Experiment M - 1
Magnetic Field Mapping of a Helmholtz Coil

Introduction

The purpose of this experiment is to measure the magnetic field inside a large Helmholtz coil and determine to what extent the coil produces a region of uniform axial magnetic field in its interior. The Concepts section should be read before the student reads the main theory reference, which begins on page 270 of "Static & Dynamic Electricity" by William R. Smythe, McGraw-Hill Books, New York (1950).

Concepts

Electrical currents produce magnetic fields. Long, straight currents generate circular magnetic field lines and circular currents produce straight magnetic field lines. This is a type of yin and yang for magnetism. The problem of calculating the magnetic field on the axis of a circular current loop is a familiar problem in introductory and intermediate courses in electromagnetism. See Figure 1.

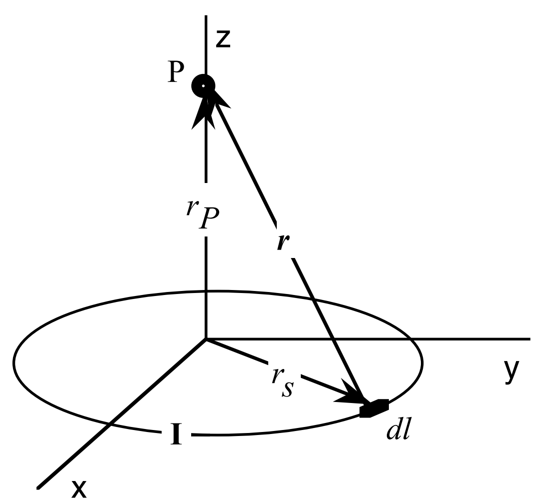

Figure 1. Circular coil of wire carrying current lies in the x-y plane.

Traditionally, the starting point is the law of Biot and Savart, Equation (1). The task is to use vector algebra to convert the geometry from cylindrical coordinates (ρ, z, ϕ) into Cartesian (x, y, z) and then integrate the effect of the source current.

$$\vec{B} = \frac{\mu_0}{4\pi}\oint \frac{Id\vec{l} \times \vec{r}}{r^3} \tag{1}$$

From Figure 1, the position vector for the field point P is $\vec{r}_P = z\hat{z}$ while the position vector for the source is $\vec{r}_s = x\hat{x} + y\hat{y}$. The r-vector points from the current element toward the field point. Hence, $\vec{r} = \vec{r}_P - \vec{r}_s$. The differential current element Idl points in the phi-hat direction so $d\vec{l} = a\, d\phi\, \hat{\phi}$. From the definition of cylindrical coordinates, $\hat{\phi} = (-\sin\phi\, \hat{x} + \cos\phi\, \hat{y})$. The integration of this term with respect to dϕ yields zero. There remains a phi integration of a function of z, which yields 2π multiplied by the function. The result is given in Equation (2).

$$\vec{B}(z)=\frac{\mu_0 I a^2}{2}\frac{1}{\left(a^2+z^2\right)^{3/2}}\hat{z} \tag{2}$$

A Helmholtz Coil is a pair of identical co-axial circular loops of wire, which carry current in the same direction. A distance equal to the coil radius separates the coils. This creates a region of relatively uniform magnetic field inside the cylinder defined by the two coils. The magnetic field on the axis of a Helmholtz coil is the sum of two versions of Eqn. (2)..

$$\vec{B}(z)=\frac{\mu_0 I a^2}{2}\left[\frac{1}{\left(a^2+z^2\right)^{3/2}}+\frac{1}{\left(a^2+(a-z)^2\right)^{3/2}}\right]\hat{z} \tag{3}$$

Choosing a field point on the coil's axis makes the integral in Eqn. 1 trivial to evaluate. However, when the field point is *off* the axis the geometry and ensuing integration becomes much more complex.

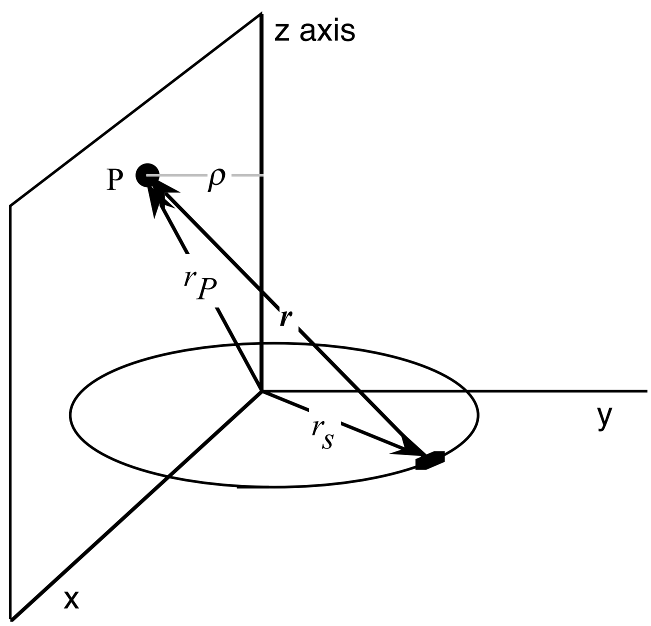

Figure 2. Geometry for mapping field points in a plane.

