

# PHY 4210-01 Senior Lab

## Lab P2: Electron Spin Resonance

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March 28, 2019

### **Abstract**

Electron spin resonance allows the structure of paramagnetic materials (such as diphenyl-picryl-hydrazyl or DPPH) to be investigated, as they have a nonzero momentum. The ratio of the magnetic moment and angular momentum is the gyromagnetic ratio of spin,  $g_s$ . Such a value for the electron was determined by applying various resonance frequencies and recording the magnetic field produced via Helmholtz coil, and determined to be  $g_s = 1.49 \pm .13$ . A resonance curve was produced by plotting voltage amplitude against the frequency seen from a resonance circuit box. The line width of the resonance signal was then calculated to be  $0.49mT$ .

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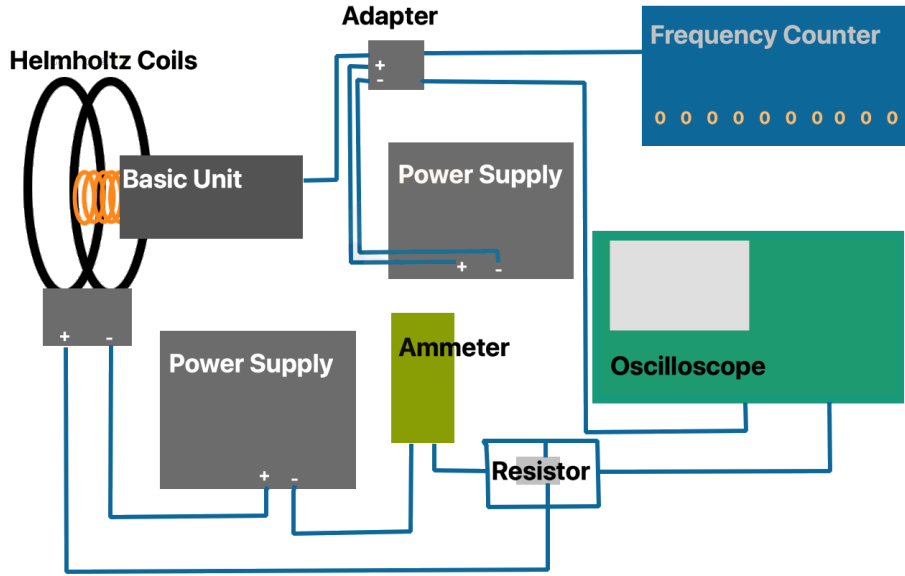


Figure 1: Schematic of equipment used in experiment

## 1 Data Analysis

### 1.1 Frequency Dependence of Resonance Field

Voltage was compared to frequency to obtain a graphical relationship for the frequency dependence of the resonance field. The amplitude voltage was obtained by measuring the peak-to-peak voltage from the oscilloscope and dividing it in half. The peak of 2 is the specific resonance frequency for the field. This value is a voltage amplitude of 1.01 V and a frequency of  $4.18 \times 10^7$  Hz. It is important to note that the electron spin resonance device divides the frequency by a factor of a thousand, and thus the Hewlett-Packard frequency counter displayed a corrected frequency. The calculations require an uncorrected frequency, i.e. the counter's frequency multiplied by one thousand.

