PHY 4210-01 Senior Lab
Lab P-5: Hall Effect in Semiconductors

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Abstract

The Hall Effect was investigated through a series of experiments using p-type, n-type, and pure Germanium, as well as pure Zinc. Experiments involved measurements and manipulation of the Hall voltage, sample voltage, current, magnetic field strength, and temperature. Temperature and current were monitored with the semiconductor mount, and field strength was measured using a Hall effect probe. Several parameters of the semiconductor materials were calculated, including the Hall constant and density of charge carriers. Because there were about 12 different experiments conducted, the results will not all be restated here. Instead, please refer to the conclusion of each respective task.

Contents

1 Theory of the Experiment

When an electric current flows through a conductor inside of a magnetic field, that field exerts a force on the charge carriers as they move. This force is the Lorentz force. The Lorentz force causes the charge carriers to accumulate on one side of the conductor, which will then balance the magnetic influence from the field and produce a voltage that can be measured across the conductor. This describes the Hall effect. The Hall voltage can be measured by a Hall probe that is placed in the magnetic field. Please note that the Hall effect is different for differing charge carriers, as the polarity of the Hall voltage differs for negative and positive charge carriers.

1.1 Deriving the Hall Voltage Equation

$$V_{(Hall)} = \frac{I * B}{n * e * t}$$

Please note that 'I' is the current through the sample, 'B' is the applied magnetic field, 'n' is the charge carrier concentration, 'e' is the elementary charge, and 't' is the sample thickness.

$$F_B = q * v * B \quad F_E = E * q = \left(\frac{V_H}{d}\right) * q \quad F_B = F_E \quad q * v * B = \left(\frac{V_H}{d}\right) * q \quad v * B = \frac{v_H}{d} \quad v_H = B * v * d \quad v = \frac{I}{n * e * A}$$

2 Hall Effect in P-Germanium

2.1 Task 1

The Hall voltage's dependence on the current through the sample was determined while holding the magnetic field and temperature constant. Please note that the manual called for the magnetic field to be held at 250 mT, but this corresponds to a current that exceeds the maximum allowed by the coils. The field was instead kept at 144 mT, which corresponds to a current of 4A.

2.1.1 Data Analysis and Results

There is a linear relationship between the Hall Voltage and the sample current, which is shown by the following equation, where α is the proportionality constant:

$$U_H = \alpha * I$$

The data was plotted in figure 1. Through the use of linear regression, the proportionality constant was determined to be $0.943 \pm 5.54 \times 10^{-3}$ V/A.
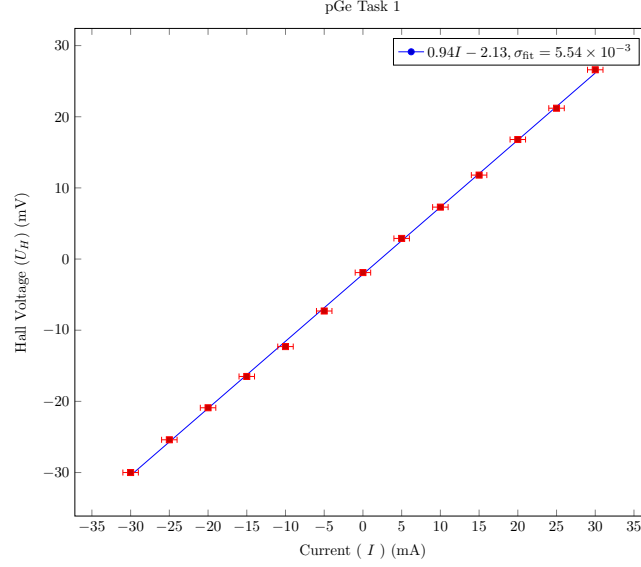
Figure 1: Hall voltage as a function of sample current in p-type Germanium. The linear fit has an R^2 value of 1.

From the equation defining the Hall Voltage, one can derive an expression for V_{Hall}/I, which represents the slope or proportionality constant α. The expression is used to obtain a value for α_{theo} below, where B is the applied magnetic field, n is the density of charge carriers, e is the elementary charge, and t is the sample thickness.

$$\alpha_{theo} = \frac{B}{net}$$
$$= \frac{0.144}{(1.5 * 10^{(}21)) * (1.6 * 10^{(} - 19)) * (1.0 * 10^{(} - 3))} \qquad = 0.6$$

2.1.2 Conclusion

The experimental constant of proportionality $\alpha = 0.943$ V/A is within 54 σ of the theoretical value $\alpha_{theo} = 0.6$ V/A. This discrepancy in the face of such a well fitting line is a sign of systematic rather than random error. Sources of systematic error include energy escaping the system or an offset in the power supply.

2.2 Task 2

The sample voltage as a function of the positive magnetic field induction was determined. The control current was held at a constant 30 mA. The resistance was computed from the sample voltage, and expressed as a change in resistance relative to the resistance with no field.

