Blackbody Radiation

EQUIPMENT

INCLUDED:

1	Prism Spectrophotometer Kit	OS-8544
1	Optics Bench (60 cm)	OS-8541
1	Spectrophotometer Accessory Kit	OS-8537
1	Aperture Bracket	OS-8534
1	Broad Spectrum Light Sensor	CI-6630
1	Rotary Motion Sensor	CI-6538
1	Voltage Sensor	CI-6503
1	Power Amplifier II	CI-6552A
1	Replacement Bulb (10 pk)	SE-8509
1	Banana Plug Cord-Black (5 pack)	SE-9751

NOT INCLUDED, BUT REQUIRED:

1	ScienceWorkshop 750 Interface	CI-7650
1	DataStudio	CI-6870

INTRODUCTION

The spectrum of an incandescent light bulb is scanned by hand using a prism spectrophotometer that measures relative light intensity as a function of angle. A Broad Spectrum Light Sensor is used with a prism so the entire spectrum from approximately 400 nm to 2500 nm can be scanned without the overlapping orders caused by a grating. The wavelengths corresponding to the angles are calculated using the equations for a prism spectrophotometer. The relative light intensity can then be plotted as a function of wavelength as the spectrum is scanned, resulting in the characteristic blackbody curve. The intensity of the light bulb is reduced, reducing the temperature, and the scan is repeated to show how the curves nest with a shift in the peak wavelength.

The temperature of the filament of the bulb can be estimated indirectly by determining the resistance of the bulb from the measured voltage and current. From the temperature, the theoretical peak wavelength can be calculated and compared to the measured peak wavelength.

THEORY

The intensity (I) of radiation emitted by a body is given by Planck's Radiation Law:

$$I(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda kT}}-1}\right) \qquad (1)$$

where c is the speed of light in a vacuum, h is Planck's constant, k is Boltzmann's constant, T is the absolute temperature of the body, and λ is the wavelength of the radiation.

Written by Ann Hanks

The wavelength with the greatest intensity is given by

$$\lambda_{max} = \frac{constant}{T} = \frac{0.002898\ m{\cdot}K}{T} \qquad (2)$$

where T is the absolute temperature of the body. The temperature of the blackbody light filament can be calculated using the resistance of the filament while it is lit. The resistivity of the tungsten filament is a nonlinear function of the temperature. A function that approximates the calibration curve (CRC Handbook, 45th edition, page E-110) for the resistivity of tungsten is used in the DataStudio setup file to calculate the temperature.

The resistance of the filament is found using

$$R = \frac{V}{I} \qquad (3)$$

where V is the voltage applied to the lamp and I is the current through the lamp. The temperature dependence of the resistance is used to determine the temperature of the hot filament. See the Appendix for an explanation of how this is done in the DataStudio file.

SET UP

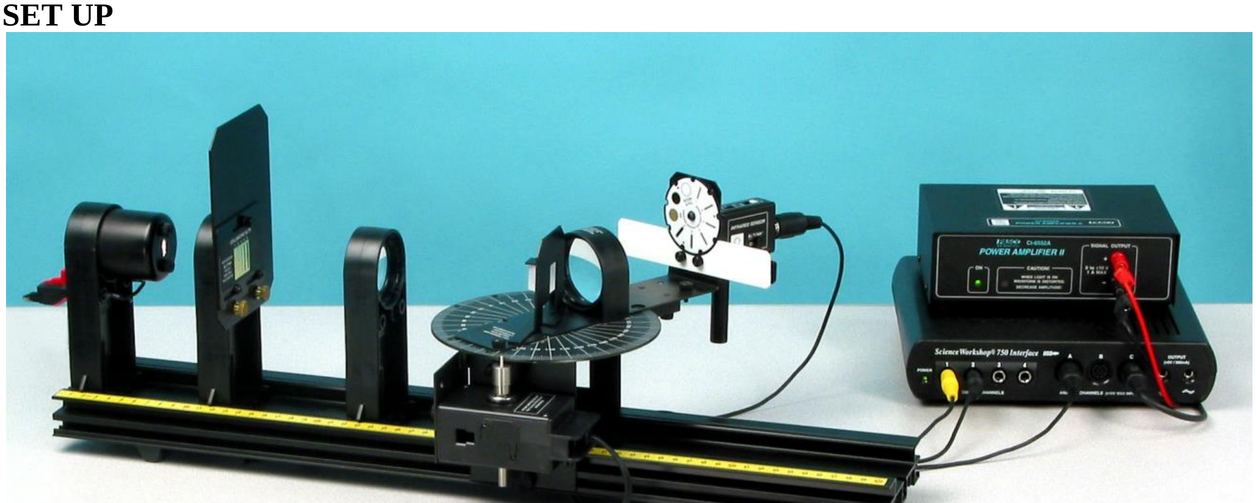


Figure 1: Complete Setup

1. Set up the Prism Spectrophotometer as shown in Figure 1. Attach a grounding wire to the spectrophotometer table as instructed by your teacher. The light sensor used in this experiment is the Broad Spectrum Light Sensor.