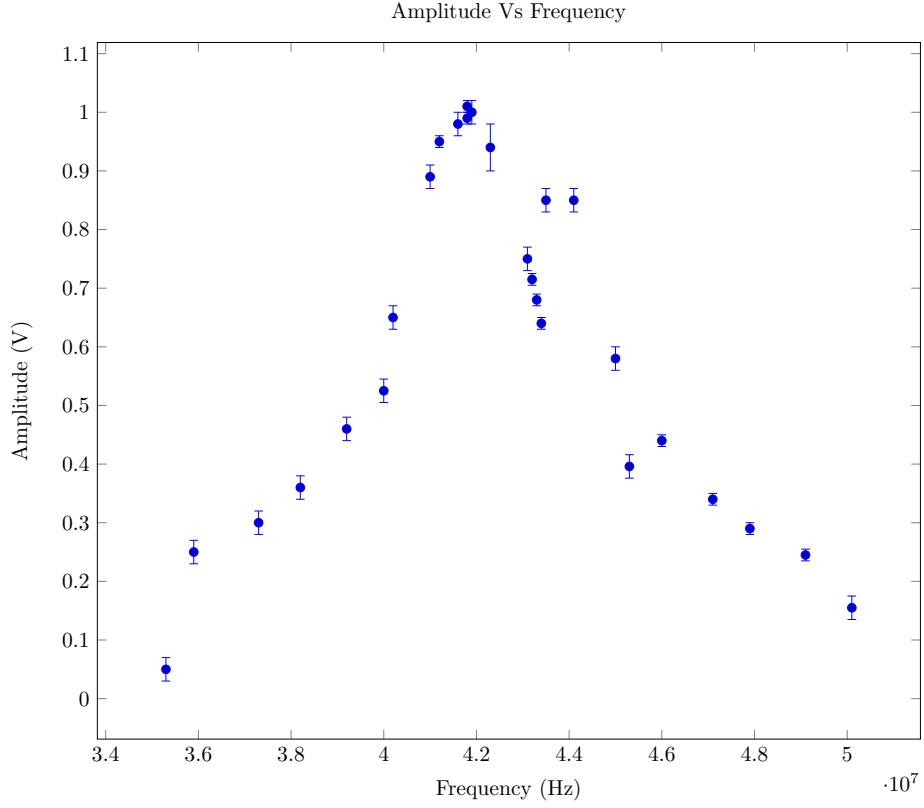


Figure 2: Graphic representation of the frequency dependence of the resonance field.

## 1.2 Propagation of Uncertainty in the Frequency Dependence of the Resonance Field

### 1.3 Experimental Value of Gyromagnetic Ratio

The gyromagnetic ratio is calculated using the following equation, where  $\nu$  is the frequency,  $h$  is Planck's constant,  $\mu_B$  is the Bohr magneton, and  $B_0$  is the magnetic field strength.

$$g_s = \frac{h \times \nu}{\mu_B \times B_0} \quad (1)$$

The magnetic field used in calculating equation 1 must be calculated as well. It is determined from the measured current using equation 2, where  $\mu_0 = 4\pi \times 10^{-7} \frac{Vs}{Am}$ , the number of turns is  $n = 320$ , and the radius of the coils is  $r = 6.8cm$ .

$$B_0 = \mu_0 \left( \frac{4}{5} \right)^{3/2} \times \frac{n}{r} \times I \quad (2)$$

Rather than measuring the current directly, the current is calculated by measuring the voltage drop across a resistor, of which the resistance is also

measured. This calculation is shown below in equation.

$$I = \frac{V}{R} \quad (3)$$

By substituting equation 3 into 2, and then substituting equation 2 into equation 1, we arrive at an expression for the gyromagnetic ratio in terms of known constants and measured quantities. This final expression is shown in equation 4.

$$g_s = \frac{h \times \nu}{\mu_B \times \left( \mu_0 \left( \frac{4}{5} \right)^{3/2} \times \frac{n}{r} \times \frac{V}{R} \right)} \quad (4)$$

#### 1.4 Propagating Uncertainty in Gyromagnetic Ratio

The error in the experimental value of the gyromagnetic ratio is determined by propagating uncertainty in equation 4. There are no uncertainties associated with fundamental constants such as  $h$ ,  $\mu_B$ , and  $\mu_0$ . It is assumed that the number of coil turns,  $n$ , also has no associated uncertainty because it was reported in the manual as such. The uncertainty in the radius is constant for all measurements, but the frequency, voltage, and resistance will differ for each measurement. Equation 5 shows this error propagation.

$$\delta g_s = g_s \times \sqrt{\left( \frac{\delta \nu}{\nu} \right)^2 + \left( \frac{\delta r}{r} \right)^2 + \left( \frac{\delta V}{V} \right)^2 + \left( \frac{\delta R}{R} \right)^2} \quad (5)$$

An example calculation for the value of  $g_s$  and its propagated uncertainty is shown below for a measurement taken with the large coil:

$$\begin{aligned} g_s &= \frac{6.626 \times 10^{-34} \times (3 \times 10^7)}{\mu_B \times \left( 4\pi \times 10^{-7} \left( \frac{4}{5} \right)^{3/2} \times \frac{320}{0.068} \times \frac{0.44}{1.7} \right)} \\ &= 1.93 \end{aligned}$$

$$\begin{aligned} \delta g_s &= g_s \times \sqrt{\left( \frac{1.00 \times 10^4 \text{Hz}}{3.00 \times 10^7 \text{Hz}} \right)^2 + \left( \frac{0.5 \text{cm}}{6.7 \text{cm}} \right)^2 + \left( \frac{0.1 \text{V}}{2 \text{V}} \right)^2 + \left( \frac{0.1 \Omega}{1.7 \Omega} \right)^2} \\ &= 1.93 \times \sqrt{(3.3 \times 10^{-4})^2 + (0.006)^2 + (0.05)^2 + (0.06)^2} \\ &= .15 \end{aligned}$$

We can calculate the discrepancy between the experimental and theoretical values as follows. Recall the theoretical value of  $g_s$  for DPPH is 2.0036, which is approximated as 2.00 due to the limited precision of the experimental value.

$$\begin{aligned} \Delta g_s &= |g_{s_{exp}} - g_{s_{theo}}| \\ &= |1.93 - 2.00| \\ &= 0.074 \end{aligned}$$

Evidently, this difference  $\Delta g_s$  is less than  $1\sigma = .15$ .

## 1.5 Rejection of Data

During the data taking process for the "big coil", a measurement at a particular frequency produced an experimental  $g_s$  value that seemed anomalous; most measurements fall between 1 and 4, but this measurement is around 13. Chauvenet's criterion will be used to determine if this datum should be discarded.