2.2.1 Data Analysis and Results

Instead of directly plotting the resistance against the field strength. A "normalized difference" was computed by dividing the total change in resistance by the resistance with no applied field, i.e. $\frac{R_m - R_0}{R_0}$, where R_m represents the measured resistance and R_0 represent the resistance at $B = 0$. The change in resistance associated with a changing magnetic field implies a change in the mean free path of the charge carriers (holes). The resultant graph in figure 2 shows a quadratic change in this expression as a function of the increasing induced magnetic field strength, which is the expected relationship between the given quantities.
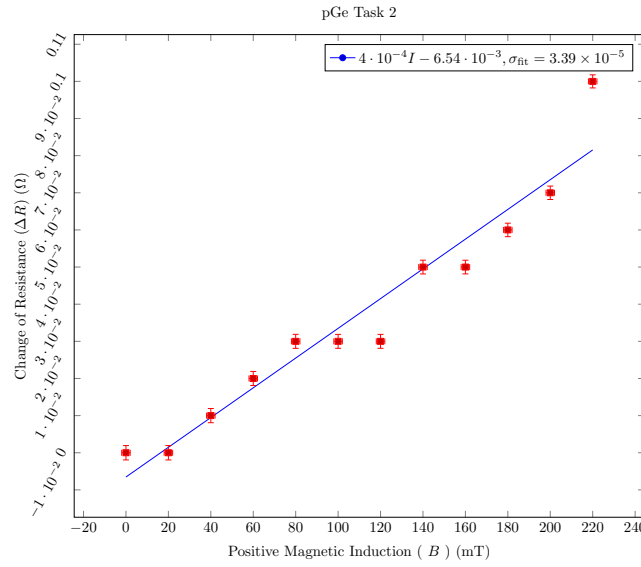

Figure 2: Normalized change in resistance against applied field strength. The determined R^2 value was 0.969

2.2.2 Conclusion

The functional relationship between normalized change in resistance and the applied magnetic field strength was quadratic, as expected.

2.3 Task 3

A constant current of 30 mA is applied, and the sample voltage is measured as a function of temperature. The magnetic field remains off for this task. The manual states that the maximum applied temperature is 140° C. In order to avoid damaging the sample, the temperature was instead limited to 110° C. Note that the sample was heated to this maximum temperature, and then the sample voltage was taken as the temperature cooled.

2.3.1 Data Analysis and Results

Sample voltage and temperature were measured, with no external field applied. The reciprocal of the voltage was plotted against the reciprocal of the temperature. The data, shown in figure 3, can be represented by a quadratic function, with a minimum occurring around 3 K^{-1}.
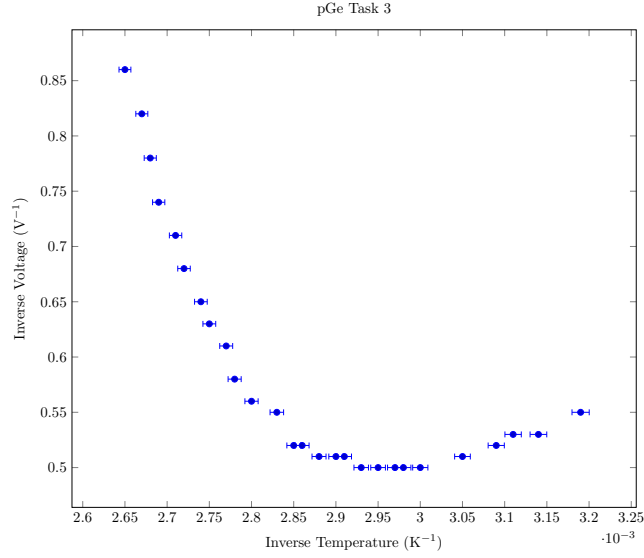

Figure 3: Inverse voltage is plotted against inverse temperature.

In order to obtain meaningful information from the data in figure 3, one can manipulate the data to uncover a linear relationship. The intrinsic conductivity can be approximated as the inverse voltage, since the temperature is constant. Conductivity is related to inverse temperature by the following function:
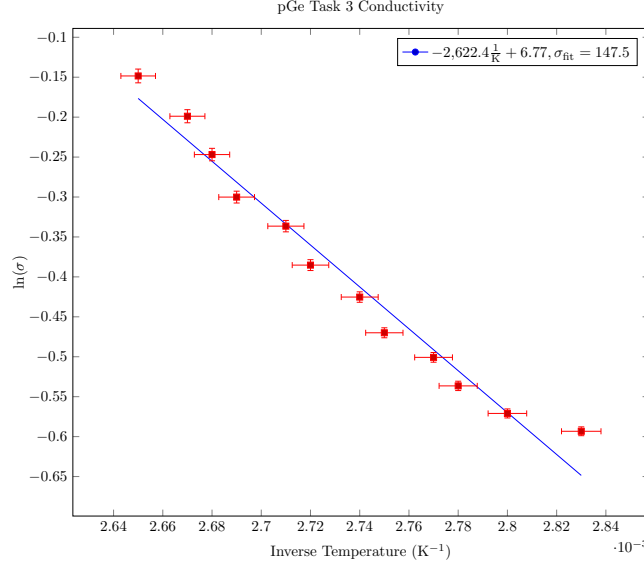
$$\sigma = \sigma_0 e^{-E_g/2kT}$$

