PHY 4210-01 Senior Lab
Lab P2: Electron Spin Resonance

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Abstract

The Lande factor, g_s, (or the gyromagnetic ratio of spin) for the electron was determined through the use of electron spin resonance and Helmholtz coils. The g-factor of a diphenyl-picryl-hydrazyl (DPPH) sample was obtained following the measurement of the frequency dependence of the resonance field. The line width of the resonance signal was then calculated.

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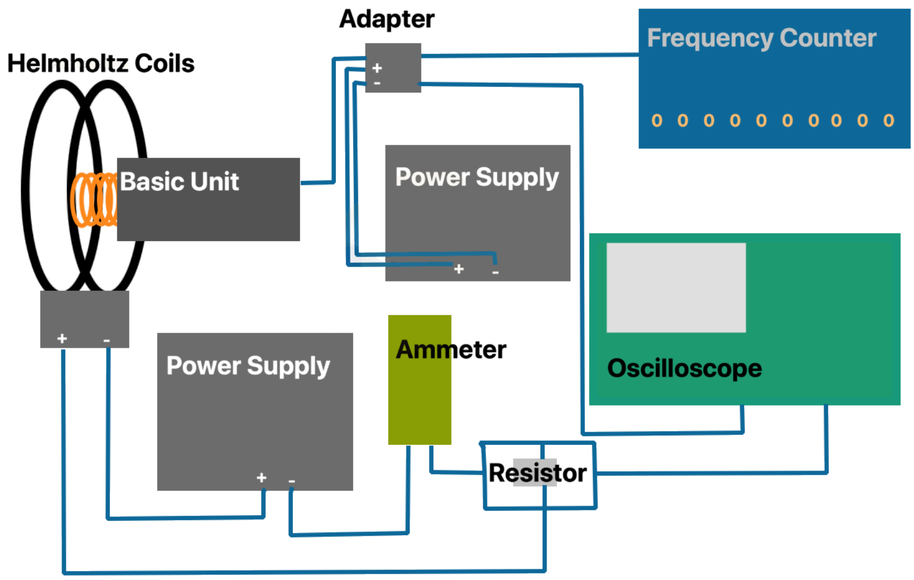

Figure 1: Schematic of equipment used in experiment

1 Data Analysis

1.1 Frequency Dependence of Resonance Field

Voltage was compared to frequency to obtain a graphical relationship for the frequency dependence of the resonance field. The amplitude voltage was obtained by measuring the peak-to-peak voltage from the oscilloscope and dividing it in half. The peak of **??** is the specific resonance frequency for the field. This value is a voltage amplitude of 1.01 V and a frequency of $4.18 * 10^7$ Hz. It is important to note that the electron spin resonance device divides the frequency by a factor of a thousand, and thus the Hewlett-Packard frequency counter displayed a corrected frequency. The calculations require an uncorrected frequency, i.e. the counter's frequency multiplied by one thousand.

