PHY 4210-01 Senior Lab
Lab M-1: Magnetic Field Mapping

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Abstract

In this experiment the magnetic field inside a Helmholtz coil was measured and compared to theoretical calculations determined from the Smythe derivation of the Biot-Sarvat Law for a plane displaced from the central axis, with coordinates z, ρ, and ϕ.When determining the magnetic field inside a Helmholtz coil, a Hall probe was used to obtain the magnitude of the magnetic field at varying positions inside the coil. Theoretically, the axial component of the magnetic field that is produced inside the Helmholtz coil is, to some extent, of uniform magnitude.

Contents

1 Objective of the Experiment

During this lab, the number of turns of wire inside a Helmholtz coil was determined for use in theoretical calculations. Then a 3-dimensional and 2-dimensional mapping of the magnetic field inside the Helmholtz coil was created in order to investigate the presence of a uniform field, running along its axial direction.

2 Theory of the Experiment

Recall for a straight current-carrying wire, circular magnetic field lines are generated around the wire in accordance with the curling right-hand rule. All the infinitesimal segments of the wire will cancel *except* for that in the axial direction. In summary, a circular current produces a linear magnetic field.

The field point of the system has before been typically placed along the axis of the direction of the magnetic field, we will call this the z-direction. This was due to the ease of solving the Biot-Savart Law under these simple conditions, as the direction and strength of the magnetic field will follow along the z-axis of the system, which is where the field point is placed. When this is applied to the co-axial coils of the Helmholtz apparatus the evaluation of the Biot-Savart Law becomes too trivial. One then chooses the field point to be placed off of the z-axis as more information about the magnetic field of the coils can be determined. This is the more general scenario and thus more complex. The off axis form can be used for any point that is off of the z-axis, while the on axis is a specific and simplified form of the general case. The general form is best represented by Smythe's derivation of the Biot-Savart Law.

$$
\begin{aligned}
B_z = \frac{\mu_0 IN}{2\pi}\Big[&\frac{1}{\sqrt{(a+\rho)^2+(a-z)^2}}\big[K_1+\left(\frac{a^2-\rho^2-(a-z)^2}{(a-\rho)^2+(a-z)^2}\right)E_1\big] \\
&+\frac{1}{\sqrt{(a+\rho)^2+z^2}}\big[K_2+(\frac{a^2-\rho^2-z^2}{(a-\rho)^2+z^2})E_2\big]\Big]
\end{aligned} \tag{1}
$$

3 Equipment Utilized

- Helmholtz coil
- Gauss meter
- Hall probe
- Meterstick
- Ruler
- Dipmeter
- Powersource
- Magnaprobe
- Multimeter

3.1 The Helmholtz coil

The Helmholtz coil consists of two concentric sets of coils, each with the same radius and separated by a distance equal to their radius. This configuration allows the contribution of each set of coils to produce a uniform field in the center of the coils. The current in each set of coils must be oriented in a particular direction so that their contributions constructively interfere. The circuit is shown in figure 1.

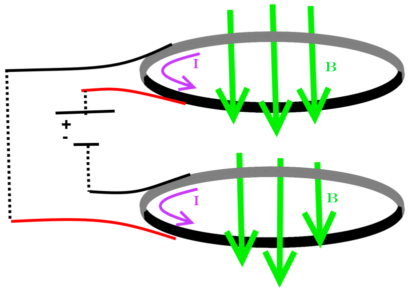

Figure 1: Flow of current through the Helmholtz coil, oriented such that the produced fields are constructive.

3.2 The Hall Effect Probe

A DC Gaussmeter (AlphaLab Model GM-1-HS) was connected to a Hall Effect Probe in order to measure the field strength inside the Helmholtz coil. The Hall Effect Probe contains a semiconductor junction that, when exposed to a magnetic field, produces a voltage proportional to the field strength.

3.3 Position Controls

The position of the Hall Effect Probe can be modified in the ρ direction by sliding the ruler bar through the acrylic cube shown in figure 2. The position can be modified in the ϕ direction by rotating the ruler bar about the central pole. However, for the sake of this experiment, this did not have to be modified because measurements were taken in a single ρ, z plane. The z coordinate was modified by sliding the acrylic cube and ruler bar up and down the central pole.

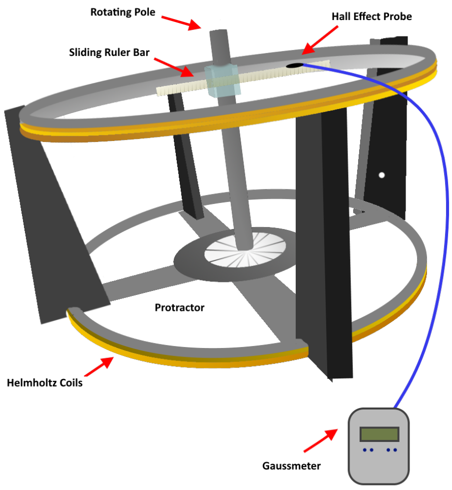

Figure 2: Two concentric Helmholtz coils separated by a distance equal to their radius. Rotating pole and sliding ruler allow for modification of the probe's position.