Figure 4: Natural logarithm of conductivity versus inverse temperature

If the natural logarithm of conductivity is plotted against inverse temperature, as shown in figure 4, it will obey a linear relationship with slope equal to $-\frac{E_g}{2k}$. Linear regression yields a slope of -2622 ± 147.5 K. Recalling that k is Boltzmann's constant, we compute the band gap energy as follows.

$$
\begin{aligned}
E_g &= -\text{slope} \times 2k \\
&= -(-2622\ K) \times 2\left(8.63 \times 10^{-5}\frac{eV}{K}\right) \\
&= 0.45eV
\end{aligned}
$$

The associated error in $E_{g_{exp}}$ can be computed from the error in the slope.

$$
\begin{aligned}
\delta_{Eg_{exp}} &= 2k\sigma \\
\delta_{Eg_{exp}} &= 2(86.3\,\mu\text{eV}\,\text{K}^{-1})(148\,\text{K}) \\
\delta_{Eg_{exp}} &= 25.5\,\text{meV}
\end{aligned}
$$

The discrepancy is computed between the experimental value of E_g and the theoretical value $E_{g_{theo}} = 0.72$ eV. The associated error in the discrepancy will be taken as the error in the experimental value, since the theoretical value has no associated error.

$$
\begin{aligned}
\delta_{E_g} &= |E_{g_{exp}} - E_{g_{theo}}| \\
&= |0.49 - 0.72| \\
&= 0.23eV
\end{aligned}
$$

2.3.2 Conclusion

The experimental energy gap $E_{g_{exp}} = 0.49eV$ is within 1.82×10^{-3} σ of the theoretical value $E_{g_{theo}} = 0.72eV$, thus the values are in agreement.

2.4 Task 4

The Hall voltage was measured as a function of the magnetic induction. This time the temperature is held constant at room temperature, and the current is held constant at 30 mA. The manual states that the applied magnetic field should reach 300 mT, however, the strength was limited to 290 mT. This was done in order to avoid applying excessive current and overheating the coils. Note that the highest current applied to the coils was 5.98 Amperes.

2.4.1 Data Analysis and Results

The Hall voltage was plotted against the magnetic field strength with both positive and negative polarities. The resultant graph, shown in figure 5, was given a linear fit, with an R^2 value of 0.999.
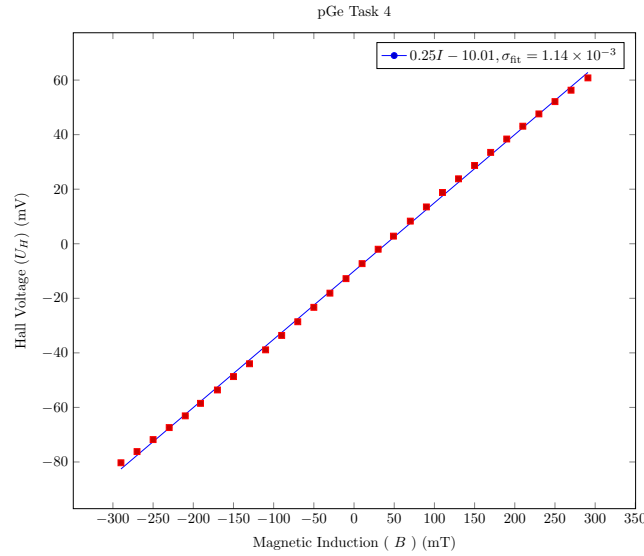

Figure 5: Hall voltage plotted as a function of applied magnetic field strength.

The experimental Hall constant is the slope, $b = 250\,\text{mV}\,\text{T}^{-1}$, of Figure (5) multiplied by $\frac{d}{I}$, where $d = 1\,\text{mm}$ is the thickness of the sample and $I = 30\,\text{mA}$ is the applied current.

$$R_{\mathrm{H}} = b\frac{d}{I}$$

$$R_{\mathrm{H}} = (0.25)\frac{(1 \times 10^{-3})}{(30 \times 10^{-3})}$$

$$R_{\mathrm{H}} = 8.33 \times 10^{-3}\,\mathrm{m}^3\,\mathrm{A}^{-1}\,\mathrm{s}^{-1}$$

Since the only variable, the slope b, is begin multiplied by the constant $\frac{d}{I} = 33.3 \times 10^{-3}$, the error in the experimental hall constant is merely the error in the slope, $\delta b = 1.14\,\mathrm{mV\,T}^{-1}$, multiplied by the constant.

