic field on the side of the wire nearest the Helmholtz coil. This would produce a systematic intrinsic error that causes the external field measurements to be overestimated. Similarly, the power supply itself may be producing a small field that could also contribute a systematic error, although the exact effect could not be determined without knowing the orientation of such a field.

## 5 Data Analysis

### 5.1 Calculating Supply Voltage

Using a multimeter, the resistance of a set of coils was measured to be  $3.4\Omega$ . In order to determine the necessary voltage to send 3A of current through the coils, we make a simply calculation using Ohm's law.

$$\begin{aligned} V &= IR \\ &= (3A)(3.4\Omega) \\ &= 10.2V \end{aligned}$$

### 5.2 Determining the Number of Turns in a Coil

Further calculations will require knowing the number of turns of wire in each set of coils. MORE TEXT TO COME HERE

## 6 Results

### 6.1 Comparing the directions of the Magnetic Field

When measuring at a probe height of  $a/2$  (16cm), where 'a' is the separation distance between the coils, the strength of the magnetic field in the 'z' direction was measured to be -3.13 Gauss. When measuring the magnetic field in the 'z' direction at a probe height of 5cm, the magnetic field strength was measured to be -3.28 Gauss. These results follow with the theory as it is expected that the magnetic field is propagated in the 'z' direction. The measured magnetic field strength for the  $\rho$  direction was -0.46 and -0.05 Gauss for a probe height of 16cm and 5cm respectively. The measured magnetic field strength for the  $\phi$  direction was -0.51 and -0.31 Gauss for a probe height of 16cm and 5cm respectively. For a probe height of 16cm the percentage for the magnitude of the magnetic field that is measured to be in the  $\rho$  direction is 14% while the percentage for the magnitude of the magnetic field that is measured to be in the  $\phi$  direction is 16%. For a probe height of 5cm the percentage for the magnitude of the magnetic field that is measured to be in the  $\rho$  direction is 1% while the percentage for the magnitude of the magnetic field that is measured to be in the  $\phi$  direction is 9%. The magnetic field produced by the Helmholtz coils should be directed along the 'z' axis. These small measured values follow the

aforementioned theory and we can determine that the magnetic field produced by the Helmholtz coil is indeed axial. Furthermore, we can determine that the magnetic field is axial along the 'z' direction.

## **7 Conclusion**