If one assumes this measurement to be valid, the resultant average and standard deviation are  $2.59 \pm 2.91$  (quite an atrocity). The measurement in question, 13.08, differs from the average by  $4.49\sigma$ . If a Gaussian distribution is assumed for the  $g_s$  values, the probability of obtaining a measurement that differs from the mean by this quantity is determined as follows:

$$\begin{aligned} Prob(\text{outside } 4.49\sigma) &= 1 - Prob(\text{within } 4.49\sigma) \\ &= 1 - .9999994 \\ &= 0 \end{aligned}$$

Since the probability of a measurement being within  $4.49\sigma$  is so high, the probability of this measurement being outside this interval is effectively zero. Therefore, we can discard the anomalous datum with extremely high confidence.

## 1.6 Determining Line Width of Resonance Signal

$\delta B_0$  is representative of an absorption line, and is obtained when the energy is measured at a fixed frequency as function of the magnetic field. The line width  $\delta B_0$  is used as an expression of the uncertainty in the energy of the transition. This is best represented by the equation  $\delta E = g \times \mu_0 \times \delta B_0$ . Using the uncertainty principle a relation is then found for  $\delta B_0$ .

$$\delta B_0 = \frac{\hbar}{2 \times g_J \times \mu_B \times T}$$

where  $T$  is the lifetime of the level and  $g_J$  is the Land  $g$  factor. Experimentally  $\delta B_0$  can be determined by the following equation:

$$\delta B_0 = B \times \left( \frac{\delta I}{I_{mod}} \right)$$

where  $\delta I$  is represented as  $\frac{\delta U}{U_{mod}} \times I_{mod} \times 2\sqrt{2}$ .

$$\begin{aligned} \delta I &= \frac{\delta U}{U_{mod}} \times I_{mod} \times 2\sqrt{2} \\ &= \frac{0.55}{2} \times 0.156 \times 2\sqrt{2} \\ &= 0.121 \end{aligned}$$

$$\begin{aligned}
\delta B_0 &= B \times \left( \frac{\delta I}{I_{mod}} \right) \\
&= 6.23 \times 10^{-4} \times \left( \frac{0.121}{0.156} \right) \\
&= 4.85 \times 10^{-4} T \\
&= 0.49 mT
\end{aligned}$$

## 2 Results: Comparing Discrepancies and Uncertainties

### 2.1 Discrepancy in Gyromagnetic Ratio

The discrepancy between the experimental and theoretical values of  $G$  for each of the coils can be calculated with  $\Delta g_s = |g_{st} - g_{se}|$ , where  $g_{st}$  is the theoretical value of  $g_s$  and  $g_{se}$  is the experimental value. The discrepancy in the value of  $g_s$  for the small coil is:

$$\begin{aligned}
\Delta g_{s,small} &= |(2.00) - (1.44)| \\
\Delta g_{s,small} &= 0.56
\end{aligned}$$

Since the standard deviation of the calculated values is  $\sigma_{g_s,small} = 0.393$ , the experimental value of  $g_s$  is  $\frac{\Delta g_{s,small}}{\sigma_{g_s,small}} = \frac{0.56}{0.393} = 1.42\sigma$  from the theoretical value.

The discrepancies for the medium and big coils were calculated in the same way, and the results are detailed in Table 2.1. The results from the three coils are then used to calculate a true average value of  $g_s$ . The uncertainty in this value is the standard deviation of all the data points across all the coils.

Table 2.1: The discrepancies between the theoretical and experimental values for  $g_s$  for the small, medium, and big coils as well as the average value of  $g_s$ .

Coil	Theoretical $g_s$	Experimental $g_s$	$\Delta g_s$	Standard Deviation ( $\sigma$ )	$\sigma \Delta g_s$
Small	2.00	1.44	0.56	0.39	1.4 $\sigma$
Medium	2.00	1.16	0.84	0.13	6.7 $\sigma$
Big	2.00	1.89	0.11	0.83	0.13 $\sigma$
Average	2.00	1.37	0.63	0.47	1.3 $\sigma$

### 2.2 Discrepancy in Line Width

The line width itself is representative as an error in the energy, as discussed above. Therefore, an uncertainty will not be calculated in the line width, as it is already a calculation involving uncertainties of constituent quantities. However,

the experimental value can be compared to an acceptable range given by literature sources. The experimental line width was determined to be  $\delta B_0 = 0.49 \text{ mT}$ . The range given for theoretical line width was  $[0.15, 0.81] \text{ mT}$ , thus the experimental value is within the acceptable range.