$$\delta R_{\mathrm{H}} = \delta b\frac{d}{I}$$

$$\delta R_{\mathrm{H}} = (1.14 \times 10^{-3})(33.3 \times 10^{-3})$$

$$\delta R_{\mathrm{H}} = 38.0\,\mathrm{cm}^3\,\mathrm{A}^{-1}\,\mathrm{s}^{-1}$$

The theoretical value of the Hall constant is $4.17\,\mathrm{dm}^3\,\mathrm{A}^{-1}\,\mathrm{s}^{-1}$. Therefore, the discrepancy between our experimental value and this theoretical value is

$$\Delta_{R_{\mathrm{H}}} = |R_{\mathrm{H}_{\mathrm{theo}}} - R_{\mathrm{H}_{\mathrm{exp}}}|$$

$$\Delta_{R_{\mathrm{H}}} = |(4.17 \times 10^{-3}) - (8.33 \times 10^{-3})|$$

$$\Delta_{R_{\mathrm{H}}} = 4.16\,\mathrm{dm}^3\,\mathrm{A}^{-1}\,\mathrm{s}^{-1}$$

The carrier concentration is found with $p = \frac{1}{eR_{\mathrm{H}}}$, where $e = 160\,\mathrm{zC}$ is the elementary charge and R_{H} is either the theoretical or experimental value of the Hall constant. The experimental value of the carrier concentration is determined by using the experimental for the Hall constant.

$$p_{\mathrm{exp}} = \frac{1}{eR_{\mathrm{H}_{\mathrm{exp}}}}$$

$$p_{\mathrm{exp}} = \frac{1}{(160\,\mathrm{zC})(8.33\,\mathrm{dm}^3\,\mathrm{A}^{-1}\,\mathrm{s}^{-1})}$$

$$p_{\mathrm{exp}} = 750\,\mathrm{\mu m}^{-3}$$

The error in the experimental value of the carrier concentration is determined with $\delta p_{\mathrm{exp}} = |p_{\mathrm{exp}}|\frac{\delta R_{\mathrm{H}_{\mathrm{exp}}}}{R_{\mathrm{H}_{\mathrm{exp}}}}$

The theoretical carrier concentration, p_{theo}, is determined in a similar manner; the theoretical value of the Hall constant is used rather than the experimental one.

$$p_{\text{theo}} = \frac{1}{eR_{\text{H}_{\text{theo}}}}$$

$$p_{\text{theo}} = \frac{1}{(160\,\text{zC})(4.17\,\text{dm}^3\,\text{A}^{-1}\,\text{s}^{-1})}$$

$$p_{\text{theo}} = 1.50 \times 10^{21}\text{m}^{-3}$$

The discrepancy between these two values is found with

$$\Delta_p = |p_{\text{exp}} - p_{\text{theo}}|$$

$$\Delta_p = |(750\,\mu\text{m}^{-3}) - (1.50 \times 10^{21}\text{m}^{-3})|$$

$$\Delta_p = -750\,\mu\text{m}^{-3}$$

2.4.2 Conclusion

The experimental hall constant NUM was withing NUM σ of the theoretical value NUM, thus the values ARE/ARENT in agreement.

2.5 Task 5

The relationship between the Hall voltage and the temperature is determined, this time using an induced and constant magnetic field. The manual calls for the magnetic field to be held constant at 300 mT. However, in order to avoid overheating the coils, we kept the magnetic field constant at 145 mT. The current was kept steady at 30 mA for the entire task. The manual also states that the temperature should reach 140° K, however our highest temperature was 110^oK, this was done to prevent the overheating of the sample.

2.5.1 Data Analysis and Results

The graph demonstrates a decrease in the Hall voltage as the temperature increases due to an increase in the number of charge carries which causes the drift velocity of the charge to decrease.
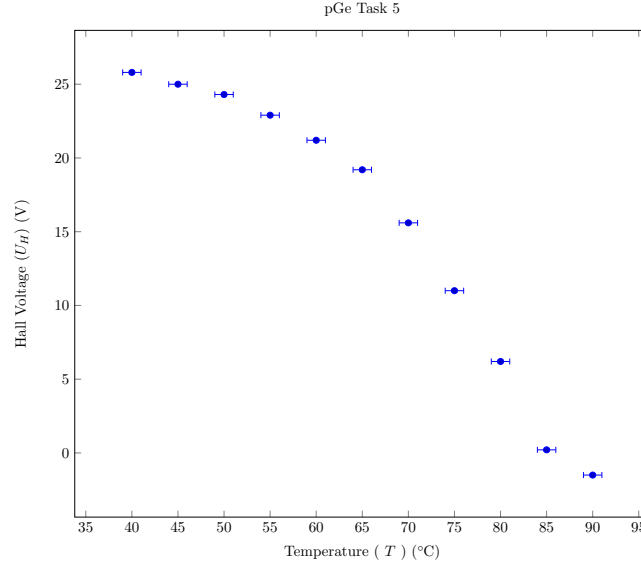
Figure 6: Hall voltage is plotted as a function of temperature.