4 Procedure

Note that, per suggestion of the laboratory manual, the procedural steps of this experiment have been omitted. The discussion section provides sufficient detail on what actions were taken.

4.1 Measuring the External Field

The Helmholtz coil is oriented such that the Earth's magnetic field is parallel to the z-axis of the coils. This allows us to produce an applied magnetic field that is exactly anti-parallel to the Earth's field. From there, we can compute the applied field by subtracting the Earth's field from the total resultant field.

Note that there was an apparent offset in the Gaussmeter reading, as the 0.36G measurement for Earth's field was consistently higher than the expected value for Earth's field of 0.24G. However, if there truly exists such an offset in the measurement device, it would appear in both the measurement of Earth's field and in the measurement of the total field inside the Helmholtz coil. Subtracting these two to obtain the strength of the applied field would cancel any contribution from such an offset.

4.2 Procedural Modifications

Upon initial inspection of the equipment, it appeared the center pole running along the z axis of the Helmholtz coil was misaligned. In order to mitigate this error and ensure that coordinates were modified independently, a cord was used to realign the pole as closely as possible to the true z axis. However, since this alignment was not quantified, it is possible that there the pole is misaligned to some degree. This would result in a systematic error intrinsic to the experimental set-up. If the pole deviates from the z-axis, the experimentally recorded z-values are underestimated, causing the experimental field strengths to trend lower than the theoretical field strengths.

The majority of field strength measurements collected for the 3-dimensional mapping were taken on the same day of experimentation. After resuming this data collection on the next day, the values appeared to be systematically higher. Possible causes of this offset were investigated. Before taking measurements and intermittently during the data collection, the hall effect probe was zeroed and observed with the power supply off in order to ensure a consistent reading of the Earth's magnetic field. The reference measurement taken at the start of this lab session was similar to those taken during the previous session (zeroed field measurements were between 0.36G and 0.4G on both days), so a discrepancy in the Earth's field strength measurement was eliminated as the source of this error. Note that any small variation in the Earth's field measurement could be due to misalignment of the probe (a systematic error in measurement that would under-report the field strength) or simply a random error in measurement due to the limited performance of the probe.

An ammeter was also used to ensure a 2A current was consistently applied on both days of data collection, thus a change in the applied current was eliminated as a source of error. Because the source of this error was ultimately not determined and eliminated, the effect had to be compensated for with a procedural modification. In order to recreate the data points from the previous lab session, the current from the power supply was modified until the field strength matched previous measurements in several locations. This ultimately required lowering the applied current from 2000mA to 1790mA.

Upon further investigation, it appeared the current from the power supply was unstable, as it would decrease and increase every few minutes. This produced a source of random intrinsic error, which was mitigated by fine tuning the current value before each measurement after the issue was discovered.

4.3 Additional Sources of Error

Because the experimental set-up was restricted to a small area, the contribution from the field produced by the power supply may be non-negligible. From the perspective of the experimenter, the power supply sits behind and to the right of the Helmholtz coil. Therefore, by the curling right hand rule, this would produce an upward magnetic field on the side of the wire nearest the Helmholtz coil. This would produce a systematic intrinsic error that causes the external field measurements to be overestimated. Similarly, the power supply itself may be producing a small field that could also contribute a systematic error, although the exact effect could not be determined without knowing the orientation of such a field.

5 Data Analysis

5.1 Calculating Supply Voltage

Using a multimeter, the resistance of a set of coils was measured to be 3.4Ω. In order to determine the necessary voltage to send 3A of current through the coils, we made a simple calculation using Ohm's law.

$$\begin{aligned} V &= IR \\ &= (3\,\mathrm{A})(3.4\,\Omega) \\ &= 10.2V \end{aligned}$$

5.2 Determining the Number of Turns in a Coil

Further calculations will require knowing the number of turns of wire in each set of coils. When a known current I is applied to a single coil, a field of strength B_{loop} is produced. A value for B_{loop} is calculated below, where a is the radius of the loop.

$$\begin{aligned} B_{loop} &= \frac{\mu_0 I}{2a} \\ &= \frac{4\pi \times 10^{-7} Tm/A \times 2A}{2 \times 0.332m} \\ &= 3.78 \times 10^{-6}\ \mathrm{T} \\ &= 3.78 \times 10^{-2}\ \mathrm{G} \end{aligned}$$

The applied field strength was then measured across the top set of coils, with the applied current of 2A mentioned above. The number of loops N could be determined by dividing the total measured field by the field calculated for a single turn of wire.

$$\begin{aligned} N &= \frac{B_{measured}}{B_{loop}} \\ &= \frac{2.75G}{3.78 \times 10^{-2} G} \\ &= 72.66\ \mathrm{G} \end{aligned}$$

The process was conducted on the top set of coils twice, for a parallel and anti-parallel field. These two measurements and calculations were then repeated for the bottom set of coils. Averaging these four values for N and rounding to the nearest integer yields an average number of turns in a coil of $N = 73$.

