Exp. M-1 Magnetic Field Mapping of Helmholtz Coils

Reference:
"Static & Dynamic Electricity" by Smyth or other text books on electricity and magnetism.

Objectives:

1. To map the magnetic fields of Helmholtz coils and to compare the measured fields with the theoretically calculated values.
2. To determine a region within which the magnetic fields are relatively constant.

Apparatus:

large Helmholtz coils, constant current power supply (0-12 V, 0-5 amps), Ammeter, Gaussmeter with Hall effect probe, dipmeter .

Theory:

The z-component of the magnetic field of Helmholtz coils as a function of ρ and z in the cylindrical coordinate system is given below: (See Ref. 1)

$$B_z = \frac{\mu_0 I N}{2\pi}\left\{ \frac{1}{[(a+\rho)^2+(a-z)^2]^{1/2}}\left[K_1 + \frac{a^2-\rho^2-(a-z)^2}{(a-\rho)^2+(a-z)^2}E_1 \right] + \frac{1}{[(a+\rho)^2+z^2]^{1/2}}\left[K_2 + \frac{a^2-\rho^2-z^2}{(a-\rho)^2+z^2}E_2 \right]\right.$$

where

$$K_1 = \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1-k_1^2\sin^2\theta)^{1/2}}, \qquad E_1 = \int_0^{\frac{\pi}{2}} (1-k_1^2\sin^2\theta)^{1/2}\,d\theta,$$

$$K_2 = \int_0^{\frac{\pi}{2}} \frac{d\theta}{(1-k_2^2\sin^2\theta)^{1/2}}, \qquad E_2 = \int_0^{\frac{\pi}{2}} (1-k_2^2\sin^2\theta)^{1/2}\,d\theta,$$

$$k_1 = \left[\frac{4a\rho}{(a+\rho)^2+(a-z)^2}\right]^{1/2}, \text{ and } k_2 = \left[\frac{4a\rho}{(a+\rho)^2+z^2}\right]^{1/2}.$$

and a is the radius of the coil.

The K and E are the complete elliptic integrals of the first and second kind. They can be evaluated using Mathcad or using the following formulas:

$$K = \frac{\pi}{2}(1+m)\left[1 + \frac{m^2}{4} + \frac{9}{128}m^4 + \cdots\right],$$

$$E = \frac{\pi}{2(1+m)}\left[1 + \frac{m^2}{4} + \frac{m^4}{128} + \cdots\right]$$

where

$$m = \frac{1-(1-k^2)^{1/2}}{1+(1-k^2)^{1/2}} .$$

Experimental Procedures:

1. Determine the direction of the earth's magnetic field using a dipmeter.

2. Align the axis of the coils with the direction of the earth's field. (Why?)

3. Measure the earth's magnetic field using a Hall probe.

4. Determine the number of turns in the coils by adjusting the current through the coils such that the total axial field (including the earth's magnetic field) at the midpoint of the coils is zero. You need to determine the number of turns for the upper coils and lower coils separately and together as a set.

5. Increase the current to about twice the value you found in the previous step when both coils are used. Then measure the field components parallel to the axis of the coils as a function of the three cylindrical coordinates so that you have good plots of the field as function of z at constant ρ and as functions of ρ at constant z.

6. Compute the theoretical values of the fields for various values of ρ and z

7. Compare the experimentally measured axial magnetic fields with the calculated values. (Similar plots should be made as those in Step 5.)

8. Determine the region where the axial components of the field is relatively constant and then obtain the magnitudes of the other two components of the field relative to the axial field within this volume.

(Original 1/14/2004, R.S. Jin, revised 1/13/07, M. Hohlmann)