The shape of the graph can be described by three differing regions. The first relatively flat area in the region of the lower temperatures depicts an extrinsic conduction. The middle section of the graph that is described by the slope shows the reduction in the drift velocity that is associated with the increase of charge carriers with an increase of temperature. The third area, which is once again relatively flat, described by the higher temperatures depicts the intrinsic conduction. While the graph as a whole shows the transition from extrinsic conduction to intrinsic conduction.

2.5.2 Conclusion

3 Hall Effect in N-Germanium

3.1 Task 1

The dependence of the Hall voltage on the current was determined by modulating current and holding the magnetic field and temperature constant. Please note that the manual called for the magnetic field to be held at 250 mT, but it was instead kept at 144 mT in order to avoid reaching the maximum amperage on the coils. The corresponding applied coil current was 4 A.

3.1.1 Data Analysis and Results

The Hall voltage and sample current maintain a linear relationship, as seen in section 1.1, such that $U_H = \alpha * I$. This is shown in figure 7.
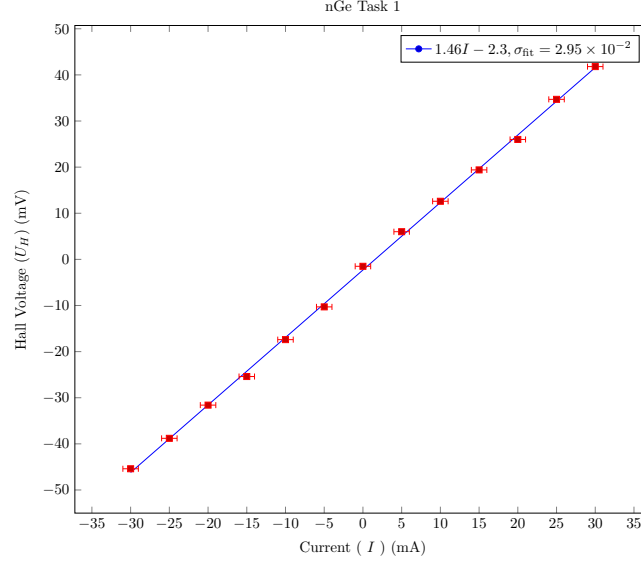
Figure 7: Hall voltage as a function of sample current in n-type Germanium. The linear fit has an R^2 value of 1.

Once again, a value for α_{theo} is determined from the slope of the plot in figure 7, where B is the applied magnetic field, n is the density of charge carriers, e is the elementary charge, and t is the sample thickness.

$$\begin{aligned} \alpha_{theo} &= \frac{B}{net} \\ &= \frac{(0.144)}{(1.5 * 10^{(21)}) * (1.6 * 10^{(-19)}) * (1.0 * 10^{(-3)})} \\ &= 0.6 \end{aligned}$$

3.1.2 Conclusion

The experimental constant of proportionality $\alpha = 1.46$ V/A is within NUMBER σ of the theoretical value $\alpha_{theo} = COPY\,FROM\,ABOVE$ V/A.

3.2 Task 2

The dependence of the sample voltage on magnetic induction was determined. The control current was held at a constant 30 mA. The resistance was computed from the sample voltage and expressed as a change in resistance relative to the resistance with no field.

3.2.1 Data Analysis and Results

Analysis was carried out in the same manner as the p-Germanium experiment. The change is resistance was normalized against the resistance with no applied field, and this was plotted against the magnetic field strength, as shown in figure 8.
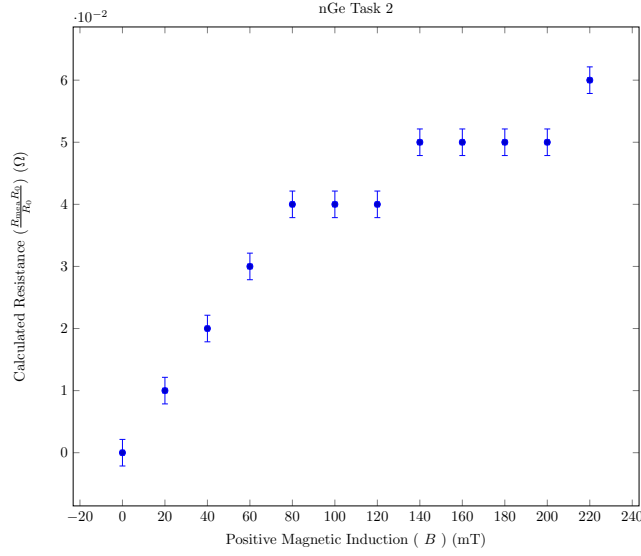
Figure 8: Normalized change in resistance against applied field strength.

3.2.2 Conclusion

There is a quadratic relationship between the normalized change in resistance and the magnetic field strength.

3.3 Task 3

A constant current of 30 mA is applied, and the sample voltage is measured as a function of temperature. The magnetic field remains off for this task. Note the maximum temperature was again limited to 110° C.