5.3 The Resultant Plots

Theoretical values for what the strength of the magnetic field ought to be within the Helmholtz coils were determined by translating Equation 1 into Julia code, shown in lines 44-95 of the first program in Appendix B.

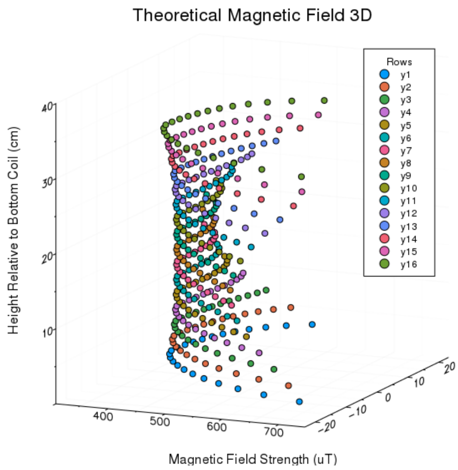

Figure 3: The theoiretical three dimensional plot calculated by using Symthe's deviation of the Biot-Savart Law.

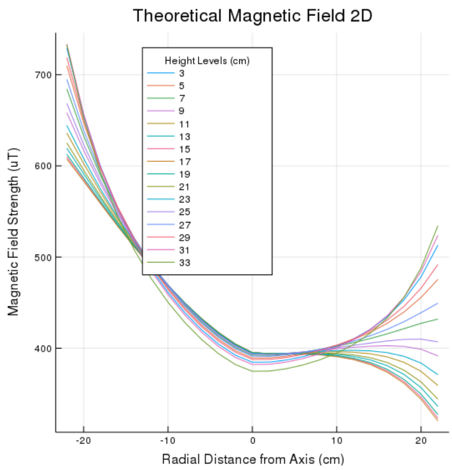

Figure 4: The theoretical two dimensional used for comparison to the data obtained during the experimentation.

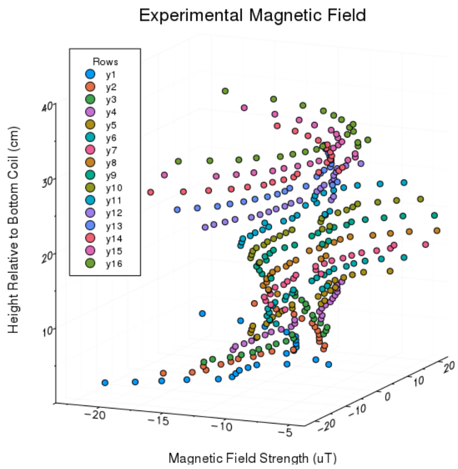

Figure 5: The three dimensional plot with the data obtained during the experimentation. Where the magnetic field of the Helmhotz coil is mapped out to provide a visual of how the magnetic field is shaped and its strengths and differing areas.

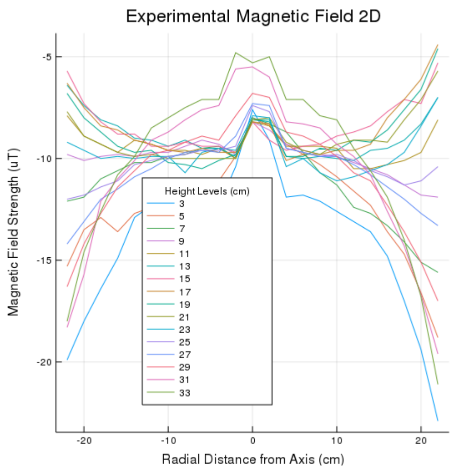

Figure 6: The two dimensional plot witht the data obtained during the experimentation. Where each line corresponds to a differing value along the z-axis of the Helmholtz coil as a function of the magnetic field strength in μ Tesla and the radial distance from the z-axis in cm.