### 3 Sources of Error

A true Helmholtz coil should have a separation distance equal to that of the radius of the coil. The width of basic unit, which holds the coil, was larger than the radius of the coil. Due to this the separation was larger than the radius of the coil meaning that the magnetic field is systematically lowered, as a true Helmholtz coil is designed to maximize the constructive interference. Furthermore, because the magnetic field is systematically lower than the prediction, the calculated  $g_s$  is systematically raised in value. Both the Helmholtz coil and the  $1 \text{ M}\Omega$  resistor heated up during the experimentation process. The heating of the equipment causes the energy in the system to be systematically lowered as the thermal energy transfers from the system to the surrounding region. This means that the value of the  $g_s$  is lowered compared to prediction. As the resistor heats up the resistance changes, however this was accounted for, and the resistance was remeasured for every trial and the measured value was used in the calculations.

### 4 Conclusion

Within the experiment, the value of  $g_s$  was calculated for small, medium, and big coils, and these values were used to determine an average  $g_s$  value. The  $g_s$  value for the small coil was calculated to be  $1.44 \pm 65 \times 10^{-3}$  with a discrepancy from the theoretical value of  $1.4\sigma$ . The  $g_s$  value of the medium coil is  $1.16 \pm 18 \times 10^{-3}$  with a discrepancy of  $6.7\sigma$ , and the  $g_s$  value of the big coil is  $1.89 \pm 210 \times 10^{-3}$  with a discrepancy of  $0.13\sigma$ . The average  $g_s$  value is  $1.37 \pm 0.05$  with a discrepancy of  $1.3\sigma$ . Additionally, the resonant frequency of DPPH was determined graphically from Figure 2 to be 41 MHz.



## 5 Appendices

### 5.1 Appendix A: Data

#### 5.1.1 Frequency & Voltage

Table 5.2: The recorded frequencies and corresponding voltages used to determine resonant frequency. These values are plotted in Figure 2.

Frequency (MHz)	Voltage (V)
$35 \pm 0.01$	$0.05 \pm 0.02$
$40 \pm 0.01$	$0.53 \pm 0.02$
$45 \pm 0.01$	$0.40 \pm 0.02$
$50 \pm 0.01$	$0.16 \pm 0.02$
$35 \pm 0.01$	$0.25 \pm 0.02$
$37 \pm 0.01$	$0.30 \pm 0.02$
$38 \pm 0.01$	$0.36 \pm 0.02$
$39 \pm 0.01$	$0.46 \pm 0.02$
$40 \pm 0.01$	$0.65 \pm 0.02$
$41 \pm 0.01$	$0.89 \pm 0.02$
$41 \pm 0.01$	$0.98 \pm 0.02$
$43 \pm 0.01$	$0.75 \pm 0.02$
$44 \pm 0.01$	$0.85 \pm 0.02$
$45 \pm 0.01$	$0.58 \pm 0.02$
$46 \pm 0.01$	$0.44 \pm 0.01$
$47 \pm 0.01$	$0.34 \pm 0.01$
$47 \pm 0.01$	$0.29 \pm 0.01$
$49 \pm 0.01$	$0.25 \pm 0.01$
$41 \pm 0.01$	$0.95 \pm 0.01$
$41 \pm 0.01$	$0.99 \pm 0.01$
$43 \pm 0.01$	$0.72 \pm 0.01$
$41 \pm 0.20$	$1.01 \pm 0.01$
$41 \pm 0.10$	$1.00 \pm 0.02$
$42 \pm 0.01$	$0.94 \pm 0.04$
$43 \pm 0.01$	$0.64 \pm 0.01$
$43 \pm 0.01$	$0.68 \pm 0.01$
$43 \pm 0.01$	$0.85 \pm 0.02$

### 5.1.2 Small Coil

Table 5.3: The data collected from measurements with the small coil and data calculated from those measurements.