3.3.1 Data Analysis and Results

The reciprocal of the voltage was plotted against the reciprocal of the temperature. The data, shown in figure 9, is represented by a quadratic function.
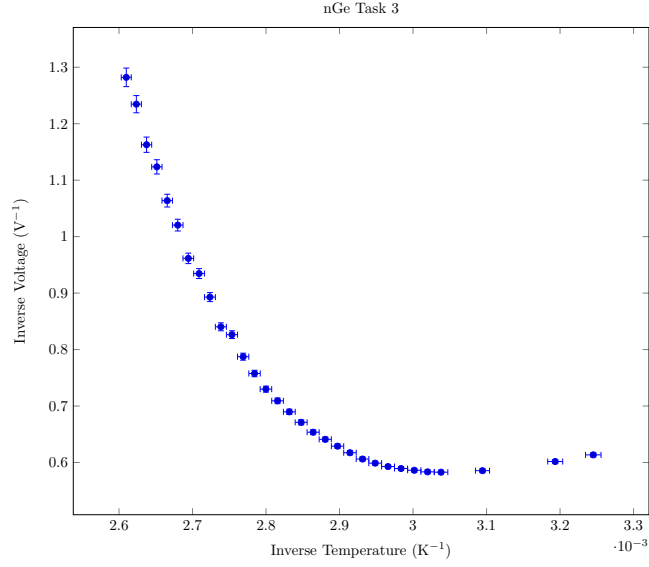
Figure 9: Inverse voltage is plotted against inverse temperature for n-type Germanium.

The intrinsic conductivity is again approximated as the inverse voltage, and the conductivity is related to the inverse temperature by the relation $\sigma = \sigma_0 e^{-E_g/2kT}$. The natural logarithm of conductivity is computed to produce the linear relation in figure 10
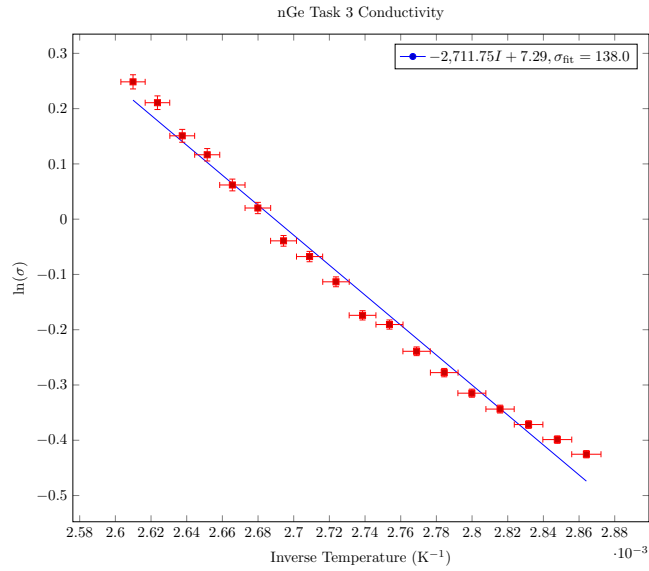

Figure 10: Natural logarithm of conductivity versus inverse temperature for n-type Germanium

Again, an experimental value for the band gap energy is computed.

$$E_g = -\text{SLOPE} \times 2k$$
$$= -(-3252) \times 2\left((8.63 * 10^(- 5)\frac{eV}{K}\right)$$
$$= 0.56 eV$$

The associated error in $E_{g_{exp}}$ can be computed from the error in the slope.

$$\delta_{Eg_{exp}} = \text{PROP ERROR IN SLOPE/2k}$$

The discrepancy is computed between the experimental and the theoretical value band gap energy. The associated error is taken as the error in the experimental value.

$$\delta_{E_g} = |E_{g_{exp}} - E_{g_{theo}}|$$
$$= |NUM - NUM|$$
$$= NUM eV$$

3.3.2 Conclusion

The experimental energy gap $E_{g_{exp}} = NUM eV$ is within SOMENUMBER σ of the theoretical value $E_{g_{theo}} = NUM eV$, thus the values ARE / ARE NOT in agreement.

3.4 Task 4

The Hall voltage was measured as a function of the magnetic induction, with constant temperature and current held at 30 mA. The field strength was limited to 290 mT.
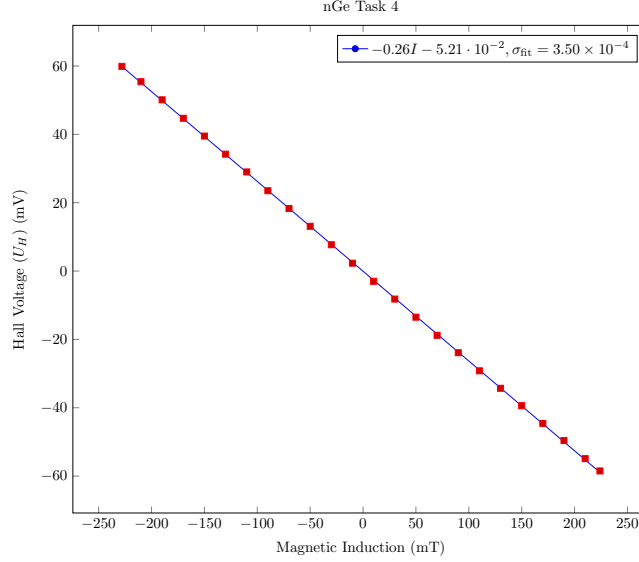
3.4.1 Data Analysis and Results

Figure 11: Hall voltage plotted as a function of applied magnetic field strength for n-type Germanium.