6 Results

6.1 Comparing the Theoretical Plots to the Experimental Plots

The experimental three dimensional plot does follow the theoretical plot in the sense that the data in the mid region should be clustered closer together

than the edge regions. This is due to the uniformity of the magnetic field in this region. Whereas the regions closest to the coils are exposed to a greater magnetic field due to their closer proximity to the coils themselves. The area nearest to the power source also draws interference from the power source and the magnetic field and thus the magnetic field is skewed to be stronger. The experimental three dimensional plot is seemingly flipped 180 degrees due to the fact the experiemental magnetic field was measured to be negative while the theoretical magnetic field was calculated to be positive.
The experimental two dimensional plot does follow the theoretical plot in the sense that the data in the mid region is clustered close together. The derivatives of this area of the graph approach a plateu, this is recognized to be the center region of the coils where the uniformity of the magnetic field is most prominent. The measured magnetic field was determined to be much weaker than the theoretical magnetic field. Our measured field was approximately an order of magnitude weaker than the theoretical. The magnetic field was once again measured to be of negative value while the theoretical magnetic field was determined to be of positive value.

6.2 Determining the Span of the Uniform Region with Margins

6.3 Comparing the directions of the Magnetic Field

When measuring at a probe height of a/2 (16cm), where 'a' is the separation distance between the coils, the strength of the magnetic field in the 'z' direction was measured to be -3.13 Gauss. When measuring t he magnetic field in the 'z' direction at a probe height of 5cm, the magnetic field strength was measure d to be -3.28 Gauss. These results follow with the theory as it is expected that the magnetic field is p ropagated in the 'z' direction. The measured magnetic field strength for the ρ direction was -0.46 and -0.05 Gauss for a probe height of 16cm and 5cm respectfully. The measured magnetic field strength for the ϕ direction was -0.51 and -0.31 Gauss for a probe height of 16cm and 5cm respectfully. For a probe height of 16cm the percentage for the magnitude of the magnetic field that is measured to be in th e ρ direction is 14% while the percentage for the magnitude of the magnetic field that is measur ed to be in the ϕ direction is 16%. For a probe height of 5cm the percentage for the magnitude o f the magnetic field that is measured to be in the ρ direction is 1% while the percentage for th e magnitude of the magnetic field that is measured to be in the ϕ direction is 9%.The magnetic field produced by the Helmholtz coils should be directed along the 'z' axis. These small measuredvalues f ollow the aforementioned theoryand we can determine that the magnetic field produced by the Helmholtz c oil is indeed axial. Furthermore, we can determine thatthe magnetic field is axial along the 'z' direction.

7 Conclusion

8 Appendices

8.1 Appendix A: Data

8.2 Appendix B: Source Code

```
# Source code for calculating the theoretical data and plotting it in both 2D
# and 3D.

# ----------BEGIN PACKAGES----------

# enable the addition of packages
using Pkg

# integration
Pkg.add("QuadGK")
using QuadGK

# plotting
Pkg.add("Plots")
using Plots

# displaying data
Pkg.add("Printf")
using Printf

# -----------END PACKAGES-----------

# ----------BEGIN CONSTANTS----------

# Permeability of Free Space Tm/A
μ0 = 1.256e-6

# -----------END CONSTANTS-----------

# ----------BEGIN BIOT-SAVART ON AXIS----------

# Biot-Savart (one loop, on axis)
B1(z,I,a) = ((μ0 * I * (a ^ 2)) / 2) * (1 / (((a ^ 2) + (z ^ 2)) ^ (3/2)))

# Biot-Savart (two loops, on axis)
B2(z,I,a) = ((μ0 * I * (a ^ 2)) / 2) *
    ((1 / (((a ^ 2) + (z ^ 2)) ^ (3/2))) +
     (1 / (((a ^ 2) + ((a - z) ^ 2)) ^ (3/2))))

# -----------END BIOT-SAVART ON AXIS-----------

# ----------BEGIN BIOT-SAVART OFF AXIS----------

```

Figure 7: The collected experimental data in Gauss. The top row represents the radial distance from the axis in centimeters, and the left column represents height levels in centimeters.