Frequency (MHz)	Voltage (V)	Resistance ( $\Omega$ )	G-Factor
$75 \pm 0.01$	$1.4 \pm 0.2$	$1.6 \pm 0.1$	$1.43 \pm 222 \times 10^{-3}$
$75 \pm 0.01$	$1.5 \pm 0.2$	$1.6 \pm 0.1$	$1.33 \pm 196 \times 10^{-3}$
$75 \pm 0.01$	$2.6 \pm 0.2$	$1.6 \pm 0.1$	$0.77 \pm 76.1 \times 10^{-3}$
$75 \pm 0.01$	$2.9 \pm 0.2$	$1.6 \pm 0.1$	$0.69 \pm 64.1 \times 10^{-3}$
$80 \pm 0.01$	$1.5 \pm 0.1$	$1.5 \pm 0.1$	$1.33 \pm 125 \times 10^{-3}$
$80 \pm 0.01$	$1.5 \pm 0.1$	$1.5 \pm 0.1$	$1.33 \pm 125 \times 10^{-3}$
$80 \pm 0.01$	$1.5 \pm 0.1$	$1.5 \pm 0.1$	$1.33 \pm 125 \times 10^{-3}$
$80 \pm 0.01$	$2.8 \pm 0.1$	$1.5 \pm 0.1$	$0.71 \pm 53.9 \times 10^{-3}$
$85 \pm 0.01$	$1.2 \pm 0.1$	$1.5 \pm 0.1$	$1.84 \pm 202 \times 10^{-3}$
$85 \pm 0.01$	$1.1 \pm 0.1$	$1.5 \pm 0.1$	$2.02 \pm 235 \times 10^{-3}$
$85 \pm 0.01$	$2.0 \pm 0.1$	$1.5 \pm 0.1$	$1.06 \pm 88.3 \times 10^{-3}$
$85 \pm 0.01$	$2.0 \pm 0.1$	$1.5 \pm 0.1$	$1.06 \pm 88.3 \times 10^{-3}$
$90 \pm 0.01$	$2.2 \pm 0.1$	$1.5 \pm 0.1$	$1.05 \pm 85.2 \times 10^{-3}$
$90 \pm 0.01$	$2.0 \pm 0.1$	$1.5 \pm 0.1$	$1.13 \pm 93.9 \times 10^{-3}$
$90 \pm 0.01$	$1.3 \pm 0.1$	$1.5 \pm 0.1$	$1.80 \pm 188 \times 10^{-3}$
$90 \pm 0.01$	$1.2 \pm 0.1$	$1.5 \pm 0.1$	$1.88 \pm 200 \times 10^{-3}$
$95 \pm 0.01$	$1.4 \pm 0.1$	$1.6 \pm 0.1$	$1.80 \pm 171 \times 10^{-3}$
$95 \pm 0.01$	$1.4 \pm 0.1$	$1.6 \pm 0.1$	$1.80 \pm 171 \times 10^{-3}$
$95 \pm 0.01$	$2.1 \pm 0.1$	$1.6 \pm 0.1$	$1.20 \pm 94.5 \times 10^{-3}$
$95 \pm 0.01$	$2.1 \pm 0.1$	$1.6 \pm 0.1$	$1.20 \pm 94.5 \times 10^{-3}$
$10 \pm 0.01$	$2.2 \pm 0.1$	$1.6 \pm 0.1$	$1.21 \pm 93.5 \times 10^{-3}$
$10 \pm 0.01$	$2.2 \pm 0.1$	$1.6 \pm 0.1$	$1.24 \pm 96.4 \times 10^{-3}$
$10 \pm 0.01$	$1.5 \pm 0.1$	$1.6 \pm 0.1$	$1.78 \pm 162 \times 10^{-3}$
$10 \pm 0.01$	$1.5 \pm 0.1$	$1.6 \pm 0.1$	$1.84 \pm 171 \times 10^{-3}$
$10 \pm 0.01$	$1.5 \pm 0.1$	$1.6 \pm 0.1$	$1.86 \pm 170 \times 10^{-3}$
$10 \pm 0.01$	$1.5 \pm 0.1$	$1.6 \pm 0.1$	$1.86 \pm 170 \times 10^{-3}$
$10 \pm 0.01$	$2.3 \pm 0.1$	$1.6 \pm 0.1$	$1.21 \pm 92.5 \times 10^{-3}$
$10 \pm 0.01$	$2.3 \pm 0.1$	$1.6 \pm 0.1$	$1.21 \pm 92.5 \times 10^{-3}$
$11 \pm 0.01$	$2.4 \pm 0.1$	$1.5 \pm 0.1$	$1.17 \pm 92.4 \times 10^{-3}$
$11 \pm 0.01$	$1.6 \pm 0.1$	$1.5 \pm 0.1$	$1.72 \pm 157 \times 10^{-3}$
$12 \pm 0.01$	$1.7 \pm 0.1$	$1.6 \pm 0.1$	$1.86 \pm 161 \times 10^{-3}$
$12 \pm 0.01$	$2.4 \pm 0.1$	$1.6 \pm 0.1$	$1.28 \pm 95.8 \times 10^{-3}$
$12 \pm 0.01$	$2.4 \pm 0.1$	$1.6 \pm 0.1$	$1.36 \pm 103 \times 10^{-3}$
$12 \pm 0.01$	$1.5 \pm 0.1$	$1.6 \pm 0.1$	$2.13 \pm 195 \times 10^{-3}$
$13 \pm 0.01$	$1.6 \pm 0.1$	$1.6 \pm 0.1$	$2.08 \pm 184 \times 10^{-3}$
$13 \pm 0.01$	$2.4 \pm 0.1$	$1.6 \pm 0.1$	$1.39 \pm 104 \times 10^{-3}$

### 5.1.3 Medium Coil

Table 5.4: The data collected from measurements with the medium coil and data calculated from those measurements.