2. Check that the prism is oriented as shown in Figure 2 with the apex facing the light source.

3. The collimating lens must be 10 cm (the lens' focal length) from the collimating slits.
4. Plug the Blackbody Light into the Power

Figure 2: Prism Orientation

Amplifier. Plug the Power Amplifier into Channel C of the ScienceWorkshop 750 interface.

5. Plug the Broad Spectrum Light Sensor into Channel A of the interface. Plug the Voltage Sensor into Channel B. Connect the leads of the Voltage Sensor to the Blackbody Light. Plug the Rotary Motion Sensor into Channels 1 and 2.

6. Open the DataStudio setup file called "Blackbody.ds". See the Appendix for an explanation of the equations in the setup file.

PROCEDURE

1. Set the collimating slits on Slit #4. Set the Light Sensor mask on Slit #4.

2. In DataStudio, click the Signal Generator window and turn the generator ON at 10 V DC. **Caution:** If 10 volts is applied to the blackbody light for an extended amount of time, the life of the bulb will be reduced. Only turn on the bulb when taking measurements.

3. Look at the light coming from the Blackbody Light Source. Observe the color.

4. Look at the spectrum on the Light Sensor screen. Are all the colors (from red to violet) present?

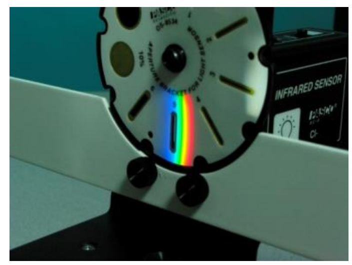


Figure 3: Spectrum on Light Sensor Mask

5. Rotate the scanning arm until it touches the stop. This will be the starting position for all the scans.

6. Set the Broad Spectrum Light Sensor gain switch to “x100” and press the tare button. Click START in DataStudio.

7. As the scan begins, check that the angle is positive. If not, reverse the Rotary Motion Sensor plugs in the interface and re-start the data run. Slowly rotate the scanning arm through the spectrum and continue all the way past zero degrees (the position where the light sensor is directly opposite the light source). The graph of intensity vs. wavelength is set to automatically stop at about 2500 nm because the glass in the spectrophotometer optics does not transmit wavelengths greater than 2500 nm.

8. There will be a peak on the intensity vs. angle graph where the light sensor is aligned with the light source because some light passes by the prism instead of going through the prism. This peak enables the initial angle to be exactly determined. Use the Smart Cursor to determine the angle from the starting position to the central peak where the light sensor is directly opposite the light source. Click the calculator in DataStudio and enter this angle called "init".

9. Repeat Steps 2 through 7 for voltages of 7 V and 4 V. On these scans, it is not necessary to scan all the way past the center as before because the angle has already been calibrated.

ANALYSIS

1. Does the peak shift toward shorter or longer wavelengths as the temperature is lowered?

2. How does the intensity change as the temperature is changed?

3. Use the Smart Cursor on the Temperature graph to find the temperatures of the filament. Calculate the peak wavelength for each temperature using Equation (2). Do these theoretical values correspond to peak wavelengths on the intensity graphs?

4. Planck's formula (Equation (1)) is already in the DataStudio setup file. Click and drag the calculation from the Data List on the left to the graph of intensity vs. wavelength. Change the amplitude in the calculator so it matches the tallest curve. Does the shape of the curve match the theoretical curve? Can the bulb really be considered a blackbody?

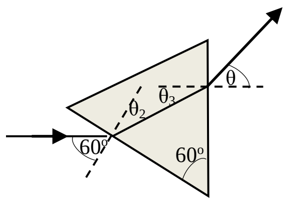
5. How did the color of the bulb change with temperature? How did the color composition of the spectrum change with temperature? Considering the peak wavelengths, why is a bulb’s filament red at low temperatures and white at high temperatures?