	-22	-20	-18	-16	-14	-12	-10	-8	-6	-4	-2	0	2	4	6	8	10	12	14	16	18	20	22
33	-4.08	-3.74	-3.54	-3.36	-3.27	-3.13	-3.08	-3.03	-2.99	-2.99	-2.76	-2.81	-2.78	-2.99	-2.99	-3.07	-3.09	-3.23	-3.34	-3.47	-3.68	-3.96	-4.39
31	-4.11	-3.85	-3.49	-3.38	-3.29	-3.19	-3.15	-3.09	-3.04	-3.02	-2.84	-2.83	-2.88	-3.1	-3.11	-3.13	-3.21	-3.27	-3.36	-3.54	-3.71	-3.95	-4.24
29	-3.91	-3.7	-3.55	-3.42	-3.34	-3.26	-3.24	-3.2	-3.17	-3.19	-3.07	-2.96	-2.98	-3.15	-3.17	-3.21	-3.27	-3.35	-3.39	-3.51	-3.64	-3.79	-3.98
27	-3.7	-3.59	-3.48	-3.43	-3.37	-3.33	-3.28	-3.25	-3.24	-3.25	-3.17	-3.01	-3.02	-3.23	-3.25	-3.27	-3.27	-3.31	-3.36	-3.42	-3.48	-3.55	-3.61
25	-3.48	-3.46	-3.42	-3.39	-3.31	-3.29	-3.27	-3.26	-3.24	-3.24	-3.22	-3.02	-3.05	-3.23	-3.25	-3.27	-3.26	-3.29	-3.34	-3.37	-3.41	-3.39	-3.32
23	-3.2	-3.24	-3.28	-3.27	-3.28	-3.27	-3.27	-3.35	-3.25	-3.23	-3.24	-3.07	-3.08	-3.32	-3.28	-3.35	-3.39	-3.37	-3.34	-3.31	-3.25	-3.12	-2.98
21	-3.05	-3.17	-3.21	-3.25	-3.27	-3.26	-3.28	-3.28	-3.28	-3.28	-3.28	-3.09	-3.08	-3.2	-3.24	-3.26	-3.21	-3.19	-3.19	-3.2	-3.09	-2.99	-2.85
19	-2.96	-3.08	-3.15	-3.22	-3.25	-3.24	-3.3	-3.31	-3.33	-3.29	-3.26	-3.09	-3.09	-3.27	-3.27	-3.23	-3.24	-3.19	-3.2	-3.14	-3.04	-2.94	-2.74
17	-2.91	-3.03	-3.12	-3.14	-3.19	-3.21	-3.24	-3.25	-3.23	-3.25	-3.25	-3.1	-3.11	-3.29	-3.26	-3.26	-3.24	-3.24	-3.21	-3.08	-2.99	-2.89	-2.72
15	-2.85	-3.01	-3.1	-3.16	-3.16	-3.22	-3.22	-3.2	-3.22	-3.22	-3.25	-3.1	-3.19	-3.24	-3.22	-3.21	-3.17	-3.15	-3.12	-3.05	-2.99	-3.01	-2.81
13	-2.92	-3.02	-3.09	-3.12	-3.18	-3.19	-3.22	-3.19	-3.23	-3.23	-3.27	-3.09	-3.1	-3.27	-3.28	-3.27	-3.28	-3.29	-3.24	-3.23	-3.19	-3.11	-2.98
11	-3.07	-3.17	-3.21	-3.25	-3.19	-3.21	-3.24	-3.24	-3.26	-3.23	-3.28	-3.1	-3.12	-3.22	-3.22	-3.22	-3.23	-3.33	-3.33	-3.31	-3.29	-3.25	-3.09
9	-3.26	-3.29	-3.27	-3.26	-3.27	-3.25	-3.25	-3.22	-3.19	-3.21	-3.26	-3.1	-3.14	-3.21	-3.24	-3.26	-3.27	-3.3	-3.33	-3.36	-3.41	-3.46	-3.47
7	-3.49	-3.47	-3.38	-3.34	-3.3	-3.3	-3.27	-3.26	-3.23	-3.22	-3.28	-3.08	-3.11	-3.15	-3.25	-3.35	-3.41	-3.52	-3.55	-3.61	-3.69	-3.79	-3.84
5	-3.81	-3.63	-3.57	-3.64	-3.55	-3.52	-3.51	-3.44	-3.43	-3.39	-3.27	-3.08	-3.12	-3.21	-3.26	-3.31	-3.37	-3.44	-3.51	-3.64	-3.75	-3.94	-4.16
3	-4.27	-4.08	-3.92	-3.77	-3.57	-3.51	-3.53	-3.55	-3.53	-3.51	-3.32	-3.04	-3.19	-3.47	-3.46	-3.49	-3.54	-3.59	-3.64	-3.76	-3.98	-4.22	-4.57

```
# the full Biot-Savart equation (two loops, off axis)
B(z,I,N,a,ρ) =
    ((μ0 * I * N) / (2 * pi) ) *
    (
        (1 / sqrt(( (a + ρ) ^ 2) + ((a - z) ^ 2))) *
        (
            K(1,a,ρ,z) +
            (
                (((a ^ 2) - (ρ ^ 2) - ((a - z) ^ 2)) /
                 (((a - ρ) ^ 2) + ((a - z) ^ 2))) *
                E(1,a,ρ,z)
            )
        ) +
        (
            (1 / sqrt(((a + ρ) ^ 2) + (z ^ 2))) *
            (
                K(2,a,ρ,z) +
                (
                    (((a ^ 2) - (ρ ^ 2) - (z ^ 2)) /
                     (((a - ρ) ^ 2) + (z ^ 2))) *
                    E(2,a,ρ,z)
                )
            )
        )
    )

# elliptic integrals of first kind (series expansion)
K(j,a,ρ,z, precision = 4) =
    (pi / 2) * (
        1 + sum([
            ((reduce(*, [n - 1 for n in 2:2:precision]) /
              reduce(*, [n for n in 2:2:precision])) ^ 2) *
            (k(j,a,abs(ρ),z) ^ n)
            for n in 2:2:precision
        ])
    )

# elliptic integrals of the second kind (series expansion)
E(j,a,ρ,z, precision = 4) =
    (pi / 2) * (
        1 - foldl(-,[
            ((reduce(*, [n-1 for n in 2:2:precision]) /
              reduce(*, [n for n in 2:2:precision])) ^ 2) *
            ((k(j,a,abs(ρ),z) ^ n) / (n-1))
            for n in 2:2:precision
        ])
    )

# the collections of variables k1 and k2
k(j,a,ρ,z) = (j == 1) ?
```

```
    sqrt((4 * a * ρ) / (((a + ρ) ^ 2) + ((a - z) ^ 2))) :
    sqrt((4 * a * ρ) / (((a + ρ) ^ 2) + (z ^ 2)))

# -----------END BIOT-SAVART OFF AXIS-----------

# ----------BEGIN CONDITIONS----------

# heights at which measurements were taken (meters)
heightLevels = 0.03:0.02:0.33

# current applied to the coils (amperes)
current = 2

# turns of wire in each coil
turns = 73

# radius of the coils (meters)
radius = 0.332

# radial distances at which measurements were taken (meters)
rhos = -0.22:0.02:0.22

# -----------END CONDITIONS-----------

# ----------BEGIN CALCULATIONS----------

# list of Biot-Savart functions for each height level
BHeights = [ futureRho -> B(heightLevel, current, turns, radius, futureRho) * 1e6
            for heightLevel in heightLevels ]

# apply the list of radial distances to each height level's function (rhos are rows)
allBs = [ map(heightLevelFunction, rhos) for heightLevelFunction in BHeights ]

# -----------END CALCULATIONS-----------

# ----------BEGIN MAKE DATA POINTS----------

# convert column number (1:23) to radial distance value (-22:2:22)
colNum2RadialDist(col) = col + (col - 24)

# convert row number (1:16) to height value (3:2:33)
rowNum2Height(row) = row + (row + 1)

# data points for a row of data at the same height level
getRow(row) = [ (rowNum2Height(row), colNum2RadialDist(col), allBs[row][col])
                for col in 1:23 ]

# data points as triples with rows as heights and columns as radial distances
allPts = map(getRow, 1:16)