The experimental Hall constant is the slope, $b = 260\,\mathrm{mV\,T^{-1}}$, of Figure 5 multiplied by $\frac{d}{I}$, where $d = 1\,\mathrm{mm}$ is the thickness of the sample and $I = 30\,\mathrm{mA}$ is the applied current.

$$R_{\mathrm{H}} = b\frac{d}{I}$$

$$R_{\mathrm{H}} = (0.65)\frac{(1 \times 10^{-3})}{(30 \times 10^{-3})}$$

$$R_{\mathrm{H}} = 8.66 \times 10^{-3}\mathrm{m^3\,A^{-1}\,s^{-1}}$$

Since the only variable, the slope b, is being multiplied by the constant $\frac{d}{I} = 33.3 \times 10^{-3}$, the error in the experimental hall constant is merely the error in the slope, $\delta b = 3.5\,\mathrm{mV\,T^{-1}}$, multiplied by the constant.

$$\delta R_{\mathrm{H}} = \delta b\frac{d}{I}$$

$$\delta R_{\mathrm{H}} = (3.5 \times 10^{-3})(33.3 \times 10^{-3})$$

$$\delta R_{\mathrm{H}} = 116.6\,\mathrm{cm^3\,A^{-1}\,s^{-1}}$$

The theoretical value of the Hall constant is $4.8\,\mathrm{dm^3\,A^{-1}\,s^{-1}}$. Therefore, the discrepancy between our experimental value and this theoretical value is

$$\Delta_{R_{\mathrm{H}}} = |R_{\mathrm{H_{theo}}} - R_{\mathrm{H_{exp}}}|$$
$$\Delta_{R_{\mathrm{H}}} = |(4.8 \times 10^{-3}) - (8.66 \times 10^{-3})|$$
$$\Delta_{R_{\mathrm{H}}} = 3.86 \, \mathrm{dm^3 \, A^{-1} \, s^{-1}}$$

The charge carrier concentration is calculated from the elementary charge and experimental Hall constant.

$$n_{exp} = \frac{1}{e \times R_{H_{exp}}}$$
$$= \frac{1}{(1.602 \times 10^{-19} As) \times (8.66 \times 10^{-3} 1/As)}$$
$$=$$

The associated error in n_{exp} is calculated as follows.

$$\delta_n = CALCULATION$$

A similar calculation is performed with the theoretical Hall constant.

$$n_{theo} = \frac{1}{e \times R_{H_{theo}}}$$
$$= CALCULATION$$

The difference in experimental and theoretical Hall constant is calculated as follows.

$$\Delta = |R_{H_{exp}} - R_{H_{theo}}|$$
$$= CALCULATION$$

3.4.2 Conclusion

3.5 Task 5

3.5.1 Data Analysis and Results

The shape of the graph can be described by three differing regions. The first relatively flat area in the region of the lower temperatures depicts an extrinsic conduction. The middle section of the graph that is described by the slope shows the reduction in the drift velocity that is associated with the increase of charge carriers with an increase of temperature. The third area, which is once again relatively flat, described by the higher temperatures depicts the intrinsic conduction. While the graph as a whole shows the transition from extrinsic conduction to intrinsic conduction.
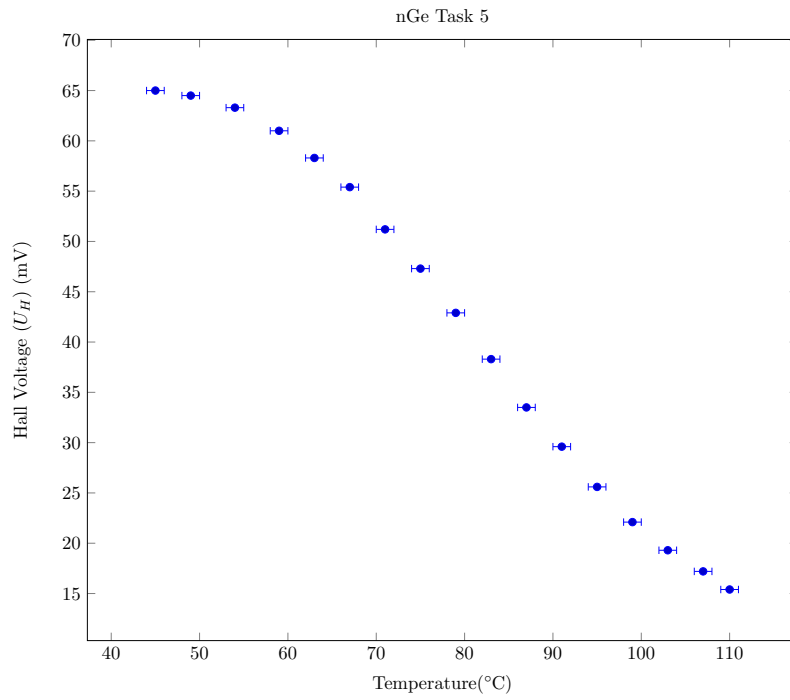
Figure 12: The plot of Hall Voltage in millivolts versus temperature in Celsius showing the three regions.

