PHY 4210-01 Senior Lab
Lab C1: Mathematical Models of Chaotic Physical Systems

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Abstract

Chaotic behavior was studied by creating mathematical models and altering key parameters. A simple chaotic model was then observed by measuring time intervals of droplet formation from a leaking faucet. A small change in initial conditions can equate to much larger changes in the behavior of a system.

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1 Period Doubling Region of the Logistic Equation

2 Chaotic Region of the Logistic Equation

3 Lyapunov Experiments

The function provided for determining Lyapunov exponents

$$\lambda = \lim_{x \to \infty} \frac{1}{n} \sum_{k=0}^{n} \ln \left(\left| \frac{df(x_k)}{dx} \right| \right)$$

has been implemented in Julia; the source code is shown in Section 8.1.1. In the regular region, $r < 3.56994$, while in the chaotic region, $r \geq 3.56994$. While the inflection point of r has been shown to be closer to $r = 3$, it is clearly seen that $\lambda < 0$ in the regular region and $\lambda > 0$ in the chaotic region.

To calculate λ for $x_0 = 0.7$, $r = 2.5$, and $n = 20$, the argument array `lyaARGS = [0.7, 2.5, 20]` is constructed and used by the program. Since r is well within the regular region, a call to the function that calculates the Lyapunov exponent, `lya(iters, x -> logFunc(r,x), initCond, r)`, produces a negative value, $\lambda = -0.853$. Similarly, for $r = 3.7$, $\lambda = 0.420$, a positive value indicative of this larger r's chaotic nature.

4 Visualization of Chaos

The difference between the regular and chaotic regions can be clearly seen by plotting the logistic function $f(x_n) = x_{n+1} = rx_n(1-x_n)$ for different values of r. Such a plot is shown in Figure 1. While the inflection point displayed in the graph is less than the theoretical $r = 3.56994$, a dramatic distinction between the two regions can nonetheless be clearly seen.

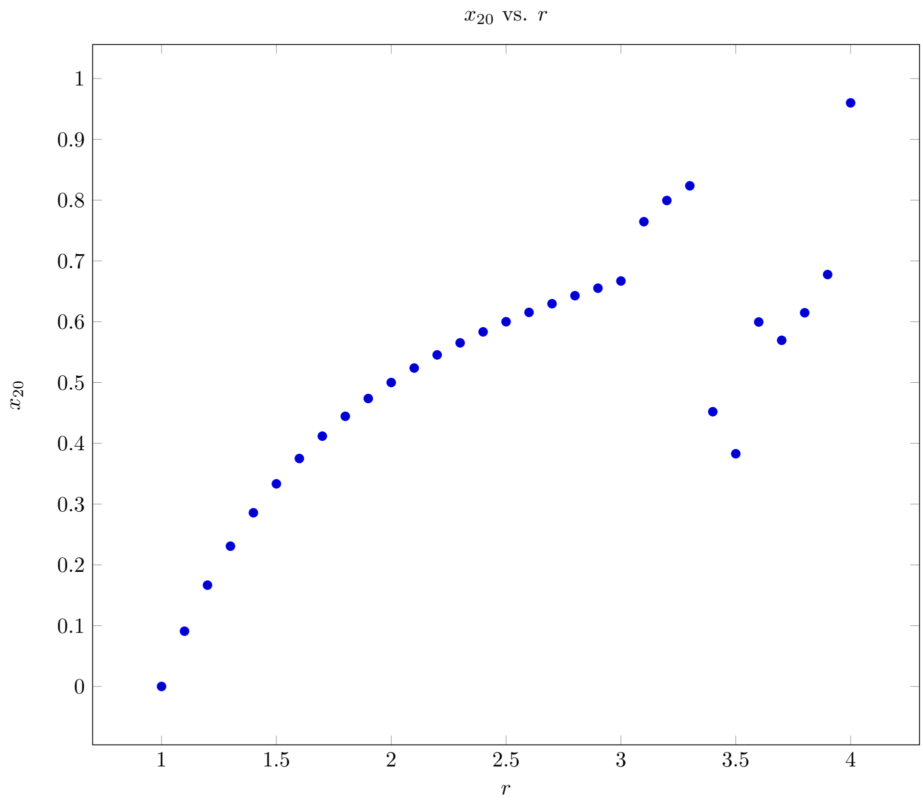

Figure 1: The plot of x_n for several values of r, where $x_0 = 0.7$ and $n = 20$. The Julia program seen in Section 8.1.2 was called with the command line arguments `0.7 20` to generate the values.