```

```julia
# -----------END MAKE DATA POINTS-----------

# ----------BEGIN FORMAT DATA----------

# split up coords into their own lists
formatRow(row) = ( [ coords[1] for coords in row ],
                   [ coords[2] for coords in row ],
                   [ coords[3] for coords in row ] )

# format all rows into form ([zs], [rs], [Bs])
listOfRows = map(formatRow, allPts)

# all these lists must be combined
tripleData = ( foldl(hcat, map(x -> x[1], listOfRows)),
               foldl(hcat, map(x -> x[2], listOfRows)),
               foldl(hcat, map(x -> x[3], listOfRows)) )

# print the data in neat columns
function displayFormatedData(data)

    # print header
    @printf("z ρ B\n")

    # iterate through each data point
    for ii in 1:length(data[1])
        @printf("%f %f %f\n", data[1][ii], data[2][ii], data[3][ii])
    end

end

# apply the data printing function onto our data
displayFormatedData(tripleData)

# -----------END FORMAT DATA-----------

# ----------BEGIN GENERATE 2D PLOT----------

# extract the Bs from the list of row triples
rowsBs = [ row[3] for row in listOfRows ]

# generate height level labels for each line
heightLevelLabels = map(string, 3:2:33)

# generate the plot
theoPlot2d = Plots.plot(listOfRows[1][2], rowsBs, label = heightLevelLabels,
                        title = "Theoretical Magnetic Field 2D",
                        size = (550,550),
                        legendtitle = "Height Levels (cm)",
                        legend = :top,
                        xlabel = "Radial Distance from Axis (cm)",
```

```
                        ylabel = "Magnetic Field Strength (uT)"
                        )

# save the plot to disk
savefig(theoPlot2d, "2DPlotTheoretical")

# -----------END GENERATE 2D PLOT-----------

# ----------BEGIN GENERATE 3D PLOT----------

# generate the plot
theoPlot3d = Plots.plot(tripleData[3], tripleData[2], tripleData[1],
                   seriestype = :scatter, legend = :topright, legendtitle = "Rows",
                   title = "Theoretical Magnetic Field 3D", size = (550,550),
                   xlabel = "Magnetic Field Strength (uT)",
                   ylabel = "Height Relative to Bottom Coil (cm)",
                   zlabel = "Radial Distance from Axis (cm)"
                   )

# save the plot to disk
savefig(theoPlot3d, "3DPlotTheoretical.png")

# -----------END GENERATE 3D PLOT-----------
```

```
# Generate 2D and 3D plots of the experimental data collected during the
# experiment.

# ----------BEGIN PACKAGES----------

# add other packages
using Pkg

# interpreter for reading CSV files
Pkg.add("CSV")
using CSV

# convenient container for data extracted from CSV files
Pkg.add("DataFrames")
using DataFrames

# plotting
Pkg.add("Plots")
using Plots

# format data output
Pkg.add("Printf")
using Printf

# -----------END PACKAGES-----------
```

```julia

# ----------BEGIN EXTRACT DATA----------

# read data from CSV into a DataFrame
data = CSV.File("2dMappingTesla.csv") |> DataFrame

# convert row number (1:16) to height value (3:2:33)
rowNum2Height(row) = row + (row + 1)

# reverse the order of the incoming heights
heightReversal(z) = z + (16 - (z - 1) - z)

# extract the data from a row into a triple representing that row
extractRow(row) = (
    fill(rowNum2Height(row), 23),
    -22:2:22,
    [(Bval + 0.000024) * 1e6 for Bval in [data[ii][heightReversal(row)] for ii in 2:24]]
)

# list of triples, each representing a row, of the form ([zs, [rs], [Bs])
listOfRows = map(extractRow, 1:16)

# -----------END EXTRACT DATA-----------

# ----------BEGIN FORMAT DATA----------

# now, a "row" is of the form ([zs], [rs], [Bs]); all these lists must be combined
tripleData = ( foldl(hcat, map(x -> x[1], listOfRows)),
               foldl(hcat, map(x -> x[2], listOfRows)),
               foldl(hcat, map(x -> x[3], listOfRows)) )

# print the data in neat columns (display only, does not need to be included)
function displayFormatedData(data)

    # print header
    @printf("z ρ B\n")

    # iterate through each data point
    for ii in 1:length(data[1])
        @printf("%f %f %f\n", data[1][ii], data[2][ii], data[3][ii])
    end

end

# apply the display function to the data
displayFormatedData(tripleData)

# -----------END FORMAT DATA-----------

# ----------BEGIN GENERATE 2D PLOT----------
```

```