In his book, *Static and Dynamic Electricity*, William R. Smythe presents a derivation for an equation of the off-axis magnetic field due to a single loop of wire carrying a current, I. The single-turn coil lies in the x-y plane. The student must examine this reference (copies are available either from the instructor or in the experiment file) and write a short summary of how Smythe obtained his equation.

Extending Smythe's work for two sets of coils, each containing N turns of wire one obtains the following for the z component of the off-axis magnetic field in a Helmholtz coil.

$$B_z = \frac{\mu_0 IN}{2\pi}\left\{\frac{1}{\sqrt{(a+\rho)^2+(a-z)^2}}\left[K_1+\left(\frac{a^2-\rho^2-(a-z)^2}{(a-\rho)^2+(a-z)^2}\right)E_1\right]+\right.$$
$$\left.\frac{1}{\sqrt{(a+\rho)^2+z^2}}\left[K_2+\left(\frac{a^2-\rho^2-z^2}{(a-\rho)^2+z^2}\right)E_2\right]\right\} \tag{4}$$

Here, K and E are complete elliptic integrals of the first and second kind respectively. The subscript 1 refers to the top coil at z = a. The subscript 2 refers to the bottom coil at z = 0. These integrals cannot be evaluated analytically and must be computed numerically. They are defined as follows where $j = 1$ or 2.

$$K_j = \int_0^{\pi/2}\frac{d\theta}{\sqrt{1-k_j^2\sin^2(\theta)}} \quad \text{and} \quad E_j = \int_0^{\pi/2}\sqrt{1-k_j^2\sin^2(\theta)}\,d\theta \tag{5,6}$$

Note that upper and lower case K's are used. Unfortunately, this is the standard nomenclature. Also, theta is a generic integration variable and is not related to any coordinates. For this experiment the lower case k's are the following collection of variables.

$$k_1 = \sqrt{\frac{4a\rho}{(a+\rho)^2+(a-z)^2}} \quad \text{and} \quad k_2 = \sqrt{\frac{4a\rho}{(a+\rho)^2+z^2}} \tag{7,8}$$

The student should feel free to evaluate these integrals any way he or she desires. Well, perhaps *desire* is too strong a word. Suffice it say, you may use any computational tool you choose. This includes using the following series expansions.

$$K = \frac{\pi}{2}\left\{1+\left(\frac{1}{2}\right)^2 k^2+\left(\frac{1\times 3}{2\times 4}\right)^2 k^4+\left(\frac{1\times 3\times 5}{2\times 4\times 6}\right)^2 k^6+\ldots\right\} \tag{9}$$

$$E = \frac{\pi}{2}\left\{1-\left(\frac{1}{2}\right)^2 k^2-\left(\frac{1\times 3}{2\times 4}\right)^2 \frac{k^4}{3}-\left(\frac{1\times 3\times 5}{2\times 4\times 6}\right)^2 \frac{k^6}{5}-\ldots\right\} \tag{10}$$

Obviously, performing the theoretical calculations, is the major hurdle in writing the report for this experiment. The student is advised to avoid procrastinating, as this will consume more time than you first estimate. Of course, the word *theoretical* must be taken with a grain of salt since these calculations will be based on an experimentally measured current.

Method

It is a good idea to spend some time thinking about the target precision for this experiment. This may help you avoid some unnecessary work. For example the aluminum and acrylic posts permit measurements of z and ρ to some level of precision. Calculate this precision and compare it to the ammeter reading. Is the current reading of a similar precision?

As of this writing, the magnetic field is measured with a relatively new digital gaussmeter. A transverse Hall Effect probe connects to this meter and a small black cavity is provided to shield the probe and then zero the meter. One unfortunate aspect of this gaussmeter is its resolution, which is ± 0.1 G. This is a large fraction of the total magnetic field of the Earth and so it would represent a large percent error if the applied field is kept in the neighborhood of the Earth's field. (This was the procedure in the past when an older but more sensitive gaussmeter was used.) Before the mapping procedures, spend time determining whether any current variations correspond to changes in the field that are significantly larger than this precision.

