PHY 4210-01 Senior Lab
Lab N4: Rutherford Scattering

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Abstract

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1 Objective of the Experiment

2 Theory of the Experiment

When an alpha particle with impact parameter b approaches a nucleus, it is scattered at an angle θ. If the impact parameter is given an infinitesimal range of [b, $b + db$], the resulting scattering angle then has a range of [$\theta - d\theta$, θ]; the impact parameter and scattering angle are inversely proportional.

Because the alpha particle can be incident within a defined range at any angle relative to the nucleus, a ring of possible incident locations is created in front of the nucleus. This ring is illustrated in Figure 1.

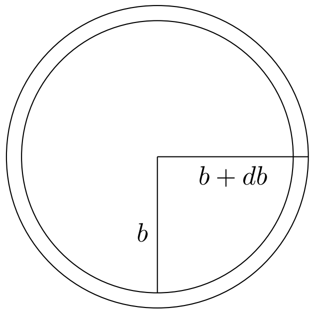

Figure 1: The ring whose area represents the possible region in which alpha particles may be incident on a target nucleus.

The area of this ring is found as the area of any ring is found:

$$\begin{aligned} A &= \pi(b+db)^2 - \pi b^2 \\ &= \pi(b^2 + 2bdb + db^2) - \pi b^2 \\ &= \pi b^2 + 2\pi bdb + \pi db^2 - \pi b^2 \\ &= 2\pi bdb + \pi db^2 \end{aligned}$$

Since db is infinitesimally small, it can be approximated to be zero. Therefore, the area of the incident ring, $\Delta\sigma$, is

$$\Delta\sigma = 2\pi bdb \tag{1}$$

Since the impact parameter b is directly proportional to the size of the cross section and the scattering angle θ is inversely proportional to the impact parameter, the size of the cross section decreases as the scattering angle increases. Therefore, the cross section experiences a negative rate of change as θ increases. Hence,

$$\Delta\sigma(\theta) = -d\sigma(\theta) \tag{2}$$

The circumference of a circle is equal to $2\pi r$, where r is the radius of the circle. In the experiment, the radius of the ring onto which the alpha particle is projected after it is scattered is $R\sin(\theta)$, where θ is the scattering angle and R, described in Figure 2, is the distance between the point at which the alpha particle was scattered and the edge of the ring.

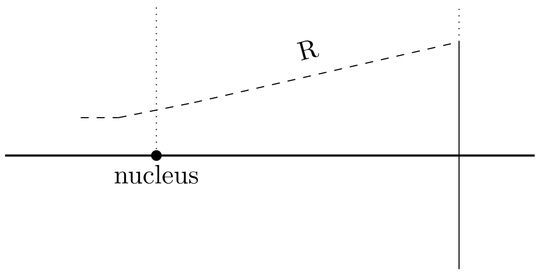

Figure 2: The path of the alpha particle as it is scattered by a nucleus. R is the path length between the nucleus and the ring of the solid angle.

The outer circumference of this ring is $2\pi R\sin(\theta)$. Since the alpha particle is incident within a range whose minimum is $\theta - d\theta$, however, the ring has a thickness of $Rd\theta$. Since the thickness of the ring is infinitesimal, the ring's area can be approximated to be that of a rectangle. Therefore, the area of the ring is $A = 2\pi R\sin(\theta)Rd\theta$.

The solid angle of the scattered alpha particles at an angle θ is:

$$\begin{aligned}\Delta\Omega &= \frac{A}{R^2} \\ &= \frac{(2\pi R\sin(\theta)Rd\theta)}{R^2}\end{aligned}$$

$$d\Omega = 2\pi\sin(\theta) \tag{3}$$

An expression for the differential cross section $\frac{d\sigma}{d\Omega}(\theta)$ can be found by multiplying Equation 2 by Equation 2 divided by itself.