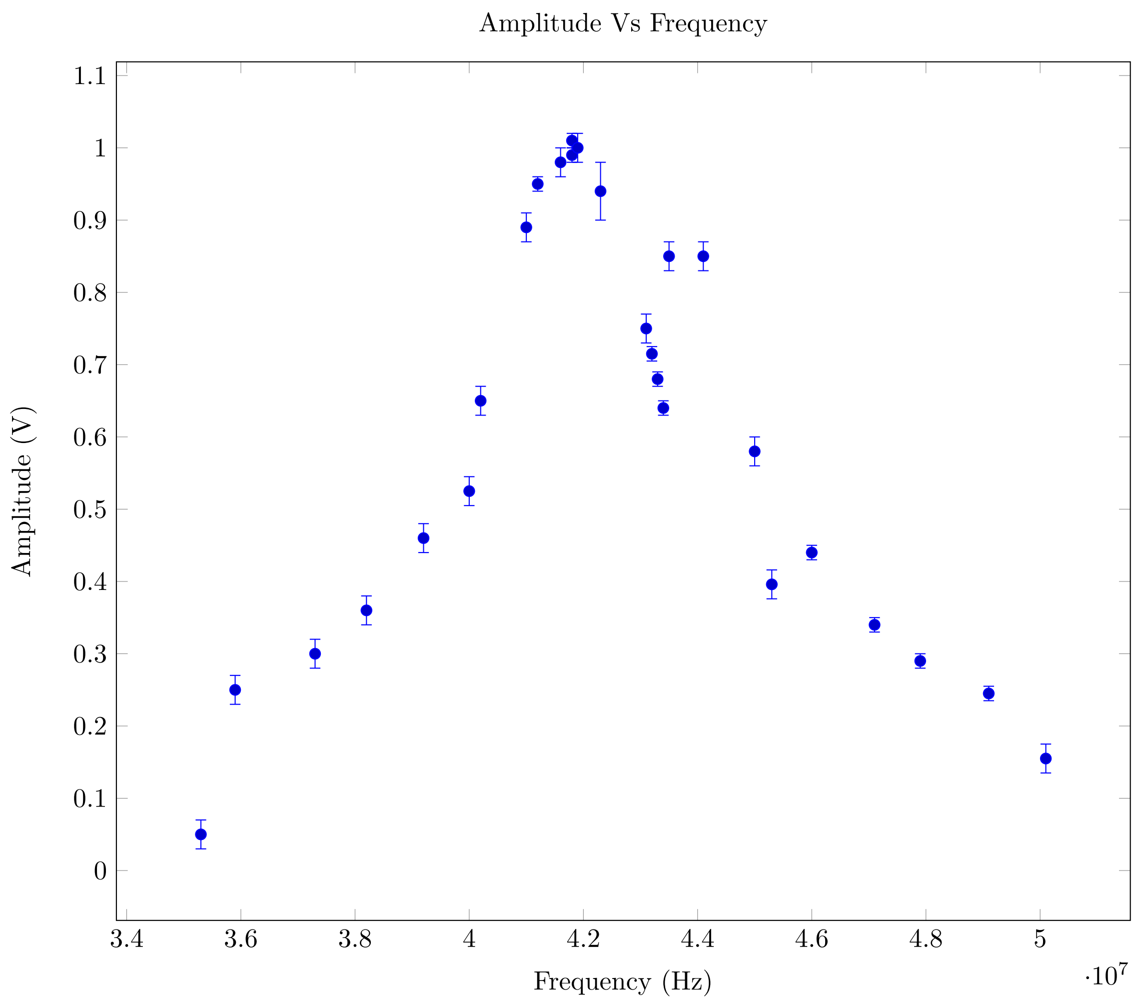

Figure 2: Graphic representation of the frequency dependence of the resonance field.

1.2 Propagation of Uncertainty in the Frequency Dependence of the Resonance Field

1.3 Experimental Value of Gyromagnetic Ratio

The gyromagnetic ratio is calculated using the following equation, where ν is the frequency, h is Planck's constant, μ_B is the Bohr magneton, and B_0 is the magnetic field strength.

$$g_s = \frac{h \times \nu}{\mu_B \times B_0} \tag{1}$$

The magnetic field used in calculating equation 1 must be calculated as well. It is determined from the measured current using equation 2, where $\mu_0 = 4\pi \times 10^{-7} \frac{Vs}{Am}$, the number of turns is $n = 320$, and the radius of the coils is

$r = 6.8cm$.

$$B_0 = \mu_0 \left(\frac{4}{5}\right)^{3/2} \times \frac{n}{r} \times I \tag{2}$$

Rather than measuring the current directly, the current is calculated by measuring the voltage drop across a resistor, of which the resistance is also measured. This calculation is shown below in equation.

$$I = \frac{V}{R} \tag{3}$$

By substituting equation 3 into 2, and then substituting equation 2 into equation 1, we arrive at an expression for the gyromagnetic ratio in terms of known constants and measured quantities. This final expression is shown in equation 4.

$$g_s = \frac{h \times \nu}{\mu_B \times \left(\mu_0 \left(\frac{4}{5}\right)^{3/2} \times \frac{n}{r} \times \frac{V}{R}\right)} \tag{4}$$

1.4 Propagating Uncertainty in Gyromagnetic Ratio

The error in the experimental value of the gyromagnetic ratio is determined by propagating uncertainty in equation 4. There are no uncertainties associated with fundamental constants such as h, μ_B, and μ_0. It is assumed that the number of coil turns, n, also has no associated uncertainty because it was reported in the manual as such. The uncertainty in the radius is constant for all measurements, ,but the frequency, voltage, and resistance will differ for each measurement. Equation 5 shows this error propagation.

$$\delta g_s = g_s \times \sqrt{\left(\frac{\delta \nu}{\nu}\right)^2 + \left(\frac{\delta r}{r}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta R}{R}\right)^2} \tag{5}$$

An example calculation for the value of g_s and its propagated uncertainty is shown below for a measurement taken with the large coil:

$$\begin{aligned} g_s &= \frac{6.626 \times 10^{-34} \times \left(3 \times 10^7\right)}{\mu_B \times \left(4\pi \times 10^{-7} \left(\frac{4}{5}\right)^{3/2} \times \frac{320}{0.068} \times \frac{0.44}{1.7}\right)} \\ &= 1.93 \end{aligned}$$

$$\begin{aligned} \delta g_s &= g_s \times \sqrt{\left(\frac{1.00 \times 10^4\text{Hz}}{3.00 \times 10^7\text{Hz}}\right)^2 + \left(\frac{0.5\text{cm}}{6.7\text{cm}}\right)^2 + \left(\frac{0.1\text{V}}{2\text{V}}\right)^2 + \left(\frac{0.1\Omega}{1.7\Omega}\right)^2} \\ &= 1.93 \times \sqrt{(3.3 \times 10^{-4})^2 + (0.006)^2 + (0.05)^2 + (0.06)^2} \\ &= .15 \end{aligned}$$