The Hall Effect is described in most Modern Physics textbooks. In brief, if you expose a junction of two semiconductors to an applied magnetic field, a voltage will develop across the junction. This voltage is directly proportional to the strength of the magnetic field. The Hall voltage will then cause a very small current to flow. At that point, most Gaussmeters and Teslameters amplify this weak signal and display a reading that has been calibrated at the factory.

Modern Gaussmeters and Teslameters, are usually only effective at measuring magnetic fields to about 0.1 or 0.2 Gauss. For weaker fields, one usually uses a magnetometer, which can measure to several nanoTesla. Off-the-shelf precision magnetometers can cost $10,000 and up in 2005 dollars.

It is worthwhile to develop an ability to mentally switch between Gauss and Tesla. 10,000 Gauss equals 1 Tesla. Perhaps it is easier to remember that 1 Gauss = 100 microTesla, so 0.01 G = 1 μT. Similarly, 1 nT = 10^{-5} Gauss or 10 μG.

It is important to keep in mind the very low electrical resistance of N turns of wire. Traditionally, power supplies are designed under the assumption that they will be connected to D.C. loads of at least fifty Ohms. So it is common to experience difficulty in controlling a current when it is being delivered to a very low resistance circuit. You will use a current between 2 and 3 A. Before beginning, examine the power supply and perform some preliminary diagnostics. First check to see if the fuses in the multimeter are good. Then check to see if it is easy to adjust the supply voltage from zero to two Volts – under an infinite load and under the low resistance load of one coil. If the voltage is not stable and easily adjustable, then you should use another power supply. Use the multimeter to measure the coils' resistance, then calculate the voltage necessary to pump 3 Amps of current through the coils. This will help you determine what type of power supply you require.

To avoid blowing fuses, monitor the current with the multimeter's 10A range. For the black, Radio Shack multimeters this means connecting the input current to the left-most banana jack. Ground connects to the center banana jack.

Procedures

Part 1 initial Set-up and Determining N

1) Read the Method section and perform all the preliminary procedures suggested therein.

2) Turn on the Gaussmeter, which is connected to a Hall Effect probe. When clamping it in the acrylic holder, cover the probe with the clear plastic cylinder provided. Only tighten the nylon screw just enough to hold the probe. The Gaussmeter reads accurately when the probe's flat face is *perpendicular* to the field. Assume the sensor inside the probe's cylinder is located 2 or 3 mm back from the end of the probe. Zero the gaussmeter.

3) Determine the magnitude and three-dimensional orientation of the Earth's magnetic field with the gaussmeter and probe. Measure and record the magnitude of the Earth's field. Use a dip meter, which resembles a protractor and a magnetized needle to help you measure the dip angle. Just for grins, calculate the value of the horizontal component of the Earth's field. Turn off the gaussmeter.

4) Align the axis of the Helmholtz coil parallel to the Earth's field. Explain why in your report. Measure the diameter and separation distance of the coils. Calculate the coil radius, a. What is the relationship between these two quantities, which creates a Helmholtz coil? All data and theoretical calculations will assume the $z=0$ plane coincides with the center of the bottom coil.

5) Remove the acrylic and aluminum support posts that mount to the base and lies along the coil's axis. Connect the power supply and a multimeter in series with the top coil. Turn on the Gaussmeter and the current.

6) Set the current to about 2 Amps and use the probe to verify that your connections create a field opposite in direction to the Earth's field. Of course, the probe detects the total field at any moment so you must add the Earth's field to each value to obtain the true field produced by the coil. If the coil's field pointed in the same direction as the Earth's, then you would have to subtract to obtain the coil's field.

7) Turn on the gaussmeter, zero it and use it to determine the number of turns of wire in the top coil.

   a) Position the probe to measure the total field at the center of the top coil where $\rho = 0$ and $z = a$. You may have to get creative with rulers, masking tape and metersticks.

   b) Measure the field with a known current and then add the Earth's field as a small correction. Use Eqn. (3) to calculate the number of turns in the upper coil. Since no current is flowing in the bottom coil you must modify Eqn. (3) appropriately.

8) Reverse the direction of current flow and repeat the previous procedure but now subtract the Earth's field. Average these two values for N.

9) Disconnect power from the upper coil and connect power to the lower coil. Repeat procedures (7) and (8). Ideally all the values for N should be approximately the same. Are they? Settle on one (integer) value to use for the analysis to follow.

Part 2 Mapping One Plane in the Helmholtz Coil

1) Connect power to both coils in series so they both produce a field pointing opposite to the Earth's field.

2) Measure the axial field component (here labeled B_z) of the Helmholtz coil in one vertical plane inside the coils. Choose a plane well away from the power supply connections. Proceed in a methodical fashion to make point-by-point measurements of B_z as a function of both ρ and z. Collect several hundred measurements. Don't forget to record the current and make sure it remains constant over time. It might be a good idea to feel the coils occasionally to make sure they are not getting warm.