Frequency (MHz)	Voltage (V)	Resistance ( $\Omega$ )	G-Factor
$43 \pm 0.01$	$0.90 \pm 0.10$	$1.5 \pm 0.1$	$1.18 \pm 153 \times 10^{-3}$
$44 \pm 0.01$	$0.80 \pm 0.10$	$1.2 \pm 0.1$	$1.11 \pm 166 \times 10^{-3}$
$44 \pm 0.01$	$0.82 \pm 0.10$	$1.2 \pm 0.1$	$1.08 \pm 160 \times 10^{-3}$
$44 \pm 0.01$	$0.82 \pm 0.10$	$1.2 \pm 0.1$	$1.08 \pm 160 \times 10^{-3}$
$44 \pm 0.01$	$0.80 \pm 0.10$	$1.2 \pm 0.1$	$1.11 \pm 166 \times 10^{-3}$
$45 \pm 0.01$	$0.96 \pm 0.04$	$1.2 \pm 0.1$	$0.94 \pm 87.2 \times 10^{-3}$
$45 \pm 0.01$	$0.92 \pm 0.04$	$1.2 \pm 0.1$	$0.98 \pm 91.8 \times 10^{-3}$
$45 \pm 0.01$	$0.80 \pm 0.04$	$1.2 \pm 0.1$	$1.12 \pm 109 \times 10^{-3}$
$45 \pm 0.01$	$0.76 \pm 0.04$	$1.2 \pm 0.1$	$1.18 \pm 117 \times 10^{-3}$
$46 \pm 0.01$	$0.96 \pm 0.04$	$1.4 \pm 0.1$	$1.11 \pm 92.1 \times 10^{-3}$
$46 \pm 0.01$	$0.94 \pm 0.04$	$1.4 \pm 0.1$	$1.14 \pm 94.6 \times 10^{-3}$
$46 \pm 0.01$	$0.80 \pm 0.04$	$1.4 \pm 0.1$	$1.34 \pm 117 \times 10^{-3}$
$46 \pm 0.01$	$0.84 \pm 0.04$	$1.4 \pm 0.1$	$1.27 \pm 109 \times 10^{-3}$
$47 \pm 0.01$	$0.96 \pm 0.04$	$1.3 \pm 0.1$	$1.06 \pm 92.8 \times 10^{-3}$
$47 \pm 0.01$	$1.00 \pm 0.04$	$1.3 \pm 0.1$	$1.02 \pm 88.3 \times 10^{-3}$
$47 \pm 0.01$	$0.82 \pm 0.04$	$1.3 \pm 0.1$	$1.24 \pm 113 \times 10^{-3}$
$47 \pm 0.01$	$0.84 \pm 0.04$	$1.3 \pm 0.1$	$1.21 \pm 110 \times 10^{-3}$
$48 \pm 0.01$	$0.84 \pm 0.04$	$1.3 \pm 0.1$	$1.24 \pm 112 \times 10^{-3}$
$48 \pm 0.01$	$0.82 \pm 0.04$	$1.3 \pm 0.1$	$1.27 \pm 115 \times 10^{-3}$
$48 \pm 0.01$	$1.00 \pm 0.04$	$1.3 \pm 0.1$	$1.04 \pm 90.0 \times 10^{-3}$
$48 \pm 0.01$	$1.00 \pm 0.04$	$1.3 \pm 0.1$	$1.04 \pm 90.0 \times 10^{-3}$
$49 \pm 0.01$	$1.00 \pm 0.04$	$1.3 \pm 0.1$	$1.06 \pm 91.9 \times 10^{-3}$
$49 \pm 0.01$	$1.00 \pm 0.04$	$1.3 \pm 0.1$	$1.06 \pm 91.9 \times 10^{-3}$
$49 \pm 0.01$	$0.90 \pm 0.04$	$1.3 \pm 0.1$	$1.18 \pm 105 \times 10^{-3}$
$49 \pm 0.01$	$0.86 \pm 0.04$	$1.3 \pm 0.1$	$1.23 \pm 111 \times 10^{-3}$
$50 \pm 0.01$	$0.90 \pm 0.04$	$1.2 \pm 0.1$	$1.11 \pm 105 \times 10^{-3}$
$50 \pm 0.01$	$0.88 \pm 0.04$	$1.2 \pm 0.1$	$1.14 \pm 108 \times 10^{-3}$
$50 \pm 0.01$	$1.00 \pm 0.04$	$1.2 \pm 0.1$	$1.00 \pm 92.6 \times 10^{-3}$
$50 \pm 0.01$	$1.00 \pm 0.04$	$1.2 \pm 0.1$	$1.00 \pm 92.6 \times 10^{-3}$
$51 \pm 0.01$	$0.95 \pm 0.10$	$1.3 \pm 0.1$	$1.16 \pm 151 \times 10^{-3}$
$51 \pm 0.01$	$1.00 \pm 0.10$	$1.3 \pm 0.1$	$1.10 \pm 139 \times 10^{-3}$