We can calculate the discrepancy between the experimental and theoretical values as follows. Recall the theoretical value of g_s for DPPH is 2.0036, which is approximated as 2.00 due to the limited precision of the experimental value.

$$\begin{aligned}\Delta g_s &= |g_{s_{exp}} - g_{s_{theo}}| \\ &= |1.93 - 2.00| \\ &= 0.074\end{aligned}$$

Evidently, this difference Δg_s is less than $1\sigma = .15$.

1.5 Rejection of Data

During the data taking process for the "big coil", a measurement at a particular frequency produced an experimental g_s value that seemed anomalous; most measurements fall between 1 and 4, but this measurement is around 13. Chauvenet's criterion will be used to determine if this datum should be discarded.

If one assumes this measurement to be valid, the resultant average and standard deviation are 2.59 ± 2.91 (quite an atrocity). The measurement in question, 13.08, differs from the average by 4.49σ. If a Gaussian distribution is assumed for the g_s values, the probability of obtaining a measurement that differs from the mean by this quantity is determined as follows:

$$\begin{aligned}Prob(\text{outside } 4.49\sigma) &= 1 - Prob(\text{within } 4.49\sigma) \\ &= 1 - .9999994 \\ &= 0\end{aligned}$$

Since the probability of a measurement being withing 4.49σ is so high, the probability of this measurement being outside this interval is effectively zero. Therefore, we can discard the anomalous datum with extremely high confidence.

1.6 Determining Line Width of Resonance Signal

δB_0 is representative of an absorption line, and is obtained when the energy is measured at a fixed frequency as function of the magnetic field. The line width δB_0 is used as an expression of the uncertainty in the energy of the transition. This is best represented by the equation $\delta E = g \times \mu_0 \times \delta B_0$. Using the uncertainty principle a relation is then found for δB_0.

$$\delta B_0 = \frac{\hbar}{2 \times g_J \times \mu_B \times T}$$

where T is the lifetime of the level and g_J is the Land $\acute{e}$ factor. Experimentally δB_0 can be determined by the following equation:

$$\delta B_0 = B \times \left(\frac{\delta I}{I_{mod}}\right)$$

where δI is represented as $\frac{\delta U}{U_{mod}} \times I_{mod} \times 2\sqrt{2}$.

$$
\begin{aligned}
\delta I &= \frac{\delta U}{U_{mod}} \times I_{mod} \times 2\sqrt{2} \\
&= \frac{0.55}{2} \times 0.156 \times 2\sqrt{2} \\
&= 0.121
\end{aligned}
$$

$$
\begin{aligned}
\delta B_0 &= B \times \left(\frac{\delta I}{I_{mod}}\right) \\
&= 6.23 \times 10^{-4} \times \left(\frac{0.121}{0.156}\right) \\
&= 4.85 \times 10^{-4} T \\
&= 4.85 \times 10^{-1} mT
\end{aligned}
$$

2 Results

2.1 Discrepancy in Gyromagnetic Ratio

The discrepancy between the experimental and theoretical values of G for each of the coils can be calculated with $\Delta g_s = |g_{s_t} - g_{s_e}|$, where g_{s_t} is the theoretical value of g_s and g_{s_e} is the experimental value. The discrepancy in the value of g_s for the small coil is:

$$
\begin{aligned}
\Delta g_{s_{small}} &= |(2.00) - (1.44)| \\
\Delta g_{s_{small}} &= 0.56
\end{aligned}
$$

Since the standard deviation of the calculated values is $\sigma_{g_s,small} = 0.393$, the experimental value of g_s is $\frac{\Delta g_{s_{small}}}{\sigma_{g_s,small}} = \frac{0.56}{0.393} = 1.42\sigma$ from the theoretical value.

The discrepancies for the medium and big coils were calculated in the same way, and the results are detailed in Table 2.1.

Table 2.1: The discrepancies between the theoretical and experimental values for g_s for the small, medium, and big coils.

Coil	Theoretical g_s	Experimental g_s	Δg_s	Standard Deviation (σ)	$\sigma\Delta g_s$
Small	2.00	1.44	0.56	0.393	1.42 σ
Medium	2.00	1.16	0.84	0.125	6.72 σ
Big	2.00	1.89	0.11	0.825	0.133 σ

2.2 Discrepancy in Line Width

3 Sources of Error

4 Conclusion

5 Appendices

5.1 Appendix A: Data

5.2 Appendix B: Source Code