6. At about what wavelength is the peak wavelength of our Sun? What color is our Sun? Why?

7. For the highest temperature, is more of the intensity (area of the intensity vs. wavelength graph) in the visible part of the spectrum or in the infrared part of the spectrum? How could a light bulb be made more efficient so it puts out more light in the visible?

Appendix: Explanations of the Calculations in the DataStudio Setup File

Wavelength Calculation: The index of refraction of the prism glass varies with the wavelength of the light. To determine the wavelength as a function of the angle, the relationship between the index of refraction and the angle is determined using Snell's Law at each face of the prism.

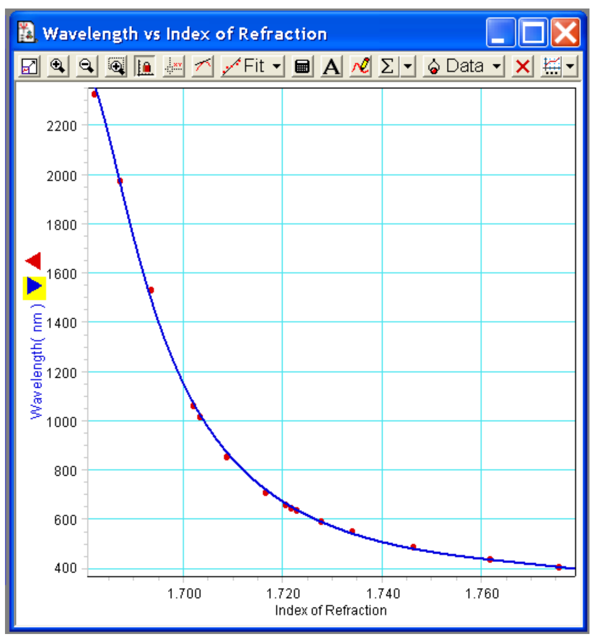


$$n = \sqrt{\left(\frac{2}{\sqrt{3}}sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

The Cauchy equation gives the relationship between the index of refraction and the wavelength: $n(\lambda) = \frac{A}{\lambda^2} + B$, where A and B are specific to the type of glass and are determined experimentally.

Solving this for wavelength gives $\lambda = \sqrt{\frac{A}{n-B}}$. However, this equation is an approximation which does not fit well in the region of interest. The following table summarizes the dependence of the index of refraction on wavelength for the prism (provided by the supplier of the prism):

Index of Refraction	Wavelength (nm)
1.68	2325.40
1.69	1970.10
1.69	1529.60
1.70	1060.00
1.70	1014.00
1.71	852.10
1.72	706.50
1.72	656.30
1.72	643.00
1.72	632.80
1.73	589.30
1.73	546.10
1.75	486.10
1.76	435.80
1.78	404.70



The wavelength is calculated using an equation derived from a curve fit for the prism, where the coefficients are determined experimentally for the type of flint glass the prism was made from. It is not necessary to change this calculation unless you use a prism different from the one supplied in the experiment.

$$wavelength = \frac{3 \times 10^3}{\sqrt{A + Bn + Cn^2 + Dn^3 + Jn^4 + Fn^5 + Gn^6 + Hn^7 + In^8}}$$

$A = -4.98552133 \times 10^7$
$B = 8.60920189 \times 10^7$
$C = -2.998332835 \times 10^7$
$D = -1.435423656 \times 10^7$
$F = 5647432.02$
$G = 1863438.86$
$H = -2719226.18$
$I = 574967.82$
$J = 835425.05$
$n = n = index\ of\ refraction$

From Snell's Law:

$$n = \sqrt{\left(\frac{2}{\sqrt{3}} sin\theta + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

This equation is expressed in the setup file as:

$$n = filter\left\{1.697, 10, \sqrt{\left(1.1547 \sin\left(\frac{Init - filter(0,20, Angle)}{Ratio}\right) + 0.5\right)^2 + 0.75}\right\}$$

where the purpose of the filters is to keep the function from getting out of range. The initial angle is determined by measuring the angle of the central maximum from the origin: $Init = Initial\ Angle = 72.4$. This initial angle will be about the same for all apparatus because the procedure calls for starting with the scanning arm against the stop. $Angle = Angular\ Position, Ch1\&2$, the measurement from the Rotary Motion Sensor.
The angle measured by the Rotary Motion Sensor is greater than the angle through which the scanning arm moves because the pin rotating against the disk acts as a gear. The ratio of the disk diameter to the pin diameter is approximately 60. However, you can rotate the disk through 180 degrees as read from the marks on the disk and compare to the reading from the Rotary Motion Sensor to get a more precise measurement of this ratio ($Ratio = 59.50$).