$$\begin{aligned}\Delta\sigma &= -\frac{d\sigma}{d\Omega}(\theta)d\Omega \\ &= -\frac{d\sigma}{d\Omega}(\theta)2\pi\sin(\theta)d\theta\end{aligned}$$

from Equation 2:

$$-\frac{d\sigma}{d\Omega}(\theta)2\pi\sin(\theta)d\theta = 2\pi b db$$

$$\frac{d\sigma}{d\Omega}(\theta) = -\frac{b}{\sin(\theta)}\frac{db}{d\theta} \tag{4}$$

Since it is known that $b = \frac{ZZ'e^2}{2E}\cot\left(\frac{\theta}{2}\right)$, it can be inserted into Equation 2. Furthermore, since b is a function of θ, $\frac{db}{d\theta}$ can also be found:

$$\begin{aligned} b &= \frac{ZZ'e^2}{2E}\cot\left(\frac{\theta}{2}\right) \\ \frac{db}{d\theta} &= \frac{ZZ'e^2}{2E}\left(-\frac{1}{2}\csc^2\left(\frac{\theta}{2}\right)\right) \\ \frac{db}{d\theta} &= -\frac{ZZ'e^2}{4E}\csc^2\left(\frac{\theta}{2}\right) \end{aligned}$$

Now that b and db have been found, the full expression for the differential cross section $\frac{d\sigma}{d\Omega}$ can be determined:

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\theta) &= -\frac{b}{\sin(\theta)}\frac{db}{d\theta} \\ &= -\left(\frac{ZZ'e^2}{2E}\cot\left(\frac{\theta}{2}\right)\right)\frac{1}{\sin(\theta)}\left(-\frac{ZZ'e^2}{4E}\csc^2\left(\frac{\theta}{2}\right)\right) \\ &= 2\left(\frac{ZZ'e^2}{4E}\right)^2\frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}\frac{1}{\sin(\theta)}\frac{1}{\sin^2\left(\frac{\theta}{2}\right)} \\ &= 2\left(\frac{ZZ'e^2}{4E}\right)^2\cos\left(\frac{\theta}{2}\right)\frac{1}{\left(2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right)}\frac{1}{\sin^3\left(\frac{\theta}{2}\right)} \end{aligned}$$

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{ZZ'e^2}{4E}\right)^2\frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \tag{5}$$

Equation 2 is the equation for calculating the theoretical differential cross-section.

The differential cross-section can also be found experimentally using a found alpha particle scattering rate at a particular angle θ. First, this relation must be constructed. The collimated beam of alpha particles begins its journey with an incident rate of $\frac{dN_0}{dt}$. This beam is then incident on a thin foil with an atomic density of $n = \frac{\rho N_A d}{A}$, where ρ is the density of the foil material, d is the thickness of the foil, and A is the atomic number of the foil material. By being incident on the foil, the alpha particles are exposed to a differential cross-section at the particular angle of $\frac{d\sigma}{d\Omega}(\theta)$. The alpha particles are scattered by the nuclei across a solid angle $\Delta\Omega = A_{\text{detector}} r^2$, where A_{detector} is the area of the detector and r is the distance between the foil and detector. Multiplying these factors together results in the scattering rate of the alpha particles incident on a particular foil at a particular angle: $\frac{dN}{dt}(\theta) = \frac{dN_0}{dt} n \frac{d\sigma}{d\Omega}(\theta)\Delta\Omega$. Since the scattering rate is determined experimentally, the equation can be rearranged for the differential cross-section:

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{\frac{dN}{dt}(\theta)}{\frac{dN_0}{dt} n \Delta\Omega} \tag{6}$$

3 Equipment Utilized

- List equipment and specifications

Figure 3: Description of schematic here

4 Procedure

4.1 Procedural Modifications

5 Data Analysis

5.1 Data Analysis I: Gold

5.2 Data Analysis II: Aluminum

6 Results

6.1 Results I: Gold

6.2 Results II: Aluminum

7 Conclusion

8 Appendices

8.1 Appendix A: Data

8.2 Appendix B: Source Code