Frequency (MHz)	Voltage (V)	Resistance ( $\Omega$ )	G-Factor
$55 \pm 0.01$	$1.00 \pm 0.10$	$1.4 \pm 0.1$	$1.28 \pm 158 \times 10^{-3}$
$55 \pm 0.01$	$0.96 \pm 0.10$	$1.4 \pm 0.1$	$1.34 \pm 169 \times 10^{-3}$
$60 \pm 0.01$	$0.96 \pm 0.04$	$1.3 \pm 0.1$	$1.35 \pm 118 \times 10^{-3}$
$60 \pm 0.01$	$0.94 \pm 0.04$	$1.3 \pm 0.1$	$1.38 \pm 121 \times 10^{-3}$
$65 \pm 0.01$	$0.90 \pm 0.04$	$1.3 \pm 0.1$	$1.56 \pm 139 \times 10^{-3}$
$40 \pm 0.01$	$0.74 \pm 0.04$	$1.3 \pm 0.1$	$1.17 \pm 110 \times 10^{-3}$
$40 \pm 0.01$	$0.68 \pm 0.04$	$1.3 \pm 0.1$	$1.27 \pm 123 \times 10^{-3}$
$40 \pm 0.01$	$0.84 \pm 0.04$	$1.3 \pm 0.1$	$1.03 \pm 92.9 \times 10^{-3}$
$40 \pm 0.01$	$0.82 \pm 0.04$	$1.3 \pm 0.1$	$1.05 \pm 95.9 \times 10^{-3}$
$35 \pm 0.01$	$0.60 \pm 0.04$	$1.3 \pm 0.1$	$1.26 \pm 128 \times 10^{-3}$
$35 \pm 0.01$	$0.60 \pm 0.04$	$1.3 \pm 0.1$	$1.26 \pm 128 \times 10^{-3}$
$35 \pm 0.01$	$0.76 \pm 0.04$	$1.3 \pm 0.1$	$1.00 \pm 92.8 \times 10^{-3}$
$35 \pm 0.01$	$0.68 \pm 0.04$	$1.3 \pm 0.1$	$1.11 \pm 108 \times 10^{-3}$
$30 \pm 0.01$	$0.60 \pm 0.04$	$1.4 \pm 0.1$	$1.16 \pm 113 \times 10^{-3}$
$30 \pm 0.01$	$0.52 \pm 0.04$	$1.4 \pm 0.1$	$1.34 \pm 141 \times 10^{-3}$
$30 \pm 0.01$	$0.60 \pm 0.04$	$1.4 \pm 0.1$	$1.16 \pm 113 \times 10^{-3}$
$30 \pm 0.01$	$0.60 \pm 0.04$	$1.4 \pm 0.1$	$1.16 \pm 113 \times 10^{-3}$

#### 5.1.4 Big Coil

Table 5.5: The data collected from measurements with the big coil and data calculated from those measurements. The omitted data point is highlighted.

Frequency (MHz)	Voltage (V)	Resistance ( $\Omega$ )	G-Factor
$30 \pm 0.01$	$0.44 \pm 0.08$	$1.7 \pm 0.1$	$1.93 \pm 369 \times 10^{-3}$
$30 \pm 0.01$	$0.60 \pm 0.08$	$1.7 \pm 0.1$	$1.41 \pm 206 \times 10^{-3}$
$30 \pm 0.01$	$0.56 \pm 0.08$	$1.7 \pm 0.1$	$1.52 \pm 234 \times 10^{-3}$
$30 \pm 0.01$	$0.50 \pm 0.08$	$1.7 \pm 0.1$	$1.70 \pm 289 \times 10^{-3}$
$25 \pm 0.01$	$0.40 \pm 0.10$	$1.8 \pm 0.1$	$1.87 \pm 479 \times 10^{-3}$
$25 \pm 0.01$	$0.44 \pm 0.10$	$1.8 \pm 0.1$	$1.70 \pm 398 \times 10^{-3}$
$25 \pm 0.01$	$0.40 \pm 0.10$	$1.8 \pm 0.1$	$1.87 \pm 479 \times 10^{-3}$
$25 \pm 0.01$	$0.50 \pm 0.10$	$1.8 \pm 0.1$	$1.50 \pm 311 \times 10^{-3}$
$20 \pm 0.01$	$0.20 \pm 0.04$	$1.9 \pm 0.1$	$3.17 \pm 654 \times 10^{-3}$
$20 \pm 0.01$	$0.40 \pm 0.04$	$1.9 \pm 0.1$	$1.58 \pm 179 \times 10^{-3}$
$20 \pm 0.01$	$0.36 \pm 0.04$	$1.9 \pm 0.1$	$1.76 \pm 216 \times 10^{-3}$
$20 \pm 0.01$	$0.40 \pm 0.04$	$1.9 \pm 0.1$	$1.58 \pm 179 \times 10^{-3}$
$15 \pm 0.01$	$0.04 \pm 0.01$	$2.1 \pm 0.1$	$13.1 \pm 3.33$
$15 \pm 0.01$	$0.48 \pm 0.08$	$2.1 \pm 0.1$	$1.09 \pm 189 \times 10^{-3}$
$15 \pm 0.01$	$0.40 \pm 0.04$	$2.1 \pm 0.1$	$1.31 \pm 145 \times 10^{-3}$
$15 \pm 0.01$	$0.12 \pm 0.04$	$2.1 \pm 0.1$	$4.36 \pm 1.47$