<u>Light Intensity Calculation</u>: The light intensity is smoothed by eight points to eliminate noise:
$Intensity = smooth(8, v) - V_o$
$v = Voltage, ChA$
$V_o = 0.00$ This is an optional offset in case you neglect to tare the light sensor before recording data.

Theoretical Intensity Calculation:

$$I(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right)$$

This equation is expressed in the setup file as:

$$Theory = \frac{(1.25 \times 10^{20})A}{x^5\left(e^{\left(\frac{1.44\times 10^7}{xT}\right)} - 1\right)}$$

where *x* is the wavelength in nanometers. To model this function, the wavelength (*x*) is varied from 300 nm to 2800 nm in 50 increments.

$x = 300\ to\ 2800, 50\ steps$

$A = 0.018$ (This is an arbitrary constant to adjust the amplitude of the function because the Light Sensor is not calibrated.)

$T = 2500$ (This is the temperature of the light bulb filament in Kelvin.)

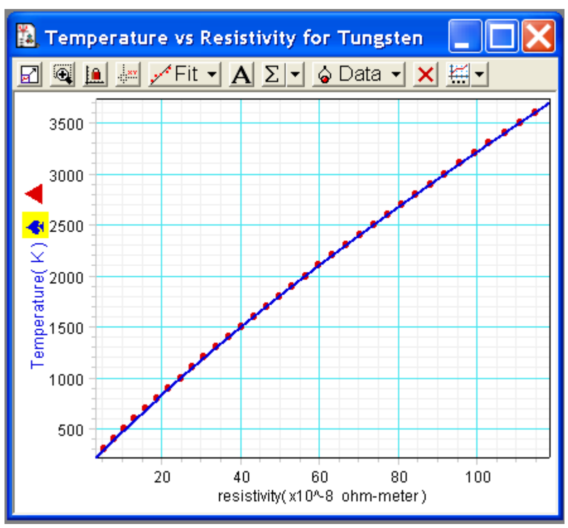
Finding the Temperature:

The temperature of the filament is calculated using the temperature dependence of its resistivity.

$$resistivity = \rho = \rho_o(1 + \alpha(T - T_o)) \longrightarrow T = T_o + \frac{\rho/\rho_o - 1}{\alpha}$$

However, the value of the temperature coefficient (α) is not constant over the range from room temperature to the temperature of the hot bulb. The table shows the temperature coefficient for a broader range of temperatures.

resistivity ($\times 10^{-8}\Omega\cdot m$)	Temperature (K)	resistivity ($\times 10^{-8}\Omega\cdot m$)	Temperature (K)
5.65	300	60.06	2100
8.06	400	63.48	2200
10.56	500	66.91	2300
13.23	600	70.39	2400
16.09	700	73.91	2500
19.00	800	77.49	2600
21.94	900	81.04	2700
24.93	1000	84.70	2800
27.94	1100	88.33	2900
30.98	1200	92.04	3000
34.08	1300	95.76	3100
37.19	1400	99.54	3200
40.36	1500	103.3	3300
43.55	1600	107.2	3400
46.78	1700	111.1	3500
50.05	1800	115.0	3600
53.35	1900	115.0	3600
56.67	2000	115.0	3600



A curve is fit to this data and a filter is used to keep the temperature calculation in the range of interest:

$$temperature(K) = filter\left(300, 4000, (103 + 38.1r - 0.095r^2 + (2.48 \times 10^{-4})r^3)\right)$$

The resistance is determined using the voltage across the filament and the current through the filament ($R = \frac{V}{I}$).

$\frac{\rho}{\rho_o} = \frac{R}{R_o} = \frac{R - R_{wires}}{R_o} \longrightarrow \rho = \rho_o \left(\frac{R - R_{wires}}{R_o} \right)$ where R_{wires} is the approximate resistance of the wires connecting the voltage source to the bulb.
In the calculation, $\rho = r = resistivity\ of\ hot\ filament\ in\ \mu\Omega \cdot cm.$

$$resistivity = 5.65 \left(\frac{\frac{V}{I} - 0.2}{R_{bulb}} \right)$$

$\rho_o = 5.65\ \mu\Omega \cdot cm$ (This is the resistivity of tungsten at room temperature.)
$V = Voltage, ChB$
$I = Current, ChC$
$R_{bulb} = 0.93\Omega$ (This is measured with a multimeter when the bulb filament is at room temperature.)