# extract the Bs from the list of row triples
rowsBs = [ row[3] for row in listOfRows ]

# generate height level labels for each line
heightLevelLabels = map(string, 3:2:33)

# generate the plot
expPlot2d = Plots.plot(listOfRows[1][2], rowsBs, label = heightLevelLabels,
                       title = "Experimental Magnetic Field 2D",
                       size = (550,550),
                       legendtitle = "Height Levels (cm)",
                       legend = :bottom,
                       xlabel = "Radial Distance from Axis (cm)",
                       ylabel = "Magnetic Field Strength (uT)"
                       )

# save the plot to disk
savefig(expPlot2d, "2DPlotExperimental")

# -----------END GENERATE 2D PLOT-----------

# ----------BEGIN GENERATE 3D PLOT----------

# generate the plot
expPlot3d = Plots.plot(tripleData[3], tripleData[2], tripleData[1],
                   seriestype = :scatter, legend = :topleft,
                   legendtitle = "Rows",
                   title = "Experimental Magnetic Field", size = (550,550),
                   xlabel = "Magnetic Field Strength (uT)",
                   ylabel = "Height Relative to Bottom Coil (cm)",
                   zlabel = "Radial Distance from Axis (cm)"
                   )

# save the image to disk
savefig(expPlot3d, "3DPlotExperimental.png")

# -----------END GENERATE 3D PLOT-----------
```

```
# Plot the difference between the theoretical and experimental data.

# ----------BEGIN PACKAGES----------

# enable the addition of packages
using Pkg

# interpreter for reading CSV files
Pkg.add("CSV")
using CSV
```

```julia

# convenient container for data extracted from CSV files
Pkg.add("DataFrames")
using DataFrames

# plotting
Pkg.add("Plots")
using Plots

# ----------BEGIN GENERATE THEORETICAL DATA----------

# Permeability of Free Space Tm/A
μ0 = 1.256e-6

# the full Biot-Savart equation (two loops, off axis)
B(z,I,N,a,ρ) =
    ((μ0 * I * N) / (2 * pi) ) *
    (
        (1 / sqrt(( (a + ρ) ^ 2) + ((a - z) ^ 2))) *
        (
            K(1,a,ρ,z) +
            (
                (((a ^ 2) - (ρ ^ 2) - ((a - z) ^ 2)) /
                 (((a - ρ) ^ 2) + ((a - z) ^ 2))) *
                E(1,a,ρ,z)
            )
        ) +
        (
            (1 / sqrt(((a + ρ) ^ 2) + (z ^ 2))) *
            (
                K(2,a,ρ,z) +
                (
                    (((a ^ 2) - (ρ ^ 2) - (z ^ 2)) /
                     (((a - ρ) ^ 2) + (z ^ 2))) *
                    E(2,a,ρ,z)
                )
            )
        )
    )

# elliptic integrals of first kind (series expansion)
K(j,a,ρ,z, precision = 4) =
    (pi / 2) * (
        1 + sum([
            ((reduce(*, [n - 1 for n in 2:2:precision]) /
              reduce(*, [n for n in 2:2:precision])) ^ 2) *
            (k(j,a,abs(ρ),z) ^ n)
            for n in 2:2:precision
        ])
    )
```

```
# elliptic integrals of the second kind (series expansion)
E(j,a,ρ,z, precision = 4) =
    (pi / 2) * (
        1 - foldl(-,[
            ((reduce(*, [n-1 for n in 2:2:precision]) /
              reduce(*, [n for n in 2:2:precision])) ^ 2) *
            ((k(j,a,abs(ρ),z) ^ n) / (n-1))
            for n in 2:2:precision
        ])
    )

# the collections of variables k₁ and k₂
k(j,a,ρ,z) = (j == 1) ?
    sqrt((4 * a * ρ) / (((a + ρ) ^ 2) + ((a - z) ^ 2))) :
    sqrt((4 * a * ρ) / (((a + ρ) ^ 2) + (z ^ 2)))

# heights at which measurements were taken (meters)
heightLevels = 0.03:0.02:0.33

# current applied to the coils (amperes)
current = 2

# turns of wire in each coil
turns = 73

# radius of the coils (meters)
radius = 0.332

# radial distances at which measurements were taken (meters)
rhos = -0.22:0.02:0.22

# list of Biot-Savart functions for each height level
BHeights = [ futureRho -> B(heightLevel, current, turns, radius, futureRho) * 1e6
             for heightLevel in heightLevels ]

# apply the list of radial distances to each height level's function (rhos are rows)
# list of lists of Bs where each internal list is a row
theoBs = [ map(heightLevelFunction, rhos) for heightLevelFunction in BHeights ]

# -----------END GENERATE THEORETICAL DATA-----------

# ----------BEGIN EXTRACT EXPERIMENTAL DATA----------

# read data from CSV into a DataFrame
data = CSV.File("2dMappingTesla.csv") |> DataFrame

# convert row number (1:16) to height value (3:2:33)
rowNum2Height(row) = row + (row + 1)