3.5.2 Conclusion

The plot in Figure (12) closely resembles what is expected. It shows the three distinct trends in voltage as temperature increases.

4 Hall Effect in Pure Germanium

4.1 Determining Band Gap in Pure Ge

For semiconductors the conductivity is a function of temperature. In this experiment, the current is held constant at 5 mA and the induced magnetic field willl remain off. The sample voltage was measured as a function of the temperature of the sample. The conductivity can be approximated as the inverse voltage. The conduc tivity is related to the inverse temperature by the relation $\sigma = \sigma_0 e^{-E_g/2kT}$.This will allow one to create a graph of the conductivity as a function of the inverse temperature. Furthermore, plotting the natural logarithmic of the conductivity versus the inverse temperature will result in a linear plot. Using the slope of this plot the energy band gap is then determined.

4.1.1 Data Analysis and Results

To calculate the energy band gap the following equation can be used, please note that 'k' represents the Boltzmann constant.

$$
\begin{aligned}
E_g &= -\text{slope} \times 2k \\
&= -(-4187) \times 2\left((8.625 * 10^{(} - 5))\frac{eV}{K}\right) \\
&= 0.72 eV
\end{aligned}
$$

The associated error in $E_{g_{exp}}$ can be computed from the error in the slope.

$$\delta_{E g_{exp}} = \text{PROP ERROR IN SLOPE/2k}$$

4.1.2 Conclusion

5 Hall Effect in Pure Zinc

5.1 Determining Hall Constant in Pure Zn

5.1.1 Data Analysis and Results

In order to determine the Hall constant R_H, one can analyze the dependence of the Hall voltage on the applied field. This was done with a constant DC current applied across the Zinc sample, with an associated random error due to the limited precision of the power supply. The error in the slope is obtained through linear regression, and the thickness prescribed by the sample specifications is assumed to have no error.

Slope, b $\left[\frac{\Omega\text{cm}}{\text{G}}\right]$	$4.11 \times 10^{-13} \pm ERROR$
Thickness, d [m]	$2.5 \times 10^{-5} \pm 0$
Sample Current, I [A]	13.5 ± 0.1

Table 5.1: Measurements and calculations used to determine the experimental Hall constant of pure Zinc

The Hall constant of Zinc is calculated as follows.

$$R_{H_{exp}} = \left(\frac{\mu_H}{B}\right)\frac{d}{I}$$
$$= (b)\frac{d}{I}$$
$$= (4.11 \times 10^{-13})\frac{2.5 \times 10^{-5}}{13.5}$$
$$= 4.11 \times 10^{-11} \text{ Vm/TA}$$
$$\equiv 4.11 \times 10^{-13} \text{ }\Omega\text{cm/G}$$

The uncertainty in the experimental Hall constant is propagated as follows.

$$\delta R_{H_{exp}} = |R_{H_{exp}}|\sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta d}{d}\right)^2 + \left(\frac{\delta I}{I}\right)^2}$$
$$= \sqrt{\left(\frac{SLOPERROR}{4.11 \times 10^{-13}}\right)^2 + \left(\frac{0}{2.5 \times 10^{-5}}\right)^2 + \left(\frac{0.1}{13.5}\right)^2}$$

A theoretical value for the Hall constant of Zinc, given by the third edition of the AIP handbook, is $R_H = 3.30 \times 10^{-13}$ Ωcm/G. This theoretical Hall constant has no given associated error, so the associated discrepancy, Δ, will simply equal that of the experimental Hall constant. The following computations are thus performed.

$$\Delta = |R_{H_{exp}} - R_{H_{theo}}|$$
$$= |(4.11 \times 10^{-13}) - (3.30 \times 10^{-13})|$$

$$\delta_\Delta = \delta R_{H_{exp}}$$
$$= \text{COPY VALUE FROM ABOVE}$$

5.1.2 Conclusion

It is evident that the discrepancy is within SOMENUMBER σ, so the experimental Hall constant $R_{H_{exp}} = 4.11 \times 10^{-13}\Omega$cm/G (AGREES / DOES NOT AGREE) with the reported theoretical value $R_{H_{theo}} = 3.30 \times 10^{-13}\Omega$cm/G.

6 Appendices

6.1 Appendix A: Data

6.2 Appendix B: Source Code