3) Make a few more field measurements. C'mon just six more! Measure B_z B_ρ and B_ϕ fairly close to the coil axis at $z = a/2$ and for one other value of z.

4) Type all 400+ data values into a spreadsheet. Recall that Maxwell's equations are substantially different in Gaussian (cgs) units compared to SI (mksA) units. Converting from one system of units to the other is not a simple task of moving a few decimal points. The equations developed in Smythe's text are based on the SI system. So convert all magnetic field values to Tesla and current to Amperes.

5) One can think of at least two equivalent ways to display the data graphically. Before computer software made it easy to produce three-dimensional surface plots, it was common to graph a function of two variables as a series of curves on one plot. In this case it would make some visual sense to produce one graph of B_z on the y-axis and ρ on the x-axis. Then each value of z would correspond to one labelled curve on the graph.

6) However, it is more intuitive (and easier in Excel) to produce a three-dimensional surface plot with values of B_z on the z-axis, ρ on the x-axis and z on the y-axis. To do this, arrange all your data in a spreadsheet with columns corresponding to different values of rho and rows corresponding to different values of z. Boy, I don't think I could have planned for a sentence to be as potentially confusing as the last one! Above the top row, enter the corresponding rho coordinates. To the left of the first column, enter the corresponding z coordinates. Select the entire table: all the data, the coordinate axes and the one blank cell in the upper left corner of the array. In this way, Excel will use the very top row and left-most column to set the axes in the graph's x-y plane. Click on the graph tool, select surface plot, and continue as usual to enter labels.

7) If you plot the data as a series of curves, you must calculate the theoretical values and plot them in the same manner. If you choose to plot the data as a surface, then set up a second spreadsheet in the same manner as the data. However each cell will now contain a lengthy formula to compute B_z as a function of rho and z.

8) Whenever you plot multiple curves **always** change the line's style so each line is visibly different from all others. Also, do **not** use colors to represent different curves, even if you own a very nice color printer. There are several reasons for this. (i) Colors that appear visible on a computer screen are not always visible in print, (ii) Some people are colorblind. (iii) The prevalence of black and white photocopying makes colored graphs a problem. (iv) Excel has a nasty habit of choosing line styles and color combinations that are difficult to see.

9) Do not underestimate the amount of time it will take to perform the theoretical calculations. Past experience shows this can be a major stumbling block to successfully

completing the report.

10) To compare the data and theory, it is probably easiest to produce a third spreadsheet that is a point-by-point subtraction of data from theory. Alternatively, you could calculate a point by point percent difference spreadsheet. Then you must use your keen powers of data analysis to boil all these numbers down into a digestible summary.

11) Determine the size of a three-dimensional region where B_z is constant to within 1% of the value of neighboring points. Express your answers as fractions of the coil radius. Duplicate this analysis for a level of constancy of 5%.

12) Compare the magnitudes of B_z B_ρ and B_ϕ, which you measured earlier. What do you notice? How fair is it to say that a Helmholtz coil produces only an axial field? Provide a quantitative answer.

13) Helpful suggestions and directions for writing your report.

a) Do not write a procedure section as this write-up presents sufficient detail. However, you should describe in writing and provide a well-labeled photo or sketch of the equipment you used to measure the axial field at the coil's center.
b) There is no error propagation required for this experiment.
c) Include a circuit diagram with your labeled photo or sketch of the apparatus.
d) Write a short paragraph explaining how the main equation is derived in the reference (Smythe, 1950). You need not duplicate the equations found in this reference. Simply write a paragraph of text to summarize the derivation.
e) In your final graphs, format the magnetic field axis in microTeslas. This leaves the values in nice whole numbers as opposed to some power of ten. However, you must carry out the calculations in Telsas.
f) We make a simplifying assumption in developing Equation (4) from the reference. What is it?

Procedures no longer used:

14) Position the probe to measure the total field at the center of the top coil where $\rho = 0$ and $z = a$. You may have to get creative with rulers and a meterstick.

15) Adjust the current until the total field is zero. Record this current and use Eqn. (3) to calculate the number of turns in the upper coil. Note that since no current is flowing in the bottom coil you must modify Eqn. (3) appropriately.

16) Repeat the previous procedure with the probe located at $\rho = 0$ and $z = a/2$.

17) Disconnect power from the upper coil and connect power to the lower coil. Repeat procedures (9).

18) Repeat procedure (9) for both coils wired together so that both coils oppose the Earth's field. Calculate an average to determine the number of turns in the Helmholtz coils.