```

```
# reverse the order of the incoming heights
heightReversal(z) = z + (16 - (z - 1) - z)

# apply corrections to data (Earth's magnetic field & correct units)
corrB(b) = ((b + 0.000024) * 1e6)

# extract the data from a row into a triple representing that row
extractRow(row) = (
    fill(rowNum2Height(row), 23),
    -22:2:22,
    [ corrB(Bval) for Bval in [ data[ii][heightReversal(row) ] for ii in 2:24 ] ]
)

# list of triples, each representing a row
expRows = map(extractRow, 1:16)

# -----------END EXTRACT EXPERIMENTAL DATA-----------

# ----------BEGIN PLOT THE DIFFERENCE----------

# list of triples, each representing a row, where the Bs are (theoB-expB)
diffRows = [ (expRows[ii][1], expRows[ii][2], theoBs[ii] .- expRows[ii][3])
             for ii in 1:length(expRows) ]

# now, a "row" is of the form ([zs], [rs], [Bs]); all these lists must be combined
tripleData = ( foldl(hcat, map(x -> x[1], diffRows)),
               foldl(hcat, map(x -> x[2], diffRows)),
               foldl(hcat, map(x -> x[3], diffRows)) )

# plot the difference between the theoretical and experimental data
diffPlot = Plots.plot(tripleData[3], tripleData[2], tripleData[1],
                   seriestype = :scatter, legend = :topright, legendtitle = "Rows",
                   title = "Difference Between Theoretical and Experimental Magnetic Field
                   size = (550,550),
                   xlabel = "Magnetic Field Strength (uT)",
                   ylabel = "Height Relative to Bottom Coil (cm)",
                   zlabel = "Radial Distance from Axis (cm)"
                   )

# save the plot to disk
savefig("3DPlotDiff.png")

# -----------END PLOT THE DIFFERENCE-----------
```