## 5.2 Appendix B: Source Code

### 5.2.1 Error Propagation and Data Processing

---

```
1  # Run calculations on the recorded data.
2
3  # -----BEGIN PACKAGES-----
4
5  # for reading data
6  using CSV
7
8  # for storing data
9  using DataFrames
10
11 # for displaying data
12 using Printf
13
14 # for getting the mean
15 using Statistics
16
17 # -----END PACKAGES-----
18
19 # -----BEGIN EXTRACT DATA-----
20
21 # get path to the data file from the command line
22 filepath = ARGS[1]
23
24 # throw the data into a DataFrame
25 dataRaw = CSV.File(filepath) |> DataFrame
26
27 # put two columns from a data frame into a list of tuples
28 getListOfPairsDF(dataFrame, col1, col2) =
29     [ (dataFrame[col1][ii], dataFrame[col2][ii]) for ii in 1:size(dataFrame)[1] ]
30
31 # put combine two lists (of same size) into a list of pairs
32 getListOfPairs(l1, l2) = [ (l1[ii], l2[ii]) for ii in 1:length(l1) ]
33
34 # get the values and corresponding errors in the voltages
35 volts = getListOfPairsDF(dataRaw, :Voltage, :VError)
36
37 # get the values and corresponding errors in the resistances
38 ress = getListOfPairsDF(dataRaw, :Resistance, :ResError)
39
40 # get the values and corresponding errors in the frequencies
41 freqs = getListOfPairsDF(dataRaw, :CorrFreq, :FreqError)
42
43 # -----END EXTRACT DATA-----
44
45 # -----BEGIN GENERAL ERROR PROPAGATION-----
46
```

```

47 # get  $\frac{\delta x^2}{x}$ 
48 sqErrRatio(dataPt) = (dataPt[2] / dataPt[1]) ^ 2
49
50 # get the uncertainty of a value calculated via multiplication of measurements
51 propUncertMult(val, meas) = val * sqrt(sum(map(sqErrRatio, meas)))
52
53 # -----END GENERAL ERROR PROPAGATION-----
54
55 # -----BEGIN CURRENT-----
56
57 # get the calculated values for the current
58 currVals = [ volts[ii][1] / ress[ii][1] for ii in 1:length(volts) ]
59
60 # get the uncertainties of the current
61 currUncerts = [ propUncertMult(currVals[ii], [ volts[ii], ress[ii] ])
62                 for ii in 1:length(currVals) ]
63
64 # combine the values and uncertainties into pairs
65 currs = getListOfPairs(currVals, currUncerts)
66
67 # -----END CURRENT-----
68
69 # -----BEGIN MAGNETIC FIELD-----
70
71 # given a current, find the resultant magnetic field using:
72 #  $B = \mu_0 \frac{4}{5} \frac{n}{r}$ 
73 magField(curr) = (4e-7 * pi) * ((4/5)^(3/2)) * (320 / 0.067) * curr
74
75 # get the value-uncertainty pairs for the magnetic field
76 magFields = getListOfPairs(map(magField, map(x -> x[1], currs)),
77                             map(magField, map(x -> x[2], currs)))
78
79 # -----END MAGNETIC FIELD-----
80
81 # -----BEGIN G-FACTOR-----
82
83 # calculate the g-factor from the magnetic field and frequency:
84 #  $\frac{h\nu}{\mu_B B}$ 
85 gFac(freq, magField) = (6.626e-34 * freq) / (9.274e-24 * magField)
86
87 # get the g-factors from frequency and magnetic field lists of equal length
88 gFacVals = [ gFac(freqs[ii][1], magFields[ii][1]) for ii in 1:length(freqs) ]
89
90 # get the uncertainties in the g-factors
91 gFacErrs = [ propUncertMult(gFacVals[ii], [ freqs[ii], magFields[ii] ])
92             for ii in 1:length(gFacVals) ]
93
94 # combine the values and errors into a list of pairs
95 gFacs = getListOfPairs(gFacVals, gFacErrs)

```

```

96
97  # the mean of the g-factors
98  meanGFac = mean(gFacVals)
99
100 # the mean of the uncertainties of the g-factors
101 meanGFacErr = std(gFacVals) / sqrt(length(gFacVals))
102
103 # -----END G-FACTOR-----
104
105 # -----BEGIN MISC DATA DISPLAY-----
106
107 # display mean g-factor info
108 @printf("%.4e %.4e\n", meanGFac, meanGFacErr)
109
110 # display standard deviation of g-factors
111 @printf("%.4e\n", std(gFacVals))
112
113 # display g-factor error
114 for fac in gFacErrs
115     @printf("%.4e\n", fac)
116 end
117
118 # -----END MISC DATA DISPLAY-----

```

---