5 Water Drop Experiment

To collect chaotic data, the frequency with which drops formed at a sink was recorded for a specified number of total drops. Six trials were conducted, and the results are plotted below. Although the results were meant to be chaotic, some of the trials show drops being produced at a fairly regular rate. The trials in Figures 4 and 5 produced more regular results than the other trials; the data points exhibit less variety than in the other plots, which, although they are less than totally chaotic, scatter fairly widely.

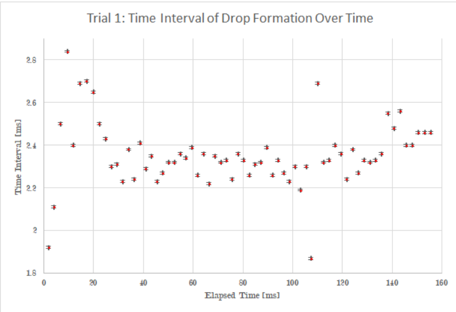

Figure 2: Trial 1

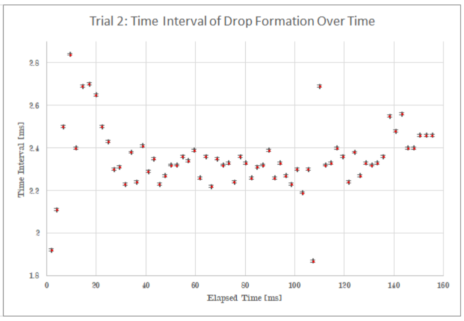

Figure 3: Trial 2

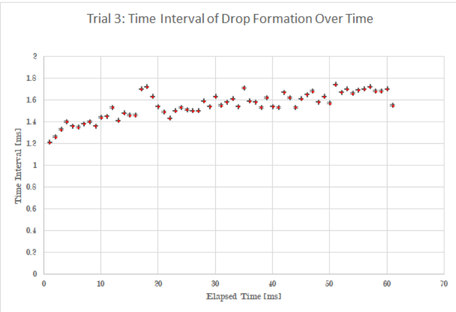

Figure 4: Trial 3

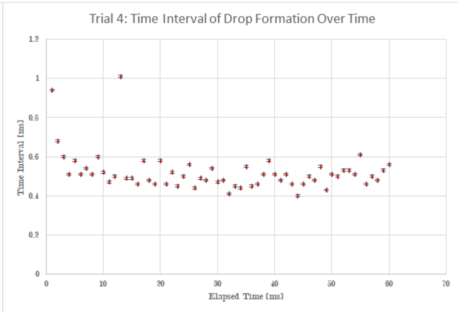

Figure 5: Trial 4

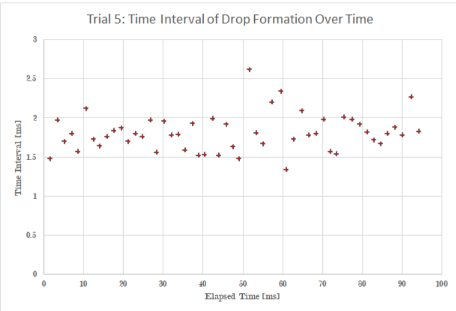

Figure 6: Trial 5

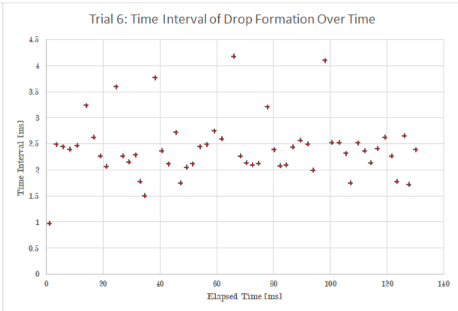

Figure 7: Trial 6

6 Sources of Error

The only source of error in the experiment is present in the times at which the water drops formed at the sink. This is random error in measurement because it is dependent upon the reaction time of the observer. It has the nominal effect of introducing slightly more chaos than there otherwise would have been.

7 Conclusion

8 Appendices

8.1 Appendix A: Source Code

8.1.1 Lyapunov Exponent Calculation

8.1.2 x_n Calculations for Different rs